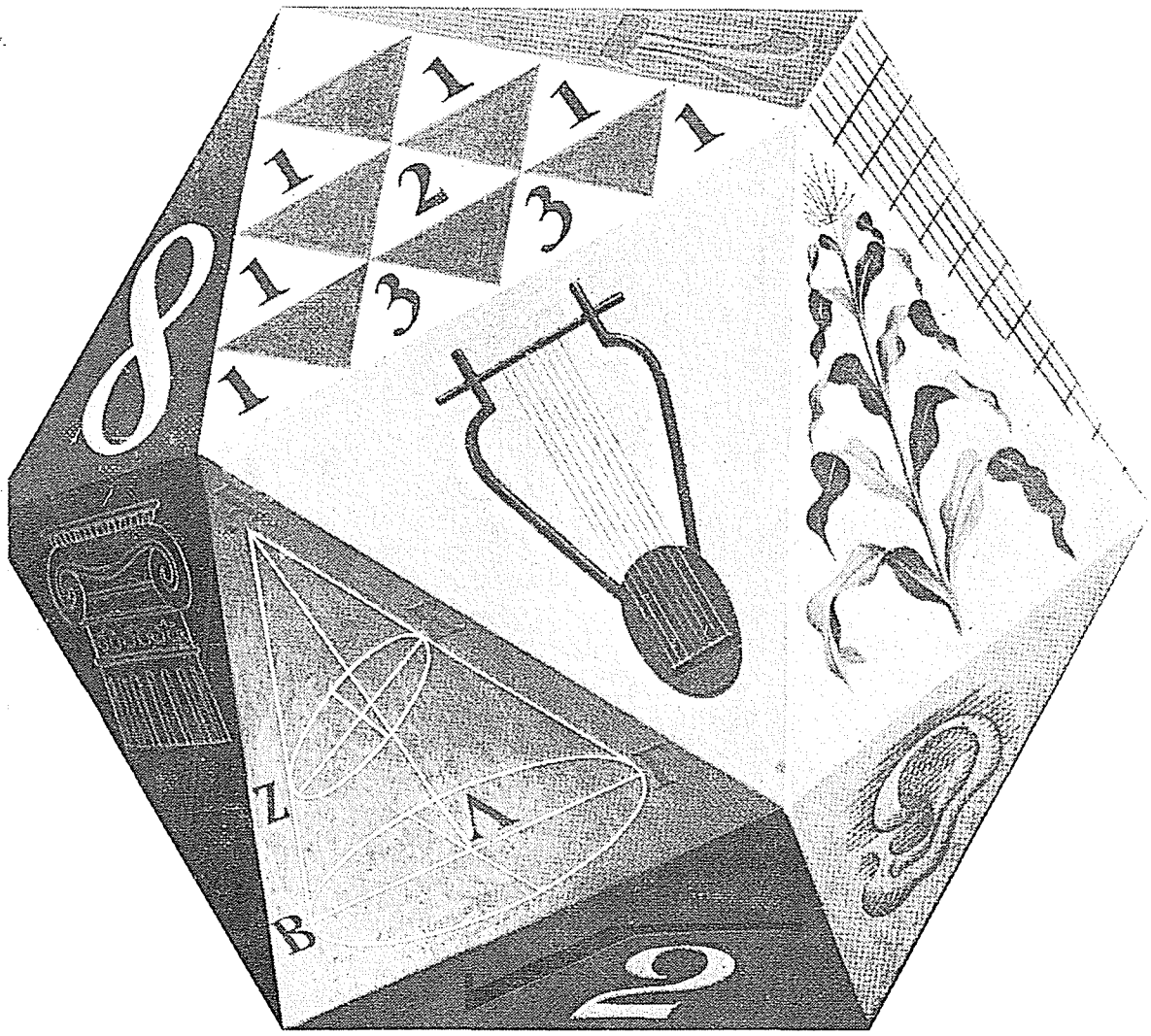
# ontario mathematics gazette

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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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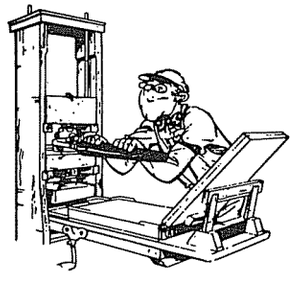
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Thought it might be appropriate to use this space this month to give recognition to some "special contributors" to the **Gazette.**

This summer I taught a graduate course in mathematics education. I had traditionally assigned a "research" paper as a major assignment in the course, but took quite a different approach this time. Since the students were all practicing teachers, many with long years of teaching experience, I thought perhaps that wealth of experience should be tapped. Many had even observed to me that frequently research articles are not written in a way that communicates effectively with practitioners about classroom implications of the studies, etc. It seemed to me that being able to write in such a way as to communicate clearly and concisely information and ideas that could be used by other teachers was an important skill for teachers to develop. Thus was born my new "major assignment" for the course. Prepare an **article** to be considered for publication in the **Gazette.** Whether finally published or not, the experience of having to "cut through it all" and produce a communicative final product had to be a valuable one. Oh sure, some had the audacity to suggest that **perhaps** I had a small vested interest in seeing articles produced for the **Gazette,** but I knew my reasoning was pedagogically sound and proceeded with my plan. Several of the articles in this month's issue, as well as some in the Sept. issue and still others to appear in March, are the results of that assignment. To my students who tolerated my experiment (with what I consider to be quite excellent results), a sincere thank you. To others who might be in a position to try a similar assignment, I encourage you to do so.

You might also have noticed that the first aim listed for OAME for 1985/86 is to encourage executive and council to contribute on a consistent basis to our publications. President Paul Lessard has responded well to this aim, with the creation of the "What's Going On" department and also contributing an article this month. Trevor Thomson had an article in before the ink was dry on the minutes listing that objective. Hopefully other councillors and executive will follow suit.

Happy Holidays.

Ralph Connelly
Sam Pravica
Editors

# TO CALCULATE OR NOT TO CALCULATE

*by*
*Mary Zeffiro*

As stated by the N.C.T.M. in An Agenda for Action, "Mathematics programs must take full advantage of the power of calculators and computers at all grade levels".

It seems a shame that this recommendation is only partly adhered to. With all the talk centering and focusing on having computers in the classroom, it is sad to see that few classrooms possess or make frequent use of the calculator.

The sad part rests in the fact that this tool is a wonderful device which is very inexpensive and above all takes the drudgery away from performing paper and pencil computations that would take much longer to perform.

It would be like using a foot-pedaled sewing machine in place of an electric one. What for?

Could the reason for not using it be that too many students and teachers still do not yet have an adequate knowledge of its use? Or could it be for fear that students would not learn their Mathematical concepts?

The truth of the matter is that if we, as teachers, are to do an adequate job in preparing the students for the adult world and the technological age of the future where the calculator plays an integral part; then we should start using this tool in schools as soon as the students have mastered the Mathematical concepts of the basic operations.

Has an adult ever calculated his Income Tax return with just a pencil and paper since the introduction of the calculator?

So with that idea in mind, then let us ensure that our students have access to this indispensable tool and try to find further ways to use it.

There should not be fear in its use and we should remember that it is a tool. The person using it definitely has to know what is being done. It is the person in junction with the calculator that arrives at the answer. This tool stresses the "what" is being done and not the "how" to do it. It allows the students the possibility to concentrate on the application of strategies learned without having the frustration of trying to concentrate on the "mechanics" of the problem.

We as teachers need to show how and when it can be used. The students must be given a positive attitude about this tool as well as the understanding that in many cases it is indeed the best means to obtain a solution.

Needless to say something that can be computed mentally should be much encouraged. However, often the student knows the theory and why he is applying it, but is negligent and makes computational errors. In this case the calculator would have been the answer because he would have had both satisfaction and success in the results.

As long as the student understands the concepts, then the calculator can and should be used because it is the understanding and internalization of these concepts that form the important basis for using this tool.

For example trying to calculate the result of $7^8$ with paper and pencil is time wasted because a working knowledge and use of the calculator, along with the theoretical concept of the problem, will provide an answer very quickly.
The time required to do the calculations with pencil, could be put to better use for more efficient learning of problem solving strategies.

Instead of giving the students 20 multiplicaon questions that would require 40 minutes to complete with little guarantee of total correct answers; let the class use the calculator presuming there is an understanding of the concept. The rest of the time could be used for more challenging work.

We know children use calculators at home, so why deprive them at school by taking a step

backward and insisting that all computations be done with paper and pencil?

It is important to keep in mind that the use of the calculator allows for the answer to be arrived at with fewer errors and as a result there is more success and eagerness displayed on the part of the student. These in turn are the catalyst for the increased and continuing performance and mathematical thinking of the students. The most wonderful part of it all is that calculators can be used in many different ways.

Estimating and problem solving are just two examples.

It is understood that in future because people will have to judge the reasonableness of computer and calculator output, estimation skills will have to be quick and sharp.

True, calculators will give the correct answers provided all the correct keys are selected. Thus the successful student must be able to judge whether the output is indeed reasonable. When a student is able to recognize that the output is wrong he or she is showing recognition that there has been an error in the processing of the problem. Students could be given practice in estimating and calculating.

The following could be a type of activity to provide the students with this kind of practice. Of course, all the operations could be used and as soon as students show improvement, then the problems can be changed to reflect what they are capable of handling.

eg.

| OPERATION | GUESS | ACTUAL | DIFFERENCE |
|---|---|---|---|
| 37 x 25 | | | |
| 513 - 19 | | | |
| 3.1 x 2.4 | | | |

Always we should encourage students to check the answer given by the calculator and to think whether it is indeed a realistic one. What is also invaluable about the calculator is that everyone can use it and by estimating and then checking, students will be more willing and eager to make the guess.

With regard to problem solving, once the student knows how to perform the basic operations, he can concentrate on the application of the proper strategy and let the calculator work out the mechanics. In this way students can take real world problems involving large numbers without the added concern of computational error. When it comes to solving the problem, the student can "write a calculator code" to find the solution so the tool then becomes part of the thought process.

The following is an example of such questions.

eg. Write a calculator code that will solve each problem. Check your code with the answer.

1. One airplane carries 96 people. How many people can 7 airplanes like it carry?
   Code: _____
   Answer: 672 people

Easy problems are not challenging nor rewarding in self-satisfaction and too difficult ones prove frustrating. So it is important to include a variety of problems that will account for all students' capabilities and interests.

The calculator is a very useful tool that will allow the student's application of knowledge in real life solution of problems with fast and accurate results. We are teachers must enforce and reinforce its use.

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# THE CALCULATOR COMPONENT
## - Calculators in your Elementary Math Program -

*by*
*Paul Lessard*

Calculator statements in the Draft Guidelines (Mathematics, Intermediate and Senior 1985) require us to review the nature of this technology within the context of our mathematics curriculum. Statements such as "The calculator has become an integral part of our way of life" or "Schools should ensure that students become proficient and discerning in the use of calculators." (Mathematics, Intermediate, Senior Division, Draft Pg. 46), encourage us to develop, implement and evaluate those calculator opportunities that will help our students in the efficient use of this educational technology.

The calculator component in a mathematics program should focus on several major goals. These goals should be highlighted under the umbrella of, "The Efficient and Proper Use" of this educational technology. This "Efficient Use" notion can be developed through several organizers.

| **ORGANIZERS** | **SKILLS AND CONTENT** |
|---|---|
| Formating | Selecting, creating and deciding the most efficient calculator messages. |
| Problem Solving | Applying calculator messages to simple problems. |
| | Applying calculator messages to multi-step problems. (Memory) |
| | Notion of estimation, checks and balances. (verify) |
| | Unique calculator problems. |
| | Creative Problem Solving. |
| Computational Math | Estimation, checks and balances. (verify) |
| | Clear key. |
| Investigating Patterns | Identifying, describing, utilizing. |
| Mental Math | Developing strategies and using the calculator to check accuracy. |

These organizers reinforce the present curricula, they extend it when appropriate and they link it in a very natural way to numerous real world applications. Lastly, this calculator curriculum should be designed to help students who live in a world of "mega" numbers make better sense of their world through the manipulation, understanding and application of number.

The calculator curriculum as a central component of a mathematics program can be featured in various ways:

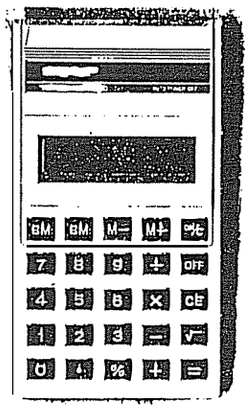
1. Dedicated opportunities through a calculator studies orientation.
2. Parallelled opportunities through reinforcement and enrichment activities.
3. Frequent "Challenges" that capitalize on this technology.
4. Project/unique investigations that use the technology to stretch the curriculum.
5. Cumulatives and reviews that stress a balanced approach to mathematics education.

This calculator direction requires creative resources that involve our students, in a very natural way, in intense activities that utilize and stretch their math senses.

**CALCULATOR TRIGGERS**
**in**
**- Predicting and Determining Patterns -**

1. Calculate with your calculator and continue the pattern for four more lines.

9 x 1 -=1 8
9 x 21 -=1 188
9 x 321 -=1 2 888
9 x 4321 -=1 38 888



CREATIVE
RESOURCES

2. Use your calculator to predict 111 111 x 111 111 = _____

1 x 1 =
11 x 11 =
111 x 111 =
1111 x 1111 =

3. (a) Use your calculator to decide the answers.

(i) $\frac{1\,2\,1}{1\,1}$

(ii) $\frac{1\,2\,3\,2\,1}{1\,1\,1}$

(iii) $\frac{1\,2\,3\,4\,3\,2\,1}{1\,1\,1\,1}$

(b) Predict the answer to this question

$$\frac{1\,2\,3\,4\,5\,4\,3\,2\,1}{1\,1\,1\,1\,1}$$

4. Determine the pattern and continue with three more lines.

81 x 9 =
882 x 9 =
8883 x 9 =

5. The number 37037 gives some interesting results when you multiply it by the multiples of 3 from 3 through 27. Use your calculator and your predictions to record the results.

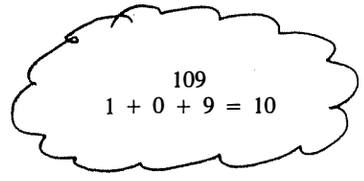
6. An interesting pattern occurs when the following calculations are made. Use you calculator for the first three lines and predict the remaining lines.

9 x 9 + 7 =
98 x 9 + 6 =
987 x 9 + 5 =
9 876 x 9 + 4 =
987 654 x 9 + 2 =
9 876 543 x 9 + 1 =
98 765 432 x 9 + 0 =

## CALCULATOR TRIGGERS
## in
## - Flowcharting, Planning and Co-operative Problem Solving -

1. How many hours have you been alive?

2. (a) How many numbers between 100 and 200 have digits that sum to 10?



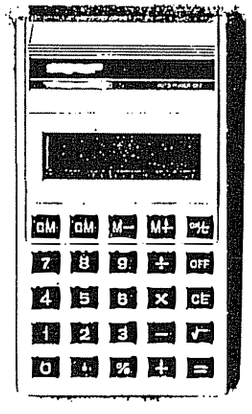

2. (b) Make a chart of these numbers and identify and describe as many patterns as you can from this chart. (Many students make several copies of their chart to help them describe their patterns).

3. Martin is 9 years old and his father is 37 years old. When will his father be double his age.

   Duelling Calculators
   Work in pairs. Use your calculators to count by 1's. Compare displays to decide when the father is exactly double the son's age.

## CALCULATOR TRIGGERS
## in
## - Multi-step Problems -

The Parent and Teacher Association of St. Columba School purchased the following computer equipment:



EFFICIENT
USERS

| QUANTITY | ITEM | COST/ITEM |
|---|---|---|
| 1 | Modem | $125.50 |
| 5 | Keyboards | $350.00 |
| 3 | Disk Drives | $250.00 |
| 1 | Color Monitor | $350.00 |
| 4 | Black & White Monitors | $125.95 |
| 1 | Softward Pkg. | $125.95 |
| 5 | Organizers | $ 49.50 |

Calculate the total cost of these purchases including provincial sales tax.

Plan your calculator work.

1. Which zeros are important?
2. Use your memory key.
3. Use your % key.
4. Do not forget the decimal point.
5. Estimate your answer.

**CALCULATOR TRIGGERS**
**in**
**- Verifying and/or Generating Questions -**

Use your calculator constants to complete the following grids.

(a)

| + | $65.50 |
|---|---|
| $ 15.45 | |
| $125.50 | |
| $ 49.95 | |
| $ 79.49 | |

(b)

| x | 46 | 50 | 16 | 18 | 92 | 903 |
|---|---|---|---|---|---|---|
| 72 | | | | | | |

Many students will multiply by 1 part way through this grid to verify their multiplication constant.

(c)

| ÷ | 2744 | 196 | 38416 | 210 |
|---|---|---|---|---|
| 14 | | | | |

The benefits of calculator learning activities are reflected most vividly through the generous range of explorations that have become a reality today through this hand-held calculator.

*Paul Lessard is the Mathematics Consultant with the Hamilton-Wentworth Roman Catholic Separate School Board and President of the Ontario Association For Mathematics Education.*

# THINKING ABOUT MATH - A "NEW LOOK" FOR PRIMARY CURRICULUM

*by*
*Merle Richards*

In its recent publication, **Shared Discovery,** the Ontario Ministry of Education (1985) confirms its support of an active, inquiry-based approach to the primary curriculum. This is consistent with the Piagetian view of learning, with its emphasis on the child's construction of knowledge (p. 5). In this view, children are thought to develop their concepts through a long process of testing their environment and reflecting upon their experiences. In particular, logico-mathematical thought is considered to result from experience with physical objects, time and space. The child does not "Take in" ideas from others, or make an internal copy of an experience. Rather, he selects from his experiences to build upon concepts already formed, modifying those concepts when they are inconsistent with new experiences. Thus, children acquire ideas about quantity, size, and space along before they come to school, largely through their spontaneous play experiences.

The implication of Piagetian theory, then, is that the basis for logico-mathematical thought lies in a rich background of activities within which the child acts upon his environment. Manipulative materials, physical movement and creative play are considered to provide the foundation for an understanding of relationships and hence of logico-mathematical thought.

A further implication is that early mathematical concepts cannot be taught. Indeed, Piaget (1976) suggested that traditional lessons often inhibit the development of mathematical knowledge. The teacher's role is therefore not to present the specific facts and skills to be acquired but rather to provide the learning environment and the experiences within which they can be discovered, mastered, and applied.

**Classroom Behaviour Settings**

In order to assess the characteristics of the classroom environment, a study was conducted based on the ecological model of Barker (1968) and Gump (1967). They describe the classroom as a "behaviour setting", with many components each influencing the members' behaviour. Each component can be analysed as a discrete segment with its own format, time, space, materials, and participants. These vary according to subject area (Stodolsky, 1984) and grade level; primary classrooms especially show a great variety of activity segments, often with several operating simultaneously.

This is due to the primary teachers' concern with individual pupils, and the need to maintain interest and progress in pupils of varying abilities. Moreover, the value of play is widely accepted as part of a developmental approach to learning, and many teachers try to include play activities in the daily curriculum.

**Method**

Ten grade-one classes were each observed for four or five non-consecutive days. Trained observers drew floor plans of each classroom and described all behaviour settings and activity segments. They also kept anecdotal records of observed incidents and activities.

**Findings**

The observations showed marked differences across subject areas. In all classes, language arts activities were more varied and numerous than mathematics activities, involving more types of materials and formats ranging from dramatic play to storytime to traditional reading lessons. Moreover, language segments occupied 40% to 60% of class time, as opposed to 10% to 20% for mathematics.

The most frequent behaviour settings for mathematics were teacher-led demonstration and drills, and seatwork assigned by the teacher. Five teachers taught math to the whole class, two used ability groups and three had individualized programs. Of the last, one used a self-paced workbook, one used individual contracts to permit some child choice, and the third provided mathematics activities for pupils to choose. Only the last scheduled no instructional time for mathematics, and had no teacher-led math lessons or specific evaluation of math knowledge.

For most of the teachers, "mathematics" implied "number work." The most frequent use of concrete materials was for counting, adding and subtracting. Children arranged numerals in sequence or used counters or Cuisenaire rods to do simple calculations. The tasks were quite tradi-

tional; they were usually directed by the teachers, either orally while supervising a group, or through worksheets or cards to be used by children at their own pace. Except for some imaginative measurement activity and a pretend "store", mathematics usually involved the use of numeric symbols and calculations. Worksheets and workbooks were the most frequent materials, although in most classes, the children were allowed to use counters when needed.

However, although the teachers acknowledged that many play activities can contribute to the development of mathematics concepts, few were adopted for this purpose. Math was viewed as work, not playful activity. Four classes had math centres availalbe at free time; only two of these included exploratory materials such as balance scales and timers as well as number cards and lines. Even though blocks and construction toys were usually available, they were not used to develop math concepts. (They were viewed mainly as "creative" materials, although one teacher used beads for sequencing exercises.) On the whole, the teachers did not apparently view math as creative, but as methods or operations for combining numbers. Evaluation was traditional, based on daily work, and in five classes, on teacher-made tests.

Only two teachers frequently used creative activities to further mathematical thinking. One of these used contracts, with children choosing activities from several provided, often working in pairs. The activities were teacher-structured, with "challenge cards" providing problems or questions (e.g. "How long does it take to jump 10 times?") This teacher had a permanent math centre with themes changing every few weeks (e.g. weight, time, etc.); it was the only math centre frequently chosen during activity time.

Some teachers did, however, make use of incidental learning to further math concepts. Behavioural routines such as lining up, disributing supplies, marking the calendar, or voting provided opportunities for "real life" arithmetic. Two teachers in particular used many group sessions in this way.

Mathematical ideas can be incorporated into science activities. However, this was rare in the present sample. Eight of the ten classes had no permanent interest centres devoted to science, and in six no science-related activities were observed. In the others, animals or plants were in the classroom, but only seed-planting provided a planned focus for observation, and no measurements were taken of the plants' growth. Objects like mirrors, lenses or prisms, which encourage math and science ideas, were not observed in any of the classrooms. If creative math was rare, creative science was non-existent in these classes. Problems of quantity, magnitude and physical properties, the basis of much mathematical and scientific thought, were either not valued by the teachers, or not considered suitable for young children. In fact, the concern for reading achievement diverted teacher attention from these areas, which could themselves have been the basis for many language activities.

One teacher, as mentioned, had no specific mathematics program in her curriculum. Aware of the Piagetian view that play is essential for the development of logico-mathematical thought, she was confident that the many creative and constructive activities provided for her pupils would lead them to the mathematical concepts usually outlined in the primary syllabus. However, her observations were not systematic enought to ascertain the level of understanding achieved by the individual pupils or to evaluate their conceptual development. Furthermore, her reliance on self-directed activity and verbal interchanges among the children reduced the opportunity for the child-adult exchanges that can enrich developing concepts (Feuerstein, 1980).

**Discussion**

Although one cannot generalize from one case, this example is instructive. This teacher had achieved a firm understanding of the Piagetian principles of active learning and self-structured knowledge. She accepted play as the basis for logico-mathematical thought. Of all the teachers in the sample, she alone refrained from verbal teaching of mathematical concepts and the use of abstract divorced from children's experience.

However, it was not evident that all her pupils actually were developing mathematical concepts. Not all the children chose activities involving constructions, relations or quantity. And the non-interventionist approach did not provide them with a vocabulary for discussing and comparing mathematical ideas, or help to focus attention on such ideas.

Thus, another element must be included in the "Piagetian" approach. Although Piaget scorned "verbal teaching" and did not consider language essential for the development of logico-mathematical thought (Furth, 1981), it remains an important organizer and aid to conceptualization.

An appropriate vocabulary helps children to understand relationships (e.g. thick, thin; equal; part; less or fewer) and quantities (much, little; many, few; one, three...). Sometimes a word can serve as the "anchor" for a concept (Vygotsky, 1962). Thus, the word "square", first simply a label for a familiar shape, later serves as a cue for recognizing or constructing similar figures and eventually for more complex operations. ("How many squares do need to make another square? What's the smallest square you can make? How do you know?) Attached to activity, language helps to develop and organize concepts, and is a necessary support for mathematical thinking. Neither activity nor language alone can provide adequate support for early learning.

**Conclusions**

One can only conclude that the nature of mathematical thinking and development in children is not well understood by teachers. In both this sample and another of 18 grade ones, it was evident that little time was devoted to exploring and testing ideas of quantity, and that mathematical symbols were often unrelated to the concepts they represent. A right answer was treated as evidence of comprehension; if children obtained correct answers, their thinking was not questioned. If they did not, the teacher usually explained the error, but in several cases, observers noted that the explanations did not result in comprehension by the pupil. Again, this appeared to be a consequence of the belief that mathematics is a highly-controlled system in which right answers imply correct methods, and in which the manipulation of symbols is an aim, rather than a means of reasoning.

The exceptional teacher described above showed understanding of the activity-based nature of mathematical thought, but failed to capitalize on the learning situation to develop interest in numbers and problems. In all cases, a narrow view of mathematics was evident, focusing on either its creative or its learned aspects, rather than on the synthesis required for true understanding.

If the Ministry of Education policy is to be implemented, exploratory activity will have to be extended to many areas of mathematics. However, it was evident from this sample that teachers would not willingly curtail the time occupied by language arts. The logical alternative is increased integration of subject areas, organized around fascinating activities that will yield learning in science, language and mathematics.

*Dr. Richards is an Associate Professor in the College of Education at Brock University with a strong interest in Early Childhood Education.*

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**Note:** This study included data from a project funded by the Ministry of Education, Ontario

# "MATHEMATICS IN ACTION"

*by*
*Jeff Shaw*

Mathematics teachers are continually investigating and searching for novel and stimulating classroom ideas, methods and teaching strategies that will enhance student motivation and ultimately their math understanding and achievement. One such method or teaching strategy that I have incorporated into my mathematics program and have found to be a very effective learning medium is having the children "participate physically" to learn and reinforce various math concepts.

Having graduated from university with a degree in physical education and teaching half-time physical education at the elementary school level, I have a personal interest and am concerned with my students' activity levels and their general physical conditions. My other half-day responsibilities include teaching the grade five core program. It is during my daily math class that I attempt to involve my students as much as the specific math concept allows in "active participation". I realize that many math concepts, because of their design, are better suited for the conventional and basic pencil, workbook and seat-work method of instruction and lesson format. Their nature and content do not "lend themselves" to active participation. However, I take advantage of the various math ideas and concepts that can be taught and reinforced through "active participation" and "physical involvement" and incorporate them into my math program. Some of these stimulating mathematics activities that "get the students out of their seats" in order to participate physically and actually "experience" the specific math concept will now be briefly described.

In geometry, my students work individually, in pairs, or as a small group and co-ordinate and align their bodies to "construct" various shapes (including circles, rectangles, pentagons, etc.), lines (including straight, parallel intersection of lines, etc.) and angles (including right, acute and obtuse). My symmetry lessons involve two students "mirroring" each other's movements and images as the third student represents the line of symmetry. Motion geometry "lends" itself well to active participation as the students perform various slides and turns.

In sequencing (be it forward or reverse, regular or irregular) we move up and down the rows and each student at his or her turn continues the sequence by "hollering" out the next number in the sequence. If the oral response is correct, the entire class congratulates the specific respondent by performing that number of stride jumps.

When reading and writing large numbers (up to nine digits), one-half of the class "contorts" and co-ordinates their bodies into various positions in order to "construct" specific numerals. The other half of the class is then responsible to guess the shaped number. If the other half of the class is correct in their guess, they are rewarded by given the opportunity to "construct" their own large number.

To reinforce the concepts of ordinal numbers, constructing tables and using time and distance measurements, my students participate in a "mini" track and field competition. The various events are planned and scheduled by the students and usually include sprint and middle distance running, long, triple and high jumps and shot put. At the conclusion of the competition, final results and statistics are analyzed and the data is summarized in a table form by the students. This tabulation includes the final "order" or standing of competitors in each event (first, second, third etc.), the times recorded for the sprint and middle distance runs (indicated in minutes and seconds) and the distances achieved in the long, triple and high jumps and shot puts (indicated in metres and centimetres).

Addition, time measurement and charting skills are reviewed and reinforced daily throughout the year by our "speed sit-ups" activities. Every school day, before morning recess, my class travels to the gym where they participate in a one-minute speed sit-ups activity. This daily sit-ups performance level is numbered in their personal charts and on Friday of each week the five numbers are totalled. In this respect, the children can evaluate their daily and weekly sit-ups performance and hopefully realize an accomplishment, improvement and progression in their mathematical and physical abilities. Each week's total number of sit-ups is also added to the previous weekly totals in order to indicate the student's cumulative achievement.

To learn and reinforce multiplication and division skills (and to show and express how each operation is the inverse of the other), my students participate in classroom games and activities designed specifically to increase the number of children and "unite" different groups of children together (eg. 4 x 3) and to separate and "breakdown" the groups (eg. 12 ÷ 3) at a given oral command or instruction. Fractions are also reinforced using this "active strategy" through commands and instructions

like "break or separate your group into halves, quarters (fourths), eighths" etc.

Real life situations, experiences and predicaments are "acted out" and performed by groups of students in front of the class. Under these circumstances, the children learn various problem solving techniques and strategies and thinking skills, working with and without numbers, as they attempt to produce a logical and workable solution to their specific problem or dilemma.

My measurement unit "lends" itself to many activities that the children can actively participate in to reinforce various measurement concepts and ideas. The number of body lengths, giant steps, standing long jumps etc. can be used to measure the length of the classroom or the width of the soccer field etc. using these examples of non-standard units. Using their bodies, groups of children devise and "construct" various shapes that are used by other class-mates to find the perimeter of their shapes. As a culminating activity to my measurement of distance and length concept, my class goes out to the soccer field and runs and "experiences" a 100 metre sprint and a 1 kilometre (1000 m) middle distance run. From their involvement in these activities, my students actually "witness first-hand and live" the concepts through their active participation.

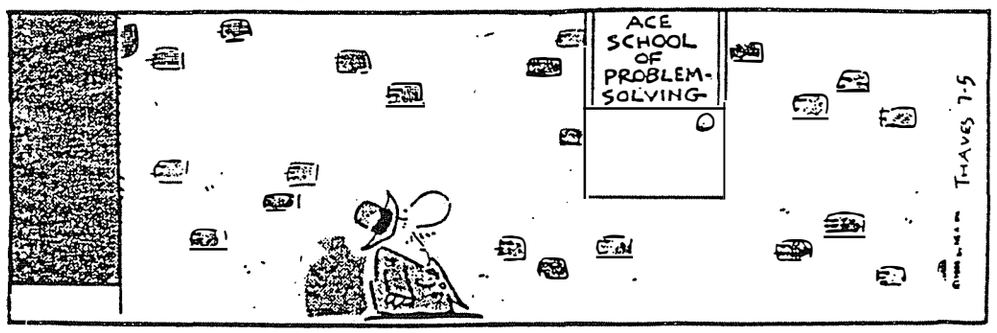
I feel my "active" math classes were successful this school year because my students anticipated the various math activities with enthusiasm and they generally achieved good to excellent mathematics results. In addition to learning and reinforcing the various math skills and concepts through "active participation", the general physical conditions of my students were likely also improved.

The various activities the children participated in to learn and reinforce their math skills, "as a bonus" also aided in developing and increasing their muscular strength and endurance (sit-ups, long, triple and high jumps, shot put etc.), their flexibility and agility ("constructing" body shapes, representing numbers etc.) and their cardio-vascular functioning (stride jumps, sprint and middle distance runs etc.).

The feedback and comments I received from students and parents regarding my "active math program" were of a positive nature because the children enjoyed participating in the various math activities, the children felt they learned the math concepts faster and remembered them longer because they enjoyed the math lessons and were motivated to participate and achieve, my math classes were viewed upon as "fun" experiences, the children were not bored and "glued" to their desks, positive social skills and relations were developed and enhanced through participation in the group settings and the children improved their general physical conditions through active participation in my math classes.

*Jeff Shaw is an elementary school teacher in Oshawa.*

Frank & Ernest



# MAKING REVIEW MEANINGFUL

*by*

*Paul Gauthier*

This article attempts to deal with an area of mathematics that is part of every curriculum; Review. Considered to be boring by many, necessary evil to some, unless it is handled well, it is often hated by students. The National Council of Teachers of Mathematics (1980) recommendation 4.3 calls for "...the inclusion of cyclic review of past topics (contents, skills, and ideas previously taught)". Very little literature, however, is readily available on how this can be effectively carried out.

To many teachers, math review is something you do to your students just before the C.T.B.S. Test so that their students can score just a little higher on the county-wide stats sheet published by the board office computer. It might also be something you subject students to in order that a real or imagined curriculum guideline be fulfilled. It also looks good on a report card. "Having completed the above topics, Mary reviewed the year's work thoroughly". This could mean that Mary not only mastered every subskill at or above grade level, but she was able to recall these skills and use them effectively and efficiently at the end of the year. A less Utopian interpretation might be that Mary managed to get her grades because she could do the tricks with numbers that the teacher told her to do. For example, she didn't know why she moved some non-existent decimal two places when working with per cent, she just knew she had to.

And when year-end review time rolled around, it's a good thing the teacher was there to remind her, because she would never have remembered that. Rules, short cuts, and tricks in mathematics can be useful to students, and review is absolutely essential, but certain factors must be considered before incorporating these into a student's program plan.

The previously described scenario about Mary illustrates two common problems that may cause a student difficulties with recall. The first problem is that the student is using rules, but does not have a clear understanding of where the rules came from or why she uses these "tricks". This is an easy problem to remedy. Math rules, short cuts, and tricks function in much the same way as mnemonics. A mnemonic can be thought of as a useful tool for achieving a task. We use a rhyme or story or letters to help us produce a related product. In the same way, we use a math trick, such as inverting and multiplying when you divide fractions, in order to achieve a related goal. A mnemonic can be a useful tool as long as:

a) the tool is remembered, and
b) the tool is clearly linked to the task.

It cannot be simply recalled in isolation. Every Good Boy Deserves Fudge must call up a clear image of a musical staff and vice versa. Similarly, seeing the algorithm ¾ divided by ½ should bring the rule "invert and multiply" to mind. A mnemonic is most easily remembered when the student is the creator of the mnemonic and understands how the mnemonic is connected to the desired function.

A student named Kevin Hudson was told about mnemonics, then was asked to create a sentence to help him remember km, hm, dam, m, dm, cm, mm, an important list in the metric system. After a brief pause, he replied using the first letter of each symbol, "Kevin Hudson Doesn't Mind Doing Centi Meters". You could argue that the mnemonic was flawed, but no one could argue its effectiveness.

Similarly, math rules should be created by the student. They will devise a rule worded in such a way that it is understandable to them. Adding 18 + 7 + 9 requires that you "carry" or "regroup". Neither the rules nor the language should be GIVEN to the student. A student should firstly have experience in working with numbers in the concrete, then they can move to the abstract. They may choose to call it regrouping, but will likely call it something else. Let them. Most importantly, lead them towards understanding the concept, then lead them slowly into using their "rule". Remember that understanding the concept is far more important than learning how to use the rule or trick.

Let us now suppose that since this is the kind of teaching Mary has been privileged with, she therefore has a clear understanding of what percent means and why she chooses to move a decimal or convert to a fraction over 100. Thus, she has a better chance of remembering how to work with percent. But let's not leave that up to chance. We can be certain that Mary remembers the concept she has acquired by addressing the second problem in our scenario and devising a properly structured system of MATH REVIEW. Before structuring math material for recall for a student, some

# A PRACTICAL CLASSROOM STUDY USING A MICRO-COMPUTER WITH LEARNING DISABLED STUDENTS

*by*

*John Forbeck*

Teachers are very often criticized for not paying any attention to educational research. As a group most teachers act upon suppositions and opinions instead of reading or conducting research. One type of research which my team member and I use quite extensively is practical classroom research.

Recently the Board of Education placed an Apple computer in the two Special Education Classrooms in our school. We saw it as a great boost to our present program. But how was it to be used most effectively? As we had done many times before in other areas of interest a brief search of educational literature and some homemade classroom research was carried out.

We looked at Education (research) in the area of Computer Assisted Instruction (C.A.I.). C.A.I., it was found, made small but significant gains in achievement scores, produced positive attitudes and reduced time for instruction. (Kulik, Kulik and Cohen 1980). Studies looking at drill and practice using C.A.I. in math and language arts found strong evidence for the effectiveness of C.A.I. over traditional instruction (Visonhaler and Bass 1972). At the elementary school level math programs supplemented with drill and practice were significantly more effective in fostering achievement over traditional methods. (Burns and Bozeman 1981)(Edwards, Norton, Taylor, Weiss and Dusseldorp 1975). Much of the research involving C.A.I. with non handicapped students was positive but not overwhelmingly. It seems that there are relatively few studies investigating C.A.I. with learning disabled students. Chaing (1978) found significant differences in favour of C.A.I. in achievement in mathematics and reading areas. Sandals (as in McDermott and Watkins 1983) used C.A.I. in arithmetic and spelling with junior high students who displayed a wide variety of learning disorders. When he compared them with non C.A.I. students he found no significant differences in the post tests. McDermott and Watkins (1983) concluded that C.A.I. held no clear advantage over traditional methods.

It became quite clear that achievement using C.A.I. was not always as successful as we had hoped it would be. Steele et al. (1983) found that significant gains were made in the area of achievement and increased computer awareness. Morris (1983) felt the greatest improvements were made by the teachers who had previously been computer illiterate.

We decided after four months of using the computer to conduct a practical, in-house C.A.I. study. In order to keep our study simple we decided against using a control group using traditional drill practice techniques. Our aim was to see how learning disabled students responded to C.A.I. for multiplication drill practice. We tried to address two questions. How effective is C.A.I. for our own learning disabled kids? And, can C.A.I. be an effective learning aid for the basic math fact program.

For the study we used eight students chosen at random from our two general learning disabilities classes. They were selected because their timetables best coincided with the computer time available for our study.

It was decided to concentrate on multiplication facts drill using mostly the nine times table. We used a program called "Basic Number Facts" by Control Data Program Company. This program is user friendly and has many good features. It involves a car race between two cars which the students name. Although this program involves any of the four basic operations, we specifically directed the students to use the multiplication section involving mostly multiplying by nine. The first race is a practice race to determine a base line time for the student. Once this is determined the race is set up between two cars. At any point a student can ask for help (a picture of groups appears) in order to get an answer correct. Each of the questions must be answered before the cars move or the student can move to the next question. At the end of the race a student is told which questions he did not get right the first time. At this point the student can choose to race again against his best time, set the time for his opponent, or play again using the same practice time. We felt this program was particularly useful for our study as it provided frequent and immediate feedback, clear presentation, motivation, assitance if needed, mastery learning and individualized pacing.

All of the students were given a pre test/post test of twenty five drill questions involving mostly the nine times table. For this we used to triple check math test of multiplication facts to 9 x 9. Each student completed twenty races using the programs over two weeks. They recorded their race time as

well as the specific questions they had wrong after each race.

**TABLE 1**

| NAME | PRE-TEST TIME | PRE-TEST ERRORS | POST-TEST TIME | POST-TEST ERRORS |
|---|---|---|---|---|
| Pete | 8:10 | 11 | 7:20 | 9 |
| Sabrina | 3:00 | 3 | 5:00 | 1 |
| Kevin | 3:10 | 4 | 4:00 | 2 |
| Doug | 2:20 | 5 | 2:40 | 2 |
| Jason | 3:40 | 8 | 2:40 | 0 |
| Rob | 4:00 | 9 | 1:50 | 8 |
| Shawn | 7:30 | 15 | 7:20 | 15 |
| Mike | :47 | 0 | :35 | 0 |

The results found in Table 1 reflect much of what the literature indicates. C.A.I. for some students proved highly successful while others only improved moderately. For table 2 we grouped the eight students into four areas.

**TABLE 2**

| | Time Up | Time Same | Time Down |
|---|---|---|---|
| **Errors Up** | | | |
| **Errors Same** | | Shawn | Mike |
| **Errors Down** | Kevin<br>Doug<br>Sabrina | | Jason<br>Pete<br>Rob |

It is interesting to note that three of the students fell into the time up, errors down block. The program which we used encourages the students to slow down and think. They must get the correct answer before they can proceed. It is this feature that may have contributed to the results of this group.

We did not set out in this study to make any other sorts of comparisons. However we were amazed when we compared Table 2 with Table 3, which we created after looking at the results.

**TABLE 3**

| Computer Active | Neutral | Computer Inactive |
|---|---|---|
| Pete | Mike | Shawn |
| Jason | Doug | Sabrina |
| Rob | | Kevin |

All eight students could be considered computer literate. Each student had spent a lot of time using the computer. After observations for six months it became quite obvious which students came in at 8:15 and left at 4:30 just to use the computer. Peter, Jason and Rob were always at the machine. In fact they preferred the computer to recess.

This study found that C.A.I. did not prove to be highly successful for all students involved. We felt that the Basic Math Facts program continued to be motivating for all students during the two week block. The program provided opportunities to meet individual needs.

Most importantly, a few hypotheses can be generated from the findings of this practical classroom

manufacturers of both hardware and software consciously direct their advertising at the male consumer.

Another blatant example of male dominated advertising suggested that anyone buying a computer could ask "father or elder brother" for any help that might be needed.

Posters advertising jobs in science, engineering and technology tend to portray the man in the dominant, responsible post while the woman, if she appears at all, waits with pencil poised, or fingers hovering over the typewriter (computer) keys, for the words of her lord and master. All these examples, and many more, help to perpetuate the divisions between males and females.

Computer Studies as a subject has lost favour with many people who have come to the conclusion that computing through Information Technology or computer appreciation classes would be more relevant to more pupils. A new syllabus for IT has been introduced into many schools and it is hoped that this course will appeal to both sexes equally.

It will still be necessary for teachers of Information Technology to actively encourage girls in computing. It will not be enough to expect them to accept the subject just because "everyone" takes it; "everyone" takes mathematics!

Schools were encouraged:
a) to develop a policy of positive action to ensure girls are not discriminated against.
b) to produce an option scheme which would not make the choice of technological subjects difficult for girls.
c) to supervise clubs to stop aggressive boys taking a disproportionate share of available resources.
d) to ensure there is no undue linking of computer technology with what are seen as boy's activities by the choice of staff, location or subject matter.
e) to develop a co-ordinated policy for the identification and training of suitable teachers.

LEA's (Local Education Authorities - A British structure similar to local School Boards here), too, recommended to
a) provide sufficient properly equipped rooms.
b) devise a coherent and comprehensive programme to attract and train the right teachers.
c) organise specific seminars for head teachers to implement curriculum change.
d) organise resource centres where good materials are easily available.
e) ask schools to provide statistics on the relative popularity of different subjects with boys and girls and the relative success in examinations.

The Information Technology syllabus for 11-14 year olds, mentioned in the Sept. **Gazette** article, is not an examinable subject. Because of this, Croydon has developed a Certificate of Competence to monitor progress. It can be useful to show levels of competency, helps in determining previous experience and can be used as an entrance requirement for extra-curricular clubs. Pupils proceed at a pace which suits them so girls, finding they do just as well as the boys, have a welcome boost to their confidence.

The Certificate is not intended to be used as an examination and should not be used in a testing environment. The teacher should be able to check off on a check list many of the points from seeing the pupils' work in normal lesson time or in a club. There are four levels in the Certificate. Each covers a different aspect of computer knowledge and allows the students to gain confidence by rewarding positive achievement over a wide range of computer skills.

The progress of pupils in Croydon schools introducing the Certificate will be closely monitored for the first six months. The numbers of boys and girls attaining the various levels will be noted and guidance given to teachers if both sexes do not appear to be achieving equal success.

**Evaluation**

In April 1983, Croydon and the Equal Opportunities Commission jointly funded a project for two years. The objectives of this initiative are to evaluate the effectiveness of the document "Information Technology in Schools" and to assess the effectiveness, in terms of girls' response, of the IT syllabus produced by Croydon and the strategies the teachers have adopted when implementing the scheme.

As part of the project, a survey testing attitudes towards computers and computer personnel was

given to over 400 Croydon pupils aged from 10-16. The results have not yet been finalised but preliminary findings suggest that the hardest sexist attitudes are held by boys in the fourth year of the junior school. Their replies tend to imply that they see women as second class citizens in the areas of technology and see no reason why they should stray into traditional male preserves. Many women teachers (and most primary school teachers are women) take an unduly pessimistic view towards their own ability to adapt to technology, particularly if they are not comfortable with mathematics, and this attitude is transmitted to the pupils.

It is interesting to note that even teachers who are totally committed to equal opportunities and are very aware of the problems faced by girls in technology still cannot totally avoid domination by boys in the classroom. Part of the report on sexism in schools by the Association of Educational Psychologists (4) refers to the fact that "even when teachers were trying to compensate the girls, they gave them less than 40% of their time yet this was perceived by all pupils in the classes as giving more time to the girls than the boys".

A cross-section of pupils hold stereotyped views which firmly place women in inferior positions with regard to careers and prefer to see them at home looking after the children. A female teacher in a junior school in Croydon, dedicated to eradicating stereotypes in the classroom, was horrified to discover the views admitted to in the survey by her male pupils. Despite a spirited class discussion led by the girls, which ended with the majority of boys agreeing that perhaps girls had the ability and the right to compete in male areas, one boy steadfastly refused to shift to the majority view. However, by bringing it out into the open and holding frank and honest discussions with pupils, teachers and parents, a climate of opinion is established which helps to place prejudicial and stereotyped positions outside the norm.

The report at the end of the two years will recommend strategies for teachers and will outline examples of good practice.

**The Future**

Employment opportunities are becoming fewer for those without skills; many of the new skills are technology based; the jobs available are changing; many of the jobs which are disappearing are those which have been traditionally female. In addition, technology is playing an increasingly important part in most people's lives. What will happen to the girls who leave school in the future if they have been allowed to ignore technological subjects and to develop feelings of inadequacy and incompetency in relation to technology? Confidence and skill in using and evaluating the use of computers is an integral part of the education of future citizens, fifty percent of whom are women.

***FOOTNOTES***

*(1) GIST, EOC Research Bulleting No. 6, 1982, EOC.*

*(2) Information Technology in Schools, Croydon, 1983, EOC*

*(3) Statistics of School Leavers - CSE/GCE Results, England 1980, 1980 DES*

*(4) Sexism in Schools, Association of Educational Psychologists, 3 Sunderland Road, Durham, 1985*

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"GIST, EOC Research Bulletin No. 6"; EOC; 1982.

"Information Technology in Schools"; L.B. of Croydon; Equal Opportunities Commission, Overseas House, Quay Street, Manchester, M3 3HN.

"Option Choice - A Question of Equal Opportunity"; John Pratt, John Bloomfield and Clive Seale; NFER-Nelson Publishing Company Limited, Darville House, 2 Oxford Road East, Windsor, Berks. SL4 1DF.

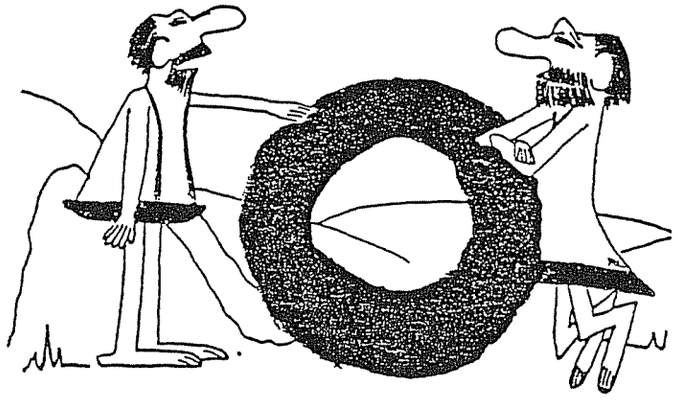
"Sexism in Schools"; Association of Educational Psychologists 3 Sunderland Road, Durham, DH1 2LH.

"Statistics of School Leavers - CSE/GCE Results England 1980"; DES, 1980.

"Teaching Technology to Girls"; Val Millman; Elm Bank Teachers' Centre, Mile Lane, Coventry.

"The Other Half of Our Future"; Women's National Commission; Cabinet Office, Government Offices, Gt. George Street, London SW1P 3Qa.

"Women and the New Technologies"; Marlaine Lockheed and Steven B. Frakt; Educational Testing Service, Division of Educational Policy Research and Services, Princeton, N.J., USA.



WELL, I CAN SEE ITS SIGNIFICANCE AS AN ADDITIVE IDENTITY IN THE REAL NUMBER SYSTEM, BUT QUITE FRANKLY I DON'T THINK IT WILL EVER HAVE ANY PRACTICAL APPLICATION!

Paul Lochner in **Mathematical Digest.**

## KENNETH D. FRYER AWARD

In Memory of the late Dr. K. D. Fryer, OAME in conjuction with the Grand Valley Mathematics Association established the Kenneth D. Fryer Award for Excellence, to be presented annually to a Mathematics Department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

The Award consists of a plaque, presented by OAME and a cash award of $200.00 for professional material, presented by GVMA. In 1984-85, the inaugural year, the award was presented to two schools

LaSalle Secondary School (Sudbury)
O'Neill Collegiate and Vocational Institute (Oshawa)

There will only be one such award presented in each subsequent year. The following inscription appears on the plaque.

> Kenneth D. Fryer was Professor of Mathematics at the University of Waterloo for twenty-five years and Associate Dean of the Faculty of Mathematics for eighteen years. He was a leader in secondary school mathematics. This leadership was evident through his editorials in the O.S.S.M.B., as a strong supporter of O.A.M.E. and willing speaker at annual conferences, as a founder of the Canadian Mathematics Competition, and as a moving force in developing the high school liaison program of the Faculty of Mathematics designed to motivate and inspire students and teachers.
>
> He practiced what he taught; while it is important to study mathematics, it is also important to develop as a complete, caring human being. He was a believer in the value of the church, family, athletics, music, drama, and student government, in the development of young people.
>
> In his memory, the Ontario Association for Mathematics Education has established the Kenneth D. Fryer Award for Excellence, to be presented to a Mathematics Department which, in addition to combining excellence in mathematics education, also contributes to the development of young people through activities beyond the classroom.

At this time OAME is requesting nominations for consideration by the committee for this year's award. It will be presented at the annual conference in London.

Please complete a copy of the form below and forward it to:
Mr. M.B. MacGregor
Sec./Treas. OAME
247 Bright St.
Sarnia, Ont.
N7T 4E9

**Deadline for nomination submission: March 31, 1986**

**Kenneth D. Fryer Award**
**Nomination Form**
**1985-86**

I/We nominate School: _____

Address: _____

Principal: _____

Math Dept. Head: _____

Nominated by: _____

Please attach a separate sheet to the nomination form outlining the reasons for your nomination.

## WHAT'S GOING ON...



*This column is included in this publication as a vehicle designed to share current information with O.A.M.E. members*

The following items of information are submitted by Paul Lessard (President of O.A.M.E.).

A. Letter To the Directors of Education & B. Leadership News

### LETTER TO DIRECTORS OF EDUCATION

Recently I wrote a letter to all Directors of Education on your behalf. The purpose of this letter was to acquaint them with the goals and practices of our association. I have enclosed a copy of this letter in this column to encourage you to utilize this information in your ongoing **promotion of our association.**

Dear Director of Education:

Greetings and welcome to a new year on behalf of the Ontario Association for Mathematics Education. This letter attempts to acquaint you with the goals and practices of our association.

The Ontario Association for Mathematics Education includes in it's membership math educators from the Public, Private and Separate Schools, from our Universities and our Community Colleges.

The major goal of our organization is to provide a forum for the exchange of information relating to all aspects of mathematics education. To this end we have planned the following activities for the school year 1985-86:

* Mini-conferences sponsored by Chapters of O.A.M.E. in seven locations throughout the province.
* The publishing of 2 professional publications.
* The administration of several MATH AWARDS/SCHOLARSHIPS
* The sponsoring of 2 major math conferences.
* The participation in effective liasion roles with the Ministry of Education; i.e. Guidelines, Reviews, Assessments.

**CONFERENCES**

| | |
|---|---|
| 1. Leadership Seminar in Mathematics | October 23-26, 1985<br>Ramada Inn,<br>Dixie Road |
| 2. 1986 Annual Conference | May 9-11, 1986<br>London |

REGISTRATION INFORMATION available through:

Mr. M.B. MacGregor
Secretary-Treasurer, O.A.M.E.
247 Bright Street
Sarnia, Ontario

**PUBLICATIONS**

Our two professional publications, the official journal of our association; the Ontario MATHEMATICS GAZETTE and our practical classroom resource newsletter; The ABACUS, result in 7 publishings each year.

We invite you to share this summary of our activities with appropriate staff members in your jurisdiction.

Sincerely

Paul Lessard
President

*This letter serves as a visible reminder to us that several unique and somewhat novel activities have formed the rich heritage of our association.*

*I encourage you to share this uniqueness with Math Educators in your jurisdiction.*

**LEADERSHIP NEWS**

Recently your executive planned, developed and delivered a three day seminar on leadership in Mathematics. Sixty participants from around the province participated in this event.

Special thanks go out to the participants and speakers for making this Fourteenth Leadership Seminar in Mathematics an overwhelming success.

I would like to take this opportunity to introduce to you the line-up of speakers that made the seminar a very special experience for all involved.

| | | |
|---|---|---|
| Richard Boswell | Director of Education | Simcoe County Board of Education |
| Nanette Deger | Teacher of Gifted | Scarborough Board of Education |
| Jay Symington | Special Education Consultant | Peel Board of Education |
| George Knill | Education Officer | Ministry |
| Bob Robinson | Mathematics Consultant | Hamilton Board of Education |
| Morris Liebovitz | Education Officer | Ministry |

| | | |
|---|---|---|
| Julian D'Angela | Computer Consultant | Hamilton-Wentworth Roman Catholic Separate School Board |
| Tom Weinstein | Computer Teacher | Hamilton-Wentworth Roman Catholic Separate School Board |
| Bill Proctor | Teacher | Waterloo County Board of Education |
| Bill Leigh | ICON Pilot Teacher | Simcoe County Board of Education |
| Doug Bell | Commissioner of the Yukon Territory | |
| Bob Tappay | Teacher | Simcoe County Board of Education |
| Don Fraser | Faculty of Education | University of Toronto |

In closing I continue to encourage you the readers to use this column to share with our members MATH ACTIVITIES in your region that highlight, "WHAT'S GOING ON...".

*May I also take this moment to wish you and your families a very Holy Christmas and a Joyful and Prosperous New Year.*

Paul Lessard
President, O.A.M.E.

## COMPUTERS IN EDUCATION

*Survey Report #1*

Thank-you to all the people who replied to the survey during the winter of 1984-85. Particular thanks to those who returned the follow-up forms which will allow us to share considerable information with readers around the country during the months and years to come.

If you have not yet become involved there is still time. Any software you have created or have made use of because someone created it, is of interest to us and will be shared with others though this O.A.M.E. publication. Just send your information to:

Bill Leigh

In future issues of the **Gazette** & Abacus we will be presenting more programs created by our members as well as reviews of other programs that we have used. Keep us informed and we will in turn pass on this data.
On behalf of the O.A.M.E. Computers' Committee,

Mickey Sandblom,
Executive Liason.



## PROBLEMS

When looking for "good" problems, I am pleased when a problem is of the type that can "cover" many grade levels by simply changing the strategy used. This month's problems, from the **New Jersey Mathematics Teacher,** are of that type. For Example, Problem one might be factored by a Senior level student, but a junior or intermediate level student could solve it by solving similar, but easier problems and looking for a pattern. Problem 2 **could** be solved many ways, from an algebraic solution to a guess and test strategy, etc. No matter how your students (or you) choose to attack them, have fun.

1. Find the value of $746384^2 - 746383^2$ without squaring the numbers.

2. I have an apple, an orange, and a peach. I "weighed" them 2 at a time with results of 140 g, 180 g, and 200 g for the three possible pairs of fruit. How much does the smallest one "weigh"?

3. Write 462 as the product of three numbers such that the sum of the three numbers falls between 40 and 50.

4. Complete the following pattern:
5---------- 4
36---------- 8
11---------- 1
53---------- 7
942--------- 14
18---------- ?
49---------- ?
371--------- ?

Answers on Page 34




Dear Editor:

In solving the problems on P. 41 of the September **Gazette,** we discovered that there is no unique solution to #2. In fact, there are infinite solutions in theory and four real solutions.

(1) $Q + D + N = 17$
$N = 17-Q-D$

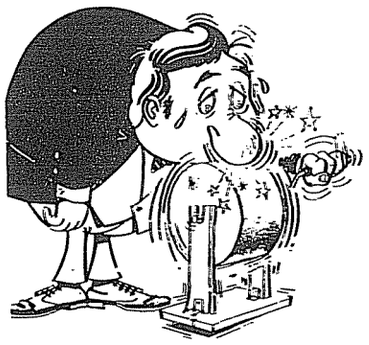
(2) $25Q + 10D + 5N = 160$
$25Q + 10D + 5(17-Q-D) = 160$
$25Q + 10D + 85-5Q-5D = 160$
$20Q + 5D = 75$
$4Q + D = 15$

| | Q | D | N | = | 17 |
|---|---|---|---|---|---|
| Real Solutions | 0 | 15 | 2 | = | 17 |
| | 1 | 11 | 5 | = | 17 |
| | 2 | 7 | 8 | = | 17 |
| | 3 | 3 | 11 | = | 17 |
| Theoretical Solutions | 4 | -1 | 14 | = | 17 |
| | 5 | -5 | 17 | = | 17 |

Note the number patterns emerging

From: Math Class, MacKenzie Senior School, Scarborough, Ont: M. Munro

P.S. We enjoy the publication and look forward to the Problem Section.



## COUCIL COMMENT

*by*
*Bob Robinson*

**Editor's Note:** *Council Comment appears in an issue of the* ***Gazette*** *following an OAME/AOEM Council meeting. Your comments are welcome and should be addressed to the Editor.*

In the September, 1985 issue of the Gazette, the names, addresses, and telephone numbers of the 1985-86 Executive and Council were listed. Please include the following with this list:

**Councillors - Term Expiring 1986**
Harry Traini

**GVMA Representative**
Stuart Telfer

**OAIP Representatives**
The following will represent OAME:
Paul Lessard - OAIP Intermediate Division Advisory Group
Harry Traini - OAIP Senior Division Advisory Group

**NOMINATION COMMITTEE**

The Nomination Committee chairman is Paul Pogue and also has Dave Davidson and Bob Robin-son as its members. Please contact any of these persons if you are interested in running for office or know of anyone seeking such a position. Here's an opportunity to take an active part in your association.

**ANNUAL MEETINGS**

**ANNUAL 1985 - SUDBURY**

Mickey Sandblom presented his report and once again thanked all those who assisted and participated in this highly successful conference.

**ANNUAL 1986 - LONDON (1986 05 09-11)**

Tom Griffiths reported that planning is progressing on schedule and· any input regarding program and other activities are most welcome by his committees. This is shaping up to be a highly educa-tional and social conference.

**ANNUAL 1987 - SCARBOROUGH**

Steve Martin explained that his group is starting to formulate the initial stages of planning.

**AWARDS**

**FATHER FAUGHT AWARD**

As reported in September, the award this year was won by Woburn Collegiate (Scarborough).

**McCARTNEY AWARD**
The winner this year was Eric Veach.

**KEN FRYER AWARD**
As reported in September, this year's winners were the mathematics departments of LaSalle Secondary School (Sudbury) and O'Neill Secondary School (Oshawa). The inscription on the plaque is to be found elsewhere in this issue of the **Gazette.** Suggestions for award winners should be sent to the Sarnia office of OAME.

**LIFE MEMBERSHIP**
The chairman of this committee is Dave Davidson and the other members are Joan Routledge and Tom Griffiths. Please refer to the announcement and requests for nominations on page 16 of the September **Gazette.**

## CHAPTER REPORTS

**COMA**
On November 14, 1985, a dinner meeting is planned for the Holiday Inn (Kent St.) in Ottawa. The topic is ''Problem Solving'' and the speaker, Dr. Lola May. In January or early February, a panel discussion will take place and focus on the new OAC's. Another publishers' display will be held in mid-April.

**GOLDEN SECTION**
A spring conference is being planned.

**GVMA**
GVMA has a new publication entitled, **Sample Grade 9 Examination Questions (Advanced).** This is the first of a new series of sample examinations based on the new guidelines. Another up-dated publication, **''What Every Frosh Should Know''** is also available. This latter title provides a convenient check-list of ideas and formulas.

GVMA is planning a summer writing project. The theme of this year's project is yet to be decided.

**NOMA**
A fall meeting was held November 1 and November 2 in North Bay.

**PRMA**
PRMA is giving a Math Award to two graduating students from Durham Alternate Secondary Education (New Adult Program) for their first graduating class.

**QSLMA**
Spring Meeting is planned for Kingston (Queen's University) on Wednesday, April 16, 1986.

**SAME**
SAME is in the midst of a most successful membership drive. Members received a large membership certificate, a wallet-size membership card and invitations to upcoming meetings including dinner meetings.

Four afternoon (4-6) P.D. sessions are also being planned for Tuesdays on Oct. 8, Nov. 19, April 8, And April 15.

SAME is actively planning for the 1987 Annual Convention.

## COMMITTEE REPORTS

**MEMBERSHIP/PUBLICITY/PROMOTION**
The Secretary-Treasurer and his wife will be attending several conferences this fall and hopefully sign up enough new members to set an all-time high. It is hoped that two new chapters will be formed in the near future with each chapter having its own membership chairman.

**COMPUTERS**
A process for reviewing and/or sharing software for the mathematics classroom is being put into place. The Council of Ministers of Education is compiling a data base of educational software. Information on good programs for mathematics education will be included in Gazette articles.

**FORMATIVE YEARS**
Communication will be sent to the Ontario Ministry of Education that we support a review of the Formative Years document and that the mathematics objectives in this document be more specific.

**WOMEN AND MATHEMATICS**
A "Women and Math" strand will be offered at the London Annual in May '86. The theme for these workshops will be: Strategies to encourage and keep girls in mathematics courses; To promote excellence in mathematics, in particular, to encourage greater participation by girls in math contests; To make teachers aware of those "unwitting" ways of discriminating "against" women in mathematics.

**PROBLEM SOLVING**
The committee will prepare a list of resources and/or projects pertaining to the teaching of mathematical problem solving.

**PUBLICATIONS**
Discussion is taking place regarding the format of our publications and guidelines for editors. Further decisions will be made in the near future.

**NOMINATIONS**
Members are encouraged to send in names of those people they would like to have run for office.

**OAIP**
It was reported that it will still be some time before the junior OAIP reaches the teachers.

**OTHER BUSINESS**

**JUNIOR MATHEMATICS OLYMPIAD**
Tom Griffiths will chair this project.

**NCTM**
The Montreal Meeting was a success and we are looking forward to the Edmonton Conference to be held October 16-18, 1986.

**ADDRESS CHANGES (see September issue)**
Mary Lou Kestell
Steve Martin

**FUTURE DATES**

**EXECUTIVE**
November 22, 1985
April 4, 1986

**COUNCIL**
February 8, 1986

# HELP!

O.A.M.E.'s problem solving committee is preparing a list of resources for improving the teaching of mathematical problem solving.

If you are aware of:

1. effective approaches to the teaching of problem solving at the school or system level,

OR

2. individuals or groups who could make a contribution to improving the teaching of problem solving by conducting professional development activities

OR

3. print or other resources which could aid in the teaching of problem solving,

Please send the information to the problem solving committee at the address below.

The results of this survey will be made available to all O.A.M.E. members.

O.A.M.E. Problem - Solving Committee
c/o Doug Henderson
146 Stonehaven Drive
Waterloo, Ontario
N2L 6C5

## ERIC VEACH RECEIVES MCCARTNEY SCHOLARSHIP

On Thursday, July 18, at the home of Morley and Mona MacGregor, Eric Veach was presented with the McCartney Scholarship. Present when Eric received his scholarship were his parents, Mr. & Mrs. R.H. Veach and three members of the Mathematics Department of the Northern Collegiate Institute and Vocational School of Sarnia namely Mrs. Debbie Heaton, Mr. Bill Strang and Mr. Al Grimwood. Eric is presently attending Waterloo University on the co-operative group study plan.

Everyone present wished Eric every success as he pursues his endeavours in the field of mathematics. I am certain our OAME members when they read this article wish him the same great success.

## ANSWERS TO THIS MONTH'S PROBLEMS

1. 1492767 (i.e., the sum of the two numbers being squared). This will work for the difference of any two consecutive numbers squared. Students who cannot factor the difference of two squares can be encouraged to consider examples like $4^2 - 3^2$; $5^2 - 4^2$; etc. and look for a pattern.

2. 60 g

3. 2 x 7 x 33 [Students can be encouraged to find the prime factorization of 462 and inspect the different ways of writing 462 as the product of 3 numbers.]

4. 18------- 8; 49------- 12; 371------- 10 (sum of the digits of the number minus 1). Here, **too** much sophistication can be a handicap-younger students will likely find the pattern sooner than older ones.

# the
# grand valley mathematics association

## G.V.M.A. PUBLICATIONS

| | |
|---|---|
| Handbook of Trigonometry (1975) | $2.00 |
| Solutions to the Handbook of Trigonometry | 2.50 |
| Grade 9 (Level 5 and 4) Sample Exams | 1.00 |
| Cumulative Testing | 1.50 |
| A Handbook of Geometry | 3.00 |
| Survey Test in Mathematics | 1.00 |
| "Mathematics for Life" (1983) | 7.00 |
| "Mathematics for Life" Problems Supplement (1983) | 1.50 |

(See the December, 1983 Ontario Mathematics Gazette for a review of this material.)

| | |
|---|---|
| Recreational Math Problems for High School Students Books I and II (Revised 1984) | 3.00 |
| 50 Grade 13 Calculus Problems | 1.50 |
| Grade 10 Sample Exams (1984) | 1.50 |
| *Math For Everyday Life (1985) | 5.00 |
| Living With Math (1985) | 7.50 |

*In the first week of July, 1984, G.V.M.A. brought together three teams of teachers to create readily implementable resources for the new grade 11 basic and general level courses. The results to date are **"Living With Math"**, and **"Math for Everyday Life"**. The general level material is currently being field tested and we hope to have it available next year.
While designed to meet the requirements of the new ministry guidelines, these two booklets contain a wealth of resource material which can be used effectively in basic mathematics courses in any grade level from 9 to 12.

**To receive these publications, send your order and payment (cheques or money orders made payable to the Grand Valley Mathematics Association) to:**

Mr. R. G. Scoins
Faculty of Mathematics
University of Waterloo
Waterloo, Ontario
N2L 3G1

# EXPLORING THE DIFFERENCE OF TWO SQUARES

*by*

*Dipendra Bhattacharya, Leah Doherty, Amitabha Tripathi*

Whatever subject matter you teach in class, motivation is one key to successful learning in the classroom. One of the problems that a teacher faces in a remedial math course is motivation, that is, how to motivate the unmotivated. This is definitely not an easy task. Some of the things we have tried in our classroom settings for the past several years have worked fairly well. We would like to share one of these class-tested cases for motivation with the reader.

Factoring of the difference of two squares is a topic, which, once learned, is usually abandoned. There are, however, several additional examples which may be given to a class to show the applications of the difference of two squares. Not only would it be possible to extrapolate from the examples given in a text, but, at the same time, it could be possible to motivate the student.

First, we show the following generalization for the difference of two squares:

$$a^2 - b^2 = (a + b)(a - b)$$

Then, we give several examples similar to the following:

$$9x^2 - 100 = (3x + 10)(3x - 10)$$

or even a more advanced example for an interested class such as:

$$x^4 - 1 = (x^2 + 1)(x + 1)(x - 1)$$

$$\text{or even } a^2 - (b + c)^2 =$$

$$[a + (b + c)][a - (b + c)]$$

After showing examples like these, most books stop. Some give further examples for rationalization, like the following:

$$(\sqrt{2} + 1)(\sqrt{2} - 1) = (\sqrt{2})^2 - (1)^2 = 2 - 1 = 1$$

It is possible to go further and show the following examples, which incorporate a level of practicality into the classroom and provide a "short-cut" to a multiplication problem. For example, the following problem could be solved using the concept of the difference of two squares: multiply the following:

64x64 - 36x36

$64^2 - 36^2$ which can be factored into the following

$(64 + 36)(64 - 36)$ which can be simplified to

$(100)(28) = 2800$

There is an additional example which could be explained as follows, again, using the concept of the difference of two squares.

$(999)(999) = (999)^2$ which may be rewritten as

$(999)^2 = (999)^2 - 1^2 + 1^2$

(both adding and subtracting one in the same step)

which simplifies into the following:

$(999 + 1)(999 - 1) + 1 = 998{,}000 + 1 = 998{,}001$

Now, let us advance a little further in exploration of the difference of two squares. Let us con-

sider $x^2 + y^2 = z^2$. If we can find three natural numbers which are relatively prime to each other, that make the above statement true, then we will call these three numbers Pythagorean Triplets. For example, the following numbers are Pythagorean Triplets:

$$3^2 + 4^2 = 5^2$$

and $5^2 + 12^2 = 13^2$

Suppose we wish to generate more of these triplets. We are aware of the fact that there are generating formulas available in books, for example George Gamow's book, "1, 2, 3, ... infinity." But we wish to look at it in a slightly different fashion:

If $x^2 + y^2 = z^2$

then $x^2 = z^2 - y^2 = (z + y)(z - y)$

Now, choose z and y such that

$$z - y = 1$$

(This means if z is even, y must be odd and vice versa) which implies 'x' must be odd.

Examples:

(1) $3^2 = 9 = 5 + 4 = (5 + 4)(5 - 4)$

Triplets: 3, 4, 5

(2) $5^2 = 25 = 13 + 12 = (13 + 12)(13 - 12)$

Triplets: 5, 12, 13

(3) $7^2 = 49 = 25 + 24 = (25 + 24)(25 - 24)$

Triplets: 7, 24, 25

In fact, for any odd number, say

$$x = 2n + 1$$

we have $(2n + 1)^2 = 4n^2 + 4n + 1$

$$= (2n^2 + 2n + 1) + (2n^2 + 2n)$$

Triplets: $2n + 1$, $2n^2 + 2n$, $2n^2 + 2n + 1$

The whole idea here is to give the students a feel for applying the concepts learned in one area to the other areas of mathematics.