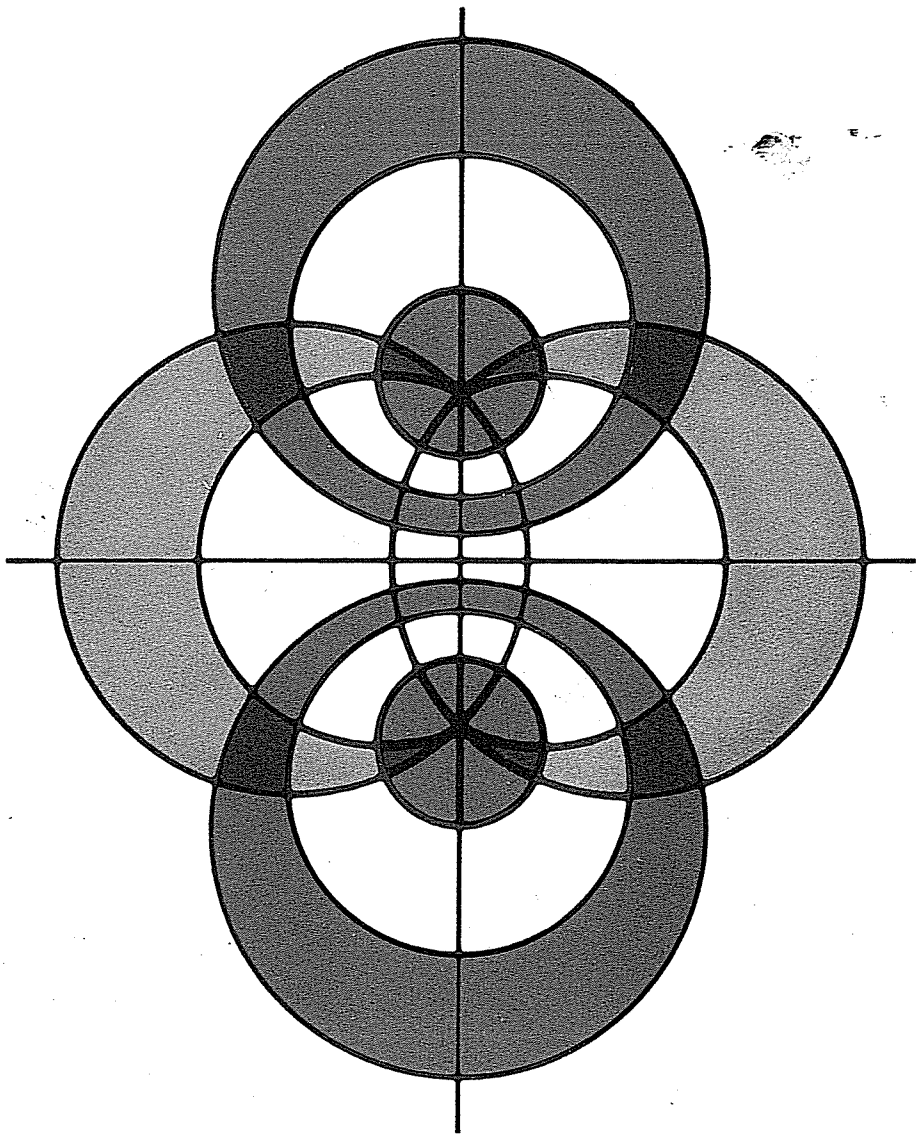
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MATHEMATICS EDUCATION

BUILDING CREATIVITY IN OUR STUDENTS

by
Donald R. Woods

Creativity - this word conjures up very pleasant and positive feelings. It is a mysterious skill that we all hope we have, skill that it would be nice to have in our students. Some say creativity is intuition. Some describe it as "the aha". Some describe it as "that sudden flash of inspiration". How can we define this elusive but valued skill, and more importantly what might we do to build that skill in our students? Although many definitions have been offered for the word creativity, in our context we will define it simply as "the ability to think up alternatives". In this paper we will look at the needs our students have for this skill, some general ideas as to how we might develop the skill and finally some specific actions we might take.

1. THE NEED FOR CREATIVITY

When is creativity needed by students and why is it important in our program? We all, including students, need to be able to see situations or problems from many different points of view. We need this creativity whenever alternatives need to be considered. We need it whenever we have to identify the factors that affect the situation. We need it when we are trying to decide what to do. In other words, we need this skill constantly at our side to be used to solve any situation that calls for a generation of many alternatives. In the classroom, students need this skill when they are trying to relate new ideas to past sets of ideas, when they are trying to answer a question such as "what alternatives do I have to perform this integration?" They need it especially when they are solving problems and are trying to see what alternative methods they could use.

Why is it important in any of our programs? Regardless of whether it is high school physics, mathematics, college or university physics or mathematics, or a course that is "strictly fundamentals" this skill is needed. It is what I would call a general thinking skill needed by all of us. Hence my stance is that no matter what the course, no matter what the subject, students should be encouraged to use and develop their creative abilities.

2. HOW TO DEVELOP CREATIVITY

Since all thinking skills are related to how our brain functions, it is useful to take a brief sidetrack to look at the characteristics of how the brain processes information and specifically how the brain retrieves information. Figure 1 is a simplified look at the short term and long term memory components to our brain. The skills that we are trying to develop relate to retrieving information from long term memory and so it is just this aspect upon which we will focus. Other details about storage of information are given elsewhere (1). The key characteristics that are pertinent to a discussion of creativity are the **linear patterns** that develop when we store information and the **control** that affects which ideas we are willing to retrieve. Considering the former, information is stored in long term memory, as a memory trace. Such memory traces have both access routes (that relate the information to other previously stored information) and intensity, richness or density (that identify the relative importance of that information). Repetition will increase the intensity or density of the information. This will provide more ready access to that particular bit of information (2). DeBono (3) uses a density line or d-line method of representing this concept of a memory trace. He shows intensity or density by the number of parallel d-lines used. He shows access routes by the number of d-lines that start as different locations but that lead to a common focus. Thus in Figure 2(a) a single experience is shown as a single d-line in memory. Although the orientation and shape of this particular d-line is arbitrary, in fact, the orientation and shape would be dependent upon previous bits of stored information. We see this by showing that all subsequent new information will be shown relative to this starting d-line. Thus, a repeated identical experience would be shown as a parallel line right beside the first one, and is illustrated in Figure 2(b). Figure 2(c) shows many related pieces of information that are all centered around an identical experience. An example is that a person is shown three geometrical figures in succession and told to memorize the shape. The resulting d-line in long term memory for item a, the vee, is shown pictorally as a straight line in Figure 3. When the second item is shown, the person recognizes it as a combination of the previous vee and a square. Thus this new chunk would have two components: a repeat symbol for the vee and another d-line representing the square. The latter overlaps the previous trace because the two

Paper presented at The American Association of Physics Teacher's Conference, June 17, 18 and 19, 1982, University of Western Ontario, London, Ontario.

symbols have a unique relationship to each other. When the final item c is shown there is further reinforcement of the vee d-line and yet another off-shoot for the circle. Thus, Figure 3(c) shows an intense memory trace for the vee with two access routes. As time passes, each individual's d-line pattern will ebb and flow so that when we try to access the information to recall alternatives or different ways of seeing things, we may not have easy access because of the clutter around one particular idea. DeBono (4) characterizes the brain as an extremely efficient linear processor but a very inefficient way of jumping from one pattern to another. Others have identified this characteristic of the brain as being **mental fixation.** That is, once we identify an area that we are interested in our memory keeps returning again and again to a single set of ideas. For example, if we are trying to recall a persons name, we might initially become fixed on the idea that their surname begins with Q; it will be extremely difficult for us to think of surnames other than Q for our forgotten surname. Knowing this characteristic of the brain will help us to understand why we do certain things when we create.

Control is a guard that allows only certain ideas to be recalled from long term memory. The control is not understood very well. We realize it is supplied by our inner needs and motives. These include the internal biological needs of hunger, thirst, rest, temperature regulation and freedom from bodily injury. Other needs are the acquired needs, such as wealth, social approval, conformity, mastery, sex, security, adventure and creativeness. Maslow (5) has suggested that these needs are structured in a hierarchy in that before one particular type of need is important, some previous need has to be satisfied. He suggests that the sequence is basic physiological needs, safety and security requirements, social needs, esteem needs and finally self-realization needs. Thus, a mature individual may have satisfied his or her social needs and thus his "control" will be functioning primarily on esteem needs. This control selects those pieces of information that will be retrieved from long term memory and will let those pass that satisfy the needs and motives of the individual. For example, if an individual feels insecure in an environment, then he will be unwilling to retrieve and verbalize an idea that may appear foolish in the eyes of his contemporaries. If however, another individual has a lot of self confidence and is functioning then at a higher level in the need structure, she may be able to retrieve and say something that is a "odd ball idea" without difficulty. Thus, for different individuals, control will limit the type of ideas that are being retrieved.

A third characteristic about individuals and how the brain functions is the combination of a limited amount of information that can be held in the short term memory and the need for social reinforcement that ideas individuals have put forward are important. These two concepts combine to make it important for us to use external storage devices such as recording ideas on paper, on tape recorders, on the blackboard, on an overhead, so that when one is trying to think of alternative ideas the idea is stored, so that short term memory is not overloaded, and so that there is acknowledgement that the idea is worthwhile. Thus, three basic background ideas about how the brain functions are useful in our exploration of creativity. These are the way memory is laid down in serial type patterns, the effect of control on what ideas will be acceptable to retrieve and verbalize, and the importance of recording via pencil and paper or some external storage means the ideas when they are presented.

3. ENCOURAGING CREATIVITY

Based on our knowledge on how our brain functions, our first task in encouraging creativity will be set up an environment in which no criticism is allowed, in which any answer in accepted. The objective is to override the control that the various individuals have so that during this identified period of time, which we might call **brainstorming,** no judgement or rationalization is needed. Ideas are to be presented but not criticized. No elaboration is necessary or even desired. Our focus will be on quantity of alternative ideas. We will not worry about the quality of the idea or even the pertinence of the idea. Adam (6) provides a listing of the types of barriers that can prevent us from letting ideas flow. Other more subtle barriers that control the flow of ideas include our inner confidence with our ability. These have been elaborated on by Miechenbaum (7). He points out the effect that negative feelings and silences have on our ability to let ideas come forward. The next characteristic of the brain, how we store information, leads us to realize that deferring judgement is not sufficient. We have to use explicit activities that will allow us to see the problem from different points of view and to move out of a single mental track. DeBono (4,5) has been probably the most visible proponent of this school of thought. He has coined the term "lateral thinking" to describe the task we have before us. We wish to be able to get our minds to move laterally and hop from d-line to d-line instead of being trapped in a single track. We describe this as the use of triggers that will restart a flow of ideas by giving us access to a different storage area within the brain. There are at least a dozen triggers available. These are summarized in Table 1.

With these specific suggestions then, how can we develop skill in deferring judgement and using triggers and at the same time provide reinforcement of the ideas and acknowledgement of acceptance of them and provide a written record to increase the capacity of the short term memory? The approach we use to develop creativity is a six hour workshop/tutorial. At the beginning we define our objectives as to think up fifty ideas in five minutes, the emphasis is on quantity, there is to be no judgement, no elaborations needed and we do not worry about repetition. The steps we will follow will be to initially define what we would like to think up ideas about and then move into the brainstorming atmosphere. In terms of complication, it is easier to dream up ideas about the properties and uses of an object than it is to tackle the more complex task of exploring situations. Thus in the workshop it is useful to start finish with developing skill in attribute identification and then move off to explore the more difficult task one of thinking about situations. To develop the skill one starts with a group of size definitely less than 40. Thus if you have a large class you would identify a small group to participate and the rest would observe. As you develop the skill you want the size of the group to shrink so that eventually the individuals will have confidence that they have the skill. The role of the leader is to preserve the atmosphere, to record ideas and basically to prevent judgement and at times to contribute triggers to keep the session rolling smoothly. The leader is really a facilitator. It is important to record the ideas as they are presented so that all can see, all can build on, and all ideas are acknowledged. With this overview I normally then do a demonstration workshop or brainstorming session on dreaming up the properties of a common everyday object such as a pen or a cup or a brick. At the end of the session, and this is a very important aspect of the development of the skill, the participants should be given the chance to record their ideas and learn from the experience. After this introduction they are then ready to try it out themselves. Under explicit instructions as to topic and who is going to be the leaders, small groups of 6 participants should try generating alternative uses for an identified, simple object. Seven minutes is sufficient for this activity. Once again, participants should record their experiences and share them with the others in the larger group. By this time the leader can comment on things he observed, suggestions as to how to improve the session, but in general introduce what might be called triggers. For attributes this will include positive and negative uses, physical uses, chemical uses, interpersonal uses and personal uses. By this time the participants should be familiar with the idea of the brainstorming atmosphere and be prepared to move into considering the more complex task of looking at situations. Several cases can be described for them. These can be within the technical context or within the social context. If they are within the technical context then usually the brainstorming session will be very restrictive and the group will be lucky if they can think of seven alternative ways of doing the task instead of the 50 ideas that represent our target. One might wish to choose an observer before this activity commences so that there is feedback to the group as to what they did when they got stuck or what happened when a brainstorming session runs out of ideas. At this stage triggers can be introduced. Usually the procedure is to define what we mean by a trigger and to tie this idea back to the concept of how the brain functions. In other words, the mental process of the participants has been going along an identified well-travelled channel and what we want to do is DeBono's so-called lateral jump. We want the participants to move away from one fixed way of thinking and move to a completely new viewpoint. This can be done by the introduction or explicit application of 1 of 13 different types of triggers or by using DeBono's CORT thinking materials. With this bit of elaboration, the groups can rotate leader and once again tackle the problem that they had just looked at. Now however they should practice using a specific trigger to get the ideas flowing. "Personal imagination" is usually a good one to start out with. "Wildest idea" is always very useful. The more difficult triggers to use are "juxtaposition" and "reversal". For these two triggers I have found it useful to explicitly illustrate these on the problem that they have been working on. After they have tried and applied several triggers the background reading material can be explained to them so that they can learn about the other triggers, on their own. I have not found it necessary to spend additional time in class introducing all of the triggers. Again after each session it is useful to record impressions and for the leader to collect information as to what they have learned from the experience.

Several other activities can be blended in with this one. One was developed by Rune Olsson (8) who uses a questionnaire built around Adam's book (6) to help identify the barriers for creativity. One way is to discuss each barrier. Alternatively, short workshop activities to illustrate the difficulty or barrier in the development. A useful and often important additional activity is the classification and analysis of ideas to illustrate how you can convert a list of 50 ideas into a practical solution. Several approaches can be taken. One that I like is to identify four different useful types of ideas, the craziest idea, and to me interesting idea. Once this selection has been made the group addresses how these can be converted into practical solutions. Usually the task is easy and the group, under the guidance of the leader, sees how to accomplish the task rather quickly.

This then completes the sets of activities for a six hour workshop on developing creative skills. Some components that are noteworthy include the teaching and learning environment used and the way information is handled. The teaching and learning environment has been a workshop style experimental learning. The role of the teacher is to be a facilitator. The written material (which is back-up providing the basic ideas) has been prepared ahead of time. That material is resource material to be read at the leisure of the student. Some work sheets are needed, especially to record ideas learned from each experience. Often I prefer to have the worksheets printed on green paper so that it is very clear for the participants which is a worksheet and which is the background resource reading. The other aspect that is important is the way information is handled. Because it is important to record the information and because we usually want to work with the ideas after they have been recorded, I prefer to have everybody work on overheead transparency sheets. This makes it easy to duplicate the sheets after the session is over yet all can see what ideas were created during the session. This does put severe constraints on the type of facilities that are needed. However I have found this method, namely using overhead projectors, to be the most successful. The resources needed are given in Table 2.

4. SPECIFIC ACTIONS IN THE CLASSROOM

There are two general areas where you can promote the development of creativity. One is how you encourage creativity just by your day to day action and secondly you can do specific things to actually develop the skill in your students. Let us consider each one in turn.

What can you do? In your day to day classroom interaction you often will want to draw from the class or students their views or some alternatives ideas for a given situation. Students learn very quickly that if you ask a question that usually if they keep silent, you will answer your own question or rephrase it into a simpler one in about 30 seconds. The first skill we need to learn, then, is to keep quiet and to not rephrase the question and to encourage them to present ideas. Next, when the ideas are presented, we have to, whether we acknowledge it or not, preserve a non-judgemental atmosphere. If we do not, then no answers will be given. This is perhaps the hardest for us as teachers to do. When a student puts forward a foolish answer our first response is to say "No, that is not a good answer". We must learn to curb this and to collect all different points of views and all different answers and then analyze them. The trick that I use is to mentally remind myself that this is what I am doing by saying to the class "Let's collect some ideas about this". Then I continue to collect ideas until I have at least the set of ideas from which I can work. If the ideas are not forthcoming then I use triggers and explicit use of those triggers to try to get them to broaden their perspective. Then when we as a group are satified that we have a broad set of ideas, we can then critique them, add judgement, and select the appropriate ones. Thus, in a simple exercise, the students see that all ideas are worth mentioning, reinforcement of how to do a brainstorming session and that brainstorming and judgement are separate activities. This style can be used in every class almost every day. As the students and you gain facility with this, we can then identify the task by its name, by saying "Let's brainstorm, let's use a trigger", "let's now apply analysis".

We can go beyond this simple daily demonstration to actually try to explicitly try to improve their ability and confidence in dreaming up alternatives. My suggestion here is to actually run a six hour workshop. Not all of us will be able to find a block of six hours. We could use a three hour period of time. Perhaps we will have to settle for a series of one hour experiences or even 40 minute experiences. A point that should be remembered is that although the ideas are relatively simple, it takes a long time to develop skill and confidence with creativity.

This description of a workshop has provided a start but before we actually start we should explicitly identify in our course outline that we are going to develop creativity as one of our objectives in our course. Just as we want them, in behavioural terms, to "understand Newton's laws" so we want them to "be creative". So the first phase is to identify the development of creativity as a learning experience, namely, identification of learning objectives. The second phase is the explicit teaching and learning experience when they try to acquire these skills. The necessary follow-up is to provide them with a self-evaluation or an evaluation tool that you use to help them to see the degree to which they have acquired the skill. With clear learning objectives and the types of definitions we have been using, such an evaluation is relatively easy. A sample problem that can be used for such an evaluation is given in Table 3.

5. SUMMARY

Creativity can be built in our students. To illustrate how we might do this we defined what we meant by creativity and elaborated on the need students have for skills with creative thinking. Some background explanation of how the brain functions provided some understanding of why we were doing certain things in developing creativity. A discussion of a sample six hour workshop to develop creativity was given. Finally, some specific actions were described for how a teacher can use this in the classroom every day and how they might build in explicit training programs in their course.

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2. Hilgard, E.R. and Bower, G.H. (1975), "Theories of Learning" 4th ed., Prentice-Hall, Englewood Cliffs, N.J.

3. deBono, E. (1973), "The Mechanism of the Mind", Penguin Books, Harmondsworth, Middlesex, England.

4. deBono, E. (1982), Workshop on "Teaching Thinking", University of Toronto, Toronto, Ontario, May.

5. Maslow, A.H. (1954), "Motivation and Personality", Harper Row, New York.

6. Adams, J.L., (1974), "Conceptual Blockbusting", W.H. Freeman, San Francisco.

7. Miechenbaum, D., Personal Communication, Dept. of Psychology, University of Waterloo, Waterloo, Ontario.

8. Olsson, R., Personnal Communication, University of Lulea, Lulea, Sweden.

7. ENRICHMENT BIBLIOGRAPHY

My favourite resources in this area of creativity are:

1. Parnes, S.J., "Creative Behavior Guidebook", Scribner's, New York, (1967).

For someone who wants to get some idea about how to run a workshop style experience. The Buffalo School uses a different definition of creativity - "The ability to think up **good** ideas". Since this implies evaluation I carefully separate these tasks into creativity - "The ability to think up ideas" and analysis "the ability to select a good idea". This subtle but important distinction does not affect the usefulness of Parnes' book on how to run a brainstorming session. The outline for a 3-day course is given on p. 112. All 16 sessions do not pertain to creativity. Rather, the full session explores a full PS strategy that he refers to as "Creative PS". The topics are:

Session 1. Settling In
Session 2. Defining the Real Problem
Session 3. Overcoming Habits Deferring Judgement* Brainstorming*
Session 4. Trigger-forcing Relationships*
Session 5. Evaluating Ideas
Session 6. Implementing Ideas
Session 7. Overview to Creative PS (p. 184)
Session 8. Fact-Finding (p. 186 to 207)
Session 9. Practice (p. 208)
Session 10. Idea Finding - Trigger Checklist* (p. 210 to 220)
Session 11. Evaluation (p. 221 to 237)
Session 12. Practice (p. 238 to 244)
Session 13. Stimulating Creativity in Others (p. 245)
Session 14. Practice (p. 252 to 253)
Session 15. Snap Decision (p. 254 to 264)
Session 16. Summary (p. 265)

An asterisk* indicates those sessions related to creativity.

2. Bob Sparks, Chemical Engineering Dept., Washington University, St. Louis, MO.

Bob's course on "Inventive Reasoning" is a classic. For inspiration and ideas to try to get a chance to hear him.

3. Prince, G.M. (1972), "The Practice of Creativity", Collier Publishing, New York.
Gordon, W.J.J., "Synectics", Harper-Row.

These books discuss synectics - a specialized technique for creativity - where a group is essential and the leader's role requires fairly extensive training. However, I use their ideas, and variations on them, to give me useful triggers.

4. deBono, E. (1969), "The Mechanisms of the Mind", Penguin Books; (1972) "The Use of Lateral Thinking", Pelican Books and (1972) "Po: Beyond Yes and No", Pelican Books.

Many mistakenly equate lateral thinking to brainstorming. Dr. deBono's contributions are in three general areas. His book "Mechanisms of the Mind" helps us to rationalize why certain techniques work. His book "Lateral Thinking" elaborates on fixation and different triggers to use to overcome it. His favorite seems to be juxtaposition. The "Po and No Po" is very similar to "Lateral Thinking". Finally he provides numerous examples of problems and imaginative or unexpected alternatives.

5. Miechenbaum, D., is a psychologist from the University of Waterloo, Waterloo, Ontario. His research is in PS as it applies to social problems. I have found some of his writings to be very useful in providing some insight into the creative processes.

6. Adams, J.L. (1974), "Conceptual Blockbusting", W.H. Freeman and Co. The emphasis here is more on the barriers that prevent us from being creative.

THE MATHEMATICIAN AT WORK RELIES ON SURPRISINGLY VAGUE INTUITIONS AND PROCEEDS BY FUMBLING FITS AND STARTS WITH ALL TOO FREQUENT REVERSALS. CLEARLY LOGIC AS IT STANDS FAILS TO GIVE A DIRECT ACCOUNT OF EITHER THE HISTORICAL GROWTH OF MATHEMATICS OR THE DAY-TO-DAY EXPERIENCE OF THE PRACTITIONERS. IT IS ALSO CLEAR THAT THE SEARCH FOR ULTIMATE FOUNDATIONS VIA FORMAL SYSTEMS HAS FAILED TO ARRIVE AT ANY CONVINCING CONCLUSIONS.

S. Feferman

TABLE 1. Summary of Triggers.

1. Personal Analogy
2. Direct Analogy (Nature)
3. Functional Analogy
4. Appearance Analogy
5. Symbolic Analogy/Simple replacement
6. Wildest Fantasy
7. Boundary scrutiny - what if
8. Juxtaposition
9. Reversal as a Trigger
10. Checklist
11. Problem Subdivision
12. Book Title
13. Morphology
14. Focus on Potential Mental Blocks
15. Awareness

TABLE 2: Facilities Needed

Facilities for Problem Solving Workshops.

Basic Room: Need a room with a flat floor. Prefer tables with chairs although we can do with tablet-chair units.

Basic A.V.: Two overhead projectors, two screens and one 35 mm slide projector with remote control near the overhead projectors.

Creativity: Basic room needed with a minimum of 2.5 m^2 of floorspace per person because small groups of six brainstorm aloud and there must be sufficient room to prevent excessive distraction from the other groups. A group of smaller sized rooms could be used.

The major challenge is power supply. At least one dedicated electrical circuit is needed for every two overhead projectors. Heavy duty extension cords, and a knowledge of the fuse or circuit breaker boxes are required.

AV: Basic AV plus one overhead and one screen per each group of 6 workshop participants. For some rooms screens are not necessary as long as the walls are flat and a light pastel color.

Materials: Nine blank acetate sheets per group and at least one water soluble transparency pen per group.

8. Integrate the following:

   $\int \sin x \cos (a-x)\, dx$

   and provide evidence with your solution, of the following:

   (a) Brainstorm by listing 50 ideas in 10 minutes as to possible methods or procedures, that might assist in the solution of this problem.

   (b) From this list, identify the craziest idea and four other different ideas. Convert all into mathematically feasible possibilities.

   (c) Take one of the possible methods and solve the problem.

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THE SECOND INTERNATIONAL MATHEMATICS STUDY IN B.C.

by
David Robitaille

During the 1980-81 school year, several thousand Grade 8 and Grade 12 mathematics students from British Columbia, along with their mathematics teachers, participated in the Second International Mathematics Study. More than twenty countries took part in various phases of the study, with Canada being represented through the participation of British Columbia and Ontario. A list of the participants is given below:

Australia
Belgium (Flemish)
Belgium (French)
Canada (British Columbia)
Canada (Ontario)
Chile
England
Finland
France
Hong Kong
Hungary
Irish Republic
Israel
Ivory Coast
Japan
Luxembourg
Netherlands
New Zealand
Nigeria
Scotland
Swaziland
Sweden
Thailand
United States

The Second International Study of Mathematics is a project of the International Association for the Evaluation of Educational Achievement (IEA). IEA is an association of educational research organizations and ministries of education whose primary goals are to conduct educational research on an international level and to assist member-states in undertaking co-operative research projects. IEA has conducted international surveys in the past, including the first mathematics study and the Six Subject Survey.

Twelve countries participated in the first mathematics study, but Canada was not among them. The international study had a three-fold purpose:

- to investigate the nature of the mathematics being taught in those countries participating in the study.
- to compare the ways in which mathematics was taught.
- to study the effects of that teaching on students' learning of and attitude toward mathematics.

The data collected in the study will enable researchers to construct an international portrait of mathematics education as it is currently being practiced in these 24 places.

Two populations of students were identified for investigation in the international study. The first, **Population A,** was defined as consisting of all those students enrolled in the grade where the majority of students have reached the age of 13 by the middle of the school year. In British Columbia this corresponds to Grade 8. The international definition for the second group, **Population B,** was intended to encompass all students in the last year of secondary school who were studying mathematics as a significant part of an academic program. In other words, students who were studying college-preparatory mathematics in Grade 12 in British Columbia were to be included in Population B, but those who were not taking any mathematics or who were taking a terminal course were to be excluded. Population B was therefore defined to consist of all students in the province who were taking Algebra 12. This included approximately 40% of the Grade 12 population.

Since British Columbia was one of the first of the participants in the international study to complete its data collection and publish a local report, few international results are available for purposes of comparison, except in the area of curriculum analysis. In addition, there has been no attempt made as yet to conduct analyses of the results to examine the relationships that may exist between the methods and resources used by teachers of mathematics and the achievement of their students.

A paper presented to the Canadian Mathematical Society on June 1, 1982.

Curriculum Analysis

- At the Population A (Grade 8) level, virtually all students in all of the participating countries are still studying mathematics. Moreover, the B.C. curriculum at that level coincides fairly well with the international grid. In other words, the B.C. mathematics curriculum at the Grade 8 level is similar in many respects to the curricula of other countries.

- In some countries, only a very small proportion of the age cohort is still studying mathematics in the final year of secondary school; less than 10% in some cases. In British Columbia, for the Algebra 12 course, this figure is just over 30%. This fact has obvious implications for the content of the Algebra 12 course, since it must be made suitable for a comparatively large proportion of the student population.

- From the data obtained regarding the amount of time available in the school program for mathematics, it is clear that, in many countries, students at the senior secondary level are able to specialize in mathematics to a much greater degree than would be possible in British Columbia.

- A major difference between the Algebra 12 curriculum in British Columbia and the comparable curricula in other places is the importance given to the study of calculus. In B.C., calculus is not part of the Algebra 12 curriculum, except as one of a sequence of optional topics for the enriched format of the course. On the other hand, the study of calculus is a major componet of the mathematics curriculum at the senior secondary level in other countries, notably in Europe.

- At both levels in British Columbia, little or no importance is given to the study of probability and statistics. This contrasts sharply with the importance given to them at either or both levels in most other countries. More importantly, given the importance of these topics in contemporary society, this situation should be examined and discussed as soon as possible.

The Teaching of Mathematics in Grade 8

- Over 20% of Grade 8 teachers teach students the British system of measurement as well as the metric system, in spite of the fact that the Curriculum Guide calls for only the latter to be taught, and that the precribed textbooks refer only to the metric system. It may be that teachers have decided that this generation of students needs to be familiar with both systems, and, therefore, both systems need to be taught. The fact that the United States now appears to be farther away from converting to the metric system than at any other time in the recent past may lend additional weight to such an opinion, and this matter should be discussed as part of the curriculum renewal process.

- Teachers rarely assume that students have mastered concepts or skills in previous grades to the point where no re-teaching or review is needed. Even topics such as the concept of a fraction, which is first introduced in the primary grades, were reported as having been taught or reviewed in almost 90% of classrooms. This may be due, in part at least, to a lack of familiarity on the part of Grade 8 teachers with the content of the mathematics curriculum at the elementary school level.

- There is a substantial lack of agreement among teachers about which formulas for the area and volume of various two and three-dimensional figures students are expected to be familiar with by the end of Grade 8.

- There are indications that in teaching computational techniques, such as multiplication of decimals, teachers emphasize the sequence of steps to be followed in using the algorithm rather than an understanding of why the algorithm works the way it does. For example, 98% of teachers agreed that they taught students how to multiply decimals by referring back to multiplication of whole numbers and giving students a rule for placing the decimal point in the answer. Similar, rule-oriented approaches to teaching were apparent for topics such as operations with positive and negative numbers, and the development of formulas.

- Although virtually all teachers reported spending some time on each of the five major areas of the mathematics curriculum covered in the five Classroom Process questionnaires, they differed significantly on the amount of time they devoted to them. For example, 10% of teachers reported spending at most eight class periods in the topic of integers while another 10% spent over twenty periods on the same topic.

- In the opinion of teachers, the principal factors which might account for a lack of satisfactory academic progress in mathematics by students were students' indifference, absenteeism, or lack of ability. Few felt that unsatisfactory progress could be attributed to a shortage of appropriate resources for teaching or to a lack of expertise on the part of teachers.

- According to teachers, whether or not a given approach to the teaching of a particular concept or skill would be easy for students to understand was the single most influential factor in determining which of a number of alternative teaching approaches to use. This factor was substantially more important than whether or not students would enjoy the approach or whether that approach was used in the textbook. Analagously, the fact that a given approach was seen as being difficult for students to understand was a powerful factor in decisions not to use a particular teaching approach.

- While teachers did not select teaching approaches primarily because they were present in the textbook, many indicated that they would be reluctant to follow approaches other than the ones in the textbook. This would seem to indicate that, if certain topics, approaches, or orientations are to be implemented in the mathematics curriculum, care must be taken to ensure that these are included either in the prescribed textbooks or in other instructional materials which are made available to teachers.

- The majority of teachers emphasized only one approach to the teaching of a given topic. While they might have used others or referred to them occasionally, they tended to emphasize only one.

- Over 25% of Grade 8 teachers of mathematics reported that, in their teaching, they seldom followed the treatment given to a topic in the textbook. Since, a large number also reported that they relied primarily on locally-developed worksheets, it may be the case that many teachers are not utilizing the prescribed textbooks to any significant degree.

- Although most Grade 8 teachers agreed that students should be permitted to use calculators, they were also strongly of the opinion that students should have a high degree of mastery of computational skills.

The Teaching of Mathematics in Grade 12

- There are very substantial differences in the amounts of time teachers spend on the different components of the Algebra 12 course. For analytic geometry, the range was from 6 to 45 periods, with 21 periods as the mean. In the case of trigonometry, the range was from 10 to 56 periods, with the mean being 29. For algebra, the range was from 7 to 96 periods, with the mean being 49.

- Approximately 40% of Algebra 12 teachers agreed with a statement to the effect that there should be a separate, semester-long course in trigonometry. Only 3% disagreed with this opinion; the remainder were undecided.

- As in the case of Grade 8, Algebra 12 teachers spend substantial amounts of time reviewing material that was covered in earlier courses.

- Relatively little time appears to be spent on developing students' ability to construct graphs of functions.

- Despite the fact that virtually all Algebra 12 teachers permitted their students to use calculators, 50% of them emphasized the computational applications of logarithms in their teaching.

- In presenting various formulas, identities, or generalizations to their students, almost all of the Algebra 12 teachers provided some form of proof or informal justification. However, they seldom expected their students to be able either to replicate such proofs or to produce them on their own. Instead, they expected students to recall the formulas and apply them.

- The use of calculators in class was permitted by 97% of Algebra 12 teachers. Moreover, 16% of the teachers said that they also made use of microcomputers in some manner.

- Results indicate that Algebra 12 teachers adhere more closely to the course outline as contained in the Curriculum Guide than do teachers of Mathematics 8. This may well be due to the fact that the course outline for Algebra 12 is much more detailed than the one for Mathematics 8.

- Algebra 12 teachers are more likely to use a particular teaching approach because they feel it will be easy for students to understand than because they think students will enjoy it. Analagously, the fact that a given approach is seen as being difficult is a stronger factor in deciding not to use a given approach to a topic than is the belief that students will not enjoy it.

- In a typical Algebra 12 class, correction of homework occupies 25% of the time, lecturing by the teacher an additional 40%, and individual seat work by students, 20%. Little time is allocated for having students work together in small groups.

- Results show that Algebra 12 teachers typically do not differentiate between levels of ability in their teaching. Most teachers teach the same content in the same way and give the same assignments to all students.

Achievement Results -- Grade 8

- Over 80% of the test items developed for use in the Second International Mathematics Study were judged to be appropriate for B.C. students in Grade 8. Students were told that the tests contained some items with which they would not be familiar, and they were advised to skip those items.

- All Grade 8 students were administered a forty-item test at the beginning of the school year, and the same test was readministered near the end of the course. Results on the posttest were 4.4 points higher than on the pretest, and improvement of about 25%.

- The range of growth scores on the forty-item core test was from -1.8 to 10.1. That is, in one class the posttest score was 1.8 points lower than the pretest score, and, at the other extreme, the performance in one class improved by 10.1 points.

- Gains in achievement from pretest to posttest were greatest on the algebra and geometry sub-tests, the two areas which consisted primarily of content presented for the first time in Grade 8.

- In a number of cases, classes performed more poorly on one or more of the sub-tests on the posttest than on the pretest. These losses occurred mainly on those subtests dealing with content from earlier grades, for example, fractions, measurement, and ratio and proportion.

- Teachers were asked to indicate whether or not the mathematics content required to respond to each item had been presented to students. Overall, there was only a moderate correlation between this Opportunity-to-Learn measure and the achievement results. This may mean that teachers tend to underestimate their students' knowledge or that they are somewhat unfamiliar with the curriculum of earlier grades. Alternatively, it may be that teachers did not interpret the Opportunity-to-Learn question in the way that had been intended. Further analyses of these results should clarify this issue.

Achievement Results -- Grade 12

- No pretesting was done with Grade 12 students. Moreover, since the test items were distributed among eight rotated forms, there was no common set of items which all students wrote.

- Of the total of 136 items on the eight test forms, 45 were judged to be inappropriate for B.C. students because they dealt with content outside the Algebra 12 curriculum.

- In constructing items for the tests, four levels of cognitive behavior were used: computation, comprehension, application, and analysis. In general, results show that achievement decreases as cognitive level increases. Similar results were found at the Grade 8 level.

- On the 91 items which were judged to be appropriate for the B.C. curriculum, the average percent correct was about 42%. Given the level of difficulty of the items, this would appear to be a satisfactory result.

Students' Attitudes

- When asked to react to a list of fifteen topics and activities in mathematics, students at both the Grade 8 and Grade 12 levels indicated that they enjoyed them, felt they were important, and did not find them to be too difficult.

- Teachers gave approximately the same ratings as their students to most of the items. Among the exceptions to this finding were that teachers felt that problem solving was more important, more enjoyable, and more difficult than their students did.

- Teachers and students both agreed that mathematics is a dynamic, growing field in which new developments are continually being made and not a static, unchanging one. Teachers were more positive in this regard than their students, and Grade 12 students were somewhat more positive than those in Grade 8.

- Students indicated that their parents considered mathematics to be an important area of study, and that their parents wanted them to do well in mathematics. Students at both levels reported that, while both parents encouraged them to do well in mathematics and believed that learning mathematics was important, their fathers enjoyed mathematics more than their mothers and were more likely to be able to assist them with assignments.

- Overall, Algebra 12 students' attitudes toward mathematics were slightly more positive than those of Grade 8 students.

- A clear majority of students at both levels agreed that it was important to know mathematics in order to get a good job; indeed, 32% of Grade 8 students strongly agreed with that opinion.

- On a sub-scale consisting of four items, Grade 8 and Grade 12 students indicated that both males and females were equally capable in mathematics, had the same need for careers, and had the same need to know mathematics.

- Over two-thirds of students at both levels were of the opinion that it was important for them to know something about computers, and that computers could be used in beneficial ways.

There was general improvement over the year as shown by the difference in the pretest and post-test scores. For example, Population A improved its mean score from 18.4 to 22.8 on 40 items. The percentage of students correctly evaluating (-2)x(-3) increased from 34% to 69%. In solving $5x + 4 = 4x - 31$, the percentage with the correct answer (-35) increased from 8% to 18.7%. However, the incorrect answer -27 was even more popular, favoured by 16.9% in the pretest and 20% in the post-test.

[The British Columbia report on the Second IMS is available from Dr. J. Matheson, Learning Assessment Branch, Ministry of Education, Parliament Buildings, Victoria, B.C. For the Ontario results, write to Dr. Les McLean, Head, Education Evaluation Centre, OISE, 252 Bloor St. W., Toronto M5S 1V6. OISE publishes the Ontario Assessment Newsletter, which deals with the Study as well as the Ontario Assessment Instrument Pool.]

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I CANNOT IMAGINE THAT I SHALL EVER RETURN TO THE CREED OF THE TRUE PLATONIST, WHO SEES THE WORLD OF THE ACTUAL INFINITE SPREAD OUT BEFORE HIM AND BELIEVES THAT HE CAN COMPREHEND THE INCOMPREHENSIBLE.

A. Robinson (1969)

WOMEN AND MATHEMATICS

by
Roberta Mura

INTRODUCTION

At the 4th International Congress on Mathematics Education held in Berkeley in 1980, there were several peresentations and workshops on the topic of women and mathematics. At the end of the conference, many participants felt that it would be very useful to have an international survey of the field. The task was undertaken by Dr. Erika Schildkamp-Kundiger in cooperation with the Second International Mathematics Study (IEA), and the survey has revently been published by ERIC/SMEAC (Ohio State University) under the title "An International Review of Gender and Mathematics".

I was responsible for the Canadian report appearing in the review. One of my conclusions is that so far, in Canada, not much attention has been paid to this topic. It still happens that when I mention my interest in the problem of women and mathematics, people react by asking: "What problem?".

One extreme aspect of "the problem" ought ot be obvious, if one thinks of the very low percentage of women among university mathematics professors. However this remarkably unequal distribution has not been particularly noticed or considered worth investigating, just as many other instances of the sexual division of labour which is still mainly unchallenged in our society. In this connection, it should be noted that in the United States - the country that has done the most research on gender and mathematics - interest in the topic was spurred by political groups who were fighting the occupational segregation existing in their country and who had identified mathematics as the critical filter preventing access to many jobs and careers.

Once one stops taking for granted the lower participation of women in mathematics, one faces a very comlex phenomenon which is still little known, let alone understood. I would like to suggest that "gender and mathematics" has the potential of becoming a whole new area of research. From an international point of view, it has already become one.

The purpose of this paper is to give an overview of this field of research. I have tried to classify the various issues and results into three main categories: describing the situation of women with respect to mathematics, explaining it, and finding ways to change it, where change is called for.

DESCRIBING THE SITUATION

What sex-related differences are there in mathematics?

(a) Participation

What is the percentage of women among mathematics students and teachers at the secondary school level? and among mathematics students and professors at the university level?

Not enough data is available in Canada to completely answer these questions. However surveys carried out in some provinces indicate that participation of women in mathematics begins to decline as soon as mathematics becomes an optional subject towards the end of secondary school. The percentage of women among secondary school mathematics teachers varies between 13% and 35% in the various provinces (excluding Ontario).

Some data about the university level has been published by Statistics Canada. Mathematics is treated as forming one category together with physical sciences. The percentage of women among full-time university teachers in this category in 1979-80 was 4.5%. Their proportion decreases as one moves from assistant professor to associate professor to full professor. Within each rank, the median salary for women is lower than that of men. In 1980-81 the percentages of women among those earning a bachelor's degree, a master's degree, or a doctorate in mathematics and physical sciences were respectively 28%, 17% and 8% in Canada. The same pattern, if not the same figures, seems to recur in several other countries.

Summary of a session presented at the annual summer meeting of the Canadian Mathematical Society, Ottawa, June 1, 1982

Of course, even if exact figures were available for all levels, they still would not constitute a satisfactory description of the situation. To give just one example, about women mathematics professors in universities, one could ask whether they are hired, promoted and tenured at the same rates as do men, whether their distribution among different subjects is the same as the men's, whether they teach the same kind and the same number of courses in the same ways, whether they do the same kind and amount of research, whether they have the same kind and amount of administrative duties, etc. On another level, one could ask: what has been their exprience? How do they live their situation of members of such a tiny minority? What internal or external factors made them decide to become mathematicians and helped them succeed? One could ask what happened to the women who had the potential of becoming mathematicians, but didn't...

(b) **Achievement**

Many studies have attempted to assess differences in mathematics achievement between girls and boys at primary and secondary school level. Results have been inconsistent: differences observed have been usually small, sometimes in favour of girls, sometimes in favour of boys.

One of the studies involving the largest number of students (about 70,000) is the Second Mathematics Assessment of the National Assessment of Educational Progress, carried out in the United States in 1977-78. Three different age groups (9, 13 and 17) were tested at four different cognitive levels (knowledge, skills, understanding and applications. The largest difference in favour of girls (1.4 percentage points) was found within the youngest group and at the lowest cognitive level; the largest difference in favour of boys (5.04 percentage points) was found within the oldest group and at the highest cognitive level.

Similar differences were found in the British Columbia and Alberta provincial mathematics assessments.

(c) **Attitudes**

Differences in attitudes towards mathematics between girls and boys have been found fairly consistently. They occur not so much in generic attitudes (liking or being interested in mathematics), as in more specific aspects. Namely it appears that: (1) boys are more confident than girls in their own ability to succeed in mathematics; (2) boys perceive mathematics as being more useful than girls do; (3) both boys and girls perceive mathematics as a male domain, but boys do so more than girls; (4) girls see mathematics as more anxiety-provoking than do boys.

(d) **Other variables**

A few researchers have begun to pay attention to some less obvius variables. An Australian study has found some evidence of differences in students' preferences of learning methods, girls preferring more cooperative methods, boys more competitive ones. The same pattern recurs in female and male teachers' preferences of teaching methods. If this is true, the higher proportion of men among mathematics teachers could lead to the hypothesis that girls avoid mathematics courses not so much because of content, as because of teaching methods.

In the United States, some intriguing results have been obtained about differences in the strategies used by girls and boys to (correctly) solve mathematical problems, and in the choices of wrong answers made by boys and girls when answering objective tests.

EXPLAINING THE SITUATION

Quite a number of factors have been considered in trying to explain why and how the differences observed arise and develop. (The distinction made here between variables describing the situation and variables explaining it is somewhat artificial. It might be more realistic - and more difficult - to view them as a system of interrelated variables interacting with each other).

(a) **Sex role perception**

The factor that is supported by the largest amount of empirical evidence is the perceived appropriateness of mathematics to one's own sex. As mentioned before, girls, as well as boys, although to a lesser extent, see mathematics as a male domain, and this affects negatively their decision to pursue the study of this subject.

(b) **Ability and attitude factors**

In spite of the sensational coverage of the topic in the popular press, sex-related differences in mathematical ability have yet to be proved. The difficulty resides both in the definition of "mathematical ability" and in the design of an experiment that would genuinely measure ability as opposed to achievement.

Some studies indicate that a difference in spatial ability in favour of boys develops during adolescence. Other studies however have not confirmed these results. Moreover the relationship between spatial ability and achievement in mathematics is still unclear.

Among attitude factors, those that have been quoted most often as being related to participation in mathematics are the self-esteem of one's mathematical ability, and the perceived usefulness of mathematics.

Other attitude factors that have been proposed as explanations of the lower participation of women in mathematics are: a higher fear of success among girls, particularly among talented ones, and a different perception of the causes of one's successes and failures. (Girls seem to have a tendency to attribute their success in mathematics to effort, luck, or easiness of the task, rather than to their own ability, while attributing their failures to lack of ability or lack of effort, rather than to bad luck or difficulty of the task; the pattern is reversed for boys. Thus girls feed less in control of their mathematics learning and are less likely to persist in its study.)

(c) **Environmental factors**

These include educational, social and cultural factors. Examples of **educational factors** identified through empirical research are:

- differences in the kind and amount of interactions between teachers and their male or female students (this may sound surprising, but it should be kep in mind that most such differential behaviour is unconcious);

- a higher rate of enrollment in science courses among boys (science courses give an opportunity of applying mathematics and this has a positive effect on mathematics achievement);

- sexism in textbooks (studies carried out in Quebec and in Manitoba, among others, prove that school textbooks give children an image of a society even more stereotyped than the real one: male characters outnumber female ones and are portrayed in five times as many different occupations, hardly any female character ever appears in a mathematics related occupation).

Social factors seem harder to quantify and have not yet come under the scrutiny of empirical research. However some researchers have pointed out that there seems to exist a conflict between the norms that school and society set out for girls (success in academic and professional life conflicting with success in private life and personal relationships).

A more obvious social factor is overt and subtle discrimination, including sexual harassment which is only now beginning to be talked about openly and whose extent is still difficult to evaluate.

Cultural factors, by their nature, probably cannot become the object of empirical studies. However they may well be the most influential factors of all. Our cultural tradition (in the form of religion, philosophy, literature, history, etc.) presents girls and women with an image of themselves that is not conducive to engaging in intellectual pursuits, especially scientific ones, nor striving for success in academic life. The most respected and admired writings of our civilization contain an astounding collection of passages which explicitely insult and ridicule women, in particular with respect to their intellectual capacity. Far from being a remnant of the past, this attitude toward the female human being keeps turning up day after day through all our media.

WHAT IS LAID DOWN, ORDERED, FACTUAL, IS NEVER ENOUGH TO EMBRACE THE WHOLE TRUTH: LIFE ALWAYS SPILLS OVER THE RIM OF EVERY CUP.

Boris Pasternak

CHANGING THE SITUATION

There exists a double link beween the task of explaining the situation and that of changing it. In fact, whether and how one thinks that the present state of affairs can be changed depends to a great extent on the kind of explanation one offers for it. Some factors can be acted upon more easily than others. For instance, the perceived uselfulness of mathematics may be easily increased by informing all students about the mathematics needed in various occupations and study programs. On the other hand, the negative effect of our cultural heritage is obviously much harder to counteract.

Thus the design of an intervention program has to be based on some theory about the origins of the differences that it tries to eliminate. Conversely, the degree of success of such a program can give some indication about the validity of the theory behind it.

Hardly anything has been done in Canada in this domain. This contrasts with the United States where there exist at least two national associations whose goal is to increase the participation of women in mathematics: the "Association for Women in Mathematics" (AWM), founded in 1971 and aimed mainly at the university level, and the more recent "Women and Mathematics Education" (WME).

Again in U.S.A., during the last few years, many different programs have been created to increase the chances that secondary school students - girls as well as boys - will continue to study mathematics. Such programs disseminate printed materials and films, and help to organize conferences and workshops to make participants aware of stereotypes about mathematics, to inform them about the importance of mathematics in different jobs and professions, to have them meet with successful women who use mathematics in their work, and to let them participate in some enjoyable mathematical activity. Some of these programs have already produced very encouraging results.

In order to improve the situation in Canada, a lot has to be done both at the individual and collective levels. On one hand, teacher trainers should develop pre-service and in-service teachers' awareness of these issues. On the other hand, elementary and secondary teachers should do their best to give their female and male students equal chances of succeeding in mathematics and of continuing to study it; this implies monitoring their own behaviour as well as finding ways to counterbalance the negative influences to which girls are subjected outside the classroom.

Issues related to women and mathematics should also be publicized in various ways by professional organizations. (At least three national conferences sepcifically about women and mathematics have already taken place in Australia.) Concurrently, research efforts should be intensified and appropriate actions (intervention programs, etc.) undertaken in various parts of the country.

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MATHEMATICAL EXPOSITIONS

MATHEMATICS AS PART OF CULTURE

by

S. Avital and I. Kleiner

Except for some applications, mathematics is usually viewed by students in isolation rather than as an integral part of our culture related intimately to such other spheres as art, science, philosophy, religion, economics and politics. Historian of science George Sarton (of this century) has advocated that in writing history, "the history of mathematics should really be the kernel of the history of culture." The contemporary American mathematician Raymond Wilder put it thus:

Mathematics was born and nurtured in a cultural environment. Without the perspective which the cultural background affords, a proper appreciation of the content and state of ... mathematics is hardly possible.

We believe that teachers should be aware of the interconnections between mathematics and other elements of culture. We present in this note a very brief historical survey of some of these interrelations in the hope that their spirit if not their detailed contents may be transmitted to students.

1. Mathematical discoveries had a significant impact on various cultural features (as we shall see). At the same time outside events undoubtedly conditioned mathematical discoveries. They may have been decisive in one instance and ineffectual in another. They did have a fundamental impact on the development of mathematics in the early civilizations such as the Babylonian, the Egyptian, the Chinese, the Mayan. When these civilizations turned to agriculture, developed trade, architecture, taxes, martial arts and other aspects of civilization, these required answers to such utilitarian questions as "how many?" and "how much?" - which led to the development of the integers, rational numbers and the elements of geometry. Herodotus, the Greek historian, conjectured the rise of geometry in Egypt as follows:

Sesostris ... made a division of the soil of Egypt among the inhabitants ... If the river carried away any portion of a man's lot, ... the king sent persons to examine, and determine by measurement the exact extent of the loss ... From this practice, I think geometry first came to be known in Egypt, whence it passed into Greece.

Whether or not this is the origin of geometry in Egypt, it is very likely that these or similar sociological factors had an impact on the development of mathematics in the ancient civilizations.

2. In ancient Greece, the interrelations between mathematics and other cultural elements were manifold. Mathematics was undoubtedly a central element of culture during the Pythagorean period (around 550-450 B.C.). The Pythagoreans believed that the key to the comprehension of the universe was mathematics, and especially number (by "number" they meant whole numbers or ratios of whole numbers). To them number ruled the universe in the sense that everything in the universe could be described and explained in terms of number. One of the first and fundamental discoveries of the Pythagoreans, which buttressed this philosophy of the universality of number, related to music. They discovered that musical harmonies depend upon simple numerical ratios of whole numbers. (Thus when two taut strings are plucked whose lengths are in the ratio 2:1, the sounds emitted differ by an octave. When the lengths are in the ratio 3:2 the shorter one gives out a note called the fifth above that given off by the longer one, e.g. C to G.) When the Pythagoreans had discovered that the diagonal of a square of unit length could not be expressed as a ratio of whole numbers, the very foundation of their faith crumbled. In effect, the Pythagorean theorem was the undoing of the Pythagorean philosophy (!) since it was with the aid of this theorem that this crucial discovery was made. The consequences for mathematics of this discovery were also far-reaching.

3. Without doubt the most fundamental contribution of ancient Greece to mathematics is the deductive method - "the explicit recognition", as mathematician and historian E.T. Bell says, "that proof by deductive reasoning offers a foundation for the structures of number and form". That this has had a deep and lasting impact on mathematics, science and other disciplines is indisputable (some of it we'll discuss in this note).

Historians of mathematics differ over when the deductive method originated in Greece. Some attribute it to Thales. (Aristotle's comment that "To Thales ... the primary question was not 'what do we know' but 'how do we know it'," gives at least an inkling of this.) Others believe the method originated about two hundred years later. In any case, it is fascinating to ask why the deductive method arose in Greece - the mathematical component of the so called "Greek miracle". Many reasons have been advanced, both within and outside mathematics. Some of the external factors which have been advanced are:

(a) The peculiar mental bias of the Greeks of classical times for philosophical inquiries. In philosophy one is concerned with ultimate questions about which one tries to reach inevitable conclusions. Such conclusions often follow from assumed premises, and in this process the deductive method is a fundamental tool. (Of course this argument only shifts the burden for the origin of the deductive method from mathematics to philosophy.)

(b) The fact that classical Greek society was based on slavery. The privileged class was supported by a large slave class that tilled the land, ran the businesses, managed the industries, took care of the households, and did both the technological and the unskilled work of the time. This fostered a separation of theory from practice, with members of the privileged class tending to prefer abstract thinking and to disdain experimentation and practical applications.

It also provided the time for contemplation - most probably a necessary ingredient in the development of abstract mathematics in any period.

(c) Democracy. This system of government originated in Greece. It required the art of argumentation and persuasion, hence encouraged logical, deductive reasoning.

4. Plato, perhaps the greatest philosopher of ancient Greece, thought very highly of mathematics. In fact, Plato's analysis of mathematical knowledge was a vital ingredient in his overall theory of knowledge. In this, Plato proved to be only a precursor of many philosophers throughout the centuries (e.g. Descartes, Spinoza, Hobbes, Locke, Leibniz, Berkeley, Kant) who found it helpful, and often necessary, to explore the nature of mathematics in order adequately to pursue their primary interests - whether in methaphysics, ethics, political philosophy or other areas. Moreover, a number of these philosophers used the deductive method a la Euclid to expound philosophical concepts (e.g. Spinoza's **Ethics** was written in a theorem-proof format). In addition, the emphasis on the moral and intellectual value of mathematics in education, as formulated by Plato and his followers, permeated educational thinking in subsequent centuries and became the basis of the educational curriculum as expressed in the "quadrivium" (geometry, arithmetic, astronomy and music), later joined by the "trivium" (rhetoric, dialectic and grammar) to form the "seven liberal arts" of medieval life.

5. The rational approach of the Greeks can be seen not only in philosophy and mathematics but also in their study of nature. Moreover, mathematics was fundamental to much of that study. Their great achievement was, in E.T. Bell's words, "the daring conjecture that nature can be understood by human beings through mathematics". We have already mentioned the Pythagoreans, whose tenet was that nature should be interpreted in terms of number and number relations. This thesis of the Pythagoreans was continued by the so called neo-Pythagoreans (early centuries A.D.), to be subsequently revived and refined in the works of Kepler, Galileo, Newton and their successors, its essence being that nature must be studied quantitatively.

Again we come to Plato, who put before the students of his Academy the problem of devising a mathematical scheme which would account for the motions of the heavenly bodies. The answer was given by Plato's brilliant student Eudoxus who constructed a complex geometrical model of the universe encompassing twenty seven spheres. This model embodied the first major astronomical theory in history and it set the stage for other major astronomical theories in Greece advanced by Aristarchus, Hipparchus and Ptolemy. Moreover, in their work in astronomy Hipparchus and Ptolemy developed the basic principles of trigonometry and computed trigonometric tables of (essentially) the sine function with remarkable accuracy.

We would be greatly remiss if we were to leave out of this account of the relationship of mathematics to science in Greece the greatest mathematician and scientist of ancient Greece, and one of the greatest of all time - Archimedes. Suffice it to say here that in his "law of the lever" and "law of buoyancy", which express physical laws as mathematical relationships, he singlehandedly created the fields of statics and hydrostatics. Moreover, he used physical considerations (especially the law of the lever) to **discover** mathematical results before proving them rigorously.

The relationship between mathematics and science, which so enriched both, was to become considerably more intimate in the "modern" period beginning in the 17th century. In fact, the great British scholar of Chinese civilization, Joseph Needham, has argued that the existence of this mathematically based tradition stemming from the Greeks was a fundamental factor which made possible the scientific creativity of modern Western Europe as compared to that of China. (This might be an opportune place to note that we will not be considering the role of mathematics in Chinese or Indian culture since these did not have a direct impact on the development of mathematics in Western civilization.)

6. A word about mathematics and art in Greece. The Greek disposition for idealizations and abstractions expressed itself not only in mathematics and philosophy but also in art. Thus, classical Greek sculpture dealt not with particular men and women but with man and woman (i.e. with ideal types). This idealization extended to standardization of the ratios of the parts of the body to each other. One of the most notable of ratios (called by Kepler the Divine Proportion) was the Golden Section, namely $(1 + 5\frac{1}{2}):2$, usually referred to by the Greeks as "the section". A rectangle with sides approximately in the golden ratio was later recognized as pleasing to the eye.

More generally, mathematics and art were but two manifestations of the Greek mind and outlook on the world. Simplicity, clarity, orderliness, finiteness - all these are common characteristics of Greek culture. The Greek temple has a clear and simple design suggesting finality, completeness, definiteness - the Gothic cathedral of medieval times seems incomparably more complex. Greek sculpture of the classical period is in large measure static, aloof, and uncluttered with elaborate dress and other frills which would detract from the main theme. The simplest type of motion for the Greeks is not along a straight line, because the straight line is potentially infinite - straight line motion is never completed. Rather, the ideal motion is circular - which is finite, changeless, completed.

7. Both the Roman civilization and the medieval civilization in Europe - encompassing a period of over a thousand years - have the distinct "honour" of producing **no** mathematics of note. A number of reasons have been advanced to explain this phenomenon. The Romans were practical people who completed vast engineering projects but disdained any scientific ideas which did not lead to immediate practical application. Their attitude toward mathematics is stated by Cicero:

The Greeks held the geometrer in the highest honor; accordingly, nothing made more brilliant progress among them than mathematics. But we have established as the limit of this art its usefulness in measuring and counting.

This attitude clearly did not encourage "mathematics for its own sake" and without this one cannot have any meaningful mathematics for scientific applications. The Romans did not heed the admonition of the British philosopher and mathematician Whitehead that

"the utmost abstractions are the true weapons with which to control our thought of concrete fact."

In contrast to the Roman era, during the early medieval period there was a singular lack of interest in the physical world. The important concerns were spiritual. Any serious study of mathematics or science which did not ultimately advance these concerns was considered unworthy.

Morris Kline, a contemporary mathematician, educator and historian of mathematics, summarizes the lack of progress in mathematics in Rome and medieval Europe, and outlines the conditions of such progress, as follows:

Apparently mathematics cannot flourish in either an earthbound or a heavenbound civilization ... it has been most successful in a free intellectual atmosphere which couples an interest in the problems presented by the physical world with a willingness to think about ideas suggested by these problems in an abstract form that makes no promise of immediate or practical return.

8. During this period of quiescence in Europe, mathematics was a flourishing part of Arab civilization. The Arabs were the first to have mathematicians assigned to the courts of the rulers - a practice continued later in Europe. Aside from their technical achievements in mathematics, the Arabs' major role was in preserving and then transmitting the Greek and Hindu works to an awakening Europe.

9. Of course, much has been written about the political and economic factors in the transition from the Medieval to the Renaissance period, a transition in which the love of a dispassionate search for truth was reborn. We will here merely indicate some cultural elements related to the development of mathematics during this period.

The emergence of commercial city-states in Italy necessitated elaborate bookkeeping systems. Following the invention of the printing press by Gutenberg around 1450 many commercial-arithmetic books were published. This encouraged the invention and standardization of arithmetical symbols (e.g. $=$, $+$, $-$, $\times$) which was accomplished in the 15th and 16th centuries, mainly by English, French and German mathematicians. (It is interesting to note the justification given by the English mathematician Recorde for the use of the symbol "$=$" to denote equality: he uses two longer lines ($=\!=$) and says that no two things can be more equal.)

MY MIND WAS STRUCK BY A FLASH OF LIGHTNING IN WHICH ITS DESIRE WAS FULFILLED.

Dante (Paradiso)

10. The Renaissance was also the age of great voyages. The new interest in seafaring raised many problems, most pressing among them being the determination of longitude. This in turn entailed laborious computations, which were also encountered in astronomical work (especially by Brahe and Kepler). Napier's invention of logarithms was motivated by the need to facilitate such computations.

11. Technological inventions raised various mathematical problems. The pendulum clock raised issues of a mathematical nature that were later tackled by Huygens. The use of cannon required study of the path of a projectile, the distance it travels, the maximum height it reaches - and these problems were tackled by Galileo. These are but two instances of the general problem of the analysis of motion, which had a fundamental impact on the rise of the concept of function in mathematics, the development of algebraic symbolism and the invention of the calculus.

12. The interaction of mathematics and art during the Renaissance benefited both. A science of perspective was created and developed by Renaissance artists and mathematicians, notably Alberti, Dürer, della Francesca, Pacioli. The specific problem which engaged the mathematical talents of the Renaissance painters was that of depicting realistically three-dimensional scenes on canvas. The artists solved this problem by introducing such fundamental goemetric ideas as perspectivity and section. All of this had a profound effect on painting. Moreover, in the course of their development of the system of perspective the painters raised questions that suggested an entirely new direction for research. The first major mathematician to explore these suggestions was the Frenchman Girard Desargues (1593-1662). His work resulted the creation of a fundamental branch of geometry, namely projective geometry. This was begun by Desargues, pursued by his contemporary Blaise Pascal, then abandoned for two hundred years, to be resurrected in the 19th century and developed into one of the most fundamental and beautiful branches of geometry.

13. Another fundamental branch of mathematics, probability theory, took form in the 17th century in response to external "demands". The Chevalier de Merē, a prominent figure at the court of Louis XIV and a reputed gambler, posed the so-called **problem of the points** to his friend Pascal. The problem asks how to divide the stakes of an interrupted game of chance beween two players, given the scores of the players at the time if interruption and the number of points needed to win the game. Pascal became interested in the problem and communicated it to his colleague Fermat. There ensued a correspondence between the two men, in which the problem was correctly but differently solved by each. This became the effective starting point for the modern theory of probability which turned out to be fundamental in such diverse areas as chemistry, physics, genetics, the insurance business, and now even literature and linguistics (in the analysis of authorship of texts).

14. Certainly one of the most fundamental interactions of mathematics with another branch of culture in the 17th century was with science. This mathematization of science gave rise to modern science essentially as we still understand it today. It was mainly Galileo and Descartes who revolutionized the very nature of scientific activity, binding it inexorably to mathematics. In brief, their plan was to seek quantitative descriptions of sceintific phenomena and express these in mathematical language, and to build up science deductively on the basis of fundamental principles. Mathematical concepts were used to describe and derive physical principles and results, and physical problems in turn motivated the introduction and development of vast mathematical theories. This intimate association of mathematics and science, especially physics, has continued unabated to modern times, to the great benefit of both. There are innumerable examples; we mention two. The calculus and its branches, especially differential equations, resolved the famous vibrating-string problem of the 18th century. (The problem consists of determining the shape of a vibrating elastic string fixed at both ends - for example, the violin string.) The problem of heat conduction, solved by Fourier in the 19th century, initiated a fundamental branch of mathematics called Fourier Analysis, which in turn had a profound impact on the description and explanation of diverse physical phenomena.

15. The mathematical explanation of physical phenomena gave rise, in the 18th century, to the so-called "Age of Reason" and the resulting doctrines of determinism, materialism and mechanism. These generated the belief that the whole future could be predicted to the minutest detail once one had the ability to divine the mathematical laws of nature. There was unbounded confidence in reason and the possibility of extending mathematical methods from the physical sciences to all fields of knowledge. Not long after Newton studied the heavens, the French physician and philosopher La Mettrie claimed to have discovered a mathematical description of the activities of the human mind and the French economist Quesnay announced equations for economic and social life. Eighteenth century intellectuals declared that all of man's problems would soon be solved.

16. From the time of Pythagoras to the nineteenth century, mathematicians and musicians alike sought to understand the nature of musical sounds and to extend the relationship between mathematics and music. The breakthrough, from a mathematical standpoint, came with the work of Fourier mentioned above, which made possible a thorough mathematical analysis of musical sounds - vocal and instrumental, simple and complex. In essence, any musical sound, however complex, can be built up from simple sounds such as those given off by tuning forks (the latter, in mathematical terms, are just sine curves of the form asinbyx). As M. Kline has put it:

It is theoretically possible, therefore, to play Beethoven's Ninth Symphony (including the Choral Ode) entirely with tuning forks. This is one of the startling implications of Fourier's Theorem.

The engineering of many components of music-reproducing and music-creating instruments such as radios, record players, and loudspeaker systems relies heavily on Fourier's analysis of musical sound. In fact, modern electronic music uses the very same principle.

17. Another fundamental mathematical development of the 19th century which had repercussions well beyond mathematics was non-Euclidean geometry. We have already mentioned that through the ages, beginning with the Greeks, philosophers concerned themselves with the nature of mathematical knowledge for the insights that this might provide into the nature of knowledge in general. Perhaps to no other philosopher was such knowledge more important than to the influential 18th-century philosopher Immanuel Kant. In addition to the two sources of knowledge -logic and experience-postulated by Aristotle, Kant argued for a third source, namely "pure intuition", and called knowledge thus obtained "synthetic a priori". This meant that the knowledge was real (as opposed to vacuous or tautological) yet independent of experience. One of Kant's chief examples of such knowledge was Euclidean geometry. As he said in his famed **Critique of Pure Reason** of 1781: "The concept of Euclidean space is by no means of empirical origin, but is an inevitable necessity of thought." That it was **not** inevitable was shown half a century later with the invention of non-Euclidean geometry. Not only was Kant's theory of knowledge found wanting, but so were most other theories which assumed that mathematical truths were absolute.

18. We mention a third groundbreaking mathematical development of the 19th century which had an impact on other areas of culture. We are referring to the creation of group theory by Galois and others. Aside from the important uses group theory was put to in physics and chemistry in the 20th century, it was the fundamental tool for the analysis of symmetry. Symmetry is all-pervasive in nature, science, mathematics and art. As the 20th-century mathematician Hermann Weyl has written, "Symmetry ... is one idea by which man through the ages has tried to comprehend and create order, beauty and perfection." Of course artists have for centuries used symmetry in their tile patterns, strip ornaments, wallpapers, arabesques and other designs, but these have only been fully analyzed and explained by the mathematical theory of groups. For example, it was only in 1891 that the Russian crystallographer Fedorov proved, using group theory, that there exist exactly seven essentially different patterns of strip ornaments, seventeen essentially different two dimensional patterns (so called wallpaper patterns), and 31 relief patterns - although artists created these for ages; thus the Moorish designers produced the seventeen wallpaper patterns in the Alhambra, in Granada, Spain.

19. In the 20th century the relation of mathematics to other elements of culture has continued with more vigor than ever. We give here the briefest of resumés. In the physical sciences mathematics has continued to play a fundamental role. Logical positivism, as a philosophical creed, has been greatly influenced by mathematics. In 1933 the renowned American mathematician George David Birkhoff wrote a work called "Aesthetic Measure" in which he attempted to apply mathematical analysis to painting, poetry, music, and pottery. In 1969 Birkhoff's son, the well-known contemporary mathematician Garrett Birkhoff, wrote a long paper on mathematics and psychology. Mathematics is being used in the analysis of language and this has resulted in the creation of the field of mathematical linguistics. Considerable work is going on in attempt to apply mathematics in the biomedical sciences. Mathematics has played a major role in this century in the social sciences, especially economics. In addition to the all-prevailing use of statistics, both discrete mathematics (e.g., matrices) and continuous mathematics (e.g., differential equations) have proved their worth. Moreover, a number of new branches of mathematics such as game theory, linear and dynamic programming, information theory, coding theory, and operations research have evolved to deal with various social, economic and military problems.

20. Finally, a word about the computer and mathematics. The theoretical underpinnings for the invention of the computer came from the works of mathematicians such as Church, Tukey, Turing, Post and von Neumann in the 1930's and early 1940's. The computer has been used in mathematics to make, test or disprove conjectures in areas involving numerical work, such as combinatorics and number theory. Its most spectacular success came in 1976 when it was used by Haken and Appel at the University of Illinois as an indispensable aid in the proof of the Four-Color Conjecture. (The famed Four-Color Conjecture, made in the 19th century, states that every map in the plane can be colored with at most four different colors so that no two "countries" with a common border are colored alike). More importantly perhaps, the computer has helped stimulate the development of new mathematical fields such as algebraic coding theory, the theory of automata, analysis of algorithms, and optimization theory, and has aided in the revival of older fields such as combinatorics and graph theory. Such developments are already having an impact on curricula in the universities, where courses in "finite" or "discrete" mathematics are beginning to vie with courses in elementary calculus as basic beginner courses in a mathematical sciences curriculum. We believe that we are not going too far out on a limb in predicting that courses in finite mathematics will be standard fare in high schools in the very near future - in fact, we support such a development.

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ALPHAMETICS IN DIFFERENT LANGUAGES

by
Marcel Danesi

Introduction

As Maxey Brooke (1969, p.3) points out, the term **crypt-arithmetic** was coined by M. Vatriquant in the May, 1931, issue of the Belgian magazine **Sphinx**. As is well known, a **cryptarithm** is an arithmetical problem in which each digit has been replaced by a distinct letter, and the solver is required to reconstruct the original numerical layout. J.A.H. Hunter (1965, p. 119) uses the word **alphametic** to designate that particular type of cryptarithm in which the letters are combined to form words, phrases or sentences, rather than just random sequences. This puzzle type is extremely popular, as witnessed by its appearance in newspapers and magazines:

```
   GET          Possible solution:      896
  HIGH                                  3183
+  ON                                 +   50
------                                ------
  LIFE                                  4129
```

Alphametics can be used in the mathematics classroom (especially at the senior public and junior high school levels) as recreational pedagogical devices. Since many students are also learning a second language at school (and possibly even a third), the use of "multilingual" alphametics can add to their appeal. The student is required not only to solve the problem numerically but also to decipher its "linguistic" meaning, thus adding an interesting verbal dimension to the problem-solving process. The purpose of this brief paper is to discuss the construction of multilingual alphametics for pedagogical purposes. The languages chosen for illustration are French, German, Spanish and Italian -- four languages studied by a large number of students.

Alphametics in the Mathematics Classroom

Before discussing the construction of multilingual alphametics, it is perhaps useful to look briefly at some pedagogical aspects connected with these puzzles. As any kind of recreational device, alphametics can be utilized to provide a fun element in the learning process. In the words of Martin Gardner (1971, p. ix), there is "no better way to relieve the tedium than by injecting recreational

topics into a course." In addition, alphametics can be employed to strengthen the student's understanding of the basic arithmetical operations. They stimulate the student to think about the "structural" patterns that exist among the digits of an operation. This is not always the case when students do rote additions, subtractions, multiplications and divisions. Moreover, since the process of solving alphametics consists in replacing letters with digits within the framework of an arithmetical operation, these puzzles can become "prototypes" of general mathematical reasoning. As Gray (1979, p. 15) observes, by using recereational devices such as these, teachers will be better able to provide young learners with an excellent opportunity to "acquire understanding of those principles, properties, and generalizations that apply to nearly all mathematics."

Multilingual alphametics will enrich the repertoire of alphametic techniques that a teacher can utilize. Since many students are studying another language at school, multilingual alphametics can become particularly interesting. They make the learning of arithmetic not only an enjoyable experience, but also an "intercurricular" one by bringing another subject into the mathematics classroom.

Multilingual Alphametics

Unless the mathematics teacher knows the language to be used, the construction of non-English alphametics may require the help of the teacher of the language in question. But many alphametics can be devised simply with the aid of a good dictionary and a rudimentary knowledge of the orthographic and grammatical patterns of the language to be used.

It is not the purpose here to describe the solutions to the alphametics. Suffice it to say that each letter corresponds to a particular but different digit. The solver is required to discover the digital values of the letters and thus reconstruct the original numerical layout. As Hunter and Madachy (1975, p. 91) point out: "No rules can be laid down for the solving of alphametics or cryptarithms. Rarely is any mathematical know-how required beyond a clear understanding of the basic facts of simple arithmetic, coupled with much clear and logical reasoning, and often a fair amount of determination and patience." A few examples of alphametics in French, German, Spanish and Italian follow, with observations on any orthographic patterns that can be of help in their construction. Only one solution is provided, although in most examples more than one solution is possible. Readers may want to verify the accuracy of the solution provided, and then try to find other solutions. Interested readers may also want to construct their own alphametics in each of the languages discussed in order to test their understanding of the orthographic patterns involved. And as an interesting advanced project, readers may want to explore the conditions under which any solution will be unique.

In French most words end in a consonant, and therefore as in the case of English alphametics most numerical layouts will contain non-identical digits in the units position: e.g., 1 **2** + 3 2 **4**, rather than, say, 1 **2** + 3 2 **2**. However, if identical digits are required then the plural forms of nouns, adjectives and determiners (articles, demonstratives, etc.) can be used because they end in **s**. Here are some examples:

D E U X	**Possible**	7601	**Meaning:** 'Two + six = eight'
+ S I X	**solution:**	+ 451	
H U I T		8052	
L E S	**Possible**	395	**Meaning:** 'The ten jumps'
× D I X	**solution:**	× 147	
S A U T S		58065	
S O N T	**Possible**	4068	**Meaning:** 'Are they friends?'
− I L S	**solution:**	− 794	
A M I S?		3274	

In German, words generally end in a consonant, as in English and French, and therefore the construction techniques are similar to these two languages:

Z W E I	**Possible**	3204	**Meaning:** 'two + two = four'
+ Z W E I	**solution:**	+ 3204	
V I E R		6408	

FÜNF	**Possible**	6986	**Meaning:** 'Five - three = two'
− DREI	**solution:**	− 1743	
ZWEI		5243	
EIN	**Possible**	526	**Meaning:** 'One + one = two'
+ EIN	**solution:**	+ 526	
ZWEI		1052	

Many Spanish words also end in a consonant, and plural nouns, adjectives and determiners end in an **s**. However, some words end in a vowel. This means that identical digits may have to be used in the units position more often than in French, German or English alphametics.

LOS	**Possible**	483	**Meaning:** 'The twelve months'
+DOCE	**solution:**	+9820	
MESES		10303	
LA	**Possible**	62	
PLUMA	**solution:**	16092	**Meaning:** 'The pen is (made) of silver'
ES		35	
+ DE		+ 83	
PLATA		16272	
TRES	**Possible**	2053	**Meaning:** 'Three + two + one = six'
DOS	**solution:**	973	
+ UNO		+ 487	
SEIS		3513	

Italian is a "vocalic" language. Almost all of its words end in a vowel. For this reason, identical digits will have to be used frequently in the units position:

OTTO	**Possible**	1771	**Meaning:** 'Eight − six = two'
− SEI	**solution:**	− 965	
DUE		806	
LA	**Possible**	21	**Meaning:** 'My house'
× MIS	**solution:**	× 341	
CASA		7161	
ECCO	**Possible**	1990	**Meaning:** 'Here is his/her hand'
LA	**solution:**	75	
+ SUA		+ 465	
MANO		2530	

Conclusion

As the above examples have attempted to show, the construction of multilingual alphametics is a fairly straightforward and interesting task. Once the basic orthographic patterns of the language have been determined, meaningful alphametics can be set up with the aid of the teacher of the language in question or of a good bilingual dictionary. Foreign language newspapers and magazines are also a source of alphametics. Multilingual alphametics not only add a recreational dimension to the learning process, but they also allow students to relate mathematics to other aspects of their education.

THE WHOLE CULTURAL WORLD, IN ALL ITS FORMS, EXISTS THROUGH TRADITION.

TRADITION IS THE FORGETTING OF THE ORIGINS.

Edmund Husserl (The Origins of Geometry)

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DEPARTMENTS

PROBLEMS? DID YOU SAY

by
Pat Yip

Notice:

Problems and solutions are solicited from readers. Came across a good one lately? SEND IT IN! (to P. Yip, Department of Mathematical Sciences, McMaster University, Hamilton, Ontario.)

PROBLEMS:

We thank Mr. Willie Yong of Singapore for sending in the following problems which P. Erdos set for a 12-year old prodigy in 1959, in Hungary.

All letters in the problems denote positive integers.

21.2.1.

Let $1 \le a_1 < a_2 < \ldots a_{n+1} \le 2n$.

Prove that (i) two of the integers have no common factor;

(ii) $a_i + a_j = a_k$ is soluble, i.e. two of the numbers add up to a third.

21.2.2.

If p is a prime and $k < p - 1$, then $1^k + 2^k + \ldots + p^k$ is divisible by p.

21.2.3

Prove that

$$\sum_{k=0}^{n} k \begin{bmatrix} n \\ k \end{bmatrix} = n2^{n-1}$$

21.2.4.

Prove that $n(n+1)(n+2)(n+3)$ is not a perfect square.

SOLUTIONS: (for problems in Volume 21.1)

21.1.1. Three Is A Wonder.

Let a $(p+1)$ digit number be represented by:

$$N = n_p 10^p + n_{p-1} 10^{p-1} + \ldots + n_1 10 + n_0 10^0$$

$$\therefore |N|_{\text{mod } 3} = \left| |n_p 10^p|_{\text{mod } 3} + \ldots + |n_1 10|_{\text{mod } 3} + |n_0|_{\text{mod } 3} \right|_{\text{mod } 3}$$

since $|10^p|_3 = 1$, $|n_p 10^p|_{\text{mod } 3} = |n_p|_{\text{mod } 3}$

$$\therefore |N|_{\text{mod } 3} = |n_p + n_{p-1} + \ldots + n_0|_{\text{mod } 3}$$

$\therefore$ N is divisible by 3, if the sum of its digits is.

21.1.2 It Seems Rather Odd.

Let $N_0, N_0+1, N_0+2, \ldots N_0+2n,$ be $(2n+1)$ consecutive numbers. Then:

$$S = N_0 + (N_0+1) + \ldots + (N_0+2n)$$

$$= (2n+1)N_0 + (1 + 2 + \ldots + 2n)$$

$$= (2n+1)N_0 + (2n+1)n$$

$$\therefore S/(2n+1) = N_0 + n.$$

For $2n$ consecutive numbers starting with N_0, the sum will be:

$$S = 2nN_0 + n(2n-1)$$

$\therefore$ S is not divisible by $2n$.

21.1.3. Out There In Space.

Label the ports by numbers in anti-clockwise directions from 1 to 7.

The ports directly connected in the anti-clockwise direction are in the order of:

1, 3, 5, 7, 2, 4, 6.

The sequence of 6 numbers for the proper docking is any cyclic or anticyclic permutation of 6 numbers beginning at any arbitrary point.

Example: 3, 1, 6, 4, 2, 7 ⇒ port 5 vacant.

or 3, 5, 7, 2, 4, 6 ⇒ port 1 vacant.

THE 23RD INTERNATIONAL MATHEMATICAL OLYMPIAD

by

E.J. Barbeau

For the second year, Canada sent a team to the International Mathematical Olympiad. Each participating country was invited to send only four members instead of the usual eight to the site of the competition, Budapest, Hungary. Canada's choice was the four top scorers in the Canadian Mathematical Olympiad, who all happened to come from Ontario:

Todd Cardno (Cardinal Newman High School, Hamilton)
Alastair Rucklidge (Toronto French School)
Charles (Cary) Timar (University of Toronto Schools)
Edward Hatton (Adam Scott C. & V.I., Peterborough)

They took part in a seminar for winners of the Canadian Olympiad sponsored by the University of Calgary. This was followed by a brief training session under the direction of Professors Geoff Butler of the University of Alberta and Ed Barbeau of the University of Toronto, who accompanied the team to Budapest. In all, 119 students from 30 countries participated. The two examinations, written on July 9 and July 10, 1982, contained six questions worth 7 points each, so that the maximum score for a team was 168. Ten countries scored over 100:

West Germany	145	Hungary	125
Soviet Union	137	Czechoslavakia	115
East Germany	136	Finland	113
United States	136	Bulgaria	108
Vietnam	133	Great Britain	103

Canada came 17th with a total score of 78, obtained as follows: Cardno (14), Hatton (12), Rucklidge (23), Timar (29). Gold medals were awarded to students with scores in the range 37 - 42, silver in the range 30 - 36 and bronze in the range 21 - 29. Ten got gold medals, 20 silver and 31, including Rucklidge and Timar, bronze.

While the hospitable Hungarians took care of all expenses in Europe, the Canadian Mathematical Society, as sponsor of the Canadian team, had the onerous task of raising travel expenses. Thanks to the generosity of a number of donors, we were able to compete, but our continued involvement in the IMO is by no means secure. The Canadian Mathematical Society expresses its gratitude to the following organizations for grants:

The Francis F. Reeve Foundation
The Waterloo Foundation
Samuel Beatty Fund
Province of Ontario
The Royal Society of Canada
The McLean Foundation
Province of New Brunswick
Confederation Life
Sun Life
Ontario Association for Mathematics Education

Ontario teachers should be aware that good students, particularly in Grade 13, can be nominated by their school principals to write the Canadian Mathematical Olympiad. Students in their 13th school year will **not** be nominated by the Ontario secretary since they are not eligible for prizes. However, the national competition will play a role in the choice of a team for the IMO, for which any student (except mature students) still in school is eligibe. Todd Cardno was such an "unofficial" participant in the Canadian Olympiad this year.

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XXIII INTERNATIONAL MATHEMATICAL OLYMPIAD
BUDAPEST

First day, July 9, 1982

1. The function f(n) is defined for all positive integers n and takes on non-negative integer values. Also, for all m, n,

$f(m + n) - f(m) - f(n) = 0$ or 1;
$f(2) = 0$, $f(3) > 0$, and $f(9999) = 3333$.

Determine f(1982).

2. A non-isosceles triangle $A^1A^2A^3$ is given with sides a^1, a^2, a^3 (a^i is the side opposite A^i). For all $i = 1,2,3$, M^i is the midpoint of side a^i, T^i is the point where the incircle touches side a^i, and the reflection of T^i in the interior bisector of A^i yields the point S^i. Prove that the lines M^1S^1, M^2S^2, and M^3S^3 are concurrent.

3. Consider the infinite sequences (X^n) of positive real numbers with the following properties:

$$x_0 = 1 \quad \text{and for all} \quad i \geqslant 0, \quad x_{i+1} \leqslant x_i .$$

(a) Prove that for every such sequence, there is an $n \geqslant 1$ such that

$$\frac{x_0^2}{x_1} + \frac{x_1^2}{x_2} + \cdots + \frac{x_{n-1}^2}{x_n} \geqslant 3.999 .$$

(b) Find such a sequence for which

$$\frac{x_0^2}{x_1} + \frac{x_1^2}{x_2} + \cdots + \frac{x_{n-1}^2}{x_n} < 4 \quad \text{for all } n.$$

Working Time, 4½ hours.

XXIIIe OLYMPIADE INTERNATIONALE DE MATHEMATIQUES
BUDAPEST

Premier jour, 9 Juillet 1982.

1. Soit f une application de l'ensemble des entiers strictement positifs dans l'ensemble des entiers positifs ou nuls, vérifiant les propriétés suivantes:

a/ Pour tout couple (m,n), $f(m+n) - f(m) - f(n)$ prend l'une des valeurs 0 ou 1;

b/ $f(2) = 0$, $f(3) > 0$ et $f(9999) = 3333$.

Déterminer $f(1982)$.

2.Soit $A_1A_2A_3$ un triangle non isocèle de côtés a_1,$a_2$et a_3 (a_i est le côté opposé à A_i).Pour tout i,$1 \leqslant i \leqslant 3$,on désigne par M_i le milieu du côté a_i,par T_i le point de contact du cercle inscrit avec le côté a_i et par S_i le symétrique de T_i par rapport à la bissectrice intérieure de l'angle de sommet A_i.Montrer que les droites M_1S_1,M_2S_2 et M_3S_3 sont concourantes.

3.On considère les suites $(x_n)_{n\in\mathbb{N}}$ de réels strictement positifs vérifiant les propriétés suivantes:

$$x_0 = 1 \text{ et pour tout } i \geqslant 0, x_{i+1} \leqslant x_i$$

a/Montrer que pour chacune de ces suites,il existe un entier $n \geqslant 1$, tel que:

$$\frac{x_0^2}{x_1} + \frac{x_1^2}{x_2} + \ldots + \frac{x_{n-1}^2}{x_n} \geqslant 3{,}999$$

b/Trouver une telle suite vérifiant:

pour tout $n \geqslant 1$, $\quad \frac{x_0^2}{x_1} + \frac{x_1^2}{x_2} + \ldots + \frac{x_{n-1}^2}{x_n} < 4$

Temps de travail: 4 h 30

XXIII INTERNATIONAL MATHEMATICAL OLYMPIAD
BUDAPEST

Second Day, July 10, 1982

4. Prove that if n is a positive integer such that the equation $x^3 - 3xy^2 + y^3 = n$ has a solution in integers (x,y), then it has at least three such solutions.

Show that the equation has no solution in integers when n = 2891.

5. The diagonals AC and CE of the regular hexagon ABCDEF are divided by the inner points M and N, respectively so that

$$\frac{AM}{AC} = \frac{CN}{CE} = r.$$

Determine r if B, M, and N are collinear.

6. Let S be a square with sides of length 100 and let L be a path within S which does not meet itself and which is composed of linear segments A^0A^1, A^1A^2, ..., $A^{n-1}A^n$ with $A^0 = A^n$. Suppose that for every point P of the boundary of S there is a point of L at a distance from P not greater than ½. Prove that there are two points X and Y in L such that the distance between X and Y is not greater than 1 and the length of that part of L which lies between X and Y is not smaller than 198.

Working time, 4½ hours.

XXIIIe OLYMPIADE INTERNATIONALE DE MATHÉMATIQUES
BUDAPEST

2e jour, 10 juillet 1982

4/ Soit n un entier strictement positif. Montrer que si l'équation $x^3 - 3xy^2 + y^3 = n$ admet une solution entière (c'est à dire un couple (x,y), solution, appartenant à $\mathbb{Z} \times \mathbb{Z}$) alors elle admet au moins trois solutions entières.
Montrer que pour n = 2891, l'équation n'admet aucune solution entière.

5/ Les diagonales AC et CE d'un hexagone régulier ABCDEF sont divisées respectivement par des points intérieurs M et N de telle sorte que: $\frac{AM}{AC} = \frac{CN}{CE} = \lambda$.
Déterminer λ lorsque B, M et N sont alignés.

6/ Soit C un carré dont la longueur de chaque côté est 100. $L = A_0A_1 \ldots A_n$ est une ligne polygonale non fermée ($A_0 \neq A_n$), sans point double, contenue dans C, telle que pour tout point P du bord de C il existe un point de L dont la distance à P ne dépasse pas $\frac{1}{2}$.
Montrer qu'il existe deux points X et Y de L tels que la distance de X à Y soit inférieure ou égale à 1 et tels que la partie de la ligne polygonale L comprise entre X et Y ait une longueur supérieure ou égale à 198.

Temps de travail: 4 h 30

BOOK REVIEWS

STUDIES IN EDUCATION

Vol. 2, Edited by Robert Morris, U.N.E.S.C.O., & Place de Fontenoy, 75700 Paris, France. Paperback. 179 pages.

This volume is one of a series titled "New Trends in Teaching" started to promote the exchange of information on mathematics. This particular volume is the result of a meeting held to review the goals of mathematics teaching. Contributors were asked to consider the question, does the teaching of mathematics correspond to the needs of the majority of pupils and the society? It was a happy surprise to find I have shared views about mathematics with several of the contributors so allow me to outline briefly their positions on the above question before a word about future directions we all might consider in our own teaching as we again face classes this month of September.

Ruth Rees, a Senior Research Fellow in Mathematics Education at Brunel University, United Kingdom, now leads a small team carrying out investigations on the problems of learning and teaching mathematics. I last met her in 1976 when she was investigating the needs of school leavers entering employment. Her article examines the growing public awareness of the need for "links" between the mathematics taught at school and that required for employment and general living. Many examples are given of teacher and industry personnel cooperation. Yet, as a result of studies read and personal experience, she returns to some basic concepts. These show that what most school leavers lack is a proper understanding of basic mathematical concepts and of the associated skills and an ability to apply them to a practical problem.

Margaret Brown is a Lecturer in Mathematics Education at Chelsea College, University of London, whom I met while she was working on the CSMS[1] project under Professor G. Matthews. Her article stems from this work and points out the difficulties children have in understanding mathematics. The CSMS study found that there is a very wide range of understanding in each topic in each year group. This suggests that text books, curriculum materials, indeed our sequential teaching of topics may well be inappropriate. "Even if the writers of a series were to aim at the median 'level of understanding' the material for the 12-year old would be 'poorly-matched' to 40 per cent of 12 year-olds, but 'well matched' to 40 per cent of the 15 year-olds!"[2] Perhaps a spiral approach to curriculum building is needed. Certainly if teachers could have better tests measuring the various levels of learning reached by individual children, then two of her other findings would change, namely that:

1. the progress of individual children is very slow.
2. the level of mathematical understanding reached by many children is relatively modest, certainly in comparison to the expectations of most curricula.

David Robitaille is Associate Professor of Mathematics Education at U.B.C., Vancouver and well known to us here in Ontario. His article stems from the B.C. Mathematics Assessment projects. But his premise is one worth repeating. In discussions and comparisons of curricula, it is important to keep in mind that implications of the goals are dependent, in large measure, upon the perspective of the observer. The three levels of curriculum goals he isolates are:

Intended Goals - those promulgated by curriculum developers. These are often set out in teachers editions of text books and listed in curriculum guides.

Implemented Goals - those that are understood and implemented by teachers in the classroom.

Realized Goals - those attained by the students, as demonstrated by their performance on tests designed to measure mastery of the intended goals.

In conclusion, he points out that there are discrepancies between the 'intended' goals and the 'implemented' or 'realized' goals. This fact is one which is perhaps all too familiar and leads us to a look at the future.

As both British Columbia and Ontario are thinking in terms of plans for curriculum revision, perhaps this UNESCO document's conclusions are relevant at this time. It would be worthwhile to take a few minutes to examine them, for the question continues to nage us as teachers, how well is your program meeting the needs of the majority of pupils and society?

1. CSMS, Concepts in Secondary Mathematics and Science, project findings reported in the text written by K.M. Hart, **Children's Understanding of Mathematics: 11-16.**

2. **Studies in Mathematics Education,** Vol. 2, Ed. R. Morris, p. 33.

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A GUIDE TO THE DIAGNOSTIC TEACHING OF ARITHMETIC

Third edition. Fredericka K. Reisman - author. Charles E. Merrill Publishing Company.

Attention: Primary, junior, intermediate and senior teachers

This is a useful book which offers the strategy of diagnostic teaching as the key to effective mathematics instruction. Diagnostic teaching is a three step approach to instruction. First it examines the student's level of performance and all the relevant information that contributes to that performance. Next diagnostic teaching looks at the material to be learned and analyses it. The culmination of diagnostic teaching is the blending of what has been learned with regard to the student's cognitive and emotional status together with the appropriate part of the task to be learned.

Professor Reisman does not suggest that this method of teaching be used only when remediation is required --though certainly this method of instruction would be of assistance under such conditions. Rather she sees this type of instruction, if used consistently, as being preventive in nature. The need for remediation would, in her opinion, be greatly diminished.

The title of the book is rather unfortunate; the content has implications for the whole field of mathematics. Indeed the teaching of any discipline could benefit from many of Professor Reisman's suggestions.

The book is divided into three parts. Part one deals with a detailed description of diagnostic teaching and a model for diagnostic teaching is presented. Part two discusses various assessment instruments such as norm-referenced tests and criterion-referenced tests. A sequence of steps for building a teacher-made diagnostic test is outlined. As well, a model for a teacher-made diagnostic screening instrument is presented. Part two continues with the powerful suggestion that for successful teaching the teacher must be fully cognizant of three areas: the child, the task to be learned, and the method of instruction to be used. Reisman examines several models or hierarchies for learning and the resultant implications for instruction. Theories presented and compared are those of Gagne, Brownell and Hendrickson and Bloom. Part two concludes with a discussion of the hierarchy of needs of the affective domain. Maslow's hierarchy of needs, Krathwohl's affective domain taxonomy, as well as theories of Rogers and Osgood are carefully detailed. Reisman emphasizes the strong relationship between learning and dealing successfully with the physiological and psychological needs of the student.

Part three examines common errors made by children in elementary school mathematics and makes some suggestions for remediating common difficulties. This is likely the least satisfactory section of the book.

Overall, this readable book is very thought-provoking. It should, as indicated earlier, be useful to all teachers of mathetics at every level. Likely time constraints would prevent individual teachers from following all of the useful suggestions. Teams of teachers would be best able to implement the ideas because the indicated workload of diagnostic teaching could be shared. However it is approached, the reading of this book will undoubtedly have some positive impact and make both teaching and learning of mathematics more profitable and enjoyable.

Olive Fullerton
Senior Consultant for Mathematics
and Author in Residence
Copp Clark Pitman

INTERVIEW
with
Bob Smith

Dr. James R. Smart is a professor of mathematics at San Jose State University in California. He began teaching at the Junior High level or, as it was called in those days, intermediate school, and then taught in El Paso before moving to California. He has been involved in the National council on various committees and is serving as the general editor for the 1981-1983 yearbooks. I know Jim as a textbook author.

Linda Silvey has taught mathematics in Los Angeles unified school district for 20 years in a Junior High School. Currently she is a mathematics resource teacher and full time counsellor with 9th graders. She has published commercially with Creative Publications.

As the 60th N.C.T.M. convention traffic flowed by us in the Sheraton Centre, Toronto, we three had a cup of coffee. I had read the 1982 yearbook "Math for the Middle Grades" the night before and was excited by it. My year at the Institute of Education in London, England had been my opportunity to examine this same topic. I felt ready to exchange views with my two guests.

RSS: Jim and Linda, you both seem very well suited for the job you took on a few years ago and which has now culminated in the text "Mathematics for the Middle Grades". We might start off with the question, "Why the concern for the middle grades at this time?".

LS: Well, the concern for the middle grades at this time has evolved for several reasons due to the fact that there seems to be a lack of specific publications in this area. While there are a lot of things written sporadically and in various areas for the middle grades we (The Publications Committee) felt it was very important that we have a publication aimed specifically at these grades. As well, children are maturing earlier. By the time they are in the 5th grade, they have very different interets from the children in the primary part of the elementary school, yet they are not adults--they need some kind of curriculum that is for them. In many cases, they do need some remedial work but at the same time they need to have activities that will involve them in the work rather than just the drill type pages which we see too much of.

JRS: That's right. The fact that it's in the middle explains it. The middle grade have been neglected from both ends. Also, the U.S. Federal Government has long been concerned with this as the most crucial period. What happens to children in those age levels really determines what's going to happen to them later on. That was where the curriculum reform began. Now, the reorganization into the middle school is again focussing attention on the crucial period. Linda was talking about the recognition that children at that age are changing very rapidly. They are neither youth nor adults nor children, they are something in between. Those who are parents understand that that is a problem.

RSS: True enough. Parents know how interesting children are at the adolescent stage. The critical concern to me when I looked at this was 'How do children learn at this stage?'. We can organize our schools in terms of the number of years they will spend in a particular division. We can organize the material and see that there is a flow from one stage to another. We have a fairly good understanding of how they learn at the early years. Education Departments state their philosophy of how children learn in the early ages, but few state how adolescent children learn. Do we know more about it now, Jim?

JRS: I think so. The yearbook doesn't attempt to explain in detail how they learn. Rather, we tend to supplement the usual ideas. We're not talking about replacing the formal curriculum but we have a lot of suggestions of how they can learn more and how they can enjoy learning more. For example, we incorporate the use of games and activity learning, instead of just sitting and working with a paper and pencil. They may be outdoors, they may be up creating something, but they are involved in mathematical learning.

LS: I think, Jim, just the fact that we have a variety of activities is good. My experience has been that youngsters at this age learn in a variety of ways. You cannot come up with a clear cut pattern or formula and say that all children are going to learn this way. Children are very unpredictable, particularly at this stage, and therefore the teacher who is dealing with this age youngster must have lots of different strategies and techniques that they can use. Very often what works with one child does not work with another. They have to try a variety of things, and I think in this yearbook we have tried to present a variety of strategies.

JRS: Some of the articles I think appeal to the intuition of the student, the intuitive idea rather than the formal idea. Others use visual materials, graphs, pictures, the sport card, any sort of interest that they might have. Rather than having mathematics confined to the classroom we want to convince them it is really a part of life, and that what goes on in the school is very closely related to what goes on after you get out of school.

LS: I think we have used the problem solving emphasis to help students see how it relates to them and their world. We have some other things where they have taken the students outside of the classroom to do activites, i.e. measurement and map making activities. We are trying to reinforce the programs and structures of the formalized textbooks.

JRS: We also have some help for teachers on how to reach students who might be turned off by the regular curriculum, who might not be able to respond to it. For example, one article is called "Help for the Learning Disabled Student in the Classroom" and there are some very good suggestions there.

LS: Even though the title does say "Help for the Learning Disabled Student" there are techniques and strategies as far as presentation of material that can be of benefit in all classrooms ... from classrooms that are remedial to those that are for honours and eventually advanced students.

RSS: Could you name one of the better enrichment articles?

JRS: "Developing Mental Arithmetic" has some suggestions that are extremely good for explaining how students do mental arithmetic in various ways. Problem solving articles give suggestions as to how the superior student might really go about solving a problem. The one on spacial visualization is one of my favourites because it shows how to teach something that originally was thought to be unteachable. You either had it or you didn't. I have always thought superior students could visualize in three dimensions but couldn't explain how they did it. This article, to me, makes extremely concrete how you might help a superior student understand a little more about how he was really able to visualize in three dimensions.

LS: Yet, I think this article can be used with some of the students that are not as able. You would work through it in a slower way, but these students can also develop the same kinds of perceptions.

RSS: Could we take a different slant? At the annual meeting we heard Professor Max Sobel declaring an emergency in terms of the number of qualified math teachers and the need to obtain suitable persons for the classrooms. I know you make no suggestions about the teacher who is going to handle this work of the middle grades. Could we take a minute to give some thought as to who are going to be the teachers in the middle grades.

JRS: Generally, we are talking about people who may not have had too many formal courses in mathematics. We visualize one use of the yearbook to be for the in-service courses. Not only for individual reading of it but also I would hope that teachers will get together and discuss the articles and perhaps supervisors will be able to encourage the use of the yearbook. I hope that it's presented in such a way that a person with relatively little formal math background can take the ideas and carry them.

LS: Another one of the strengths of this yearbook is the many articles written by teachers who talked with their students and reported what happened in their classroom. We have teachers writing from their experiences about things they enjoyed doing. Not only are they teaching mathematics but they are interacting with the students in a humanistic way as individuals, and they are trying to develop them as positive human beings.

JRS: In reading some of the articles the actual enthusiasm within the classroom comes through, you can almost hear it - as if there was a soundtrack with the book. We were really fascinated with some dialogue-type articles and I think these turned out to be some of the best articles.

RSS: Two editors for whom I have much respect, David Fielker and David Wheeler, say articles sent to them should contain some dialogue with children. That's a good policy, so I'm glad to see the stock market article, "Teaming Up On the 20's" using dialogue.

LS: This was an article we almost didn't get. We had accepted the article on our first round, and then I sent a letter of encouragement to some thirty-eight authors. One of those got lost in the mail. Fortunately, I phoned, he accepted the rushed deadline and produced that delightful article.

RSS: Perhaps one last point. You mentioned earlier in our conversation that you hadn't met all the authors personally. At today's session I was impressed by Carla Thompson and her ideas on computer use. Perhaps you would just like to say a word about her article "Integrating Computer Literacy into the Curriculum.".

MIDDLE SCHOOL COMPUTER LITERACY
MATHEMATICS CURRICULUM INTEGRATION MATRIX

		Mathematics Content Areas								
	Computer Literacy Areas	Numeration systems	Real number operations	Mathematical applications	Sets and number theory	Elementary algebra	Measurement and geometry	Tables and graphing	Logic and probability
Direct Computer Application	Operating calculating devices								
	Operating computer hardware								
	Programming and software								
	Computer applications								
Computer Awareness	Information processing								
	History of computing								
	Using algorithms								
	Computing implications								

Shading indicates **major objectives**
Lack of shading indicates **minor objectives** (assignment)

LS: It is organized to increase computer awareness either in the presence or absence of hardware and software. Also, she has taken the standard strands that you think of and organized these according to the computer topics, along with primary objectives and secondary objectives. It's a very well written article.

JRS: As we look across the country now, we see something about computer literacy coming into the middle grades in almost every school, but I think the crucial word in her title is "integrating". The point she's trying to make is that you shouldn't stop what you're doing, teach computer literacy for two weeks, and then go back to something else.

RSS: Thank you very much. I'm going to ask one more thing before you leave, please autograph my copy of "Mathematics for the Middle Grades".

LS: We would be delighted to.

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THE SCIENCE AND EDUCATION STUDY OF THE SCIENCE COUNCIL OF CANADA

On May 31, 1982 at the meeting of the Canadian Mathematical Society in Ottawa, Dr. Graham Orpwood, the project officer, and Dr. William Higginson, of Queen's University, were invited to comment on the Science Council's Science and Education Study, now approaching its closing stages. Dr. Orpwood explained that the idea for this study grew from public criticism, articulated in particular by Tom Symons who in his 1975 report stated that Canadians are unaware of Canadian achievement and David Suzuki who felt that the schools were turning out two estranged groups, one alienated from science and the other immersed in jargon. Basically, critics were saying that science education is not directed to the most important objectives and it was not achieving its objectives.

The questions addressed by the study are: (1) what are the present aims of science education? what is actually happening? (2) how has the present state been achieved? (3) what should be the aims? The strategy is to foster a "deliberative enquiry" to stimulate and inform two processes, national debate on alternatives, and political and professional commitment to change. The first stage, already complete, is to identify the issues. Secondly, it is necesary to have a reliable database about the context of change; this has been largely done. Finally, there are to be deliberative conferences, at least one in every province, to discuss proposals and begin the process of reform.

In addition, a number of reports have been commissioned from outsiders to describe the Canadian situation, to provide social prospective, to give an engineer's viewpoint and to discuss scientific thinking and literacy. Other resources utilized by the Study include major research projects, policy documents from the ministries of education, surveys of teachers' objectives, analyses of textbooks and reports from visits to schools.

The scope of the Study does not include mathematics, computer science and social and environmental studies, in part because these are separate organizationally from the traditional sciences. However, the Study is attempting to maintain contact with the mathematical community in its final stages. Dr. Orpwood posed three questions to his mathematical colleagues: (1) Are criticisms of science education valid for mathematics? (2) In what ways, given that there are problems in science education, is mathematics teaching part of the problem and part of the solution? (3) If changes occur in mathematics education, what are the implications for mathematics?

Dr. Higginson explained his involvement with his study. The Canadian Mathematics Education Study Group and the Canadian Mathematical Society, dismayed at the low profile of mathematics in the Study, sent letters to the Science Council. It was decided to commission a background paper, authored by Higginson, on the role of mathematics in science education. The speaker took the audience through his paper, emphasizing a number of points, including the widening chasm between mathematics and science and the lack of adequate training for many new teachers. He deplored the lack of any historical dimension in most school teaching so that students do not appreciate the origins of their discipline and the failure of mathematics textbooks to treat any science at all. He was not optimistic about the future relationship of mathematics and science in the schools.

William Higginson's paper, "The Role of Mathematics in Science Education," bulletins on the Science and Education study and monographs commissioned by the study are available for the Science Council of Canada, 100 Metcalfe Street, Ottawa K1P 5M1 (613-995-0806). At least one copy of these important documents should be housed in the mathematics division of every board of education. The Canadian Mathematical Society is planning to sponsor a further session on the Study at its meeting in Toronto in December, 1982. Further details can be obtained from Ed Barbeau, University of Toronto. (416-978-8601).

AN INFORMAL NEWSLETTER — THE INTERMEDIATE/SENIOR MATHEMATICS CURRICULUM COMMITTEE

(Mathematics Information Newsletter #1) 82 10 01

The Ministry has begun work towards a single mathematics guideline, Grades 7 to 13. (Until a decision is made to the contrary we are assuming Grade 13 will remain).

Dave Alexander, Faculty of Education, University of Toronto, has been seconded by the Minstry for two years to assist in this project.

The organization of the project is as follows:

A Project Committee - responsible for preparing Guideline and consisting of:

Moe Liebovitz, Project Manager
Dave Alexander, Project Leader
Art Abramovitch, McArthur High School, Ottawa
Dave Davidson, Collingwood C.I., Collingwood
Kevin Doran, Peel Board of Education, Mississauga
Marta Johnson, Assumption College School, Windsor
John Milliken, Midnorthern Regional Office, Sudbury
Mike Silbert, Hamilton Board of Education, Hamilton

This group will be expanded by an Advisory Committee of five mathematics educators named by O.T.F., to assist the Project Committee by providing detailed reaction. (The names of members of the Advisory Committee will be listed in a future newsletter).

Additional resource persons will be approached for specific help on details of the Guideline.

The intention is to have as much input as possible from interest groups such as: Ontario Association for Mathematics Education; Ontario Mathematics Coordinators Association; Ontario University Mathematics Chairmen Group; Deans of Technology, CAAT's; Deans of Business, CAAT's; as well as interactions through professional development days etc. with teachers.

The first meeting of the Project Committee took place on September 29, 30, 1982.

Some of the questions which are of immediate concern to the Project Committee are:

What are appropriate objectives for general level courses?

What are examples of units which have been successfully used in general level courses?
Should there be a Grade 13 course designed specifically for those proceeding to social sciences, etc.?
What are examples of units which have been successfully used with Grade 13 students not proceeding to a mathematics related program?
What has been the reception of such courses by universities?

Any help that you can give with these questions would be appreciated.

The Project Committee is of the opinion that there should be a separate general level mathematics course in each of grades 11 and 12 for students preparing for Technology courses in the CAAT's. What is your reaction?

We will try to keep you informed as this project continues through further issues of MAinfo. This informal newsletter will be limited in distribution and will be sporadic in frequency. It must not be construed as official Ministry policy, but will reflect the current thinking of the Project Committee for discussion and reaction. Limited numbers of copies will be sent to OTF, OAME, OMCA, Regional Offices, representatives of CAAT's, and representatives of universities. Those receiving MAinfo are invited to share with others.

Members of the Project Committee would appreciate the opportunity to meet with groups of interested mathematics educators during professional development conferences, etc. to obtain feedback. Contact members directly or Dave Alexander.

Any comments related to the I/S Mathematics Guideline should be conveyed, preferably in writing, to:

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Education Officer
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COMA-AMOC
THE MATHEMATICS INTERFACE PROJECT

J. Byrnes, B. Nicoll, F. Maskell, R.M. Batley and J. Starrs present to the Canadian mathematical Society at their Ottawa meeting on May 31, 1982, "A panorama of activities: a fast-paced look at some exciting experiments". These resulted from a letter from H.K. Fisher, Deputy Minister of Education and Colleges for Ontario to Peter Ullrich when he was president of the Carleton-Ottawa Mathematics Association, in which Dr. Fisher said that from a previous interface study, "there was a clearly expressed need for improved communication and co-operation between secondary schools and the universities and colleges".

The Ottawa-Carleton Mathematics Interface Project, operated by a board with membership from the Ottawa Board of Education, the Carleton Board of Education, Algonquin College, Carleton University and the University of Ottawa with grants from the province, is due to terminate in June 1983. There are a number of activities in progress:

(a) **University visits** (K. Hardy): There were visits to five universities to clarify and study opinions on interface concerns. A report is being prepared.

(b) **The Examining Board** (B. Nicol): This project, partly funded by the Interface Committee, has the aim of setting up a confidential examining service for students who are to use it. Algonquin College supported a pilot phase of the Examining Board at the Grade 12 level in 1980 and 1981, and Phase I was carried out in the spring of 1982.

(c) **The Differences Project** (S. Blakeman, R.M. Batley). This is designed to test differences in method of learning a small body of knowledge. With awareness of difference approaches students may be better prepared to tackle university or college methods of presentation or teaching styles. A follow-up study is planned.

(d) **SERP and Mathematica at the Interface (SMIF)** (B. Hammond, D. Fox, A. Jones): This examines mathematical course outlines with respect to content and methodology on both sides of the interface, and will provide a basis for changes in the Grades 7 to 12 course, if Grade 13 is deleted.

(e) **A Day of Hearings** (J. Byrnes): Planned for the autumn of 1982, this will enable the mathematical/educational community to present briefs on SERP, Intermediate Guidelines, contests, conferences and other areas of concern.

(f) **Content Survey on Relations & Functions:** A 20-item questionnaire for Grade 13 students soliciting their opinions on their grasp of techniques and on the interest and value of the course was circulated at the meeting.

The Chairman of the Steering Committee is Dr. Joe Byrnes, special assignment teacher with the Carleton Board of Education (613-820-1820, ext. 311), who can be contacted for further information.

JOHN OMIELAN WINS THE McCARTNEY

On July 26, 1982, John Omielan was the recipient of the $400.00 John McCartney Scholarship. John who lives at R.R.#3 Ingersoll, Ontario was chosen from among the 85 Grade XIII students who applied for the scholarship. The judges were Paul Pogue of Barrie and James Boyd of Kingston. John is the son of Mr. & Mrs. Stanley Omielan. John attended the public schools of Dunn, Dickson, Harris Heights and Victor Morris before entering Ingersoll District Collegiate Institute. John in 1982 was also the winner of the Silver Medal in the Descartes competition. Next year he intends to enter the University of Waterloo in the Co-Operative Applied Mathematics Course. He believes that he will upon graduation go into Scientific Research that will require a great deal of Mathematics. The Head of Mathematics at the Ingersoll Collegiate is Lonnie Hudson and this year John studied under Mr. S. Walker, Mr. Fenell and Mr. Rachich.

The John McCartney Scholarship is presented annually by the Ontario Association for Mathematics Education to a Grade XIII student who shows excellence in Mathematics. John McCartney was a prominent educator who at the time of his passing was doing research for Science Research Associates.

COUNCIL COMMENT

by

Bob Robinson

Editor's Note:

Council Comment appears in an issue of the Gazette following O.A.M.E./A.O.E.M. Council Meetings. The Editorial Panel hopes that you find this section of interest. Your comments are welcomed and should be addressed to:

The Editor
Ontario Mathematics Gazette
c/o Department of Mathematics
McMaster University
Hamilton, Ontario
L8S 4K1

The October Council Meeting was held on Saturday, 1982 10 16 at the Ramada Inn (Airport West). The names of members of council are found on pages 41-43 in the September, 1982 issue of the Gazette.

NOMINATING COMMITTEE

A set of guidelines was approved by Council. A request for nominations for the positions of President Elect, Director and Councillor was presented in the November issue of the Abacus.

SIZE OF COUNCIL

The size of Council will be discussed at the November meeting of the Executive and a report will be made to Council in February.

O.A.I.P. JUNIOR

All the geometry items have been field tested except for 2 booklets. Results of the field test have not been received as of this meeting of Council. It is hoped that the Arithmetic Items and Measurement items will be in the schools for field testing by December, 1982.

O.A.I.P. INTERMEDIATE

A report if to be published (Ed. Note: Report is probably available at the time of this issue of the Gazette).

LEADERSHIP SEMINAR

At the time of writing these Council Comments the Leadership Seminar was taking place at the Ramada Inn (Airport West). The general co-ordinators of this seminar are Morley and Mona MacGregor who from all reports have organized another excellent leadership function of the O.A.M.E. The program committee included George Knill, J. Symington, Mona MacGregor and Morley MacGregor. O.A.M.E. is one of the leaders on this continent with regard to the operation of this type of Mathematic Education event. Next year's Leadership Seminar is planned for 1983 10 19-22 at the Ramada Inn (Airport West) in Toronto. Members are encouraged to submit suggestions to the Sarnia office of O.A.M.E.

1983 ANNUAL MEETING

We are all looking forward to our Tenth Annual Conference at the University of Waterloo on 1983 05 13-15. See the ad elsewhere in this issue.

O.E.C.A.

A report is to be made to Council in February regarding the review of Geometry tapes produced with the assistance of O.A.M.E.

N.C.T.M.

Councillors were reminded of the proper ordering procedure for N.C.T.M. publications which was outlined in the September issue of the Gazette on page 44. O.A.M.E. receives a rebate from N.C.T.M. if you follow this procedure. A special rate for the Arithmetic Teacher ($10.00) was also discussed. If there is any project or special program going on in your area that you would like mentioned in the "What's Going On?" section of the Arithmetic Teacher let Paul Pogue or Bob Robinson know. Information pertaining to the Publications and N.C.T.M. membership should be directed to Dr. Ralph Connelly, N.C.T.M. Representative, College of Education, Brock University, St. Catharines, Ontario.

CONSTITUTION

The constitution of O.A.M.E. will be examined this coming year. Copies of the constituion may be obtained by writing to Bob Robinson, Mathematics Department, Hamilton Board of Education, 100 Main Street West, Hamilton, Ontario L8N 3L1.

PUBLICATIONS

Councillors were reminded of the Call for Editor for the Gazette on page 48 of the September issue. Detailed information regarding a Canadian wide publication will be discussed at the February Council meeting.

MEMBERSHIP

An active campaign to recruit new members is underway. Councillors were encouraged to promote membership in O.A.M.E. at educational functions.

PUBLIC RELATIONS

O.A.M.E. is presently in the process of collating ideas/information with respect to our goals (p. 44-September issue). A product proposal will be ready for the February Council meeting.

SENIOR GUIDELINES

A committee has been established at the ministry level to examine Mathematics 7-13. The first Mathematics Information Newsletter (MA info) was distributed to councillors. A copy of MAinfo may be obtained by writing:

Dr. D.W. Alexander
Education Officer
Ministry of Education
Senior and Continuing Education
17th Floor, Mowat Block
Queen's Park
Toronto, Ontario M7A 1L2

(See also this issue of the Gazette).

O.A.M.E. has established a 7-13 guideline committee to give input to this revision of the Ministry guidelines. Members wishing to provide input to this important committee should contact:

Tom Griffiths
Chairman, O.A.M.E. Guideline Committee
30 Conifer Court
London, Ontario
N6K 2X4

PROBLEM SOLVING

A steering committee pertaining to a study of problem solving was established.

BILL 82

The Bill '82 Committee is currently studying Bill '82 as it pertains to the subject of mathematics. Members should send position papers or other pertinent information pertaining to Bill '82 to O.A.M.E. Information pertaining to mathematics programs for the learning disabled student and the gifted student should be sent to Elaine Harvey, 137 Inverness Crescent, Kingston, Ontario K7M 6G9.

COMPUTER COMMITTEE

The following position of O.A.M.E. was approved. "It is the position of O.A.M.E. that, as of 1982 all students have the right to use computers throughout their education JK to 13 and that we must overcome obstacles such as budget restriction, resistance to change and lack of teacher training".

It was also approved that O.A.M.E., through its executive, approach the ministry regarding O.A.M.E.'s position for Computer Education and to clarify the Ministry's position on the following: (1) Compatability of hardware and software, (2) establishment of a pool of resources personnel across the province (3) allocation of funds for (1) and (2).

CHAPTER REPORTS

(Ed. Note: More detailed information may be obtained from Chapter Representatives listed on page 43 - September issue).

C.O.M.A. reports that the executive for 1982-83 is established and they are continuing to support the following activities: Secondary-Post Interface Project, Arux Mathematicoum, and Examining Board. The major thrusts for the year are (a) membership (b) mathematics for the general level student (c) mathematics avoidance and anxiety and (d) mathematics in computer education.

Golden Section plans a conference of math workshops and seminars dealing with enrichment activities, the gifted child and computers. The dates are Friday evening, 1983 03 11 and Saturday morning 1983 03 12. The cost is $2.00 for members and $3.00 for non-members. Contact Julian D'Angela. Primary, Junior, Intermediate and Senior sessions will be provided.

NOMA

Held one of its most successful conferences in Sudbury. About 150 teachers attended with a majority being from the elementary panel. Plans for holding the conference at the same location next year are already in progress.

PINE RIDGE	is planning an evening of activities on 1983 04 05 and a full day of activities on 1983 06 27. This chapter also publishes the Durham Math News Letter twice a year.
QSLMA	held their fall meeting on 1982 10 25 at They are planning their next meeting for Wednesday, 1983 04 06 at R.M.C. in Kingston.
GVMA	is busily preparing for our Annual Conference in May and the first brochure was distributed and mailed with the November
SAME	held their mini conference at Tebor Vocational School on 1982 10 28. The executive of SAME is involved actively in assisting the Heads of Mathematics with the preparation of the mathematics program for the two professional development days scheduled for 1983 02 10-11. These workshops are open to all secondary mathematics teachers and all interested elementary teachers.
TORNY	was actively involved with their function on 1982 10 16 (meeting of O.A.M.E. Council) and therefore no formal report was presented.

CALENDAR OF EVENTS

The following conferences are sponsored by the O.A.M.E.

SAME	Dates: 1983 02 10-11 Place: Scarborough Contact: Iris Brown
GOLDEN SECTION:	Dates: Friday evening, 1983 03 11 Saturday evening, 1983 03 12 Place: McMaster University
PINE RIDGE:	(1) Date: Tuesday evening, 1983 04 05 (2) Date: Monday, 1983 06 27 Contact: Kumar Pasricha
OSMLA	Date: Wednesday, 1983 04 06 Place: RMC, Kingston Contact: Elaine Harvey

NOTICE TO AUTHORS

Articles submitted for publication should be typewritten or reproduced from type written copy, double-spaced on 8½'' x 11'' paper with wide margins, and on one side of the paper. Figures and diagrams are to be drawn on a separate sheet of paper in india ink. Authors should take the greatest possible care in preparing the original manuscript. Two copies should be submitted to the editor and one kept by the author as protection against loss. The authors full address and affiliation must appear at the end of the manuscript.

NOTICE AUX AUTEURS

Les articles proposés pour publication doivent être dactylographiés ou photocopiés seulement dans une face, á double interligne, sur des feuilles 8½'' x 11'' et avec larges marges. Les figures et les diagrammes doivent etre faites dans un papier séparé et à l'encre de chine. Prière de préparer soigneusement le manuscrit original. Deus copies doivent être soumis a l'éditeur et une restera avec l'auteur. L'adresse et l'affiliation de l'auteur doivent etre indiquées dans l'article.

Editorial Correspondence and Articles:
to -

H.P. Heinig
Department of Mathematical Sciences

SUBSCRIPTION RATES

Members of O.A.M.E./A.O.E.M. receive the *Gazette* and *Abacus* free. Single copies $3.50; one year $10.00.

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