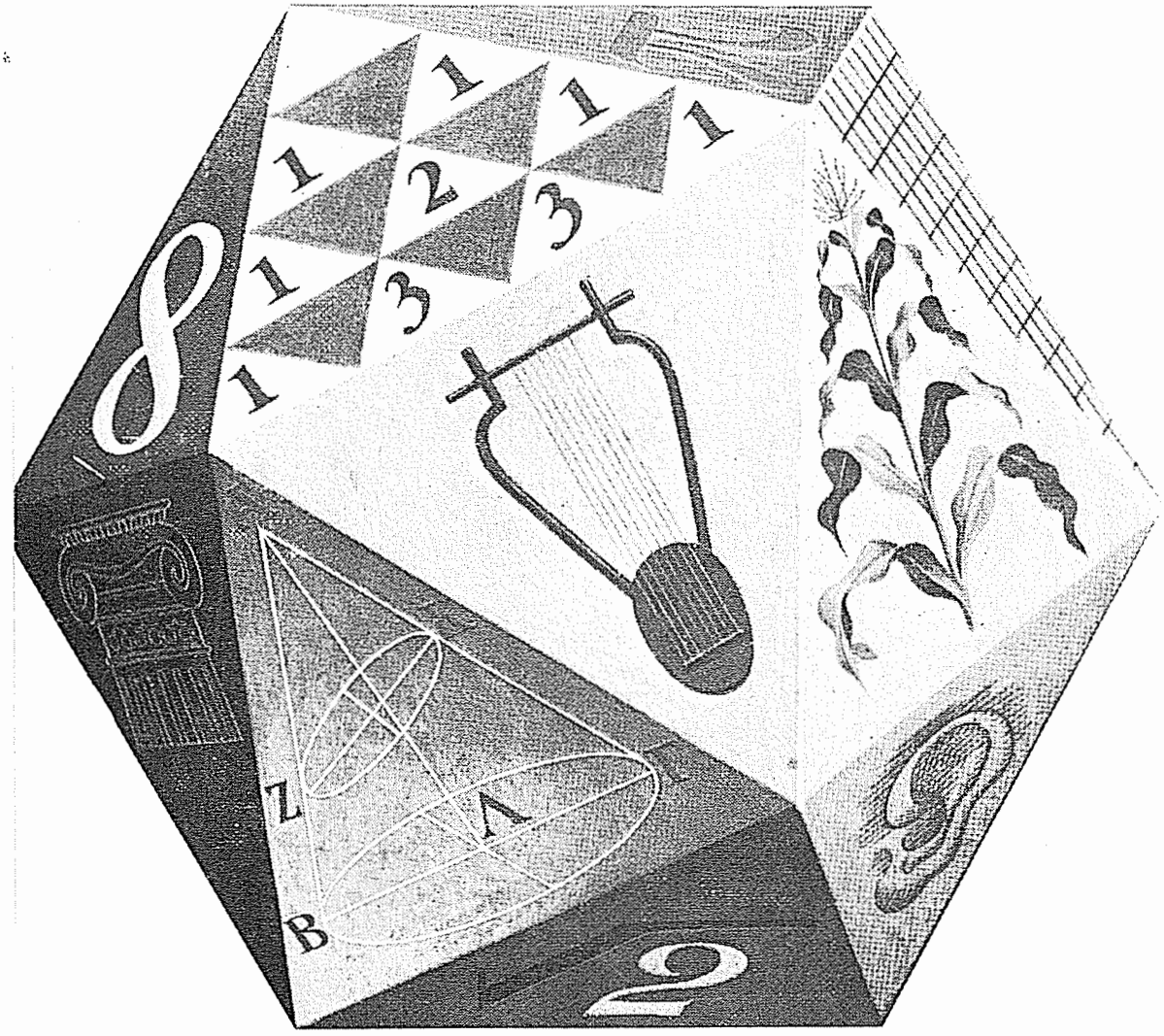
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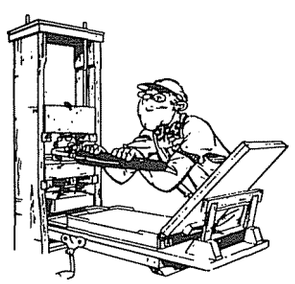
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Well, it's hard to believe another school year is upon us already. (It's particularly hard to believe since I'm writing this in July so it's not true! However, since I bored you with an editor's warped sense of time in the last editorial I won't dwell on it here.)

Since this is our last year as editors of the **GAZETTE** we thought a new cover design was in order. Changing the **outside** of a journal is easy. The real challenge is having an impact on the **inside** of the journal. Sam and I began our tenure as editors with great hopes, ideas, and expectations. "Maybe we could have a special issue devoted to....."; "We **definitely** need to get more balance - the "secondary slant" is way too heavy", etc., etc., etc.

However, all too soon harsh reality set in. People who were contacted and agreed to do articles on topics we thought would be of high interest to our readers didn't come through. We're sure the reasons for this were as legitimate as they were varied, but nonetheless it meant some of our "greater dreams" have not come to fruition. We have been receiving and continue to receive many excellent articles, and for this we are indeed grateful, but the "flow" is still excrutiatingly slow. Our "options" continue to be limited.

In this our final year as your editors, we hope **you,** our readership, will think seriously about the things you'd like to see in the **Gazette** and then **TAKE ACTION!** Either write something yourself or "be a missionary" and ask someone who you felt had something that would be of benefit to our readership to submit it. For those from whom we've received "verbal commitment" for a contribution, make this the year that you act on it.

We're looking forward to a great last year as your editors. Hope your year goes well.

Ralph Connelly
Sam Pravica
Editors

THE CHALLENGE OF CHANGE

by
Trevor Thomson

"That sounds great in theory, have YOU ever actually tried it?"

"That might work for you (or in your school), but my situation is different!"

These, and many other similar comments are often used in response to a situation when a teacher suggests an approach which is not conventional. We teachers of mathematics are very conservative in our teaching styles and classroom practices and this can be both a strength and a weakness.

Because of our tendancy to assess each suggestion with great care before we consider trying it, we avoid jumping on a lot of bandwagons which may not have been going anywhere in the first place. At the same time we may reject suggestions from the risk takers among us --- and only when we are prepared to take risks can we expect to see a significant change in our results.

After a traditional teaching career: classroom teacher, assistant head, head consultant of mathematics I began to be convinced that we had to find better ways of approaching many of our students. All of us are products of University programs. Our curriculum reflects this, but most of our students will not be moving in this direction. In the past year, I finally mustered the courage to respond personally to my concerns about mathematics instruction.

Crestwood Vocational Secondary School! The words strike the cords of terror and challenge. The students are the product of special education classes. Attendance, attitude and behaviour problems are the norm. After experiencing eight years of failure in mathematics, students are not likely to want to be in my class. All of these thoughts passed through my mind, especially in the final week or two of August. But I had made a decision, and I was going to stick with it. Besides, I knew many teachers of mathematics in similar circumstances for whom I had a great deal of respect. I also had a principal who was supportive and encouraged me to try whatever experiments I wanted. We both knew that I would have some dismal failures, but that there might also be some successes that would make it all worthwhile.

Well, the year is now over. The time for assessment is upon me --- and all of my expectations have been met. I did experience a number of failures, but none matched my fears. I did find it necessary to drop some of the ideas I had hoped to try, some now seemed impractical, others premature. And thank goodness, I did experience some successes --- but perhaps not as dramatic as my fondest dreams!

The experience has definitely been a good one for me. I have been forced to re-examine my educational philosophy. When I fell into the trap of telling my principal that what I was doing in his school I would not do in a Composite school, he challenged my educational philosophy and forced me to recognize that what was appropriate in a Vocational school is most appropriate in any Basic Level classes and worth trying in General Level classes as well. This re-evaluation, by itself, makes the change worth the challenge. The experimentation and occasional successes are pure bonus.

I am more convinced than ever that it is necessary for us to put ourselves in new situations from time to time, just to stimulate personal growth. This is even more important in present times when relatively few inexperienced teachers are joining our ranks. The idealism and energy of novice teachers have contributed to the development of many innovations in the past; now we must find ways of generating that enthusiasm from within.

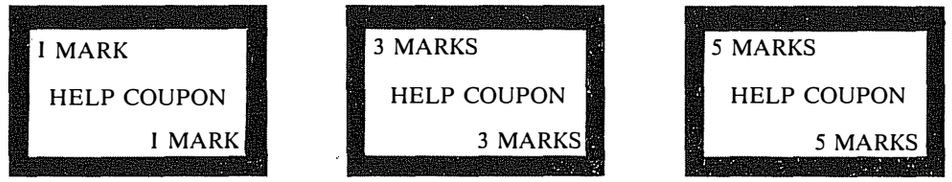
"So what did you do that was so different?", you might ask. That is definitely a fair question, and the honest answer is "Not much"! However, let me give you one small example.

As I prepared for the year, during the summer, I knew that the capabilities and needs of my students would be different from any I had experienced in the past. As long term failures, they needed success. As people soon to join the labour force, they desparately needed to learn responsibility --- regular and punctual attendance, the habit of arriving prepared to work and with all the necessary tools and a sense of fair play. I had built all of this into my evaluation scheme. However, I did not anticipate the challenge of testing.

These students had such a poor history of test results that they didn't care. They would talk out to

me and to each other, argue and refuse to try --- problems I had never experienced before. The refrain "This is a test, that's where I ask the questions and you give the answers" given in personal frustration did nothing but increase the frustration which students felt. Problems would start as soon as students found a question that they could not do and that was bound to happen somewhere on the test.

In my frustration, I remembered that a colleague, Jack Weiner, had once used Monopoly money in the marking of Income Tax and thought that maybe there could be some way of using this approach. Thus the HELP COUPON was born.



The Help Coupons are designed to allow the students to ask questions during a test. If a student runs into a problem they can not handle, they are allowed to ask for help and I will provide it by working out the actual question which they are working on. The value of the cashed coupon will, however, be deducted from the final mark on the test. Clearly, for the student who seeks help only once there are only gains. The lost mark for the cashed coupon will be balanced by the mark(s) gained for the question I have done for them. Seeing that question done may be enough to enable them to continue on the test, thus frustration is reduced. Some decision making practice is occuring ("Is it worth cashing the 3 Mark Coupon for that question?")

For me as well, there were significant gains. As student frustration levels are reduced so are mine. Students perceived this as an attempt on my part to play fair and responded in kind. For some students this produced "teachable moments" which did not seem to occur at any other time in the course. For once, the theory of testing being used for teaching as well as evaluation was being put into practice, admittedly in a restricted way, but it was better than nothing.

There are drawbacks too. It is harder to watch the other students while helping one. The evaluation aspect of my testing was weakened. My practice is not universally respected by other teachers.

In balance, I'm glad I did this, and a number of other equally small things. Together, I think they made things better for my students and for me. I can't guess what next year will hold in store for me, however, I hope that there will be continuing changes which will stimulate challenges, experimentation and growth.

Trevor Thomson, as you can tell from his article, is presently teaching at a vocational high school in Hamilton. He is also an OAME Councillor.

MCCARTNEY SCHOLARSHIP

This year's winner of the McCartney Scholarship is Eric Veach from Northern Collegiate Institute and Vocational School in Sarnia. Eric had marks of 100, 100, 99, and 99 in the four mathematics courses he was taking this year. He is a member of this year's Canadian Mathematics Olympiad team, and has been a consistently high finisher in many Math Contests, including a 1st place in the Canadian National Math League Contest with a perfect score.

This scholarship is awarded by OAME to a top Gr. 13 student from entries received from public and private high schools in Ontario. OAME supplies the funds, with support from SRA for whom Jack McCartney worked prior to his passing. Judges for this year's selection were Jack LeSage (Barrie) and Doug Henderson (Waterloo).

A VIEW OF MATHEMATICS TEACHING FROM THE OTHER SIDE

by
Tom Griffiths

This is my self-funded leave year, and in order to obtain the most benefit from it I decided that I should do something different. So I am teaching mathematics for a change. However I am teaching university level mathematics instead of high school mathematics.

One of the purposes for my doing this is to obtain some perspective of the student's experiences in first year university so that I might better advise and prepare my university bound students in high school in later years.

My observations regarding the students may be of assistance to teachers preparing students for university, to the students themselves and to students at university and in trouble academically. Some of these observations are general and some specific to mathematics.

From the point of view of high schools we could improve our preparation of our students in several ways, although I must comment that many of our students are very well prepared and achieve most commendably. One of these ways is to give them more freedom in grade 13. We do need to coddle and spoon feed them less, and let them take the responsibility for their own learning more. We could allow them to miss some classes without any other penalty than its obvious effect on their results. From this they should learn that too much class time missed does not work well towards academic success, and it should improve their self responsibility. I suspect it would have a startling effect on the failure rate in grade thirteen, which would increase over the next few years, but would improve considerably the results of our students who do qualify for university. From a parents point of view, I think that failure in grade 13 and hence redirection to a non-academic career, or retaking grade 13 with a more mature attitude, would be preferable to enforced success in grade 13 and subsequent expensive failure and waste in University. This waste being in both time and money. The time might be more profitably spent in learning some employable trade, than in failing academically. Above all my strongest impression of those students who are not succeeding is that they have not "got it together". They are the ones who, for example, do not realize that the quiz is at the same time as another important engagement. When they come for help they come too late, and have no specific questions. They seem constantly disorganised and disorientated.

For grade 13 teachers in general my main observation is that some students need to come to the university lecture hall with more learning skills. Many of them do not seem to be able to learn from the text and notes, but seem to need individual coaching and personal assistance. Although this is possible to some extent, the speed at which university courses cover material makes it impossible for a student to succeed who is too dependent on others for their learning. In grade 13 we probably need to remove such crutches as "make up tests", and in general give less individual allowance to students, and insist that they follow the class norm on deadlines and "do dates". We must remember that grade 13 is specifically a university preparation year.

For the mathematics teacher, especially of grade 13, my main observations are as follows. I have talked to a lot of first year students in my capacities as lecturer and councillor, and have found that the students with the best chance of success in mathematics are those with a fundamental understanding of basic concepts. The ones who succeed do not rely on formulae and algorithms, but understand the deduction and development of ideas. They do not learn by rote. Those who have learned mathematics rather than mathemagic have a great advantage. So my suggestion is to teach and test proof and understanding from the lowest grades, as well as teaching the usual algorithmic material. Give the students the ideas and logic chains so that they can see how knowledge develops. Further we would do our best students a great favour if we insisted on them learning problem solving techniques, so that they would develop into independent autonomous learners.

It may well be worth mentioning that the topics which I have observed give the students the most problem are as follows: Above all, the techniques for sketching graphs seems to be lacking in most students. Linear transformations seem to give problems; to be more precise, finding the transformation which changes from a sequence such as 150, 200, 250, 300, 350 to the sequence -2, -1, 0, 1, 2. Solving equations which have mixed linear and quadratic terms gives trouble. Manipulating radicals is still difficult and frightening for many students. Visualising three dimensional situations gives difficulties, with specific reference to volumes of rotation. In Calculus the definition of derivative is not well understood by many students, nor is the chain rule.

Many of the students I have come across are not aiming for a career in science but are taking science and mathematics courses as prerequisites for other courses. Many of them are really surprised to be taking mathematics, do not want to do so and above all are frightened of it. This is one major problem which I think we should come to grips with, and that is the problem of math anxiety. It is very real to many people and precludes them from success in mathematics courses and therefore many possible jobs. *(Editor's Note: See more on this topic elsewhere in this issue)*

I believe that it is also important for mathematics teachers to understand that the mathematics courses taught at university are not just for the sake of the academic material involved. The mathematics itself is important and interesting, however some of it is in courses for the benefit of other non-mathematics courses. For example the half year Finite Mathematics course is a prerequisite for several social science courses. This is for two reasons. Firstly the material such as Statistics is used in these courses, but secondly it has been decided that mathematics is one of the best disciplines to use for screening students. The correlation between success in mathematics and success in other subject areas is high. The most obvious advice I can request to be passed on to next year's first year students is simply to attend lectures. I have been most surprised to find that the attendance level at lectures runs at twenty five to seventy per cent, with the lower attendance occurring mostly on Fridays or when assignments are due. I think the problem is that students do not realize that their lecturers spend many hours preparing lectures, in order to help their students through the maze of ideas in the simplest way. If the students do not attend the lectures, then they must find their own way, and with their lack of experience this will take considerably longer than for the lecturer. In other words not attending lectures is simply a waste of time.

It should also be stated that students at university are expected to learn material on their own. The purpose of a lecture is to guide the student on the correct path, not to give all the required materials. It is therefore most important that the studetns work from their texts, both for concepts and exercises, and that they consider their lecture notes as a starting point and not a complete coverage of the course.

The next piece of advice I have to offer is to keep up with the work. If the material presented in a lecture is not reinforced within twenty four hours, then it must be relearned almost from scratch, and this is an unnecessary waste of time, time which could be spent in other ways. I would also advise students who do not understand work in lectures or from the text book, to go and see the lecturer during office hours. I have found that some students do not realize that office hours exist until it is too late for them to remedy their problems. We all have office hours and they are for the use of the students. One final piece of advice, which may be asking a lot of young people, is to take a few minutes before a lecture to read through the relevant section of the text. This tunes the student in to the concepts and alerts him or her to any potential problem areas. My students who have taken this advice, and some do, have found it to be advantageous.

Overall the advice is that to succeed at university one must take responsibility for one's own learning, and adopt effective study habits and strategies from day one. Do not look for rescue just before the examinations. As a first year councillor I have found that I have been giving the same advice to almost every student who is having problems with his or her work. In fact some of us who felt like stuck records collaborated and wrote out a sheet listing what we considered the essential and most effective strategies, and we hand these to lost souls when they come for guidance. This saves a lot of time and repetition, as well as giving the student something specific in the way of suggestions to consider and follow up at a later time.

A copy of this sheet follows.

HAVING PROBLEMS WITH YOUR MATHEMATICS COURSE
HAVE YOU TRIED THE FOLLOWING

1. Have you decided that you really want to do well and have you committed yourself to spending the necessary time?
2. Are you going to lectures regularly?
3. Are you attending tutorials regularly?
4. Do you know the prerequisite material thoroughly?
5. Are you keeping clear, concise notes or study sheets for important results, and have you studied them?

6. Have you made a thorough effort to understand the techniques used? Do you read the theory several times while making notes in your own words? Have you made use of the text book?
7. When stuck on a problem, do you come back to it several times at intervals of two or three days?
8. When stuck on a problem, do you try to solve it using different techniques?
9. Have you made a careful written list of what are your problems?
10. Have you got together with other students in your class for self help sessions? This is relatively easy if you live in residence. If you live off campus, you should make an effort to do this. How about asking your professor if he or she would make an announcement for all students interested in a Math pizza party to meet you after the lecture?
11. Do you know where solutions are posted, and have you consulted or copied them?
12. Do you know where old exam and test papers can be seen?
13. Do you know where the library is, and can you find relevant books there? Sometimes a different book will give you a new and understandable perspective on a problem.
14. Have you seen your instructor for help? Do you know when his or her office hours are? When you do, take your written list of difficulties and your attempted solutions with you. This makes assisting you easier.

Much of the advice on study habits, and the use of university facilities is given to the students during the first week of the year. This is the week when first year students are undergoing orientation activities. The riotous tone of this week is not conducive to the serious consideration of such instruction and advice. It is not an appropriate time to have young people consider and absorb such important matters.

It is most important to state that many of the students entering first year university are extremely well prepared and produce excellent results. It really is a great pleasure to see them learning.

Overall I must say that I am having a great time at university, and have been very well accepted by my colleagues here, who have been extremely helpful, friendly and patient. I believe that both systems could benefit by more exchange of personel and ideas, and that such exchanges should be encouraged and instigated much more than is presently the case. We are, all of us, on both sides of the interface, interested mainly in the success of our young people, and can improve their chances greatly by making this interface as small an obstacle and as little a cultural shock as possible.

Tom Griffiths, past-president and present Councillor of OAME, teaches high school in London, Ontario. He was one of the leaders of this year's Canadian Mathematics Olympiad team.

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288 is too gross

Arithmetic teachers really know how to make the little things count.

Exponents are not on the level

TEACHERS: ARE WE DEFEATING MATH ANXIETY OR DEVELOPING IT?

by
Sandra Folk

Sweaty palms, churning stomachs and headaches are some of the physical symptoms which accompany the psychological characteristics of fear, distress, tension, and feelings of inadequacy that a math anxious person experiences when confronted with a mathematical situation. This may occur in the classroom learning mathematics or in mathematics related situations such as balancing a chequebook or calculating a tip.

Math anxiety afflicts both males and females. As Kelly and Tomhave (1985) noted, "The literature suggests that both men and women can be afflicted by math anxiety." Being math anxious does not necessarily mean that one is generally anxious in nature, nor does it mean that a person is inferior in intelligence or inherently incapable of learning mathematics. As Sheila Tobias (1978) explained, "Math anxious people are not math incompetent. Most are bright people who enjoy school and success in other subjects, but they openly fear or avoid math." This is substantiated by Janet Morris (1981) who wrote that, "Among adults, some highly successful executives are math anxious."

For many mathophobics, the problem usually begins as a single negative experience during early school years. Then, aggravated by a series of frustrations and failures in the classroom and in mathematics related situations, it mushrooms into a "full-fledged syndrome of anxiety." (Tobias, 1978).

Since mathematics is a subject which is cumulative in nature, concepts must be firmly established before further understanding or meaningful development of new ideas can occur. Without this, students experience difficulty and are limited in their achievement. Frequently, this is the situation for math anxious students. Also, these students lack effective problem solving skills, persistence, and are too inflexible to be able to consider alternatives in the process of their work, or to be reflective. They have little faith in their own intuition. "If an idea comes into their heads, or a strategy appears to them in a flash, they will assume it is wrong." (Tobias, 1978). For them, mathematics becomes a series of memorized "what-to-do's"; a collection of unfamiliar words and rote ideas.

A variety of factors such as learning styles, attitudes, math myths, counsellors, teachers, etc., and their relationship to math anxiety have been identified and examined in the literature. Because teachers are "the most important educational influence on students' learning mathematics," (Fennema, 1980), and they spend countless hours in direct contact with children, their relevancy to this phenomenon merits further consideration.

As mathematics educators, we are aware that the quality of mathematics teaching in the elementary grades has a direct influence on the future mathematical success of most students. Therefore, it is of paramount importance that teachers at this level be skillful and confident in the subject area to ensure that mathematical concepts are established in a logical, meaningful way and that positive attitudes are transmitted.

Yet, there are many teachers who themselves are math anxious. As Kelly and Tomhave (1985) concluded from their study on math anxiety, "Math anxiety seems to be exhibited by two categories of people: students learning mathematics and prospective teachers who will be teaching mathematics." Because of their feelings of inadequacy, these teachers teach mostly by the book or resort to rote-level instruction. There are other teachers who either lack interest in the subject area, who are insufficiently trained, or who lack the necessary competence for providing appropriate experiences for their students. This is substantiated by Frank K. Lester Jr. who wrote: "During my career as a teacher educator, I have taught mathematics methods courses to well over a thousand preservice elementary teachers. Anyone who has taught such a course is well aware that, as a group, elementary teachers' understanding of mathematics is weak. Furthermore, their mathematics deficiencies are compounded by generally negative attitudes or even anxiety toward mathematics." (Lester Jr., 1984).

Typical elementary school teachers are required to teach many subjects. This limits the amount of time that they can devote to any one subject, especially to one which is least favoured. As mathematics is classified as one of these "least favoured" subjects, Anderson's (1981) findings in a study completed at his institution suggest that children will not be adequately prepared.

Conversely, there are mathematics teachers who are knowledgeable and well-trained. Too often, they present mathematics as a theoretical body of knowledge in abstract form which students cannot comprehend. Unfamiliar terminology and complex formulae are used without clear explanation or understanding. They emphasize the importance of having the right answer. Solutions are given in final form, and students are not given the opportunity to see the trial and error process or the guessing and testing that might have been used in working these out. It is these teachers that make students believe that mathematics is mystical and perpetuate the myth of the "math mind."

As concerned educators, it is our responsibility to adopt an appropriate course of action for dealing with the aforementioned issues in an effort to ameliorate the problem of math anxiety. The following suggestions are proposed. Workshops should be set up for math anxious teachers to help them understand and deal with conquering their fear. Exposure to mathematical concepts and processes in a non-threatening environment is one way Kelly and Tomhave have suggested for this. In-service programmes to enhance teachers' understanding of mathematical concepts and to provide new techniques for making mathematics education come alive for children would be helpful.

Prospective teachers in university training programmes must be well prepared. We need to ensure that a positive attitude toward teaching mathematics is developed; that they achieve appropriate understanding of concepts and that they acquire the necessary skills for knowing how to present the concepts in a meaningful and non-threatening way. These students must enter the classroom as teachers who are confident and competent in mathematics.

It is important to identify prospective teachers who might be math anxious. Having done this, help, in the form of support groups, should be provided to encourage them to overcome their fears of mathematics.

It is crucial that we, as classroom teachers, judiciously examine our techniques and procedures. As concerned educators we must do the following in order to help alleviate the problem of math anxiety: create a positive supportive classroom atmosphere; encourage students to take risks in working out solutions and in problem solving; encourage discussion of ideas and accept all suggestions; stress understanding the thought process; endeavour to dispel the "math mind" myth by reassuring students that teachers need to try again when a problem arises; employ physical models to support an abstract idea; reinforce understanding of one concept before introducing another new one; exhibit enthusiasm; reduce tension and pressure by having students work in groups when possible; emphasize the relatability of mathematical ideas; and above all, make learning mathematics appealing.

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Sandra Folk, a teacher and independent math consultant, is currently working as an author for Addison Wesley.

A REVIEW OF MATH ANXIETY - IMPLICATIONS FOR THE FEMALE STUDENT

by
Jeff Fraser

> Whether the incident occurred while learning work problems in the sixth grade, coping with equations in high school, or first confronting calculus and statistics in college, failure came suddenly and in a very frightening way. An idea or a new operation was not just difficult it was impossible. (Tobias, 1978).

Mathematics is a subject which demands a quiet untroubled mind for its proper absorption as well as for its ensuing communication. Social uncertainty, anxiety, examination apprehension, etc., affect performance in mathematics more immediately than in most other subjects (Howson, 1972).

The proven connection between math anxiety and its more than frequent occurrence in women has been well documented throughout the literature. Findings indicate that males dominate the field of mathematics both from an academic and participation perspective. To illustrate, a 1972 survey of the amount of high school math taken by incoming freshmen at Berkeley, California, revealed that while 57 per cent of males had taken four years of high school math, only 8 per cent of the females had the same concentration of math (Tobias, 1978). Having completed less than four years of high school mathematics, the females in question would be ineligible for university courses involving calculus, chemistry, physics, geology, economics, pharmacy, etc. In effect, this would inevitably void approximately 30 per cent of the career choices available to them.

Math anxiety or mathophobia can be perceived as both a cognitive and psychological problem. Cognitive in terms of a "disturbance of the mind regarding the mental uptake of math" and psychological as a "tense emotional state characterized by fear and apprehension without apparent justification." Besides the obvious factors of student interests and attitudes, researchers have also stressed sex differences, ethnicity, fathers' and mothers' education, parental attitudes and socio-economic status. Indirectly each factor effects the elements of a mathematics classroom, be that of course organization, teaching techniques, instructional materials, etc.

Quite often math anxiety is associated with less than positive attitudes, which are experienced as feelings of panic, nervousness, mental disorganization, uneasiness, accompanying headaches and upset stomaches. These feelings and emotions rise to the surface as an air of non-confidence for many females. Fennema (1982) defines confidence in mathematics as "the belief that one has the ability to learn mathematics and to perform well on mathematical tasks."

Female math anxiety is caused by variables linked to 1) socio-culture; 2) perception; 3) attitudes; 4) education and 5) biological considerations. All these variables overlap and compound each other in creating math anxiety in females. For instance, stereo-typing can be derived from a female's perception, attitude, education and even socio-culture. Fennema and Sherman (1978) indicate that attitudes and perceptions of stereo-typing, i.e. Math is a Male Domain, have been delineated and personified by teachers and parents for years. Text books even play a stereo-typing role, as literature has proven, through sexual bias. Many researchers have found that females as a group perceived math as being less useful to them. In part this may be due to females receiving from home and school less encouragement to pursue math courses thus creating a career barrier for themselves. There has been a hypothesis circulating amongst authors that women show a fear of success towards mathematics. Hoffman (1974) cites Homer as suggesting that "a motive to avoid success exists in women because of their anticipation of negative consequences in the form of social rejection or loss of femininity." This information is indeed interesting since 1) various forms of media have impressed upon us equality of the sexes and 2) there seems to be no concrete evidence indicating a female cannot do as well as a male in mathematics.

As mentioned previously, female math anxiety can be linked to educational and biological variables. In relation to the latter, it has been put forth by various authors that males excel in mathematics due to 1) inherited recessive X chromosomes, 2) low levels of the hormone estrogen and 3) brain lateralization (Burton, 1978). All these attributes combine to increase the male's ability to spatially visualize and, in some cases, as Tobias (1978) points out, increase the male's ability to complete complex restructuring tasks. If this is the case, this would undoubtedly create amongst the female student population poor performance, frustration and inevitably math anxiety. The implication is that males and females do not think nor solve problems alike. It is hard to imagine that if a

person has a low level of the female hormone estrogen, then one's mathematical spatial ability increases. This does however bring about an interesting theory. During puberty and adolescence, females have relatively high amounts of estrogen in their bodies. It is also at this time that females frequently begin showing signs of math anxiety. There are clearly many possible explanations for this, but perhaps biological ones cannot be completely discounted.

Contrary to popular belief, math anxiety is not apparent in grade school. A study (Wiggan et al, 1984) using the data from the Second National Assessment of Educational Progress, which sampled over 70 000 students aged nine to thirteen revealed little overall difference between males and females in mathematics. Most authors set the emergence of math anxiety and therefore math avoidance at Grade 12. The Wiggan et al (1984) study of approximately 30 000 students from grades 9 to 13 attending 27 Toronto Board High Schools indicates that through grades 11, 12 and 13 the female avoidance percentages are 0%, 11% and 22% respectively. There seems to be general agreement with Fennema (1978) who noted that math anxiety among females increases substantially as the learner progresses from grade 6 to grade 12.

The math curriculum, textbooks, and teachers all contribute to math anxiety. Teachers themselves, especially at the elementary level, create and promote math anxiety. A study conducted by Kelly and Tomhave (1985) assessing math anxiety in pre-service elementary teachers established that a high proportion of female education majors were math anxious. They go on to state "If the results of our study are representative of pre-service teacher education, then women elementary school teachers may be perpetuating math anxiety with the young girls in their classroom." Math teachers often insist or regiment correct answers invariably to be calculated and deciphered with haste thus inevitably creating an environment of pressure situations. Combine this with math's cumulative nature and what results are feelings of frustration and helplessness that are compounded daily. This in part creates what is known in math circles as low math-esteem.

Current trends, however, do indicate that changes are near. In today's elementary classrooms, some teachers are emphasizing problem solving, stressing processes rather than the immediate answers. Some classrooms have adopted mastery learning and various forms of invitational education to increase self-esteem and therefore lower math anxiety levels. It would be interesting to see the results from an ability-group study where the classrooms were totally comprised of female senior math students. A secondary school in Toronto is currently experimenting with such a programme. This particular study could prove useful in determining if math ability-anxiety differences actually exist between the sexes in a controlled environment. As can be expected, recommendations are endless. However, educators must show foresight and act immediately, as the data in the latest Carnegie Report on Policy Studies in Higher Education would suggest "By the year 2000 post secondary institutions will enrol more women than men."

Summary

Anxiety will always be a component of mathematics. In this present review, student attitudes and perceptions, socio-culture, biological considerations, sex, teachers and the curriculum were all identified as contributors to math anxiety. Throughout the literature, there is a strong positive correlation between females and math anxiety. While math anxiety cannot be totally eliminated, it can be diffused. Improved areas of student attitudes and perceptions along with accompanying positive educational and parental views can contribute to both preventing math anxiety and diminishing stereo-typing. Lessening math anxiety begins at the school, where changes are needed in the mathematics curriculum, teaching techniques and teachers themselves.

Biological considerations need further research and study to establish correlation among the elements of sex, math ability and anxiety. Future research in math anxiety most likely will continue to be aimed at sex differences. Undoubtedly this research and theory will have to catch up to cures and solutions being implemented already. Research in the 1980's should be continued and directed on the hows and whys of female math anxiety-avoidance at the crucial yet accessible barrier : grade 12.

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Jeff Fraser teaches at a private high school in Thorold, Ontario.



Ode to Q.E.D.

Hail to thee
O Q.E.D.—
At the end of each proof you are found.
While mathematics to me
Was a turbulent sea,
You represented dry ground.

My inadequate brain
Strove to conjure again
The steps that the author deleted,
As you played double role
(Both inspirer and goal)—
A Greek phrase for "mission completed."

Quite early in youth
I discovered the truth—
I've no mathematical leaning.
But years of exposure
Have brought me composure,
And you, Q.E.D., a new meaning.

You're the first thing I see
That's familiar to me,
Though the last thing that catches my eye;
Then, hail to thee!
O Q.E.D.
Greek for "God only knows why!"

Luella Ellington

GIRLS AND COMPUTING

by

Robin Ward

Editor's Note: *Mrs. Ward is the Assistant Director of Britain's Information Technology Project at the Davidson Centre in Croydon, a borough of London. Although the article was written relative to the curriculum in England and the concerns regarding the participation of females in technology, it had enough applicability to the situation here in Ontario to warrant its inclusion here.*

Traditionally men and women have been divided into stereotyped roles and this has carried on into the world of technology. In far too many cases, boys are taught they must fit into the mould of leader and provider, while girls are shown that they should remain in the background, giving support from an inferior position. By the time students reach the age of secondary education, most of them have already been stamped with the prejudices and ideas of society and their places in it. It is up to the teachers to prove to the girls that they are just as capable as their male counterparts of competing successfully in the sphere of technology. Employment opportunities in the future will require new knowledge and the social implications of technology are great, but students must be prepared to develop their potential to enable them to cope with the needs and responsibilities of their future lives.

Because Information Technology is a new subject in the school curriculum, it is the ideal time to influence the sex role ideas of the students - and indeed some of the teachers. When Computer Studies was introduced in the schools, the teaching seemed to gravitate towards the Mathematics department, the equipment was stored in the "Maths Block" and the teachers prepared to tackle the new subject were almost invariably mathematicians. Since mathematics has always been a male-dominated subject, the implied link with computers discouraged many girls from taking the course. Many of those who did brave the lions' den, found within quite a short time, that they wished they had not. Statistics show that far too many girls drop Computer Studies altogether or fail to sit the final examination.

There are many reasons why this might be so:-

1. Girls are traditionally more passive and allow the boys to push them aside when computer time is needed. This means they are sometimes unable to complete required assignments.
2. The boys in the Computer Studies classes usually outnumber the girls. This means that, for various reasons such as fear of being laughed at for making a mistake, girls neither ask nor answer questions.
3. The computer rooms available before and after school fill up quickly with boys and the girls feel intimidated enough not to enter.
4. Many of the teachers, predominantly male and often with inborn predjudices, are unaware of the special needs of the girls and appear to ignore them to some extent. The girls, not as confident in their own ability as the boys and not willing to make mistakes, fade into the background. They eventually find themselves so far behind that they lose interest altogether.
5. The syllabus seems to be aimed at the interests of the boys and helps alienate the girls.

The solutions to most of these problems, once they are recognised, seem to be quite obvious. The main thing is to make the teachers aware of what is happening. Once they accept that many of the problems stem from the nature of the girls themselves and also from the style of teaching, then they can begin to overcome them.

Possible Suggestions for the Teacher

1. Materials - search your teaching materials carefully to ensure that they have general appeal. If possible, focus on non-traditional roles and achievements of women. Check posters and illustrations and avoid using those showing stereotyped characters.
2. Examples - think carefully about the examples you pick to illustrate a point. They should interest both sexes equally.
3. Questions - try giving open questions to the whole class and notice who answers. If you find the boys are dominating the answers, as is often the case, then be specific with your questions, being sure to involve the girls as much as the boys. If the class knows that everyone is to be included in questioning, then the pupils will expect a mixed contribution and should not be so wary of giving a poor answer.

4. Discussion - watch your usual discussions carefully and then direct further ones to avoid dominance by either sex. It is often easy for a teacher to allow a spirited and interesting discussion to carry on for some time without realizing that only boys are taking part.
5. Jargon - choose your vocabulary and be careful to avoid any unexplained use of specialist jargon. This, more than almost anything, will alienate girls.
6. Hidden Curriculum - the reasons for the differential uptake of certain subjects by girls and boys go beyond the content of the formal curriculum.
   i. be sure that other staff and parents are aware of what Information Technology is and why it is included in the timetable.
   ii. arrange seating in the classroom. Allow girls to group together for moral support, but be sure they are near the front and not on the fringe of any activity. Girls will assert themselves much more when other girls are present to support them.
   iii. during extra-curricular activities, try to ensure an even spread of the sexes. If your computer clubs are dominated by boys, try running single-sex clubs to allow both sexes to establish their interest. Consider a simple proficiency test to determine entrance to the club. If the pupil can demonstrate competent and confident use of the machine, then she/he should gain access to the club. Girls can achieve this as easily as boys. (ie. Certificate of Competence)

The new Information Technology Project under the directorship of Trisha Strong (IT Unit, Davidson Centre, Croydon), is designed to produce a syllabus for pupils from 11 - 14 years old. The course is being taught by a wide variety of teachers from many backgrounds. It is important that teachers who have communication skills and a general interest in Technology, rather than in the devices themselves, should introduce the new subject.

The twelve booklets and the accompanying software are being produced to appeal to girls and boys equally. Because of this, and the fact that all pupils in the first three years of secondary education will take the course, girls should soon recognize that computers are for everyone. The numbers of boys and girls will be relatively even and, as long as the teacher makes sure the "hands-on" time is equally divided, girls should gain in confidence and competence.

By the time the students choose their options at the end of their third year in secondary school, both sexes should have a better idea of what Information Technology is all about and be able to decide whether or not to continue with a Computer Studies course. For many, it will be enough to have had the experience of handling and becoming familiar with computers in Information Technology classes. It is the fear of computers and the lack of knowledge about their capabilities which must be overcome. The students must gain a realistic appreciation of the power of computers and their relevance to future work and leisure activities. We, as educators, must be sure that we do not ignore the interests and needs of the female half of the population.

The Scarborough Association for Mathematics Education (SAME) is planning a mini-session at Howard Johnson's on Markham Rd. in Scarborough at 4:00 p.m. on Tuesday, September 24, 1985. Woody Sparrow (Nelson's) will demonstrate an elementary school software package.

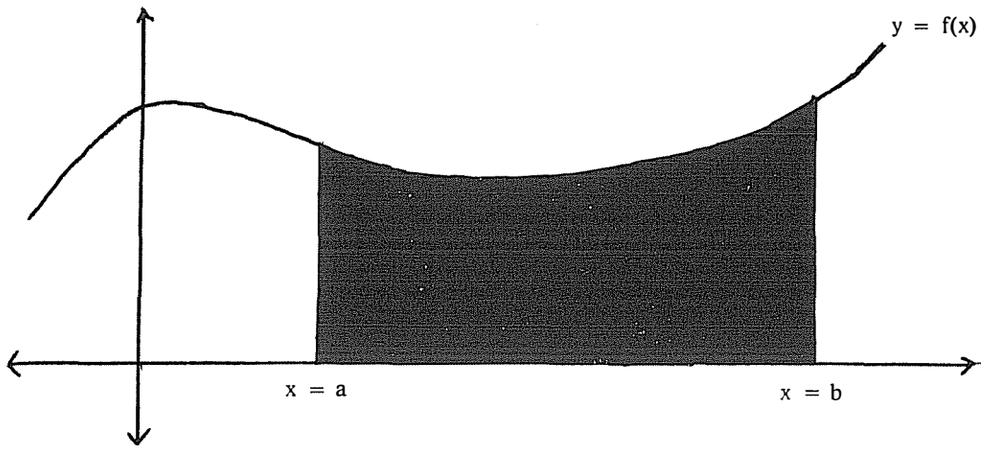
The SAME Fifth Annual Mini-Conference will be held at Maplewood on Tuesday, October 22, 1985.

CALCULUS FORMULA SIMILARITIES

by
David R. Duncan and Bonnie H. Litwiller

A well known calculus integration formula is used to determine the area bounded by the graph of a positive function y = f(x), the x-axis, and the lines x = a and x = b. Figure 1 depicts this situation.

Figure 1



The area formula is $A = \int_a^b f(x)dx$. If you have forgotten how this formula is derived you may wish to consult your calculus book.

Near the end of the chapter (of your calculus book) concerning uses of integration, another formula will be used to calculate the amount of work done in moving an object from point a to point b along a line. See Figure 2.

Figure 2



If an object is visualized as being moved from a to b, then for each value of x in [a, b], let f(x) be the force necessary to move the object when it is at point x.

In general, physicists use work to denote the product of a force which is exerted and the distance over which it is exerted. If the force required is a constant (k) for each value of x, the work necessary to move the object from a to b is then k(b - a). If the force is not a constant on [a, b], then the work performed can be computed by $\int_a^b f(x)dx$. Note that this is the same formula which was used to compute the area of Figure 1.

Why the similarity? One way at looking at this similarity is to visualize an object on Figure 1 being pushed along the x-axis from a to b. The area of the figure may be regarded as a weight which retards the motion and which must be overcome by exerting force.

A reasonable supposition is that for any location of the point x, the weight resting on that point is proportional to the height of the curve above the point - namely f(x). If the weight and distance units are adjusted properly to make the constant of proportionality 1, we have described the situa-

tion in which a force of f(x) is necessary to move a point located at x. The use of the formula $\int_a^b f(x)dx$ to compute either area or work is certainly not proven by this analogy, but is at least made intuitively plausible.

Profs. Duncan and Litwiller teach in the Mathematics Department of the University of Northern Iowa. They are frequent contributors to the ***Gazette.***



LIFE MEMBERSHIP

In the March, 1985 **Gazette** (Council Comment, pp. 44-45) information was given with respect to procedures for nominating someone for Life Membership in OAME. To qualify for Life Membership in OAME/AOEM a candidate shall: (i) have ten or more accumulated years of membership in OAME/AOEM, and (ii) have demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular.

If there is someone you wish to nominate for life membership in OAME/AOEM, you should submit his/her name and any information supporting the nomination to:

Dave Davidson, Life Membership Committee
7 Ferguson Road
Collingwood, Ontario L9Y 2Z9

Nominations should be submitted to Dave by **November 15, 1985.**

MANIPULATIVES: A VEHICLE FOR CREATIVE PROBLEM SOLVING

by
Ron Cole

It is common knowledge among educators that our main objective in mathematics is to teach children how to solve problems. Some strategies used in doing this are drill, guesses, estimation, inference and conjecture. In examining the literature, there seems to be some agreement on the following 7 characteristics of a good problem solver:

1. The student has superior ability and knowledge of the overall mathematics at whatever grade level he/she is in.

2. The student's verbal and general reasoning ability is average to above average. Expression of ideas and thoughts are therefore done freely.

3. This student has a very good overview of the problem. His/her spatial ability to look and visualize the situation is above average.

4. Our problem solver has a good positive attitude about him/herself and has a positive self image. This enables him/her to problem solve more freely for he/she does not lose confidence if he/she fails the first time.

5. The good problem solver is very resistent to distraction. Their concentration on task is very high.

6. The problem solving student is independent.

7. Finally, the problem solver is a divergent thinker. Having all of the characteristics from 1 to 6, the problem solver is able to draw from experience, background of the situation, factual knowledge, etc. and attempt to solve the problem possibly in a new way or to a further degree. This person therefore has the ability to think creatively, solving problems.

If we are to develop students that are divergent, creative problem solvers, we must make sure they are efficient in all 7 areas. How do we do this?

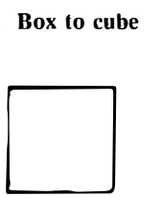
The most organized and developmental method of teaching to develop creative problem solvers is through the use of computers. Computers and their use will increase through the 1980's. With the better understanding of 3rd. generation software, all the above areas will be drilled, strategies developed and positive feedback practiced on the computer. Hopefully, we will develop younger students with better problem solving skills through the use of computers. Therefore, students with superior problem solving skills should be able to solve even more difficult problems for their ages. But the computer cannot do it all. Problem solving in the form of process problems, real life situations and hands on activities are among some of the experiences that the super computer problem solver must now adapt his expertise to. It is this transfer of problem solving strategies to job and real life situations that we as educators must help teach.

Manipulatives in the classroom are one way of applying problem solving skills to situations at any grade. Here are some examples of activities that are open ended and add another perspective to students' problem solving skills. All activities are geared for the junior-intermediate divisions of the elementary school.

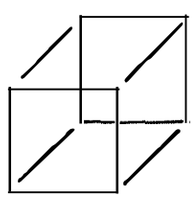
1. TEACHING CHILDREN TO THINK AND VISUALIZE IN 3 DIMENSIONS

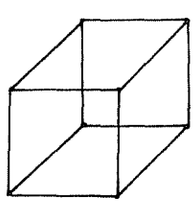
Objective: Reinforcement of spatial and visual awareness.

ACTIVITY 1: Box to cube



Box





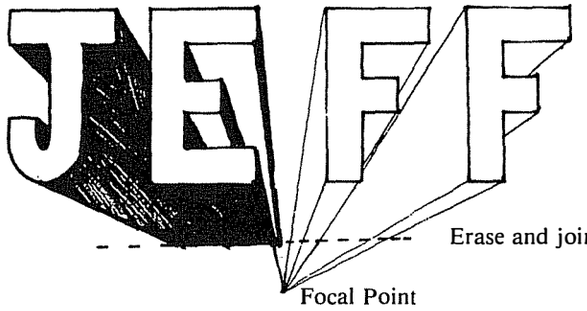
3D Block or Cube

EXTENSION: Triangle to Pyramid

*Note: This activity should be done on graph paper.

ACTIVITY 2: 3 D Name

1. Print your name on graph paper.



2. Pick a focal point
3. Using your pencil, join all edges to focal point.
4. Decide whether or not you want the lines going to dot. If you don't, erase at a point and join letters there.
5. Now colour each letter in differently and each background in a different solid colour.

*Always join letter lines from the bottom first.

EXTENSION: Adapt lettering to project cover.

2. **COIN COLLECTING**
   Objective: Increase knowledge of his home/family.

   (1) Set up booklets with 30 years of coins to be filled in.
   (a) If you have a penny collection how much will it cost: _____
   (b) If you have a nickel collection how much will it cost: _____
   (2) Have students find coins for each year.
   (3) Student then researches in coin book to find appraised value of the coin.

EXTENSION: Student now goes home and finds a personal event for each year. e.g. Parents got married, Dad's first car etc.

3. **COLOURING DOODLES**
   Objective: Spatial/Visual awareness.

   (1) Give out simple doodles - students colour in
   (2) Give out spatial 3D doodles. - students must organize colour scheme. Repeat colour pattern.

EXTENSION: Students create an object, spacecraft, building, by adding people, wheels, doors. Cut out and put class pictures together in a mobile. Give it theme title: eg. Doodles from outerspace.

4. **CODES AND CIPHERS:**
   Objective: General reasoning.

   (1) Introduce encoding/decoding.
   (2) Set up secret decode book with alphabet/numbers.
   (3) Practice from some given examples.

EXTENSION: Students divide into groups of two. Create code, write messages back and forth in code.

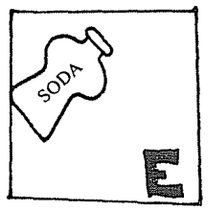
ENCODE: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
DECODE: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

ENCODE: M A T H IS FU N
DECODE: 13 1 20 8 9 19 6 21 14

5. **BRAIN TEASERS**
Objective: General reasoning: Inferences.

(1) Student is given pictures that mean something.

FLOWER



ANSWER: POPPY.

(2) Student must connect visual and verbal (telegraphic message) to solve problems; e.g. FLOWER (verbal part) + SODA (visual) + E = POPPY

EXTENSION: Create new code for Activity 4

ACTIVITY: (A)
6. **STRUCTURED LEGO BLOCK PLAY:**
Objective: Creative Problem solving - Divergent Thinking.

(1) Student is given period to play and familiarize themselves with blocks.
(2) Give picture of a car. Students must create exact car using blocks. Repeat car #2, #3. Now student has a good idea of car.
(3) Student now creates his own idea of a car.

EXTENSIONS: Using graph paper he draws out his car example. Colours it in.

ACTIVITY: (B)
(1) Student is given period to build house from pictures.
(2) What is the perimeter of your house?
(3) What is the area of the rectangle?
**Remember one hole in the block = 1 Unit
(4) Create your own modern house.
(5) Draw it out on graph paper.
(6) Find area and perimeter of the building.

EXTENSION: Starting off using graph paper draw (a) a house, office building, (b) race car, truck, etc. Give the building, or vehicle a name. Colour drawing. Construct actual model.

In this article you have been given six different ways to promote divergent thinking. The bibliography that follows lists several sources of activities of this type. Remember students of all ages enjoy games that promote creative thought. It is our job to foster this.

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Graphitti

MATH is a 4-letter word

Numerals are what you call numbers when you're in school.

Algebraic symbols are used when you do not know what you're talking about.

ALL WORK DONE ON PREMISES
(no work done on conclusions)

CONTEXT EFFECTS ON MATHEMATICAL PROBLEM SOLVING

by
Henk Nobel

I. Introduction

Current mathematical practise seems to presume that students learn mathematics by internalizing a set of procedural rules and applying these to similar problems.

Many students themselves seem to presume that learning mathematics involves internalizing procedural rules. During the learning of a new skill their primary concern is how to do the various subprocedures as opposed to considering when the new skill should be employed. This type of incorrect thinking is promoted by the organization of mathematic textbooks and their exercises designed to help the student through practice.

This paper intends to outline an experiment done with Grade 11 and 12 students in order to illustrate the above statement and to test the hypothesis that mathematical problem solving is influenced by the context of the other problems in which the problem is embedded.

II. Method

The experiment that was done was based on the review of a mathematical topic previously introduced to the students: factoring. It was considered ideal since it is taught, reviewed and used in all levels of high school mathematics and is considered difficult by many students.

In order to test the hypothesis that mathematical problem solving is influenced by the context of the other problems in which the problem is embedded, two forms of review were designed: "blocked" and "mixed".

The blocked format was designed to be similar to the homogenous problem sets that students encounter in class by the textbook after a new topic is taught. All the problems within the same set are of the same type but become progressively more difficult. The blocked design actually included two homogenous drills - one to remove the highest common factor (e.g. $2x + 2y$) and then one to take the difference of squares (e.g. $x^2 - y^2$). Each drill began with its own instructions.

The mixed format was somewhat similar to what is usually given as "end of unit" tests or mixed drills that occasionally are found at the end of a unit in some mathematic texts. However, even in these mixed drill reviews each question may be different from the previous one, but the problems within each question are still the same. In this "mixed" format there was a single instruction that included instructions for both factoring out the highest common factor(s) and for taking the difference of squares. The problem sequence maintained the progressions in problem difficulty but it randomly mixed the different problem types (common factors and the differences of squares).

The test which was to follow both versions of the review was mixed in design and intended to bring out differences between the two review groups. Although it is not normal in classroom testing, it was decided that in addition to factoring and difference of squares questions, questions that were similar to these two types, but which were impossible to simplify would be included (e.g. $x^2 + 16$). It was hoped that the inclusion of "impossibles" would further bring out differences in the ability to pick out discriminating features in both types of problems. In addition to considering final scores on the test it was decided to ask the students to predict how well they though they would do on the test and to record how long it took each subject to complete both the review and the test.

It is to be remembered that this study was designed to test the hypothesis that context affects mathematical problem solving. Given the two groups of student, 1) those who did the blocked review, and 2) those who did a mixed review, we thought that the context effects would be shown in differences in the amount of time taken to do the review, predictions of how well each group would do on the test, how much time was required to do the test, and how well each group actually did on the test. Specifically we though that the "mixed" design group would take longer on the review, would predict that they would do less well on the review, but would actually take less time and do better on the test than the "blocked" design group.

The students tested were 23 advanced grade 11 students and 14 advanced grade 12 students. Both classes had covered the topic of factoring earlier in the school year. The test was introduced to the students as one which was designed to take a closer look at the effectiveness of certain teaching

methods. The students were told that they would be asked to work through a review sheet of two types of factoring: 1) removing the highest common factors, and 2) taking the difference of squares. Answers to the review sheet were also given to the students and they were told that this sheet was to assist them in their review. At this point the teacher worked out a simple example of each type on the blackboard. The students were told that the review would immediately be followed by a test covering the materials just reviewed. The review and answer sheets were distributed with every second student receiving a blocked design review sheet and the rest a mixed design review sheet.

When the student began they recorded their start time and when finished they noted their finishing time and exchanged their review sheets for the test.

They then read the test instructions after which they predicted the number they felt they would have correct out of the 25 test questions. Again start and finish times were recorded.

III. Results

The results of the experiment are outlined in Figures #1 to 4. Figures #1 and 2 indicate that the grade 11 block design group had a mean time of 28.5 minutes for the review while the mixed design group had a mean of 34.7. Grade 12 had 28.1 and 37.7 respectively. In contrast to the review times the block design group took longer on the test than the mixed design group.

Figures 3 and 4 indicate the mean predicted and actual performance on the test for both the blocked and mixed groups. All the scores tended in the hypothesized direction.

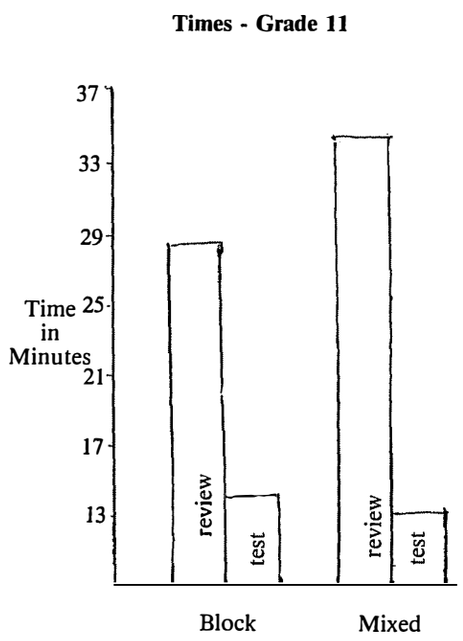


Figure #1

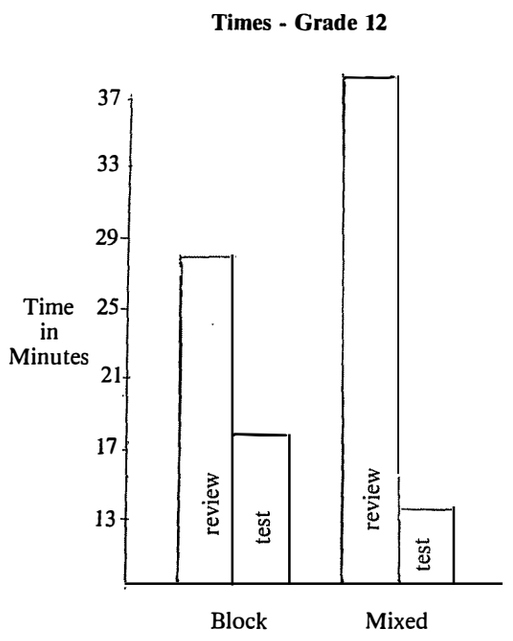


Figure #2

Phase of Experiment

Times - Grade 11

Times - Grade 12

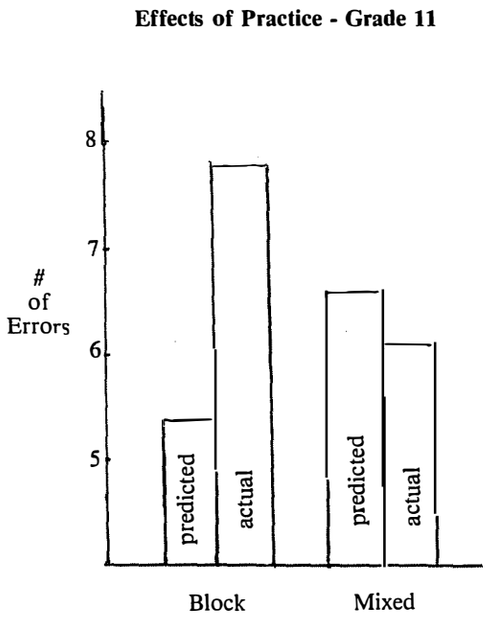


Figure #3

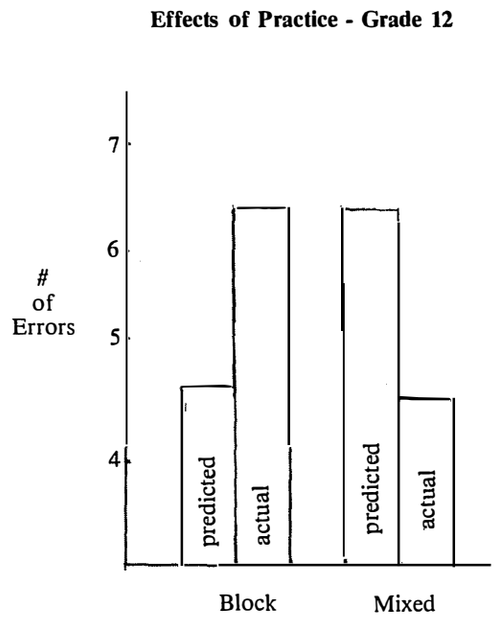


Figure #4

Performance on Test

IV. Conclusions

This experiment does lend support to the notion that context affects the learning and the perception of learning of factoring. All results tended in the hypothesized directions; students in the block design on average did work through the review more quickly than those in the mixed condition but took longer on the test and did less well. Students in the block design also predicted that they would do better on the test than they actually did, while mixed design students on average predicted they would perform less well on the test than they actually did.

Of course, an experiment as outlined above can hardly be conclusive but it does tend to support many criticisms heard today about the structure and organization of mathematic textbooks and do suggest practical implications in the field of education. Especially in the area of mathematics, classroom practice often follows a blocked pattern (because of the textbook) whereas testing is very often "mixed design" in its organization. Block design students predicted they would do better than they did. This suggests overconfidence prior to the test and these feelings may keep the individual from engaging in much needed review. Test results poorer than one expects may lead to an incorrect attribution of the cause of failure. Math teachers often have to contend with sayings such as "I'm no good at math anyway so why should I bother even trying?" Students who continually do poorer on tests for which they thought they were well prepared are often those who prematurely give up on the discipline.

In addition to predicting that they would do better on the test than they actually did, students who did the blocked review actually did do worse than those who had received the mixed review. In the classroom it is most often the case that the introduction of a new skill is followed by homogenous drill and only at the end of the unit is any time spent on a mixed drill type of review. The implication is that some of the notoriously poor test results in mathematics might be alleviated with changes in teaching strategy. Including more mixed drill throughout the whole unit may result in overall better performance on tests. It might also affect a student's ability to apply previously learned material to more sophisticated problems. It is assumed that once a skill has been taught a student will be able to apply it in a wide range of situations. Much frustration results because this does not appear to be the

case but it may be that with more mixed drill students will be better able to recognize when a particular skill has to be applied to solve a problem and also be better able to apply it. The fact that problem context influences problem solving also has ramifications for mathematics remediation. Since there appears to be more to problem solving than the internalizing of procedural rules, remediation should go beyond the teaching of applicable subrules and should possibly include mixed drills in which the problem types are embedded with other problems with varying degrees of similarity to the remediated problem type, although this should not be restricted to remediation only.

While there are many questions which remain unanswered, the findings of this study do challenge the appropriateness of descriptions of problem solving that emphasize the exclusive role of the characteristics of a problem in determining a student's ability to solve the problem. Characteristics of the external context in place during the learning of a particular problem solving skill and at the retrieval of this skill do seem to influence problem solving ability. The reputation of mathematics could be bolstered if teaching strategies were modified to include more mixed drill on a regular basis.

Henk Nobel teaches high school in Hamilton, Ontario

The Educational Materials Committee of the National Council of Teachers of Mathematics is now seeking manuscripts for the 1988 NCTM Yearbook, "Algebraic Concepts in the Curriculum, K-12". The Yearbook Editor is Professor Arthur Coxford of the University of Michigan. The Yearbook Advisory Panel is interested in papers addressing important issues in the teaching of algebra at all levels in the curriculum, and is also particularly interested in relatively short papers reporting proven classroom practices in teaching specific algebraic topics. Guidelines for the preparation of manuscripts are available from the General Editor, Albert P. Shulte, Oakland Schools, 2100 Pontiac Lake Road, Pontiac, MI 48054.

THE MAGIC BOX: A MATH CURRICULUM WITH A DIFFERENCE

by

Janet Jull

After discussing math curricula with colleagues in the last few weeks, I decided that the curriculum Brant County has developed for grades KP-6 is worthy of being shared with others.

About five year ago a team of interested elementary teachers, along with the Curriculum Assistant involved with Mathematics, decided the need was there to revise an aging math curriculum.

Using "The Formative Years", "Education in the Primary & Junior Divisions", and other current Ministry documents, the basic contents for the curriculum were outlined.

One important factor to be considered in the work was the integration of the three main strands of Mathematics: Geometry, Measurement and Arithmetic. The team decided that too many classrooms were heavily involved for the main part of the year doing mechanical arithmetic and only doing measurement and geometry activities in June if the other components were mastered. A new outline for each grade from KP-6 was produced where each of the three strands was being covered each term.

Another factor thought to be important by the writing team was that the use of math texts was too restrictive to the program. The curriculum was then designed to eliminate the need for texts at all. It was to be a collection of ideas, resources, and activities that each classroom would have access to. The team felt strongly enough that the use of activities and centres should be an integral part of this curriculum that the Board of Educaton was petitioned for funding so that each school could afford to purchase back-up materials. Each school as a result was able to purchase class sets of calculators, geo-boards, miras and metric measurements kits.

During the writing and development of the final product, teacher volunteers from all over the county piloted various units being considered for inclusion in the final product. In the spring, preceding its final implementation, each KP-6 classroom teacher was released from class duties for a half day in-service session to explain the concepts and the way the curriculum was to work. Resources were shown to the teachers and activities were tested out. Many teachers were able to really enjoy geometry using miras and geoboards. Calculator activities were an eye-opener to many as well.

Later that spring, each teacher was given his/her own "Red Box". The primary and junior curricula were split into two sections. Each one consisted of a red tool box about 40 cm by 20 cm by 30 cm high. Inside on the lid was an index listing all the files inside and where it could be located in the box. Each teacher was also given an individual file for the grade they were teaching that gave a suggested order of study of the topics.

Each file consisted of two sets of sheets. For example, under the topic of two dimensional geometry, there was a set of topics, objectives for each and suggested resources and activities to use to accomplish these objectives. In most files the actual seatwork assignment or task card was included in the file.

In the years since the box was put into the classrooms of Brant County, new files have been created and added to the box. Some files have been removed. Many new resources, like the computer file and problem solving resource books, have been introduced with the aid of more in-service sessions.

Workshops have been conducted to introduce games to the primary and junior divisions. These were of the Make-and-Take variety. Each teacher went back to his/her classroom with about twenty ready-to-use games. One of the most recent additions to the math box has been Domino Activities. Inspired by the use of a "toy", the children are able to learn many arithmetic concepts using the "black blocks with dots".

Another unique feature of the box of files is its instant facility for the teacher to meet the individual needs of his/her pupils. For enrichment activities in a grade 4 class, she/he only has to pull the related file on a topic at the grade 5 or 6 level. For remedial work on a given concept, the teacher may use a grade 3 file. Given the nature of the activities, a child never feels she/he is doing "dummy" work.

The Primary and Junior Curricula were evaluated on a grand scale at the end of the first year of its implementation. The writing team visited each school. With enough supply teachers in tow to allow classroom teachers to leave their jobs for periods of up to one hour, they arrived. Teachers were given a chance to praise, criticize, and/or make suggestions about the sections and grade level units they had used. The writing team followed up on these ideas and changes were made to the boxes.

In the four years since the box joined our classrooms, I have had a chance to use both the Junior and the Primary box. It is indeed the answer to a harried classroom teacher's prayers. All the organizing on a topic is done. Resources, films, TV programs, even dittos and task cards in many cases are supplied. It leaves the teacher free to do his/her job of teaching and observation of pupils!

My only hope, since I am now teaching Grade 7, is that I will soon find a "red magic box" in my room again. I will be able to take its philosophy and many of its related activities with me. But the hard job of organizing the materials into the Grade 7 curriculum will once again be up to me.

Brant County has started something super with its Mathematics Curriculum for Grades KP-6. I hope it continues this trend upward through the grades. The Board with its budget backing and the seriousness of the Curriculum Assistant and the writing team, has ensured that the emphasis on Mathematics will not soon fade in Brant County schools.

Janet Jull teaches in Brantford, Ont. She has taught primary & junior grades and, as you might gather from her article, will be teaching Grade 7 this year.

References

Mathematics Curriculum, Brant County Board of Education

Personal involvement with piloting the "Red Box"

Discussion with writing team

Once upon a time there was a super-smart horse that could add, subtract, multiply, divide and even extract roots. Someone suggested that the horse should try a bit of analytic geometry (ordered pairs, graphing, etc.). The horse promptly died.

The moral of this story is still frequently heard today. You can't put Descartes before the horse.

GEOBOARD ACTIVITIES FOR JUNIOR GRADES

by
Karla Tessaro

The geoboard is one of the more versatile teaching manipulatives that has been used successfully with children in elementary grades. In a workshop in Hamilton, teachers made their own geoboards from 25.5 cm by 25.5 cm squares of ½" aspenite (chipboard) or particle board. On each board, 25 Escutcheon pins were nailed in a 5 × 5 array, spaced 5 cm apart. There is a 2.5 cm border which allows for several boards to be placed together for larger constructions. Simple problems can be made more complex by enlarging the boards. A wide variety of geoboards (transparent, circular and triangular arrangements) are also commercially available. Given a number of coloured rubber bands, students of all ages and abilities can use the geoboard.

Inservices, like this one, can be used to make teachers aware of the number of different types of materials available (hundred boards, Belmer Box, pattern blocks, Cuisenaire material) their capabilities and extensive use across the curriculum. It is hoped that teachers will be able to make use of diverse instructional strategies and materials to develop and review concepts or skills as well as to provide opportunities for students to discover and inquire.

Geoboard activities can be performed individually, in pairs, in small groups or with the whole class. They can be undertaken on a geoboard, pegboard or transferred to dot paper. As a means to solve problems the geoboard's use is limited only by the teacher's and students' imagination and creativity.

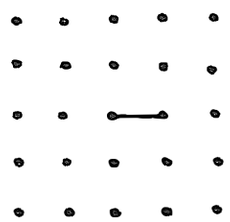
The following problems would only give the teacher a starting point. They are designed to develop learning outcomes implied by some of the grade 4-6 objectives set forth in the Hamilton Board of Education mathematics curriculum. However, this division is merely suggestive, and many of the problems could be extended to levels above and below.

Many examples and much experimentation are needed to gain understanding. The problems outlined below do not begin to exhaust all the possible suggestions for the use of the geoboard. Teachers and children alike can design many problems of their own.

Some of the activities described are open-ended and can be interpreted in various ways. Allowing pupils to give a variety of responses permits not only creativity but also gives the teacher some insight into individual understanding.

Line Segments

1. What is the shortest line segment you can make on your geoboard? (Allow it to be the unit for length)



2. What is the longest line segment you can make on your geoboard?
3. How many possible segments of length 1 unit are on the geoboard?
4. Construct pairs of intersecting lines.
5. Construct an angle, estimate and then use your protractor to find the measure of each angle.

Polygons

1. Construct a shape (polygon, figure) with 5, 6, 7, 8 sides. Are any of the sides equal? Make shapes with as many equal sides as you can find.

2. Make a shape with as many sides as you can. How many different ones can you make with that same number of sides?

3. Make as many different triangles as you can on a 3 × 3 geoboard.

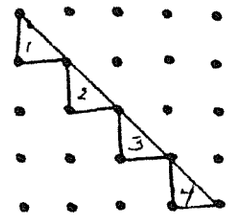
4. Can you make a triangle that has one square corner (angle) and one corner larger than a square corner?

5. (a) Construct 2 quadrilaterals of the same size but different shapes.
   (b) Cnstruct three quadrilaterals of the same shape but different sizes. Describe each.

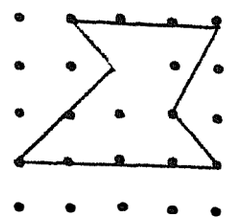
6. With a single movement, can you make a parallelogram
   (a) into a square?
   (b) into a pentagon?
   (c) into a rectangle?

7. Construct a rectangle 1 unit by 2 units.
   Cut it into (a) 4 right triangles
   (b) 3 right triangles
   (c) 6 right triangles
   (d) 8 right triangles

8. Place these four right triangles together to form different shapes.

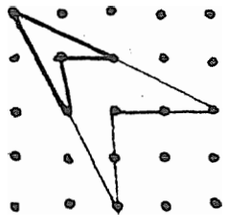


9. Divide this shape into four triangles and one square.



10. On a 3 by 3 geoboard, can you make the following:
    (a) an equilateral triangle?
    (b) an octagon?

11. Make a shape using one corner and then enlarge it. Check on the length of sides and the area.

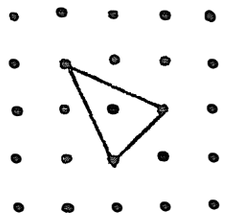


Transformations

When working with geoboards, where one shape is compared to another, one must be able to determine whether or not one shape can be placed on the other by a slide, flip or turn.

1. Make all segments (1 unit length) that are the slide (turn or flip) images.

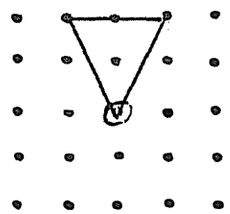
2. (a) Make all triangles congruent to this one.



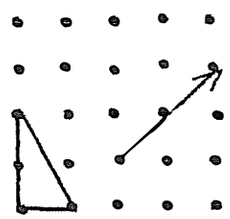
(b) Describe how each can be obtained from the first by a turn.

(c) Describe how each can be obtained from the first by a flip.

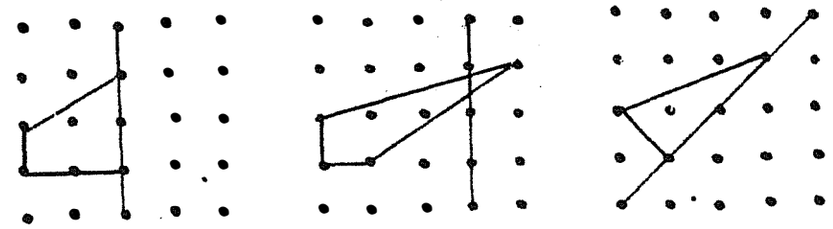
3. Make half turn images of the shapes, using the circled nail as the centre.



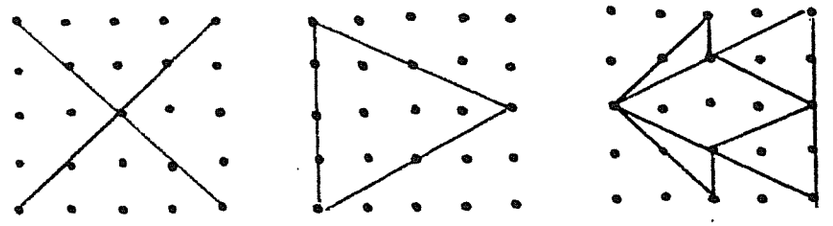
4. Slide the figure the direction indicated by the arrow.



5. Make mirror images of shapes in the mirror lines.



6. Put in the mirror line in these pictures.



7. Can you cover your geoboard with a number of similar shapes? (tessellations)

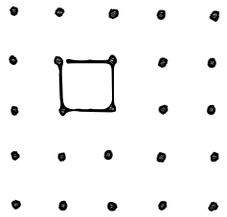
Perimeter

1. What is the perimeter of the largest hexagon you can make?
2. Make two rectangles of different lengths on your geoboard each with a perimeter of 10 units.
3. Can you construct a triangle whose perimeter is equal to that of a square?
4. Construct a rectangle that has a perimeter of 10 units and an area of 4 square units.

Area

1. Find as many shapes as you can, each of which has an area of 2 square units. (The basic square

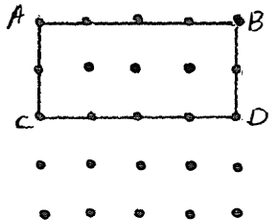
unit is shown in A1)



2. Construct a rectangle and a square so that they both have equal area. Which has the smallest perimeter?

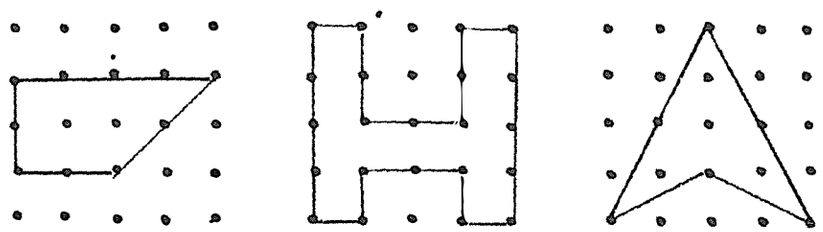
3. Construct a shape(s) that would have an area of ½ square unit. What is its name?

4. Divide the rectangle ABCD in half. Do the halves look alike? What is the area of each half?



5. Construct a parallelogram having an area of three square units.

6. Find the area of each of the following polygons.



7. Make 5 different shapes with 4 nails on the boundary and only 1 nail inside the boundary. Find the areas of each.

8. Design shapes to help you complete this table. All shapes will have 4 nails on the boundary.

Nails inside	1	2	3	4	5	6
Area of shape	2	3				

Fractions

1. (a) Divide the board into two halves with one straight line.

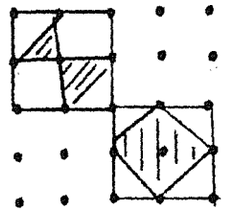
(b) Divide the board into two halves, each of which is the same shape, and the line does not have to be straight.

(c) Divide the board into two halves, so that they are each different shapes.

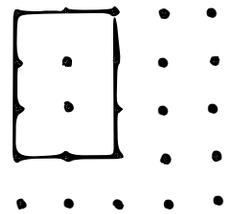
2. Construct a square 2 units by 2 units. Build 1/2, 1/4 and 1/8 in as many ways as you can.

3. Construct a rectangle 4 units by 2 units. Divide the rectangle into 1/2, 2/4, 4/8 and 8/16.

4. Add the following fractions.



5. Construct a rectangle ABCD. What is 1/2 × 1/3? First build the 1/3. Enclose 1/2 of that 1/3. The shape you just enclosed is what fractional part of the whole rectangle?

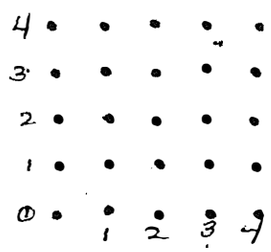


Ordered Pairs

Use rubber bands to connect the following nails:
(a) (1,1) (1,3) (3,3) (3,1) (1,1)

(b) (1,1) (1,3) (3,3)

Describe each shape



These activities used by teachers in a workshop suggest the potential of a geoboard as a teaching strategy. This device is by no means the end to teaching a mathematical concept. As one kind of manipulative, it does, however, provide teachers with one way to help children understand concepts and make the route to abstraction smoother and more effective. Similarily, geoboard activities can give students a sound foundation in basic ideas that are being utilized in their later experiences in mathematics. It is hoped that after teachers have had experience solving problems on a geoboard,

they will take these ideas back to the classroom and will adapt them to their students.

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Morrow, Lorna J. **Selected Geoboard Activities, Grades 4-8,** Willowdale, North York Board of Education.

The Board of Education for the Borough of North York, Department of Mathematics, **Geoboard Activities.** 1971.

Karla Tessaro teaches elementary school in Hamilton, Ontario.

What do you get when you cross a pigeon and a zero?
Answer: A Flying nun

An ape and a mathematician?
Answer: A Harry Reasoner

What do mathematicians sleep on?
Answer: Matrices, of course

What math is discussed between seabirds?
Answer: Intergull calculus

What do you call a mermaid's undergarment?
Answer: An Algae bra



Dear Sir:

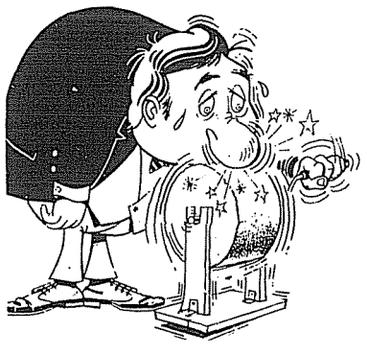
Please be advised that I am now retired from the teaching profession. This could be considered as my resignation from OAME.

I have enjoyed being a member of the past years and your publication and the OAME meetings have helped greatly in the classroom.

I wish the Association the very best in the future.

Sincerely,

Capt. William J. Hockin, C.D.
Oakville, Ont.



COUNCIL COMMENT

by
Bob Robinson

Editor's Note: *Council Comment appears in an issue of the Gazette following OAME/AOEM Council meetings. Your comments are welcome and should be addressed to the editor.*

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CHAPTER REPORTS

COMA (Central Ontario Mathematics Association)
A dinner meeting in honour of Fred Maskell was held at the Ottawa Curling Club on 1984 12 13. Brendan Kelly gave an excellent talk on "Leadership In Mathematics".

***Editor's Note:** *We are very sad to report that Fred Maskell passed away in January.*

On 1985 03 07, an evening of mathematical bloops, bleeps and blunders was held at the Ottawa Curling Club. The evening included mini-puzzles posed by the mathematics departments of Carleton University and Ottawa University. The results of first year calculus exams at Carleton and Ottawa Universities were presented.

On 1985 05 01, a very successful publisher's display was held at the Embassy West. COMA has been requested by both teachers and publishers to have a similar event next year using the same format.

A bus to the "OAME Annual" in Sudbury was organized by the Math Project of the Carleton Board. Several COMA members from neighbouring Boards travelled along with the Carleton teachers. Next year this shall be a COMA project if they can get the support of the Boards in their area. From all reports, a fantastic time was had by all!

The final event of the year was a wine and cheese social at the home of Duff Butterill. An update on the new OACs was given by Richard Rancourt. It is also reported that some of Duff's old classroom projectuals were raffled off ... collectors' items, no doubt.

GOLDEN SECTION
The annual OAME evening seminar was finally held on 1985 04 17 at Brock University. The format of the evening was a speaker, dinner and a series of workshop speakers dealing with the theme "Changes In The New Curriculae". The evening was well attended and the speakers and topics were well received. Plans are now being made for another evening meeting centred around a dinner and a main topic speaker.

GVMA (Grand Valley Mathematics Association)
About 200 people attended the spring general meeting. There are no special writing projects planned for this year. Watch announcements of the next meeting planned for 4:30 on Thursday, 1985 11 07 at Eastmount Secondary School. The tentative topic is "Mathematics and the General Student".

PRMA (Pine Ridge Mathematics Association)
A very successful mini conference took place on 1985 04 16 at General Vanier School in Oshawa. Seven different workshops were offered followed by a buffet dinner. About 130 people attended.

A P.A. day is being planned for 1985 11 15. PRMA is very pleased and honoured that the Mathematics department of O'Neill C.V.I. in Oshawa was chosen as one of the recipients of the Ken Fryer award at OAME '85 in Sudbury.

QSLMA (Quinte St. Lawrence Mathematics Association)
A fall get-together is being organized. No date had been set at the time of writing.

Their meeting of the past spring was very successful.

NOMA (Northern Ontario Mathematics Association)
Now as the exciting OAME '85 in Sudbury is fondly remembered, NOMA is actively planning a fall conference for September 27, 28, 1985.

The mathematics department of Lasalle Secondary School in Sudbury was selected as a recipient of the Ken Fryer award.

SAME (Scarborough Association For Mathematics Education)
SAME had a most successful 1984-85 school year. Among the events were the fifth annual October P.D. afternoon-dinner-evening session; four afternoon P.D. presentations; a fine workshop on calculators-in-the-classroom; and the P.D. day project on 1985 02 15 was successful thanks to careful planning, efficient management, an experienced conference team, and, most of all, interesting, enthusiastic speakers.

The executive has begun planning for their 1985 October P.D. afternoon-dinner-evening, for four afternoon P.D. sessions, and for the P.D. day to be held 1986 02 14. Watch for these important dates and locations.

A membership drive has commenced. Encouragement is being given to join OAME.

Aims/Objectives for 1985-86

The following aims/objectives of OAME for 1985-86 were approved:

1. To encourage executive and council to contribute on a consistent basis to our publications.
2. To continue to work in close liaison with the Ontario Ministry of Education.
3. To encourage each councillor to increase the publicity of OAME in their respective chapter, region or school.
4. To increase our membership to 1,500.
5. To have the executive complete a review of the present OAME Guidelines.
6. To encourage the nominations committee to continue to promote a high percentage return of ballots.
7. To initiate projects that will promote the name of OAME.
8. To provide a forum for the exchange of information relating to all aspects of mathematics education in the province.

ANNUALS

1985 ANNUAL - SUDBURY
A very successful excellent annual conference. Mickey Sandblom will make a more detailed report

at the next council meeting.

1986 ANNUAL - LONDON (1986 05 9-11)
Tom Griffiths reports that their planning is progressing well. The '86 Annual will run from Friday morning until Sunday noon. The committee is planning their sessions so that there will be ample time to mingle and chat. An interesting innovation is the plan to repeat many of Friday's sessions on Saturday so that some sessions need not be missed. If any readers have suggestions for topics and/or speakers, they should forward them to Tom Griffiths.

1987 ANNUAL - SCARBOROUGH
Niagara Falls and Scarborough each made excellent presentations as to why they should host the 1987 Annual Conference. In a very close vote, Scarborough was selected as the site. Niagara Falls was encouraged to resubmit for the 1988 Annual.

1990 ANNUAL - HAMILTON
It was recommended and approved that Hamilton be the site of the 1990 Annual which will be held in conjunction with a NCTM Regional meeting.

NCTM REPORT
Ralph Connelly has been appointed the Canadian member of the Conventions and Conferences Committee (CCC) of NCTM. Ralph will continue as NCTM representative for OAME as well.

LEADERSHIP SEMINAR
There is someone in your school board which has information pertaining to the Fourteenth Annual Leadership Seminar. Ask your director, superintendent, mathematics co-ordinator or consultant. The dates are 1985 10 23-26 and the location is the Ramada Inn - Airport West in Mississauga. An outstanding program has been organized. The registration fee includes accommodations and meals. A very worthwhile seminar. For more information contact:

Mr. M. B. MacGregor
Secretary/Treasurer OAME
247 Bright Street
Sarnia, Ontario
N7T 4E9

KEN FRYER AWARD
The Ken Fryer award was won by the mathematics department of Lasalle Secondary School (Sudbury) and O'Neill Secondary School (Oshawa).

FATHER FAUGHT AWARD
This year's winner is Woburn Collegiate (Scarborough).

LIFE MEMBERSHIP
Life membership qualifications were described in March, 1985 issue of the **Ontario Mathematics Gazette.** The following have been appointed members of the Life Membership Committee: Dave Davidson, Tom Griffiths and Joan Routledge.

COMMITTEE REPORTS

Publications Committee
A copyright statement has been formulated for the **Gazette** and **Abacus.** We are actively seeking articles for publication. If you wish to contribute, contact:

Dr. Ralph Connelly, Editor
Ontario Mathematics Gazette
College of Education - Brock University
St. Catharines, Ontario L2S 3A1

Membership Committee
The membership committee is anxious to increase membership in OAME. If you have any ideas,

send your suggestions to:

Robin Hill
Chairman - Membership Committee

Formative Years Committee
This committee will continue to work in the anticipation that the Formative Years may be revised in the not-too-distant future.

Computer Committee
Last year's survey continues to be compiled and analysed. The committee will attempt to find out what software teachers find useful and are using successfully.

Problem Solving Committee
This committee will locate problem solving materials used in Ontario and evaluate them in order to recommend those that are worthwhile.

Women and Mathematics Committee
The work of this committee will focus on solutions to the existing problems.

FUTURE DATES

Executive
September 6, 1985
November 22, 1985
April 4, 1986

Council
September 28, 1985
February 8, 1986



PROBLEMS

This month's problems all deal with a topic dear to many of us - **MONEY!** I would suggest that you use real money in working out the solutions and that you feel free to send in those real-money models of your solutions!

1. A vending machine takes any combination of pennies, nickels, and dimes. How many ways can you pay for a 35 cent candy bar? If the machine also took quarters, how many ways would there be?

2. Michael received his $1.60 allowance in quarters, dimes, and nickels. He got 17 coins. How many of each did he get?

3. A man bets $30 and gets back his original bet plus $90. He spends one-sixth of his winnings on food, and one-third of his winnings on travelling. If he started out with $200 and spends nothing else, how much will he take home?

4. An "oldie" but still good. I have 55 cents in **two** coins. One of them is **not** a half-dollar. What are the two coins?

Answers are on page 43

WHAT'S GOING ON...

This column is for you the readers to tell the membership about some exciting math events in your Board

The following two events have been submitted by the President of O.A.M.E. (Paul Lessard) as two examples of unique math activities recently initiated by the Trustees of the Hamilton-Wentworth Roman Catholic Separate School Board and the teachers of the Hamilton Unit of the Ontario English Catholic Teachers Association.

A. Junior Math Challenge. & B. Intermediate Math Challenge.

JUNIOR MATH CHALLENGE

(A Team is composed of two students from each of Gr. 4, Gr. 5 and Gr. 6)

This second annual Junior Math Challenge was an overwhelming success. Thirty-one schools registered their **6 member teams** to participate in this novel math event. This resulted in 186 students supported by their parents and teachers, to converge on St. Eugene's School on Saturday, February 16, 1985 as early as 8:15 a.m.

The organizing committee consisted of representatives of, the Curriculum and Planning Department of the Board, the Local Unit of O.E.C.T.A. and Teachers selected jointly by the Mathematics Consultant and an Executive Member of O.E.C.T.A. This committee did an excellent job of developing and delivering this education opportunity to junior division students in our system.

The day was opened by Paul Lessard the Board's Mathematics Consultant. Paul's main task was to help the contestants start, "Thinking Math" through non-threatening highly motivating co-operative problem solving activities.

At this point each team was ushered to their first challenge site by intermediate students from St. Eugene's School. Students were greeted at each challenge site by a teacher(s) responsible for the presentation and supervision of the challenge and collection of the answers. The answers were marked against the answer key by grade eight students who then placed the results in an envelope and delivered them to the challenge desk. The challenge desk was managed by three teachers. The contestants were then ushered to the next challenge site by the same intermediate students. Each team was cycled through 4 "Challenge Sites" before lunch and 2 after lunch.

The Challenge Sites were as follows:
(1) Mental Math (3) Estimation/Calculators (5) Puzzle Corner

(2) Measurement (4) Sequences (Patterns) (6) Problem Solving

These "Challenge Sites" were selected by the Committee to emphasize the issues of mathematics education in the 80's. The committee was responding to the notions of accuracy, understanding, and logical though while continuing to emphasize the importance of a well balanced math program.

The "Puzzle Corner" was a highly motivating feature of this Junior Math Challenge. This challenge site was really a "Challenge Penny Sale". Students placed their answers to puzzles in shoe boxes and glass jars, etc. These puzzles required quessing, estimating, predicting and calculating. There was a prize table at this site that further established this "Penny Sale" flare. The Valentine's theme in evidence at this site resulted in Sacred Heart School winning the Valentine's candies.

The honour of this day went to each team member as they received a Junior Math Challenge crest in recognition for their enthusiastic participation. Three teams were furthered recognized for their special achievement.

First Place ..... St. Ann's/Father Loftus

Second Place ..... St. Francis Xavier

Third Place ..... St. Anthony's

Each member of each of these three teams received a small trophy as a memento of their achievement. These three schools received a penant to remain in the school and a large trophy to reside at the school for one full year as further acknowledgements of their teams accomplishments at this educational event.

This year, we expanded our Junior Math Challenge Day by including a competition for the best math exhibit or math display. Trophies were awarded for the best 3 exhibits.

First Prize ..... St. Anthony's Gr. 4, Gr. 5 for Math Shapes

Second Prize ..... St. Margaret Mary's Gr. 6 for Math Poster

Third Prize ..... St. Cecilia's Gr. 5 for Tessellations

The committe is most grateful to the 45 teachers and the 15-20 principals who came out to support this Junior Division Activity. The parental support that was in evidence throughout the day surely will be long remembered by youngsters **who wrote six,** twenty minute tests to complete their challenge.

INTERMEDIATE MATH CHALLENGE

(Our Board uses the well established **GAUSS CONTEST** as their **Intermediate Math Challenge**).

Thirty of our schools registered over 1200 Grade 7/8 students to write this challenging contest on June 11, 1985. The efforts and interest of all participants were honoured as each participating teacher presented the contest writers with the University of Waterloo Participation Certificate.

The top Grade 7 and top three Grade 8 students in each school have received a Sr. Math Contest Crest for their special achievement in this challenging math event. A sample problem from the twenty-five problems, which competitors had sixty minutes to solve, is question 11;

11. If the average of the five numbers 4.9, 3.1, 7.7, k and 9.3 is 5.1, then k equals

(a) 0.5 (b) 4.6 (c) 5.0 (d) 5.1 (e) 6.25

SOLUTION: The average is $\frac{4.9 + 3.1 + 7.7 + k + 9.3}{5} = 5.1$

$$\frac{25 + k}{5} = 5.1$$

$$25 + k = 25.5$$

$$k = 0.5$$

The top Grade 7 student and top Grade 8 student in each participating school were further recognized for their unique achievement in this contest by being placed on our system honour roll.

A Sample Honour Roll is enclosed for your consideration.

GRADE EIGHT

INTERMEDIATE CHALLENGE HONOUR ROLL

The GRADE EIGHT students listed in this honour roll achieved the highest score on the GAUSS CONTEST in their school.

Christ the King - Paul Silva
Corpus Christi - Robert Cassiani
Father Loftus - Anjie Bellini
Immaculate Heart - Julie Maggi
St. James the Apostle - Tanya Jablonski
St. Jerome - Peter Ushold
St. John the Baptist - Bradley Kelly
St. Joseph - Danielle Heathfield

We are happy to announce that **JOSEPH BIELLO** of **ST. TERESA OF AVILA SCHOOL** is the first student to write a perfect paper in our System Math Challenge.

For complete information on the Gauss Contest write to:

The Candian Mathematics Competition
Faculty of Mathematics
University of Waterloo
Waterloo, Ontario
N2L 3G1

/cs

Paul Lessard
Mathematics Consultant

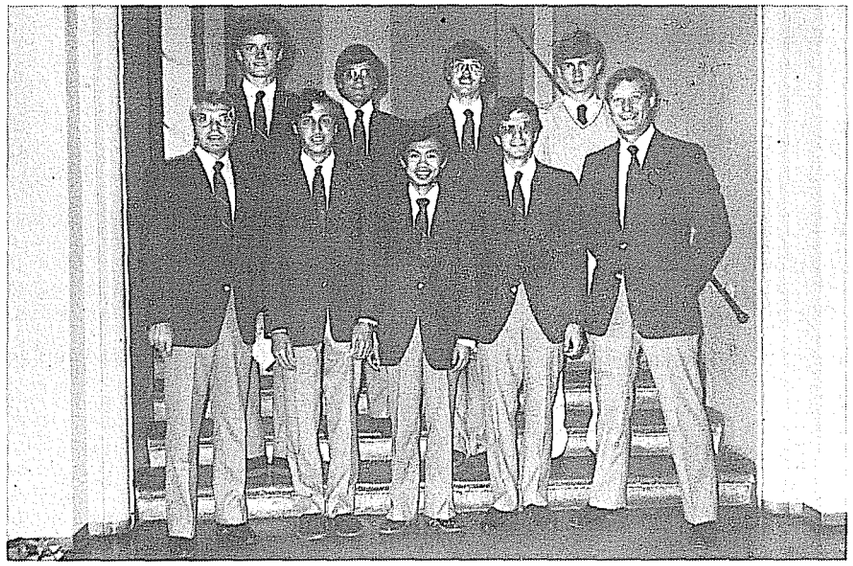
ANSWERS TO THIS MONTH'S PROBLEMS

1. With pennies, nickels, and dimes, there are 20 ways. With quarters as well, there are 24 ways.

2. 2 quarters, 7 dimes, 8 nickels. **NOTE:** the question **implies** that at least one of each coin is given. Otherwise, 15 dimes and 2 nickels would also be a solution.

3. $245

4. A half-dollar and a nickel. (The nicket is **not** a half-dollar.)



1985 INTERNATIONAL MATHEMATICS OLYMPIAD
JOUTSA FINLAND

This year the twenty sixth International Mathematics Olympiad was held at a resort complex in Joutsa Finland. Joutsa is a small town two hundred kilometres north of Helsinki. The jury met at Heinola, one hundred kilometres north of Holsinki, and the awards ceremony took place at the University of Helsinki.

There were 38 countries represented this year, and 209 competitors, the largest number ever in both cases. A full team consisted of six competitors.

This year for the first time the Canadian team trained at the University of Waterloo for two weeks before departing for Finland. Involved in the training session were also prespective candidates for the 1986 I.M.O. to be held in Poland. Many thanks are due to the many members of the University of Waterloo Mathematics department who assisted in the training of the team. Special thanks are due to Prof. R. Dunkley who was director of the training session. Much appreciated also was the team uniform, and other financial assistance provided by the Canadian Mathematics Competition Foundation. Only the Australian team and ourselves wore team uniforms, and very positive comments were made by many people on the appearance of both teams. All travel expenses for the team were paid by the Canadian Mathematics Society, for which we express our gratitude.

The team members were chosen by considering the results of various contests including the Euclid, the Descartes, the Canadian Olympiad, and the U.S. Olympiad, which several of our team were invited to write. For two of our team, Minh Tue Vo and Frank D'Ippolito, this was their second I.M.O. and for one, Martin Piotte, his third. Regretfully we once more had no girls on the team.

The Canadian team finished in twelfth position this year, which is our best showing since the first time we entered a team at the Washington Olympics in 1981. In which we placed seventh. The team also won one silver and four bronze medals, which was a most commendable showing. Medals are awarded as follows, the top twelfth of the contestants receive gold medals, the next sixth silver, and the next quarter bronze.

The team members were:-

Frank D'Ippolito	Sudbury, Ontario	Bronze medal
Moses Klein	Toronto, Ontario	Bronze medal

Martin Piotte	Montreal, Quebec	Bronze medal
Guisseppe Russo	Sudbury, Ontario	
Eric Veach	Sarnia, Ontario	Bronze medal
Minh Tue Vo	Montreal, Quebec	Silver medal

The leaders were:-

Prof. R. Scoins	University of Waterloo
Tom Griffiths	London

The order of placement of the first fifteen teams was as follows:-
Romania, U.S.A., Hungary, Bulgaria, Vietnam, Soviet Union, East Germany, West Germany, France, Great Britain, Australia, Canada and Czechoslovakia tied, Poland, Brazil, Israel.

Students from Romania and Hungary achieved perfect scores on the extremely difficult paper.

The question paper is as follows:-

FIRST DAY
Joutsa - July 4, 1985

1. A circle has centre on the side AB of the cyclic quadrilateral ABCD. The other three sides are tangent to the circle. Prove that AD + BC = AB.

2. Let n and k be given relatively prime natural numbers, 0 k n. East number in the set M = 1, 2, ..., n-1 is coloured either blue or white. It is given that
   (i) for each i M, both i and n-i have the same colour, and
   (ii) for each i M, i k, both i and i - k have the same colour.
   Prove that all numbers in M must have the same colour.

3. For any polynomial $P(x) = a_0 + a_1x + \ldots + a_kx^k$ with integer coefficients, the number of coefficients which are odd is denoted by w(P). For $i = 0, 1, 2, \ldots$ let $Q_i(x) = (1 + x)^i$. Prove that if $i_1, i_2, \ldots, i_n$ are integers such that 0 i_1 i_2 ... i_n, then

$w(q_{i_1} + Q_{i_2} + \ldots + Q_{i_n})$ $w(Q_{i_1})$.

Time allowed: 4½ hours
Each problem is worth 7 points.

SECOND DAY
Joutsa - July 5, 1985

4. Given a set M of 1985 distinct positive integers, none of which has a prime divisor greater than 26. Prove that M contains at least one subset of four distinct elements whose product is the fourth power of an integer.

5. A circle with centre 0 passes through the vertices A and C of triangle ABC, and intersects the segments AB and BC again at distinct points K and N, respectively. The circumscribed circles of the triangles ABC and KBN intersect at exactly two distinct points B and M. Prove that angle OMB is a right angle.

6. For every real number x_1, construct the sequence $x_1, x_2, \ldots$ by setting $x_{n+} = x_n \times (x_n + 1/n)$ for each n 1. Prove that there exists exactly one value of x_1 for which 0 x_n $x_{n+\$}$ 1 for every n.

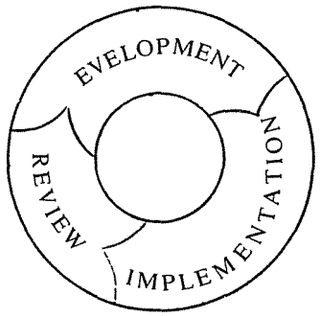
Time allowed: 4½ hours
Each problem is worth 7 points.

Life membership qualifications were described in March, 1985 issue of the **Ontario Mathematics Gazette.** The following have been appointed members of the Life Membership Committee: Dave Davidson, Tom Griffiths and Joan Routledge.

OPEN LETTER

Thoughts on the Implementation Process: A Response to "Play It Again, Sam - A Theme from the Trenches"

The Ministry of Education in Ontario approaches curriculum change as a three-fold cyclical process of develpment, implementation, and review.



(Diagram on the cover of **A Guide to Implementation: Suggested Planning and Operating Procedures for the Ministry of Education.)**

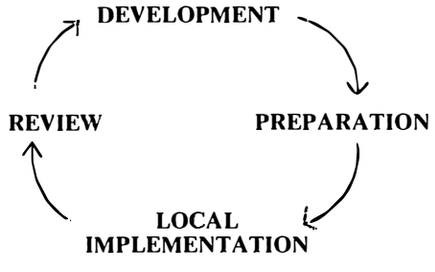
It is clear that the actual implementation of a new or revised curriculum takes place in classrooms and is the primary responsibility of local teachers. We are the ones who do the final interpreting and translating of the official curriculum into meaningful activities, experiences and exercises for students. But before any change or implementation actually takes place in the classroom, there is a crucial period of planning and activity that is a very significant part of the whole curriculum change process. This period between the completion of the development of the new guidelines and the actual implementation of those guidelines in classrooms across the province is a time of preparation for classroom implementation.

At least two different types of activity take place during this phase of the process.

(1) The Ministry of Education faces a province-wide communications task, aimed at enabling thousands of teachers to gain a deep understanding of the new curriculum.

(2) Every classroom teacher works on translating and fashioning this curriculum into appropriate activities and lessons for the students.

It is clear that this preparatory time is a vital part of the whole implementation process, but is also a distinct and separate phase. It marks a transition period, where the primary responsibility for the curriculum change process passess from the provincial level (Development) to the local level (Classroom Implementation). It is a complex interactive phase, since all levels of education (provincial, regional, local) are involved in different ways in this preparatory task. It might be advantageous to adapt the official curriculum change model to incorporate this key phase as a separate stage in the process.



This subdivision stresses both the importance of the Preparation period, and the differences with the Local Implementation phase. It forces educators at all levels to pay very careful attention to the distinctive nature and tasks of each period.

What's Really Changed in the New Curriculum?

I would like to focus on the first type of activity that takes place in the Preparation period - the communications task. The stated aims of the Ministry of Education are:

> (a) to disseminate the policy statements...
> (b) to create an awareness and understanding... of the change in beliefs, teaching approaches, and materials incorporated in the new policy;
> (c) to stimulate and facilitate implementation of the policy at the local level;
>
> (**A Guide to Implementation**, 1982, p. 6)

What is described here is not a simple transfer-of-information task, but a delicate communications and understandings task involving values, meanings, and clarifications, as well as information. It is the Ministry's responsibility to help teachers understand and appreciate the developers' intentions, underlying assumptions, and major changes incorporated in the new curriculum. What is required is not a one-way proclamation or declaration, but an interactive dialogical process.

Let us look at this interaction in the light of the current changes. One way in which the curriculum developers can encourage and stimulate such a teacher-learning process is by describing and clarifying the significant changes in beliefs, teaching approaches, and materials incorporated in the new curriculum. After studying the draft guidelines and participating on a board validation committee, I am still unsure of the exact nature and extent of the changes proposed. The content descriptions of the courses tend to give the impression that we're mainly shuffling the same old pieces around. However, process sections like Applications and Problem Solving seem to suggest a changing style and methodology for teaching mathematics. Please, someone, spell out as clearly as you can, the nature and significance of the proposed changes, as you see them.

Only then can we teachers in the trenches respond, react, reflect, and retort. Only then can we involve ourselves in really wrestling with the heart and guts of the new (or is it a revised) curriculum. Don't just pass a written document to us, but enable, encourage, and challenge us to get at the meanings and assumptions that are behind, under, and around the printed words. A deep understanding and feeling for the given curriculum is a necessary first step in preparing for classroom implementation. Let's take time to think, question, listen, reflect, and talk about the new curriculum - and not just rush to put it superficially into our classrooms.

Roy Baumgart
Math teacher
Roland Michener Secondary School
Timmins, Ontario

Reference:

Ontario Ministry of Education. **A Guide to Implementation: Suggested Planning and Operating Procedures for the Ministry of Education.** Toronto, 1982.