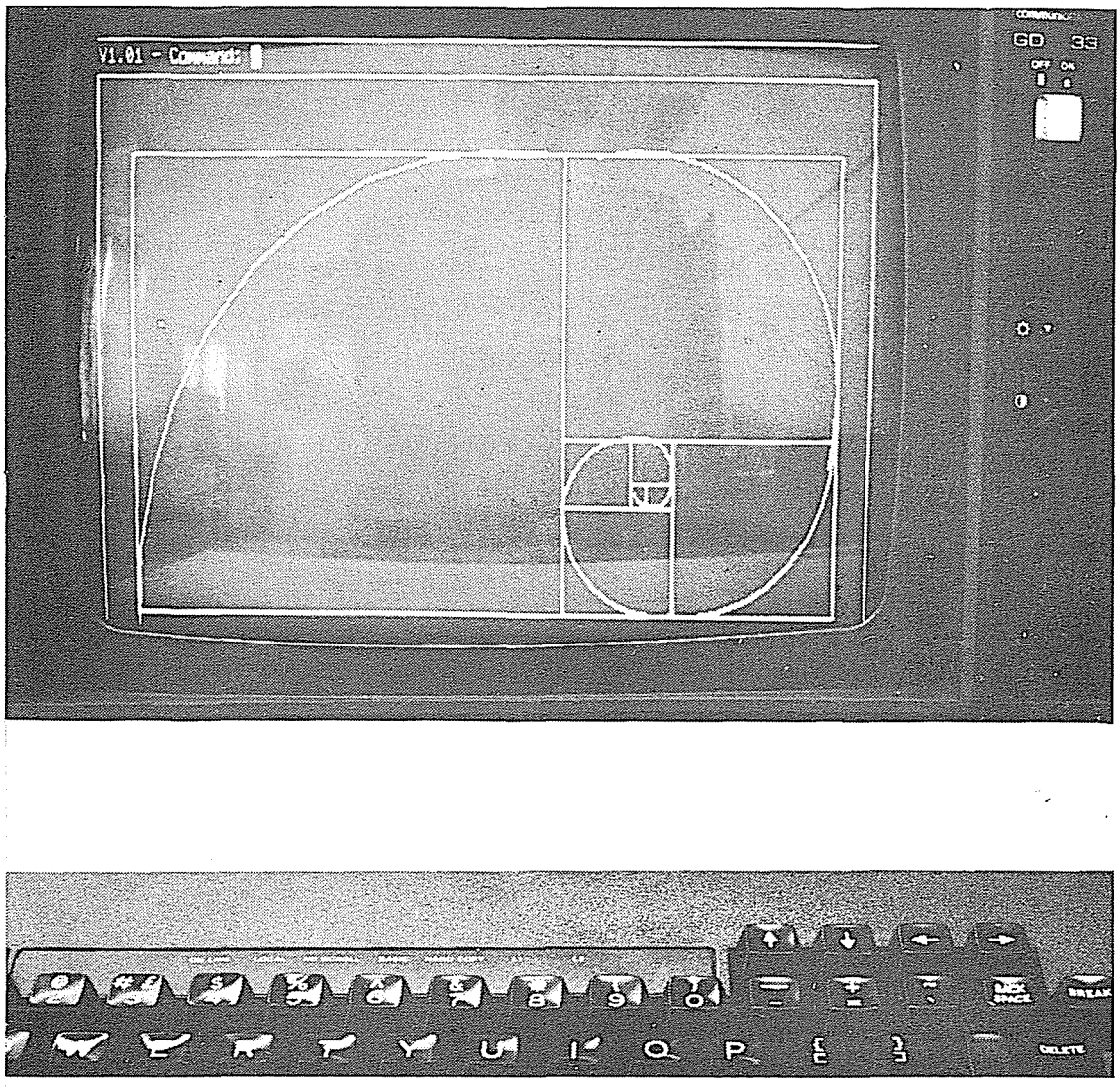
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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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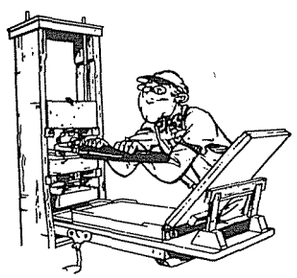
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In his "parting" editorial, Hans Heinig observed that the editor(s) "need the support of the community of mathematics educators in Ontario" to function effectively. As we begin our term as editors, we give a resounding "Hooray" to that sentiment.

In addition, we are struck with the "let's try something new" syndrome; a compulsion that seems to be generated whenever one assumes a new responsibility. This need may be self-imposed. More likely it is a way of, hopefully, generating new and continued interest in the OMG in the target milieu.

Some changes will be quite obvious; others, subtle. We hope the cover design has caught your attention. It suggests that applications of computer technology can no longer be ignored and perhaps should be integrated with the teaching techniques of traditional and not so traditional mathematics. In a subtle way, the design also suggests that this technology lacks a desired clarity of definition in the output; something we teachers can still do better.

We are particularly concerned with the dearth of submitted articles aimed at the Primary and Junior levels. While both of us feel very strongly that the **OMG** should strive for a balance of articles across the mathematics curriculum, clearly we can only publish what we receive. In attending various "P.D." days, OAME and other conferences, etc., it's clear that there are many Primary and Junior Level teachers with **fantastic** ideas for the teaching of mathematics. We'd sure love to receive some of them to share with readers of the **OMG.** "What can **I,** mere mortal member of **OAME,** do?" Glad you asked (and even if you didn't, we're going to tell you anyway). When you see or hear (or **have!**) ideas that would lend themselves to an **OMG** article, encourage the person to put those ideas down on paper and submit them. This applies to **all** levels, of course, and not just Primary/Junior, it just happens that the Primary and Junior levels are the least well represented at the moment.

The **OMG** is a unique journal for teachers of mathematics in Ontario. We feel it should, therefore, reflect the views and opinions of those concerned about mathematics education in Ontario. Perhaps it can be a vehicle for carrying the readers views in an open letter format to the Minister for the purpose of soliciting an open letter response from the Minister. The first of hopefully a series of letters appears in this issue and addresses some of the concerns surrounding the activities of the "new" Intermediate/Senior Division Mathematics Revision Committee.

As we begin our term, we know we'll be making mistakes. We encourage your comments and suggestions. The **OAME** Publications Committee will be preparing a "Reader Survey" which will be included in a future issue of the **OMG,** but your input is welcome anytime.

Many people have worked very hard in the past to make the **OMG** the excellent journal it has become. We hope we can continue to make it prosper and grow, and with your help we're confident that it will.

Ralph Connelly
Sam Pravica



PRESIDENT'S MESSAGE

by
Dave Davidson

It was 1973 - Saturday, May 12, 9:30 a.m. "A Time for Decision" - the decision was made and O.A.M.E. was born in North Bay on that famous weekend with Dr. David Alexander serving as our first president in the 1973-74 year. Ten years have passed, many changes have taken place and we have seen much growth in our organization. I would like to take this opportunity to say thank you to the members of our association for allowing me to serve as your eleventh president. I shall do my best to live up to the trust you have placed in me.

A very special vote of thanks must go to our previous Gazette editor, Hans Heinig, for the excellent quality journal with which he has provided us over the past two years. Many thanks to you and your committee, Hans. And now to put a plug in for Ralph and Sam, our new editors. There are many talented members in this Association and these two gentlemen are ready and willing to receive your articles for publication. Please share your talent and ideas with others. It will make our editors' job much easier and give you the satisfaction of having served better the cause of Mathematics across the province.

By the time you receive this publication, the year will be well under way. We are hoping to raise our membership to 1,500 this year. With your help we can do it. Let your colleagues know what OAME/AOEM is and what it does. Sure, we have a convention and they'll enjoy the hospitality but OAME/AOEM is much more than that. Our conventions boast some of the best presentations seen anywhere and our journals, the Gazette and Abacus are second to none. Don't forget to mention the Leadership Seminar held in October and our various Chapter meetings. And when asked, "What do I get for my $15.00?" you can say, "It's not what OAME can do for you but what you can do for OAME". When you participate in this way, all of us will benefit.

This year we hope to get more publicity for our organization. We have done and will continue to do much to improve mathematics education from K to Post-Secondary across the province. Let's take the time to tell our colleagues and the public what we are doing. Our Public Relations/Publicity Committee is working hard in this regard.

Our Gazette will have a new feature later this year - Colour pages. The Publications committee will be making a survey to see if you, the members are getting the kind of publications you wish. Please take time to respond.

By October we should have a position paper on Bill 82. The Senior Guidelines Committee will continue to provide input to the Ministry of Education on the revision of the 7-12/O.A.C.'s Mathematics Curriculum.

At the request of our Computer Committee I have recently written to Lorne Smith, who is chairman of the Computers in Education group to request that a member of our council be on the Ministry Committee that will decide what future software projects will be funded. As of this date, I have received no written reply but am homeful that we can make our mark on this project. We need your assistance. Write to me or any council member providing us with any input you can.

The Problem Solving Committee has been studying problem solving for children with difficult and different learning styles. We have also initiated a committee to investigate whether in fact there is a serious Mathematics Teacher shortage in Ontario. Hopefully we can gather sufficient information in order to make a report by our May Convention which will be held in Kingston this year.

Hope you had a good summer and that a great year lies ahead. Please become involved, submit an article for publication and introduce a friend to OAME/AOEM.

COMPUTERS - CABBAGES OR KINGS?

by
Alex Norrie
Peel Board of Education, Mississauga, Ontario

This discussion is not presented from the viewpoint of a computer expert or from the viewpoint that I am against computers in education. Clearly computers have a role to play in the educational system.

However, the question of what is 'computer literacy' at least has a tenuous answer. In my thinking a definition from a recent article about Family Studies serves our purposes. True computer literacy implies knowing when it is appropriate to make use of computers and computing based on one's sense of intuition, values and judgements.[1].

The question of whether computers are cabbages or kings still remains. We know that cabbages are "a kind of cultivated vegetable with round heart or head used mainly for boiling." While kings are "male sovereigns especially hereditary - rulers of independent states." Let's look at some descriptors of each:

Cabbages	**Kings**
- need slow cooking on a back burner with lots of ventilation	- on occasion rule by divine right
- are great as a-wrap-around for various tasty meat fillings	- rise above the common man/woman
- are great in coleslaw only if the correct dressings and spices are added	

There seems to be little or no intersection between the two sets. So are computers cabbages or kings? Maybe they are simply one of the madhatters in the wonderland of education - you have to decide!

To raise more questions and probably find few answers, I would like to explore five issues related to computers and education. Let me set the scene for further discussion.

Our culture is "O.D."ing on change, particularly technological change. We are shifting in the 1980's to a post-industrial economy. By 1990 more than 50% of our labour force will be directly involved with the information processing sector.

The entry of new technologies will mean the creation of new jobs related to:

- telecommunications/information processing	- automation/robotics
- materials/synthetics substitutes	- alternative energy/propulsion
- biophysics/genetic engineering/life extension	- laser/holography
- bionic electronics	- recycling: natural/human/fiscal/technical information[2]

Mathematics and students fit into this scenerio like this:

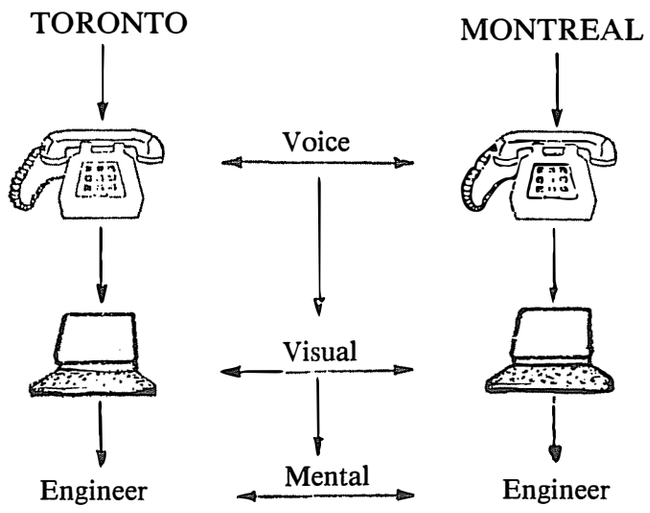
Students without core skills - primarily able to do some arithmetic	Limited opportunity - potential for low level jobs
Core students - a balance of arithmetic, geometry and measurement	Employment and opportunities are predictable, further education is open
Beyond core students - higher level skills, good problem solvers, a desire to learn	Potential leaders - employment and educational choices will - continue to increase

I realize that I am in a minority when I suggest that the final part of this scenerio has to be that computers will not revolutionize education. Also one could suggest that we are not doing the correct things with the computers we now have in education.

There are two main reasons for this last comment. Educational systems are notorious for resisting change. Our institutions are numbered among the most change resistant components in society. Also past technological trends such as television and teaching machines have made little impact on the teaching of mathematics. Yes, even calculators have created very little change. Ask yourself how you have changed your teaching to absorb electronic calculators?

The first issue to discuss is whether or not schools ought to try to keep pace with technological change. I would contend that elementary and secondary schools will find it impossible to keep up with business and society and in fact we shouldn't try to keep pace.

An example of what is out in the world of business came to me recently when I was having lunch in a restaurant. Several business men were at a nearby table and were discussing a project that was six months ahead of schedule.



For awhile they had been able to link offices in Toronto and Montreal by voice and visual using telephones and micro-computers. As I understand it the break through that was six months ahead of schedule was a communication link which allowed either engineer to change a computerized diagram and the change would instantaneously appear on the other computer screen. This is not 2001, it is reality and in my schools the computer people are still recommending Commodore Pets.

We cannot keep up with technological change, the best we can do is teach students the learning and literacy skills necessary to handle rapid technological change. Teaching programming as a main objective will not achieve this goal.

Teacher training is another issue of concern. One of the current educational bandwagons in the Province centres around computer courses for Ministry certification and Board sponsored computer courses mainly for elementary teachers.

Consider the possibilities! A grade 4 or 5 teacher at the Superintendent's suggestion takes a short computer course. At the end of the course the teacher can keyboard enough to access one kind of computer and has written one short program.

Back at the school there is one computer in the learning resource centre and the teacher has 30 children in the classroom. When the children were surveyed 25 indicated that they had a micro-computer at home, 10 children had a second machine at the cottage and 5 children could use a mainframe computer at an industrial location. At a school several blocks away in a grade 4 class only one child had a cheap micro-computer.

Where are the courses to help the teachers manage the parameters of these kinds of social situations?

Issue #3 occurs when we realize that many of the students particularly those in our secondary schools are talented in computers and technology far beyond the current computer courses and teachers of those courses.

Consider the student who is 17 years of age. He owns a micro-computer complete with disk drives and printer. His computer has been rebuilt and upgraded twice in the last 18 months using new components obtained by computer club members. This club includes engineers, lawyers and a police inspector. All members regularly use several computer languages and are also skilled enough in technology to by-pass regular computer functions and manipulate the chips in the machine.

What does his school offer? Two credits in computer science which are essentially programming courses taught by mathematics teachers using a Pet computer.

There is very little that this secondary school can offer for students such as the one described above. He got his last computer credit for setting up all the administrative computer programs for the school. The revolution won't come from computer's but rather from our students.

Within a short period of maybe 5 years a computer literacy course in grade 9 or 10 will probably be redundant. Also many of the proposed objectives for senior computer courses will be obsolete before the final guideline is printed. New crystal and light-wave technology from the National Research Council already outdates the technology courses.

When the issue of computers in the elementary school arises most educators admit that no one yet knows the best way to use computers in the learning process.

Jean Piaget presents strong evidence for giving children up to age 11 and beyond systematic opportunities to experiment and discover mathematical principles.[3] He further suggests that the lack of success in mathematics is due to the approach not the subject. Teachers continue to insist on children learning with words or symbols instead of using concrete materials.

How then should we use the computer to teach mathematics? Information on a computer screen can never be concrete. Computer processed information is not reality, it is an abstraction of reality. In fact, the computer is a metaphor through which we conceptualize reality.[4]

Even Papert with his "Mindstorms" comments may have misinterpreted Piaget because in transferring his turtle from the Kindergarten floor to the computer screen he leapfrogs some very important developmental stages.

My final issue for discussion arises from the fact that computers need software to produce symbols for learner interaction. Aside from one or two secondary school graphing programs and the odd logic program like Apples-Maptown, I haven't seen any software that does anything that can't be done with paper and pencil. However, "logo-type" languages do have some potential for creativity.

Again I would suggest that we should use computers to teach appropriate mathematics and we should use good software when it becomes available.

Here are some guidelines for checking software:[5]

GUIDELINES FOR CHECKING SOFTWARE

1. Students control the pace, level and content.
2. Each answer does not require bells and whistles.
3. Appropriate directions for grade level.
4. Too much text is a no - no!
5. Does the program have flexible application - more than one grade level - more than one subject application.
6. Is there a good management component?
7. Does the program meet the stated objectives?
8. Is the program an effective learning media or would another approach be more productive?

Once you have the software and access to a computer use this checklist to get a good mathematics program with a positive computer component.

A Checklist for Using Computers in the Classroom

How often did computers get students to ...

1. remember specific ideas or details
2. present and explain an idea or point of view
3. create novel and imaginative designs
4. generate and organize unique solutions to problems
5. classify and explain
6. apply previously acquired knowledge to new problems
7. compare or contrast two or more different situations
8. predict outcomes logically
9. ask relevant questions
10. evaluate and logically discuss a situation

Whether computers are cabbages or kings remains for you to answer. We must however reflect on the issues. The computer should never rule the teaching of mathematics by divine right and computers will only benefit learning when the correct dressings and spices are added.

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CIRCULARITY IN SOLVING A WORK PROBLEM

by
Gerald R. Rising and Dipendra N. Bhattacharya
Suny at Buffalo

A classroom dialog:

Teacher: All right, class, turn in your books to page 427 and let's try exercise 1. **(Reads:)** "Find two numbers whose sum is 30 and whose product is 216." Doris, how do we proceed?

Doris: Let the numbers be x and y. We can get the equations:

$$x + y = 30$$
$$xy = 216$$

(The teacher records these on the chalkboard.)

Teacher: Good start, Doris. Now to solve, Sam?

Sam: In the second equation $y = 216/x$. Substitute that for x in the first equation.

Teacher: Fine, Sam. **(Writes:**

$$y = 216/x$$
$$x + 216/x = 30)$$

And then, Mary?

Mary: Multiply the last equation by x and you get

$$x^2 + 216 = 30x$$ **(Teacher records this on board.)**

Teacher: Good, Mary. Now, Jane?

Jane: That's a quadratic so we rewrite it

$$x^2 - 30x + 216 = 0$$ **(Teacher records this.)**

Set two parentheses equal to zero with x in the front of each. **(Since this choice language carries meaning in the class, the teacher writes:**

$$(x \quad) \quad (x \quad) = 0.$$

Teacher: Good Jane. Now, Billy.

Billy: Let's see. We need two numbers whose sum is 30 and whose product is 216 ...

Cut. Hold it. Wait a minute! Haven't we seen that question before? Indeed we have. In fact this dialog begins and "ends" (although clearly it would continue smoothly if we had not cut it off) with exactly the same question!

In how many thousands of classrooms does such a dialog occur, with students and teachers lulled into a sense of progress by the **formalization** of the question into algebraic terms.

Bob Wirtz has made a related point in a series of pattern exercises he gives to middle school youngsters. He starts by writing on the chalkboard:

7	*		
2	8	11	4
5	3	7	4
10	-		

Then he points at the position marked * and asks what students think should go there. Many will agree 11. He writes that in place of *. Next students arrive at 24 for ___, and so on. Now with the students all alert to the pattern he adds exercices.

7	11	18	8	10			19		30
2	8	11	4		15	13		and	
5	3	7	4	6			19	finally	
10	24	77	16		45	52			216

At the end of this sequence these students **who have studied no algebra at all** attack our old friend -- sum 30, product 216 -- directly and solve it with relative ease.

Wirtz' point, I believe, is that we often confuse simple straightforward mathematical ideas with language and notation. We agree with him on this entirely, but our concern in this paper is slightly different. We urge all teachers to include **first** in their list of equation solution techniques

by inspection.

If a student can look at an equation or a system of equations and produce an answer or a set of answers he should be rewarded. (We note, of course, that we do not include here "by inspection of his neighbor's work!")

There are other perhaps more persuasive reasons for encouraging this first step in equation solution. What we have persisted in calling "the check" for a solution found by other means is in fact **the** acid test for an acceptable value. A value that does not check is unacceptable no matter what the procedure that led to that value.

Consider, for example, how we might attack a simple linear equation

$$3x + 2 = 11$$

The standard attack on this exercise is to develop (by one means or another) a series of simpler equivalent equations, in this case usually

$$3x = 9$$

and then

$$x = 3$$

Most students and teachers call this series of steps "the solution." We do not adopt the purist's posture that this is not a solution at all for indeed among other things it does not solve the student's problem - that of satisfying his teacher that he has performed satisfactorily. But we do suggest that only

$$x = 3$$

is the solution to the equation and that the direct test by substitution in the original equation (or check) is the means for determining if this result is indeed correct.

In fact we all know the problems that arise later when students "lose" roots in equations like

$$x^2 = x$$

by dividing each side by x, or "gain" roots in equations like

$$x = -3$$

by squaring. Now, that check that was an extra throw-in earlier becomes an important part of the solving procedure. But a part that our earlier careless pedagogy leads them to miss. Since the check earlier in the learning sequence was always an extra duplicative task, students continue to see it as superfluous now when it is needed. With direct trial up front this is not so much the case.

Note that we do not claim that the original equation cannot be solved by direct means that avoid circularity. In fact

$$x + y = 30$$
$$xy = 216$$

may be solved by the following means

$x - y = \pm \sqrt{(x + y)^2 - 4xy}$ from the identity
$(x - y)^2 + 4xy = (x + y)^2$
$x - y = \pm \sqrt{(30)^2 - 4(216)} = \sqrt{36} = \pm 6$

Now

$$x + y = 30 \qquad x + y = 30$$
$$\text{and}$$
$$x - y = 6 \qquad x - y = -6$$

yields the solutions by straightforward means (addition and subtraction, for example).

The original equation may also be solved by completing the square

$x^2 - 30x + 216 = 0$
$x^2 - 30x + 225 = 9$
$x - 15 = \pm 3$
$x = 18$ or 12.

or by the Hindu (bombardment) method:

$x^2 - 30x + 216 = 0$
$4x^2 - 120x = -864$ (multiply by 4a)
$4x^2 - 120x + 900 = 36$ (add b^2)
$2x - 30 = \pm 6$ (square root)
$2x = 36$ or 24
$x = 18$ or 12.

In either case the solution circularity is avoided, but in these cases the complications of technique are beyond the level of instruction usually - and quite rightly, we believe - employed at this early stage of algebra instruction. (The method of completing the square is, of course, of major importance later.)

Too often we teachers who make such an issue of circular reasoning in our geometry classes, fall into exactly the same trap in our own pedagogy. We have suggested here: (1) that you be on the look-out for these teaching slips, and (2) that you adopt a positive attitude toward "solution by inspection" as a way around the problem in this instance.



Did you hear of the mathematics professor who received a biology award for inventing the square root?

SQUARES ON THE RIGHT JUSTIFIED MULTIPLE TRIANGLE: DIFFERENCE PATTERNS

by
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University of Northern Iowa, Cedar Falls, Iowa

Middle school and junior high school teachers are always looking for activities which can be used to practice computational skills and to discover number patterns. We shall present several such activities on the right justified Multiple Triangle.

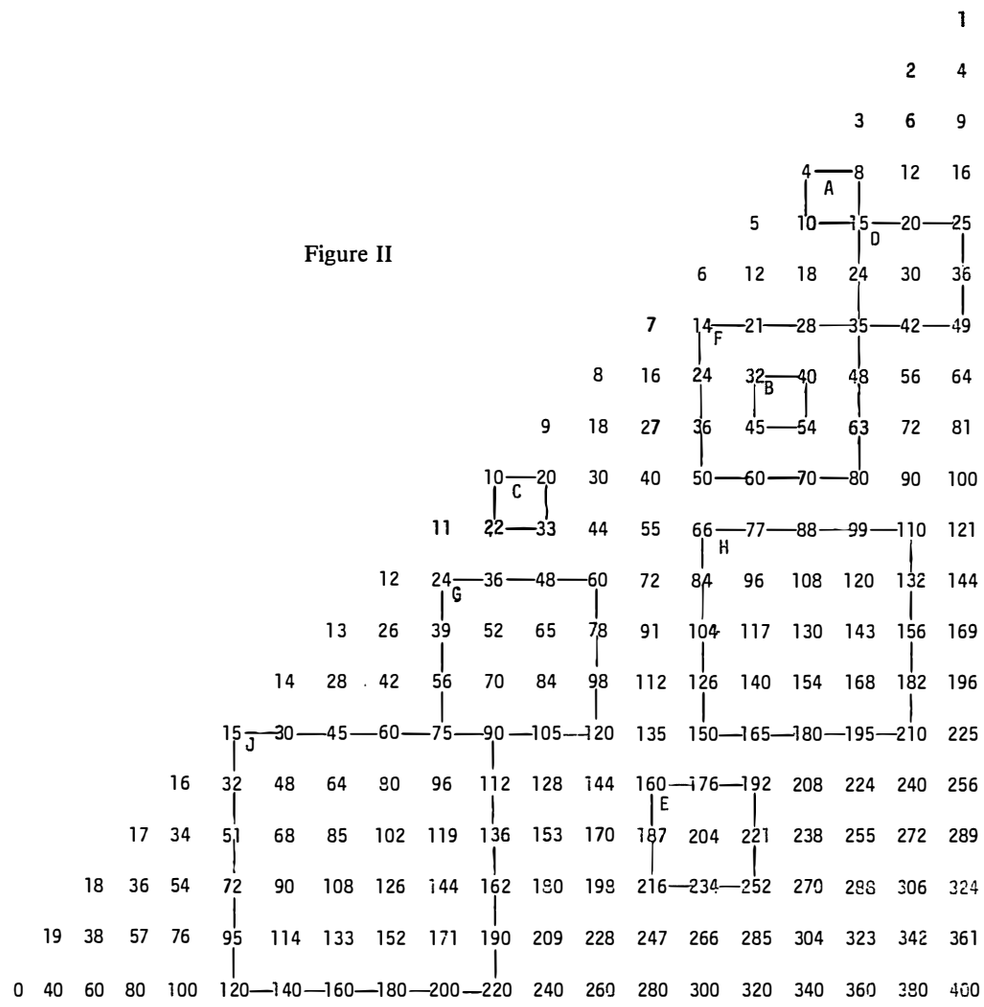
The Multiple Triangle is constructed by listing in consecutive rows the first multiple of 1, the first two multiples of 2, the first three multiples of 3, ... Figure I displays the first 20 rows of the right justified Multiple Triangle.

																		1
																	2	4
															3	6	9	
													4	8	12	16		
											5	10	15	20	25			
										6	12	18	24	30	36			
									7	14	21	28	35	42	49			
								8	16	24	32	40	48	56	64			
							9	18	27	36	45	54	63	72	81			
						10	20	30	40	50	60	70	80	90	100			
					11	22	33	44	55	66	77	88	99	110	121			
				12	24	36	48	60	72	84	96	108	120	132	144			
			13	26	39	52	65	78	91	104	117	130	143	156	169			
		14	28	42	56	70	84	98	112	126	140	154	168	182	196			
	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225			
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256			
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289		
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361
0	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	390	400

Figure I

On Figure II, squares of various sizes have been drawn. Observe that the sides of the squares lie on the rows and columns of the triange.

Figure II



Call squares A, B, and C 1 by 1 squares since each one has one unit per side. Similarly, squares D and E are 2 by 2 squares, F and G are 3 by 3 squares, H is a 4 by 4 square, and J is a 5 by 5 square.

Activity I

Using the squares of Figure II, find the sum of each pair of opposite vertices. For A, 4 + 15 = 19 and 8 + 10 = 18. Note that the difference is 1. For B, the sums are 86 and 85; the difference is also 1. For C, the sums are 43 and 42; again the difference is 1.

For each of the remaining squares, find the difference between the sums of the opposite vertices. Table I shows the results of the computations.

Table I

Square	Size	Sums of Opposite Pairs of Vertices	Difference
A	1 by 1	19 and 18	1
B	1 by 1	86 and 85	1
C	1 by 1	43 and 42	1
D	2 by 2	64 and 60	4
E	2 by 2	412 and 408	4
F	3 by 3	94 and 85	9
G	3 by 3	144 and 135	9
H	4 by 4	276 and 260	16
J	5 by 5	235 and 210	25

A pattern emerges. For the examples given, an n by n square has a "sum-difference" of n^2. Draw other squares of various sizes and positions, and verify that this pattern holds. You may wish to extend the triangle by adding the 21st row, 22nd row, etc.

Activity II

For each square of Figure II, find the product of each pair of opposite vertices. In square D, 15 x 49 = 735 and 35 x 25 = 875; these differ by 140. Table II displays the product-difference computations for all the squares.

Table II

Square	Size	Product of Opposite Pairs of Vertices	Difference
A	1 by 1	80 and 60	20
B	1 by 1	1800 and 1728	72
C	1 by 1	440 and 330	110
D	2 by 2	875 and 735	140
E	2 by 2	41 472 and 40 320	1152
F	3 by 3	1750 and 1120	630
G	3 by 3	4500 and 2880	1620
H	4 by 4	16 500 and 13 860	2640
I	5 by 5	10 800 and 3300	7500

Upon first inspection, there seems to be no pattern; however, let us generate patterns by considering special sliding sequences of squares. Figure III contains a set of 1 by 1 squares (a, b, c, d, e, and f) which are in successive vertical positions in the same two columns.

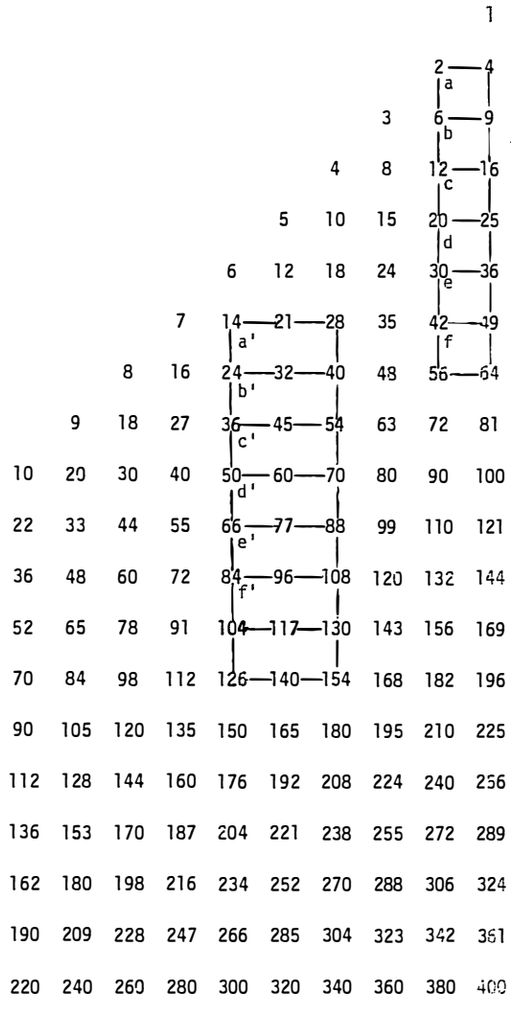
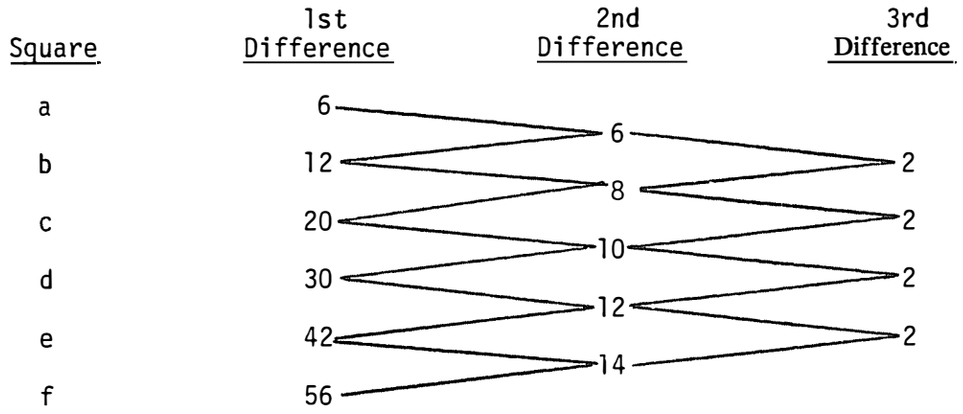


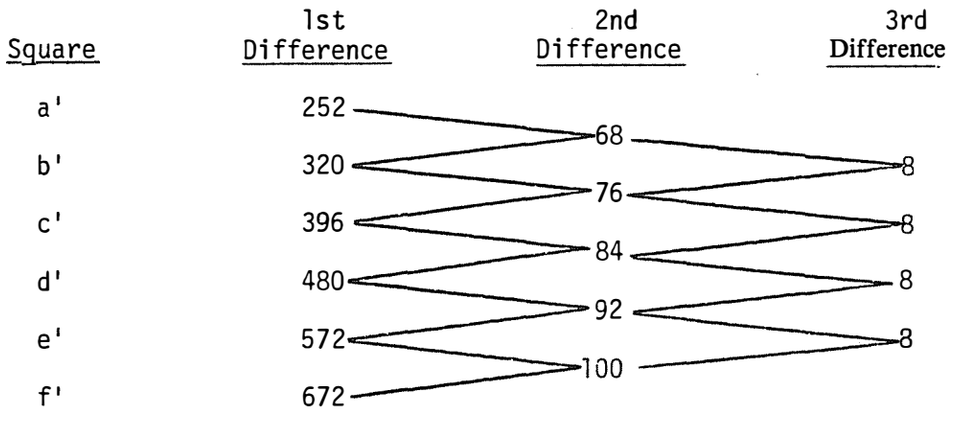
Figure III

Find the difference between the products of the opposite pairs of vertices. They are as follows:



Verify that the third set of differences is the constant 2 when a vertical series of 1 by 1 squares is drawn anywhere on the triangle.

This pattern may be extended by sliding a vertical series of n by n squares. For example, a', b', ..., f' represent a vertical sliding of 2 by 2 squares. Again find the product-differences.



A vertical sliding of squares of size 1 by 1, 2, by 2, 3 by 3, 4 by 4, ..., n by n, ... yields a pattern. The third differences of the products of opposite vertices are 2, 8, 18, 32, ..., or $2 \cdot 1^2$, $2 \cdot 2^2$, $2 \cdot 3^2$, $2 \cdot 4^2$, ..., $2 \cdot n^2$, ... Observe that this is twice the sequence of first differences of Activity I.

Figure IV is similar to Figure II except that the sliding of squares is horizontal. Again, find the product differences.

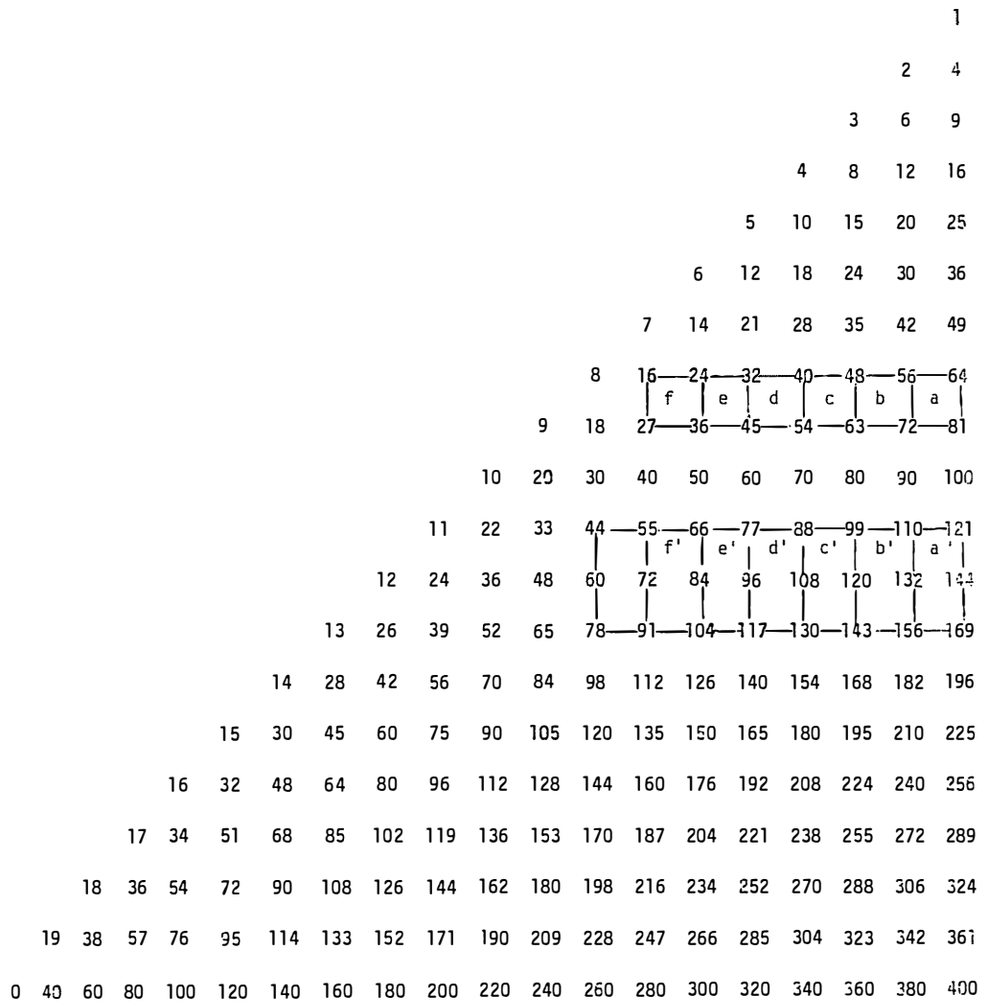


Figure IV

For 1 by 1 squares:

Square	1st Difference	2nd Difference
a	72	
		0
b	72	
		0
c	72	
		0
d	72	
		0
e	72	
		0
f	72	

For 2 by 2 squares:

Square	1st Difference	2nd Difference
a'	572	
		0
b'	572	
		0
c'	572	
		0
d'	572	
		0
e'	572	
		0
f'	572	

Observe that the second differences are all 0. If a series of n by n sliding squares were drawn at different places on the triangle, the first differences would be a constant and the second set of differences would be 0. Verify this.

Figure V depicts a series of 1 by 1 and 2 by 2 squares drawn diagonally.

Find the product differences.

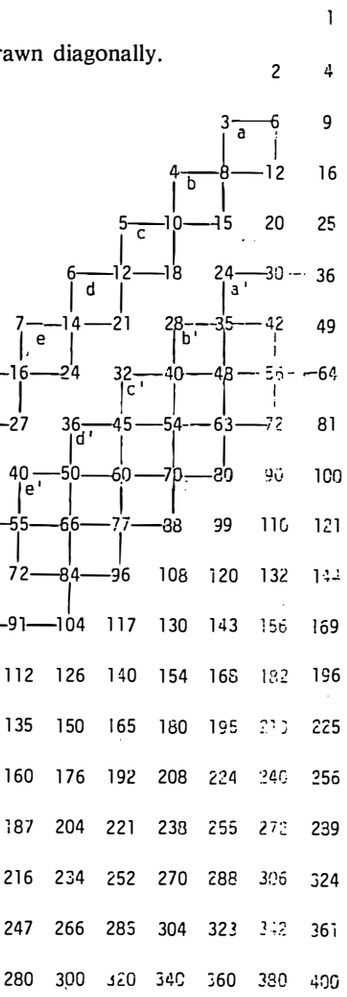
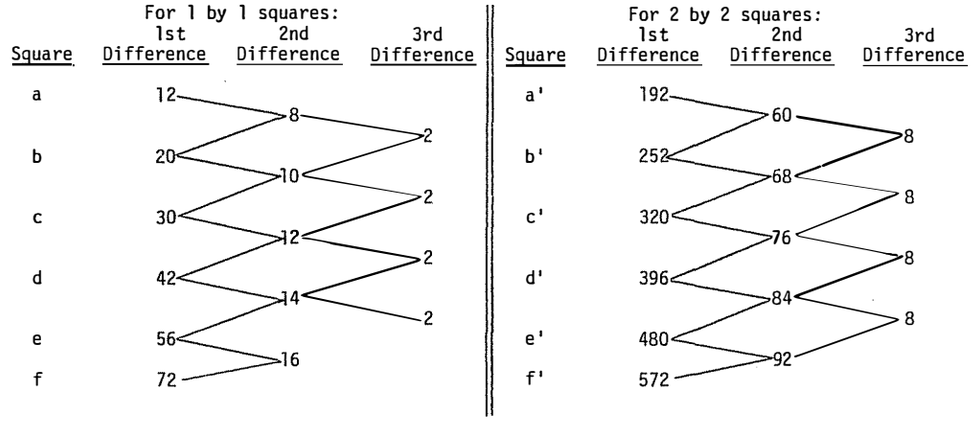


Figure V



If n by n squares are slid diagonally, the pattern generated by the product differences is: $2 \cdot 1^2$, $2 \cdot 2^2$, $2 \cdot 3^2$, ..., $2 \cdot n^2$, ... Note that the same pattern is generated if the squares are slid in the "opposite diagonal pattern."

Challenges for the reader:

1. Draw diamonds upon the right justified Multiple Triangle and look for patterns concerning sum-differences and product-differences.

2. Redraw the triangle so it is left justified as shown:

```
1
2  4
3  6   9
4  8  12  16
```

   Again look for patterns including sum-differences and product-differences for squares and diamonds.

3. Draw other numerical arrays and look for number patterns.

INNOVATION AT THE A.C.T. MATHEMATICS CENTRE

by
Dr. Neville de Mestre
Mathematics Department, Royal Military College, Duntroon, A.C.T. 2600, Australia

Early in 1977 the doors of the A.C.T. Mathematics Centre were opened to students in upper elementary school classes (ages 8 - 11) from any school in the Australian Capital Territory, or from any school in the other six states if its pupils were visiting the national capital (Canberra). Since then the age group has been extended right up to the end of high school (17 years), and more than 60 000 students have visitited this centre.

Teachers can make a booking to visit the Centre with their class for an 80 minute session of which there are three per day. During a session the children work in pairs and communicate mathematically only within their pair or to the Centre teacher (i.e. not with their class teacher). The Centre teacher hands out to each pair an attractively designed container in which there is an instruction card plus some "hands-on" material associated with a specific mathematical task.

These mathematical tasks are designed to occupy about 15 minutes each on the average. If a pair is initially successful with a task - in say 5 minutes - then they will be given a harder task next. The students take the task to a working table and when they have completed the task or when they are "stuck for further ideas" they bring the task to the Centre teacher who remains at a central desk during the whole session.

The Centre teachers, of which there are two, are vital to the whole project. They assist in task construction and have to know each task in detail so that they can draw out the mathematical concepts involved in the problem. During a session they have to know whether to give a hint so that students will not become frustrated, whether to tell some pairs to persevere, or whether to inform some students to begin trying. In these days of instant alternatives or instant answers at the back of a book the Centre is encouraging students to perceive that time and application is needed to solve most problems. The Centre teacher emphasizes to all students the importance of reading the instruction card carefully, as most children merely want to use the concrete materials without reading what is wanted.

We now have more than 400 tasks at the Centre, from which only about 30 are chosen for each session. These tasks are based on ideas gathered from books, magazines, journals, toy-shops, observation and day-to-day activities. Some of the ideas are familiar as pencil-and-paper tasks, but we have developed a new way of looking at them through the concrete or "hands-on" approach. We do not have mathematical games at the Centre because games encourage a retention of strategies, whereas we seek a sharing of strategies.

The criteria that we used in choosing suitable tasks were as follows:

(i) each task should isolate one particular mathematical idea,
(ii) the task should be solved best of all using "hands-on" material,
(iii) the equipment must be eye-catching and durable,
(iv) the wording on the instruction card should be brief, simple, and precise,
(v) it should be possible for an average pair at that level to complete the task in about 15 minutes

As an illustration we describe some of the tasks that have been developed by the Centre teachers and myself. Others are available in the reference quoted, at the end of the article.

TASK 1 **Geometry category**

Equipment: Modelling clay, a table knife and a cutting board.

Instructions: Make the clay into a flat cake.

Use the knife to make a straight cut across the cake, but do not move the pieces apart.

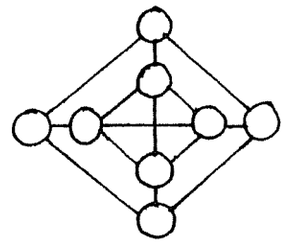
Make 2 more straight cuts anywhere across the cake to get the greatest number of pieces possible. (The pieces may be any size).

What is the largest number of pieces you can get after 4 straight cuts?

Mathematical Idea: (Primary level) A logic problem through observation.
(Secondary level) Development of a pattern using concrete materials.
(Tertiary level) Leads to different equations. (e.g. How many after n cuts?)

TASK 2 **Number category**

Equipment: The task has 9 wooden counters with the numbers 1 to 9 on them and a board painted as in the diagram.



Instructions: Place 8 of the counters on the board so that if you add the numbers

(i) around the big diamond
(ii) around the small diamond
(iii) along the straight line across the board,
(iv) along the straight line down the board

you will get 20 every time.

Mathematical Idea: There are 4 lots of pairs of numbers which add up to 10, and so 5 has to be eliminated.

TASK 3 **Number category**

Equipment: 9 rods numbered, 3, 6, 7, 9 10, 11, 13, 14, 17 plus a block with a 3 x 3 array of holes.

Instructions: Active uranium is carried in a lead box to prevent radioactivity escaping.

The rods of uranium are placed into the holes in the box.

Each rod has its radio-active strength written on it.

The level of radio-activity is dangerous if the total strength of any line, including the two diagonals, is greater than 30.

Make the box safe.

Mathematical Idea: Magic square variation.

TASK 4 **Measurement category**

Equipment: A ruler, an airline's map of Canada and a board with an outline of Canada drawn to the same scale. On this board is a number of dots representing certain cities.

Instructions: Make any measurements you like to determine the names of the cities associated with the dots.

Carefully measure the distances on the board between various assigned cities.

Calculate the actual distances using a given scale.

Mathematical Idea: Careful measurement and scale.

TASK 5 **Logic category**

Equipment: Two wooden cubes painted with blackboard paint, chalk and blackboard eraser.

Instructions: The two cubes are to be made into a calendar. One cube alone or both cubes together are to be used to show the number of any day in a month.

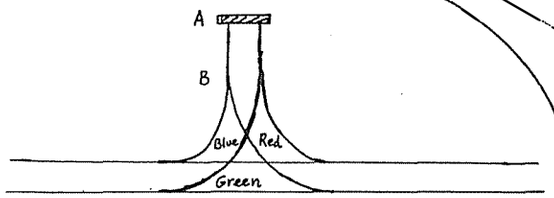
Write the numbers needed on the faces of the cubes.

Mathematical Idea: Logic associated with frequency of use.

TASK 6 **Logic category**

Equipment: Thick wooden board with grooves cut in it to represent railway tracks, an engine with wooden wheels that fit into the grooves, a blue carriage and a red carriage.

Instructions: The railway siding tracks are as shown in the diagram.



Place the blue carriage on the blue section, the red carriage on the red section and the engine on the green section.

The engine does not fit in the section AB but each carriage does. Use the engine to shunt the carriages around so that the blue carriage finishes on the red section, the red carriage finishes on the blue section, and the engine returns to the green section.

Mathematical Idea: Topology.

There are many educational advantages to be obtained from establishing a mathematics centre of this form. Here are some of them!

1. The students learn to work with others and share ideas.
2. Practice is obtained in verbalizing mathematics.
3. Because the task material is attractive and eye-catching the students commit themselves to solving the problem.
4. Problem-solving strategies are learned.
5. Students are encouraged to persevere with a task where they may experience the satisfaction of finally solving the problem.
6. Success is possible for everyone because the tasks are distributed according to ability.
7. The class teacher can observe the class in a different mathematical environment and from a different position.
8. Mathematics is an extremely suitable subject for generating 15 minute problems with concrete materials.
9. The students recognize that mathematics can be fun and stimulating.

REFERENCE

Neville de Mestre and Bea Duncan, "Sixty tasks from the ACT Maths Centre", Curriculum Development Centre, Canberra, Australia (1980).
Price: $5.95 (Aust.) plus postage.

GEOMETRY IN SCHOOL - EUCLID OR KLEIN?

by
P.R. Scott
University of Adelaide, South Australia

Editor's Note:

On first reading, it was felt that the views of the author in this article were perhaps directed toward changes in Australia's curriculum rather than Ontario's and hence not suitable for publishing in the O.M.G. But, wait a minute. Aren't we wrestling with the same problem; what to do with the present Geometry curriculum?

The paper does present some pointed arguments that touched of the occasional "pedegogical nerve". Should this be the reason for giving it a "nyet"!? We want the O.M.G. to stimulate reader reaction. It is our feeling that this article has the potential of doing that. How relevant are Scott's comment's to our curriculum problems? Do some of his comments strike your pedagogical nerve? Let's hear from you. We would be very interested in your reactions.

In recent years in South Australia there has been a serious questioning of the place of geometry in the school curriculum. Changes in syllabus have taken place, although I am not sure that all the basic issues have been resolved (or are ever likely to be!). The present paper is an attempt to think through some of these basic issues.

I have been involved in school mathematics for some years now, but two events have been particularly provoking. In 1973, while on study leave, I became a teacher, as my children did their mathematics by correspondence. I was literally appalled at some of the material which was being presented under the name "new mathematics". I may not be a bright mathematician, but I think I ought to be able to understand my children's primary mathematics! Things got to the stage where I would say to Karen, "Look, the right answer is this, but I think you are expected to write down this and this," and the answers were seldom the same. In 1975, I noticed that the students in my third year university geometry course were particularly weak in geometry. I believe this was the first geometry class that I taught, which had come through under "the new mathematics". I may be generalizing a little, but my impression was that this class knew no Euclidean geometry. For example, corresponding angles and any relationship to parallel lines were quite unknown. A tutorial question about concyclic points remained unanswered - what does 'concyclic' mean? Clearly the basic geometrical facts which used to be learned and retained, which became second nature to us, were no longer being passed on. The unfortunate thing about this class was that they know no transformation geometry either; they could not tell me even the most fundamental things about reflections, translations, similarities. That is, these top mathematics students had retained practically nothing of their school geometry. Of particular interest and contrast was a weak but older student who began the course with a significant advantage because of his background knowledge.

These then are two reasons why I became concerned about the state of the new mathematics, and in particular about the state of geometry.

We all know that there have been tremendous changes brought about in recent years in mathematics, and in particular in geometry. It seems to me that a lot of this change has been introduced by *academics*. These academics produce marvellous, polished mathematics which cannot be flawed; unfortunately it has one drawback, and that is it is unteachable. Another group involved in establishing the new courses is made up of *teachers* and *educators*. (I have a particular dislike of the pseudo-educator, who pretends to know everything about everything, but who in fact knows nothing about anything!) I think it is probably fair to say that many in this group have no deep insight into where the mathematics is leading, or what the deeper points are. So from this group of people we get very teachable material which may or may not be mathematics. Wouldn't it be ideal if these two groups of people could get together! It does occasionally happen, but rather infrequently, I think, in Australia.

Whoever was to blame, the new courses came out; the changes in the curriculum were made. There were big changes in geometry, and the old course on Euclidean geometry was replaced by something else which tended to be mainly transformation geometry. Let us begin with a brief survey of Euclidean geometry - its good points and its bad points.

EUCLIDEAN GEOMETRY

Good points:	Bad points:
DEDUCTIVE	INFLEXIBLE
VISUAL	UNTEACHABLE
EXPERIMENTAL	NOT EXAMINABLE
ESTABLISHED TRADITION	NOT RIGOROUS
PRACTICAL	
CHALLENGING	

What are the good points in Euclidean geometry? The first thing we can say is that Euclidean geometry is a *deductive system.* Perhaps this is its best mathematical attribute. We are given certain facts; from these we deduce, using logical reasoning, certain other facts. My own view is that Euclidean geometry is the best means for teaching this sort of reasoning. You may disagree with that, but I know of no other part of mathematics which has shown the same possibilities for teaching deduction.

Secondly, Euclidean geometry is a *visual* discipline. It has to do with points, lines and planes. I would like to return to this important point later, because the visual aspect is common to most branches of geometry.

Euclidean geometry is also *experimental.* It involves the student in doing things - well, it didn't when I was at school, but it could have. Did you know that Durell's *Elementary Geometry,* printed in 1925, has a large number of oral exercises which are extremely practical? "Take your ruler. Measure the distance from the wall to your desk. Measure the height of your desk. What angle does the wall make with the floor?" If he had called them "discovery exercises", he would be right up to date! As I remember, my teacher always ignored the oral exercises, so this whole aspect of geometry was lost.

We can say something too about the *established tradition* of Euclidean geometry. We know that this geometry consists of a certain body of factual material. There is a fixed sequence of presentation, and while this has its drawbacks, at least as teachers we knew *what* we were supposed to be teaching, if not *why*! Thus there is a fixed body of material which constitutes the subject.

As well as this, we know that Euclidean geometry is a *practical* subject, in the sense that it is the geometry of the real world. It might even have real applications (!), and it does apply to real life situations.

Finally, we can say that Euclidean geometry is *challenging.* Many of us will remember the exercises or riders which followed a given theorem, the challenge of spending time on them, and the achievement (perhaps!) of getting some of them out.

You may be able to think of other good points, but let us now consider some bad points.

First of all, it cannot be denied that Euclidean geometry is certainly rather *inflexible.* Because of the fixed body of material and the fixed sequence of presentation, there is not much room for variation in a course. It is not easy, for example, to make allowance for the weaker student.

Some people claim that Euclidean geometry is *unteachable;* this presents teachers with a real problem if it is true? To what percentage of students is it unteachable? Is it the subject matter or the presentation which is at fault? And if the top 25%, 30% or even 35% of students enjoyed Euclidean geometry and profited from it, might it not have been possible for them to continue with the course? My idealism coming through!

Another bad point is that Euclidean geometry appeared to be *non-examinable.* We all know that if a student sat for an examination in Euclidean geometry, then he got either 0 or 100, and it was very hard to give gradations between these two. One of the problems here was the time factor - we seem to need more time for an examination in Euclidean geometry. And this raises a most interesting question. Of all the parts of mathematics which we teach, why is it that this claim on non-examinability applies only to Euclidean geometry? Is there in fact something unique here which was being tested, something which is not tested in any other part of mathematics?

Finally, we get the accusation that Euclidean geometry is *not rigorous.* This is certainly true - we require something like 16 to 23 axioms to present the subject rigorously. We are thus subject to the valid criticism that we leave out axioms, use incomplete arguments, and so on. I think myself, that the criticism is irrelevant; if we can teach sound principles of logical reasoning to our students, then the sophistication and depth of argument being queried here can be filled in later.

There may be some other bad points, but notice that except for the question of rigour, the listed bad points all have to do with teachability. That is, Euclidean geometry has been largely thrown out of the school for pedagogical and not mathematical reasons. And we have substituted in its place transformation geometry, which is highly teachable. We have eliminated the defects of the Euclidean course, but I fear we have not asked too many questions about the mathematical virtues of transformation geometry. Surely it is dangerous to throw out something which has been tested and tried and proved for centuries, and replace it with something new, untested, untried? The decision may be the right one, but it requires fairly careful thought.

Some pertinent questions come to mind. First, a question about *the past.* Is it true that Euclidean geometry has always been unteachable? If so, how did it come to last so long? What about our *students?* Are we teaching a different group of students now? There are certainly more of them! But my guess is that in past times it is more likely that the students were from well-to-do families, than that they were studying because of their mathematical ability. What about our *teachers* - their attitudes, their training? Is our teacher training adequate? And lastly, what about *the present*? We live in an instant everything society - instant coffee, instant tea, instant results. Are we afraid to have problems which do not have instant solutions? Perhaps there have been changes in society which have influenced the education process. And perhaps geometry itself is a victim - a hard problem, discarded because the solution was not instantly forthcoming.

Let us continue by looking at transformation geometry.

TRANSFORMATION GEOMETRY

FLEXIBLE
TEACHABLE - MODELS
- INTERESTING
- VISUAL
- EXPERIMENTAL
EXAMINABLE

We start with the good points. Transformation geometry as it is now taught is a very *flexible* subject. It does not really matter which subject material is presented; we can progress at our own rate. Again, of all parts of mathematics, especially in the hands of a good teacher, transformation geometry is probably the most *teachable.* It is a subject in which we can use models, it interests the students, it is certainly visual, and it is a subject which involves the students in experimental work -all a tremendous 'plus' for transformation geometry. Finally, in the absence of any complaints, I guess transformation geometry is *examinable* - or at least as examinable as any other part of mathematics. From the teaching point of view then we are way ahead with transformation geometry.

But surely, if transformation geometry has no mathematical virtues, then all this is a waste of time. We might as well let the kids play baseball or checkers, and have a good time, unless there is some *mathematical purpose,* some *mathematical goal,* some end. *What are we actually trying to achieve* in the teaching of transformation geometry? This is the question to which we turn now. Let us consider again some of the points made about Euclidean geometry, and see how well transformation geometry scores.

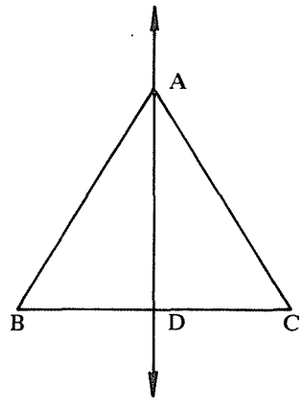
We begin with the *established tradition* mentioned earlier. Now transformation geometry does have some sort of tradition - the subject began with Klein in 1872. But in the schools there is no fixed body of material. For example, I understand that glide reflections do not form part of the New South Wales syllabus. It seems unimaginable to me that a whole quarter of the isometries can be arbitrarily deleted. The fact is that nobody is really quite sure just what should be in the course. We have the attitude that "anything goes" and as a result I fear that everything is going. The students are having a good time - it does not really matter much what they learn - besides, I am short of time for my maths - something has to go - goodbye geometry! Someone in authority needs to set down a fixed body of material so that we know exactly what it is that makes up a transformation geometry course.

Geometry (in all its forms) is the only branch of mathematics which presents a *visual aspect*. The importance of this was brought home to me some years ago by my son, aged 4 at the time. You may know that it is possible to buy jigsaws in a frame; when the pieces are tipped out, the indentation of the shapes is left behind. Now Nicholas was a whizz at doing these jigsaws - he could complete them faster than I could! When I do a jigsaw, I tend to match colours or patterns. Nicholas matched shapes. He would pick up a piece and put it in isolation in the middle where it matched the indentation. I then realized that with all my mathematical training and know-how, here was a piece of information that I was not using. (Admittedly, Nicholas got into serious trouble when he progressed to harder jigsaws without a background!) But isn't it true that in our geometry there is a visual aspect which makes the geometry not just an isolated discipline, but rather an integral part of the whole of mathematics? I consider the teaching of geometry to be important because it gives this unique insight into mathematics. If our students do not receive this, then they are the poorer throughout the whole of their mathematics.

Now we take the *non-rigorous* point. Transformation geometry does not score very well in terms of rigour. My main quibble with transformation geometry as it is currently taught, is that it is never very clear what is being assumed. For example:

Definition: *isosceles triangle* - a triangle which has an axis of symmetry,

$AB = AC$, $BD = DC$, $\measuredangle B \cong \measuredangle C$.



It is as though you get a bulk package - the whole thing arrives at once! There is no sense of deduction. In fact, it is not clear to me immediately what an axis of symmetry is, or if I happen to know, why this particular triangle has one. The starting point is quite obscure. Contrast this with Euclidean geometry where we have "Given, To Prove, Proof", and even if the proofs are incomplete, at least we have a logical basis that can be applied to the whole of mathematics.

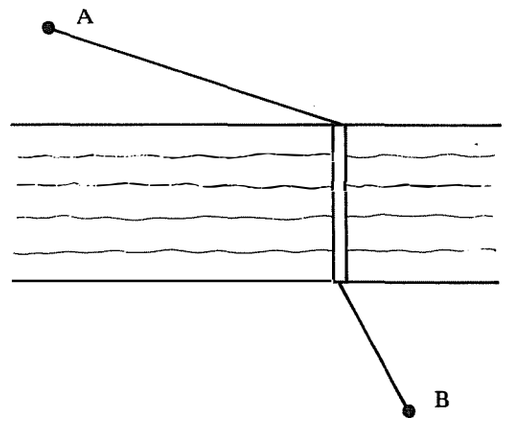
Related to this is the fact that we have very *little deduction* in transformation geometry. Obviously it is hard to deduce something from an unknown starting point. Actually it is possible to make transformation geometry rigorous and deductive. We begin with reflections and their properties and define the other isometries in terms of these. Coxeter's *Introduction to Geometry* gives an outline of this approach.

Thus it is possible to introduce deduction into transformation geometry, but again we have to decide whether it is worthwhile doing this. Notice that while we can use the transformation to prove "an isosceles triangle has various properties", it is much harder to deduce Euclidean theorems such as "$AB = AC$ implies that $\measuredangle B \cong \measuredangle C$". Such proofs are possible, but often require more ingenuity than the original Euclidean proofs. This then is the dilemma we face: we feel we must retain the traditional Euclidean theorems (and I believe there is good reason for doing so); we wish to teach transformation geometry, because it is good to teach; we are then unable to use the transformations satisfactorily to establish the Euclidean results.

Here are three examples of problems which are best suited for deductive solution using transformation geometry.

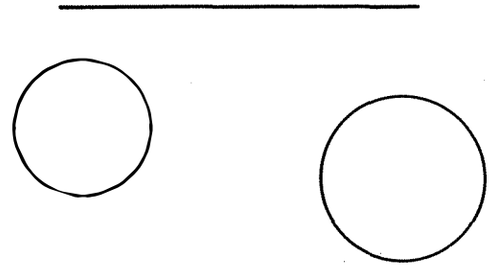
(1)

Two cities A and B are separated by a straight river. A bridge is to be built at right angles to the parallel banks. The problem is to minimize the path from A to B.



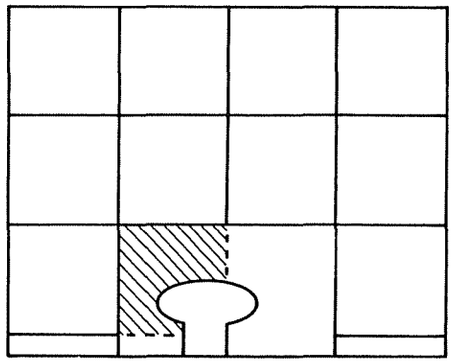
(2)

Two circles are given, and a line segment of given length and direction. The problem is to determine constructively how the line segment can be placed with an endpoint on each circle.



(3)

Square tiles are being laid on a bathroom floor. Is there an easy method for determining how to cut the tiles to fit around (say) the base of a pedestal basin?



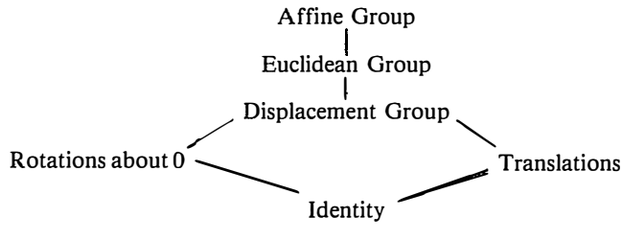
Each of these examples is a translation problem. In fact, because of its practical nature, example (3) would make a good introduction to the subject of translations. Unfortunately the selection of problems of this sort is fairly limited, and many, like example (2), are rather unmotivated.

A final question in connection with transformation geometry concerns *endpoint*. What are our goals? Where does the subject lead? I can see three profitable endpoints. The first is the study of *matrices* which should follow on naturally, sequentially from transformation. Thus, our transformations are of the form

$$x' = Ax + b$$

where A is 2 by 2 and non-singular. Surely this is an ideal and practically useful way of introducing matrices.

The second topic that should be introduced is *groups* or at least *algebraic structure*. Klein based his whole system on a certain basic group and its subgroups. For example:



The study of structure and invariance is a natural outcome of the transformation method. It makes no sense to stop short of some of the most useful applications.

The third topic is *complex numbers.* Having studied transformation, why not regard

$z \to z + a$	as a translation
$z \to \bar{z}$	as a reflection in the x - axis
$z \to az (\lvert a \rvert = 1)$	as a rotation

and so on?

SUMMARY

(1)
I am sure that there is room for *improvement:* either we have to improve the teaching of Euclidean geometry, or we have to improve the mathematics in transformation geometry. The present situation is not satisfactory.

(2)
Some sort of decision must be made. Should we teach Euclidean geometry in a new way, or should we teach transformation geometry? Perhaps we should teach both; if so, how is this to be done? Or perhaps we should teach some other sort of geometry altogether.

(3)
In order to find answers to these questions we need to have certain broad *goals.* What is the purpose of teaching geometry? What am I trying to achieve? What are the goals, the endpoints?

(4)
Finally, my own personal plea. I believe that it is vitally important that geometry be retained, because in geometry there are insights, challenges, ways of thinking which are not present in any other branch of mathematics, and our students will be much the poorer if they miss out on this viewpoint.

A NOTE ON THE IRRATIONALITY OF $\sqrt{2}$

by
Claire Zeller
Peel Board of Education, Mississauga, Ontario

Attending a conference is always worthwhile as long as one has received a few (possibly one!) good ideas or strategies. The success of the 1983 Annual Meeting of the NCTM in Detroit hinged, at least in part for me, on the following insight provided by Ivan Niven.

When demonstrating the irrationality of $\sqrt{2}$, one usually depends on some argument concerning divisibility (or evenness). More particularly, assume $\sqrt{2} \in Q$.

Then, $\sqrt{2} = \frac{a}{b}$ with a, b $\in$ I, and (a, b) = 1

and, $2 = \frac{a^2}{b^2}$ by squaring

and, $2b^2 = a^2$ by multiplying
forcing a to be even, say $a = 2p$,
then, $2b^2 = 4p^2$ by substitution
and, $b^2 = 2p^2$
forcing b to be even, say $b = 2q$,
then 2 is a factor of a and b, and (a, b) $\neq 1$
forcing a contradiction, and requiring $\sqrt{2} \in Q$

Invariable the home assignment asks for a demonstration that $\sqrt{3}$, $\sqrt{5}$, ... are also irrational. These proofs require arguments concerning the divisibility (or non-divisibility) by 3, 5, ... which are not usually known, or easily developed. Dr. Niven observed that strengthening the characterization of $\sqrt{2}$ to $\sqrt{2} = \frac{a}{b}$ with a, b $\in$ I, (a, b) = 1, and b **least** such integer can be done without loss of generalization, and eases the extension to other radicals of the same form.

For example:

assume $\sqrt{2} = \frac{a}{b}$, b least

more specifically **b is the least integer for which $\sqrt{2}$ b is also an integer**
examine: $\sqrt{2}$ b - b it is clearly less than b ($\sqrt{2}$ = 1.414 ...)
examine: $\sqrt{2}$ b - b it is clearly an integer
now, $\sqrt{2}(\sqrt{2}\,b - b) = 2b - \sqrt{2}\,b = b - a$, an integer
thus, b is not least, forcing a contradiction
and, b $\in$ Q must be concluded.

For $\sqrt{3}$, ($\sqrt{3}$ b - b) is chosen.
For $\sqrt{5}$, ($\sqrt{5}$ b - 2b) is chosen.
For $\sqrt{11}$, ($\sqrt{11}$ b - 3b) is chosen.

AND SO ON ...

FAST WAY TO GRAPH AN ELLIPSE

by

Glen A. Sherman, 3rd Year Math and Music, Queen's University

The problem of graphing a relation defined by $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is in general a lengthy task. The standard procedure involves a change of variables by trigonometric substitution to eliminate the xy term and another change of variables to eliminate the linear terms. The resulting relation is graphed and the change of variables is interpreted as a rotation and transformation to graph the original relation.

The following **special case** of the problem may be dealt with more efficiently. Suppose we wish to graph the relation defined by

(1) $$ax^2 + bxy + cy^2 = d \quad d \neq 0$$

There is a procedure which involves diagonalizing a 2 x 2 matrix but this still employs a change of variables which must be interpreted as a rotation. The following method is somewhat faster and relies on a simple geometric idea.

Returning to (1), if $b = 0$ the problem is easy (consider setting x or y equal to zero) so assume $b \neq 0$. Then the following 3 cases are known to arise.

$b^2 - 4ac < 0 \rightarrow$ ellipse if a, d have the same sign
0 otherwise

$b^2 - 4ac = 0 \rightarrow$ two parallel lines if a, d have the same sign
0 otherwise

$b^2 - 4ac > 0 \rightarrow$ hyperbola

The second case can be solved by factoring so we concentrate on the other two.

From the form of (1) we see that the origin is the center of the ellipse. [The equation is symmetric about the origin, so also is the ellipse.] To graph the ellipse it suffices to determine the four points which lie both on the ellipse and on its axes. At each such point the distance L from a point (x, y) on the ellipse to the origin is either maximized or minimized.

We can write L as a function of m, the slope of the line through (0, 0) and (x, y) using (4) below.

(2) $$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + (mx)^2} = \sqrt{\frac{d(1 + m^2)}{a + bm + cm^2}}$$

Set $\frac{dL}{dm} = 0$. After some arithmetic we get

(3) $$m^2 + \frac{2(a - c)m}{b} - 1 = 0$$

To graph the ellipse, obtain two roots m_1, m_2 from (3). These are the slopes of the axes and as expected $m_1 \cdot m_2 = -1$. For each of m_1, and M_2 obtain two points (x, y) by

(4) $$x^2 = \frac{d}{a + bm + cm^2} \text{ and}$$

(5) $$y = mx, \text{ 4 points in all.}$$

Note that (4) is just (1) with $y = mx$. The hyperbola is graphed in a similar manner except that (4) is impossible for one of the Mi. The slopes of the asymptotes are found by solving $a + bm + cm^2 = 0$ unless one of the asymptotes is the y-axis. [look at (2), what slopes m will give an L tending to ∞?].

LETTER TO THE MINISTER

by
Sam Pravica
Co-editor of O.M.G.



Dr. Bette Stephenson
Minister of Education
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The editors of the O.M.G. would like to adopt an open letter policy with your Ministry that will deal specifically with issues affecting mathematics curricula and the possible effects they may have on the teaching/learning processes in Ontario schools. The reason this is being suggested is largely due to my growing concern that curriculum changes currently underway have not been sufficiently articulated to those who will ultimately be required to implement them. As a member of a Faculty of Education, one is able to view issues from an observer's point of view, and it is clear that a conflict exists between the actions of the Ministry and how those actions are perceived (usually negatively) by members in the field. My observations are that teachers generally feel they are given little or no opportunity to be part of the decision making processes in matters that ultimately affect them. Yet, as a past member of Ministry - organized committees I am aware that the Ministry feels it does everything it can to do just the opposite. Perhaps a reason for this arises from the fact that the Ministry communiques deal with broad issues from which generalizations need to be made while teachers look for information that is subject specific. This notion is supported in the "Response to 'S.E.R.P. Discussion Paper' " by the Faculty of Education, U.W.O. where it states, ". . . that many sections of the "Discussion Paper" are simply too general to either support or reject intelligently. We recommend, consequently, that the final recommendations are accompanied by more detailed discussion and evidence of a concrete nature to support them." Perhaps, then, the open letter format will afford the Ministry an avenue to address issues that are of a concern to educators and that deal specifically with mathematics curricula.

It would seem most appropriate, then, that the first letter should deal with the most current focus affecting mathematics curricula; the activities of "The Intermediate/Senior Mathematics Curriculum Committee" chaired by Dave Alexander.

It is unfortunate that this committee began its task before the Intermediate Mathematics Guideline - 1980 was fully implemented. I commend the Minister for coming to grips with the issues of Secondary School Education in Ontario; a difficult decision had to be made but a necessary one. The Ministry is now committed to implementing the plans for educational renewal in the Secondary Schools as outlined in the R.O.S.E. (the Renewal of Secondary Education in Ontario) report. Our task as teachers of Mathematics is to help in the development and implementation of a sound curriculum guideline that clearly reflects the needs of a changing society and the spirit of ideas outlined in the report. The first step in achieving these ends is to identify a model for curriculum development and implementation. Such a model may be in existence, but unfortunately has not been made available for public scrutiny. Yet, the Ministry is very much aware of the value of a working model, ". . . planning must have two major components: the need to establish the stable, fundamental foundations on which secondary education can be based; then, the need to provide carefully connected processes by which curriculum change and renewal can occur." (ROSE p(7)).

How does the Minister propose to ensure that these two components are part and parcel of the Mathematics curriculum development model?

The R.O.S.E. report outlines a number of concerns related to meeting the needs of students whose education must prepare them for living in future society.

Specifically,

(i) "Micro-technology will change our understanding of what some learning is, as well as how it must occur . . ." (p. 7 R.O.S.E.).

How will the Mathematics guideline address this concern specifically?

(ii) "Research will gradually force educators to adjust their expectations about how young people learn, producing changes in instructional strategies, materials and standards." (p. 9, R.O.S.E.).

What are the Ministry's plans for ensuring that these notions are successfully translated into classroom practice?

Curriculum implementation often appears as the final component of a curriculum development model. Perhaps it is one of the reasons why so little attention is given to this phase. Yet, research suggests that without an adequate implementation strategy little or no change takes place with respect to the expectations in the new document. There is clear evidence of this in the Provincial Review Report, Volume 3, November 4, 1979-1980.

"A significant part of the core program is not being taught in grades 11 and 12 in the sample. Not one of the fifty-five core topics in the general program is taught by all schools. Only 13% of the core topics in the advanced level are taught by all schools." (taken from the Provincial Review Report, Volume 3, Number 4, 1979-1980.

In his book, Future Shock, Alvin Tofler suggests that the only constant we can look forward to is change, and that the most important aspect of change is the ability to adapt to it. "Rate of change has implications quite apart from and sometimes more important than, the directions of change . . . Any attempt to define the content of change must include the consequences of pace itself as part of the content." (p. 3)

The R.O.S.E. report reflects this concern.

(i) "The teachers of the 80's will be older and more experienced and naturally sensitive to changes which they do not value. Staff development to provide new understanding, attitudes and skills will be essential to any changes which education might wish to make." (p. 9)

Ministry of Education curriculum project management teams will be responsible for:

(ii) Monitor(ing) requirements for human resources and new skills. As a component of implementation, staff development programs dedicated to orientation, understanding and program planning are essential." (p. 5). It it clear that the Ministry has ignored the basic tenant of research in this area; that the essential role of the change agent is to prepare their audience well in advance of the implementation date.

Does the Ministry intend to continue with its traditional implementation policy; a policy which has yet to succeed? If not, what is the new policy? I'm sure teachers would welcome a detailed plan of action.

A guideline should ideally:

(iii) "Provide direction to the faculties of education as well as to other professional groups involved in the training and development of teachers." (p. 23)

How will the Minister monitor the procedures to ensure that both the policies and the spirit of the new guidelines will be reflected in programs and courses offered to prospective teachers? As yet there is no evidence that planning to accommodate for these changes is taking place. I have always been puzzled by the fact that the Faculties of Education are rarely involved on a collective basis (subject specific) in the curriculum development and implementation phases by the Ministry. Yet, they are supposed to ensure that student teachers are properly schooled in these matters.

Issues surface from time to time for which inadequate responses to questions of concern are provided. The consequence has been apathy to new ideas resulting in a lack of committment to the implementation phase. Perhaps this open letter format may help us deal with change by becoming better informed, where issues can be aired and clarified. Our intention, as the editors of the O.M.G., is to publish a journal that is very much committed to being part of helping develop Mathematics excellence in education in Ontario.

I look forward to your early response. I am also looking forward to responses from our readers both in a form of reactions to this new idea and in the form of contributions for open letter publications to you.

Sincerely,

Sam Pravica,
Co-editor of O.M.G.

INTERVIEW WITH DAVID ALEXANDER

by
Bob Smith

Dr. David Alexander is Professor of Mathematics Education at the Faculty of Education, University of Toronto. He has been seconded to the Ministry of Education for a two year period as project leader on the Intermediate/Senior Mathematics Curriculum Committee. It was my pleasure to visit with Dave outside the Math and Computer Building at the University of Waterloo after the OAME '83 Curriculum Implementation Seminar.

RS: Dave, take a moment to ouline perhaps two or three of the main reasons why this project committee was set up.

DA: The prime reason for getting involved with the project was that the revision of the Senior Guideline was long overdue. The intermediate guideline revision had been delayed but finally came out in 1980. The intent was that a senior division guideline revision would be undertaken immediately. However, the SERP activity delayed that revision and as a result of the SERP recommendations, the mandate of the committee was extended beyond the senior division to encompass the intermediate division.

RS: In you December Mathinfo letter you mentioned that an advisory committee had been set up naming the personnel to it but I wasn't clear on what its purpose was.

DA: The advisory committee concept flows from the past history of the development of curriculum documents which has always included within the process designated representation from the Ontario Teachers Federation. In this particular revision, the decision was made that the project team would be kept reasonably small. To encompass the concept that the OTF would have input, the advisory committee was formed. The advisory committee is composed of the project committee with five additional members determined by the OTF. This Committee provides a sounding board for consideration of input from the field. We've had a series of meetings of the advisory committee considering the submissions from the field as we make revisions on the guidelines.

RS: The intent of the Mainfo Newsletter was certainly to involve persons in the field in the ongoing attempts to come to some policy decisions as well as a device for gathering information about timing and topics, etc. In fact, what have you learned from Mainfo, particularly, say the grade 7 and 8 questionnaires?

DA: In the initial questionnaires that we sent out with the grade 7 material we found that our perception that some of the material could well be delayed to grade 8 was not the perception of teachers in the classroom. Many of the topics which we were suggesting could be deleted, they asked to have returned to the curriculum.

RS: Could you give me an example?

DA: Integers is a major example. This we had suggested might well be delayed and they quite rightly pointed out that students coming in from grade 6 had a background with integers and the topic should be kept alive. I think that sort of input was valuable in keeping the committee in tune with the field. However, in doing so we were left with a secondary problem. How could one realistically envision core curriculum being taught in the allotted 120 hours, while leaving some time for optional topics. So the second wave of questionnaires involved timing and we are just in the process of analyzing that. One of the major problems we have to deal with are the tremendous ranges in the amount of time people perceive as being spent on the different topics. The encouraging fact is that there are some modes that seem to stand out. I think that we will be able to come up with a realistic impression of what a fair number of teachers feel are appropriate times to be spent on these topics. The next problem will be to see whether these fit together to be encompassed within 120 hours and still allow time for appropriate optional work to be taken.

RS: You mentioned at the Curriculum Implementation Seminar that you're very much aware of a fidelity approach and an interactive or adaptive approach in the writing of this curriculum policy material. I wasn't quite sure where you stood in terms of your own beliefs in the writing of the Math Curriculum. Where do you stand?

DA: All right. For the terms adaptive and fidelity approach I am indebted to Mike Fullan of O.I.S.E. These terms have allowed me to put a label on what I was doing. My belief is that there are some things that we, as a mathematics education community, can come to conclusions about in terms of what we would like to teach to students and also have fairly firm convictions about the behavioural objectives that are attached to those topics. One of the disadvantages of the 1980 guideline was a lack of this specificity. This flowed from a philosophy that had permeated the system that guidelines should not be specific but rather should be allowed to be interpreted at the classroom level. This lack of specificity made the task of any OAIP committee very difficult because of the lack of identification of what should be tested.

RS: In fact, at the end of the junior division in The Formative Years it simply says slides, flips and turns. Now where are you going to start in grade 7 when you have no idea of what specificity is required of a student at the end of grade 6?

DA: We have tried to describe that for slides, flips and turns the student should be expected to sketch images - we would think in terms of templates being moved and sketching of images after those templates have been moved. We have suggested that students should be asked to identify from diagrams what is the image under a certain slide, flip or turn. In the most recent draft we have labelled items as either review, extension, or new work. Where we feel that the intent of the formative years is that work has been covered in grades 1 - 6 we tried to clarify that. There may be some misgivings throughout the province as to whether that indeed should be our role, but we felt that we had to make some statement of this sort in order to have a base from which to begin since the mandate that we are working under is to be specific. We are trying to declare what we see as being the base from which we are operating by indicating that some things indeed are review. Now there are some people who feel that we should go even further, break down some of the things that we are describing into even more specific terms and be much clearer as to what has come from grade 6 or before and what we see as new rather than use the less definitive work extension which can be interpreted in a variety of way. My own commitment is that I still believe that the teacher or teachers are professionally capable of making such interpretations themselves and the Ministry role is to be reasonably clear but not be as prescriptive as that. I don't want to see us return to the day of when someone could go into any classroom in the province and say, aha, this should be taught on this particular day.

RS: So you're giving us a baseline from which to operate. Yet in using problem solving as your example are you suggesting that you don't wish to get to the strategies or techniques to go about teaching a particular topic?

DA: I wouldn't say that I don't want to get into the strategies and techniques. What I would say is that I don't think that we are ready or indeed perhaps ever will be ready to lay down specific strategies of teaching for all teachers to use throughout the Province of Ontario. I think that is a professional decision that should be made by the individual teacher with the individual students that they are dealing with. Hopefully, in fact, with individual groups of students within their own classroom. But I think we can certainly try to be reasonably specific about what we see are possible strategies which might, in fact, improve the experience that students are having in some classrooms. From my knowledge of the classrooms across Ontario, both as a member of the Faculty of

Education and from the role that I now have, I'm convinced that in the majority of classrooms we have concerned teachers who are doing their best to provide for students experiences that are worthwhile. The philosophy on which those experiences grow, the beliefs that teachers have, differ widely but I think that we have extremely successful teaching going on in the majority of the classrooms. As far as the processes (the 1980 document refers to them as emphases) that students undergo in the learning and the processes which we are attempting to teach, problem solving is one of the prime exemplars. If we look across the world there is an effort in most countries to do more with problem solving but the actual implementation of what is done varies greatly within countries, within states, within provinces, and I would say within different schools within a school board. Much of it is good. We are still searching for what we really mean definitively as good problem solving techniques. Teachers have to grow in terms of realizing what are good problem solving techniques and I would say curriculum developers have to grow also. So here is a case where I think, in Fullan's terms, we are dealing with an adaptive approach. What the Ministry should provide is a means whereby further experimentation can take place. We should use it in order to spur further development, but I don't think it is the role of the Ministry to make unnecessarily restrictive demands either on teachers or on students in terms of what is expected with respect to specific student behaviours related to problem solving. We are still in the developmental phase here and probably will be for 10 or 15 years or longer.

RS: It comes though very strongly, Dave, that you have a fundamental belief in the professional nature of the teacher and that you are going to hold to that and not even the Ministry of Education is going to shake you from it.

DA: Well, you know, I really don't think that the Ministry of Education in fact has any other approach towards the teachers. The more I'm with the Ministry the more I appreciate that there are many different pressures on the Ministry from many different directions and, in fact, the pressure for being specific about the curriculum is coming from teachers. They are the ones that are making statements about being more specific and, in some ways, this disturbs me because I think that they are drawing back from the role that I feel they should be fulfilling which is a professional role of making professional decisions.

RS: **Education in the Primary and Junior Divisions** provides an extensive philosophical basis and rationale for the program of the early years in school. For some years we have been promised a similar document for the intermediate division. How close is it to completion?

DA: Well, I must say that having been with the Ministry less than a year, there are some aspects of it I am not aware of. I'm certainly not aware of that document. On the other hand it may be on the verge of being published for all I know. I think that one of the difficulties I perceive from my experience within the Faculty of Education and my knowledge of the research that is involved, is that we still have a very incomplete view of development at this level. I think the reasons for it are quite clear. Adolescents are very difficult to work with in terms of really exploring the depth of their understanding. They have developed defensive mechanisms that don't exist in the younger age groups and I think the fact that much of Piaget's work was done with the younger age is probably due to the fact that they are much easier to study, they are much more open in expressing themselves. The work that I did with adolescents in terms of trying to explore their knowledge with algebra was quite successful with a fair number of students but there were some that just completely clammed up so I was not able to explore their thinking. The people that I got the most out of were those that were totally confused and knew it and that were ready to admit it and those that felt they were in complete control of the situation even though that was not necessarily true. There was a great middle ground where the people obviously were in semi-control of the concept and were somewhat apprehensive about their knowledge. When I started to ask more searching questions, they simply would not respond. Well, I think that sort of difficulty in terms of exploring the adolescent concepts means that we are a long way from knowing exactly what the changes are that go on in these years. There are various theories but I think we have to accept them as theories, draw what we can from them, but not base all our work on them.

RS: So its going to remain an area of challenge for any teacher who wishes to take up the work of the intermediate division. The Third National Assessment of Educational Progress report states that: "The mathematics needed today is not the mathematics that was needed a century ago, but for the most part that is the mathematics still being taught in schools." This quote seems to be referring to the emphasis on arithmetic as opposed to that which is needed in this age of the computer. How are you approaching or attacking this problem in your writing of the new curriculum guidelines.

DA: What I perceive is that with all the emphasis on the computer, something has got lost. Five to ten years ago we were talking about the possibility of involving the calculator in our teaching and suddenly we have jumped to the computer age. I really think that what we should be doing is implementing the calculator age. People can now carry around a calculator on their wrist and we should recognize that fact. For K-6 children this does not necessarily mean that we depend entirely on the calculator but certainly as we enter the intermediate division, I think that the calculator should be build into the program. We are making it core, with the expectation that teachers will be teaching the proper use of the calculator, which means involving estimation, as well as other procedures that are adapted to the calculator and to the problem solving parts of the course. We are trying to be definitive about how far students should be expected to tackle a skill such as addition, subtraction, etc. For instance, we believe that the student has demonstrated all that needs to be demonstrated in terms of say multiplication, if they can multiply a four digit number by a three digit number, that it shouldn't be demanded of them that they go through the process at a more complex level. So, if our guideline is, in fact, accepted as policy in the long run, it follows that evaluative processes such as OAIP would contain instruments which were limited to that sort of skill where paper and pencil was involved and that if there were further demands to be made, then the use of a calculator would be appropriate. In saying this, we recognize that there are also demands made for entry into the workforce, there are tests that students have to go through, skills they are asked to demonstrate that in fact they will never use on the job. On the job they will probably be handed a calculator and told, "Here use this!". But they are expected to go through an initiation to enter the lodge as it were. I think society has to change as much as the school system and recognize that we have entered a new age and we should make use of the technology where it seems appropriate.

RS: What do you hope to accomplish in the next year?

DA: We are hoping in September to have further guidelines out, particularly the Advanced and Mathematics for Technology. The Basic program should be available shortly thereafter. We will also be turning our attention over the summer to writing the various preambles that will attack some of these issues we have been talking about in terms of process, teaching suggestions, etc. The long term goal is that there be a document that is reasonable complete by December so that the process of fine tuning it for printing can get underway and that the complete Guideline will be out in the schools by next June.

RS: Very good but you have a tight timeline in front of you, Dave.

DA: I might add that the committee isn't as enthusiastic as I am about the timeline.

RS: You've had a fair response to the Math for Business directions that you put out and its interface with the CAAT's. Please give us a statement about it.

DA: With the general level program at the senior division, we faced the problem of whom it should be designed for. The statement that we have from the Ministry's response to the SERP report was that the general level courses should be designed for entry into the college of applied arts and technology and for entry directly into employment. There was some concern among the members of the committee, that for mathematics the colleges of applied arts and technology have been turning toward advanced level students in their admissions. The Ministry policy is that we should provide, within the general level program, math courses which are delivering the type of prerequisites that the colleges are identifying through such processes as the Linkage II program. The Math for Business turned out to be fairly easy to design because it was possible to look on the material within it as being useful to informed citizens. We view such a course as appropriate for students who are entering the workforce as well as students who are entering the college of applied arts and technology. There have been some concerns expressed about the level of attention being given to some of the basic skills that the colleges have identified. I hope we can clarify that in further drafts.

As far as the Mathematics for Technology is concerned, this is a much harder nut to crack. The demands of technology for math are high and what I think we are looking at is creating a course which in some ways will draw students away from the advanced level programs. In particular those students who are not heading for university. The figures show that at least half of the students currently enrolled in grade 13 do not enter the universities and there is almost half who neither enter the universities nor the colleges. The SERP report suggests that these students would be better directed toward courses that were preparing them for employment or for the college of applied arts and technology.

RS: Dave, we can't leave without a word about curriculum implementation and I would repeat John Bates words, 'We are great writers of curriculum but we've done a damn poor job of implementing it'. I'm sure you have some thoughts on implementation.

DA: The basic problem with implementation I think has been identified and that is, if you are looking for change, first, it is not a short term project and, secondly, it demands a great deal of activity with people as opposed to activity through the printed word. I agree, I think our record has been very poor in terms of implementation and those areas where we have been successful, we have been successful probably because of the readiness of teachers to accept the new curriculum. That's the role I see for the current revision. The review of senior division mathematics made it reasonably clear that people were happy with the work involved, except for the general level programs, so it appeared to me that our major thrust should be at the general level. It would have been nice to think of that thrust going back into the intermediate grades and we have received some submissions that suggest we should do just that. On the other hand, the 1980 Intermediate Guideline is just in the process of being implemented. There have been comments made by publishers and by Boards to a fair amount of work in implementation and so really I think we are better off not to look at massive change at the 9 and 10 level, although perhaps we can point the way for further development. The role of the Ministry in terms of implementation I think is clarifying what is to be implemented and the role various parts of the educational system are to play in implementation. I think there is a recognition that the guidelines have not been implemented successfully. There is a model for implementation being generated now that holds promise in terms of looking at development, implementation and review as being a cyclical process. The Curriculum Project Management team for Mathematics will meet in September to begin planning implementation strategies.

This particular committee, I hope, will be able to generate plans and the necessary funding so that implementation can proceed in a way which has a better chance of success. When we come right down to it, the amount of funding and the amount or organization at the Ministry level, really has very little to do with implementation. The actual implementation comes about by the interaction of teachers in their own schools and at the board level. While the Ministry can encourage such things, I'm afraid it cannot impose such interaction.

RS: I have been heartened, Dave, by listening to you. You see much evidence that there are positive things going on in the system and that, in fact, we have come a long way from the introduction of the 1980 intermediate and 1972 seniour math guidelines. I'm encouraged. Thank you very much, Dave, for talking with us.

DEPARTMENTS

BOOK REVIEW OF TEACHING STRATEGIES IN SCHOOLS THROUGHOUT THE WORLD

by
Vic Barnett
Published by the International Statistical Institute

Teaching Statistics in Schools Throughout the World is a survey of what is presently happening in the presentation of statistics around the world. It answers some of the common questions that might be asked, such as: Where is statistics taught? When is the teaching of statistics begun? What content is common to most countries? Would you be surprised to learn that there is a wide variation from country to country, or even within a country, as to what is taught? Would you be surprised to learn that there is a general lack of coordination - everybody does his own thing? Two basic facts come out again and again - there is a lack of systematic training for the teaching of statistics and there is a lack of good, graded statistics books. Barnett notes that there is a rapidly growing interest in and recognition of the importance of statistics because the rise of statistics is becoming more and more pervasive in all disciplines.

One of the interesting sidelights of this book is the description of the various school systems around the world. (For instance, I finally figured out what an ''O - level'' in England is.) Writers from the various countries describe the weakness - is of their own systems in teaching statistics. General comments recur frequently: that there is too much mechanistic manipulation of formulae; the teaching approaches are too formal; there are not enough real-life situations studied. There is little reference in the book to the misuse of statistics, especially by the media.

Another feature of **Teaching Statistics in Schools Throughout the World** is the description of typical problems in teaching and testing in various countries. Although not all countries provided these details, the ones that did, offer a good sampling of elementary to secondary to post-secondary problems that can be found. It is a good source of questions.

For anyone who is going to be writing curriculum involving statistics (and probability), whether for a whole course or for selected topics in some other course, this book would be a very handy aide. 'It will help to suggest topics, emphasis, and areas to find suitable problems.' The description of the wide use of statistics in so many areas underlines the need to increase our awareness of this important topic and to do a good job of teaching it to our students.

Reviewed by John Warnebe
Scott Park Secondary
Hamilton, Ontario

INTERVIEW WITH KATH HART

Kath Hart is Senior Research Fellow at the Centre for Science and Mathematics Education, Chelsea College, University of London.

Once again I have renewed acquaintance with her, this time at the R.C.D.P.M. meeting in Bowling Green, Ohio where she gave a report on S.E.S.M. Strategies and Errors in Secondary Mathematics.

RS: In September 1978 issue of O.M.G. our interview outlines your work during the years 1975-1980 in the C.S.M.S. project. The results of this project formed the basis for your book **Children's Understanding of Mathematics: 11-16.** I found the book very useful, helpful as well as interesting reading. What has been its reception in England?

KH: The book was written by the C.S.M.S. Mathematics team and contained information on the research into the understanding of a number of secondary mathematics topics. It has sold very well and the research was mentioned favourably in the Cockcroft report. The appearance of this report has stimulated an awareness of the differences between children of the same age as far as their understanding of mathematics is concerned. There have been a number of Cockcroft Conferences for teachers and we have been involved in speaking about the C.S.M.S. results at some number of them.

RS: You obtained further funding and are now working on a project called S.E.S.M. Do give us some details about the new project.

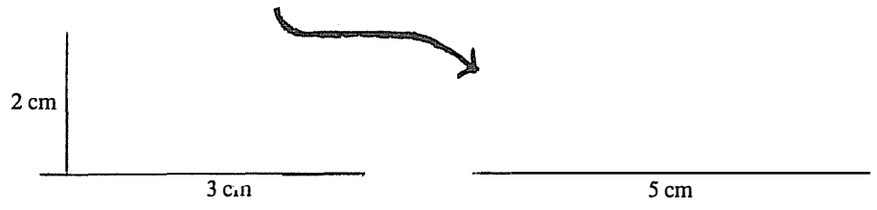
KH: The data from the large C.S.M.S. survey enabled us to form hierarchies of understanding in each topic investigated and also showed us the incidence of certain errors. We had used the C.S.M.S. word problems to interview children on the methods they were using and often it had been possible to identify a wrong answer occurring as a direct consequence of an incorrect method. These erroneous answers had been coded when the survey tests were marked so we had an idea of their prevalence. S.E.S.M. was designed to look in depth at those errors which were occurring often (perhaps on 40% of the papers) in order to find the reasons why children were producing these answers and then to suggest ideas on how the situation could be improved.

We had noticed in the C.S.M.S. work that children often described methods of solving problems which were not teacher-taught and which in the book I call "child-methods". These tended to produce correct answers on the easier questions but were insufficiently generalisable for successful use on harder items. For the S.E.S.M. work we (two researchers and three students) took errors in Algebra, Ratio, Graphs, Fractions and Measurement and bearing in mind the child-methods, tried to find out what was going wrong.

RS: Perhaps you could illustrate the methodology through your concern about the teaching of ratio.

KH: In the C.S.M.S. survey on the understanding of Ratio and Proportion we found that on four questions the same type of incorrect answer appeared very often. It was also found that a third of the total 1976 sample of 2257 was consistently giving this type of answer on the four questions, so that it seemed the error was probably the result of a planned strategy. Karplus and Piaget had both identified an incorrect method used by children when trying to solve ratio problems, in this the child added a difference a-b rather than multiplying by a factor a/b so that in this question:

what is the new height to give an enlargement?



the child would reason that 3 cm became 5 cm by an addition of 2 cm so the new height would be 4 cm. The erroneous answers at the 40% level were of this type.

The methodology we used to obtain further information on the errors was to (i) recruit children aged 13, 14, 15 in five schools, who were deemed to be of 'average ability', (ii) test this group with the relevant C.S.M.S. paper and then (iii) identify those who were committing the specific errors. This subsample was then interviewed to find the methods used not only when these erroneous answers were obtained, but also when answers were correct. Later the children were interviewed again using different questions and with the interviewer trying to find their reactions when confronted with the logical outcome of their methods.

The interviews on Ratio resulted in a general description of methods used by the children giving these specific wrong answers. The 'adders' solved 'easy' Ratio questions by some form of doubling and halving or found answers in small segments and then added these sub results. On the harder items they continued the operation of addition but added a fixed amount ('a - b' described earlier). Fractions other than 'one half' were avoided at all costs and no attempt was made by 'the adders' to find a scale factor which could be used to multiply.

RS: How are you developing modules for classroom teaching?

KH: Having found what appeared to be lacking in the methods used by the children we attempted to build a short teaching programme which was aimed at those identified deficiencies. Ideas were first tried out with small groups of adders, using the classic research format of pre-test, treatment then immediate and delayed post tests. The first four groups I taught, were given an introduction to the multiplication of fractions and practice on using the rule, besides having stress placed on the operation of multiplication rather than addition. After six lessons the children abandoned the incorrect addition strategy but on the delayed post-test their general knowledge of fractions proved inadequate to the task of getting the questions right.

The modeule was modified so that the fraction techniques were replaced by the use of the calculator. This approach assumed that the understanding of Ratio and Proportion could be distinguished from an ability to handle fractions. The work sheets which formed the module were then tried with another group of adders and later in half-classes where of course there were children who correctly solved the items, some who were adders and yet others who gave different incorrect answers.

The final phase was to offer the worksheets and general instructions to volunteer teachers for use with whole classes of 'average' children. The results were promising and the incorrect addition strategy was to a large extent eradicated but although performance has improved considerably, some children still did not succeed.

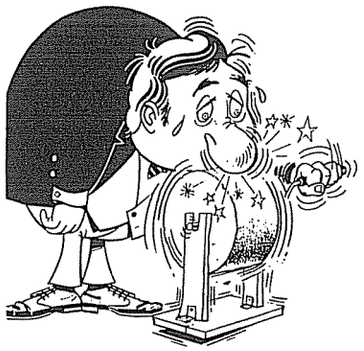
RS: How are teachers going to obtain this information?

KH: We are publishing a series of five research monographs, one on each of the S.E.S.M. topics, with NFER/Nelson. Algebra and Ratio should be available this autumn.

RS: And what is next?

KH: We now have funding for two year's research to look at how primary school children make the leap from practical or introductory work to a generalization or rule. For example how difficult is it to go from counting squares to find the area of a rectangle, to the formula a x b.

RS: It has been great talking to you once again. Perhaps this interview will spark enough interest in Ontario to have you as a speaker at one of our conferences.



COUNCIL COMMENT

by
Bob Robinson

Editors Note:

Council Comment appears in an issue of the Gazette following OAME/AOEM Council Meetings. We hope that you find this section of interest. Your comments are welcomed and should be addressed to:

The Editor
Ontario Mathematics Gazette
College of Education, Brock University
St. Catharines, Ontario L2S 3A1

The June Council meeting was held on Saturday, 1983 06 11 at the Ramada Inn (Airport West).

1983-84 OAME/AOEM

Executive

Past President:	Sharon McPhail
President:	Dave Davidson
President-Elect	Paul Pogue
Director:	Ralph Connelly
Director:	Bernadette Laframboise
Director:	Paul Lessard
Secretary-Treas.	Morley MacGregor

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Dino Dottori

Tom Griffiths

Elaine Harvey

Mary Lou Kestell

George Knill

Alex Norrie

J.Symington

Trevor Thomson

Chapter Representatives

NOTE: These representatives are acting voting councillors.

COMA (Central Ontario Mathematics Association):
* Representative to be named in Fall '83.

GOLDEN SECTION
Julian D' Angela

GVMA (Grand Valley Mathematics Association)
Bill Proctor

NOMA (Northern Ontario Mathematics Association)
Mickey Sandblom

PINE RIDGE
Kumar Pasricha

QSLMA (Quinte St. Lawrence Mathematics Association)
Norah Wilson

SAME (Scarborough Association for Mathematics Education)
Steve Martin

TORNY (Toronto-North York)
Patricia Delgaty

Council Meetings 83/84	Executive Meetings 83/84
1983 06 11	1983 05 27
1983 10 15	1983 09 09
1984 02 11	1983 11 18
	1984 04 06

COMMITTEE CHAIRPERSONS AND EXECUTIVE LIAISON

Publications

O. Fullerton (Chairperson)
R. Connelly (Executive Liaison)

Senior Guidelines

T. Griffiths (Chairperson)
P. Pogue (Executive Liaison)

Computers

G. Knill (Chairperson)
P. Lessard (Executive Liaison)

Problem Solving

C. Carlow (Chairperson)
P. Lessard (Executive Liaison)

Membership

D. Dottori (Chairperson)
D. Davidson (Executive Liaison)

Public Relations/Publicity

K. Pasricha (Chairperson)
S. McPhail (Executive Liaison)

Bill 82

E. Harvey (Chairperson)
B. Laframboise (Executive Liaison)

There are, as well, four standing committees namely:

Conference Liaison Committee
Nominations Committee
Leadership Seminar Research Committee
Mathematics Teacher Shortage Committee

AIMS AND OBJECTIVES

The following aims and objectives were approved by Council:

1. To increase the publicity of OAME/AOEM and its members.
2. To initiate projects that will promote the name of OAME/AOEM.
3. To increase the membership of OAME/AOEM to 1500.
4. To evaluate the effectiveness of our publications.
5. To conduct a study to determine the availability of qualified mathematics teachers in the secondary schools in Ontario.
6. To have a constitution review complete for presentation at the 1984 Annual Meeting in Kingston.
7. To promote mathematics from Kindergarten through post secondary in the Province of Ontario.
8. To adopt and publish a position paper on Bill 82.

1985 ANNUAL CONFERENCE

The 1985 Annual Conference will be held in Sudbury.

1984 ANNUAL CONFERENCE

Plans for the 1984 Annual Conference in Kingston are proceeding well. Sessions are being scheduled for Thursday afternoon with a keynote speaker on Thursday evening. Friday will have sessions all day with the banquet on Friday evening. There will be Saturday morning sessions followed by the closing luncheon and Annual Meeting. The dates are 1984 05 10-12. Further inquiries can be directed to Elaine Harvey.

1983 ANNUAL CONFERENCE

Council congratulated Waterloo (GVMA) for the excellent conference held at the University of Waterloo in April.

JUNIOR OAIP

The results of the Arithmetic and Numerals materials will be evaluated during the summer. Jim MacLean's retirement from the Ministry will necessitate a new appointment to lead the project.

OTF CURRICULUM FORUM

Paul Pogue is the OAME/AOEM representative to the OTF Curriculum Forum.

NCTM

Ralph Connelly was reappointed as the association's representative to the NCTM. Orders for NCTM should be prepaid (payable to NCTM) and directed through Ralph Connelly at Brock University. In this way OAME can benefit from a 20% rebate provided that orders are sent in this manner. Bob Robinson has been appointed the Board of Director member of NCTM Educational Materials Committee.

LEADERSHIP SEMINAR RESEARCH COMMITTEE

The chairperson, J. Symington, is awaiting input from the membership regarding the future direction of the Leadership Seminar.

FATHER FAUGHT AWARD

The Father Faught Award will be given to the winner of the Fermat Contest.

PUBLICATIONS

Trevor Brown and Lorna Morrow have been appointed co-editors of the Abacus commencing with the November 1983 issue. The Council expressed their appreciation, on behalf of the membership, to Claire Zeller for his fine work in producing the Abacus over the past 2 years. Readers of the journals will be surveyed to see if they are providing what the mathematics education community wants to read.

MEMBERSHIP

The membership committee has set itself a goal of increasing membership by a substantial amount.

SENIOR GUIDELINES

A letter of response from the Minister of Education pertaining to OAME/AOEM recommendation with respect to the senior division mathematics curriculum was distributed.

PUBLIC RELATIONS/PUBLICITY

This committee is looking at ways of publicizing OAME/AOEM and obtaining other sources of income.

COMPUTERS

OAME/AOEM is seeking ways to provide input on the Ministry of Education committee that will decide which future software projects will be funded.

BILL 82

This committee expects a formal position paper by September. Any member of the association should contact Elaine Harvey for further information.

PROBLEM SOLVING

The committee is studying problem solving for children with different learning styles. A meeting was scheduled for June when problems will be finalized for the committee to test.

EDUCATIONAL TELEVISION

The workbook of student activities is now available for distribution by T.V. Ontario to accompany the "Landscapes In Geometry" series which was written by members of OAME/AOEM.

CHAPTER REPORTS

GVMA Had a busy year due to hosting the 1983 Annual. Bill Proctor thanked the council for their support and indicated that an October mini-conference is likely.

NOMA Is holding their annual meeting on 1983 09 30 and 1983 10 01. September 30 is a P.D. day in their region. The president of NOMA is Doug Henderson and the Vice President is Bob Tuck.

PINE RIDGE A successful meeting was held in April with Dave Alexander being the after dinner speaker. An afternoon programme was held on 1983 06 28 in Whitby where 8 workshops and a book display were provided. Another full day of activities is being planned for 1983 11 16 which coincides with the annual P.A. day.

QSMLA A very successful meeting was held in April at RMC. They are well on their way with the plans for the 1984 Annual in Kingston.

SAME Plans are under way for the annual mini-conference at Maplewood Vocational School in Scarborough on Tuesday, 1983 10 18. A future math conference is being discussed for K-13 teachers in February.

TORNY Reports that they held two successful conferences last school year and are looking forward to another productive new school year.

NEXT MEETING OF COUNCIL - SATURDAY, 1983 10 15

PLUS + + +

As an aid to communication among mathematicians, and mathematics educators across Canada, this short magazine is to be inserted in mathematical education journals throughout the country. We hope to tell you about important events, research, curriculum development and items in the literature of national interest. For this, we need **your** help in sending information. Letters to the editor are also welcome. Out of consideration for our host journals, be brief.

Material (in either English or French) should be sent to:

Dr. Don Kapoor, Editor
Professor of Education
University of Regina
Regina, Sask. S4S 0A2

3.2.1. ICMI-JSME REGIONAL CONFERENCE ON MATHEMATICAL EDUCATION

This conference is to take place 10 - 14 October 1983 in Tokyo. "School Mathematics in and for Changing Societies" is the theme of this conference organized by the Japan Society of Mathematical Education and the International Commission for Mathematical Instruction. The principal working languages of the conference will be English and Japanese. The conference fee of about $120 is in addition to travel, meals, and accommodations. Request the second notice from T. Sawada; Secretary of the ICMI-JSME Regional Conference on Mathematical Education; c/o National Institute for Educational Research; 6-5-22, Shimomeguro; Meguro-Ku, Tokyo; 153 JAPAN. Include your name, address, professional position, and your suggestions for the program.

3.2.2 ICME 5 SOLICITS SPEAKERS

The U.S. Commission on Mathematical Instruction has been asked to submit names for consideration as potential speakers and contributors for the Fifth International Congress on Mathematical Education (ICME 5). **The congress will be held 24 - 30 August 1984 in Adelaide, Australia.**

For more information, write immediately to:

Don Hill
Mathematics Department
Florida A. & M. University
Tallahassee, Fl. 32307

3.2.3 **MATHBEC 1983**

The Quebec Association of Mathematics Teachers and the Department of Mathematics of Concordia University are hosting a conference designed for teachers of mathematics at the elementary, junior, senior secondary levels and at CEGEP and university. The main thrust of the meeting will be mathematics education in the present and in the future with emphasis on the Microcomputer in the Instructional programme.

Date: September 23 - 24, 1983

Place: Concordia University
Place: Loyola Campus, Montreal, Quebec

3.2.4 **TENTH MATHEMATICS SUMMER WORKSHOP**

This workshop is sponsored by British Columbia Association of Mathematics Teachers and National Council of Teachers of Mathematics.

Date: Tuesday and Wednesday, August 30 and 31, 1983

Place: Sutherland Secondary School, North Vancouver, B.C.

For more information, call or write:

Jack Taylor
c/o Argyle Secondary School

3.2.5 **PME - NA ANNUAL MEETING**

International Group for the Psychology of Mathematics Education, North American Chapter is holding its annual meeting on September, 29 - 30 and October 1, 1983 at Montreal, Quebec.

For more information regarding registration, write to:

Dr. Nicolas Herscovics
Department of Mathematics
Concordia University

3.2.6. **SCIEMATICS CONFERENCE SET FOR NOVEMBER IN REGINA**

The Saskatchewan Mathematics and Science Teachers Societies are holding a joint Sciematics conference on November 9, 10, and 11, 1983. The theme of this year's conference is "Towards the 21st Century." The conference will be held at Campbell Collegiate, 102 Massey Road, Regina.

The keynote speaker will be William Watson Purkey, Professor of Education, University of North Carolina at Greensboro.

The conference will feature morning and afternoon sessions on a wide variety of topics, exhibits, tours, workshops, guest lecturers and social events.

For further information, contact:

Gerry Goski



CALENDAR OF EVENTS

The Scarborough Association for Mathematics Education (S.A.M.E.) will be offering its annual Mathematics Mini-Conference on Tuesday, October 18, 1983, from 3:45 p.m. to 8:00 p.m. at Maplewood Vocational School in Scarborough.

For further information, contact:

Steve Martin, President
Scarborough Association for Math Education
140 Borough Drive
Scarborough, Ontario
M1P 4N6

McCartney Scholarship

The winner of the McCartney Scholarship for the year 1983 was:
Neale Ginsburg,
125 Westmoreland Rd.,
Kingston, Ont.
K7M 1J6
He was a pupil of Mr. James Boyd, long time member of Ontario Association of Teachers of Mathematics, The Quinte St. Lawrence Mathematics Assocation Chapter of Ontario Association for Mathematics Education. Details of the presentation will be published in the December issue of your Gazette.

Re Gazette

Your December Gazette and your registration form for your 1984 Annual O.A.M.E. conference will be mailed in one package during the month of January 1984.

Problem-based Learning (PBL) Oct. 18 to 21, 1983 Hamilton, Ontario (Introduction plus enrichment - general)	Faculty of Health Sciences McMaster University. Introduction to PBL. May 3. Plus 3 parallel sessions, 3 day workshops on specific topics: the role of the tutor in small group learning, the design of evaluation tools and systems, and the use and design of resources for PBL. Details: Margaret Jenkins P.E.D. McMaster University Health Sciences Centre
Creativity Oct. 17 to 21, 1983 Greensboro, N.C. (intro-industrial applications)	Five-day workshop on Creative leadership for R&D managers by the Center for Creative Leadership. Details: P.A. Wegner
Research Papers on PS April 23 to 27, 1984 New Orleans (research)	American Education Research Association annual meeting. AERA 1230 Seventeenth Street N.W. Washington, DC 20036

PROBLEMS

Editor's Note: *With the current emphasis on problem-solving at all levels, this section will be devoted to presenting novel & interesting problems that teachers might find useful for their classes. Readers are invited (urged, begged!) to submit any problems as problem sources they come across which might be of interest.*

This month's problems come from a **Reader's Digest** *excerpt from the book* **Alice in Puzzle-Land. A Carrollian Tale for Children Under 80,** *by Raymond Smullyan (distributed in Canada by Gage Publishing, Ltd.). We will publish the answers in the December issue.*

1. The salt had been stolen! Well, it was found that the culprit was either the Caterpillar, Bill the Lizard or the Cheshire Cat. The three were tried and made the following statements in court:
CATERPILLAR: Bill the Lizard ate the salt.
BILL THE LIZARD: That is true!
CHESHIRE CAT: I never ate the salt!
As it happened, at least one of them lied and at least one told the truth. Who ate the salt?

2. "Here's another," said the Red Queen. "A little girl named Alice had a brother named Tony," Alice interrupted. "I don't have a brother named Tony."
"I wasn't talking about *you*," the Red Queen replied sharply. "I was talking about another Alice!"
"Oh," replied Alice.
"Now," continued the Red Queen, "Tony has as many brothers as sisters. Alice has twice as many brothers as sisters. How many boys and how many girls are in the family?"

3. "Here is one," said the White Queen. 'I once had to post four letters. Well, I had the four letters written, and I had the four envelopes correctly addressed, but I was careless and put some of the letters into the wrong envelopes. However, I put only one letter in each envelope. As it happens, either I got exactly three of them right, or I got exactly two of them right, or I got exactly one of them wrong. How many did I get right?"

4. "If a grandfather clock takes thirty seconds to strike six, how long does it take to strike twelve?" asked the Red Queen.
"Why, sixty seconds, of course!" exclaimed Alice. "Oh, no," she suddenly realized, "that was wrong!" What is the correct answer?

5. Alice practically stumbled on Tweedledum and Tweedledee, who were grinning under a tree right by their house. "I'm afraid I can't very well tell you apart without your embroidered collars," remarked Alice.
"You'll have to use *logic,*" said one of the brothers. At this point he pulled out a playing card from his pocket — it was the queen of diamonds — and showed it to Alice. "As you see, this is a red card. Now, a red card signifies that the one carrying it is telling the truth, whereas a black card signifies that the speaker is telling a lie.
"Now, my brother there is also carrying either a red or a black card in his pocket. He is about to make a statement. If his card is red, he will make a true statement, but if his card is black, he will make a false statement. Then your job is to figure out whether he is Tweedledee or Tweedledum."
At this point, the other brother said, "I am Tweedledum, and I am carrying a black card." Who was he?