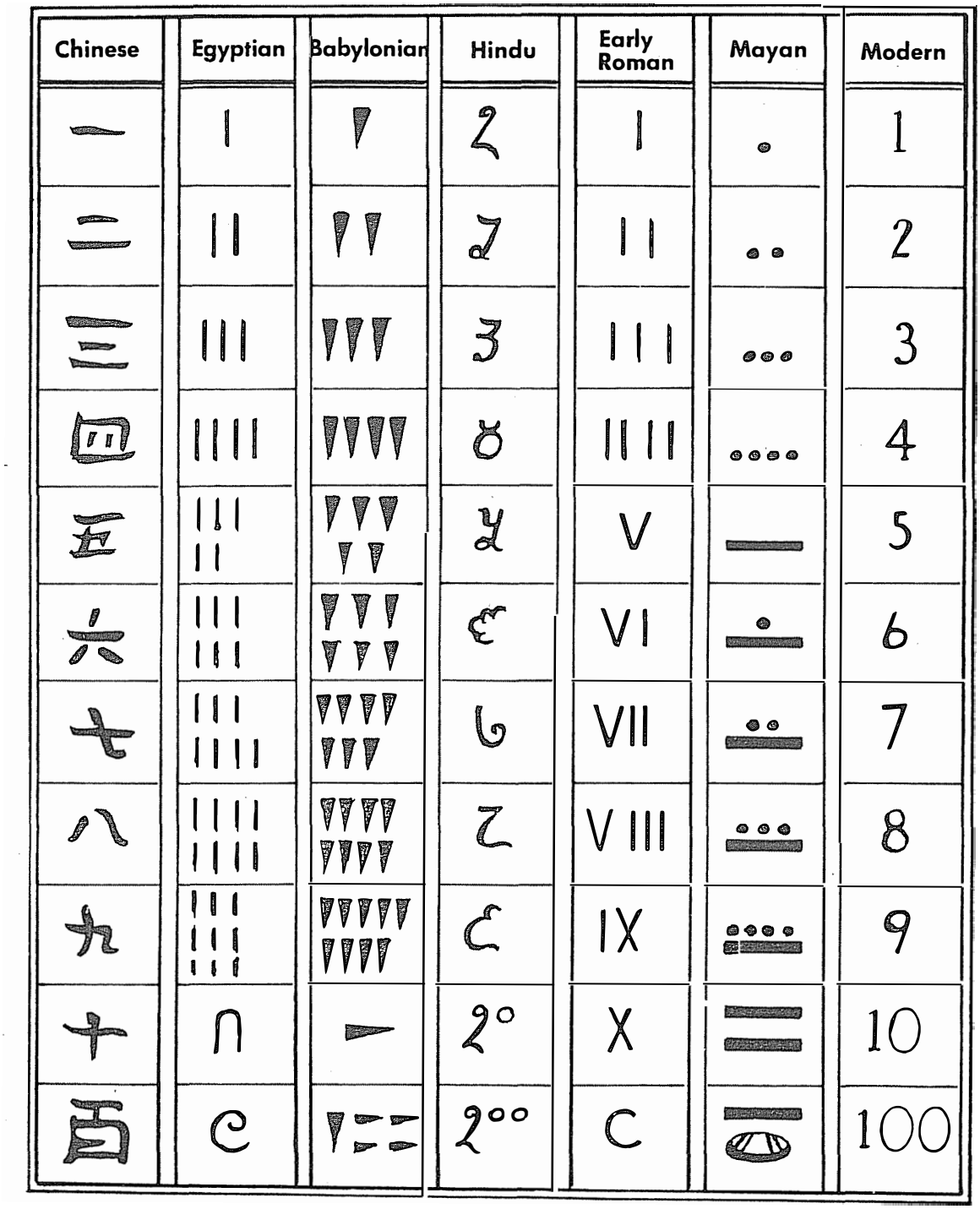
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VOLUME 25 NUMBER 3 MARCH 1987



Chinese	Egyptian	Babylonian	Hindu	Early Roman	Mayan	Modern
一	𓏺	𒁹	९	I	•	1
二	𓏺𓏺	𒁹𒁹	८	II	••	2
三	𓏺𓏺𓏺	𒁹𒁹𒁹	३	III	•••	3
四	𓏺𓏺𓏺𓏺	𒁹𒁹𒁹𒁹	४	IIII	••••	4
五	𓏺𓏺𓏺 / 𓏺	𒁹𒁹𒁹 / 𒁹	५	V	—	5
六	𓏺𓏺𓏺 / 𓏺𓏺𓏺	𒁹𒁹𒁹 / 𒁹𒁹𒁹	६	VI	•—	6
七	𓏺𓏺𓏺 / 𓏺𓏺𓏺𓏺	𒁹𒁹𒁹𒁹 / 𒁹𒁹𒁹	७	VII	••—	7
八	𓏺𓏺𓏺𓏺 / 𓏺𓏺𓏺𓏺	𒁹𒁹𒁹𒁹 / 𒁹𒁹𒁹𒁹	८	VIII	•••—	8
九	𓏺𓏺𓏺 / 𓏺𓏺𓏺	𒁹𒁹𒁹𒁹𒁹 / 𒁹𒁹𒁹𒁹	९	IX	••••—	9
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TABLE OF CONTENTS

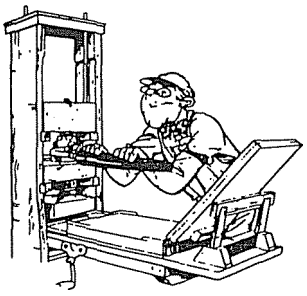
FEATURES:

Editorial 2

The President's Corner 3
Geometry , symmetry and relations between Dedekind zeta functions 4
Putting the byte on the Bard 6
The agenda for action in Canada 8
Student computer literacy an outmoded teaching idea 13
Canadian mortgages 14
GAIM: An interview with Alice Onion 16
"Transforming" a difficult problem 20
Reflections on teaching and learning mathematics 21
SMPY at ISU 23
La regle de T2: Une application des logarithmes 26
Logo in London: A mathematics enrichment project 28
"Macpainting" the inverse trig functions 30
"Let x = ..." 36

DEPARTMENTS:

Women and Mathematics
  Editorial 38
  Girls and Mathematics: Part I 39
  A Bibliography 40
History and Philosophy of Mathematics 42
Across The Editor's Desk 44
Problems 46
What's Going On 49
Book Review 53
Council Comment 54



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OS:IS and Mathematics: **CHALLENGE and OPPORTUNITY**

OS:IS is a renewal of our mathematics teaching and presents an excellent opportunity for growth and rejuvenation.

We have a tremendous opportunity for rejuvenation of our curricula, our methodologies, our spirit, our enthusiasm. Seize every opportunity to enjoy this renewal. How about starting with this year's annual conference at Scarborough and sharing the excitement?

Reactionaries are threatened. They would have us believe that we will not cover the curriculum, that we will be involved in more testing, that mathematics education will suffer. Don't be deluded by these Cassandras.

Persons concerned with the harmful effects of OS:IS should realize the benefits of this broader view of an educated person. We do not teach mathematics. We teach people through the medium of mathematics. We are teachers and must be concerned with the healthy development of the whole student, not just intellectual development but also social, emotional and physical growth. This new emphasis on process and people replaces our narrow concern solely with results; remember the "good old days" when teachers spent half the year reviewing old examinations so that students would score well on the departmental examinations. Well, learning how to learn and enjoying learning are important life skills and are now officially recognized as such.

Improved evaluation skills will be demanded by this new approach - formative and summative evaluation. We should use a variety of tools, not just testing, but methods such as projects, seminars, oral presentations, peer and self-evaluation. Sharing experiences with members of other departments can prove invaluable; Mrs. Vera Taylor and her associates will be discussing evaluation of mathematics at our annual conference this year.

Greater involvement of the community is requested under OS:IS; many companies will provide speakers and handouts for your classes. Involve your students in securing these community resources; the students have many contacts and will often provide expert speakers. Students can be given assignments or projects that require them to get involved with the community; for example, banking or real estate assignments. Some of these vehicles are scheduled for presentation at Scarborough '87.

More varied teaching methodologies will be required. Our OAME council is preparing a "survival handbook" for mathematics teachers that will outline some of these techniques, but we can share these ideas with each other - our successes and our failures - by writing articles for this Gazette, by attending chapter meetings, by attending the annual conference. Lets work together.

"Mathematics is the language with which God has written the universe!" - Galileo. Lets continue to spread the good news.

Arnie Carefoote, Guest Editor
Assistant Head, Mathematics
Albert Campbell CI
Scarborough, Ontario



THE PRESIDENT'S CORNER

by
Dino Dottori

The last council meeting, on Valentines Day, concluded the formal meetings of this council, but not its work. Many of the initiatives that were taken on by the council will continue either because like the publications committee, they are ongoing or because timelines were set that carried our work into another year.

As the voice of mathematics teachers in the province, it was most fitting that the Primary/Junior Committee presented a position paper as the OAME response to the Early Primary Education Project (EPEP). The position is that our organization supports the philosophy as expressed in the report of May 1986. The committee further saw fit to send three recommendations to the Ministry of Education on your behalf - these related to a planned mathematical program, recognition of play as a natural way of learning, and appropraiate primary education qualifications for teachers. More on this will appear elsewhere in the publications. The members of the Primary/-Junior Committee and Susan Stuart, Chair, are to be commended for their fine work and for the quality of the response.

The Intermediate/Senior Committee, chaired by Doug Henderson, has had a wide variety of tasks this year. The members of this committee have been very busy representing OAME at TVO, the OAIP and the Calculus Committees at the Ministry of Education, as well as the curriculum committee at OSSTF. A very interesting subcommittee had been struck to develop a ways and means to recognize exemplary practice in the teaching of problem solving. Look for some writings from Chester Carlaw in an upcoming issue of the Gazette on this one.

Speaking of recognition of exemplary practice, Tom Griffiths who has done a lot of work with contest teams, is heading up a special group that will develop a plan to hold a poster contest to stimulate interest in unsolved problems. More on this contest for students in an upcoming issue.

The Membership Committee, with Chair Betsy Friar, has worked very hard to develop strategies to maintain the membership and to introduce new people to OAME. It is through the efforts of this committee that as an organization, we have been able to gradually increase our numbers so that we now have about 1500 people enjoying membership. As mentioned in an earlier "corner" much of the work of the OAME is carried out through the council or in the chapters.

At the time of this writing there has been some activity in the North West and South West of the province to organize chapters. The executive through the membership committee and council supports these activities and looks forward to welcoming new chapters. So remember to boost OAME - it's good for math education.

Paul Pogue and the Futures Committee have reported that they are ready to proceed with the documentation of the first hundred years of the unbroken line of mathematics education associations in Ontario (OATMP, OATM, The Math Commission, and now the OAME). As well, this committee recognizes the number of experienced teachers, new to mathematics, that will have math classes for the first time. Assistance in the form of a survival binder is being assembled; help is coming!

As a result of the work of the Women and Math Committee, chaired by Elaine Harvey, assistance in identifying quality speakers has been provided to the program committees in the chapters. Considerable progress has been made by this committee because, when I looked at the nominations for the upcoming OAME election, I saw that the new council would reflect the membership that we have.

This has been a very rewarding year for me as president of OAME. I feel that together as a council and an organization we have grown. The level of achievement of council and the chapters is very high this year. But then it has been a very good year where realistic objectives were set, and a lot of people worked in a collaborative style towards these goals. The quality of the projects taken on has been very high, with several of these carrying over into following years. I would like to thank the executive, council, Morley and Mona and the membership in general for their continued support. You are all very much appreciated. I would also like to wish the incoming president, my friend Ralph Connelly, the best for the next year - when you need my support, Ralph, you have it. In closing, may I just say a very sincere thank you for letting me serve this great organization - OAME.

GEOMETRY, SYMMETRY AND RELATIONS BETWEEN DEDEKIND ZETA FUNCTIONS

by
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Arithmetic, or more correctly number theory, is the study of number fields. The agricultural parlance is a little misleading. The fields of complex numbers, C , you will recall, is the set of symbols, $x + iy$, with the formal properties that $i^2 = -1$ and that x, y are real numbers. We add and multiply in C by the rules

$$(x + iy) + (x' + iy') = (x + x') + i(y + y')$$
$$(x + iy) \cdot (x' + iy') = (xx' - yy') + i(xy' + i(xy' + yx').$$

A number field, K, is any subset of C which is made in the following manner. Take a finite set $x_1, x_2, \ldots, x_n$ of complex numbers which are solutions of rational polynomials; for example, $x_1 = \sqrt{2}$ is a solution of $0 = t^2 - 2$. Now form all the sums of rational multiples of products of powers of the $\{x_i\}$. This set will form a number field, K. The sum or product of two numbers in K is again in K and K contains all the rational numbers plus all the $\{x_i\}$. These properties characterise K, which is written $K = Q(x_1, x_2, \ldots, x_n)$. An elementary, but important, family of number fields is given by the quadratic fields, $K = Q(\sqrt{d})$, where d is a rational number.

The idea behind the study of number fields is that arithmetic varies according to the field in which you do it. For example, the famous "Last Theorem" of Fermat asks about solutions in the rational field, Q, of the equation $x^n + y^n = z^n$ when $n \geq 2$. It is known that for a fixed n there are at most finitely many such solutions and that for infinitely many n the only solutions are the obvious ones such as $(x, y, z) = (1, 0, 1)$. However, for any fixed n, if we take a large enough K it is easy to find solutions; for example, take $K = Q(\sqrt[n]{2})$, $x = y = 1/\sqrt[n]{2}$, $z = 1$ so that we have $x^n = y^n = \frac{1}{2}$ and $z^n = 1$. The need for studying arithmetic in larger fields, K, is that it can be the first step to solving a problem in the arithmetic of the rational numbers or of establishing the impossibility of a solution.

Along the same lines Sherlock Holmes might have said, "Elementary, my dear Watson. First suppose there is a crime, next show that the culprit is a man, then a Chinese, then a blonde - what do you deduce from that? Why, that there never was a crime at all!"

Each number field, K, possesses a zeta function, $\zeta_K(s)$. This is a function, in the sense of calculus, of the complex variable, s. The most famous is the zeta function studied by B. Riemann (1826-66),

$$\zeta_{\mathbb{Q}}(s) = \sum_{n=1}^{\infty} 1/(n^s) = \prod_{p \text{ prime}} 1/(1-(1/p^s)) .$$

The infinite sum and infinite product converge (that is, they actually make sense) if the real part of $s = x + iy$ satisfies $x > 1$. For other values of s the definition is more complicated, but we will not require it.

$\zeta_K(s)$ had many staggering successes because it embodies the arithmetic of K. For example, by studying $\zeta_Q(s)$ and $\zeta_{Q(v)}(s)$ (where $v = \cos(2\pi/m)$) P.G.L.Dirichlet (1805-59) proved that there are infinitely many prime numbers in the arithmetical progression $\{a, a + m, a + 2m, a + 3m, a + 4m, \ldots.\}$. A further major success was the determination by R.Dedekind (1831-1916) of the leading term of $\zeta_K(s)$ when expanded as a power series about the value $s = 1$. This is called the Analytic Class Number Formula; the class number of K being a very important number associated with K. The class number can, for example, tell us of many cases when Fermat's Last Theorem is true.

In 1922/2 Emil Artin (1898-1962), building on the work of Weber (1842-1913) began to study the problem of factoring one $\zeta_K(s)$ as a product of other zeta functions. To do this he invented L-functions, which generalise the zeta functions, and permitted him to exploit the symmetries of the field, K, to derive relations between zeta functions. A symmetry of a number field is a rule to attach to each number, x, in K, a second number, f(x), also in K, in such a manner that $f(x) = x$ for each rational x and also $f(x) + f(y) = f(x + y)$, $f(x) \cdot f(y) = f(xy)$ in general. For example, if $K = Q(\sqrt{d})$ such an f may be defined by $f(a+b\sqrt{d}) = a - b\sqrt{d}$ where a,b are rational numbers. A famous family of fields, called the icosahedral fields, were analysed by the great German mathematician, Felix Klein. Their set of symmetries (or Galois group, as it is called after its discoverer, Evariste Galois) is the same as the set of symmetries of the icosahedron or dodecahedron.

Geometrical shapes (perhaps higher than three-dimensional), as well as number fields, have sets of symmetries. Building upon results of Artin I gave an explanation, involving geometrical symmetries, by which to determine all

the relations between zeta functions. I will describe how the method works in the case of an icosahedral field, L.

Consider the dodecahedron in the figure below (the icosahedron formed by joining the centres of its adjacent faces would serve us just as well).

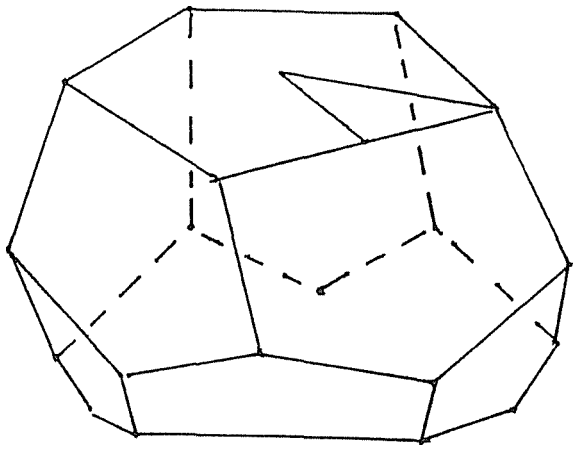


FIGURE 1

By a symmetry of the dodecahedron we mean an operation which picks up the shape in figure 1, turns it about and puts it back, occupying the same space, with the vertices and faces possibly permuted. For example, we may rotate the shape through 72 degrees about the vertical axis through the centre of the top (horizontal) face. There are 120 symmetries of figure 1 and they are exactly the same set as the Galois group of an icosahedral number field.

By successively applying these symmetries each point on the surface of the dodecahedron can be moved to a unique point in the triangle shown on the top face. Our algorithm now proceeds as follows. For each point, P , in the triangle we can look at the number field, inside L , consisting of those complex numbers fixed by the **same** symmetries that leave P fixed. In our example this process results in a triangle decorated with number fields-L_1, L_2, L_3, L_4, L_5 - each one within L , at the vertices, along the edges and in the interior.

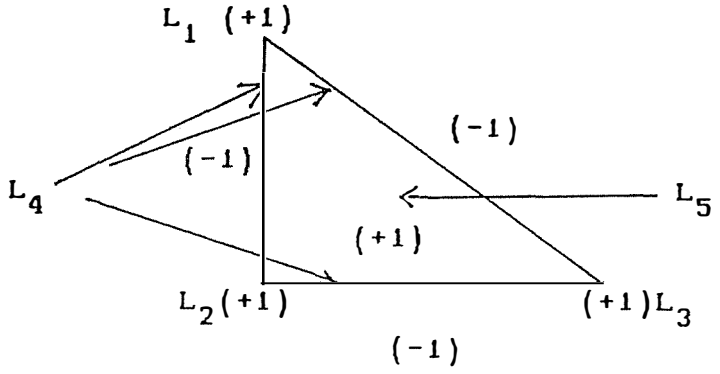


FIGURE 2

In addition, to each of these labelled segments of the triangle is assigned an integer - called the Euler characteristic - which is shown in figure 2. The relation that one associates to this labelled figure is

$$\zeta_L(s) = \frac{\zeta_{L_1}(s)\zeta_{L_2}(s)\zeta_{L_3}(s)\zeta_{L_5}(s)}{\zeta_{L_4}(s)^3} \quad .$$

In general, relations between zeta functions can be very complicated and the geometry by which they are found will be correspondingly complex. However, the principle that the number theoretic equations relating zeta functions are governed by geometrical symmetries is an easy principle to understand.

This illustrates very well what mathematics is all about - the discovery of simple principles which control very complicated processes.

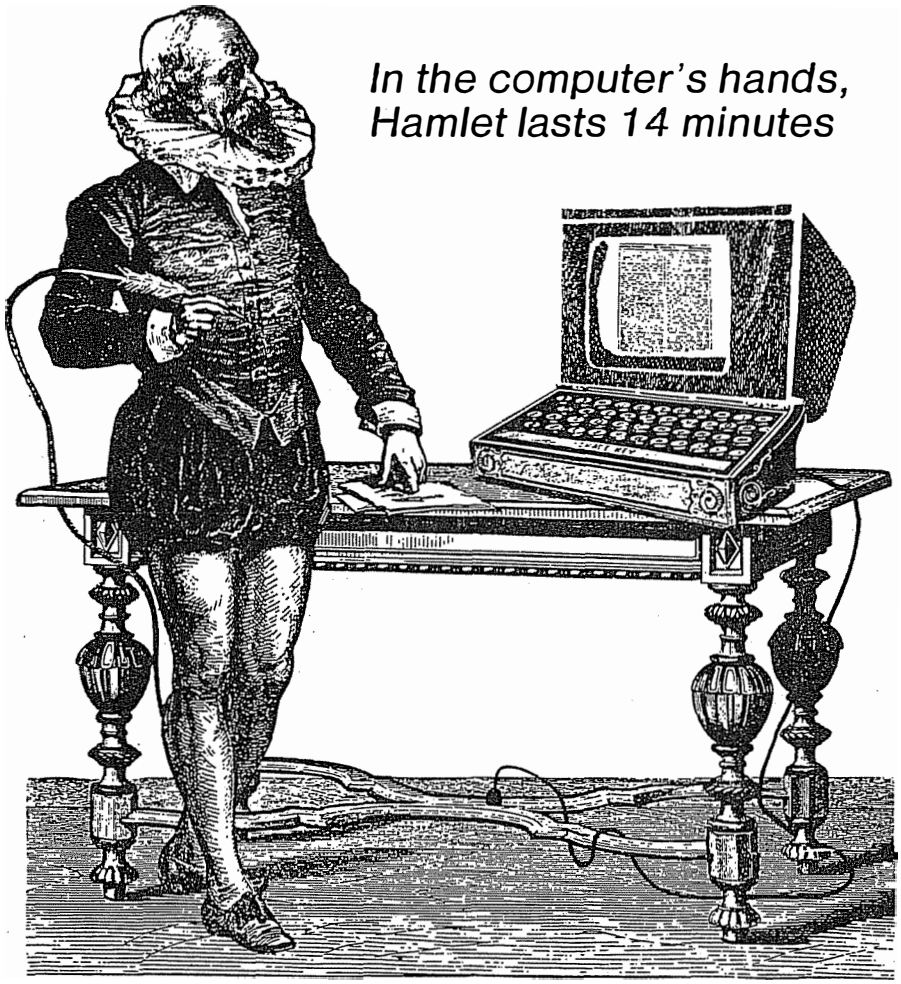
In conclusion I would like to thank Tom Griffiths of A.B. Lucas Secondary School, London, Ontario for his helpful comments concerning the first draft of this article.

PUTTING THE BYTE ON THE BARD

by

Charles Burress

Editor's Note:** This article is reprinted with the author's permission,* ***SanFrancisco Chronicle, *April 20, 1986.*



Shakespeare's greatest tragedy wasn't *Hamlet.* It was not having a computer.

Computers have come a long way since the Stone Age of the microchip 20 years ago, when they were used for such raw displays of brute technology as hurling men to the moon.

Today, the computer is a creature of sophisticated finesse, shooting for the moons of the mind. One result is a revolution in the art of writing, a transformation unmatched perhaps since adverbs first emerged from the pre-lingual ooze.

The breakthrough consists of a masterpiece of word-processing software known modestly as a style-checker. Like a jeweler's lens, it can reveal a seemingly perfect gem of writing to be a rough-hewn landscape of blemishes. You put in the prose, the computer spits out the mistakes. But its crowning achievement is the next step: It composes improvements.

This brave new world, however, has not been Tempest-free. While style-checkers are winning friends on campuses and in offices they have met stubborn resistance from the battlements of literature.

Indignation still simmers over what a Bell Laboratories style-checker did to the Gettysburg Address a couple of years back. Lincoln's first sentence — *Fourscore and seven years ago, our forefathers brought forth upon this continent a new nation, conceived in liberty and dedicated to the proposition that all men are created equal* — was improved to read:

Eighty-seven years ago, our grandfathers created a free nation here.

With Lincoln, however, the style-checkers were just flexing their cursors. They were preparing the eventual assult on the Mt. Everest of literature — Shakespeare.

That sublime peak was claimed recently when a Berkeley scientist revealed he had successfully trained his computer to sniff out Shakespeare's flaws. Dr. C.J. Wallia — a Stanford Ph.D. and consultant in electronic publications — turned his customized style-checker loose on Hamlet's "To be or not to be" soliloquy. The computer coughed up 34 errors, found the language "obsolete" and "overwritten," and gave this 15-word alternative:

Is it better to live with bad luck or end it all and have nightmares?

There we have it, the high-water mark of the computer as a young artist. But were Shakespeare lovers grateful?

"I think it's hideous," said Jerry Turner, artistic director of the Oregon Shakespearean Festival, the 50-year-old company that has performed Shakespeare for more people than any other theater in America.

"It's absurd," he added. "Shakespeare's work is the standard of the best literature there is. Any attempt to say it can be improved is presumptuous."

Turner's not alone. A chorus of ridicule greeted Wallia's effort. But let us not be too hasty to join the herd. There's little profit in literary lemminghood.

If truth be told, the glare of Shakespeare's fame often blinds us to his actual merit. When someone says "Shakespeare," we genuflect from habit. To praise Shakespeare

or to bury him — that is not the question. The issue is, no matter how great Shakespeare is, can he be improved by computer?

If so, the world has suffered an immeasurable tragedy. Millions of readers died knowing only a Shakespeare who did not fulfill all his potential — a stunted Shakespeare. Our highest standard of literature has been but a poor shadow of what it could be.

In short, the crown jewels of writing are riding on Wallia's experiment. Let us then remove the literary chastity belts from our minds and consider the possibility that Shakespeare wasn't perfect. It's helpful to recall that other Elizabethan giant, Ben Jonson, one of Shakespeare's ardent but not fawning admirers. Jonson wrote:

The players have often mentioned it as an honor to Shakespeare, that in his writing he never blotted out a line. My answer hath been, "Would he had blotted a thousand."

Such a view, of course, is merely a generalization. The real test must be to examine the text itself. This means casting an uncowed eye on the Hamlet speech, as composed without a computer:

To be or not to be—that is the question.

Already we have a problem. "To be or not to be" is not a question. But let's not quibble. Hamlet is clearly torn between living and dying — or at least it appears that way until the second sentence:

Whether'tis nobler in the mind to suffer the slings and arrows of outrageous fortune, or to take arms against a sea of troubles and by opposing end them.

Let us ignore the metaphoric indigestion of taking arms against a sea. Here the choice that divides Hamlet is not life or death, but passive suffering vs. active opposition.

We naturally go to the third sentence to find out what Hamlet's talking about, and run into this:

To die, to sleep — no more, and by a sleep to say we end the heartache and the thousand natural shocks that flesh is heir to.

Now he's back on the death trip. No wonder Hamlet's confused. On top of that this sentence is not a sentence but a fragment without proper subject and verb, and thus not a complete thought. Moreover, try saying it out loud. It hardly rolls trippingly on the tongue.

From there it's downhill at a gallop. We hit a *bodkin* and some *fordels* and phrases like *the spurns that patient merit of the unworthy takes,* and other such stuff as headaches are made on.

One can rummage through the play and find numerous examples of that country from whose bourne no comprehension returns. Here is a typical Hamlet remark from later in Act III:

Let the galled jade wince, our withers are unwrung.

The meaning of this sentence may not leap out at first glance. Luckily, we have the footnote in Professor G.B. Harrison's widely used tome, "Shakespeare: The Complete Works." The sentence translates:

"Let a nag with a sore back flinch when the saddle is put on; our shoulders feel no pain."

This example makes one thing clear: society owes a large debt to Shakespearean scholars, who have kept the old Bard afloat on a sea of footnotes.

Think of Wallia's computer as Galileo's telescope. First comes the shock of heresy. Then acceptance of Shakespeare's not being the center of the literary universe. Finally we enjoy the discovery's benefits.

For example, if Hamlet's 265-word soliloquy can be trimmed to 15 words, then the same rate of improvement can reduce the entire four-hour play to a 1980's-size bite of culture — 14 minutes. Add drums and electric strings, and imagine Shakespeare born anew for today's world: *Hamlet, the Rock Video.*

Call Shakespeare a casualty of progress, a moldy scribbler, an emperor unclothed — but do not call him to account. He's not to blame. How could he have known our vocabulary and attention spans would become much slimmer thanks to the quick-thrill diet of modern entertainment? The fault, dear William, is not in ourselves, but in our stars — Joan Collins, Mr. T, Boy George...

THE AGENDA FOR ACTION IN CANADA

by
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Editor's Note: *A Report to the Annual Conference of the Mathematics Council of the Newfoundland Teachers Association April 28, 1986.*

The National Council of Teachers of Mathematics in 1980 published **An Agenda for Action** through which the council presented its viewpoint of the directions mathematics programs should be taking in the 1980's. The Agenda outlined a set of eight recommendations based on a series of studies funded by the National Science Foundation and an extensive survey conducted through the Priorities in School Mathematics (PRISM) project.

NCTM published and widely distributed the Agenda among its 65,000 members and 190+ affiliated groups of mathematics teachers associations across North America, and both encouraged and challenged leaders in mathematics education to implement the recommendations. While mathematics education in Canada may or may not reflect problems, concerns, and priorities in American mathematics education, the Agenda has meaning for mathematics teachers in Canada.

The Mathematics Council of the Newfoundland Teachers Association chose as the theme for its 1986 annual conference, **"Half-way through the 80's: Half-way through the Agenda for Action???"** As the Canadian Representative to the Regional Services Committee of NCTM, I was asked to speak to the theme and to reflect a Canadian perspective. Wilbur Boone, Newfoundland Mathematics Consultant, addressed the Newfoundland scene. The Mathematics Council is to be complimented on its choice of theme, its concern for mathematics education, and for its recognition of the value of self-examination.

This presentation can not and will not be a definitive analysis of the state of mathematics education in Canada. It will reflect a compilation and organization of information provided by the provincial and territory departments or ministries of education in response to a short questionnaire. The mathematics curriculum consultants and the presidents of the mathematics teachers association in each province and territory of Canada were asked to outline actions that exemplify attempts to achieve each recommendation and actions that would likely inhibit the achievement of each recommendation of the Agenda. I was delighted to receive responses from both territories and all but one province, particularly in light of the very short time period available to respond to the questionnaire and to prepare this address. Thanks to each respondent.

Each of the eight recommendations will be addressed in general terms and a composite of the questionnaire responses will be presented. I will not attempt to cover all of the issues and specifics of the Agenda, rather I would encourage you to get your own copy of the Agenda and read it carefully (at least once a year).

Recommendation 1. The National Council of Teachers of Mathematics recommends that problem solving be the focus of school mathematics in the 1980's.

Problem solving is the focus? Is that true, or has "PROBLEM SOLVING" become a hollow catchphrase for the eighties? Every respondent indicated that his/her curriculum had accepted problem solving as an integral part, the teaching of problem solving strategies as being important, and applications as being the media through which mathematical reality might best be delivered to students. Most provinces have either developed their own materials or have identified and provided resource materials that support this approach. Inservice programs have been developed and presented to math teachers in many provinces. Some attention has been given to the special problems associated with evaluating student progress when problem solving is intergrated in mathematics programs.

At first glance one might conclude that this recommendation is in place and achieved in Canadian mathematics education. However, a further reading of the Agenda, and a look a the supplementary recommendations, may call this to question. The Agenda suggested that mathematics curriculum be reorganized around problem solving. Have curricula been reorganized, or have we just added a unit of study called "problem solving strategies"? Is there now a topic for those dreadful last two periods on Friday afternoon, "Hey kids, let's solve some problems". The agenda suggested that students must learn to "formulate key questions, analyze, define a problem and goal, discover patterns, seek appropriate data, experiment, transfer skills and strategies to new situations, and draw on their background knowledge." None of these can or should be confined to a single unit of studies on problem solving. Rather, the learning of these concepts should permeate all studies in mathematics.

The Agenda specifically noted the tendency to define problem solving as solving "word problems". Whether a student is asked to add 2 + 2, or "How much is 2 cats and 2 cats?", simple computation is the objective. Problem solving is much broader. Several respondents noted that teachers often interpret problem solving in this restrictive way. Several bemoaned the tendency of publishers to disguise a purely algorithmic approach in a flurry of words giving the illusion of solving problems. To make problem solving basic to the teaching of mathematics one must take a middle

ground between two extremes. On one extreme is the model that teaches all of the mathematics, the basic skills, the concepts, and the algorithms, prior to involving the child in doing mathematics in the mistaken belief that they need to be prepared for what's coming. Have you noticed that for most students IT never comes. On the other extreme is a totally unstructured model that says that given enough opportunities for learning and enough problems, students will recreate the mathematics themselves. As with most issues in Education, good sense should place us somewhere between these two extremes.

A number of the respondents noted the difficulties inherent in changing teachers' attitudes and methods. Improved teaching materials, a good range of applications at all levels, further research, and extensive teacher training and retraining are actions that need to be considered through the remainder of the decade. Math teachers look out. There is a rumour running rampant that Social Studies teachers, Science teachers, and others may have recently discovered PROBLEM SOLVING. I would encourage all teachers to address the process of problem solving whatever their discipline.

It is my feeling that our curriculum leaders in Canada sincerely believe what they have said in their curriculum documents. A problem solving approach is where we should be. It is my hope that PROBLEM SOLVING will not become just a meaningless catch-phrase, another bandwagon. It represents a fundamental change in philosophy, one which expects and anticipates that children can and will think, that children will take some responsibility for their learning, and that the teacher as coach, facilitator, and resource to learning will be the norm.

Recommendation 2. Basic skills must encompass more than computatioanl facility.

Do you remember the **Back to the Basics** movement? The Agenda, through this recommendation, attempted to meet those concerns head-on. The Agenda endorsed the list of ten basic skill areas identified by the National Council of Supervisors of Mathematics which included: applying mathematics in everyday situations; alertness to the reasonableness of results; estimation and approximation; appropriate computational skills; geometry; measurement; reading, interpreting, and constructing table, charts, and graphs; using mathematics to predict; and computer literacy. In addition, the Agenda encourages us to see basic skills as being dynamic not static, to incorporate estimation activities in all areas of the program, to provide opportunities for students to communicate in mathematics, and to include the development of higher-order thinking skills as basic to the application of mathematics.

This is not a **"The mind is a muscle, so exercise it"** approach to mathematics instruction. You know the muscle approach - solve a quadratic equation once, get the method down pat, and then do the same thing 50 more times to be sure you got it.

The Canadian respondents reflected a much broader definition of basic skills than mere computational facility. A number specifically mentioned the NCSM list as being an accepted definition of basic skills in their provinces. When the responses are considered as a group, each point raised by the Agenda is included. At least one respondent noted that Canada has traditionally had a broad definition of what is basic to mathematics.

On the negative side, some teachers were again cited as being less than comfortable with a broad definition of basic skills. Teacher evaluations tend to emphasize the "right answer". Some resource materials still show pages and pages of rote exercises in isolation, or are cookbook style workbooks. Concern was raised that the Canadian Tests of Basic Skills, a popular standardized test, places too much emphasis on computational prowness.

It would appear that, at least in theory, mathematical basic skills in Canada encompass a wide range of activities. In practice, that may or may not be true. It is my feeling that the pressure to return to a limiting focus on computation has not been strong in Canada, and that, in general, our students are being given a comprehensive experience in mathematics. Having said that, I am concerned that through ill-conceived computer software based on the worst parts of the stimulus-response movement some sense of credibility is being given to glorified "electronic workbooks", and that the drive for accountability is leading educators to embrace standardized evaluation tools that also reflect a very narrow view of that which a child should know. More about these concerns later.

Recommendation 3. Mathematics must take full advantage of the power of calculators and computers at all grade levels.

Through this recommendation, the National Council of Teachers of Mathematics has clearly stated its position on the use of calculators and computers within the mathematics program. Some attention must be given to teaching children how to use the technology, but clearly the major emphasis is on using these devices to support and enhance the teaching of mathematics. Children and mathematics is the focus, not the machines.

The agenda recommends that students have access to calculators and computers throughout their mathematics programs, that the use of these tools be integrated into the core of the curriculum, and that materials be developed that use the technology in diverse and imaginative ways. The agenda also stresses the need for computer literacy both among our students and ourselves. As with basic skills, the definition of computer literacy cannot be static. As a matter of fact I would propose a new definition in this year of 1986. Computer literacy should now mean: finding the appropriate materials on the resource shelf, getting access to a computer on which it will run, finding the right slot to put the disk in, finding the right switch to turn on, and get-

ting on with learning mathematics.

Each province in Canada that responded to the questionnaire indicated that calculator use in mathematics is expected, encouraged, and supported with resource materials. A number of departments have developed guidelines and inservice programs for teachers related to calculator use. One province stated that calculator use is encouraged at all levels for problem solving, for rigorous, many-digit calculations, and as a tool to introduce concepts.

On the negative side respondents indicated that common practice is less than ideal, that teachers are in some cases reluctant to embrace the technology, and that parents and the general public fear that children will rely on the calculator and be math illiterate. One person expressed the concern that schools are tending to bypass the calculator and go directly to the computer and that teachers sometimes misunderstand the value of the calculator as an instructional tool.

Computers in the mathematics classroom is not a fait accompli, at least as I read the comments on the questionnaire. Some provinces encourage computer use within mathematics, some are actively pursuing quality software, and most are attempting to define the appropriate role of the computer in mathematics.

During the period of time from approximately 1980 to 1984, the machine itself was given a major focus in discussions and in practice. The Agenda may be misinterpreted as advocating an isolationist view of the computer in mathematics. However, in the past 18 months to 2 years a change of direction and focus has taken place. Integration is the call-word. I believe the focus of attention has shifted back to the child and the rich mathematics curriculum. The microcomputer is beginning to be seen as a powerful instructional tool. Software is improving. Truly unique and innovative applications of the microcomputer are becoming evident. The "electronic workbook -PRESS RETURN TO CONTINUE - page flippers" are not catching on. Software that promotes creative expression, critical thinking, and problem solving is on the rise.

It is my belief that the impact of microcomputing in mathematics is still before us. I only hope we will get there before the disillusionment that has accompanied other educational technological innovations overcomes our initiatives.

Recommendation 4. Stringent standards of both effectiveness and efficiency must be applied to the teaching of mathematics.

This recommendation concerns itself with the process of learning that is going on in the mathematics classroom and the instructional organization and methodology being used. NCTM has suggested that with problem solving as a focus, the use of time in the classroom must be rethought. A child involved in higher-order skills in problem solving will need more time to reflect and to allow what he/she has learned to sink in, as opposed to a child involved in lower-order, mechanistic skills. The Agenda indicates that external forces, administrators and parents, must respect and support the teacher's attempts to keep learners on-task and involved. It is suggested that instructional methods be diverse and creative, and that resource materials should support a variety of teaching techniques.

Not all of the respondents interpreted this recommendation in this way. Rather, several responses addressed the general question of teacher evaluation. Those responses will be included in the discussion of recommendation 7. Of the remainder, one province has done an elementary school study of the cognitive level of responses relative to teacher expectations. Another has initiated a program of teacher effectiveness, awareness and training. Most indicated that a variety of teaching methodologies are encouraged. One person felt that effectiveness and efficiency in mathematics teaching was not, and has not been in decline. Several respondents commented on the tendency of math teachers to use a "chalk and talk" approach almost exclusively, and to rely on a text-based methodology.

It is my belief that time is ripe for mathematics educators to look carefully at what we believe about how children learn mathemataics, and to reconsider our teaching approaches in light of those deliberations. I am fascinated by the holistic movement in language arts, one that is based on a psycholinguistic theory of the development of a child's ability to communicate meaning and to derive meaning from language. This is a philosophy of language instruction that parallels our focus on problem solving processes and learning through the context of applications. I believe we have much to learn from this child centered approach to learning.

Recommendation 5. The success of mathematics programs and student learning must be evaluated by a wider range of measures than conventional testing.

During the Delegate Assembly at the NCTM 1986 Annual Meeting in Washington, D.C., fully one-third of the resolutions dealt in some way with the issue of student assessment and standardized testing. Mostly, the resolutions were calls to the NCTM to take a stand against the restrictive minimum competency testing that is prevalent in the United States today. The Agenda, in 1980, made a clear statement of NCTM's position. It included comments such as: evaluation of learning should include the full range of the program's goals, including assessment of problem solving skills, minimal competencies should not be construed as an adequate measure of mathematics achievement; parents should be informed and involved in the evaluation process; teachers should use a wide-range of techniques, particularly in respect to diagnosis; and evaluation must begin with the goals of the program and assess performance against these goals. The agenda warned about the potential effect minimum competency testing might have on programs. It would appear, based on the resolutions at Delegate Assembly, that these warning were not heeded. In fact, NCTM recently appointed a task force to take a detailed look at the issue of evaluation.

The Canadian scene is quite different from the American with respect to this issue. We have not placed a major emphasis on standardized testing. Even though there has been a return to province wide examinations in a number of places, the emphasis in Canada is classroom level evaluation. Most of the respondents indicated that efforts have aimed at improving the evaluation techniques used by classroom teachers. Suggestions and guidelines have been developed. One province has established an Assessment Instrument Pool, a growing collection of examples of various types of questions from which teachers may draw. Another province has begun a series of inservice programs aimed at improving evaluation by teachers to better reflect curriculum changes. One respondent suggested that if we would enrich mathematics instruction, evaluation would of necessity improve.

On the negative side, teachers tend to want a pencil and paper approach to testing. Not enough alternative methods are used. Examinations seem to be becoming the major determiner of success in mathematics. Evaluation is summative rather than formative. Canada seems to be in a good position relative to competency testing, but that position places additional responsibilty on the classroom teacher to do a good job of assessing student, progress and communicating that assessment to parents and administrators.

Recommendation 6. More mathematics study must be required for all students and a flexible curriculum with a greater range of options should be designed to accommodate the diverse needs of the student population.

The Agenda, through this resolution, is addressing the significant problem of students not opting for mathematics electives through their high school years and thus closing doors on their futures. The studies that provided background data for the Agenda noted that significantly less time is given to mathematics instruction in US schools than in other industrialized countries. This recommendation also addressed the options that are available to students of mixed abilities and interests. It noted that the most neglected child in mathematics education is the gifted student of mathematics.

The recommendation included the suggestion that schools increase the amount of time students study mathematics, that the secondary curriculum become more flexible and permit a greater number of options, that a preparation of all students for the calculus may not be appropriate, that attention be give to the developmental sequence best suited to achieving process goals, that all concerned assist students, especially females and minorities, to have a positive attitude towards the study of mathematics, and that students with special needs not be overlooked in the emphasis on problem solving.

There is quite a range of compulsory mathematics requirements from province to province, everything from one math credit required each year to the end of the eleventh grade, to no math required beyond grade 8. It should be noted that the province that requires no math beyond grade 8 still has almost 90% of all students still taking mathematics in grade 12. Most provinces offer a variety of courses in mathematics at the secondary level, with anywhere from 110 to 140 hours of instruction per course. Small schools are not able to offer a wide range of options, and with declining enrollments, smaller schools may be the norm in the future. It is interesting that little mention was made of accomodating students with special needs in the secondary program. Several respondents noted special programs offered at the junior high level. However, a review of several curriculum documents provided to me by the respondents indicated quite a lot of attention to students who had difficulty with mathematics. The curriculum for these students reflected the emphasis on problem solving and applications that has been suggested by the Agenda. If the practice is as stated in the provincial documents, then it would appear that students of varying abilities are being supported.

Recommendation 7. Mathematics teachers must demand of themselves and their colleagues a high level of professionalism.

The Agenda began discussion of this recommendation by acknowledging that there are many well-prepared and effective teachers in our profession, that there are those who are less prepared but are willing to work hard to improve, and that there are some teachers functioning at less than a professional level. It also noted that in 1980, the shortage of qualified mathematics teachers was becoming a crisis. The Agenda recommended that every teacher accept responsibility for maintaining their own competence through coursework, inservice programs, and by being active in a professional organization. Officials at school boards should attempt to staff their mathematics programs with competent, qualified teacher. MCTM through this agenda has called on teacher training institutions to incorporate the problem solving emphasis within their training programs and courses. It suggests that certification standards be amended to reflect the recommendations in this document.

In discussing this recommendation of professionalism, almost all respondents to the questionnaire mentioned activities carried out by the mathematics teachers association in their province. These associations of leaders in mathematics education perform a very valuable service to education in Canada, conducting inservice programs, organizing conferences, and serving on curriculum committees, to name a few.

While those who get deeply involved in their professional organization gain tremendous benefits, not all math teachers take advantage of the opportunities. Across Canada approximatley 5,000 teachers are members of one of the 12 mathematics teachers associations and about 1,300 teachers are members of NCTM. There are approximately 30,000 to 35,000 secondary mathematics teachers in Canada.

Several respondents mentioned the problem of unqualified teachers

being placed in mathematics positions. This is generally a result of declining enrollments. Teachers with seniority may replaced a more qualified candidate with less seniority. A related problem was noted, the aging of our profession. With few openings for new, young teachers, the average age of our profession is growing. This adds an even greater necessity for opportunities for professional development.

We have a good group of professionals teaching mathematics in Canada and an excellent network of professional organizations across the country. With an association in each province, there is not a math teacher in Canada who does not have access to an organization of mathematics educators. An additional strength for the associations outside Quebec and Ontario is that they are special interest groups of the provincial teachers union or federation within the province. This connection gives us a larger and louder voice with respect to education. All 12 associations are affiliated with NCTM as well.

Recommendation 8. Public support for mathematics instruction must be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

NCTM has called, through this Agenda For Action, on the general public to respect the professionalism of mathematics teacher, and, in turn, has called on mathematics teachers to live up to that respect. This recommendation assumes that communication and cooperation between the school and the home, as well as the general public, is vital to the growth of quality mathematics education. The Agenda lists a number of obstacles that inhabit the teaching/learning process: more time required for unproductive record keeping, many unmotivated, disciplined students, lack of parental support, ambivalence in government regulations, shifting societal priorities, and the lack of agreement on out-of-school study habits.

The Agenda noted that these and other obstacles were making the professional role of the educator almost impossible, and the recruitment and retention of competent educators difficult. NCTM recommended that society provide incentive that will attract and hold competent, prepared, qualified mathematics teachers; that all concerned establish new and higher standards of cooperation; and that government at all levels operate to facilitate the attainment of goals agreed upon cooperatively.

The respondents did not comment to any great extent on this recommendation. Those that did comment seemed to feel that public support for mathematics education in Canada was high. An interesting observation was made about the perception of mathematics in our northern territories-math doesn't seem as important in a land-based economy, but as the north displays a growing dependence on a wage-based economy the attitude towards mathematics is changing. In several provinces parent pamphlets that have been written and distributed.

NCTM has declared April **MATHEMATICS EDUCATION MONTH**, and has produced a number of resource materials that can be used by various groups to promote mathematics education. A number of provinces promote **EDUCATION/MONTH** but few of the math teachers associations in Canada take an active role. While the public support for mathematics education in Canada may be good, I believe we need to be more active and more visible to the public eye. Your Newfoundland Council is to be complimented on the work and promotion that it has done for Education Week this year.

In summary, the Agenda for Action is six years old. Is it still valid? Was it ever valid in Canada? Has it been implemented? My feeling is yes, yes, and partially. It has been a refreshing exercie to go through the Agenda carefully again. The authors of the document did an excellent job of defining many of the problems mathematics education has and is facing. The recommendations are pointed, reasonable, insightful, and implementable.

While the details behind some of the recommendations may not be duplicated in Canada, each of the the recommendations have something of value to us. Problem solving should be our focus, basic skills should be broad, calculators and computers should be used to support the curriculum, and so on. While one of the respondents felt the Agenda was one of the weaker statements put out by NCTM, I feel very positive about the recommendations and I would feel very good about the state of mathematics education in Canada if they were implemented.

Where are we on implementation? My guess is that we are well along. The respondents to the questionnaire are the very people charged with responsibility for the directions ofmathematics education in Canada. The curriculum they have described is in line with the recommendations of the Agenda. There is some question about how closely the practices in mathematics classrooms parallel the prescribed curriculum, but that is to be expected. With support and direction from our provincial leaders, the task of improving instruction will progress.

I feel good about mathematics in Canada. The quality, dedication, commitment, insights, and good common sense of the mathematics teachers that I have met across Canada support that conclusion. I hope that we can continue to work together to the benefit of the young minds that are entrusted to our care.

STUDENT 'COMPUTER LITERACY' AN OUTMODED TEACHING IDEA

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SPEAKING OF computers — and who isn't thinking or talking about them today — isn't it amazing how quickly our attitudes towards them have changed? When I first started to report on computers on radio and television about 10 years ago, all of the emphasis was on explaining how computers work. These days, the innards of the microelectronic revolution are of little concern. All of our attention is focused on *what* they can do. That's fair enough. In the beginning, you had to have a sense of what the limits and potential of the technology were because there was a good chance you might have to fix it yourself. But now there are all kinds of specialists for you to get help from and it's no more important to know how the computer works than it is for the average driver to understand how a car's automatic transmission operates.

It's very clear that computers are here to stay and we're already oblivious to them in our social landscape. At airports we take advanced seat selection for granted. We can get boarding passes for connecting flights a continent away, something impossible before computers. We accept that when "the computer is down," we have to wait because human workers can no longer do things by hand. My wife, who teaches writing at Harvard, reports that the most frequent excuse for missing a deadline for a paper is problems with the computer. In my day, it was sickness or personal problems.

Only a couple of years ago, parents were panicked by predictions that their children would probably require computers in their jobs in the future. Parental hangups and fears about computers led many to send their children to summer computer schools, to buy computers for their kids and to lean on schools to get more computers. That cry for computer literacy has been, in my view, one of the biggest cons ever foisted on the school system.

First of all, schools should not get into the position of trying to anticipate the future needs of the workplace. If history is any guide, our ability to predict requirements there, become very fuzzy beyond a couple of years. But surely it is not the obligation of schools to prepare youngsters for the job market, at least not beyond the universal skills of thinking, reading, calculating and so on.

There are those who see in the computer a panacea for teachers which opens the possibility to teach children how to think through a series of problem-solving challenges. Nonsense. That is a human activity that demands human guidance and understanding. Nothing will ever replace the flexibility and originality of the human mind. No machine will ever duplicate a good teacher. And do we want to involve our children in what is basically a solitary activity at a time when they should be acquiring social and language skills?

Many of the people who now look to the computer as the great gift to education are the very same people who were trumpeting the promise of video technology a decade ago. They are technocrats who jump on the superficial benefits without consideration of the broader ramifications or costs.

In our rush to ensure that our children are computer literate, we have forgotten to ask what we are trying to accomplish in the schools. After all, the computer is simply a machine. Is it any different from an automobile or television set? We didn't change the entire curriculum of schools to accommodate those technologies when they were new. Nor would we think of training are youngsters to drive a car using a Model A Ford, yet in terms of what today's youngsters will use when they grow up, present-day computers will seem pretty crude indeed. Back in the 1960s, I struggled to learn Fortran, the computer language of the time, and I can tell you that the ease with which we now use computers staggers me. Computers will continue to get easier and easier to use.

So what's the panic? Children will grow up accepting computers as a part of the landscape of their culture and will use them as the need arises. The Ontario Ministry of Education has embarked on an ambitious program to try to confer computer literacy on all teachers in that province. This has all of the earmarks of the illfated program to make all civil servants bilingual. But what's the point? The young teachers coming along will all take computers for granted. Teachers who find a real pedagogical use for computers will overcome their temerity. If they see no use for a computer, why force them?

The computer provides access to vast amounts of information with remarkable speed. But one of the biggest problems educators face today is that youngsters already have an overload of information, especially from television. Is the computer's *faster* access to even *more* information going to help the teacher? No. Because the challenge is to decide what information is worth anything and how to make use of that information. That is taught best by teachers. I think we are misguided in our panic about computers. And diverting scarce funds for primary school teaching into such technology only exacerbates the problems we have dealing with technology. I will have more to say about this in future columns.

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CANADIAN MORTGAGES

by
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1. **INTRODUCTION.** If you ever tried to re-create an amortization table you may have started full of confidence, only to run into a "Canadian" snag.

The simplest type of mortgage in common use involves equal monthly payments over a given time (the amortization period) at a fixed interest rate. If the interest rate were compounded monthly the calculation would be straightforward: the monthly rate would be one-twelfth of the annual rate. However, Canada allows compounding "at most semi-annually"[1], and most mortgages use semi-annual compounding.

So what monthly rate **do** you use? If you have an amortization table for your mortgage, you might find in its heading a so-called "interest factor", and if you use it as the monthly rate, you will be able to reproduce the table. But where does this "factor" come from? A quick survey of the commonly used grade 13 mathematics textbooks sheds no light, since they only discuss cases where the compounding period and the payment period are the same.

A college-level text[2] finally told me how to calculate the factor. In essence, the reasoning is given as follows: at the monthly rate **i**, in one year a **deposit** is multiplied by $(1 + i)^{12}$, while at the annual rate **R** compounded semi-annually, it would be multiplied by $(1 + R/2)^2$; therefore, equating these expressions gives the appropriate value of i to use when the compounding is semi-annual.

Indeed, the above calculation gives the correct value for i, but I find the deduction less than satisfactory for two reasons:

a) since the balance is adjusted every month, the expression "compounded semi-annually" seems to be a contradiction; at best, it can be interpreted as a "trade name" for the correct description;
b) more serious, the above reasoning refers to a **deposit** collecting interest, which is certainly not the case when we deal with mortgages.

Since it would be nice to show our students how to handle this "real life" problem, a self-contained treatment is presented in the following sections.

2. **BALANCE FORMULA.** As a preliminary step, I will discuss a generalized problem: a loan of **A** dollars has to be repaid in **N** equal payments of **F** dollars each; the interest rate per payment period is **I**. When the j-th payment ($j = 1, 2, 3, \ldots, N$) is made, the balance is reduced by a part of F which I call P_j, and it becomes B_j. I will obtain an expression for B_j in terms of j, such that $B_N = 0$ (loan repaid). Notice that $F - P_j$ is the interest payment at the end of the j-th payment period.

$$\text{Since } F = B_{j-1} I + P_j \qquad \text{(j-th payment period),} \tag{1}$$

$$\text{and } F = (B_{j-1} - P_j) I + P_{j+1} \qquad \text{((j + 1)-th payment period),} \tag{2}$$

comparing (1) and (2) we obtain

$$P_{j+1} = (1 + I) P_j,$$

and repeated application of this recursive formula gives

$$P_j = (1 + I)^{j-1} P_1. \tag{3}$$

Since the sum of all the payments P_j repays the loan,

$$A = \sum_{j=1}^{N} P_j \quad ; \text{ evaluating this geometric series gives}$$

$$A = P_1 \frac{(1 + I)^N - 1}{I}, \text{ which can be rewritten}$$

$$P_1 = \frac{AI}{(1 + I)^N - 1} \tag{4}$$

The n-th balance B_n is given by

$$B_n = A - \sum_{j=1}^{n} P_j \qquad \text{(loan minus all payments to date).}$$

Using (3) and (4) we obtain

$$B_n = A - \frac{AI}{(1 + I)^N - 1} \sum_{j=1}^{n} (1 + I)^{j-1},$$

and, evaluating the series, (5)

$$B_n = A\left[\frac{1 - (1 + I)^n - 1}{(1 + I)^N - 1}\right]$$

3. **INTEREST FACTOR.** All of the above formulae assume that the compounding period equals the payment period. For semi-annual payments and compounding, I = R/2, where R is the annual interest rate. If the payments and compounding are monthly, (5) still applies, with N = amortization period in months, and I = R/12. However, because we are supposed to use "semi-annual" amortization, I is adjusted so that the balance **at the end of each semester** is the same as if we had made semi-annual payments, compouned semi-annually. In other words, the monthly payments will keep the loan "in step" with hypothetical semi-annual payments and semi-annual compounding, over the same amortization period.

Applying the above condition to (5) produces N/6 equalities:

$$\frac{(1 + I)^6 - 1}{(1 + I)^N - 1} = \frac{(1 + R/2)^1 - 1}{(1 + R/2)^{N/6} - 1} \qquad \text{(1st semester),} \qquad (6)$$

$$\frac{(1 + I)^{12} - 1}{(1 + I)^N - 1} = \frac{(1 + R/2)^2 - 1}{(1 + R/2)^{N/6} - 1} \qquad \text{(2nd semester),} \qquad (7)$$

etc.

Clearly, $(1 + I)^6 = 1 + R/2$ is a solution of all N/6 equations; that it is the **only** solution can be shown by comparing (6) and (7), to obtain

$$\frac{(1 + 1)^6 - 1}{(1 + R/2)^1 - 1} = \frac{(1 + I)^{12} - 1}{(1 + R/2)^2 - 1}$$

$$= \frac{[(1 + I)^6 - 1]\,[(1 + I)^6 + 1]}{[(1 + R/2) - 1]\,[(1 + R/2) + 1]}$$

and, simplifying,

$$(1 + R/2) + 1 = (1 + I)^6 + 1, \text{ q.e.d.} \qquad (8)$$

Explicitly,

$$I = (1 + R/2)^{1/6} - 1; \qquad (9)$$

this is the mysterious "interest factor" mentioned in the heading of the amortization tables. It can be seen to depend only on the yearly rate R.

The financial effect of using (9) can be seen as follows: from (8) we obtain

$$R/12 = I + (5/2)\, I^2 + (10/3)\, I^3 + (5/2)\, I^4 + I^5 + (1/6)\, I^6;$$

therefore $R/12 > I$, i.e. the "interest factor" I is lower than the monthly amortization rate R/12, to the advantage of the mortgagee. To evaluate this advantage, we obtain an approximate rational expression for I from (9) using the binomial formula

$$(1 + x)^b = 1 + bx + (b(b - 1)/2)\, x^2 + \ldots \; (b \in Q, \, |x| < 1);$$

retaining up to the quadratic term and neglecting the rest, (9) gives

$$I \doteq (R/12) - (5/72)\,(R/2)^2. \qquad (10)$$

Formula (10) shows that the difference between I and R/12 increases with R; e.g., for R = 12%, I ≐ .00975 instead of .01, a 2.5% difference, while for R = 6%, I ≐ .0049375 instead of .005, a 1.25% difference.

References

[1] *P. Zima and Robert L. Brown, Mathematics of Finance, p. 105, McGraw-Hill--Ryerson, 1983.*

[2] *Ibid., p. 93.*

GAIM: AN INTERVIEW WITH ALICE ONION

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In the December, 1984 issue of O.M.G., my interview was with Margaret Brown of Chelsea College, London, England. We discussed the aims and objectives of GAIM - the Graded Assessment in Mathematics Project. This interview provides an update on the project.

Chelsea College has become King's College London University of London, Chelsea Campus, as a result of a reorganization of Colleges within the University of London. The location remains the same, as does the excellent work done in Mathematics Education. I interviewed Alice Onion who had taught in a Secondary Comprehensive School and had done much inservice work with teachers.

RS: What were the objectives of GAIM?

AO: We were originally set up to devise (under the title "Graded Assessments in Mathematics") an assessment frame work that would be suitable for the entire 11 to 16 age range.

RS: What have you acheived in the last two years?

AO: We have launched a pilot scheme of the assessment frame work that we devised which includes the testing schemes, the pupil work and the means of leveling it. Classrooms and resources have been set up. Pupils have gone through a try out period for a whole year. They have now received certificates.

RS: Unlike many schemes where assessment may well mean pencil and paper tests, perhaps computer marked, your scheme looks at a wider variety of tasks. What are those tasks?

AO: I think the linch pin in our assessment scheme is criteria referenced items. In other words when the pupil achieves a certain part of mathematics, perhaps a very small part; that is credited to them. It doesn't matter whether they have demonstrated that in conversation with the teacher, through a piece of written work or through practical demonstration. The whole of the assessment frame work could not be done through just writing. There are practical assessments as well.

RS: Please give us an illustration of a practical assessment.

AO: It is a different way of thinking about assessment, so I can't answer that question directly. There are criteria and the students can demonstrate those criteria in the course of their normal classroom work. So really what we are looking at, is continuous assessment. Let me give you an example. The assessment criteria might be adequate use of the tape measure. Hopefully, during the course of the school year the pupil will demonstrate competent use of the tape measure. If not, then the teacher themselves will devise an activity that will give the pupil an opportunity to demonstrate competence.

RS: It seems there would have to be some kind of arrangement for recording achievements as they occur. Is this a lot of book work for the teachers?

AO: It is a lot of ticking off of criteria on pupil profiles. We don't want to involve the teacher in a lot more paper work. We have suggested that as a result of a good practical lesson where the pupils have spent their times going around the room measuring things, maybe taking surveys, doing statistical experiments or other practical work of this kind, the teacher might well think critically about what the children have actually done, and then will mark off appropriate achievements on their profile. We are hoping that this will replace some of the book marking and the routine exercises the pupils used to do and not add to the teacher's burden.

RS: I noticed in the '86 Spring Term News letter, a very interesting article on children's self-assessment. This is almost foreign in our formal assessment work. We hope that children will try to decide what they know and what the don't know. How do you go about actually encouraging and recording the mathematics self assessment.

AO: This is a field that we are only just starting on. Can I just say a little bit about the reason for this?

RS: By all means do.

AO: In Britain there is no stipulated maths curriculum as yet. Although there is a lot of common agreement about what goes into the math curriculum, the individual school student's experience can be very mystifying. They don't know the aims of the course they are on, they don't know what the aims of the particular examination board are. So one of main things we hope self assessment will do, will be to involve each student in the process of their own education. They will be able to say, yes, I achieved these aims in this lesson, this week, this half term. That was the motivation. Now having said that, we are just starting our trial period. It means that we are in the businss of discussing it with teachers, of drawing up alternative ways of doing it. We do have profile formats that pupils themselves fill in and we have some of the criteria translated down from "teachers speak" to "students speak" which a couple of schools have been using.

RS: So it is in the trial period.

AO: Absolutely, the self assessment is very much a trial.

RS: Have you been involved yourself?

AO: Not directly with school students but I have been indirectly involved with two teachers. I was in fact involved in some self assessment with college students before I started in the GAIM Project, which is one of the reasons why I took over that particular area when I started with GAIM. I had a list of achievements in mathematics that I hoped the college students would achieve and I would sit with them towards the end of the year in an interview situation. We would discuss the list, item by item to determine whether they thought they had achieved them. Recently I have been involved with two teachers who have tried out self assessment with their school students. One asked his students to do an open report on how they thought they were doing in mathematics. That was particularly revealing because it was completely open ended. It was interesting to see what the students themselves chose to write about and in fact they focused on other things that may be a teacher wouldn't include in self assessment, for example the resources, the room and the arrangement of the furniture, how they were getting along with their friends and what mood they were in that day. The other teacher was working with a form which was based on translations of the GAIM assessment criteria. So she was working much more directly with the sort of assessment that we would need for GAIM and she gave each of her third year students a form to carry around with them where they could tick off each time they thought they had done something in maths. She reported back to us that this provided great motivation. Some students, as a result requested her to do something in measurement or with the computer so that it could be ticked off on the report form.

RS: Well I can't think of education in any better way than that. A student himself decides what he wants to learn or what he wants to attack or what he wants to have practice in.

Any graded assessment must be hierarchial. In investigations (open ended project type work), how are levels stated? Perhaps you could illustrate it for us.

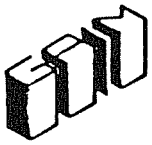
AO: Roofs is an investigation which enables the very lowest attainers to achieve something. It is one of the difficulties that we originally had with investigations that they were all very well for children of maybe just below average, or above average ability, but the lowest attainer very often couldn't get over the first hurdle of generating the results that you needed to investigate with. So roofs is one of the investigations we found worked well.

The work card is given on the following page along with the three earliest levels of achievement which may be expected.

For level one the pupil can draw at least three correct roofs. We are here looking at the ability to follow the instructions. The work lists five different roofs, two of which in fact won't work when the student follows the instructions.

For level two, all roofs on student material are either correctly drawn or are stated to be impossible. This means that the student has succeeded in working through all five examples on the sheet and knows what work has been done.

The third level gives a choice of achievement but it requires the student to arrive at some sort of mathematical conclusion. The first mathematical conclusion, is that the first and the third numbers are the same. You will only get a roof if the first and third numbers are the same. They are, if you like, the slopping sides of the roof.

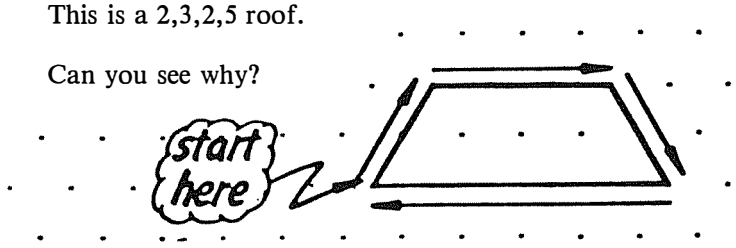


You will need triangle dot paper. 1003

Roofs

This is a 2,3,2,5 roof.

Can you see why?



Try drawing these roofs (2 are impossible)

1) 1, 2, 1, 3
2) 2, 2, 2, 4
3) 2, 1, 2, 2
4) 3, 1, 3, 4
5) 2, 3, 1, 4

Draw some more roofs of your own. Make up sets of 4 numbers and try to draw them.

Investigate which roofs are possible and which are not.

Can you predict which ones will not work?

Can you find any rules?

Level 1:

Has drawn at least three correct roofs.

Level 2:

All roofs on student material either correctly drawn or stated to be impossible.

Level 3:

Any correct mathematical statement; Example:
a) "The first and third number are the same"

b) "The last number is the same as adding the first and second".

The other possible conclusion is that the last number is the same as adding the first and the second numbers together. That may sound a more complicated conclusion to come to, but, looking at the arrangements of numbers quite often students come to that conclusion first. The more obvious conclusion, that the first and third numbers are the same escapes them. We found that the majority of students can get to level three on this investigation. In other words, they have the experience of working through something and drawing their own conclusion from it.

RS: So far we have been talking about graded assessment and the work that has been done so far in the PILOT Project. There seems to me to be a very interesting second aim for this whole GAIM Project. What would you say it is?

AO: It is to do with teachers and inservice training. In order to develop their own work, to be critical of what they are doing, and to generate new ideas, teachers have to have something they find motivating. Every working weekend we have the teachers involved in some mathematics. They do investigations, they get going on a project and they go back to their own school filled with enthusiasm. I think this is really a very large part of our work. If we did not generate teacher enthusiasm, we would never get the GAIM task done.

RS: Agreed. From the beginning, GAIM was funded in part by the University and in part by Nuffield but

also by support personnel moving in and out from ILEA as well as some of the surrounding counties. There was a real reason for involving the local boards of education.

AO: Yes, in the beginning, the only educational authority to give us any funding and people was ILEA but then some counties outside of London joined in.

RS: Where is GAIM going now?

AO: We hope, that the development will continue up to the level equivalent to the top of GCSE. Top of GCSE is highest level normally an offer to 16 year olds. University Entrance exams are normally taken at age 18 and GAIM has no intention (yet?) of covering that field.

RS: Do you feel your objective of graded assessment for all levels has been met?

AO: We haven't finished the task yet. I think the levels we are working with are succeeding. At the moment higher attainers don't have any graded assessments. We haven't set out to devise the graded assessments for them as yet. Yes we are happy with what we have done.

RS: You have tackled the problem that was revealed in the Cockcroft Report. So many of the children going through your comprehensive schools really were never recognized as achievers. Now they can receive a statement of their achievement in mathematics.

AO: This is partly a problem with the way the examination system was devised. It was in fact only intended for the top 60% and in some subjects 80% do pass, so we can't really complain that the bottom 20 or 40% fail. The system was designed that way. I think more recently, people have decided that this isn't altogether fair and it would be better to advise a scheme for everybody.

RS: That is actually what GAIM has been trying to do.

AO: Absolutely.

RS: Thank you so much Alice, it has been a pleasure talking with you.

AO: Thank you Bob.

GAIM sends out a newsletter. If you are interested in their work write to

GAIM Project
King's College
552 King's Road
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or better still, visit when you are in London, England.

"TRANSFORMING" A DIFFICULT PROBLEM

by
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Each month I publish two newsletters that I send to all secondary math teachers in our system. In the second issue, I always include a PROBLEMS CORNER, for teachers to encourage their kids to get involved in the problem solving process.

Last month I included a routine problem to test students' analytic geometry prowess. The problem was as follows:

"Determine if the centroid, circumcentre and orthocentre of a triangle are collinear."

I expected, and received a number of solutions spanning three to four pages. Most students found the equations of medians, right bisectors and altitudes followed by their points of intersection. These points of course were shown to be collinear by equating slopes.

One student however, Igor Ryzer, surprised me with a rather unique approach. Unfortunatley he did not follow the proof successfully to the finish, but he prompted me to complete what I think is a rather elegant and precise solution to the problem. It also illustrates the beauty of transformation geometry.

Our solution is as follows:

A is the intersection of the altitudes (orthocentre)
B is the intersection of the medians (centroid)
C is the intersection of the right bisectors (circumcentre)

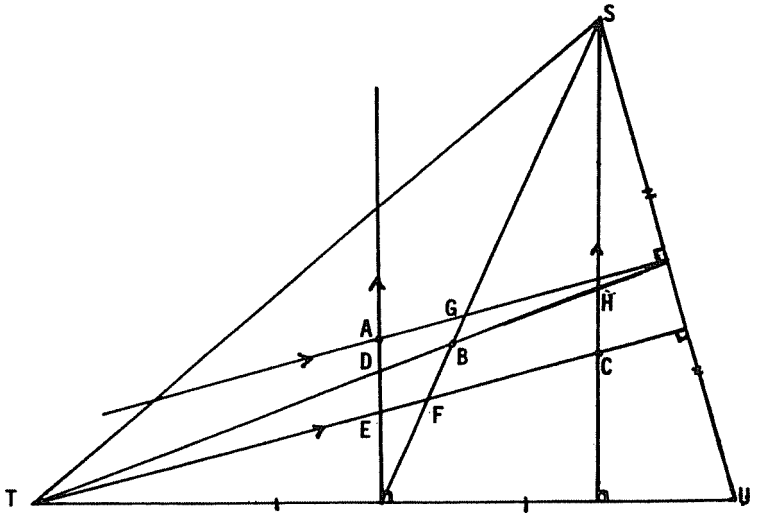
Required to Prove: A, B, C are collinear.

Consider quadrilateral BGAD and quadrilateral BFCH

$\angle$ DBG = $\angle$ FBH (VOAT)
$\angle$ AGB = $\angle$ BFC (alt. angles)
$\angle$ ADB = $\angle$ BHC (alt. angles)
$\angle$ DAG = $\angle$ HCF (axiom of subtraction)

So quadrilaterial BGAD is similar to quadrilateral BFCH.
Quadrilateral BGAD can be rotated 180° about B (since DBH is a straight line) and a similarity transformation can be applied to BGAD so that this quadrilateral will correspond to quadrilateral BFCH.

$\therefore$ [B,G,A,D] $\longrightarrow$ [B,F,C,H]
So BA $\longrightarrow$ BC (property of a rotation)
$\therefore$ ABC are collinear



REFLECTIONS ON TEACHING AND LEARNING MATHEMATICS

by
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Reason in man has, itself, biological roots, and mathematical thought is born of the spirit's need to simulate external reality.

René Thom

Teaching high school adolescents for 12 years and adults in a regular high school setting for 4 years have made me realize the following: although we will always have students exhibiting weaknesses in certain areas of mathematics (such as the inability to recall or recognize some standard result), **the major difficulty is a totally wrong conception of the nature of mathematics, and of the approach to it.**

René Thom states "The real problem which confronts mathematics teaching is not that of rigour, but the problem of the development of 'meaning' of the 'existance' of mathematical objects".[1] **We have been conditioning our students into accepting results and not asking about what made one think of that idea or that process.** For example, many students come to us in Grade 9 knowing the formulae for the circumference of a circle and the area of the corresponding disc. A proof of these is not given to the student until the study of the Calculus (if at all); yet, with a knowledge of the area of a triangle, the change in area under a dilatation (which can be done in Grade 9) and the square of a binomial, we can give some very convincing proofs of these formulae. Obviously, this would lead to discussions on volume, and with the machinery developed before, we can easily tackle the surface area of the sphere and the volume of the corresponding ball. This leads to discussions on the concept of a limit which can be used to show more succinctly the utility of algebra. Two excellent sources for these topics are: *S. Lang, **Math!: encounters with high school students,** (New York, Springer Verlag, 1985). S. Lang and G. Murrow, **Geometry: a high school course,** (New York, Springer Verlag, 1983).*

It would not be entirely incorrect to state that the existing situation is that we teach routines. We are all aware that present day microcomputer programs can do most of the routines we teach in high school. The basic quality of those students studying mathematics for its own sake (and these are very few indeed) is an **ability to generalize,** to take an active role and call on his/her individual spirit of enterprise. Rene Thom believes that only those topics which have a quality of play have educational value and of all such games, "Euclidean Geometry with its constant references to underlying intuitively understood fundamentals, is the least gratuitous and the richest in meaning."[2] For these students we must bring back the Euclidean Geometry that has been dropped from the curriculum. We have to realize that geometry is no longer a subject - **what is important is a geometrical way of looking at mathematical situations.** I believe that Euclidean Geometry will impart to our students the intellectual excitement associated with geometrical discovery and the shared enjoyment of understanding the discoveries of others.

The basic quality of the rest of the students should be judgement - the ability to decide how far a mathematical process, which is nearly always a simplification of a complex reality, is near enough to the facts to be satisfactory. At every opportunity we must show the application of mathematical symbolism to physical and non-physical ideas.

W.W. Sawyer emphasized the following to his students when he taught at the Faculty of Education of the University of Toronto:

1. The student must acquire, before entering university, the habit of reading mathematics for himself, attacking problems for himself and achieving intuitive insight into mathematical concepts.

2. There should be continuous practice with simple questions in translating statements into symbolic form. This is the only way algebra can be appreciated as a mathematical language.

3. As a teacher, you should always **explain** the problem to the student, for students can only attack problems if they understand what they are.

4. There should be continual emphasis of simple, concrete pictorial representations and geometrical interpretations of mathematical results.

5. There should be close integration of mathematical and scientific studies, so that each reinforces the other. What is the use of telling students that the physical interpretation of the cross product has to do with moments if the students do not know what a moment is and the teacher does not take a moment to explain it? (No pun intended).

What students need then is a very strong and secure centre of mathematical understanding. They need to associate

mathematical symbols with the practical situations to which these refer (e.g. in calculus d^2s/dt^2 and acceleration). This will help them translate practical situations into symbolic form. It might also enable them to distinguish mathematical sense from nonsense (e.g. $(x + y)^2 = x^2 + y^2$ and $1/x + 1/y = 1/(x + y)$, both of which we see occassionally).

I believe it is time that our textbooks reflect these points. Instead of attempting to bring unity to the subject most texts seem to go out of their way in presenting the subject without showing any interrelations. Both authors and editors must share the blame for the lack of decent texts. I do not see any quick solutions to these difficulties. Now that we have new guidelines we have an opportunity to improve the texts. We must make sure they are student oriented (technically accessible and intellectually nontrivial). This is the first and most crucial step because as studies have shown, **the textbook determines what is to be taught, how it is to be taught, and when it is to be taught.** If the teacher is intellectually curious, he will supply the why - if not, the why never gets asked or answered.

There are some long term solutions. These deal with teacher training and retraining.

There is a need for an on-going development of teachers during the entire course of their professional lives. The boards of education can play an important role here. The need for co-ordinated and developmental continuing education programs that are more than just isolated workshops does exist. More in-service programs should be made available and attendance should be strongly encouraged.

The selection process the faculties of education have developed must take into account the fact that knowledge of mathematics is a necessary but a non-sufficient condition for effective teaching in secondary school. They must also accept that this applies to elementary schools. The faculties must select candidates who do not dislike or fear mathemataics. Deliberate attention must be given to assuming that teachers develop competencies in the art of teaching, **including the ability to translate their own understanding of mathemataics and their ability to perform mathematical operations, into instruction that is meaningful to children,** together with the knowledge of when and how to employ different teaching strategies.

Prof. Abe Shenitzer states that a long term goal is "the intellectualization of teacher training as the key to mathematically significant education for **all** students"[3] and that our "immediate aim should be to gain university support for extensive experimentation involving courses and curricula with a significant critical and historical component".[4] A concerted effort by the universities, the boards of education, the Ministry of Education and the various mathematics associations can bring this about.

Endnotes

[1] *René Thom, "Modern Mathematics: Does it Exist?" A.G. Howson (ed), **Developments in Mathematical Education,** Cambridge, Cambridge University Press, 1973), p. 202.*

[2] *Rene Thom, "'Modern' Mathematics: An Educational and Philosophic Error?", **American Scientist** 59, (1971), p. 696.*

[3] *Abe Shenitzer, "Some Thoughts on the Teaching of Mathematics", **The Mathematical Intelligencer** 8, No. 1. (1986), p. 24.*

[4] *Ibid, p. 24.*

SMPY AT ISU

by
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Researchers at SMPY (The Study of Mathematically Precocious Youth) have been dedicated for the past 15 years in the quest to better understand the bases of exceptional mathematical talent. Julian Stanley and his associates have used the Scholasitc Aptitude Test (SAT) in talent searches to identify extremely talented youths. Every year approximately 100,000 gifted students aged 13 or less now take the SAT in the U.S. The SAT is a test of developed ability, not primarily learned subject matter, designed mainly for approximately the abler half of twelfth-grade students throughout the country (Angoff, 1971).

One part of the SAT, called SAT-V, is verbal. The other major part, SAT-M, consists of 60 multiple-choice items to solve in 60 minutes and tests mathematical reasoning ability. In order to solve the items, only fundamental knowledge of algebra and geometry is necessary. It is an especially good measure of mathematical reasoning when given to intellectually talented youths aged 13 or less, because they have had little experience with abstract mathematics (Benbow and Stanley, 1983).

Results from these studies utilizing the SAT have yielded some highly variant and somewhat controversial correlates of extreme mathematical talent. While environmental factors have been posited to underly extremely high mathematical reasoning ability, little research on possible biological correlates has been carried out (Benbow, 1986a). In response, researchers at SMPY have begun to investigate some such possible factors. Four physiological correlates of this ability have been identified to date: left-handedness, symptomatic atopic disease (allergies), myopia, and gender (Benbow, 1986a). Some or all of these correlates may be related to the influence of fetal hormones. In this article we will summarize SMPY's findings regarding physiological correlates. A more complete coverage can be found in Benbow (1986a, b).

Left-handedness

Recent evidence suggests that left-handers may show high performance levels on tasks mediated by the brain's right hemisphere, including spatial and musical ability (Burnett et al., 1982; Deutsch, 1980). Also, research has shown that among university mathematics teachers and students, music students, artists, astronauts, and architects, higher frequencies of left-handedness have occurred (Annett & Kilshaw, 1982; Deutsch, 1980; Mebert & Michel, 1980). Contrary to computational ability, mathematical reasoning ability is probably more strongly under the influence of the right hemisphere (Benbow, in press a; Gardner, 1983; Troup et al., 1983). Left-handers, then, would seem likely to have an advantage on such reasoning tasks.

Thus, left-handedness (and mixed-handedness) was studied among students with extreme mathematical talent (Benbow, 1986a; Benbow & Benbow, 1984). Using the Edinburgh Handedness Inventory of Oldfield (1971), we found that extremely mathematically talented students were more than twice as likely to be classified as left-handed by this instrument than individuals in the general population (15.1% vs. 7.2%). The extremely talented were also overrepresented in the category of mixed handedness. Moreover, such young adolescents who had scored extremely high (at least the top 1 in 10,000) on the SAT-M were much more frequently left-handed when compared to their parents, siblings, and a comparison group of the same-age students (Benbow, 1986a).

Symptomatic Atopic Disease

As with left-handedness, symptomatic atopic disease (allergies) was twice as likely to be found in extremely precocious mathematical reasoners (approximately 53%) as compared to members of the general population (approximately 20-25%) when surveyed by an allergy questionnaire (Benbow, 1986a). In this study, extremely talented students were more frequently reported by their parents to have allergies than the parents themselves, their siblings, or a comparison group of students. This finding is consistent with widely held stereotypes about the gifted; however, no other locatable studies have formally reported this relationship.

Myopia

Karlsson (1973, 1975) and Sofaer and Emery (1981) have found myopia to be correlated with higher general intelligence. A dramatically increased rate of myopia was found among the extremely high mathematical reasoners, as reported by Benbow (1986a). It was found that extremely high ability students were about four times as likely to be myopic as were high school students in the United States and were much more frequently myopic than their siblings and a comparison group. Although hereditary factors (Dunphy, 1970; Karlsson, 1973, 1975) and environmental factors (Green, 1980) have been implicated in producing myopia, the basis for the relationship of myopia to extremely high mathematical reasoning ability is uncertain.

Gender

A controversial topic under current debate is whether extremely high mathematical reasoning ability is a correlate of an individual's sex. Many studies have reported sex differences in mathematical aptitude and achievement (Deaux, 1985: Fennema, 1974). Dougherty et al. (1980) have even reported sex differences for seven to nine-year-olds. Yet it must be emphasized that no sex differences have been found in computational ability or the ability to apply already learned concepts or in mathematics course grades.

In a series of studies conducted by researchers, at SMPY, sex differences in mathematical reasoning ability have been reported among highly talented pre-adolescents (Benbow & Stanley, 1980, 1983). Several hundred thousand students have been tested over a 14-year period revealing a consistent sex difference in magnitude of about one-half standard deviation. The significant result here, though, is that there were many more extremely mathematically talented boys than girls (e.g., a ratio or about 12 to 1 at the top 1 in 10,000 level). Moreover, and perhaps most importantly, this large sex difference seems to have important longterm consquences (Benbow & Stanley, 1982).

To date a totally environmental explanation of the sex difference among the intellectually talented has not been given scientific support (Benbow, in press b). For instance, there were no sex differences in attitudes toward mathematics (Benbow & Stanley, 1982; Fox, Brody, & Tobin, 1985) and course-taking (Benbow & Stanley, 1980 1982) among students who took the mathematical reasoning test. Further, there was no evidence on the basis of sex of differential encouragement by parents for mathematics study (Raymond & Benbow, 1986; Fox, Brody, & Tobin, 1982).

Hormones

Much hypothesizing has taken place regarding male and female hormones and their role in influencing high mathematical performance. The hypotheses dealing with prenatal exposure to hormones and their effect on brain development have found some scientific support. Reinisch et al. (1979) have reported that progesterone exposure enhances numerical ability. Moreover, Geschwind and Behan (1982) postulated that left-handedness and immune disorders, such as allergies, are related to high level testosterone exposure in fetal life or to high fetal sensitivity to testosterone. They have suggested that testosterone indirectly enhances the development of the right hemisphere, while simultaneously affecting immune development through its influence on the thymus gland. In view of the fact that left-handedness and allergies were related to extremely high levels of mathematical reasoning ability, testosterone may be another physiological correlate of that ability. Since testosterone is the male hormone, this also suggests a connection between it and the sex difference in mathematical reasoning ability (Benbow, 1986a).

Bilateralization

Findings previously mentioned above regarding left-handedness also have implications for brain functioning. Left- and mixed-handers and right-handers with left-handed relatives have been found more frequently to have bihemispheric representation of cognitive functions (Bradshaw & Nettleton, 1983; McKeever et al., 1983). It may be then, that bilateral representation of cognitive functions (rather than greater specialization of the hemispheres) is associated with extremely high mathematical reasoning ability. Preliminary data were consistent with this view (Benbow & Benbow, 1986).

In this regard it is also intriguing to note that Witelson (1985) has found that the corpus collosum (the main fiber tract connecting the two cerebral hemispheres) is larger in left- and mixed-handers than in right-handers. Thus, the "greater bihemispheric representation of cognitive functions in left- and mixed-handers may be associated with greater anatomical connection between the hemispheres" (Witelson, 1985).

Conclusion

This summary of our research concerning the etiology of extremely high mathemataical reasoning ability has presented four physiological correlates [i.e., left-handedness (and mixed-handedness), immune disorders, myopia, and gender]. Two other correlates of mathematical reasoning ability have been hypothesized: prenatal hormones and bihemispheric representation of cognitive functions. These last two possible correlates may also offer a partial explanation for the sex difference in mathematical reasoning ability.

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LA RÈGLE DE 72: UNE APPLICATION DES LOGARITHMES

by
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C'est lors d'une conversation avec un agent d'assurance, il y a quelques années, que j'ai entendu parler de "la règle de 72". Cette règle permet à un individu de vite calculer combien d'années environ avant que son placement double en valeur. Pour faire la calculation, il faut simplement diviser 72 par le taux d'intérêt. Par exemple, un placement à un intérêt composé de 9% p.a. doublera en environ $\frac{72}{9} = 8$ ans.
On peut vérifier le résultat en nous servant de logarithmes pour résoudre l'équation exponentielle appropriée. On vera donc que le résultat est 8.05 ans quand le taux d'intérêt est 9% p.a.

Cette règle m'a beaucoup intéressé et j'ai décidé de l'enseigner aux élèves quand les mathématiques de placement sont considérées. Ça leur donne un bon moyen facile de vérifier leurs réponses et de développer l'ingénue de déterminer si leurs résultats sont raisonnables. Naturellement je voulais savoir **pourquoi** la règle marchait si bien. En réalité, c'est très simple et le pourquoi peut certainement être apprécié par les élèves ainés de l'école secondaire.

Prenons d'abord l'équation de placement de un dollar à un intérêt composé de **i**% p.a. Si le placement doit doubler on devrait avoir \$2 à la fin de **t** ans. On peut donc écrire l'équation,

$$(1 + .01i)^t = 2$$

et ensuite le logarithme naturel de l'équation

$$t \log_e (1 + .01i) = \log_e 2$$

$$t = \frac{\log_e 2}{\log_e (1 + .01i)}$$

Pourquoi les logarithmes naturels plutôt que les logarithmes simples? Pour répondre à cette question, il faut examiner la table des logarithmes. Bien que les calculations sont vite faites par machine à calculer, la table est meilleure pour nous montrer la formule. La table 1 illustre très bien le fait que $\log_e(1 + .01i) \approx .01i$ et donc les logarithmes naturels nous permettent de simplifier l'équation de la façon suivante:

$$t = \frac{0.693}{.01i}$$

$$t = \frac{69.3}{i}$$

Alors pourquoi la règle de 72 et non la règle de 69.3 ou même 70? Il faut d'abord examiner l'utilité de la règle. Les facteurs premiers de 72 nous indiquent que 72 est divisible par 2, 3, 4, 6, 8, 9, 12, et 18. Ces chiffres représentent les taux d'intérêts les plus fréquemment utilisés même en temps d'inflation. De plus, on a fait des calculations approximatives pour rédiger la table alors la différence entre 69.3 et 72 est sans conséquence. Chose étonnante, la règle de 72 est presque exacte. (Voir table 2).

Les professeurs de mathématiques savent que c'est très difficile d'enseigner un sujet tel que les logarithmes dans un âge de machines à calculer électroniques. Cette application donne à l'élève une règle qu'il pourra utiliser aussi bien que la réalisation que même si les machines à calculer peuvent faire les calculations routines mieux que les logarithmes, les logarithmes sont néanmoins nécessaires pour plusiers calculations.

i	$\log_{06e}(1 + .01i)$
1	0.00995
2	0.01980
3	0.02956
4	0.03922
5	0.04879

Table 1 - Approximation en se Servant de Logarithmes Naturels

Taux d'Intérêt	Doubler (La Règle de 72)	Doubler (Calculation Exacte)
2	36	35.5
3	24	23.4
4	18	17.7
6	12	11.9
8	9	9.0
9	8	8.0
12	6	6.1
18	4	4.2

Table 2 - Comparaison de la Règle de 72 aux Calculation Exacte

LOGO IN LONDON: A MATHEMATICS ENRICHMENT PROJECT

by
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OBJECTIVES

My vision in this project is to create a gently learning environment in which the child and teacher are partners in a positive environment. The computer is simply ONE TOOL TO ENHANCE LEARNING within the existing mathematics curriculum and LOGO SIMPLY MAKES THIS TOOL EASIER TO USE. The teacher is a facilitator rather than a lecturer in this exploration of mathemataics and problem solving concepts.

THE FIRST STEPS

After personally checking with teachers and principals during the Summer of 1986, I selected the following schools. An attempt was made to select various types of classes at various grade levels from the 4 school districts in London. I selected:

4 primary classes
8 junior classes including 1 junior gifted class
4 intermediate classes including: 1 intermediate GLD class
1 grade 9 general level math class
1 grade 9 basic level math class
1 senior high school mathematics class

Most of the teachers had been my students in the Computers in the Classroom LOGO sections at Althouse College over the past few years so we had a 'flying start'.

CLASSROOM MATERIALS

Each of the 8 junior and 4 intermediate classes, was provided with 15 student texts ... **Learning Math With Logo** ... Logo Publications 1986. Many of the activities in the texts were done in pairs. 1 teacher resource book and a class set of 'robot turners' to help in understanding turns was provided for each of the teachers.

Each of the classes was provided with Commodore 64 LOGO software.

The primary class teachers were provided with **Kinderworld** by Sandra Reisch and the **OISE Microworld Project** materials.

The number of computers available ranges from 1 to 15 per class. Many of the activities are first completed away from the computer. Hence teachers have managed well with few computers.

PEDAGOGICAL THRUST

Too many students in today's classrooms simply wait for someone else to give an answer. There is too much emphasis on THE CORRECT ANSWER RATHER THAN ON **WORKING TOWARDS AN ANSWER.**

In the LOGO LEARNING ENVIRONMENT students and teachers feel 'in control' of their learning. We attempt to create an atmosphere which encourages creativity in problem solving.

We did not introduce the computer to junior classes until the end of October. Students 'played robot'. This is an essential step. The ABSTRACT CONCEPT BECOMES CONCRETE THROUGH WALKING THROUGH IT.

We have 'programmed' the process of THINKING THROUGH, then WRITING DOWN on paper before testing a concept by TELLING A ROBOT TO 'DO IT'. This is a must if the computer is to be used as a TOOL WITHIN CURRICULUM. It also allows a large class to work with LOGO even though only a few computers are available.

FURTHER STEPS

August/86 ... an 'information workshop' for participating teachers
Sept.-Nov. ... emphasis on Junior and Intermediate classes
... deliver materials ... LOGO program, student text, robots
... present weekly or biweekly lessons
... begin monthly sharing sessions for project teachers
Dec., Jan. ... begin LOGO work in primary classes
... KINDERWORLD ... part of the monthly sharing session #2
... LOGO FOR YOUNG CHILDREN OISE PROJECT ... Saturday workshop

REACTIONS

We expected and are getting a very enthusiastic response. Students in various types of classes are being helped in their problem solving techniques by using concrete material and 'chunking' activities. At least they are 'committed to the problem' ... a first step in problem solving.

IN SYSTEM ... our monthly sharing meetings, Saturday workshops and information worksessions have been attended by teachers of schools who are not in the project. We are receiving requests for initiating this project in their classes.

OUT OF SYSTEM ... requests for workshops from many school systems, provincial conferences and consultant groups have been received. Monthly sharing sessions have been attended by many educators out of system. We welcome them. For meeting dates and information call the London Board of Education office.

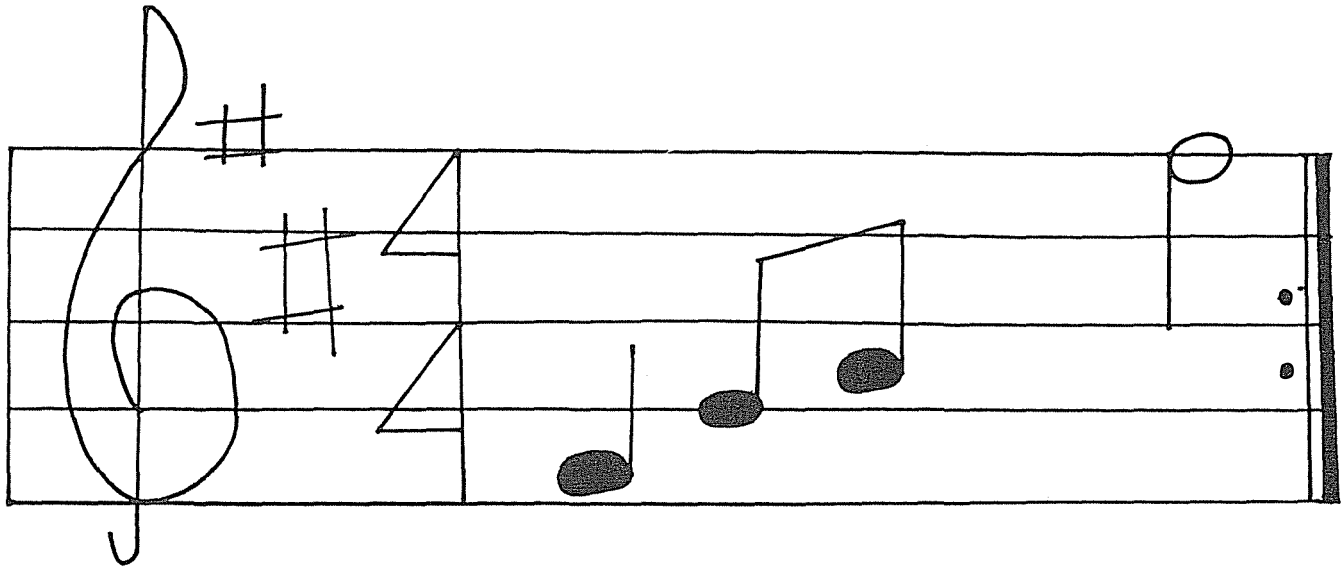
THE FUTURE ... A VISION

I see LOGO being used first in mathematics K-13 and then later in language arts, science, geography and music throughout this system. LOGO IS THE LANGUAGE OF LEARNING. It is simply a tool to enhance learning in PARTS OF SUBJECTS ACROSS THE CURRICULUM.

The power of the vision is undermined if a meaningful exchange of ideas does not occur. The present monthly meetings are a good beginning. Over the next few years, we must resist the temptation for opening the project up to all comers. We must form a core of people who are clear on WHERE the project is headed and WHY.

PLANS ... FEBRUARY, MARCH, APRIL ...

Begin monthly workshops to prepare teachers for future LOGO projects. Begin LOGO as a tool within the senior mathematics curriculum.



Math leaves you on a high note.

Linda Lee, Grade XI
Birchmount Park CI
Scarborough, Ontario

"MACPAINTING" THE INVERSE TRIG FUNCTIONS

by
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Guelph, Ontario N1G 2W1

This is the second in a series of articles in which the graphics capabilities of the Macintosh computer have been used to create worksheets to facilitate the presentation of various topics.

This time, I would like to share worksheets which introduce the inverse trigonometric functions. I use the "FLIP & ROTATE" method of easily obtaining a picture of an inverse function, and a particularly neat way of remembering the restricted or principal ranges of the inverse trig functions.

This class would follow an earlier discussion on inverse functions. In that preceding class, we would stress the idea that in an inverse relation, the roles of x and y are reversed. We would then find the inverse relation of several functions, using the standard two-step recipe:

1) interchange x and y;

2) solve for y.

I find it helpful to reintroduce here the still relatively scary exponential and log functions, as examples of inverse functions. With $f(x) = e^x$ and finding $f^{-1}(x) = \ln(x)$ (using the two steps), we would then consider

$$f^{-1} \circ f(x) = f^{-1}(f(x)) = f^{-1}(e^x) = \ln(e^x) = x, \text{ and}$$
$$f \circ f^{-1}(x) = f(f^{-1}(x)) = f(\ln(x)) = e^{\ln(x)} = x.$$

That formula, "$e^{\ln(x)} = x$", gives students such heartburn, that I use any chance I can to reintroduce it, reinforce it, and, I hope, eventually make it easier on the digestion.

With several examples, it has become clear that for an inverse function to exist, the original function must be injective, that is, one-to-one.

Now, on to the inverse trig class.

I begin with a little grade 11 geometry. With the first worksheet, we take a point P(x,y), draw a line from P to meet the line $y = x$ at 90°, and then extend this line an equal distance to a point Q. We then discover that the coordinates of Q are (y,x). Thus, a pictorial method of finding an inverse function (or relation, for that matter) is to look at the **M**irror **I**mage of the function in the line $y = x$. Domains and Range are, of course, interchanged. On to the next worksheet.

The students draw the graphs of $y = e^x$ and $y = \ln(x)$, and fill the appropriate boxes regarding domains, ranges, and asymptotes. They note the mirror image of $y = e^x$ gives the picture of $y = \ln(x)$ which they have drawn in below.

Now the fun begins. Students are told to flip over this worksheet, hold it up to the light, and rotate it so that in the graph of $y = e^x$, the positive y-axis appears where the **positive** x-axis should be and the positive x-axis appears where the **positive** y-axis should be.

Behold: there for all to admire is the graph of $y = \ln(x)$. Why? Because by flipping and rotating as described, we have interchanged the roles of x and y.

The students love this. Invariably, someone asks, "Is it okay to do this on a test?"

Let me relate, friends and colleagues, a true and tragic tale. One year, I told my students that on the final exam, in the first question, they would be required to graph an inverse function, based on the graph of the function given. The big day comes. One thousand nervous bodies, two hundred belonging to me, fill the gymnasium. The registrar's representative completes his opening instructions and the exam period begins. Almost simultaneously, 200 exam papers are hoisted in the air, the better to let the light shine through. The registrar's rep. has a Kinipshin!! Then, a severe mental breakdown. "What the hell's going on!?!", he cries, near tears.

Well, he subsequently recovered from his anxiety attack. And I was told, very slowly, very clearly, and seemingly without much warmth,

"**Don't** (pause) **do** (pause) **that** (glare) **again!**"

Back to work. The third worksheet displays the graphs of the six trigonemetric functions. I explain to the students that we would like to find inverse functions. Rather than stating an angle and asking for the corresponding ratio, the inverse functions will input a ratio and output the corresponding angle.

But there is a serious problem. For example, if the sine ratio is 1, the angel could be $\pi/2$, or $5\pi/2$, or $-3\pi/2$, and so on. We must restrict the domains, of the trig functions, in order that the inverse **functions** will exist.

The students get to play deity at this point. I pose the question, "What would you choose as an appropriate restriction of the sine function's domain so that the range will include each value from -1 to 1 exactly once?" Someone always gives me the answer I want: $-\pi/2 \leq x \leq \pi/2$.

We then subtitle this page, "Restricted Trig Domains", fill in boxes for the sine function, and shade the portion of the curve where $-\pi/2 \leq x \leq \pi/2$.

Next question: "What does the inverse sine function look like?" No problem: just flip and rotate. Now that students get out the final worksheet. One area is set aside for the function y = arcsin(x).* The students can now fill in the graph, the domain, and the range, from the picture they have just flipped and rotated.

We repeat this procedure for y = cos(x). The students agree readily that $0 \leq x \leq \pi$ is the appropriate restricted domain. They then F & R (not "Functions and Relations"!!!) to obtain y = arccos(x).

One of the major problems students seem to have with this material is remembering the ranges of the inverse trig functions, that is, the restricted domains of the trig functions. Here is a trick that works well for my classes.

On the trig function worksheet, think of the left column as the **SIN** dominated column. We have

y = **SIN**(x),
y = tan(x) = **SIN**(x)/cos(x), and
y = csc(x) = 1/**SIN**(x).

For each of these three **SIN** dominated trig functions, use $[-\pi/2, \pi/2]$ as the restricted domain. Of course, with y = tan(x), we drop the end points. For y = csc(x), $x \neq 0$.

Similarly, the right column is **COS** dominated so each domain is, more or less, $[0, \pi]$.

Thus, the students do not have to memorize six strange new graphs, plus domains and ranges. For each arctrig function, they need only the graph of the corresponding trig function and to know whether it's **SIN** or **COS** dominated.**

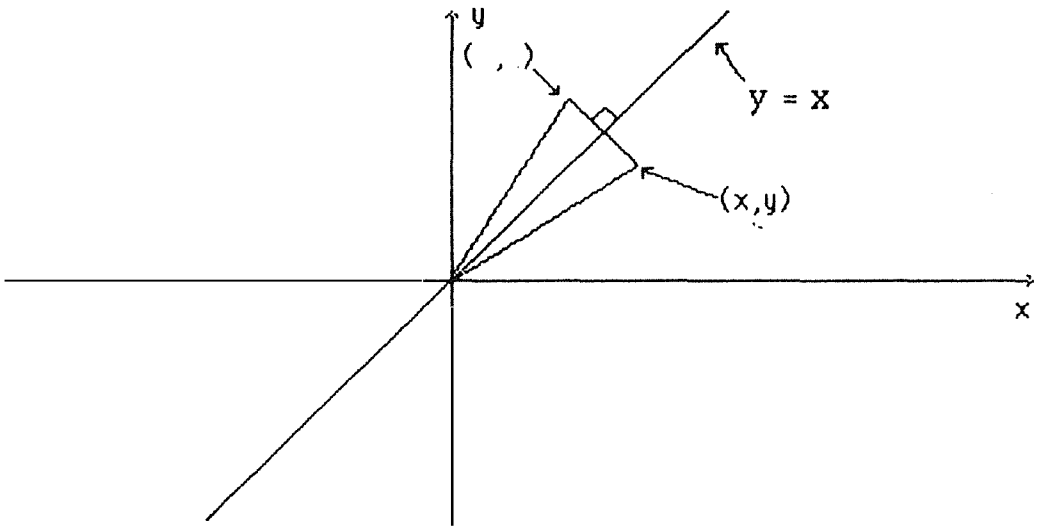
This makes the students happy with one qualification. "Yeah, but how do I remember what the trig functions look like?"

Ah, what is life without challenge?

*I take a couple of minutes at this point to explain why many texts (including the one my students use) write "**arc**sin", rather that "$\sin^{-1}$" to denote the inverse sine function. It is also worth mentioning that some texts distinguish between the "**a**rcsine" **relation** and the "**A**rcsine" **function.**

**Many texts use $[-\pi, -\pi/2) \cup [0, \pi/2)$ as the range of y = arcsec(x). By doing so, the derivative formula simplifies to $y' = 1/[x\,(x^2 - 1)^{1/2}]$. With the range used in this article, $[0, \pi/2) \cup (\pi/2, \pi]$, we have $y' = 1/[|x|(x^2 - 1)^{1/2}]$

INVERSE FUNCTIONS



Suppose $y = f(x)$ is an injective (or "one-to-one") function; that is,

If $f(a) = f(b)$, then	$=$

Then the inverse function, $y = f^{-1}(x)$, exists. It is the

M _____	I _____

of the function, $y = f(x)$, in the line [ ]

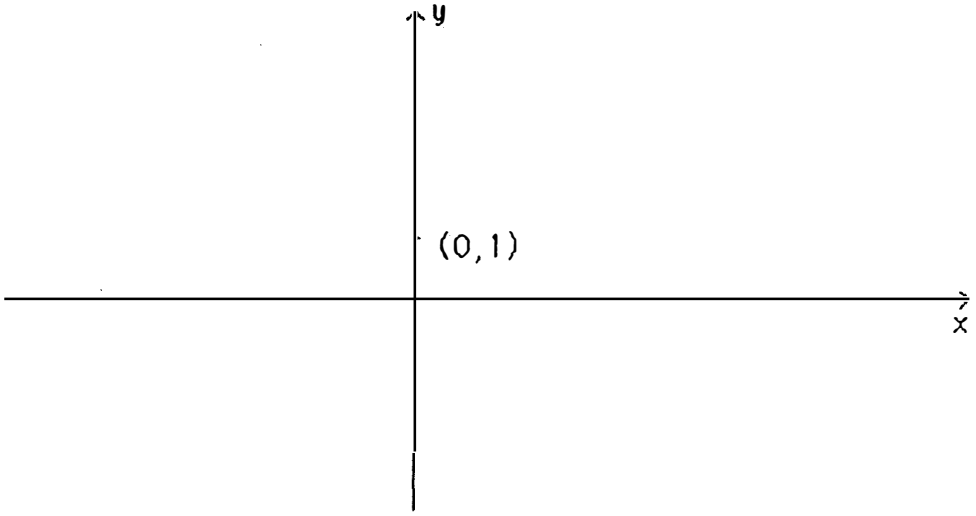
DOMAIN (f^{-1}) =
RANGE (f^{-1}) =

Given an injective function, $y = f(x)$, you can algebraically find the inverse function, $y = f^{-1}(x)$, by

interchanging [ ] **and** [ ] **in the original equation and then** [ ] **for** [ ] .

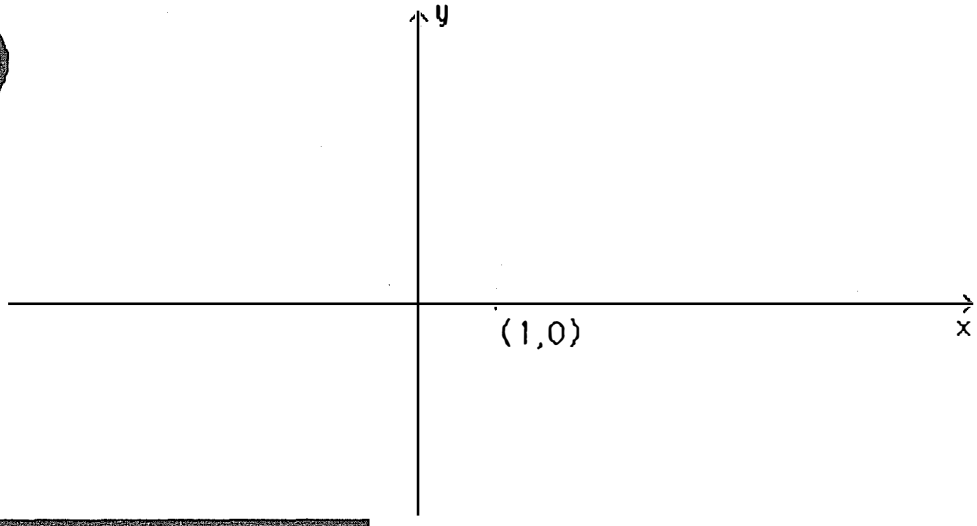
INVERSES: e^x and $\ln(x)$

$y = e^x$



DOMAIN (e^x) =
RANGE (e^x) =
HORIZONTAL ASYMPTOTE:

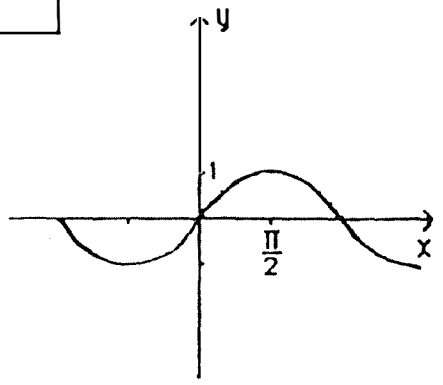
$y = \ln(x)$



DOMAIN $(\ln(x))$ =
RANGE $(\ln(x))$ =
VERTICAL ASYMPTOTE:

TRIG + FLIP + ROTATION = ARCTRIG!!

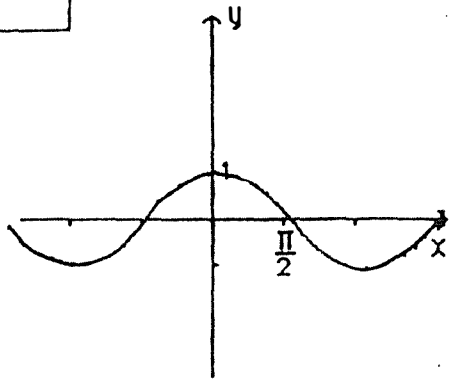
y=sin(x)



D =

R =

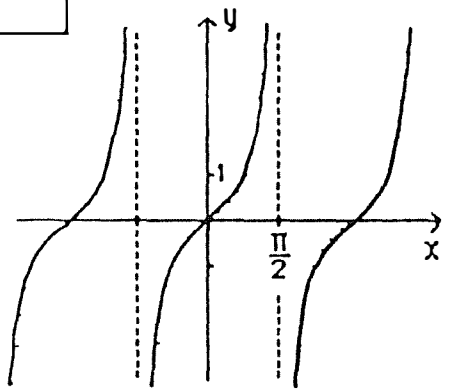
y=cos(x)



D =

R =

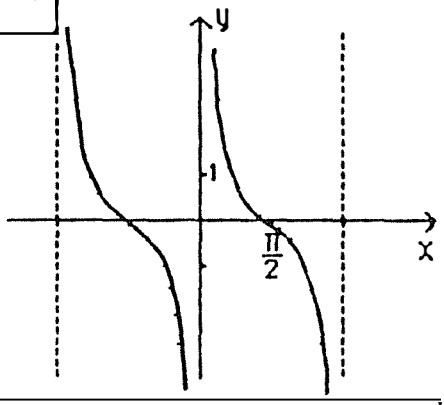
y=tan(x)



D =

R =

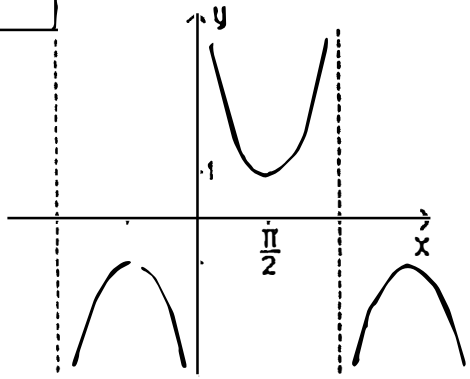
y=cot(x)



D =

R =

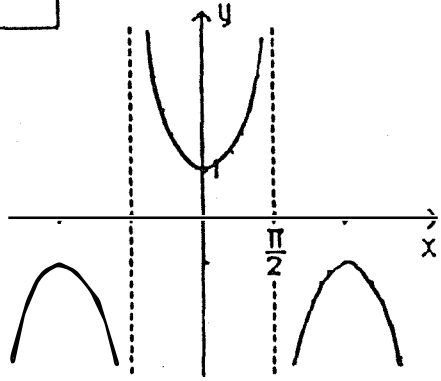
y=csc(x)



D =

R =

y=sec(x)

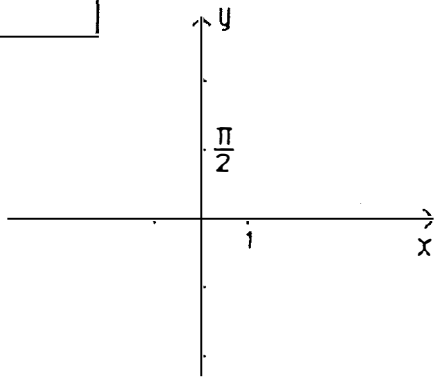


D =

R =

TRIG + FLIP + ROTATION = ARCTRIG!! (page 2)

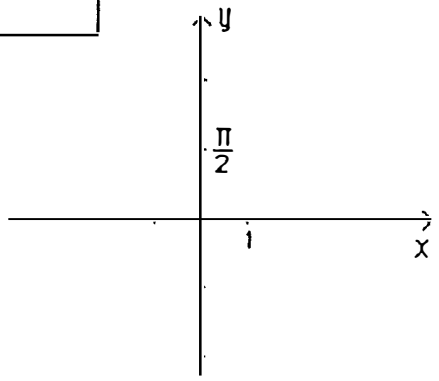
$y = \arcsin(x)$



D =

R =

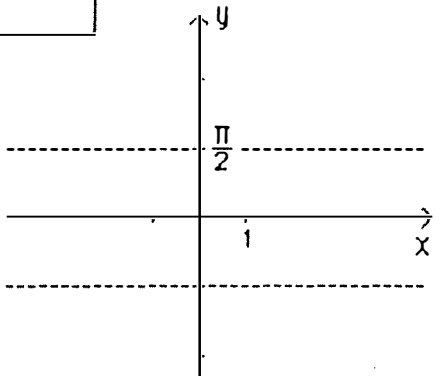
$y = \arccos(x)$



D =

R =

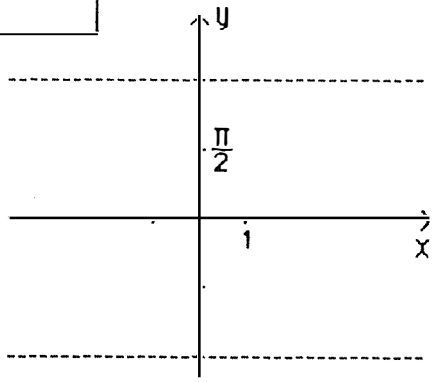
$y = \arctan(x)$



D =

R =

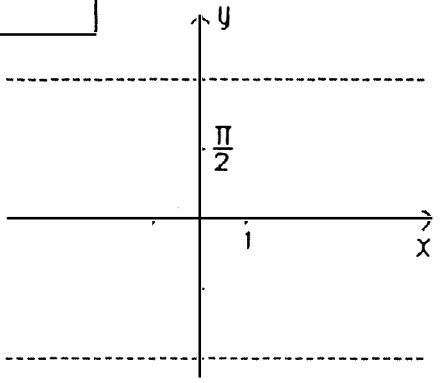
$y = \operatorname{arccot}(x)$



D =

R =

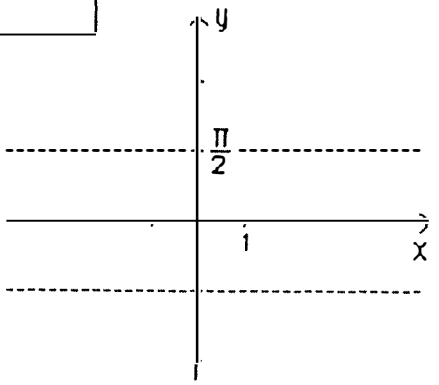
$y = \operatorname{arccsc}(x)$



D =

R =

$y = \operatorname{arcsec}(x)$



D =

R =

"LET X = ..."

by
Pat Rogers
Faculty of Education and Department of Mathematics
York University
Toronto, Ontario M3J 1P3

Variations of the following word problem can be found in most intermediate text books:

Farmer Brown rears chickens and pigs. Altogether there are 52 eyes and 68 legs.
How many of each animal does she have?

I frequently use this problem in my courses in mathematics for teachers to motivate a discussion of some of the reasons why problem solving is such a problem for so many children. I was reminded of it forcefully the other day when I overheard the following exchange between two teachers:

First person: Do you think that solutions to word problems should always start with the statement:

"Let x = ..."?

Second Person: Yes. It gets them thinking in the way you want them to think.

This exchange raises a number of important questions. Does starting with "Let x = ..." get them thinking at all? In what way **do** we want them to think and **should** we be attempting to have so much control over the way they think? The suggestion that solutions to word problems should begin with the statement "Let x = ..." is without doubt enormously helpful to some students. It does get them started and assists them in organizing their work. But insisting on this approach can also inhibit and cripple a student's natural creativity and contribute to producing the despair evident in the cry: "I know the answer but I don't know how I got it!"

A solution of the kind envisaged above might begin with the statement:

Let x = the number of chickens.

This introduces a complication possibly not anticipated by the material covered in the pages of the text prior to the exercise. There are, after all, two unknowns. This is easily taken care of by letting y = the number of pigs. The unknowns are connected, of course, because we know that there are 52 eyes and hence 26 animals altogether: so, $y = 26 - x$. But students do not always see this and the result is a system of equations in two unknowns:

$$2x + 2y = 52 \quad \ldots (1)$$
$$2x + 4y = 68 \quad \ldots (2)$$

Subtracting equation (1) from (2) gives $2y = 16$, and hence $y = 8$. Thus, there are 8 pigs and 18 chickens.

Any reader who had difficulty remembering whether y represented the number of pigs or the number of chickens in the solution given above is not alone. And that brings me to my first criticism of this approach. There is nothing sacred about x and y, particularly in a context such as this. When labelling the horizontal and vertical axes of a graph it might be considered perverse not to adopt the usual convention. But for this problem a more natural choice might be p for the number of pigs and c for the number of chickens.

A more serious criticism of this approach is that it forces children into a rote logical framework for solving problems, causing them to abandon their own ideas in deference to authority.

I have posed the pigs and chickens problem to a variety of adults including pre-service and in-service teachers. Most solve it using trial and error. They usually arrive at the correct answer but, when asked to describe how they found it, they are reluctant to divulge their method and often react with embarassment and nervous laughter followed by a disclaimer: "Well, it's not mathematical. I didn't really use math at all!" What they have actually done is some version of the following:

> I know there are 26 animals because there are 52 eyes and each animal has 2 eyes. So suppose there are 13 of each. Pigs have 4 legs, giving 52. Chickens have 2 legs, giving 26. So altogether we have 78 legs. That's too many. So, try fewer pigs: say 10 pigs and 16 chickens. And so on, until the solution is determined.

The method used here is, in fact, a variation of an ancient one with a noble tradition: the Egyptians called it "the rule of false position". But it is a route immediately ruled out by the restriction that solutions to word problems should always start with: "Let x = ...".

When asked what a mathematical solution would look like there is always someone in the group who is able to begin the standard argument above. However this has usually been abandoned either because of inability to solve the resulting equations or because it seemed like using a sledge hammer to crack a nut.

If our purpose in posing such problems is to give our students experience using a particular method to solve an applied problem we should be very careful to choose one which genuinely requires the use of the prescribed method. For the problem under discussion, the trial and error method of solution is probably the most sensible approach to take. In this sense the pigs and chickens problem is not a real problem at all: it presents no significant challenge. A problem is a problem only when the answer is not obvious at the outset. For exercises like these where the answer often is obvious, the student who is lucky enough to guess the answer straight away may learn nothing at all.

Here is an 'off the wall' solution to the pigs and chickens problem I discovered it in the middle of a class where I was encouraging the students to come up with different methods of solution.

> We see immediately that Farmer Brown has 26 animals. Imagine they are all lined up in an unfinished stage of creation, waiting for their legs. Farmer Brown walks around with a basket full of legs distributing them to the animals. After giving them 2 legs each she finds that she has only 16 left (68 - 2 $\times$ 26 = 16). Whose legs are they? They must belong to the pigs! How many pigs are there? 8!!!

The reaction of the class to this was interesting. Everyone understood the method I employed immediately and preferred this way of visualizing the problem to any other we had discussed. Genuine surprise was expressed that I, a mathematician, would consider this a real solution to the problem. Many were not satisfied until I had convinced them that my solution could be written up in a reasonable and logical form (possibly a relic from a time when someone had proclaimed that, while their answer might be correct, it could not be written down).

The point I am trying to make here is that there is more than one right way to solve a problem and that our students have useful ideas of their own. By insisting that they formulate their solutions **our** way we may be inhibiting other creative ways of thinking. Furthermore, even when a student has successfuly escaped the control we have tried to exert, as my experience with the pigs and chickens problem has shown, they remain firmly convinced that there is something un-mathematical about the way they choose to solve problems.

Students' perceptions of mathematics are coloured by our reactions to their early attempts to create meaning for themselves. If we teach them that our way is best and their way is wrong (or at best clumsy), they **will** learn to follow the rules but they may also begin to suppress and devalue their own ideas, to the point where the method they used to solve a problem is concealed from themselves as well as from their teachers. (Hence the cry: "I know the answer but I don't know how I got it!")

What I have tried to describe is one example of the implicit training some children receive in the mathematics classroom which leads them to view mathematics as an area in which there is only one 'right' answer with only one 'right' way to find it; an area in which they find they have very little contribution to make. This perception of mathematics remains with them in their adult lives as the following amply illustrates:

> And on the eighth day, God created mathematics. He took stainless steel, and he rolled it out thin, and he made it into a fence forty cubits high, and infinite cubits long. And on this fence, in fair capitals, he did print rules, theorems, axioms and pointed reminders. "Invert and multiply." "The square on the hypotenuse is three decibels louder than one hand clapping." "Always do what's in the parentheses first." And when he was finished, he said, "On one side of of this fence will reside those who are good at math. And on the other will remain those who are bad at math, and woe unto them, for they shall weep and gnash their teeth."
>
> Math does make me think of a stainless steel wall -- hard, cold, smooth, offering no handhold, all it does is glint back at me. Edge up to it, put your nose against it, it doesn't take your shape, it doesn't have any smell, all it does is make your nose cold. I like the shine of it -- it does look smart, intelligent in an icy way. But I resent its cold impenetrability, its supercilious glare.[1]

Is this the way mathematics is? Or is it just the way we present it? Any restrictions we place on our student's way of thinking contribute in a significant way to shaping this perception.

Endnotes

[1] *Buerk, Dorothy (1981). Changing the conception of mathematical knowledge in intellectually able, math avoidant women (Doctoral dissertation), SUNY at Buffalo.* ***Dissertation Abstract International,*** *42(1), 119A.*

WOMEN AND MATHEMATICS DEPARTMENT

EDITORIAL

by

M. Elaine Harvey, Women's Editor

Why **Women and Mathematics?**

Yes, there have been women mathematicians. There was Hypatia and Agnesi and Kovalevskaya and Young and Noether and du Chatelet and Germain and Somerville. But it is the names of the men mathematicians that we hear, study and remember.

Almost all of the recognized body of mathematics is ascribed to men. In this field women have not been heard. As a matter of fact, women have had little to say because until very recently women have been denied the opportunity to learn this particular branch of recognized and revered knowledge. Except for the few women who have somehow been able to break into the male sphere, women have contributed little to the field of mathematics.

Almost everyone would recognize that there are *some* differences between men and women. That a body of knowledge should have been created by one segment of the poplulation without the benefit of any significant contribution from the remaining population would suggest that that body of knowledge could be enlarged and enriched with the participation of that silent segment. This is particularly true when the silent segment has some biological and social characteristics that are noticeably different from those whose knowledge is recognized. Mathemaics has been viewed from one perspective only. Certainly it has benefited by the various points of view of those within the contributing set of males who belong to its various subsets--German, French, English, Indian, Russian, etc.--but it has not had or created the opportunity to benefit from that other major perspective of women.

The time has come. Women are gaining the foundations of male mathematical knowledge in a way that has previously been denied them. They are beginning to find their own mathematical voice.

To gain the female perspective in mathematics it is not only necessary for women to find their own voice. They must also be heard. Here is a forum for that voice. It is a voice that women are beginning to develop. But it is also a voice which some men as well as women are beginning to hear. It is a voice which may have the dulcet tones of the soprano or the resonances of the baritone. It is a voice that may be found in a female or a male body. It is a voice without which the subject of mathematics can never be complete.

GIRLS AND MATHEMATICS CONTESTS: PART I

by
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Editor's Note: This is the first of a series of articles on ***Girls and Mathematics***

Although mathematics contests have utilitarian functions as instruments for 'talent search' and partial determinants of university admissions and scholarships, they are at most levels, as the name of the Mathematical Olympiad suggests, a form of recreation. The analogy with amateur athletic events is extremely apt, in the training and discipline required to achieve one's best performance, the joy of a successful 'run', the feeling of frustration when one 'misses', and the pressures of team competition. But despite the extremely serious nature of math contests, amateur athletics, music festivals, and pumpkin growing competitions, in our society these are not part of the 'main business' of life: they are recreations, they are mostly optional, and they exist for people who enjoy them.

Moreover, while mathematics contest scores may be statistically correlated with success in university mathematics programs*, they are hardly infallible indicators of 'mathematical ability', a characteristic which is, after all, highly complex and only partially understood. We all know people who have recognizable mathematical talent and creativity who have never excelled in contest writing.

For these reasons it is probably not necessary to be very concerned about girls and mathematics contest, except to take care that the impression is never given that (a) the contests are primarily for the boys or (b) mathematics is only or pirmarily for the contest buffs.

Nevertheless, in our analysis of results by sex of the Descartes and Euclid contests in recent years, certain patterns emerge repeatedly, as they do in other similar studies. Since it is interesting and possibly important to speculate on the reasons for these, we will list them here, and follow them up with a more detailed discussion in Part II.

1. Girls participate in math contest far less frequently than boys, although their participation rate seems to be increasing.

   (Are there many girls who would enjoy participating and are not being encouraged to try, at least in the right way? Or are many girls with high ability simply more interested in other recreations?)

2. Girls don't win math contests.

   (Actually, they do occasionally, as any avid watcher of the recent contest scene will know. Are girls doing less well than they could with training, practice, and the right sort of attitude?)

3. Girls who try the contests leave questions untouched more frequently on the average.

   (This sex difference is most marked in questions involving geometry: is this a symptom of difficulty with spatial vizualization - often not needed to any great extent in the actual solution - or simply a failure of confidence? Even after years of experience, we too feel a little prickle of fear upon first confronting a problem with a complicated diagram. Do both sexes feel this equally?)

4. In multiple choice questions girls choose the 'distractor' answers more frequently.

   (Distractor: an incorrect response reachable by a simple but fallacious argument which is expected to lead a substantial number of competitors astray.)

   Is this a symptom of:
   A. impulsiveness B. inexperience C. panic D. despair E. lack of interest F. a more holistic thought process?

We would be happy to have thoughts from our readers on any of the above.

** We have consistently found significant correlations between Descartes contest scores and first year averages in the University of Waterloo Mathematics Faculty.*

WOMEN AND MATHEMATICS: A BIBLIOGRAPHY

by
M. Elaine Harvey

Alic, Margaret. **Hypatia's Heritage: A History of Women in Science from Antiquity to the Late Nineteenth Century.** London: The Women's Press, (1986). [$13.25*]

This gives the best historical overview of women mathematicians that I have found. Of particular interest is Chapter 11: "The Nineteenth-Century Mathematicians: The Mathematical Contributions of Sophie Germain--Ada Lovelace and the Beginnings of Computer Science--The Mathematical Mind: The Story of Sophia Kovalevsky"

Bleier, Ruth. **Science and Gender: A Critique of Biology and Its Theories on Women.** The Athene Series, New York: Pergamon Press, (1984). [$19.95*]

The **Athene Series** is an international collection of feminist books that focuses on the construction of knowledge and the exclusion of women from the process. "This book is concerned with the role of science in the creation of an elaborate mythology of women's biological inferiority as an explanation for their subordinate position in the cultures of Western civilizations."

- Preface, page vii

Bunt, Lucas N. H. et al. **The Historical Roots of Elementary Mathematics.** New Jersey: Prentice-Hall, Inc. (1976) [$12.54*]

This is a good historical resource for teachers written from the traditional non-feminist perspective.

Cajori, Florian. **A History of Mathematical Notations: Volume 1: Notations in Elementary Mathematics.** La Salle, Illinois: The Open Court Publishing Company, (1928, reprinted 1974) [$7.15*]

This classic on mathematical notation provides interesting historical nuggets for the classroom teacher. Topics are Numeral Symbols and Combinations of Symbols; Symbols in Arithmetic and Algebra: groups of symbols used by individual writers, topical survey of the use of notations; Symbols in Geometry: ordinary elementary geometry, past struggles between symbolists and rhetoricians in elementary geometry.

Fox, Lynn H. et al., ed. **Women and the Mathematical Mystique: Proceedings of the Eighth Annual Hyman Blumberg Symposium on Research in Early Childhood Education.** Baltimore and London: Johns Hopkins University Press, (1980). [$5.00 second hand*]

I. Female Mathematicians. II. Sex Differences in Mathematics Achievement and Course-taking. III. Facilitating Women's Achievement in Mathematics.

Gilligan, Carol. **In a Different Voice: Psychological Theory and Women's Development.** Cambridge, Mass.: Harvard University Press. (1982) [$7.95*]

This book has become a classic in four short years!
"My goal is to expand the understanding of human development by using the group left out in the construction of theory to call attention to what is missing in its account. Seen in this light, the descrepant data on women's experience provide a basis upon which to generate new theory, potentially yielding a more encompassing view of the lives of both of the sexes." - page 3-4

Jacobs, Judith E., ed. **Perspectives on Women and Mathematics.** ERIC Clearinghouse for Science, Mathematics and Environmental Education, College of Education, Ohio State University, 1200 Chambers Road, Third Floor, Columbus, Ohio 43212, (1978) [$1.75*]

Papers presented in the Women and Mathematics strand of the 1978 conference of the National Council of Teachers of Mathematics held in California form the core of this book.

Kaseberg, Alice et al. **Use EQUALS to Promote the Participation of Women in Mathematics.** Math/Science Network, Lawrence Hall of Science, University of California, Berkeley, California 94720. (1980) [US$9.00*]

The EQUALS program is part of the Math/Science Network's elementary and secondary focus. This book contains resources for classroom projects, teaching strategies, problem solving activities, career information, resource materials, model workshops and bibliographies.

Newman, Louise Michele, ed. **Men's Ideas/Women's Realities: Popular Science, 1870 - 1915.** The Athene Series, New York: Pergamon Press, (1984). [$21.75*]

"Before sanctioning the proposal to subject women to a system of mental training... it is needful to consider whether this can be done without serious injury to her health and strength." -- Henry Maudsley
It is statements such as the above that impelled Newman to put together scientific writings of the turn of the century with commentary.

Perl, Teri. **Math Equals: Biographies of Women Mathematicians + Activities.** Don Mills, Ontario: Addison-Wesley Publishing Company, (1978). [$9.45*]

This book is a must for teachers and students. The biographies are interest, and the related activities imaginative.

Resnikoff, H. L. and R. O. Wells, Jr. **Mathematics in Civilization.** New York; Dover Publications, Inc. (1973). [$14.95*]

This is a well-written book dealing with Mathematics in Antiquity, the Adolescence of Computation, the Rise of Geometrical Analysis, and Twentieth-Century Mathematics. It contains some excellent ideas for the secondary teacher. It is a traditional work without a feminist perspective.

Rothschild, Joan, ed. **Machina Ex Dea: Feminist Perspectives on Technology.** The Athene Series, New York: Pergamon Press, (1984). [$19.95*]

Of particular interest in this book is Chapter 3: "Mathematization of Engineering: Limits on Women and the Field".

Spender, Dale, ed. **Feminist Theorists: Three Centuries of Key Women Thinkers.** New York: Pantheon Books, (1983). [$17.25*]

"This then is a glimpse of some women's intellectual traditions over the last three centuries. It is a glimpse however, which patriarchy would prefer us to do without." - page 7

Spender, Dale. **For the Record: The Making and Meaning of Feminist Knowledge.** London: the Women's Press, (1985), [$13.25*]

This is an account of the work of feminist writers and theorists of the last twenty years. It will be of particular value for those who want an overview of recent feminist thinking.

Spender, Dale. **Invisible Women: The Schooling Scandal.** London: Writers and Readers Publishing Cooperative Society Ltd. (1982). [$8.95*]

This book is a must for teachers. Read about The Old, Old Problem; The Knowledge of Males; In the Classroom; The World According to Men; Women's View.

Spender, Dale. **Man Made Language.** London, Routledge & Kegan Paul, (1980) [$13.46*]

"For women, language is man-made, for it encompasses the meanings of men, who have arrived at their definitions of the world from a position of dominance, a position which women as a group do not occupy; for women, these meanings are partial, and false. In this book Dale Spender presents an exciting, wide-ranging feminist study of language, its rules and uses. - back cover

Spender, Dale, ed. **Men's Studies Modified: The Impact of Feminism on the Academic Disciplines.** The Athene Series, New York: Pergamon Press, (1981). [$23.50*]

"Fundamental to feminism is the premise that women have been 'left out' of codified knowledge, so that the world has been explained in terms of men but not women. Essays on the following academic disciplines explore not only how this happeded but why: language, literary criticism, philosphy, history, sociology, political science, anthropology, economics, media studies, **education**, law, medicine, **biology**, and **the scientific ethic.**

Spender, Dale. **Women of Ideas (and What Men Have Done to Them).** London: Ark Paperbacks (Routledge & Kegan paul plc) (1982), 800 pages, [$12.50*]

"Until Dale Spender undertook her amazing research into three hundred years of the voices of women with ideas struggling to be heard their very names were barely known."

Thompson, Jane. **Learning Liberation: Women's Response to Men's Education.** London & Cangerra: Croom Helm, (1983) [$19.50*]

"More recently the education system, constructed historically by the ruling class and managed by the middle class to reflect the interests and requirements of dominant groups, has been portrayed as an important regulator of social and economic class relations and a powerful ideological instrument in the battle for the hearts and minds of dutiful workers, who need to be conformed to the rules of order required by class domination if that oppression is to be continued."

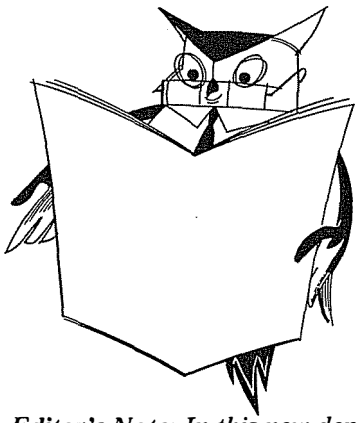
- Chapter 3 "The Schooling of Girls", page 29

"Women's Development and Education". **Journal of Education** 167, no. 3 (1985), Boston University, 605 Commonwealth Avenue, Boston, Mass. 02215.

Of particular interest in this women's issue is Dorothy Buerk's article, "The Voices of Women Making Meaning in Mathematics.

Women's Education des femmes. This is a billingual quarterly journal published by the Canadian Congress for Learning Opportunities for Women. Enquiries: CCLOW, 47 Main Street, Toronto, Ontario M 4E 2V6, 416-699-1909. [Single copy, $4.25--newsstand price*]

* These amounts are offered to give you some indication of the retail price. All you can be sure of is that each was the price at one time and one place of purchase.



HISTORY AND PHILOSOPHY OF MATHEMATICS

by
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Editor's Note: *In this new department of the Gazette, we want to open a dialogue with our readers about the whole mathematical enterprise, especially on matters of history, philosophy and heuristics.*

Philip Davis and Reuben Hersh have just come out with a successor to their book, **The mathematical experience.** Entitled **Descartes' Dream** and published by Harcourt, Brace and Jovanovich, it discusses the impact that mathematics has in other areas. The attempt to mathematize the world has its roots in the work of Descartes, who dreamed of a universal method which would yield in a rational and systematic way the solution of any human problem whatsoever.

The curriculum for the OAC course in Calculus includes the approximation of solutions to equations $f(x) = 0$ by Newton's Method. Assuming that $f(x)$ has a derivative, we begin with a number u which is close to the required solution. Then, under suitable hypotheses, a closer approximation to the solution is given by

$$u - \frac{f(u)}{f'(u)}.$$

Often a geometric explanation for this method is provided. If we sketch the curve $y = f(x)$, we can see that if will cross the x-axis at the point $(c, 0)$ where c satisfies the equation $f(x) = 0$. Taking a point u near c, draw the tangent to the curve through the point $(u, f(u))$. The x-intercept of this tangent is $(u - \frac{f(u)}{f'(u)}, 0)$; this point may well be closer to $(c, 0)$ than $(u, 0)$ is. If so, it can be used for the basis of a better approximation.

Is this in fact how Newton himself justified the method? In 1669, Newton wrote a tract in Latin whose titled translated into English is "Of analysis by equations of an infinite number of terms". Then, as now, publishers were concerned about the size of the market for their works, so it was not until 1708 that the tract was actually printed. Eventually, in 1745, it was published in English, and a facsimile version is now available as a Johnson Reprint.

Newton's purpose was to expose a general method for finding the areas under curves. He begins with the observation that, if the curve has equation

$$y = ax^{m/n}$$

then the area under the graph from the orgin to the abscissa x is given by the formula

$$\frac{an}{m + n} x^{\frac{m + n}{n}}$$

From this, it follows that for a curve in which y is a sum of monomials ax^r, the area under it is the sum of the corresponding antiderivatives of its monomial summands. Since he applies this to infinite sums, and since a wide variety of curves can be expressed by equating y to a power series, this method for finding areas can be applied very widely indeed.

However, not every example we are given has its equation in the required form. Sometimes, as in the cases

$$y = \frac{a^2}{b + x} \quad \text{or} \quad y = \sqrt{a^2 + x^2}$$

(Newton's examples), there is an algorithm for expressing y as a power series in x. In other cases, such as

$$y^3 + a^2y - 2a^3 + axy - x^3 = 0,$$

one has to solve for y as a series in x.

To illustrate his technique, Newton first describes it for a numerical equation involving y. Consider the equation

$$y^3 - 2y - 5 = 0.$$

A first approximation to the solution is 2. Accordingly, put $y = 2 + p$. This leads to a new equation

$$p^3 + 6p^2 + 10p - 1 = 0.$$

Since p is small, an approximation to this equation is $10p - 1 = 0$, whereupon p is about equal to 0.1.

The next stage is to set $p = 0.1 + q$ and obtain

$$q^3 + 6.3q^2 + 11.23q + 0.061 = 0.$$

"And since $11.23q + 0.061 = 0$ comes near to the Truth, or since q is almost equal to -0.0054, ..., and then supposing $-0.0054 + r = q$, I substitute this as formerly, and the operation is continued as far as you please." And at each stage, in the equation to be solved, all terms of degree higher than 1 in the variable are neglected.

How do we bring this into line with what you and I understand as Newton's Method? Let, as before, the equation to be solved be $f(x) = 0$, and let the true solution be the (unknown) number c. Suppose u is the approximation. By Taylor's Theorem, we can write

$$0 = f(c) = f(u) + f'(u)(c - u) + \tfrac{1}{2}f''(u)(c - u)^2 + \ldots$$

where f', f'', ... denote the first, second, ... derivatives of f. If u is chosen close to c, then the powers $(c - u)^2$, ... are negligible and we have approximately

$$0 \doteq f(u) + f'(u)(c - u)$$

or, solving for c,

$$c \doteq u - \frac{f(u)}{f'(u)}.$$

In Newton's example, $f(x) = x^3 - 2x - 5$, $f'(x) = 3x^2 - 2$, $f''(x) = 6x$, $f'''(x) = 6$, $u = 2$, $c - u = p$. The equation for p is

$$0 = f(2) + f'(2)\,p + \tfrac{1}{2}f''(2)\,p^2 + (1/6)f'''(3)\,p^3$$

$$= -1 + 10p + 6p^2 + p^3.$$

The method is now adapted for literal equations, with the strategy of constructing approximate equations by neglecting "higher order" terms being adopted. Thus, for

$$y^3 + a^2y - 2a^3 + axy - x^3 = 0,$$

when $x = 0$, we have the solution $y = a$. Accordingly, set $y = a + p$ and obtain

$$p^3 + 3ap^2 + axp - x^3 + 4a^2p + a^2x = 0.$$

This equation is to be replaced by an approxiamte equation for p. To help us identify the terms to be retained, we plot points (u, v) in a Cartesian plane whenever $p^u x^v$ occurs in the left side of the equation: thus, we plot (3, 0), (2, 0), (1, 1), (0, 3), (1, 0), (0, 1).

Imagine a line passing through the point on the x-axis nearest the orgin: (1, 0). Rotate this line clockwise from the x-axis until it first contains at least one other point (u, v). The values of (u, v) which occur on this line correspond to the terms of the equation which are to be retained. In this case, the line passes through (1, 0) and (0, 1), so we consider the approximate equation

$$4a^2\,p + a^2\,x = 0.$$

New set $p = -\tfrac{1}{4}x + q$, and continue on.

To briefly consider a second example, suppose the equation

$$y^6 - 5xy^5 + x^3y^4 - 7x^2y^2 + 6x^3 + x^4 = 0$$

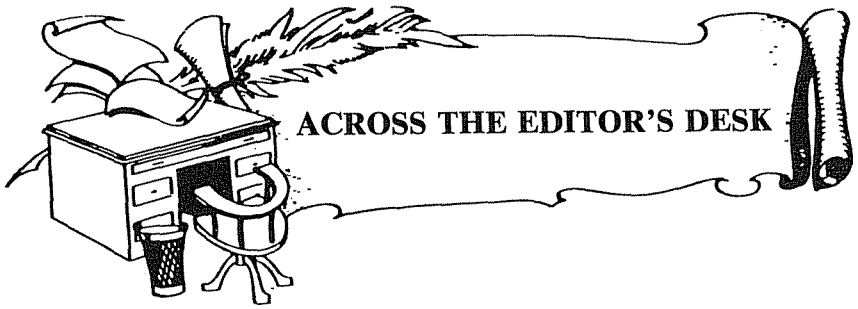
is given, in which y is to be solved in terms of x. We plot the points (6, 0), (5, 1), (4, 3), (2, 2), (0, 3), (0, 4). The point nearest to the origin on the x-axis is (6, 0) and the line through this point which interests us also passes through (2, 2) and (0, 3). As a first approximation, we have the equation

$$y^6 - 7x^2y^2 + 6x^3 = 0$$

or $$(y^2 - x)(y^4 + xy^2 - 6x^2) = 0.$$

Thus, y is approximately $x^{1/2}$, so we take $y = x^{1/2} + p$, and carry on.

These ideas of Newton are alive today, not only in the area of solving equation, but also in modern number theory. But that is a story for another time.



1987 03 26

Clifford Gravelle, Editor
College of Education
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Dear Mr. Gravelle:

As an ex-editor of the Gazette, I was initially pleased to see the new format. However I have several comments about the content (September 1986).

a) page 4. "The pen problem" perpetuates the myth that, given any initial terms of a sequence, the rest of the sequence is uniquely determined. If $t_1, \ldots t_m$ are given, then one may define an **infinite** number of sequences which have these initial terms.

[i.e. $S = t_1, \ldots t_m, q_1, \ldots q_n \ldots$ where $q_1, \ldots q_n \ldots$ is any sequence]

b) page 7. Why use the ambiguous term 'billion' when 10^9 is precise? This appears part of the 'sympathetic magic' syndrome that naming an object somehow leads to understanding [It was not until entering an elementary school some ten years ago that I discovered the words 'subtrahend' and 'minuend, yet I'd been able to subtract without needing extraneous words].

c) page 30. I am in sympathy with the letter about OSIS and the OAC's. I, too, am disturbed. The Ministry should have started by asking the universities what they require from entering students, then designed a school curriculum that, for students intending to attend universities provides them with adequate knowledge.

Incidentally I'm also concerned with the modern text books which seem to contain much extraneous material. Surely the emphasis should be on mastery of content without padding on historical and biographical material.

Yours sincerely,

J. S. Griffith
Dept. of Mathematical Sciences

IN MEMORIAM

Mr. James W. Fencott

Born March 19, 1920 - Died January 22, 1987



Jim is survived by his wife, Edith, four daughters, Jean, Betty, Marie and Veronica, and four sons, Robert, Peter, Anthony and Patrick.

Jim was born in Burma and served in the Burmese military during World War II, attaining the level of staff lieutenant. He also served in the Burma Civil Service from 1945 to 1948.

He began his teaching career in 1952 in London, England, and came to Canada in 1956. His first teaching assignment was at Uxbridge High School from 1956 to 1961. He joined the Scarborough Board of Education in 1961 and was Mathematics Head at R.H. King Collegiate Institute and Stephen Leacock Collegiate Institute. Jim was appointed Co-ordinator of Mathematics for Scarborough in 1972, when Jack R.I. McKnight retired. Mr. Fencott retired as a Co-ordinator in 1985 but continued to be active in Mathematics. In 1985-86, he spent a year in Gambia instructing Mathematics teachers through the Canadian Organization for Development Through Education (C.O.D.E.).

Jim was extremely respected by all his colleagues in the province and by Mathematics teachers who worked with him across Canada, as well as in Scarborough. He was a very compassionate and very generous man - he would help anybody. He co-authored numerous texts, initiated the first Scarborough chapter of O.A.M.E. (S.E.Y.M.A. and S.A.M.E.), started Saturday morning enrichment classes, won a Descartes Gold Medal for his contribution to Mathematics education and served on numerous Planning Committees for the O.A.M.E. and N.C.T.M. Conferences.

Jim leaves a challenge to all Mathematics educators to help deliver the best possible Mathematics program to teachers and students.

He will be missed.

Paul Zolis
Co-ordinator of Mathematics
Scarborough Board of Education
Scarborough, Ontario M1P 4N6



PROBLEMS

by
Ronald Lancaster, Problem's Editor
Oakville-Trafalgar High School

1. For each of the following, expand and simplify.

   a) $x(x + 1)$

   b) $x(x + 1)(x + 2)$

   c) $x(x + 1)(x + 2)(x + 3)$

   d) $x(x + 1)(x + 2)(x + 3)(x + 4)$

   e) $x(x + 1)(x + 2)(x + 3)(x + 4)(x + 5)$

   f) $x(x + 1)(x + 2)(x + 3)(x + 4)(x + 5)(x + 6)$

2. Discuss your results for Q1 and use your observations to expand and simplify.

   $x(x + 1)(x + 2) \ldots (x + 8)(x + 9)(x + 10)$

3. Prove that

$$\sqrt[3]{2 + \sqrt{5}} + \sqrt[3]{2 - \sqrt{5}} = 1$$

CORRECTIONS FOR PROBLEMS FOUR AND FIVE, VOLUME 25, NUMBER 2, DECEMBER 1986

4. There is no need to use the method of indirect proof in this question. The proof given was not quite right anyway, as the equality case was not dealt with. The result can be proved as follows.

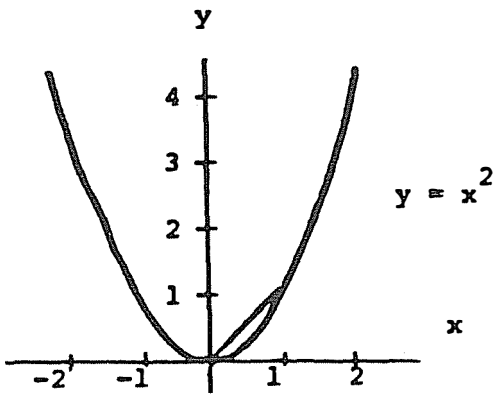
   Note that

$$\begin{aligned} & 101^{50} - 99^{50} \\ &= (100 + 1)^{50} - (100 - 1)^{50} \\ &= 2\left\{\binom{50}{1}100^{49} + \binom{50}{3}100^{47} + \ldots + \binom{50}{49}100\right\} \\ &> 2\left\{\binom{50}{1}100^{49}\right\} \\ &= 100^{50} \end{aligned}$$

   Therefore $99^{50} + 100^{50} < 101^{50}$ as required.

5. There were some typographical errors in Q5. A correct copy is given below.

Consider the curve $y = x^2$ and the two points (0, 0) and (1, 1). Join (0, 0) to (1, 1) with a straight line.



Note that the distance between (0, 0) and (1, 1) is less than the length of the curve from $x = 0$ to $x = 1$.

Thus
$$\sqrt{2} < \int_0^1 \sqrt{1 + 4x^2}\, dx$$
$$= \frac{\sqrt{5}}{2} + \frac{\ln(2 + \sqrt{5})}{4}$$

ANSWERS TO THIS MONTH'S PROBLEMS

1. a) $x^2 + x$

   b) $x^3 + 3x^2 + 2x$

   c) $x^4 + 6x^3 + 11x^2 + 6x$

   d) $x^5 + 10x^4 + 35x^3 + 50x^2 + 24x$

   e) $x^6 + 15x^5 + 85x^4 + 225x^3 + 274x^2 + 120x$

   f) $x^7 + 21x^6 + 175x^5 + 735x^4 + 1624x^3 + 1764x^2 + 720x$

2. My Grade 10 class recently worked on Q1 and they developed the triangular array of numbers shown below for expanding these things.

			1	1			
		1	3	2			
	1	6	11	6			
1	10	35	50	24			
	1	15	85	225	274	120	
	1	21	175	735	1624	1764	720
1	28	322	1960	6769	13132	13068	5040

The above array of numbers is very similar to Pascals Triangle. In the nth row, the first number is 1 and the last number is n! The numbers in between can be obtained from the row above. (multiply the number above and to

the left by n - 1 and add to this the number above and to the right). For example, in the seventh row, the number 322 was obtained from multiplying 21 by 6 and adding to this 175. These numbers are called Stirlings Numbers of the First Kind. The above array of numbers can easily be extended to obtain the expansion called for in Q2.

3. My Calculus class recently worked on this problem and I received the following two proofs.

Solution 1

$$\sqrt[3]{2 + \sqrt{5}} + \sqrt[3]{2 - \sqrt{5}}$$

$$= \sqrt[3]{\left(\frac{1 + \sqrt{5}}{2}\right)^3} + \sqrt[3]{\left(\frac{1 - \sqrt{5}}{2}\right)^3}$$

$$= \frac{1 + \sqrt{5} + 1 - \sqrt{5}}{2}$$

$$= 1$$

Solution 2

$$\text{Let } x = \sqrt[3]{2 + \sqrt{5}} + \sqrt[3]{2 - \sqrt{5}}$$

$$x^3 = 2 + \sqrt{5} + 3\sqrt[3]{(2 + \sqrt{5})^2}\sqrt[3]{2 - \sqrt{5}} + 3\sqrt[3]{2 + \sqrt{5}}\sqrt[3]{(2 - \sqrt{5})^3} + 2 - \sqrt{5}$$

$$= 4 - 3\sqrt[3]{2 + \sqrt{5}} - 3\sqrt[3]{2 - \sqrt{5}}$$

$$= 4 - 3x$$

$$\therefore \quad x^3 + 3x - 4 = 0$$

$$(x - 1)(x^2 + x + 4) = 0$$

Thus $x = 1$ as required.



WHAT'S GOING ON

SCARBOROUGH '87

Mayor Gus Harris will declare May 3-10, 1987 "Mathematics Awareness Week" in the City of Scarborough.

14TH ANNUAL O.A.M.E. CONFERENCE: "Of Mice and Math ...", Scarborough Association for Mathematics Education is pleased to host the fourteenth annual conference of the Ontario Association for Mathematics Education at Centennial College, Progress Campus, on Thursday 7 May to Saturday 9 May 1987. We hope that you will count yourself in on the excitement and the enthusiasm.

IDEAL LOCATION: The location is ideal for our purposes because Centennial College, Howard Johnson's, and the Cultural Centre are the only buildings on a cul-de-sac and we are using all three. Centennial College is easy to reach; simply drive along highway 401 and exit at Markham Road south, turn left at the first stop light (Progress Avenue) and you're at Centennial College and free parking. The staff at Centennial are enthusiastic, cooperative partners in this conference. Relax Inn and Venture Inn are most convenient, merely short walks, across Markham Road from the Convention Headquarters. Centennial College has dining facilities available, as well as the local hotels, and there are many fine dining establishments nearby; such as, The Keg and Chi-Chi's restaurants. Families will appreciate the close proximity to the Metro Toronto Zoo, Scarborough Town Centre Shopping Mall, Ontario Science Centre, Bluffers Park. Scarborough is the seventh largest city in Canada and offers a unique mixture of urban and rural lifestyles within a few kilometres of Centennial College. Scarborough offers something of interest for everyone.

GREAT PROGRAM: Peter Crippin, Shirley Fairfield, and Murray Major have all been busy preparing our 1987 program with 135 different speakers and 150 sessions offered. The "Murr" provides us with some teasers: "A First Class Lesson, Lynda Colgan and "Microcomputer snacks", Frank Ebos and "Turn-ons for students", John Egsgard and "The last 5 minutes", Neil Graham and "OS:IS evaluation", David R. Johnson and "Salesmanship, showmanship, and surprise", Jack LeSage and "Testing tips", Richard Lucas and "Friday afternoon", Chris Monk and "Math and the car", Bill Nimigon and "Pool hall math", Catherine Rellinger and "Math anxiety", Liz Smythe and "Gifted girls in math", Jack Weiner and "Math Missteaks". We are excited by Thursday's dynamic kenote speaker, David R. Johnson, chairman of mathematics and science departments at Nicolet High School, Glendale and by Friday's outstanding banquet speaker, Dr. Peter Hilton, distinguished professor of mathematics at State University of New York at Binghamton, Binghamton, NY, past president of Mathematical Association of America. Our program committee has prepared several strands; special education, contests and enrichment, general level, evaluation, computers, women and math, school visitations - Scarborough computer lab (Highbrook), gifted programs (Woburn, CI, Churchill Hts.), vocational school (Scarborough's Eaton SS, super lunch included) and the ever popular "Make it, take it"; note that Scarborough will again offer the "Trading Post", but at two levels - elementary and secondary.

GREAT HOSPITALITY: Enjoy yourself at the 14th conference. Bob Wilson and Iris Brown have arranged receptions on Thursday and Friday afternoons, the Thursday night sock hop, breakfast for Friday morning and the Friday president's banquet, plus MATHEMATICS NORTH (look out Sudbury/Ottawa).

ATTRACTIVE CARDIGANS: Many conventioneers at London stopped by our Scaborough Booth and inquired about purchasing the cardigans our team was wearing. Anyone who wishes to purchase one of these attractive white cardigans, crested with SAME / Scarborough '87 / OAME; simply indicate your desire to do so when you register, indicate your size and prepay ($30); the cardigan will be available when you pick up your registration package.

INFORMATION: If you have any questions, please contact our conference chairmen: STEVE MARTIN, Danforth Gardens PS, 20 Santamonica Blvd, Scarborough, M1L 4H4, telephone 416-265-1411; PAUL ZOLIS, MATH CENTRE, ST. ANDREWS JR. PS, 60 Brimorton Drive, Scarborough, M1P 3Z1, telephone 416-431-5644.

THANKS: Thanks for all the help with publicity from a terrific team: Lainey Ankenman Lorraine, Sir A. Mackenzie Sr; Jan Cornwall, Math Centre; Bill Haehnel, Bendale SS; Darlene Stephenson, Lynngate Jr; Paul Zolis, Math Co-ordinator.

ARNIE CAREFOOTE
PUBLICITY CHAIRMAN
OAME '87

ALBERT CAMPBELL CI
1550 Sandhurst Circle
Scarborough, Ontario M1V 1S6
416-298-0130

TRADING POST: O.A.M.E. '87

Bring 100 copies of your favourite idea, teaching tip, best lesson, pet project, ace assignment, etc.

Submit your package to the secondary or elementary desk and receive your participation ticket.

Return to that desk 4 p.m. - 5 p.m. Friday; present your ticket and receive 1 copy of each submission - for your panel.

"PARTICIP - ACTION"

THE

ONTARIO MATHEMATICS GAZETTE WANTS YOU!

Every mathematics teacher has special teaching techniques, useful worksheets, or other neat items for the mathematics classroom. These should be shared!

Here is your chance at stardom. Turn those super ideas into articles for possible use in the Gazette or other journals.

INTRODUCING

THE O. M. G. AUTHORING ROOM

The O.A.M.E. Publications Committee has reserved space at the S.A.M.E. Conference from 10:00 A.M. to 4:00 P.M. on Friday, May 8. The room (to be announced) will be stocked with Macintosh and Commodore 64 microcomputers and staffed with Gazette editors to help you **efficiently** and **painlessly** put those ideas into a publishable package.

DON'T BE SHY. COME AND SHARE!!

For more information, please contact:

Jack Weiner, O.A.M.E. Publications Committee
Department of Mathematics and Statistics
University of Guelph
Guelph, Ontario, N1G 2W1
(519) 824-4120, extension 2157 (message at 2155)



BOOK REVIEW

by
Book Review Editor, R.S. Smith, Faculty of Education
University of Western Onario, London, Ontario N6G 1G7

Studies in Mathematics Education, Volume 3, Edited by Robert Morris, U.N.E.S.C.O. 7 Place de Fontenoy, 75700 Paris, France, Paperback. 258 pages.

This volume is one of a series prepared as a part of Unesco's program for improving mathematics instruction. Volume 3 (Primary) as well as a further volume four (Secondary) concerns itself with "the weakest link in the chain" Teacher Education.

This book is organized in sections so you may wish to read only the section which is particular to your interests. The early section deals with comtemporary trends and their implications for teacher education, with a specific chapter on calculators and computers. The middle section of four chapters is concerned with concepts. The last section of five chapters is devoted to pre-service and in-service education and the book closed with examples of support programs that are in operation.

Fey, in summarizing studies done by the National Science Foundation about achievement in mathematics, goals, and methodology, was led to conclude that there is a consistent pattern of great differences between the apparent reality of mathematics education in most schools and the recommendations of practices of many prominent teachers, supervisors and professional organizations. He has also written that many elementary school teachers act as though they believed that their sole responsibility in mathematics teaching is to develop facility in arithmetic computation.

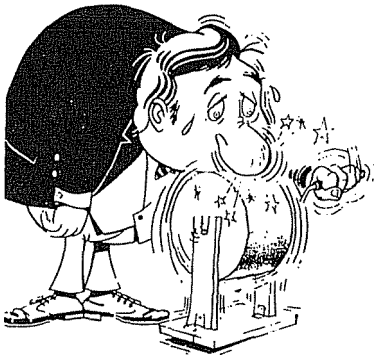
After those most discouraging observations, it is encouraging to find support in one's views by such statements as:

> We teach mathematics in order to help people to understand things better--- Based on the fundamental idea that there is no difference between the nature of a child's thinking and that of a mahematician, a tendency is slowly growing to replace the learning of mechanisms and their applications to standard problems by activities in which the child demonstrates research and inventiveness.

Implications of both the good news and bad news are obvious. Teacher education needs much improvement. Some guidelines and some models are developing. Experience in the United Kingdon suggests that teachers involved in groups devising curriculum guidelines and activities are much more likely to exemplify these beliefs. In the Netherleands, Freudenthal writes that subject matter learned by the future teacher must be close to that to be taught in his future career... that developing a mathematical attitude as a means of developing a good educational attitude is more important than the quantity of mathematics taught. From the United States comes Polya's pragmatic conception of teacher training.

> The best way to learn is to do, to ask, and to do.
> The best way to teach is to make students ask, and to do.
> Don't preach facts - stimulate acts.
> **The best way to teach teachers is to make them ask and do what they in turn will make their students ask and do.**

From the above items comes the distinct message that improvement in mathematics education cannot proceed in isolation in Faculties of Education although there may be a primary need for a leadership role to be taken by them. This book will provide further examples and opinions of directions to solve the problems. Do examine a copy.



COUNCIL COMMENT

by
Bob Robinson

Editor's Note: Council Comment appears in an issue of the **Gazette** *following an OAME/AOEM Council Meeting. Your comments are welcome and should be addressed to the Editor.*

CHAPTER REPORTS:

COMA/AMOC

COMA's "Geometry for All" workshop, featuring Trevor Brown, was a resounding success, with over 150 registered. It was also good to see that over 50% of the attendees were from the elementary panel.

Plans for COMA's 2nd Leadership Conference, being held Oct. 2-3, 1987 at the Glenn House in Gananoque are well underway.

Preparations for hosting OAME's 1989 Annual have already begun, under the direction of a Committee co-chaired by Dave Hunter and Ron Bender.

GOLDEN SECTION

The Annual Spring Fling is tentatively slated for April 23, 1987 at Brock University featuring Peter Taylor.

OAME '88 is progressing well, with keynote speakers obtained and several "minicourses" already being developed.

GVMA

Summer writing session is tentataively planned to develop resources for the basic level course **Math for Work and Home,** Gr. 9.

NOMA

NOMA has a Fall Conference planned for Sept. 25-26 at Lasalle Secondary School in Sudbury. C. McNabb is Chair for that Conference.

A certificate recognizing outstanding work in mathematics is under development.

PAME

PAME's fall MATHFEST was a great success, with about 200 in attendance.

Membership in PAME continues to grow, with active involvement from teachers at all grade levels.

PINE RIDGE

Fall P.A. day was very successful, with about 150 in attendance.

A mini-conference of 9 workshops at different levels will be held on April 14, 1987 at General Vanier Secondary School in Oshawa from 4-7:30 p.m.

QSLMA

Fall Conference was successful. Another meeting is planned for April 8, 1987.

SAME

SAME has held several successful meetings and P.D. sessions and continues to plan actively for OAME '87.

Nominations for the "Jim Fencott Memorial Award" (news of his recent sudden passing was relayed) will be solicited in late March.

Elections will be held in late April. Interested members are asked to contact Arnie Carefoote at Albert Campbell Collegiate Institute.

Annual Meeting Updates

1987 Everything is ready to roll for OAME '87. If you haven't already registered, be sure to do so soon. You won't want to miss it!

1988 Keynote speakers and several "minicourses" are already set. OAME '88 at the Brock Hotel Complex in Niagara Falls, May 5-7, 1988 promises to be a superb conference.

1989 COMA has already formed a committee to start planning for OAME '89 in Ottawa, May 11-13, 1989.

NCTM

Mary Lou Kestell reported on her recent attendance at an NCTM Leadership Conference in Minneapolis. She brought back much information to share with Memberhip, Publicity, and others.

Ralph Connelly was named OAME's delegate to this years's Delegate Assembly, with Dino Dottori as the alternate.

Members are reminded of the NCTM Regional meeting being held in Halifax, Nova Scotia August 19-22, 1987. The program is set and looks *great!* Plan now to attend.

Assuming approval of the Board of Directors, OAME will be hosting an NCTM Regional Conference in Hamilton in May of 1990.

Position Paper on EPEP

The Primary/Junior Committee has prepared an OAME position paper on the Early Primary Education Project which was approved by Council. The paper will appear elsewhere in this issue.

COMMITTEE REPORTS:

Publications Committee

Ron Ripley (Queen's) was appointed as the new editor for the ABACUS. His term will begin with the Sept. 1987 ABACUS.

The Publications Committee will staff an "Authoring" room at OAME '87, to help attendees turn their great teaching ideas into articles. Details can be found elsewhere in this issue.

Primary/Junior Committee

Position paper on EPEP can be found in this issue.

Committee is pursuing the obtaining of information from the Research on the Training of Fractions Project in Alberta.

Intermediate/Senior Committee

The Committee announced that Chester Carlowe is sponsoring an award for innovative approaches to teaching problem-solving. More details will follow.

The Committee is also looking into ways of dealing with the problem of teachers with insufficient math backgrounds being called upon to teach Senior level mathematics.

Futures

The Committee is investigating the possibility of putting together a book on the History of OAME.

NOTE: ANYONE WITH ANY MATERIAL RELATED TO THE HISTORY OF MATHEMATICS EDUCATION IN ONTARIO WITH PARTICULAR EMPHASIS ON THE ROLE OF MATH TEACHERS' ORGANIZATIONS IS ASKED TO SEND IT TO: Joan Routledge
Box 194
Aurora, Ontario L4G 3H3

A possible sourcebook for people with weak math backgrounds teaching math is being pursued.

Women In Math Committee

The Committee experienced success in encouraging women to seek office in OAME.

Nominations Committee

A slate of candidates was submitted. These will be sent to the membersip. Please remember to vote. Much time was spent in developing the vote by mail system now in place so that **every** member of OAME has the chance to cast a ballot for OAME's officers.

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
PRIMARY/JUNIOR COMMITTEE

Response to the Early Primary Education Project

OAME

- Supports in principle the philosophy as expressed in the Report of the Early Primary Project, May 1986.

and

- Recommends that the following three areas be recognized as being critical to the effective mathematical development of young children:

1. A planned mathematical curricular component must be a part of every early primary program.*

   This program must be planned, organized sequentially, and must involve sound evaluation techniques in order to maximize learning.

   Recognizing the fact that the understanding of measurement and geometry are critical to a child's understanding of numerical concepts, this planned program must reflect an appropriate balane of measurement, geometry and number activities.

2. In order for Boards of Education to implement the major recommendations of EPEP by establishing classrooms where play is recognized as the child's natural and legitimate way of learning, significant monetary support will be needed for:
   a) Teachers' inservice training in appropriate philosophy and methodology as well as subject - specific teaching skills and knowledge.
   b) Suitable manipulative materials made available to all JK to Grade 3 classrooms. As well as assisting Boards financially with these purchases, the Ministry should prepare a document similar to Circular 14, identifying the best concrete and print resources on the market each year.

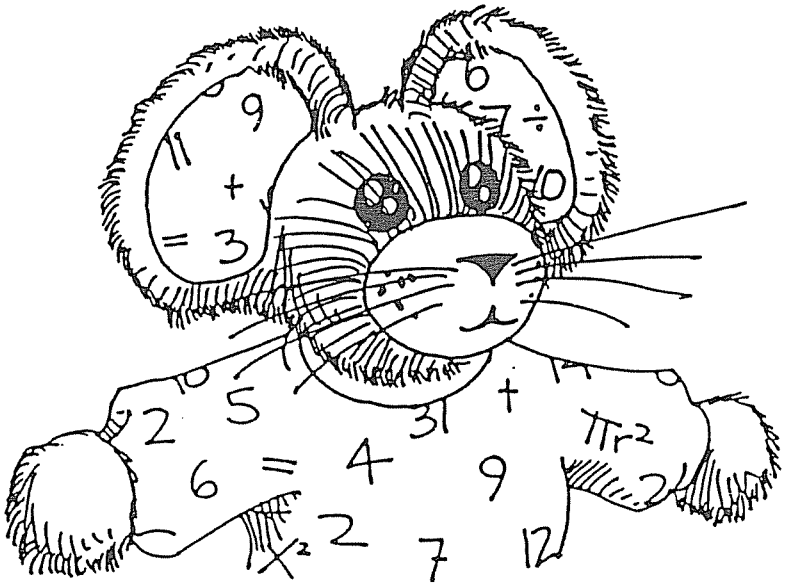
3. We support EPEP's recommendation that certified teachers employed in the Primary Division hold appropriate Primary Education qualifications. O.A.M.E., however, urges the Ministry to make available Additional Qualification courses leading to a Specialists Certificate in Primary Mathematics.

   It is also important that it be clearly stated how, when, and where upgrading can be obtained by teachers presently in the profession and that a reasonable timeline for this upgrading be provided.

* Mathematics in Early Primary Education, 1984-11-30; Report to EPEP from Ontario Mathematics Coordinators Association, p. 7.

Ontario Association for Mathematics Education

14th Annual Conference



Of Mice and Math...

May 7 - 9, 1987

Centennial College of Applied Arts and Technology
Progress Campus, Scarborough, Ontario

Hosted by the SAME *Scarborough Association for Mathematics in Education*

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Weiner, Jack, Dept. of Mathematics & Statistics, Univ. of Guelph

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GOLDEN, Haines, Edward

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NOMA, Kenny, Ann

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