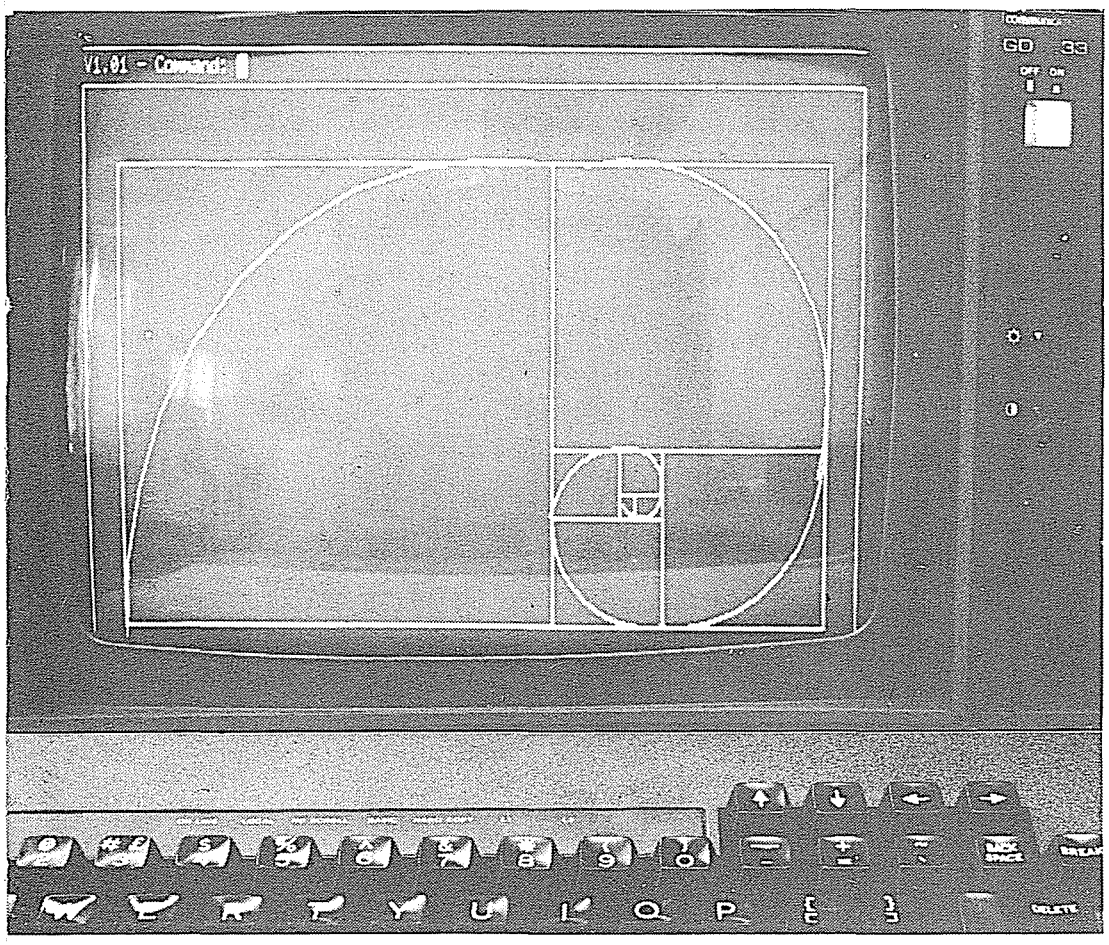
ontario mathematics gazette

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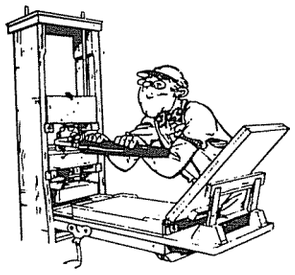
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We think you'll find some exciting things in this issue to "kick off" the new school year. Although the number of submissions received is starting to increase, it is still quite slim, especially at the elementary level. We hope the situation continues to improve thoughout this year.

You will find enclosed with this issue a survey prepared by the Publications Committee of OAME asking for your responses to a few questions regarding the **GAZETTE.** We hope you will immediately take the few minutes required to complete the survey and return it. As we've been saying for the past year, the **GAZETTE** is **your** journal and we need to hear from you to help it grow and become better. The ballot returns for the past election were quite disappointing (only 106 returns were received). We look forward to a high percentage return on the surveys.

OAME's Kingston Conference is now history, and the conference was a great success. Photos of some of the "magic moments" appear throughout this issue.

Ironically, as I sat putting this editorial together, I received a phone call from an OAME member. The caller indicated that he hoped he could look forward to some "scathing" editorials this year, and not just reference to the articles, etc. in that issue. With that in mind, I've been sitting here now for 2 hours wondering who or what I'm supposed to "scathe." I've decided that I will instead end this editorial with optimism, leaving you, the readers, **un**scathed. I will optimistically assume that the response to the survey **will** be high; that items for the Problems section **will** come in this year (none did last year); and that readers **will** take the opportunity provided for them in the "Open Letter" section (none have for the last two issues); and that readers **will** react to the articles, OAME committee reports, etc. appearing in the **Gazette.** Should all that optimism prove misguided, there'll be time enough later for "scathing." Have a good school year.

Ralph Connelly
Sam Pravica
Editors, **OMG**



PRESIDENT'S MESSAGE

BY

PAUL POGUE

This year, as others have been, promises to be a busy and challenging one for your executive and council. It seems that new projects are always waiting in the background, ready to replace those that have been completed. Facing these new challenges year after year has been the strength and lifeblood of our organization, and I am sure that this year's group will continue to serve the needs of mathematics education in Ontario in our usual outstanding manner.

It is my pleasure to serve as your president and I thank you the membership, for allowing me to do so. I am prepared to expend whatever time and energy I must, to fulfill the duties of this office. I am honored to be the twelfth president of O.A.M.E. and will always cherish the memories of my eight years on your executive or council. The camaraderie, the kibitzing, the heated discussions, the painstaking decisions, but most of all the friendships that have been made, make it all worthwhile. May I encourage you, the general membership, to run for office. It is a committment, but a very rewarding one.

Each year at the June council meeting, committees are formed and a plan for the year, including aims and objectives, is established. Please refer to Council Comment on page 41 for a complete list of councillors, committees, aims and objectives, etc. I know this will give you a sense of the overall direction we are heading and will limit my comments to those I feel require some clarification, or emphasis.

With the chance of sounding like a broken record I would encourage each and every one of you to consider submitting an article to the editors of our publications. The Gazette and Abacus are outstanding publications and will continue to be if the editors receive quality articles for publication. Many of you have a favourite way of presenting a topic, a challenging problem, and exciting proof, an interesting application; please share these with us through your journals. In this way we will all benefit and continue to provide the students of Ontario with (in my mind) the best mathematics education in North America.

Sometime during the year the Senior Guidelines Committee will complete its work. Validation of the guidelines will take place during the fall. The Ministry will distribute the document to boards during August and is requesting board submissions by the end of December. I can assure you they will receive a submission from O.A.M.E. at that time. If you have any concerns you wish to voice through O.A.M.E. please submit them to David Handley (before the October council meeting if possible).

With the completion of the Intermediate and Senior revisions, a review of the K-6 curriculum is underway. We have formed the Formative Years committee, Bill Proctor, chairman, to represent the interests of O.A.M.E. in this review. I expect this committee to be an ongoing one as curriculum review does not occur in one year. They will have a report prepared by the end of November outlining concerns and or suggestions of O.A.M.E. re revision of the Formative Years.

Another new committee is the Women and Mathematics committee co-chaired by Lorna Wiggan and Irv Pupko. This committee will initially assist the conference committee to get speakers on this topic. Further plans will be printed in Council Comments following the October meeting.

No president's report would be complete without a plea for increasing our membership. It is, and will always be, a battle to keep our membership at a level where we are able to provide the leadership that we have exhibited throughout our existence. You can help. Our goal for this year is 1500 members (slightly over 200 more than last year). If we are to achieve this goal we need your help.

When was the last time you:
- discussed O.A.M.E. with a non-member colleague
- shared an article from one of our publications with a non-member collegue
- encouraged colleagues to join O.A.M.E. at a mathematics meeting in your board
- encouraged colleagues to attend the Leadership Seminar, a chapter meeting or the O.A.M.E. annual conference

If one member in six is able to recruit a new member we can reach our goal.

Once again may I thank you for your past support and encourage you to promote the outstanding organization of which you are a member.

IN MEMORIAM

Dr. Kenneth D. Fryer



Born 13 June 1924 - Died 19 May 1984

Ken is survived by his wife Pat, four daughters, Cathy, Pam, Laura, and Carol, and one son Paul.

He was born in St. Thomas, received his B.A. at Western (1946), where he was a Dr. Kingston Gold Medallist, his M.A. at Toronto (1947), and his Ph.D. at Toronto (1952). He was named a 1984 Distinguished Teacher Award recipient at Waterloo. He taught at Toronto, Royal Military College, Queen's, and Waterloo. He co-authored numerous texts, served as Associate Dean of the Faculty of Mathematics, Chairman of the Descartes Scholarship Committe, Chairman of the Waterloo Mathematics Foundation, committee member of the Canadian Mathematics Competition, and in numerous other capacities.

He will be remembered by former students as one of the truly outstanding instructors, with a genuine concern for their development, and by teachers across the province for his leadership in bridging the secondary school-university gap; but he will be remembered most as a truly inspirational person with a zest for life and the enjoyment of mathematics. He leaves a challenge to all mathematics educators to continue the many projects in which he was the driving force.

He will be missed.

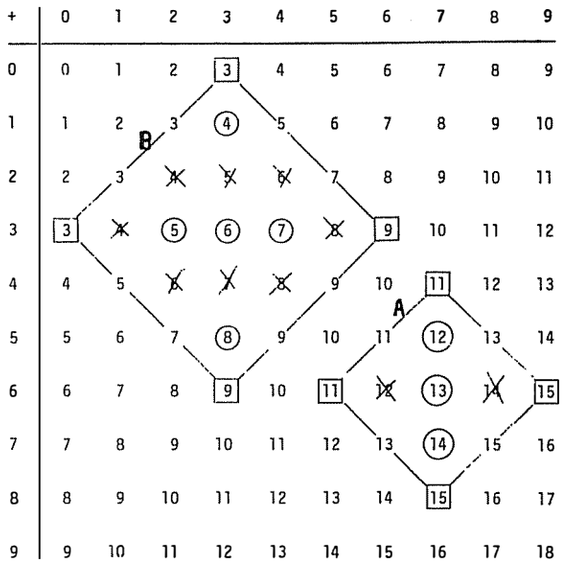
RHOMBI (SQUARES) AND RATIOS ON THE EXTENDED ADDITION TABLE: CONJECTURE AND PROOF

Bonnie H. Litwiller and David R. Duncan
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

Middle school and junior high mathematics teachers often wish to organize activities which provide both for computational practice and the discovery of number patterns. Many such activities may be developed using an addition table.

Figure 1 is the standard addition table which displays the 100 basic addition facts. Rhombi A and B are drawn upon the entries of the table. For each rhombus, note that each pair of opposite vertices lies in the same row or in the same column.

Figure 1



For rhombus B:

1. Box the four vertex numbers. They are 3, 3, 9, and 9.
2. Sum the four vertex numbers; $3 + 3 + 9 + 9 = 24$.
3. In the interior of the rhombus, circle one of the interior numbers, 4. Now cross out all of the numbers which are diagonal to the circled number, 4, in either direction. We crossed out 4, 4, 6, and 8.
4. Circle one of the remaining interior numbers, 5. Again cross out all remaining numbers diagonal to 5. These are 7 and 5.
5. Circle one of the remaining interior numbers. We circled 8. Cross out all the remaining numbers diagonal to 8. We crossed out 6 and 8.
6. Circle one of the remaining interior numbers. We circled 6. Again, cross out all the remaining numbers. This time, however, there are no number left on the diagonals.
7. Only one number remains, so circle it. Thus, we circled 7.
8. Sum the circled numbers; $4 + 5 + 8 + 6 + 7 = 30$.
9. Find the ratio of the sum of the circled numbers to the sum of the boxed numbers:

$$\frac{30}{24} = \frac{6 \cdot 5}{6 \cdot 4} = \frac{5}{4}$$

Observe that there were 5 circled numbers and 4 boxed numbers.

If the same procedure is followed with rhombus A, but with fewer steps since it is smaller, the ratio is $\frac{39}{52} = \frac{13 \cdot 3}{13 \cdot 4} = \frac{3}{4}$. Observe that there were 3 circled numbers and 4 boxed numbers.

Conjecture: If steps 1, 2, 3, ... are performed on a rhombus of any size, the ratio of the sum of the circled numbers to the sum of the boxed numbers is equal to the ratio of the number of circled numbers to the number of boxed numbers. Have your class investigate this conjecture by drawing rhombi the same size as A and B, but place them elsewhere on the addition table.

If larger rhombi are drawn, it is useful to extend the addition table. Figure 2 displays the interior of an extended addition table. Rhombi C, D, and E were drawn upon the table and steps 1, 2, 3, ... were performed.

Figure 2

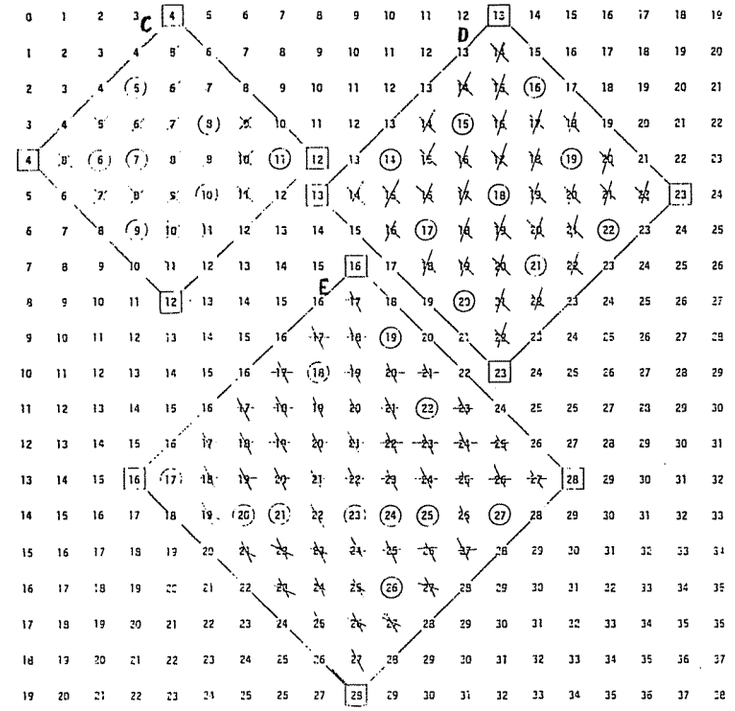


Table 1 reports the results of the computations.

Table 1

Name of Rhombus	Size of Rhombus	Sum of Circled Numbers	Sum of Boxed Numbers	# of Circled Numbers	# of Boxed Numbers	Ratio of Column 3 to Column 4	Ratio of Column 5 to Column 6
A	2 by 2	39	52	3	4	$\frac{39}{52} = \frac{3}{4}$	$\frac{3}{4}$
B	3 by 3	30	24	5	4	$\frac{30}{24} = \frac{5}{4}$	$\frac{5}{4}$
C	4 by 4	56	32	7	4	$\frac{56}{32} = \frac{7}{4}$	$\frac{7}{4}$
D	5 by 5	162	72	9	4	$\frac{162}{72} = \frac{9}{4}$	$\frac{9}{4}$
E	6 by 6	242	88	11	4	$\frac{242}{88} = \frac{11}{4}$	$\frac{11}{4}$

The conjecture appears to be correct, based upon the cases considered Again, have your students draw rhombi of the same size as C, D, and E but placed in different positions on the extended addition table to check the conjecture.

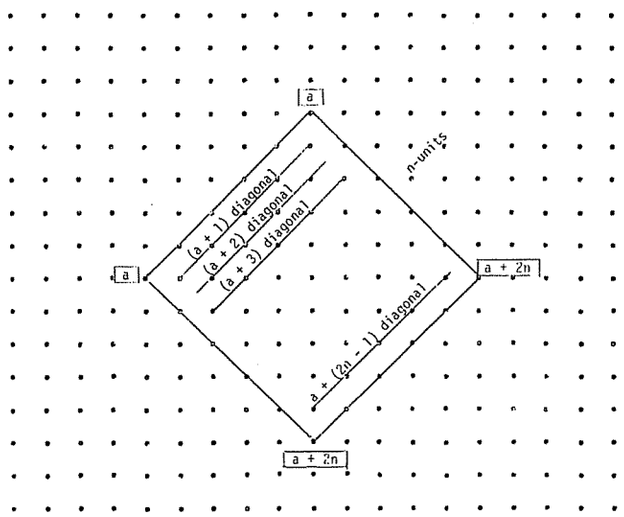
How might this conjecture be verified?

Observe the following:

1. On rhombus D, the entries on an "upper left to lower right" side are 13, 15, 17, 19, 21, and 23. In general, the entries on an "upper left to lower right" side of an n by n rhombus are a, a + 2, a + 4, a + 6, a + 8, ..., a + 2n.
2. On rhombus D, nine "lower left to upper right" diagonals in the interior of the rhombus can be identified. In each of these diagonals the entries are all the same, namely 14, 15, 16, ..., 22. In general, for an n by n rhombus, there are 2n - 1 "lower left to upper right" diagonals in the interior of the rhombus. In each of these diagonals, the entries are all the same, namely a + 1, a + 2, a + 3, ..., a + (2n - 1).

Figure 3 represents an n by n rhombus descibed in the two preceding observations.

Figure 3



Observations 1 and 2 dictate that the 4 boxed numbers are a, a + 2n, a + 2n, and a. Their sum is 4a + 4n = 4(a + n).

The process of steps 1, 2, 3, ... will always yield exactly one circled number in each of the a + 1 diagonal, a + 2 diagonal, a + 3 diagonal, ..., a + (2n - 1) diagonal. The sum of these circled numbers is thus

$$(a + 1) + (a + 2) + (a + 3) + \cdots + (a + (2n - 1)) =$$

$$(2n - 1)\,a + \frac{(2n - 1)(2n)}{2} =$$

$$(2n - 1)\left[a + \frac{2n}{2}\right] =$$

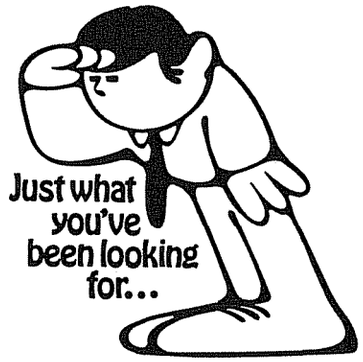
$$(2n - 1)(a + n)$$

The ratio of the sum of the circled numbers to the sum of the boxed numbers is

$$\frac{(2n - 1)(a + n)}{4(a + n)} \text{ or } \frac{2n - 1}{a + n}$$

This is the same as the ratio of the number of circled numbers to the number of boxed numbers.

Challenge: Draw rhombi on extended subtraction and multiplication tables. Does the ratio conjecture hold for each table?



CONSERVATION

by Bonnie Richardson

Editor's Note:

This article comes from a booklet ***"Hand's On" mathematics for the Primary Division,*** *prepared by Bonnie Richardson and distributed by F.W.T.A.O. This excerpt is reprinted with the author's permission.*

Definintion

Before the child can understand the simplest addition and subtraction exercises, he must have repeated experiences of the **constancy of number.** He must find out for himself, in many different ways, through many activities, that number remains constant despite changes in the spatial arrangements of groups of objects. This is basic number concept work....

Only through experience can he learn about constancy of amount. There are two types of materials in the environment which serve to assist the child in his learning. Piaget talks of "Continuous" quantity and "discontinuous" quantity. "Continuous" materials are things like sand, water, clay, or playdough - substances which run together, forming "lump sums". Groups of counters, stones, pegs, or nails are "discontinuous" quantities - separable into definable and stable units. A group of four counters can be separated into four single counters. By contrast, 100 cc of water is a "lump of water" which can be put into variously shaped containers.

Most children understand that continuous substances remain constant despite shape deformations before they can assert constancy of discontinuous quantities....A square container can be the same volume (or, in child terms, "hold the same water") as a round one. A short, wide jar can be equal in volume to a taller, narrower bottle.[13]

Objectives

Through many experiences in actively manipulating both "continuous" and discontinuous" materials, the child will:

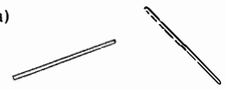
1. "acquire an understanding of the concept of conservation of quantity".[14]

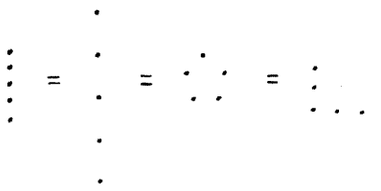
[13]Chris Nash, **The Learning Environment: A Practical Approach to the Education of the Three-, Four-, and Five- Year Old,** (Toronto: Methuen, 1976), pp. 65 - 68

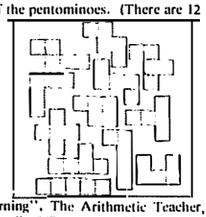
[14]Ontario Ministry of Education, **Education in the Primary and Junior Divisions,** p. 6

Learning Outcomes	Teaching Strategies/Learning Activities
Understands that continuous substances remain constant despite changes in shape.	- works at the sand table or water centre with materials which have been selected to provide feedback to the child regarding the objective: (i) three or four clear plastic containers, each a differnt shape, but all of equal volume; (ii) funnels; (iii) coloured water or dry sand: - **Teacher asks:** "Is there the same amount of water/sand in this container as there is in that one? Why?"
	- works with playdough, clay, or plasticine to discover that he or she can make the original 'blob' into many different shapes, and even return it to the original 'blob' - and the amount of the playdough, clay, or plasticine remains the same

Learning Outcomes	Teaching Strategies/Learning Activities
Understands that continuous substances remain constant despite changes in the number of containers.	- works at the sand table or water centre with materials which have been selected to provide feedback to the child regarding the objective: (i) three clear plastic containers, one large one exactly equal to two small identical ones; (ii) funnels; (iii) coloured water or dry sand; - Teacher asks; "Is there the same amount of water in these two small containers as there is in the big one? Why?"

Learning Outcomes	Teaching Strategies/Learning Activities
	- works with sticks and blocks of various sizes and shapes at a block centre and learns through exploratory play (e.g. constructing a tower) that: (i) blocks retain their size whatever their position; (a) (b)  (ii) two short sticks (or four even shorter sticks) may be substituted for 1 longer one; (c) (d)  - Teacher asks: "Are these (c) as long as this (d)? Why?" "Is (a) as long as (b)?"

Learning Outcomes	Teaching Strategies/Learning Activities
Understands that discontinuous substances remain constant despite changes in location/position.	- works with buttons, blocks, shells, counters, etc., assembling and reassembling matching sets so that he or she can see that the one-to-one correspondence is preserved even when the sets have different configurations:  - uses straws or pipecleaners to match one-to-one if he or she is having difficulty with the above - Teacher asks: "Are there the same number of black buttons as white buttons? Why?"

Learning Outcomes	Teaching Strategies/Learning Activities
	- works with tangrams and tangram puzzles - works with **Pentominoes:** A pentomino is a plane figure that is formed by grouping five congruent, square shapes together so that every square has at least one of its sides in common with at least one other square...Two pentominoes are different if they are not congruent; that is, if one cannot be made to match the other by flipping or by rotating. A good classroom exercise is to have your students find out and cut out of graph paper all of the pentominoes. (There are 12 possible shapes)[15] 

[15] Richard A. Cowan, "**Pentominoes for Fun and Learning**", The Arithmetic Teacher, Volume 24, Number 3, (Reston, Virginia: National Council of Teachers of Mathematics, 1977), p. 188.

Learning Outcomes	Teaching Strategies/Learning Activities
Understands that the weight remains constant despite changes in the shape of the object or in the number of containers.	- works with either continuous substances or discontinuous substances plus a balance scale: (i) weigh a ball of playdough, (ii) removes it from the scale and changes its shape, (iii) returns it to the scale for reweighing, OR (i) weighs a large container full of water, (ii) removes it from the scale and puts the water into the two small containers which hold an amount equivalent to the large container and then (iii) returns the two small containers to the scale for weighing



A CLEAR PICTURE OF WORD PROBLEMS

by Robert Sovchik

For quite some time elementary teachers have attempted to develop strategies for teaching word problems (Brueckner, 1935; Brueckner and Grossnickle, 1953). A common approach is to develop a list of questions that are similar to these:

1. What is the given information?
2. What is the unknown?
3. What operation or operations should I use?
4. What number sentence should be written?
5. How do I solve this number sentence?
6. How can I recheck my work?

This "recipe" type of teaching word problems has been useful because it allows for practice at each stage of the list. For example, students may be given several word problems and they may be asked to identify the given information. At another time, students are asked to identify the unknown in a set of word problems. By working on individual items in the aforementioned list the task of solving story problems is made more manageable and systematic.

A clever way of individualizing this procedure is to construct a file folder activity which helps students identify word problem skills. This file folder activity helps children to sequence skills by using cards which turn over to form a picture.

Here is an example of this activity. On the left interior side of a file folder write a word problem and directions (see figure1).

Fig. 1

WORD PROBLEM:

We need five players for each team. There are 30 students. How many teams can we have?

Directions:

Using skill cards, solve this problem by putting correct skill cards in order.

1. What is the given information?
2. What do you want to find?
3. What operation should you use?
4. What is the number sentence?
5. What is the answer?

When finished, close the folder and hold it tightly. Now flip it over and open the back. If your solution of the problem is correct, you will have a complete picture.

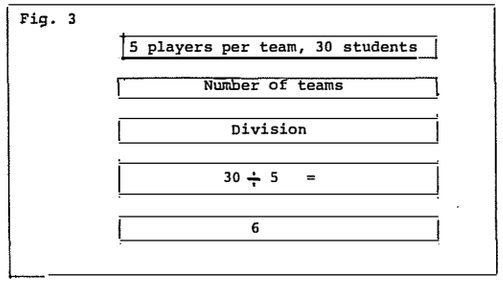
On the right interior side of the file folder, make a pocket to contain the skill cards (see Fig. 2).

Fig. 2



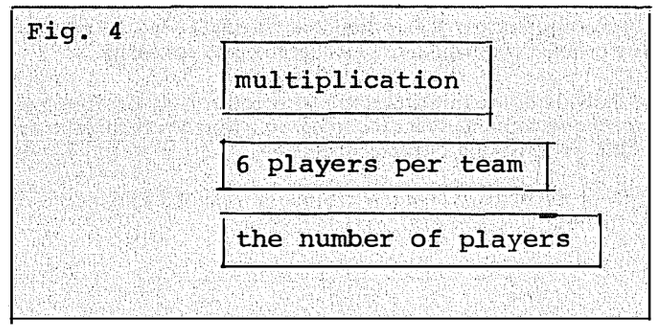
Finally, construct your skill cards. This is easily done by dry mounting an interesting photograph or picture on tag board. Next cut the picture into five equal sized strips. On the back of each strip put you preferred answers. For example, the cards selected for this story problem have the following statements (see Fig. 3).

Fig. 3



After you have constructed your preferred set of answers, create three distractors. This is done by using the extra tag board and cutting it to the same size as your preferred answers. The distractors selected for this word problem are a card containing the word multiplication, a card containing the incorrect information of six players per team, and a card incorrectly stating the unknown as the number of players (see Fig. 4).

Fig. 4



All eight cards are placed in the pocket and students are asked to select the appropriate answers for the question listed on the interior left side of the file folder. If the students have correctly answered the cards they are rewarded by the sight of a complete picture. If one or more of the distractors are chosen then flipping over the cards will yield an incomplet picture, containing at least one tag board colored strip.

In conclusion, elementary teachers attending workshops have enjoyed making this activity and have often constructed several file folders to be used at learning centers. This activity is a useful and enjoyable supplement to word problem teaching strategies. Hopefully, elementary mathematics students will get the picture!

Brueckner, L. J. "Diagnosis in Arithmetic" in **Educational diagnosis,** the Thirty-Fourth Yearbook of The National Society for the Study of Education, Bloomington: Public School Publishing, 1935.

Brueckner, L. J. and Grossnickle, F. E. **Making Arithmetic Meaningful,** Philadelphia: Winston, 1953.

Dr. Robert Sovchik, University of Akron, Ohio 44325

THE CASE FOR STUDENT ESSAYS

by E.J. Barbeau

It is the custom of a few teachers in the province to assign to their mathematics classes a project or an essay. While the effort required of both student and teacher is very great, there is a corresponding benefit to each. For the student, it offers him the chance to get into a small area of mathematics, understand it and put his ideas into order. For the teacher, it is another window into the capabilities of his students and an opportunity to get involved in mathematics other than that on the curriculum. Students in high school often appear to be in a romantic stage of their mathematical development, eager to explore the limits of concepts, notation and techniques; essays are a good outlet for this and the results can even challenge the notions of the teacher himself.

There are sound pedagogical reasons for assigning essays. The first is the need for students to be more articulate and discriminating. Many students leave high school with an inability to express themselves effectively in mathematics. They do not know the range of applicability of a technique or a result, have only a fuzzy perception of basic concepts, and are unable to express with any coherence their progress towards the solution of a problem. Writing an essay forces the student to go through the exercise of working with an idea until he can make it intelligible to a reader. Indeed, for the less able student, an essay offers the opportunity to come to grips with some mathematics, clarify his ideas over a longer time period than is possible in a test or an ordinary assignment and, perhaps, even find a sense of achievement which may elude him elsewhere in his studies.

A second reason for the desirability of essay-writing is that it is another way to help a student get a sense of his abilities. The usual round of class assignments and tests may not only provide an incomplete view, but in fact may be downright misleading. University mathematics courses are strewn with the corpses of students who did not realize that their high school grades were due to a good memory or effective cramming techniques rather than an understanding and appreciation of mathematical concepts and reasoning. Preparing an essay tests the author's interest, motivation and skill at weaving ideas.

Finally, writing mathematics at length is a useful occupation for a gifted student. Such students are often left to gobble up the high school curriculum long before their graduation year. What should happen then? Should they go on to more advanced mathematics, to take university courses? Such an idea should be viewed with extreme caution. The student's ability has to be very carefully assessed to make sure acceleration is not a route to disaster. It is better to enrich his high school curriculum to include useful topics which are not part of the university core. For example, traditional higher algebra, Euclidean geometry, finite differences, basic information theory, combinatorics are a few fields involving tools accessible to a good high school student. He might be encouraged to become a problem-solver, and even submit solutions to journals which publish problems. Finally, he should be encouraged to indulge in some mathematical explorations and present his results in the form of an essay. In short, rather than rush through a lot of work he will ultimately see anyway if he proceeds in mathematics, the gifted student should be encouraged to read, solve, write and above all think.

The nature of the essay. As a rough guide, the student should be asked to write about ten pages. High standards of exposition, involving complete English sentences, correct spelling and grammar, should be insisted upon. A great deal of emphasis should be placed on the essay having a strong focus and on the need for the student to carefully work through his material and express it in his own words. Failure to do this will result in vague generalities on the nature of mathematics or cut-and-paste jobs in which the student includes about twenty poorly-digested ideas of rubber-sheet topology with no common thread except that they were all taken from the same popular book. If the student is relying completely on material from outside himself, he should look for ways of presenting it on his own terms, perhaps by creating his own examples or by describing what aroused his interest in the topic. All outside sources of information should be acknowledged with reference to a bibliography at the end of the essay. Jargon should be avoided as much as possible, and unfamiliar terms should be defined; the essay should be self-contained from the point of view of any student in his class. Particular care should be taken by the marker to ensure that any reasoning in the essay is sound; passages with faulty reasoning should be returned for revision.

It might be suggested to the student that the student work through one main idea, and imagine that he is writing a short account which might be read by or to his classmates.

Topics. Possible topics fall into several categories. The beauty of an essay is that the choice of topic can take into account the interests, skill and ability of the writer.

(a) **Historical topics.** The essay might be written on an individual mathematician or the mathematical achievements of a certain era (for example, Arabic mathematics, the rise of arithmetic and algebra during the Renaissance, applications of mathematics in the eighteenth century). The danger is that the student will simply lift passages wholesale from the nearest encyclopedia, so for such a topic, the marker will have to strongly grade the essay on originality and the ability to crystallize the key points from a mass of data.

(b) **Description of mathematical ideas or results.** The student should be encouraged to read a book, absorb some mathematics and then express it in his own way. No original work is expected, but again the student should try to grasp the key ideas and perhaps come up with some suggestive example. He might write on game theory, some mathematics of coding (e.g. trapdoor functions), algorithms for evaluating polynomials or inverting matrices, the real number system, infinity, interpolating values by polynomials, the mathematics of pitch.

(c) **Applications of mathematics.** The student might look briefly at how mathematics can be applied in different areas, such as the study of biological populations, deciphering codes, studying an economic system, predicting planetary motions.

(d) **Original research.** The possibilities in this area are more numerous than you probably realize. The student's best ally here is a teacher who does his own reading and doodling, and is alert to the possibilities inherent in some of his own explorations. A good idea is th direct the students to a problem journal like **Crux Mathematicorum** to see if he can find a problem which can be generalized.

Having run an essay contest for four years, I am not ignorant of the demands made on the teacher by assigning an essay. They are considerable. But the results can be very gratifying. I will conclude with an appendix suggesting resources and possible topics.

APPENDIX

A. **The Samuel Beatty Essays** The Samuel Beatty Fund supported for four years a high school mathematics essay contest, and the winning essays have been published in two volumes prices respectively at $4.00 and $3.50 (or $7.0 for the pair). They can be obtained from E.J. Barbeau, Department of Mathematics, University of Toronto, Toronto, Ontario M5S IA1. Please make cheques payable to "The University of Toronto". There is money available to publish further volumes, and teachers are encouraged to send to Prof. Barbeau the best from their classes. These volumes will serve to illustrate what high school students are capable of. It is significant to note that the best essays did not always come from the students who distinguished themselves by winning contests; some apparently came from students of no exceptional mathematical ability who did possess a combination of perseverance and enthusiasm.

B. **Crux mathematicorum.** This is probably the best source of ideas for original work (as well as a good source of problems to preserve the gifted from boredom). $22 pays for a year's subscription of 10 issues, obtainable from the managing editor, F.G.B. Maskell, Algonquin College, 200 Lees Avenue, Ottawa, Ontario K1S 0C5 (cheques payable to "Crux mathematicorum"). This enjoys the sponsorship of the Carleton-Ottawa Mathematics Association, but appears to be surprisingly little used by Ontario teachers.

C. **Extra reading.** There are some excellent short books written particularly for high school students which introduce them to more advanced work at an elementary level. Two series to look at are the Houghton Mifflin Mathematics Enrichment Series (Houghton-Mifflin Company, Boston) and Topics in mathematics (D.C. Heath and Company, Boston). One of the best series is the New Mathematical Library, published by the Mathematical Association of America, 1529 Eighteenth Street NW, Washington, D.C. 20036, from whom a catalogue can be obtained. If you are in Toronto, you might visit Book World, 118 Avenue Road (M5R 2H4, 925-5657) or Progress Books, 71 Bathurst Street (M5V 2P6, 368-5336), who handle English-language books from the U.S.S.R. The books in the Little Mathematics Library (Mir Publications, Moscow) are excellent and dirt cheap. There are a host of other books; the only one I will mention here is **The mathematical experience,** by Philip J. Davis and Reuben Hersh. It is available in both hardback (Birkhauser) and paper editions.

D. **Topics.** Here are a number of possible topics, many suitable for investigation. Some of the problems are unsolved, but you need not tell that to the students; they were chosen because they offer the scope of finding partial results.

1. A game is played as follows. n dots are given in the plane and each pair of dots is connected by an edge, so that there are ½n(n - 1) edges in all. Players P and Q move alternately; P goes first. A move consists of removing three edges which form a triangle (i.e. all the edges involving pairs of some set of three points). The first player who is unable to make a move loses. Give upper and lower bounds for the number of moves required to bring the game to a conclusion. Does either P or Q have a winning strategy? Look at small values of n, and work up; when $n = 3$, obviously P wins on the first move.

2. $720 = 27^2 - 3^2 = 28^2 - 8^2 = 29^2 - 11^2$, so that 720 differs from the next three greater squares by a square. Show that there are infinitely many numbers like 720 in this respect. Are there any number which differ from the next four squares by a perfect square? five?

3. Given a natural number n, what is the maximum number of integers which can be chosen from the set $\{1, 2, 3, \ldots, n\}$ in such a way that for each pair $\{a, b\}$ in the chosen subset, the product ab is not a multiple of $a + b$? Clearly, this maximum exceeds ½n (take all the odd integers). Can it be substantially bigger than ½n? Pick on some specific values of n and get the computer working on it.

4. Is it possible to find six number such that the sum of any pair is a perfect square? Replace "six" by a lower number, if necessary.

5. Let m and n be any positive integers. In how many ways can the numbers $\{1, 2, 3, \ldots, mn\}$ be arranged in a rectangular array with m rows and n columns, such that, in each row, each number is larger than the one to its right, and in each column, each number is larger than the one below it. An example of such an array when $m = 3$ and $n = 4$ has the respective three rows: (12, 11, 7, 3), (10, 8, 5, 2), (9, 6, 4, 1).

6. Let $p < q < r$. In how many ways can q numbers be chosen from $\{1, 2, 3, \ldots, r\}$ so that no p of them are consecutive?

7. Certain geometric figures can be dissected into a finite number of disjoint congruent pieces, each similar to the given figure. How many such figures can you find?

8. Let a, b, c, d be the lengths of four sides of a quadrilateral (listed in cyclic order). Since the quadrilateral can be deformed without changing the lengths of its sides, the area is not uniquely determined. let m and M be, respectively, the smallest and largest possible areas. What estimates can be given for m and M? An early approximation formula for the area is the product of $\frac{1}{2}(a + c)$ and $\frac{1}{2}(b + d)$. When is this exact? Is it always true that this approximation lies between m and M? What happens if the quadrilateral is concyclic?

9. A conjecture of Erdos states that, for every positive integer $n \geq 3$, the rational number 4/n can be written as the sum of three distinct integer reciprocals; thus, $4/n = 1/a + 1/b + 1/c$, where a, b, c are positive integers. Examine the evidence for this conjecture. See how many cases you can dispose of. What is the smallest number of reciprocals to sum to 5/n and 6/n? Make some conjectures.

10. Let b be any integer base of numeration. Find those numbers, which when written to base b, are the sums of the squares of their digits. Do the same problem with cubes.

11. Everybody is familiar with the tiling of the infinite plane by squares, equilateral triangles or regular hexagons (honeycomb). What other regular or nonregular polygons allow a tiling by congruent copies of itself? Suppose you are allowed to use two different polygons for your tiling instead of one.

12. It is commonly asserted that three pieces of information are required to specify (up to congruence) a triangle. Is this true? Is a triangle determined by specifying the lengths of its three medians? its angle bisectors? its altitudes? the relative positions of its orthocentre, centroid and incentre? If these three bits of information are provided, is it always true that there exists a triangle filling the specifications? You can look at such questions from the point of view of geometry or trigonometry. Look at other possibilities for the three items of information.

13. **Colouring problems.** Examples: (a) Is it possible to colour the points (x, y) with integer coefficients in the plane with three colours so that each colour appears infinitely often in infinitely many lines parallet to the x-axis and no three points, one of each colour, are collinear? (b) If all the points of the plane are to be painted so that no two points at unit distance receive the same colour, what is the minimum number of colours that can be used? At least give upper and lower bounds. (c) Six points are such that no three are in a line. Each of the fifteen line segments joining them in pairs is coloured, some red and some blue. Prove that some triangle formed by the segments has all its edges the same colour. Generalize.

14. Consider the following functions under iteration: i.e. form the sequence n, f(n), f(f(n)), f(f(f(n))), ... Look at numerical cases, make conjectures and obtain partial results.
(a) $f(n) = \frac{1}{2}n$ (if n is even), $f(n) = 3n + 1$ (if n is odd);
(b) $f(n) = 1$ (if n is 1 or prime), $f(mn) = mf(n) + nf(m)$ in general;
(c) f(n) is the sum of the kth powers of the digits of n to some base b.

15. A latin square is a square array of n^2 numbers such that each entry is one of the numbers {1, 2, 3, ..., n} and each of the numbers appears exactly once in each row and each column. Two latin squares are orthogonal if, given any pair a, b of numbers between 1 and n inclusive (not necessarily distinct), there is a position which contains a in the first square and b in the second. For example,

$$\begin{matrix}1&2&3\\2&3&1\\3&1&2\end{matrix} \quad \text{and} \quad \begin{matrix}1&3&2\\2&1&3\\3&2&1\end{matrix}$$

are orthogonal. Construct latin squares and pairs of orthogonal latin squares of various orders. Try to draw some general conclusions.

16. Let p(t) be a given polynomial and study the remainder when p(t) is divided by $(t - a)^r (t - b)^s \ldots (t - h)^u$. Consider special cases of the divisor, for example a single power of a linear polynomial, or a product of distinct linear polynomials all raised to the power 1.

17. The factorial powers of t are defined by $t^{(1)} = t$; $t^{(2)} = t(t - 1)$; $t^{(3)} = t(t - 1)(t - 2)$; ...; $t^{(k)} = t(t - 1)\ldots(t - k - 1)$;...
Investigate the coefficients when factorial powers are expressed as polynomials in ordinary powers, and when ordinary powers are expressed as "polynomials" in factorial powers. Investigate the sum $\sum_{k=1}^{n} k^{(r)}$ and use your results to obtain information about the sums.

18. Let d and r be integers with $d \geq 2$, $r \geq 1$. Suppose p is a polynomial with real coefficients, and degree $p < r$. Show how to partition the set $S = \{p(0), p(1), p(2), \ldots, p(d^r - 1)\}$ into d disjoint subsets whose union is S, such that the sum of each subset is the same. Look at special cases: the partitions you are looking for should work when p(t) is constant or t^k. Consider the special case $r = 1$. Do the case $r = 2$, any d. Do the case $d = 2$, any r. Generalize.

19. Consider the following recursion, where x is a variable:

$$u_0 = 0 \ , \ u_1 = 1$$

$$u_n = xu_{n-1} - u_{n-2} \qquad \text{when} \quad n \geq 2$$

Thus, we get a sequence of polynomials:
0, 1, x, $x^2 - 1$, $x^3 - 2x$, $x^4 - 3x^2 + 1$, and so on.
When we take x - 2, the sequence becomes 0, 1, 2, 3, 4, 5, ... and when x - 3, it is a portion of the Fibonacci sequence 0. 1, 3, 8, 21, 55, ... Look for patterns in the sequence of polynomials which generalize those in the special cases. For example, examine factorizations of the polynomials. Look at the running total of the odd indexed. terms.

20. (Junior topic) What squares can be written as the sum of two other squares? three other squares? look for patterns and families.

21. (Junior) How long an arithmetic progression can you find whose members are all prime numbers? Example: 5, 11, 17, 23, 29.

22. (Junior) What is the largest number of integers which can be chosen between 1 and 100 inclusive so that no two of them add up to a perfect square. Do the same problem replacing 100 by other integers.

23. (Junior) Write four positive numbers at the vertices of a square. Form a new square array of numbers as follows: each is the absolute value of two adjacent numbers of the original square. Repeat the process. What eventually happens? Why? Examine many examples.

24. (Junior) Devise an arrangement whereby a class of 15 students can be marched to chapel each day for a week in five files of three students each in such a way that any two of the students are in the same file on exactly one of the days. Can the same problem be solved for files of three or 9 students over four days, 21 students over ten days, 27 student over thirteen days, etc.?

25. Does it make sense to define the logarithm of a negative number?

26. Incommensurability.

27. Regular solids of the fourth and higher dimensions.

28. Semi-regular three-dimensional solids.

29. Space-filling curves.

30. Fractals. The dragon curve. (See Scientific American, April, 1978)

31. A description of a winning strategy for some game.

32. Encoding and decoding.

33. Mathematics applied to biology. Population growth. Interaction of species.

34. Computational aids: logarithms, slide rule, algorithms, planimeters.

35. Computer techniques for (a) testing for primality of integers, (b) testing for irreducible polynomials, (c) factoring polynomials, (d) evaluating polynomials.

36. The mathematics of map-making. Projections.

37. How does a pocket calculator work?

38. Navigation on earth and/or outer space. Determination of latitude and longitude on the high seas.

39. (Junior) A biography of a famous mathematician: Newton, Euler, Lagrange, Gauss, Hilbert, von Neumann.

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RADIOACTIVE TRIGONOMETRY

by Douglas Brown

A nuclear reactor needs a lead shield in order to keep the radioactivity on the inside where it is helpful. The question that arises is how thick must this lead wall be to guarantee a desired level of safety.

The search for the answer to this question crystalizes into a neat piece of applied trigonometry. I develped it for use in my grade 12 Computer Science class as an example of a simulation model. It could be used equally as well in a senior level mathematics course.

One of the basic premises of mathematical modelling is to simplify the problem so as to get an initial grasp of the process involved. With this in mind let us develop a simplistic model of the problem.

Assumptions:

i) the wall is t cm. thick.
Each particle that enters the wall collides with other particles in a totally random manner but to simplify this event it will be assumed that:

ii) there is a fixed distance that each particle moves between collisions. This intercollision distance will be d cm.

iii) each particle will lose its energy (and thus no longer move about in the wall) after n collision.

With these three assumptions it is now possible to determine the final position of each particle that enters the wall. Figure 1. shows the initioal movement of a single particle. It enters the wall at an angle of 0_1 where $-90 < \theta_1 < 90$ and travels a distance. At this point it collides with another particle and again travels a distance d in a new direction. Figure 2. summarizes the particle's travels so far.

Figure 1

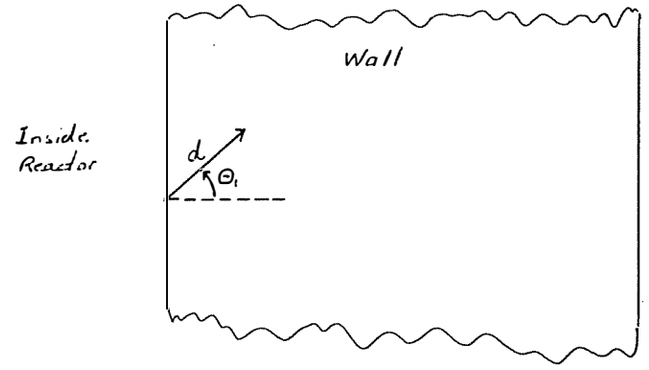
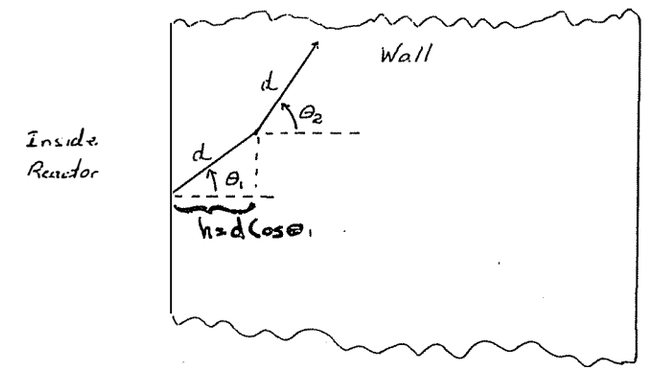


Figure 2



The actual itinerary of the particle is not our concern. Our only interest is the horizontal movement of the particle. Some simple trigonometry will help at this point. Figure 2. illustrates the obvious. In general; $h_i = d \cos 0_i$ where d is the intercollision distance and 0_i is the angle at which the particle moves during the i'th move.

$$0° < \Theta \leq 360° \quad i > 1$$

It would seem simple then to generate the following formula to determine the ultimate position of the particle where H represents the horizontal distance travelled relative to the inside of the wall.

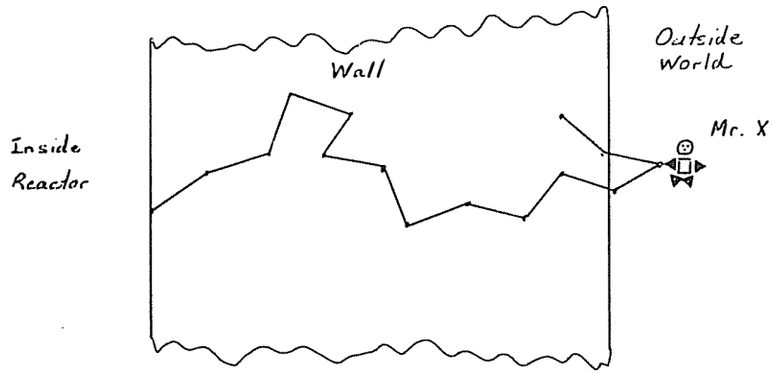
$$H = \sum_{i=1}^{n} h_i$$

$$H = \sum_{i=1}^{n} d \cos \theta_i$$

$$H = d \sum_{i=1}^{n} \cos \theta_i \quad \text{----- (A)}$$

But life is never quite so simple. The first concern can be classified as a "But what if?" concern. Some sharpy in the class will raise the question: "But what if a particle moves back towards the reactor? The horizontal distance travelled will be negative and the result should be a subtraction." CAST rule to the rescue! A quick review of the fact that $\cos \theta < 0$ if $90 < \theta < 270$ will help the student realize that d cos θ will end up as a negative quantity at the correct times.

Figure 3



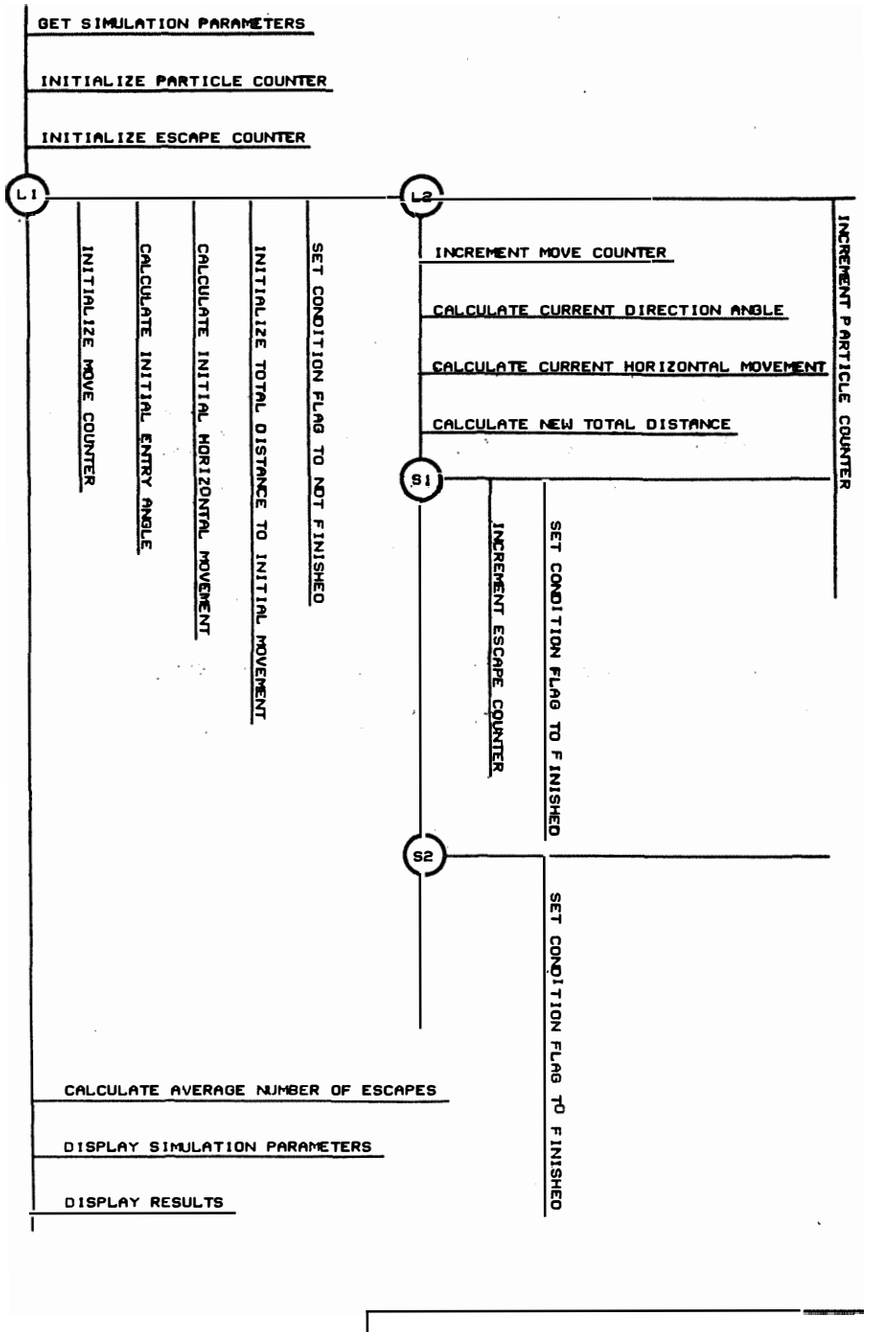
The second concern is a more major one. Formula A only determines the ultimate position of the particle. There is a danger here. Figure 4. illustrates the concern. It would be of little consolation to Mr. X that the ultimate position of the particle was 12 cm. and the wall thickness was 13 cm. It is not enough to determine the ultimate position. Rather, it is necessary to constantly monitor the movement of the particle until it either escapes the wall, dies out in the wall or re-enters the reactor. Algorithm to the rescue!

Appendix A outlines the necessary algorithm for the model and appendix B is a computer program of the model written in Waterloo Basic for the PET. It will run as is on any CBM4016 or CBM4032 with Waterloo Basic.

References: Mathematical Models 2: Descriptive and Casual Models TV Ontario (Video Tape)

Douglas Brown, Computer Liaison Office Waterloo County Board of Education

APPENDIX A



L1 : LOOP UNTIL ALL PARTICLES ARE SIMULATED

L2 : LOOP UNTIL ALL MOVES COMPLETED OR CONDITION FLAG INDICATED FINISHED

S1 : IF TOTAL DISTANCE > WALL THICKNESS

S2 : IF TOTAL DISTANCE IS NEGATIVE

APPENDIX B

```
110 REM *************************  NUCLEAR REACTOR  **************************
200 SYS9*4096:REM WATERLOO BASIC
205 YES=1:NO=0
310 CALL INPUT
320 CALL SIMULATE
325 CALL SUMMARY
390 END
400 REM ***********************************************************INPUT***
410 PROC INPUT
415  PRINT"███             NUCLEAR REACTOR"
420   INPUT"█         WALL THICKNESS(CM)=";T
430   INPUT"█INTERCOLLISION DISTANCE(CM)=";D
440   INPUT"█          NO. OF COLLISIONS=";N
450   INPUT"█           NO. OF PARTICLES=";P
490 ENDPROC
500 REM ********************************************************SIMULATE***
510 PROC SIMULATE
512 ITER=1
513 ESCAPES=0
514 LOOP
516  PRINT"████████████████              PARTICLES    ████";ITER
520  MOVE=1
540  THETA=RND(-RND(0))*180-89
550  H=D*COS(THETA*π/180)
560  DIST=H
565  FINISHED=NO
570  LOOP
575   PRINT"████████████████████                  MOVE    ████";MOVE
580   MOVE=MOVE+1
590   THETA=RND(-RND(0))*360
600   H=D*COS(THETA*π/180)
610   DIST=DIST+H
620   IF DIST>T
630    ESCAPES=ESCAPES+1
640    FINISHED=YES
650   ELSEIF DIST<0
670    FINISHED=YES
680   ENDIF
690  UNTIL MOVE>=N OR FINISHED=YES
700  ITER=ITER+1
710 UNTIL ITER>P
715 PRINT"██████♥♥ SIMULATION COMPLETED ♥♥♥"
716 FOR WT=1 TO 500:NEXT WT
720 ENDPROC
800 REM *********************************************************SUMMARY***
810 PROC SUMMARY
815  PRINT"███             NUCLEAR REACTOR"
820  PRINT"█                  SUMMARY"
822  PRINT"█         WALL THICKNESS : ";T;"CM"
824  PRINT"█INTERCOLLISION DISTANCE : ";D;"CM"
826  PRINT"█COLLISIONS PER PARTICLE : ";N
830  PRINT"███       NO. OF PARTICLES : ";P
840  PRINT"█         NO. OF ESCAPES : ";ESCAPES
850  PER=ESCAPES/P*100
860  PRINT"█     PERCENTAGE ESCAPED : ";PER;"%"
890 ENDPROC
READY.
```

ONTARIO STUDENT ACHIEVEMENT IN INTERNATIONAL PERSPECTIVE RESULTS FROM THE SECOND INTERNATIONAL MATHEMATICS STUDY

by Dennis Raphael, & Merlin Wahlstrom

Editors Note:

Previously results were reported for Ontario Gr. 8 (Dec. '83 Gazette) and Ontario Gr. 12/13 (Mar. '84 Gazette). This report puts the Ontario results into the framework of the international results. We felt the results were interesting and worth publishing, but are plagued by the question, "now what?" What are the ***teaching & curricular implications*** *of these results? Are there implications here for the Senior Guidlines Committee to consider? For O.A.I.P.? We hope that the publication of these results will produce meaningful dialogue & changes where needed.*

Ontario Student Achievement in International Perspective:[1] Results from the Second International Mathematics Study

Ontario's participation in the Second International Study of Mathematics (SIMS) was described in two recent issues of the Gazette (1, 2). Participants in the study included the following nations:

Australia	France	Netherlands
Belgium (Flemish)	Hong Kong	New Zealand
Belgium (French)	Hungary	Nigeria
Canada (British Columbia)	Irish Republic	Scotland
Canada (Ontario)	Israel	Swaziland
Chile	Ivory Coast	Sweden
England	Japan	Thailand
Finland	Luxembourg	United States

The present report provides data related to international comparisons in student achievement including Ontario's standings as compared to other SIMS participants. Before presenting these data a brief backgroud to the study is provided.

The Second International Mathematics Study

The Second International Mathematics Study, a project of the International Association for the Evaluation of Educational Achievement (IEA), targeted two age groups or populations of students for study.

Student Populations

Population A was defined as those student enrolled in the grade where the majority of students had reached the age of 13 by the middle of the school year. In Ontario, British Columbia and the United States this corresponded to Grade 8. Population B was defined as students in the normally accepted terminal year of secondary school who where studying mathematics as a substantial part (approximately five hours a week or more) of their academic program. In Ontario the group best fitting this definition were those Grade 13 students enrolled in two or more Grade 13 mathematics courses, while in British Columbia and the USA, the Population B students were in Grade 12.

The two populations were selected as representing participating countries' school systems at significant junctures. In most countries, including Ontario, all Population A students were studying mathematics with little or no tracking or streaming taking place. Students in Population B, on the other hand, represented a highly selected group, albeit self-selected in Ontario. Most were bound for university, and the amount of mathematics they generally reflected preparation for engineering, science or other professional careers.

Cognitive Instruments

Following a lengthy validation procedure, 180 Population A and 136 Population B items were finally selected and classified in a content by behaviour grid. At the Population A level the five main content areas were Arithmetic, Algebra, Geometry, Probability and Statistics and Measurement. The seven Population B areas were Sets, Relations and Functions, Number Systems, Algebra, Geometry, Analysis, Probability and Statistics and Finite Mathematics. For both populations, items

were also classified into one of four "behavioural" levels--computation, comprehension, application and analysis.

International Comparisons: Population A

Eighteen of the original 47 subsets of achievement items defined for Population A were chosen for international comparisons. Table 1 presents descriptive statistics for the 20 nations which contributed student achievement data at the Population A level[2] and the Ontario scores and standard error of these scores for the eighteen subsets.

POPULATION A: INTERNATIONAL AND ONTARIO SUB-SCORE RESULTS

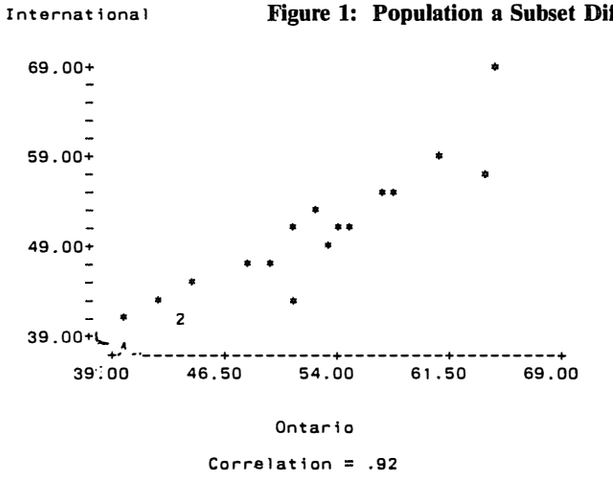
Sub-Score ID	Sub-Score	International		International			Ontario	
		Mean	S.D.	QI	Median	Q3	Mean	Sem.
A01	Estimation and Approximation	54.3	9.7	60.1	55.4	48.2	57.9	0.9
A04	Algebra (Computation)	45.3	9.3	51.9	44.8	39.6	44.4	0.9
A06	Arithmetic (Computation)	57.0	8.1	63.1	59.4	50.8	63.6	1.0
A07	Proportionate Thinking	49.1	8.1	55.9	49.6	44.9	53.6	0.9
A09	Whole Numbers	58.1	8.0	64.5	58.8	52.4	60.9	0.8
A15	Arithmetic	50.5	7.5	57.3	50.8	45.5	54.5	0.9
A16	Arithmetic (Other than Computation)	46.6	7.6	53.2	47.7	41.5	49.2	0.9
A19	Equations and Inequations	41.8	8.3	46.8	42.1	38.9	43.8	0.9
A20	Algebra	43.1	8.6	49.7	43.0	38.6	42.0	0.8
A21	Algebra (Other than Computation)	41.2	8.4	47.8	40.6	37.9	39.8	0.8
A26	Plane Figures	47.2	7.9	50.0	47.7	43.6	48.0	0.9
A28	Informal Transformations in Geometry	43.5	10.3	50.9	43.9	38.5	50.8	1.1
A29	Geometry	41.4	7.8	44.8	42.5	37.9	43.2	0.8
A34	Probability and Statistics	54.7	9.1	59.8	57.3	51.9	57.0	0.9
A36	Standard Units of Measurement	68.5	13.1	80.8	69.2	56.9	64.2	1.0
A38	Measurement	50.8	8.8	57.5	50.5	47.3	50.8	0.9
A43	Non Verbal	51.7	8.7	57.5	50.9	46.5	53.7	0.9
A44	Verbal	53.0	8.0	59.6	52.6	48.4	52.8	0.9

Table 1: Population A: International and Ontario Sub-score Results

As shown, the International Mean for the subset of items concerned with Estimation and Approximation was 54.3 percent correct, the upper and lower quartiles of the distribution of national scores was 60.1 and 48.2 percent correct respectively, and the International Median was 55.4 percent. Ontario's average score for this subset of items was 57.9 percent correct which was above both the International Mean and Median.

Ontario scored above the International Median for 13 of the 18 subsets and was at or very close to the upper quartile for two subsets: Arithmetic (Computation and Informal Transformations. Ontario was below the International Median for only five areas with only one, Standard Units of Measurement, showing more than a one percent difference. Figure 1 shows the marked correspondence between Ontario and the International participants in patterning of achievement results.

Figure 1: Population a Subset Difficulties



International Comparisons: Population B

The 136 Population B items were assigned to one or more of 27 subsets, 17 of which are being used for International comparisons. Table 2 presents descriptive statistics for the 15 nations contributing data at the Population B level and the Ontario scores an standard error of these scores for the 17 subsets.

Ontario scored above the International Median for seven subsets, below the median for six subsets and was at the International Median for five areas. Except for the area of Sets, Relations, and Functions, where Ontario's scores fell a few points below the Upper Quartile, Ontario's scores did not fall near the Upper or Lower Quartiles of the distribution of participating nations' scores. Figure 2 shows that Ontario results deviate nominally from International scores indicating that Ontario student achievement was at the International median.

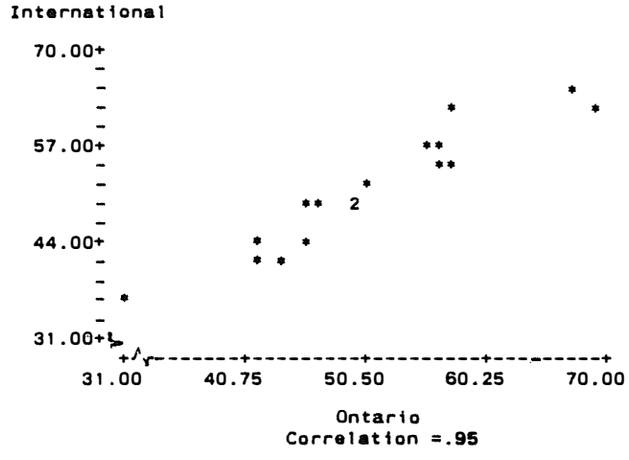
POPULATION B: INTERNATIONAL AND ONTARIO SUB-SCORE RESULTS

Sub-Score ID	Sub-Score	International		International			Ontario	
		Mean	S.D.	Q1	Median	Q3	Mean	Sem.
B01	Sets, Relations and Functions	61.6	12.7	71.8	61.4	51.2	69.1	1.0
B02	Number Systems	49.5	12.9	59.4	46.6	40.3	46.6	0.9
B03	Number Systems Excluding Complex Nos.	56.4	12.1	66.6	54.1	48.1	55.1	0.9
B04	Complex Numbers	36.9	15.9	47.0	35.1	23.5	31.1	2.0
B06	Algebra	57.4	11.6	66.0	56.7	46.9	56.7	1.0
B07	Algebra Computation	65.2	11.9	74.8	64.9	53.7	66.8	1.1
B08	Algebra Other than Computation	52.6	11.6	60.5	51.5	42.1	50.5	1.1
B09	Equations and Inequations	61.3	10.5	69.2	59.6	52.9	57.3	1.0
B10	Geometry	42.4	10.4	48.5	41.8	32.6	41.5	0.7
B11	Analytical Geometry	49.3	12.4	58.2	47.0	39.7	49.2	1.0
B13	Trigonometry	49.0	12.6	57.4	49.5	33.8	49.5	1.4
B14	Analysis	44.1	14.5	54.6	45.5	28.3	45.5	1.0
B15	Functions	55.2	12.4	63.1	56.3	42.4	56.3	0.8
B18	Calculus	40.3	15.6	49.3	41.9	23.2	43.2	1.1
B19	Probability and Statistics	49.5	13.2	63.7	45.6	38.4	46.1	1.2
B25	Behaviour Level Analysis	44.4	10.5	49.4	44.8	33.0	41.8	0.8
B27	Behaviour Level Computation	54.8	12.1	65.1	55.7	43.1	57.2	0.8

Table 2: Population B: International and Ontario Sub-Score Results

The population B results must be interpreted in relation to the proportion of the age cohort enrolled as Mathematics Specialists within a participating nation. In Ontario the proportion of the age cohort enrolled in two or more Grade 13 mathematics courses is approximately 18 percent, a figure which preliminary analysis suggests is appreciably greater than that of most other participating nations. However, Ontario's specialists are enrolled in their thirteenth year of schooling, while in virtually all nations Population B students were enrolled in their twelfth year of schooling.

Figure 2: Population B Subset Difficulties



Summary and Conclusions

The data presented in this report suggest that for Population A, Ontario grade 8 students' mathematics performance tends to fall above the median in most cases. Performance in the areas of Arithmetic (computation) and Informal Transformations is noticeably above the International Median, close to the Upper Quartile of participating nations. Performance is noticeably below the Median for Standard Units of Measurement.

At the Population B level, except for the Sets, Relations and Functions items, Ontario Mathematics Specialists' scores are not distinctively divergent from the International Median. The latter results must be considered in light of the relatively larger cohort of students enrolled as mathematics specialists in Ontario and the difference in years of schooling among the participating nations.

References

(1) Raphael, D., Wahlstrom, M., & McLean, L. Intentions and attainments in the teaching and learning of Ontario Grade 8 mathematics: Results from the Second International Study of Mathematics. **Ontario Mathematics Gazette,** December 1984, **22,** no. 2

(2) Raphael, D., Wahlstrom, M., and McLean, L. Intentions and attainments in the teaching and learning of Ontario Grade 12/13 mathematics: Results from the Second International Study of Mathematics. **Ontario Mathematics Gazette,** March 1984, **22,** no. 3.



[1]This research was funded under contract by the Ministry of Education, Ontario. This report reflects the views of the authors and not necessarily those of the Ministry.

[2]Some of the original 24 nations participated in only the curriculum analysis, others at only one of the two Population levels.

COMPUTERS AND THE IMPENDING REVOLUTION IN MATHEMATICS EDUCATION

by John Poland

Editor's Note:

Prof. Poland makes several provocative predictions & conclusions in this article from a talk he gave at O.A.M.E.'s annual meeting in Kingston. While we don't share some of Prof. Poland's visions, the article presents an excellent framework for discussion and reaction.

Do you know of a nice formula for the sum

$$1^5 + 2^5 + \ldots + n^5 = \sum_{j=1}^{n} j^5 ?$$ No?

Well I will return to this later then. Perhaps a better beginning, to set you in the mood of my topic, would be to quote from an article by Sue Dolezal in the 1983 Newsletter of CIMSE:

Let me share one of my favourite fantasies. I enter my classroom at 8:30 a.m. Before me the bright faces of 20 senior math students eagerly await the lesson. I step to the corner of the room and switch on my microcomputer which is attached to a "big screen" TV. I announce the topic of the day's lesson - studying trigonometric graphs of the AsinBx+C form. I type sinx onto the computer keyboard and a blue sine wave graph appears on labelled axes. Next I type 2sinx and a red graph twice the amplitude of the first overlays the screen. This process continues as I vary all three coefficients A, B, C, a number of times. A student suggests I try 2/5sinx/3 + 1/8. Almost instanteously this graph appears in orange. In 50 minutes time I have completed the lecture effectively and elegantly. The students ask to stay after class so they can create their own graphs.

What are the technological prospects in the near future, a microcomputer the size of a pencil case that folds open to display a keyboard below and a screen for graphics above? Maybe with the capabilities of a portable telephone to dial up and interact with a classroom or home computer? We already have easy access to high speed exhaustive numerical methods and accurate high resolution colour graphics in 2 and 3 dimensions on the current microcomputers. We are just seeing the beginnings of the ability to do symbolic algebraic calculations, of which I will have much more to say, and of course all of this can be done on small machines that have virtually unlimited memory and access to enormous data bases. Most importantly these machines are becoming cheaper and the programs are becoming more and more interactive. The convergence of microcomputers and handheld calculators has begun. No wonder the National Science Foundation in its 1983 report on K-12 curricula talked of "the vast potential to enrich and enliven mathematics teaching", and the NCTM publication **Computing and mathematics: the impact on secondary school curricula** [1984] talks of "striking opportunities". But in most of these discussions there is a wide distance between their prognoses and detailed practical suggestions. Would large screen TV's in the classroom be practical? The cost and maintenance and possible vandalism make it doubtful. And won't these new technologies be far more demanding on the teachers? Why not just assume that there will be little change in the mathematics classroom in the next 10 years? Overhead projectors have not proved as revolutionary as everyone predicted years ago.

What are the forces that will compel us to change? There is the widespread use of computers in business and government, and the daily contact with spectacular computer generated graphics on television. Our students play computer games so they are well aware of the sophisticated methods and results available with computers. The new guidelines for Grades 10, 11 and 12 in Computer Science will make the students even more curious about the relationship between mathematics and computers. And finally of course there is the fact that we face increasingly complex unsolved problems; it is as though we have solved all the simple problems. To handle these larger problems we will have to use the help of a computer in analysing the data and various strategies.

At the university level one of the major changes that the computer has brought about is not so much in the way we are now teaching mathematics but in the mathematics that we actually teach - the curriculum. There is a strong demand for mathematics courses on the language and background necessary for studying computer science, particularly discrete mathematics. Numerical analysis, optimization, estimation, and approximation are important topics. The effect this is having on undergraduate mathematics teaching is discussed in my joint paper with B. Hodgson in the November 1983 issue of the **Notes of the Canadian Mathematical Society.**

I see three impending revolutions in secondary school mathematics teaching. The first is already in progress: the widespread use of decimal notation. The second is at our elbow: the use of graphics to display shapes, modify them, shift view-points and apply various transformations. The third is on the horizon: the new programs to do symbolic algebraic calculations such as finding the roots of quadratics without using decimal notation.

What will we do in 10 years time when
- Euclidean geometry looks completely irrelevant to **the** geometry of computer graphics,
- our students demand earlier teaching of topics in mathematics like symbolic logic and Boolean operations, induction to check that recursive programs are correct and graph theory for their computer science courses,
- and software will enable at least 85% of **all** presently taught intermediate and senior mathematics to be done on the micro, including word problems and proofs in deductive geometry?

Will students go to Jane's house to do their homework using her grade 11 math diskette?

The future of geometry teaching looks to be more visual, more experimental and completely reoriented. In the beginning there was Euclid and trigonometry to measure angles. Then with Descartes we had analytic geometry for measuring lengths, and later, vectors to see physics. The latest emphasis has been upon transformations. Clearly transformations, vectors, analytic and coordinate geometry and trigonometry are all possible on the computer with some very relevant computer graphics. But Euclid? forget it: the need for proof is elsewhere and ruler and compass seem completely irrelevant. I realize this wipes out large portions of the new advanced grades 9, 10 and 12 geometry curricula in Ontario and requires moving graphing techniques earlier than grade 12 but this change seems inevitable. And there is a desperate need to teach the power of reasoning and rational thought in a more relevant situation than the geometry of Euclid.

In algebra we have been under pressure to reduce the time spent on the simplification of radicals, factoring trinomials, and working with rational expressions. There will be even more pressure now because of muMath[1], software available on a floppy disk to do symbolic algebraic calculations. With muMath the square root of 12 is twice the square root of 3, not the approximation 3.464102. Do you remember that problem I asked at the beginning of this article? With muMath, you type in SIGMA {J^5,J,1,N} and get the answer $[N^6/6 + N^5/2 + 5N^4/12 - N^2/12]$. The computer does this by guessing the right 6th degree polynomial, finding the coefficients using small values of N; we have to verify it using induction. Here are some more relevant examples of what can be done algebraically using muMath. Type in $2y(y^2 - z) + 2z(y + z)$ and muMath will expand and simplify to respond with $2y^2 + 2z^2$. Enter SOLVE $[x(3 + x^2) = 4x(1 + c^2)-x; x]$ and muMath will display the solution set $\{x = 0, x = 2c, x = -2c\}$. Enter FCTR $[6x^2y - 4xy^2/z]$ and muMath returns $2xy(-2y + 3xz)/z$. Ask for $1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7$ and get the reply 363/140. Try expanding $(x + 2y)^3$ and the answer comes $12 + y^2 + 6x^2y + x^3 + 8y^3$. Mathematics has held an advantage over many sister disciplines because it is so exact with its emphasis upon calculation and the strong distinction between right and wrong answers. Our colleagues in history and French have envied our position, but now these very aspects make us vulnerable to computer automation of our procedures.

Jerry Johnson, writing in **Computing and Mathematics,** suggested we try teaching quadratic equations in 3 stages now. First we discuss how quadratic equations arise in science {projectiles} and in non-linear functions in economics; then we discuss solving quadratics by guessing and testing solutions, first by hand to get some feeling for the process and then using the computer to aid in the search or draw a graph. In the last stage we can use muMath for the formal solution and discuss the formal analysis. The advantages of this method are that we proceed from the concrete to the abstract, we begin with good motivation, the first method we teach (trial and-error) is the one most likely to be used later in life, and by the third stage we have naturally selected the better students. Writing on muMath, Z. Usiskin in **The Mathematics Teacher** {February 1984} said "Some skills are clearly necessary but ... too much else should be learned about mathematics to waste time in practising obsolete skills. Mathematics is getting easier [with muMath]. We will not be able to keep this secret from our students forever."

The National Science Foundation recommended in 1983 that at the elementary and intermediate levels calculators and computers be used in the classroom at the earliest grade possible, to enhance understanding of arithmetic and geometry and to learn problem solving. Substantially more emphasis should be placed on developing the skill of mental arithmetic, estimation and approximation.

Substantially less emphasis should be placed on paper and pencil execution of arithmetic operations. Students should gain direct experience with collecting and analysing data. [The statistical functions to analyse these data are very widely available on calculators and micro computers.] At the senior level they recommended streamlining the traditional curricula to make room for discrete mathematics, computer science and statistics and probability.

Many of us will have worries about reducing our teaching of algebraic skills. For example, if we delete a treatment of (a + b) (C + d) what will we do when we come to teach the complex numbers later on? And there is the age old question of how much skill must precede conceptual understanding and problem solving. Don't students gain understanding and proficiency only from more complicated algebraic manipulations and applications? Or should we follow the example of the suggested impact of the calculator on arithmetic: emphasize what can be done mentally and deemphasize paper and pencil execution? There is a strong need for more study and thought on this.

If much of what we presently teach in secondary school mathematics is obsolete what should we be teaching? What is mathematics? It is known as one of the oldest and most impressive disciplines of human knowledge: do we do it justice with our present methods of teaching? Can we do it justice? What should we be telling our students?

Lastly, we must think about, and tell our students about, the limitations of computing. We are all familiar with the problem that the computer will only answer the questions it has been programmed to consider. These and many other problems of relying upon computers are discussed in a book I would highly recommend: **Computer power and human reason: from judgement to calculation** by Joseph Weizenbaum.[2] I can do no better than to close by quoting some of his concluding remarks in that book.

I want them to have heard me affirm that the computer is a powerful new metaphor for helping us to understand many aspects of the world, but that it enslaves the mind that has no other metaphors and few other resources to call on. The world is many things, and no single framework is large enough to contain them all, neither that of man's science nor that of his poetry, neither that of calculating reason nor that of pure intuitition. And just as a love of music does not suffice to enable one to play the violin - so is it not enough to love humanity in order to help it survive. The teacher's calling to teach his craft is therefore an honorable one. But he must do more than that: he must teach more than one metaphor, and he must teach more by the example of his conduct than by what he writes on the blackboard. He must teach the limitations of his tools as well as their power.

It happens that programming is a relatively easy craft to learn. Almost anyone with a reasonably orderly mind can become a fairly good programmer with just a little instruction and practice. And because programming is almost immediately rewarding, that is, because a computer very quickly begins to behave somewhat in the way the programmer intends it to, programming is very seductive, especially for beginners. Moreover, it appeals most to precisely those who do not yet have sufficient maturity to tolerate long delays between an effort to achieve something and the appearance of concrete evidence of success. Immature students are therefore easily misled into believing that they have truly mastered a craft of immense power and of great importance when, in fact, they have learned only its rudiments and nothing substantive at all. A student's quick climb from a state of complete ignorance about computers to what appears to be a mastery of programming, but is in reality only a very minor plateau, may leave him with a euphoric sense of achievement and a conviction that he has discovered his true calling. The teacher, of course, also tends to feel rewarded by such students' obvious enthusiasm, and therefore to encourage it, perhaps unconsciously and against his better judgement. But for the student this may well be a trap. He may so thoroughly commit himself to what he naively perceives to be computer science, that is, to the mere polishing of this programming skills, that he may effictively preclude studying anything substantive.

If the teacher, if anyone, is to be an example of a whole person to others, he must first strive to be a whole person. Without the courage to confront one's inner as well as one's outer worlds, such wholeness is impossible to achieve. Instrumental reason alone cannot lead to it. And there precisely is a crucial difference between man and machine: Man, in order to become whole, must be forever an explorer of both his inner and his outer realities. His life is full of risks, but risks he has the courage to accept, because, like the explorer, he learns to trust his own capacities to endure, to overcome. What could it mean to speak of risk, courage, trust, endurance, and overcoming when one speaks of machines?

[1]muMath is available from Microsoft Consumer Products, 10700 Northrup Way, Bellevue, Washington 98004. Ph. (206) 828-8080

[2]Weizenbaum, Joseph. **Computer Power and Human Reason:** From Judgment to Calculation. Freeman; San Francisco, 1976

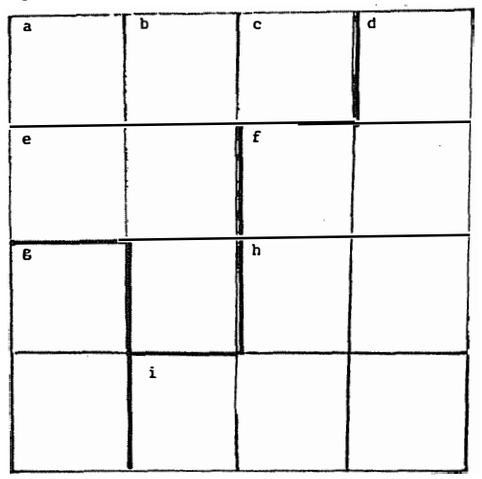


CROSS-FIGURE PUZZLE #4

by Cathy Graeber

Editor's Note:

In the March '84 ***Gazette*** *a short article appeale explaining cross-figure puzzles, including 3 puzzles. Four more of the puzzles, with their solutions, are included in this issue.*



ACROSS

a (a down)^2

i (a down) + (a across)

DOWN

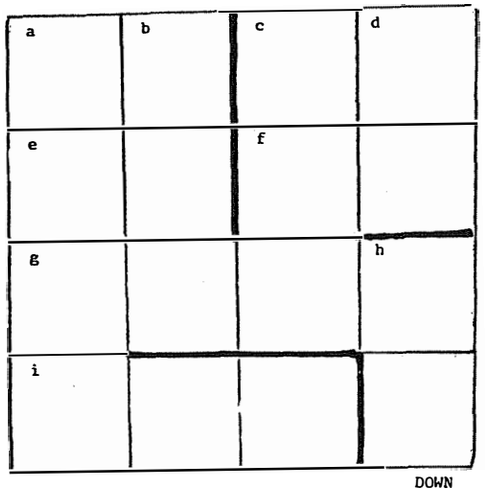
b (h across) x 15

c (g down)^2

d (e across) x (f across)

CROSS-FIGURE PUZZLE #5

by Marilyn Hardwick



ACROSS

a (e across) x 2

c one tenth of (b down)

e digital sum of (a down)

g (c across)^2 x 9

DOWN

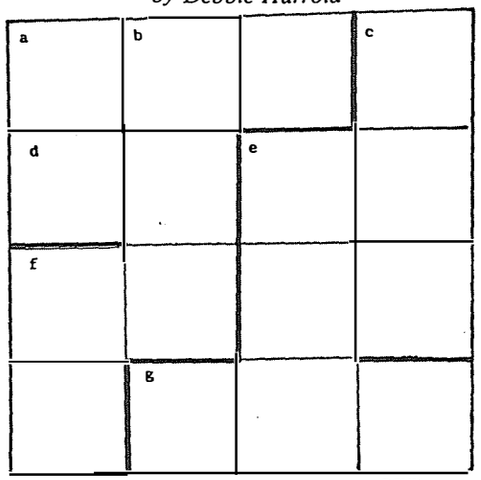
d (a across) x 2, and also a cube

h (e across) + (f across)

i palindromic, and twice the cube of a prime

CROSS-FIGURE PUZZLE #6

by Debbie Harrold



ACROSS

a (g across) + (b down)

d (e across) + (f down)

DOWN

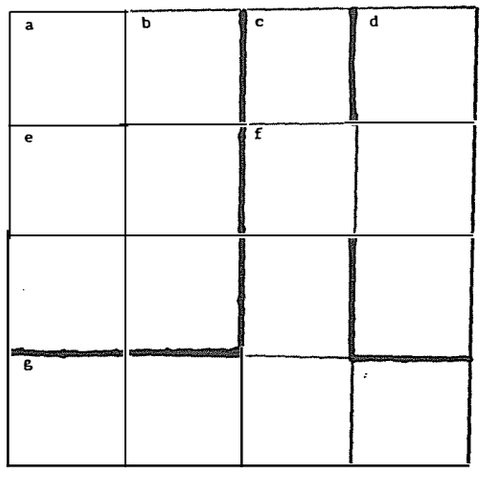
a (c down) ÷ 3

b (a across) - (g across)

e (c down) - (e across)

CROSS-FIGURE PUZZLE #7

by Debbi Heighway



ACROSS

e (a down) ÷ 2

f a prime which is (a across) reversed

g (d down) x (a across)

DOWN

a $(\text{a across})^2$

b 5(e across) - 6

c (g across) - (d down)

d (a across) + (e across)

SOLUTIONS

#4

a 1	b 9	c 6	d 2
e 4	0	f 5	0
g 8	0	h 6	0
1	i 2	1	0

#5

a 3	b 2	c 2	d 6
e 1	6	f 2	4
g 6	0	8	h 4
i 6	8	6	0

#6

a 6	b 4	0	c 2
d 9	7	e 1	0
f 8	0	9	7
7	g 1	7	0

#7

a 1	b 4	c 1	d 1
e 9	8	f 4	1
6	4	5	2
g 1	5	6	8



INTERVIEW WITH HILLARY SHUARD

by

Robert S. Smith

Hilary Shuard is the Deputy Principal of Homerton College, Cambridge, England. I first came to know Miss Shuard through her book **Primary Mathematics Today** (1) by Williams and Shuard; this contained a preface by W. H. Cockcroft. In 1978, she was asked to serve on a committee of inquiry set up by the government of the day under the chairmanship of Dr. Cockcroft, to inquire into the teaching of mathematics. The report, **Mathematics Counts** (2), was published in 1982, and Miss Shuard spoke to the 1984 N.C.T.M. meeting in San Francisco using the title "Cockcroft and After". As her talk came immediately after my own, I attended, enjoyed, requested an interview and thus the following.

RS: Hilary, tell us about what you do in England.

HS: I am a teacher educator. I work at Homerton College, Cambridge. I have been working in teacher education for about twenty-five years - more with primary teachers than secondary teachers. I suppose that my major interest at the moment is in the primary mathematics curriculum and in the use of language in mathematics.

RS: What exactly did the Cockcroft Committee have to do?

HS: The Cockcroft Committee was asked to enquire into the teaching of mathematics in the primary and secondary schools in England and Wales paying particular attention to the needs for mathematics in further and higher education, employment and adult life generally, and to make recommendations.

RS: I am sure you spent some interesting days not only reading but hearing evidence. Give us two or three of the main findings that came out of your work.

HS: The research projects we commissioned confirmed our feelings that the majority of the adult population suffer from mathematics anxiety. They know only too clearly that they cannot do mathematics and they do not much like it. It became very important to us to look for causes of that attitude and to discover anything that we might be able to do to change the situation. That was one

of our major preoccupations. The other one was that what we teach in school mathematics very often does not seem to be very effective - people use the mathematics they have understood for themselves rather than what we taught to them in school.

RS: It seems to me that any inquiry finishes by making some recommendations, to try to remedy the problems they have discovered. What were some of those recommendations?

HS: We did not make specific recommendations. We wrote a report which incorporated an argument about the problems and ways of tackling them. The major points of concentration were about teaching styles, about the curriculum for both primary and secondary schools and about the use of calculators and computers.

RS: Since you are in teacher education and must have felt a fair responsibility to implement some of the recommendations, take a moment to outline one or two ideas that you are working on.

HS: We regarded inservice teacher education as extremely important. It is perfectly clear that one cannot educate a teacher for a teaching career which may span forty years during preservice education. So we put a great deal of our emphasis on inservice teacher education, and particularly in trying to help teachers who are in positions of responsibility. We are trying to identify and train one teacher in primary school who will be able to act as the resource person and the leader in developing the mathematics teaching in that school. I am particularly concerned with courses for those teachers.

RS: In those courses, would you be spending more of your time on mathematics content or on a variety of strategies and teaching styles?

HS: We are trying to persuade teachers to implement a variety of strategies. It seems to me that most teachers have a style and they stick to it - whether it is appropriate or inappropriate to what they are doing. One of the recommendations in the report that has caused most attention is a list of six teaching strategies that teachers at all levels should try to employ. The first of these is **exposition** (not lecture) - Not exposition by question and answer but exposition supplemented by a rather more balanced language exchange, more an equal dialogue between teacher and pupil, or indeed between pupils themselves. The language exchange is **discussion**. However, I have found great difficulty in finding examples of real discussion between teacher and pupils, and I am now beginning to think that discussion very rarely happens in mathematics classrooms. You do sometimes hear discussions between pupil and pupil - especially when there is a computer around. Quite often using a computer produces very interesting discussions.

RS: We have heard about two of the strategies. Perhaps you could go on with the other four.

HS: The next one is **appropriate practical work.** We felt that in almost all mathematical topics practical work is appropriate at the beginning of the topic. Pupils of every age need to get to grips in some way with the situation as it is; then they can move on to representation, and then to symbols. We firmly believe that practical work has a part to play in mathematics at all levels. The fourth strategy is **practise of skills and routines.** I would not want to say any more about that because all mathematics teachers know a great deal about it, and it figures prominently in most classrooms. The fifth is **problem solving** and **making applications to everyday situations.** Problem solving is becoming more important in mathematics, but it is very often only interpreted as rather limited word problems; whereas many of the most interesting problems for children are those in which they apply their mathematics to everyday situations. The sixth teaching strategy is what we have come to call **investigations,** though this term is not widely used outside Britain. An investigation is an open-ended mathematical problem that can usually be tackled at a number of different levels by learners at different states. For instance, a young child might just be able to find a pattern, another pupil might be able to express the pattern in a formula, and a more sophisticated mathematician might be able to prove the formula, but all three will have had considerable success at their own level in that problem. There is now quite a wide development of suitable problems for investigation, and this kind of teaching is beginning to appear in more mathematics classrooms.

RS: I believe the Inner London Education Authority is now publishing a magazine titled **The Investigator** (3). I wonder if you could just spend a moment now on two controversial things. One part of the report says that for certain, logarithm tables are done - they are gone. The calculator is doing that job. What are you doing with the calculator in the primary schools?

HS: We are trying to encourage the use of the calculator. The committee felt that the calculator has considerable potential in the primary school, but this was not yet being realized. Some of the more obvious uses of the calculator are to enable children to discover patterns and to use it as an aid for the understanding of place value, mathematical operation and how numbers work. The calculator is **not** just a machine for doing calculations that are too tiresome for you to do by hand. It is also a machine that embodies the structure of the decimal number system, so children can learn a lot about that structure by using their calculators. Calculators also enable pupils to deal with realistic numbers. For instance, it is now possible to find the average height of a class, where it would have been far beyond the capabilities of most pupils before. However, I should say that there is considerable resistance, both among parents and some teachers, to the use of the calculator. The problem of trying to develop a curriculum to take account of the fact that most calculations are now done mentally or by calculator rather than by pencil and paper, has yet to be addressed.

RS: Interestingly enough, hidden away in one of the recommendations was a time bomb. "Long division should be done away with, as it is normally done by calculator". Do you have any comment?

HS: That is not a time bomb, that is a fact of life. No one, except students in school, has done long division for about the last five years; everyone uses their calculators when they need to do long division.

RS: Why then is it still in your syllabus?

HS: Because all school systems are conservative forces and slow to change. The curriculum that we have, in its numerical aspects, is still the 19th century curriculum, written at a time when we needed a great many clerks who could calculate accurately with pencil and paper.

RS: Or quill pens.

HS: That's right.

RS: But we moved quickly from the straight pen and nib to the ball point that we all use today. Then, five years down the line, do you see the same movement from pencil and paper long division to calculator long division?

HS: Outside school the move from pencil and paper to calculators has already happened. I think it will take longer than five years in schools. Perhaps it will not take five years for pencil and paper division to die but it certainly will take longer than that for 'subtraction with exchange' to die, even though no one goes through the exchanging process except when they are in school.

RS: I would like to finish off the interview by talking some more about your concern about classroom discussion. What have you been doing?

HS: I have been working with a group who are sitting in classrooms with tape recorder trying to find innovative styles of mathematics teaching. We have been singularly unsuccessful. It seems to us that it is still believed that the teacher's job is to lead the exposition, and the kids very soon find that their job is not to put forward ideas of their own, but to guess what the teacher is thinking of. I think this is a limiting factor in the development of the curriculum, and it is one that we need to overcome if we are going to move the mathematics curriculum to one which encourages pupils to think mathematically and to understand what they are doing, rather than to learn the methods and the algorithms; traditionally it has been the teacher's job to pass on the algorithms rather than to encourage thinking.

RS: Agreed. Tell us about your research into language & mathematics.

HS: I have done quite a lot of work over the years on the influence of language on mathematics teaching and learning. The started with an interest in children's reading difficulties in mathematics, and I did some work in that area. Since then I have gone on to try to understand more about the oral language of the primary classroom and in mathematics. I have been working with a group of members of the British Society for Psychology of Learning Mathematics who have been doing similar things. We have discovered that even in primary classrooms, even with children between the ages of 7 and 9 much of the teaching in mathematics is very didactic and allows little opportunity for

children to engage in any form of discussion with the teacher, even when the work is of a practical nature. Ther is such an expectation in mathematics that teachers almost automatically operate in a didactic way. I think this is very sad because I believe strongly in the importance of language as a means of thinking, as a tool of thinking, and it seems to me that we need to find many more opportunities to allow children to use language for themselves in their mathematics learning.

RS: Hilary, I understand you have a new book coming out called "Children Reading Mathematics" (4). Let me give you a slight indication of something that happens I'm sure in many parts of the world. As I visit schools I automatically go into the libraries to see what books are available. They are few and far between. At the beginning years, picture books about number and a bit about geometric shapes in the middle years but more often the only offerings are old textbooks. You might find the odd copy of Flatland which of course, stretches the imagination, but the title of your book "Children **READING** Mathematics" is an encouraging title. Tell me a bit about this new book coming out.

HS: Well, children in England are often expected to read their mathematics workbook or textbook. Many teachers have attempted to individualize the work so that children can learn at their own rate. Because of the children's different rates of learning, they have found that they needed to provide written materials as the stimulus for learning. So there has been a considerable proliferation of textbooks and workbooks which children are expected to work through individually, acquiring knowledge of what they are meant to do through their reading, and only going to the teacher for help when they are stuck. This makes the learning of mathematics very dependent upon reading, and reading difficulties can have a serious impact on the child's mathematics learning.

RS: Very true. We have a small test of readability in our country, and I'm sure you have it here, called the Cloze Procedure. Using it, many texts are found to be beyond the children's reading level. What are you doing about it in this country?

HS: In the Cloze procedure, you obliterate every seventh word from a passage and give it to a child to fill in the gaps. There has been a great deal of research on this test, particularly in the United States, and it would suggest that if a child can't put back about 40% of the obliterated words correctly, the book is likely to be too difficult for him to read even when he has the teacher around to give him assistance. In the work that has been done on primary mathematics books in this country, it has been found that the reading level, as measured by the Cloze procedure, is too high for the average reading level of the children. However, this isn't the only difficulty; the Cloze procedure is designed to be used with consecutive prose but mathematics texts are not consecutive prose. There are many other difficulties in the mathematics texts such as graphs and symbolism and pictures which may embody rather subtle shades of meaning. However, if the child can't read the prose, he is unlikely to get to grips with the mathematics.

RS: Let us turn back to your study about discussion among children, or even a more balanced discussion between pupil and teacher. Have you found many classrooms where there is a large amount of pupil discussion? Has there been much of this in the classrooms you've been studying?

HS: No.

RS: Surely you must have found some teachers, or heard of some teachers, who may not be using a didactic style and who do encourage talk.

HS: Yes. That happens more when children are encouraged to work together on a task, talking to one another as they do it and then reporting and discussing with the teacher what they have found out, rather than the teacher leading. I think we need to do a lot of thinking about activities which will encourage this sort of pupil talk and then pupil with teacher talk. It seems to me that problem-solving and investigations are a fruitful field here, and so is suitable work with the computer - not the drill and practice type of work where an individual pupil uses the computer to reinforce skills, but problem solving programs which are designed to be operated by a small group of children.

RS: Derek Ball of the University of Leicester was visiting us last month and from talking with him, I understand that he is using the computer in this way, as a facilitator for discussion. I'm sure there are other people doing this as well. Please tell us a little bit about how they are using the computer.

HS: Well, Anital Straker of the MEP Primary Project has written quite a lot of investigational, problem solving programs which are designed to be used in this way. We at Homerton have been doing

some work with LOGO in primary school and we have been encouraging and supporting teachers in using LOGO with children from 5 upwards; and have one group of teachers that is even using it with preschoolers. Certainly this is a very fruitful source of exploration and discussion.

RS: I would like to turn to another topic. In the Cockcroft report, I found you wrote the appendix discussing the differences in Math performance between girls and boys. Yet you made no specific recommendations in the appendix as you did in the rest of the report. Why not, Hilary?

HS: We did make recommendations on maths and girls and we did suggest some strategies for improvement.

RS: Is there one major recommendation for the education of girls in mathematics you would underline?

HS: I think that we should try and persuade them to be more confident about what they are doing, to enjoy it better, and to realize that it's as relevent for them as it is for boys.

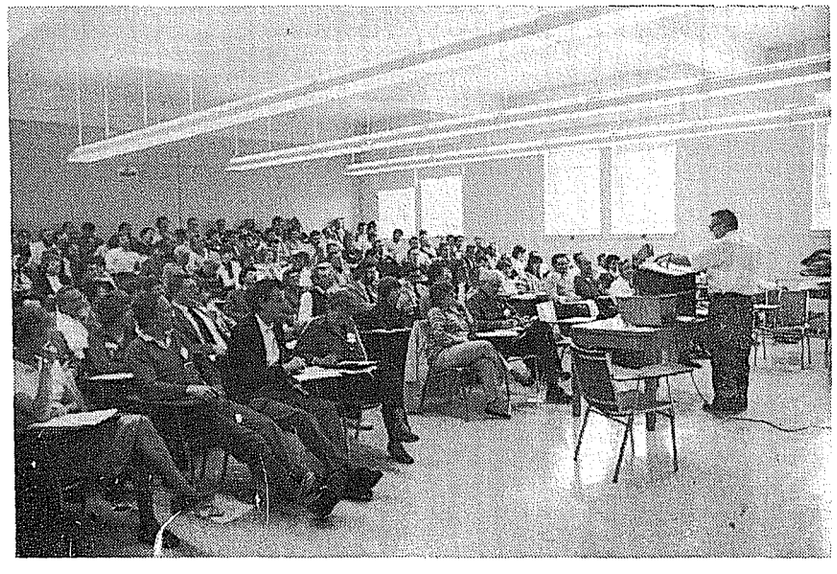
RS: The range of books you have authored impresses me. I have beside me books for primary teachers, a book on calculus and now Children Reading Mathematics. Where do you find all the time?

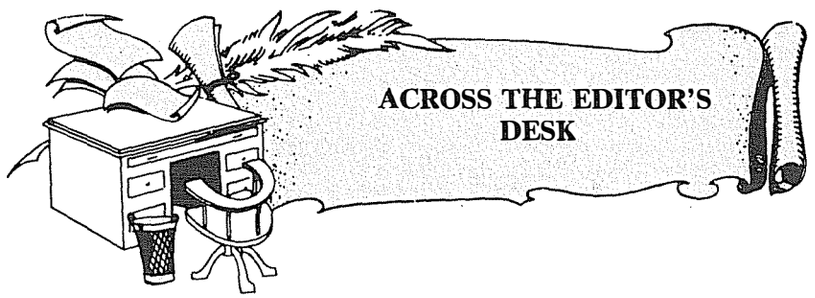
HS: Oh, I do it when I'm not doing anything else.

RS: Thank you so much, Hilary, for an interesting interview.

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1. Williams, C. M., & Shuard, H. **Primary Mathematics Today** (3rd Ed. for the Age of the Calculator), Harlow, Longmans, 1982.
2. **Mathematics Counts,** Cockcroft, W. H., H.M.S.O., London, 1982, L5.75
3. **The Investigator,** I.L.E.A., Ladbroke Mathematics Centre, London.
4. Shuard, H., & Rottiony, A. (Eds.) **Children Reading Mathematics,** London: John Murray, 1984





ACROSS THE EDITOR'S DESK

Dear Editor:

The article on Summing Sequences of Cubes by Bonnie H. Litwiller and David R. Duncan Vol. 22 No. 2 Dec. '83 prompts me to proceed one step further to establish formulae for the sums of first powers, squares, cubes, fourths, etc. of consecutive integers.

First Power

		A_n
1	=	1
1 + 2	=	3
1 + 2 + 3	=	6
1 + 2 + 3 + 4	=	10

$A_n = 1 + 2 + 3 + 4 + \ldots\ldots + n$

This is an Arithmetic Progression and $A_n = \frac{n(a + 1)}{2}$, l the last term

$$= \frac{n(n + 1)}{2}$$

Squares

		B_n	1st diff	2nd diff	3rd diff
1^2	=	1			
			4		
$1^2 + 2^2$	=	5		5	
			9		2
$1^2 + 2^2 + 3^2$	=	14		7	
			16		2
$1^2 + 2^2 + 3^2 + 4^2$	=	30		9	
			24		
$1^2 + 2^2 + 3^2 + 4^2 + 5^2$	=	55			

As the 3rd differences are all equal

$$B_n = an^3 + bn^2 + cn + d$$

$B_1 = 1$
$B_2 = 5$
$B_3 = 14$
$B_4 = 30$

Form simultaneous equations and solve for a, b, c, d

$$B_n = \frac{n(n+1)(2n+1)}{6}$$

Cubes

	C_n	1st diff	2nd diff	3rd diff	4th
1^3	= 1				
		> 8			
$1^3 + 2^3$	= 9		> 19		
		> 27		> 18	
$1^3 + 2^3 + 3^3$	= 36		> 37		> 6
		> 64		> 24	
$1^3 + 2^3 + 3^3 + 4^3$	= 100		> 61		> 6
		> 125		> 30	
$1^3 + 2^3 + 3^3 + 4^3 + 5^3$	= 225		> 91		
		216			

As the 4th differences are all equal

$$C_n = an^4 + bn^3 + cn^2 + dn + e$$

$$= \left[\frac{n(n+1)}{2}\right]^2$$

Set your pupils to determine D_n, E_n, F_n, G_n .....

Garnet J. Greenbury
Brisbane, Australia

John F. Ross C.U.I.
18 March 1984

Dear Sir:

In the March '84 issue of the **Ont. Math Gazette** the two letters on pages 34 and 35 seem to be connected.

The former on mathematical rigour (sic) leads us to say the latter on $y = x^{\frac{1}{x}}$, $x > 0$ has an error in its statement that the function has a max. value at $x = 3$. Really, it's at $x = e$.

Sincerely,
D.E. Kennedy

Dear Editor:

Re: THE IMPOSSIBLE DREAM OF BARRINGTON MILLER, GAZETTE, MARCH 1984, Page 34.

Barrington Miller t'would seem
Has dreamt the impossible dream
It's at $x = e$
And not $x = 3$
x to the 1 over x is extreme.

A. Zerkante (1904 - 1961)

OR TO MAKE MATTERS EVEN WORSE

x raised to the 1 over x
Graphically makes me think "SEX"
For just like big Zee
It's sagging by 3
Beyond 3, it never erects.

Mrs. A. Zerkante (1944 -)

Yours in mathematics,

Mrs. A. Zerkante.

Editor's Note:

The above letter was printed ***as received*** *and does not reflect the opinions (or* ***performance****) of the editors.*

BOOK REVIEW

Reviewed by R. S. Smith

Childrens Strategies and Errors Ratio: Kathleen M. Hart
Childrens Strategies and Errors Algebra: Lesley R. Booth
Childrens Strategies and Errors Fractions: Daphne Kerslake
Childrens Strategies and Errors Graphs: ?
Childrens Strategies and Errors Measurement: ?
N.F.E.R. - Nelson Publishing Co. Ltd., Darville House, 2 Oxford Rd. E, Windsor, Berkshire SL4 1DF England L8.95.

The above books are the outcome of three years of research into the misconceptions underlying certain widespread errors. These errors were all shown to be commonly made by children aged 12-16 in the earlier research project 'Concepts in Secondary Mathematics and Science' (CSMS). The study was conducted in two parts. In the first, individual children were interviewed to establish the reasons for the errors they were making. In the second, the interview data were used to develop and trial a short teaching module designed to help children avoid the misconceptions which led to the errors. Illustrative extracts from the interviews are included, along with the full teaching module, for direct use or adaptation by teachers. The teaching experiments involved both small groups and whole classes.

As in the earlier book, (Children's Understanding of Mathematics: 11-16) I found the section labelled "Implications for Teaching" most interesting and helpful, with the Class Trials section probably most useful to teachers. We all have recognized the problem; now we have a series of books which give the results of teaching experiments to overcome the problem. I cannot think of a more useful series.

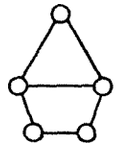
R. S. Smith
Faculty of Education
The University of Western Ontario



Problems

Editor's Note: *We are still anxiously waiting for examples of interesting problems that we know many readers have and use. September's problems came from a packet of problems received at an O.A.M.E. Conference, but I don't know from whom. Answers will appear next issue.*

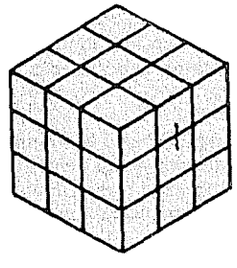
1. Place each of the numbers 1, 2, 3, 4, 5 in one of the circles in this figure so that no number is connected by a line to the preceding or following number. (For example, 3 must not be connected to 2 or 4.)



2. You have two equal rolls of pennies. You do not know how many pennies are in each roll, but you do know that on a balance scale, two rolls of pennies plus 3 loose pennies will balance 13 loose pennies. How many pennies in each roll? How many if 2 rolls and 5 loose pennies balance one roll and 17 loose pennies? Draw a picture.

3. We wish to measure 4 L of water but we have only a 5 L container and a 3 L container. How can it be done if neither container is marked for measuring?

4. Visualize a three-centimetre cube painted red. Assume the cube is cut into 27 one-centimetre cubes, as shown.



a) How many 1 cm cubes will have red paint on none of the sides

b) on one face only

c) on two faces only

d) on three faces only

e) on more than three faces

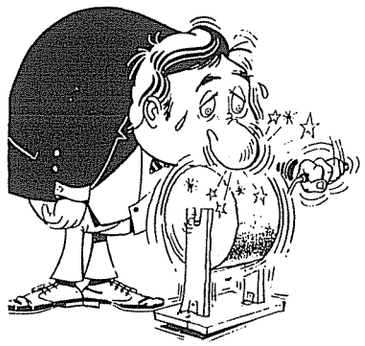
ANSWERS TO MARCH PROBLEMS

1. a) T is next in the series
   b) T is next in the series
   c) unfortunately this sequence contained an error (just wanted to see if you were paying attention?) The **correct** sequence is -2, 4, -12, 48, -240 and then the correct answer for the next term is 1440.

2. a) 240
   b) 6
   c) 1

3. 3 darts (45, 22, and 34)

4. $245.00



COUNCIL COMMENT

by
Bob Robinson

Editor's Note: *Council Comment appears in an issue of the Gazette following OAME/AOEM Council meetings. Your comments are most welcome and should be addressed to:*
The Editor
Ontario Mathematics Gazette
College of Education
Brock University

The June Council meeting was held on Saturday, 1984 06 09 at the Ramada Inn (Airport West).

Future meetings of the Executive and Council are:

Executive	Council
1984 09 07	1984 10 13
1984 11 16	1985 02 09
1984 04 05	

Here is your 1984-85 OAME/AOEM Executive and Council. Contact someone who is a representative from your area to provide input to Council.

Executive
Past President: Dave Davidson

President: Paul Pogue

President-Elect: Paul Lessard

Director: Ralph Connelly

Tom J. Griffiths
Elaine Harvey
Frontenac S.S.

Councillors - Term Expiring 1985
William Bisset
Chester Carlow
Dominic DiFelice
Robin Hood
Linda Marshall
Kenneth Mason
Roland Morrow
Poce
Robert Robinson
Jeffrey Shifrin

Phone: (Home) 416-493-1931 (Business) 416-633-4012

Councillors - Term Expiring 1986
Dino Dottor
Mary Lou Kestell
George Knill
Dr. David Lenchyshy
Steve Martin
Alex Norrie
J. Symington
Trevor Thomson

Councillors - Term Expiring 1987
Trevor Brown
David Handley
Robin Hill
William Leigh
Michael Sandblom
Ronald Sauer
Angela Walsh
Lorna Wiggan

Phone: (Home) 416-767-1424

Chapter Representatives
COMA (Central Ontario Mathematics Association)
David Thorsell
GOLDEN SECTION
Edward Haines
GVMA (Grand Valley Mathematics Association)
William Proctor
NOMA (Northern Ontario Mathematics Association)
Jim Turcott
PINE RIDGE
Kumar Pasricha
QALMA (Quinte St. Lawrence Mathematics Association)
Norah Wilson
SAME (Scarborough Association for Mathematics Education)
Robert Wilson
TORNY (Toronto-North York)
Irving Pupko
COMMITTEE CHAIRPERSONS AND EXECUTIVE LIAISON

Publications
T. Thomson (Chairperson)
R. Connelly (Liaison)

Formative Years
B. Proctor (Chairperson)
P. Lessard (Liaison)

Problem Solving
C. Carlow (Chairperson)
D. Davidson (Liaison)

Senior Guidelines
D. Handley (Chairperson)
T. Griffiths (Liaison)

Membership/Publicity
M. Sandblom (Chairperson)
P. Pogue (Liaison)

Computers
K. Mason (Chairperson)
P. Pogue (Liaison)

Women and Math
I. Pupko (Co-chairperson)
L. Wiggan (Co-chairperson)
E. Harvey (Liaison)

***Nominations:** If you or anyone you know is interested in seeking election to the Council of OAME/AOEM, contact any of the following members:
D. Davidson
G. Knill
B. Robinson

AIMS AND OBJECTIVES

The following aims and objectives were approved by Council:

1. To encourage each councillor to increase the publicity of OAME/AOEM in their respective chaper, region, or school.
2. To initiate projects that will promote the name of OAME/AOEM.
3. To continue to work in close liaison with the Ontario Ministry of Education.
4. To increase our membership to 1,500.
5. To evaluate the effectiveness of our publications.
6. To establish committees on: (a) Formative Years
   (b) Women In Mathematics.
7. To have the executive complete a review of the present by-laws.
8. To encourage the nominations committee to promote a better percentage return of ballots.

TRIBUTE TO KEN FRYER

The meeting began in a sombre and reflective manner. Paul Pogue spoke on the passing of Ken Fryer, a "pillar" in mathematics and mathematics education. He will be missed by us all.

NCTM REGIONAL SERVICES REPRESENTATIVE

Fred Crouse, the Canadian Member of the Regional Services Committee of the NCTM, attended our meeting and took part in the Computer Committee discussion. Fred was the President of the Nova Scotia Mathematics Teachers' Association, is presently a Vice-Principal of a Secondary School in Wolfville, Nova Scotia, and has been appointed recently as the Computer Consultant for the Department of Education in his province.

CHAPTER REPORTS

COMA (Central Ontario Mathematic Association)

A successful meeting was held on 1984 05 29 at Carleton University Provincial examination was the main topic.

GOLDEN SECTION

A fall meeting is being planned. More information will be forthcoming.

GVMA (Grand Valley Mathematics Association)

Bill Proctor spoke of the sad loss of Ken Fryer. In part, Bill stated:
"Most councillors will be fully aware of the positive influence Ken had on the mathematics community of Ontario and beyond. Ken was a long-time supporter of GVMA activities and a frequent speaker at our general meetings. His expertise, encouragement and humour will be sorrily missed."

Summer Writing Project

During June and July, three teams of teachers were working on a resource development project sponsored by the GVMA. Three courses from the proposed Ministry Guidelines were chosen and the objective was to identify currently available resources and develop additional resources which would be helpful in teaching selected themes and topics in these courses.
The courses chosen were:

Grade 11	Mathematics for Everyday Life
Grade 11	Mathematics for Work and Home
Grade 11	General Business Mathematics

Fall General Meeting

The Fall General Meeting will be held at Eastwood C.I., Kitchener on Thursday, 1984 11 08 with registration beginning at 4:15 p.m. The keynote speaker will be Professor John Moore of the Faculty of Engineering University of Waterloo. His talk is entitled, "You Ain't Seen Nothin' Yet".

NOMA (Northern Ontario Mathematics Association)

A fall conference is planned for 1984 09 29-30 in North Bay under the direction of Bob Tuck.

PINE RIDGE

A successful mini-conference was held on 1984 04 03 at Harwood High School, Ajax. Twelve workshops were offered and 125 people attended. A film and video session was held on 1984 06 26. Pine Ridge will continue its participation in the Math and Science lecture series for 1984-85

They are also starting preparations for their annual P.A. day activities on 1984 11 14.

QSLMA (Quinte St. Lawrence Mathematics Association)

QSLMA hosted our highly successful annual meeting in Kingston and are planning to promote some projects in partnership with COMA. More information regarding these projects will be announced.

TORNY (Toronto-North York)
Meetings are being planned for the upcomming year and efforts will be made to have more TORNY members attend the Annual Meeting.

SAME (Scarborough Association For Mathematics Education)
An appreciative group of elementary teachers attended a workshop co-sponsored by SAME and Collier-MacMillan.

The entire workshop series, co-sponsored by various publishers, was extremely well received and attracted many participants from both elementary and secondary panels. Another series of similar workshops will be offered during 1984-85.

A Math Mini-Conference is tentatively scheduled for 1984 10 18.

Another major project for next year is the K-13 P.A. Day Program on 1985 02 15. This full day event will be held at the Holiday Inn, Don Valley, and a full slate of workshops, a keynote speaker, publishers' displays, and a luncheon will be offered.

OAME ANNUAL CONFERENCE 1984

Elaine Harvey, Conference General Chair, spoke of the passing of Dr. Norman Miller, a life member of OAME, and paid tribute to this excellent mathematics educator.

The council commended the QSLMA in general and Elaine, in particular for their hard work in presenting a very successful "annual". Attendance was quite high late Friday and Saturday which is a credit to the very fine program presented. Special mention was made of the certificates designed by P. Battey Pratt.

OAME ANNUAL CONFERENCE 1985

OAME '85 will be held in Sudbury from noon Thursday, 1985 05 09 through noon Saturday, 1985 05 11. Tentative highlights are already planned and the registration and lodging costs will be close to those of Kingston. We are all looking forward to OAME '85 to share once again that great "Northern Hospitality".

OAME ANNUAL CONFERENCE 1986

OAME '86 will be held in London from Friday, 1986 05 09 through noon Sunday, 1986 05 11.

NCTM REPORT:

Ralph Connelly was appointed NCTM representative for 1984-85.

Bob Robinson has been elected to the Executive Committee of the NCTM Board of Directors and also serves as the Board liaison on the Conference and Convention Committee.

Here are the dates for future "annuals" of NCTM (held in April) and NCTM Canadian Regional Conferences:

Annual	**Regional**
San Antonio '85	Winnipeg '84 (Oct.)
Washington '86	Montreal '85 - '86
Anaheim '87	
Chicago '88	
Orlando '89	

LEADERSHIP SEMINAR

The 13th Annual Leadership seminar will be held 1984 10 24-27 at the Ramada Inn (Airport West). This highly successful seminar will provide up-to-date topics, outstanding speakers, and a host of "extras". Contact your Director, Superintendent of Curriculum or Mathematics Co-ordinator for further information. For more information, contact Paul Pogue, Dave Davidson, or Morley MacGregor. Don't miss it!

WOMEN AND MATHEMATICS

Speakers will be suggested to the program committee of the OAME '86 Annual in Sudbury.

PUBLICATIONS

The 1983-84 Editorial Board has been reappointed for the 1984-85 year.

FORMATIVE YEARS

This committee will contact the Ministry for further clarification regarding the Formative Years Review. A report is to be prepared outlining concerns or suggestions regarding the revision.

MEMBERSHIP/PUBLICITY

This committee is committed to increasing the membership especially at the elementary level and a specail drive directed to the mathematics teachers at the C.A.A.T. colleges will be undertaken.

COMPUTERS

A computer survey will be sent out with the November issue of the **Gazette.**

PROBLEM SOLVING

A set of issues was distributed to each councillor with a request to submit any further suggestions. It is hoped that this committee will promote wide participation across the province in the area of problem solving and develop increased sensitivity to this vital area of mathematics education.

NOMINATIONS

Anyone interested in running for office in the association should contact any of the three members of this committee.

SENIOR GUIDELINES

A final meeting of this committee is being planned in the fall.

NEXT MEETING OF COUNCIL - SATURDAY, 1984 10 13



OAME '84
KINGSTON

Did you miss the '84 O.A.M.E. Conference in Kingston? Could you have gone but didn't because, like many, you thought the new Thursday to Saturday format would not work? Well if you did, you, like not too many, made an error in judgement. It just happened that you may have missed one of the best attended, best run conferences in some time.

Elaine Harvey and committee deserve considerable applause. Those who had attended the conference had nothing but praise for the sessions; their variety and their excellent quality. In the sessions that I attended leaders ran out of handouts..."didn't expect such a marvelous turnout", they said. And to top it all off, the banquet was extremely well attended as was the dance. I felt that the equation $E = kC$ was definitely in effect where, E the enjoyment of the music was proportional to the amount of liquid refreshments consumed, C, and k the coefficient of enjoyment. I thought the music was great! I noticed that some even stayed up to watch the sunrise!

The '85 conference is to be in Sudbury. The committee for this affair has a big act to follow. We did get a sneak preview of things to come... a contest that I think will be of great interest is to be a highlight. Details in the next issue...look for it!

Although they have a long row to hoe, the committee members feel they can outdo the events of the Kingston affair. Judging by their enthusiasm and the support of the membership at the '84 O.A.M.E. I think that they just might...don't miss it!!

Sam Pravica

ANNOUNCEMENT

The Ontario Educational Association will be holding its Annual Awards Luncheon late in 1984. The Awards to be presented will be the Greer Award, the Elementary Teacher Awards, and A Friend to Education Award. The latter is to be given to "a non-practising educator, volunteer or lay person who has made a significant contribution to education".

The Greer Award, established in 1947 in memory of the distinguished Ontario educator, Dr. V. K. Greer, is awarded annually to an individual or organization prominent in education. In addition to a certificate, the recipient is presented with a cheque for $1,000.00.

The Elementary Teacher Award will be made to a practising classroom teacher at the elementary level "in recognition of an outstanding classroom educator". In addition to a plaque, the recipient will receive a cheque for $500.00

Your nominations for each of these three awards with supporting reasons will be greatly appreciated by our Association as well as the teaching profession as a whole. All submissions will be acknowledged, and should be received in this office no later than FRIDAY, OCTOBER 26, 1984, addressed to:
Mrs. Else Davies
Office Secretary
Ontario Educational Association