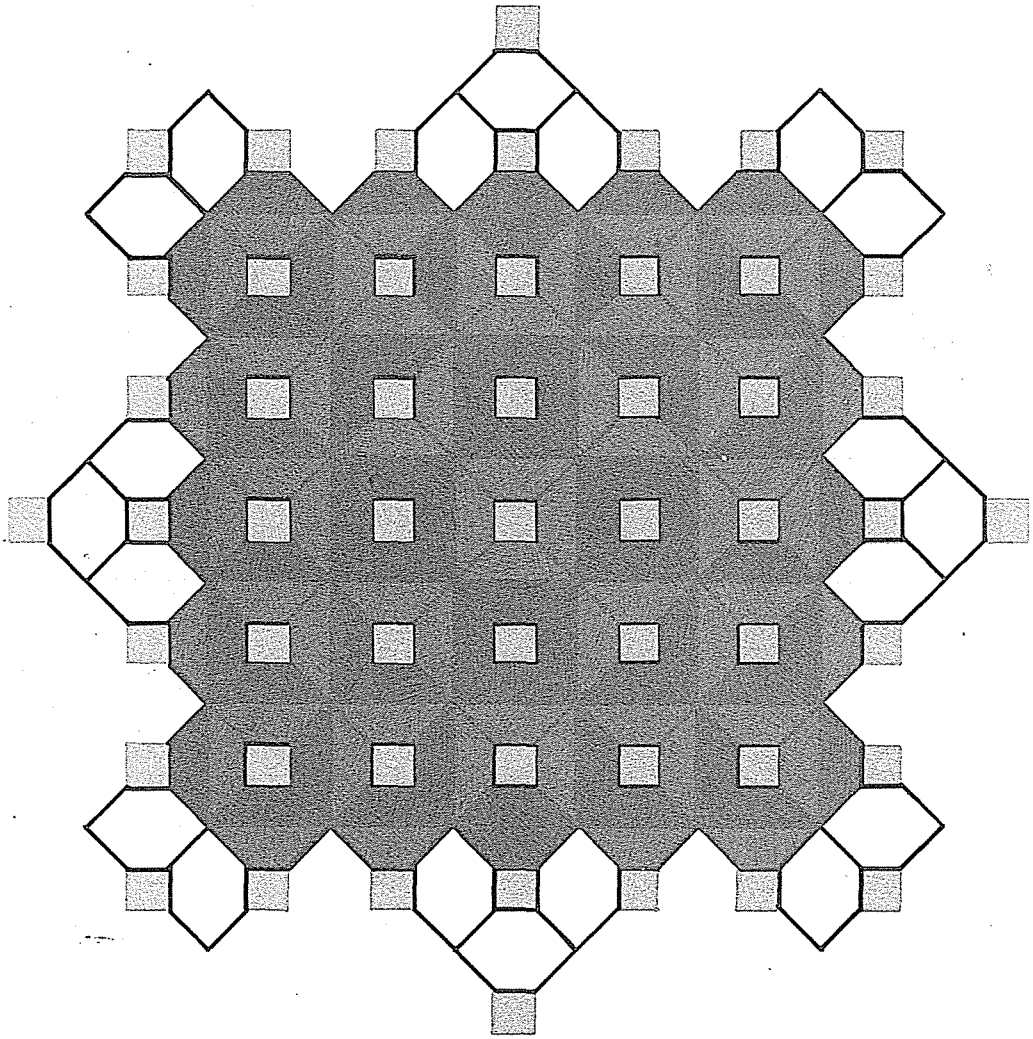
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EDITORIAL

The Editors Look at the "Pogue Report"

In the last issue we looked at the life and work of Jean Piaget, whose research and theoretical writings have helped teachers to understand better the need for an experiential approach to learning, one based on the stage of maturation of the child. Now we move on the look at the implications of such findings for the Ontario curriculum, especially the Intermediate Division (grades 7-10, ages 11-15).

For readers who are not familiar with the Ontario system, in the mid-sixties the Ministry of Education phased in a set of general guidelines for the Intermediate and Senior (grades 11-13) Division, based on what some liked to call "the new math". Since then the Ministry has adopted a policy of cyclic review; last year a new document, **The Formative Years,** was published for Kindergarten-grade 6, which placed new emphasis on "basics", and which, in mathematics, stressed geometry, especially symmetry and transformations.

It has been generally agreed that the Intermediate guidelines are very badly in need of up-dating. The Ministry took the sensible step of contracting research on local and foreign programs, with the purpose of receiving recommendations for the Intermediate Division. In 1976, the resulting report, **Recommendations for Intermediate Division Mathematics in Ontario,** was published by University of Ottawa Press. The principal investigator was Patrick Babin and the project director was Paul Pogue. Throughout this issue the document will be referred to, for brevity's sake, as the Pogue Report.

There are a number of articles on curriculum in this issue. Some of the members of the editorial staff discuss the report. A brief resume of the main content proposals follows. Our Book Review editor gives an overview of several curriculum reports, including the Pogue document. Tom Griffiths, who is a member of the Ministry Intermediate committee, give us a progress report on the shaping of the new guidelines. SWOAME Council studied the report in depth; a brief account of their meeting with Mr. Pogue is found in **Chapter Items.**

In this writer's opinion the three most important aspects of the report are (i) a strong commitment to an experiential approach, with emphasis on development of problem solving skills through "real life" examples, (ii) the inclusion of statistics in the core and (iii) continuing emphasis on transformations in geometry. Your editor happens to agree with all three of these proposals. Strange to say, since transformations have been a particular interest of mine, the third one gives me the most trouble.

The problem is implementation. We need more specialist teachers of mathematics and science in the Intermediate Division, especially if we expect the report proposals to be translated into effective action in the schools. My fear is that due to lack of understanding, transformations will be taught too abstractly, ("shadow rather than substance"), or will be ignored.

The report itself has a pretty good set of recommendations, which, whether you agree with them fully or not, give a basis for thoughtful consideration. I was disappointed with the theoretical foundation for these recommendations, partly because some favourite writers of mine were ignored: Polya and Dienes for a couple. There is good body of scholarly work on mathematics education available and it would make me feel better if we were right for the right reasons.

Carrying on, Neil Williamson, our president-elect, presents O.A.M.E. recommendations for the Senior Divisions, which should now be in the hands of the Ministry. Here's hoping that they continue the policy of having official O.A.M.E. representation on mathematics committees. Also, let's cut out the "indecent" haste. Give the committees time to do their work properly. Our best guidelines are now the Senior Division ones; that's partly due to the dedicated work of George Scroggie, and partly because the committee had adequate time to develop them. However, with the above minor complaint, let me close by saying that the policy of contracting research, reviewing curriculum cyclically, seconding practising teachers full-time to the committees, and consultation, is an excellent one, as long as election politicking doesn't interfere.



A SUMMARY OF THE RECOMMENDATIONS OF THE "POGUE REPORT"

1. Statistics and Probability should be implemented as a core topic throughout the Intermediate Division.

2. Flow charts should be included as an integral part of the mathematics curriculum.

3. A study of computers should be integrated into the mathematics curriculum. This study should be concerned with both computer awareness and programming techniques. Maximum use as a problem-solving tool should be encouraged.

4. Relations and functions should be introduced early in the programme.

5. Estimating, Approximating and Reasonableness of answers should become a significant part of the programme.

6. Transformations should become a topic considered essential for all students.

7. The following topics should be deleted from the core of the Intermediate Mathematics Division:
   - division of polynomials
   - factoring
   - operations and manipulations with rational algebraic expressions
   - complicated word problems

8. Geometry should be studied from a variety of points of view: transformation, vector, concrete models, measurement, traditional Euclidean, and when feasible should involve a mix of synthetic and analytic techniques.

9. Measurement should receive much more attention that it does in our present programme.

10. Trigonometry should be introduced as a core topic at the grade 10 level.

11. The heavy emphasis on structure should be eliminated from the programme. Concepts of structure should appear in the Intermediate Division, but no attempt at formalization should be made and the use of words such as commutative, associative, etc., should be de-emphasized, if not discontinued.

12. Computational skills should be related to, and motivated from, problems, applications, and games rather than be an end in themselves.

A CRITIQUE

We have addressed our look at the **Recommendations for Intermediate Division Mathematics in Ontario** to the **"Rationale for Recommendations".** It is our belief that prior to looking at a report of recommendations, one must be knowledgeable in the reasons for the report being made.

The rationale for these recommendations is based on the philosophy that proposed Recommendations for Intermediate Mathematics will be "more believeably more ultimately effective if they are based on a sound rationale".

The writers have set out in their model 5 variables: Goals and Objectives, Program Content, Learning Experiences, Approaches and Methods and Evaluation. Of the five variable, goals and objectives and evaluation are the control variables. The model illustrates the influence of the Goals

and Objectives on all other variables. Although there is this influence on content, learning experiences, approaches and methods, a successful interaction of these three variables is essential to an efficient program. It is suggested that, "the aim of this interaction is to achieve maximum effectiveness and efficiency in reaching the objectives".

It is the intention of the writers that the interaction of the 5 sections of the model, effectively manipulated and utilized when and where needed within the mathematics program will provide a pattern through which each teacher's individual program might be totally evaluated, and then continued as is, or altered to more effectively meet the needs of the students. In this model, the teacher is allowed to look at the variables independently of each other. This allows for change or alterations in only one of the modular modes, or to make wholesale changes.

Regardless it should be noted that the "on going evaluation" allows for complete knowledge of program effectiveness by the teacher.

David & Sharon McPhail

PUBLICATIONS AVAILABLE

TO O.A.M.E. MEMBERS

	Regular Price	O.A.M.E. Member Price
Handbook of Trigonometry (1975)	$2.00	N/C
A Preparedness Test for Grade Nine Students (1973)	2.00	N/C
Sample Grade 13 Examinations	1.00	N/C
Grade 11 (Level 5) Sample Examinations	1.00	N/C
Reprints of 1974 & 1975 Gauss Contest Papers (Elementary)	.10	N/C

To receive these please contact:

Mr. R. G. Scoins,
Faculty of Mathematics,
University of Waterloo,
WATERLOO, Ontario.

Boxes, Squares and Other Things (Elementary)		
A leather guide for a unit in informal geometry	1.80	N/C

Available from: Mr. M. MacGregor,
Secretary-Treasurer O.A.M.E.

IMPORTANT NOTICE TO SUBSCRIBERS

Due to rising costs it has been necessary to raise the subscription rate for the Gazette. BEGINNING OCTOBER 1, 1977, the annual rate will be $6.00 and individual issues will cost $2.50 each.

THANKS

The editorial staff of the Gazette wishes to thank all contributors for their excellent articles; you make our magazine a success. Next year the Gazette will be published in September, December and March. Deadlines for submissions are August 1, November 1 and February 1.

During 1974-76 Pam Ostrom, U. W. O. did the mathematical typing. Commencing with the December 1976 issue this job has been done by Janet Williams, Department of Mathematics, U. W. O. Many thanks to both Pam and Janet for their excellent work.

We should like also to pay tribute to our genial printer Jamie Pole and his hard-working staff, especially Lynn Evans, Shirley Gammon and Laurie McCormick.

SEE YOU IN OTTAWA!

LETTERS TO THE EDITOR

Dear Arn,

Would you please send me a copy of the proof you have received for Fermat's Last Theorem.

I have been curious about this since my last course in Number Theory.

Sincerely,
Joe Bachewsky

Dear Arn:

I read with much interest the December issue of the Ontario Mathematics gazette, particularly the information on Piaget. Keep up the good work.

Please send me a copy of the "new proof" of Fermat's Last Theorem.

Yours sincerely,

Brian Kennedy,
Head of Mathematics Dept.
Seaforth District High School

Dear Professor Harris:

Some years ago I read of a mathematical game, called the "New Year Game". The object is to write numerals for 1 to 100 using all of the digits of the year number, in that order, and using any basic operations. Our algebra class enjoys doing this every year, although this year was a little tougher, having to contend with a double digit.

The students who had the most to do with completing this were Ian Burbidge, Rosalee Mitchell and Edward Chechak. We wonder if any other schools are doing this, and if they have found any numerals we missed.

Yours truly,

Don Pickersgill,
Head of Mathematics
Port Dover Composite School

THE NEW YEAR GAME 1977

$1 = 1^{9} + 7 - 7 = +1^{977}$

$2 = 1^{9} + 7 \div 7$

$3 = 1 \times \sqrt{9} + 7 - 7 = [\sqrt{1+9}] \times 7 \div 7$

$4 = 1 \times \sqrt{9} + 7 \div 7 = [\sqrt{1+9}] + 7 \div 7 = -1 - 9 + 7 + 7$

$5 = 1 + \sqrt{9} + 7 \div 7 = 19 - 7 - 7$

.
.
.
.

$96 = 1 + 97 - [\sqrt{7}] = 19 + 77$

$97 = 1 \times 97 \times [\sqrt{\sqrt{7}}]$

$98 = -1 + 97 + [\sqrt{7}] = [\sqrt{197}] \times 7$

$99 = 1 \times 97 + [\sqrt{7}]$

$100 = 1 + 97 + [\sqrt{7}]$

Editor's Note: Mr. Pickersgill's class has at least one correct solution for 1 - 100. The rest will be published in the next issue. They also used the factorial symbol. Interesting solutions are welcomed.

NIFTY NUMBER NINE: AN HISTORICAL NUMBER

David and Sharon McPhail

The number nine has an interesting history that rates as far back as ancient civilization. It has even been considered a rather mysterious number by many.

In Greek civilization, the number nine was looked upon as a perfect number. When placed in the form of an array, 9 dots would be 3 x 3 and constituted a perfect square.



The earliest known magic square was the chinese "lo-shun". It was known as early as 1000 B.C. and can be found today engraved on charms worn in the east. This magic square was based on the nine consecutive natural numbers.

The ancient people discovered that this mysterious number repeatedly appeared in the products of multiplication problems where it was used.

$$\begin{array}{r} 6 \\ \times 9 \\ \hline 54 \end{array} \quad (5 + 4 = 9)$$

$$\begin{array}{r} 541 \\ \times 9 \\ \hline 4869 \end{array} \quad (4 + 8 + 6 + 9 = 27 \text{ and } 2 + 7 = 9)$$

A present day check for accuracy, "Casting Out Nines" was discovered in ancient times through the use of counting boards.

There are many references to the number nine throughout history:

- 9 days of wonder;
- 9 heavens;
- 9 days of mortification;
- etc., etc.

We use expressions today that involve this number also; 'a stitch in time saves nine'.

The number nine has not lost its curiosity to people in our civilization. It poses many interesting problems for our students to explore presently.

Consider some of the following exercises for your students to investigate:

(1) a) Think of a number (any size) 465371
b) Rearrange the digits 634751
c) Subtract the smaller number from the greater number 634751 − 465371 = 169380

d) The answer can always be divided by 9.

(2) Can you arrange three NINES so they will equal TEN?

(3) Take a digit out of one of the rectangles and place it in another so that all three rectangles will add up to 15.

789	456	123

(4) The nine times table can be explored for its patterns.

PATTERN 1			PATTERN 2
	9 x 1 = 09	0 + 9 = 9	09
	9 x 2 = 18	1 + 8 = 9	18
examining products	9 x 3 = 27	2 + 7 = 9	27 examining products
	9 x 4 = 36	3 + 6 = 9	36
			45
			54
			63
			72
			81
			90

PATTERN 3 - examining the digit multiplied by NINE plus the tens digit of the product

1 x 9 = 09	1 + 0 = 1
2 x 9 = 18	2 + 1 = 3
3 x 9 = 27	3 + 2 = 5
4 x 9 = 36	4 + 3 = 7

PATTERN 4 - examining the digit multiplied by 9 plus the ones digit of the product

9 x 1 = 09	1 + 9 = 10
9 x 2 = 18	2 + 8 = 10
9 x 3 = 27	3 + 7 = 10
9 x 4 = 36	4 + 6 = 10

(5) Mathamusement:

Write a two digit number (tens digit smaller than ones digit:

26

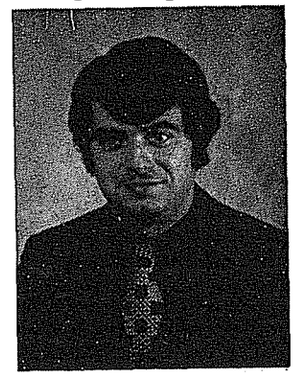
Multiply the difference of the two digits by NINE:

4 x 9 = 36

Add the product to the original number:

$$\begin{array}{r} 26 \\ \underline{36} \\ 62 \end{array}$$

Reverse these digits and you have your original number. (6)



Dave is a Director of O.A.M.E., former chairman of the Publications Committee, as well as an assistant editor of the Gazette

These are only a very few exercises that might be used with a class; perhaps they willwhet your students 'math appetite' to find other questions with "nifty number nine".

ANSWERS (2) $9 + \frac{9}{9}$

(3) 789 456 123

shift the nine

A MODEL FOR INDIVIDUALIZING INSTRUCTION AND AN INVESTIGATION ON NUMBER OPERATIONS (pupils aged 11 - 14, London, England)

Robert Smith, Department of Mathematics, Faculty of Education, University of Western Ontario

Professor Smith spent the 1975-76 school year on study leave at the Institute of Education, University of London.

Part 1 of a 2 part article. It introduces a project called SMILE begun in I. L. E. A. (Inner London Educational Authority) in 1973 as a result of concern for the teaching of classes of mixed ability. This is one experiment which is trying to help teachers cope with pupils with varying backgrounds,

but more basically, is attempting to help pupils individually as they move from the concrete operational to the formal stages of learning Mathematics.

Part two is a brief report on **Number 1,** a part of the **Concepts in Secondary Mathematics and Science** project which continues Prof. Geoffrey Matthew's link with Piaget started during the Nuffield years.

Introduction

As a Canadian visitor to England interested in Mathematics teaching and the way pupils aged 9-13 learn, I found I had to visit Junior Schools (8 - 11) Secondary Schools (11 - 16) and a new type of school called Middle Schools (9 - 13). From my reading I felt I was familiar with the changes brought about by the British Infant School Movement. But I was not prepared for the stress on active learning, the embedding of Mathematics into the total programme nor the emphasis on language of this movement that has carried over into the Junior School. Many schools are Mixed Infant and Junior Schools so I really should not have been surprised. Prof. Matthews and Miss Edith Biggs led the way in Jr. Schools with an emphasis on the unifying themes of Mathematics; the former with the Nuffield Guides, those excellent handbooks for teachers, full of practical suggestions and samples of children's work; the latter with her unbounded enthusiasm and innumerable workshops for teachers.

In the Secondary Schools it was Prof. Bryon Thwaites who led the way with not only a new syllabus but a series of text books (S.M.P.)[1] which caught the interest of the new era. At the same time the Labour Government began its move to comprehensive schools, to bring elitism to an end in the state schools. When pupils in traditional courses did not perform well, when pupils were no longer streamed, both the courses and the method of teaching was challenged. Prof. D. Crawford has written about the programmes M. M. P. and M. M. C. P. The following article on SMILE is a way that was developed to deal with challenge of meeting individual differences.

SMILE  SMILE

(Secondary Mathematics Individualised Learning Experiment)

History

SMILE began in Nov. 1962 at Ladbroke Mathematics Centre with a group of concerned secondary school teachers who were trying to teach mathematics to mixed ability classes. Neither class teaching nor the existing work cards seemed to be the answer. Some teachers present had been working with Bertie Banks and materials that later became known as the Kent Mathematics Project. The group present decided to adopt the structure followed by K. M. P. but write their own programme. In 1973 some 20 secondary schools (11 - 16) became involved. Now, more than 60 schools are SMILE schools. The writing team still meets each Monday at Ladbroke Mathematics Centre and the materials are centred there. The network of tasks, work cards, games, booklets, and work chosen from existing material is either written or decided upon by the writing team, given a SMILE number and printed. This network is not the result of a preconceived scope and sequence chart but rather it has grown from including all the work that the team felt suitable to give to children and then trying to place it roughly in the right place on the network. The network thus grows each year.

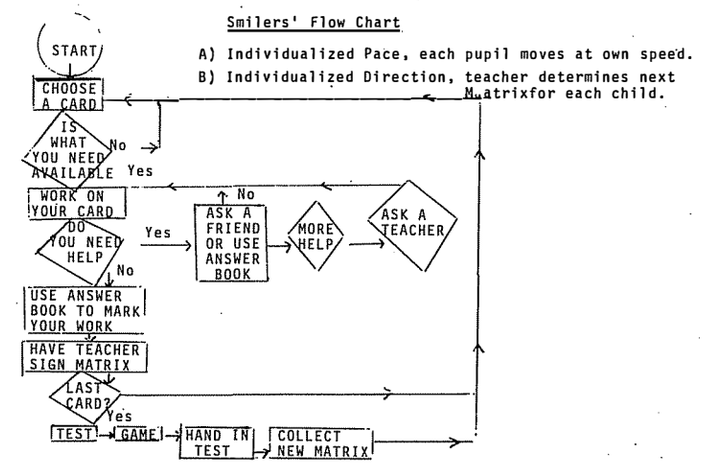
1. School Mathematics Project.

It extends over the five years of secondary school (11 - 16) and a Mode III C.S.E. 'O' Level syllabus was drawn up and presented to the examination board during 1976.

Smilers' Flow Chart

A) Individualized Pace, each pupil moves at own speed.

B) Individualized Direction, teacher determines next Matrix for each child.



Not only do the pupils move at their own rate but they also take responsibility for tackling the task, for organizing materials needed and for getting on with it and for marking the correctness of each task. The teacher marks the test when the ten tasks on the matrix are completed and on this basis decides the next ten tasks to be done, the direction the child will go. Notice that in the flow chart there is a place for discussion with a friend and with teacher. So teacher aids when there is trouble and in practice I saw teachers doing small group teaching.

A SMILE CLASSROOM

On entering a SMILE classroom, the immediate feeling of disorganization is overwhelming. Perhaps it is because it is so unlike that traditional math class you are accustomed to, where pupils engage in a chalk and talk lesson followed by the busy hum of pencils as questions from the text book are worked on. Here the responsibility for finding his work card, for obtaining the necessary materials, for getting on with the task as well as the opportunity to work with a neighbour is entirely with the child. Thus most children are doing different things. The teacher is freed to attend to individual questions or for discussion with temporarily formed groups.

Tom Brown's Matrix

Name **T. Brown** class **Ia** SMILE

Base Three Subtraction 0012	Sum and Product 0074	Getting to School 0274	Desk Cal. 4 0084 R.M.G. 5/10/74
Frog's Puzzle 0126 R.M.G. 6/10/74	Area 2 0023 R.M.G. 30/9/74	12 Squares 0180	Which Set? 0291
Co-ordinates 1 0261	Angles 1 0038	TEST Go 0140	

The tasks in the matrix have been selected as suitable for Tom Brown. Tom may do them in any order, his choice being restricted only by the availability of the work card, or material needed. When that task is finished, he consults the answer book to check his work. He next has the teacher initial his matrix in the appropriate space. When all ten tasks are completed, he tries the tests

which determine what has been learned from the matrix. While visiting a SMILE classroom at Drayton Manor Secondary School, I observed a 13 yr. old boy who, while trying the test, quickly returned to reread and do again the work on nodes, regions and arcs when it was obvious he was ill prepared. The end test consists of ten short tests, one for each task done, so this lad completed the tests after he had clarified the problem on networks. I was surprised at the variety of topics in the SMILE program geared to 12 year olds. Much work was done on number sequences and patterns, dissections, bearings, tessellations, networks, using the hand turned calculator, other bases, polyominoes as well as work with fractions, decimals and operations with whole numbers.

Teacher's Responsibility

As each child finishes a task, the teacher initials the matrix signifying completion of that card. As each child finishes a matrix test, the teacher must mark the test and on the basis of the results determine the next matrix. A teacher's manual gives both correct answers to the tests and suggestions for the next stage. However as a teacher becomes more familiar with the network these suggestions are often ignored and a decision of what next for that particular pupil becomes an individual professional decision.

Matrix Record Name **Tom Brown** Class **Ia**

	1	2	3	4	5	6	7	8	9
	10010	20011	30012						
	10027	10029	20074						
	10080	20212	20274						
	10081	20272	30084						
	20123	20124	30126						
	30185	10022	10023						
	30292	10265	10180						
	30295	20008	30261						
	30293	40284	30291						
	30300	20283	30038						
	TEST	TEST	TEST						
	0.125	0.130	0.142						
M.A.L.	2.1	2.0	2.4						
Date	3/9/74	27/9/74							
Test	56/93	71/89							
Remarks	Number O.K. Area Antony								

Mean Attainment Level reflects the work done in these ten cards. Over a period of time it shows the level of work the pupil is doing. Thus for the teacher there is some record keeping, marking certainly, but most important of all is the time free for the teacher to discuss the individual problem brought by a child to him for help, the opportunity to introduce a new topic when a small group is ready; the chance to clarify. Let me give you an example told to me by Ronnie Goldstein, the advisory teacher to the SMILE project.

I began by asking him during our interview if in fact the card does help the child in this difficult time when he/she is moving from the concrete operation stage to the formal operation stage. He replied that he did not feel this most fundamental learning stage does happen through the card at all but happens perhaps because of the atmosphere created by the SMILE situation, by the socializing with, and/or by the interaction with the fellow pupils. The card might well suggest that a particular abstraction is sensible in a particular situation. Questions, discussion, with fellow pupils and with the teacher play a big part at this point. He went on to tell me about a pupil doing a dissection question just a day or so before. (See below) She had done it very well. It was a perfectly good square. But the child was not convinced. "Please sir, Is this a square?" she asked. "It is not the same as in the answer book." That question came from her, about a very concrete thing that she had just finished doing.

"How can we decide if it is a square?" Ronnie asked.

"Well it does have four sides," she replied. "And the sides are straight."

At this point Ronnie drew a rhombus and asked, "Is this a square?"

And so the discussion went on about a very abstract thing - the definition **of an abstraction - a square.**

S M I L E another way to individualize instruction.



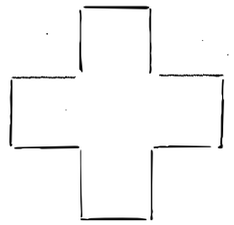


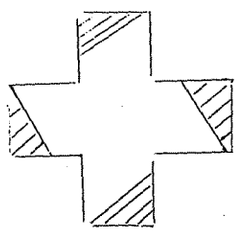
S M I L E Task No 0050 Level 1

Materials required: pencil, paper, ruler, scissors, squared paper, glue, coloured pens.

Dissection 1

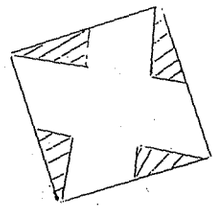
The shape on the right is called a Greek cross.
Copy it on paper.





Draw a line from the middle of each side to a corner as shown. Colour the 4 triangles. How many edges does the cross have? What is the length of each edge? What is the distance all round the cross?

Cut out the cross. Cut off the 4 triangles. You now have 5 pieces. Paste these pieces into your book in the shape of a square. What is the length of each edge? What is the distance all round the square?







BACK (?) TO BASICS - RATIO AND RATE

(continued)

Doug Henderson, Master Teacher of Mathematics, Sudbury Board of Education

In the December issue of the Gazette, Mr. Henderson discussed the place of ratio and rate in the school program, and gave a set of nine experiments on ratio, which have been used successfully in grades seven and eight.

He has also produced ten experiments on rate, five of which are given below. The remainder will appear in September, 1977.

RATE EXPERIMENT #1

Equipment: (i) Piece of string with a weight supported by a retort stand to form a simple pendulum.
(ii) stopwatch

Procedure: (i) Set the pendulum length at about 50 cm. Put the pendulum in motion and complete the following table:

Number of swings	Time (nearest tenth of a second)
5	
10	
15	
20	
25	
30	

(ii) Display your results on a graph. What pattern appears on the graph? This pattern indicates that the swings have been occurring at a constant rate. What is the average number of swings in one second? (Your answer will likely be a fraction.) This is the rate in swings per second.

What is the average time required for one swing? This is the rate in seconds per swing.

(iii) Write the relationship between the number of swings and the time in seconds.

(a) $\frac{\text{number of swings}}{\text{time in seconds}} =$

(b) number of swings $= ___ \times$ time in seconds

(c) $\frac{s}{t} =$

(d) $s = ___ \times t$

(iv) The form used in parts (c) and (d) is called a formula. Use the formula to find the number of swings in 100 seconds.

Problem

If a pendulum swings 14 times in 35 seconds,

(a) how many times will it swing in 50 seconds?
(b) how long will it take to swing 63 times?

Teachers' Hint

For goodness sake do not try to vary the length of the pendulum. Most kids are having trouble enough understanding rate as it is.

RATE EXPERIMENT #2

Equipment: A ball of string, a stopwatch, a metre stick.

Procedure: (i) Measure out 5 m of string. Find the time required for each member of the group to re-wind the string.

(ii) Example: if Elmo wound 5 m of string in 3.7 seconds, then he wound an average of $\frac{5.00}{3.7}$ of 1.35 m each second. Thus his rate of winding string is 1.35 m/s.

Find the average rate of winding string for each member of the group.

(iii) This table is for Elmo. Complete a similar table for yourself.

time(s)	string(m)
0	
1	
2	
3	
4	
5	

(iv) Draw a graph of your table, placing time on the horizontal axis. What pattern do you notice in the graph.

(v) State how to find the length of string if you know the time in seconds.

(a) in words

(b) as an equation string = ——— × time

(c) in algebra s = ——— × t

(vi) Use the formula of part (c) to find the amount of string you could unwind in one minute.

Problem

The elevator at the CN tower in Toronto will rise from the ground to the top (about 600 metres) in 3 minutes and 20 seconds.

(a) What is its rate in metres per second?
(b) Write a formula connecting distance and time.
(c) How far will the elevator go in 63 seconds?
(d) How long will it take to rise 150 metres?

RATE EXPERIMENT #3

Equipment: (i) water tap
(ii) device for measuring volume of a liquid
(iii) stop watch

Procedure: (i) Turn on the tap so that the water is flowing slowly. Measure the time required to fill a container (for example a 500 ml beaker).

(ii) In ——— seconds the flow is 500 ml
In 1 second the flow is ——— ml.

Your last answer is the rate of flow in ml/s.

(iii) Complete the table.

Draw a graph with time on the horizontal axis.

Time (seconds)	Amount of water
0	
1	
2	
3	
4	
5	

(iv) Write a formula

(a) as a fraction $\frac{\text{amount of water}}{\text{time}}$ =

(b) as an equation $\frac{\text{amount}}{\text{of water}}$ = ——— × time.

(c) Rewrite (a) and (b) in algebraic notation.

Let w represent the amount of water and t represent the time.

Problems

1. If a spring can deliver 2.7 litres in 5 minutes, how much water can it provide in one day?
2. If a storm sewer can handle 250000 litres per hour, how long would it take to get rid of 100000 litres of excess rainfall?

Teachers' Hints

(i) The water should flow very slowly. A healthy drip is best.

(ii) Rates may be calculated in many other situations such as burning candles or tapers. Burning dynamite fuse usually gets their attention.

RATE EXPERIMENT #4

Equipment: (i) pieces of metal bar of uniform diameter but different lengths
(ii) ruler and weigh scales

Procedure: (i) Complete the table and draw a graph. What pattern do you see on the graph?

length(cm)	weight(g)

(ii) Calculate the weight of a piece of this iron bar which is one cm long. This will be the rate in g/cm.

(iii) Use the rate of part (ii) to find the weight of a piece of this bar which is 21.4 cm long.

(iv) Write a formula which connects the weight of a bar (w) with its length (l).

Problems

1. If a uniform bar weights 4.1 g/cm, find the weight of a piece of this bar 11.4 cm long.
2. If a uniform steel beam 4.5 m long weighs 312 kg, find the weight of a piece of this beam which is 1.9 m long.

Teachers' Hint

The bar must be cut exactly. The machine shop at the local secondary school should help.

RATE EXPERIMENT #5

Equipment: (i) stop watch
(ii) paper and pencil

Procedure: (i) Using the stop watch find the length of time required for each person to write one line of zeroes on a page. Calculate the rate of writing zeroes in characters per second. Have each person complete the following table:

Time	Number of zeroes
0	
1 sec	
2	
3	
4	
5	

(ii) Each person complete step one using his other hand.

(iii) Each member of the group is to construct his own graphs, placing time on the horizontal axis and number of zeroes on the vertical. Graphs for both right and left hands should be put on the same set of axes.

(iv) For each hand find a formula connecting the number of zeroes (z) with the time in seconds (t).

Problems

1. Approximately how many zeroes could you write in one year?

2. How long would it take you to write 10^{10^2} in decimal notation? How long would it take to write $10^{10^{10}}$?

Teachers' Hint

The questions at the end of the experiment provide a good exercise in approximating and working with exponents.

% PERCENT %

By J. John Kere, St Catharines, Ontario

Mr. Kere, before his retirement, was Coordinator of Mathematics for the St. Catharines schools.

Percent, without a doubt, is present in the student's immediate environment. Students see the percent symbol on the T.V. screen, in daily newspapers, in windows of local stores, and in the scores they achieve on school tests. Since the word percent and its symbol (%) is almost constantly in their daily life, one might conclude that the concept of percent and its applications would NOT involve difficulties for learners. Unfortunately, experience in the classroom indicates that this is not the case - both the teaching and learning of percent has been and continues to be one of the most troublesome parts of Intermediate mathematics.

Why? It is not the mathematical computations in percent problems that is the source of difficulty. Percent concept and symbol, or percent problems, do not require mathematical ideas that learners have not already studied, when percent is introduced. The ambiguities and difficulties in percent terminology and symbolism, thus in teaching and learning of percent, are caused by attempts to indoctrinate learners that percent is a **ratio**; "a special class of ratios"; "particular ratio"; "a percent is an ordered pair of numbers"; "percent is a rate - it is also a ratio - but it is not a numeral for a rational number"; etc.

It is encouraging, that a well selected committee, when discussing the Intermediate Curriculum, came to the following conclusion:

"9) **Percent:** Percent should remain in the grade 7 and 8 program and should also be included at the grade nine level in applications for commercial topics. The content matrix indicates that **we should change our approach to this topic.** Of the program studied, ours is the only program that approaches it from a ratio point of view. With the advent of calculators it is recommended **that we return to the fractional definition** and do most of this work using decimal notation." (Underlining is mine)

My intention is to present one way of the meaning of "...**that we return to the fractional definition...**".

As it is now, the teaching of percent is artificially separated from the teaching of other mathematics topics. Students are asked to memorize special "proportions" for each type of percent problem. Questions involving percent usually are left as the last chapter in a textbook, and are considered especially hard problems. Besides, these problems are in many cases complicated by printing percent rates never used in the real world, for example, 5 15/19%. All this makes the topic of percent in students' eyes and in some teachers' opinion, an extremely complicated part of mathematics.

Even in the "new mathematics" SMSG - School Mathematics Study Group - Revised edition, 1963, texts, nothing has been done to show the simplicity of the percent concept, and to adjust its teaching and learning with respect to the needs of the practical business world.

What can be done in the future?

In the courses of study, or in texts, the percent concept does not need to be treated as a special theoretical topic. In theory, percent is nothing more than a fraction, always expressed in hundredths, identified by a special name and labelled with a special sign. Therefore, parallel to the development and learning of fractional number concept, and exercising operations with them, the idea of percent should grow painlessly. I found only one Canadian Textbook awarding consideration in a section topic, percent beside fractions: "Percents, Fractions and Decimals."

When learners do problems regarding 1 cent = 1/10 of a dime, 1 cent = 1/100 of a dollar, and similarly, we should also show them that there is a different symbol for 1/100,

ie. %, or 1/100 = 1%. It is more convenient to write or print 7% than 7/100. That way the percent idea can be associated with the hundreths concept - a special fraction - and nothing else.

In the formative years, we should not only do problems on how to find 1/2, 1/3, 1/5, 1/10, etc., of a whole number, but also questions on how to find 1/100 or 1%. For example: In a marathon, of the 300 runners, 1% were disqualified. How many runners were disqualified?

Later when learners do problems to find any fraction of a whole number, for example, 5/12 of 180, we should include questions like: 6% of $300.00, 14% of 2000 kg. These questions would not cause any additional difficulties for this topic, moreover, it will make this topic wider, more interesting, and closer to the real world problems. At the same time, learners get an earlier chance and work longer with the percent concept, not only in grades 7, 8, and 9.

Indeed, the meaning of percent fits completely into the meaning of fractional numbers, only the name and the symbol is different: "How many percent - so many hundredths. How many hundredths - so many percent."

To establish the understanding of the percent concept- % represents "hundredths of" - nothing helps more than the following type verbal problems.

(a) Susan earned $150.00 from baby sitting. She spent one hundreth of that to buy a gift for the baby.
Find: How much did the gift cost?
Solution: $\frac{1}{100}$ of 150 = $\frac{150}{100}$

= 1.50 The gift cost $1.50.

(b) John's class collected $150.00 during the recent Pollution Drive. If they spent 1% of that money to buy stamps, how much did they pay for the stamps?
Solution: 1% of 150 = $\frac{1}{100}$ of 150

= $\frac{150}{100}$

= 1.50 They paid $1.50 for stamps.

(c) The works department planned to repair a 760m long road in a week. How many metres of the road was repaired, if the workcrew repaired 30% of the road? 50%? 100%?

(d) Solve the following problem in <u>two</u> ways: A supermarket sold 1,280 kg of grapes in 3 days. In the first day it sold 45% of the total sale, in the second day only 25%. How many kilograms were sold on the third day?

To put more light on per cent concept, that is:

$$\% = \frac{1}{100} \quad \text{or} \quad \% = 0.01,$$

it is worthwhile to do some problems, where:

$$\frac{1}{1000} = \text{o/oo},$$

that is <u>mill,</u> has been used.

Very few texts introduce "per TEN" before presenting per CENT, using the symbol: o/ or /o = $\frac{1}{10}$.

If a learner can replace the percent symbol with a fraction 1/100, then the percent problems, in principal, do not differ from the same type fractional number questions: In a school marathon 10% gave up after the first kilometre, and only 1/3 of the rest finished the 5 km run. How many finished the marathon if in the beginning there were 70 runners?

Solution: Since $10\% = 10 \times \frac{1}{100} = \frac{1}{10}$

and $\frac{1}{10}$ of 70 = 7,

therefore 70 - 7 = 63 gave up

Since $\frac{1}{3}$ of 63 = 21,

therefore 21 runners finished the marathon.

It is NOT an advantage, during the teaching of the understanding of the percent concept, to express the given percent <u>as a decimal</u> fraction, for example: 5% of 20 = 0.05 × 20. The <u>decimal</u> fraction 0.05 fails to clarify the idea, that 0.05 itself means 5 × 0.01, and therefore the symbol % is merely replaced by the

decimal fraction 0.01.

The form of common fraction emphasizes the meaning "hundredths" more than the decimal form of the same fraction. Furthermore, it should be stressed that the percent symbol (%) is another, more convenient, numerial for $\frac{1}{100}$, thus

$$\boxed{\% = \frac{1}{100}}$$

When this is really established, only then should be stated:

$$\% = 0.01$$

For example: 9% of 15 = 9 × 0.01 × 15

or = 0.09 × 15 = 1.35

0.3% of 15 = 0.3 × 0.01 × 15

or = 0.0003 × 15 = 0.045

125% of 15 = 125 × 0.01 × 15

or = 1.25 × 15 = 18.75

When learners are able to express fractions in the simplest form, and find the common denominator, then the association of fractional numbers and percent can be deepened. The work relies on the concept of percent: how many percent - so many hundredths; how many hundreths - so many percent.

For example:

(a) $60\% = 60 \times \frac{1}{100}$

$= \frac{60}{100}$

$= \frac{3}{5}$

Therefore $60\% = \frac{3}{5}$

(b) $\frac{1}{20} = \frac{1}{20} \times 100 \times \frac{1}{100}$

$= 5\%$

Therefore $\frac{1}{20} = 5\%$

(c) Which is larger: $\frac{2}{25}$ or 4%?

Solution: $\frac{2}{25}$ or 4%

Since $\frac{2}{25} = \frac{4}{50}$ | but $4\% = \frac{4}{100} = \frac{2}{50}$

Therefore $\frac{2}{25} > 4\%$

In conclusion, fractional numbers and percent are identical in their meanings, only the symbols are different. The percent symbol is more convenient than the symbol for hundredths.

A SYNTHETIC TEST FOR MAXIMA, MINIMA OR POINTS OF INFLECTION IN POLYNOMIAL EXPRESSIONS

(Continued)

W. H. van der Velde
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Mr. C. Pinnegar wrote an article under the above title in the December 1975 issue. In it he makes the conjecture that if the multiple roots of $y^1 = 0$ are of even order, there exists a point of inflection and not an extremum at the corresponding critical point, but if the multiple root is of odd order, there is an extremum but no point of inflection at that point. I like to prove this as a theorem.

Theorem: If $f(x)$ is a polynomial of degree $k = n+m+1$ and $x = a$ is a real multiple root of $f'(x) = 0$ of order n, then for n even there exists a point of inflection at $x = a$ and for n odd there exists an extremum at $x = a$.

Proof: Let $f'(x) = (x-a)^n q(x)$ where $q(x)$ is a polynomial of degree m and $q(a) \neq 0$. Then $f''(x) = (x-a)^{n-1}[nq(x) + (x-a)q'(x)]$. Let $r_1 r_2, \ldots r_j$, be the real roots of $q(x)$, disregarding multiple roots, where $j \leq m$ and $r_1 < r_2 < \cdots < r_j$. Then a lies in one of the intervals $(-\infty, r_1)$, $(r_1, r_2), \ldots (r_j, \infty)$.

When n is even, $f'(x)$ will have the sign of $q(x)$ in the indicated intervals because $(x-a)'' > 0$. Hence, in the interval containing a, there will be no change of sign about the point $x = a$, and consequently no extremum by the first derivative test. Since $f''(x) = 0$ at $x = a$, we have a point of inflection.

When n is odd, $(x-a)^n < 0$ for $x < a$ in the interval containing a and $(x-a)^n > 0$ for $x > a$ in the same interval. The sign of $f'(x)$ will be opposite about the point $x = a$. By the first derivative test the graph will have an extremum at $x = a$. Q.E.D.

INTERMEDIATE DIVISION CURRICULUM REVISIONS 1977 ONE COMMITTEE MEMBER'S VIEW

By Tom Griffiths, President, O.A.M.E.

Since October 6, 1976, when the Honourable Thomas Wells started the ball rolling on the curriculum review for the intermediate division, there has been frantic activity in the Curriculum Division of the Ministry. The ripples of that activity have spread far and wide, even to your peace loving president closetted in the quiet back waters of South Western Ontario. Having been invited to represent O.A.M.E. on the curriculum review committee, I have been involved since our frenetic first meeting on November 23rd.

Our first month or so was almost schizophrenic in its approach to the task at hand. Were we to prepare a "core only" document as implied by the press reports of the minister's speeches with a back to basics thrust, or could we find some method of including this and write a document that would follow in from **The Formative Years'** lead, and include the mathematics that would take Ontario mathematics education into the 1980's? Was there some method whereby we could fulfill both roles? Just what are basics? Should we follow the "Pleasing all with New and Innovative Curriculum", the PANIC method of curriculum development, or could we extend or modify our time-lines to enable us to produce something worth while?

With Paul Pogue and Peter Weygang on the committee, and with the "Pogue" and "Coleman" reports hot from the press, we really had no choice; we had to plan to produce a lasting and future-directed document. However, we are still obliged to finalise some kind of curriculum statement before February 1977, and obviously something which teachers with their present background and training, can teach with only the present textbooks to work from.

With the realisation that we had to fulfill this double task, a pattern of development emerged which I find extremely exciting, and hope that the ministry adopts. Basically it was decided to recommend that evolution of the curriculum, rather than revolution, would be our course and that we should set up the machinery to evolve a worthwhile guideline by 1980 whilst starting at the present state of the art. To achieve this, Claire Zeller suggested that we produce a changeable document in loose-leaf form. With this format it would be feasible to modify sections of the document at any time, simply by sending out replacement sheets. With this philosophy a document can be produced now which has the still desirable and valuable features of the I12 series as its core. This was as requested by the minister, and is also the only possible starting point with present teacher knowledge and available resources. However, in this document can be stated the nucleus of our proposed 1980's curriculum in the form of optional and supplementary topics. Over the next few years these optional and supplementary topics can be gradually updated to core, while the present core will be absorbed to some extent in the new topics. For example, we expect that the skills we now require the students to achieve by the study of number systems will later be acquired by the use of numbers as a tool in problem-solving. This development will also enable teachers to get a view of the future, and give them time to acquire the needed knowledge and skills before the newer material is core. We hope that the teachers will experiment with these topics with their students, and, since the material is not yet prescribed, they will be free of the pressure of specific goals, and be able to take a couple of years to develop their programmes at a comfortable pace.

Another side effect we are hoping for is that teachers will communicate their ideas and opinions concerning the material to the curriculum committee so that we can incorporate these ideas into the developing curriculum. We would also hope to be able to give the individual teachers credit for any of their suggestions used. The ministry has now given permission for the names of the curriculum committee to be listed in the document. Hopefully we will be able to extend this to including names of teachers submitting ideas. My Utopian dream is that eventually the document be composed entirely of the ideas of many classroom teachers from all over the province.

Another advantage of this suggested procedure is that the textbook writers will have longer to produce new material than has been the case in the past. Hopefully this will enable them to spend more time on research and validation of their material than they had at the last session of curriculum changes.

At the same time as coming to grips with a feasible method of implementing our ideas on what a good mathematics education should include, we were, of course, considering what our educational objectives were, and with what mathematical approach they could best be achieved.

One point on which the committee is in agreement is that the best way to learn mathematics is through the investigation and solution of problems. This is in contradiction to much of the present approach in the intermediate division. For example, the I12 document for grade nine specifies that part of the course consists of studying the real number system broken down into its intrinsic parts, such as rational numbers. Use of each of these parts is then practised and finally problems tackled using these types of numbers. This approach leads to exercises of skill-developing questions for their own sake, with problems being tucked on almost as an afterthought, and being of an unrealistic nature. This is not a very interesting way of studying mathematics, and does not in any way do justice to the subject. It does not allow the students the pleasure of investigating and analysing the world about them using mathematics as the very powerful tool we know it to be. It does not allow the development of the natural abilities to mathematise and algebraetise as is mentioned in the "Coleman" report. Some of this skill-building is necessary, but let it be learned as required, not be studied for its own sake. Incidentally, I would hope that teachers will enrich the studies of their potential pure mathematicians with some of the better traditional problems; those which are good just for their own sake. By this means, the better students should gain the best of both worlds.

Through the study of problems we hope that students will acquire their number skills as a side line, much as a student of woodwork learns to use tools by making objects. For example, the study of statistics should give students ample opportunity to practise their number skills, in such tasks as finding means and medians.

Our major problem is deciding what to include in the guidelines was one of having so much excellent material, that we had to prune it down to a practical amount. We will try to present the topics in such a way that they will be integrated with each other, and not learned in isolation. It is very difficult to produce a list of topics, and yet at the same time present them as an indivisible whole. This was of course one of the stated aims of the "New Maths", which was unfortunately not properly implemented. Somehow we have to communicate the message that we really intend to have the topics intertwined. For example, this can be achieved in the study of perimeter and area in mensuration by approaching the topic through physical measurement, with numbers and shapes on paper, through construction and finally by generalisation into a mathematical or algrebraic.

What should or should not be included and how the curriculum should be written to demonstrate the connection between topics is not yet finalised. We have, however, some strong favourites, apart from the traditional topics. These are as follows:-Statistics, experimental in nature, rather than as taught in university. One model question suggested in this field by Peter Waygang was "What would happen to our school if 100 new houses were built in this area?" This example, I hope, gives the image of experimental, non-formal and open ended approach. Other topics to be treated in the same sort of way are: probability, transformation geometry, the use of flow charts as a way of thinking and solving problems, shopping lists, stock lists, tables of values and other forms of matrix, and finally the building of algebraic models. One decision that has had an influence on our thinking is the acceptance of the calculator as an every day item to be owned by all our students. With the acceptance of this, all kinds of real problems can be tackled without fear of the associated complicated calculations involved.

In all the topics we have considered, I have found to my pleasure, and sometimes surprise, that the concepts to be learned are not difficult or esoteric, and that most of them would be a lot of fun to teach. I found myself leaving committee meetings looking forward very much to trying out some of the ideas on my students.

This curriculum, should be in your hands sometime before the end of the academic year. I hope it will assist you in giving your students a better feeling towards mathematics and a better knowledge of what it is and how to use it.

For those who would like to read more on the topic I highly recommend "Recommendations for Intermediate Division Mathematics in Ontario" by Patrick Babin, Paul Pogue, and Robert Knoop, and Chapter four of the report "Mathematical Sciences in Canada."

AN OLD LOOK AT TANGENTS AND ASYMPTOTES

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Introduction

In discussing tangents and asymptotes to plane curves in high school, and indeed introductory university mathematics courses, the emphasis has for some time been on calculus of limit concepts, and quite rightly so. Nevertheless it seems highly likely that most instructors will not resist the temptation to refer to asymptotes as "tangents at infinity". It is the purpose of this article to show that it might be both feasible and pedagogically useful to exploit the latter intuitively appealing idea in a formal way at the high school level. The ideas here are very old, coming from classical algebraic geometry, but they yield valuable geometrical insight, since tangents and asymptotes are here seen to be the same objects. The approach is algebraic, and elementary, presupposing only a little analytic geometry and some ideas about polynomials.

Tangents to Algebraic Curves

We recall the intuitive idea of a tangent to a plane curve C at a point p: this is a line L with the property that it intersects C at p, but at no other points sufficiently close to p, and such that, if L were rotated ever so slightly about p, a new point of intersection p' with C would arise (cf. fig. 1).

Of course, we must in this treatment restrict ourselves to so-called algebraic curves in the plane: these are curves with an equation of the form $f(x,y) = 0$ (in rectangular cartesian coordinates), where $f(x,y)$ is a polynomial in x and y. For example, (1) $f(x,y) = xy - 1 = 0$ (fig. 1: rectangular hyperbola); (2) $y^2 = x^3$ (fig. 2); (3) $y^2 = x^3 + x^2$ (fig. 3), are some algebraic curves, as are all conic sections.

To carry out an algebraic formulation of the notion of tangents to algebraic curves we consider the intersections of a line L with the curve C. Suppose $p(x_0,y_0)$ is a point of intersection of L and C (fig. 1). Then (x_0,y_0) satisfies the equations of both L and C: if C has equation $f(x,y) = 0$, then $f(x_0,y_0) = 0$, while if L has equation $ax + by + c = 0$, with, say, $a \neq 0$, it can be represented parametrically by the equations $x = x_0 + bt$, $y = y_0 + at$, t any real number. Of course, by varying a, b we obtain the equations of all lines through $p(x_0,y_0)$. Thus a point (x,y) will belong to both C and L if, and only if, $f(x_0 + bt, y_0 + at) = 0$; in other words, the points of intersection of L, C are given by those values of t such that

$$F(t) \equiv f(x_0 + bt, y_0 + at) = 0 \ldots \qquad (1)$$

Of course, F(t) is a polynomial in t, and certainly has zero t = 0, which yields the point of intersection $p(x_0,y_0)$ postulated at the outset. Furthermore, the degree of F(t) as a polynomial in t is the same as that of f(x,y).

Suppose, for example, that C is the parabola $f(x,y) = x^2 - y = 0$. We know the line L: $y = 0$ is a tangent to C at $p(x_0,y_0) = (0,0)$. Consider $F(t) = f(0 + bt, 0 + at) = b^2t^2 - at = t(b^2t - a) = 0$. This is a polynomial equation of degree 2 in t and naturally has t = 0 as a root. In fact, t = 0 is a root of multiplicity 1 of the equation F(t) = 0: recall that t = 0 is called a root of multiplicity k of F(t) = 0 if t^k is a factor of F(t) while t^{k+1} is not, where k is a natural number.

Definition 1. The line L cuts C at $p(x_0,y_0)$ with multiplicity k if the corresponding equation F(t) = 0 has t = 0 as a root of multiplicity k.

Thus we see in the example of the parabola that any line L through (0,0) (that is, for any choice of a, b) intersects C: $y = x^2$ at (0,0) with multiplicity at least one.

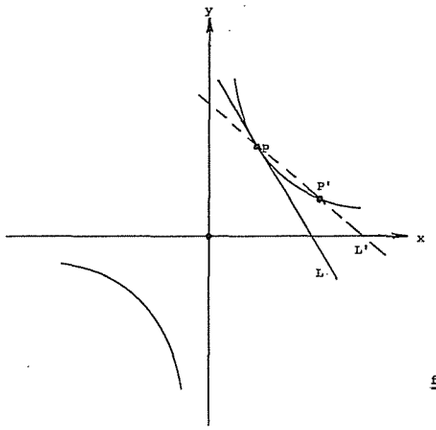


fig. 1: xy = 1

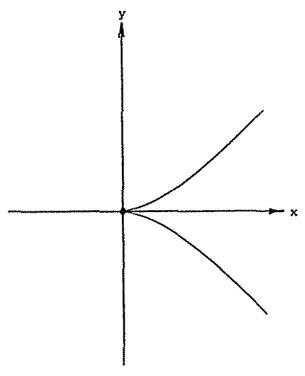


fig. 2: $y^2 = x^3$

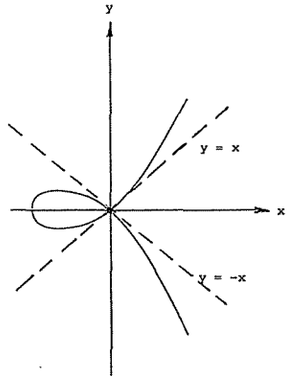


fig. 3: $y^2 = x^3 + x^2$

However, exactly one line cuts C at (0,0) with multiplicity 2; namely, the line for which a = 0, in which case $F(t) = b^2t^2$. Of course this is the line with (parametric) equations x = bt, y = 0, i.e., the x-axis, or, y = 0, as we would expect. This example motivates the following algebraic definition of a tangent to an algebraic curve:

Definition 2. Suppose p is on C. Then there is a positive integer k such that every line through p cuts C at p with multiplicity at least k, but not all lines cut C at p with multiplicity k + 1. Then any line cutting C at p with multiplicity at least k + 1 is called a tangent to C at p.

Example. The curve $C : y^2 = x^3 + x^2$ (fig. 3). Consider $p(x_0,y_0) = (0,0)$. $F(t) = f(bt, at) = t^2(a^2 - b^3t - b^2)$.

All lines cut C at p with multiplicity at least 2, while lines with $a^2 - b^2 = 0$ cut C there with multiplicity 3. These are the two lines with $a = \pm b$, i.e., $y = \pm x$, as shown.

It is not hard to convince oneself that the algebraic definition of a tangent given above yields all lines which are tangents by the intuitive definition given earlier (see, for example [2]). However, the former also yields lines which would not ordinarily be called tangents. For example the "curve" with equation $x^4 + y^4 + x^2 = 0$ has tangent the line $x = 0$, even though the "curve" consists of only one point in the plane. An explanation of this "paradox" may be found in [2].

Asymptotes and the Projective Plane

Recall that when we say the line $y = 0$ is an asymptote to the rectangular hyperbola $xy = 1$, we are inclined to say that it is a tangent to this curve "at infinity". Unfortunately, there is no point in the plane R_2 "at infinity". The natural temptation is to enlarge R_2 by adding the necessary points at infinity. The resulting set of points is called the projective plane, which we may denote as P_2. One way of constructing this set, bearing in mind that we want it to "contain" R_2, is as follows. Since points in R_2 are given by pairs of numbers (x,y), we introduce new variables X, Y, Z, put $x = X/Z$, $y = Y/Z$, and stipulate for the moment that $Z \neq 0$. The set of all such triples (X,Y,Z) can of course be visualized as Euclidean three-space R_3 less the plane $Z = 0$, and each point (x,y) in R_2 yields many points in this space, namely any triple (X,Y,Z), with $x = X/Z$ and $y = Y/Z$. However, if two triples (X_1,Y_1,Z_1), (X_2,Y_2,Z_2) are obtained as above from the same point (x,y) in R_2, then as $x = X_1/Z_1 = X_2/Z_2$, $y = Y_1/Z_1 = Y_2/Z_2$. it follows that $(X_2,Y_2,Z_2) = ((Z_2/Z_1)X_1, (Z_2/Z_1)Y_1, (Z_2/Z_1)Z_1)$. In other words (with $c = Z_2/Z_1$ here)

$$(X_2,Y_2,Z_2) = (c \cdot X_1, c \cdot Y_1, c \cdot Z_1), \text{ some } \quad c \neq 0, \quad (2)$$

Thus, in order to get one triple (X,Y,Z) for each point (x,y) of the plane R_2, we agree to call two triples the "same" if they satisfy (2). In otherwords, the point (x,y) in the plane corresponds to all triples (cx,cy,c), $c \neq 0$, and all are regarded as being the same point in P_2.

Definition 3. The projective plane P_2 is the set $\{(X,Y,Z) \mid X,Y,Z \in R$, not all 0, and with $(cX,cY,cZ) \equiv (X,Y,Z)$, $c \neq 0\}$.

Any triple (cX,cY,cZ), $c \neq 0$ is called a set of homogeneous coordinates for (X,Y,Z). Notice that the definition does not require $Z \neq 0$; triples (X,Y,0) are called points at infinity. Furthermore P_2 "contains" R_2 if we associate with (x,y) the triple (x,y,1), or alternatively, the line in R_3 containing this point; that is $\{(cx,cy,c \mid c \in R, c \neq 0\}$. In other words, points in R_2 become lines in R_3 (less the origin (0,0,0)), but are called points of P_2.

Having suitably enlarged R_2 to P_2 we can obtain curves in P_2 from algebraic curves $C:f(x,y) = 0$ in R_2 as follows: put $x = X/Z$, $y = Y/Z$ in the equation of the curve and clear denominators by multiplying by a suitable power of Z. For example, if $f(x,y) = xy - 1$ (hyperbola), we obtain the equation $X/Z \cdot Y/Z - 1 = 0$, or $f(X,Y,Z) = XY - Z^2 = 0$, as the equation of the corresponding curve in P_2. This is an homogeneous equation in X, Y, Z because each term has the same sum of exponents. More generally an algebraic curve in P_2 is the locus of any homogeneous polynomial equation in X, Y, Z. In particular the straight line $ax + by + c = 0$ in R_2 becomes the curve (line) with equation $aX/Z + bY/Z + c = 0$, or $aX + bY + cZ = 0$, in P_2. This can of course be visualized as a plane through the origin in R_3. The special line $Z = 0$ in P_2, consisting of all points at infinity, is naturally called the line at infinity.

With the correspondences established above, we can now consider the problem of asymptotes to algebraic curves in R_2 by considering tangents to the corresponding curves

in P_2; for, the points at infinity at which asymptotes are tangent to the curve are now <u>accessible</u>, i.e., <u>in</u> P_2. First, however, we must examine the concept of a tangent to a curve in P_2. It is at this point that the algebraic definition for a tangent to a curve is especially useful, for, we can carry it over unaltered to tangents in P_2. Suppose $aX + bY + cZ = 0$ is the equation of a line L in P_2. As mentioned earlier this can be visualized as a plane in R_3 through the origin, and so is determined by two distinct points different from the origin. If (X_0,Y_0,Z_0), (X_1,Y_1,Z_1) are these points on L, then the equation for L can be parametrically written as

$$\begin{aligned} X &= sX_0 + tX_1 \\ Y &= sY_0 + tY_1 \\ Z &= sZ_0 + tZ_1 \end{aligned} \qquad (3)$$

where s, t are real numbers not both zero. That is, as s, t run through all real numbers, all points on L are generated, and only points on L. If $s = 1$, $t = 0$, we obtain the point (X_0,Y_0,Z_0). Now since $\left(\frac{1}{s}X, \frac{1}{s}Y, \frac{1}{s}Z\right) = (X,Y,Z)$ (as points in P_2), all points of L except (X_1,Y_1,Z_1) are generated by the equations

$$\begin{aligned} X &= X_0 + tX_1 \\ Y &= Y_0 + tY_1 \\ Z &= Z_0 + tZ_1 \end{aligned} \qquad (4)$$

when t runs through all real numbers, and furthermore, all points (except (X_1,Y_1,Z_1)) of <u>all</u> lines L through (X_0,Y_0,Z_0) can be generated by equations (4) for some choice of X_1, Y_1, Z_1.

Now suppose that $p = (X_0,Y_0,Z_0)$ is a point lying also on an algebraic curve $C: f(X,Y,Z) = 0$ in P_2. Then all the points of intersection of L and C (except (X_1,Y_1,Z_1)) are given by those t such that

$$F(t) = f(X_0 + tX_1, Y_0 + tY_1, Z_0 + tZ_1) = 0 \qquad (5)$$

Of course t = 0 is a root of equation (5) since it yields the point (X_0,Y_0,Z_0) on L and C. Thus, as in definition 1, we can say that L cuts C at $p = (X_0,Y_0,Z_0)$ with multiplicity k if the equation (5) has t = 0 as a root of multiplicity k. Then the definition for L to be a tangent to C at p is exactly definition 2, suitably interpreted.

Examples

1. Consider, for example the hyperbola C:xy = 1 whose corresponding equation in P_2 is

$$f(X,Y,Z) = XY - Z^2 = 0 \qquad (6)$$

Since a curve in R_2 can have asymptotes only if it has points at infinity, we begin by finding these points: they are the intersections of (6) with the line at infinity Z = 0, and hence must satisfy XY = 0. Homogeneous coordinates for two distinct points in P_2 with this property are (1,0,0) and (0,1,0).

Take $(X_0,Y_0,Z_0) = (1,0,0)$, and consider the intersections of any line L through this point with the curve in P_2 whose equation is (6). From equation (5) we must consider the multiplicity of t = 0 as a root of

$$\begin{aligned} F(t) &= f(1 + tX_1,\ 0 + tY_1,\ 0 + tZ_1) \\ &= (1 + tX_1)tY_1 - (tZ_1)^2 \\ &= t(-tZ_1^2 + tX_1Y_1 + Y_1) = 0. \end{aligned}$$

Clearly all such lines cut C at (1,0,0) with multiplicity 1, but the lines with $Y_1 = 0$ cut C with multiplicity 2, since the equation then reduces $F(t) = -t^2Z_1^2$. The lines in P_2 with $Y_1 = 0$ have parametric equations

$$\begin{aligned} X &= 1 + tX_1 \\ Y &= 0 \\ Z &= tZ_1, \end{aligned}$$

and since y = Y/Z, the line y = 0 in R_2 corresponds to a line in P_2 with these equations, and hence to a line in

P_2 tangent to (6) at the point at infinity (1,0,0). In other words, $y = 0$ is an asymptote to $xy = 1$.

2. We conclude by showing that the parabola $y = x^2$ has no asymptotes. For, the corresponding equation in P_2 is $f(X,Y,Z) = X^2 - YZ = 0$, which intersects the line at infinity $Z = 0$ in points satisfying $X^2 = 0$. Homogeneous coordinates for the point of intersection are thus (0,1,0), and any line through this point has equations

$$X = tX_1$$
$$Y = 1 + tY_1$$
$$Z = tZ_1,$$

for some X_1, Y_1, Z_1. In this case $F(t) = f(tX_1, 1 + tY_1, tZ_1) = t(tX_1^2 - tX_1Z_1 - Z_1)$ so all lines through (0,1,0) intersect the curve with multiplicity 1, while only lines with $Z_1 = 0$ intersect with multiplicity 2. In other words, the only possible asymptotes are lines whose equations in P_2 are

$$X = tX_1$$
$$Y = 1 + tY_1$$
$$Z = 0.$$

This is of course the line at infinity, which does not correspond to any line in R_2. It follows that $y = x^2$ has no asymptotes.

References

(1) J. L. Coolidge: **A Treatise on Algebraic Plane Curves.** New York, Dover Publications, 1959.

(2) W. E. Jenner: **Rudiments of Algebraic Geometry.** New York, Oxford University, Press, 1963.

(3) R. J. Walker: **Algebraic Curves.** New York, Dover Publications, 1962.

Mary had a little lamb
With pretty woolly ears
Its age in weeks is just a half
Of Mary's age in years
Five years ago her mother says
Young Mary's age was three
So what's the age of Mary's lamb
In weeks what must that be?

INDUCTIVE PROBLEM SOLVING

Hugh Ouellette, Winona State College, Winona, Minnesota

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It is often said that the development of a student's problem solving is not the working of the typically assigned exercises that require the completion of identical skill development exercises that have been demonstrated by the teacher of fully explained in the text. Although these types of exercises are necessary, they ignore the creative aspect of mathematics. Problem solving is the finding of a solution to a novel situation and emphasized thinking and reasoning instead of completing a familiar task.

Probably the best way to develop problem solving techniques is to attack the problem situations and then study the processes that were successful in finding solutions. Although there is no one procedure that will solve all problems, this article will illustrate one method for solving certain types of problems.

A criterion task--a challenging question of sufficient difficulty that a generalization must be found in order to answer the question--is used to set up the problem situation. The following are examples of criterion tasks:

1. If $\frac{2}{111}$ is expressed in decimal form, what digit appears in the 101st place?
2. What is the unit's digit in the computed form of 2^n?
3. What is the number of lines of symmetry of a regular polygon containing k sides?
4. Find the sum of
$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \frac{1}{7\times 9} + \cdots + \frac{1}{999\times 1001}.$$

Notice that to insure that a student makes a generalization to complete the criterion tasks above, either a large number or a variable was used in the statement of the task. The choice of using a large number or a variable depends on the student's mathematical maturity; if the student is not comfortable with variables, then large numbers should be used.

Consider the fourth criterion task. A student may try many different ways of attacking the problem. One procedure may be to find the sum of simpler but related problems, i.e.,

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} = \frac{6}{15} \text{ or } \frac{2}{5}$$

$$\left(\frac{1}{1\times 3} + \frac{1}{3\times 5}\right) + \frac{1}{5\times 7} = \frac{2}{5} + \frac{1}{35} = \frac{15}{35} \text{ or } \frac{3}{7}$$

$$\left(\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}\right)+\frac{1}{7\times 9}=\frac{3}{7}+\frac{1}{63}=\frac{28}{63}\text{ or }\frac{4}{9}$$

After finding the sums he may organize the data in a table as follows:

LAST TERM	$\frac{1}{1\times 3}$	$\frac{1}{3\times 5}$	$\frac{1}{5\times 7}$	$\frac{1}{7\times 9}$	. . .	$\frac{1}{999\times 1001}$
SUM	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{7}$	$\frac{4}{9}$		$\frac{500}{1001}$

Once it is felt that a pattern has been discovered he should check a few more cases to determine if he is correct. Often a student will need to compute additional sums to help him find a generalization to solve the task. It should be noted that a student often derives a sense of satisfaction from the realization that his discovered pattern can be used to solve a difficult problem and the solution does not require a long, complex computation. It must be emphasized that no formal proof of the correctness of a proposed solution to the criterion task has been given. Whether or not a verification (proof) should be given will depend on the student's mathematical maturity.

In the spirit of problem solving it seems proper to leave you with the following sequence of criterion tasks:

1. What is the sum of
$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+\cdots+\frac{1}{1492\times 1493}\text{ ?}$$
2. How many diagonals can be drawn from one vertex of a regular polygon containing 1003 sides?
3. What is the sum of the measures of the 1003 interior angles of the polygon in number 2?
4. Find the total number of diagonals possible for the polygon in number 2.
5. What day of the week comes 180 days after Wednesday?
6. Compute $(111\ldots1)^2$ (49 ones)
7. If 15 points are placed on a circle, what is the maximum number of segments and regions that could be formed?

Ontario Association for Mathematics Education

MATH IN 77

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REGISTRATION INFORMATION

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A.) NOTES:

1.) The official conference is Friday night through Sunday noon - however, there is a full day Friday program with Thursday night accommodation facilities available.
2.) The accommodation included in the packages below, is in residences which are approximately 25 metres from all lectures, presentations, sports facilities, and meals. (no walking!)
3.) The University of Ottawa used for number two above is in the downtown centre of Ottawa therefore, several hotels, main shopping areas, many points of interest (Parliament Buildings etc.) and excellent restaurants are all within walking distance.
4.) Rooms will be available at convention rates (approx. $20 - $30 single, $25 - $36 double) in the following hotels, on a ''book yourself'' basis:
Lord Elgin Hotel, 100 Elgin Street, Ottawa;
Park Lane Hotel, 111 Cooper Street, Ottawa.
5.) Registration will be available from Thursday noon through Saturday noon til 11:30 P.M. each evening.
6.) Thursday evening (for those who arrive) will be your ''night on the town''.
7.) For those not opting for package B 2 or 3, meals are available on a cash basis in the ''close - at - hand'' cafeteria.

B.) ALL INCLUSIVE OPTIONS: (Select appropriate package(s) below; AND CIRCLE ON REGISTRATION FORM.)

	MEMBERS		NON-MEMBERS	
	before Mar. 15/77	after Mar. 15/77	before Mar. 15/77	after Mar. 15/77
1 REGISTRATION FEE only: includes: -all presentations and workshops Friday, Saturday and Sunday.	$10.00 person	$10.00 person	$10.00 person	$14.00 person
2 SINGLE PACKAGE: includes: -registration as above, Friday and Saturday in residence accommodation, sports pass for 2 days, Sat. & Sun. breakfast and lunch, parking 3 days, Fri., night welcoming party (see extras Part ''C'')	$50.00 person	$55.00 person	$60.00 person	$65.00 person
3 DOUBLE PACKAGE -as above for number two, except 2 per room	$40.00 person	$45.00 person	$50.00 person	$55.00 , erson

C.) ''SUPER-PRICED'' EXTRAS: (Select appropriate package(s) below AND CIRCLE ON REGISTRATION FORM)

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2) THURSDAY NIGHT PACKAGE: includes: -Friday's program, accommodation extra night, Friday breakfast and lunch, sports pass for extra day, parking extra day (night on the town - your cost).	$15.00 person	$17.00 person
3) FRIDAY NIGHT PARTY:* includes: -entertainment, hors' d'oeuvres, dancing, cash bar. * included in packages 2 and 3 part B	$1.50 person	$2.00 person
4) MEMBERSHIP IN O.A.M.E.: includes: -Membership until Aug. 31/77, 3 issues of math magazine ''Gazette'', 4 issues of ''Abacus'', reduced fees at conference, voting rights at annual meeting,	$10.00	$10.00

ATH IN 77

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5) SATURDAY NIGHT PRESIDENT'S DINNER - DANCE PARTY: includes: -hip o' beef dinner, complimentary wine, orchestra, disc jockey, casino, ''centre of activities'' lounge, bridge tournament, cash bar, and transportation from residence, to and from party site. President's dinner limited to 400	$14.00 person	$15.00 person
6) SATURDAY NIGHT PRESIDENT'S DANCE PARTY ONLY: -as above for number Five, except NO dinner	$8.00 person	$9.00 person

D.) NEEDED INFORMATION: (CIRCLE THE APPROPRIATE NUMBER FROM BELOW ON THE REGISTRATION FORM)

1) I will be attending Friday's sessions.
2) I am interested in mainly: (a) primary, (b) Junior, (c) intermediate, (d) secondary (e) post-secondary.
3) Send me a program booklet when available.
4) I am now a member.
5) I am including membership fees.
6) I am not and do not wish to be a member.

— — — — — — — DETACH AND RETURN WITH CHEQUE — — — — — — —

O.A.M.E. 77 05 13 - 15 REGISTRATION FORM

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3 ($1.50/$2.00)
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5 ($14.00/$15.00)
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3
4
5
6

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O.A.M.E. 77 Ottawa May 13 - 15

Synopsis of Program

A.) Friday: An exciting program including: **Keynote Speaker: Dr. Eric McPherson;** ''Present Day Anxieties in Mathematics'', Workshops for P. J., in French & English; to be led by **Dr. Lola May;** ''''What to do on Tuesday after Teaching Monday''; **Dr. Derek Woodrow** (from Manchester, England); **Yzette Ferland,** and **Roberte Legris, (BLOCS LOGIQUES);** visits to National Research Council, interdisciplinary Science studies, to French Language or French Immersion Mathematics Classrooms; up to date **speakers, discussion & panel,** re: ''Examination & Standards'', ''K - 13 Mathematics Continuum''; **Mathematics** Topic Speakers: **David Ireland, Jacques St. Aubin, Roger Fischler.**

B.) Saturday: a full day program with something for everyone from kindergarten teacher to University professor: **Dr. Lola May** (take note elementary teachers!) speaks on ''What every mathematics teacher ought to know'', also **Keynote Speaker: Peter Hilton;** ''Causes & Consequences of Anxieties in Teaching & Learning Mathematics'', **Jean Paul Collette, Paul Pogue** et al (Recommendations for Intermediate Division Mathematics), and many others with a full range of workshops and panel discussions. All the exhibitors and publishers will be in attendance, complete with complimentary coffee and donuts.

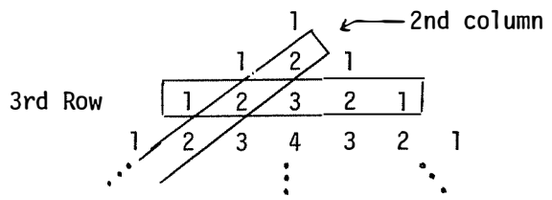
The all-important annual election and meeting will be from 11:30 - 12:30 - members let's VOTE!!

C.) Sunday: **Keynote Speaker: David Robitaille;** ''Identifying & Correcting Learning Difficulties in Mathematics'', and a morning filled with multi ''shorts'' for everyone - 10 min. - 30 min. presentations on just about every topic you can imagine - something to use the very next day - Monday!! or - clear the head from the night before - Saturday!!

***N.B.: Program booklets will be finalized by March - register NOW!!

8. Find an easy method for computing the product (444,444,444,444,444) × (999,999,999,999,999).
9. Given a set of 101 points on a plane, no three of which lie on the same straight line, what is the maximum number of lines which could be formed?
10. How many pairs of vertical angles are formed by 1,001 concurrent lines?
11. What is the greatest number of points of intersection possible for 101 lines in a plane?
12. If $f(X) = |3 - X|$, find $f(f(f...(f(1))...))$ (1492 f's)

Consider the "integer triangle" given below:



13. How many terms are in the 2000th row?
14. In the 1000th row, what is the 1990th term?
15. Find the sum of the integers in the 100th row.
16. What is the middle term of the 69th row?
17. What is the mean (average) of the 100th row?

When a student becomes actively engaged in finding solutions to these criterion tasks, he is given a chance to experience the inductive nature of mathematics, a chance to make a true mathematical discovery, and a chance to find that mathematics is interesting and can be enjoyable.

GAUSS BICENTENNIAL SYMPOSIUM

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E. Forbes (Edinburgh) - Astronomy
G. Garland (Toronto) - Geo - magnetism
K. O. May (Toronto) - History of Mathematics
A. Selberg (Princeton) - Numbers Theory
D. A. Sprott (Waterloo) - Statistics

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University of Toronto

ON CONGRUENCES AND THE CHINESE REMAINDER THEOREM

William J. Bruce, Department of Mathematics, University of Alberta.

The purpose of this article is to suggest to elementary teachers material that can be gleaned from number theory and treated at various grade levels for skill development motivated through problem solving.

Consider two sets of natural numbers, A = {2,3,5} and B = {7} in which neither includes unity and such that in pairs all elements are relatively prime. Suppose that we are to find the set N of natural numbers such that counting by any of the elements of set A leaves 1 as a remainder, whereas counting by the element of set B gives a zero remainder. This problem is a special case of the Chinese Remainder Theorem known to Sun-tse in the first century A.D. [1].

By counting only, pupils of grades 3 and 4 can solve this problem to find at least the smallest number. All they have to do is count by 7's until they find a number such that counting by each of 2's, 3's, and 5's will give a remainder of one. Additional counting by 7's will lead to another, and so on. The smallest number is 91 and the next one is 301, which is 91 + 7(30). It is not unreasonable to expect that some bright pupil even at this level might discover that all such numbers are given by 91 + k(30), where k is 0 or is formed by counting by 7's, and 30 is the product of 2, 3, and 5 in this example.

Another approach to the solution of this problem by elementary students might be to write down the system of congruences for the sets A and B, namely,

$$n = 2a + 1$$
$$n = 3b + 1$$
$$n = 5c + 1$$
$$n = 7d$$

and attempt to find the smallest n by trial and error selection of a, b, c, d or by using a more systematic approach such as given by Shockley [1]. A modification of this is

considered next.

Let n be the natural number sought. The elements of set A are factors of their LCM, namely 30. But the element 7 of set B is not a factor of 30. It follows that we must have

$$n > 30 + 1 = 31.$$

Then

$$n = 31 + m(30),\ m \geq 1.$$

But n/7 must be a whole number, hence $\frac{31 + m(30)}{7}$ must be a whole number. Clearly m = 2 is the smallest number for which this is true. Thus 91 is the smallest number that satisfies the conditions of the problem. From the above discussion we see that the general solution is

$$n = 91 + k(30),$$

where k is divisible by 7.

Suppose now that A = {4,6,7,9}, in which the elements in pairs are not necessarily relatively prime, and B = {11}, in which the elements of A in pairs with the elements of B are relatively prime. We proceed as in the first example. The elements of the set A are factors of their LCM, namely 252. But the element 11 of set B is not a factor of 252. It follows that we must have

$$n \geq 252 + 1 = 253.$$

Then

$$n = 253 + m(252),\ m \geq 0.$$

The smallest number m for which n is divisible by 11 is m = 0. Ths 253 is the smallest number that satisfies the conditions of the problem. The general solution is

$$n = 253 + k(252),$$

where k is divisible by 11.

Consider now the example in which A = {2,3,5} and B = {8}. in this case the elements of A paired with the element 8 of B are not all relatively prime in pairs, namely 2 and 8 are not relatively prime. Proceeding as before, we must have

$$n > 30 + 1 = 31.$$

Then

$$n = 31 + m(30),\ m \geq 1.$$

But 31 + m(30) is always an odd number so can never by divisible by 8. Thus the problem has no solution.

In the example in which A = {2,3,5}, B = {9} we see that 3 and 9 are not relatively prime and to solve the problem we have to show that 31 + m(30) is divisible by 9. But to be divisible by 9 this must also be divisible by 3. Clearly m(30) is a multiple of 3 but 31 is not. Hence 31 + m(30) cannot be divisible by 9. Again the problem has no solution.

It appears that for the examples that we have chosen, namely those that yield remainders of one when counting by the elements of set A, that a solution always exists as long as only relatively prime pairs are obtained when pairing the elements of A with the element of B, otherwise there is no solution. The condition from which it follows that this is true in general is given by Shockley [1].

Teachers are encouraged to construct their own sets such as A = {2,3,4,5,6}, B = {7}, the solution of which is 301 + k(60), where k is divisible by 7, and A = {2,3,5,7}, B = {11}, the solution of which is 2101 + k(210), where k is divisible by 11, and so on. By grade 7 most students should be equipped to handle all of the types suggested or those chosen by the teacher.

Reference

(1) Shockley, James E. **Introduction to Number Theory.** New York: Holt, Rinehart and Winston, Inc., 1967, pp. 45 - 49.

A REAL TWISTER

If my three were a four
And my one were a three,
What I am would be nine
Less than half what I'd be.

I'm only three digits,
Just three in a row.
So what in the world
Must I be? Do you know?

PROJECT ZERO ZERO SEVEN

Bruce Ewen

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Here is an exercise in cryptogramming that introduces a very sophisticated James Bond technique, but at a simple level, from which an avid student can make further, more secure, methods of hidden writing.

A	0
B	1
C	2
D	3
E	4
F	5
G	6
H	7
I	8
J	9
K	10
L	11
M	12
N	13
O	14
P	15
Q	16
R	17
S	18
T	19
U	20
V	21
W	22
X	23
Y	24
Z	25
□	26
,	27
.	28

Encoder:

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

The equipment is shown in the box at left. It consists of matching sets of symbols and numerals and the encoder. Box (for word space), comma and period may not be used but complete a set of 29 symbols, a nice prime number. These sets do not have to be orderly, as mine are, it is only necessary that the sender and the receiver of messages use the same matching. The encoder (call it a matrix if you like, but don't dwell on it) for the beginner should be chosen so that the difference between the products of the diagonals make one, in my example, 3(2)−5(1)=1. (You mathematicians would say it: the determinant of the matrix is one.) After they have a few successes, the students will want to know how it works when this is not so, and they will be able to find it for themselves.

Upon the security of the equipment depends the security of the message. Even the simple method used in my example will be difficult to 'crack,' and in so doing my adversary must use many trials until he/she hits upon the correct combination of matching sets and encoder; some cryptograms can be broken by logical processes, these will not.

Now let's go to work. Here is a message that any self respecting agent will need sometime. It ends with a period just to make an even number of symbols:

SEND MORE MONEY.

Using the box for word spaces, break it up into two parts and call it a matrix if you like, this one having two rows and eight columns, or a two by eight matrix.

$$\begin{bmatrix} S & E & N & D & □ & M & O & R \\ E & □ & M & O & N & E & Y & . \end{bmatrix}$$

Now using your matching sets, trade the symbols for numerals, and set the encoder in front as if to say, I'm going to multiply.

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \bullet \begin{bmatrix} 18 & 4 & 13 & 3 & 26 & 12 & 14 & 17 \\ 4 & 26 & 12 & 14 & 13 & 4 & 24 & 28 \end{bmatrix}$$

Now multiply. Multiplication here means that a two by two matrix multiplied by a two by eight matrix will give you a new two by eight matrix. The element that appears in the first row, first column of the new matrix is made from the first row of the encoder and the first column of the message, this way: 3(18)+1(4). The element in the second row, seventh column of the new matrix is made from the second row of the encoder and the seventh column of the message in the same way: 5(14)+2(24). Our new matrix is:

$$\begin{bmatrix} 58 & 38 & 51 & 23 & 91 & 40 & 66 & 79 \\ 98 & 72 & 89 & 43 & 156 & 68 & 118 & 141 \end{bmatrix}$$

This is now reduced by subtracting (or adding the inverse) multiples of 29 so that every element is less than 29.

$$\begin{bmatrix} 0 & 9 & 22 & 23 & 4 & 11 & 8 & 21 \\ 11 & 14 & 2 & 14 & 11 & 10 & 2 & 25 \end{bmatrix}$$

This easily translates, withthe aid of the matching sets, to

$$\begin{bmatrix} A & J & W & X & E & L & I & V \\ L & O & C & O & L & K & C & Z \end{bmatrix}$$

This is stretched out and sent off, our agent is sure that nobody will read it.

Within the fullness of time, our agent will get a reply. He/she eagerly reads JXTXCFNEADVAJNZI – and gets out his/her equipment. He/she begins by setting out the message in matrix form.

$$\begin{bmatrix} J & X & T & X & C & F & N & E \\ A & D & V & A & J & N & Z & I \end{bmatrix}$$

This is translated into a message of numerals, and the decoder is placed in front in readiness for multiplication. The decoder is made from the encoder by switching the top left element for the lower right, and making the other two elements negative. (Mathematicians will recognize the multiplicative inverse of the encoding matrix so long as the mentioned determinant is equal to one.)

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \bullet \begin{bmatrix} 9 & 23 & 19 & 23 & 2 & 5 & 13 & 4 \\ 0 & 3 & 21 & 0 & 9 & 13 & 25 & 8 \end{bmatrix}$$

Now multiply in the same way, being careful about the signs. Again, to find the element in the second row, fifth column of the new matrix combine the second row of the decoder with the fifth column of the message thus: (−5)(2)+3(9). We now have the matrix:

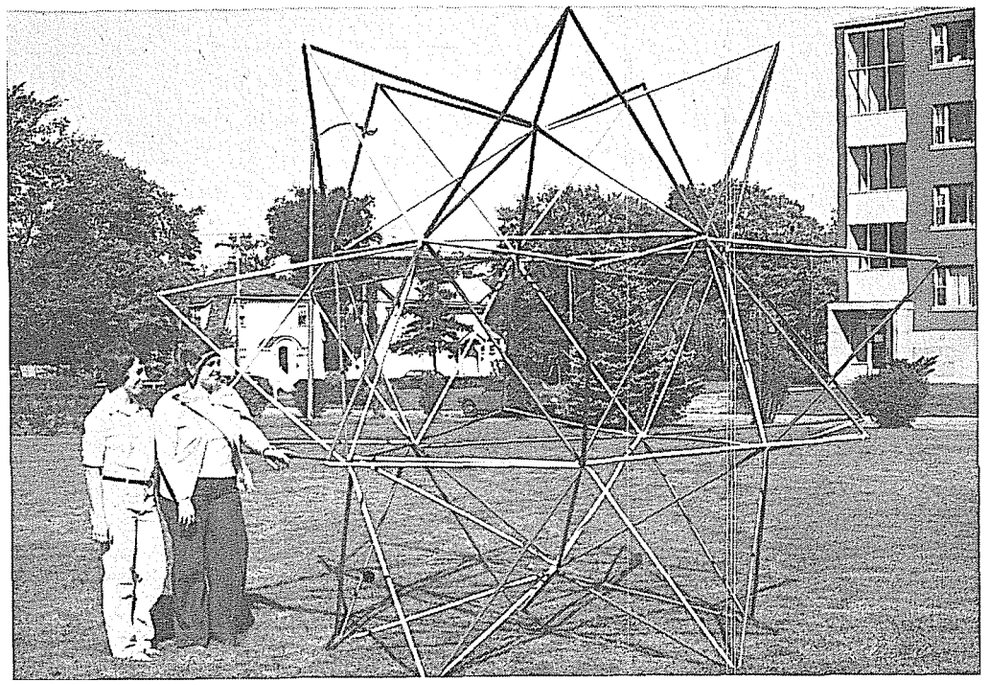
$$\begin{bmatrix} 18 & 43 & 17 & 46 & -5 & -3 & 1 & 0 \\ -45 & -106 & -32 & -115 & 17 & 14 & 10 & 4 \end{bmatrix}$$

Each element is then reduced or increased as necessary until each is a positive number between zero and 29, using multiples of 29.

$$\begin{bmatrix} 18 & 14 & 17 & 17 & 24 & 26 & 1 & 0 \\ 13 & 10 & 26 & 1 & 17 & 14 & 10 & 4 \end{bmatrix}$$

Now our agent can translate and read it, maybe with a sigh.

$$\begin{bmatrix} S & O & R & R & Y & \square & B & A \\ N & K & \square & B & R & O & K & E \end{bmatrix}$$



STELLATE IS BEAUTIFUL -- Student teachers Sandy Beckwith and Wendy Wile pose proudly on the grounds of Nova Scotia Teachers College with a stellate icosahedron constructed with 125 cm wooden rods, a project in mathematics education for their instructor, OAME member H. D. Allen.

Assembled in the small hours of the morning -- "to avoid hecklers" -- the structure brought mathematics to the front page of the community paper.

Truro Daily News Photo.

SS3 - A STRUCTURED STUDENT LANGUAGE

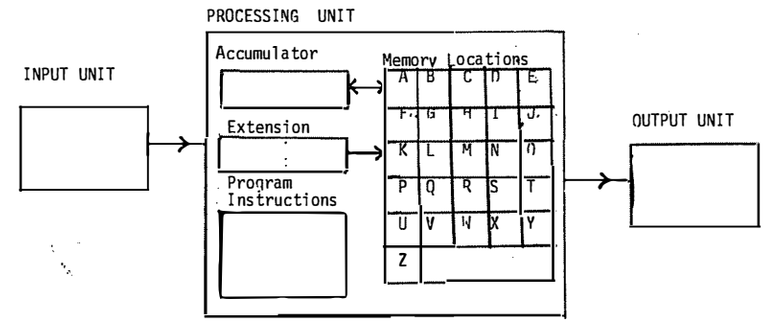
John F. Leightell, Pickering College, Newmarket

Introduction

This article outlines an introductory student programming language. Ths object of the language is to enable the student to understand the operation of a simple computer and at the same time to provide a disciplined language to introduce the concepts of programming.

Most high level languages are designed to give maximum scope to the professional programmer. Consequently they give too much scope to the beginner. The novice tends to use features he is not yet ready for and sometimes he is forced to use features designed to cover many possibilities (e.g. the Read, Write, Format statements in Fortran). What is needed, initially, is a restricted language or at least a subset of an existing language. The flexibility of most programming languages has the disadvantage, for the novice, of allowing for individual creativity. The beginner needs a disciplined approach if he is not to write programs which are so individualistic that they are impossible to decipher. It is for this reason that the language presented here is a disciplined (or structured) language. This forces the student to write programs in a prescribed format. It is also much easier for the teacher to follow the flow of the program and to spot errors.

It is desirable that students have a simple understanding of the basic operations of a computer. For this reason SS3 is designed to simulate a simple computer. By using this language the student is able to perceive and, hopefully, understand the basic operations of the computer. The diagram below outlined the required properties.



The computer consists of an Input unit (card reader), an output unit (line printer) and the processing unit. This contains the accumulator (AC), the extension register (EXT), a storage area for the program instructions and 26 memory locations with addresses A to Z respectively. The processing unit can perform the operations of addition, subtraction, multiplication and division in the AC and EXT registers. Data can be moved from the input unit to the memory locations and from the memory locations to the output unit. The AC is linked to the memory locations so that data can be moved between them in both directions. The EXT is linked to the memory locations in such a way that data can only be moved from the EXT to the memory locations and not in the reverse direction.

Structured Programming

Before describing the language it will be useful to say a few words about structured programming. Although there seems to be some difference of opinion as to the exact definition, the essential feature seems to be the elimination of the 'GO TO' instruction. This means that the flow through a program is 'vertical' and no 'horizontal' transfers are permitted. i.e. transfer cannot be made from one section of the program to some distant section. A structured program can be thought of as a collection of modules arranged in a vertical order and transfer takes place only from a module to the module immediately below it.

Description of the Language

An instruction consists of an operation code and an operand. In most cases the operand is the address of a memory location. In some cases it is permissible to use a numeric operand. The examples below illustrate this.

1. Input three numbers and print out their average

```
IN  ABC          Read three numbers from the same data card and store in
                 locations A,B and C.
CA  A            copy contents of A into AC
AD  B            add c(B) to AC.     c(B) means contents of location B.
AD  C            add c(C) to AC
DV=      3       divide c(AC) by 3
TM  D            transfer c(AC) to memory location D
PR  D            print out c(D)
```

2. Input two numbers. Divide the first by the second. Print out the quotient and the remainder.

```
IN  AB
CA  A
DV  B            divide c(AC) by c(B).Integer division,quotient in AC,
                 remainder in EXT,
TM  C
XL  D            c(EXT) to D
PR  CD           print c(C) and c(D) on the same line  of output
```

3. Convert a Fahrenheit temperature to Celsius

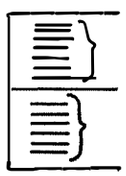
$$\left(C = \frac{5F - 160}{9} \right)$$

```
IN  F       location F contains the fahrenheit temperature
CA  F
MY=      5
SB=    160
DV=      9
TM  C
PR  FC
```

Decision Blocks

A decision block must contain three instructions in a prescribed order, namely;

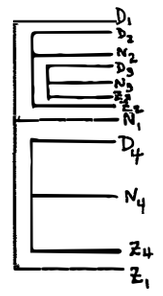
1. The decision statement followed by the instructions to be executed if the decision is 'true'.
2. The instruction 'NO' followed by the instructions to be executed if the decision is false.
3. The instruction ZZ' to signify the end.of the decision block.



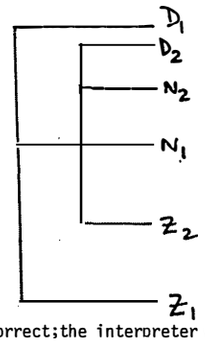
There are three decision instructions; LS, GT and EQ for less than, greater than and equal to respectively. They operate as explained below:

LS G if the contents of the AC is less than the contents of location G. then the instructions immediately following the decision are executed and the instructions following the 'NO' are skipped. If not, then the instructions following the 'NO' are executed. The decision instructions may be used with a numeric literal, eg. LS = 6798. The use of the equals sign signifies to the interpreter that a number will follow. Hence LS = 6798 means 'if the AC is less than 6798'.

Decision blocks may be nested to a depth of 10. To ensure execution the inner blocks must be completely contained between adjacent arms of the outer block as shown in the diagram (D stands for a decision statement).

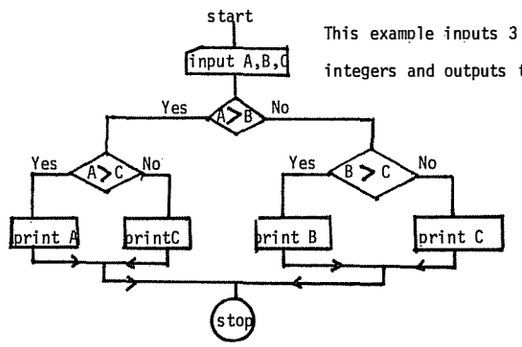


correct



incorrect;the interpreter will detect this invalid structure and prevent execution.

This structuring of decision blocks give a fast and easy way for students to check the logic of their programs.



This example inputs 3 non-equal integers and outputs the largest.

```
IN  ABC
CA  A
GT  B
CA  A
GT  C
PR  A
NO
PR  C
ZZ
NO
CA  B
GT  C
PR  B
NO
PR  C
ZZ
ZZ
```

In a flow diagram the inner blocks must always be closed before the outer ones. It is possible to check the validity of the decision blocks by drawing in the decision frames alongside the coding. If the decision frames intersect, there is an error in the structure of the blocks.

Loops

There are two kinds of loops in SS3. The first causes a series of instructions to be repeated a specified number of times (fixed loops). The second allows a series of instructions to be repeated until a specified variable has attained a stated value (variable loops).

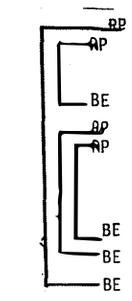
Fixed Loops

To control this kind of loop, two instructions are required. At the beginning of the loop, the instruction RP = (numeric literal) indicates the number of times the loop is to be repeated and the instruction BE signifies the end of the loop.

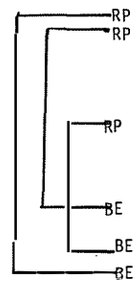
```
Example   Find the sum of 100 terms of the series  3+5+7+...
          CA=    3
          TM  T              T contains the number 3.T is the term
          CA=    0
          TM  S              S contains 0.S is the sum.
          RP=  100           repeat the following instructions 100 times.
          CA  S
          AD  T
          TM  S              these instructions repeated 100 times
          CA  T
          AD=    2
          TM  T
          BE                 end of loop
          PR  S
          FI
      END
```

FI is the code for the logical end of the program and END is the physical end.

Fixed loops may be nested to a depth of 10 but they must not intersect.

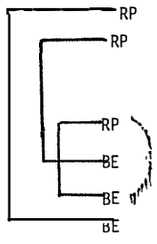


A- correct

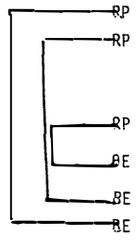


B- incorrect

Although diagram B represents an incorrect structuring of loops, it does not produce a detectable error. It would be interpreted as shown below.



intended



interpreted

Variable Loops

This second type of loop is used when the number of repititions may vary. This situation occurs frequently e.g. in finding the mean of a set of marks, we would require the program to work for any number of marks.

Two instructions are required for this kind of loop. At the beginning of the loop the instruction UC A indicates that the loop is to be repeated until the contents of location A have reached a value specified by the final instruction in the loop. This is of the form VFn B or VFn= (numeric literal) where n may have the following values and meanings.

n	meaning
0	less than
1	equal to
2	greater than
3	less than or equal to
4	greater than or equal to

Hence the loop is repeated until the location specified at the beginning of the loop has attained the value specified at the end of the loop. This may be either the contents of a location or a specific number.

Examples

1.
```
UC  A
 ---
 ---   } instructions to
 ---     be repeated
VF4=  675
```
in this example the loop would be repeated until the contents of location A are greater than or equal to 675.

2.
```
UC  J
 ---
 ---   } instructions to
 ---     be repeated.
VF0  B
```
in this example the loop would be repeated until the contents of location J are less than the contents of location B.

3. For the following series, 2 + 4 + 8..., 3 + 6 + 12..., 4 + 8 + 16..., ..., n + 2n + 4n..., find the sum of each series and the total sum of all the series. Each series contains 10 terms.

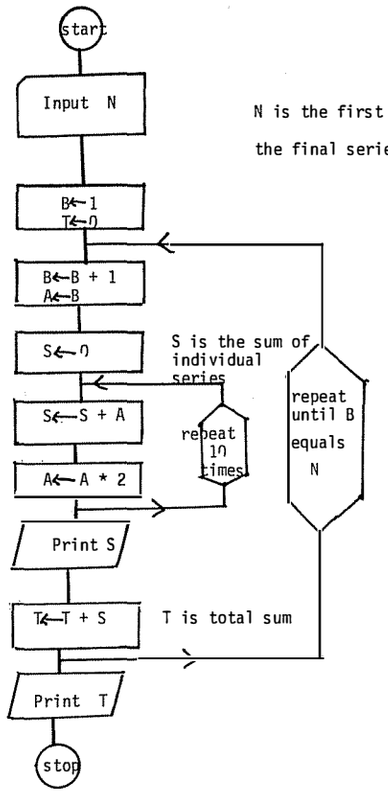
Example 3 illustrates the nesting of loops. Note that fixed loops can be nested within variable loops. Variable loops may be nested within fixed loops.

It is obvious that variable loops can be used to fulfill the function of fixed loops. Fixed loops were included in the language because of their simplicity. It is much easier to introduce the concepts of loops because students can focus their attention on the details of the loop and not have to worry about creating and incrementing a counter. Eventually students are encouraged to use variable loops because of their greater flexibility.

Other Details

The codes for the language were chosen so that no two instructions have the same first letter and no two instructions have the same second letter. This enables an op-code to be recognized on the identification of only one of the two letters. This is a very useful feature when students are forced to use optical cards.

The interpreter contains a test which limits the student programs to 2000 operations (an operation is defined to be the execution of one instruction). This is to guard against infinite loops and other typical student errors which lead to endless programs. This limit can be altered in situations where this is necessary.



N is the first term of the final series

The inner loop sums the individual series.These sums are added to the overall total(T).This process is repeated until the required number of series have been processed.

```
      IN  N
      CA=   1
      TM  B
      CA=   0
      TM  T
   ¬C  B
      CA  B
      AD=   1
      TM  B
      TM  A
      CA=   0
      TM  S
   ¬RP=  10
      CA  S
      AD  A
      TM  S
      CA  A
      MY=   2
      TM  A
   ¬BE
      PR  S
      CA  T
      AD  S
      TM  T
   ¬VF1 N
      PR  T
      FI
END
```

The Interpreter

The interpreter consists of a compilation section and an execution section. In the compilation section each instruction is checked for possible errors. Where these occur a diagnostic message is printed directly under the instruction containing the error. The interpreter is programmed to correct errors where possible. In the execution section, the instructions are carried out (providing that no compilation errors have occurred). The interpreter is written in Fortran IV (on an IBM 1130) and stored on disc in core image. Each student program must begin with the control card //XEQ SS3. This causes the interpreter to be loaded into main memory (loading time about 4 secs.). Further details concerning the language and the interpreter can be obtained by contacting the author.

Half of me is just one more
Than seven ninths of fifty-four
So now perhaps you'd like to try
To answer this: "What am I?"

EXPERIENCE '76: A COMPUTER APPLICATIONS PROJECT

Don Fox, Ministry of Education, Ottawa Valley Region

The Ontario Youth Secretariat was, in the summer of 1976, again responsible for co-ordinating the Ontario Government's employment program known as Experience '76. Now in its fourth year of operation the program was administered by providing funds to the various Ministries, with the Youth Secretariat providing information and publicity.

The Ministry of Education operated program 60 in co-operation with local school boards by supplying grant monies to undertake a wide variety of projects. Two criteria were used in approving individual projects for funding (to a maximum of $5,300 each)-in addition to providing direct educational benefit to the community, a project must provide the students with an opportunity for a learning experience. In many cases the experience played an important role in helping students to make career decisions.

In the Ottawa Vally Region, 26 projects were approved, and 119 ;secondary or post-secondary students were hired as team members by the board, or as coordinator or project leaders by the Ministry. The ''Computer Applications'' project was one of the three operated by the Renfrew County Board of Education. The two students involved have both been enrolled in the Computer Science courses at Fellowes High School in Pembroke. In June 1976, Jonathan completed the (approved) year 5 experimental credit course at Fellowes.

For further information please feel free to contact Renfrew Board Advisor Bill Britton, Dan Russell of the Ministry's Data Processing Branch at Queen's Park (965-4540) or the undersigned in Ottawa(225-2230).

STUDENT REPORT TO RENFREW COUNTY BOARD OF EDUCATION

The main objective of our project was to produce a package of many computer programs which would facilitate easier access and faster job turn around for students using computers in the province of Ontario. Thisobjective was accomplished in different ways.

Firstly, a good number of OMR (optical mark read) interpreters were written. Under normal circumstances, one inputs information to the computer via the use of punched cards. These cards are usually punched on a keypunch which produces a pattern of holes unique to each character punched and costs in the neighbourhood of $2500. As a result, keypunches in schools across the province are few and far between. Even at schools that are fortunate enough to have such machines, the rate of student use is low, as administrative duties usually occupy the machines a good percentage of the time. Therefore, through the use of these programs at any computer installation that supports IBM assembler and OMR card readers (most do), students may achieve their goals much more easily. They simply use OMR cards to code their programs on and choose the appropriate coding by filling in bubbles on the card with magnetic lead pencils (eg. HB). The OMR interpreters then take this student job which has been 'marksensed' and translate it into the proper language configuration just as if it were a punched card. Included among the OMR interpreters are those for FORTRAN, COBOL, SP/k, SPECTRE, SPECTRE MAP and general utilities (eg. a program which allows marksensed input for the SGIS input stream and for the Supermarket Management game).

Secondly, in the writing of the above programs, a great deal of design was put into the construction of OMR cards. Each of these cards was designed with the student in mind. Therefore, they were easy to read and had the ability to be quickly marked with the occurence of errors being relatively low. These cards, once designed, were ordered and placed on stock for extensive student use.

Thirdly, a good deal of design work was put into the translation of the Huntington Computer Education package, a group of educational programs spanning many educational fields. These programs were written originally in a language called BASIC and were translated into FED, a FORTRAN subset supported at a few Ontario schools, last summer on an Experience '75 grant.

This summer, however, these programs were translated into WATFIV, a broader FORTRAN subset supported at many installations across the province. Thus these programs will now be made available to a great deal more students across the vast province of Ontario.

Fourthly, we did a great amount of work this summer on designing and redisigning computer simulation games such as the Purdue Supermarket Management Game and the Pollution Game. Such computer simulation games are very beneficial to student and teacher alike, as they allow the student to experience and the teacher to supervise a very realistic situation, whether it be in the business world or the realm of science. Therefore, through some modifications and revamping, we have rendered such computer simulation games more compatible to computer systems in schools throughout the province. These modifications have increased the efficiency of job turn around in many ways, such as allowing for marksense cards in the input job stream and saving paper in the output stream, which proves to be a good measure in these times of financial difficulty.

Finally, we engaged ourselves in a project we did mostly in our own spare time. Most computer installations throughout the province support SPECTRE MAP, a small language which leads one vaguely into the massive IBM assembly language upon completion. Many students following this course of study (SPECTRE MAP to IBM assembler) find that somewhere between the two a definite step is missing. This step, however, does exist. It is known simply as WATMAP and was developed at the University of Waterloo several years ago to fill this gap. But, because of the surprisingly high cost, schoolboards across Ontario have yet to procure this language compiler. Therefore, we decided to write out own version of this popular introductory computer language. This design project was named FAULTI, an acronym derived from "Fellowes Assembly Utility Language and Training Interpreter". This language incorporates all of the features offered by the WATIAC computer and introduces a few new ones. In short, this small language which we have designed will save the provincial school system a good deal of money, as it runs on any IBM oriented machine and will be ready for student use by Christmas of this year.

In conclusion, therefore, our major project this summer has been to improve computer usage in schools throughout the province. Hopefully we have succeeded, but we will let our fellow students be the judges of our efforts.

Respectfully submitted,
MIKE LEONARD
JONATHAN HARRIS
EXPERIENCE '76

students of Fellowes High School,
Pembroke, Ontario

INTERVIEW

Taped Talks with Bob Smith

• • • EDITH BIGGS

Many teachers in Ontario will remember Miss Biggs for the Math workships she ran during 1968 and some will have read her books. I interviewed her on July 6, 1976 at her home in the London suburb of Ealing.

R.S. Even in retirement, I hear you are continuing your work. Where have you been?

E.B. Well, to begin with, I have been to India to do some volunteer work, then to Africa for U.N.E.S.C.O. working in Botswana, Lesotho and Swaziland. After that I spent some time in Autralia, but, I have been told by the University of London that I will have my wings clipped as I must keep University terms as a Doctoral candidate. However, next August, I am off to Australia again.

R.S. I am interested in your research study. What is it to be?

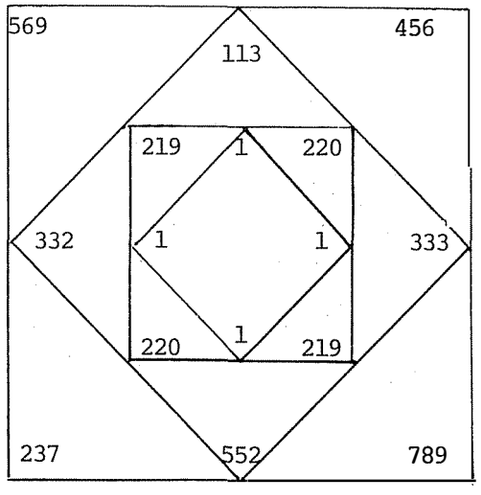
E.B. The project is basically to compare two patterns of In-Service education in Mathematics for teachers of children age 5 - 13. One will be on-site, working with all the teachers, including the head; the other will be off-site, where teachers from a group of schools will meet at a teachers' centre. I feel part of the difficulty we have experienced - why changes were not so great after 18 years of intensive work, is that when teachers return to the classroom after workshops they really do not know what is involved. I also feel rather strongly that heads (headmasters) must become involved.

R.S. Have you already started to work on the study?

E.B. Very definitely. And for the first time I am working with teachers in on-site schools who have a very negative attitude towards mathematics. In fact, some of the teachers met me at the door of the school and stated that they had always hated mathematics and they don't like it now and they don't think anything anyone will do will make them change their mind.

R.S. It looks as though you have a task before you. May we change the topic?

E.B. Certainly, but first I would like to say something about number knowledge. Because I am concerned that teachers seem to be confused as to whether the children should know their number facts or not. Teachers seem to be far more concerned over multiplication facts when they should be worried about the subtraction facts, which are on the whole not well known. This is an area where we should be doing a good deal of work. I would put the emphasis on patterns. Let me give you an example of a good practice activity. The child, to begin with, thinks of 4 numbers less than 10 and places one at each corner of a square. Then he finds the difference of each pair and puts these at the mid points of the edges. He joins the new numbers. He continues this because in Mathematics you always continue until you have to stop. The interesting part is that you can take numbers up to 1000 as I did with children today and you will find that there are a certain number of stages you go through before you have to stop. In other words, there is a limit and this can be proved by simple algebra. By the way, this can be tried with triangles, pentagons, hexagons or any shape. It is a serious activity and I am not going to tell you what the answer is but to assure you that it is tremendous fun (and a thank you to Mary Dolly of the U.S. for this activity).



R.S. Thank you, Miss Biggs, you always have a suggestion for the classroom teacher.

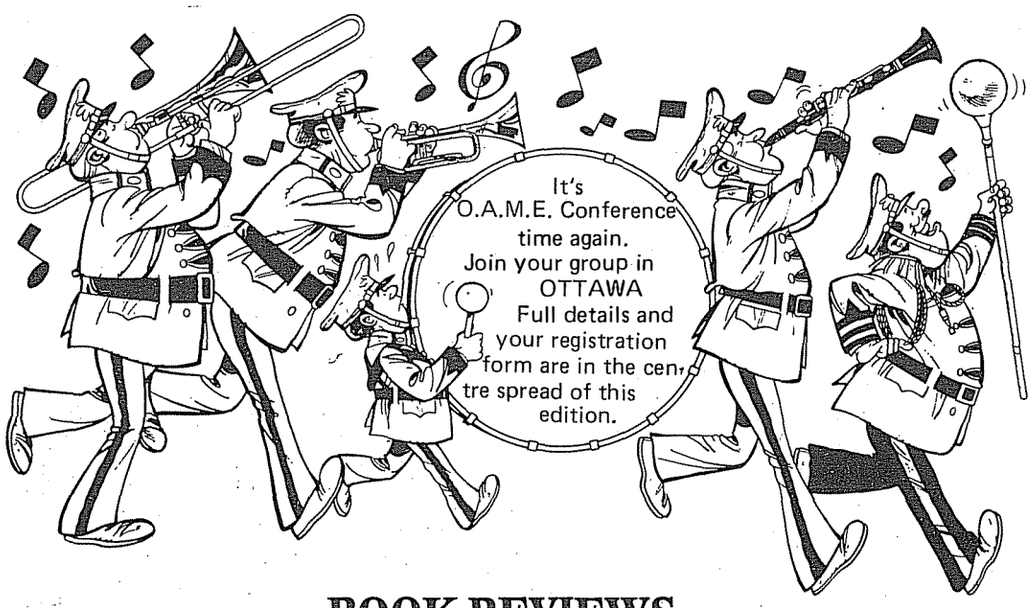
I am a pair of one-digit numbers. Multiply me, subtract me, add me. My difference subtracted from my product equals my sum. What numbers am I?

Question: How did the Romans add and subtract roman numerals?

Answer: Simple - they used an abacus. The Roman version of this bead - and frame device had seven long bars and seven short ones. Four beads were strung on each long bar and one each on the short. The short bars were marked V for 5, and the long bars were graduated 1, X, C, M up to the millions. To write the number 135, for instance, you pushed up one C bead, three X beads and the corresponding V bead. If you needed to subtract 22 from 135, you pushed down two X beads, the V bead, and moved up three I's, leaving 113. Extra bars were added for problems involving fractions.

As you might expect, all mathematical calculations were clumsy by modern standards until the development of Arabic numerals and other symbols, though roman numerals were prominent for nearly 2000 years. Somehow it was easier for the average user to see three in III than in 3.

(Seen in Canadian Magazine, **You Asked Us,** January 1976).



BOOK REVIEWS

Observations on the Gelfand Club of Ontario, Eureka, and the 'Russell' and 'Pogue' Reports.

Dr. William Higginson, Queens University.

Even at the best of times it is difficult to reach any definite conclusions about the state of an enterprise as complex as mathematics education in the province of Ontario. These are not the best of times. Rapid changes in cultural values and expectations, combined with significant shifts in the composition of school and university student bodies and the continual expansion of mathematical knowledge, should induce a sense of caution in any would-be commentator or forecaster. However, with eyes firmly fixed on circulation figures and related advertising dollars, members of the media apply the fundamental rule that 'sensation and simplicity sell' and proceed to accentuate the bizarre, and flamboyant and the idiosyncratic. The general public is accordingly treated to letters to the editor from irate chemistry professors, hot-line conversations with disgruntled taxpayers, and editorials filled with visions of wastelands of atrophied intellect caused by the premature and excessive use of hand calculators. The point is not that these people are wrong - the 'everything's wonderful' rebuttal which sometimes comes is no less primitive - but rather that is very seldom that a balanced picture is presented. Even in the rare case where comments are made about serious studies, misinterpretations and misplaced emphases are too common. (The treatment of the Coleman Report **The Mathematical Sciences in Canada** would be an example of this.)

In a minor attempt to reverse this trend. we review here four publications which, even within professional circles, soon to be less well-known than they deserve to be. It seems possible that these low-profile projects may, in their own way, be better indications of the state of mathematics education in the province than some of the other more widely reported efforts.

The first two publications have several similarities. Both are problem-centred periodicals and both are the efforts of small, dedicated groups. Dr. E.J. Barbeau of the University of Toronto has been running the Gelfand Club of Ontario for over five years. The Club is intended to provide a means of putting outstanding high school mathematics students in touch with university mathematicians. (The club is named after the famous Russian mathematician who worked extensively with high school students.) Each month during the academic year, a series of problems on some mathematical theme is circulated to all members of the club. The themes chosen are usually either extensions of some high school work such as 'Matrix Algebra and Linear Programming' (March, 1973), or attractive enrichment topics such as 'Inversion in a Circle' (December, 1971). The problems, selected by Dr. Barbeau and a few colleagues, range in difficulty. Of the nine problems on the January 1977 paper on 'Squares', for example, we find:

"P2. $1 + 2 + 3 + 4 = 1^2 + 3^2$

$1 + 2 + 3 + \ldots + 8 + 9 = 3^2 + 6^2$

$1 + 2 + 3 + \ldots + 15 + 16 = 6^2 + 10^2$

State and prove a generalization.

P8. Let a,b,c be three positive integers such that $a + b$, $a + c$, $b + c$ and $a + b + c$ are all perfect squares. Show that either a,b,c are all multiples of 4 or that two of a,b,c are multiples of 8 and the remaining integer leaves a remainder of 1 upon division by 8. Find all such triples (a,b,c) for which two of the numbers do not exceed 100."

Not the sort of question one would assign to the average student. But then the Gelfand Club is not intended for average students. Students who do try some of the problems are encouraged to send their solutions, complete or incomplete, to Professor Barbeau for comment.

Eureka, a problems journal edited by Leo Sauve, is sponsored by the Carleton-Ottawa Mathematice Association (COMA) and is published by Algonquin College. Volume one of **Eureka** appeared in 1975. Some indication of the scale and rate of growth of this publication can be deduced from the fact that the ten numbers of Volume one occupied 102 pages and the ten numbers of Volume two took up 234 pages. Attractively laid out, charmingly bilingual, and overflowing with tempting problems, this is an extremely impressive publication. Most of the problems could be attempted by an able undergraduate or an unusually capable high school student. As mental 'keep fit' exercises for the mathematics teacher, they are first class. The editorial comments on proposed solutions to the problems are very thorough and well documented. In the August-September 1976 number, 2(7), for instance, we get seven pages of excellent discussion of the following well-known poser:

Let f be the operation which takes a positive integer n to ½ n (if n is even) and to $3n + 1$ (if n is odd). Prove or disprove that any positive integer can be reduced to 1 by successively applying f to it."

It is gratifying to see that the work of M. Sauve and his colleagues is being recognized in the world of mathematics problem-solvers. (The names of Trigg, Meyers, Conrad, and Dodge have begun to appear with regularity.) The task of producing a publication of this quality is a major one. **Eureka** needs and deserves more support from the Ontario mathematics community, so take out a subscription (see information at the end of this review) and start solving.

In 1975 the Ministry of Education published the results of a large-scale investigation of the characteristics of elementary school mathematics programs in the province. The investigation had been carried out by a team led by Professors Howard Russell and Floyd Robinson of OISE. The project team found that "the instructional methodology practised in most schools and most types of schools was more traditional than the flexible open style advocated in P1J1". Other observations were that many teachers were not familiar with, or made little use of, ministerial guidelines, that the learning materials important to the implementation of the program were often not readily

available and that geometry and problem-solving were two of the areas least adequately handled.

The 'Russell Report' is comprehensive and well-organized and would be an invaluable document for anyone interested in elementary mathematics education in this province. Given that the observations about scarcity of concrete materials and unfamiliarity with guidelines were made prior to the introduction of **The Formative Years,** it would seem that the suggestion of the elementary school principals that more teacher training and guidance in mathematics teaching would lead to better mathematics programs needs particular consideration.

A second research project of the Ministry of Education was carried out by Dr. Patrick Babin of the University of Ottawa, and Mr. Paul Pogue of the Simcoe County Board of Education, from July, 1974 to August, 1975. In 1976 the final report of this project was published as **Recommendations for Intermediate Division Mathematics in Ontario.** Mr. Pogue and Dr. Babin examined a large number of mathematics programs for 12-15 year olds in other parts of the world and carried out many personal interviews. On the basis of this work, the 'Pogue Report' (as we shall somewhat inaccurately refer to it) makes a number of recommendations. It was felt that the "major goal of intermediate mathematics education be to provide the learner with mathematical concepts, facts, and skills, that develop his ability to think logically, through interdisciplinary problem solving, to enable him to function affectively in a technological society". The report endorses an emphasis on problem-solving and applications. Topics to be emphasized include statistics and probability, flow charts, transformations, computer awareness, estimating and approximating, and measurement. It is recommended that relations, functions, and trigonometry be introduced earlier than is now the case. Factoring, rational algebraic expressions, complicated word problems, division of polymonials, and the heavy emphasis on structure are recommended for deletion.

Given the recent decision of the Ministry of Education to produce much more substantial guidelines for the mathematics curricula of the Intermediate Division, the Pogue Report is an important document for teachers of both elementary and secondary school mathematics. In the opinion of this reviewer, the recommendations in the report are essentially sound. The method of arriving at these recommendations, however, appears to be somewhat arbitrary. The extensive influence of a few mathematics educators (particularly Bell and Usiskin) and the virtual exclusion of others (such as Davis, Polya, Sawyer, and the members of the Association of Teachers of Mathematics in Britain) seem hard to justify. Nor is it obvious that the considerable space given to the 'curriculum model' component of the report is well invested; as, for instance, in "The structure of the discipline 'mathematics' should be organized to facilitate the instruction and development of the learner. It consists of a series of related concepts and principles which constitute the domain of the discipline. The domain is then the content of organized knowledge. This knowledge Schwab (1970) calls the substantial structure of a discipline; it is that part of content that has to be understood by students. When Bruner (1963) proposes his spiral curriculum, he is concerned with one of Taba's main concerns ..." (pp. 37-38).

The difficult part of the Pogue Report, however, will be its implementation. To say that we need more applications and more problem solving is one thing. To find sources and examples is quite another. It is a worthy task and one which members of OAME should take upon themselves.

The Gelfand Club of Ontario
c/o E. J. Barbeau
Department of Mathematics
University of Toronto
Toronto, Ontario M5S 1A1.

Has been appearing monthly during the university academic year. Available for the cost of postage and stationery. This was $2.50 per annum but it may increase.

Eureka sponsored by COMA.
Publie par le College Algonquin
Content communications: Editor,
Leo Sauve Mathematics Department
Algonquin College, 281 Echo Drive
Ottawa, Ontario K1S 1N3.

Published monthly except (July and August). 6.00 per annum. Back copies of combined Volumes 1 and 2 for $10. Back issues $1.00 each. Subscription information from Mr. F.G.B. Maskell, Secretary-Treasurer COMA,
Algonquin College,
200 Lees Avenue,
Ottawa, Ontario K1S 0C5.

Current Ontario Elementary School Mathematics Programs:
A Study of the Characteristics of Current Elementary School
Mathematics Programs in Ontario: H. Howard Russell, Principal Investigator, Floyd G. Robinson, Co-Investigator.
Toronto: Ministry of Education, Ontario, 1975. 305 pp.
Available from OISE Publications Sales, 252 Bloor St. West,
Toronto, Ontario M5S 1V6. $6.00

Recommendations for Intermediate Division Mathematics in Ontario:
Based Upon a Study of Local and International Programs.
Patrick Babin, Principal Investigator, Paul Pogue, Project Director. Toronto: Ministry of Education, Ontario, 1976. 180 pp. Available from OISE Publications (see address above) or the Ontario Government Bookstore, 880 Bay Street, Toronto, or from The University of Ottawa Press, 65 Hastey Avenue, Ottawa, Ontario K1N 6N5. $4.00

Briefly Noted

An excellent 'attitude changer' for elementary mathphobes is Marilyn Burns' **The I Hate Mathematics! Book**, 127 pages of intriguing questions (How Many Sides **Does** a Banana Have?) and activities. Attractively designed and humorously illustrated, this book would appeal to students in the junior and intermediate divisions. Their teachers would probably enjoy it too. (Boston: Little, Brown and Company, 1975. 0316117412. $4.50 PB.)

Martin Gardner's latest collection to be published is **The Incredible Dr. Matrix: The World's Greatest Numerologist**, New York: Scribner's, 1976. 068414669X. 256 pp. $8.95 HB. Eighteen chapters of numerical madness which have appeared in Scientific American over the last sixteen years. Addicts have no choice; novices are warned that Gardner is habit forming.

The Proceedings of the 'Euclid' conference on Basic Mathematical Skills and Learning (October, 1975) are available at no charge from Edward Esty, Mail Stop 7, National Institute of Education, Washington D.C. 20208. This conference was organized by Peter Hilton and Gerald Rising. Thirty-three of the most able mathematics educators in the U.S. were asked to draft short position papers on the question of 'basic skills'. These position papers and the reports of the working groups constitute the two volumes of the proceedings. The paper by Hilton and Rising called **'Thoughts on the State of Mathematics Education Today'** is particularly good. These proceedings should be required reading for any person interested in the 'basic skills' area.

Texas Instruments Incorporated is preparing to move into elementary mathematics in a major way. Some idea of the materials and approaches they have prepared can be seen in their brochure **Mathematics Learning with Calculators** which is available from TCI, 13500 N. Central Expressway, P.O. Box 5012, M/S 54 Dallas, Texas, 75222.

The Ascent of Man, the book of the television series of the same name by Jacob Bronowski has now appeared in a paperback edition (Boston: Little, Brown, 1976, $10.). This should bring his very fine chapter on mathematics ('The Music of the Spheres', Chapter Five) to an even wider audience.

Dr. Kenneth Iverson, the creator of the very powerful and increasingly popular computer language APL has gone into the publishing business with the formation of APL Press. (Iverson produced A Programming Language for his Harvard doctorate. A Canadian by birth he is also a graduate of Alberta and Queen's) Of special interest to high school mathematics teachers will be his **Algebra: An Algorithmic Treatment** and **Elementary Analysis** and D.L. Orth's **Calculus in a New Key.** Prices and further information are available from APL Press, Box 27, Swarthmore Pa. U.S.A. 19081.

Teachers looking for sources on the use of hand calculators will find a five-page bibliography prepared by the National Council of Teachers of Mathematics to be of interest. Entitled **Minicalculator Information Resources** it is available from the NCTM at 1906 Association Drive, Reston, Virginia, U.S.A. 22091.

For Science Fiction Buffs:

One of the benefits of retirement is the opportunity to do some "idle" reading. I have never been a science fiction fan; however Isaac Asimov's collection of his best stories caught my eye in the public library recently. I especially recommend **Mirror Image**, written in 1972, an interesting crime puzzle involving two mathematicians, and **The Billiard Ball** (1967), another crime drama about general relativity and "anti-gravity". Asimov, Isaac: **The Best of Isaac Asimov**, Doubleday & Co. Inc., New York, 1974)

A.H.

DIVERSIONS

By T. G. Friday

"X"s AND "O"s

1	2	3
4	5	6
7	8	9

The traditional "X"s and "O"s can be modified as follows.

First modification: Each player plays in turn 3 moves, and then no new "X"s or "O"s can be added. Instead each player in turn can slide their X (or O) into an adjacent square. I.e. 4→5 or 4→1 or 4→8 or 4→7 or 4→2, but 4 could not be moved to 3,6 or 9. First player to have 3 in a row wins.

Second modification: Either player can play "X" or "O" at any turn. Again first person to have 3 in a row, wins.

	O	
X	O	X
	X	

eg. A B
O in 5 X in 2
X in 6 X in 8
X in 4 then any play that B makes results in a win for A.

Thus for second modification - a refinement, first person cannot use the centre first.

David Boyle,
A.B.-Lucas Secondary School,

London, Ontario

PROBLEMS

These 10 problems were presented to the Senior Students of London in one of the recent inter-school Mathematics contests.

1/ What is the number of intersection points of 4 planes if no two are parallel and no three intersect in a straight line?

1 minute.

2/ In how many ways is it possible to make up 28c using Canadian currency?

3 minutes.

3/ A circle is inscribed in a square of side 2. A square is inscribed in that circle. A circle is inscribed in that square, and so on ad infinitum. What is the sum of all the areas of all the squares?

3 minutes.

4/ Given that $a+b\sqrt{2}$ with $a, b \in R$ is a closed system under multiplication, what is the reciprocal of $3 - 2\sqrt{2}$? in the form $a+b\sqrt{2}$

1 minute

5/ What digits can odd squares end in in base 8?

2 minutes.

6/ The numbers $Fn = 2^{(2^n)} + 1$ for $n = 0, 1, 2, 3, \ldots$. are called Fermat numbers, after the great French mathematician Pierre Fermat. Approximately how large is the 6th. Fermat number in scientific notation?

3 minutes.

7/ What is the reciprocal of 3 modulo 5?

1 minute.

8/ On a twelve hour clock, how often are the minute and hour hands at right angles in 12 hrs?

3 minutes.

9/ Given the following multiplication table, what is the value of 7/5?

x	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

1 minute.

10/ What is the number of intersection points of 5 planes if no two are parallel and no three intersect in a straight line?

3 minutes.

INTERESTING FORMULA

Submitted by Kernie Houser, Friendsville High School, Tennessee, to **The Mathematics Student,** April, 1974.

The Volume (V) of a regular icosahedron in terms of the length of an edge (e) is:

$$V = \frac{5e^3}{12 \cos 54^\circ} \sqrt{3(\sin 54^\circ + 1)^2 - \cos^2 54^\circ}$$

Editor's Note: Readers and their students might be interested in trying to prove this result.

MATH PROBLEM ANSWERS

Solutions submitted by Dave Boyle, A. B. Lucas Secondary School, London

15-2-1 :

The man must transport fox = f, goose = g and corn = c. f eats g and g eats c.

f,g,c		
f,c	g→	
f,c	←	g
c	f→	g
c	g←	f
g	c→	f
g	←	cf
	g→	cf
		cfg

OR

f,g,c		
f,c	g→	
f,c	←	g
f	c→	g
f	g←	c
g	f→	c
g	←	f,c
	g→	f,c,g
		cfg

15-2-2

	x	y
z	A	C
w	D	B

A(area) B(area) = xz · wy = xzwy

D(area) C(area) = wx · zy = wx zy

$\therefore$ AB = CD

15-2-3

$$x + yz = 2 \quad 1$$
$$y + zx = 2 \quad 2$$
$$z + xy = 2 \quad 3$$

By symmetry

$w + ww = 2$

$w^2 + w - 2 = 0$

$(w+2)(w-1) = 0$

$w = -2$ or $w = 1$

$\therefore \{(x,y,z)\} = \{(-2,-2,-2), (1,1,1)\}$

LOGIC PROBLEMS

**George Lawson, Department of Mathematics, Faculty of Education,
University of Western Ontario**

One type of problem that is of interest to both elementary and secondary school students is the "Logic Problem." This type of problem which requires no special knowledge usually calls for deductive reasoning with little or no numerical calculations. These problems contain a number of clues from which certain inferences can be drawn and the reader is required to play the role of a detective and to sift through the clues in order to arrive at a solution. These problems vary in degree of difficulty, from the very easy ones that can be solved in one or two steps, to the more difficult types which often require a systematic approach in order to arrange and sort the clues. The easier types provide an excellent introduction to informal deductive reasoning at the elementary school level while the more difficult problems will provide a challenge to the most able student at the senior levels. Almost all students enjoy struggling with this type of problem and readily accept the challenge which they present.

Although there is no one set method for attacking these problems there are some techniques which help when organizing and sorting the clues. A few sample problems, with solutions, to illustrate some of the possibilities are presented here. For maximum enjoyment, try each problem before examining the solution.

Example 1



1. Tom always tells the truth.
2. Dick sometimes tells the truth.
3. Harry never tells the truth.

Who is Who?

Solution: From (1) the middle man cannot be Tom; hence the statement from the man on the left is false and so he cannot be Tom. Thus the man on the right is Tom and since he always tells the truth, the middle man in Harry and third man is Dick.

Example 2

Alice, Mary and Jane are in the same class.

1. Alice has blond hair.
2. Mary spends less time on homework than the smartest of the three.
3. The dumbest of the three spends the most time on homework and she also spends a lot of time at the beauty salon having her dark hair attended to.

Who is the smartest and who is the dumbest?

Solution: From (2) Mary is not the smartest. From (3) Mary is not the dumbest since the dumbest spent the most time on homework. Then Alice is either the smartest or the dumbest. She cannot be the dumbest since from (3) the dumbest has dark hair and from (1) we know that Alice is blond. Hence, Alice is the smartest and Jane is the dumbest.

Example 3

Four married couples are eating together in a restaurant. In alphabetical order the wives' names are Alice, Chris, Mary and Rose, while the husbands' names are Bob, Dave, Ian and Tom.

1. Alice is married to Ian.
2. Mary is Dave's sister.
3. Tom and Mary were once engaged but the engagement was broken off when Mary met her present husband.
4. Rose has four brothers but her husband is an only child.

Who is Chris's husband?

Solution: Although this problem can be solved by sorting out the clues as in the previous examples, it is often useful, when a problem contains a number of clues, to use some system to organize the data. A schematic way of listing the clues to this problem is to use a matrix and a form of binary notation. This scheme not only provides a means of organizing the data but also displays the various relationships in an easily readable form. If we know that a man and woman are married, we place a 1 in the corresponding square of the matrix and if they are not married, we place a 0. Each row and column of the matrix will have exactly one 1. Following this scheme (1) allows us to place a 1 in the square corresponding to Alice and Ian, and using (2) and (3) an 0 in the corresponding squares as shown. Since each row or column has exactly one 1, we can now proceed to fill in more squares in the matrix and thus display other relationships.

	Bob	Dave	Ian	Tom
Alice			1	
Chris				
Mary		0		0
Rose				

	B	D	I	T
A	0	0	1	0
C			0	
M		0	0	0
R			0	

	B	D	I	T
A	0	0	1	0
C			0	
M	1	0	0	0
R			0	

	B	D	I	T
A	0	0	1	0
C	0		0	
M	1	0	0	0
R	0		0	

If we can place one more 1 we can complete the matrix. From the final matrix we note that Rose is married to either Dave or Tom and (4) tells us that her husband is an only child. Since by (2) Mary is Dave's sister, then Rose is married to Tom and the matrix can now be completed. In the final form the matrix reveals the marriage relationship that exists between these people. Thus Chris is married to David.

	B	D	I	T
A	0	0	1	0
C	0	1	0	0
M	1	0	0	0
R	0	0	0	1

In some problems it may be necessary to use two or more matrices to list the information and draw the conclusions. The following problem which appears in Gardners[1] second book of puzzles and diversions can be organized using two matrices.

Smith, Jones and Robinson are the engineer, brakeman, and fireman on a train but not necessarily in that order. Riding the train are three passengers with the same three surnames, to be identified in the following premises by a "Mr." before their names.

1. Mr. Robinson lives in Los Angeles.
2. The brakeman lives in Omaha.
3. Mr. Jones long ago forgot all the algebra he learned in high school.
4. The passenger whose name is the same as the brakeman's lives in Chicago.
5. The brakeman and one of the passengers, a distinguised mathematical physicist, attend the same church.
6. Smith beat the fireman at billiards.

Who is the Engineer?

Example 4

Two women, Arlene and Cheryl, and two men, Burton and Donald, are musicians. One is a pianist, a second is a violinist, a third a flutist, and a fourth is a drummer. On a day they were seated around a square table.

1. The person who sat across the table from Burton was the pianist.
2. The person who sat on Arlene's left was the violinist.
3. The person who sat across from Donald was not the flutist.
4. The person who sat on Cheryl's left was not the drummer.
5. The flutist and the drummer were of the opposite sex.

Determine the instrument played be each person.[2]

This type of problem can be approached by setting out the possible seating arrangements for the four people, imposing the given conditions and testing for any contradictions.

(4) not drummer

(3) not flutist — B — Drummer

C D

(2) violinist

A

(1) pianist

contradicts (5)

(2) violinist
(4) not drummer

(3) Not flutist — B — flutist

A D

drummer

C

(1) pianist

contradicts (4)

B

C A

D

(1) pianist
(2) violinist

(1) and (2) produce a contradiction

B

D A

C

(1) pianist
(2) violinist

(3) not flutist
(2) violinist

flutist

B

A C

D

(1) pianist
(4) not drummer

contradicts (5)

B

(2) violinist D C (3) not flutist drummer

A

(1) pianist
(4) not drummer

Following are a few sample problems gathered from a variety of sources. Try them in your classroom.

1. A boy and a girl are sitting on the front steps of their commune.

"I'm a boy," said the one with black hair.
"I'm a girl," said the one with red hair.
If at least one of them is lying, who is which?

2. Assuming that a statement is either true or false, determine the truth or falsity of the following four statements.

    1. The even-numbered statements are false.
    2. The odd-numbered statements are false.
    3. The square-numbered statements are false.
    4. Most of these four statements are false.

3. Use the following information to determine who plays the various positions on this baseball team.

    Andy dislikes the catcher. Ed's sister is engaged to the second baseman. The centre-fielder is taller than the right-fielder. Harry and the third baseman live in the same building. Paul and Allen each won $20 from the pitcher. Ed and the outfielders play poker together. The pitcher's wife is the third baseman's sister. All the battery and the infield except Allen, Harry, and Andy are shorter than Sam. Paul, Andy and the shortstop lost $50 each. Paul, Harry, Bill and the catcher lost to the second baseman at pool. Sam is getting a divorce. The catcher and the third baseman each have two children. Ed, Paul, Jerry, the right-fielder, and the centre-fielder are bachelors. The others are married. The shortstop, the third baseman and Bill won $100. One of the outfielders is either Mike or Andy. Jerry is taller than Bill. Mike is shorter than Bill. Jerry, Mike and Bill are each heavier than the third baseman.

4. Five boys took part in a swimming race. At the end of the race five judges made the following announcements.

    1. Harry was second and Mark was third.
    2. Tom was first and Peter was second.
    3. Peter was third and Tom was fifth.
    4. Harry was second and Wally was fourth.
    5. Mark was first and Wally was fourth.

    If we know that one part of the statement made by the judges is true and the other part of the statement made by each judge is false, find the correct order of the five boys who finished the race.

5. Three prisoners, Mack, Harry and Walter, came up for parole and the Warden found that he was only able to grant parole to one of them. Since they all had equal qualifications for the parole, he could not decide on the basis of their records and so he elected to test the prisoners on a problem with the winner to gain the parole. He told the prisoners that he would place five small hats, three red and two green, in a room and that each man would be blindfolded and taken to the room to select a hat. Prior to taking the men down to the room the Warden removed the two green hats without telling the men. The prisoners were taken to the small room where they selected a hat and then they were returned to the original room, with their hats on. The men were seated so that when the blindfolds were removed each prisoner could see the hat worn by the other two but could not see the hat on his own head. The prisoner who could tell the color of his hat and give a logical explanation for his answer was to be the winner.

    After sitting for about ten minutes Mack declared, "I give up - I cannot tell the color of my hat." After a further five minutes Walter announced, "I give up." When this happened Harry said, "I have a red hat on." How did he explain his answer?

References

1. Gardner, Martin. **Mathematical Puzzles and Diversions,** New York, Simon and Schuster, 1961.
2. Phillips, Hubert. **My Best Puzzles in Logic and Reasoning,** New York, Dover Publications, Inc., 1961.

NINTH CANADIAN MATHEMATICAL OLYMPIAD - 1977

Patron: The Governor-General of Canada

Date of Examination

The Ninth Olympiad will be written on Wednesday, May 4, 1977 from 9 a.m. to noon at schools throughout Canada. The examination papers will be marked in May and the results will be circulated to schools as soon as possible thereafter.

Conditions of Examination

All examination materials, whether submitted for marking or otherwise, are the property of the Canadian Mathematical Congress. Decisions of the examiners are final. Cash prizes and scholarships may be awarded at the discretion of the examiners, who may choose to subdivide or otherwise alter the terms of awards.

Eligibility of Candidates

Candidates must be Canadian citizens or landed immigrants, and must be registered in a high school or its equivalent. (In Ontario the competition is open to students in Grade XIII or earlier).

Nothing in the foregoing is intended to preclude the student's enrolment in advanced mathematics courses, at his own institution or elsewhere.

Nomination of Candidates

All candidates of the Olympiad must be nominated by the chairman of a provincial competition or by a school principal. Copies of nomination forms can be obtained from:

Professor G.H.M. Thomas
Chairman, Organizing Committee
Department of Mathematics,
University of Saskatchewan,
SASKATOON, Saskatchewan S7N 0W0.

(The Ontario chairman is Prof. K. D. Fryer, Department of Mathematics, University of Waterloo, WATERLOO, Ontario, N2L 3G1 and the Ontario quota of candidates if 50).

School principals may nominate not more than three candidates who are unable to participate in a provincial contest. Each nomination by a school principal must be accompanied by an entry fee of $5.

The First Five Olympiads

A booklet containing the questions of the first seven Olympiads, solutions to the questions, and the names of the prize winners, has been prepared and may be obtained from:

Secretariat,
Canadian Mathematical Congress,
3421 Drummond Street,
MONTREAL, Quebec H3G 1X7

at a cost of $2, including postage.

Prizes

First prize - $1500., second prize - $1000., third prize - $500., fourth prizes - 5 prizes of $200., fifth prizes - 10 prizes of $100.

A few additional prizes may be awarded to those submitting especially elegant solutions to one or more problems.

A certificate of honourable mention is presented to approximately 25 candidates whose rank in the examination follows the 18 prize winners.

ONTARIO ASSOCIATION

FOR

MATHEMATICS EDUCATION

1976-77 EXECUTIVE

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Roma Acheson,
Head of Mathematics,
Grey Highland Secondary School

SECRETARY-TREASURER

Morley B. MacGregor

RECOMMENDATIONS FOR THE SENIOR DIVISION MATHEMATICS CURRICULUM

Neil Williamson, President-Elect, O.A.M.E.

Mr. Williamson is a Mathematics Consultant for the North York Board of Education. He has taught both elementary and secondary school, serving as a principal and a chairman. For the past two years he has been chairman of the O.A.M.E. Senior Division Cyclic Review Committee.

At the outset, I should like to thank the members of O.A.M.E. Council who served on the Review Committee:

1974 - 75: Tom Griffiths, Bill Higginson, Ross Leigh, Ron Scoins, Norm Viire.

1975 - 77: Roma Acheson, Andy Czempinski, Frank Ebos, Tom Griffiths, Fred Maskell, Ed Silcox.

I have enjoyed working with the concerned members of O.A.M.E. during the past six years and I look forward to a busy and exciting year in 1977 - 78.

We will need the co-operative efforts of the O.A.M.E. Council and many of the 1200 O.A.M.E. members to tackle the numerous issues facing us this year.

Thank you for giving me the opportunity to serve you.

See you in Ottawa on May 13, 14, 15/77.

Neil J. Williamson



Neil J. Williamson

This report has been prepared for the Ministry of Education by the Ontario Association for Mathematics Education for consideration during their review of the Senior Division Mathematics Curriculum.

The OAME considers the present senior division guidelines to be basically sound curriculum statements. We also believe that the present format should be maintained in future revisions.

We offer the following recommendations in order to improve the senior division curriculum and facilitate its eventual implementation.

RECOMMENDATIONS

1. **It is recommended that the core topics and skills for a particular course be clearly identified so that greater understanding and mastery can be achieved.**

This recommendation does not conflict with the spiral approach in the present curriculum but allows for the establishment of priorities within that approach.

2. **It is recommended in principle that externally prepared provincial common examinations be available for students proceeding to post secondary educational institutions. Consideration should be given to the establishment of examining boards involving participation by universities, colleges, professional organizations, the Ministry of Education, the Ministry of Colleges and Universities and local school boards.**

These examinations would complement the evaluation within the school and aid in establishing provincial standards.

3. **It is recommended that the Ministry consider the establishment of a mathematics course for those students who plan to study Social Sciences after secondary school. This course shall be composed of topics selected from the present guidelines.**

The focus of this course would be applications of mathematics within the social sciences.

4. **It is recommended that the Ministry provide detailed support documents for the following:**

(a) "A Mathematic's Style Guide" outlining current notation, correct use of symbols, and concise, logical proofs.
(b) "Problem Solving" requiring the use of many concepts to stress mathematical interrelationships.
(c) "The History of Mathematics" relating both the history and people of mathematics to specific topics.
(d) "Mathematics and the Social Sciences".
(e) "A Mathematics Bibliography".

5. **It is recommended that the Ministry identify the prerequisite skills required for each Senior Division Mathematics course.**

The identification of the required knowledge and skills will enable the student to make more rational course selections. This will also enable teachers to identify topics which need review and consolidation within a course of study.

6. **It is recommended that the Ministry authorize the provision of opportunities for high ability students to realize and exercise their full potential in mathematics.**

This involves the encouragement of special courses for special individuals, and the encouragement of school boards to provide classes for fewer than 15 students, when appropriate.

7. **It is recommended that the Ministry contact the Ontario Association for Mathematics Education to assist with the implementation of revisions within the Senior Division.**

Editor's Note: It is encouraging to note that Tom Griffiths represents O.A.M.E. on the Intermediate Division committee.

SEVEN CHAPTERS OF OAME

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CHAPTER ITEMS

GVMA: General Meeting 1 was held on November 24 at the University of Waterloo with George Scroggie as guest speaker on the new Intermediate curriculum. The date of the Spring meeting is not yet available.

New Publications: A Report on County-Wide Exams;
A Solution Booklet to accompany the Trigonometry booklet will be available soon;
Reprints for the 1976 Gauss paper (elementary)

More publications are forthcoming: e.g. **A Numerical and Algebraic Approach to Deductive Geometry.**

SWOAME: By publication time the March 5 Windsor Conference will have taken place. Theme- **Mathematics, Yesterday, Today and Tomorrow;** guest speaker - Dr. A. J. Coleman, Queens University, Kingston.

The new Executive has taken office and under the leadership of Gary Peters, President, has held two workshop meetings of the Council to study in depth the so-called Pogue Report. The Council of approximately 25 members represents each educational jurisdiction in Southwestern Ontario. On February 5, Paul Pogue met with the Council to clarify the rationale and recommendations of the Report, which is briefly summarized in this issue.

Members of the Council agree in the main with the recommendations of the report. Afew are concerned that the recommendations for deletion do not necessarily allow time for the new content topics proposed. Several members were critical of the lack of clarity and relevance of the theoretical justification for the proposed changes. The question of "how to implement" also arose.

Mr. Pogue gave an excellent overview of the report and its probable impact on the Intermediate Division Committee, of which he is a full-time seconded member. He is a practising teacher with experience at the elementary and secondary level. His main task in the Study was to study local and international programs, interview mathematics educators and make recommendations. Dr. Patrick Babin, a curriculum specialist, provided the theoretical framework.

The curriculum model used was one of objectives, approach, methodology, content and evaluation. The program envisaged is a total one for grades 7 - 10, with a common core for all four and five year students.

The main recommendation is for a new emphasis on approach: an experiential one with stress on development of problem-solving skills through the selection of "real-life", meaningful problems, and a consequent drastic reduction of emphasis on the structure of the discipline until grade 10 at the earliest. The most significant content proposals have to do with statistics and probability, transformation geometry and the use of calculators.

In a brief summary of the new curriculum, the first phase of which will be available soon, and the long-range proposals for 1980, Mr. Pogue said he thought that eventually the curriculum would be in very close agreement with the main proposals of the Report.

A more detailed account of the SWOAME reaction to the Pogue Report will be sent to the O.A.M.E. Executive and the Ministry Committee at a later date. SWOAME Council had an excellent discussion with Mr. Pogue and congratulates him for his part in the development of a major, milestone and document.

INTERNATIONAL MATHEMATICS EDUCATION

Did **no-one** in Ontario attend the Karlsruhe Conference?

N.C.T.M. 55th Annual Meeting: Cincinnati, Ohio, April 20 - 23, 1977. Why not attend?

NATIONAL METRIC WEEK IN U.S.A.

9 - 13 MAY 1977

WHEREAS, The United States and Canada are now adopting the metric system of measurement; and
WHEREAS, Today's children, as adults, will live in a largely metric world; and
WHEREAS, It is the responsibility today of our schools to educate their students for the future; and
WHEREAS, Parents should have an awareness and an understanding of their schools' metric program;

Now, therefore, the National Council of Teachers of Mathematics declares that the week of 9 - 13 May shall be designated National Metric Week; and encourages each person to develop and maintain an awareness of the metric system; and admonishes all persons to use the metric system where applicable in all phases of daily life.

Single copies of a list of Community Awareness Activities for National Metric Week are available free from the National Council of Teachers of Mathematics Headquarters Office, 1906 Association Drive, Reston, Virginia, 22091.

MATH'S THE SUBJECT

Faith M. Grant
Haddon Hts., NJ

Reprinted with permission from the New Jersey Mathematics Teacher

Some people say that math is a bore,
  They stop and refuse to learn any more.
They never give it a chance to prove
  That it's a subject that really can move.

First you start with some elementary thoughts,
  Then expand them to be where we ought.
Take for example a simple function -
  Let another cross it - you've got a junction.

Next march on to the world of trig,
  Relax, sines and cosines aren't very big.
But remember all these special entities -
  They're the basis for all those long identities.

Lastly examine the calculus derivative
  Whose discoverer must have been very creative.
So many forms and ways of manipulation
  That students often lose their patience.

To end this poem, I'll use some math,
  Just follow along the beaten path.
Take the derivative of a constant ... zero.
  That's a math student's (and major's) hero!

NOTICE OF MOTION

The following notice of motion was presented at the Annual Meeting of the Ontario Association for Mathematics Education at Brock University, St. Catharines on the fifteenth day of May, 1976.

That Article I of the Constitution which reads
"The name of the organization shall be Ontario Association for Mathematics Education."
be amended to read
"The name of the organization shall be in the English language Ontario Association for Mathematics Education and in the French language **Association Ontarienne d'Education Mathematique.**"

This Notice of Motion will be duly considered at the Annual Meeting of the Ontario Association for Mathematics Education at Ottawa on the fourteenth day of May, 1977.

Dated at Sarnia on December 15, 1976.

M.B. MacGregor
Secretary Treasurer



Morley and Mona MacGregor enjoying their work.

Editors Note: Professor Phil Casaubon of the Department of Modern Languages, Faculty of Education, U.W.O., has suggested to me that a better translation of "Ontario Association for Mathematics Education" would be "Association Ontarienne pour l'Education en Mathematiques". Francophones should contact the Executive on this point.

OAME DATES FOR 1977

March 4 Executive Meeting
March 5 SWOAME - Windsor
April 2 - Council Meeting
May 13 - 15 Annual Conference Ottawa
May 27 Executive Meeting
June 11 Council Meeting
Sept. 9 Executive Meeting
Oct. 1 - NOMA - Kirkland Lake
Oct. 19 - 22 Leadership Seminar
Oct. 28 Executive Meeting
Nov. 5 Council Meeting

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Orders for subscription and single copies should be sent to the Secretary-Treasurer.

M.B.MacGregor

REVIEW OF SENIOR DIVISION CURRICULUM

The Council of OAME invites responses to the report of the Committee published in the September, 1975 Gazette. They should be directed to Neil Williamson, 1754 Walnut Lane, Pickering, Ontario. L1V 2X3.