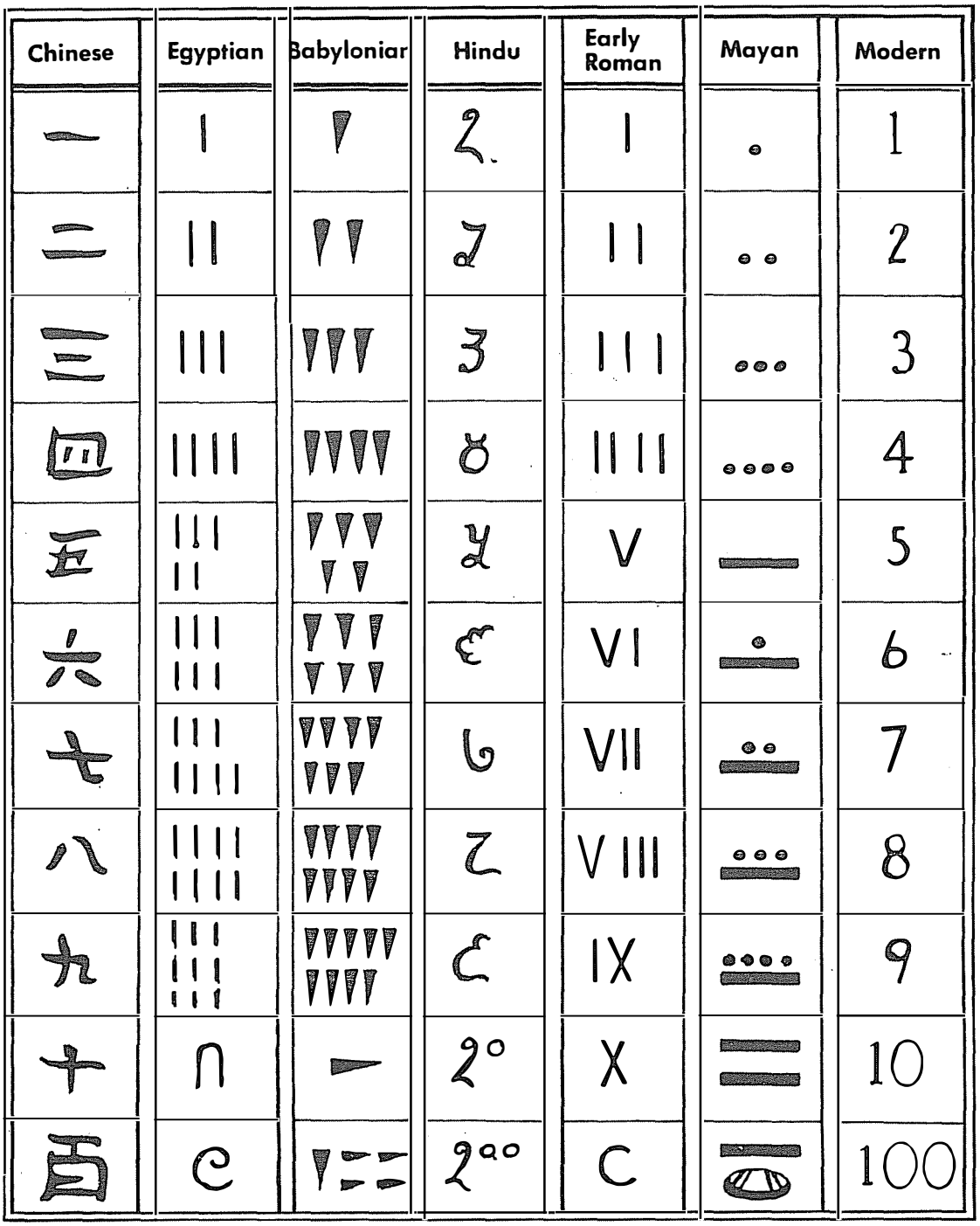
ontario mathematics gazette

VOLUME 26 NUMBER 2 DECEMBER 1987



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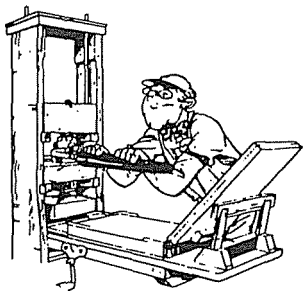
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PRESIDENT'S MESSAGE

by
Ralph Connelly

This month's message was one of the most difficult I've ever had to write.

Clifford Gravelle, our **GAZETTE** editor for the past two years, passed away as this issue was going to press. He had not been feeling well, but fought off his illness and put this issue together before going into a Toronto hospital for tests and treatment. He died a week later.

Cliff was one of the most organized people I've ever met, and one of the hardest working. His diligence and organization were immediately obvious in the **GAZETTE.** He was extremely successful in getting people to agree to submit articles, and following up on them so that the articles would come in. Thanks to his efforts, not only was this issue ready to go, but he had sufficient articles "in the pipeline" for the March issue as well. Since I was the previous editor I know how difficult a task this was, and could only marvel at Cliff's success. Cliff was a fairly new "presence" in OAME and didn't know a lot of people in the organization, making his accomplishments with the **GAZETTE** even more remarkable. Anyone who came into contact with this kind and gentle man could not help but respect and admire him.

OAME has lost one of its most productive members, and I've lost a trusted friend.

On behalf of OAME I would like to dedicate this issue of the **GAZETTE** to the memory of Clifford Gravelle. He will be sadly missed by all of us.



I have a lovely - sized smorgasbord of articles for your mid winter enjoyment - a bit of this and a bit of that from hither and yon and points in between.

Bob Robinson has contributed his informative council comments. John Savage takes a look at a new method to factor quadratics.

Jack O'Neill wants his students to carry out investigations in mathematics, while John C. Egsgard has an interesting introduction to sequences and series. Eric R. Muller would like his classes to take a hitch-hiker's guide through the binomial distribution.

Stuart Telfer is interested in the uses of rally mathematics in our cirriculum, while John Partridge reminds us to divide and multiply. In addition to these excellent articles, we have OAME '88, and Eric Wood, our new Book Review Editor.

On behalf of the Educational Board and the Publications Committee I extend to you our best wishes for the Holiday Season, and continued success in the New Year.

NOTICE!

With Clifford Gravelle's passing there are clearly a lot of communications to and from contributors that have been lost in the shuffle. As soon as possible we will try to notify contributors as to the status of their articles, etc., but we ask your indulgence as we endeavor to cope with the void left by Clifford's death.

A NEW METHOD TO FACTOR QUADRATICS

by
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When teaching the section on "radicals" to my grade 10 enriched class I threw in the concept of "i", just for fun. They, of course, were not too willing to accept $i^2 = -1$ but we had a lot of fun playing with it and seeing that expressions like $x^2 + 9$ could be actually be factored .... $(x + 3i)(x - 3i)$.

I asked them to try to factor $x^2 + x + 1$ and of course they were unable to think of two numbers whose sum is 1 and whose product is also 1.

Since the sum of the numbers is 1, the average of the numbers is $\frac{1}{2}$. The two numbers may then be represented by

$$\frac{1}{2}+a \text{ and } \frac{1}{2}-a$$

thus $$(\frac{1}{2}+a)(\frac{1}{2}-a) = 1$$

giving $$\frac{1}{4}-a^2 = 1$$

$$a^2 = -\frac{3}{4}$$

$$a = \pm\sqrt{\frac{3}{2}}\, i$$

therefore $$x^2+x+1 = (x+\frac{1}{2}+\sqrt{\frac{3}{2}}\, i)(x+\frac{1}{2}-\sqrt{\frac{3}{2}}\, i)$$

They weren't really sure this was correct until they multiplied the factors and **VOILA!**

I actually hit on this method by accident. I had planned just to give the class

$$\frac{1}{2}+\sqrt{\frac{3}{2}}\, i \text{ and } \frac{1}{2}-\sqrt{\frac{3}{2}}\, i$$

and ask them to add and multiply in order to verify that they were the numbers we were looking for.

This method that we used got me thinking that any quadratic could be factored this way and ultimately the quadratic formula derived.

Example: Factor $x^2 + 14x - 207$

For many students, finding two numbers with a sum of 14 and a product of -207 is a difficult task.

Let the numbers be $7 + a$ and $7 - a$ (ensuring a sum of 14)

Therefore $$(7 + a)(7 - a) = -207$$

$$49 - a^2 = -207$$

$a^2 = 256$ and $a = 16$ (use + ve root)

The required numbers then are $7 + 16$ and $7 - 16$

Therefore $$x^2 + 14x - 207 = (x + 23)(x - 9)$$

This method may be applied very nicely to quadratics which a student may use the "decomposition" method on...like $2x^2 - 23x - 39$

Here we would be looking for numbers whose sum is -23 and whose product is -78.

By using $\frac{-23}{2} + a$ and $\frac{-23}{2} - a$ the numbers are easily found.

Quadratic equations with irrational roots may easily be solved using the same factoring method.

Example: Solve $2x^2 - 3x - 8 = 0$

$$x^2 - \frac{3}{2}x - 4 = 0$$

To factor we use $\frac{-3}{4} + a$ and $\frac{-3}{4} - a$

Solving the "product equation", we find $a^2 = \frac{73}{16}$ and $a = \sqrt{\frac{73}{4}}$

Therefore $x - \frac{3}{4} + \sqrt{\frac{73}{4}} = 0$ or $x - \frac{3}{4} - \sqrt{\frac{73}{4}} = 0$

and $$x = \frac{3 \pm \sqrt{73}}{4}$$

The Quadratic Formula may be derived as follows:

$$ax^2 + bx + c = 0$$

$$x^2 + b/ax + c/a = 0$$

To factor the left side we want two numbers whose sum is b/a

Use $\frac{b}{2a} + k$ and $\frac{b}{2a} - k$

product $= \frac{b^2}{4a^2} - k^2 = \frac{c}{a}$

Therefore $k^2 = \frac{b^2 - 4ac}{4a^2}$ and $k = \frac{\sqrt{b^2 - 4ac}}{2a}$

Therefore $(x + \frac{b + \sqrt{b^2 - 4ac}}{2a})(x + \frac{b - \sqrt{b^2 - 4ac}}{2a}) = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This method of finding two numbers with a given sum and product could be taught at the beginning of the section on factoring quadratic trinomials. This would give the student an easy, surefire method of finding the numbers. If the factors are integral, the equation to be solved...$a^2 = \ldots$ will always involve a perfect square. A calculator with square root capability is handy here. The same method, as seen above, can be used throughout the factoring and quadratic equation sections....right up to the development of the quadratic formula. The method never changes and if students are reasonably adept at adding two fractions, the questions do not get much harder to deal with. As you have also seen, equations with non-real roots can also be handled before the student has a knowledge of the quadratic formula....introduce them to "i" and away they go!!!

FALSE COINS

by
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In a problem which apparently dates from the First World War, two weighings on a beam balance without weights are to determine whether eight coins are all good, or whether there is a unique bad coin among them, on the assumption that any bad coin would necessarily be deficient in weight.

In a problem which apparently dates from the Second World War, three weighings are allowed for twelve coins, when a bad coin may be either light or heavy. The legend has it that this problem has been planted during the war by enemy agents, since many man - hours have been spent on its solution.

Both problems belong to the category of Recreational Mathematics. For their solutions see [1] and [2]. We shall solve the problem of twelve coins using the Theory of Information, namely the *entropy.*

A basic assumption of information theory is that information can be measured. Information is conveyed by choosing one among a set of possible messages. The choices occur with probabilities, since some of the messages are more frequent. Intuitively, the information content of a message is greater the less the message is anticipated. A message which occurs often would be known to occur next and nothing new would be gained from it. The rarer the event the greater is its information content. Thus it would seem reasonable that the information of a message increases as its probability decreses. The logarithm function satisfies this property and consequently the amount of information we measure by the logarithm of the number of choices. The information, when there are only two choices, is proportional to the logarithm of 2 to the base 2. But this is unity, so that a two choice situation is characterized by information of unity. This unit of information is called a bit (binary digit). If one has available say 16 alternative messages among which one is equally free to choose, then since $16 = 2^4$ so that $\log_2 16 = 4$, one says that this situation is characterized by four bits of information. We define information as the logarithm of the number of choices.

If one relay can handle unit information, then three relays can handle three times as much. For three relays are capable of responding to $2^3 = 8$ choices, which symbolically might be written as 000, 001, 011, 010, 100, 110, 101, 111.

The expected value of information content of n independent events which occur with probabilities $p_1, p_2, \ldots, p_n$ is given by

$$-[p_1(\log p_1) + p_2(\log p_2) + \ldots + p_n(\log p_n)]$$

Often what we referred to above as information content is actually called, in the field of information theory, the entropy content and information content is defined as its negative. Thus the entropy of n mutually exclusive events is defined by

$$H(p_1, \ldots, p_n) = -[p_1(\log p_1) + p_2(\log p_2) + \ldots + p_n(\log p_n)],$$

$$p_1 + p_2 + \ldots + p_n = 1.$$

Suppose that we are choosing between two possible messages whose probabilities are p_1 for the first and $p_2 = 1 - p_1$ for the other. It turns out that H has its largest value, one (1), when the two messages are equally likely, that is, when one is completely free to choose between the two messages. When one message has high probability, the value of H is very small. When one probability is one (unity), and all the others zero, then H is zero, that is, there is no uncertainty at all, no freedom of choice, no information.

Consider an event B with 24 possible outcomes, since every one of 12 coins may be a counterfeit coin either heavier or lighter than a good coin. All these outcomes have equal probability. Therefore, the entropy H(B) of the event B is equal to log 24, which means that log 24 units of information should be obtained. Just as we can get from a complex event $Z_w = A_1A_2 \ldots A_w$ consisting of w weighings, an information no better than w log 3 = log 3^w, and $3^3 = 27$, so we can say from the first glance that it is probably possible to locate the counterfeit coin in three weighings and to determine whether it is heavier or lighter than a good coin.

Suppose we put m coins on each side of the balance for the first weighing [event $A_1^{(m)}$]. If they balance (outcome P of the event $A_1^{(m)}$), then the counterfeit coin must be one of 12 - 2m coins not being on the scale, which corresponds to 2(12 - 2m) equally possible outcomes out of the total number of 24 outcomes of the event B. If the right side of the balance goes down (outcome R), then the counterfeit coin is either heavier and one of m coins on the right side, or it is a lighter one and one of the m coins on the left side. These are 2m cases of the event B. If the left

side of the balance goes down (outcome L), then we have again 2m cases of the event B. The three outcomes of the event $A_1^{(m)}$ have the following probabilities:

$$2(12 - 2m)/24 = (6 - m)/6, 2m/24 = m/12, \text{ and } m/12.$$

It follows immediately that out of six events $A_1^{(1)}$, $A_1^{(2)}$, $A_1^{(3)}$, $A_1^{(4)}$, $A_1^{(5)}$, $A_1^{(6)}$, the event $A_1^{(4)}$ has the greatest entropy, its three outcomes being equally possible, with the probabilities of ⅓. Therefore, we receive the best information from the event $A_1^{(4)}$, that is, we must divide the group of twelve coins into three groups of four. Let us consider separately two cases.

FIRST CASE. Suppose there is an equilibrium after the first weighing. In this case the counterfeit coin is one in the group of four which has not been put on the balance. In the remaining two weighings we must determine the coin, and whether it is heavier or lighter than the others. Since there are (2)(4) = 8 possible outcomes left of the event B, and 2(log 3) = log 9   log 8, it is to be expected that this is indeed possible. Now, if we put on each side of the balance one of our remaining four coins [event $A_2^{(1)}$], and produce equilibrium then we need two more weighings to solve the problem. If we put two coins on each side [event $A_2^{(2)}$], then one side will be heavier, but we need again two more weighings to solve the problem. It seems to be impossible to solve the problem in three steps. Nevertheless we should not jump to conclusions. We still have 4 + 4 = 8 coins which can take part in the second weighing. Actually, we have two better variations of the event A_2 to consider. Let $A_2^{(m,n)}$ represent the event of putting m coins from our set of four coins on the right side of the balance, and n   m coins from the same set on the left side of the balance. Let p(P), p(R), p(L) represent the probabilities that in the event $A_2^{(m,n)}$ the sides will balance, the right side will go down, and the left side will go down respectively. It is easy to calculate these prob-abilities. They are the ratios of the outcomes B, where $A_2^{(m,n)}$ has an outcome P, R, or L, to the total number of outcomes B (which is 8). Since m + n   4, it is easy to consider all possibilities. The probabilities p(P), p(R), and p(L) are given in the following table, as well as the entropy $H[A_2^{(m,n)}]$ of the event $A_2^{(m,n)}$, equal to
$H = - p(P) \log p(P) - p(R) - p(L) \log p(L)$:

m	n	p(P)	p(R)	p(L)	$H[A_2^{(m,n)}]$
1	1	1/2	1/4	1/4	0.452
1	0	3/4	1/8	1/8	0.320
2	2	0	1/2	1/2	0.301
2	1	1/4	3/8	3/8	0.470
2	0	1/2	1/4	1/4	0.452
3	1	0	1/2	1/2	0.301
3	0	1/4	3/8	3/8	0.470
4	0	0	1/2	1/2	0.301

From this table of entropies we can see that the events $A_2^{(2,1)}$ and $A_2^{(3,0)}$ have the greatest entrophy. Therefore, to obtain the best information in the second weighing we must put on one side of the balance two coins from the group of 4 suspicious coins, and on the other side of the balance one suspicious coin and one which is certainly a genuine coin, or, on one side of the balance three suspicious coins and on the other side three genuine coins. It is easy to see that in both cases it is possible with the third weighing to determine the outcome B. If the event $A_2^{(2,1)}$ or the event $A_2^{(3,0)}$ has the outcome P, then the counterfeit coin is the single suspicious coin not taking part in the second weighing. Weighing this coin against any one of 11 genuine coins as the third weighing will determine whether the counterfeit coin is heavier or lighter than a good coin. If the event $A_2^{(2,1)}$ has the outcome R, then either the counterfeit coin is one of the two suspicious coins and certainly heavier than the others, or, the counterfeit coin is the one suspicious coin on the other side of the balance and certainly lighter than the others. Weighing two suspicious coins against each other (third weighing) we find the outcome B. If they balance, then the counterfeit is the single suspicious coin on the other side of the balance. Otherwise it is the heavier coin of the third weighing. If the event $A_2^{(3,0)}$has outcome R, then the counterfeit coin is one of the three suspicious coins and heavier than genuine coins. Weighing two of these three coins against each other (third weighing) we find out the outcome B. If they balance, then the counterfeit coin is the third coin from the group of suspicious coins. Otherwise it is the heavier of the two coins. In a similar manner we can argue the cases when the events $A_2^{(2,1)}$ and $A_2^{(3,0)}$ have the outcome L.

SECOND CASE. Suppose that the first weighing shows that one side (say the right side) of the balance is heavier than the other. In this case either one coin from the group of four coins on the right side of the balance is the counterfeit coin and heavier than the others, or, one coin from the group of four coins on the left side of the [illegible]nce is the counterfeit coin and lighter than the others. In the second weighing we can put on the right side of the balance m_1 'right' coins and m_2 'left' coins, and on the left side of the balance we can put n_1 'right' coins, n_2 'left' coins, and $(m_1 + m_2) - (n_1 + n_2)$ definitely genuine coins from the group of coins which did not take part in

the first weighing. Note that $m_1 + m_2 \quad n_1 + n_2$. We call this event $A_2^{(m_1,m_2;\, n_1,n_2)}$. We could try again to make the table of entropies, but the number of variations is simply too great.

Notice that the information which we can obtain about the outcome B in the case of the third weighing (event A_3) does not exceed log 3 (since $H(A_3) \quad \log 3$) and after two weighings we do not have more than three possible outcomes of the event B. Otherwise the event A_3 does not give the possibility to determine uniquely the outcome B. It follows that the number of suspicious coins which do not take part in the second weighing must not exceed B. We have

$$8 - (m_1 + m_2 + n_1 + n_2) \leq 3,$$

$$m_1 + m_2 + n_1 + n_2 \geq 5,$$

and since $m_1 + m_2 \geq n_1 + n_2,$

we have $m_1 + m_2 \geq 3,$

and $n_1 + n_2 \geq 5 - (m_1 + m_2).$

Therefore, if the outcome of the event $A_2^{(m_1,m_2;\, n_2,n_2)}$ is R, then either one of m_1 'right' coins from the right side of the balance is the counterfeit coin and heavier than the others, or, one of the n_2 'left' coins from the left side of the balance is the counterfeit coin and lighter than the others. Using the same argument it is possible to see that in the case of outcome L the counterfeit coin is one of m_2 'left' coins on the right side of the balance or one of the n_1 'right' coins on the left side of the balance. From this we obtain two more inequalities:

$$m_1 + n_2 \leq 3,$$

and $m_2 + n_1 \leq 3.$

It is obvious that the following inequalities are also true:

$$m_1 + n_1 \leq 4,\ m_2 + n_2 \leq 4,\ (m_1 + m_2) - (n_1 + n_2) \leq 4.$$

If we count all the cases which satisfy our requirements we can put them in the following table:

m_1	m_2	n_1	n_2	p(P)	p(R)	p(L)	$H(A_2)^{(m_1,m_2;\, n_1,n_2)}$
2	1	2	1	1/4	3/8	3/8	0.470
2	1	2	0	3/8	1/4	3/8	0.470
2	1	1	1	3/8	3/8	1/4	0.470
1	2	1	2	1/4	3/8	3/8	0.470
1	2	0	2	3/8	3/8	1/4	0.470
1	2	1	1	3/8	1/4	3/8	0.470
3	1	1	0	3/8	3/8	1/4	0.470
1	3	0	1	3/8	1/4	3/8	0.470
2	2	1	1	1/4	3/8	3/8	0.470
2	2	1	0	3/8	1/4	3/8	0.470
2	2	0	1	3/8	3/8	1/4	0.470
3	2	1	0	1/4	3/8	3/8	0.470
2	3	0	1	1/4	3/8	3/8	0.470

In this manner we see that not only two variations of the event A_2 contain the best information as was the case in the previous table of entropies, but all 13 variations of the event A_2 in this case give the best information, the entropy being 0.470 in

being 0.470 in the relation to the event B. It is obvious that in this case the information $I(A_2, B)$ is equal to the entropy $H(A_2)$. Any one of the events A_2 proves to give sufficient information to determine the event B using just one more (the third) weighing. For example, in the case of the event P of the event $A_2^{(2,1;\, 2,1)}$, the counterfeit coin is one of two coins which did not take part in the second weighing of 'left' coins. Since we know that this counterfeit coin is lighter than a good coin, to isolate it, it is enough to compare these two coins by comparing one of them with the genunie coin. In the case of the event R, either the counterfeit coin is the heavier one of the two 'right' coins on the right side of the balance, or, the counterfeit coin is the single 'left' coin on the left side of the balance and being lighter than a good coin. Therefore, to identify the counterfeit coin it is sufficient to compare the two suspicious 'right' coins. The same argument is valid if the event $A_2^{(2,1:\, 2,1)}$ has the outcome L. This concludes the analysis of the strategy for twelve coins.

Let us now consider the general case of c coins and w weighings. First of all, the entropy of the event B under consideration (the outcomes of which we consider, as always, are equally possible) is equal to log 2c and the en-

tropy of the event $Z_w = A_1A_2 \ldots A_w$, which consists of w weighings, is not greater than $w \log 3 = \log 3^w$. We must have $2c \leqq 3^w$, that is, $c \leqq 3^w/2$. We also have $c \leqq (3^w - 1)/2$ and $w \geqq \log_3(2c + 1) = \log(2c + 1)/\log 3$.

For example, if $c > (3^3 - 1)/2 = 13$, it is impossible to solve the problem in three weighings. It is not difficult to see that in the case of $c = (3^w - 1)/2$, w weighings are not always enough to solve the problem. In general, if $c \geqq (3^w - 1)/2$, the w weighings may not be enough. If $c < (3^w - 1)/2$, that is, if $c \leqq (3^w - 3)/2$, that is , if $w \geqq \log_3(2c + 3) = \log(2c + 3)/\log 3$, then w weighings suffice.

Let us use the mathematical induction to prove this. The statement is obviously true for $w = 1$. Let us assume that it is true for some w and show that in the case when

$$(3^w - 1)/2 < c \leqq (3^{w+1} - 1)/2,$$

then $w + 1$ weighings suffice.

For the first weighing put x of c original coins on one side of the balance and put $x - 1$ of c original coins plus one coin which is for sure a genuine one on the other side of the balance. This leaves $c_1 = c - (2x - 1)$ unused coins. We choose number x so that $2x - 1 \leqq 3^w$, or $c - (2x - 1) \leqq (3^w - 1)/2$, that is, $3^w \geqq 2x - 1 \geqq c - (3^w - 1/2$. Clearly, this is possible when $c \leqq (3^{w+1} - 1)/2$, since $c - (3^w - 1)/2 \leqq (3^{w+1} - 1)/2 - (3^w - 1)/2 = 3^w$. If the balance remains in the state of the equilibrium, then we look for the counterfeit coin in the group of $c_1 \leqq (3^w - 1)/2$ coins and this is possible in w weighings by the force of the induction hypothesis. If there is no equilibrium, then we have $2x - 1 \leqq 3^w$ suspicious coins. If the counterfeit coin is one from the group of s coins, then it is lighter than the others, and if it is from the group of t coins ($s + t \leqq 3^w$), then it is heavier than the others. (If the first pan is heavier, then $s = x - 1$, $t = x$; if the second pan is heavier, then $s = x$, $t = x - 1$.) Therefore, in this case it is also possible to isolate the counterfeit coin.

Let us now go back to our original number $c \leqq (3^w - 3)/2$ of coins. One of them is different. Put on each pan $(3^{w-1} - 1)/2$ coins. The number of unused coins is

$$c_1 = c - 2(3^{w-1} - 1)/2 \leqq (3^w - 3)/2 - (3^{w-1} - 1) = (3^{w-1} - 1)/2.$$

In the case of the equilibrium the number of suspicious coins is $c_1 \leqq (3^{w-1} - 1)/2$. We have $3^{w-1} - 1$ definitely genuine coins and in accordance with what has been proved above the next $w - 1$ weighings will isolate the counterfeit coin and determine whether it is heavier or lighter than a good coin. If there is no equilibrium then we have $3^{w-1} - 1 < 3^{w-1}$ counterfeit coins and we know that if the counterfeit coin is one from $s = (3^{w-1} - 1)/2$ coins then it is lighter than the others, and if it is from $t = (3^{w-1} - 1)/2 (= s)$ of the others then it is heavier than a good coin. In this case we can also solve the problem by using the next $w - 1$ weighings.

References

[1] *O'Beirne, T. H.*, ***Puzzles and Paradoxes***, *Oxford University Press, London, 1965.*

[2] *Pedoe, D.*, ***The Gentle Art of Mathematics***, *The MacMillan Company, New York, 1959.*

INVESTIGATIONS IN MATHEMATICS

by
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Here are two opportunities for Intermediate students to carry out investigations in mathematics. The students can be encouraged to work on these problems over a period of days. The computer can be used in both investigations to extend them. The computer can "crunch" numbers that the students could not hope to deal with using pencil and paper.

Both investigations require the students to carry out algorithmic calculations, to form hypotheses and test these hypotheses.

HAILSTONE NUMBERS

Investigate the following:

1. Start with any number.
2. If the number is odd, then triple it and add one.
3. If the number is even, then divide it by two.
4. The result each time is a new number to which you apply either step 2 or step 3.
5. Keep on doing this for a large number of steps.
6. Do the numbers tend to grow larger or smaller? Do they converge to some particular value or do they diverge towards infinity? How many steps are required?
7. A large number of initial values need to be investigated before any tentative conclusions can be drawn - perhaps all initial values from 1 to 27 could be done before trying to draw conclusions.
8. Those who are learning computer programming can write a computer program to aid in the investigation.
9. The computer program included here can be used to extend the investigation.

ANSWER SHEET FOR HAILSTONE NUMBERS

STARTING NUMBER	NUMBER OF STEPS	MAXIMUM VALUE REACHED
1	3	4
2	1	2
3	7	16
4	2	4
5	5	16
6	8	16
7	16	52
8	3	8
9	19	52
10	6	16
11	14	52
12	9	16
13	9	49
14	17	52
15	17	160
16	4	16
17	12	52
18	20	52
19	20	88
20	7	10
21	7	64

22	15	52
23	15	160
24	10	24
25	23	88
26	10	40
27	111	9232

OTHER ANSWERS

STARTING NUMBER	NUMBER OF STEPS	MAXIMUM VALUE REQUIRED
54	112	9232
97	118	9232
649	144	9232
703	170	250 504
1819	161	1 276 936
20 895	255	50 143 264
77 671	231	1 570 824 736

NOTES FOR TEACHERS:

1. No proof has been found, but in all cases ever tested, the series of numbers eventually ends up at 1.

2. Path lengths in the 350 range have been found.

3. Maximum values can reach into the billions.

4. The term "hailstone numbers" is appropriate because, in one series, consecutive values rise and fall repeatedly before descending to the value 1 (Real hailstones rise and fall repeatedly in the atmosphere before reaching the ground). Students can graph a given series.

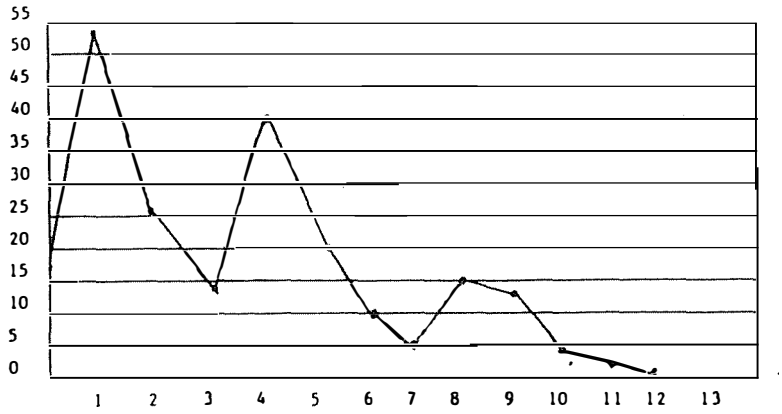
5. The computer program included will quickly list all numbers in a series, even very large values.

6. Students who make inferences before they try 27 as a starting value may begin to wonder if 27 will follow the pattern of the previous numbers.

GRAPHING

Students can get a better appreciation of the "hailstone-like" behaviour of these numbers by graphing one or more series.

GRAPH FOR HAILSTONE SERIES BEGINNING AT 17



NUMBER OF STEPS

COMPUTER PROGRAM

```
 2 PRINT "HAILSTONE NUMBERS": PRINT
 5 C = 0
10 INPUT X
20 y = X/2
30 Z = 3 * X + 1
40 IF Y      INT (Y) THEN GOSUB 1000
50 IF Y = INT(Y) THEN GOSUB 2000
55 IF X = 1 THEN GO TO 70
60 GO TO 20
70 PRINT "ENDS"
75 PRINT "SERIES LENGTH = "; C
80 GO TO 5
1000 PRINT " "; Z;
1010 LET X = Z
1020 C = C + 1 : RETURN
2000 PRINT " "; Y;
2010 LET X = Y
2020 C = C + 1
2030 RETURN
```

Reference: Scientific American, Jan. '84, Computer Recreations

AN AREA AND VOLUME INVESTIGATION

Investigate the volumes of various open-topped boxes that can be made by cutting a square piece from each corner of a piece of paper and then folding the paper to make a box.

As you proceed, prepare a chart that shows the volume of each box that you have made (in cm^3) and the area of the paper used to make the box (in cm^2).

Try to discover which of the boxes has the largest volume. You may be mistaken if you guess that the box with the largest volume is the one that uses the most paper (i.e. the paper with the largest area).

Helpful Hints:

You can find the area of the paper used by using centimetre grid paper and counting the number of squares. One method for finding volume is to fill the box with centicubes (these are 1 centimetre cubes) and to count them. The first box that you make should be the one that results from cutting out a 1 centimetre square from each corner of the paper. Start with a 15 cm $\times$ 15 cm square paper.

Area of Paper Used (cm^2)	Volume of Box Made (cm^3)
221 cm^2	169 cm^3

Next the "box" is changed by cutting out a square of size 4 cm^2 from each corner, and a new box is made and the new volume is determined. The area of paper used for the new box and its volume are added to the chart.

Area of Paper Used (cm^2)	Volume of Box Made (cm^3)
221 cm^2	169 cm^3
213 cm^2	242 cm^3

All other possible boxes that can be made from a 15 cm by 15 cm square of grid paper are made.

If you have access to a computer, you can type in the following simple program and when you run the program the display will show the results for all possible boxes.

```
100 PRINT "AREA", "VOLUME"
110 FOR H = 1 TO 7
120 LET L = 15 - 2 * H
130 LET W = 15 - 2 * H
140 LET V = L * W * H
145 LET A = 225 - 4 * H * H
150 PRINT A, V
160 NEXT H
170 END
```

Here are the results for the investigation for a 15 cm × 15 cm sheet of paper:

VOLUME AND AREA INVESTIGATION
Type in the length of the paper: 15
Type in the width of the paper: 15

HEIGHT	AREA	VOLUME
1	221	169
2	209	242
3	189	243
4	161	196
5	125	125
6	81	54
7	29	7

The activity can be extended by looking at different beginning sizes of grid paper e.g. a 12 cm or a 20 cm × 20 cm piece of grid paper. Also, initial grid sheets that are rectangles can be investigated e.g. 20 cm by 16 cm.

Below is a more flexible computer progarm that can be used to find the areas and volume for any beginning size sheet of paper.

```
 10 PRINT "VOLUME AND AREA INVESTIGATION"
 11 PRINT "**************************"
 12 PRINT
 50 INPUT "TYPE IN THE LENGTH OF THE PAPER:"; M
 55 PRINT
 60 INPUT "TYPE IN THE WIDTH OF THE PAPER:"; N
 65 PRINT
 70 IF N   M THEN ? "NO!  NO!  WIDTH MUST BE LESS THAN LENGTH": GO TO 60
 99 PRINT
100 PRINT "HEIGHT", "AREA", "VOLUME"
105 PRINT
110 FOR H = 1 TO INT (N/2 - .1)
120 LET L = M - 2 * H
130 LET W = N - 2 * H
140 LET V = L * W * H
145 LET A = M * N - 4 * H * H
150 PRINT H, A, V
160 NEXT H
170 END
```

Students with some programming experience can be encouraged to modify the program so that boxes whose sides are not whole numbers are included in the display of results.

You may wish students to complete their investigation by reporting on their results by making a visual display that includes all the various boxes made, and their numerical results.

This could be an excellent display entry for a Math/Science Fair.

Following are other typical results for sheets of paper with dimensions:

12 cm × 12 cm
20 cm × 20 cm
25 cm × 30 cm
64 cm × 57 cm

Computer Result: Don't try to complete 64 cm × 57 cm with paper and scissors.

VOLUME AND AREA INVESTIGATION

Type in the length of the paper: 12
Type in the width of the paper: 12

HEIGHT	AREA	VOLUME
1	140	100
2	128	128
3	108	108
4	80	64
5	44	20

VOLUME AND AREA INVESTIGATION

Type in the length of the paper: 20
Type in the width of the paper: 20

HEIGHT	AREA	VOLUME
1	396	324
2	384	512
3	364	588
4	336	576
5	300	500
6	256	384
7	204	252
8	144	128
9	76	35

VOLUME AND AREA INVESTIGATION

Type in the length of the paper: 25
Type in the width of the paper: 30

HEIGHT	AREA	VOLUME
1	621	529
2	609	882
3	589	1083
4	561	1156
5	525	1125
6	481	1014
7	429	847
8	369	648
9	301	441
10	225	250
11	141	99
12	49	12

VOLUME AND AREA INVESTIGATION

Type in the length of the paper: 64
Type in the width of the paper: 57

HEIGHT	AREA	VOLUME
1	3644	3410
2	3632	6360
3	3612	8874
4	3584	10976
5	3548	12690
6	3504	14040
7	3452	15050
8	3392	15744
9	3324	16146
10	3248	16280
11	3164	16170
12	3072	15840
13	2972	15314
14	2864	14616
15	2748	13770
16	2624	12800
17	2492	11730
18	2352	10584
19	2204	9386
20	2043	8160
21	1884	6930
22	1712	5720
23	1532	4554
24	1344	3456
25	1148	2450
26	944	1560
27	732	810
28	512	224

AN INTERESTING INTRODUCTION TO SEQUENCES AND SERIES

by
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Recently I read an article in the March, 1984 issue of **The College Mathematics Journal.** The article contained a description of the snowflake curve together with some of the sequences associated with the curve. I decided that I would use the curve as an introduction to sequences and series the next time I taught that topic. The students found the snowflake curve so fascinating that they appeared more eager than usual to learn more about sequences and series. I hope that the following description of the curve will tempt you to use it too. In class I had each student roughly construct the first three or four stages of the curve. As they did this we filled in the summary chart on the blackboard.

The snowflake curve was first produced by the Swedish mathematician Helge Von Koch in 1904. The snowflake begins with an equilateral triangle of side 1 unit and, hence, perimeter 3 units.

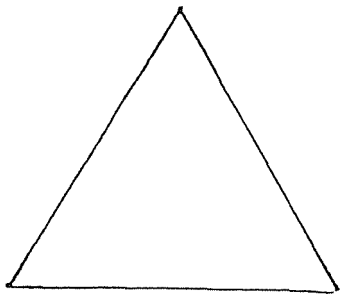


FIGURE 1a

The first stage in the construction divides each side into 3 equal parts, constructs an equilateral triangle in the middle part and deletes that middle part. Figure 1b, shows this construction. Notice that each side of length 1 produces 4 new sides each of length 1/3. Hence the number of sides increases from 3 to $3 \times 4 = 12$. Since each of the 12 sides has length 1/3 the new perimeter is $12 \times 1/3 = 4$

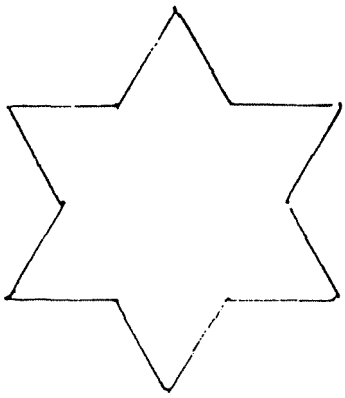


FIGURE 1b

Stage 2 in the construction repeats stage 1 for each new side, that is, each side is divided into 3 parts, the middle part is omitted and 2 sides of an equilateral triangle are drawn in place of the middle part. The construction for stage 2 on one of the original sides is shown in Figure 2a Stage 2. Again each side produces 4 new sides with each new side having length $1/3 \times 1/3 = 1/9$.

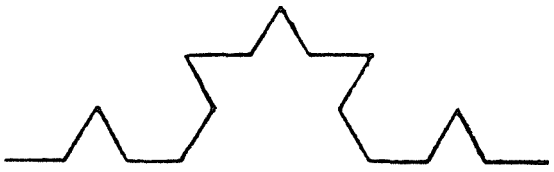
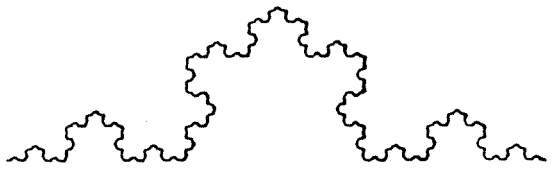


FIGURE 2a Stage 2

Since the number of new sides is now $4 \times 12 = 48$ the new perimeter is $48 \times 1/9 = 16/3$.

The construction continues in the same manner indefinitely. In each stage of the construction every side is replaced by 4 new sides with length 1/3 of the length of the side that was replaced. The construction on one of the original sides for Stage 5 is shown in Figure 2b

FIGURE 2b



The following chart summarizes some of the information about the snowflake.

	number of sides	length of each side	perimeter of snowflake
start	3	1	3
after stage 1	12	1/3	4
after stage 2	48	1/9	16/3
after stage 3	192	1/27	64/9
after stage 4	768	1/81	256/27

The snowflake construction produces 3 sequences. The first 5 terms of each sequence are as follows.

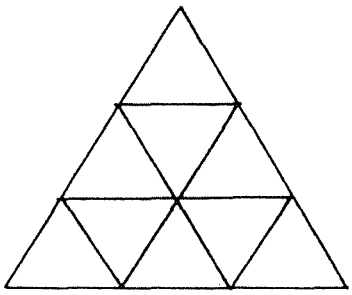
sequence 1	3, 12, 48, 192, 768
sequence 2	1, 1/3, 1/9, 1/27, 1/81
sequence 3	3,4, 16/3, 64/9, 256/27

The students will observe that each term in sequence 1 to 4 times the preceding term. This should lead them to conclude that if the construction is repeated through many, many more stages the numbers in sequence 1 will grow larger. They should also see that each term of sequence 2 is obtained by multiplying the preceding term by 1/3 and that the numbers become smaller and get closer and closer to being equal to 0.

They should also note that the numbers in sequence 3that represent the perimeter of the snowflake get larger and larger because each number in the sequence is 4/3 times the preceding number. That is, each number in the sequence is larger than the one that precedes it. This shows the students an astonishing thing about the snowflake curve - the perimeter is infinitely large!

A fourth sequence associated with the area of the curve makes the latter fact even more exciting for the student. This sequence leads to the conclusion that the area of the curve is finite.

FIGURE 3a



3b

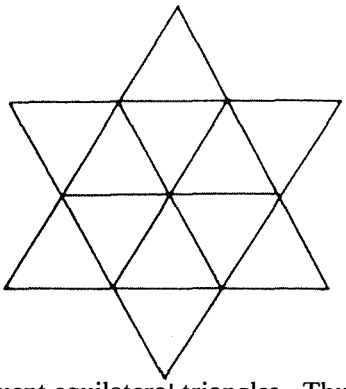


Figure 3a shows the original equilateral triangle divided into 9 congruent equilateral triangles. Thus, if the area of the original triangle is 1 unit^2, then the area of each of the nine triangle is 1/9 unit^2. Figure 3b shows that the area of the snowflake after stage 1 is

$1 + 1/9 + 1/9 + 1/9 = 1 + 3(1/9) = 1 + 1/3 = 4/3$ units^2.

It should be clear that after stage 2, the area of the snowflake will be

$1 + 3(1/9) + (12)(1/81) = 1 + 1/3 + 4/27 = 40/27$ units^2.

Proceding in this manner the students will learn that the area of the snowflake after stage 5 is

$1 + 1/3 + 4/27 + 16/243 + 64/2187 + 256/19863 = 1 + 1/3 + 4/3^3 + 4^2/3^5 + 4^3/3^7 + 4^4/3^9$.

The terms of the sequence of numbers added to give this area are

$$1, 1/3, 4/3^3, 4^2/3^5, 4^3/3^7, 4^4/3^9.$$

These numbers are getting smaller and smaller! Indeed the values of these terms are approaching the value of 0. But the area at any stage is calculated by adding the corresponding term of this sequence to the area of the previous stage. Thus, at the 21st stage, the term added is $4^{20}/3^{41}$ 0.000 000 030 141, which hardly increases the area at all! Using this fact, and taking the area of the original triangle to be 1 unit^2, Koch was able to prove that the area of the snowflake gets closer and closer to, but never exceeds, 8/5 units^2.

Figure 4 shows the snowflake curve growing through several stages.

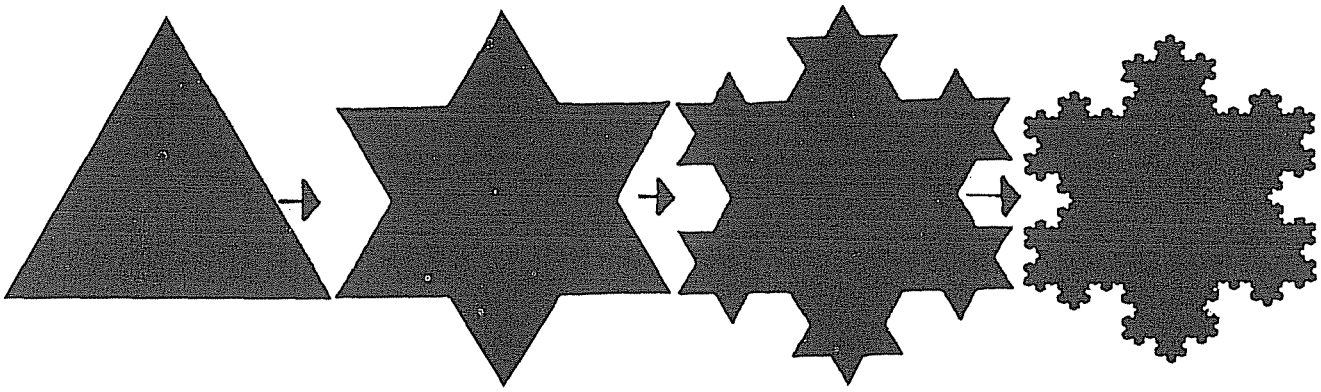


FIGURE 4

One of the fascinating observations that Koch loved to make about the snowflake is that although the perimeter of the snowflake is infinite, the area of the snowflake never grows beyond 8/5 ths of the area of the original triangle. In other words, if you had a grass lawn in the shape of a snowflake you could never get enough fence to enclose it. Indeed, if you could find some way to walk along the outside edge of the lawn it would take you forever to go all the way around. Nevertheless, you could cut the grass on the lawn in a short time!

The following program in **TERRAPIN LOGO** will show the growing. It is interesting to see the apparent speed of the pen decrease as the number of stages increase due to the fact the distance travelled in one loop increases.

```
TO SNOWFLAKE
  SNOWFLAKE1
  SNOWFLAKE2
  SNOWFLAKE3
  SNOWFLAKE4
  SNOWFLAKE5
  HT
END

TO SNOWFLAKE1
  PU SETXY ( - 110 ) ( - 30 ) PD
  RT 30
  REPEAT 3 [FD 150 RT 120]
END

TO SNOWFLAKE2
  PC 2
  MAKE "S 150
  REPEAT 3 [DIV.SIDE1 :S RT 120]
END

TO DIV.SIDE1 : S
  FD :S / 3 LT 60 FD :S / 3 RT 120 FD :S / 3 LT 60 FD :S / 3
END

TO SNOWFLAKE3
  PC 3
  MAKE "S :S / 3
  REPEAT 3 [DIV.SIDE2 :S RT 120]
END

TO DIV.SIDE2 :S
  DIV.SIDE1 :S LT 60 DIV.SIDE1 :S RT 120 DIV.SIDE1 :S LT 60 DIV.SIDE1 :S
END

TO SNOWFLAKE4
  PC 4
  MAKE "S :S / 3
  REPEAT 3 [DIV.SIDE3 :S RT 120]
END

TO DIV.SIDE3 :S
  DIV.SIDE2 :S LT 60 DIV.SIDE2 :S RT 120 DIV.SIDE2 :S LT 60 DIV.SIDE2 :S
END

TO SNOWFLAKE5
  PC 5
  MAKE "S :S / 3
  REPEAT 3 [DIV.SIDE4 :S RT 120]
END

TO DIV.SIDE4 :S
  DIV.SIDE3 :S LT 60 DIV.SIDE3 :S RT 120 DIV.SIDE3 :S LT 60 DIV.SIDE3 :S
END
```

DEVELOPING STATISTICAL THINKING: AN ACTIVITY

by
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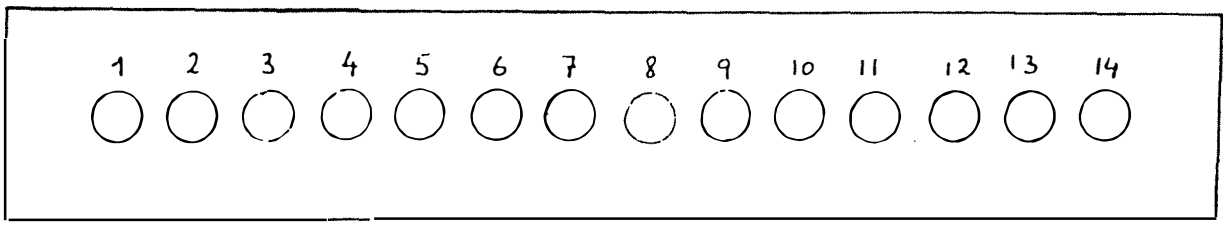
1. Introduction

The **1985 Intermediate and Senior Division Mathematics Guidelines** contain sections on statistics and probability at most of the grade levels. There is no doubt that the development of statistical thinking is an important educational goal. Many political arguments, advertising decisions, positions in wage negotiations, determination of levels of alcohol in the blood for police tests, etc. etc., are supported by statistical arguments. H.G. Wells in 1946 wrote "Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write."

For many, statistical thinking appears very different from mathematical thinking. The statistical activities seem far removed from the algebraic, geometric and other traditional mathematical activities that we teach. Nevertheless statistics is placed in the mathematics curriculum because of its dependence on mathematical models. These models tend to be quite sophisticated and we are left with a rather unsatisfactory situation where few inferences or decisions can be tested. In other words the vital link between probability (models) and statistics is never made. What we need to develop are some stimulating activities where uncertainty is present, but quantifiable, and where the optimal decisions can be demonstrated to be superior. I have used the following activity successfully for grades 6 through 13 as enrichment. I hope you will find it useful in your own situation. The activity is divided into three levels. Level 1 is generally accessible for grades 6 to 8, level 2 for grades 9 to 12 and level 3 for grade 13.

2. Materials

You will need a board marked with circles or shallow holes to place chips. I have a large number of boards with different number of circles. The board illustrated below has 14 positions numbered 1 to 14.



You will need 2 regular six-sided dice — it is also useful to have available dice constructed from the other four regular polyhedra (4, 8, 12 and 20-sided). These can be found in many commercial games.

Finally, you will also need a sufficient number of chips of two different colours, for example red and blue.

3. Level 1 - General description of play

Two teams to a board — I've found that teams of two students work well as each student has a partner to discuss strategies. We generally play with the rule that teams do not discuss strategies with other teams. Explain to the teams that a pair of six-sided dice will be rolled and the sum on the faces will be computed — for example if a 5 and 3 are rolled the circle on the board marked with an eight will be a winner. Before any die is rolled teams take turns in selecting what they believe are the most favourable positions. To do this one of the teams is selected to place a chip (red) in one of the remaining thirteen positions. This procedure alternates between the two teams until either (a) all positions on the board are filled or (b) one team no longer wishes to place any of its chips, then the other team may occupy all vacant positions. The board is now set and remains unchanged for a game consisting of a large number of rolls of the pair of dice — choosing 25 rolls as a start works well. Each time the dice are rolled the team which has a chip on the position corresponding to the sum on the two dice records a point. The team with the most points at the end of 25 rolls wins that game. All chips are now taken off the board and a new game may be started.

The objective for each team is to find a winning strategy, that is, a strategy for selecting the positions of their chips which will provide the best chance for winning. One **must** resist the temptation to provide solutions and I respond as follows:

When a team approaches me with a strategy which it believes is the best I will change the team's opponents or materials to either

(a) expose the possibility of a better strategy, or
(b) reinforce the team's strategy.

My aim in this first level is to get the students to estimate probabilities summarized by the following objectives:

(1) students are to observe which outcomes (sum on the two dice) are possible and conclude that these outcomes are not all equally likely;

(2) students are to quantify the uncertainty, that is, estimate the probabilities of each outcome. For a pair of fair dice I reluctantly give you the answer — please don't pass it on to the students!

$$P(1) = P(13) = P(14) = 0,\ P(2) = P(12) = \frac{1}{36},$$

$$P(3) = P(11) = \frac{2}{36},\ P(4) = P(10) = \frac{3}{36},\ P(5) = P(9) = \frac{4}{36},$$

$$P(6) = P(8) = \frac{5}{36},\ P(7) = \frac{6}{36}.$$

(3) students are to develop the strategy which selects those positions whose sum of the corresponding probabilities is maximum.

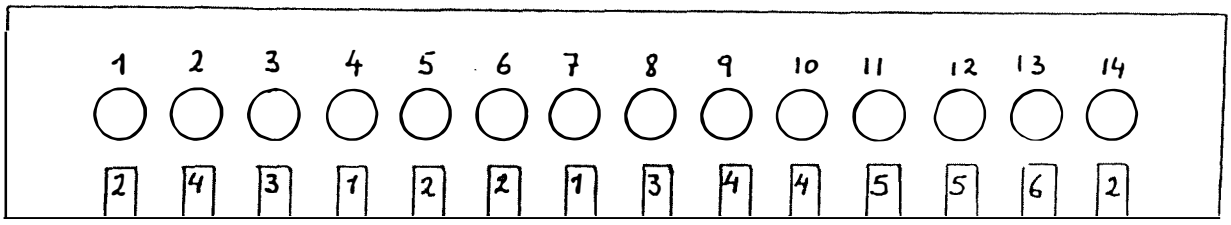
To reinforce winning strategies I supply the team with one six-sided and one eight-sided die, or, a 12 and a 20-sided die and a different board. To expose a non-optimal strategy I select an opposing team which has found the optimal strategy.

4. **Level 2 - More specialized play**
In this activity we want the students to explore the concepts of

(i) random variables, that is, we introduce the possibility that different positions on the board are worth different number of points, and

(ii) expectation value.

To do this I add the value of the position on the board below in the square. So my boards look like this



Where in this particular board position 10 is worth 4, and position 5 is worth 2, etc. Thus if we roll a six and a four and our chip is on position 10 we would get 4 points. The game proceeds exactly as before but now the strategy of selecting the positions for the chips is different. The team with the most points after 25 rolls of the dice wins the game. (The winning strategy is obtained by maximizing the expectation value, that is, a set of positions such that the sum of products — probability times values of position — is greater than that for the opposing team). The best position on the above board is the 9 with expectation value 4*4/36, the next best is the 8 with expectation value 3*5/36. I have a number of boards, each with a different sequence of points. By switching boards you can either reinforce an optimal strategy or expose one which is not optimal.

5. **Level 3 - A Hitch-hiker's quide through the Binomial distribution!**
This level is more of an activity than a game. Our aim is to guide the students through a number of experiments to show that as the number of trials is increased the probability of winning the game with an optimal strategy is increased. In levels 1 and 2 we rolled the dice 25 times — what happens if we rolled the pair only 3 times or 11 times, would our conclusions be as decisive?

We experiment as follows:

Each team now works individually and is given a board. Chips are placed for an optimal level 1 strategy, for example positions 7, 6, 5, 4, 3, 2, and these are retained for all experiments. An experiment consists of rolling the dice n times, and the experiment is repeated 20 times.

Case 1, roll dice once (n = 1), record a point and an automatic win only if the total corresponds to a position occupied by a chip, repeat experiment 20 times. Count number of wins. Calculate the relative frequency of wins $= \frac{\text{Number of wins}}{20}$.

Case 2, roll dice three times (n = 3), if number of points is greater or equal to 2 record a win, repeat experiment 20 times. Count number of wins.
Calculate relative frequency of wins.

Case 3, roll dice five times (n = 5), if number of points is greater or equal to 3 record a win, repeat experiment 20 times. Count number of wins.
Calculate relative frequency of wins.

etc. sufficient cases to reinforce the concepts and yet not too many to make it boring. The mathematical model behind these experiments is the Binomial distribution.

The Binomial Distribution has the following properties:

1) n identical trials, that is, n rolls of the pair of dice.

2) each trial results in one of two outcomes, a point if the total on the dice is equal to the position of a chip and no point otherwise.

3) the probability of getting a point, p, in a single trial remains the same from trial to trial, that is, the chips are not reset between rolls, probability of not getting a point q = 1 - p.

4) trials are independent, that is, the result of one roll does not depend on the result of previous rolls.

Then the probability of exactly x points is given by

$$C(n, x)\, p^x\, q^{n-x}$$

To win we must get more than half the points, this probabilty is given by

$$\sum_{x=\left\lceil \frac{n}{2} \right\rceil}^{n} C(n, x) p^x\, q^{n-x}$$

where $\left\lceil \frac{n}{2} \right\rceil$ is the smallest integer greater than n/2.

Students are asked to verify that the experimental relative frequencies are close to the theoretical values obtained from the Binomial probability distribution, namely

$$n = 1 \quad P = \sum_{x=1}^{1} C(1, x)\, p^x\, q^{1-x} = p$$

$$n = 3 \quad P = \sum_{x=2}^{3} C(3, x)\, p^x\, q^{3-x} = 3p^2q + p^3$$

$$n = 5 \quad P = \sum_{x=3}^{5} C(5, x)\, p^x\, q^{5-x} = 10p^3q^2 + 5p^4q + p^5$$

Positioning the chips for an optimal strategy, the starting team will have

$$p = \frac{21}{36}$$

giving for n = 1 - P = 0.5833

giving for n = 3 - P = 0.6238

giving for n = 5 - P = 0.6534

I normally ask students to compare the graphs of the model against the graph of the corresponding values obtained

in their experiment. Each group then has to (i) propose another experiment where the binomial conditions are satisfied (ii) carry out the experiment (iii) compare the results theory vs experiment. If many groups are involved a discussion ensues as to which group has the best fit of experimental results to the model — can we develop an indicator which would determine which is the best fit? — I ask the groups to develop some. An informal discussion of the Chi-square and Kolnogorov-Smirnov test can be presented. (A description of these tests can be found in any introductory Probability and Statistics text.)

Conclusion

At the 1983 meeting of the **Canadian Mathematics Education Study Group** held in Vancouver a working group recommended the following goals for the development of statistical thinking for all:

1. To develop critical attitudes towards conclusions based on commonly used statistical arguments.
2. To develop those skills of data exploration necessary for achieving first goal.
3. To develop an awareness of the statistical arguments.

I have found that the activities proposed here can be used successfully to achieve these goals with enrichment classes. Hopefully you will find something you can use in your classes when teaching probability and statistics.

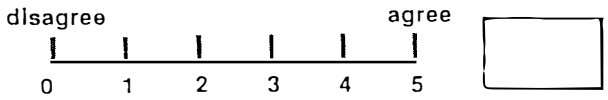
PROBLEM SOLVING: TEACHER PROFILE

by
Gary Flewelling, Mathematics Consultant
The Wellington County Board of Education
Guelph, Ontario N1E 6K2

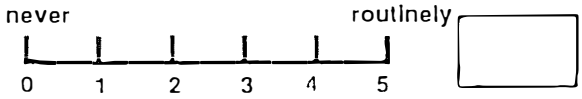
Your response to the 20 statements below will give you an indication of the degree to which you, the teacher, bring problem solving into your classroom.

(Running) TOTAL

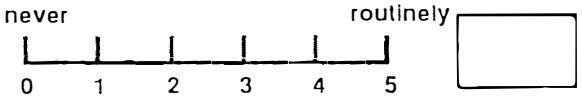
- "Learning to solve problems is the principal reason for studying mathematics."



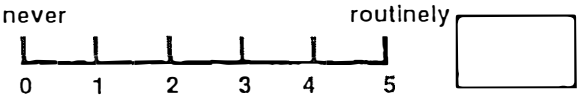
- "I teach my students general P.S. strategies."



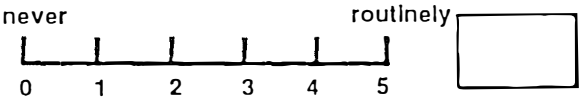
- "I allow my students to work on problems in small groups."



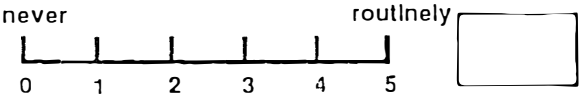
- "I introduce math topics through problems."



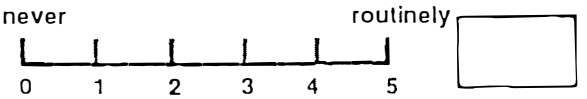
- "I encourage students to write out their solution 'plan'."



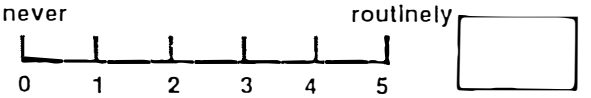
- "I permit the use of calculators in P.S. activities."



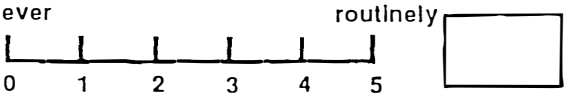
- "I stress the process of solving a problem over getting the answer to a problem."



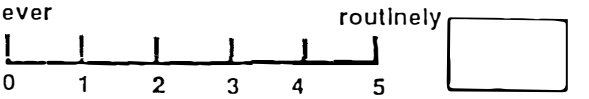
- "I assign problems that have extraneous or missing information."



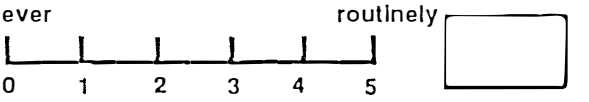
- "I assign ambiguous problems or problems that require an assumption to be made before a solution can be found."



- "I assign problems that permit more than one method of solution."



- "I assign problems that have more than one (or no) solutions."



- "The solving of a problem usually leads to my students or myself asking, 'What if ...'"

never routinely
0 1 2 3 4 5

- "I have my students practice 'rule selection'."

never routinely
0 1 2 3 4 5

- "I don't have enough time to teach the course and teach P.S. too."

agree disagree
0 1 2 3 4 5

- "My students solve problems of their own creation."

never routinely
0 1 2 3 4 5

- "P.S. represents a significant part of my students' marks."

disagree agree
0 1 2 3 4 5

- "I integrate P.S. throughout my courses."

never routinely
0 1 2 3 4 5

- "Problem solving is for every one."

disagree agree
0 1 2 3 4 5

- "I enjoy solving problems with my students."

disagree agree
0 1 2 3 4 5

- "My students enjoy problem solving."

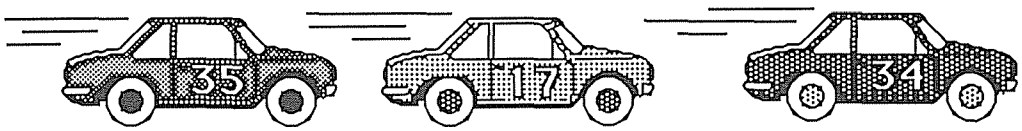
disagree agree
0 1 2 3 4 5

PROBLEM SOLVING SCALE

INEFFECTIVE				HEALTHY					EXEMPLARY
0 10	20	30	40	50	60	70	80	90	100

Rally Math - a painless drill activity

by
Stu Telfer, Mathematics Consultant
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Kitchener, Ontario N2G 3X5



Rally Math is a popular drill activity that I have used many times. Students, usually working in pairs, are given a road map of Ontario and try to be the first team to reach the final (undisclosed) destination. To find this destination, students must correctly answer mathematical questions so that they do not stray off the intended route.

The students have really enjoyed this activity but the large maps that were used did not have a long life expectancy. Also, too many basic level and general level students found the road maps too difficult to read and *very* difficult to fold-up again at the end of the class! One solution has been to create my own map of a fictitious town. One such city map is included for the town of **Mathton**. Students still "rally", but now the map is simpler to read and is also disposable so that they can display their route directly on the map.

That following activity sheets are included with this article:

- 2 sample Rally Exercises
- 1 map of Mathton
- 1 blank map (so that you can create your own rally!)

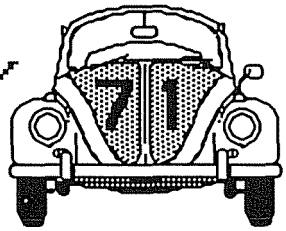
The two sample exercises included both follow the same route. However, one is a numerical exercise (suitable for intermediate basic level or general level classes) and the other is an algebraic exercise (suitable for grade 11 or 12 advanced). The exercises are easily changed to match the abilities of your students.

Here are few suggestions when creating your own activity:

- try the sample rallys given to get a sense of the questions that can be asked
- the last question in the rally should be a challenging problem solving activity so that good students do not complete the rally too soon and discourage others from completing
- have rewards for the 1st, 2nd and 3rd place finishers
- stress that winners must correctly answer ALL questions along the route (they can't just jump to the end!)
- monitor weaker teams and tell them when they go astray
- don't be hesitant to put challenging mathematical questions in the exercise. Students who normally avoid difficult questions, will struggle endlessly as they get 'caught-up' in the excitement of finding the destination.
- this is an excellent activity for "Friday last period" and for "the day before Christmas Holidays". The students are engaged in a productive, self-motivating review activity.

I enjoy creating and monitoring a rally. After a rally period, students have commented "I didn't think Math was supposed to be fun!". Other maps and rallys have been created and can be made available for future publications.

Rally Math



Note: • a "T" intersection is one where the road you are on ends and you must turn either left or right
• "turn right at 3rd opportunity" means make the third possible right turn

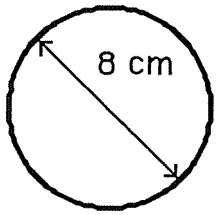
When the timer says go, follow the instructions in numerical sequence and fill in all required information. Find your final destination and solve the final problem.

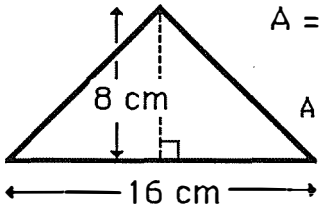
1. You are presently in your car in front of the PHIL INN heading east on Power Drive.

2. To begin the rally, solve : $c = 18 - 4x2 + 3^2$, c = _____
   If c = 20, turn left at the first opportunity
   If c < 20, turn left at the second opportunity
   Otherwise, turn left at the third opportunity

3. Write the name of the street you are on in capital letters _____

4. Find value of **n**: $\mathbf{n} = \sqrt{5^3 - 11^2}$ .Turn right at the **n**th opportunity (answer to **n** above). If you were to stop here for lunch, what kind of food would you buy? _____
   Drive along this street and stop at the first intersection. What did you pass on your right hand side? _____

5. 

$A = \pi r^2$
A = ____


$A = \frac{1}{2}bh$
A = ____
   If the area of the circle is greater than the area of the triangle, then turn right at the interesection. Otherwise, turn left.

6. Continue to drive along this road until you are immediately next to an area where you hear "love 15". What is the third letter in the name of the street that you just crossed over? _____
   Continue to drive and turn left at your first opportunity and stop at the T intersection. What building do you see in your rear-view mirror?
   _____

Ask for Page 2 ! !

7. If $(-8 - 4)(10 + 12 \div (-2)) = -8[-36 \div (-6)]$, turn right, otherwise turn left.

8. Continue to drive until you reach a T intersection. There was a building you passed that could be called Tut's Hut. Name that building.

_____

9. Turn left and drive along this road. On your left is a store. Write down 5 different **4 letter words** by using only the letters found in the store owner's name. _____ _____ _____ _____ _____

10. Turn left at the 'fork' in the road.
If **1.5% of $240** is less than **120% of $3.00**, turn left at the second opportunity. Otherwise, turn left at the first opportunity. Record the name of the river you crossed by writing it **upside-down** in the square.

11. Continue to drive until you reach a T intersection. What building is directly in front of you? _____

12. Solve:

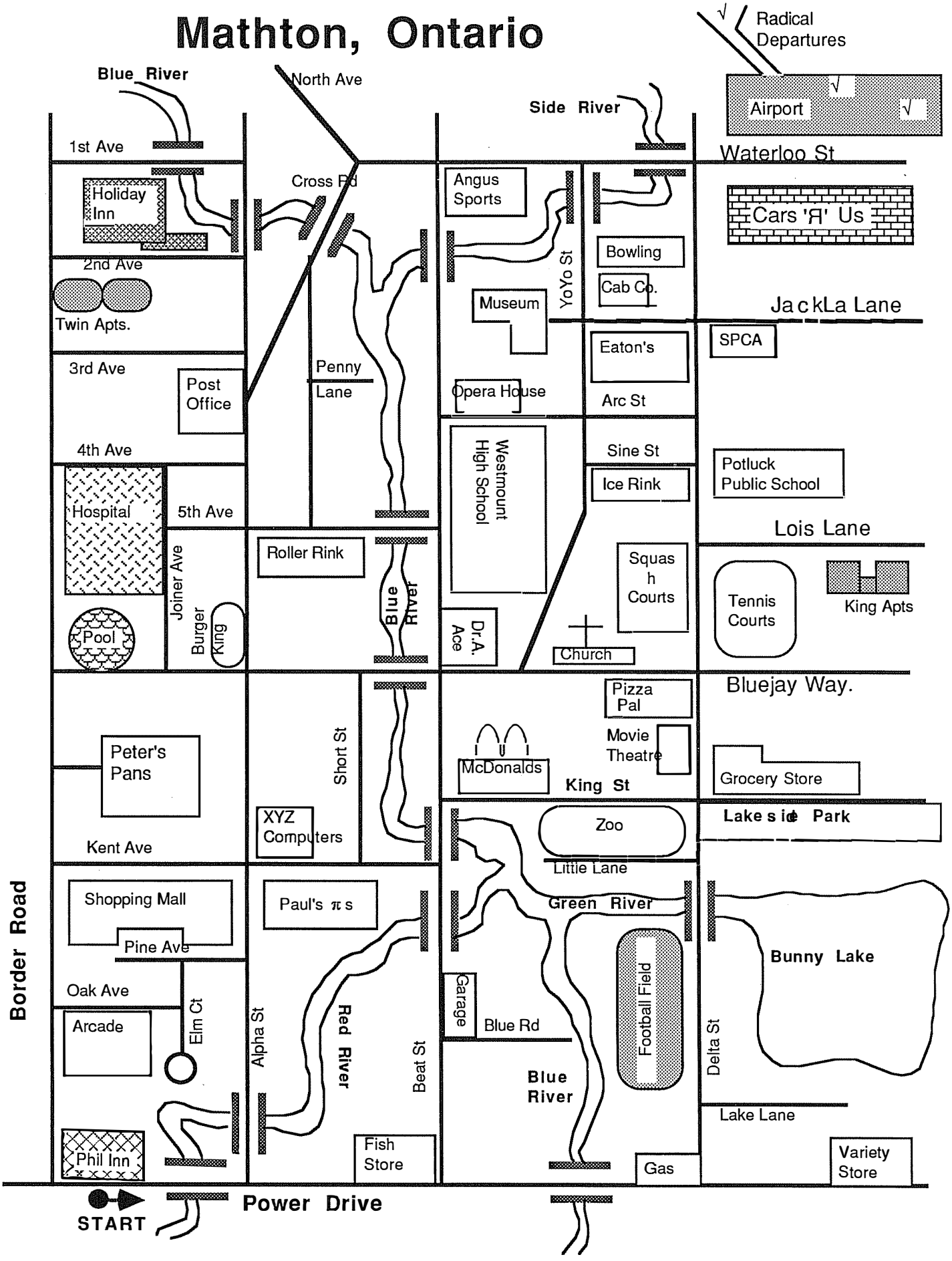
$$x = \left(\frac{5}{9} + \frac{3}{4} - \frac{5}{6}\right) \div \frac{4}{9} \text{ , } x = \text{ _____}$$

If **X** is larger than 1, turn left. Otherwise turn right.
Stop at the first intersection. Name the building directly in front of you.

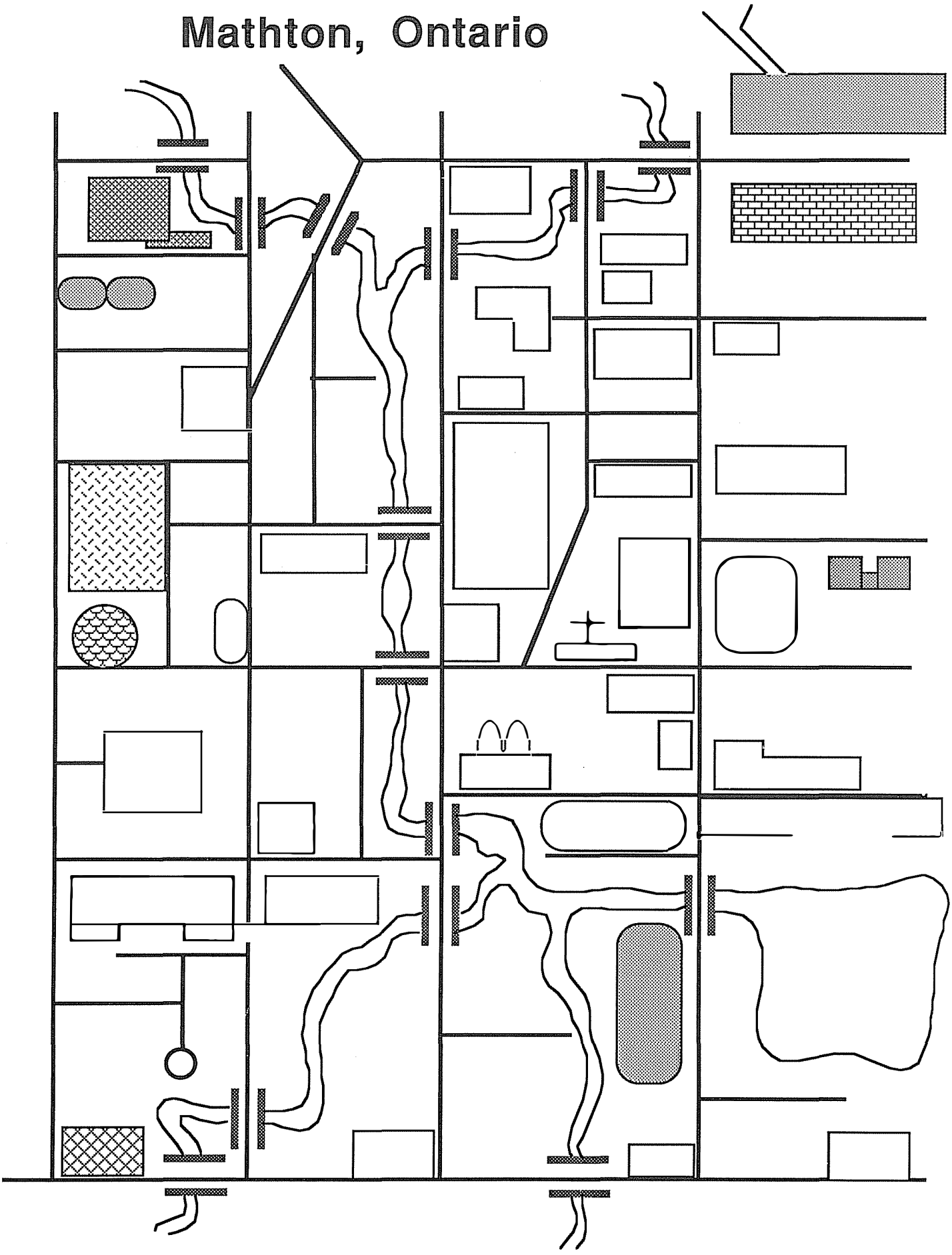
_____

13. Adjacent to the building named above is another buiding. Scramble **all** the letters in the name of this building to form one word. This word can be found on your map and this is your final destination. From your present position, mark on your map a route to your final destination , BUT you must cross **EXACTLY 4 bridges** and it must be the **shortest** possible route.

14. Hand in your map and rally sheets with your team names written on the back.


Mathton, Ontario
√ Radical Departures
Airport
√
√
Blue River
North Ave
Side River
Waterloo St
1st Ave
Holiday Inn
Cross Rd
Angus Sports
Cars 'Я' Us
Bowling
Cab Co.
YoYo St
2nd Ave
Twin Apts.
Museum
Jac kLa Lane
SPCA
Eaton's
3rd Ave
Post Office
Penny Lane
Opera House
Arc St
4th Ave
Sine St
Potluck Public School
Hospital
5th Ave
Westmount High School
Ice Rink
Lois Lane
Joiner Ave
Roller Rink
Blue River
Squas h Courts
Tennis Courts
King Apts
Burger King
Pool
Dr.A. Ace
Church
Bluejay Way.
Pizza Pal
Movie Theatre
Peter's Pans
Short St
McDonalds
King St
Grocery Store
Kent Ave
XYZ Computers
Zoo
Lake s id e Park
Little Lane
Shopping Mall
Paul's π s
Green River
Pine Ave
Bunny Lake
Oak Ave
Elm Ct
Garage
Football Field
Arcade
Alpha St
Red River
Blue Rd
Delta St
Beat St
Blue River
Lake Lane
Phil Inn
Fish Store
Gas
Variety Store
Border Road
START
Power Drive

Mathton, Ontario



LUCKY WINNERS ONLY LUCKY AFTER THEY DIVIDE AND MULTIPLY

by
John Partridge

Editor's Note: This article is reprinted with the author's permission, ***The Globe and Mail,*** *May 15, 1987.*

The next time a voice at the other end of your telephone says, "All you need to do to win our super-bonanza grand prize is answer a skill-testing question, and is now a convenient time for me to ask it?" — or words to that effect — you might want to offer a little prayer of thanks to the frayed nerves and fractured eyeglasses of Ernest Ranger.

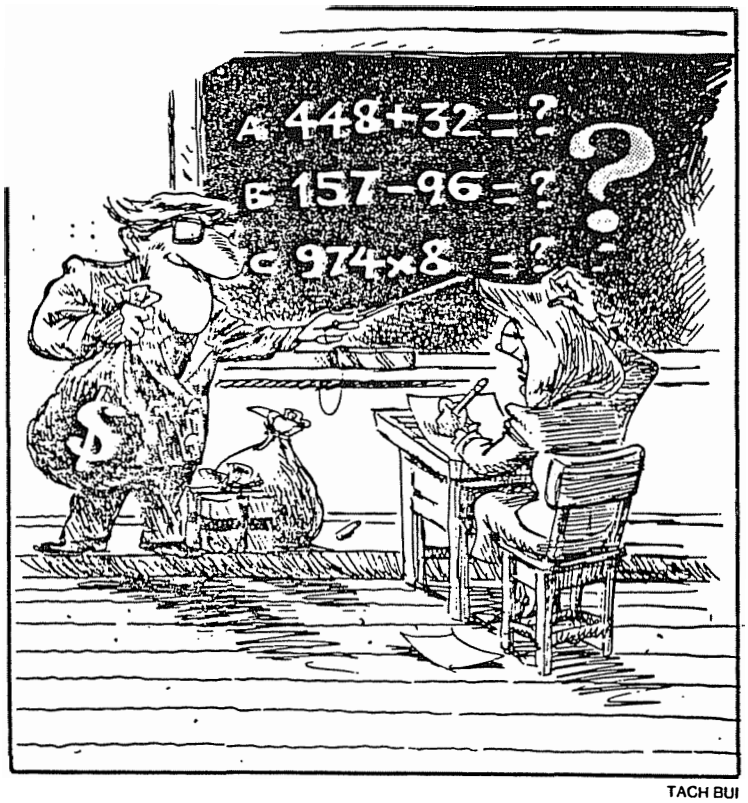
Mr. Ranger was a trucker from North Bay, Ont., who in 1970 forever altered the arcane rules governing the skill-testing questions that determine the winners of hundreds of promotional contest Canadians enter each year. He did so by successfully suing Peter Jackson Tobacco Ltd. and Herbert A. Watts (Que.) Ltd. for $10,000.

The reason for Mr. Ranger's suit? The cigaret company and its contest manager — Watts — had denied him a $10,000 prize in a promotion they were running when he failed to provide the correct answer to the following skill-tesing exercise: multiply 24 by 6; add 388; divide by 7; subtract 38.

The correct answer was — and, arithmetic being one of life's few relative immutables, still is — 38. But when Mr. Ranger hastily did the sum, aided by his wife and 12 year-old son, he came up with four incorrect responses, the last being 114.

There were two reasons for the trucker's gaffe, his impassioned lawyer told Ontario Supreme Court Justice Edson Haines. The first was that his glasses were broken. The second was that, scenting victory but aware he'd be required to answer a question of any sort, he was in an emotional state.

Or, as Judge Haines admonished the defendants: "Mr. Ranger was in a complete tizzy... He saw $10,000 slipping out of his fingers (and) his last answer shows he was going round in circles. He was fumbling completely. You had him on the ropes."



TACH BUI

To keep themselves off the legal ropes, contest management companies such as Watts and A.C. Nielsen Co. of Canada Ltd., both based in Toronto, have since tried not only to make it perfectly clear to contestants that they will be required to answer a skill-testing question, but also to give them a reasonable choice as to when they will answer it.

"That's so the individual has the opportunity to get calm and collected," says A.C. Nielsen's lawyer Jack Whiteside, adding with a chuckle, "and have his spectacles ready."

That still, of course, leaves the question of why tens of hundreds of Canadians who enter contests every year — including those vying for the chance to run the Olympic torch a kilometre apiece across Canada to Calgary next winter — have to answer a skill-testing question in the first place.

For that, we have to thank at least indirectly the puritanical legislators who, back in 1892 drafted what is now Section 189 of the Criminal Code. Taking their cue from the British Gaming Act of 1802, they decided that lotteries, games or contests whose outcomes depended solely on chance would henceforth be illegal.

The fact that a later generation of legislators has empowered governments to conduct multi-million-dollar cash lotteries without fear or favor or anything more than mere chance as the determing factor is just another of the wonderful paradoxes that make the world go around.

However, some bright spark figured out somewhere along the line — the exact date is not known — that adding an allegedly skill-testing question as the final step in choosing a winner would allow contest sponsors to skate around Section 189. Thus could they save themselves from the embarrassment

of committing an indictable offence and risking the two years' imprisionment for which the section calls.

But what, precisely, constitutes skill when it comes to skill-testing questions?

Perhaps the pithiest summation of that — and, indeed, the first judgment confirming that mathematical questions can be used to satisfy the skill requirement — came in a decision rendered in an Alberta coutroom in 1984. Ruling against the Crown in its appeal of a verdict that had exonerated Canada Trust Co. on a charge of running a lottery in the form of a mortgage-burning promotion, Mr. Justice Kenneth Plomp cited a number of precedents as to what did and did not add up to a test of skill.

"A reading of the cases," he said, "shows that skill is required to predict temperatures and attendances at fairs or future dates... but shooting turkey at 50 yards, peeling potatoes, answering questions from (advertising) flyers, memorizing answers given on the back of receipts...do not contemplate the degree of skill which would prevent the scheme from being a lottery."

Serious turkey-shooters and those who pride themselves on being potato-peeling wizards might well protest this derogation of their talents. Nevertheless, the judge went on to conclude that the problem Canada Trust had set — multiply 228 by 21; add 10,824; divide by 12; subtract 1,121 — was acceptable.

"It may or may not be true that the majority of residents of Alberta could answer the mathematical question," he said, "but it would appear from the Oxford (dictionary) definition of skill that in order to correctly answer the question, it would require precision and certainty, practical knowledge in combination with ability."

Contest organizers generally hire psychologists to draw up the questions, partly because they can gear them to the skill level of the audience at which the contest is aimed. And to cover their corporate backsides even more securely, they often require entrants to sign a declaration stating they have answered the question without the aid of mechanical devices such as calculators, or of other people.

All this would seem to put the organizers of the contest for Olympic torch bearers, sponsored by Petro-Canada, out on a bit of a limb — despite the fact that the only reason those selected will have to answer a skill-testing question is that the oil company plans to award each a track suit to wear while running their kilometre and then keep as a memento.

Petrocan spokesman Sandy Hunter says the company was to begin mailing the skill-testing question to potential winners today, and he says the contest does indeed meet all the rules and regulations.

But, at the same time, he's made it clear that the company doesn't want anyone to miss out on his or her chance to carry the torch just because he or she can't answer the question. As a result, as far as Petrocan is concerned, runners can get help from family, friends or even tax accountants to do their sums correctly.

It's unlikely anyone would be mean-spirited enough to kick up a stink in an attempt to deny any of citizens who've sent in a staggering 6.7 million entries to the torch relay their shot at shin splints, blisters and a small place in Olympic history. But, it's easy to see why people such as Mr. Ranger get upset when the stakes are much, much higher.

But event the experts are hard pressed to estimate how much prize money rests on giving correct answers to skill-testing questions in Canada in a given year.

Some, such as Wayne Mouland, vice-president of A.C. Nielsen's promotion services division, won't even hazard a guess. "As a representative of Nielsen," he says with a chuckle, "I can't afford to guess."

INTERVIEW WITH BARRY ONSLOW

by
Robert S. Smith
Faculty of Education
The University of Western Ontario
London, Ontario N6G 1G7

RS: It's Saturday, March 7th. It is an unusually warm day and Dr. Barry Onslow and I are sitting in the sun at my kitchen table. I first met Barry in Nottingham in the spring of 1984. He was a doctoral student working with Dr. Alan Bell of the Shell Centre for Mathematics Education. What is the title of your thesis Barry?

BO: The title is "Overcoming Conceptual Obstacles Concerning Rates: Design and Implementation of a Diagnostic Teaching Unit". I had been concerned about the difficulties children experience when examining rate problems. I wanted to devise a diagnostic teaching module which would improve their level of understanding.

RS: You completed your Ph.D., last spring. What are you doing now?

BO: I am vice-principal of an Elementary and Junior High School in Ralston, Albeta. Ralston is a small part of a National Defence Base, near Medicine Hat.

RS: What do you actually do at Ralston School?

BO: I spend about 60% of my time teaching mathematics to students in grade 6 through 9, and the rest of my time is spent on administration.

RS: So you are putting into practice that which you wrote in your doctoral thesis. Let us start with your opinion about pupil errors. For example 1/4 + 2/3 = 3/7.

BO: I think there are certain deeply embedded misconceptions Bob, which are neither random nor careless. Often teachers assume that children are answering carelessly, but by asking well designed questions, it becomes obvious that this is not the case. Very often there are certain conceptual obstacles which children find very hard to overcome and normal teaching strategies do not seem to overcome these obstacles.

RS: If that's so, perhaps you could give us some examples. If an answer of 8 is given for the question 5 + 4 it may well be careless or it might not be. It seems to me this type of error is a skill type error.

BO: Yes. These skill type errors can very often be atributed to careless mistakes or random errors. However, conceptual errors are different. Many of these have been identified by Kathy Hart of the Chelsea Group in London, England and by Dr. Alan Bell and Molcolm Swan at the Shell Centre, University of Nottingham, England. Typical examples, might be 5 ÷ 10 or seen as $10\overline{)5}$. Very often children will give the answer 2. They might well be able to compute a long division question but when faced with the above example, answers are frequently incorrect. During interviews, many students declare, "You always divide the big number by the small number". Other children think that the two division symbols are synonymous. If a question reads 5 ÷ 10; it can be read as 5 goes into 10 $5\overline{)10}$, or $10\overline{)5}$ can be read as 10 divided by 5 (10 ÷ 5). Another example is 10 divided by 1/2 (10 ÷ 1/2). Children confidently say, "It must be 5 because 1/2 of 10 is 5". However, if asked. "How many halves in 10?" even young children will say quickly that the answer is 20.

RS: Let us look at a different example. Which is largest 0.6, 0.75 or 0.429?

BO: This was an experiment that was done by Malcolm Swan and also by Kathy Hart. In the Hart study many children put 0.429. Most teachers assumed that children were forgetting the decimal point. 429 is certainly bigger than 75 or 6 so they chose that answer. However, Malcolm Swan discovered that 0.6 was an equally frequent answer, but was more difficult to explain. During interviews a consistent explanation from the children choosing 0.6 (which, by the way, were very often children with a good mathemtical background) was that tenths were bigger than hundredths or thousandths, therefore 0.6 had to be larger. Six **tenths** when compared with 75 **hundredths** or 429 **thousandths,** must be the larger decimal, since everyone knows that tenths are bigger than hundredths or thousandths!

RS: Surely, children grow out of the childlike concepts as they progress through secondary schools.

BO: Not so! In my study of rates across the different age groups from about 11 to 15, there was very little

difference in many of these deeply embedded misconceptions. In fact, by graphing these results, we found almost the same percentage of students getting questions correct at age 11 as was getting them correct at 15. This is one of the most disturbing facts arising from the research. There were far more variations within one class than across the grades.

RS: Do adults do any better?

BO: In the U.K., Ruth Rees and George Barr did some interesting research with adults, and their findings showed that the average adult has the same misconceptions as the average 13 year old.

RS: That's frightening!

BO: It certainly is! If the present educational methods do not produce very much change from 11 to 15 and there is very little change among adults, then these deeply embedded misconceptions could be permanent for many of our students.

RS: So you decided to do something about it. Your diagnostic teaching unit sets out three stages.

Opening Activities

Conflict Discussion

Consolidation Study

Let us look at each of these in turn.

BO: For the **opening activity,** let us use the examples that we talked about earlier, e.g. 10 divided by 1/2 and 2/3 + 1/4. It does not have to be anything very complicated as long as it focuses upon the misconception. The whole purpose of the **opening activity** is to try to arouse conflict within the child. The child has to think, "I have a problem and I don't know the answer. I don't know how to solve it, but I want to know what is wrong and why." The teacher's role is to develop an active learning environment. Explanation, demonstration and practice have proved to be insufficient stimuli for overcoming strongly held beliefs.

RS: The essence here is not that students had to be working with materials but that they had to be involved mentally to a point where they were prepared to stand and argue that their answers 5 or 3/7 were correct. Standing up for their answers was, in fact, the necessary involvement.

BO: Yes, Bob, that is correct. In this regard the correct level of difficulty was crucial. When we aimed a lesson too far below the children's abilities, then they answered quickly and easily. There was no challenge. If questions were too far above, then there was silence and confusion and the children didn't understand what we were talking about. Correct diagnosis leading to relevant examples was most important.

RS: In the design, your control group was definitely taught by good teachers and classes were of average ability. The teachers followed normal teaching practice of good clear exposition followed by examples.

BO: That's right. We used good teachers in average classrooms for all of the groups, both control and experimental.

RS: In your experimental classes, the teachers used an element that you call "conflict discussion".

BO: In the **conflict discussion stage,** the children had to justify their answers. Whether they were correct or not did not matter at this point. The teacher's role was crucial, closer to that of a chairperson, who used what might best be called 'a socratic approach', i.e., asking questions and always trying to get a deeper understanding from the children. Teacher location was very relevant here. When the teacher was at the front of the room directing the lesson the children were waiting for the answer. It took us some time to realize that it was better to stand to one side. In that position the students felt we were a part of the class and they were much more willing to participate in the discussion.

RS: During the discussions it must have been very difficult not to take charge when the persons arguing for the wrong answer began convincing the rest of the class.

BO: Yes, a new attitude had to be developed. What we tried to do was discuss the incorrect answers first.

We found that when we discussed the correct answer, especially when given by a student considered to be good at mathematics, then that would be the end of the discussion. Instead we put the various answers on the blackboard in an attempt to discuss them with a certain amount of anonymity. We required only one rule during all discussion. The children must respect the teacher, and each other, but the teacher must also respect the students' answers. This mutual respect is one of the key ingredients of the approach and if this respect does not exist in the classroom then this method of teaching will probably be a dismal failure.

RS: So to summarize, the teacher should be a chairperson not a giver of knowledge, should stand to one side physically rather than being looked on as the authority at the front, should demonstrate respect for children, and should examine wrong answers as much as correct ones.

BO: That's right Bob. Errors are not something of which to be ashamed. They can be used in positive ways. We can learn from our mistakes. For teachers it is very gratifying to hear correct answers being given, so we often indicate our approval. It is difficult to do, but teachers must learn to conceal these signals which can destroy a profitable discussion.

RS: Is this to suggest, then, that if the students were to take a vote and they all agreed that a wrong answer was the right one, you would accept this?

BO: No! That is not it at all, Bob, and that would be very worrying. If it were a social studies classroom and we were discussing values then maybe I could say, "Yes". Everyone has a right to his own opinion but we didn't want children walking out of a classroom having examined 5 ÷ 10 saying, "I thought 2." "I thought 4," or "my answer is just as good as yours." No, we didn't want that at all! It is the teacher's role to keep the pupils on track, but rather than simply telling students that they are wrong s/he should guide them back onto the right track by helping them to think logically. Children **do** think logically, and once they understand mathematical questions, they realize that math is not a collection of tricks, but rather a set of very coherent relationships. One of the most satisfying experiences for me was the occasion when a child had been explaining why her wrong answer was correct. Suddenly during that explanation the child's eyes lit up as she realized that she was actually explaining why it was wrong. She had discovered the correct concept, and done it by herself! She would never forget. This perhaps is the true meaning behind the words of Archimedes, "Eureka! I have found it."

RS: The last stage is called the **consolidation study.** What happens in this stage?

BO: **Conflict discussion** brought the problem areas to the surface. We would have been very misguided had we believed that merely through discussion, children would resolve all of these deeply embedded misconceptions. Thus the consolidation exercises were designed to allow students to reflect back on their earlier misconceptions, their discussion, and their newly acquired thought processes. In the 30's William Brownell said that a problem was not truly solved until the child understood what he had done and why he had done it. This is most relevant to our consolidation exercises. They were not a collection of twenty similar problems for the child to work through algorithmically. Rather they were comprised of a few problems designed so that the child had to examine the original question from different view points.

RS: You have a most appropriate quote to illustrate this idea. What is it?

BO: This quote from **The London Association for the Teaching of English** is equally as relevant to mathematics as it is to English.

Children learn by talking and listening and should be given more opportunity to talk. Children talking in small groups are taking a more active part in all their work. Tentative and explicit talk in small groups is a bridge from partial understanding to confident meaningful statement. **Present talk is future thinking.**

RS: That last sentence really strikes home!

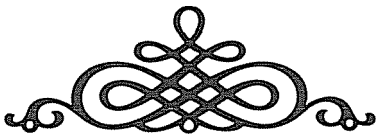
BO: I think that is what this learning strategy is all about.

RS: Barry, we must continue to spend much of our time teaching the skills of the arithmetic courses in our Elementary Schools.

The most commonly used style of teaching succeeds well for algorithms but is not good enough to help

children with conceptual problems, particularly when it comes to moving children from deeply embedded misconceptions. Your diagnostic teaching model should be a help.

Thank you Barry for sharing your ideas with us.



IN MEMORIUM TO DON FOX

It was with sadness and deep regret that the members of OAME learned of the death of Don Fox.

Don was a founding member of the Carleton-Ottawa Mathematics Association (COMA) and throughout his career had exhibited considerable leadership in OATM (OAME's predecessor organization), COMA and the entire Ontario mathematics community.

Dino Dottori gave a moving tribute to Don at the October OAME Council meeting and a minute of silence was observed in his honour.

Don will be sadly missed by all who had the pleasure of knowing and working with him. OAME extends its deepest sympathy to his wife, Isabel, and his family.



NATIONAL COUNCIL OF
Teachers of Mathematics

October 15, 1987

C. Gravelle, Editor
College of Education, Brock University
St. Catharines, Ontario L2S 3A1

Dear Mr. C. Gravelle:

The Advisory Panel for the 1990 Yearbook of the National Council of Teachers of Mathematics has just completed development of the guidelines for potential authors. We believe some of your readers may be interested in contributing to this yearbook.

The Panel would be very appreciative if you would run the enclosed "call for manuscripts" in an issue of your journal.

Thank you for your cooperation in this matter.

Sincerely yours,

Christian R. Hirsch
General Editor, 1990-92 Yearbooks

AUTHORS SOUGHT FOR NCTM YEARBOOK ON TEACHING AND LEARNING IN THE 90s

Potential authors are invited to submit manuscripts for the 1990 Yearbook of the National Council of Teachers of Mathematics which is tentatively titled, **Mathematics Teaching and Learning in the 1990s.** The thrust of the Yearbook will be to provide a thought-provoking analysis of forces that potentially alter the teaching and learning of mathematics in grades K-12 during the next decade.

The teaching and learning of mathematics will be subjected to a number of different forces during the 1990s, including; developing theoretical/philosophical and research-based perspectives on how mathematics is learned, changes in the demographics and economics of our society, emerging models for effective instruction, and accelerating technological advances in calculators, computers, and telecommunications. Analyses of these forces and their implications for mathematics instruction in the elementary, middle school, and high school grades will be the focus of the Yearbook.

The Yearbook editor is Thomas J. Cooney, University of Georgia, Athens, Georgia.

Guidelines for authors, which include a more complete description of the topics to be addressed and instructions for preparing manuscripts, may be obtained from

Christian R. Hirsch
General Yearbook Editor
Department of Mathematics
Western Michigan University
Kalamozoo, MI 49008

The deadline for submission of manuscripts is March 1, 1988.

October 23, 1987

C. Gravelle, Editor
College of Education, Brock University
St. Catharines, Ontario L2S 3A1

Dear Mr. C. Gravelle:

I have recently "discovered" a simple procedure which give students a surefire method of finding two numbers with a **given sum and product.** Students encounter this problem, of course, when trying to factor quadratic trinomials.

Example: to factor $x^2 - 7x + 12$ students would look for two numbers with a sum of -7 and a product of 12. For the vast majority of students, this presents no great difficulty. However....

Example: to factor $x^2 + 14x - 207$ students would look for two numbers with a sum of 14 and a product of -207. For many, this is now a **problem.** The following outline shows the procedure which can be used to find these numbers using an easy "mechanical" process. This method may **not** be new, but I have never encountered it before, nor have any of my colleagues. I have explained how I came upon this method and how it may be used to solve quadratic equations with irrational roots, develop the "quadratic formula" and investigate trinomials with "nonreal" factors.

Yours truly

John Savage, Mathematics Coordinator
Sudbury Board of Education
Sudbury, Ontario P3E 5K3

PROBLEMS

by
Ronald Lancaster, Problem's Editor
Oakville-Trafalgar High School
Oakville, Ontario L6J 3L6

1. On the first day back to school this year, I gave my Grade 11 Mathematics class an assignment to do. One of the problems that they had to work on is given below.

   a) When you go home tonight, get out a deck of cards and follow the instructions given below. In carrying out these instructions, you will end up freely choosing a card. Your chosen card will get mixed into the deck and without touching or even seeing the deck, I will find your card!

   Start off by removing all the face cards (Jacks, Queens, Kings) and put them back in the case. You will not be using these cards.

   Now please remove all the RED cards below the value of 5. Place these cards face down on your left side. Now remove all the BLACK cards above the value of 5. Place these cards face down with the others that you have just removed.

   Put the remainder of the cards face down to your right. Pick up the cards to your left and mix them up as much as you would like. When you are done, hold the cards face down and look at the top card and remember it. Put this card back on top and place these cards face down on the table to your left. Take the cards on your right side and place these on top of the cards on your left side.

   The card you have selected is now in the pack and unknown to me. In a few minutes you will see how my magic can overcome the distance that separates us.

   Pick up the pack of cards and deal a pile of cards one at a time onto the table. This pile is to contain twice as many cards as spots on your chosen card. For example, if your card is an 8-spot, you must deal 16 cards. Now TURN OVER the remainder of the cards in your hand and drop them on top of the dealt cards.

   Pick up the pack and deal again. This time, if your card is a HEART or a SPADE deal 20 cards; if it is a CLUB or DIAMOND, deal 24 cards. As before, TURN OVER the remainder of the cards in your hand and put them on top of the dealt cards.

   Deal once again, if your card is RED, deal 13 cards; if it is BLACK, deal 18 cards. Again, TURN OVER the remainder of the cards and place them on top of the dealt cards.

   I think that you'll agree that your card is lost in the pack. But I can find it by offering you a very special welcome back. Pick up the cards and deal a card from the top of the pack for each letter in the sentence given below.

   WELCOME BACK FROM R.L.

   When you have finished the spelling, look at the top card of the pack. (it might be face up or face down) It is your chosen card!

   b) Explain in detail how this trick works.

   c) Modify the instructions so that the trick works for the sentence given below.

   IT IS NICE TO SEE YOU AGAIN

Solution

1. b) This trick is rather amazing, but we should be able to figure it out by looking at each step very closely.

   Note that when the face cards are removed (there are 12 of them) we are left with 40 cards. We are then asked to remove all the red cards below the value of 5 and all the black cards above 5 and to set these cards to the left. The remaining cards are set down to our right. A quick count shows that there are 18 cards in the left pile and 22 cards in the right pile.

Note that the selected card ends up face down on top of the cards in the left pile. The cards from the right pile are placed on top of the cards to the left. Therefore, the selected card will **always** end up being eighteenth up from the bottom.

With a deck in your hand, you should be able to see that all of the counting and turning over of the cards does **absolutely nothing.** At the end, the selected card is **still** eighteenth up from the bottom. Look closely at the sentence "WELCOME BACK FROM R.L." Note that there are 17 letters in this sentence. After the spelling is done, the next card will be the one that was originally eighteenth up from the bottom. That card of course is the selected card!

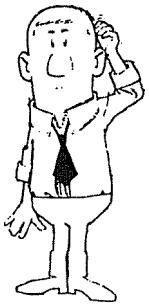
c) The sentence "IT IS NICE TO SEE YOU AGAIN" has 22 letters in it. So we need to have 22 cards in the pile on the left. There are many ways of doing this, but the simplest way is just to have the card selected from the pile on the right, instead of the left. None of the other instructions need to be changed.

Comments

The point of this exercise was to give my students some practice at sorting through some instructions and trying to make sense of it all. It was an excellent exercise in problem solving and it was also a terrific way of starting off the year!

Correction for Q 3a, Volume 26, Number 1

Joe Byrnes, from Osgood Twp. H.S. in Metcalf, Ontario recently wrote to me and pointed out that Q 3a and Q 3bii were the same. Q 3a should have had ¼ instead of ½. As Joe pointed out, the given solution is incorrect with the ½ in, and become correct when ¼ is used.



WHAT'S GOING ON

LIFE MEMBERSHIP

To qualify for Life Membership in OAME/AOEM a candidate shall: (i) have ten or more accumulated years of membership in OAME/AOEM, and (ii) have demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular.

If there is someone you wish to nominate for life membership in OAME/AOEM, you should submit his/her name and any information supporting the nomination to:

Mr. Jack Lesage, Life Membership Committee
9 Mount Baton Road
Barrie, Ontario L4M 1T4

Nomination should be submitted to Jack on or before February 1, 1988.

VERY IMPORTANT NOTICE

A school who has an OAME membership in the name of the school, the library of the school or the mathematics department of the school is entitled to the reduced fee for an OAME Conference for **ONE** teacher and **ONE TEACHER ONLY.**

If more teachers of that school who are not individual members of OAME attend they are required to pay the full registration fee.

JOHN G. McCARTNEY SCHOLARSHIP

Mona and Morley MacGregor are pleased to announce the results from this year's competition. After careful scrutiny and much deliberation, Rocky Lee from Woburn Collegiate in Scarborough has been declared the 1987 winner of the John G. McCartney Scholarship.

Rocky graduated from Woburn CI in June 1987 after an excellent academic career and outstanding achievements in Mathematics. Rocky has been an active participant in the various mathematics contests throughout his high school career and an enthusiastic cheerleader and coach of his fellow classmates. Rocky has coached the Junior Mathematics team at Woburn CI for two years and this last year Rocky coached the championship Senior Woburn Mathematics Team. Rocky's own achievements in Mathematics this year are nothing short of spectacular! Rocky was third in Canada on the 1987 Descartes Mathematics Competion; Rocky was in the top eight on the U.S. Olympiad competition and won a trip to Washington, D.C. for his efforts; Rocky was selected to be a member of our own Canadian Olympiad team for the second straight year and earned a Bronze medal in the International Olympiad Competition.

Typically of Rocky, when he discusses the International Olympiad, he points out that the Canadian Team scored its highest total ever in international competition and that teammate Ravi Vakil won a gold medal in the competition and that two other teammates were just one point shy of earning bronze medals. Rocky says "he is lucky" for all his successes and points to all the help he received from teachers Peter Crippin, Larry Rice, Paul Zolis. But we all know that "luck" is just hard work and preparation meeting up with opportunity to display this talent. Rocky is full value for all his successes and no doubt will earn many more honours in his university career and beyond. Rocky is an outstanding young man, a pleasure to know, and a joy to teach.

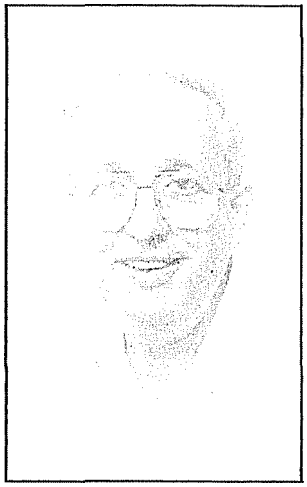
We wish Rocky continued good luck and success as he pursues a career in medicine at Harvard University, on scholarship.

O.A.M.E. extends congratulations to Rocky Lee, our top graduating Ontario mathematician for 1987.

Arnie Carefoote O.A.M.E. Director
1987-09-03

DONALD ROBERTS: SOCIETY OF DESCARTES MEDALLISTS

Editor's Note: This excerpt is reprinted with the author's permission, Ronald Dunkley, Chairman of the Rene Descartes Foundation, June 6, 1987.



Donald Roberts received his early education in Toronto and after attending the Teachers College there taught in Toronto, Pembroke, North York, and Scarborough. He returned to school in 1964 at the University of Windsor, graduating with a BSc.. Since then he has taught with the Lincoln County Board of Education. He is presently head of the Department of Mathematics at Beamsville, where he has carried on the tradition of excellence established there by a previous medallist, Bob McNeil, and ensures that the school will continue to send well-prepared students on to further study.

In addition to his excellent teaching, he is a primary force in the mathematics community of the Niagara Peninsula where he is one of the principal organizers of student workshops and professional development activities. He has served the area as a regional director of OAME, was co-chairman of the OAME conference in St. Catharines, and is the co-chairman of the program for the 1988 conference in Niagara Falls.

In private life he devotes time to his family, his church, minor sports, and personal recreation, including trap shooting. He spent the summer of 1986 as a gold assayer at a mine site in Northern Ontario. He is a quiet, modest man who is a pillar of strength to his colleagues in the Niagara area.

The Rene Descartes Foundation is pleased to invite medallist, Geoffrey Steel, to welcome Donald Roberts to the Society of Descartes Medallists, and to present to him a Rene Descartes Medal.

INTERNATIONAL MATHEMATICAL OLYMPIAD

Editor's Note: *This report is reprinted with the author's permission, Bruce Shawyer, Chairman, Canadian Mathematical Society International Mathematical Olympiad Committee.*

The 28th International Mathematical Olympiad was held in La Habana, Cuba, from Sunday 5th July, 1987 to Thursday 16th July, 1987 at the V.I. Lenin Vocational School. Forty-two countries from around the world participated in this year's IMO. Canada was represented by a team of six High School students, Stephen Fry, Woburn Collegiate, Scarborough, Ontario, David Lee, Even Hardy Collegiate, Saskatoon, Saskatchewan, Rocky Lee, Woburn Collegiate, Scarborough, Ontario, Gavin MacBeath, St. Johns Ravenscourt School, Winnipeg, Manitoba, Colin Springer, Waterloo Collegiate, Waterloo, Ontario, Ravi Vakil, Martingrove Collegiate, Etobicoke, Ontario. Accompanying the team were the Leader, Bruce Shawyer, the Deputy Leader, Ron Scoins, and an Observer, Ron Dunkley. The students were chosen on the basis of their performance in various tests, such as the Canadian Mathematics Olympiad and the Canadian Mathematics Competitions, as well as a correspondence program. From 15 June 1987 to 30 June 1987 before the trip to Cuba, the team attended a training seminar in Waterloo, sponsored by the Canadian Mathematical Society and the University of Waterloo.

In Cuba, the Leaders of all the delegations were sequestered for the construction of the examination papers. (Each examination paper consists of three questions and are written on two successive days. All students are expected to try to solve all the questions.) Meanwhile, the students and the Deputy Leaders were given time to prepare themselves for the competition and were entertained to some cultural, sports and recreational activities.

The first examination day (Friday July 10th) began with the opening ceremony, presided over by Sr. J.R. Fernandex, the Minister of Education and Deputy President of the Council of Ministers of Cuba. The ceremony was quite short and the students immediately went to the examination rooms for the first paper. It is an exciting time to see the marks entered on the main scoreboard. This is done immediately after each country has had a question marked. There is much speculation as to who will do what in the remaining questions, especially as some questions are more difficult than others. Although, like the Olympic Games, there is no official Team Championship, there is considerable interest in National Scores. Who will come out on top? Will Canada beat Australia? And so on. There is always an interested chattering crowd in front of the main scoreboard.

The final marking was on Monday 13 July. But the competition was not over until shortly after lunch. Romania had "won" with 250 out of 252 possible points and West Germany were second with 248 points. Australia had squeked past Canada's 139, although our students had obtained about 20 points above the score that we had hoped to obtain. Our individual scores were:

Ravi Vakil	: 42 points - a perfect paper and a Gold Medal;
Colin Springer	: 33 points - a Silver Medal;
Rocky Lee	: 19 points - a Bronze Medal;
Stephen Fry	: 17 points - one point short of a Bronze Medal;
Gavin MacBeath	: 17 points - one point short of a Bronze Medal;
David Lee	: 11 points.

There was a little sadness in the Canadian camp since two students missed a Bronze by only one point.

That evening, a buffet reception in the Havana Convention Centre was hosted by the Minister of Education. He led the reception line and did a walk about amongst the delegations. The Guide for the Canadian Team was very keen for his charges to be introduced to the Minister. This gave him an opportunity to display his translation skills in front of a very important Cuban person. The Minister wished all our students success in their future endeavours.

On the morning of Tuesday 14th July, the closing ceremony was held in the Theatre of the Cuban Army. This was the only time that we saw much evidence of the military. We were instructed not to take cameras or bags with us. (Photography of military installations, bridges, airports, harbours and bridges is forbidden). We were saddened that we could not take pictures of our students receiving their awards although official photographers were present. Hopefully we will receive official photographs later. There had been some speculation that Fidel Castro would be present at the closing ceremony. This turned out to be impossible because of the opening of the Cuban parliament. The Cubans were sad because they told us that Fidel likes to attend functions such as the IMO awards ceremony. The Minister of Education chaired the ceremony. All students from a country who received a medal were called up together. All students were photographed in groups. There were special cheers for the Chinese girl, the only girl to get a Gold Medal and for the eleven your old Australian boy who received a Silver Medal. The Cuban medal winners were presented last of all to a standing ovation.

The final event of the IMP was a day long excursion that ended at a beach resort hotel at the Bay of Pigs. Features of the excursion were a visit to a crocodile farm and the Museum commemorating the "First Major Defeat of Imperailism". We saw kilometer after kilometer of citrus orchards with residential high schools where the stu-

dent give three hour work per day on the citrus plantation as well as six hours per day in school. There were a couple of sugar mills and a place called Australia! The resort hotel at Playa Giron was typical of what can be expected in Cuba. Sandy beaches with shade offered by the magnificent Royal Palm trees. There were also a few conifers and brilliant Flamboyant Trees with orange coloured blossoms. Many people enjoyed a swim in the 28° C. water of the ocean. The constant easterly trade winds ensured that the 35° C. daytime high temperatures were bearable in the shade.

We cannot say enough in praise of our Cuban hosts. Everyone concerned made great efforts to ensure our comfort and convenience. The event was organized with great care. The Cuban committee deserve much praise for their efforts.

We were unable to leave Cuba until Saturday 18th July. Most of Thursday was spent relaxing on the terraces of the Riviera Hotel. The students joined the delegates to be entertained at a party at the Canadian Embassy. The Ambassador himself came to the party and talked at length with the students and delegates. We are very grateful to the Ambassador and his staff for this hospitality. We had to get up at 2:00 a.m. for 2:30 a.m. transportation to the airport. The instruction from Cubana Airlines was to be at the airport by 3:45 a.m. Our Cuban hosts had taken this instruction seriously and had arranged our transportation accordingly. However, Cubana Airlines flight 484 did not leave Havana until 6:30 a.m. to fly over the Bahamas, New York City, Syracuse, Ottawa and into Mirable. It was a long day before we were all safely home.

It was an experience of a lifetime for the students and the coaches alike. It is hard work for the students, especially the concentration required for two **four and one half hour** examinations. But it is also hard work for the Delegates, who must not only work in multiple languages to construct the examinations, but must struggle to understand the students' writings and interpret them to the Markers. Add the high temperatures and humidity of Cuba and you find an exhausting experience. None-the-less, it was a highly rewarding experience and I am delighted to have been granted the opportunity to be a participant.

Bruce Shawyer, Head
Department of Mathematics and Statistics
Memorial University of Newfoundland
St. John's, Newfoundland A1B 3Y1

CALL FOR NOMINATIONS

At the Annual Meeting of OAME/AOEM in Niagara Falls in May 1988 we shall announce the result of the elections for 1 President Elect, 3 Directors and about 10 Councillors.

If you would like to suggest your name for consideration by the nomination committee or wish to nominate another OAME member for nomination Committee's consideration for any of these positions, please complete the following nomination form and send it to

Dino Dottori, 640 Lancelot Ave., North Bay, Ontario P1A 3A8

A summary of the qualifications of any one who wishes to place his name before the nomination committee must accompany this form.

This summary of qualifications of those nominees who are selected to stand for election will be published in March.

In OAME/AOEM the procedure of voting by mail is followed.

NOMINATION FORM

I nominate_____ (name)

_____ (address)

_____

for the position of ____ President Elect

____ Director

____ Councillor

Name & Address of Nominator _____

To be valid this nomination form must be received by Mr. Dino Dottori on or before 4 p.m. January 20, 1988. Nomination forms received after the specified closing hour and date, regardless of postal markings at the point of mailing or if courier hand delivered, will not be considered.

ATTENTION ELEMENTARY SCHOOL TEACHERS

AWARDS FOR THE TEACHING OF PROBLEM SOLVING

If you are an elementary school teacher who has developed "neat" and effective ways and means of improving teaching of problom-solving in your classroom, school, or district, then this announcement is for you.

Chester Carlow, in cooperation with OAME, is sponsoring awards for the improvement of teaching problem-solving in the elementary schools: **$300 for first place, $200 for second place, and $100 for third place.**

WHAT IS REQUIRED:

Teachers are asked to submit a report which details the innovative instructional strategy(ies) which were used, and the results relative to the improvement in students' abilities to solve problems.

Teachers should select an instructional strategy which looks promising in terms of previous theory and research on problem solving. This will promote steady progress in the area.

Bibliography to get started:

Berry, Lois Langfor, **Problem Solving In The Elementary School Curriculum,** ED 253-416, 46 pages

Morrow, Lorna, **The Classification of Mathematics Problems By Young Children: An Exploratory Study,** Doctoral Dissertation, O.I.S.E., 1988

Riley, Mary S. and others, **Development of Children's Problem Solving Ability In Arithmetic,** ED 252-410, 47 pages

Complete information about the structure of the reports can be obtained from:

Ralph Connelly
11 Berkwood Place
Fonthill, Ontario
L0S 1E0
1-416-688-5550

J. Symington
11570 Yonge Street
R.R. #1
Richmond Hill, Ontario
L4C 1X7
1-416-884-4802

Chester Carlow
87 Risebrough Avenue
Willowdale, Ontario
M2M 2E2
1-416-223-2932

Anyone wishing to be considered for the awards must have their information in by April 1, 1988.

DON'T KEEP THOSE EXCITING IDEAS A SECRET ANY LONGER, LET US HEAR FROM YOU!

EXCELLENCE IN MATHEMATICS AWARD CERTIFICATE

Place your order now for these OAME certificates

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Please indicate ENGLISH or FRENCH

BOOK REVIEW

by
Eric Wood, Book Review Editor
Althouse College, London, Ontario N6G 1G7

Diagnosis and Prescription: Some Common Math Problems. Ruth Rees and George Barr. Harper and Row, London, 1984.

Rees and Barr have delved into the middle of both children and adults in an effort to uncover some of the major obstacles hindering a person's mathematical development. Having described their findings they then proceed to suggest various ways to eradicate these difficulties.

Each chapter outlines a specific topic e.g. decimal fraction. The authors explain that effective diagnosis is more subtle than it at first appears. For example, in chapter 3 they point out that if one asks questions such as 0.4×0.4 or $\sqrt{0.25}$, the students' assume that the correct is understood. However, if the questions are altered slightly, 0.3×0.3 or $\sqrt{0.4}$ typical answers are now 0.9 and 0.2, demonstrating a serious misconception. Thus effective diagnosis triggers the teacher's awareness of these pitfalls in children's thinking. Rees and Barr also point out that there might exist a learning-teaching-learning cycle, since it was observed that teachers in training (including some specialist mathematics teachers) often demonstrated uncertainty when presented with some elementary mathematical tasks. Thus misconceptions, as well as attitudes towards mathematics, could be inadvertantly transferred from one generation to the next.

The latter part of each chapter offers a prescriptive remedy. The basic premise is that teachers must LOOK, LISTEN and LEARN with the students, while being AWARE, EXPLICIT and FLEXIBLE so as to promote the development of CONCEPTS with COMPETENCE and CONFIDENCE. Although I would agree with the authors that these traits will likely lead to a more confident and numerate society, these sections of the book are somewhat weaker than the diagnoses and could offer more practical suggestions for overcoming the well illustrated difficulties.

Several of the misconceptions are not unique to this book. There has been extensive research in this field over the past few years. However, the findings are thoroughly documented and easy to digest, making this an excellent source of reference. Some stimulating points for discussion (both implicity and explicit) arising from the text also make it a valuable resource for both pre-service and in-service education.

Note: Book reviewed by Barry Onslow, Althouse College, London, Ontario N6G 1G7

MATH GREMLINS

by
Clifford Gravelle

Editor's Note: *Please correct this gremlin*

Volume 26, Number 1, September 1987, Page 39.

BOOK REVIEW
by
Eric Wood, Book Review Editor
Althouse College, London, Ontario N6G 1G7

15 ANNUAL CONFERENCE
Niagara Falls - May 5, 6, 7, 1988

Program

A blend of the traditional conference program together with new approaches which have proved popular at other major mathematics education conferences.

(i) Nationally and internationally sought after keynote speakers with a wide range of mathematics education experience.

Leroy Callahan
Peter Hilton
Carolyn McCann
Ralph Stanton
David Wheeler

(ii) At other prime time, nineteen concurrent sessions,

4 Primary
4 Junior
4 Intermediate
4 Intermediate/senior
2 Special Interest

(iii) Mini Courses - limited preregistered enrolment, half day or full day topics. Work thoroughly through a given topic or area. Get to know other teachers, their problems, their concerns, their approaches and their solutions. (Additional fee to cover lunch and/or materials).

(iv) Visit to schools in the area - find out how computers have been integrated at all grade levels.

Accommodation

A range of facilities are offered to cover varying budgets and tastes. Book early to ensure the accommodation of your choice (see September Gazette flyer for addresses and telephone numbers). These facilities are aware that you may wish to make this a family event and may extend your stay through to Sunday.

Other Attractions

All conference locations are within walking distance of the Falls.

Join in the Friday night banquet and/or the dance with "Jeffrey and the Juniors".

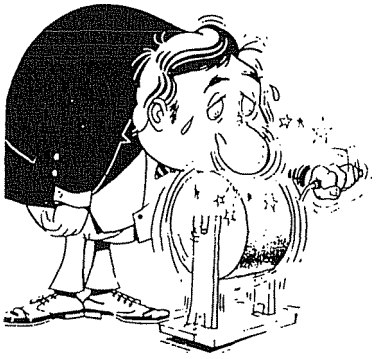
Each hotel will have their tour organizers on hand to help you select from the many available tours and to assist you in the choice of entertainment.

It's the start of blossom season in Niagara.

OAME
15th Annual
Conference

Outstanding program
Great accommodation
Ideal surroundings

May 5-7, 1988
in
Niagara Falls



COUNCIL COMMENT

by
Bob Robinson

Editor's Note: *Council Comment appears in an issue of the* **Gazette** *following OAME/AOEM Council meetings. Your comments are welcome and should be addressed to the editor.*

Don Fox

We are all saddened by the passing of our dear friend and mathematics colleague, Don Fox. Dino Dottori spoke eloquently of Don and his leadership in mathemtics education. He is so fondly remembered.

History Booklet AD HOC Committee

A committee consisting of:

J. Symington
Dave Alexander
John Del Grande
Secretary-Treasurer of OAME

were appointed to assist Joane Routledge in this interesting project.

COMMITTEE FOR THE POSITION OF SECRETARY-TREASURER

A committee has been formed to recommend names for the position of Secretary-Treasurer of the Association due to the upcoming retirement of Morley MacGregor. The members are J. Symington, Mickey Sandblom, and Dave Davidson. If you are interested in applying for this important position or know of anyone who might be interested, please contact any of the committee members.

Nominations Committee

The members of the Nominations Committee appointed by the Executive are Dino Dottori, George Knill, Paul Lessard. Anyone interested in running for councillor should contact any member or the Secretary-Treasurer in Sarnia.

NCTM

Members are reminded of the Education Materials list published by NCTM. This catalogue is written in Canadian funds and can be obtained by writing: NCTM at 1906 Association Drive, Reston, VA 22091

or by contacting Don Attridge or Shirley McIntyre.

ANNUAL MEETINGS

1987 Scarborough

Their financial statement was presented and we reaffirmed our appreciation of everyone's efforts in making this an excellent conference.

1988 Niagara Falls

Mark in your calendar the dates of May 5, 6, and 7, 1988. These are the dates of our next "Annual" which will be hosted by the Niagara component of the Golden Section Chapter. A flyer appeared in the last issue of the Gazette and a program synopsis will appear in the next issue. Sessions and accommodations will be provided at the Ramada Hotel-Fallsview, Best Western - Murray Street, Old Stone Inn and Mount Carmel Carmelite Community.

1989 Ottawa

Initial plans are underway; the chairpersons have been appointed. There is full regional mathematics education involvement through the four local boards, two universities, the community college, as well as private schools. A logo has been designed. Keep the dates and place in mind: Ottawa University - May 11, 12, 13, 1989

1990 Hamilton

Our 1990 Annual will be held as an NCTM Regional Meeting. The Council appointed Bob Robinson to be the Conference "Chair" and Ralph Connelly, Alex Norrie as the Program "Co-Chairs".

Leadership Seminar
At the time of writing, 67 + participants are expected to attend. The theme is "Futures". Watch for their report in the next "Council Comment".

Ontario Teacher's Federation
Mary Lou Kestell is OAME's representative to OTF. Any concerns you have pertaining to mathematics education and OTF can be directed through Mary Lou.

CHAPTERS

COMA/AMOC
COMA's second Leadership Conference took place at the Glen House Resort in Gananoque, on October 2 and 3, 1987. About 60 people attended. The facilities were excellent and all sessions were well received. The theme of leadership was very appropriate and the three keynote speakers - Brendan Kelly, Gary Tushingham, and Hal Willis were all different but provided practical illustrations of what leadership is. Congratulations are extended to the organizing committee - Dave Hunter (Chair), Ron Bender, Jerry Lunney, Bruce Muller, Fran Outerbridge, and Bob Payne.

The new executive is planning a January event, as well as the annual Publisher's Display in April.

COMA sadly misses Don Fox, who passed away. Don was a founding member of COMA and a continual supporter of Mathematics teachers in Eastern Ontario. He will be missed.

GOLDEN SECTION
Golden Sections efforts have been concentrating on the preparation of the 1988 annual meeting in Niagara Falls, on May 5, 6, 7.

G.V.M.A.
The fall conference will have taken place by the time you read this column. A report will be forthcoming.

"Grade 12 General Business Math", a new resource booklet is available now from GVMA.

Plan ahead - the G.V.M.A. Spring Conference is being held Thursday, April 14, 1988.

NOMA
The Executive of NOMA are:

Chairman George McNabb
Secretary Jerry Maloney
Treasurer Mickey Sandblom
NOMA Representative to OAME Ann Kenny
Program John Savage
Susan Stuart
Harve Tremblay
Publicity and Promotion Jim Sutherland
Brian Charbonneau
Bob Tuck

The fall conference held at La Salle Secondary School in Sudbury was a success. The attendance doubled from last year. The conference brought in about 15 **new** memberships to OAME and several others newed. Mona and Morley MacGregor were delighted! They are planning for an even larger conference next year.

PAME
The fall conference has been held and a report will be presented later. The keynote speaker was Claire Zeller, who spoke on "Thinking About Thinking".

PAME is making plans to run a math contest across the region for students in grades 9 and 10. This would be held in early December. They are hoping to make this an annual event and expand to give coaching to both teachers and students involved in preparing for math contests.

PAME is actively involved in planning a mini-conference on Peel's P.D. Day in February. They hope to pass the 300 mark in attendance as they have done in the past.

There has been a steady growth of people getting actively involved in this one of our newer chapters of OAME.

PINE RIDGE
The P.A. Day on November 13, 1987 will have been held by the time this column is read. The keynote speaker is

Father Stan Buzuska of Boston College. He was to make two presentations entitled, "The Teacher at the Threshold of the 21st Century", and "Problems From the Frontiers of Number Theory that Utilize the Calculator and/or Computer".

Linda Bain, from O'Neill C.V.I. and her committee has prepared a booklet for the preparation for the Gauss, Descartes, and Euclid Math contests.

QSLMA

Their full conference was held in late October in Gananoque. There were such topics as "board-wide exams" presented, as well as Joan Routledge speaking on "Key In The Brain" - calculations in the classroom.

Plan ahead for March when their Spring conference will be held on a Friday night in the Officer's Mess of RMC.

SAME

The unfortunate "strike" forced the cancellation of SAME's Fall Conference. They are planning a P.A. Day in February, as well as a Spring Mini Conference. They are "on hold" at this time to "regroup" after the strike.

York Mathematics Association

This is our newest addition attempting to achieve chapter status. The representative to OAME is Bill Nimigon. They have created a 20 member committee and are planning to attend a Wine and Cheese social on November 19. They may even be called "Y4MA" representing the York boards (Public and Separate), North York, and York University. Good Luck with your initial activities.

SWOAME

Todd Romiens is attempting to reactivate the South Western Ontario Association for Mathematics Education. He and others are in the early stages of a membership drive to generate enthusiasm. It would be wonderful to see this chapter thrive once again in the South Western region of Ontario. Best wishes and may this chapter grow and grow.

COMMITTEES

Women and Math Committee

Speakers have been suggested for the May annual. Names have been submitted to the Nominations Committee. They are concerned about female representatives on committees and will investigate this further.

Primary And Junior Committee

A Problem Solving Project '87 - '88 has been initiated. Look for announcement elsewhere in this issue of the Gazette. The Problem Solving Project Committee consists of Chester Carlow, Ralph Connelly, and J. Symington.

The Evaluation Team for submissions will be Chester Carlow, Shirley Fairfield, Lorna Morrow, and J. Symington.

The Committee was very pleased with the **Geometry: Junior Division - Curriculum Ideas for Teachers** support document for the Junior Division teachers written by Joan Routledge. They suggest that a similar document would be of great value to the teachers of mathematics at the grade 7 and grade 8 levels as well.

Intermediate and Senior Committee

This committee is discussing bill 262. Look for an OAME response in the not-too-distant future.

The Calculus Evaluation Study by OISE is underway. We hope it will be a worthwhile study for teachers.

Packets are being prepared for the Grades 11 and 12 Senior OAIP pilot evaluations. No contracts have been let for the OAIP at the OAC's level at this time.

This committee is discussing the concern of non-mathematics specialists teaching mathematics.

The committee is studying the use of board wide tests. Dialogue will continue in this area.

Publications Committee

The editors of the **Gazette** and **Abacus** are seeking elementary school level articles. Submissions should be sent to Ron Ripley (Abacus) and Ralph Connelly (Gazette).

Membership and Publicity Committee

A newly revised membership form is available. Contact your chapter representative, OAME Councillor, or the

Secretary-Treasurer for a copy. This committee is proud to announce that our membership is just over 1500 and still growing. Come On...Be Part of the Action!!

Mathematics Coordinators and other interested parties are invited to order Mathematics Achievement Certificates for this school year. Many positive responses have been received regarding this certificates.

Futures Committees

This committee is working with OSSTF to produce a "Mathematics Teacher Survival" binder. More news will be provided at a later date.

They will also look at what constitutes a "a special project" for funding purposes, as well as funded committee for special projects will be appointed.

The process of recommending persons to NCTM committees will be investigated, as well as review the entire operation of the association and make appropriate recommendations to Council.

Future OAME Meeting Dates

Executive	**Council**
1987 11 20	1988 02 13
1988 04 ?	

RESEARCH CLIPS

Elementary School Children's Attitudes Toward Mathematics

Classroom teachers can take pride in the fact that most elementary school children have positive attitudes toward mathematics, yet they can further enhance attitudes by allowing pupils to be more active learners in the instructional process, particularly by providing increased peer-related activities.

Most boys and girls on the elementary school level maintain positive attitudes toward the study of mathematics through Grade 6 and believe mathematics to be the most important academic subject. Yet, children at this level perceive few opportunities for interaction among themselves relative to the mathematics studied in school. Data from questionnaires and interviews with children indicate that teachers can promote still more positive attitudes toward mathematics by (1) providing more laboratory-type activity and greater opportunity for sharing ideas, and (2) complementing teacher-dominated lessons with more pupil-to-pupil activity, such as peer tutoring and small-group work sessions. Furthermore, with approximately 3/4 of the elementary school children surveyed expressing a special sense of pleasure at being able to solve a difficult mathematics problem without direct intervention on the part of the teacher, both pupil attitudes and instructional objectives might well be enhanced if small groups of children were permitted greater opportunity to engage in problem solving as a joint venture.

References

Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Lindquist, M. M., and Reys, R. E. (October 1980). "Students' Affective Responses to Mathematics: Results and Implications from National Assessment." *Arithmetic Teacher*, Vol. 28, pp. 34-37, 52-53.

Corbitt, M. K. (April 1984). "When Students Talk...." *Arithmetic Teacher*, Vol. 31, pp. 16-20.

Suydam, M. N. (November 1984). "Research Report: Attitudes toward Mathematics." *Arithmetic Teacher*, Vol. 32, p. 12.

RESEARCH CLIPS

Teaching Estimation Skills in the Mathematics Curriculum

Teachers can improve pupils' estimation skills by sharing effective strategies, including their own, providing for sustained practice, and including these skills within the testing program.

All students, not just the weaker ones, require systematic instruction in estimation skills. Being a good estimator requires a willingness to take risks, a trait not necessarily characteristic of even those students most proficient at computation tasks. Classroom teachers at all grade levels must introduce strategies that foster estimation skills. Such strategies may be selected from the literature and from the teacher's own repetoire of effective techniques. For example, in estimating the height of a basketball net, students might be advised first to make reference to the height of some particular person. In estimating the length of a side of a room, they might be advised first to estimate the lengths of particular parts of that side. Or, in estimating the answer to some arithmetic problem, students might first alter the numerical data to facilitate the computational aspects of the problem. Teachers must highlight the benefits of possessing a variety of strategies for estimating the answer to a given problem and encourage discussion of the strategies that their students devise. They must provide sustained practice of the estimation skills being developed and include them within the testing program. In this way, students may come to see estimation skills as crucial instructional goals, with more practical application to everyday, real-world situations than even paper-and-pencil algorithms.

References

Benton, S. E. (1986). "A Summary of Research on Teaching and Learning Estimation." In *Estimation and Mental Computation*, 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, Va.: The Council, 1986.

Coburn, T. G. (March 1987). "Estimation and Mental Computation." *Arithmetic Teacher*, Vol. 34, pp. 24-25.

Lindquist, M. M. (March 1987). "Estimation and Mental Computation." *Arithmatic Teacher*, Vol. 34, pp. 16-17.

Reys, B. J. (September 1986). "Estimation and Mental Computation: It's 'About' Time." *Arithmetic Teacher*, Vol. 34, pp. 22-23.

Reys, B. J. (1986). "Teaching Computational Estimation: Concepts and Strategies." In *Estimation and Mental Computation*, 1986 Yearbook of the National Council of Teachers of Mathematics. Reston, Va.: The Council, 1986.

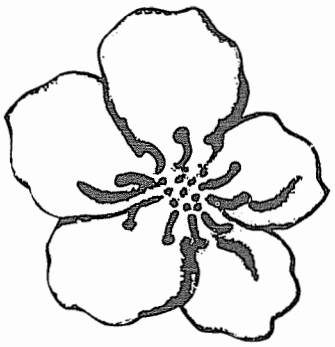
Reys, B. J., and Reys, R. E., (March 1986). "One Point of View: Mental Computation and Computational Estimation - Their Time Has Come." *Arithmetic Teacher*, Vol. 33, pp. 4-5.

Reys, R. E. (November 1985). "Testing Mental-Computation Skills." *Arithmetic Teacher*, Vol. 33, pp. 14-16.

Suydam, M. N. (December 1984). "Research Report: Estimation and Computation." *Arithmetic Teacher*, Vol. 32, p. 35.

Trafton, P. R. (1978). "Estimation and Mental Arithmetic: Important Components of Computation." In *Developing Computational Skills*, 1978 Yearbook of the National Council of Teachers of Mathematics. Reston, Va.: The Council, 1978.

15th ANNUAL CONFERENCE



O.A.M.E.

"Mist and Math"

Niagara Falls

May 5, 6 & 7, 1988

Hosted by: The Niagara Component of the O.A.M.E.
- Golden Section Chapter

THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHEMATIQUES

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