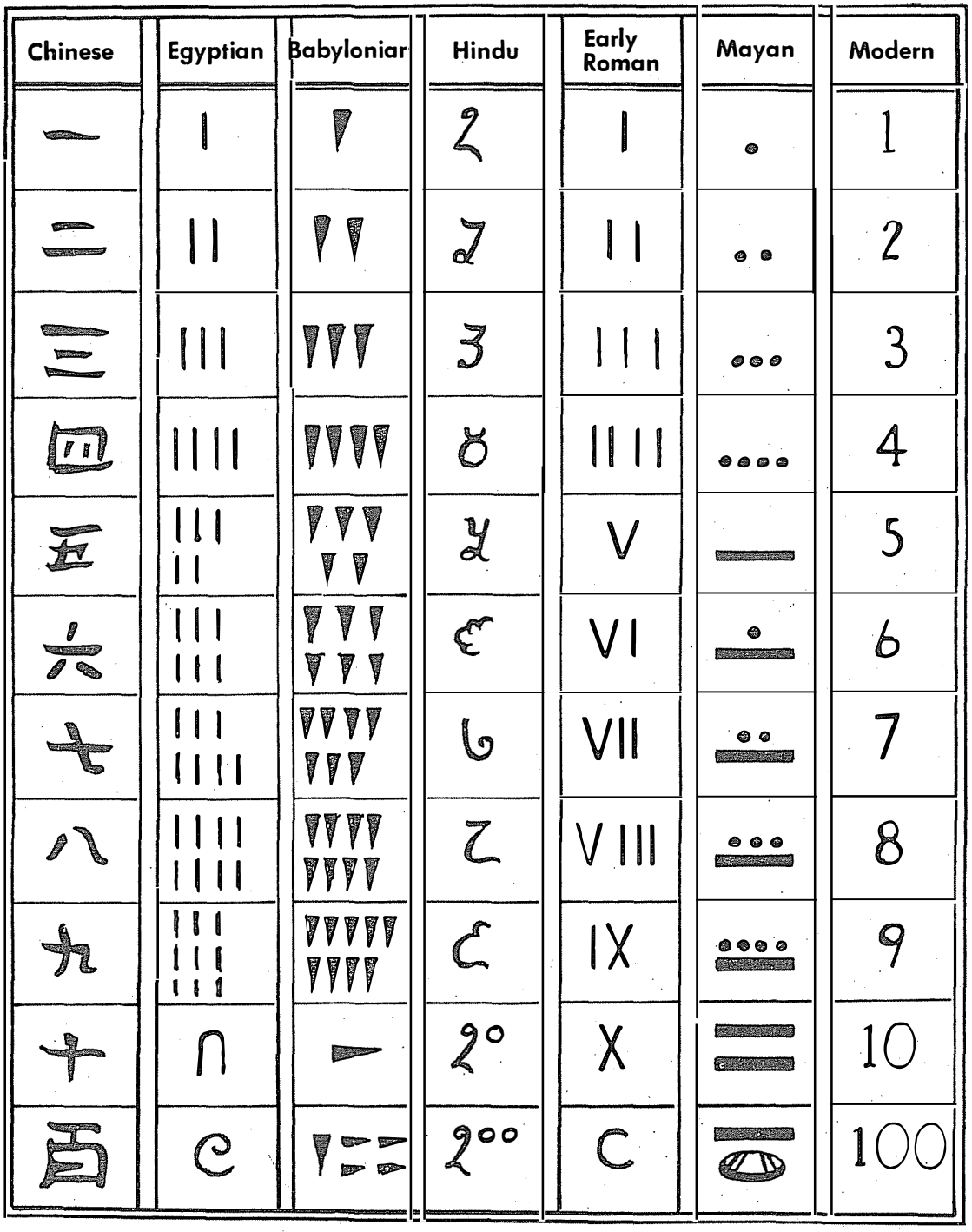
ontario mathematics gazette

VOLUME 25 NUMBER 2 DECEMBER 1986



Chinese	Egyptian	Babyloniar	Hindu	Early Roman	Mayan	Modern
一	𓏺	𒁹	९	I	•	1
二	𓏺𓏺	𒁹𒁹	७	II	••	2
三	𓏺𓏺𓏺	𒁹𒁹𒁹	३	III	•••	3
四	𓏺𓏺𓏺𓏺	𒁹𒁹𒁹𒁹	४	IIII	••••	4
五	𓏺𓏺𓏺 / 𓏺𓏺	𒁹𒁹𒁹 / 𒁹𒁹	५	V	—	5
六	𓏺𓏺𓏺 / 𓏺𓏺𓏺	𒁹𒁹𒁹 / 𒁹𒁹𒁹	६	VI	•—	6
七	𓏺𓏺𓏺 / 𓏺𓏺𓏺𓏺	𒁹𒁹𒁹 / 𒁹𒁹𒁹	७	VII	••—	7
八	𓏺𓏺𓏺𓏺 / 𓏺𓏺𓏺𓏺	𒁹𒁹𒁹𒁹 / 𒁹𒁹𒁹𒁹	८	VIII	•••—	8
九	𓏺𓏺𓏺 / 𓏺𓏺𓏺 / 𓏺𓏺𓏺	𒁹𒁹𒁹𒁹 / 𒁹𒁹𒁹𒁹	९	IX	••••—	9
十	∩	𒌋	१०	X	=	10
百	𓍢	𒌋𒌋	१००	C	=—	100

THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

EDITORIAL BOARD

EDITOR:
Clifford Gravelle, College of Education, Brock University

ASSOCIATE EDITORS:
Ralph Connelly, Brock University
David Handley, Hastings County Board of Education
Marg Ives, Lincoln County Separate School Board
Eric Muller, Brock University
Judi Palmer, Peel Board of Education
Stuart Telfer, Waterloo Board of Education
Trevor Thomson, Hamilton Board of Education
Robert Smith, University of Western Ontario
Jack Weiner, University of Guelph

PUBLICATIONS COMMITTEE

Robert Robinson, Chairperson
Ralph Connelly
Tom Griffiths
David Handley
Sam Pravica
Stuart Telfer
Jack Weiner

O.A.M.E./A.O.E.M. EXECUTIVE

Past President: Paul Lessard
President: Dino Dottori
President Elect: Ralph Connelly
Director: Mary Lou Kestell
Director: Michael Sandblom
Director: Jay Symington

NOTICE TO AUTHORS

Articles submitted for publication should be typewritten or reproduced from type written copy, double-spaced on 8½" x 11" paper with wide margins, and on one side of the paper. Figures and diagrams are to be drawn on a separate sheet of paper in india ink. Authors should take the greatest possible care in preparing the original manuscript. Two copies should be submitted to the editor and one kept by the author as protection against loss. The authors full address and affiliation must appear at the end of the manuscript.

AVIS AUX AUTEURS

Les articles soumis pour publication doivent être dactylographiés à double interligne et avec de grandes marges au recto de feuilles 8½" x 11". Les figures et les diagrammes doivent être dessinés à l'encre de chine sur des feuilles séparées. Les auteurs sont priés de préparer leur manuscrit avec soin et de soumettre deux copies à l'éditeur, tout en conservant une copie en cas de perte. L'adresse et l'affiliation de l'auteur doivent accompagner le texte.

Editorial Correspondence and Articles to:

Clifford Gravelle
College of Education
Brock University
St. Catharines, Ontario L2S 3A1

SUBSCRIPTION RATES

Members of O.A.M.E./A.O.E.M. receive the *Gazette* and *Abacus* free. Single copies $4.00; one year $12.00 in Canada, $14.00 in United States, and $16.00 outside Continental North America.

Orders for subscription should be sent to:

M.B. MacGregor
Secretary-Treasurer, O.A.M.E.
247 Bright Street
Sarnia, Ontario N7T 4E9

Unless otherwise indicated in the article, permission to photocopy material from the **Ontario Mathematics Gazette** is granted to classroom teachers for instructional use, to authors of scholarly papers and to librarians who wish to place a limited number of copies of articles on reserve. In all such instances due credit should be given to the author(s) and OAME. Permission must be sought for any commercial use of the content from this journal. Other uses of OAME journal material should be reported to the OAME.

DECEMBER 1986

ISSN 0030-3011

CONTENTS

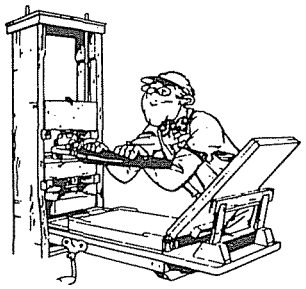
TABLE OF CONTENTS

Editorial 2

The President's Corner 3
Mathematics: To change or not to change 4
Mathematics and poetry 7
The poetry factor 9
Teaching the need for proof 11
"Macpainting" the area under a curve 16
Just put it in a box: I'll take it with me 20
Computing determinants 29
Understanding the power of the chess pieces through percent 31

DEPARTMENTS:

Women and Mathematics
Editorial 33
Ms. Math...A unique mathematics program for girls 34
Don't be discouraged: Don't be discouraging 35
Real women don't do math! 38
Mathematics is a (wo)man's subject 44
History and Philosophy of Mathematics 47
Across The Editor's Desk 48
Problems 50
What's Going On 51
Book Review 55
Council Comment 56



The Ontario Mathematics Gazette is produced by:
POLE PRINTING
Box 69, Forest, Ontario
(519) 786-5112



This year marks a change in the editorship of the Gazette. It is my hope to maintain the tradition of inquiry that has been preserved and nurtured by Ralph Connelly and the other editors over the past years. Teachers often ask me if the Gazette has a particular theme for each issue. We do not, but we do have a permanent theme, which is the transmittance of our love of mathematics to the students within our classrooms.

In this edition our authors have chosen to write about a spectrum of ideas. President Dino Dottori has decided to outline his "state of the OAME" message, and Bob Robinson has contributed his informative council comments. Stanley Bezuszka takes a look at us, the teachers of mathematics and some elements that must prevail and be enhanced if mathematics is to make a positive impact on students.

Peter Taylor and W.J. Barnes see some similarities between mathematics and poetry. James O'Keefe has a few suggestions about calculating determinants. Jack Weiner has experimented with the "Macpaint" program to eliminate one of the difficulties of teaching, and Eric Wood wants his students to think rather than blindly accept what seems obvious.

James Boyd is interested in the uses for operation boxes in our curriculum. Dipendra Bhattacharya and Leah Doherty remind us to use the concept of percent from mathematics to examine the power of the chess pieces.

In addition to these fine articles, we have mice and mathematics at OAME '87, and R.S. Smith, Book Review Editor, has reviewed a book by Daphne Kerslake, while Eric Wood has perused a text for prospective teachers of mathematics. Ron Lancaster sees problems as a way of stimulating teaching.

I would like to call your attention to our new departments. M. Elaine Harvey, Women's Editor, wants teachers and students to think of mathematics as a (wo)man's subject, and Pat Rogers takes a look at the fact that real women don't do mathematics. John Hindle has some suggestions about how teachers can become more encouraging in the classroom, and Bonnie Taylor Gillings is in favour of involving us actively in a mathematics program for girls.

Also, E.J. Barbeau, History and Philosophy Editor, wants teachers to investigate the historical and philosophical aspects of mathematics.

On behalf of the Editorial Board and the Publications Committee, I extend to you our best wishes for the Holiday Season, and continued success in the New Year.

Clifford J. Gravelle

THE PRESIDENT'S CORNER

by
Dino Dottori

At the time of writing, we find ourselves at the halfway mark in the fall term. Our secondary schools are either nearing the middle of the first semester or preparing for November exams, while the elementary schools are getting their first look at the new OAIP instruments for Junior Division. Copies of new textbooks, new computer software, and a variety of other learning materials have reached the schools, generating a lot of local activity evaluating these learning tools. Many teacher groups are heavily involved in updating present courses and preparing new courses to reflect the changes identified in the Intermediate/Senior Guideline, or in the Early Primary (EPEP) document and the Junior Division Review which were recently released by the Ministry of Education. In our post-secondary institutions, the anticipation of meeting the new students is over. These students have once again arrived at their new schools with varied backgrounds and levels of achievement in mathematics. But there is hope! I am informed that one of the results of the many O.A.M.E. chapter meetings that take place in the province throughout the year is a better understanding of the students' needs - and this translates into a smoother transition for them.

I am pleased to report to you that all of our chapters are functioning well under the general OAME umbrella. Each group has an executive in place, and the regional meetings and conferences that are organized by the chapters are providing direct service to members, while the provincial council meets a more general need. One strength in our association is the manner in which the annual conference is organized by the local committees at the chapter level. The Scarborough Association for Mathematics Education (SAME) has an excellent program, both educational and social in place for the upcoming 1987 annual on May 7, 8, 9. In 1988, the annual conference will be hosted by our chapter in the Niagara peninsula, the Golden Section, in beautiful Niagara Falls. At the October council meeting, Peter Lenczewski, COMA representative, and Jerry Lunney, councillor from Ottawa, made a proposal that the Carleton Ottawa Mathematics Association host the 1989 conference. This offer was accepted unanimously, so that after twelve years, we are returning to our capital for the annual. There is considerable behind the scenes activity to host another NCTM Regional Conference in conjunction with our annual conference -Hamilton is the preferred site for the early nineties.

Speaking of NCTM, the executive wishes to congratulate councillor Bob Robinson upon his appointment as chairman of the elections committee. Ralph Connelly continues as the OAME general representative on NCTM while on November 15, Mary Lou Kestell attended a NCTM workshop in Minneapolis for leaders in the association.

During the summer months and in the fall, OAME councillors have been active on our behalf. Paul Lessard and Ed Haines attended several meetings at the Ministry in their capacities as OAME representatives on the OAIP subject advisories for Intermediate and Senior Mathematics respectively. In this way, OAME has direct input at the development stage of the pools. George McNabb has attended meetings on the OAC Calculus project as the OAME representative and is participating in the pilot study. Mary Lou Kestell has been appointed as our representative on the OTF curriculum committee. These are some of the ways in which OAME is able to represent mathematics education in the province beyond our organization.

The Futures Committee has identified two major projects. This group recognizes that there are a number of teachers in secondary schools, who, while not having a strong background in mathematics, are assigned mathematics classes in their timetables. Materials in the form of a "survival binder" are being collected to provide a resource for teachers of mathematics. The mathematics community in Ontario has been organized under several names, including The Ontario Mathematics Commission (OMC), Ontario Association of Teachers of Mathematics and Physics (OATMP), and The Ontario Association for Mathematics Education (OAME), for the past ninety-six (96) years. The Futures Committee is now in the process of receiving a proposal to document the first 100 years of our organization. In the next issue, we should be able to provide more information on this project. In the meantime, anyone having any information, photographs, or memorabilia relating to the proud history of mathematics education in Ontario should communicate with the chairman of the Futures Committee: Paul Pogue, 43 Cynthia Court, Barrie, Ontario L4M 2X3.

In closing, I would like to make an appeal for even more sharing of ideas on the teaching/learning of mathematics and for collaborative projects among groups in OAME. I would very much like to thank those who have already had their work published in the new **GAZETTE**, and to invite others to participate in this exchange of information.

On behalf of Morley and Mona MacGregor, the executive, and the councillors, I extend to you our best wishes for the upcoming holiday season, and continued success in the new year.

MATHEMATICS: TO CHANGE OR NOT TO CHANGE

by
Stanley J. Bezuszka, S.J.
Mathematics Institute, Boston College
Chestnut Hill, Massachusetts 02167

History confirms that in times of change, whether slow or rapid, there is an underlying conviction among all people

--- that there always is a stratum of stability and permanency,

--- that not everything changes, that not everything should change,

--- that change must be balanced by the heritage and wisdom of a tried and accepted tradition.

And what are some of these permanent attributes of mathematics? Mathematics, in its earliest origins, was conceived by the mind of man struggling for daily survival -- but it was nurtured by the profitless dreams of the mind of man in the challenge to count the grains of sand on the seashore or number the limitless stars in the heavens. This dual aspect of mathematics:

the practical, the applicable, the economically indispensable, and the fanciful, the imaginative, the inspirational,

have always been characteristic of mathematics.

Our concern in the past, in the present and for the future lies with these problems:

-- Is there a way to teach and transmit the splendid humanistic traditions of mathematics without compromising sound educational principles?

-- Can the wonders of mathematics be taught with dignity and professionalism?

-- Can teachers share the conviction that mathematics is useful, both in intellectual order and for the necessities of a technological era, and still avoid a meaningless idealism or an all pervading vocationalism?

-- Can one make the mathematics class a joyful, meaningful, personal experience without engaging in the folly of extreme permissiveness?

-- If mathematics is everywhere around us, then mathematics should be for everyone.

-- Can a teacher reach all mathematics students and not just the elite, thus meeting the social responsibilities of teaching as well as educational goals?

Can these questions be answered to benefit the professional life of the teacher and to achieve the desired intellectual development of students? I am convinced that true master teachers have always known what their students need.

-- What is needed is tenderness for what the child is and respect and wonder for what the child may become.

-- For you cannot touch a child and not leave fingerprints on the child's mind and heart.

-- You should not take the complex, sensitive, reactive body of a child and treat it like a robot, preparing the child only for robot tasks.

Actually, human nature has not changed drastically over the many centuries. The inherent curiosity of the ancients is still there in every exploratory touch and grasp and look of young children. The imagination that built the seven wonders of the world is still with us in the seven million wonders in the eyes of a child. And eyes, whether old or young, can always be filled with awe, amazement and wonder.

We do agree with those who urge reform that change must be a fact of life in the schools. But in addition to alterations that are necessitated by technological developments and our particular social reformations, the structure and content of a mathematics curriculum for the future should also contain some stable, lasting elements of good mathematics.

Some elements that should remain constant to ensure effective mathematics education follow. These elements must prevail and be enhanced if mathematics is to make a positive impact on students.

1. Priority in mathematics teaching must be given to motivation.
   Not too long ago I visited a 5th grade math class in one of our public schools. I showed the class several delightful problems in number theory, geometry and the history of the great mathematicians. After class, one of the boys came to me and asked: "Do you love mathematics"? "Of course", I answered. He paused and

quietly said: "Then teach me, please teach me the kind of mathematics that makes you love it". The boy did not ask for the mathematics that would help him pass tests, or get him higher grades or ensure a promotion to the next grade. He wanted mathematics that he could enjoy and love. Perhaps we have neglected this aspect of mathematics for too long. Perhaps it is time for those in mathematics education research to examine just what it is that turns students to the love and enjoyment of mathematics. Maybe we should let students learn to love mathematics and then they will love to learn mathematics.

2. Patterns should be a focal point of mathematics.
The search for patterns is not only the heart of mathematics, it is also a powerful motivational element. Patterns lead to generalizations, and generalizations of particular instances are what makes mathematics interesting, challenging and enjoyable. Recall how the mathematical genius Karl F. Gauss was recognized by his teacher after discovering the pattern for summing the series

$$1 + 2 + 3 + 4 + 5 + \ldots + n = \frac{n(n+1)}{2}$$

Throughout the history of mathematics, mathematicians have maintained
"Wherever there are patterns, there one finds mathematics and wherever there is mathematics, there one finds patterns".
The great English mathematician G. H. Hardy wrote: "A mathematician, like a painter or a poet, is a maker of patterns".

Encouraging students to find, continue, and generalize patterns should take precedence over having them drill for mechanical proficiency in skills.

3. Algorithms change and are replaceable.
Because of the increased use of computers in the schools, existing algorithms must be modified and new algorithms are needed to solve problems. One cannot impress students with the need for new algorithms until they are apprised of the many unsolved problems that abound in our mathematics. Many will argue that unsolved problems in mathematics are necessarily abstruse and beyond the understanding of young people. Some problems do fall in this category. However, many unsolved problems in mathematics are simple to understand and are not beyond the grasp of young students. For example, one unsolved problem is: find an efficient and rapid means of getting the divisors of composite numbers and determine whether or not a number is a prime. In fact, there are a variety of understandable unsolved problems in mathematics. Students can experience the challenge of the unsolved.

4. Number Theory is an exceptional motivator.
The gift of numbers, like the gift of fire, has made the life of man much brighter. Number theory provides excellent motivational material for the enjoyment and love of mathematics. It affords innumerable opportunities for discovery. Number theorists are constantly on the alert for new and more powerful algorithms that will help resolve some of their unsolved problems. It is highly improbable that students will ever make a hobby of factoring trinomials or computing square roots the long way. However, numbers have fascinated and intrigued the young and the old, the amateur and professional mathematician. In no other branch of mathematics is a novice apt to make a new contribution or rediscover old theorems than in number theory. The thrill of the amateur who rediscovers a theorem in number theory is as great as that of the professional mathematician who finds a new theorem. It is regrettable that this subject with all its potential for motivating students makes so little impact on the curriculum.

5. The history of mathematics is not to be overlooked.
It has been said that those who neglect the history of their subject are doomed to repeat its errors. Some aspect of the history of mathematics should be an integral part of each class day. Now, text books often contain excerpts on the lives of prominent mathematicians or details on the origin of the symbols of mathematics. This information can easily be shared with our students and serve as a catalyst for additional explorations.

6. Geometry is a basic skill.
No topic in school mathematics receives more attention than geometry. There are powerful factions clamoring to throw out Euclidean geometry from the curriculum. Others who have already done away with the Euclidean approach are ready to give it a second chance. Geometry can be a challenging and absorbing subject. It contributes to the intellectual development of students in ways that other subjects do not or cannot. Hopefully, the geometry that students take in the future will not be so narrow in scope. Admittedly, there have been numerous advances in geometry in the past two thousand years that need to be heard in the classroom. The ideal program should embrace elements of coordinate geometry, transformational geometry and vector geometry with the Euclidean geometry. Courses in Geometry should be available for a broad range of students. The student who

is capable of proof can benefit from analyzing and comparing proofs in the various approaches to the subject. Other students can learn from a more informal exposure. All students should be aware of the applications of geometry.

7. Problem solving is a major factor.
   Problem solving is mathematics and has been and will continue to be a major force in the curriculum. A computational approach to solving word problems using a calculator and computer favors the student who has trouble bridging the gap between algebra and arithmetic. Better students should be able to compare the advantages of algebra and calculus over some of the slower computational algorithms for the solution of problems.

The list above is a beginning. There are certainly other elements pervasive to mathematics. What would you add to the list? As you make your choice, be guided by the objective:

LET STUDENTS LEARN TO LOVE MATHEMATICS AND THEN THEY WILL LOVE TO LEARN MATHEMATICS.

MATHEMATICS AND POETRY

by
Professor Peter Taylor
Department of Mathematics and Statistics
Queen's University
Kingston, Ontario K7L 3N6

Mr. Gravelle's invitation to write this article was waiting for me when I returned home from attending the summer meeting of the CMESG (Canadian Mathematics Education Study Group). This is a rather fine organization consisting of mathematicians and educators from about thirty Canadian universities. It's a small group with about 60 members, some 40 of whom get to each meeting. We meet for 4 days and follow an intensive study group format so the meetings are quite intimate. At this meeting I was the co-leader of a group on **affective** aspects of learning mathematics: the idea is that how you feel about what you're doing has a large effect on how well you're able to do it. I guess we all know that this is often an important factor in the mathematics classroom; for a lot of students there's far more anxiety then joy.

So the invitation came at an interesting time because during the course of the meeting, I had thought more than once about the Math and Poetry course we run at Queen's. You see one of the big ideas we explored in the workshop was that one way to free the student from anxiety and open him or her up to the joy of learning was to replace the normally cool, competitve and rather mechanistic atmosphere in the mathematics classroom by one that was more human, open and supportive. And that's certainly one thing the poetry did in the Math and Poetry course. When you've shared a poem with your fellow students and your math teacher, and talked about how beautifully it captures some essential human experience, then perhaps you can more easily accept them as people to share in your struggle to do mathematics. Certainly, on looking back at last year's class, it was an open and responsive community we formed; I was aware of very little anxiety, and there were moments of true exaltation.

In order to give you a feeling for what the course is about, let me quote a few sentences from our "advertising" sheet.

> A poet who wished to entice his listeners towards the rewards of poetry would give them a short, tempting poem and have them read it, listen to it, play with it, study it, talk about it, and seek to understand how it is that the poet has achieved his remarkable effect.
>
> And this is also the way a mathematician should entice his listeners. The "poems" he should choose are elementary problems or intriguing patterns which cry out to be played with and understood.
>
> But this is rarely done in mathematics courses. Too often the student is presented with a comprehensive set of lectures about a fixed topic which exposes the heart of the matter whether he is ready for it or not - rather like a systematic review of all Milton's poems. That's fine for the committed scholar, but probably not for anyone else.

In this course, each three hour class will consist of two equal parts. In one, a poem will be presented, savoured, and discussed, and in the other the process will be repeated with a pattern of numbers or shapes. The emphasis will be on student exploration and discussion. The objective is to given the student greater understanding and appreciation, and increased technical proficiency, both as poet and as mathematician. Each student will be required, for each of the two subjects, to submit 3 or 4 papers which present a discussion or analysis similar to that done in class.

The course was conceived about four years ago, out of the idea that the activity of a poet wasn't fundamentally different from that of a mathematician, at least on the level of things like language, pattern, technique and beauty. I thought it might be interesting to explore that idea with a group of adventurous students. Not explicitly, of course: I never had any desire to talk about similarities and differences. I wanted simply to juxtapose the two activities in the same classroom and see what happened. I also had the feeling that while the mathematics would mostly be directed by me, the poetry could be something that came from the class; thus we'd each have something to offer and thereby the glaring asymmetry between student and teacher (more glaring, I think, in the natural sciences than in the social sciences or humanities) might be softened. So in that first year, we did mathematics for some 2 hours each week, and the students took the 30 minutes at the end to take turns reading selected poems, occasionally their own. And it worked pretty well, I guess. Well, the students though so, and that counts for something. Certainly it was interesting to watch the science and humanities students interact (there were about half of each).

But I was dissatisfied. I felt more leadership was needed with the poetry. I wanted to make the same kind of

"progress" in coming to grips with the poetry, as I was aiming to make with the math. And for that I felt we needed more authority. [And here I have always felt the classroom to be too authoritative - alas!] I wanted the math and poetry to be completely parallel. I was aware that it would mean abandoning the idea of the peotry originating from the students, but I thought it would be worth it.

So I looked for a colleague, and with a real stroke of luck, I met Bill Barnes of the Department of English. I was not five minutes into my description of what I was up to, and why and how, before he signed on. We have now taught the course together twice, and each time I have learned a huge amount.

One thing we try to do is to approach the class in the same manner, seeking to unite the two disciplines, not in terms of content but in terms of methodology. We entice the listener with a simple example that has some immediate appeal, but which hides something surprising, mysterious, or deep. The students talk about it, play with it, and hopefully discover, with a gentle nudge here or there, one or two of its subtleties.

One after another, each Monday evening, we do our own thing. I have learned an enormous amount about poetry from him, but more than that, from being a student of his, a student who is also a fellow teacher, I have learned a lot about teaching, mainly about the importance of involving students as intimately as possible in what's going on.

But this has led to some frustration too. Consider the picture: my fellow teacher has produced a moving poem about love and loneliness, and has spent an hour wrestling with it, and getting the students to read parts of it in their own way, to relate it to their own feelings, to try to say it themselves, to suddenly see how it is that the poet has created a sharp impression with just the right placing of rhythm or word. And when the hour is up, we have all shared something that has belonged to each one of us but is somehow magnificently bigger than us all. And then it's my turn.

How can I follow an act like that? Even the most highly polished geometrical gem, even the most perfect of perfect numbers, must surely pale beside that shared moment of love and loneliness. Or will it? Is mathematics so different after all? Is it not also something that belongs to each one of us but is somehow magnificently bigger than us all? But is that how it is for them? I stand presumptuously at the front, watching them return with their cardboard cups of vendamatic coffee. They sit and stare quizzically at me, wondering if I, or perhaps my stuff, will be up to scratch. And sometimes I wonder too.

Certainly my experience with this course has led to some real changes in my view of what constitutes progress in the classroom, and of the relation between content and process. A lot of this I have discovered by studying how poetry is taught, and the urgency of these changes has come from the challenge it has laid at my feet. Mathematics and Poetry: are they colleagues or competitors? I'm still not sure. Perhaps both.

Editor's Note: This article is reprinted with the author's permission, The Queen's Review, May-June, 1986.

1986 ASUS Teaching Awards

Peter Taylor (Mathematics) and Bill Barnes (English) are the co-winners of this year's Arts and Science Undergraduate Society (ASUS) Teaching Excellence Award.

John Rider, chairman of the ASUS selection committee, says Professors Taylor and Barnes were chosen for their "genuine desire to promote achievement and learning in an innovative and interesting manner. These professors showed they love and respect their students and their profession."

Prof. Taylor and Prof. Barnes have collaborated since 1982 on a course called Mathematics and Poetry. Says Prof. Taylor, "It is rare that you find arts and science majors in the same class, equally excited about the same questions."

THE POETRY FACTOR

by
Associate Professor W.J. Barnes
Department of English
Queen's University
Kingston, Ontario K7L 3N6

The 17th century poet, John Donne, once said, in a sermon: "God loves renovators, not innovators." Much as I love Donne and his poetry, I have long believed that this particular pronouncement contained a needless, or needlessly absolute, disjunction. There is, it seems to me, no inherent reason why those of us who "bring in new things" cannot, precisely by so doing, "make things new." I realize that Donne, perhaps anticipating our 20th century dismay at observing primary or secondary school systems through which educational fads and fashions blow almost continually, was anxious to avoid the dangers of spiritual (in our context read: educational) fickleness. But at the university level, at least in some areas, we are often in urgent need of renewal, and should not fear useful, positive innovation.

Something like this attitude was in my mind and heart when Peter Taylor asked me to consider taking on the main responsibility for the poetry side of this new course. I accepted his invitation almost immediately (as he says), though not without fear and trembling, since I had not done any serious work with mathematics since high school, apart from a course in Boolean logic which I took as a second year philosophy student in the fifties.

I accepted partly because I relished the intellectual challenge: I wanted to see whether my mind--accustomed to little more in the way of "math" than paying household bills or counting the lines in a sonnet or tapping out the metre of a lyric--could enter what seemed the arcane world of number theory and grasp at least **something** of the reality under discussion, even if my practical skills and basic knowledge in mathematics were almost certainly wanting. There was a conscious recklessness in this decision: no one wants to appear foolish in public, and many of the students adept at math would soon enough perceive my inadequacies, however much I might try to hide them behind a certain metaphysical loftiness of approach. But I reflected that these very limitations of mine would perhaps create a bond of authentic fellow-feeling between me and the weaker mathematicians as we together struggled to follow Peter's lead, on the one hand, and between me and the science-oriented students coming to the sophisticated study of poetry with some measure of apprehension and wariness, on the other. As things turned out, my hunch was confirmed: the students and I gradually, and less and less self-consciously, acknowledged the truth of Dr. Samuel Johnson's 18th century dictum: "Every man has a natural right to be ignorant."

I accepted partly because of the pedagogical challenge: always the evangelist for poetry, I wanted to see whether I could gently lead non-English majors and non-Arts students to realize that poetry's excitement arises not only from its appeals to our capacity to think, to feel and to imagine (what they all at some level already knew) but also from its technical side, **how** the poet manages the resources of language--from punctuation marks to sounds to the very texture or tactile "feel" of a work--in order to accomplish his "meaning," which can never be divorced from technique (what they were often skeptical about in the beginning, though most were more or less converted by the end).

In addition, I relished the proferred opportunity to work with students--and a professor--whose interests were wide-ranging and interdisciplinary, for I have often though that most university systems do far too little to encourage and foster such students in their attempts to make sense, inside and outside their formal studies, of the diverse and apparently conflicting "worlds" they experience. In the event, I was privileged to meet, for example: a biology major doing classics courses, a history major interested in chemistry, a physics student who wrote fairly decent poetry, a math student who painted in her free time, a music major interested in computers, a law and education student discovering a first-time interest in poetry. These cases were the norm, not the exception, in this course. The rewards of working with such students have been many and profoundly satisfying.

My method is (and will probably remain) to give a general talk at the outset about the kinds of issues mentioned above--**what** poetry typically does and **how** it usually goes about doing it. Afterward, to the end of the term, I bring to class handouts of individual poems of varying degrees of difficulty, worthy of our intellectual and emotional attention but also illustrating or highlighting one or more of the technical elements enumerated in my opening lecture. Because of the time constraint, the poems tend to be short lyrics, the shortest being two lines long (Frost's "The Span of Life"), the longest perhaps fifty (Al Purdy's "Further Deponent Saith Not"). I try to choose poems that are fairly accessible and appealing, but I make no significant concessions in terms of difficulty, remembering George Whalley's words: "With poetry, there are no lessons for beginners." One cannot simplify the inherently complicated.

The poems tend usually (but not exclusively) to be modern or contemporary; and I include an appropriate Canadian component. Occasionally I use a poem of my own or of a student, provided I think that it will stand up under sustained scrutiny and analysis. Students quickly learn, for example, to discard the intentional fallacy (the notion that the poem's meaning must be restricted to what the poet intended to mean by it) when they find that they see--

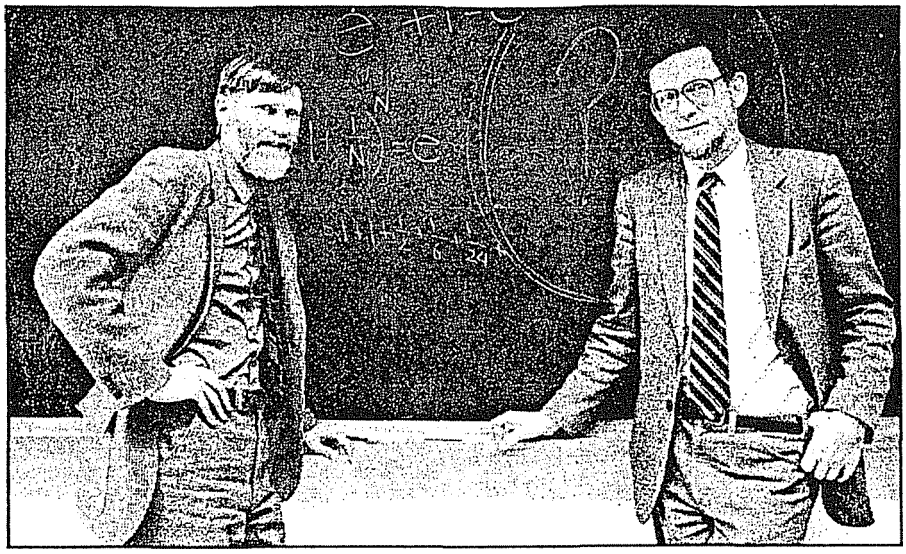
and properly see--meanings or suggestions or overtones in the poem which I or their fellow undergraduate did not notice when writing it, and was not aware of "putting into it." And there are many other good reasons for having the poet in the classroom when the poem is being discussed. So far, I have not sensed the need for a text (though the Norton **Introduction to Poetry** might serve effectively if one were setting forth on a project like this) or for a glossary of critical terms. In the latter case, I believe that one must steer a middle course between not providing any traditional terminology at all and providing a maze of labels and forms so complex and intricate that the newcomer turns away in discouragement or incredulity.

After analysis, synthesis. Having pulled the poem apart in several directions, having noticed how its images or its sound effects or its stanzaic form or its structure contribute to its "meaning," we try to arrive at a "re-integrated reading" of it, a fresh and more comprehensive exposure to the total experience which the poem offers us. Though such a full, and fully alert, reading of a peom is never actually possible, we regularly hold up to ourselves this ideal; and we try to reach it by reading the poem aloud. This ritual "closure" (or closing ritual) is performed not just by me or Peter but often by the students themselves. Then, together we watch and listen, fascinated, as the poem, unlike Humpty Dumpty, jumps back together into its whole organic life, after our necessary and non-lethal dismemberment of it. The magic which is poetry dances itself out again.

Finally, a few words in response to Peter's. First, I think that he has been too modest in his assessment of his own "performances" in this course. In both my roles, as student and colleague, I have seen him agonize over his contribution to the class, observed him struggle on the spot to adjust his purposes or his materials to the "psychodynamics" of that particular session. I have watched a resourceful teacher draw in history ("Pythagoras knew about this." "You're asking an 11th century question, there."), bring to class pine cones or sunflowers (apparently the pattern of the arrangement of their parts conforms to that of a Fibonacci number sequence) or figuratively stand on his hands in order to catch and hold our students' attention. Moreover, his own deep interest in the writing of poetry and drama means that I often feel that he grasps more of my "stuff" than I grasp of his (though in fairness to me it must be said that he requires little in the way of new, specialized language in order to cross over into "my" territory).

Second, it's worth reading that because neither of us is wedded to a rigid set of lectures, we can respond, in both content and style, to what the other has just done or has announced he will do next week. I believe that this feature of the course accounts for a substantial part of the students' enjoyment of it: they see that two minds deeply engaged can induce lively "sparks" in each other, and that a genuine intellectual **discipline** can paradoxically leave room for thought to "do a wheelie" out and around, and then come back, refreshed from what **seems** a digression, to the original point. This, as I experience it, is the way a poet works; and I'm beginning to perceive (though the tools of my insight are but crude) that this how the mathematician works as well. For both, as for us and our students, the process is attended by a certain elegant delight.

Successfully mixing English and Math



Lisa Lowry

ASUS teaching award winners Peter Taylor, Mathematics (left), and Bill Barnes, English. The two professors collaborate on a unique course.

TEACHING THE NEED FOR PROOF

by
Eric Wood
Althouse College, University of Western Ontario
London, Ontario N6G 1G7

The new advanced level mathematics guidelines contain some interesting discussions about what are called "process components". In particular, there is no doubt that the notion of proof is a significant and important strand. Although the document does admit that not all students are ready for some of the subtleties of proof it does suggest that

> the processes of proof should be developed by challenging students to justify statements, to search for all cases, to look for counter examples, and to demonstrate the existence of a number of geometric figure with certain properties.[1]

This article provides the teacher with a number of problems and paradoxes that do precisely this; challenge the students to think rather than blindly accept what seems obvious. In addition, the fact that difficulties do arise for students who assume that their intuition is as good as a proof helps to establish the **need** for proof. I have found that students are much more likely to try to prove things once they realize that it is not just an empty exercise.

Problem: It is 300 km from Cocoa Beach to St. Petersburg.
If you drive from Cocoa Beach To St. Pete's at 100 km/h and back at 80 km/h, what is the average speed for the trip?

Intuitive Answer: 90 km/h.

To really make the point about this question it is necessary to pursue the answer after the students offer it. Rather than commenting on the correctness of the answer let us proceed to check it.

Time to drive to St. Pete's is $\frac{300}{100} = 3$ h.

Time to drive to Cocoa Beach is $\frac{300}{80} = 3.75$ h.

If the total trip took 6.75 hours, then at an average speed of 90 km/h we would have covered $6.75 \times 90 = 607.5$ km.

Clearly there is a problem somewhere, perhaps the intuitive answer is not correct after all. Let's do the problem a little differently:

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{600}{6.75} = 88.9 \text{ km/h.} \quad \text{m/h.}$$

This is in fact the correct answer to the question and serves to show that unless we make careful calculations, or prove our statements, the obvious answer may be incorrect.

It is possible to get even more out of this question if we ask under what conditions (if any) the intuitive answer is correct.

If the speeds of the two trips are s_1 and s_2 and the times required to complete a journey of D km are t_1 and t_2 respectively, then assuming that both of the previous methods are equivalent we get

$$\frac{\frac{s_1 + s_2}{2}}{\frac{2D}{t_1 + t_2}}$$

$$\frac{s_1 + s_2}{2} = \frac{2D}{\frac{D}{s_1} + \frac{D}{s_2}}$$

Dividing out the D, adding fractions and cross multiplying yields

$$(s_1 + s_2)^2 = 4 s_1 s_2$$

$$(s_1 - s_2)^2 = 0$$

Therefore the "obvious" method only works if the speeds (or equivalently the times) are the same.

Problem: Two containers each have 1000 ml of liquid in them. The first contains water, the second vinegar. A teaspoon of water is taken from the first jar, placed in the second jar and mixed well. Then one teaspoon of this water-vinegar mixture is put into the water jar and mixed in. Both containers now hold 1000 ml of liquid again. Is there more water in the vinegar jar or more vinegar in the water jar?

Intuitive Answer: Since the spoonful from the vinegar jar was slightly diluted by the water that was previously added, there is not a full teaspoon of vinegar in the water jar. Therefore there is more water in the vinegar jar than vinegar in the water jar.

Once again, rather than trying to convince the students that they have the right logic or the wrong logic let's work out the question to prove that our intuition is correct. Just make the work a little easier let's assume that a teaspoon holds 5 ml of liquid. After the teaspoon of water is added to the vinegar jar the ratio of vinegar to water is 1000 : 5 or 200 : 1. We can assume, since it was well mixed, that the same proportions will be present in the teaspoon of mixture that is now returned to the jar. This means that this teaspoon will contain $\frac{200}{201}$ X 5 ml of vinegar and $\frac{1}{201}$ X 5 ml of water. Therefore after all the changes have been made Jar #1 will contain $995+\frac{5}{201}$ or $\frac{200000}{201}$ ml of water and $\frac{1000}{201}$ ml of vinegar. On the other hand, Jar #2 contains $1000-\frac{1000}{201}$ or $\frac{200000}{201}$ ml of vinegar and $5-\frac{5}{201}$ or $\frac{1000}{201}$ ml of water. The amounts of foreign substances in each jar can be seen to equal!

Even though our intuitive answer is incorrect it is worth noting, especially to the students, that there is absolutely nothing wrong with using intuition in Mathematics. As a matter of fact it is to be encouraged. However, an intuitive guess should always be investigated by more precise methods because unless a guess is proved to be true we can never be sure of its accuracy.

Both of the previous examples are drawn from Algebra, but Geometry also provides us with a rich source of paradoxes. In figure 1 is an accurately drawn figure. If it is carefully traced with carbon paper onto thick cardstock and then cut out it can be used for a most interesting problem. Incidentally, do not use a photocopier to magnify these diagrams as I have found there is some significant amount of distortion introduced by the optics of the machine.

The pieces of the puzzle are arranged as shown in the original diagram and the base and the altitude of the triangle are measured. Calculate the area of cardstock using these values. Now re-arrange the pieces as shown in figure 2. Measure the base and altitude again and it can be seen that they are exactly the same as they were before! Since the area of the two triangles is the same, where did the cardstock that used to fill the hole disappear to?

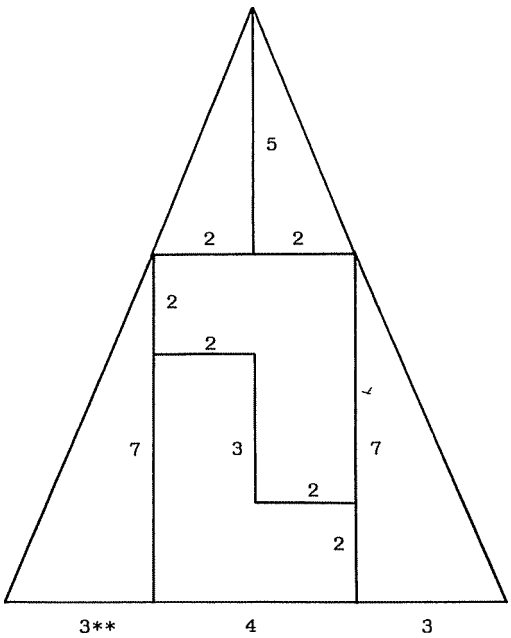
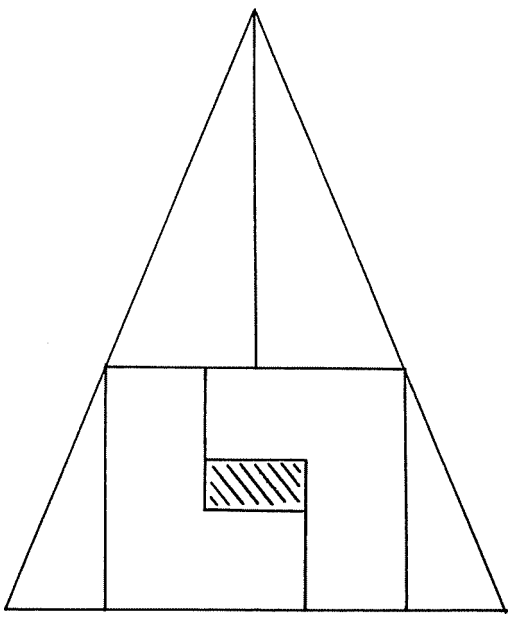


Figure #1


Figure #2

** All dimensions are in cm in this diagram

This puzzle and many others like it can be found in a delightful little book by Martin Gardner called **Mathematics Magic and Mystery.** The puzzle is quite intriguing and only after some time would I have students sight along one side of the "triangle". Placing a ruler along one side of the original "triangle" will also show the problem. The figure is not a triangle at all, it only appears to be so when given a quick glance. In addition, we are expecting it to be a triangle so we naturally assume that it is! The second figure has the area of the hole distributed over its surface and since its shape is almost the same as the first figure, it is not noticed. The point to be made once again is that things that appear to be obvious do in fact require proof. If we neglect to prove things it can result in contradictions to beliefs that we know are true, for example, cardstock does not just disappear into thin air.

Another nice geometric contradiction is illustrated in figure 3. If quarter inch graph paper is used to draw the square it will made a very good demonstration. When the original square is cut along the lines drawn and reformed to make a rectangle as shown in figure 4, the area has grown by 1 square unit!

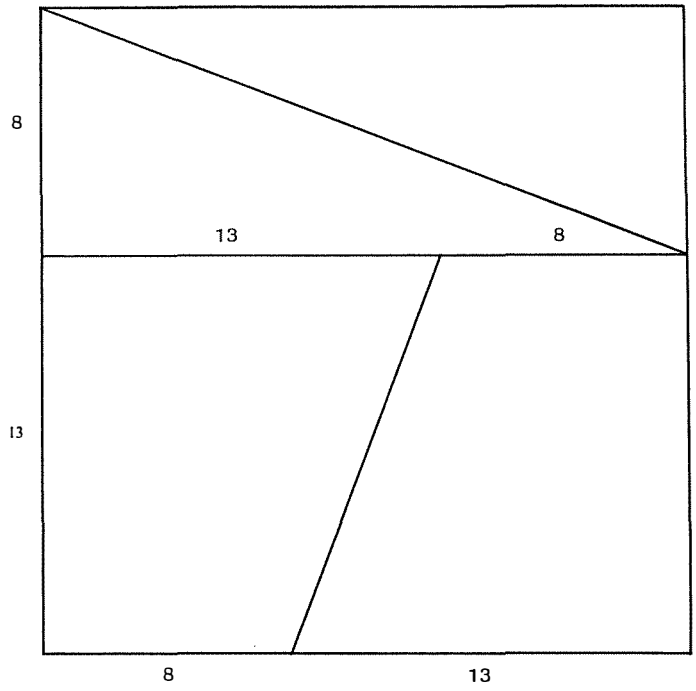


Figure #3

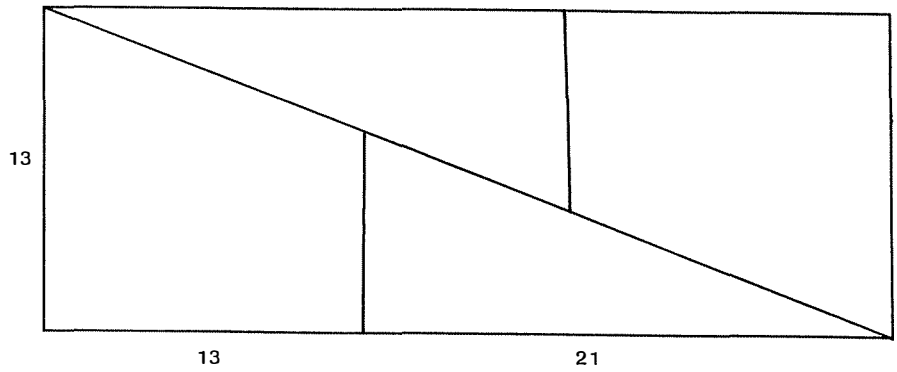


Figure #4

In this case there is actually a very thin parallelogram shaped space along the "diagonal" of the rectangle. However, since the total area of this space is 1 square unit compared to the 441 square units of cardstock it takes to make the original square, it is not noticeable. This type of puzzle has often been published using an 8 by 8 square that becomes a 13 by 5 rectangle. I believe that the dimensions given here are even better, however, since the proportion of space is so small.

Although this is interesting, what is its relevance to the value of proof? Again it is important to try to get the students to pinpoint what the problem is and to try to establish what assumptions are being made that could possibly be invalid. In this case we assume that the "diagonal" of the rectangle that is formed is a straight line and it turns out that this is not the case.

This can be established quite nicely by use of the cosine and sine laws. Considering the triangles in figure 5 it can be seen that in $\triangle$ DCE

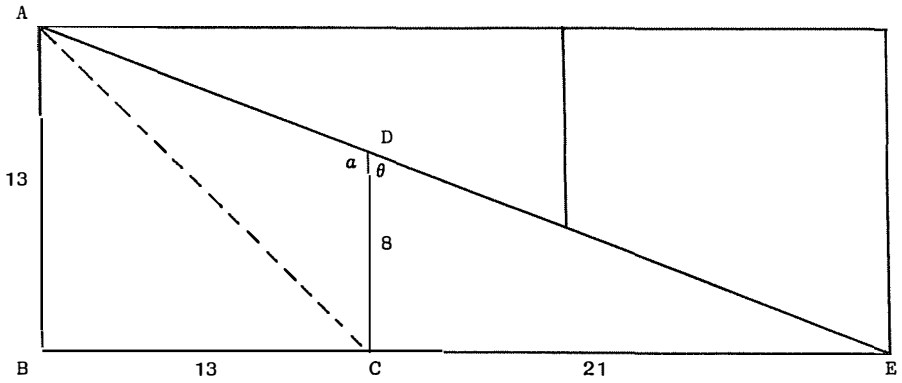


Figure #5

In ΔABC, $AC = 13\sqrt{2}$, $\angle ACD = 45^{\circ}$ so the cosine law yields

$$AD^2 = (13\sqrt{2})^2 + 8^2 - (2)(13\sqrt{2})(8)\left(\frac{1}{\sqrt{2}}\right)$$

$$AD = \sqrt{194}$$

Now using the sine law on ΔADC we get,

$$\sin\alpha = \frac{(13\sqrt{2})(\sin 45^{\circ})}{\sqrt{194}}$$

$$= 0.9333456$$

$$\alpha = 68.962489^{\circ} \text{ or } 111.03751^{\circ}$$

Therefore $\angle ADE = 69.14552^{\circ} + 111.03751^{\circ} = 180.18303^{\circ}$.

This proves that the diagonal is **not** a straight line; but it can also be used to calculate the area of the parallelogram of space contained between the two halves of the figure. In figure 6 the parallelogram is redrawn by itself (not to scale of course) with letter labels as they were in the previous diagram. Since the area of a parallelogram is the product of the base and the altitude it is necessary to calculate the values of AX and DE.

$\angle XDA = 180.18303 - 180 = .18303^{\circ}$. Hence $AX = \sin(.18303^{\circ})(\sqrt{194})$
$= 0.0444941$. Also by the Pythagorean theorem $DE = \sqrt{505}$.
Hence the area of the parallelogram is given by $0.0444941 \times \sqrt{505}$
$= 0.9998807$ square units.

Without a shadow of a doubt we have found the extra 1 square unit of area, and the mathematics has **proven** that

there is no contradiction in a way that the eye never can.

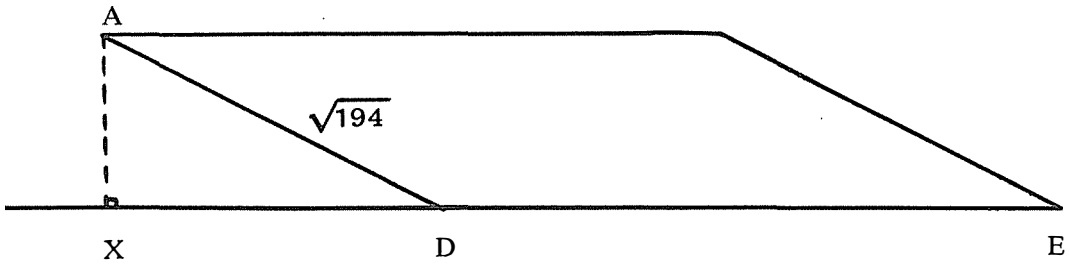


Figure #6

Speaking of using our eyes, measure the two lines in figure 7. Amazing isn't it? This is a particularly effective strategy to use with students because they are so absolutely dumbfounded that they will check and re-check their measurements rather than admit that their senses are fooling them. Once again the fact that we cannot make unjustified assumptions is emphasized. Saying that two lines are equal in length because they "look" equal is certainly not sufficient since our perception is greatly influenced by the size of the other things around the particular thing that we are looking at.

I am not suggesting for a moment that doing these exercises with your students will immediately improve their ability to do geometry proofs. I do think, however, that they will help students to appreciate that proving statements is a **necessary** part of the mathematical process and not just an exercise for its own sake.

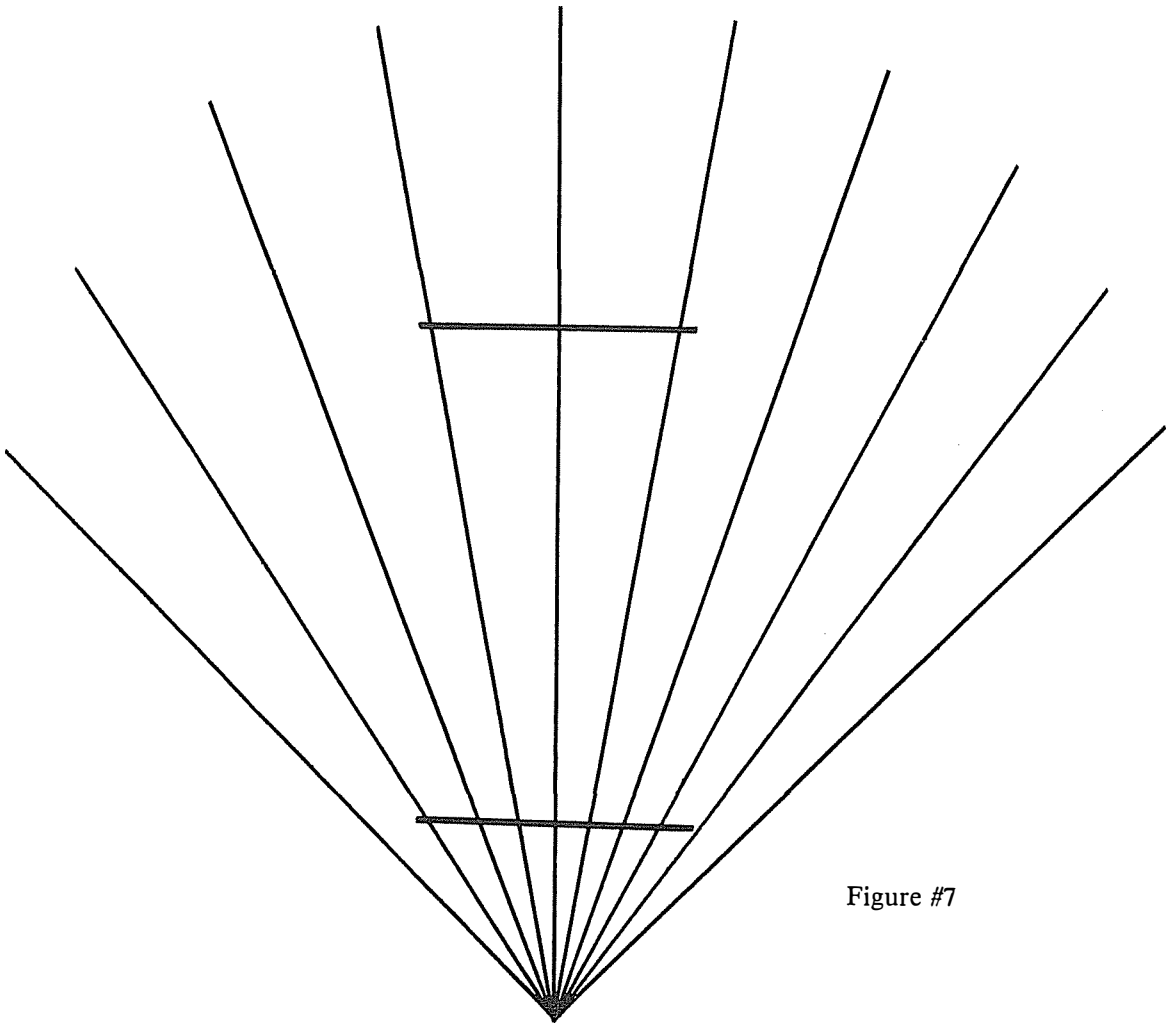


Figure #7

References

1. Mathematics, Intermediate and Senior Divisions, 1985, Ontario Ministry of Education, Toronto.

2. Gardner, Martin. **Mathematics Magic and Mystery.** New York: Dover Publications Inc., 1956.

"MACPAINTING" THE AREA UNDER A CURVE

by
Jack Weiner
Department of Mathematics and Statistics
University of Guelph
Guelph, Ontario N1G 2W1

One of the joys of a Macintosh computer is the ease with which it eliminates one of the difficulties of teaching.

Often, when presenting a new, interesting topic, a teacher may become bogged down with notation, words, and gobbledygook (spelling?). In doing so, the teacher would expect the impossible of students. They must simultaneously

(i) take down copious notes,

(ii) listen carefully to what **THE INSTRUCTOR** is saying,

(iii) answer questions,

and, of the utmost importance,

(iv) catch the teacher's inevitable mistakes.

(Incidentally, I find it very useful at the beginning of the semester to emphasize to my students that I will intentionally make an error at least once per class. I explain that it is the duty of the class to catch and correct all such mathematical or arithmetical transgressions. This tactic serves two purposes. First, the students have extra incentive to keep up to date and actively participate in the class. Second, I cover my, uh, well, ..., for this article, let's say the tactic serves one purpose.)

Of the four tasks outlined above, the first is not only the least important, but also the most responsible for preventing students from effectively accomplishing tasks (ii), (iii), and (iv). They are too busy writing! Do you remember, in your student days (long, long ago), thinking, "I don't know what she (or he) is talking about. I'll sort it out at home later!" Later comes at 3:00 a.m., when you can't sort out the notes because the teacher made an error. Unfortunately, you're not exactly sure where or what it is. "If only I was in class now, I'd know exactly what to ask to figure this stuff out!"

Enter the Macintosh. The Macpaint program makes it very easy, and a lot of fun, to prepare visually appealing worksheets, with matching transparencies. These are designed with two objectives.

(1) Eliminate the need to write down all that stuff that must appear in the notes, but adds little to the students' understanding of the material.

(2) Place boxes, ovals, or circles, to be filled in by the students as the topic is developed, where writing down an answer will facilitate their grasp of the ideas.

The students are not so busy copying that they cannot listen. You are not standing in silent vigil at the board, waiting patiently for your group to catch up to you. You cover the material more efficiently; the students listen more effectively; and with careful questioning, you get interactive participation, even with those students who shy away from verbally answering your questions. They lightly write their answers in the spaces provided, making any necessary corrections as you fill in the transparency.

This is the first of a series of articles, each of which will provide two or more of these Macpainted worksheets. I hope you find them useful and effective. Often the worksheet will be self-explanatory. Other times, I may include some suggestions for their use.

In this issue, I present a two page development of the area between $y = x^2$ and $y = 0$, from $x = 0$ to $x = 2$. After step 5, "chalk and talk" your students through the formation of the sum of the areas of the rectangles into a neat (preferably still factored*) formula. Finally, let n embark on its long trek towards $+\infty$.

If you use this material and have any suggestions for improvements, **PLEASE** let me know.

*The sum of the rectangular areas, after applying the formula for the sum of the squares from 1 to n, is

$$8(n)(n+1)(2n+1) / (6n^3) . \qquad (1)$$

Some students want to expand the numerator, and then divide by $6n^3$, eventually obtaining

$$\frac{8}{3} + \frac{12}{n} + \frac{4}{3n^2} . \qquad (2)$$

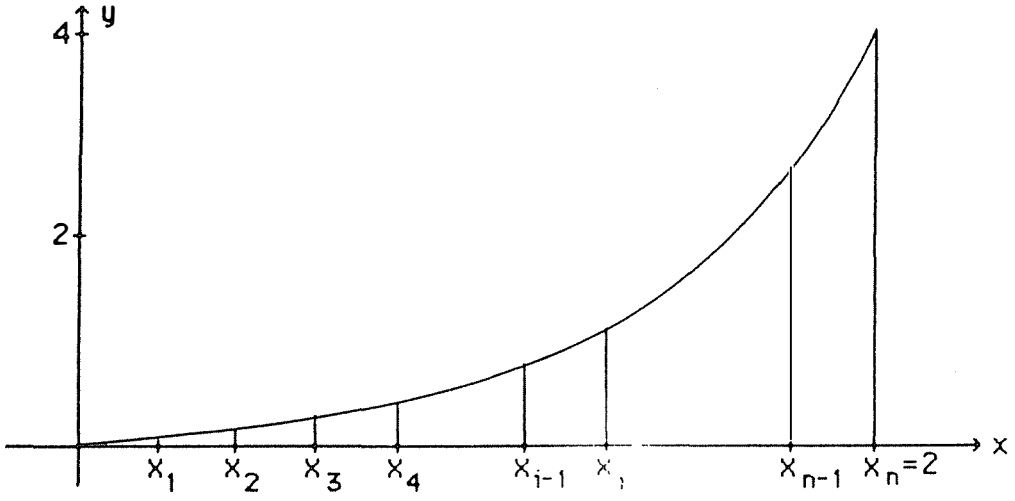
I find it much more pleasant to divide the three factors of n in the denominator into the three factors in the numerator containing n, yielding

$$\frac{4}{3}(1)(1 + 1/n)(2 + 1/n) . \qquad (3)$$

Taking the limit as n approaches positive infinity is a simple affair in either expression (2) or (3). But the trip from (1) to (3) is, I think, mathematically more scenic than the journey from (1) to (2).

AREA UNDER A CURVE

PROBLEM: Find the area between $y=x^2$ and $y=0$ from $x=0$ to $x=2$.



STEPS

1. Divide [0,2] into n subintervals, each of length

$$\frac{2-0}{n} = \frac{2}{n} , \quad n \in N.$$

2. Label the endpoints of the subintervals:

$x_0 = 0$, $x_1 = 2/n$, $x_2 = 2(2/n) = 4/n$,

$x_3 = 3(2/n) =$ [ ], $x_4 =$ [ ] $=$ [ ],

..., $x_i =$ [ ], ..., $x_n =$ [ ] $=$ [ ].

3. Choose a special number z_i in the i'th subinterval.

$z_1 \in [0, 2/n] = [x_0, x_1]$ = first subinterval,

$z_2 \in [2/n, 4/n] = [x_1, x_2]$ = second subinterval, ...

$z_i \in [2(i-1)/n\ ,\ 2i/n\ ] = [x_{i-1}, x_i]$ = i^{th} subinterval, ...

$z_n \in [2(n-1)/n\ ,\ 2\ ] = [x_{n-1}, x_n]$ = n^{th} subinterval.

FOR OUR EXAMPLE, CHOOSE
z_i = The Right Endpoint of $[x_{i-1}, x_i] = x_i$

Therefore, $z_1 =$ [x_1] $=$ [$2/n$],

$z_2 =$ [ ] $=$ [ ], ...

$z_i =$ [ ] $=$ [ ], ...

$z_n =$ [ ] $=$ [ ].

4. Form n rectangles.

First: Base $= x_1 - x_0 = 2/n$; Height $= f(z_1) = (z_1)^2 = (2/n)^2$,

Second: Base = ☐ = ☐

Height = ☐ = ☐ = ☐ , ...

i'th: Base = ☐ = ☐

Height = ☐ = ☐ = ☐ , ...

n'th: Base = ☐ = ☐

Height = ☐ = ☐ = ☐ .

5. Determine the area of these n rectangles.

$A_1 = f(z_1)\Delta x_1 = (2/n)^2(2/n)$,

A_2 = ☐ = ☐ , ...

A_i = ☐ = ☐ , ...

A_n = ☐ = ☐ .

(Δx_i = width of i'th rectangle or subinterval)

JUST PUT IT IN A BOX: I'LL TAKE IT WITH ME

by
James Boyd
Retired Head of Mathematics
Loyalist Collegiate and Vocational Institute
Kingston, Ontario

The first task in preparing this paper was to choose a title. I first considered "Why can't people be more like computers"? Then I decided it should be "Why can't computers be more like people"? So I abandoned both and used the above title.

The idea that I'm going to outline first occurred to me, when I began to learn a little about computing, back in the days when key punches and Hollerith cards were the latest thing.

The statement (in FORTRAN) X = X + 3 was well calculated to cause a mild form of apoplexy for most Mathematics teachers, to whom it represented either an impossible equation or an abuse of the equals sign.

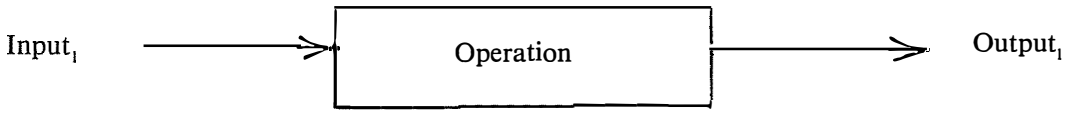
However, if we consider the following conversation between a talking computer and the user, we can see the correct meaning of this statement.

User to computer: Remember that X I told you about?
Computer to user: Yes, at present it is indexed at zero.
User to computer: Well, I would like you to reassign that value by adding 3.
Computer to user: Fine, it is now equal to 0 + 3.

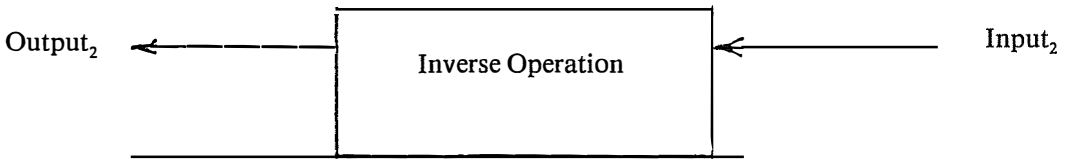
Round 1 of the conversation concludes with both parties happy. This gripping tale can be extended to a serial with n or fewer episodes.

Based on this idea we introduce the idea of an operations box.

First we consider the fundamental operation box



Then we consider the inverse operation box

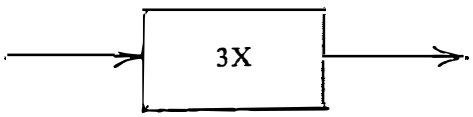


The boxes will represent inverse boxes if when we equate output$_1$ and input$_2$ we get output$_2$ = input$_1$.

Consider the following operation boxes.

Fundamental Operation

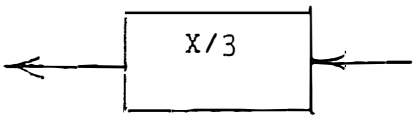
Multiplication



Box instructs "user" to take input X and output 3 times X for all acceptable X's.

Input	2	a	t	x/3	-1/2
Output	6	3a	3t	x	-3/2

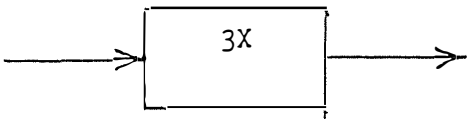
Question: What operation would use the above output to produce the above input? Obviously a division box.

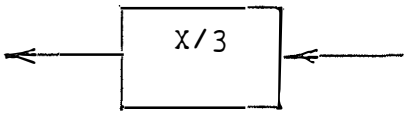


Input	6	3a	3t	-3/2	X
Output	2	3	t	-1/2	X/3

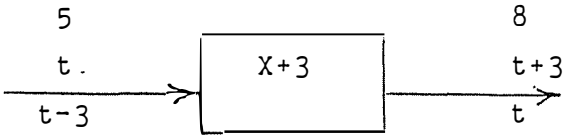
Fundamental Operation **Inverse Operation**

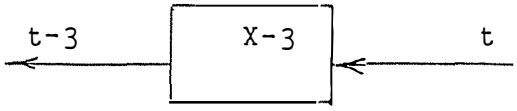
Multiplication



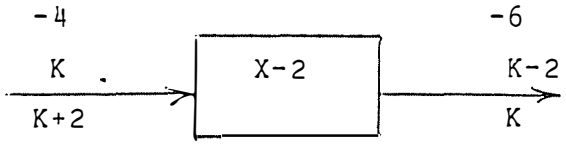


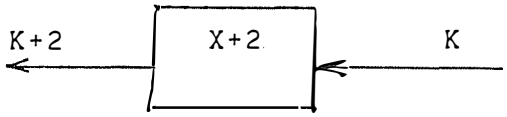
Addition



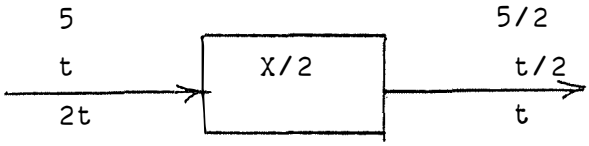


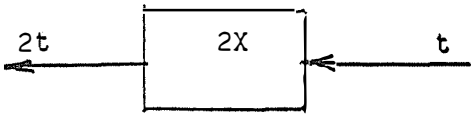
Subtraction



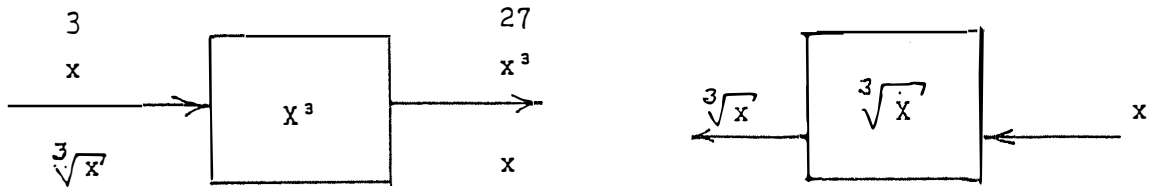


Division

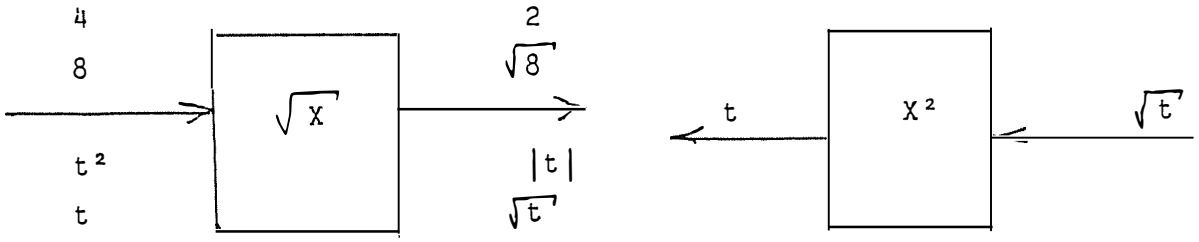




Power



Root



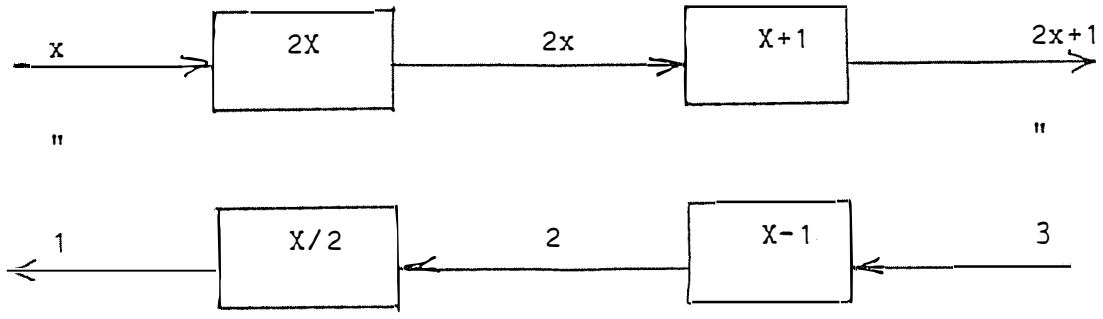
Obviously with the root box, we will need to decide which root or both? Since we are not computers we will be able to decide in context.

Not surprisingly the inverse operations match up as usual.
Addition ⟷ Subtraction
Multiplication ⟷ Division
Power ⟷ Root

Where can this idea be put to use in the Mathematics curriculum? I will deal with five topics.

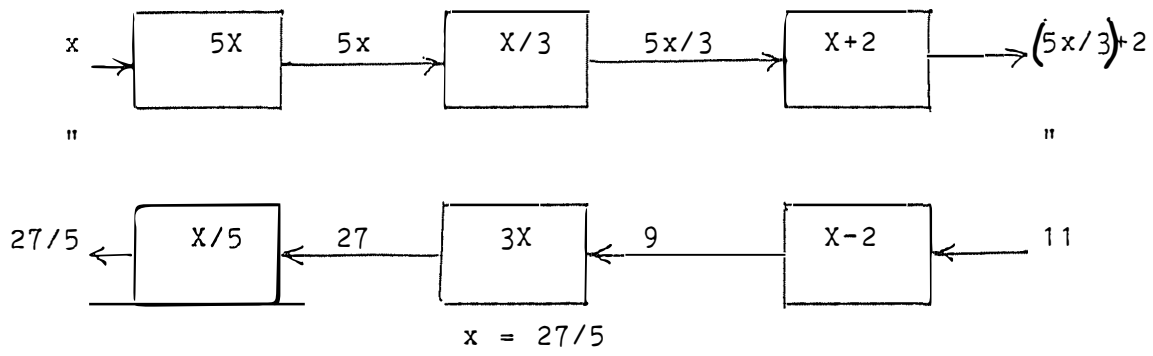
1. Solving linear equations.
The use of boxes motivates an understanding of "undoing" or "inverse" operations and has worked well with general level and basic students. I will illustrate the method with examples.

Example 1. A numerical example of a x + b = c.
2 x + 1 = 3.
(1) build up the left side with operation boxes (two).
(2) replace left side with right side.
(3) pass right side through inverse boxes (in reverse order).



If 2 x + 1 = 3 then x = 1.

Example 2. $(5/3)x + 2 = 11$

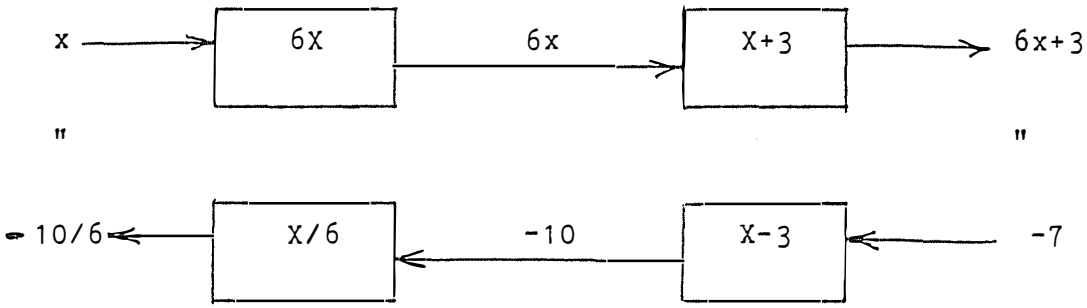


To illustrate further and also to partially show the transition back to traditional equation solving consider example 3.

$5X-7 = 11x + 3$ This is not of form $ax+b=c$.

Proceed as follows: Underline "largest" x term,
✓ Check mark term to "move".

$5x-7-5x = 11x+3-5x$
$-7 = 6x+3.$



x = -5/3

My purpose here is not to deal exhaustively with equation solving. The use of the box is meant to introduce the topic and to motivate concepts of equation solving such as:

1) Equal treatment of both sides as guaranteed by the Bill of Rights.
2) Inverse or undoing operations to achieve desired simplifications.

Placing the stages of such a solution in a vertical arrangement, reversing one set of arrows will accomplish this.

i.e. In example 1 $2x + 1 = 3$
$2x = 2$
$x = 1$

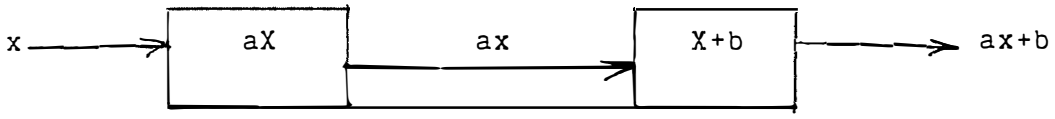
example 2 $5x/3 + 2 = 11$
$5x/3 = 9$
$5x = 27$
$x = 27/5$

example 3 $5x - 7 = 11x + 3$
$-7 = 6x + 3$
$-10 = 6x$
$-10/6 = x$

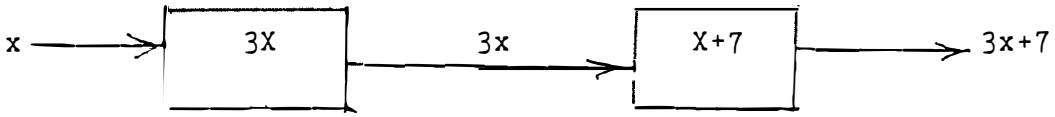
If for example 4 I choose 2x/3 - 7 = 5x/4 + 2/5, I might easily motivate multiplication by 60 as my priority operation.

2. Evaluation of some functions (relations).

(a) Linear functions y = a x + b.



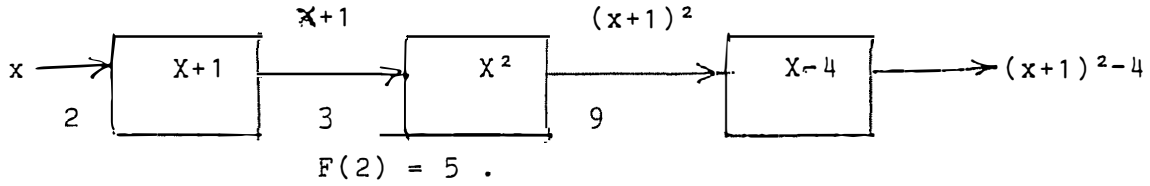
Example 1. f(x) = 3x + 7.



Use an input 2, -4, 2/5, 3t, k/4, g(x). etc. to discover f(2) = 13; f(-4) = -5; f(2/5) = 41/5; f(3t) = 9t + 7; f(k/4) = 3k/4 + 7; f(g(x)) = 3g(x) +7* *We intend to use this result later.

Since this is very similar to the equation solving application I will not discuss further examples of linear functions.

(b) Quadratic function f(x) = x^2 + 2x-3. We again encounter the problem of operations which cannot be uniquely ordered. If we change x^2 + 2x-3 to $(x + 1)^2$-4 we can proceed as before.



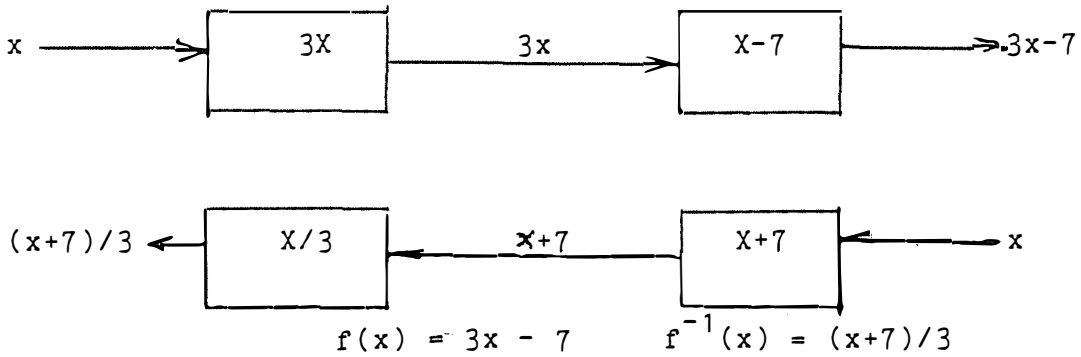
This particular application is not really too useful or creative.

3. The inverse of a function.

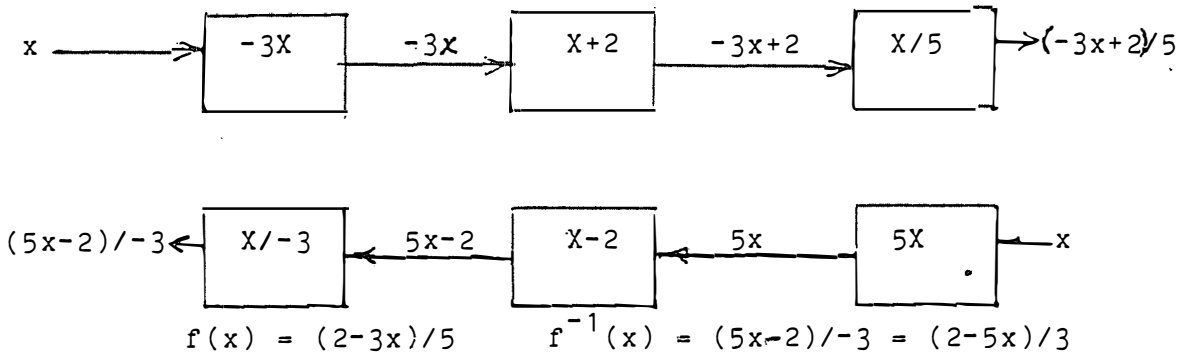
The inverse of a function whose operations may be uniquely ordered can be found by using the boxes in two different ways.

Method 1. Build up the function with operation boxes.
2. Pass x through the inverse boxes in the reverse order.

Example 1. f(x) = 3x - 7 Find $f^{-1}(x)$.



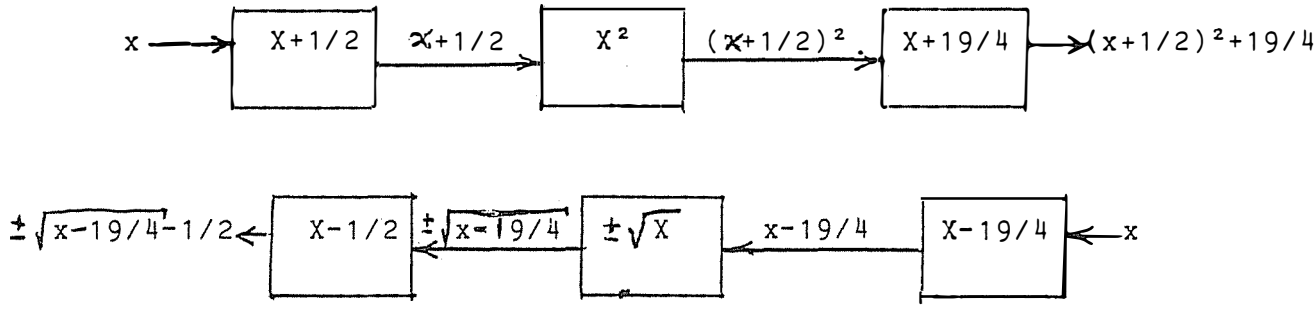
Example 2. $f(x) = (2-3x)/5$ find $f^{-1}(x)$.



At this time I might note that numerical values can be passed through the boxes in order to evaluate things like $f^{-1}(-3)$ or $f^{-1}(2)$ etc.

For example 2. If we pass -3 through inverse boxes we get 17/3. Check that if we pass 17/3 through the function boxes we get -3.

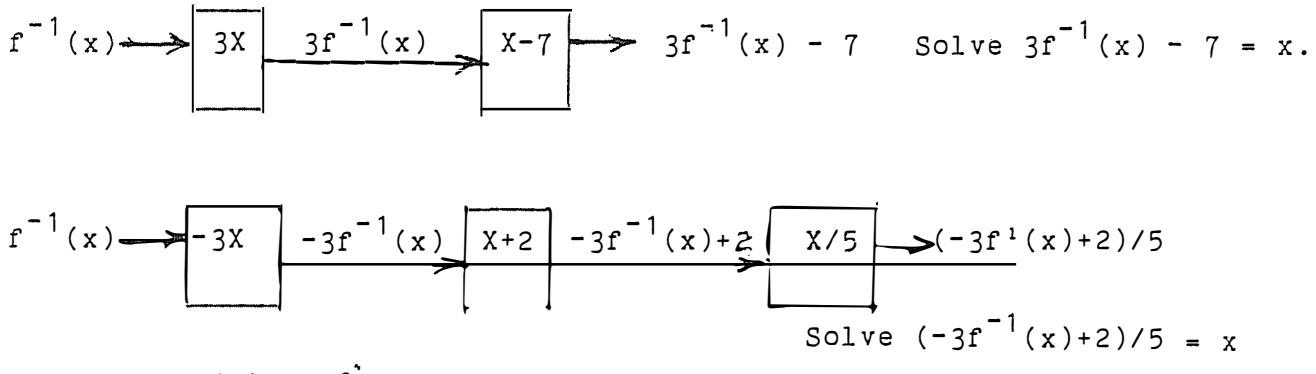
Example 3. $f(x) = x^2 + x + 5 = (x+1/2)^2 + 19/4$.



We know the inverse of this function is not a function but can be split into two functions by using ± with the square root and correctly choosing domains and ranges.

Method 2. Many tests define f^{-1} by using
$f(f^{-1}(x)) = f^{-1}(f(x)) = x$. This can be applied by passing $f^{-1}(x)$ through the function boxes, equating the output to x and solving the resulting equation.

Example 1. $3x - 7 = f(x)$ **Example 2.** $f(x) = (2-3x)/5$



For $f(x) = x^2 + x + 5$ simply use

$[f^{-1}(x)]^2 + f^{-1}(x) + 5 = x$ and solve using the quadratic formula

$f^{-1}(x) = (-b \pm \sqrt{D})/2a$ where $a = 1,\ b = 1,\ c = 5 - x$

$D = b^2 - 4ac = 4x - 19$

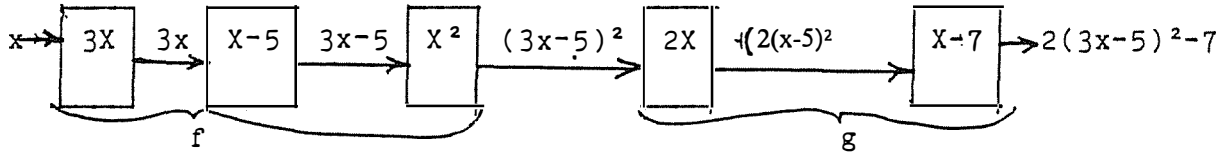
$f^{-1}(x) = (-1 \pm \sqrt{4x-19})/2$.

4. Composition of functions.

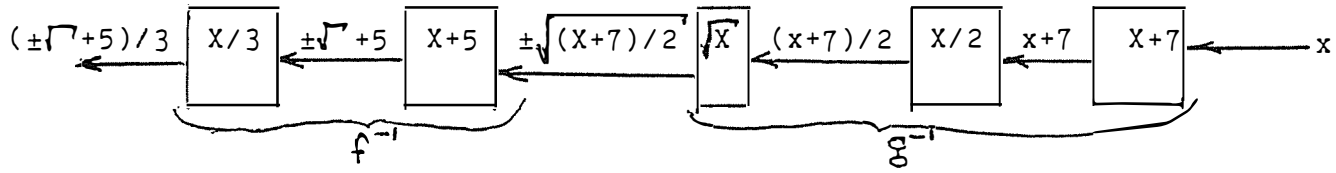
I use the boxes here for introduction and condense the method later. I also get a good appreciation of the inverse of a composition by reversing the chain through inverse boxes, also clearly illustrating the relationship $(gof)^{-1} = f^{-1}og^{-1}$.

For my example I will use: $f(x) = 3x - 5$ (linear) and $g(x) = 2x^2 - 7$ (Quadratic, but operations are uniquely ordered).

fog means f after g and gof means g after f.

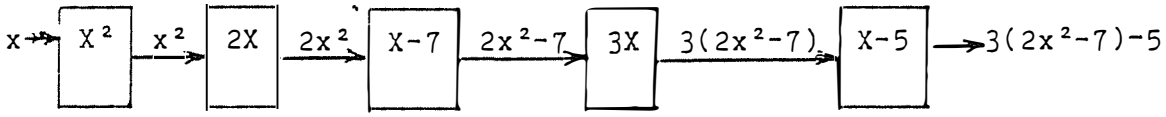


$gof(x) = 2(3x-5)^2 - 7$

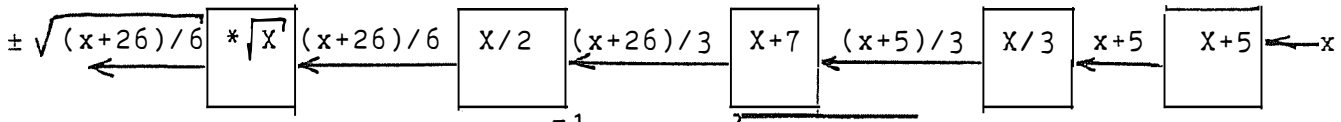


Passing x through the inverse boxes in the reverse order produces $(gof)^{-1}(x) = (\pm\sqrt{(x+7)/2} + 5)/3$.

Note how diagram shows that $(gof)^{-1}$ is $f^{-1}og^{-1}$.



$(fog)(x) = 6x^2 - 26$

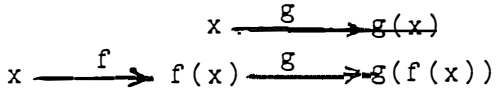


$(fog)^{-1}(x) = \pm\sqrt{(x+26)/6}$

Again by bracketing the operations belonging to each function or inverse we can see $(fog)^{-1} = g^{-1}of^{-1}$.

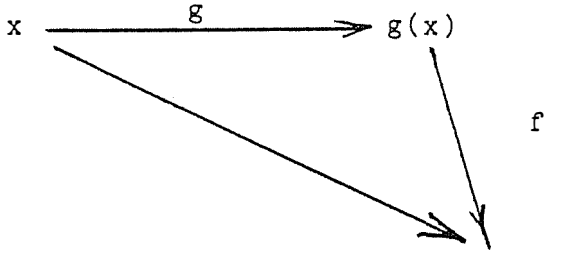
Aside: To complete composition I use either the "layered look" or "around the corner" to condense the composition.

The Layered Look for gof



$(gof)(x) = g(f(x))$

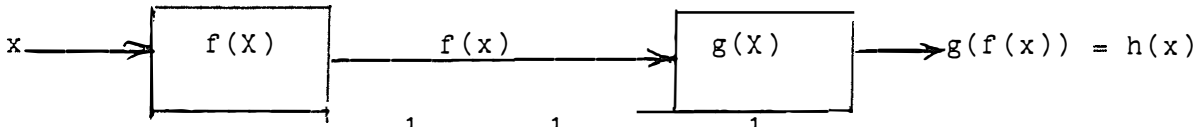
Around the corner for fog



5. Derivatives of functions requiring the "chain" rule.

Method Build up the composite function by a "chain" of operation boxes. For the derivative, follow the boxes in reverse order, taking the derivative of each "box" and evaluating at its input.

Consider: $h(x) = gof(x) = g(f(x)) = h(x)$



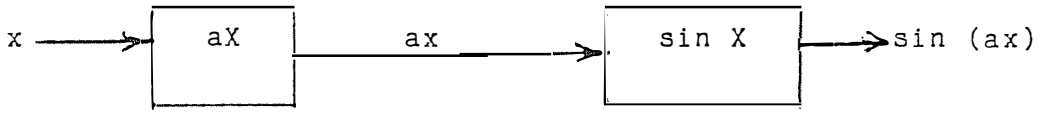
The chain rule says $h^1(x) = \underbrace{g^1(f(x))}_{1} \cdot \underbrace{f^1(x)}_{2}$

1. Derivative of content of box 2, evaluated at its input;

times

2. Derivative of content of box 1, evaluated at its input.

Example 1. $y = \sin(ax)$, $a \in R$

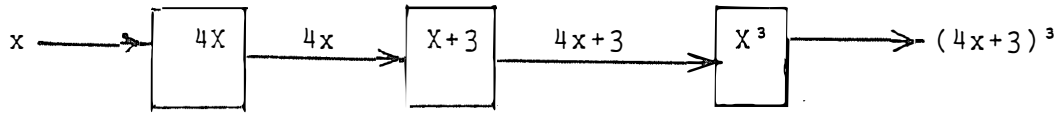


Derivative is: Derivative of sin X evaluated at (ax)

times derivative of aX evaluated at x.

$\frac{dy}{dx} = [\cos(ax)] \cdot a = a \cos(ax).$

Example 2. $y = (4x+3)^3$

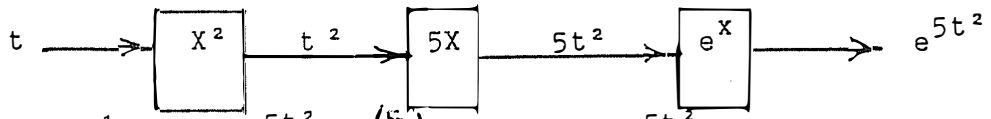


Derivative is $3X^2$ evaluated at $4x+3$

times 1 evaluated at $4x$ $\quad y' = 3(4x+3)^2 \cdot 4$

times 4 evaluated at x $\quad = 12(4x+3)^2$

Example 3. $f(t) = e^{5t^2}$

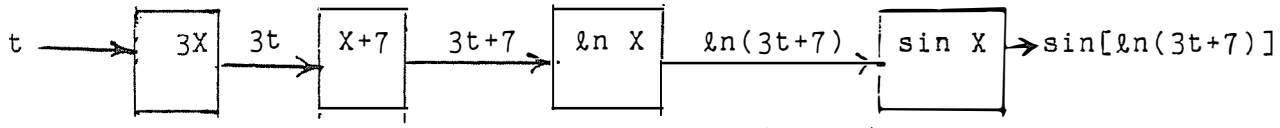


$f^1(t) = (e^{5t^2})\ (5)\ (2t) = 10te^{5t^2}$

Derivative is: (e^X evaluated at $5t^2$) times (5 evaluated at t^2)

times (2X evaluated at t).

Example 4. $y = \sin[\ln(3t+7)]$



Derivative is: ( ) evaluated at ( )

times ( ) evaluated at ( )

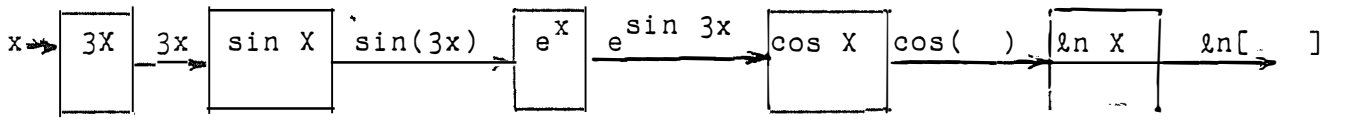
times ( ) evaluated at ( )

times ( ) evaluated at ( )

Fill the blanks to obtain

$$y' = \frac{3}{3t+7} \cos\,[\ell n(3t+7)]$$

Example 5. $f(x) = \ell n\,[\cos e^{\sin 3x}]$



Derivative is: ( ) at ( ) times ( ) at ( )

times ( ) at ( ) times ( ) at ( ) times ( ) at ( )

$$f'(x) = \frac{1}{\cos\left[e^{\sin 3x}\right]}\left[-\sin\left[e^{\sin 3x}\right]\right]\left[e^{\sin(3x)}\right]\left[\cos 3x\right][3]$$

For a final example try $f(x) = (\sqrt{x^2 + 2x + 5} + 1)^2 - 3$

using $x^2 + 2x + 5 = (x+1)^2 + 4$ and a chain of 7 boxes.

$$f'(x) = \frac{2(x+1)\left[\sqrt{(x+1)^2 + 4} + 1\right]}{\sqrt{(x+1)^2 + 4}}$$

I have illustrated several uses for operation boxes in our curriculum and I hope that you may have found one or two of them interesting and/or useful. I'm sure there must be other places where this idea could be used.

COMPUTING DETERMINANTS

by
James O'Keefe
Computer Science Teacher
St. Peter's High School

Calculating determinants by hand can be a very tedious and time consuming task. Therefore, computerizing the algorithm makes a lot of sense.

The algorithm I used is based on the theorem which states that if a matrix is upper or lower triangular, the deter-minant is the product of the elements of the main diagonal. The following computer program therefore uses elementary row operations to reduce a given matrix to lower triangular form and then calculates the determinant by calculating the product of the elements of the main diagonal.

I won't attempt to explain triangular matrices or elementary row operations since there are several good books which could do that better than I. The book I used as a reference was **Introductory Linear Algebra With Applica-tions** by Bernard Kolman (2nd edition, Macmillan Publications).

A few words are in order about the program listing. It was written on a Commodore 128 with BASIC V7.0 but it would not be difficult to translate it to other versions of BASIC. The colons at the beginning of some lines serve no computational purpose and were used to allow indenting to improve readability.

The program is easy to use. Simply type the matrix into the data statements at the end of the program. When the program is run it will ask for the number of rows in the matrix (the number of rows must equal the number of col-umns). Type the appropriate number and hit RETURN. The program does the rest and displays the answer on the screen. You may want to change the display routine to suit your own needs, particularly if the matrix contains en-tries of differing lengths.

```
100 REM  JIM O'KEEFE
110 REM  ST. PETER'S HIGH SCHOOL (MATH DEPT.)
120 REM  PETERBOROUGH, ONTARIO
130 :
140 REM  COMMODORE 128     BASIC V7.0
150 REM PURPOSE--> TO CALCULATE THE DETERMINANT OF THE MATRIX
160 REM             FOUND IN THE DATA STATEMENTS
170 :
180 DET=1
190 :
200 REM INPUT THE NUMBER OF ROW IN THE MATRIX BEING USED
210 INPUT"HOW MANY ROWS ";NR
220 DIM A(NR,NR),O(NR,NR)
230 :
240 REM READ IN DATA
250 FOR I=1 TO NR
260 :    FOR J=1 TO NR
270 :        READ A(I,J)
280 :        O(I,J)=A(I,J)
290 :    NEXT J
300 NEXT I
310 :
320 REM REDUCE MATRIX TO LOWER TRIANGULAR FORM
330 FOR I=2 TO NR
340 REM THE ELEMENT IN COLUMN K OF ROW I MUST BE REDUCED TO ZERO
350 :    FOR K= 1 TO (I-1)
360 :
370 :        REM INTERCHANGE ROW K WITH ROW I IF NECESSARY
380 :        IF A(K,K)=0 THEN BEGIN
390 :            FOR J=1TONR
400 :                TEMP=A(I,J)
```

```
410 :                A(I,J)=A(K,J)
420 :                A(K,J)=TEMP
430 :             NEXT J
440 :             DET=-DET
450 :          BEND:ELSE BEGIN
460 :          REM ELSE MULTIPLY ROW K BY M AND SUBTRACT IT FROM ROW I
470 :          REM USE THE RESULT TO REPLACE ROW I
480 :             M=A(I,K)/A(K,K)
490 :             FOR J=1 TO NR
500 :                A(I,J)=A(I,J)-M*A(K,J)
510 :             NEXT J
520 :          BEND
530 :
540 :    NEXT K
550 NEXT I
560 :
570 REM CALCULATE DETERMINANT BY MULTIPLYING THE DIAGONAL ELEMENTS
580 FOR I=1 TO NR
590 :    DET=DET*A(I,I)
600 NEXT I
610 :
620 REM DISPLAY RESULTS
630 FOR I=1 TO NR
640 :    PRINT"  |";
650 :    FOR J=1 TO NR
660 :        PRINT O(I,J);
670 :    NEXT J
680 :    IF INT(NR/2+.5)=I THEN PRINT"|  =  ";DET:ELSE PRINT"|"
690 NEXT I
700 END
710 :
720 REM DATA STATEMENTS CONTAIN THE MATRIX BEING USED
730 DATA 4,2,2,0
740 DATA 2,0,0,0
750 DATA 3,0,0,1
760 DATA 0,0,1,0
```

SAMPLE OUTPUT

```
|  4   2   2   0 |
|  2   0   0   0 |  =   4
|  3   0   0   1 |
|  0   0   1   0 |

| -2   3  -1 |
|  1   2  -1 |  =  -2
| -2  -1   1 |

|  0   3  -1 |
|  0   2  -1 |  =   0
|  0  -1   1 |

|  1   2 |
|  3   4 |  =  -2

|  1   2  -3   4 |
| -4   1   3   5 |  =   48
|  3   0  -3   6 |
|  2   0  -2   3 |

|  1   2  -3   4   5 |
| -4   1   3   6   7 |
|  3   0  -3   7   8 |  =   0
|  2   0  -2   3   8 |
|  0   0   0   0   0 |
```

UNDERSTANDING THE POWER OF THE CHESS PIECES THROUGH PERCENT

by
Dipendra N. Bhattacharya
Clarion University of Pennsylvania
and
Leah Doherty
Suny at Buffallo

About a century ago, Binet made a study of the connection between chess and mathematics. After interviewing a large number of leading chess masters, he found that ninety percent were very good mental calculators. He also pointed out that although mathematicians are often interested in chess, very few become top chess players, one notable exception being Anderson.

Binet felt that both chess and mathematicians had a common direction and the same taste for combinations, abstraction and precision. One characteristic of chess which was missing from mathematics was the combat characteristic, in which two intellects contend for mastery with all the qualities required of generals in the battlefield, which was an important distinction.

What Binet did not say: Emanuel Lasker showed a precocious skill at arithmetic, he became a famous mathematician and the world chess champion. Also, it must be noted that Max Ewe, a former chess champion, was an excellent mathematician.

The purpose of this note is not to elaborate on chess and mathematics, but to use the concept of percent from mathematics to examine the power of the chess pieces.

After a few days of play, even a beginner will realize that the Queen is the most powerful piece on the chess board. Following her are Rook, Bishop, Knight, and finally the Pawn. We wish to explain some rationale behind this thought to qualify them.

The power of the chess pieces depends on the mobility on the chess board, or the space it commands. For understanding, let us consider the most simplest situation, that is a lone Queen placed at the center of the board, see diagram #1.

Diagram #1

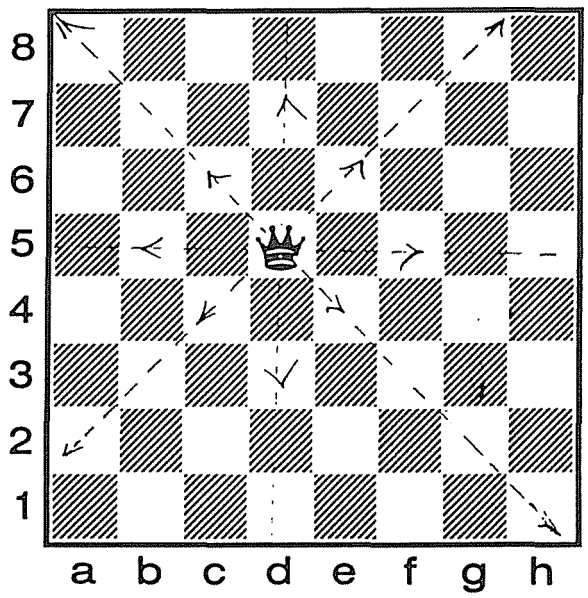
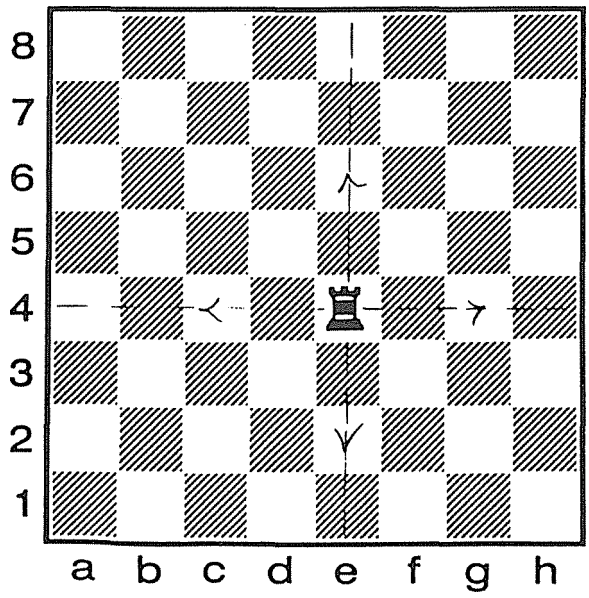


Diagram #2



We have placed a Black Queen on d5. We count the total number of squares the Queen is controlling: d1, d2, d3, d4, d6, d7, d8; a2, b3, c4, e6, f7, g8, h1, g2, f3, e4, c6, b7, a8. Please note that we are not counting the square on which the Queen is sitting. We see that the Queen is controlling 27 squares out of a possible 64 squáres on the chess board. Therefore, the percent of the board that the Queen is controlling is $\frac{27}{64} \times 100 = 42\%$ (approx.). Now, consider a Black Rook on e4: let the student count all the squares controlled. It is: e1, e2, e3, e5, e6, e7, e8; a4, b4, c4, d4, f4, g4, h4, for a total of 14 squares. The percent of territory controlled by the Rook is $\frac{14}{64} \times 100 = 22\%$ (approx.).

Diagram #3

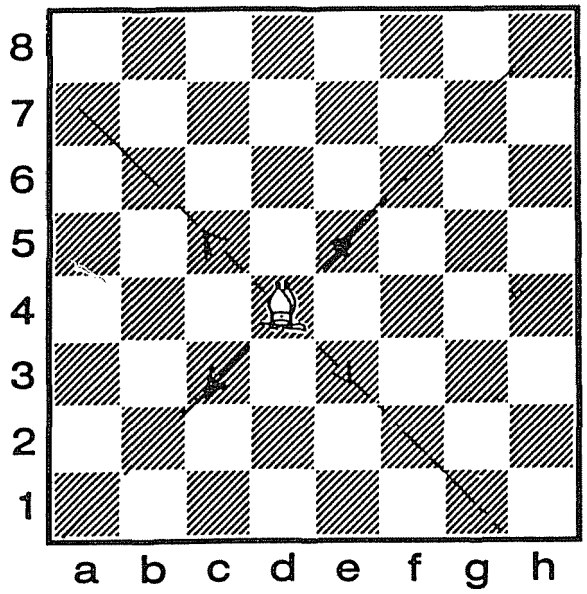
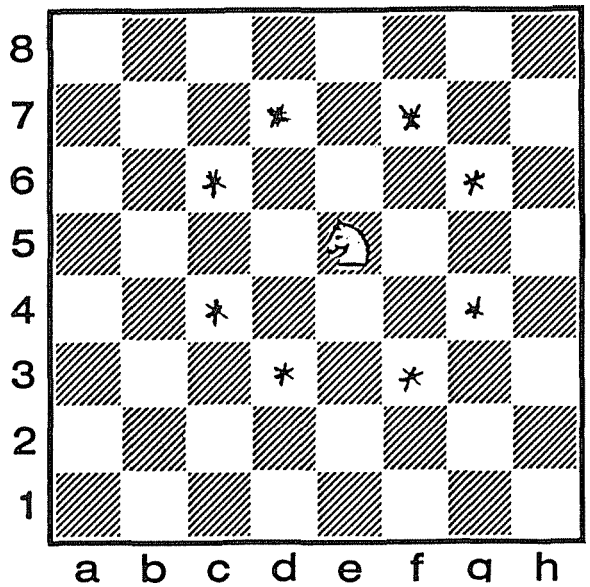


Diagram #4



Next, consider a White Bishop on d4. The student will be able to see that the Bishop is controlling a1, b2, c3, e5, f6, g7, h8, plus a7, b6, c5, e3, f2, g1, for a total of 13 squares. The Bishop will therefore control 13/64x100 = 20% (approx.).

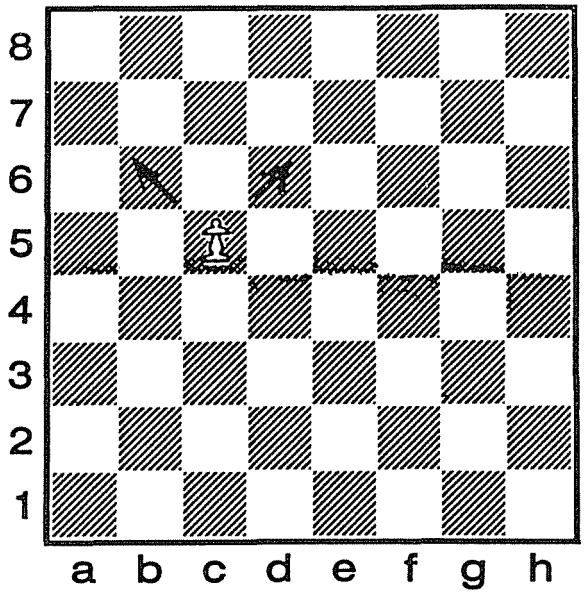
Next, consider a White Knight on e5. He is controlling the following squares: c4, c6, d7, d3, f7, f3, g6, g4, and the percent of control is as follows: 8/64x100 = 13% (approx.).

Next, a Pawn on c5 controls b6 and d6. Therfore, a Pawn controls $\frac{2}{64}$ x 100 = 3% (approx.) of the territory.

From these a student will have a good idea how the values are assigned to a particular piece. One thing must be mentioned here, that the value of a particular piece depends on the other pieces present on the board. It is in general, a dynamic thing, for example, we only considered here one piece, at a time, on an empty board and this is generally not the case. Because of that, our calculation of the percentage of the territory controlled by the Bishop came a little higher than it should be. Generally speaking, a Bishop could be blocked by a Pawn or a piece of the same color.

We believe, this kind of activity might bring some joy in the classroom and might motivate the unmotivated.

Diagram #5



Reference:
*Sunnucks, Anne, **The Encyclopaedia of Chess** (New York: St. Martin's Press, 1970).*



WOMEN AND MATHEMATICS DEPARTMENT

EDITORIAL

by

M. Elaine Harvey, Women's Editor

Editor's Note: M. Elaine Harvey is a councillor of OAME and the founder of its Women and Mathematics Committee.

If a long title for a department in the **Gazette** were as desirable as a short one, this section would not be headed **Women and Mathematics** because it is neither exclusive to women nor to mathematics. In fact, you might find that a more appropriate but less convenient title might be **(Women's Mathematics) U (Men's Mathematics) U (Teaching Male Students) U (Teaching Female Students) U (Women Learning Mathematics) U (Men Learning Mathematics).** I have purposely chosen the symbol for union in this title because it implies the inclusive "or". It does not mean that we must choose either Women's Mathematics or Men's Mathematics because the option is there to choose both.

However, in my struggle to communicate the field of interest of this department, I have used sets which emphasize the uniqueness of their members and the differences between them. Unfortunately set theory does not allow members to be fuzzy or indistinct, or sets to have borders that vary from one time to another.

Let me try again to re-phrase the title: **Mathematics, Learning and Teaching, on a Gender Continuum.** If you are not sure what this means, I'm not either. Why not try reading/corresponding/contributing to this department, and together we will create the meaning for such a title.

With this issue the Gazette introduces **Women and Mathematics** as a regular feature. Don't read it *because* you are a woman. Don't avoid it *because* you are a man. Read it because it may help you to understand the worlds of mathematics, of teaching and of learning from the broader perspective of a continuun of gender--a continuum between female and male--which has been lacking in the education of modern man (and woman).

MS MATH ... A UNIQUE MATHEMATICS PROGRAM FOR GIRLS

by
Bonnie Taylor Gillings, Vice-Principal
Lady Spencer Churchill School
St. Catharines, Ontario L2T 3J6

Why is it that 20 to 25% fewer females take math at an advanced level? Why is math perceived to be a male domain?

A group of concerned educators and parents formed a committee in the spring of 1984 to address the problem of girls not pursuing careers in math, and thus closing the doors to many career opportunities. A grant was received from the Secretary of State to conduct a search of the literature. Findings indicated that, for a variety of reasons, many young girls were not taking math at the secondary level, and therefore were being limited in their career selection.

The Ms Math program was developed as a direct intervention strategy with the following objectives:
a) to reach girls in the elementary grades where attitudes to math are being formed;
b) to develop positive attitudes to math;
c) to replace math anxiety with math proficiency;
d) to encourage girls to continue taking math courses at the secondary level;
e) to appreciate the importance of math as a key to many career choices;
f) to improve skills in spatial visualization and orientation;
g) to be exposed to female role models in non-traditional careers.

The course is taught by qualified teachers on eight Saturday mornings at Brock University. Parents are invited to the first session, where they view a video about the program which explains the purpose and outlines the curriculum. Parents are led through a typical session, and given information about myths and realities in relation to math, and are encouraged to stress the importance of math with their daughters.

Ms Math is designed for girls in grades 6 to 8 who will soon be making course selections at the secondary level. Many girls are experiencing math anxiety at this time; their confidence in math needs to be increased. The perceived usefulness of math appears to be correlated to participation in math in secondary education.

The program focuses on three main areas: creative problem solving, spatial skills, and role models. A different strategy in creative problem solving is taught each week. The girls are taken, step by step, through the strategy.

Special care is given to the teaching methods used by the teacher. The girls are encouraged to take risks, to be creative, to use the strategy, and to enjoy what they are doing. Part of the uniqueness of the program is the organization of many math problems into the eight strategies. The teacher and pupils do several examples together to reinforce the usefulness of the strategy. Then the girls break into small groups and work on activity cards based on the strategy taught that day. This gives them an opportunity to consolidate the learning and to interact in a small group setting.

Another important aspect of the program is the development of skills in spatial orientation and visualization. This appears to be an area where girls consistently do not perform as well as boys. This could be a consequence of limited social opportunities as a child. Little boys and girls are often socialized in different ways, from the toys they play with, to parental attitudes and expectations, to the influence of the media. Many females do not automatically learn the total concept of space: direction, distance, perspective, movement, and relationships of objects to each other in space. This part of the Ms Math program has been adapted from **Spatial Encounters,** a set of exercises in spatial awareness developed in New Mexico.

The third part of the program has to do with role models, and the influence that they can have. Women in non-traditional roles are invited to speak to the girls and to show how they use math in their jobs. Much of the research points to the importance of role models in affecting attitudes. Often girls who have pursued a career related to math indicate that they were influenced by a female math teacher in school, or a female already in the job.

From the lack of females pursuing careers in the math-related field, it is apparent that some form of intervention is necessary. Hopefully the Ms Math program will make an impact in this area. A kit is available and can be adapted to meet the needs of students in a classroom setting.

DON'T BE DISCOURAGED - DON'T BE DISCOURAGING

by
John Hindle, Special Assignment Teacher
Carleton Board of Education
Nepean, Ontario K2C 2K3

"We live in a decaying age. Young people no longer respect their elders. They are rude and impatient. They inhabit taverns. They have no self-control."

This inscription on a 6,000 year old tomb alludes to one of the many facets of the age old relationship between adults and children, teachers and students. This inscription reflects discouragement, the discouragement of the young people mentioned and the discouragement of the speaker. All teachers and students of mathematics have experienced some form of personal, academic, and/or professional discouragement. The signs of discouragement are as obvious, as familiar and as consistent through the ages as this inscription implies. The fact that we as teachers unwittingly contribute to this discouragement is the important point.

It has become apparent through recent research, that a growing number of students, especially young women, are withdrawing from mathematics after obtaining only the minimum compulsory credits required for a graduation diploma. Set aside for now such influences as parental and peer pressure, factors over which the teachers have little influence. The goal of this article is to discuss the damaging effects of discouragement on all students, how teachers contribute to this discouragement, and what they can actively do now to remedy the situation.

Spencer said, "Education has as it's object the formation of character," implying that the character formed is good, has a strong self-esteem, and makes worthwhile contributions to our society. Only a highly encouraged person can meet these criteria. Ontario schools create these highly encouraged individuals, but the proportion of encouraged students to discouraged students can be improved.

For some young people, what schools produce are students that are so discouraged that they choose misbehaviour as a means of gaining recognition. So often what teachers do to correct this misbehaviour **is responsible for them not improving.** Teachers of these misbehaving students participate in an unwritten contract where the student agrees to misbehave, and the teacher agrees to try ineffective discipline techniques discouraging both sides even further.

The teachers enter into this discouraging contract because of a lack of experience with alternative approaches to discipline and encouragement, while simultaneously they themselves may be experiencing some form of personal discouragement. A. H. Maslow stated, "If the only tool you have is a hammer, you tend to view everything as if it were a nail." Most teachers have had little instruction on techniques for combating discouragement or little time to practise them. Most of the formal teacher-training dealt with the day to day classroom management strategies and not with the increasingly complex skills of 'person development'. In addition, most of the teachers in Ontario now fall in the category of "experienced teachers", meaning that their initial training occurred more than fifteen years ago. The need for additional training is indicated by the high drop-out rate of students, and the severe dislike for mathematics of a great number of students who remain in mathematics only because it is needed for a future programme.

Why do the students enter this discouraging contract? The discouragement causes them to seek a mistaken goal through misbehaviour. The goal of this misbehaviour is one of four things; attention, power, revenge, or withdrawal.

All people require some form of encouragement, and students are no exception. When students become discouraged, research shows that there are four misbehaviours that may result. The first misbehaviour that students may display is that they try to obtain some form of attention. Most teachers recognize students who wave their hands so hard to answer a question, and when asked, don't know the answer, or students that are continuously late for class. If this attention-seeking misbehaviour does not gain the students some form of mistaken recognition they will usually enter into a power struggle with the teacher. If still further discouraged students usually resort to revenge. Lastly if discouragement continues, such students withdraw, often sitting passively in class, skipping classes, dropping the subject, or even withdrawing from school completely. The levels of misbehaviour - first, attention seeking; second, power struggles; third, revenge; and fourth, withdrawal - are linearly regressive. If other research tells us that our students are withdrawing at an alarming rate, then we can see now how deeply discourage these student have become.

What is discouragement?

If discouragement is at the root of student misbehaviour then what is discouragement? Discouragement is a reduction of confidence, a deprivation of courage, a persuasion to refrain, a reduction of hope, a debilitator, a

result of humiliation, indulgences, neglect and/or failure, a result of perfection, a result of our mistake-oriented society, and a forerunner of misbehaviour.

Self-esteem is one's favourable belief in one's own character and abilities, and discouragement is a destroyer of one's self-esteem. Consider the following formula.

$$\text{Self-esteem} = \frac{\text{number of successes}}{\text{number of successes hoped for}}$$

It is a teacher's responsibility to insure that students have sufficient successes so that they can continue to hope for successes. If students are continually discouraged they will attempt to protect their self-esteem. Since the number of successes they have is for the most part not within their control, to protect their self-esteem they can only stop hoping for successes. What a discouraging attitude - not even to hope for success!

How do teachers contribute to discouragement?

What actions contribute to the discouragement of students? Contrary to popular belief, competition is not encouraging. Granted that competition is motivational and rewarding for those few who are the winners, for most students competition is a futile and discouraging task. Some school critics believe that a student's most enduring lesson is that their primary task in school is to avoid embarrassment. If this is minutely true and for most students competition is an embarrassing ordeal - then competition is itself discouraging.

Praise is seldom encouraging. Praise emphasizes unwittingly that the student is more acceptable because he/she did well. Praise recognizes only completed tasks and not the effort or improvement. Praise like 'Your the best' focuses on superiority rather than contributions. Praise feeds only the need for attention, which in itself is a mistaken goal. Thus a cycle is created which, when removed, leaves the student doubly discouraged.

If one listens carefully to conversations in school staffrooms, a most discouraging technique used by teachers is revealed. They fail to separate the student from the misbehaviour, fail to separate the deed from the doer. The most encouraging teachers find it is possible to disapprove of a student's actions without disapproving of the student. This is a trainable skill and with practice can be used by all teachers.

Success is not a goal. It's a by-product of encouragement. If schools are to be successful, then teachers must make them into more encouraging places to visit, to learn, and to teach.

Both discouragement and encouragement are contagious; it's the choice of each school and each teacher in that school. If the school in general must adapt to a newer and more demanding life, and the teachers in particular must improve their ways of being encouraging, then what must happen for a change to take place?

What are the criteria for change?

Each teacher must become aware of his/her own present style. The teacher must show a willingness to change. Learning is a volitional discovery process and changing old habits can not be mandated. Training must take place that raises the teacher's awareness of alternative styles. Nothing is permanent, and change should be a natural expectation. Opportunities for practice must be arranged when the teacher can practise and improve these skills, in an encouraging environment. Each teacher must be able to risk, and the ability to exhibit courage is a prerequisite of a successful change. The teacher must give themselves permission to struggle with the techniques, and to be forgiving of their mistakes. Lastly each teacher must exercise one of their most frequently called upon qualities, patience.

How can teachers be more encouraging?

Teachers can be more encouraging by the following methods:
First, reflect on the following quotes:

- "When you cease expecting, you have all things." - Buddha
- Demand of your students only what you demand of yourself
  - improvement, not perfection.
- "If I had to choose between pain and nothing, I would choose pain." - Falkner
- "The years teach much which the days never knew." - Emerson
- "If I had my life to live over again, I'd try to make more mistakes." - 85 year old
- Remember that 74% of all communication is non-verbal.

- We should give the child attention... but not when he/she demands it.

- One's worth is not measured by success.
  - A mistake is not a failure.

- It is only with the heart that one can see rightly;
  - what is essential is invisible to the eye.

Next, analyze the way they interact with their students, and consider their present style in relation to the list of encouraging actions given below: warm tone of voice, sharing feelings, laughing, listening, sharing responsibility, smiling, acknowledging feelings, eye contact, being interested, being there, trusting, challenging, caring, asking opinions, refraining from criticising, mentioning the effort, stressing the uniqueness, regular routines, building a sense of belonging, talking.

In addition teachers may include in their vocabularly more encouraging words and phrases like: yes, wow, enjoy, I agree, looks nice, fantastic, that was fun, tell me more, terrific, that is a help, beautiful, let's both do this question, I am pleased, I hear you, super, you can do it, that's great, I'm glad you're here today, try it, fine work, swell, good job, looks great, well done, I'm impressed, you do a good job of..., you have improved so much in...

Teachers need to be more encouraging. They need to encourage their students more, encourage their colleagues more, but most of all, encourage themselves more, each and every day. It is vitally important to so many, that each teacher, be as encouraging an element in the whole school as possible.

And finally, consider the wisdom of Janis Joplin, particularly during the first few weeks of a new term: **"When I walk on stage, all that I can give them is me".**

REAL WOMEN DON'T DO MATH!
A Mathematics Camp for Grade 10 Girls

by
Pat Rogers
Faculty of Education and Mathematics Department, York University
Toronto, Ontario M3J 1P3

There are many reasons why girls avoid mathematics: factors such as general ignorance of the importance of mathematics to future career choices, and social pressure to limit themselves by not competing in what many still believe to be a "man's world" are extremely important. But I would argue that a more important determinant may be the very nature of the mathematical experience itself - by this I mean not the nature of the mathematics but the way it is so frequently experienced. For many students mathematics is received not created, "out there" rather than part of their own experience, an area over which they have very little control except through withdrawal or avoidance.[1]

Concern about girls' poor enrolment in grade 12 and 13 mathematics courses has given rise to a variety of initiatives aimed at attracting them to mathematics. For the past two years, York University has offered "Real Women Don't Do Math!", a three-day residential mathematics conference, to grade 10 girls from North York schools. The conferences were designed primarily to give the girls a collaborative experience in the process of creating mathematics.

Ideally, such a conference should be a collaborative effort between a university, college or school and a local Board of Education. In our case, Hugh Beattie, the North York Mathematics Coordinator, facilitated my consulations with interested mathematics teachers in order to define criteria for selecting the girls and organized their selection. Who to include and how many are important questions to answer. We agreed that, in only three days, the conference was unlikely to have much effect on the attitudes of girls who were weak at mathematics. So the participants chosen by their mathematics teachers were of average ability in mathematics: even girls who are quite able in mathematics are known to be at risk of dropping it too soon. Since we wished to reach as wide an audience as possible, the girls were also selected on the basis that they were respected by their peers and hence likely to have some influence with them if they came home excited and enthusiastic about their experience.

Funding was needed mainly to pay for accommodating the participants because, in order to provide equal access to the opportunity, we invited all the girls at no cost. The main source of our financial support was the Ontario Women's Directorate in 1985 and Petro-Canada Inc. in 1986. Our experience was that speakers and workshop leaders were willing to donate their time. We were able to offer student assistants nominal honoraria and York University provided space, materials and administrative assistance.

Numbers were restricted by funding. However, we also restricted the size of the group to provide occasion for the intimacy which can develop between the girls from the different schools and the resulting ease with which they are then able to engage in and contribute to the various activities. Our conference involved thirty girls from about six different schools and this seems to be a manageable number.

The primary aim of the conferences was to engage girls in mathematics; the secondary aims also are important:

1. to provide participants with an experience of university life, hence the residential nature of the conference;

2. to show them that mathematics opens doors by providing them, through panel discussions and workshops, with career information and statistics demonstrating the negative results of dropping mathematics too soon;

3. to involve them, through a variety of activity sessions, in unfamiliar and interesting areas of mathematics which challenge their pre-conceived notions about the nature of mathematics; and

4. to counteract their impression of mathematics as a "male domain" by exposing them to undergraduate women successfully pursuing courses of study which are heavily concentrated in the mathematical sciences. This final point is very important. Research indicates that the best "role models" for teenage girls are women not too much older than themselves; women of the same generation as their parents, no matter how well meaning and successful, are likely not to be as effective.[2]

An outline of the conference programme can be found at the end of this paper.

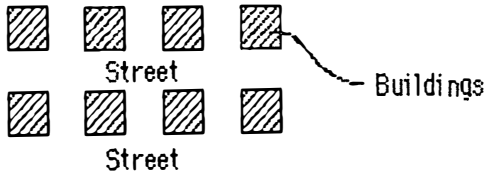
The Problem Solving Workshop

The Problem Solving Workshop is the central feature of the conferences. It has four components:

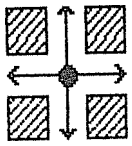
1. **Orientation workshop on mathematical problem solving skills and strategies**

This session is a key one for it establishes the ground rules and procedures for the small group sessions. In this workshop the girls are introduced to collaborative learning strategies and group work skills; creative problem solving strategies are developed in the context of solving real life problems and these strategies are then used to investigate mathematical situations. An example of the kind of problem which lends itself well to this approach is the following.[3]

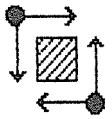
Imagine a city, rather like Toronto, whose streets form a square grid each side of which is 100 m long:



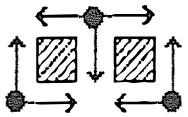
Suppose that a police officer is standing at a street corner and that she can spot a suspicious person at a distance of 100 m. Then she can survey a maximum of 400 m of street length like this:



For a single block, we need 2 police officers:



Two blocks in a row will require 3 police officers:



How many police officers are needed for 3 blocks? 4 blocks? 5 blocks? What about blocks in squares? In rectangles?

2. **Small-group problem solving sessions**

For these the girls are divided into groups of five or six, each with two leaders: "the expert", in our case a mathematics professor or graduate student; and "a guide", in our case an undergraduate student. Working with an "expert" can be less daunting than one might imagine, as one girl wrote afterwards:

> Math was much more "glorified" when talking with a Math Ph.D. I felt very thankful for such an experience and I wanted to converse with such "geniuses" as much as possible, and I found that these "geniuses" are people too, and such a position can also be available to me.

"The expert" need not meet with the group itself, but she is responsible for choosing a suitable problem; meeting with the undergraduate leader prior to the conference to discuss the problem and any background mathematical ideas that might be required; and, ideally, she should be available during the conference for consultation if required.

The groups meet for a total of approximately six hours in three sessions. Students rarely experience sustained activity of this kind in mathematics and it is important not to skimp on the amount of time allowed. In the first session, the problem is introduced and the group brainstorms possible questions and strategies for answering them. It is in the second session that most of the hard work is done solving the problem. And in the third session the groups "fine tune" their solutions and prepare and rehearse their final presentations.

In this model "the expert" acts as facilitator and resource person - she is an authority but not in control. We

have found that this format works well and does empower the girls, putting them in charge of "the experts" and to some degree of their own learning.

The importance of choosing good problems or investigations cannot be over-emphasized: they must be amenable to group solution and sufficiently open-ended to allow for a variety of approaches. Some sample problems that have worked well are available later in this article; another rich source of problems is the What-if-not? strategy of Stephen Brown and Marion Walter.[4]

3. **Discussion groups**
The role of feelings in learning mathematics is crucial. A person who for one reason or another has a very low estimate of her ability to do well in mathematics will not function well in the group unless these feelings are shared and put in perspective. Small group discussions, involving at most ten students and led by an undergraduate student, serve as a forum for sharing fears and doubts, for discussing one's experiences and discovering that one is not alone. These discussions are an integral part of the workshop and should take place each day.

4. **The final "mini-conference"**
The workshop culminates with each group's presentation of their work in a final "mini-conference" - the climax of the whole programme. Its purpose is to give the girls themselves the experience of being "the expert" and the confidence which might follow from giving a successful presentation.

Naturally, the girls will be very nervous at the though of giving a talk of their own and, initially, many may even refuse to do so. However, we have found that as the conference progresses and the girls' confidence and ease with each other increases, most of them join in quite readily.

It is more than likely that some of the presentations will not be understood by girls from other groups but this is probably not important. Indeed, it is possible that solving a hard problem together and presenting it to other students who have difficulty comprehending the solution may actually increase their sense of personal accomplishment.

Elvis and the Tower of Benares
In this section I describe the results of one group's work together. The problem given them to solve was an ancient one, posed in the following form:

> In the great temple of Benares there is a large brass slab on which there are three diamond needles each a cubit high and as thick as the body of a bee. At the beginning of the world God placed 64 gold discs with holes in their centres on one of these needles, the largest resting on the brass slab and the others decreasing in size to the top. Day and night priests transfer the discs from needle to needle without deviating from the immutable laws of Brahma. The priests may move only one disc at a time and they must place this disc only on a free needle or on top of a larger disc. When the 64 discs have been transferred from the original needle to one of the other needles, the temple will crumble and the world will end. If the priests transfer one disc each second, what is the lifetime of the world?[5]

Session 1: The girls spent the first session, in pairs or alone, playing the "tower game" with three, four and five discs. This route suggested itself after some initial discussion and frustration about the size of the problem and the impossibility, given time restrictions, of ever finishing it - the group estimated that the world would probably end in just under a year. (The final answer is surprising!) The idea of "minimizing" the game by reducing the number of discs came to the girls when they reflected on the morning's orientation workshop: with a little probing they were able to apply techniques learned there to gain more insight into the nature of the problem. No model was provided, so the girls created their own with coins, pieces of coloured paper, playing cards and straws. By the end of this session charts had been constructed describing the game for 3, 4 and 5 discs and some girls were beginning to realise that a different approach would be needed to solve the original problem.

Session 2: This session took place the following morning. The charts were reproduced and examined for common features and patterns and before too long most girls were convinced they could accurately guess the number of moves for a game with 6 discs. This was not verified empirically. Rather it followed from an analysis of the mechanics of the game and the observation that each game was embedded in the next so that the number of moves in one game was one more than twice the number of moves in the previous game.[6] The girls were quick to realise that working through this table to obtain the number of moves for the 64 disc game, while possible, was not the most elegant way to answer the question. So they began to look for an alternative approach. It took a surprisingly long time for them to come up with the pattern in the third column of the following table:

No. of discs	No. of moves	Pattern
1	1	$2^1 - 1$
2	3	$2^2 - 1$
3	7	$2^3 - 1$
4	15	$2^4 - 1$
5	31	$2^5 - 1$
6	$63 = 2 \times 31 + 1$	$2^6 - 1$

Once this pattern was established the final answer came quickly. The 64 discs would take $2^{64} - 1$ seconds to transfer from the original needle to one of the others and this was estimated to take more than 5000 million centuries! (The exact answer is 5 845 420 460.9 centuries.) Some discussion followed as to the nature of the final presentation: how to divide the work up; what materials would be required; what models would have to be made; how to involve the audience.

Session 3: By this time, the final morning of the conference, the girls were very tired but fortunately most of the work was done. Two models of the game were prepared: one was a concrete representation made from cardboard and straws and intended for use by the audience; the other was used to demonstrate the game on the overhead projector and was made from acetate sheets using different coloured pens. A poster was prepared outlining the problem and other posters and overheads were prepared to explain the girl's solution. While all this was going on the girls discussed and rehearsed their parts in the presentation. There was very little input from either of the two group leaders in this session.

"Mini-conference": The girls gave a talk that any group of student teachers would be proud to have given. It was professional and lively, with a variety of useful and attractive visual aids and lots of audience participation and involvement. But what impressed me most was the ownership of the problem and solution which the group evidently experienced and which was demonstrated by their presentation of the original problem. In the formulation which follows, the context is completely changed but the spirit and the essential ingredients of the original problem remain. Their ability to alter the conditions of the problem in this way is also clear evidence of the extent to which the girls must have understood the problem and the details of its solution.

In the glitter city of LA, Elvis Presley collected platinum copies of his 64 greatest hits and piled them on a silver pin in the order in which they were released, the most recent on top. He set this record tower between two empty silver pins. In his last will and testament, he left the following instructions for his fans:

"Transfer all the records to one of the other pins obeying the following rules:

a) Move only one record at a time;
b) Do not put any record on a later release;
c) Each fan may move only one record and then only once."

How many fans does Elvis have? If each move takes 1 second, how long will the fans take to complete the process?

Sample Problems

Two excellent sources for mathematical investigations are:

Tansy Hardy, Anne Haworth, Eric Love and Alistair McIntosh, **Points of Departure 1,** (Derby: Association of Teachers of Mathematics, 1986).

John Mason with Leone Burton and Kaye Stacey, **Thinking Mathematically,** (London: Addison-Wesley, 1982).

1. How many rectangles can you find inside a 3 x 4 rectangle? Inside a 4 x 5 rectangle? Inside an m x n rectangle?
What about a 2 x 3 x 4 box? How many boxes can you find inside an m x n x p box?

2. You are faced with twelve balls and a balance scale (which tells you only whether the right pan's or left pan's contents are heavier). All but one of the balls are identical in weight; this oddball is either heavier or lighter than all the rest. You are allowed three weighings to discover which is the oddball and whether it is heavier or lighter. How do you do it?[7]

3. Jaime Poniachik...said he had a friend...with a curious social security number: its nine digits include every digit from 1 through 9, and they form a number in which the first two digits (reading from left to right) make a number divisible by 2, the first three digits make a number divisible by 3, and the first four digits make a number

divisible by 4 and so on until the entire number is divisible by 9. What is the number?[8]

Some Additional Remarks

The problem solving sessions are essential to achieving the conference's aims, but the environment in whcih they occur is also a factor.

"Sleep-over" The "sleep-over" element of the conferences is vital. It increases the intensity and the intimacy of the experience and allows plenty of time for developing community. This is achieved through sports and recreational activities, and evening entertainments such as skits, charades, stargazing and movies. Of course, the residential nature of the conference brings with it problems of an entirely different kind. The term "sleep-over" is a euphemism; we learned to accept that the girls simply would not sleep. We hired student chaperones to look after them, in addition to two teachers who were involved each day and stayed each night. Inevitably the girls become very tired to we tried not to schedule activites too early in the morning and to allow time in the afternoon for recreation or rest. We also provided a variety of activities for the evenings that were not too structured.

In the second year, we organised a "Math Scavenger Hunt". This consisted of a search for numbers answering like the following:

1. A kangaroo was running with a rabbit that had a head start of sixty jumps. The kangaroo took six jumps to the rabbit's nine. Three of the kangaroo's jumps are equal to seven of the rabbit's. How many jumps did each animal take before the kangaroo caught up to the rabbit?

2. A pencil, an eraser and a notebook together cost $1.00. A notebook costs more than two pencils, and three pencils cost more than four erasers. If three erasers cost more than a notebook, how much does each cost?

The questions varied in difficulty and were given to the girls on the first evening. They worked on them in their school teams during the conference and there were prizes at the end for the team with the most correct answers. This was the only aspect of the conference that was competitive and the girls engaged in it with an energy and enthusiasm which we found quite surprising.

Teachers Our reason for inviting teachers to take part each day was not simply to provide experienced and highly skilled chaperones. We wanted to see and evaluate what we were doing. By being involved in this way they were able themselves to provide follow up and support for the girls when they returned home.

Parents No programme such as this can hope to be successful unless parents are also involved. For this reason families were invited to spend the first evening with their daughters and we planned the evening specially for them.

Evaluation

Whether conferences like this one or the "Expanding Your Horizons" conferences are successful in the long term remains to be determined. They certainly raise issues and consciousness and ensure that the problem of math avoidance by able girls is not forgotten. Short term effect is easier to assess. We employed pre-conference and post-conference evaluation questionnaires and the results indicate that the experience was an extremely positive one for everyone: for some girls the first positive experience they had ever had in mathematics. It was also interesting to dicover that the reason most girls gave for finding the conference fun and enjoyable was that the mathematics they engaged in was interesting and challenging. Another reason given was meeting "real women" doing mathematics, as one girl remarked in a radio interview afterwards:

> Well, I thought...I...I've never been offered any kind of opportunity like this and, I thought this was a really interesting way to get involved in mathematics and, plus, to meet women professors. I've never met a women professor before, and I really wanted to see what they were like. I thought they were these kind of women who were, you know, brief case in hand and walked in and had glasses on! But they were real people and I...was really surprised at that! And so, I kind of enjoyed the whole thing.

Outline of the Programme

Evening of Arrival: Parents' evening

Day 1 morning: Problem Solving Orientation Workshop, Discussion Groups
afternoon: Small Group Problem Solving Session 1, Recreation and free time
evening: Panel Discussion: Mathematics, Careers and Marriage.

Day 2 morning: Small Group Problem Solving Session 2, Discussion Groups

afternoon: Mathematical Activity Sessions, Recreation and free time
evening: Entertainment: Math Skits/Charades/Math Scavenger Hunt Awards

Day 3 morning: Small Group Problem Solving Session 3, Mathematical Activity Sessions
afternoon: "Mini-conference": Student Presentations

Endnotes

*[1] Pat Rogers, "Overcoming Another Barrier: Real Women Don't Do Math - With Good Reason!" **Canadian Woman Studies Journal** 6, No. 4. (Winter 1985), p. 82-84.*

[2] Elizabeth Fenemma, International Organization of Women in Mathematics Education Discussion Group, Tenth International Conference of Psychology of Mathematics Education, London, England, July, 1986.

*[3] Tansy Hardy, Anne Haworth, Eric Love and Alistair McIntosh, **Points of Departure 1,** (Derby: Association of Teachers of Mathematics, 1986).*

*[4] Stephen Brown and Marion Walter, **The Art of Problem Posing,** (Philadelphia: The Franklin Institute Press, 1983).*

*[5] Stanley Gudder, **A Mathematical Journey,** (New York: McGraw-Hill, Inc., 1976).*

*[6] For a more detailed discussion of this discussion of this problem see John V. Trivett, **...And So On: New Designs for Teaching Mathematics,** (Calgary: Detselig Enterprises, Ltd., 1980), p. 13-23; or write to me for a copy of a paper I am preparing on the use of this problem in teacher education.*

*[7] Saul X. Levmore and Elizabeth Early Cook, **Super Strategies for Puzzles and Games,** (New York: Doubleday and Co., 1981), p. 43-46.*

*[8] Martin Gardner, "Mathematical Games." **Scientific American,** (December 1978), p.23.*

MATHEMATICS IS A (WO)MAN'S SUBJECT

by
M. Elaine Harvey

I would like you to consider this article as a wine-taster would taste a wine. Approach your glass with an open mind and no pre-conceived opinion, sniff the bouquet and enjoy the fragrance of this new aroma, risk taking a sip and swirl it around in your mouth savouring the experiencing of a new flavour. But if you favour the old flavours, don't dismiss it on a wim; give it a second chance by checking it out another day; or if you are enthusiastic, yes, go ahead and order a case right away!

With the awareness that has resulted from the women's movement of the last twenty years or so, most people would recognize that western society is patriarchal. It is men who have held most of the powerful positions, and it is men whose history has been written. Women have remained relatively silent.

It is men who have shaped our educational institutions. Until recently women were excluded not only from the administration of these institutions but also from the student bodies of institutions of higher learning. Even yet women have little impact on the decision-making regarding education at any level. Those women who have power have had to pay the price of buying into the patriarchal system.

The patriarchal system has as its basic structure hierarchy. Christian, Jewish and Moslem religions have been structured as hierarchies with God (a male God) at the top. The democratic political system is hierarchical with the Prime Minister at the top. The education system of Ontario is hierarchical with the Minister of Education at the top, and within the school, the principal at the top. When there is a top, there is also a bottom, and the students are found at the bottom of the hierarchy. This, when you think of it, seems a little strange since the education system is purportedly set up for the sake of the student.

Women's way is not hierarchical. Of course it is not hierarchical, because women in the hierarchical structures which exist at the present time are almost never at the top. They are generally at or near the bottom. The oft repeated phrase, "Women and children first," lumps women with children, and children are at the bottom of our hierarchy and are given very little respect from others. It is assumed for instance that young children do not know the difference between truth and falsehood, and cannot be trusted to testify in a court of law. Even parents are frequently reluctant to believe a child who suggests that s/he has been sexually manipulated by an adult. It is more likely the case that children give more truthful testimony than adults. Women have had to buy into the patriarchal structure/hierarchy in order to survive in a culture in which they have been as powerless as children.

Now, however, there is evidence of structures which are not hierarchical but are instead based on co-operation. They are frequently called collectives. The **Toronto Women's Bookstore** is a collective. The women's monthly journal, **Broadside,** is operated by a collective. There is a **Women's Health Collective** in Kingston. Some schools have attempted to adopt a modification of this kind of structure; however, it is difficult to do so within the superstructure of the educational system of Ontario. Lord Elgin Secondary School in Toronto, Bayside S.S. in Trenton and Bayridge S.S. in Kingston have all attempted to structure themselves without department heads. However, they have not been able to eliminate the hierarchical position of principal, and because of the difficulties of operating within the hierarchical superstructure, most of these experiments have failed and/or been modified in a way which moves them closer to the patriarchal principle than to the female principle.

Because men have had the control of education through the centuries, it is they who have defined and created knowledge as it is recognized in the schools. Not all knowledge is given the dignity of being identified as worthy of study. Knowledge which we refer to as "street smarts" has not been dignified by being recognized as part of the curriculum. The "language of the gutter" has not been recognized by the curriculum, nor has the language of children or of teenagers. Neither has most of women's knowledge. Domestic science was first promulgated by Catherine Beecher in the United States and Mrs. Beeton in England in the 1840's, but it must be noted that the scientific approach was a male creation even though it was applied to housework. It is only recently that Women's Studies have begun to be recognized by the universities, but these courses have the constant struggle of trying to exist within the hierarchical institution with its rules and regulations which are contrary to the collective/co-operative approach of women.

Men have defined and created mathematics. I do not have to convince readers of the **Ontario Mathematics Gazette** that mathematics exists. Even women readers would have to admit to that. We must, however, base the belief of the existence of mathematics on the assumption that there is a natural order to the universe, for if there were no natural order, then we could not create laws or rules that must, of course, apply in more than one case. Without a natural order, two_{10} plus two_{10} may not always equal $four_{10}$. However, it is unlikely that women would have defined and classified mathematics, or indeed any academic subjects, in the way that men have done.

Mathematics is structured in an hierarchical manner. The Ministry of Education has divided its Grade 7 Mathematics Curriculum Guideline (1985) into two sets:

I. Numerical Methods and Algebra
II. Measurement and Geometry

Set I contains nine subsets--beginning with (1) Calculator Skills, (2) Whole Numbers and Decimals--each of which contains from three to eleven members, and Set II contains four subsets each of which contains from seven to twelve members. As the Ministry has done, so have most textbook authors classified Mathematics by sets (chapter headings) and subsets (sections). Mathematicians have specialized in one branch (one set) of mathematics or another. The organization of the subject then is hierarchical/patriarchal. This establishing of an hierarchy in mathematics is done by classifying; and classification involves breaking down into parts, and studying and examining each part separately. This was the basis of Descartes' "Method", and it is truly part of the patriarchal heritage.

Women, had they had the chance, would have considered mathematics in context. They would not have identified mathematics as an academic discipline nor would they have classified mathematics into branches such as analytic geometry, arithmetic, topology, etc. They do not break things down into parts, but consider them holistically. They would, if they had not been sucked in by the patriarchal system, have considered mathematics in context.

Women **do** consider mathematics in context. Those women (and men) who are mathematically anxious do not live in a world devoid of mathematics. They have learned to cope with mathematical problems in context in their everyday lives often without identifying them as mathematical problems and certainly without separating them off from the contextual situation.

Women, even the most feminine of women and even those severely afflicted with math anxiety, have dealt with mathematical problems associated with money, recipes, quilt making, knitting, home decorating, etc., frequently without identifying the mathematics of the situation. They have taken the holistic approach. They have looked at mathematical situations in context.

Context independence is a patriarchal approach. Pure mathematics is context independent; so pure mathematics is patriarchal. Note that there is a large dose of pure mathematics in our advanced courses at the secondary level, particularly at the senior level. And pure mathematics, which is mathematics taken out of context, is more in line with the male approach than the female approach. It is no doubt true that one of the reasons that women students in secondary schools have a higher drop out rate from advanced level mathematics courses than men students do is that the subject matter is taken out of context, is not seen as relevant to the lives of the students, and is more in line with the male/patriarchal/hierarchical approach than with the female/contextual/co-operative approach.

It is not only women who suffer from math anxiety and who drop out of mathematics courses. Those men who also suffer the anxiety affliction and who struggle along or give up have not found the competitive/win-or-lose hierarchical/patriarchal system appropriate to their needs either. Men have a need for the female principle of co-operation/context awareness as women have a need for the male principle of hierarchy/context independence, but the balance between the two is out of kilter in our society. The women's movement has created the awareness that there is another perspective. This awareness is important so that society can slide back and forth along the continuum that exists between the absolute patriarchal principle and the absolute female principle, neither of which alone can provide the kind of reality that leads to true fulfillment/completion/understanding.

You may ask, then, why women do not excel in the secondary school general level mathematics courses since these have traditionally consisted primarily of applied mathematics. And I will ask you to look at the applications. The Ministry Guidelines (1985) for the Grade 12 course, MATHEMATICS FOR BUSINESS AND CONSUMERS, has six sections, four of which could be considered applications: Mathematics for Investment, Mathematics for the Informed Consumer, Mathematics for Small Business, and Mathematics and Current Issues. Of those four topics, only Mathematics for the Informed Consumer would appear to be relevant to the futures of teenaged women students. Yes, you may say that many women invest money, many women run small businesses, and many women are involved in current issues, but traditionally these have not been women's topics. And most teenaged women have bought into the traditional life-style. They dream of marrying a successful businessman and living happily ever after in the suburbs with a two-car garage, four children and a dog. They do not see that these topics are relevant in the lives that they envisage for themselves. They do, however, see themselves as consumers. Nevertheless, the three sections of Mathematics for the Informed Consumer--1. Personal Budgets, 2. Credit Purchases, 3. Mathematics and Accommodation--are all areas that the traditional dominant-male husband will supervise. For the teen-aged woman student who has bought into the traditional patriarchal system (and most of them have), there are no applications in this Grade 12 course that are relevant to the future that she envisions for herself. The name of the other Grade 12 General Course, MATHEMATICS FOR TECHNOLOGY, continues to be of concern to many educators because technology takes us immediately into the male realm in which many women, and especially teenagers feel uncomfortable and perhaps unwelcome. Basic Level Courses with units on tiling and brickwork also

imply that mathematical applications are to be found primarily in the male world. Most of the applications in both General and Basic Level Courses are taken from the world of men. This is not to say that the world of women is non-mathematical. But the mathematics in the world of women has not been taken out of the contextual situation to be identified as mathematics. We do not yet have the framework for identifying the female applications.

The reason that women students do not excel in mathematics to the same extent as men students in senior secondary and post-secondary schools has nothing to do with differing abilities of the sexes. It has everything to do with the fact that the subject matter has been defined by men and for men and applied to the lives of men. It is for this very reason that the women's movement, while considering science, literature and literary criticism, history, law, economics, psychology, medicine, etc. from a female perspective, has neglected to tackle mathematics. It is the hierarchical, context-free, abstract nature of mathematics as it has been defined by men that makes it difficult to consider from a women's point of view.

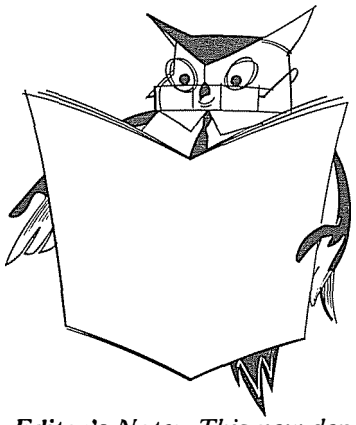
Those primary teachers who teach **Mathematics Their Way** have discovered an holistic approach to mathematics. Teaching and learning is co-operative; teacher and students together participate in both teaching and learning. Just as the teacher expects respect from the students so also does the teacher give the students respect. No answer is wrong/turned down. All answers are accepted, but some may require more explanation than others. If the methods of **Mathematics Their Way** were adapted for use in later grades, applications relevant to the lives of students, both female and male, would be presented by the students themselves, and the teaching methods would be more appropriate than the competitive sink-or-swim attitude that dominates many mathematics classrooms.

Women students will relate better to mathematics that is offered through and investigative approach once they overcome the expectation of the traditional mathematics classroom which they have developed throughout most of their schooling. Problems or investigations with the following characteristics could be chosen:

1. They can be approached in many ways.
2. They do not bring to mind a formula or model from traditional classroom mathematics.
3. There is a diversity of methods to reach the answer(s).
4. They build confidence since most people (with encouragement) do have a response.
5. They have some ambiguity which will allow students to interpret them in ways that may yield more than one answer.
6. Some new problems grow out of the initial problem.

Both women and men students will enjoy the creative possibilities and the sense of responsibility for their own learning that such an approach offers.

We cannot nor do we want to throw out the patriarchal approach to knowledge or to mathematics in particular. We can, however, enrich our knowledge by an awareness of the female perspective of our subject, and enhance our students' interest, understanding and success in mathematics by combining the patriarchal with the woman's approach.



HISTORY AND PHILOSOPHY OF MATHEMATICS

by
E.J. Barbeau, Editor
University College
University of Toronto
Toronto, Ontario M5S 1A1

Editor's Note: *This new department of the Gazette will deal with matters that come under the heading of history and philosophy. We hope the readers will send in queries, comments and short articles for consideration.*

The best general introduction to this area is a book by Philip Davis and Reuben Hersh entitled **The mathematical experience,** published by Birkhauser in 1981. It is in reality a collection of essays treating such matters as mathematical proof, what sort of reality mathematics deal with and the nature of the community of mathematicians.

One issue which has occasionally divided mathematicians is what constitutes an acceptable proof. For example, in the early part of the present century, the Intuitionists sought to avoid proofs which purported to show the existence of some quantity without providing some process which would display it. Although their movement was initially overwhelmed by the success of mathematical arguments which made liberal use of infinite sets, the axiom of choice and the law of the excluded middle (if **A** can be shown to be false, then **not A** must be true), E. Bishop tried to revive the constructivist approach in the 1960s through a book which reworked basic results in analysis.

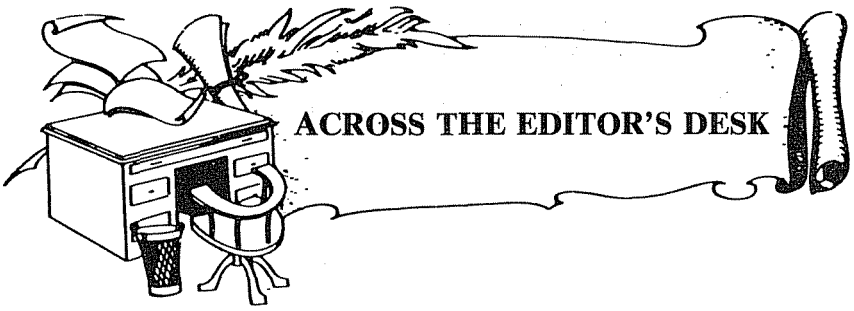
Here is a result which illustrates the point nicely. We wish to show that there are nonrational numbers a and b for which the exponential quantity a^b is rational. Consider $\sqrt{2}^{\sqrt{2}}$. Either this number is rational or it is not. If it is rational, then taking $a = b = \sqrt{2}$ provides the desired example. However, if it is nonrational, then taking $a = \sqrt{2}^{\sqrt{2}}$, $b = \sqrt{2}$ provides the example. In either case, we obtain suitable a and b.

While the traditionalist would not find fault with this argument, it somehow seems unsatisfactory. The reader is left with two possibilities, with no way of deciding which actually works. He cannot say what a and b actually **are.** Can something be said to exist if you cannot lay your hands on it?

Another way to look at the situation is structure the proof as a contradiction. Suppose, if possible, that numbers a and b as required do not exist. Then, if $\sqrt{2}^{\sqrt{2}}$ is rational, we are led to a contradiction. And if $\sqrt{2}^{\sqrt{2}}$ in nonrational, we are led to a contradiction. Are these contradictions enough to demonstrate the existence, or is there a middle way between existence and nonexistence? Further, how can we test whether $\sqrt{2}^{\sqrt{2}}$ is rational? Can we say that $\sqrt{2}^{\sqrt{2}}$ is either rational or nonrational in the absence of a well-defined procedure to accept one possibility and reject the other?

The way around these nagging questions would be either to demonstrate the character of $\sqrt{2}^{\sqrt{2}}$ or else to determine in some other way nonrationals a and b with a^b rational. Perhaps some reader would like to have a try at it.

In a somewhat different vein, what do readers make of the following chestnut? It is required to find the probability that a chord chosen at random within a circle of radius 1 has a length exceeding $\sqrt{3}$. Observe that $\sqrt{3}$ is the side length of the inscribed equilateral triangle. Consider the following three arguments:
(1) With no loss of generality, we can consider one end of the chord fixed at P. Consider the angle that the chord is to make with the tangent through P. In order to exceed $\sqrt{3}$ in length, the chord must pass into the interior of the inscribed equilateral triangle with one vertex at P, and so must make an angle between 60° and 120° with the tangent. Since all angles between 0° and 180° are possible, the required probability is ⅓.
(2) Without loss of generality, we can consider chords perpendicular to a fixed diameter. Such chords exceed $\sqrt{3}$ in length if and only if they cross the diameter no further distant than ½ from the centre of the circle. Thus, half the points on the fixed diameter are crossing points of successful chords, and the required probability is ½.
(3) With the exception of diameters, which we can ignore since the probability of a chord being a diameter is 0, there is a one-one correspondence between chords and points of the circle with each chord associated with its midpoint. Midpoints of chords exceeding $\sqrt{3}$ in length lie within a circle of radius ½ concentric with the given circle. This circle has ¼ of the area of the larger circle, and the required probability is ¼.



November 12, 1986

Mr. C. Gravelle, Editor
College of Education, Brock University
St. Catherines, Ontario L2S 3A1

Dear Mr. C. Gravelle:

I am attempting to complete my collection of Ontario Mathematics Gazettes, and was wondering if you or any of the readers might be able to assist me in this.

I am of course expecting to pay for any issues that are sent to me.

I am missing Volumes 1 - 8 and Volume 13 number 1.

I would appreciate your help in this matter.

Yours sincerely

Tom Griffiths

Tom Griffiths

November 15, 1986

The Ontario Association For Mathematics Education
Mr. M. MacGregor, Secretary-Treasurer
247 Bright Street
Sarnia, Ontario N7T 4E9

Dear Mr. M. MacGregor,

CUSO is an international development organization which places Canadians in Third World countries in response to requests for personnel assistance.

One of the requests we frequently receive is for math/science teachers at the secondary school level. For instance, we are currently providing Canadian volunteer assistance for Nigeria's community schools program. We anticipate continued placements in this sector.

We would appreciate any assistance you could provide in our search for Canadians to assist with the development of Third World schools and communities.

Thank you for your attention,
Sincerely,

B. Kuehs

B.K. for Carol Shepard
Education Officer, Resource Group

cc: London Local Committee

MATH, CHEMISTRY AND PHYSICS TEACHERS

DESCRIPTION:

Due to their traditional distance from technology, students in developing countries often have difficulty with the 'scientific thinking' required for math, physics and chemistry.

In addition, few schools have the money to equip a laboratory or to purchase teaching aids. Thus students do not receive the 'hands on' science experiences which are an integral part of learning difficult concepts.

CUSO receives ongoing requests for Canadian math/science teachers to work in secondary schools and teacher training colleges. Teachers need a sound knowledge of their subject area and the ability to transfer their knowledge to others. They are often required to make their own equipment, to modify science curriculums to include the teaching of basic skills which were missed, and to improvize to make learning a practical rather than a theoretical experience.

QUALIFICATIONS:

Minimum: B.Sc. in math, chemistry of physics
Preferred: B.Sc., B.Ed. and proven teaching ability.

TERMS OF SERVICE: CUSO is a private, non-profit organization which places a variety of qualified technical and professional Canadians in developing countries on two-year contracts.
Salaries are sufficient to cover the cost of living in the host country. CUSO provides transportation and medical/life insurance, and other benefits.
Couples will be considered if there are positions available for both people. Opportunities for families with children are more limited.

APPLICATION: CUSO is represented across Canada through Local Committees and Regional Offices where you can obtain the necessary information and, if eligible, begin application.

INTERESTED? Then consult your phone book for the CUSO office nearest you.



PROBLEMS

by
Ronald Lancaster, Problems Editor
Oakville-Trafalgar High School
Oakville, Ontario L6J 3L6

1. The Ultimate Word Problem

   This problem is from an excellent book, **Amusements in Mathematics** by H.E. Dudeney. (Dover Publications, page 8).

   The combined ages of Mary and Ann are forty four years, and Mary is twice as old as Ann was when Mary was half as old as Ann will be when Ann is three times as old as Mary was when Mary was three times as old as Ann. How old is Mary?

2. Prove that the thirteenth day of the month lands on a Friday more often than any other day.

3. Straighten out and make sense of the following:

   3 times 5 is 4 times 6 plus the same as 2.

4. Without using a calculator, show that

$$99^{50} + 100^{50} < 101^{50}$$

5. Without using a calculator, prove that

$$\sqrt{2} < \frac{\sqrt{5}}{2} + \frac{\ln(2 + \sqrt{5})}{4}$$

Answers on Page 60



WHAT'S GOING ON

KENNETH D. FRYER AWARD

In Memory of the late Dr. K. D. Fryer, OAME in conjunction with the Grand Valley Mathematics Association established the Kenneth D. Fryer Award for Excellence, to be presented annually to a Mathematics Department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

In 1984-85, the inaugural year, the award was presented to two schools:
La Salle Secondary School (Sudbury)
O'Neill Collegiate and Vocational Institute (Oshawa)
There will only be one such award presented in each subsequent year. In 1985-86, it was awarded to Overlea Secondary School (East York).

At this time OAME is requesting nominations for consideration by the committee for this year's award. It will be presented at the annual conference in Scarborough.

Please complete a copy of the form below and forward it to:
Mr. M.B. MacGregor
Sec./Treas. OAME
247 Bright Street
Sarnia, Ontario
N7T 4E9

Deadline for nomination submission: March 31, 1987

Kenneth D. Fryer Award
Nomination Form
1986-1987

I/We nominate School: _____

Address: _____

Principal: _____

Math Dept. Head: _____

Nominated by: _____

Please attach a separate sheet to the nomination form outlining the reasons for your nomination.

SCARBOROUGH '87

14TH ANNUAL O.A.M.E. CONFERENCE: "Of Mice and Math...", Scarborough Association for Mathematics Education is pleased to host the fourteenth annual conference of the Ontario Association for Mathematics Education at Centennial College, Progress Campus, on Thursday 7 May to Saturday 9 May 1987. Count yourself in on the fun and sharing of ideas.

THURSDAY'S DYNAMIC KEYNOTE SPEAKER: David R. Johnson, chairman of mathematics and science departments at Nicolet High School Glendale, Wisconsin is the author of "Every Minute Counts: Making your Math Class Work", co-author "Guidelines for the Tutor of Mathematics", and author of several articles for "Mathematics Teacher" and "Arithmetic Teacher". He is a dynamic, entertaining, informative speaker: one who is constantly in demand. Mr. Johnson was a speaker at the 64th NCTM in Washington 1986, the keynote speaker at NCTM Region in Columbus Ohio in December 1985, speaker at NCTM Regional in Milwauke Wisconsin in November 1985, speaker at NCTM Regional in Winnipeg in October 1984, speaker at the 61st NCTM in Detroit 1983, speaker at the 60th NCTM in Toronto 1982. This outstanding classroom teacher of 28 years was Wisconsin's "Outstanding Magician" in 1966 and is currently president of National Council of Supervisors of Math. David R. Johnson will display "Showmanship, salesmanship, and surprise - specific ideas for teaching with these elements". What you do and say in the classroom makes a difference. Being both effective and efficient in your mathematics teaching is a real skill. Strategies for developing classroom techniques that will increase classroom involvement and student attentiveness will be presented. Ideas relating to the art of questioning and obtaining student involvement will be discussed."

FRIDAY'S OUTSTANDING BANQUET SPEAKER: Dr. Peter Hilton, distinguished professor of mathematics at State University of New York at Binghamton, Binghamton, NY, past president of Mathematical Association of America, author of 15 books and over 300 papers, member on a number of committees with national scope, speaker at 63rd NCTM San Antonio 1985 ("What are polygons and polyhedra? Why should we study them?"), banquet speaker 61st NCTM Detroit 1983 ("Cryptography and the early history of computing: In World War II, a group of British mathematicians were brought together to decipher enemy codes. The methods involved, effectively, the invention of the computer"), speaker 60th NCTM Toronto 1982 ("Cryptography in the second World War: A description of personal experiences at Bletchley Park - 'Churchill's secret weapon'. Tribute will be paid to Turing's genius. Conclusions are drawn about the employment of mathematicians.") Dr. Hilton will entertain and inform us on his favourite topic.

SUPERIOR PROGRAM: The "Murr" (Murray Major, Mowat CI) promises an excellent program this year. The conference is again starting on Thursday and finishing with meaningful, interesting presentations on Saturday afternoon (about 3 p.m.). Peter Crippin, Shirley Fairfield, and Murray Major have all been busy preparing our 1987 program. They have prepared several strands: special education, contests, general level, evaluation, computers, women and math, and the ever popular "Make it, take it"; note that Scarborough will again offer the "Trading Post". Computer facilities and faculty are provided by Centennial College an active partner, eager to host our conference.

VISITOR ATTRACTIONS:

SCARBOROUGH'S METRO ZOO:

Explore the continents of the world ... meet Brum the Siberian tiger ... Indo-Malayan orangutans ... African foxes ... Chinese leopards ... all waiting for you in their climate-controlled natural habitats! More than 3000 of the "most famous animals in the world" live in this 710-acre (287 ha) zoo-geographical park, considered one of the finest in the world. An electric train can be boarded to view the park, for a nominal cost. Lunch can be purchased while you roam the grounds. Wear comfortable walking shoes!

ONTARIO SCIENCE CENTRE:

Take a fascinating expedition to the Ontario Science Centre. Metro's famous "hands-on" museum of the 21st century, whose motto is "please touch!" Challenge wits with a computer in a game of tic-tac-toe, participate in a simulated space-docking operation inside an astronaut's capsule, watch a laser beam cut through a brick ... these and over 500 mind-bending exhibits appeal to all ages.

Families will appreciate the cose proximity to both urban and rural recreations. Shop at Scarborough Town Centre Shopping Mall. Picnic at Bluffers Park. Ride on the Light Rail Transit system (Scarborough's "Toonerville Trolley"). Tour Teleglobe Canada and Pickering Nuclear Power Plant. Jog the trail from Centennial College through the ravine down to Scarborough College.

ADVICE AND INFORMATION: If you have any questions or advice to offer, please contact our conference chairmen: STEVE MARTIN, Danforth Gardens PS, 20 Santamonica Blvd., Scarborough, M1L 4H4, telephone 416-265-1411; PAUL ZOLIS, MATH CENTRE, ST. ANDREWS JR. PS, 60 Brimorton Drive, Scarborough, M1P 3Z1, telephone 416-431-5644.

ARNIE CAREFOOTE
PUBLICITY CHAIRMAN
OAME '87

ALBERT CAMPBELL CI

MR. LANCASTER COMPARES CALCULUS TO DRIVING

Editor's Note: This excerpt is reprinted with the author's permission, Sally Armstrong, "Super Teachers", Canadian Living September 6, 1986: 40.

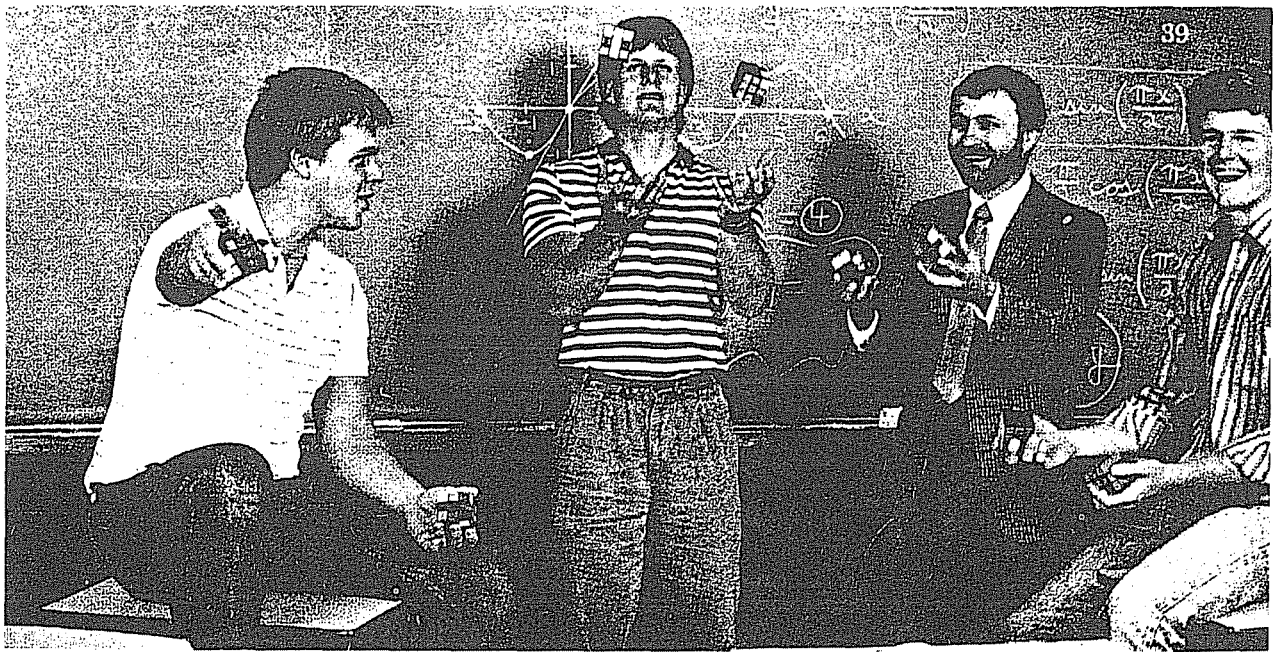
One of the things we remember Ron Lancaster doing when he taught math at Oakville Trafalgar High School was relating it to practical applications. He could make math easy for everyone to understand. For instance, he would compare calculus to driving a car through an S-turn. He used the radio dial and the Richter scale to teach logarithms. He started with the basics, used lots of exercises and repetition and got his point across -- every time. He also made learning fun. He ran a Rubik's cube contest one year at our school and was always organizing math contests and using magic tricks.

We repected him because he was knowledgeable in his field and liked him because he was so interested in others. He always had time to help. He was soft-spoken, and no one can ever remember him getting upset in the classroom. He played no favorites. He obviously liked all the kids -- not just the brainy ones.

He understood the pressures of exams and tried to defuse them. There were often cartoons drawn on our exam papers. And one time, he left chocolate mints on each desk so that when students arrived to write the exam, the first thing they saw was candy.

He's the best.

Jamie, Bill and David Moffatt,
Oakville, Ont.



Math teacher Ron Lancaster of Oakville Trafalgar High School in Oakville, Ont., says he uses magic tricks to capture his students' imaginations. "I have as much fun as the students in my classes. I love teaching them and watching them grow". Three of Lancaster's fans, David, Bill and Jamie Moffatt, remember that part of the fun of math classes were Rubik's cube contests.

JOHN G. McCARTNEY SCHOLARSHIP

Mona and Marley MacGregor and judges Jack LeSage of Barrie and Debra Heaton of Sarnia are pleased to announce the results of this years's competition. After careful scrutiny and much deliberation, Guiseppe Russo of Lasalle Secondary School in Sudbury was named the 1986 winner of the John G. McCartney Scholarship.

Morley and Mona MacGregor journeyed to Sudbury in mid July where we met with Guiseppe and his parents to make the official presentation. Guiseppe graduated from Lasalle in June 1986 with marks of 100 in Calculus, Functions and Relations and Problem Solving and 98 in Algebra. An active participant in various Mathematics contests , he scored 78 on the Descartes (8th best in Canada), tied for 6th in the CNML and the AHSME score of 121. He also was first in the McMaster contest and second in Carleton's.

In trials for the Canadian Olympiad, he tied for first and thus became a member of our International Olympiad Team for the second year in a row. As an I.M.O. training session participant he placed first overall for six problem sets.

In the fall of 1986 Guiseppe has enrolled as a student in Mathematics at the University of Waterloo. O.A.M.E. extends its congratulations to Guiseppe on his past achievements as well as best wishes in his future endeavors.

M. Sandblom, O.A.M.E. Director



NEEDED: Editor(s) for the ABACUS
(16 page centre-section of the Gazette)
for a two year term,
beginning with the September, 1987 issue

Apply to: M. MacGregor by Feb. 1st. 1987
247 Bright Street
Sarnia, Ontario
N7T 4E9

INTERESTED! BUT UNDECIDED
For information call: Sam Privica 519-623-3600
Galt Collegiate,
200 Water Street, North
Cambridge, Ontario
N1R 6V2

CANADIAN MATHEMATICS TEACHER

In the registration package for the Sudbury Conference an opportunity was given for the purchase of the Canadian Mathematics Teacher. To date no magazine has been received. If any one who wishes the $3.00 refund please send a SELF addressed envelope to OAME, 247 Bright, St., Sarnia, Ontario N7T 4E9. No requests received, at the above address, after January 30/87 will be honored.

BOOK REVIEW

by
Book Review Editor, R.S. Smith, Faculty of Education
University of Western Ontario, London, Ontario N6G 1G7

Fractions: Children's Strategies and Errors, Daphne Kerslake. N.F.E.R. - Nelson Publishing Co., Darville House, 2 Oxford Road, E., Windsor, Berkshire SL4 10F England.

In the September 1984 issue of O.M.G. I outlined a series of books which were the results of the Strategies and Errors in Secondary Mathematics Project. The book being reviewed is one of this series.

As the title implies, this book is most useful for teachers attempting to help children remedy their errors. In particular, it focuses on three aspects of fractions. From interviews, the three areas of difficulty found were: (1) that the fraction a/b can be interpreted as a $\div$ b (2) that fractions are numbers and (3) simple equivalent fractions. The teaching sequences are outlined and the worksheets are contained in the appendicies.

The most worthwhile part of the book comes in the summary chapter where the findings are discussed and suggestions are given concerning the implications of these findings. Briefly the findings can be stated for the three areas as:

1. The only model of a fraction that was familiar to all the children who took part in the study was the "part of a whole" one;

2. The division aspect of a fraction was not familiar nor readily accepted;

3. Many children found it difficult to accept the fact that a fraction is a number.

For the following encouraging statement "a short period of teaching intervention can prove beneficial", there is a discouraging observation "the number of instances of children's avoidance of fractions is remarkable." Apparently the geometric 'part of a whole' view of a fraction is an inhibiting presence and sets up a cognitive conflict that children have difficulty resolving when teachers attempt to extend the concept of a fraction. A problem still!

This book provides many insights into pupil's interpretation of basic ideas concerning fractions, but better yet, it gives examples of teaching sequences that may help the pupil overcome problems.

BOOK REVIEW

by
Eric Wood, Faculty of Education
University of Western Ontario, London, Ontario N6G 1G7

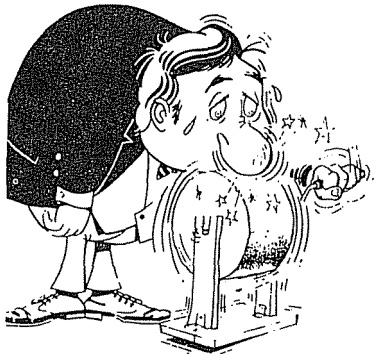
Teaching Secondary School Mathematics, Alfred S. Posamentier and Jay Stepelman. Charles E. Merrill Publishing Company, Toronto, 1986.

As the title implies, this book is a text for prospective teachers of mathematics. It is, however, more than its name suggests, as it also contains a selection of enrichment topics that are extremely relevant to the level of material taught in secondary school mathematics.

The book is well written and covers the traditional topics of pedagogy such as lesson planning, questioning, evaluation and motivation in a thorough and practical fashion. Examples and illustrations are employed to clarify the theoretical discussions of the text and show how to apply the general principles that have been presented. In addition, new emphases in mathematics education such as problem solving and computer and calculator use are discussed at some length. Each chapter ends with a list of suggested readings, most of which are from sources readily available to mathematics teachers. Consequently, even experienced teachers can gain some fresh insights from these initial chapters.

The second sectin of the book would be particularly useful to a practising teacher, whether they teach a gifted group of students or whether they need a few extra ideas with which to challenge a good student. In total the book presents 119 actual enrichment topics complete with objectives, teaching strategies and suggestions for evaluation. These topics run the gamut from geometry and ancient methods of representing numbers to using computers to solve mathematical problems.

This book would make a worthwhile addition to the professional library of any secondary school mathematics department.



COUNCIL COMMENT

by
Bob Robinson

Editor's Note: *Council Comment appears in an issue of the* ***Gazette*** *following an OAME/AOEM Council meeting. Your comments are welcome and should be addressed to the Editor.*

CHAPTER REPORTS:

COMA/AMOC

An excellent Leadership Conference was held on September 19 and 20 at the Opinicon Lodge at Chaffeys' Lock. Some 49 Heads of Mathematics, Assistant Heads, and invited guests from Ottawa-Carleton and surrounding areas were joined by 19 contributors. The conference centred around leadership requirements at the department level and the implementation of the new Mathematics Courses. Feedback was extremely positive especially with regard to its early date in the school year. Preparation has begun already on next year's version which will be expanded by half a day to accommodate an additional 25 participants. Watch for further details.

Two future events are being planned for this school year. The first will take place in the third week of January and will consist of a dinner-speaker evening. COMA will be repeating its 3rd Annual Publisher's Display. It will be held from 3:00 p.m. until 7:00 p.m. at the Embassy West Hotel in Ottawa on Tuesday, April 7, 1987. A cash bar and some appetizing food (courtesy of COMA) will be provided for the participants while they browse through the exhibits.

GOLDEN SECTION

They are continuing to prepare for the 1988 Annual Meeting.

GVMA

The new booklets advertised in the September **Gazette** are now ready for distribution.

NOMA

A good turnout was reported at the NOMA Conference held during the last weekend in September. An even bigger conference is being planned for next September; once again in Sudbury.

PINE RIDGE

The annual regional P.A. day was held on November 14, 1986 with three general sessions in the morning and five workshops in the afternoon.

Pine Ridge's scholarship fund has been expanded to include funding for some mathematics conferences.

QSLMA

The Fall Meeting was held at Jones Falls in conjunction with Lanark County's P.D. Day.

The Spring Meeting will be held at RMC and will feature excellent food, hospitality, ambience and the usual high quality mathematical presentations. The date is Wednesday, April 8, 1987.

SAME

This chapter is continuing to grow. Memberships are reported from Scarborough, Durham, York County, Metro Separate, North York, and Toronto. Current members are renewing their memberships and many are now choosing to become members of the parent organization, OAME.

A dinner meeting was held at Maplewood Vocational School from 4:00 p.m. to 9:00 p.m. where fifteen workshops were offered at all levels.

Four afternoon P.D. sessions are being offered at Howard Johnson's from 4:00 p.m. to 6:00 p.m.

Date	Session
November (third week)	Junior
November (fourth week)	Senior
March (last week)	Intermediate
April (first week)	Primary

SAME is planning to issue a newsletter every two months or so.

SAME will again award a $150 scholarship in the name of Jim Fencott to a deserving student graduating from a Scarborough High School. SAME also presents plaques of achievement to high ranking students in Scarborough's J.I.R. McKnight problem-solving contest.

PAME

PAME hosted a major dinner and P.D. session on Monday, October 20. Workshops were conducted for teachers in all four divisions. Over 200 participants attended sixteen different sessions.

Future plans involve a mini-conference on a Peel Regional P.D. Day in February as well as another event in the Spring of 1987.

There will be a major membership drive to acquire members from the Dufferin Peel Roman Catholic Separate School Board and other neighbouring boards.

ANNUAL MEETINGS:

1986 Tom Griffiths presented the financial report of the London Conference and received the congratulations from Council on behalf of the 1986 "Annual" organizers.

1987 SAME is actively planning for the 1987 Annual Conference under the leadership of Steve Martin and Paul Zolis. Centennial College is eager to assist to make our '87 Conference successful. Two excellent keynote speakers have been announced: David Johnson on Thursday and Dr. Peter Hilton on Friday.

1988 Plans are progressing as the Golden Section gets ready to host the '88 Annual at the Brock Hotel Complex in Niagara Falls on Thursday through Saturday, 1988 05 5-7.

1989 COMA will host the 16th Annual in Ottawa from May 11-13, 1989.

NCTM A very successful Region Conference was held in Edmonton and excitement is growing about the Halifax Meeting which will be held August 19-22, 1987. People are starting to make plans for this exciting summer meeting. The publishers are planning some outstanding activities.

The NCTM is in the midst of a new membership campaign where memberships are available at 50% off for qualified individuals and many prizes are also available. Contact Ralph Connelly for more information.

Bob Robinson has been appointed chairman of the Elections Committee for the Council. Next year a slate to elect four directors-at-large will be presented.

Members of NCTM should note that one of our OAME members, Lorna Morrow, is running for Director in the upcoming election. Watch for your ballot coming with your copy of the News Bulletin.

AWARDS:

Father Faught Award

The Toronto French School is this year's recipient.

McCartney Award

G. Russo of Sudbury is the award winner this year.

Ken Fryer Appreciation Award

See the advertisement elsewhere in this issue of the **Gazette.**

COMMITTEE REPORTS:

Women and Mathematics Committee

This committee has been working in co-operation with the program committee of the 1987 Annual Conference to prepare a strand of speakers on Women and Mathematics. Individuals are encouraged to submit articles on women and mathematics to the editor of the **Gazette.** A speaker's list which could be distributed to the chapters of OAME will be discussed at their February meeting.

Publications Committee

Our editor was commended for his excellent inaugural issue of the **Gazette/Abacus** and a heavier cover paper was suggested. We are phasing out subscriptions to the **Gazette** in favour of memberships. This has been a successful campaign so far. OAME members are encouraged to send in suggestions for editorial topics. These should be sent, or submitted, to the editor.

Membership and Publicity Committee

An updated membership brochure was distributed to each councillor. If you wish copies of this new document, contact a Councillor or the Secretary-Treasurer. Certificates of Merit will be made available to the membership. Watch for further information. Membership in OAME will be promoted at a North West Conference in Thunder Bay on February 27, 1987. It is a hope of Council that a new vibrant chapter can be created in North West Ontario.

Intermediate/Senior Committee

David Wing is OAME's representative on the Calculus Committee of the Ministry. The OAIP at the Senior level has completed the grade 12 review and is now working on the grade 11 advanced and general. The Intermediate OAIP screening will begin in November.

Here are some other concerns or directions for the committee. What do you, our readers, think?

- How can we best serve the teachers of the Intermediate/Senior Divisions?
- What about province-wide testing?
- Semestering: Is it an issue that is affecting mathematics content?
- Should we have a leadership seminar centering on the Intermediate/Senior Divisions?
- Can we do a better job of teaching mathematics to the university bound student?
- What should be said about the grade 7 through 9 interface? Should grade 9 be taught in our grade schools? Should grade 8 be taught in our high schools?
- What are some successful teaching strategies used at the general level?
- What are effective and realistic ways to teach problem solving?
- How do computers help to enhance the teaching of mathematics?
- What should be said about non-mathematics specialists teaching mathematics at the secondary levels?
- Should the Ministry consider offering mathematics specialist programs to teachers at the grade 7/8 levels?

Primary/Junior Committee

The committee discussed its concern about the "Primary Report" and will no doubt draft a response to be endorsed by Council at a future meeting.

Futures Committee

A history of OAME will be compiled in order to be available for our 100th Anniversary in 1991.

A video or slide presentation highlighting OAME activities and benefits will be investigated.

Nominations Committee

Members are encouraged to run for the positions of Councillor or Director. Contact Paul Lessard, Paul Pogue, Bob Robinson, or write the Sarnia Office.

OTHER:

Canadian Olympiad

OAME will design and have produced blazer badges for the Canadian International Olympiad Team.

Posters

The Executive will investigate a contest for posters illustrating unsolved problems in mathematics open to students of Ontario.

Future OAME/AOEM Meetings

Executive	Council
November 21, 1986	February 14, 1987
April 24, 1987	

1987 Annual Conference -- May 7-9, 1987 (Scarborough)

CALL FOR NOMINATIONS

At the Annual Meeting of OAME/AOEM in Scarborough in May, 1987 we shall announce the results of elections for 1 President-Elect, 3 Executive Members (Directors) and about 8 councillors.

If you would like to fill one of these positions or wish to nominate someone else, please complete the following and send it to:

Paul Lessard, 60 Leinster Ave. South, Hamilton, Ontario L8M 3A1

This nomination **must** be accompanied by a summary of qualifications of the nominee. These summaries, together with the names of members willing to run for office will be published in March. In OAME/AOEM the procedure of voting by mail is followed.

I nominate _____(name)

for the position of _____ President-Elect

_____ Director

_____ Councillor

Name of nominator: _____

FINAL DATE: January 30, 1987

ANSWERS TO THIS MONTH'S PROBLEMS

1. Mary is 27½ years old. (Ann is 16½ years old)

2. We need only consider one cycle of 400 years in order to find the relative frequency of the days of the week for a given day of the month. From January 1, 1900 to January 1, 2300, there are 4800 occurences of the thirteenth of the month. The distribution is given below:

Sunday	687	Monday	685
Tuesday	685	Wednesday	687
Thursday	684	Friday	688
Saturday	684		

Thus the thirteenth day of the month lands on a Friday more often than any other day.

3. 4 times 5 is the same as 3 times 6 plus 2.

4. Suppose that $99^{50} + 100^{50} > 101^{50}$

Then $101^{50} - 99^{50} < 100^{50}$

But $101^{50} - 99^{50}$

$= (100 + 1)^{50} - (100 - 1)^{50}$

$= 2\{\binom{50}{1}100^{49} + \binom{50}{3}100^{47} + \ldots + \binom{50}{49}100\}$

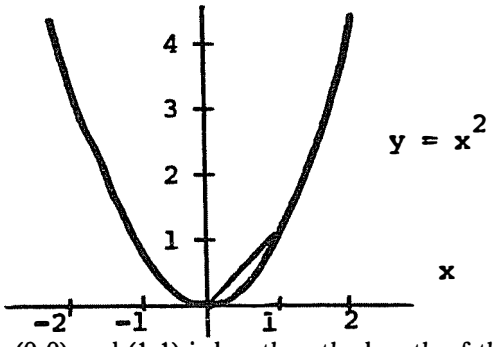
$> 2\{\binom{50}{1}100^{49}\}$

$= 100^{50}$

Thus $100^{50} < 100^{50}$ (contradiction).

Therefore $99^{50} + 100^{50} < 101^{50}$ as required.

5. Consider the curve $y = x^2$ and the two points (0,0) and (1,1).
Join (0,0) to (1,1) with a straight line.



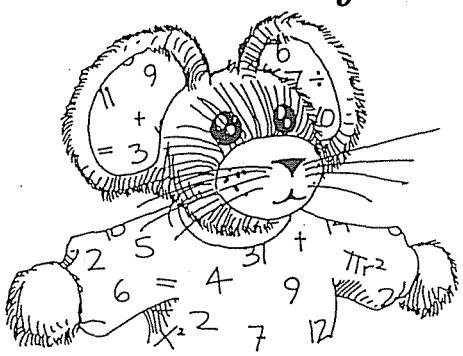
Note that the distance between (0,0) and (1,1) is less than the length of the curve from $x = 0$ to $x = 1$.

Thus $2 < \int_0^1 \sqrt{1 + 4x^2}\, dx$

$= \frac{\sqrt{5}}{2} + \frac{\ln(2 + \sqrt{5})}{4}$

Ontario Association for Mathematics Education

14th Annual Conference



Of Mice and Math...

May 7 - 9, 1987

Centennial College of Applied Arts and Technology
Progress Campus, Scarborough, Ontario

Hosted by the SAME *Scarborough Association for Mathematics in Education*

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
1986-87 EXECUTIVE AND COUNCILLORS

Past Pres. - Paul Lessard

Pres. - Dino Dottori

Pres. Elect - Ralph Connelly

Director - Mary Lou Kestell

Director - Michael Sandblom

Director - J. Symington

Councillors Term Expiring 1987

Handley, David

Hill, Robin

Hudson, Lonnie

Leigh, William

McNabb, George

Sauer, Ronald

Wiggan, Lorna

Rasmussen, David

Councillors Term Expiring 1988

Carlow, Chester

Collins, Eileen

Davidson, Dave

Friar, Betsey

Griffiths, Tom J.

Harvey, M. Elaine

Henderson, Douglas

Morrow, Roland

Pravica, Sam

Robinson, Robert

Shifrin, Jeffrey

Councillors Term Expiring 1989

Carli, Enzo

Knill, George

Lunney, Jerry

Martin, Steve

Pogue, Paul

Stuart, Susan

Weiner, Jack

Chapters and Representatives

COMA, Lenczewski, Peter

GOLDEN, Haines, Edward

GVMA, Telfer Stuart

NOMA, Kenny, Ann

PAME, Jones, Robert

PINE RIDGE, Pasricha, Kumar

QSLMA, Whitley, J.A.

SAME, Carefoote, Arnold

OAME Office - M&M MacGregor, 247 Bright St., Sarnia N7T 4E9 519-337-8382

Gravelle, Clifford Editor - OAME Gazette, Faculty of Education, Brock University, St. Catharines, Ont. L2S 3A1