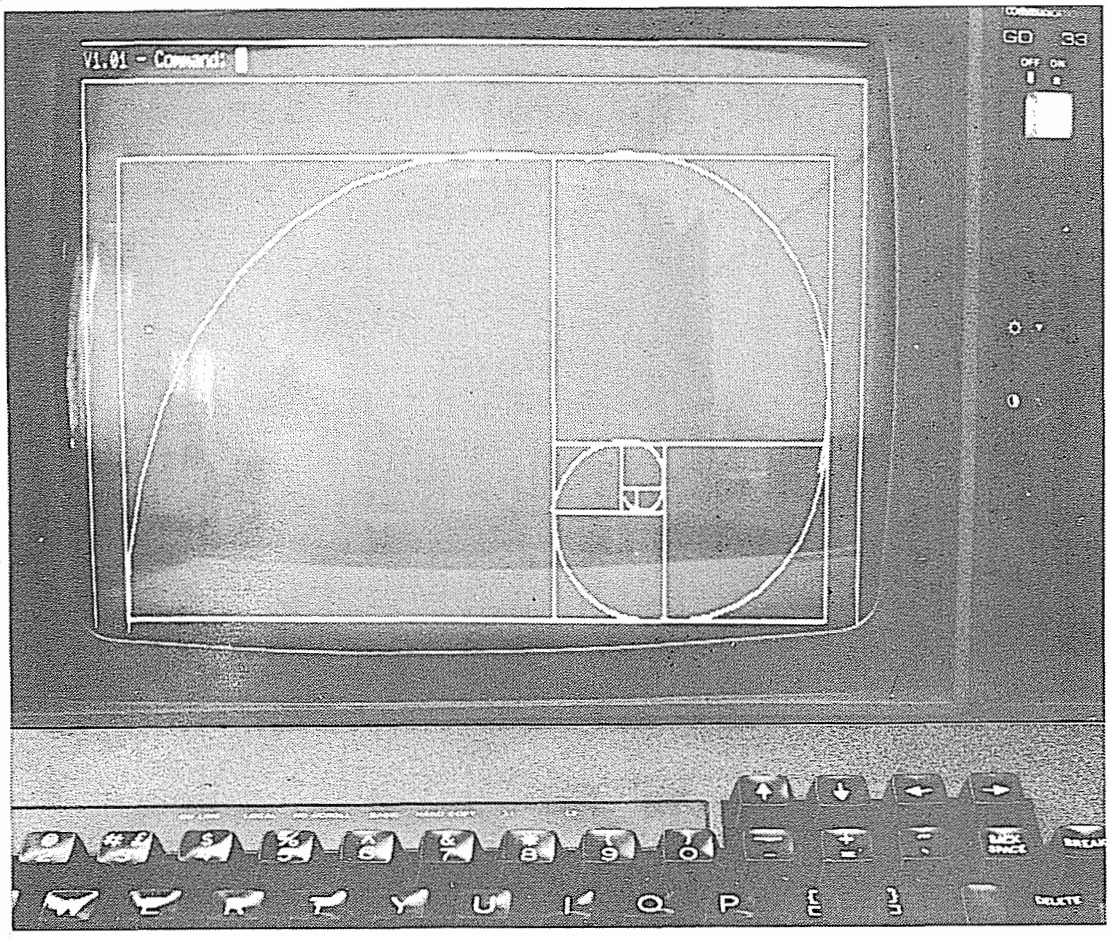
ontario mathematics gazette

VOLUME 22 **NUMBER 3** **MARCH, 1984**



THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

EDITORIAL BOARD

Editor:
Sam Pravica, Galt C.V.I., Waterloo County Board of Education
Ralph Connelly, Brock University, College of Education

Associate Editors:
Trevor Thomson, Hamilton Board of Education
André Ladouceur, University of Ottawa
Wes Keener, University of Western Ontario
Walker Schofield, University of Western Ontario
Olive Fullerton, Faculty of Ed., University of Toronto
Monica Stevenson, Lincoln Co. Board of Education, St. Catharines, Ont.
Eric Muller, Brock University
Robert Smith, University of Western Ontario
Clifford Gravelle, Brock University, St. Catharines, Ont.
David Lenchyshyn, Niagara South Board of Education

PUBLICATIONS COMMITTEE

Olive Fullerton
Ralph Connelly
Robert Smith
Carol Poce
Robin Hood

O.A.M.E./A.O.E.M. EXECUTIVE

Past President: Sharon McPhail
President: David Davidson
President Elect: Paul Pogue
Director: Sister Bernadette Laframboise
Director: Ralph Connelly
Director: Paul Lessard

MARCH, 1984 ISSN 0030-3011

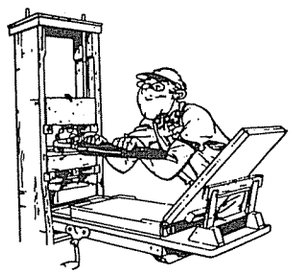
CONTENTS

Editorial 2

Patterning ("Hands-on" Mathematics for the Primary Division)
by Bonnie Richardson 3
Classification ("Hands-on" Mathematics for the Primary Division)
by Bonnie Richardson 5
High School Grades Versus Diagnostic Tests as Predictors of Achievement in First Semester University Calculus *by G.R. Chapman and Jack Weiner* 10
Inversion *by G.R. Mager* 13
Cross-Figure Puzzles *by William Ewbank* 16
Intentions and Attainments in the Teaching and Learning of Ontario Grades 12/13 Mathematics: Results from the 2nd International Math Study
by Dennis Raphael, M. Wahlstrom, and L. McLean 18

DEPARTMENTS

Interview with David Robitaille *by R.S. Smith* 28
Across the Editor's Desk 34
Book Review 36
Problems 38
Council Comment 39
Report on Teacher Shortage 43



The Ontario Mathematics Gazette is produced by:
POLE PRINTING
Box 69, Forest, Ontario
(519) 786-5112



It's hard to believe that with this issue we conclude our first year as editors. In this issue you'll find continued reporting on results from the 2nd International Math Study, both in Raphael et. al.'s article and in the Interview with David Robitaille. We are fortunate that Bonnie Richardson consented to let us reprint some of her work so we at last have material we hope Primary teachers (as well as teachers at other levels) will find of interest. With the other articles in this issue the **Gazette** is beginning to take on a better balance. We hope others will see their way clear to submit articles so this trend might continue.

As educators, all of us are aware of the vital importance of **FEEDBACK!** Our students need our feedback to continue to learn and improve; everyone needs feedback from those they work with to aid and reassure them. Lonely **GAZETTE** editors need feedback, too! No one has reacted as yet to our request for submissions to the "Problems" section; no one responded to last issue's Open letters from Dave Alexander and from Doug Henderson. The reports on the ICON and on Bill 82 have drawn no response.

Whether it's just a pat on the back or your target is a bit lower than that, give us your **FEEDBACK.** We need your submissions of articles, letters, problems, reactions, etc. to make the **GAZETTE your** journal.

Have a good remainder of the School year and summer. See you in September.

Ralph Connelly
Sam Pravica
Editors, **OMG**

Editor's Note: *This article comes from a package prepared by Bonnie Richardson and distributed by FWTAO. The whole booklet is entitled* ***"Hands-on" Mathematics for the Primary Division.*** *This excerpt is reprinted here with permission of the author.*

PATTERNING

by
Bonnie Richardson

Definition

Pattern is the underlying theme of mathematics. The skill of recognizing and using patterns is a valuable problem solving tool for a child to learn to use for it can have a profound effect on the development of a child's mathematical understanding.[5]

Objectives

Through experiences in patterning in the primary division, the child:
1. will "recognize spatial patterns in the environment;"[6]
2. will develop the ability to observe similarities and differences; and
3. will have many opportunities to create, reproduce, and extend patterns.

Learning Outcomes	Teaching Strategies/Learning Activities
Gains an awareness of the way in which the parts of his/her body relate and work together, and the way his/her body relates to the world and its objects.	- makes patterns with his/her body in response to the teacher's command: "Make yourself as tall as you can." "Make yourself into a ball." "Roll like a log." "Balance on one point."
Understands spatial concepts such as: behind in front of next to over under through between middle forewards backwards sideways	- makes patterns with his/her body in response to the teacher's command: "Climb over...." "Hide behind...." "Stand next to...." - plays "follow the leader" through an obstacle course - plays "Simon Says": "Simon says hop forewards." "Simon says touch your right ear."
Makes own patterns with his/her body. Duplicates patterns made by others using the body.	- plays "Rhythmic Clapping"[7] The children sit on the floor and imitate a pattern made by the teacher or another pupil doing some combination of clapping hands, snapping fingers, and slapping knees, e.g. CLAP-SNAP, CLAP-SNAP, CLAP-SNAP.... CLAP-SLAP-SNAP, CLAP-SLAP-SNAP, CLAP-SLAP.... SNAP-SNAP-CLAP, SNAP-SNAP-CLAP, SNAP-SNAP....

7 Mary Baratta-Lorton, **Mathematics Their Way**, p. 21.

5 Mary Baratta-Lorton, **Mathematics Their Way** (Don Mills, Ontario: Addison-Wesley, 1976), p. 20.

6 Ontario Ministry of Education, **The Formative Years**, p. 7.

Learning Outcomes	Teaching Strategies/Learning Activities
Discovers the possibilities and limitations of the patterning materials through play.	- makes his/her own patterns, designs, pictures, etc. with buttons, beads, shells, wooden cubes, unifix or multilink cubes, parquetry blocks, pattern blocks, attribute chips, tiles, 'junk' materials
Duplicates patterns which have been presented in concrete, then pictorial form.	- works with a partner, one making the pattern and the other copying it - reproduces patterns presented on cards using the concrete materials listed above
Translates patterns created with concrete materials to pictorial form.	- creates a pattern using concrete material, then copies or draws it onto a card for others to use
Reverses a given (linear) design.	- takes a pattern and makes his/her pattern by starting at the end of the given pattern and proceeding backwards

Learning Outcomes	Teaching Strategies/Learning Activities
Understands spatial concepts such as: up, down, top, bottom, middle, left, right, and uses these concepts to locate and organize geometric patterns.	- the teacher works with small groups or individual children to determine each child's understanding of spatial concepts: (each child is provided with a pegboard and pegs) "Put a red peg in the **top** row of holes." "Put a blue peg in the **bottom** row of holes." "Put a yellow peg in the row of holes on the **left**." "Put a purple peg in the row of holes on the **right**." "Put a green peg in the **middle** of the board." "Move your red peg to the **first** hole in the top row." "Move your blue peg to the **last** hole in the row." - makes own patterns with pegboards, geoboards, etc. - reproduces patterns presented by another child or on cards (the teacher should watch the way in which each child tackles the problem; some children will need help in arriving at a strategy (e.g. counting holes/rows to determine where the pattern begins and how large it is - creates a pattern using concrete materials - pegboards or geoboards - and translates this pattern onto blank dot to dot sheets for others to use

Bonnie Richardson

Editor's Note: *This article comes from a booklet, "Hands-on" Mathematics for the Primary Division, prepared by Bonnie Richardson and distributed by FWTAO. This excerpt is reprinted with the author's permission.*

CLASSIFICATION

by
Bonnie Richardson

Definition

"All classification involves a grouping or sorting of objects into classes according to some rule or principle."[8]

Objective

Through experience in matching, sorting and classifying objects, the child will acquire a basis for logical and mathematical concepts.

Classification serves as a basis, psychologically speaking, for the development of both logical and mathematical concepts.[9]

Children see numerals and even learn to write them long before they understand their meanings. However, it is only after many first-hand experiences with classifying and ordering related to quantity and number that they are ready to learn a system of notation.[10]

Learning Outcomes	Teaching Strategies/Learning Activities
Given material with which he/she is unfamiliar, the child explores the material to discover its properties and the ways in which it can be used.	- plays with a collection of 20 or more common objects or commercial materials to make patterns, pictures, or 3-dimensional structures: buttons nuts shells small toy animals or vehicles leaves geometric solids or cardboard containers attribute chips
Given a collection of objects, the child can match objects to containers having the same attribute.	- puts objects back where they belong after use, such as scissors in the scissor rack, crayons in the crayon box - puts red objects in a red box and blue objects in a blue box; rough objects on sandpaper and smooth objects on smooth paper; round objects in a round container and square objects in a square container; wooden objects in wooden boxes, and metal objects in metal boxes
Knows the labels applied to the attributes and dimensions used in sorting and classifying.	- small group, teacher-directed activity: - provide a collection of common objects - each child selects and describes an object, e.g., "a pretty blue button" - the activity can be made more challenging by asking the child to use three, or four, or five words to describe the object as "a small thin, smooth, red triangle" - the teacher may need to draw attention to attributes/dimensions not mentioned by the children

8 Hans G. Furth and Harry Wachs, **Thinking Goes to School: Piaget's Theory in Practice** (New York: Oxford University Press, 1975), p. 210.

9 Richard W. Copeland, **How Children Learn Mathematics: Teaching Implications of Piaget's Research** New York: McMillan Publishing Co., Inc., 1979), p. 61.

10 Ontario Ministry of Education, **Education in the Primary and Junior Divisions**, 1975, p. 67.

Learning Outcomes	Teaching Strategies/Learning Activities

Dimension	**Attributes**
shape (3-D)	cube, sphere, prism, pyramid
(2-D)	circle, square, rectangle, triangle
colour	red, yellow, blue, etc.
size	large, small
	long, short
	wide, narrow
	thick, thin
texture	rough, smooth
weight	heavy, light
buoyancy	sinks, floats
appearance	pretty, ugly
emotional appeal	likes, doesn't like

Given a collection of objects, the child identifies one group of articles having a common attribute but does not sort the entire collection. This is known as **consistent sorting.**	- selects objects possessing a particulr attribute from a collection of 20 or more common objects or commecial materials - names the attribute selected
Given a collection of objects, the child sorts the entire collection according to a single dimension. This is known as **exhaustive sorting.**	- sorts an entire collection of objects that differ with regards to two or more attributes (red, yellow, blue) within one dimension (colour) - names the attributes and the dimension selected - the teacher could introduce new concepts such as **negation** through suggestions such as: "Find all the things that **do not** roll."

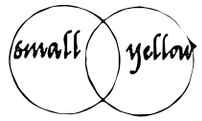
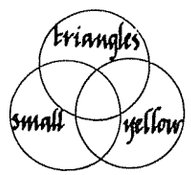
Learning Outcomes	Teaching Strategies/Learning Activities
Given a collection of objects which she/he has sorted according to a single dimension, the child resorts the entire collection according to a second, then a third dimension. This is known as **horizontal reclassification.**	- resorts a collection already sorted according to colour, according to size, and then according to shape - the teacher could prompt the resorting with: "Sort your shells another way." "Which ones sink and which ones float?
Uses the above classification skills in a game situation: - identifying and naming the attributes of an object - identifying the similarities and differences between objects	- small group game using attribute chips: - each member of the group (4) chooses a shape and then finds all the chips having that shape - each child builds a structure with his/her chips - they take turns hiding one of their chips while the others guess which piece is missing, as: "The big red square." - this game can also be played where the children choose to work with colour (3 per group), size, texture, or thickness (2 per group) - the one-difference game: "One child puts down any piece out of the set (of attribute

chips). The next child puts down a piece which is different from the first piece in one attribute only. This difference might be size only, or thickness only, or colour only, or shape only. The next player, which would be the first player if there are only two children playing, puts down a third piece, which differs from the second piece in one attribute only. The game goes on in this way until all, or nearly all, the pieces in the set have been placed in a row.

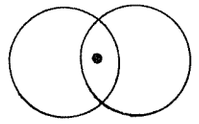
Each player has the right to challenge the previous player. This means that if a player believes that the previous player put down a piece which did not differ in one attribute only from the piece before, he can say so. If he wins his challenge, he gains a point. If he wrongly challenges a previous player, he loses a point. Each successful, i.e. unchallenged placement of a piece gains a point.... The child with the largest number of point wins the game."[11]

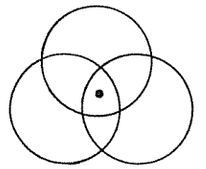
- the two-difference game:
  - this game is played in the same manner as the one-difference game except that the pieces differ from each other in two attributes only
  - it can be extended to a three-difference or a four-difference game

11 Z.P. Dienes and E.W. Golding, **Learning Logic, Logical Games** (London: University of London Press, 1966), p. 19.

Learning Outcomes	Teaching Strategies/Learning Activities
Identifies those objects which possess the attributes of both groups, i.e. the members of the **intersecting set.**	- sorts objects from two different dimensions, e.g. puts the yellow objects in one hoop and the small objects in the other hoop - the teacher asks: "Where do the objects which are small and yellow belong?" - the child realizes that they belong in both hoops and the only way to accomplish this physically is to overlap the two hoops:  - sorts objects according to three different dimensions, identifies those objects possessing attributes of two of three of the groups, and places them in the intersecting sets: triangles 

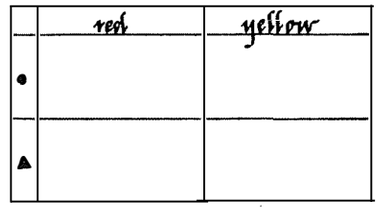
Learning Outcomes	Teaching Strategies/Learning Activities
	- determines the appropriate principal of classification, given the members of the intersecting set, and completes the sorting:[12]



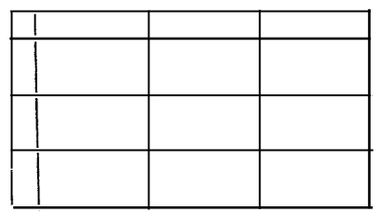


Classifies an object or objects in two or more sets simultaneously, i.e. **multiple or cross-classification**

- sorts attribute chips on a matrix according to two dimensions:



sorts attribute chips on a matrix according to three dimensions:

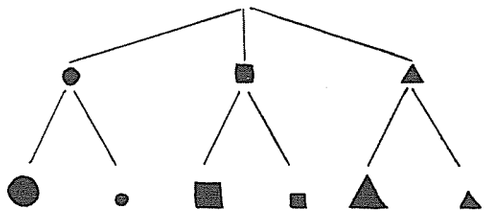


pupils may either complete a given matrix, as in the top diagram, or construct their own, as in the blank grid

12 Furth and Wachs, **Thinking Goes to School**, p. 218.

Learning Outcomes	Teaching Strategies/Learning Activities
Reclassifies objects on the basis of an increasing number of characteristics with each sub-group a part of the preceding group, i.e. **vertical classification** or decision trees.	- makes a decision tree using a collection of common objects or commercial materials:

attribute blocks



Bonnie Richardson

HIGH SCHOOL GRADES VERSUS DIAGNOSTIC TESTS AS PREDICTORS OF ACHIEVEMENT IN FIRST SEMESTER UNIVERSITY CALCULUS.

by

*G.R. Chapman and J. Weiner**

I. **Introduction.** It is the practice at many Universities to administer a test to freshmen students embarking on a first semester university course in calculus. Such a test is intended to measure the students' level of preparedness for the course which is to follow. It is given in the first week of classes, and although it is commonly refered to as a "diagnostic test", it actually has three main purposes.

The first is in fact diagnostic. From the test, a student may learn of areas in which he is weak, and then take steps to remedy the situation before these weaknesses harm his prospects.

The second purpose is one of placement, in that students are allowed (to a limited extent) to switch within the different calculus courses offered.

Finally, it is possible that the test has a certain predictive value. Each student receives a ranking relative to his peers, and one might expect this ranking to approximate the ranking given by the grades actually achieved in the first semester calculus course. Our object here is to study this correlation. In particular, we want to investigate whether the grades obtained in this diagnostic test correlate better with subsequent performance in the course than do high school calculus grades.

High school grades take into account student performance over the entire year, and represent careful judgements made by teachers who have had considerable contact with their students, whereas the diagnostic test judges performance in a single hour. Moreover the student has little or no preparation for the diagnostic test, and little incentive to perform well (since the results count nothing toward the final grade). Thus one might expect high school grades to be the better predictor. On the other hand, the diagnostic test might be construed to have an advantage, because the people who set the diagnostic test also conduct the subsequent course. Further, the fact that there is no official means of ensuring uniformity across the province in high school grading procedures might detract from the predictive value of high school grades.

There have been many studies of factors which may be used as predictors of performance in university calculus courses[2, 3, 4, 5, 6, 7, 8]. Many of these studies were done in the U.S., where external assessments of student attainments and aptitudes are available - for example SAT, ACT, etc.. In some of these studies[3, 4, 5, 6, 7], data is presented pertaining to high school grades and performance on external tests. It is interesting to note that in all these cases, high school achievement appears to be the better predictor of performance in first semester university calculus (though in some cases, the sample sizes are so small that little statistical significance can be put on the results). Howlett[3], presents evidence to suggest that high school rank is the best of the numerous predictors he considers. In fact it is argued that high school rank, when taken together with scores on external tests like ACT, SAT, yield such good predictions that "first week" tests (such as described in this paper) are unnecessary, at least from the predictive viewpoint.

2. **Procedure.** The introductory honours calculus course at the University of Guelph, 63-120, is designed for students of engineering, honours science, and mathematics. Equal stress is placed on mastery of techniques, problem solving, and theory. Students are expected to have grade 13 Calculus and Functions and Relations (or their equivalents) as prerequisites. On evaluations, students rate the course as "slightly" to "much" harder than the average University of Guelph first semester course.

The course consists of three 50-minute lectures and one 50-minute lab. per week, for thirteen weeks, and the final grade is calculated on the basis of 3 tests (5 marks each), 1 essay (5 marks), 2 midterms (20 marks each) and a compulsory 40% final. Students are given the option of a 100% final, if this proves favourable.

The one hour diagnostic test has been designed to cover many of the basic topics in the grades 9 to 13 mathematics curriculum. It is a multiple choice, computer marked test, with the questions grouped by topic. For example, there is a section on basic algebra, and another on evaluating definite integrals. Copies of the test are available on request.

In the Fall 1982 semester, when this study was undertaken, the diagnostic test was written in the first lab. hour, which (for organizational reasons) occurred in the second week of classes. Thus the students had experienced some re-exposure to mathematics before taking the test.

After the final grades were tabulated, we considered only students for whom the following three marks were available: grade 13 Calculus, the diagnostic, and a 63-120 final grade. Many of the 240 students enrolled in the course lacked one or more of these marks (due, for example, to having attended a high school out of the province, or to electing not to write the diagnostic, or to missing the final examination) and our eventual sample size was 159. Averages and correlation coefficients were calculated, and standard tests for statistical significance were applied to the latter[1].

3. **Results and Conclusions.** The 159 students had an average mark of 63% in the course 63 - 120, whereas their average in grade 13 calculus was 78.7%. This drop of about 15% agrees with educational folk lore. The worst drop experienced by an individual student was 64% (probably due to some kind of culture shock), while a handful of students increased their grade, one by 20%.

For our sample, the correlation coefficients for grade 13 Calculus marks versus 63 - 120 marks (r_{13}) and for diagnostic test marks versus 63 - 120 marks ($r_{D.T.}$) were

$$r_{13} = .6220,\ r_{D.T.} = .7329.$$

Both of these are statistically significant positive correlations. Interestingly, the correlation r_{13} is higher than any of the correlations between performance on external tests (e.g. SAT, ACT) and first semester university calculus marks given in the references.

Suppose we had other samples of students entering our engineering, honours science and mathematics programs. Would we frequently find $r_{D.T.} > r_{13}$? A test of significance (based on the sample size of 159) does not permit us to conclude with 95% confidence that the true value of $r_{D.T.}$ exceeds that of r_{13}. More data is required.

Performing linear regression on our data we find

$$Y = 1.079x_1 - 21.91 \qquad (1)$$

is the line of best fit for 63 – 120 marks versus grade 13 Calculus marks. The standard error, which can be regarded as a measure of how good a predictor this equation is, is 14.83. Some implications of this are presented in the following table, which displays, for a seletion of grade 13 marks and confidence levels, the range in which one could expect a student's 63 - 120 mark to fall. For example, if a student's grade 13 mark is 80, one could say with 60% confidence that his 63 - 120 mark will lie between 52 and 77.

		Confidence Level.		
		60%	75%	90%
Grade 13 Mark.	50	19 - 45	15 - 50	7 - 57
	60	30 - 55	26 - 60	18 - 68
	70	41 - 66	37 - 71	29 - 78
	80	52 - 77	47 - 82	40 - 89
	90	63 - 88	58 - 92	51 - 100

Similarly,

$$y = 3.534x_2 + 2.466 \qquad (2)$$

is the line of best fit for 63 - 120 versus the diagnostic test. In this case, the standard error is 12.88, which means that for this sample the diagnostic test was the better predictor.

Finally,

$$y = .4696x_1 + 2.722x_2 - 20.57 \qquad (3)$$

gives the best fit for 63 - 120 versus both grade 13 (x_1) and the diagnostic test (x_2). The standard error is 12.28.

An F-test was used to determine whether the predictions obtained via (3) were significantly more accurate than those from (2). It was found that the improvement in accuracy was significant, so that benefit is gained by considering high school marks in addition to the diagnostic marks.

A commonly told tale in recent years concerns the high school student with good marks who is nevertheless totally unprepared for university. The evidence here suggests that such students are rare. The drop of 15% from the grade 13 average is reasonable, considering the level of difficulty of 63 - 120. This, together with the high correlation r_{13} indicates that our students were, on the whole, well-equipped for university calculus.

REFERENCES

1. Alder H. and E.B. Roessler, *Introduction to Probability and Statistics (6th Edition),* San Francisco, W.H. Freeman and Co., (1977), pp. 160 - 163, 228 - 240.

2. Anderson J., and R.G. Whittemore, Predictive utility of certain criteria for advanced freshman mathematics courses. *Math. Teacher,* 60, (1967), pp. 619 - 620.

3. Howlett J.L., A study of placement methods for entering college freshmen in the proper mathematics sequence at Michigan Technical University. *Math. Teacher,* 62, (1969), pp. 651 - 659.

4. McKillip W.D., The effects of high school calculus on student's first-semester calculus grades at the University of Virginia. *Math. Teacher,* 59, (1966), pp. 470 - 472.

5. Robinson W.B., The effects of two semesters of secondary school calculus on student's first and second quarter calculus grades at the University of Utah. *J. Res. in Math. Ed.,* Vol 1, No. 1, (1970), pp. 57 - 60.

6. Scannicchio T.H., Student achievement in college calculus, Louisiana State U. 1967 - 8. *Research in Education,* Jan. 1972.

7. Schimizu M.T., Achievement in senior advanced mathematics and first-year college mathematics. *Math. Teacher,* 62, (1969), pp. 311 - 315.

8. Wampler J. F., Prediction of achievement in college mathematics. *Math. Teacher,* 59, (1966), pp. 364 - 369.

G.R. Chapman, J. Weiner
Department of Mathematics and Statistics
University of Guelph

* The authors thank Prof. B. Allan for advice and comments.

INVERSION

by
G. R. Magee, Professor Emeritus, The University of Western Ontario

Introduction

In the Euclidean plane the term "inversion" may be used to indicate a method of mapping the plane into itself by means of a circle with an arbitrarily selected centre and arbitrary radius. If this process were to be considered, the mapping would be labelled "circular inversion" and the circle would be "the circle of inversion".

During the Golden Age of Greek Mathematics (generally considered to have ended in the fifth century A.D.), geometers possessed the knowledge required to establish, with straight edge and compasses a (1,1) point correspondence in the Euclidean plane, using circular inversion. A millenium later the advent of Cartesian coordinates made it possible to accomplish this mapping by algebraic methods in which the actual use of a circle of inversion fades into the background. All that is really required is the selection of an arbitrary point 0 in the plane and an arbitrary real number, say k^2. (These primary elements are the relics of inversion with respect to a circle with centre at 0 and radius k.) The point 0 may be referred to as the centre of inversion.

In this article it is proposed to discuss inversion without focussing attention on a circle of inversion. From the point of view of algebra, the maps ("inverses") of conics (with the exception of circles) have representative equations of order greater than two in Cartesian coordinates and at one time were called "higher plane curves". Nothing that is new will arise from the following example of inverting the parabola--hopefully the purpose is to assist in arousing interest in the history of mathematics. Some higher plane curves were developed in efforts to solve the Three Famous Problems of Antiquity. In the matter of maintaining interest in the history of mathematics, tribute should be offered to a former editor of the Ontario Mathematics Gazette, namely Professor Hans P. Heinig of McMaster University, Hamilton.[1]

The Mapping Process

As indicated in the introduction, it is necessary to choose a point 0 as centre of inversion and a positive number k^2. If P is any point in a Euclidean plane, other than the point 0, define the inverse of P to be the point P' on the ray OP such that the produce $OP \bullet OP' = k^2$, or equivalently

(1) $$OP' = \frac{k^2}{OP} .$$

This relation describes a process which maps the plane into itself. The point 0 has no corresponding point (or image) in this plane. It is convenient to accept the convention that a single "ideal" point, called P_∞, shall be added to the Euclidean plane and that this ideal point be regarded as lying on every line of the plane; further, that P_∞ shall be regarded as the inverse of the centre of inversion 0. A Euclidean plane thus augmented is called an "inversive plane". Using this convention, relation (1) represents a mapping $P \leftrightarrow P'$ having coordinates (x,y) and (x', y') respectively. Then

$$\frac{x}{x'} = \frac{OP}{OP'} = \frac{OP \cdot OP'}{OP'^2} = \frac{k^2}{x'^2+y'^2} , \quad \frac{y}{y'} = \frac{OP}{OP'} = \frac{k^2}{x'^2+y'^2} .$$

Consequently inversion with respect to 0 in the inversive plane may be represented algebraically by

(2) $$x = \frac{k^2 x'}{x'^2+y'^2} , \quad y = \frac{k^2 y'}{x'^2+y'^2} .$$

If P(x,y) is a member of a set of points comprising a curve C, then equations (2) will map *C* onto its inverse *C'*. Any alteration in the value k^2 merely establishes a central dilation about 0-- no loss of intrinsic geometrical properties of *C* vis-a-vis *C'* will occur.

With a suitable choice of coordinate axes an equation of a parabola may be written in a standard form by

(3) $$y^2 = 2ax.$$

It will be assumed that $a > 0$, so that the parabola (3) is symmetric about the positive x-axis. The canonical form (3) gives the information that the focus of the parabola is the point (a/2,0) and the latus rectum is 2a. Employing equations (2), a map of (3) is given by

$$k^2y'^2 = 2ax'(x'^2 + y'^2).$$

Suppressing the primes and choosing $k = 2a$, the inverse of the parabola (3) is represented by

$$(4) \qquad y^2 = \frac{x^3}{2a - x}, \quad 0 \leq x < 2a.$$

Equation (4) may be displayed as the cubic equation

$$(4') \qquad 2ay^2 - x(x^2 + y^2) = 0.$$

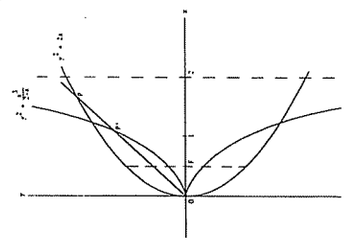
Equation (4) or (4') is an algebraic representation of a curve now called the Cissoid of Diocles (circa Second Century B.C.). At least a century earlier than the lifetime of Diocles, conics were known to Greek geometers as sections (slices) of cones. Undoubtedly Diocles constructed an outline of the cissoid by applying circular inversion (with straight edges and compasses) to the parabola. His purpose was probably an attempt to solve one of the Three Famous Problems of Antiquity, namely the Delian Problem. The source of this problem is buried in Greek mythology; the problem is really that of finding a cube root of a real number using straight edge and compasses, in particular, the number $\sqrt[3]{2}$.[2] This problem is an out-growth of finding square roots of numbers, as demonstrated by Pythagoras four centuries before Diocles.

It is unlikely that Diocles and his contemporaries knew the over-all appearance of the cissoid. Equation (4) gives the immediate information that the curve has a vertical asymptote, namely $x = 2a$. As would be expected from the symmetry of the "parent" parabola, the cissoid has the same axis of symmetry. A pair of self-corresponding points in the mapping of the parabola onto the cissoid occurs where $x = (\sqrt{5} - 1)a$. Also, the cissoid meets the parabola at the vertex of the parabola (the origin of the coordinate system) but this point is not a self-corresponding point in the mapping--according to its postulated existence P_∞, as a point on the cissoid, corresponds to the vertex of the parabola. Note that the finite self-corresponding points lie in that region of the plane which is bounded by a vertical line through the focus of the parabola and the asymptote of the cissoid. If circular inversion had been employed the self-corresponding points are on the circle of inversion with centre at 0 and radius $k = 2a$. As a matter of passing interest, this circle (lurking in the historical background) is tangent to the asymptote of the cissoid.

To obtain further information about the cissoid, modern methods would apply differential calculus to (4) and thus obtain

$$(5) \qquad \frac{dy}{dx} = \frac{(3a - x)x^2}{(2a - x)^2y}.$$

Recalling that $0 < x < 2a$, it will be observed from (5) that tangent lines at points along that portion ("branch") of the cissoid which lies above the x-axis have positive slopes; along the branch which is below the x-axis the tangent lines have negative slopes. Hence the cissoid is concave upwards above the x-axis and concave downwards below the x-axis. The calculus method of measuring slopes of tangents yields an indeterminable result at the origin. However, methods developed in the study of higher plane curves produce the information that tangent lines along the two branches of the cissoid merge into the line $y = 0$ as the contact points meet at the origin. Thus it is known that the cissoid has a cusp at the origin, with the x-axis serving as cuspidal tangent.[3] A diagram follows, showing the cissoid as a map of a parabola.



Epilogue (a brief historical sketch)

In the introduction, reference was made to Professor Heinig's listing of "well-known conjectures and problems from various branches of mathematics ranging through the centuries". Some of these problems, in particular the Three Famous Problems of Antiquity, are now known to be unsolvable under the original specifications (straight edge and compass only). Nevertheless, the efforts of many early mathematicians, mainly geometers working in Athens, on Greek Islands, and at Alexandria, have borne fruit in the sense that through their interests much has been contributed to the development of mathematics over many centuries. It seems appropriate to refer briefly to the works of some Greek geometers who preceded Diocles.

Out of the school founded by Plato at Athens near the beginning of the Third Century B.C. came the discovery of conics as sections of a cone--this is usually credited to Menaechmus, an associate of Plato. Menaechmus knew that parabolas of varying "focal widths" (latera recta) could be made by positioning the cutting plane relative to the vertex of a right circular cone.[4] During the last half of the Fourth Century B.C., Hippocrates, working on a Greek Island called Chios, considered the problem of finding two mean proportionals between a given line segment and another with length twice the first segment. In modern symbolism, if x and y represent the desired line segments, then

(6) $$\frac{a}{x} = \frac{x}{y} = \frac{y}{2a}.$$

Equations (6) imply

(7) $$y^2 = 2ax, \quad x^2 = ay.$$

Thinking now in terms of Cartesian coordinates it is clear from (7) that an intersection point of two parabolas with a common vertex and mutually perpendicular axes of symmetry is the point with coordinates $a\sqrt[3]{2}$, $a\sqrt[3]{4^5}$. Note that the latus rectum of one parabola is twice that of the other parabola. The Delian Problem required the determination of the value of the real number $\sqrt[3]{2}$. Clearly this is not as simple as the problem of determining a line segment equal to $\sqrt{2}$, which is the length of the hypotenuse of a right-angled triangle with sides of unit length. The straight edge and compasses construction of a right angle has, of course, been known since the days of Euclid (Third Century B.C.).

From available records, Diocles may have been a pioneer in the point-to-point mapping of a plane. While his objective, namely a search for $\sqrt[3]{2}$, was unobtainable by geometrical methods, his construction of at least a portion of the cubic curve called the Cissoid inspired a number of later mathematicians to invert conics using centres of inversion geometrically related to whatever conic was to be mapped. For example, if an ellipse is inverted using one of its focal points as centre of inversion the inverse curve is a heart-shaped curve with a cusp on the major axis of the ellipse; this curve is called a "cardioid" and was of interest to Pascal (1623-1662). A mid-Twentieth Century edition of the Encyclopedia Brittanica lists more than sixty special plane curves, some of which may be generated by inversion. It would appear that Sir Isaac Newton (1642-1727) was interested in the legacy of Diocles, for Newton constructed a simple linkage of rods which would trace out a Cissoid.

Acknowledgement: The author wishes to thank Professor B.L.R. Shawyer for the meticulously drawn diagram.

REFERENCES

1. Hans P. Heinig, O.A.M.E. Gazette, Vol. 21 (1983), pp. 27-31.

2. Carl B. Boyer, A History of Mathematics, pp. 71, 75; John Wiley and Sons, 1968.

3. A. B. Bassett, An Elementary Treatise on Cubic and Quartic Curves, pp. 11 - 12; Bell and Cambridge (1910) (probably out of print).

4. See Boyer, p. 103.

5. Ibid., p. 105.

6. Ibid., p. 103.

G. R. Magee
c/o Department of Mathematics
University of Western Ontario

CROSS-FIGURE PUZZLES

by

William A. Ewbank

Editor's Note: *These three puzzles are part of a collection submitted by Dr. Ewbank. If reaction is favourable, others will be included in future issues.*

Cross-figure (or cross-number) puzzles, as a companion to crossword puzzles, have been in use for a long time. Usually they are a form of concealed drill, and as such they do have the merit of being self-checking. Others have clues of a social, literary, or environmental nature, such as "golf course number" for "18", or "number of Snow White's little admirers" for "7". But cross-figure puzzles do not seem to have the fascination of crosswords, possibly because they lack the challenge.

A much less used type of cross-figure puzzle is the one used in the following puzzles. It has a 4 x 4 configuration using 16 digits, fenced by bars to form two-, three-, or four-digit numbers; and it appears to be insoluble. No definitive clues are given, and all clues seem to be circular, that is, they refer to each other. The puzzles do not seem to have a place to start. Puzzles of this kind are miniature models of real-world problem solving: often there is doubt about where to begin; clues must be examined; evidence must be sifted; trial and error must be employed; and sometimes there may be no single correct solution.

The following puzzles were invented by my students.

Short puzzles of this nature are very easy to invent, but giving just the right number of clues is difficult. Clues can conveniently use the following concepts:

- The four fundamental operations
- Powers and roots, multiples and divisors
- Digital totals (e.g., 203, 14, and 1004 each have a digital total of 5)
- Palindromic numbers (e.g., 121, 2002, and 3773)
- Prime numbers
- Perfect numbers (e.g. 28 is "perfect" because the sum of its proper divisors, 1, 2, 4, 7, and 14, is exactly 28)

Puzzles of this nature could be assigned to upper middle school pupils and those in higher grades as homework or for extra credit. If you assign the job of inventing them to the whole class, however, you will have a big editing job on your hands! A puzzle that is too easy or too difficult defeats its object, and many puzzles will fall into this category at first. After assigning a simple puzzle or two to the whole class, you might want to invite the more gifted and talented members, or anyone else who seems particularly interested, to invent their own as homework, perhaps solving each other's in a small group. It is also a good pursuit for members of a mathematics club.

Challenging cross-figure puzzles can be of any size or format from 2 x 2 upward, can be of any shape, and can use bars or blocked-out squares, as in the more familiar crosswords.

CROSS-FIGURE PUZZLE (1)

by Joann Aeschliman

ACROSS

a a perfect number

b (b down)^2

e prime with digital total 17

f 10 x a cubed number

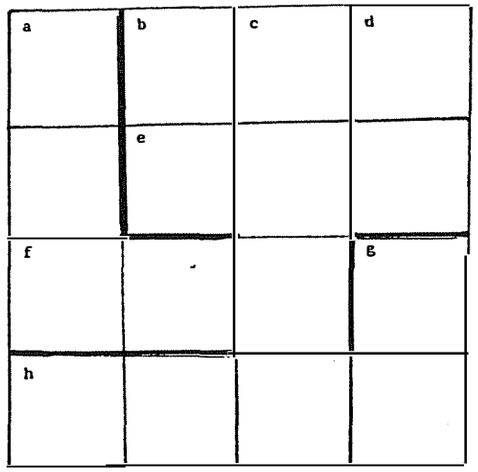
h (d down) x (g down)

DOWN

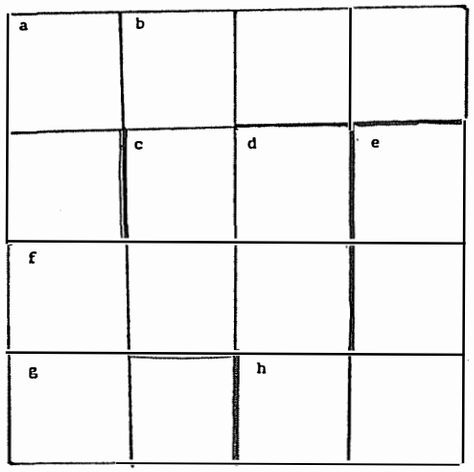
a (f across) - (b down), and is palindromic

c (f across) x (g down)

g has same digital total as, and is a factor of (h across)



CROSS-FIGURE PUZZLE (2)
by Kathryn Cabush



ACROSS

a (a down) + (e down)

c (g across) x 2

f a prime with 2 digits the same

g a prime which is one more than a square number

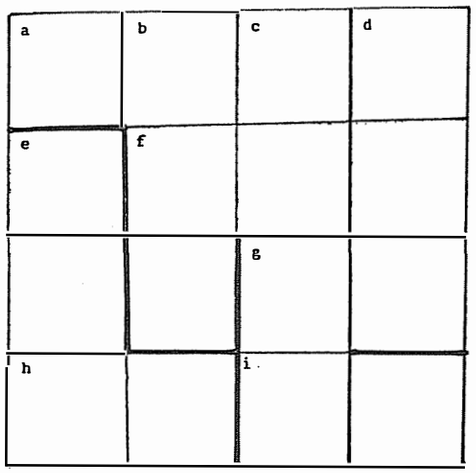
DOWN

b (c across) + 800

d (b down) - (f across)

e $(\text{h across})^2$

CROSS-FIGURE PUZZLE (3)
by Amy Fuller



ACROSS

a $(\text{g across})^2$

g digital total 10

h a palindromic prime

i first two digits of (f across)

DOWN

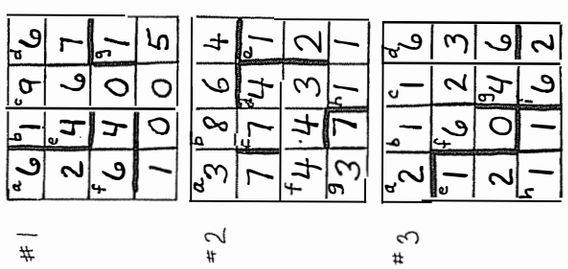
b 10 times $(\text{a down})^4$

c (f across) x 2

d (f across) + 13, and is palindromic

e $(\text{h across})^2$

SOLUTIONS



INTENTIONS AND ATTAINMENTS IN THE TEACHING AND LEARNING OF ONTARIO GRADE 12/13 MATHEMATICS:

RESULTS FROM THE SECOND INTERNATIONAL MATHEMATICS STUDY

by
Dennis Raphael, Merlin Wahlstrom, Les McLean
Educational Evaluation Centre
Ontario Institute for Studies in Education

Editor's Note: *The Grade 8 information from this study was included in the December* ***Gazette.*** *Interested readers should also refer to the* ***Interview*** *in this month's* ***Gazette*** *for a summary of the B.C. results. While this article is long and a bit "heavy" on the research, the content* ***should*** *raise some important issues. We need reader reaction (e.g. do the proposed new Senior Guidelines seem to be addressing themselves to the learning shortcomings identified in this study?)*

During the school year 1981-1982, as part of Ontario's participation in the Second International Mathematics Study (SIMS), several thousand Grade 8, 12 and 13 students, along with their teachers and school principals contributed information concerned with the teaching and learning of mathematics in the Province of Ontario. Similar data were collected in over twenty nations. (For study purposes, Ontario and British Columbia were regarded as 'nations'.)

Design of the Study

The design of SIMS had three components. A curriculum analysis examined the context in which teaching and learning was taking place within each participating nation. Prime focus was upon the mathematics content expected to be conveyed to the students enrolled in mathematics classes--the "intended Curriculum." Second, teachers supplied many details of their concepts of mathematics, their priorities and their teaching methods--the "implemented Curriculum." The unit of study was one of the teacher's classes within the school, the "target class." Finally, students answered questions designed to measure their mastery of the mathematics content and their attitudes towards mathematics and the learning of mathematics--the "Attained Curriculum."

Two age populations were studied in SIMS. Population A was the grade during which the majority of students had reached the age of 13. In Ontario, British Columbia and the United States this corresponded to Grade 8. Population B was those students in the normally accepted terminating year of secondary school who were studying mathematics as a substantial part (approximately five hours a week or more) of their academic program. In Ontario the group best fitting this definition were those Grade 13 students enrolled in two or more Grade 13 mathematics courses, while in British Columbia and the USA, the population B students were in Grade 12. In Ontario, data was collected from Grade 12 classrooms to allow comparisons to Grade 12 students in other countries.

The Ontario study was funded by the Ontario Ministry of Education and conducted by the Educational Evaluation Centre (EEC) of the Ontario Institute for Studies in Education. Internationally, SIMS was a project of the International Association for the Evaluation of Educational Achievement (IEA). An earlier report in the Gazette (1) provided details concerning the design of the study, the mathematics educators participating in the study as members of the Ontario Steering Committee (OSC), and results from the Ontario Grade 8 study. Other reports summarizing aspects of SIMS and the Ontario results have been published in Orbit (2,3) and an analysis of the teaching and learning of Geometry in Ontario Grade 8 classrooms is available (4).

Present Report

The present report summarizes the Ontario Grade 12/13 achievement findings analyzing the correspondence between the Intended, Implemented and Achieved curriculum in the Province. Focus will be upon the three main Grade 13 courses of Relations and Functions (called Relations in this paper), Calculus and Algebra and an analysis of the Intended, Implemented and Attained Curriculum in the Grade 12 Foundations of Mathematics course. Finally, achievement levels of Ontario Grade 13 Mathematics Specialists (those comparable to IEA samples in the other nations) will be examined.

Validating the SIMS Achievement Items for Ontario

There were 136 SIMS achievement items at the Population B level. These items were placed, for administration purposes, into 8 forms which were rotated through a class. Within each class therefore, a student completed one of eight forms at a sitting. Using this matrix sampling approach, responses to the full 136 items were obtained with minimal testing time expended by each individual student.

Each student completed two different forms over two testing periods near the end of their course. The first step in validating the SIMS items for use in Ontario classrooms was the rating by the Ontario Steering Committee as to whether the content of the items were likely to be covered in a typical Grade 12 and/or 13 Ontario mathematics class. Raters felt that over 90% of the items were covered in either the Grade 12 Foundations or Grade 13 courses. Detailed breakdown of the validity of items for the different courses is in a following section of this report.

In a similar way, items were rated as to whether the content required to answer the questions were covered in widely used Ontario textbooks. Analyzing the content of textbooks for the Grade 12 Foundations and the three Grade 13 courses revealed that virtually all items (94%) were covered somewhere in these texts. Sixty-eight per cent of items were covered or assumed as prerequisite knowledge in a widely used Grade 12 Foundations of Mathematics textbook, 39% in a Relations text, 29% in a Calculus text, and 36% in an Algebra text. The items were seen, therefore, as appropriate for administration to Ontario Grade 12/13 students.

Sample Selection at the Population B Level

By international definition, Population B included students in the normal final year of secondary school for whom the study of mathematics constituted a significant portion of their course content. In Ontario, this was taken to include those Grade 13 students enrolled in (or had already taken) two or more of Relations, Calculus, and Algebra. Schools in Ontario containing Grade 12 and 13 classes were stratified by (a) size of community (b) school type (private, French, public), (c) ratio of number of Grade 12 to Grade 13 students (a socioeconomic index) and (d) geographical region of the province. Five schools were selected from each of 17 strata for a total of eighty-five schools.

Within each school one class of Relations, Calculus, Algebra and one Grade 12 Foundations of Mathematics class was chosen to participate in the study. A random choice was made by the OISE team wherever there was more than one of any type of class in a school. The Foundations class was added because Grade 12 is the last year of secondary school for some students and because it corresponds somewhat to British Columbia and U.S. Grade 12.

Grade 12 and 13 students were administered the achievement measures only at the end of their mathematics courses. For semestered courses, testing took place for some classes in December and for others in May. Full year courses were tested in May.

Principals provided school data at the beginning of the school year and teachers completed questionnaires over the duration of the mathematics course. In all, data were collected from 85 principals, 267 Grade 12/13 teachers, and 6078 students. Of these students, 3214 were or had taken two or more Grade 13 Mathemtics courses and were included in the Ontario International Population B sample. The care with which the sample was chosen allows the results to not only describe characteristics of the studied schools, teachers and students but allows for the description of the Intended, Implemented and Attained curricula in the Province as a whole.

The Intended, Implemented and Attained Curriculum

The 136 SIMS population B achievement items were organized by IEA into 20 content areas. Twelve of these content areas were particularly relevant to the Ontario Grade 12/13 curriculum and will be the focus of this report. These content areas are: Sets, Relations, Number Systems (excluding Complex Numbers), Complex Numbers, Analytic Geometry, Algebra (computation), Algebra (other than computation), Equations and Inequalities, Functions, Trigonometry, Differentiation, Integration and Probability and Statistics.

The Intended Curriculum in Ontario

The Intended Curriculum in Ontario was the Ministry of Education Senior Division Mathematics Guideline. The content of the Guideline is well known and will not be detailed here. Members of the Ontario Steering Committee (OSC), based upon their knowledge of the Guideline, rated each achievement item as either appropriate, that is, the mathematics required to successfully answer the item should have been taught prior to or during the Grade 12 or 13 course, or was not covered and therefore inappropriate. The OSC estimates of appropriateness of items constitutes the Intended Curriculum in this report.

The Implemented Curriculum in Ontario

Teachers of each target class were asked to indicate whether the mathematics required to correctly answer each achievement item had been taught during the school year. If it had not been taught, teachers indicated whether it was due to a) the content having been taught prior to the particular course; b) is taught in another course; c) not in the curriculum at all or d) other reasons. In all 267 (66 Grade 12 Foundations of Mathematics, 69 Relations, 65 Calculus, and 67 Algebra) teachers contributed this data and their reports constitute the Implemented Curriculum in Ontario.

In this report, the expression "items were covered" should be read as "the mathematics required to correctly answer the item had been covered." Results reported are the average percentage of items in a content area covered by the Grade 12/13 teachers.

The Attained Curriculum in Ontario

Though each content area consisted of items of varying difficulty, it was possible to compute an average difficulty for items within a particular area. The Attained Curriculum was represented by the average percentage of students correctly answering these items in a SIMS content area. For example, the statement "49% correct for the Sets, Relations content area" would indicate that the items in this area were on average answered correctly by 49% of students. The data presented in this report were contributed by students in the 79 schools and 270 clases which contributed achievement data.

Results from the Grade 12/13 Study

An important issue is the extent to which the SIMS achievement items map upon the Ontario Grade 12/13 mathematics courses. As noted earlier, virtually all SIMS items are covered in one of the four courses of interest, yet the degree to which the SIMS Population B content area items, are covered in any one course varies greatly. As shown in Table 1, a majority of the Sets, Relations, Number Systems, Analytic Geometry, Algebra (both content areas), Equations and Inequalities, Functions and Trigonometry items are seen as appropriate for Grade 12. Similarly, the Differentiation and Integration content area items are seen as appropriate for the Calculus course. The items seen as appropriate for the Relations and Algebra courses cut across virtually all the SIMS content areas making it difficult to identify specific SIMS content areas apprpriate for these courses.

Table 1

OSC Estimates of Percentage of SIMS Population B Content Areas Covered by Ontario Courses

Content Areas	No. of Items	Grades 11,12	Relations	Calculus	Algebra
Sets, Relations, Functions	7	100	43	0	29
Number Systems	11	72	18	9	18
Complex Numbers	6	0	50	100	50
Analytic Geometry	6	83	17	0	33
Algebra (computation)	10	80	0	0	30
Algebra (other than computation)	16	88	25	0	25
Equations and Inequalities	13	100	8	8	23
Functions	17	76	35	12	6
Trigonometry	8	88	38	0	0
Differentiation	14	7	0	78	0
Integration	12	0	0	92	0
Probability and Statistics	6	67	33	17	33

The Plan of Presentation

Since there appears to be a reasonable match between the Grade 12 Foundations of Mathematics and Calculus courses and selected SIMS content areas, analysis for these courses will be made between the Intended, Implemented and Attained Curricula for these two courses. The Relations and Algebra course reports focus upon the Intended and Implemented Curricula only since it is difficult to examine achievement levels of students enrolled in these courses as the SIMS content areas map poorly upon individual courses. Also, students enrolled in one Grade 13 course are usually enrolled in another Grade 13 course and may be learning the content of items from these other courses. This phenomenon is not a problem for the Grade 12 and Calculus courses. However, achievement levels of mathematics specialist (those having taken two or more Grade 13 courses) will be examined to provide indirect measures of the Attained Curricula for all three courses.

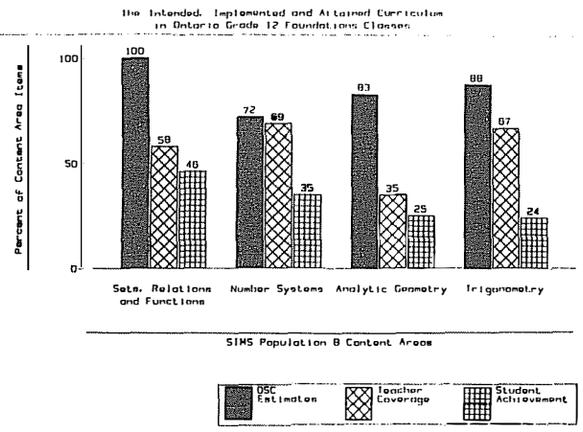
The Grade 12 Foundations of Mathematics Course

The Grade 12 Foundations of Mathematics course contains topics on Geometry, Relations, Quadratic Functions and Equations, Analytic Geometry, Trigonometric Functions, Exponential and Logarithmic Functions, Sequences and Series, Mathematical Induction and Statistics and Probability. Each topic contains core and optional subtopics.

The Intended, Implemented and Attained Grade 12 Curricula

The OSC noted eight SIMS content areas as containing items appropraite for the Grade 12 mathematics courses (see Table 1). Since Grade 12 teachers indicated whether the mathematics required to answer the items had been taught, and students enrolled in the course attempted to answer the items, comparison between the Intended, Implemented and Attained Curriculum can be made. Results for these relevant content areas for the Grade 12 Foundations course follows.

Figure 1



Sets, Relations

- of the seven items, the OSC rated 29% appropriate for Grade 11 and the remaining 71% as appropriate for the Grade 12 Foundations of Mathematics course. All items (100%) therefore were seen as appropriate for Grade 12 students.

- on average, Grade 12 Foundations teachers reported covering the mathematics associated with 40% of the items. An additional 18% of items were seen as taught prior and therefore assumed known by students. Overall then, 58% of the items were seen as assumed, known or taught.

- sets, Relation items were, on the average, answered correctly by 46% of students enrolled in Grade 12 Foundations of Mathematics.

Number Systems

- of the 11 items, the OSC rated 63% as appropriate for the Grade 11 and an additional 9% as appropriate for Grade 12 for a total appropriateness rating of 72%.

- on average 28% of Number Sysems items were covered during Grade 12 with an additional 41% assumed taught prior. Total coverage was therefore 69%.

- items were, on the average, answered correctly by 35% of Grade 12 Foundations students.

Analytic Geometry

- of these 6 items 83% were seen by the OSC as appropriate for Grade 11 Foundations. No items were seen as appropriate for the Grade 12 Foundations course.

- on average, 16% of the items were taught during Grade 12 with an additional 19% assumed taught prior. Total coverage therefore was 35%.

- Analytic Geometry items were, on the average, answered correctly by 25% of students.

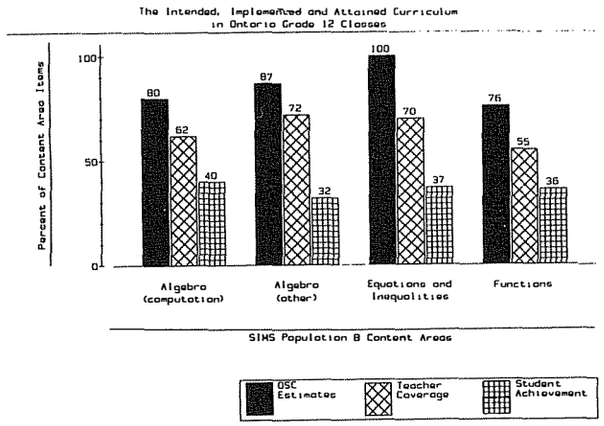
Trigonometry

- of the 8 items concerned with Trigonometry 88% were rated as appropriate for the Grade 12 Foundations course.

- teachers reported covering, on average, 64% of the mathematics required to answer these items. An additional 3% was assumed taught prior, for a total coverage of 67%.

- items were on average answered correctly by 24% of Grade 12 Fondations students.

Figure 2



Algebra (Computation)

- of the 10 items involving computation aspects of Algebra, 60% were rated as appropriate for Grade 11 Foundations and an additional 20% for Grade 12 Foundations. Eighty percent of items were therefore seen as appropriate.

- on average, teachers reported covering 47% of items during Grade 12 with an additional 15% assumed taught prior. Total coverage was therefore 62%.

- items were, on the average, answered correctly by 40% of students enrolled in the Grade 12 Foundations course.

Algebra (Other than Computation)

- of the 16 Algebra items involving either Comprehension, Application or Analysis, 56% were rated as appropriate for Grade 11 and 31% for Grade 12. Therefore, 87% of items were rated appropriate.

- on average, Grade 12 teachers reported covering 46% of items during Grade 12 and assumed that an additional 26% were taught prior for total coverage of 72%.

- items were answered correctly, on average, by 32% of Grade 12 Foundations students.

Equations and Inequalities

- the OSC rated all 13 items as appropriate. Eight-five percent were seen as taught during Grade I I and an additional 15% during Grade 12.

- on average, Grade 12 Foundations of Mathematics teachers covered 39% of items. An additional 31% of items wre assumed taught prior to Grade 12 for a total coverage of 70% of items.

- items were, on the average, answered correctly by 37% of students.

Functions

- the OSC rated the 17 Function items as follows: 76% were appropriate for Grade 12; none were felt to be taught during Grade 11.

- on average 48% of the items were reported covered during Grade 12 and an additional 7% assumed to have been taught prior.

- items, on average, were answered correctly by 36% of Grade 12 students.

Analysis: The Intended, Implemented and Attained Grade 12 Foundations Curriculum

Figures I and 2 show the correspondence between the Grade 12 Foundations, Intended, Implemented and Attained Curricula. For the Implemented Curriculum, figures include both teacher report of teaching the items as well as assuming the content had been taught prior. For all SIMS content areas, Grade 12 teachers, on average report less coverage than would be expected according to the OSC estimates. While this divergence is minimal for the Number Systems and the two Algebra content areas, this is not the case for the other SIMS content areas. For Analytic Geometry, the divergence is striking. Reasons for the findings may be that Grade 12 teachers assume that the content will be covered in a Grade 13 course. The average percentage of items not covered for this reason is as follows: Sets, Relations (24%), Number Systems (20%), Analytic Geometry (40%), Trigonometry (25%), Algebra (both 15%), Equations and Inequalities (10%) and Functions (22%). Grade 12 teachers, in summary, cover less content than might be expected from an anlysis of the Grade 12 course Guideline.

Concerning achievement, the Sets, Analytic Geometry, Algebra (computation), and Funtions content areas scores correspond reasonably to the extent of teacher coverage. Number System, Trigonometry, Algebra (other) and Equations and Inequalities achievement levels merit some attention as most of these items (83% and 85%) are rated by the OSC as taught in Grade 11.

All-in-all, Grade 12 student findings are as follows: a) teachers cover fewer items than was expected; b) student performance is lower than would be expected considering the content of the course syllabus; and c) students do especially poorly on items appropriate for Grade 11.

The Grade 13 Calculus Mathematics Course

The Grade 13 Calculus Course includes topics of Slopes and Simple Derivations, Applications of Differentiation, Equations of the Form Dxy = f(x), and Complex Numbers and Polar Coordinates. All of the Calculus topics are considered core topics. The following section focusses upon the SIMS Differentiation and Integration content area; the Complex Numbers items are considered later.

- Grade 13 Relations teachers cover 12% of items and assume 8% taught prior. Sixty percent of items are assumed to be taught in a different Grade 13 course; 11% not in the curriculum and 9% were not taught for other reasons.

- Grade 13 Calculus teachers cover 20% of the items and assume knowledge by students of an additional 8%. Forty-three percent of items are assumed to be taught in another course, 13% not in the curriculum and other reasons were given for not covering 16% of items.

- Algebra teachers on average, cover 28% of items and assume 7% taught prior. Thirty-three percent of items are assumed taught in another course, 20% not in the curriculum and 11% are not covered for other reasons.

- items are answered correctly by 32% of Ontario Grade 13 Mathematics Specialists.

Figure 6

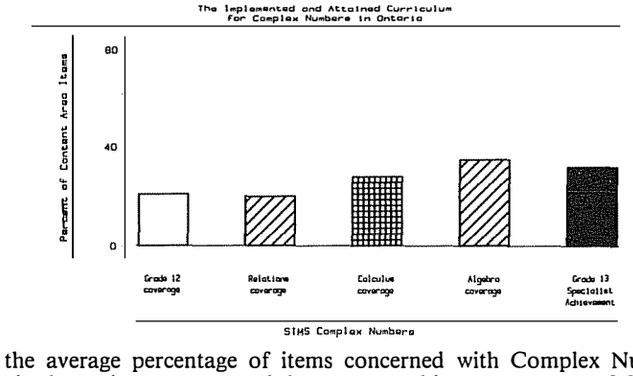


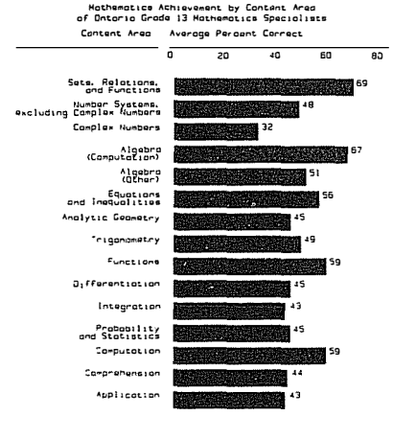
Figure 6 shows the average percentage of items concerned with Complex Numbers reported covered by teachers in the various courses and the average achievement scores of Ontario Grade 13 Specialists. (It should be noted that the Complex Numbers achievement scores as a function of course do not differ from the specialist achievement levels).

Coverage of Complex Numbers is sporadic. Many Grade 13 teachers (in each of the different courses) report that the topics is covered in another course. Not surprisingly, student achievement in this area is very low.

Grade 13 Mathematics Specialists Achievement

Grade 13 Specialist (those enrolled or completed two or more Grade 13 mathematics courses) scores were highest for the Sets, Relations and Functions, Algebra Computation and Equations and Inequalities content areas. See Figure 7. Considering the lack of coverage in Grade 13 courses, it was not surprising to observe poorest performance on Complex Numbers, where less than one-third of the items were correctly answered by Ontario mathematics specialists.

Figure 7

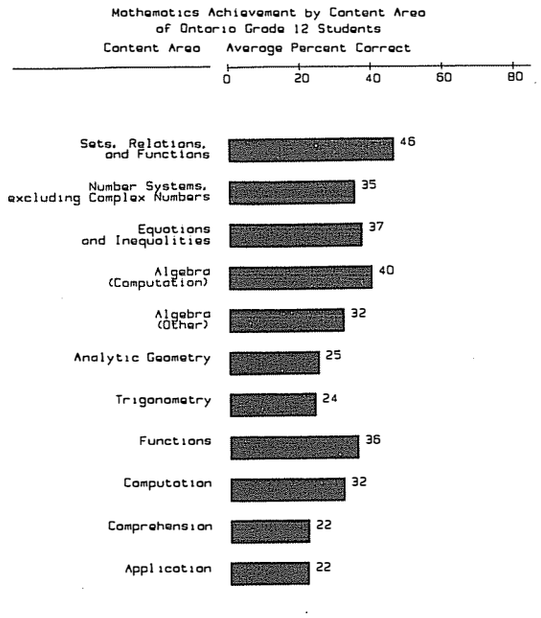


Interpretation of these findings will be made by Ontario mathematics educators and Ministry officials according to their expectations and in the light of results from the other participating countries.

Summary and Conclusions

The current revisions being undertaken of the Ontario Secondary Program and the associated Grade 13 mathematics courses requires careful study of these findings. Ontario Grade 12 students do not do especially well on items designed by International Mathematics Educators (under the auspices of SIMS) to measure mathematics content taught in the normal terminating year of secondary school.

Figure 8



Interestingly, the Relations and Algebra courses contain content cutting across a wide domain of mathematics knowledge. Those concerned with rewriting the Guideline may wish to reorganize the content of these courses with the aims of focussing specific content within particular courses in order to assure adequate coverage of important areas of mathematics knowledge. Analyses of the curricular organization of other participating SIMS nations and their associated mathematics achievement may prove helpful in these tasks.

DEPARTMENTS



INTERVIEW WITH DAVID ROBITAILLE

by
R. S. Smith

Dr. David Robitaille is the Head of the Department of Mathematics and Science Education at the University of British Columbia's Faculty of Education. I first met David in San Diego at an N.C.T.M. convention a few years ago and solicited an article for the journal which he wrote in English and French. Today we are in his office in the Scarfe Building on a beautiful August day just before the BCAMT convention.

RS: David, a word about your background, before we start on your involvement with the Second International Study of Mathematics.

DR: I taught in Montreal for quite awhile and eventually became the math coordinator there. I left in 1971 to take a job at the faculty here in British Columbia as an assistant professor, was promoted to full professor in 1982 and became head of the department a year ago, July 1, 1982 just before going on leave.

RS: And during the lst couple of years you have been very much involved with the International Association for the Evaluation of Educational Achievement (IEA)?

DR: Yes, they're the group that began by conducting an international survey of mathematics achievement in twelve countries in the early 60's. Canada didn't take part in that study but you may remember something about it. Most of the headlines concerned the fact that the United States did so poorly. That study turned into pretty much of a horse race.

RS: But Canada did become involved in the second study.

DR: Yes, the data were collected between 1980 and 1982 in different countries.

RS: Specifically, the B.C. Ministry of Education became involved and you were asked to help.

DR: Not exactly. I contacted the Ministry and urged them to fund B.C.'s participation in the study. The Ministry provided the money for the study, selected the schools and was instrumental in getting the schools to agree to cooperate. I was asked to coordinate the study here in B.C., to act as a liaison between the province and the organization, and to supervise the operation and the writing of the B.C. report.

RS: Is the report finished now?

DR: Yes. The report was published in the fall of 1981. It concerns B.C. strictly.

RS: It was early findings in that other countries hadn't completed their data collection so you really couldn't compare results?

DR: That's correct.

RS: What were the purposes of the second IEA study?

DR: The three purposes of the study were:

1. to compare curricula at the secondary school level in mathematics in the 24 countries taking part in the study;

2. to compare teaching practices used by teachers of mathematics at the secondary school level;

3. to compare the results of the whole process by looking at students' achievement and students' attitudes towards mathematics.

RS: Which groups of students were studied?

DR: There were two groups of students. The first one was defined as thirteen year olds, which is Grade 8 here in B.C. the first year of secondary school. The second group was defined as students in the last year of secondary school taking a significant amount of mathematics which turned out to be the Grade 12 level in our case.

RS: Well, since you've already written the report, perhaps you could take a moment to talk about the major findings in let's say the first area - curriculum analysis.

DR: The curriculum analysis internationally shows that there is a great deal of comparability of math curricula at the 13 year old or Grade 8 level. Many of the same topics are common to the vast majority of countries and the exceptions come in places like France and Belgium, for example, where there is considerably more emphasis on geometry than there is in other places and it's transformational geometry rather than plane Euclidan geometry. Also, some places such as Japan where the course at that level appears to be more oriented to algebra than to arithmetic as it is here. On the whole, however, there is a great deal of comparability.

RS: Is that also true in the last year of secondary school?

DR: No it isn't at all. There's quite a change at that level and it has to do, I suppose mostly with the selectivity or what Howard Russell of OISE has called retentivity of systems.

RS: How does he define that word?

DR: B.C., Ontario and the United States retain more students studying serious mathematics than anywhere else. In B.C., 40% of the students in grade 12 are following what could be called a pre-calculus math course. The proportion is very similar in Ontario and the U.S. In Ontario at the grade 13 level, if I'm not mistaken, the proportion is even higher of students that are taking one or more grade 13 level math courses. In other countries, England for example, you're talking at that level of only 6% of the students.

RS: That's in line with their greater selectivity of students going on to university and a wider variety of opportunity in trades and in their post secondary technical schools.

DR: I don't know all of the reasons, but the fact is that we seem to retain a lot more students in mathematics than is the case in most other countries. Japan is a notable exception, Japan also retains a very high proportion of students.

RS: Interesting. Let us look at the second area, that is what really happens in the classroom when the teacher stands in front of a group, teaching math. Did you find any particularly surprising findings?

DR: Looking at the B.C. results we didn't find anything surprising. Math teaching at the secondary school level is very much a stand up and talk exercise, a Chalk and Talk exercise. It's too early yet to talk about any international comparisons because data are just becoming available now from the other countries and I haven't had time to look at them in sufficient detail to see if there are any interesting trends or not. I hope there will be some interesting differences and we hope to try to relate any differences that we do find to student achievement to see what connections there are there.

RS: So there is a group looking at this and they will probably be publishing a report.

DR: Yes, I'm part of a group of seven or eight people that's responsible for producing a volume comparing teaching practices in secondary school mathematics. We hope that the report will be published in about a year.

RS: The third part of the IEA study was concerned with students' attitudes. What did you find?

DR: In B.C., the attitudes toward mathematics among grade 8 students are still fairly positive and even more so among the grade 12 students but, of course, the grade 12 students are electing to take mathematics so that's not particularly surprising. We didn't find any strong sex stereotyping of attitudes among students, although I understand from people at OISE looking at the Ontario data there are some differences, some major differences.

RS: It's my understanding that a significantly larger portion of the male population is taking math

2. The Teaching of Mathematics

In Grade 8

..Over 20% of Grade 8 teachers in B.C. teach students the British system of measurement as well as the metric system in spite of the facts that the Curriculum Guide calls for only the latter to be taught and that the prescribed textbooks refer only to the metric system. Teachers may have decided that this generation of students needs to be familiar with both systems, and, therefore, that both systems need to be taught. That the United States seems farther away from converting to the metric system than at any other time in the recent past lends additional weight to such an opinion, and this matter should be discussed as part of the curriculum renewal process.

..Teachers rarely assume that students have mastered concepts or skills in previous grades to the point where no reteaching or review is needed. Even topics such as concept of a fraction, which is introduced in the primary grades, are reported as having been taught or reviewed in almost 90% of the classrooms. This may be partly due to a certain lack of familiarity on the part of Grade 8 teachers with the content of the mathematics curriculum at the elementary school level. On the other hand, there is evidence that students perform relatively poorer on items dealing with content teachers have not reviewed.

..There are indications that in teaching computation techniques such as multiplication of decimals, teachers emphasize the sequence of steps to be followed in using the algorithm rather than an understanding of why the algorithm works the way it does. For example, 98% of teachers agree that they teach students how to multiply decimals by referring back to multiplication of whole numbers and giving students a rule for placing the decimal point in the answer. Similar, rule-oriented approaches to teaching are apparent for topics such as operations with positive and negative numbers, and the development of formulas.

..In the opinion of teachers, the principal factors that account for students' lack of satisfactory academic progress in mathematics are students' indifference, absenteeism, and lack of ability. Few feel that unsatisfactory progress can be attributed to a shortage of appropriate resources for teaching or to a lack of teachers' expertise.

..According to teachers, whether or not a given approach to teaching a particular concept or skill is easy for students to understand is the single most influential factor in determining which of a number of alternative teaching approaches to use. This factor is substantially more important than whether or not students enjoy the approach or whether that approach is used in the textbook. That a given approach is seen as being difficult for students to understand is a powerful factor in decisions not to use a particular teaching approach.

..The majority of teachers emphasize only one approach to teaching a given topic. While they might use others or refer to them occasionally, they tend to emphasize only one.

..Although most Grade 8 teachers agree that students should be permitted to use calculators, they are also strongly of the opinion that students should master computational skills.

In Grade 12

..There are very substantial differences in the amounts of time teachers spend on the different components of the Algebra 12 course. For analytic geometry, the range is from 6 to 45 periods, with 21 periods as the mean. In the case of trigonometry, the range is from 10 to 56 periods, with the mean being 29. For algebra, the range is from 7 to 96 periods, with the mean being 49.

..As in the case of Grade 8, Grade 12 teachers spend substantial amounts of time reviewing material covered in earlier courses.

..Despite the fact that virtually all Algebra 12 teachers permit their students to use calculators, 50% of them emphasize the computational application of logarithms in their teaching.

..The use of calculators in class is permitted by 97% of Algebra 12 teachers. Moreover, 16% of the teachers also use microcomputers in some manner.

..Algebra 12 teachers are more likely to use a particular teaching approach because they feel it will be

easy for students to understand than because they think students will enjoy it. That a given approach is seen as being difficult is a stronger factor in deciding not to use a given approach to a topic than is the belief that students will not enjoy it.

..In a typical Algebra 12 class, correction of homework occupies 25% of the time, lecturing by the teacher an additional 40%, and individual seat work by students, 20%. Little time is allocated for having students work together in small groups.

3. Achievement Results

In Grade 8

..All Grade 8 students were administered a 40-item test at the beginning of the school year, and the test was re-administered near the end of the course. Results on the posttest were 4.4 points higher than those on the pretest, and improvement of about 25%.

..Gains in achievement from pretest to posttest were greatest on the algebra and geometry subtests, the two areas that consisted primarily of content presented for the first time in Grade 8.

..In a number of cases, classes performed poorer on one or more of the subtests on the posttest than on the pretest. The losses occurred mainly on the subtests dealing with content from earlier grades; for example, fractions, measurement, and ratio and proportion.

In Grade 12

..In constructing items for the tests, four levels of cognitive behaviour were used; computation, comprehension, application, and analysis. In general, results show that achievement decreases as cognitive level increases. Similar results were found at the Grade 8 level.

..On the 91 items judged to be appropriate for the B.C. curriculum, the average percent correct was 42%. Given the level of difficulty of the items, this seems a satisfactory result.

4. Students' Attitudes

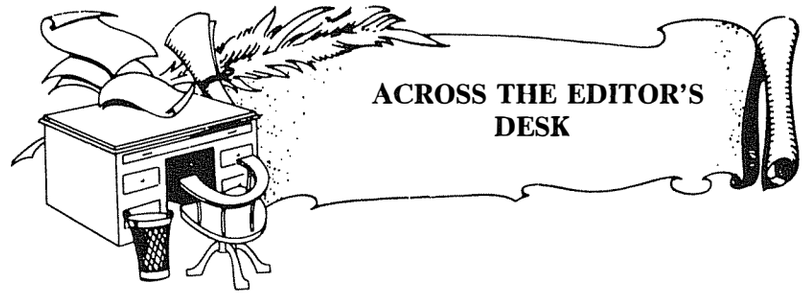
..When asked to react to a list of 15 topics and activities in mathematics, students at both the Grade 8 and Grade 12 levels indicated that they enjoy them, feel they are important, and do not find them too difficult.

..Teachers gave approximately the same ratings as their students to most of the items. However, teachers feel that problem-solving is more important, more enjoyable, and more difficult than their students do.

..Teachers and students agree that mathematics is a dynamic, growing field in which new developments are continually being made and not a static unchanging one. Teachers are more positive in this regard than their students, and Grade 12 students are somewhat more positive than Grade 8 students.

..Students indicate that their parents consider mathematics to be an important area of study, and that their parents want them to do well in mathematics. Students at both levels report that, while both parents encourage them to do well in mathematics and believe that learning mathematics is important, their fathers enjoy mathematics more than their mothers and are more likely to be able to assist them with assignments.

..Over two-thirds of students at both levels believe that it is important for them to know somthing about computers, and that computers can be used in beneficial ways.



ACROSS THE EDITOR'S DESK

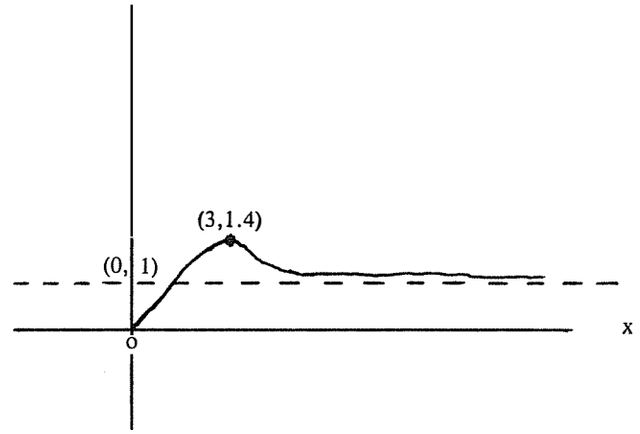
Dear Editor:

I made a very interesting observation by working with the equation $y = x^{1/x}$, $x > 0$. The value of y reaches a maximum when $x = 3$ and a minimum point when $x = 0$. As the value of x increases it pushes the value of y close to one.

It would be very interesting if this fact became common knowledge to everyone because of the simplicity of the value of y. Regardless of the magnitude of a number that is greater than three. If it is raised to it's reciprocal power it's value would approach one.

I would certainly feel happy if I saw a note pointing this fact among the logarithm and trigonometric tables in the back of my textbooks.

Graph of $y = x^{1/x}$, $x \geq 0$



I would appreciate it if you could publish this letter in your journal as a first step towards making this dream a reality.

Yours Sincerely

Barrington Miller

Dear Editor:

In reading your article by P.R. Scott on "Geometry in School-Euclid or Kelin?", I was reminded of a passage I had just read in P. Beckmann's amusing book **A HISTORY OF PI** (4th edition, 1977, pp. 48-9) and which you might like to share with your readers.

"The major part of the Elements was probably known before Euclid. But the importance of Euclid's work is not in what the theorems, as such, say. The great significance of the Elements lies in their methods. ... Euclid is not the father of geometry; he is the father of mathematical rigor. The Elements consist of twelve books, and the elementary geometry taught in the world's high schools corresponds, in one form or another, to the first four. Alas, the reason why it should be taught is very often grossly misunderstood. Except for a few elementary theorems, Euclidean geometry is of little use for modern science and engineering: One can usually go much further and faster by trigonometry and analytical geometry. The real significance of Euclidean geometry lies in the superb training it gives for logical thinking. .... Euclidean geometry teaches the difference between **if** and **if and only if**, and the difference between **one** and **one and only one.** If somebody teaches you how to bisect an angle by the use of compasses without insisting on the proof that the construction is correct, he has thrown out the golden kernel and handed you the dead garbage. Who needs to bisect an angle, anyway?"

Sincerely,

John Poland

GOLDEN SECTION - ST. CATHARINES AREA - TRIES SOMETHING NEW

A number of new activities have been tried this year, with the attention directed at the student. In December an evening conference for grade 11 and 12 girls attracted 90 participants from area schools. Ten different women speakers addressed the topic "successful careers with mathematics". The girls were divided into groups and met with four different speakers -- two before a pizza supper and two after. Speakers emphasized the importance of mathematics as a means of keeping career options open. Speakers were mathematicians, economists, biophysicists, computer scientists, engineers, business women, etc. We thank the women speakers from Canada and the U.S. for their time and presentations. In February an evening conference was directed at students interested in writing the Descartes contest. An anticipated 40 participants mushroomed to 70. One general session, for students and teachers, was held before the pizza supper. Afterwards the students were divided into two sessions and a special session was held for teachers. We thank Ron Scoins, Ron Dunkley and Jack McKay from the University of Waterloo for running the sessions.

Both student evenings were organized by Don Roberts, Beamsville Secondary and Eric Muller, Brock University. We thank Herb Kornelson for helping with the facilities at Lakeport School.

In October a very popular evening conference for teachers considered the new guidelines. Dave Alexander and the "gang of three" discussed the "where is it at now" in the mathematics guidelines. Because of limited facilities, numbers were restricted to 100 participants. Unfortunately many applicants had to be turned away. We thank Dave Alexander, Tom Davison, Mike Silbert and George Knill for their presentations. The evening, held at Brock, and organized by Warren Sebben, Eric Muller, Don Roberts and Dave Lenschychyn ended sociably with a great "hip of beef" dinner.

BOOK REVIEW

THE HANDBOOK OF ARTIFICIAL INTELLIGENCE, VOLUME III

Edited by: Paul R. Cohen and Edward A. Geigenbaum, William Kaufman, Inc. Los Altos, California, 1982.

Reviewed by: Mark Green

This is the final volume in the Handbook of Artificial Intelligence. The first volume of this handbook concentrates on the basic techniques used in artificial intelligence and the second volume deals with its major applications. This volume covers five topics that don't fit into the previous two volumes. These topics are: models of cognition, automatic deduction, vision, learning and inductive inference, and planning and problem solving. The articles in this volume are largely independent of those in the other volumes and can easily be understood with a basic knowledge of artificial intelligence.

Like the previous two volumes, this volume consists of a chapter for each of the topics covered. Each chapter contains a general introduction to the topic, followed by a number of short articles covering key issues and showing how the techniques have been used in practice. This organization allows the reader to select the level of detail at which he approaches the topic.

Ideas from artificial intelligence have occasionally been used as the basis for theories of human intelligence. Over the past two decades there has been considerable discussion about the validity of this approach to human intelligence. The first chapter of this volume, models of cognition, addresses this controversy and describes some of the models that have been developed to explain human behaviour. Since these models often lead to computer programs it is possible to test them in ways that are not practical with more traditional psychological theories.

The third chapter of this volume deals with computer vision. Computer vision is concerned with the processing and interpretation of pictures. In most cases a television camera is used to convert the pictures to machine readable form. Based on this digital representation of the picture a vision program must find the regions of the picture corresponding to real world objects. From these regions, descriptions of the objects are constructed and the objects identified. Since in most cases these programs only have intensity or colour information to work with this is not an easy task. In this chapter some of the basic ideas and current research areas in computer vision are presented. A large number of computer vision systems are described showing how their techniques and sophistication have evolved over time.

The remaining three chapters in this volume deal with how computers can reason and solve problems. The chapter on automatic deduction describes some of the techniques that have been used in computer based theorem proving. Some experts have argued that the key to truly intelligent computer systems is the availability of sophisticated theorem proving programs. This chapter reviews the progress that has been made in this area and gives several examples of the types of theorems that can currently be proved by computer.

The chapter on learning and inductive inference deals with how programs can learn from their environment. This is probably the most misunderstood area of artificial intelligence due to the confusion between machine learning and human learning. In most cases machine learning deals with the recognition of patterns and distinguishing different objects. This chapter covers the different approaches that have been used in machine learning. It also presents several existing systems that illustrate the power of current techniques.

The final chapter, planning and problem solving, is devoted to the techniques that have been used for solving nontrivial problems. The solution of a nontrivial problem requires several major steps that cannot be predetermined. The approach used to solve this type of problem is called planning. In this approach the program develops a plan or description of how the problem can be solved. This plan can then be examined by the program to determine if there are any mistakes or problems with it. Once the plan has been examined it is used to guide the solution of the problem. If the solution is not successful the plan is modified to produce a new solution. This chapter covers some of the techniques that have been used in planning and describes some of the programs that have been developed for problem solving.

This volume completes the Handbook of Artificial Intelligence. This handbook is one of the major works in artificial intelligence, and is by far the best single reference source for the field. With the current popularity of artificial intelligence a good reference source is very important. The one major problem with the handbook is its size. Even though it is well written, it takes a considerable amount of time to read all three volumes. The editors have tried to reduce this problem by making each chapter relatively independent of the others. This allows the reader to skip the chapters that are not of interest to him. Unfortunately this solution introduces a degree of redundancy that further increases the size of the handbook. Despite its size, this book is recommended reading for anyone with a serious interest in artificial intelligence.

Reviewed by

Mark Green
Unit for Computer Science
McMaster University
Hamilton, Ontario



Problems

Editor's Note: *For a couple of years a while back MENSA Canada, in conjunction with* ***TODAY*** *magazine, sponsored a "Superbrain Test". Many of the questions on those tests provided some superb problem-solving opportunities. March's problems were selected from those tests.*

1. Continue the series by adding one term in each case:

   a) B Z E Y I W N T _____

   b) A E D I H N M _____

   c) −1 4 −12 48 −240 _____

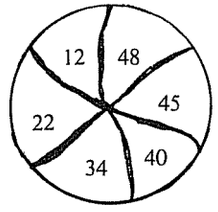
2. What is the next number in each series?

   a) 2 4 12 48 _____

   b) 7 9 7 8 7 7 _____

   c) 81 49 25 9 _____

3. On this unusual dartboard, what is the least number of darts that could be used to score 101?



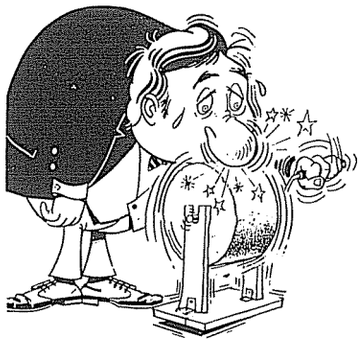
4. A man bets $30.00 and gets back his original bet plus $90.00 He spends one-sixth of his winnings on food and one-third of his winnings on travel. If he started out with $200.00 and spends nothing else, how much money will he take home?

ANSWERS TO DECEMBER PROBLEMS

1. The question derives from a Cambridge College paper of 1815, but was still in school texts of 1894. B is 25 miles from London.

2. The sailing rates were 5 and 7 miles per hour. The distance between Dover and Calais, 22 miles.

3. **A** shot 6 birds and **B** shot 4 birds.



COUNCIL COMMENT

by
Bob Robinson

Editor's Note: *Council Comment appears in an issue of the Gazette following OAME/AOEM Council meetings. We hope that you find this section of interest. Your comments are welcomed and should be sent to:*

The Editor
Ontario Mathematics Gazette
College of Education
Brock University
St. Catharines, Ontario
L2S 3A1

The February Council meeting was held on Saturday, 1984 02 11 at the Ramada Inn (Airport West).

CHAPTER REPORTS:

• COMA

Conducted a meeting recently to breathe new life into the association. A partial slate of officers has been elected.

• GOLDEN SECTION

Is planning a date in late September or early October to conduct a mini-conference. The tentative theme is "Computers at the Elementary Levels".

• GVMA

Is very pleased with the success and attendance figures for their fall general meeting. Over 300 people attended. Frank Ebos was the keynote speaker and the after dinner speakers were Dave Davidson, Mary Thompson, Bev Marshman, Larry Rice, Paul Zolis, and Jack Wiener.

The final report of the highly successful 1983 OAME Annual in Waterloo has been forwarded to the executive.

The current membership of GVMA is 392 with 188 of these joint members of GVMA/OAME.

A spring general meeting will be held at Eastwood C.I. in Kitchener on Thursday, 1984 04 12 beginning at about 16:30. The keynote speaker will be Barny Lawrence whose topic will be "You and the Law". After dinner speakers include Brendan Kelly and Jack McKay whose topics are "Mathematics Applications" and "Lotteries".

Planning is underway for the Fall '84 general meeting. This meeting wil be held on Thursday, 1984 11 08 and the keynote speaker will be John Moore from the University of Waterloo whose talk title is "You Ain't Seen Nuttin' Yet".

• NOMA

Is busy planning for OAME '85 to be held in Sudbury.

• **PINE RIDGE**

Had a successful P.A. day on 1983 11 16. About 200 people attended with 22 workshops being offered. A Career Night was held on 1983 11 08 for Mathematics and Science. An evening mini-conference will be presented on 1984 04 03 in Harwood High School in Ajax, where 12 different sessions will be offered. As part of a Mathematics/Science lecture series, a mathematics lecture is set for 1984 02 28 at Durham College, Oshawa. Dr. Ron Mullin, University of Waterloo, will speak on "Computer Security".

Four scholarships have been awarded to teachers for upgrading their mathematics qualifications.

• **QSLMA**

Conducted a very successful meeting in Kingston last November offering sessions from kindergarten through post-secondary. The membership of QSLMA is over 100 with 95 of these being members of OAME/AOEM.

• **SAME**

Sponsored the third annual mini-conference on 1983 10 18 at Maplewood Vocational School. Teachers, numbering 175, attended from Scarborough, York Region, Durham Region, North York and Etobicoke. A workshop series was initiated this year co-sponsored by three different publishing companies. The first workshop was presented by Sharon and David McPhail on 1983 11 29. The second featured Larry Ridge on 1984 01 17, while the third is on 1984 04 03 and presents Don Fraser.

On 1984 02 10, a successful K-13 professional activity day was held at the Holiday Inn - Toronto Don Valley and featured Lorna Wiggan as the keynote speaker.

• **TORNY**

Held its first activity on 1984 01 31 and 1984 02 01 at the Harbourfront Computer Centre in Toronto. This computer workshop was planned for one day but because of the large response, a second day was added. There was a very positive response to these hands-on sessions using Apples, C-64s, Olympics, and Hyperions. The teachers experienced hardware and software which they had not previously seen and gained many ideas for the classroom and field trips.

A Tax Planning and Investment Workshop is scheduled for early May.

1983 LEADERSHIP SEMINAR:

The 12th annual leadership seminar was another success. It was noted that some of the participants were already in leadership roles. Thanks are extended to Sharon and David McPhail for the organization of this important event.

1984 LEADERSHIP SEMINAR:

Four meetings have been held already and Dave Davidson and Paul Pogue are the organizers for our next leadership seminar. Your input and suggestions would be appreciated.

1984 "ANNUAL" (KINGSTON):

The registration forms are now available in your schools. If you haven't seen any, contact your curriculum department or appropriate superintendent or co-ordinator. Elaine Harvey and her colleagues promise an exciting and varied educational and social experience. Place these dates on your calendar: Thursday, 1984 05 10 through Saturday, 1984 05 12.

1985 "ANNUAL" (SUDBURY):

Mickey Sandblom reports that plans are well underway. The dates will be Friday, 1985 05 10 through Sunday, 1985 05 12. The organizers know that once they get you to Sudbury you will thoroughly enjoy yourselves. The theme will be "Mathematics Across the Curriculum". A visit to "Science North" and the underground mines are also being planned.

1986 "ANNUAL" (?):

If your area is interested in hosting an OAME "Annual", contact any member of the Executive or Council. Their names are listed in the September, 1983 **Gazette.**

CONSTITUTION REVISION:

This project is coming along very well and should be available for discussion early during the next school year.

NCTM:

Each councillor was given a copy of the position statements of NCTM. The "Annual" for the NCTM is to be held in San Francisco from 1984 04 25 through 1984 04 28. Our delegate to this year's annual will be Ralph Connelly and the alternate Paul Lessard.

FORMATIVE YEARS:

An advisory committee is to be formed to make recommendations to the Ministry of Education on issues pertaining to mathematics at the primary and junior levels.

POSITION STATEMENTS:

In the future, any position statement of OAME/AOEM must receive positive vote of 2/3 majority of the councillors present.

HIGHLIGHTS OF OAME/AOEM

1973 - 1984

Mona and Morley MacGegor have produced an excellent document which names all past presidents, directors, and councillors for the past ten years. The highlights of the last decade are reviewed. A most worthwhile addition to anyone's collection of memorabilia. This booklet took hours and hours to write by our outstanding secretary-treasurer and his wife. For your free copy, write to:

OAME/AOEM HIGHLIGHTS
247 BRIGHT STREET
SARNIA, ONTARIO
N7T 4E9

PUBLICATIONS:

Look for a flyer in your September, 1984 issue of the **Gazette,** which will ask you if our publications are meeting your needs. Are we publishing the type of journal you want? Be sure to fill out this important questionnaire when it arrives.

MEMBERSHIP:

Our membership is about 1300. If you have any ideas for increasing membership, write to:

Dino Dottori
640 Lancelot Avenue
North Bay, Ontario
P1A 3A8

SENIOR GUIDELINES:

Tom Griffiths reports that they are meeting with the University Chairman group to present our recommendations. Final draft copies of these recommendations will be sent to councillors when completed. Their task has been an onerous one, but we have all benefited from their work. Council encouraged the continuation of their endeavours and approved in principle the formation of a committee to study curriculum development, the philosophy of mathematics education in Ontario, formation of an external examination available for university entrance.

PUBLIC RELATIONS:

A very interesting new information and public relations brochure has been produced. This committee will organize a contest for a conceptual design poster; investigate the colour, size, quality, and cost of production; approach graphic designers for some designs and cost; seek funding from different organizations in order to produce and distribute.

COMPUTERS:

An interesting article appeared in the December, 1983 issue of the **Gazette** which related to the ICON. With your last issue of the "Abacus", a green questionnaire pertaining to computers was attached. Be sure to fill it out and return it to:

Trevor Thomson
Mathematics Department
Hamilton Board of Education

PROBLEM SOLVING:

A steering committee has been established by OAME to promote the **systematic** investigation of mathematics problem solving in Ontario. This committee chaired by Chester Carlow has put together a paper which outlines five issues with suitable background information. If you wish to assist in this investigation, contact:

Chester Carlow
Curriculum Department
Ontario Institute for Studies in Education

NOMINATIONS:

The nominating committee chaired by Sharon McPhail has put together a very impressive slate of candidates. All are winners! Be sure to vote when you receive your ballot.

MATHEMATICS TEACHER SHORTAGE:

Each councillor received a copy of "Report on Teachers of Mathematics Shortage/Ontario 1983-84" which was prepared by Alex Norrie. It will appear in this issue of the **Gazette.** Watch for it! Your comments will be appreciated.

BILL 82:

OAME/AOEM's position statement on BILL 82 appeared in the December 1983 issue of the **Gazette.** Further information and articles will be forthcoming in future issues. A new publication entitled **Consumer's Guide to BILL 82** (Special Education in Ontario) is available for OISE Publications.

REPORT ON TEACHERS OF MATHEMATICS SHORTAGE/ONTARIO 1983-84

prepared by
Alexander L. Norrie
Coordinator of Mathematics
Peel Board of Education
and Kristian Kirkwood
Chief Research Officer
Peel Board of Education

1984-02-06

"Mathematics and science teachers are in high demand nationwide, in part, because teacher salaries do not compete with those offered by private industry and because many states have required additional courses in these subjects for graduation." This statement was presented in January, 1984 and billed as a moral issue by Florida newspapers claiming that "children who are not given adequate math and science training are 'destined for failure'."

Recent achievement test scores and results on international studies have raised concerns about the teaching of mathematics in school districts all across our neighbour to the south. Reports such as "A Nation At Risk" indicate that in many states students can receive a high school graduation diploma without taking mathematics.

It appears that the answer to the aforementioned problem in the United States is to provide access to more and better qualified mathematics teachers. This is where the actual 1983 statistics become rather frightening - there are some facts state by state:

"Currently, 114 students are in mathematics teacher education programs throughout the state of California. The Los Angeles District alone could use over twice this number."

"The University of Nebraska at Lincoln received requests for 464 new mathematics teachers, but the school graduated only 18 new teachers."

"37,275 students in New York State are being taught mathematics by non-mathematics teachers."

Examples such as these influenced the members of the Executive of the Ontario Association for Mathematics Education to raise the question of a mathematics teacher shortage in the Province of Ontario.

Subsequently, Alexander L. Norrie was asked to conduct a study of the problem in the school year 1983-84 with a report being due to the O.A.M.E. Council in the Spring, 1984.

The study was supported by previous National Council of Teachers of Mathematics data; Mr. R. Robinson, Director of N.C.T.M.; Morley MacGregor, Secretary-Treasurer O.A.M.E. and the Peel Board of Education.

METHOD

A two page questionnaire (Appendix A) was prepared and distributed in October, 1983. Approximately 425 questionnaires were distributed province-wide by O.A.M.E. Council members, Chapter representatives and members of the Ontario Mathematics Coordinators Association. No attempt was made to include all Secondary Schools in the Province but all major geographical regions were represented. Public Boards, Separate Boards, Large Boards and Small Boards were included. Although French language schools were included no private schools were sampled.

The returns from 165 schools were analyzed by the Research Department of the Peel Board of Education.

RESULTS

The results for the Ontario Association for Mathematics Education Survey on teacher supply and demand is presented in three distinct sections. The first section will deal with the number of full time

teaching positions in mathematics, the number of teachers hired and the qualifications of these teachers who were given the positions for teaching of mathematics. The second section deals with the difficulty of filling the mathematics positions with qualified mathematics teachers both at the current time and three years hence. The third section deals with the three open ended questions which the respondents were invited to complete.

Table 1
The Number of Full-time Teaching Positions in Mathematics

	Mean	Min.	Max.
1. How many mathematics teaching positions (full-time equivalents) are there in your school?	8.12	1.00	15.00
2. How many mathematics teachers did you hire for September 1983?	.76	0.00	5.50
3. How many applicants did you have for these positions?	10.79	1.00	56.00
4. How many of the applicants were specialists in mathematics?	6.26	1.00	40.00

Section 1

Table 1 presents the individual question in the first section and the average number of teachers responding to each of the questions. In addition, the range is given with the minimum and maximum number of teachers presented. When asked how many math teachers were currently in their respective schools, the respondents indicated a range from 1 to 15 with an average of 8 full-time math teachers in each school. On the average, less than 1 teacher for mathematics was hired for September, 1983.

Related to hiring were the numer of applicants who applied for September positions. There were approximately 10 applicants for each position. However, the range was from 1 to 56 applicants. Of those applicants applying approximately 6 out of every 10 were specialists in mathematics.

Table 2
The Number of Teachers Hired

	Mean	Min.	Max.
1. How many of your vacancies for September 1983 were filled by teachers who were:			
a) a type A in mathematics	0.69	0.00	3.00
b) certified to teach mathematics but not type A	.94	0.00	5.00
c) certified for secondary school but not in mathematics	.68	0.00	6.00
2. How many full-time staff positions in your Department are filled by teachers not certified in mathematics?	.88	0.00	11.0

Section 2

Table 2 dealt with the qualification of teachers currently applying for vacancies in the teaching of mathematics. The results revealed that an average of at least 0.69 Type A's filled each vacancy in the 165 schools responding. However, there were schools that indicated that no Type A's filled their vacancy. One school indicated that 3 Type A's were hired to teach.

An average of almost one certified but non-type A teachers filled vacancies where Type A's were not hired. One school hired five certified for mathematics but not Type A.

A single school indicated that six vacancies were filled by teachers not certified in mathematics but it was not clear whether these are full or part-time mathematics positions. However, an average of 0.68 positions are being filled by teachers not certified in mathematics.

Finally when asked how many full-time staff positions were filled by teachers not certified in mathematics, the respondents indicated that approximately one individual per school was filling a position and did not have the appropriate qualifications in mathematics.

The second question deals with the department heads rating the difficulty of filling mathematics positions with qualified mathematics teachers for September, 1983. Interestingly, there is an approximate 50-50 breakdown with one half finding it not difficult and the remaining finding it average to very difficult to fill the positions. In terms of whether or not it would be difficult in three years to fill vacant positions with qaulified math teachers, 2 out of 3 respondents indicated that it would be either average or very difficult to fill the positions. Thirty-six percent indicated that it would not be difficult to find qualified math teachers.

Section 3

In this section comments are presented from the remaining three questions. It is of interest to note that for all three questions approximately 50% of the respondents made either general or specific comments.

For the first question the department heads listed a number of plausible causes related to their perception of what is causing the shortage of qualified math teachers. Reasons included such explanations as better job opportunities outside of education, problems associated with declining enrolment, also difficulties with finding qualifed people to teach mathematics and a shortage of French language instructors.

Those suggesting better job opportunities outside of education, cited specific suggestions such as computer careers, higher pay and alternate careers several times. In the area of declining enrolment, the respondents indicated problems such as job security, the bumping process, senority, tenure, limited advancement, as well as diminishing graduates from faculties of education and a general surplus of teachers. Finally, other issues were noted by individual respondents.

These included such topics as lack of turnover, lack of mathematics specialists, the unpredicatability of future needs, hiring problems, loss of attraction to the profession, and no mathematics teachers for alternate schools or settings. The next question asked the respondents "If you predict a shortage over the next few years, what caused you to make this prediction?" Interestingly, the majority of people who responded to this question indicated that they believed there would not be a shortage over the next few years. However, for those who were predicting a shortage, several reasons were cited. These included the fact that Canada usually has a five year time lag behind the United States and that the Americans were currently experiencing a critical shortage of mathematics teachers. They felt that this phenomenon would also occur in Canada. Other reasons included a modest rise in the birth rate, a lack of teacher vacancies, lack of opportunities for advancement, and the notion of teachers being overworked and underpaid and the decline of supply teachers in the specific area of mathematics.

The final question asked the respondent to comment on any relevant aspects related to the issue of supply and demand of qualified math teachers. The largest number of teachers responded to this item. However, the comments included for this open ended question mirror the comments of the two preceding questions. From this one general theme emerged. The respondents lamented the fact that there were too many non-specialist teachers teaching at the secondary school level. This comment was reiterated time and time again.

CONCLUSIONS

The data and analysis presented in this report does not support the thesis that there is or will be a shortage of qualified mathematics teachers in the province of Ontario. However, there clearly arises two other major problems related to appointments of qualified mathematics teachers to vacancies in mathematics departments.

The small sample of French language schools supports the fact that it is difficult to find qualified mathematics specialists for these specialty schools.

A larger province-wide problem exists in the phenomenon of more and more teachers from other subject areas being assigned to teach mathematics. This trend is supported by declining enrolment, the K to 10 certification process and recent O.S.I.S. credit alignments. The trend has potential for creating a negative influence on mathematics teaching in Ontario.

* * * MEMBERSHIP RENEWAL NOTICE * * *

All 83/84 Memberships expire on April 30, 1984
Membership Fee: $18.00
OAME offers you four ways to renew your membership

1. Include your membership with your registration when you are registering for the Annual Conference at Kingston

2. Purchase your membership **personally** at the OAME desk at Kingston

3. If you are not attending the convention, please send your cheque to OAME, 247 Bright St., Sarnia, Ont. N7T 4E9
   Include your mailing address (home address preferred) and indicate a chapter if you so desire

4. All members who have not renewed by May 14 will be sent a renewal notice. This method is not the preferred method due to costs involved. Hope everyone can use one of the first three methods.

M. MacGregor

OAME '84

11th ANNUAL CONFERENCE

KINGSTON

MAY 10-13

Have you registered for the 1984 Conference in Kingston? The Keynote Speaker will be Dr. John Coleman, of world wide repute, now retired from the headship of the Department of Mathematics and Statistics, Queen's University. Dr. Coleman's topic will be "Bicycles, Mathematics and Computers". Last date for mailing your registration is April 19. Otherwise register in person at the conference. See you there!