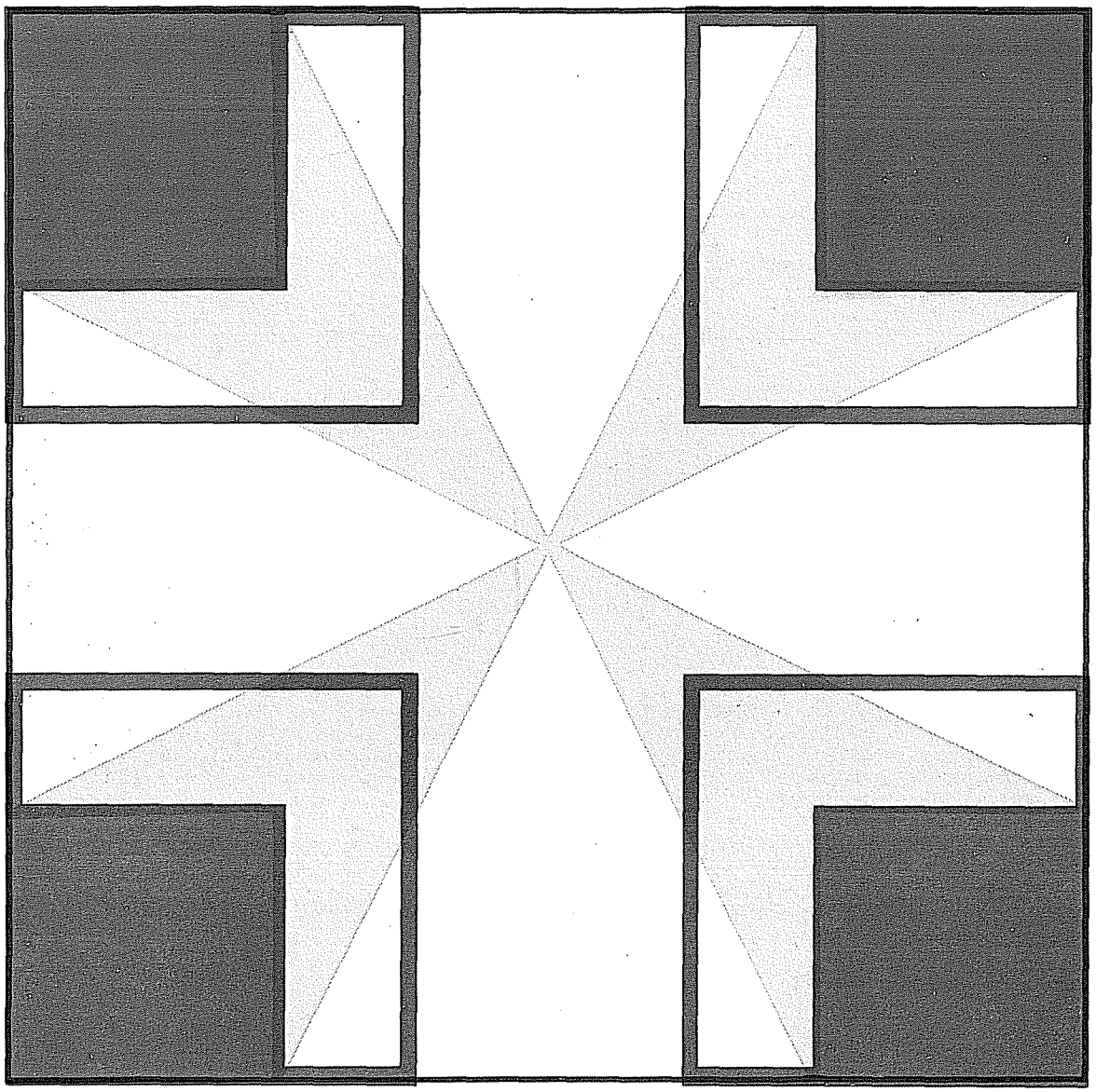
ontario mathematics gazette

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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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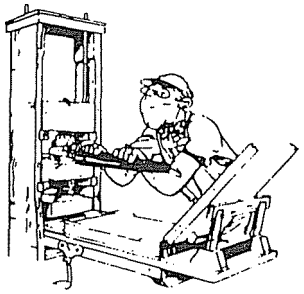
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by
Eric Wood

For as long as I can remember, September has been a time to look forward to new experiences. When I was a student, it brought a new teacher and class; as a teacher it brought new courses and faces to teach; and, now it brings a new role as guest editor of the *Ontario Mathematics Gazette*. The look of the Gazette too is new this fall as we bring our first theme issue to the readership. In addition, the colours have been changed to match those of the host of the annual conference, in this case McMaster University.

Although many things change, many others stay the same. Ask any group of students (or adults for that matter) whether it is better to get three 12 inch pizzas or one 24 inch pizza for the same money and most will assert that three 12s are the better buy. Such a misconception is due to the fact that students do not understand the formula for the area of a circle as a statement about how the area of a circle is related to the radius; and, how changes in one affect the other. Students often employ a linear thinking model which assumes doubling the radius doubles the area and yet, because the area depends upon the square of the radius, a doubling of the radius produces a quadrupling of the area. Consequently one 24 inch pizza is the same as four 12 inch pizzas, a fact that is often difficult for students to understand. The root problem here is that we look upon the *calculation* of the area of a circle as simply that -- no more, no less. The whole concept is more useful however, if looked at as a functional relationship between two quantities, rather than as a formula for computation.

The guidelines make suggestions along these lines but the texts that drive so much of our teaching do not seem to mirror the spirit of the suggestions.

Many other examples of students' poor understanding of mathematical relationships could be cited; indeed, the problem is not a new one. However, despite our best efforts students seem to continue to have a poor idea about relationships in mathematics and how changes in one quantity produce changes in another.

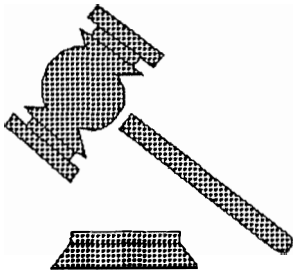
The idea of functions and relationships was chosen as a theme for this issue because of this continuing difficulty and because of the primary place that patterns hold in the field of mathematics. Godfrey Harold Hardy (1877 - 1947) is credited with saying that a "A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs it is because they are made of ideas." It is unfortunate that in many cases the patterns that are so important are lost in a sea of calculations and washed ashore on a deserted beach where no one is apt to find them.

In these more modern times the fundamental ideas of mathematical relationships still retain their place of importance. The new NCTM document entitled *Curriculum and Evaluation Standards for School Mathematics* (1989) lists relationships, patterns and functions as one of the key topics in elementary, middle and secondary school curricula.

In this issue we explore a number of areas of mathematics that deal with relationships: Robert Chisnal illustrates how mathematical concepts are related to knots; Barry Onslow looks at relationships from a graphical perspective and Ron Reesor discusses a number of functional relationships that can be examined experimentally. Ron Lancaster's problem section (which we can all relate to) is back, along with a provocative viewpoint about the direction of mathematics education in Ontario written by Dave Hunter. Lorna Wiggan provides some thoughtful writing about the issues surrounding females and mathematics and regular departments such as Book Reviews and Council Comments provide a familiar look to the journal.

I warmly wish that this September will provide for each and every reader the kind of positive challenges that we all need to grow; and, that the new direction of the Gazette is at least a small part of that process. Whether we succeed or fail, please let us know!

PRESIDENT'S MESSAGE



Recently I sent cards greeting the new OAME executive, cards saying "It is not the years that make us grow - but the changes." We are constantly required to react to the complexions and needs of our customers. We have to implement change in curriculum, in the structure and make-up of classes, in our delivery methods, and in our evaluation techniques - even though we sometimes feel neither ready nor committed to doing so. Keeping the needs of the students in the forefront of our thoughts is very important to the educational process and to us as well. Politicians, parents, industry, community colleges, and universities all have ideas on directions for change. Many of these are good ideas whose time has come - ideas that will motivate changes in the way we educate everyone. In the final analysis, though, we must be motivated most by the needs of the children, adolescents, and adults we teach - our clientele.

With the numbers of new teachers joining us in the classrooms we have big responsibilities to face, NOW! New teachers need to be coached in how to deal with the requirements of guidelines that conflict with aims of OS:IS. They don't know how to react to students that come from weak family structures - students who have no support and no encouragement to do well in school. They will need help in implementing cooperative and collaborative learning models in the classroom. Just as previously they had trouble learning how to stand and deliver, they now will need help to learn to make the new ideas and models work. They will have trouble understanding young people with weak commitments to the future - a future to these young people is but a moment away. I think when we try to get perspective on this issue we need to remember MIT'S study that reports, "information these days has a life-span of about four years". Delayed gratification is no longer reason enough to do homework. The material has to have relevance today. We must learn to operate as our best selves at all times because the future is so soon upon us.

The equity issue is one that, as mathematicians, we cannot ignore. Statistics (which we teach) show that as years go on, fewer females stay in mathematics, science and technology courses. When we match these figures with ones that show graduates from business and secretarial training make half as much as graduates from science and technological training, we see that we must address the issue in order to protect our standard of living. We must make access to learning available to all even if it means filling in gaps in educational background for our immigrants and fighting family attitudes towards preparation of females for the work force. *Everybody Counts*, from the National Research Council of the USA, points out that we do well in preparing white males in mathematics, but that white males will form only 15% of the replacement work force in the years from 1985 to 2000. The equity issue translates to non-white males and non-white females.

We have to work together to build a better system of education, one that is changing as fast as the clientele. The one common goal teachers from all levels and all grades share is that all want to help students build a strong foundation for their lives. We need to help them find self esteem and confidence so that they will go forth to build strong families and rich communities. We must show them that they do matter - each and every one of them.

Mary Lou Kestell
June 1989



TRIBUTE TO MORLEY AND MONA MACGREGOR

Morley and Mona MacGregor have been associated with the Mathematics Associations since Morley's election to the council of O.A.T.M. in 1964. Shortly after election to the executive in 1968, he became Secretary-Treasurer, a work which he and Mona have cheerfully and capably carried out ever since.

Morley entered the teaching profession in 1938 in a rural school as was customary at that time. He later moved to Sarnia where he became principal. In the early 1950s he became the first principal of Bluewater School, a model for integrating the education system of native Indians into a local Board of Education. He is remembered fondly and with much respect by the native Indian community in Lambton County.

Morley was very active politically. He was president of Sarnia Liberal Association for 25 years and campaign manager for successful members of both the provincial and federal parliaments. In retirement he has served the province as a commissioner of the Blue Water Bridge Authority in Sarnia.

Mona and Morley MacGregor are blessed with seven children and nineteen grandchildren, a family they are very proud of. They have lived at 247 Bright Street, Sarnia since 1943. It is an address we all know well; for it is there that most of the work was done for out organization. From building the membership to packaging and mailing of the *Ontario Mathematics Gazette*, they were always aware of the budget. Presidents of the organization came and went, and behind each were Morley and Mona, helping to keep OAME as strong an organization as we have today.

At Friday night executive meetings, their sincerity, frankness and honesty were always greatly appreciated. At the vast number of council meetings, they both kept the beliefs and traditions of the organization before the councillors and often outworked their co-workers at the meetings.

Dr. Samuel Johnson said this about poetry:

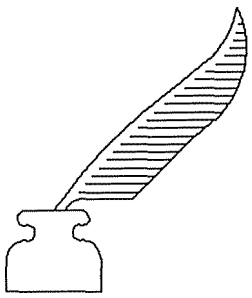
> "Poetry is the art of uniting pleasure with truth by calling imagination to the aid of reason."

Morley and Mona have, over the years, united pleasure with truth and most certainly were consistently calling everyone's imagination to the aid of reason, that O.A.M.E. might prosper.

Thank you both.

May you enjoy a healthy and happy retirement.

LETTERS TO THE EDITOR



April 2/89

135 Overlea Blvd.
Don Mills, Ontario
M3C 1B3

Dear Editor,

Over the last few months I have eagerly followed the discussion of the proposed changes to the way that mathematics will be taught and learned. While I applaud the efforts of leaders in mathematics education to humanize the learning process in the high schools, I have strong reservations that it will be possible to implement these changes. Our teaching process is almost entirely teacher centred and curriculum bound. It usually consists of motivation, explanation, drill and problem solving. Remediation, if it exists at all, usually involves more explanations of algorithmic procedures, more drill practice and less problem solving experience. The assessment of student performance is usually summative and consists of quizzes, unit tests and final examinations.

Ontario mathematics teachers have been expertly trained to maintain rigid standards of excellence by efficiently transmitting the mathematical information contained in the Ministry curriculum guidelines in the allocated time. Process objectives and formative evaluation techniques, already stipulated in the guidelines, are most often ignored.

Content and credit requirements have pigeon holed the students and teachers. Students are slotted into their level where they are fed bite sized pieces of information by teachers according to the structure of some Ministry mandated textbook. In this context open ended investigations are seen as a frivolous waste of time and deviations from the prescribed algorithmic solution are interpreted as threats to the teachers authority.

How then can mathematics teachers take advantage of the synergistic effect of group dynamics, the power of calculators and computers to develop new problems and problem solving approaches, and enhance the learning of mathematics by encouraging each individual to actively construct their own mathematical concepts?

Helmut Paddags

As this year marked the retirement of Mona and Morley McGregor from the office of secretary-Treasurer of OAME, the Life Membership Committee considered presenting Life Membership to Mona. Morley is a Life Member, and Mona's long-standing membership in the association and valuable contribution to it make her very worthy of the award.

Although receiving Life Membership is a great honour, we decided that we would give Mona a more unique honour, and so we have established a new award in her name. The **Mona McGregor Award** will not necessarily be given every year. Recipients may be, but may not necessarily be, members of OAME, nor will they have to be math educators. They will be people who have made an outstanding contribution to this association and who deserve special recognition for their service.

The plaque reads:

"The Mona McGregor Award for Outstanding Contribution to the Ontario Association for Mathematics Education"

Mona proudly accepted this honour as its first recipient.

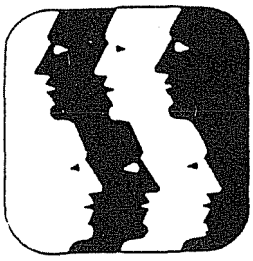
Eileen Collins (Chairman)
Life Membership Committee 1989

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I ♥ Math buttons promote the appreciation of mathematics among students and teachers. They are available in French and English at 40¢ each, $18.00 for 50.

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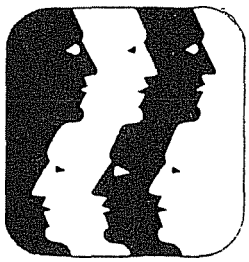
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Viewpoint

WHERE ARE WE AND WHERE DO WE GO FROM HERE?

by
Dave Hunter
Mathematics Consultant
Ottawa Board of Education



The implementation of OS:IS has almost been completed and the new OAC courses in mathematics will be implemented fully in the school year 1989-90. Very little has changed in the mathematics curriculum at the Advanced Level as a result of these changes. The only major change has been in the General Levels at Grade 11 and 12 where a distinction has been made between Mathematics for Technology and Mathematics for Business and Consumers.

The time has come to sit back and reflect. Exactly what are we trying to accomplish in the mathematics classroom? The following issues need to be addressed:

1. The so-called spiral approach that exists in our guidelines.
2. The volume of material that is identified as core content.
3. The lack of applications relevant to the student in our courses.
4. The lack of attention given to the problems of learning mathematics. We still do not know the most effective time to teach the various topics.
5. The number of associated issues that must be addressed in the mathematics classroom such as language development, developing problem solving ability, career information, reducing mathematics anxiety, and integration of computers.
6. The failure of the system to adequately recognize and to give methods for coping with individual learning needs in recognition of the various degrees of preparation and readiness on the part of the students.
7. The total time available for the teaching of mathematics.

Many of the above problems are interrelated and the changing of one variable will have impact on the others. Let me briefly address each of the above issues.

1. SPIRAL APPROACH

The so-called "New Mathematics" in the late 50s and early 60s saw the introduction of a spiral approach to the various topics of mathematics. A student would spend a short time each year on a topic, to be reviewed and expanded upon in each succeeding year. The intended result was that students would see mathematics as a unified subject rather than a series of unrelated topics. I suspect that a closely related assumption was that a short exposure to a topic and re-visit to that topic in successive years would improve retention along with the integration to other topics. This building-block theory of learning was strongly supported by cognitive psychologists who try to develop some explanation of how students learn.

Consequently, a topic like Analytic Geometry was spread over a five year span in high school, Euclidean Geometry over grades 9-12, and Relations and Functions introduced in grade 10 and studied each year through to grade 13. The new guidelines published in 1985 have maintained this philosophical approach. I say "philosophical" because no one has ever proven that it is effective in its current format.

However, the feedback received from teachers and from my own experience in attempting to cope with a curriculum organized in this fashion, leads me to the conclusion that there are severe problems associated with this approach. The general consensus to this organization of curriculum is that it is neither successful nor efficient. Most students do not see mathematics as a unified subject nor do they possess the technical skills demanded at the university or college level when they have completed their high school mathematics.

With the current organization of the curriculum, insufficient time is available for any one topic and there are too many different topics, causing their relationship with each other to be lost in obscurity. The spiral

structure fails to recognize that learning mathematics, although cumulative and somewhat hierarchial, demands sufficient time to assimilate vocabulary and concepts, as well as time to practice with the related ideas of the topic before they can become integrated into long term memory. In simple terms, we do not spend enough time on each level of the spiral.

The length of time to become familiar with the concepts of Euclidean Geometry and the approach of writing a formal proof is much longer than is permitted in any of the current guidelines. Consequently, many schools experience dismal failure with this topic. In our own board, we have stated simply that we do not expect students to write out a proof of a theorem or a difficult deduction on examinations because they have not had sufficient time to practice this skill during the course. One might ask why we are bothering to teach Euclidean Geometry at all?

Another example to illustrate the failure of the spiral approach is associated with the topic of Analytic Geometry. Prior to the curriculum revisions in the 60s, there existed a single course in Grade 13 on Analytic Geometry. Students spent an entire year becoming familiar with an approach to geometry, at the end of which they had acquired a set of skills and some sense of appreciation for the relationship of algebra with geometry. This material has now been spread over the grades from 10 to 13. Most students are not prepared for the abstract nature of this material at the earlier grades, and as a result are not very successful with this topic. By the time they reach the senior grades some are still confused about finding the equation of a straight line.

Without going into more detail, the conclusion is that as a result of the spiral approach to curriculum, too many of our students end up knowing a little of this, a little of that, but not mastering very much in the process.

2. CONTENT OVERLOAD

Who should decide what content is appropriate for students in high school? Much of the content in our current advanced courses is a direct result of post-Sputnik changes suggested by mathematicians from universities, many of whom never have had to teach adolescents who struggle with elementary mathematics. If one reads the history of this development it becomes clear that several very forceful individuals influenced the whole movement (Moon, 1986). Much of this input was theoretical in concept and the curriculum was never validated in the sense of having a full testing and validation review. Although some of the more abstract ideas initially brought into the curriculum have been discarded (i.e. set theory, different number bases, group theory), the curriculum still demands a measure of maturity that is not evident in many of our students.

Closely related to the nature of the content is the volume of material that appears in the Advanced courses. Unless we truly isolate the top 20% of our students, the volume of material is excessive. If one examines the content of the new grades 11 and 12 advanced courses in mathematics, and writes down all the specific objectives included, it becomes evident that there are far too many different concepts for most students to master in the time that is available. The very bright will have little difficulty, but we do not have only very bright students in our advanced classes.

It is difficult to know exactly what is a realistic expectation in today's society where most students have part-time jobs, many are coping with family breakups, and television provides an added distraction from homework. However, most teachers agree that if more time was devoted to fewer topics, then a greater sense of accomplishment would exist.

Thus we see that the content issue is really three-sided. First there is the question of how much content is realistic, secondly is the question of what exactly should we be taught, and thirdly is the question of when it should be taught. The international studies that have been done are less than reassuring in answering any of these fundamental questions. It becomes difficult to compare what is happening in Ontario with any other jurisdiction because the populations that are being compared are so diverse. One is left with the impression that no one is quite sure of what should be taught to whom or when.

3. APPLICATIONS

A serious defect in the advanced level courses from my perspective, is the lack of applications of content relevant to the experiences of the students. This is largely a result of the abstract nature of some of the topics. The new texts now being produced are beginning to address this problem; however, the Ministry

guidelines still emphasize the theoretical rather than the applied. Using trigonometry as an example, the emphasis on the function approach is appropriate for a mathematician, but not to most students who could better relate to solving problems with triangles related to surveying or the applications to statics and forces.

The whole concept of the function is a wonderful unifying theme from the perspective of a mathematician. However the average adolescent is probably more interested in knowing how it relates to some practical problem. In general, the abstract theoretical approach is not appropriate to the age and maturity level of most of our students. Considerable resources should be directed to developing a curriculum that attempts to integrate the mathematics with the science, technology, and social sciences studied in the other courses. For those few students who can appreciate the theoretical framework of mathematics, the enriched level is available in some of our schools. The tragedy is that too many of our students are being exposed to a mathematics curriculum that is not appropriate to their needs or interests. The lasting effect is a distaste for mathematics.

The new general level courses in mathematics at the grade 11 and 12 level are more appropriate for most of our students. The difficulty with these courses is the social stigma that is attached to them as well as the fact that some community colleges do not accept them as prerequisites for some of their courses.

In our area, approximately 80% of our students are taking Advanced Mathematics courses supposedly preparing students for university mathematics. No wonder that we have so many students experiencing difficulty and anxiety.

4. PROBLEMS IN LEARNING MATHEMATICS

The time has come to examine the emphasis in the mathematics classroom. Are we devoting too much time and energy to the teaching at the expense of the learning? The two are hopefully related, but there is no doubt that in many classrooms the delivery of the content is dominated by the teacher with students passively participating. The major reason for this mode of delivery lies with the sheer volume of the content along with the fact that this is the way that most of the teachers were taught. The teacher feels compelled to teach, teach, teach, - otherwise the course will not be covered and students will not be prepared adequately for the next grade.

Some classroom teachers are experimenting with group work, individual projects, and discovery lessons. The rewards are immense and these departures provide some recognition for the various learning styles that are present within any class. However, most teachers feel that such departures from the traditional Socratic approach will result in less coverage of content. Whether or not this is the case is quite immaterial if one adopts the philosophy that the purpose of education is to create an independent learner.

More teachers of mathematics must begin to address the issue of providing opportunities for students to become actively involved in their own learning, to examine the process by which they learn, and to develop the skills they will need to survive in a world that will demand flexibility and ingenuity. These skills are not learned by passively listening to a lecture. No one out there in the "real world" is sitting in rows and moving at the sound of a bell every 40 or 50 minutes.

If content is preventing teachers from addressing the process of learning, then the content must be reduced. There is far more mathematics to be learned than can ever be taught. We must sample the content that is appropriate, and use it as the vehicle to teach self discipline, self reliance, and confidence is one's ability to become an independent learner. The new Ministry guidelines certainly support this philosophy, but unfortunately the content objectives prevent their implementation.

5. NON CONTENT ISSUES

The new guidelines emphasize many issues that should be addressed by the classroom teacher. If we examine just a few of these, we find that the volume of material to be taught prevents much time or effort being devoted to them.

We might first examine the whole issue of language development. Listening, speaking, reading, and writing skills should be an integral part of the mathematics program. This could partially be addressed if more cooperative learning was to occur in the

classroom. If students were made responsible for class presentations, short research projects, and brief historical summaries related to mathematics, then these language skills could be enhanced. There is potential here to cooperate with the language teachers. However all these diversions take time from the classroom teaching, and consequently rarely occur.

Secondly we might examine the whole issue of problem solving. Everyone will admit that this is an ambitious and worthy objective. However it seems to rarely occur. In attempting to cover the core material of the new grade 11 and 12 courses in mathematics, most teachers find little time available to even consider problem solving activities. As a matter of fact, there is insufficient time to teach the core if you allow for testing, review, examinations and days lost for various student activities.

Thirdly we might consider the problem of mathematics anxiety that is displayed by many of our students. I might suggest that there is also considerable anxiety created in our teachers when we reflect on the other issues discussed in this paper. However, let us concentrate on mathematics anxiety in our students. Why do we not hear of science anxiety, English anxiety, of history anxiety? There must be something peculiar about the subject *or* the way the subject is taught. I rather suspect that it is both, but when something is as well documented as mathematics anxiety, then clearly the process of teaching mathematics deserves close scrutiny. As mentioned earlier, a considerable proportion of the anxiety might be the result of the fact that we have so many students taking advanced mathematics courses when they are inadequately prepared for the work involved.

When we combine the factors of inappropriate content, high volume of material to be covered, and inadequate preparation of the student, it becomes obvious why students experience anxiety and why failure rates in mathematics are higher than in any other subject.

I need not expand on other issues such as integrating the computer into the classroom; discussing careers involving mathematics; incorporating values issues into the curriculum; stressing applications; and increasing student participation through experiential approaches, to emphasize that the mathematics teacher is being asked to perform an impossible task using current methodology.

6. STREAMING STUDENTS

If we assume for a moment that ability to learn mathematics is somewhat normally distributed, then it becomes obvious that something is drastically wrong with the enrolment structure in mathematics. The Ministry has attempted to design Advanced Mathematics courses that will prepare students for the requirements of university mathematics and science. Since approximately 25% of our students proceed to university and only about half of these take university mathematics, it might be argued that at most 15-20% of the students need the mathematics at the advanced level. These courses were designed for this select group of students.

However, in our area, 80% of our students are still in the advanced programme in mathematics by the end of Grade 12. Does this mean that students are brighter than students elsewhere? This obvious mis-match of students with programme creates many problems.

Those schools that are able to stream their students into enriched and advanced classes meet the Ministry expectations in their enriched classes. They teach a modified course in their advanced classes. Other schools are simply adjusting the curriculum to justify average failure rates at about the 15% level. The results of our city-wide testing programme verify this. With summer school make-up programmes, many of the students who fail at one level mange to be promoted to the next level usually, but not always, no better prepared. These students need more time than the regular 110 hours to learn the material of the course.

Streaming to create enriched level classes does provide a partial solution in those schools with sufficient numbers to justify the creation of a single section class, but in most schools this is not a viable option. The argument against streaming among the advanced classes in most schools is that the lower group will dictate the coverage because they all must write a common examination. You might just as well leave the classes as a heterogenous mix of ability and hope the better students will help motivate the others.

The reality is that if you want to keep 80% of the students in the Advanced programme then

you must change the programme. This is exactly what is happening.

7. TIME AVAILABLE FOR MATHEMATICS

The evidence obtained through our city-wide testing programme at the grade 12 level in mathematics over a three year period indicates that there is insufficient time available for our students to master some of the concepts of the course. If we really expect our students to master some of these concepts and methods, then they must have more practice. This translates into more time.

Semestering seems to aggravate this problem even more. Students in semestered schools do not have the same amount of time to absorb concepts as students in non-semestered schools as they are rushed to cover the material in a shorter period of time. Needless to say this presents some learning difficulties to many students. Short-cuts are taken and there is little time to review, pause, wander off topic, or enjoy a divergence into problem solving. In short, the semester system is a pressure system for both student and teacher. Since the trend across the province seems to be to semestering, this problem will simply increase. Now that some of the problems confronting mathematics teachers have been outlined, it is time to look for possible solutions. My suggestion for a long term solution is that the Ministry establish some criteria for success in mathematics at various stages throughout the curriculum. There are clearly two choices: either reduce the amount of content so that it can be taught in a realistic fashion, or give the teachers the support they need to guarantee that only qualified students are admitted into the advanced courses.

Students, and teachers, do not have a standard other than the one created by the individual teacher or school. This creates great variation across the province and even within a school system. There will always be variation in teaching and in student performance, but as long as universities demand over 80% in mathematics in order to be accepted into mathematics related courses, then the students in our advanced courses in Ottawa who obtain 80% should be roughly equivalent to those students in Toronto obtaining 80%. Today we cannot make this statement with reliability.

Why not have a provincial examination at the end of grades 8, 10 and 12? This would create a standard to be attained by students wishing to remain in the Advanced programme. These examinations could be marked locally and the examination would count a certain percentage (say 35%-50%) towards the final mark. Such a system would accomplish the following:

a) It would define a level of achievement for all students and teachers in the province. Expectations of performance would be clearly defined.
b) It would provide a common measuring instrument so that individual variation in evaluation would be reduced. (It will never be eliminated)
c) It would help reduce the inflation of marks because it is much easier for an external authority to set a standard that it is for the individual classroom teacher.
d) It will provide a clear message to students and parents that high marks need to be earned through consistent effort and performance.
e) It will provide an incentive to incorporate problem solving into the curriculum as the Ministry would establish minimum standards in this area. My personal preference would be to make a problem solving folder an integral part of each advanced course.

Most other jurisdictions in the world have clearly defined objectives along with some sort of external examination process in mathematics to demonstrate for students the expectations and the level of achievement needed to be successful. For some reason, Ontario has not followed this obvious process. City-wide common examinations have been successful in meeting some of these needs, but if the final marks on which university acceptance is determined are lowered in one jurisdiction as a result of this testing process, then these students are at a disadvantage with students from another location where there is no common standard. It is for this reason that some universities have rated various schools and adjust their marks accordingly.

In conclusion, I think the mathematics curriculum needs a re-examination so that the spiral approach is revised, the content, where possible, is more closely related to applications, problem solving is truly integrated, and clearly defined student outcomes are evaluated by an external province-wide testing scheme so that only those students who have demonstrated the willingness and ability to work at the advanced level are admitted to the advanced programme. Until these changes

occur, our students will continue to receive a confusing message about mathematics and about their expectations beyond the high school.

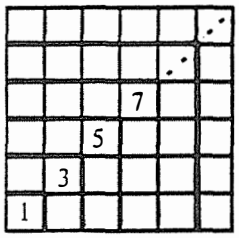
References

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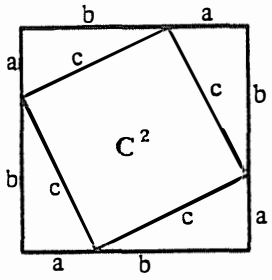
Teaching Tips

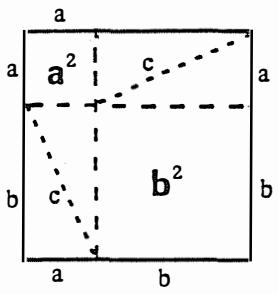
UNWRITTEN PROOFS OR PROOFS WITHOUT WORDS

I have been playing around with some unwritten proofs. I have come up with these. Perhaps you can think of others. (Proofs without words) Send them in.

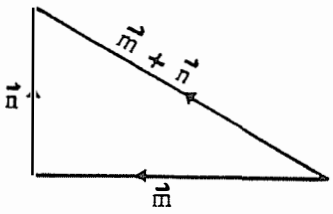


$$1 + 3 + 5 + 7 + \text{---} + (2n - 1) = n^2$$





$$a^2 + b^2 = c^2$$



$$|\vec{m} + \vec{n}| \leq |\vec{m}| + |\vec{n}|$$

Suggested by:
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TEACHING MATHEMATICS WITH KNOTS AND ROPE

by
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There are many variations on the tale of the Gordian Knot, all of which extol the cleverness of Alexander the Great in untying a puzzling tangle of rope. Here is one:

> In the town of Gordian there was a very old and famous farm wagon. The pole of this wagon was attached to a wooden ox-yolk by a pin and a strange knot. The knot was unbelievably complicated and tied so cunningly that it was impossible to find a free end. The people believed that the wagon once belonged to King Midas. They also told a story that the person who could loosen the knot would become ruler of all Asia.
>
> One day a young general visited Gordian. He had just won a great battle and had ambitions to become the ruler of many countries. The local people told him the legend of the knot. A man with such great hopes could hardy ignore the wagon, the story, or the knot. But it was a terribly puzzling knot with no free ends.
>
> The young general did not hesitate. Instead of attacking the knot, he slipped the pin free and pulled the ox-yolk out through the loops of the knot. After that, to everyone's surprise, the knot fell apart into many separate pieces.
>
> The young general went on to be a very mighty conqueror and king. And if he did not exactly rule all Asia, he came closer to it than any other man before and since.
>
> He is known to history as Alexander the Great.
>
> (Horwood, 1969, p.2)

Other stories would have Alexander solving this problem in a less subtle manner with the aid of his broad sword. However the original event happened, if at all, the point is well made. A knot can be a source of intellectual challenge.

Nowadays, knots are important to us in more ways than just the securing of shoelaces. Many recreationists - including sailors, mountaineers, spelunkers, campers, and fishermen - depend on a variety of knots to perform essential tasks (Chisnall, R. 1987.). Knots are not just antiquities to professionals either. Bookbinders, butchers, firemen, leather workers, linemen, riggers, steeplejacks, and surgeons are but a few who use knots in their respective occupations (Budworth, G. 1985). Even forensic investigators analyze knots found at the scene of a crime to implicate likely suspects (Budworth, G. 1982).

Robert Chisnal is a graduate student and instructor at Queen's, founding member of the International Guild of Knot Tyers, consultant and rock climber with a keen interest in mathematics and conundrums. Adventure and experiential education are the focus of his research.

Far from the historical romanticism of conquering generals, but not so distant from the structure of recreational and occupational knots, lies the world of topology. Mathematicians are concerned with geometrical properties that remain unchanged by continuous deformation. The concept of neighbourhood and continuity are important. A topological knot, a concept that can often be modeled by a length of cord, is defined as a one dimensional curve situated in three-space. It has no ends and does not intersect itself. Powerful techniques have emerged from the study of topology that have applications in biology, physics and chemistry (Lipkin,R. 1988).

This article is not about topology per se; rather it is about how everyday, practical knots and bits of rope can be employed as educational tools in the classroom. The metaphoric value of practical knotcraft in mathematics is the focus of the discussion.

Familiar knots, like the Reef Knot (see Figure 5 to follow) or Figure Eight Knot (see Figure 4, to follow), are very aesthetic and concrete items with which mathematics students can work. Knots are three-dimensional patterns. They can be icons for many mathematical abstractions.

Many topics in mathematics could be illustrated with knots, but only a small selection is offered here: plane geometry, sequences, transcendental functions, transformations, group theory, logic, and the nomenclature of invariance. The following activities are presented as basic outlines for instructing students. The reader is invited to adapt these ideas as needed and create new ones. The intention is to inspire imagination and reveal the pedagogical potential of knots.

PLANE GEOMETRY

At first one might ask how practical, three-dimensional knots can be utilized to motivate students to learn about polygons, congruency, and the summation of angles. Here is one activity that has been adapted from a popular knotting trick *(Ashley, C. W. 1944)*.

Tying a Pentagon

Tie a standard Overhand Knot in a long strip of paper or piece of ribbon to form a pentagon, as illustrated in Figure 1.

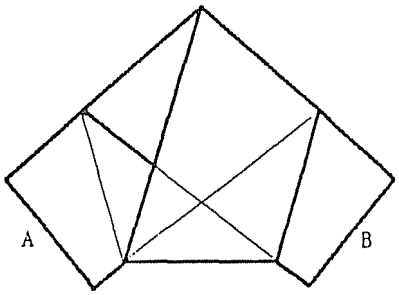


Figure 1. A Pentagonal Overhand Knot.

The Overhand Knot (Figures 2. and 3.), often called the Trefoil or Thumb Knot, is the simplest knot that can be tied and this activity serves as a good warm-up for more complicated investigations.

The letters A and B in Figure 1. indicate the working ends or wends of the original strip of paper. The thicker lines depict the visible edges of paper while the thinner lines represent those edges that are concealed from view. It is important to use a strip of paper or ribbon that is a regular rectangle (i.e. having parallel sides) and this point should be emphasized to students because it is vital to the proof that the resultant structure is a regular pentagon. It is also important to use a strip long enough to permit tying. Strips measuring 30 centimetres by 3 centimetres work well.

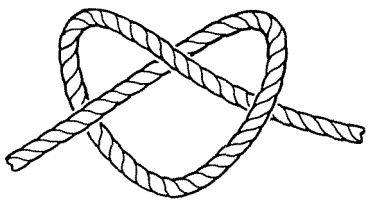


Figure 2. A Standard Overhand Knot Tied in cord.

With care, a perfect pentagon can be fashioned by folding the paper or ribbon at each bend in the Overhand Knot. Now the student is faced with proving that this structure is in fact a regular pentagon. One tack is to examine congruent parallelograms.

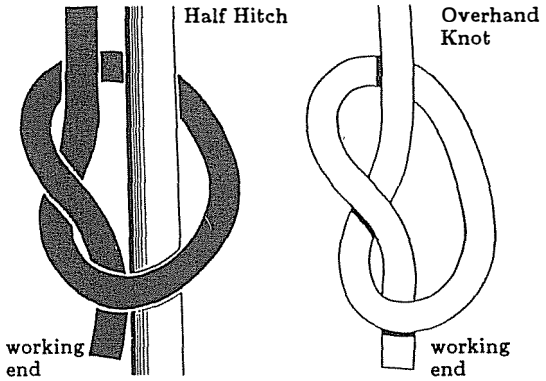
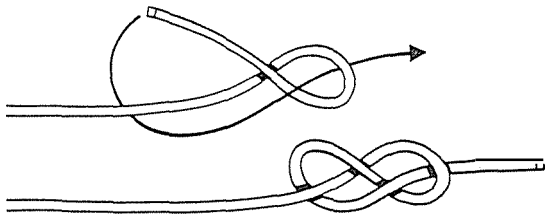


Figure 3. Half Hitch and Overhand Knots

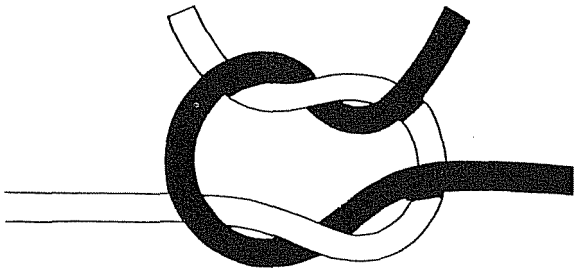
Further inquiry can be generated by asking the following questions. Are there any other ways of tying regular polygons in rectangular strips of paper? Are there possibilities for creating irregular polygons? If so, how can one prove that these knotted strips of paper form regular or irregular polygons



4. Figure Eight Knot.

Of course there are numerous knots which will generate polygons. For example, the Figure Eight Knot (Figure 4.) will form an irregular hexagon, and a Reef Knot (Figure 5.) will yield a regular hexagon. This can be discovered with a little experimentation.

Finally, can any generalizations be made regarding knots, polygons and inherent patterns? This kind of complex question does not lend itself easily to a simple or closed investigation. Some students may thrive on the open-endedness of such questions while others become frustrated. Use discretion in delimiting the activity.



First hitch: right over left and under.
Second hitch: left over right and under.

Figure 5. Reef Knot.

MATERIALS: An ample number (10 per person) of paper or ribbon strips (30 cm. x 3 cm.), plus rope or cord. (See Ashley, Clifford W., 1944.)

SEQUENCES

The following activity provides students with an introduction to some basic topological ideas, but its main purpose is to demonstrate the power of mathematical nomenclature in representing generalizable patterns. The Mobius strip is known to knot enthusiasts as well as topologists and it affords the student a hands-on opportunity to learn about sequences.

Investigating the Mobius Strip.

This is an old favourite for many math teachers. Named after the German mathematician and astronomer, August Ferdinand Mobius, a Mobius strip or band is formed by turning one end of a rectangular ribbon 180 degrees and then fastening it to the other end.

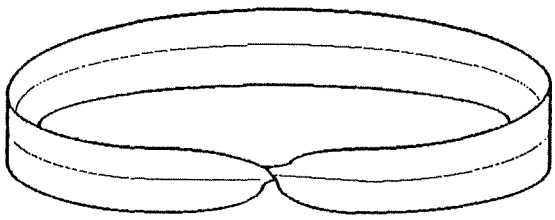


Figure 6. Mobius Strip: A One-sided Ring With One Twist.

The resulting ring has one side only, as illustrated in Figure 6. Students are usually intrigued by this and become quite inventive in trying to prove or disprove that there is only one side. The Mobius strip offers some exciting possibilities for classroom investigation.

Although the Mobius strip is primarily a topological oddity, it has some practical applications. Quite often factories will employ a drive belt which models the Mobius strip. This ensures that both 'sides' of the belt will wear equally over pulleys. The reason for this will become apparent in the following activity description.

Draw a straight line along a strip of paper, as depicted by a thin line in Figure 6. Join the ends with tape to make a Mobius strip; then cut along that line all the way around the ring. A solitary ring is produced which has several twists and two sides.

What happens if the ends of the strip of paper are joined without any twists and a cut is made along a central line, shown in Figure 7?

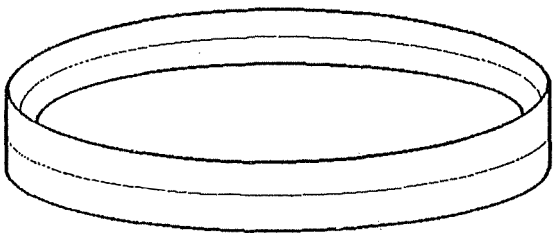


Figure 7. A Two-sided Ring With No Twists.

Two unlinked rings are produced of course. If the ends are joined after three 180 degree twists, a central cut transforms the original ring into a larger single ring (i.e. with greater circumference) in which an Overhand Knot is tied. This discovery can often spark student interest in extending the investigation.

By varying the number of twists, rings of different characteristics can be created. Even more surprises develop when a central cut is made in each one. Patterns quickly emerge. Tables 1. and 2. summarize several of these basic patterns. Students should be encouraged to analyze the emerging patterns and summarize them algebraically, thereby facilitating predictions about subsequent rings.

Ti	Tc	Rn	Si	Sc	F
1	4	1	1	2	1 ring
3	8	1	1	2	1 overhand knot
5	12	1	1	2	1 double overhand knot
7	16	1	1	2	1 triple overhand knot
n	2n+2	1	1	2	1 (n-1)/2uple overhand knot

Table 1.

An interpretation of the rings generated by the central cut is somewhat difficult to make when higher numbers of twists are used. It is particularly challenging to determine the number of twists in the resulting rings. When examining the sequence of rings produced by unevenly-numbered 180 degree turns, one must ensure that the twists are not incorporated into the actual Overhand Knots, wherein they would be obscured. Make sure the twists have been pushed into the unknotted portion of the strip before counting.

Ti	Tc	Rn	Si	Sc	F
0	0	2	2	2	2 separate rings
2	2	2	2	2	2 single-linked rings
4	4	2	2	2	2 double-linked rings
6	6	2	2	2	2 triple-linked rings
n	n	2	2	2	2 (n/2)-uple-linked rings

Table 2.

Legend
Ti = The number of 180° twists in the initial strip of paper.
Tc = The number of 180° twists in the resulting ring(s) or member(s) after the central cut.
Rn = The number of rings or members present after the central cut.
Si = The number of sides or surfaces on the initial ring of paper.
Sc = The number of sides or surfaces on the ring(s) of paper after the central cut.
F = The form of the resulting members(s)

This may take practice and students should be cautioned to use patience and care in making and analyzing their rings. This is an excellent activity for group work.

MATERIALS: At least 20 strips of paper or lengths of ribbon for each student (40 cm. x 3 cm. works well). Scotch tape and scissors are required as well.
(See Stewart, Ian. 1987).

TRANSCENDENTAL FUNCTIONS

Although high school students are exposed to a variety of functions during their mathematics education, probably very few encounter or know what a transcendental function is. Although the following activity does not involve actual knots, rope and other hands-on materials are required. This exercise should be reserved for more advanced students.

CATENARIES AND PARABOLAS

If string, cable, rope, or chain - anything of uniform structure and flexibility - is suspended freely between two attachment points, the curve formed is a catenary. The actual shape will vary with the size and weight of the cord or cable and with the amount of slack between the suspension points.

The seventeenth-Century mathematician and physicist, Jacob Bernoulli, discovered the formula of the catenary:

$$y = \frac{a(e^{x/a} + e^{-x/a})}{2}$$

where **a** is a constant and **e** is Euler's number.

The catenary is a transcendental curve, not algebraic, in that the function is the tractrix, sometimes called the Tractrix of Huygens.

The parabola is closely related to the catenary. If a parabola rolls along a straight line, its focus traces out a catenary. Also, suspended rope can be changed into a parabola by hanging evenly-spaced, equal weights on it. The more uniform the weights, the more continuously they are distributed, and the heavier they are compared to the weight of the rope, the more closely the drape of the rope approaches a perfect parabola. The cable of a suspension bridge approximates a parabola and there are other examples in similar commonplace structures.

Equipped with these facts - or any portion thereof - the student can begin an investigation of suspended cords. The amount of information furnished will depend on student ability and the emphasis desired. For example, it may be appropriate to have a group of students determine the value of e given a known constant a for a particular suspended rope.

Utilizing a lamp, a student can carefully trace the silhouette of a suspended length of pliant cord onto a large piece of graph paper and analyze it. Such an analysis could be performed as a class exercise on a manageably larger scale (say with a 5 metre length of cord in a gymnasium). Students could even attempt to design their own method of analysis. For very gifted students, have them conduct an investigation of the curve without prior knowledge of catenaries or any hints about a method of analysis.

The shape of the curve can be varied by moving the suspension points in or out to vary the analysis. Weights can be suspended on the cord to create approximations of parabolas for a comparative analysis. As well, the notion of measurement error should be mentioned prior to the analysis.

MATERIALS: Rope or cord that does not kink (1 to 3 metres, depending on the scale desired), electric lamp, large sheets of graph paper, uniform weights, suspension points (retort stands will do). (Horwood, R. H. 1969).

TRANSFORMATIONS AND GROUP THEORY

There are numerous lab materials that are commercially available which facilitate the instruction of geometrical transformation concepts. Knots afford a unique and relatively inexpensive alternative. Many knots could be used in the following activity, but some fairly simple and common ones are used in the following description.

Trambles, Transformations and Symmetry.

Some mention of *trambling* is in order prior to any description of the actual activity. The word *tramble* has been coined by persons involved with practical knotcraft. A tramble is a way of converting a given knot into another similar knot with a simple modification. For example, Figure 8. illustrates how a Reef Knot can be transformed into an Oblique Sheet Bend. The working end, or wend, of the white cord is removed from the bight formed by the black cord. The working end of the white rope is then tucked in behind itself.

Whole families of knots can be related in this way. The Reef Knot, Jinx Knot, Tumbling Thief Knot, Granny Knot and Carrick Bend can be *mapped* (not in the mathematical sense) together by a sequence of tucks and trambles (Budworth, G. 1982). A Useful exercise for students might be to have them denote these changes utilizing a system of symbols or operators.

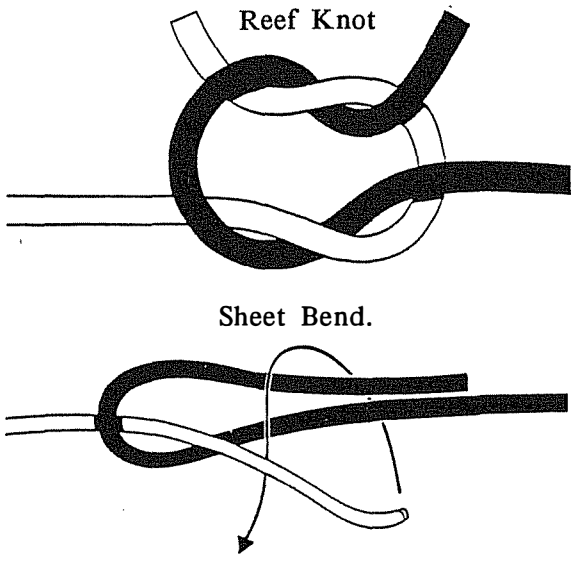


Figure 8. Trambling a Reef Knot to a Sheet Bend.

Of particular interest in this activity are the following structurally similar but behaviourally

(i.e. regarding strength and security) dissimilar knots: Reef Knot, Granny Knot, Tumbling Thief Knot, and Jinx Knot. If these knots are depicted in two dimensions, the structural relationship can be observed a little more easily (Figures 9. and 10.). The Reef Knot is illustrated in Figure 5. (also see Figure 9. 1a, 1b, 2a, and 2b). Note that the working ends are on the same side as are the standing parts (the greater portions of the cords left untied). Arrange the Reef Knot so that the wends and staring parts are diagonally opposite, and it becomes the Tumbling Thief Knot (see Figure 9. - 3a, 3b, 4a, and 4b). A steady tug on the two standing parts will reveal how the latter knot acquired its name. Similarly, the Granny Knot, which is depicted in Figure 11. can be changed into a Jinx Knot (see Figure 10. wherein the right-hand knot in each cell is a Jinx Knot). Situate the working ends and the standing parts in diagonal opposition. The Jinx Knot should be even more insecure than the Tumbling Thief Knot.

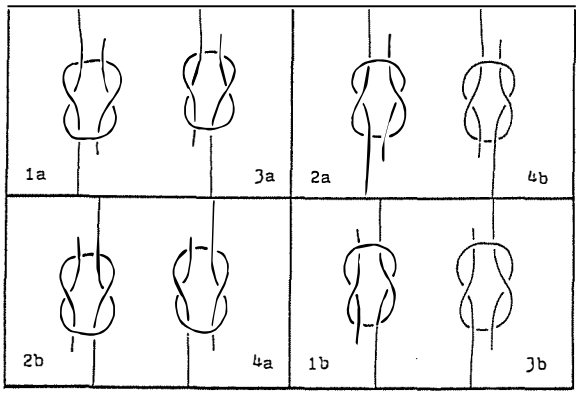


Figure 9. The Family of Reef and Thief Knots.

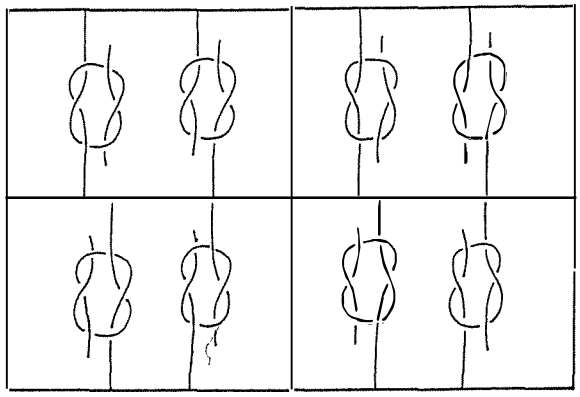
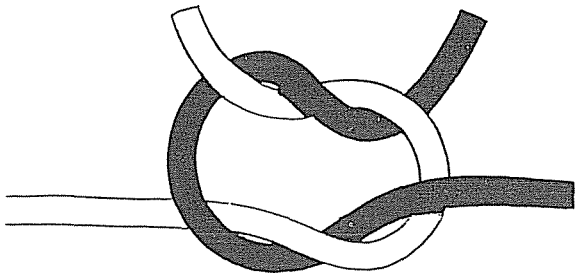


Figure 10. The Family of Granny and Jinx Knots.

The tug test is a good method of distinguishing Thief and Jinx Knots from Reef and Granny Knots respectively.



Both hitches are right over left and under.

Figure 11. Granny Knot.

By simply changing crossovers or switching working ends with standing parts, the knot tier can tramble any of these four knots into one of the other three. Here is a preliminary problem for students. How could these operations be represented mathematically?

There are sixteen basic trambling possibilities overall and these are illustrated in Figures 9. and 10. Figure 9. illustrates Reef and Thief Knots. Figure 10. depicts the family of Granny and Jinx Knots. How can their be sixteen possibilities when there are only four basic knots? Since these are topological structures, transformation geometry can be applied. Congruent structures can be identified by rotating and flipping each variation.

Actually there are eight truly different knots overall: a dextral and sinistral version of each of the four basic knots mentioned above. In Figure 9. knot number 1a. is a Left-Handed Reef Knot. A 180-degree rotational transformation produces knot number 1b. These are congruent structures with different orientations. Knot 2a. is a Right-Handed Reef Knot. Another rotational transformation demonstrates that 2a and 2b are congruent. Tramble 2b to generate 3b, a Tumbling Thief Knot. Transformations involving reflection and rotation can generate the remaining three Tumbling Thief Knots, 3b, 4a, and 4b.

The same kind of analysis can be applied to Figure 10. and the two families can be related by trambling.

Hence, the concept of reflection and symmetry can be illustrated quite nicely with this exercise. As well, group theory can be introduced. Students may choose to draw or tie the knots they analyze. Overhead transparencies help as well.

MATERIALS: Several lengths of cord for each student, onion skin, coloured pencils, overhead transparencies. (See Ashley, C. W., 1944; Lietzmann, W., 1965.)

LOGIC

Knots offer plenty of opportunities for teaching logic. The following example is offered because it is somewhat different from the other activities and it is grounded in something practical.

Highway Interchanges Analysis

The shape and complexity of cloverleafs and highway interchanges offer a unique opportunity for mathematical investigation. Basically, the problem can be described in terms of providing connecting roads which fulfil the following expectations in the simplest and most efficient manner:

1. Motorists who intend to stay on one of the major highways expect to be able to drive straight ahead.

2. Motorists who wish to turn from one highway to another expect to do so without interrupting or being interrupted by other traffic flows.

3. Motorists who wish to make a U-turn expect to be able to turn around within the confines of the interchange.

The initial step in analyzing this problem is to recognize that each major highway consists of two separate roads running parallel. These are restricted to conducting traffic in opposite directions. With all of these rules clearly identified, the student has a conceptual foundation from which to design cloverleaf intersections which involve the crossing of two or more highways. It might be valuable to have students create their own precise rules in defining the limits of a cloverleaf. The three expectations cited take a motorist's perspective.

There are only a few possibilities for a two-highway cloverleaf, but by increasing the number of major highways converging at the interchange, complex knot-like structures emerge. The form and pattern can be examined mathematically. The problem of representation and nomenclature then becomes evident and essential to the design procedure. Students will discover that basic rules can be derived most easily from simpler systems and subsequently applied to more complicated ones. Topological comparisons can be made as well to determine whether two similar designs are congruent or not. The students can be asked to summarize their findings algebraically or symbolically and perhaps apply them to similar situations, like house wiring or plumbing, for example.

MATERIALS: To enhance the presentation of this activity, the following items might be utilized: road maps, aerial photographs, civil engineering highway plans, overhead transparencies, models and/or model-making materials. (See Holden, Alan. 1983.)

THE NOMENCLATURE OF INVARIANCE

As mentioned initially, topologists are concerned with the invariance of geometric characteristics during continuous deformation. Attendant to this is the problem of nomenclature; how to label structures and represent operations.

Aficionados of practical knotcraft have been struggling with the lack of uniformity throughout the knotting literature regarding names and terminology. A knot can have several names. For example, the British Square Knot is something quite different from the American version. Some knot experts have endeavoured to classify practical knots using crossing points, taxonomies, or function. More recently, symbolic and mathematical systems for representing tying methods, trambles, and so forth have emerged.

Both the topologist and the practical knotsperson appreciate the complexity of the task. How might students approach the problem?

Mathematical Nomenclature for Knots.

Enthusiasts of practical knotcraft have struggled to find a method of knot classification that can be applied universally, one that is simple to learn and use. Most have found that classical topology is too esoteric for their purposes. Among the innovations and

systems developed since the time of the Incas, several numerical methods have evolved. These are available from the sources cited.

Introduce the notion of numerical knot classification to students and present then with a few simple knots to which they can apply their own systems. Keep the scope of their investigation narrow so as to avoid confusion and frustration. The Reef, Granny, Jinx, and Tumbling Thief Knots are excellent starting points. See what they can devise. Add several new and slightly more complex knots so that they can test the limits of their systems and introduce modifications. Adding the Sheet Bend (Figure 8.) to the initial knot list usually will not throw students too much, but the introduction of something like the Double Fisherman might (see Figure 12.). This activity readily lends itself to modifications according to student progress and need.

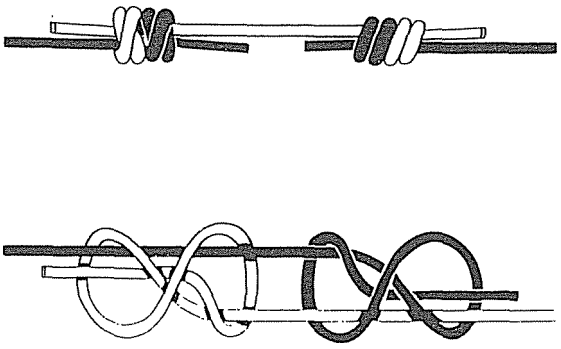


Figure 12. Double Fisherman.

MATERIAL: Lengths of differently coloured rope for each student and perhaps a few simple knot references or drawings.
(See Brown, F. D. 1984. Budworth, G. 1985.)

CONCLUSION

This is just a sampling of what can be done with knots in the mathematics classroom. The references will provide many more creative ideas. Here are some other avenues to pursue:

1. The structural patterns of Single-Line Turk's-Heads.

2. Structure and pattern in weaving, knitting, and macrame.

3. Pulleys, belays, friction, and ratios.

4. A statistical comparison of knot strengths.

5. A statistical analysis of shoelace tying habits.

6. Knots themselves as symbols, calendars, and calculators.

7. Isotopy in *equivalent* knots like the Figure Eight and Pretzel Knots (see Figure 13.).

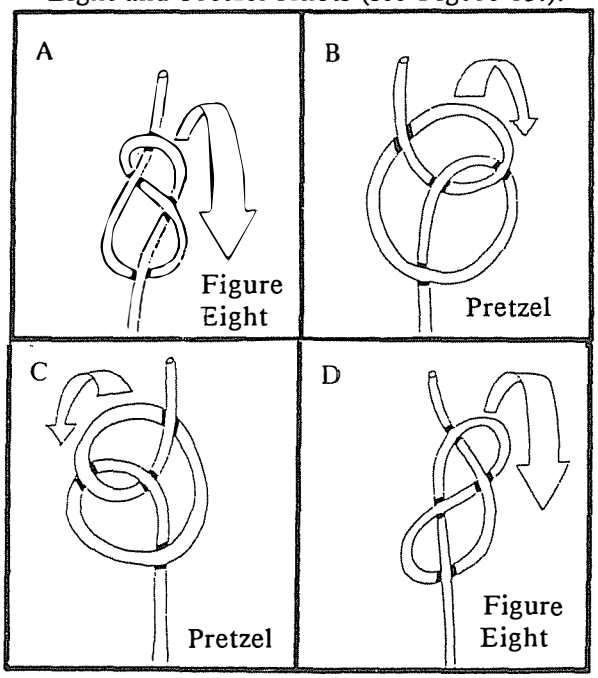


Figure 13. Isotopy of the Figure Eight and Pretzel Knots.

A parting word of caution is in order. For the student who finds mathematics frustrating, the intricate spatial nature of knots may be just another thorn to endure. Whereas some students pick up knots quickly, others are all thumbs. Knots should be introduced in an entertaining manner if possible, and no pressure should be placed on the students to memorize a plethora of knots and jargon. Experience demonstrates that two is the maximum number of significantly different knots a student can successfully learn and recall in one class without too much anxiety.

The mathematical potential and practical appeal of common knots makes them a rich teaching resource. Knots are icons and artifacts for many unusual and exciting

pastimes, occupations, structures, and processes. There is something to stimulate almost any student.

QUIET, CALM DELIBERATION

DISENTANGLES EVERY KNOT

W.S.Gilbert

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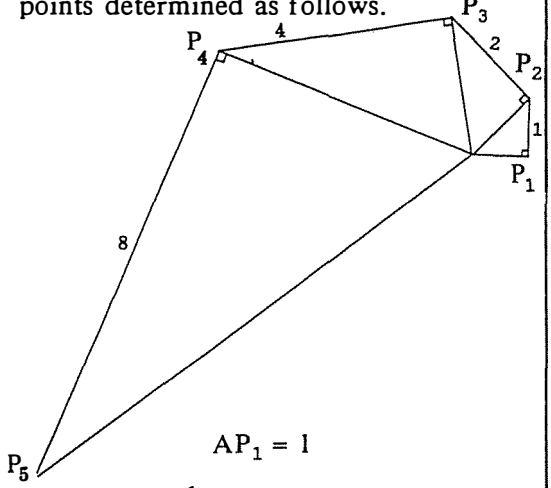
PROBLEMS

by
Ronald Lancaster
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1. Prove that if the roots of a quartic polynomial are consecutive terms of an arithmetic sequence, then the roots of its derivative are also consecutive terms of an arithmetic sequence.

2. Let A, B and C be any three non-collinear points.
Let P, Q and R be points outside of $\triangle ABC$ for which:
$\angle PBA = 45°$ $\angle PAB = 30°$
$\angle QBC = 15°$ $\angle QCB = 15°$
$\angle RCA = 45°$ $\angle RAC = 30°$
Prove that $\angle PQR = 90°$

3. Let P_1, P_2, P_3... be a sequence of points determined as follows.



$AP_1 = 1$

$P_nP_{n+1} = 2^{n-1}$ and $\angle AP_nP_{n+1} = 90°$

for n = 1,2,3...

Find $\lim_{n\to\infty} \angle P_nAP_{n+1}$

SOLUTIONS AND COMMENTARY

1. Dr. Ian McGee and Dr. Peter Ponzo, two excellent professors at the University of Waterloo, recently passed this problem on to me. I plan on putting this question on an assignment for my Calculus students this fall. I think it would be best if I start them off by first giving them a specific rather than a general sequence to deal with.

The first thing that I did was to draw a graph and this helped me to see that the result was a reasonable one. I have outlined several solutions below, if you come up with something different please send it along to me.

Solution 1a

Let the roots of the quartic polynomial be
a, $a + d$, $a + 2d$ and $a + 3d$
and let
$f(x) = (x-a)(x-a-d)(x-a-2d)(x-a-3d)$

I then found $f'(x)$ and simplified my answer. (Yes, it was a mess!) At first I wondered how I could possibly solve the cubic equation $f'(x) = 0$. I tried the cubic formula and gave up. Several days later I woke up in the middle of the night and realized that a root of the cubic equation was $a + \frac{3}{2}d$ (the arithmetic mean of the four roots of the quartic). I then divided $f'(x)$ by $x - a - \frac{3}{2}d$ and solved the resulting quadratic equation. The three roots of $f'(x)$ are given below and you should note that they are indeed consecutive terms of an arithmetic sequence.

$$x_1 = a + \tfrac{3}{2}d - \sqrt{5/2}$$

$$x_2 = a + \tfrac{3}{2}d$$

$$x_3 = a + \tfrac{3}{2}d + \sqrt{5/2}$$

Solution 1b

Let the roots of the quartic polynomial and $f(x)$ be as given in solution 1a. This time I started off by assuming that the roots of $f'(x)$

were consecutive terms of an arithmetic sequence.

I let the roots be:

$$a + \tfrac{3}{2}d - k,\ a + \tfrac{3}{2}d, \text{ and } a + \tfrac{3}{2}d + k.$$

I also let

$$g(x) = (x - \tfrac{3}{2}d + k)(x - \tfrac{3}{2}d)(x - \tfrac{3}{2}d - k)$$

I then found $f'(x)$ as in solution 1, but did not simplify my result. Assuming that g is indeed the derivative of f, it follows that $f'(a) = g(a)$. (Other conclusions can be made, of course.)
From this I was able to show that $k = \sqrt{5}/2$.

Before you object to my assumption about f', I should point out to you that I integrated g, obtained f, and thus showed that everything was in order.

I recently attended a Mathematics and Computer Conference in Exeter, New Hampshire and had access to three sophisticated pieces of software, *Derive, Maple, and Mathematica.* All three programs handled this problem in seconds!

2. Jeff Martin, Mathematics Program Advisor for the Etobicoke Board of Education recently passed this problem on to me. Jeff is well known throughout the province for his work in the area of problem solving. I would highly recommend that you give your students this problem to play with. In order for you to have a chance to work on this for awhile, I will run a solution in the near future. (I have not yet been able to solve it...)

3.

Note that

$$P_nP_{n+1} = 2^{n-1}$$

Also
$$AP_n = \sqrt{1 + (1^2 + 2^2 + 2^4 + \ldots + 2^{2n-4})}$$
$$= \sqrt{1 + \frac{2^{2n-2} - 1}{3}}$$

Thus $\tan(\angle P_nAPn+1) = \dfrac{P_nP_{n+1}}{AP_n}$

$$= \frac{2n-1}{\sqrt{1 + \frac{2^{2n-2} - 1}{3}}}$$

Using the fact that

$$2^{n-1} = \sqrt{2^{2n-2}}$$

you should be able to show that

$$\lim_{n\to\infty} \tan \angle P_n AP_{n+1} = \sqrt{3}$$

therefore

$$\lim_{n\to\infty} \angle P_nAP_{n+1} = 60°$$

This past semester I gave this question to my Grade 12 Mathematics class on their trigonometry test. I liked the problem because it combined two areas of mathematics; trigonometry and sequences and series. I allowed the students to take the question home with them and hand in their work several days later. Because it was a Grade 12 class I did not use the limit notation. The instructions were simply to investigate what happens to the sequence of angles P_1AP_2, P_2AP_3, P_3AP_4... My students really liked the problem too. Most of them took the time to draw an excellent diagram on a large sheet of paper. (Try it yourself and you will understand why I underlined the word large.) Incidentally, I had not yet worked out a solution to the problem, when I assigned it and I was surprised that the limit came out to be 60°.

Finally I would encourage you to generalize the problem. Let $P_nP_{n+1} = r^{n-1}$ and see what happens. (The result is pretty neat!)

Note:
In the last issue of *The Gazette* I included a geometry problem (see Question 3) and promised that a solution would be run in this issue. This problem has been well received by people all over the province and I am still receiving solutions to it. In the next issue I will run a summary of what I have received. If you have not yet looked at it, go back and do so now!

LOCAL INITIATIVES TO EFFECT CHANGE

by
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York University

Editor's note: This article originally appeared in Mathematics 4 Girls, published by the Ontario Educational Research Council in 1988. It is reprinted here with permission.

In January 1982, the Science Council of Canada published *Who Turns the Wheel?* (the proceedings of an invitational workshop) which expressed concern at the under-representation of Canadian women in occupations requiring a knowledge of mathematics and science. A few months later, the Toronto Board of Education published *The Invisible Filter - A Report on Math Avoidance, Math Anxiety, and Career Choices* which presented findings from a study based on data collected during the previous school year on the enrolment of female students in mathematics and computer science courses in 27 Toronto secondary schools. These two publications marked early stages in a growing awareness of the lesser participation of girls in mathematics and science courses and women in mathematics- and science-based careers. Both documents included an examination of factors contributing to the situation and recommended strategies for change.

In the past six years, a spate of events, programmes and initiatives has made it evident that energetic and committed groups of educators throughout Ontario are seeking to encourage schoolgirls to retain mathematics in their timetables. This article will provide an overview of the kinds of interventions that educators and career women have undertaken to promote greater participation of female students in mathematics courses. If the article reflects a Toronto bias, this is due to the strength of the network of dedicated workers in this region and the numerous projects that they have carried out.

Lorna Wiggan is a mathematics consultant with the Toronto Board of Education. She is presently seconded to the Faculty of Education, York University.

Why is it that so many girls drop mathematics as soon as it ceases to be a compulsory school subject? The causes are complex, overlapping, and interrelated. At the heart of the problem is the insidious sexism that pervades the culture; evident in the way boys and girls are socialized, the organizational structures in the school system, and the ways in which traditional expectations function to channel male and female students into different programmes which inevitably lead to different occupations. A review of the literature reveals that many high school girls are unaware of the usefulness of mathematics in careers, perceive mathematics as more suited to their male classmates, lack confidence in their own mathematical ability, and lose interest in mathematics as they advance through the senior high school grades. These findings may be a reflection of methods used in the teaching of mathematics.

What has been done to improve the situation? In the first few years, dissemination of information was of the utmost importance. The Science Council of Canada made copies of *Who Turns the Wheel?* and pamphlets on the necessity of taking steps to improve the participation of women in science and technology available without charge to interested individuals or groups. The Ontario Ministry of Education distributed copies of *The Invisible Filter* in all schools having grades 7 to 12 Ontario Academic Credits. These documents drew the attention of the public in general and educators in particular to the need for action to effect change in the sex-ratios in senior mathematics classes. Many school boards organized workshops for their mathematics and science teachers and guidance counsellors. In some cases, it was the mathematics and science departments that took responsibility for organizing these events, but sometimes the initiative came from an affirmative action committee or women's studies group. Occasionally, it was individual teachers or interested feminists who organized workshops on the

topic of the importance of mathematics in the career development of young women.

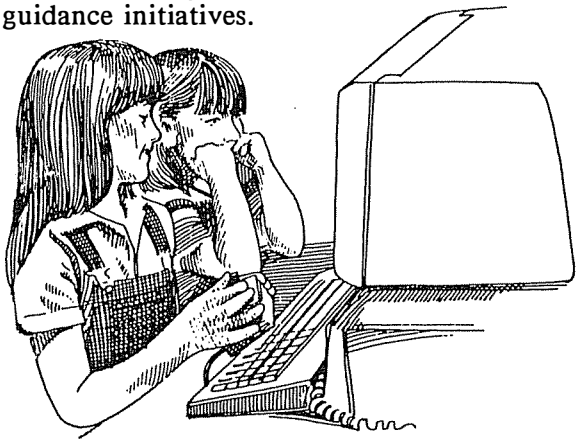
Although the problem of underrepresentation of women in scientific, mathematical and technological occupations could have been approached in different ways, it seemed most urgent to begin work with female students themselves. Many of the earlier initiatives sought to provide schoolgirls with opportunities to meet same-sex role models in workshop settings or as guest speakers at conferences. Others were designed to broaden curriculum content and process by highlighting more creative problem solving, hands on experience, practice in the use of spacial skills, and career information concerning the application of mathematics. The general aim was to provide a more informal and supportive mathematics classroom environment. Field trips to workplaces in business and industry and to universities and colleges of applied arts and technology assumed a different focus to more clearly reflect the relevance of mathematics to women's working lives. One unique programme dramatically changed the high school mathematics classroom environment by creating segregated classes for selected grades. Most of these interventions and innovations were not undertaken as separate and discrete entities but rather in combination with other activities having more diffuse goals. It is, therefore, difficult to evaluate their effectiveness.

Career Conferences

Nevertheless, career conferences for schoolgirls became popular starting points for student interventions. The concept was flexible enough to allow for a variety of modifications to meet local conditions and take advantage of local resources. Modelled after the "Expanding Your Horizons in Science and Mathematics" conference (which was originated in the Lawrence Hall of Science, Berkeley, California), the goals of the conference were to foster the interest of female students in mathematics- and science-related fields, and to provide opportunities for schoolgirls to meet career women working in nontraditional fields. The workshops that were initially presented to students in grades 7 to 12 involved high interest, hands-on activities showing the applications of mathematics and science in many occupations, as well as career information. For many school-girls, it was a brand new and heady experience to attend a full-day event organized exclusively for them with only women presenters and unfolding an unfamiliar world laden with new choices. Enthusiastic educators organized conferences in Toronto, North York, Sarnia, Windsor, York, Peterborough, Sudbury and other areas.

Schoolgirls were not the only ones to benefit from these career conferences. Committed women and men put forth time and energy to turn an idea into reality. Planning and working together brought such people closer when they realised that they shared a common cause. Organizations such as the Canadian Association of Women in Science (CAWIS), Women in Construction, Women in Science and Engineering (WISE), Ontario Hydro, government departments such as the Ontario Ministry of the Environment, and many other groups worked tirelessly to meet the requests from school boards for conference speakers and workshop leaders. The Ontario Women's Directorate provided materials to display at conferences or include in the participants' kits, and female scientists from universities and colleges were generous in donating their own time and encouraging their students to serve as panel members at career sessions or guides on student tours.

Conferences can be costly; as much as $15 000 can be spent on a one-day conference for 400 students. In addition, conferences consume a great deal of time and energy. There are those who question whether the costs are worth the uncertain benefits. Moreover, in a school system of thousands of female students, career conferences can only reach a small percentage. It is perhaps for these reasons that some schools have opted for other kinds of career guidance initiatives.



Several secondary schools in Scarborough hold annual career assemblies for girls in grades 9 and 10. They listen to guest speakers before completing their option sheets for the next grade's subject choices. A few schools in Toronto have also organized sessions for their grade 7 and 8 girls. Also, the Toronto Board of Education is considering an incentive plan to encourage schools to offer equity career workshops for their students in the hope that a greater number of girls will become aware of the importance of mathematics in women's careers.

One particularly interesting version of the career conference idea was a career day held in Toronto school and attended by children in grades 1 to 6. The majority of the presenters were students and staff from George Brown College but some were parents of the children. The presenters came dressed in their working clothes and carrying tools and equipment. They brought materials to show the children or to have them handle. I can remember the wide-eyed excitement of the children as they worked with the truck driver, the chef, and the carpenter - all women - and with the male nurse and the male typist. It was a wonderful learning experience for the children.

Innovative Programmes

"Ms Math" and "Real Women Don't Do Math" are examples of programmes designed to provide female students with a more stimulating and creative experience in mathematics. "Ms Math", a problem-solving programme organized by the mathematics department of the Lincoln County Board of Education, involved a series of sessions offered outside of school hours to interested girls in grades 5 to 8. Parents were also invited to attend some of the sessions. A kit of problem-solving activities was developed to support the programme.

"Real Women Don't Do Math" was a cooperative venture of the York University and North York Board of Education. The primary goal of this three-day, residential programme was to engage female students in creative mathematical activities, but it also provided a taste of university life, career information and the opportunity for discussion. An important element was the exposure of the participants to women graduate students in mathematics programmes.

In the 1984-85 school year, the North York Board of Education initiated sex-segregated classes in mathematics for certain grade 10 students and, in subsequent years, the programme was extended to include grades 11 and 12. The purpose of this pilot project was to improve the attitude of high school girls to mathematics and to increase their enrolment in mathematics courses in the senior high school grades. A research report completed in November 1986 suggests that, for some female students, sex-segregated classes had a positive effect. However, the perceptions of the majority of female students and their parents were that neither attitudes nor achievement had been affected by participation in a single-sex mathematics class. Among the positive features reported were the supportive classroom environment and the comfort of the girls in responding to and posing questions. Teachers and students alike commented on the atmosphere of the classroom. Enrolment of girls in grade 13 courses in algebra and calculus did increase more substantially in the school where the pilot project was carried out than in the school designated as a control school and the sex-segregated mathematics programme was perceived as having had an influence on the rest of the school.

Initial attempts to inform teachers about the need for encouraging girls to retain mathematics in their timetables included the dissemination of published materials, the use of posters and other media, and combining teacher and parent workshops with student activities concerning girls and mathematics. At many of the Horizons conferences, certain sessions were provided for teachers and parents. Secondary school teachers unfamiliar with the issues were encourages to gather data in their own schools as a way of comprehending the extent of the problem in their own setting. The 1985 Intermediate and Senior Mathematics Guideline brought a sense of legitimacy to the concern. In this document, the Ontario Ministry of Education (for the first time) highlighted "math anxiety" and "careers and mathematics" as "cross-curricular components". Attention was drawn to the lesser participation of female students in senior level courses in mathematics and a list was included showing what teachers "should be" doing.

Focus on Teachers

Recently, more attention is being directed to interventions specially designed for teachers because each teacher has the potential to reach hundreds of students. The Women and Mathematics Committee, a standing committee of the Ontario Association for Mathematics Education (OAME) Council, organizes a strand of related workshops for teachers at their annual conferences, conducts a search for suitable women interested in seeking nomination to Council, and publishes articles in the OAME journal, *The Gazette*. Also, the Math-Science Network of the Toronto Board of Education, as its main function, organizes equity workshops and programmes for teachers. One of its programmes, "Job Site Visits", plans teacher visits to workplaces participating in the programme. The teachers then take students on visits to these workplaces where they take part in activities involving mathematics and science and have opportunities to talk with women about the usefulness of mathematics and science in their careers. This year, business, industrial, and government organizations (eg. McLeod Young Weir Limited Toronto Transit Commission, Warner-Lambert Pharmaceutical, Northern Telecom, Esso Petroleum, and the Hospital for Sick Children) welcomed the teachers and their students.

School boards (such as East York, North York and Toronto) and teachers' organizations (such as the Ontario Secondary School Teachers' Federation and the Federation of Women Teachers Association of Ontario) have encouraged the development of teacher expertise in the mathematical education of girls by providing funds for sabbaticals, scholarships, short courses, and other professional development activities. An example is the conference "Women and Mathematics" which was held by FWTAO for about a hundred of its members in the spring of 1987.

Family Involvement

Work with parents is slowly gaining momentum and specific plans and projects are beginning to surface face. In 1983, the Scarborough Board of Education mailed awareness materials to the parents of all grade 8 students. The Toronto Board has recently initiated three activities: a parent workshop, a



video *Math Counts, Science Matters*, and a programme called "Family Math". A great deal of thought and planning, as well as field testing, was put into the parent workshop as well as the video. The purpose was to enable parents to feel comfortable in discussing with their daughters the issues related to mathematics course-taking.

"Family Math" is designed to empower parents to encourage, support, and share in the mathematical activities of their children. A product of the EQUALS programme at the Lawrence Hall of Science in California, "Family Math" is usually offered to parent and their children in four sessions. Those sessions include high-interest, hands-on activities in number, geometry, measurement and the use of a calculator. Several Toronto schools are preparing to offer this programme to their communities. Also the East York Board of Education is offering a "Family Math and Science" programme in its continuing education classes.

Comment

The need for a stronger sense of what has been done, where it has been done, and what works is being presented at several levels. Economy, efficiency, and a reluctance to reinvent the wheel are forcing educators to assess their accomplishments and to plot future directions. The Toronto Board Math - Science -Guidance Co-ordinating Committee was formed in 1987 to bring together subject co-ordinators and representatives from different subject areas including women's studies and affirmative action, superintendents, and parents to monitor the interventions and ensure that they fit into an overall plan. At the provincial level, a group of elementary, secondary, and post-secondary educators based in Toronto are in the process of initiating a project to support Ontario teachers interested in encouraging girls and women to study mathematics, science and computer technology. The plan is to first conduct a literature review of interventions that have been tried and then to conduct a feasibility study. In the late spring of 1988, a study

group will consider the findings and determine the specific nature of a project to be launched. At the national level, the Canadian Teachers' Federation is in the process of collecting national data on programmes that have been initiated to reverse the practice of female students dropping mathematics. These data will be analyzed and compiled in "The IDEA Book: a Guide to Successful Programmes for Improving Female Student Participation/ Success Rates in Math, Science and Technology."

After six years of activities, it is time to look back and reflect on the progress that has been made. The many programmes and initiatives in place testify to educators' efforts and the raised consciousness in the community. Evaluations of the various innovations and events have tended to be informal and do not meet the rigour of formal research projects. Although the clients make positive comments, and although it seems reasonable to assume that they are now better informed, the extent of their attitude or behaviour change remains unclear.

Nevertheless, data collected suggest some gains. The mathematics enrolment in one North Toronto school is reported to have jumped by a hundred students in one year after a mathematics teacher and a guidance counsellor worked co-operatively to help math anxious students and to counsel students and their parents about the importance of mathematics.

A comparison of the enrolment figures for 1985-86 with those of 1980-81 in grade 13 mathematics courses at the schools of Toronto Board of Education showed that female enrolment had increased by 4 per cent in functions and relations (41% from 37%) and 7 per cent in calculus (41% from 34%), and remained constant in algebra (27%). A Statistics Canada review of 1984-85 and 1980-81 data on the enrolment of women in full-time undergraduate courses in Canadian universities reveals increases such as 2 per cent in engineering (10% from 8%), 2 per cent in physics (12% from 10%), 6 per cent in medical studies and research (43% from 37%), 7 per cent in dentistry (28% from 21%) and 4 per cent in chemistry (34% from 30%).

Small as these gains are, they are encouraging and suggest that our work is not in vain. It makes intuitive sense that teachers may be the most direct avenue to change because teachers have the power to change their classroom environment and their own teaching strategies. A review of literature on interventions designed to increase the enrolment of female students in mathematics was carried out by Lantz. She concluded that many of the successful remedial programmes involved classroom techniques and strategies used by teachers in their own subject area. Lantz found that neither the teacher's sex nor the male/female ratio of the students in the class was an important factor, but that a fertile area for change was in the area of increasing teacher competence in mathematics content and teaching methods. These findings certainly confirm educational thought and direction here in Ontario.

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L.F.W.

THINKING THROUGH PROBLEM SOLVING IV
Functions and Relationships

by
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When I reflect on my early experiences of graphing functions in secondary school I have memories of completing tables of values, plotting points, then joining these points with straight lines or curves. I have little recollection of understanding that a graph was an abstract representation of a relationship between two quantities until much later in my school career.

When a large random sample of British students were given the question shown in figure 1, less than twenty five percent of fifteen year olds were successful (Hart, 1981, p.129). Most students either omitted the question or provided incorrect spatial responses such as (a) "went along then turned left, then turned right", (b) "going north-east, then north-west", (c) "climbing a mountain" or "going up, going down, then up again" (Hart, 1981). When I have asked similar questions of Canadian students, the results have been no better, and although I can only hypothesize on my own understanding at this age, I suspect that I may well have answered such questions incorrectly.

Many students view line graphs as a picture rather than as a representation of a relationship; and, earlier school experiences e.g. circle graphs, may well have reinforced this interpretation. If mathematics is a language which assists communication then surely we need to spend time in our classrooms discussing and interpreting graphs from a qualitative, as well as quantitative, perspective.

The problems in this article are from classroom research and teaching materials developed at the Shell Centre for Mathematical Education, Nottingham, England (1985) based on work by Janvier [1] (1978) and Swan (1980). This material was developed to encourage thoughtful reflection and discussion of graphical representations, and to facilitate students' attempts to understand and communicate mathematical ideas. The questions are applicable to many different age and/or ability levels, and students' interpretations of the graphs should be discussed in both small and large group settings.

OUTLINE FOR A LESSON

(See *Gazette*, September 1988 p.30, for a detailed rationale.)

Understanding the Problems.

Initially teacher and students will need to discuss each problem to ensure that students understand the task. **School Reports** (problem 2) is probably the least difficult question for easing students into graphical interpretation.

Developing and Completing a Plan

Groups of two, three or four work best. Create an environment in which students feel comfortable taking risks and sharing ideas. As students work on the problems in their groups, tour the room and listen to their explanations. Conflict is healthy. Allow students time to resolve the conflict for themselves. Interject

Which of the graphs represent journeys? Describe what happens in each case.

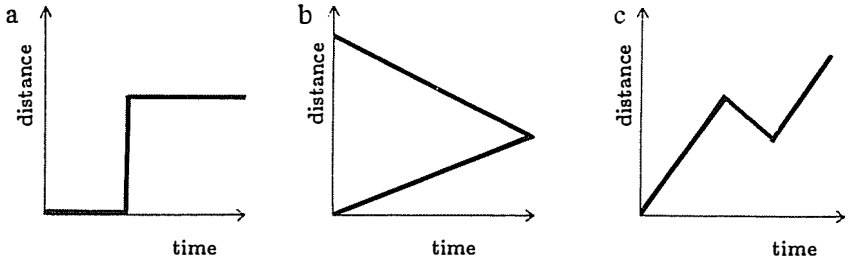


Figure 1

only when necessary and ask questions which encourage the students to re-examine the problem from another perspective.

Analyzing why solutions are incorrect is often more beneficial than finding a correct answer. When hints are offered, do not take ownership of the correct solution from the students. Thoughtful reflection takes time.

Looking Back

Discussing different strategies with the whole class reinforces the view that there is often more than one method for finding a solution.

Encourage students to critically analyze each other's work, but at the same time to RESPECT each person's opinion.

A valuable extension to these problems is the development of similar questions by the students for their peers. Such an activity allows for creativity and deeper conceptual understanding of the situation.

1. Claude Janvier is a mathematics education professor from the University of Quebec, Montreal, who completed his research while on sabbatical leave at Nottingham. It seems unfortunate that we, in Canada, have made little use of his research, while the British have incorporated his ideas into their curriculum and national assessment program.

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Janvier, C. (1978). The interpretation of complex cartesian graphs: Studies and teaching experiments. Unpublished Ph.D. thesis, Shell Centre for Mathematical Education, University of Nottingham, Nottingham, England.

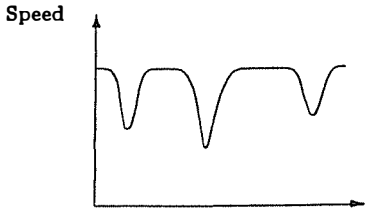
Shell Centre for Mathematics Education. (1985). The Language of functions and graphs. Shell Centre for Mathematical Education, University of Nottingham, Nottingham, England. (Also available in North America from Dale Seymour Publications).

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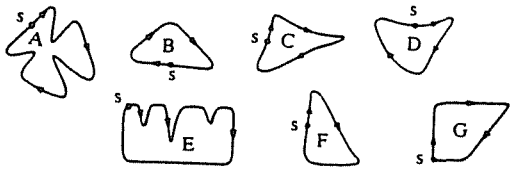
1

The Race Track

The graph shows how the speed of a racing car changes during the second lap of a race.



Which of these circuits was it going round?
* s is the start of the race.



Discuss this problem in your group.
Write down your reasons each time you reject a circuit.

Hint: What is happening when the graph rises/falls

Extension: Invent a racing track of your own, then sketch the graph to show how the speed varies, and see if a neighbour can reconstruct your graph.

Solutions:

- The car was travelling around circuit C.
- A, E and G have too many bends.
- B and D are not correct because the second bend should be the most difficult.
- F is not correct because the longest "straight" should occur between the second and third bends.

2 School Reports



Alex has been extremely lazy all term and this has resulted in an extremely poor examination performance



Suzy is a very able pupil, as her exam mark clearly shows, but her concentration and behaviour in the classroom are very poor. With more effort, she could do extremely well in this subject.

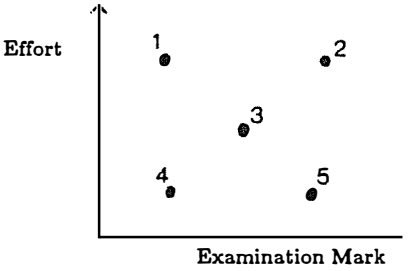
Catherine has worked well and deserves this marvelous examination result. Well Done!





David has worked reasonably well this term and has achieved a satisfactory examination result.

Each school report is represented by one of the points in the graph below. Label four points with the names Alex, Suzy, Catherine and David. Make up a school report for the remaining point.



Hint: Where will people exerting most/least effort be located?

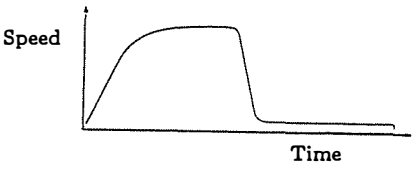
Extension: Describe another topic for which this type of graph is appropriate. Write a scenario and draw the graph.

Solution:

1. This person has put forth a lot of effort, but had a poor exam.
2. Catherine.
3. David.
4. Alex.
5. Suzy.

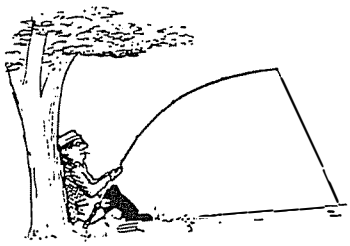
3 Which Sport?

Which sport will produce a graph like this?



Choose the best answer from the following and explain exactly how it fits the graph.

Write down reasons why you reject alternatives.



Fishing
Pole Vaulting
100 metre Sprint
Sky Diving
Golf
Archery
Javelin Throwing
High Jumping
High Diving
Snooker
Drag Racing
Water Skiing

Hint: What is happening when the slope is steep/horizontal?

Extension:
Draw the graphs for some of the other sports.

Note:
In a sport such as javelin throwing students need to discuss why the speed decreases as the javelin rises through the air and increases as it descends.

Sky diving is one possible solution.

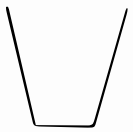
- vertical acceleration increases as diver falls.
- speed becomes constant as wind resistance becomes equal to gravitational pull.
- deceleration as parachute opens.
- constant speed again.
- zero speed on contact with the ground.

4 Which Container? Which Graph?

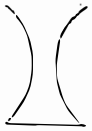
Here are 6 containers and 8 graphs.
Water is poured into each container at a constant rate.
Choose the correct graph for each container.
Explain your reasoning clearly.
For the remaining 2 graphs, sketch what the containers should look like.



Beaker



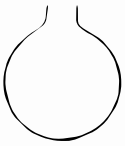
Bucket



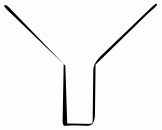
Vase



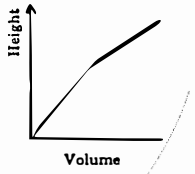
Conical Flask

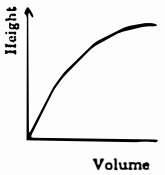


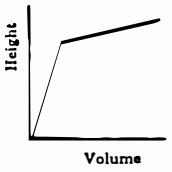
Evaporating Flask

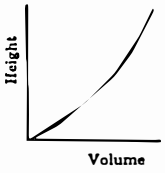


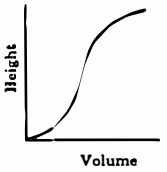
Plugged Funnel

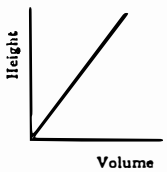


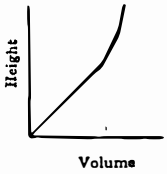


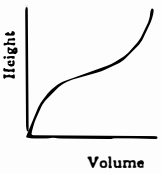












Hint: Since this is the most difficult of the problems, students will need to watch demonstrations of different containers being filled. Have students fill wide/narrow containers, and note when the height increase rapidly/slowly.

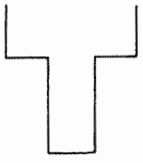
Note: Students often believe graph (c) corresponds to the plugged funnel since a straight edge on a container must correspond to a straight line on the graph. A discussion as to why there are straight lines or curved lines on the graphs is important.

Extension: Invent your own container and sketch the graph on a separate piece of paper. Exchange graphs with a neighbour and construct the container from the graph.

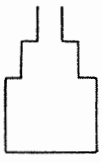
Solutions:

Beaker	-	(f)
Bucket	-	(a)
Vase	-	(e)
Conical Flask	-	(d)
Evaporating Flask	-	(h)
Plugged Funnel	-	(b)

(c)



(g)



COCKROACH MODEL - PART THREE
THE LINE

by
Ron Reesor
Faculty of Education
University of Western Ontario

Lite, Lite, Lite - this is not a commercial advertising barley sandwiches, but a commercial asking you to consider change, asking you to reflect on your teaching, and asking you to read this paper with a critical, challenging mindset.

Although the latest math guidelines (Ontario Ministry of Education, 1985) stipulated a change in the mathematics content of our courses, it should be pointed out that changing or rearranging this content will not necessarily result in improved learning of mathematics. The rote learning style prevalent today is a product of the factors time, content and methodology. One of the major concerns of teachers is that there is not enough time to be innovative/creative (to deal with the process components) if the course content is to be covered. It is time to get a little Lite headed - it is time for a change - it is time that this issue be faced in a responsible, professional manner. Too long we, as teachers, have readily accepted the mandates handed down to us. The intent of this article is not to focus on this concern, but it is left to the reader to initiate whatever actions are deemed appropriate. For this paper let me make the assumption that it would be difficult, if not impossible, to remove content, and hence allow more time for fewer topics. If one is concerned about improving the effectiveness of learning situations, then the only factor left to change is the methodology. It is impossible to separate the content and the methodology, but this paper will attempt to present a problem solving approach for the study of the straight line. H. Freemont, in his book *How to Teach Mathematics in Secondary Schools* lists five principles that should guide the organization of classroom experiences:

1. The life cycle of a mathematical idea dictates that learning must proceed from active involvement with concrete objects toward analysis with abstractions.

2. Throughout the process students must be free to reflect and draw their own conclusions.

3. Opportunities for adventurous thinking in spurts and starts should precede analytical, logical thought.

4. Children can abstract a mathematical principle if presented with a broad variety of situations in which the principle is inherent.

5. Visual images are necessary for students to be able to understand and use abstract concepts (Freemont, H. 1969).

With these principles in mind, I have established five stages in the development of a particular unit of work. Each of these stages will be demonstrated in the unit on the straight line. Group work would be used extensively throughout the unit, with the students developing a practised ability of explaining solutions to their peers.

THE LINE

INITIAL (experimental) STAGE

In this stage students conduct experiments, set up tables of values, search for patterns, and solve problems. The teacher should set the scene for a question (or questions) that will be presented to the students. Instead of giving the problem in a straight forward manner, one can sometimes create more interest by setting the scene through a very short story. This could be done by:

- having the teacher read the story.
- having the teacher read the story while several students act it out.
- letting the students be responsible for the entire presentation.

A copy of the story and *relevant* facts should be given to all the students. The story of the Sasquatch catches the imagination of both young and old, and would make an excellent introduction for the work on rates as well as on the straight line.

SASQUATCH

Once upon a time long ago there lived two prospectors, by the names of Chip and Bark, in a little cabin high up on a plain in the Rocky Mountains. One day Chip and Bark were panning for gold in the stream that ran close to their cabin. Without any warning there appeared at the edge of the tree line a hairy-shaped, man-like monster. According to Chip and Bark, this monster had abnormally long arms, with a wide flat nose, walked upright, and was more than 7 feet (213 cm) tall. It didn't notice the two men and was content to eat the berries at the edge of the forest. But Chip and Bark panicked, throwing their pans in the air. Yelling and shouting they scrambled out of the water, falling over each other in their attempt to reach the safety of their cabin. Without uttering a word they slowly raised their eyeballs above the window ledge and peered across the stream. They turned and stared at each other - the monster was slowly making its way across the stream, heading for the cabin. With the calmness and thoughtfulness of two desperate men, they managed to load the old musket and shakily aim it out the window at the advancing creature. BANG - the gun went off and when the smoke had cleared the monster lay dead in the clearing beside the cabin. They didn't venture from the cabin the rest of the day - hoping that this was just a bad dream. As darkness descended, Chip and Bark settled down for a restless night's sleep. Suddenly, crash, bang, bang, crash - it was hailing outside, not from rain, but from rocks that were being hurled at the cabin. They were under attack from unknown assailants. Chip and Bark scurried under the bed just as a boulder crashed through the roof and landed on the floor. Finally silence riveted the night air and it wasn't until dawn that our two fearless prospectors emerged from under the bed. The creature's body had disappeared and all that remained were many giant footprints in the soft earth. The largest of these prints was 41 cm, long and 15 cm wide. That day Chip and Bark gave up prospecting to search for these hairy creatures. Today they are old men, but they still have the obsession to find their beloved Sasquatch. If you wish to assist the Bark Chip expedition, please send your donations to Big Foot, Morris Mountain, B.C.

PROBLEM: If the Sasquatch have human proportions, then how tall would the creature have been?

Relevant facts about the Sasquatch (hairy men).

- a legendary tribe of aboriginal giants who were almost exterminated in battle many years ago.
- known originally to the Indians as Saskehavas (wild men).
- characteristics
  - live solitary, wandering lives inhabiting remote mountain caves, coming together only to start families, and raise the young.
  - almost entirely vegetarian.
  - nocturnal.
  - hibernate in the winter.
  - strong swimmers.
  - shy, non-aggressive towards humans.
  - mass - 500 to 1000 lbs (225 - 450 kg).
  - stride - 50 - 55 inches (127 - 140 cm).
- 1811 is earliest date they are mentioned by white settlers.
- the first reported sighting in Canada was at Harrison Lake, 60 miles from Vancouver.
- 1967 - amateur photographer filmed a Bigfoot.
- Big Foot News is published monthly in The Dalles, Oregon.

Teacher note:

- Divide the class into groups with 3-4 students per group.
- Explain that you have hints to give them if they need help.
- Be sure and allow sufficient time for this group work.
- You should circulate around the classroom and answer only information-type questions (don't give hints).
- You should be aware that the experimental calculations may not form a uniform pattern, and therefore some teacher guidance and interpretation may be required.
- Allow approximately one period (70 min.) for the problem.
- Discuss with your class the criteria for evaluating this type of experimental (problem solving) work.

Hints:

Establish a relationship between a person's height and his foot length. (This will help the student to better understand the problem, but it also provides a focus for determining whether particular strategies are appropriate.)

Complete a table of values

height (H)	foot length (L)

Graph the results obtained from table.

Determine a relationship between the height and the foot length (either from the graph or the table). Answer:

$$H = kL,\ 6 < k < 7$$

Use the relationship to predict an answer to the problem. (The answer could also be predicted from the graph.)

Extensions:

Animal	Height(cm)	Front Paw (hoof)length
Red Fox	40	6.3
Deer	34	2.5
Buffalo	167	12.7
Wolf	76	12.0
Mountain Goat	42	8.9
Moose	198	15.0
Mountain Lion	71	7.6
Black Bear	90	11.4

Table 1

From Table 1, determine which animals have the same (approx.) relationship between height and foot length. Can you suggest a reason for this?

[Fox-Wolf; Deer-Moose-Buffalo]
[Same Habitat.]

Other relationships that could be investigated:

- height to waist
- circumference of head to wrist, fist to foot length, a cup to its height.
- height of plant to length of roots
- stretch of a spring for different masses
- distance travelled at a fixed speed
- perimeter of a square or a rectangle with a specific length
- population growth of a town

SECOND STAGE

The students are removed one step from the experimental, hands on experience. At this stage, the results of experiments are given in the form of tables. This has two advantages:

- results can be given for experiments that may be difficult to carry out.
- time is saved in giving the experimental results.

① N. O. Brains, the spectacular stuntman, falls from the top of the CN Tower into one metre of water. His velocity is given as indicated in the following table.

Time(s)	0	1	2	3	4
Vel(m/s)	0	10	20	30	40

a) Graph the results.
b) If he is in the air 10 seconds, then what is his velocity when he hits the water? Change the answer into km/h.
c) Will he survive?

② With reference to the last question, suppose this same act took place on the moon and that our hero was shot downward from a cannon with an initial speed of 10 m/s. His velocity is given by the following table:

Time(s)	0	1	2	3	4
Vel(m/s)	10	11.5	13	14.5	16

a) Graph the results.
b) If he is in the air for 21 seconds, then what is his velocity when he hits the water?
c) Why is he in the air longer on the moon than on earth?

③ An experiment is conducted where different masses are attached to a spring, and the stretch of the spring is recorded.

mass(gm)	0	1	2	3	4
str(cm)	0	2.5	5	7.5	10

a) Graph the results.
b) Determine a relationship between the mass and the amount of stretch.
c) What would be the stretch for a 9 gm mass?

d) What mass would produce a stretch of 8cm?
e) Are there any restrictions on the mass or the amount of stretch?

④ The experiment in #3. is conducted again, only this time the masses are compared to the total length of the spring to produce the following table:

mass(gm)	0	1	2	3	4
total length(cm)	4	6.5	9	11.5	14

a) Graph the results.
b) Determine a relationship between the mass and the total length of the spring.
c) How long was the spring before the experiment began?
d) What is the total length of the spring is a 2.25 gm mass is applied?
e) What is the mass that will produce a total spring length of 16 cm?

⑤ During a thunderstorm you always see the lightning before you hear the thunderclap. The following table relates the distance you are from the storm to the time difference between the lightning and the thunder.

time diff(s)	0	1	2	3	4
dist(m)	0	330	660	990	1320

a) Graph the results.
b) Determine a relationship between the time difference and the distance.
c) How far away is a storm if the time difference is 6.4 secs?
d) If the storm is 1100m away, then what will be the time difference?

⑥ Teenagers are notorious for spending a lot of time in the shower. It will take 3.5h for a hot water take, which holds 152L, to heat cold water to the recommended hot water temperature. The service manual states that if cold water is put in the tank, then after 1h the temperature is 23°C, after 2h it is 40.5°C, and after 3h it is 58°C.

a) Graph the results.
b) Determine a relationship between the time and the temperature.
c) Find the temperature of the water after 2.5h of heating.
d) How long would it take the water to reach a temperature of 50°C?
e) At what rate does the heater heat the water?
f) What is the temperature of the cold water that is put in the tank? Where does this value appear on the graph?
g) Is there a maximum water temperature? Explain.
h) If a teenager has a 20 min shower, and uses water at a rate of 3.5 L/min, then how long will it take a full tank to be at the required temperature?

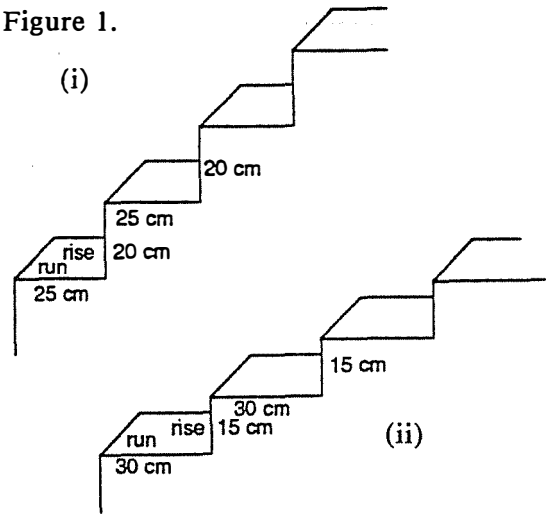
THIRD STAGE

At this stage problems are introduced which the students will be unable to complete because additional skills and/or strategies are necessary. Hopefully the students will now see a need for learning new skills. The teacher must now take the time to 'teach' these skills and provide reinforcement to solidify student understanding. Routine problems can then be given as further applications for these new skills. Any algebraic and/or geometric patterns can also be established either by looking back at the previous problems or introducing new ones.

① Two flights of stairs are shown in the figure below (Figure 1.).

a) Which are the 'safest' to climb?
b) How can one measure this 'safety' index?

Figure 1.



Teacher notes:

- Instead of using the diagram, stairs in the school could be used.
- Have students work in pairs.
- The word slope and its definition should be introduced. Some of the students should formulate this in b).

② Draw a diagram of stairs with a slope of:
i) 1 ii) 2 iii) 2/5 iv) 0

2b. In your town there is a building code which indicates the safety standards by which buildings are constructed. What would be your limit on the slope of stairs in the building code?

③ In 1982 the city of CITY had a population of 200 000. During the next five years the population grew at an average rate of 10 000/year.
a) Draw a graph to illustrate the results of this population growth.
b) How is this growth rate shown on the graph?
c) In 1989 the population was 300 000. What was the average growth gate from 1987 to 1989?

Teacher notes:

- Make sure the students are familiar with the term 'growth rate'.
- Slope is now defined as $(y_2 - y_1)/(x_2 - x_1)$ or $\Delta y/\Delta x$.
- The slope can be either a ratio or a rate (this difference should be pointed out to the students at some point during this third stage).

④ The yearbook committee in your school has determined that if 100 yearbooks are sold the cost will be $2000, and if 500 are sold the cost will be $8000.

a) Graph the results.
b) Determine a relationship between the number of yearbooks sold and the cost.
c) Find the slope of this line.
d) What quantity does the slope represent?
e) What would be the cost if 350 yearbooks were sold?
f) The line intersects the vertical axis at a certain value. What meaning does this intercept have?

Teacher notes:

- The next activity would be a series of routine questions where the students would calculate the slopes of lines from ordered pairs, and would find intercepts.
- From this should come the concept that lines can have slopes that are either positive, zero, or negative. These properties would be illustrated on graphs to show their differences.

Note: Questions #5-7 are not required for the general level course.

⑤ Refer back to the following questions:
Initial Stage - problem
Second Stage - #3,4,5,6
Third Stage - #4

What information was required to establish the relationship between the two variables that resulted in a straight line graph?

5b. Suppose that A(3,4) and B(-1,1) are joined to form a straight line. What is the relationship between the variables x, and y for a point P(x,y) on the line?

Teacher notes:

- It is at this point that the two point form of the equation of a line should be developed.
- Practice examples should be done to reinforce the concept (include horizontal and vertical lines).

⑥ Instead of being given two points on a line, suppose you are given a point and the slope of a line. Is this enough information to establish the relationship between x and y for P(x,y) on the line?

Hints:

Try this example - point M(2,5) is on the line with slope 3. Find the relationship between x, and y for P(x,y) on the line.

Draw the line for the above example.

Teacher notes:

- It is at this point that the slope point form of the equation of a line should be developed.

- Some time should be spent having the students graph lines given the slope and a point.
- Practice examples should be done and this would lead into question #7.

⑦ Find the equation of the line through:
a) (0,4); slope of 3.
b) (0,-2); slope of -5.
c) $(0,\frac{1}{2})$; slope of 2/3.
d) (0,b); slope of m. [Hint: use the pattern from a), b), c).]

Teacher notes:

- Part d) gives the slope, y-intercept form.
- Practice examples should be done. These would include giving the equation and asking the students to determine the slope and the y-intercept.
- As an extension, have the students find the slope - y-intercept form or the x-int. - y-int. form.

⑧ In the arcade game BOOMER, a Munchy is gobbled up by a Tongue, if the Tongue can score a *direct hit* on the Munchy. In this game a direct hit means hitting the Munchy from the side at 90° (perpendicular). A Munchy can travel in any direction, but only in a straight line across the screen. The Tongue must manoeuvre in order to hit them at 90°.

If the Tongues know the slope of the line a Munchy is travelling on, what direction (slope) must they aim to intersect the Munchy for a direct hit? (Assume that the Tongues can vary their speed in order to arrive at the point of intersection at the correct time for a direct hit.)

Hints.

Establish a relationship between the slopes of the paths of the Munchies and the Tongues for a direct hit.

Draw perpendicular lines whose slopes you know.

Set up a table of values

slope L_1	slope L_2

Establish the relationship from the table.

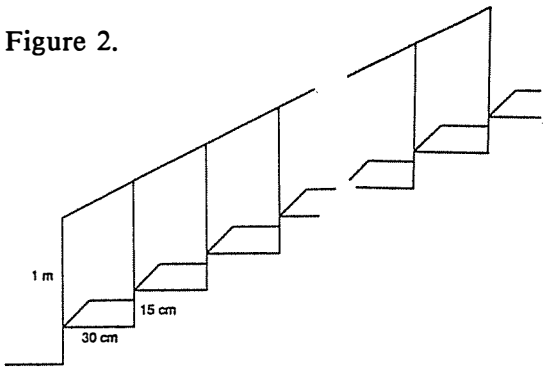
Teacher note:

- Ensure that the word *perpendicular* and the perpendicular notation are familiar.
- Establish the pattern for slopes of parallel lines by:

a) a question similar to #8
b) a guided discovery exercise
c) a Socratic approach

⑨ The stairway in a concert hall has 56 steps, as shown in Figure 2. The railing is fastened 1m above the steps. How long is the railing?

Figure 2.



Hints:

Simplify the problem by finding the length of the railing from one step to the next.
Use the relationship of the sides of a right angled triangle.

Teacher note:

- From this question, establish the formula for the length of a line segment by introducing coordinate axes on the diagram to establish ordered pairs.
- Practice examples should be done.

10. With the overcrowding at airports, planes are required to *stack up* while waiting to land. The rules for stacking are:
a) Planes must travel in a circular path with the airport at the centre.
b) Only two planes are allowed at a particular altitude.
c) The two planes in the same orbit must always be at opposite ends of the diameter.

The radar grid in front of the air traffic controller is designed with the airport located at the point (20,20). If one plane is at the location (35.3,8.1), then what would be the location of the other plane in the same orbit?

Teacher note:

- From this question the formula for the midpoint of a line segment should be established.
- Practice examples should be done.

FOURTH STAGE

This stage involves summarizing the results, generalizing, and reviewing. The students should have gained an informal understanding of the differences between dependent, and independent variables. The students will have used interpolation/extrapolation techniques, although these words were not used. One part of the review would be to apply the concepts of slope, and distance to verify the properties of plane figures.

FIFTH (Evaluation) STAGE

1. A problem similar to the one in the INITIAL STAGE (maybe choose one of the alternative investigations) is done in pairs.

2. Routine questions with the emphasis on solving problems and understanding the concepts (multiple choice questions may be part of this evaluation).

3. A non routine type question (at your discretion).

4. Group work evaluation throughout unit.

   - teacher observation
   - self evaluation
   - peer evaluation

References

Ontario Ministry of Education. (1985). Curriculum Guidelines, Mathematics. Intermediate and Senior Divisions.

Freemont, H. (1969). How to Teach Mathematics in Secondary Schools. W.B. Saunders.

Kenneth D. Fryer Award for Excellence

The award is presented by

O.A.M.E.

to Departments in High Schools that,
in addition to combining excellence
in mathematics education, also contribute
to the development of young people
through activities beyond the classroom.
This year the award went to eighteen
members of the mathematics department of

Northern Collegiate, Sarnia.

1989 RECIPIENTS:

John Clarke (Dept. Head),
Janet Allan, Kathy Beck, Craig Bowie, Roberta Buchanan,
Al Grimwood, Wendy Harris, Debbie Heaton, Bruce Houtby,
Marc Lachance, Al Lozier, Barth Marcinkou, Peter Mousseau,
Sue Parish, Gayle Roger, Angie Smith, Bill Stran, Phil Warner.

Kenneth Fryer was a Professor and Associate Dean of Mathematics
at the University of Waterloo.

NON MATHEMATICS SPECIALISTS TEACHING MATHEMATICS

Recommendations from the Intermediate/Senior Committee Feb/89.

1. Team Teaching

   Assign an experienced member of the math department who is teaching the same course as the non math specialist as a mentor or tutor. This will make the outside member feel part of the department as well as having someone in the math department that he/she can consult with their difficulties and obtain reference material.

2. Assignment of Courses

   Assign only one new course in each semester or each year to non math specialists.

3. Department P.D.

   Arrange to have an experienced member of the department present some ideas for teaching topics at the Grade 9 and 10 level. (basic, general, advanced)

4. Upgrading Courses

   Promote within the board courses on methodology for teachers teaching outside their disciplines.

   O.A.M.E. should solicit the Ministry of Education to offer summer upgrade courses for teachers involved in mathematics education for the first time.

5. Course Meetings

   Organize meetings within the department to discuss strategies for teaching upcoming units. This will encourage the non math specialist to familiarize with the upcoming material and provide them with an opportunity to discuss problems and difficulties that might arise or that they have encountered.

6. Course Guidelines

   In the preparation of school based course outlines, include timing, methodology, and suggested homework as well as resource reference material.

7. Survival Kit

   O.A.M.E. to develop a teaching strategy package for new teachers. This kit could include games, puzzles, reference material and resources on various topics especially at the grade 9 and 10 level (basic, general, advanced).

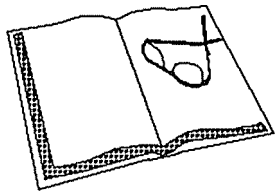
Life Membership in the Ontario Association for Mathematics Education is granted each year to a long-standing member who has demonstrated outstanding leadership in mathematics both locally and provincially, thereby making a significant contribution to mathematics education in Ontario.

The recipient for 1989 is **Dino Dottori.** Dino is a Past President of OAME, and has also served as a Director and as Counsellor for many years. He has presented numerous sessions at math gatherings across Ontario as well as in other provinces and in several states. He has served as an ambassador for mathematics in his role as teacher, department head, textbook author, Education Officer, and presently as Acting Director of the Ministry of Education in North Bay.

Dino is a worthy recipient of the honour and accepted it proudly.

Eileen Collins (Chairman)
Life Membership Committee 1989.

BOOK REVIEWS



HELP YOUR CHILD WITH MATHS

Angela Walsh (Ed.) Cambridge, England: B.B.C. Books. 1988

Help Your Child With Maths provides an excellent collection of math activities for children aged 4 - 11 and is designed for parents who want to help their children learn mathematics in a meaningful way. Many parents want to facilitate their children's mathematical development but recognise great differences between the mathematics taught today and that of their own childhood years. Text book writers, such as Houghton Mifflin, are now beginning to recognise the importance of parental guidance, and suggestions for parents can often be found in the teacher's text. Angela Walsh's book supplements these suggestions with a superb collection of games and activities which will stimulate and enhance children's mathematical understanding.

Working with their children allows parents to see beyond mere computation. Relationships can be examined through pattern recognition, allowing the child to build logical structures. Many parents express a negative attitude towards mathematics and this attitude can be transferred to their children, especially from mother to daughter. Working together through activities in this book can build positive, healthy feelings towards a subject that might previously have been seen as merely following rules and doing 'sums'.

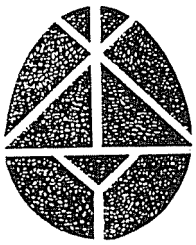
The book is divided into two main sections: In the Home and Outdoors. Each section is subdivided into activities suitable for either primary or junior children. The required materials can be found in most homes and special preparation is minimal.

Many useful hints are offered to the parent, such as talking and listening to the child as much as possible and helping the child to find many diverse solutions to a problem.

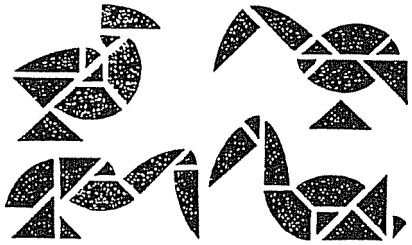
Activities include: finding geometric shapes in the kitchen cupboard; examining patterns and predicting outcomes; contemplating symmetry and nature; classifying with buttons, laundry, money, Christmas cards etc.; logical reasoning through the playing of games; estimating using grains of rice, buttons, toothpicks; conservation of capacity using different sizes of containers in the bath.

HUMPTY DUMPTY
Shape/Size

This is the famous Chinese Egg Puzzle.
Cut out the shapes carefully.



Move the shapes around and see how many different birds you can make.



Try to put the egg together again.

A few of the activities are typically British, e.g. pub sign math and the petrol pump, but the majority are practical examples for children of any nationality. Angela Walsh has collected a stimulating set of over 1000 ideas. I recommend, without reservation, *Help Your Child With Maths* to any parents wanting to help their children find joy and mathematics in their world, and prepare them to meet the challenges of the twenty first century. To meet these challenges we need a generation of open minded, divergent thinkers who can reason logically and are prepared to take risks. Angela Walsh's book offers such opportunities to our youth and their parents.

Reviewed by Barry Onslow
Division of Curriculum Studies
Faculty of Education
University of Western Ontario

TRIGONOMETER MATHS

Peter Shannon. Edinburgh Mathpack. Edinburgh, Scotland: John Bibby. 1987.

Trigonometer Maths is comprised of a Pupil Workbook, Pupil Worksheets, and a Trigonometer. The Trigonometer is made of a sturdy fibreboard with a clear plastic circle mounted on a swivel over a co ordinate axes. A red line on the plastic is the hypotenuse (length 10) for all angles in standard position. A circle has been drawn on the grid and angles are marked on the exterior of the circle in units of 10 from 0° - 360° (also included are 45°, 135°, 225° and 315°). There is a teacher's Trigonometer that can be used on an overhead projector.

This package would provide a very good hands on approach to the study of trigonometry on the coordinate axes. It is self-contained and could be used as an individualized instruction unit.

The sine, cosine, and tangent ratios are defined and there are numerous exercises throughout the unit for student practice. The Trigonometer gives the student a practical hands on experience to determine various ratios and establish patterns, although the proposed two decimal accuracy may be overly optimistic. Further development and exercises include relating angles to acute angles, establishing the CAST rule, solving trigonometric equations, and graphing the three basic trigonometry curves.

The workbook and the worksheets are well coordinated with the instructions being clear and precise. The diagrams are clearly labelled and of good size. Unfortunately, too much material has been placed on each page and this gives a very busy, cluttered appearance to the booklets.

This is a very worthwhile package and one that the students would enjoy working with in learning basic trigonometry concepts.

Reviewed by Ron Reesor
Division of Curriculum Studies
Faculty of Education
University of Western Ontario

MATHEMATICS FOR THE 1990s CONFERENCE

The Faculty of Education
The University of Western Ontario
London, Ontario
Saturday, October 28, 1989

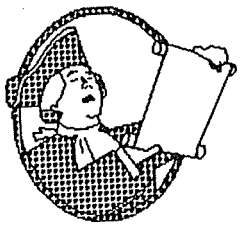
KEYNOTE SPEAKER: Dr Jim Schultz, Ohio State University

Dr. Schultz is an internationally recognized mathematics educator and a member of the writing team that produced the NCTM Standards document.

- sessions and panel discussions for all levels from kindergarten to OAC.
- registration cost (including gourmet lunch) is $25 before October 2 and $30 afterwards.

Register now!

For more information contact Doug Edge, Faculty of Education, UWO.



COUNCIL COMMENT

by
Bob Robinson

These comments are based on the Council Meeting held at the Ramada Inn - Airport West, on June 10, 1989.

PRESIDENT'S MESSAGE

Mary Lou Kestell introduced the members of Council and briefly discussed the work to be done on the constitution (Dave Davison, Warren Sebben); council committees their terms of reference and duties of the chair, board liaison and reporting; membership issues; and financial considerations.

CHIMO

The Chimo Project Director, Jo Ann Wilton, discussed CHIMO, a computer mediated communications research environment for education. An attractive informative brochure is available from: Dr. Jo Ann Wilton, CHIMO Project Director, Peel Board of Education, 5650 Hurontario St., Mississauga, ON L5R 1C6. Ph.(416)890-1099 X 2516, Fax:890-6747.

ANNUAL MEETING '89 - OTTAWA

Peter Lenczewski reported on behalf of the Ottawa organizers. Council expressed their appreciation to them for a "capital" conference. The organization and operation of the '89 annual were superb. Some felt that it was the "biggest and best yet". Congratulations.

ANNUAL MEETING '90 - HAMILTON

A successful planning meeting was held in Hamilton on March 31 and April 1, 1989. The meeting will begin on Wednesday evening, May 9 and conclude at noon hour on Saturday, May 12, 1990. Any person interested in helping at the conference or presiding at sessions should contact George Knill, Bob Robinson (conference co-chairs), or Ralph Connely, Alex Norrie (program co-chairs).

ANNUAL MEETING '91 - PEEL

In May, 1991, CHAMP (Credit Humber Association for Mathematics Promotion) will host our "91 Annual" at the Constellation Hotel in Etobicoke. Committees have been established and the "chairs" are in place. Program superintendents from four area boards have met with the organizers to discuss their support for the meeting.

NCTM REPRESENTATIVES

Janet and Barry Schully were appointed NCTM representatives for our association.

NCTM MATERIALS ORDERS

When you order NCTM materials please order through your Ontario representatives listed below. Your order helps OAME since OAME receives a commission on all materials ordered. Place your order as follows:

Members

1. Use the order form found in the Catalogue of Educational Materials.
2. Include membership number, expiry date, address used for membership purposes.
3. Subtotal your order, then deduct 20%. This is one of the benefits of being an individual NCTM member.
4. Send the order form and cheque made payable to NCTM ONTARIO in Canadian funds, to: Janet and Barry Schully, NCTM Ontario Representatives, 329 Sussex Ave. Richmond Hill, ON L4C 6B4.

Non members

1. Use the order form found in the Catalogue of Educational Materials.
2. Subtotal your order. Send the order form and cheque made payable to NCTM-ONTARIO in Canadian funds, to your Ontario Representatives.

If you have any questions about ordering materials or about becoming a member of NCTM please call us at (416)737-0817.

OAME HISTORY

Are you an old mathematics teacher? (Or a descendant of one?)

Joan Routledge (416-727-4167) and Jack LeSage (705-728-8958) have been asked by the Ontario Association for Mathematics Education to produce a history of Math ematics Associations in Ontario to celebrate 100 years.

If you have any information, convention programmes, pictures, minutes, vignettes, etc. involving any past Mathematics organizations in Ontario (OEA, OATMP, OATM, OMC, etc.) please phone Joan or Jack or write Jack at 9 Mountbatten Rd., Barrie, ON L4M 1T4.

Note: Use the last 4 digits of Joan's phone number and find the difference between the largest number and the smallest number that you can form. Now do the same thing for Jack's number and repeat until you get Joan's 4 digits. (That's why they were chosen to do this project!)

CHAPTER REPORTS

CHAMP (Credit Humber)

Over 80 members have renewed their membership for 1989-90. Plans are well under way for **MATHFEST'89** to be held on Oct. 18. The **MARCH WITH MATH** event, held March 2 was a success. The display of NCTM materials was well accepted.

Prizes were presented at the Peel Math Elites gathering, the Dufferin Peel Mathematics banquet and the Etobicoke Math Awards to the winners of the P.A.M.E. Math Contest. This is the second year for the contest which had been expanded to include Grade 11 as well as Grades 9 and 10. The math contests committee is already

organizing the 1989 contest which is to include Grade 8. A subcommittee is investigating a group problem solving version of the contest. Samples of contests from other chapters/boards would be welcome. They are working on a constitution and would appreciate any guidelines from other chapters.

The Executive is:
Past-President: Marg Warren, President: Janet Mitchel, President-Elect: Walter Curran, Secretary: Nancy Weinman, Treasurer: Helen Baker, Membership: Diane Martin, Chapter Rep. to OAME: Lis Henry.

COMAC (Carleton Ottawa Mathematics Assoc.)

The Annual COMA meeting was held May 29, 1989. The new executive officers are: Past-President: Dave Hunter, President: Del Smith, Vice-President: Barbara Sherriff, Secretary: Francoise Hutcheson, Treasurer: Fran Outerbridge, Membership: Dianne Martin, Chapter Rep: Peter Lenczewski.

Fresh from the success of OAME '89, COMA will return to its tradition of hosting a Fall Workshop/Conference. The format will be oriented toward the classroom teachers at all levels. The conference will be a joint venture, co-hosted by COMA and QSLMA. It will take place Oct. 4 & 5, 1989 at the Opinicon Lodge, Chaffey's Lock. The theme will be "Towards 2001".

A short history of COMA has been written by Duff Butterhill, one of the original members of the Founding Council.

GVMA (Grand Valley Mathematics Assoc.)

The April meeting was held 9 am. through 5 pm. on a Saturday rather than Thursday evening. The semi-annual meetings (Oct/89, Apr/90) are each set for a Thursday.

Three writing teams have been preparing sample exams for each of the three Ontario Academic Mathematics credits. These will be available in Oct. '89 and will be advertised in a future issue of the Gazette.

NOMA (Northern Ontario Mathematics Assoc.)

They are planning a 1.5 day conference in Sudbury beginning Thursday evening, Oct.26 through Friday, Oct 27. It will be held in conjunction with a Sudbury schools' professional activity day.

To promote OAME and NOMA throughout N.E. Ontario, and to recognise student excellence, they have begun, this year, to sponsor cash awards from $30 to $50 to zone champions in the Canadian MAthematics Competition. Awards have been presented to grade 9, 10 & 11 winners. Students from Sudbury, New Liskeard, North Bay, and Sault Ste. Marie were winners. Grade 12 & 13 awards will be presented shortly.

NWOAME (Northwestern Ontario Assoc. for Math. Ed.)

NWOAME published its first Newsletter in June '89 and all Councillors are enthusiastic. Current membership is 52 persons. NWOAME '89 will be held Sept.29 at Confederation College, Thunder Bay. NWOAME members look forward to publishing the Sept.'90 Abacus.

PRM (Pine Ridge Mathematics Association)

A very successful Mathematics Award Night was held June 8th. Over 100 students from Grades 6 to 13 received awards for various Mathematics competitions from the CNML Grade 6 Elementary Grades Contest to the Grade 13 Descartes Contest. The Durham Mathematics Competition involving three students from 10 high schools was held May 24 & 30. The Grade 9, 10 & 11 students competed in a "Reach for the Top" contest format. The winning school was Henry Street High School led by Andrew Hull, Grade 11.

QSLMA (Quinte St. Lawrence Mathematics Assoc.)

The Spring Conference was held at the Royal Military College (Kingston) on April 6. One hundred thirty two teachers attended sessions on topics ranging from "Fraction Fun for Everyone" to "Teaching the O.A.C.'s".

SAME (Scarborough Assoc. for Mathematics Educ.)

The Jim Fencott Award was presented to Rene Ho of Albert Campbell C.I. and Kent Newton of W. A. Porter C.I. at the 25th annual Mathematics Awards ceremony, on June 6 at the Scarborough Civic Centre.

On April 13 we held a very successful "MATH CIRCUS" at Maplewood High School. The event combined publishers' displays, presentations, displays of learning centres and MAKE AND TAKE.

Election results are: Past-President: Shirley Fairfield, President: Bill Haenel, Vice-President: Jan Cornwall, Secretary: Bonnie Taylor, Treasurer: Richard Kinney.

Lastly, plans are under way for an Oct. 17 Mini-conference at Maplewood High School.

SWOAME (Southwestern Association for Math. Ed.)

The May meeting had over 100 people in attendance. The next meeting will include a Friday night dinner and sessions (Dec.1 & 2,'89) The theme is "Using Computers and Calculators in the Classroom".

Y4MA (York)

This chapter is now 1 year old. A mini course is planned for November. They are investigating their committee and contest organizations.

COMMITTEE REPORTS

Primary/Junior - Sue Stuart (chair), Roland Morrow (Executive Liaison).

The committee will monitor developments in the Ministry of Education in the primary and junior divisions. How to encourage more participation in the problem-solving award will also be investigated.

Intermediate/Senior - Anne Jones (Chair), Jeri Lunney (Executive Liaison).

Developments in the Ministry of Education that have an impact on mathematics education in the Intermediate and Senior divisions will be monitored by the committee. They will seek ways to encourage mathematics departments to be involved in the competition for the Ken Fryer Award. OAME will become pro-active in the non streaming issue and search for the non-streamed class of the future. A survey will be sent out shortly to ascertain if there is a shortage of qualified mathematics teachers in the province. Jeri Lunney and Paul Lessard serve on the Provincial Review committees in mathematics.

Membership/Publicity - Lis Henry (Chair), Mickey Sandblom (Executive Liaison).

A concerted effort to promote OAME throughout Ontario will take place as council strives to increase membership. Our membership brochure will be up-dated and further ways to distribute information to teachers and potential teachers will be explored.

Futures - Peter Lenczewski (Chair), J. Symington (Executive Liaison).

The Committee will evaluate (OAME) policies and regulations and recommend revisions when appropriate. They will also be the "Watchdog" for new legislation from the ministry that would affect mathematics education and alert the executive and the proper committee about policies to be reviewed and responded to.

Council is awaiting a report regarding the OAME's "survival binder". The Committee is accepting submissions to host our 1992 annual meeting. If you want your area to act as host, contact the chair of the Futures Committee.

Women in Mathematics - Lorna Wiggan (Chair), Mary Lou Kestell (Executive Liaison).

The committee objectives are:
1. To emphasize the need for programs, teaching strategies and other kinds of interventions which may help to encourage females at all grade levels to study mathematics.
2. To encourage and organize workshops and meetings focusing on women and mathematics at the OAME annual conference.
3. To publicize and highlight relevant issues, research and interventions using the Gazette as a means of reaching the OAME membership.
4. To promote leadership among women in OAME as well as in the mathematics education community at large.
5. To facilitate and encourage chapters to pursue our common goal.
6. To liaise with other organizations to promote our common goal.
7. To encourage educators to introduce career awareness activities to students in the formative years and to distribute information on non traditional careers for females.
8. To identify and heighten awareness of gender inequalities when they occur and to recommend positive methods of overcoming these inequalities.

A very successful breakfast was provided at the Ottawa Conference with about 125 members in attendance. An excellent presentation on "Gender Sensitivity in the Classroom" was given by Heather-Jane Robertson from CTF. The committee plans to make this an annual event with the next one to be held at the Hamilton Annual Meeting in May 1990.

They are investigating ways to reach the people in the chapters. A list of speakers (male and female) who can address women and mathematics issues will be compiled. Your suggestions should be forwarded to Lorna Wiggan.

A CEMSAT (Coalition for Equity in Mathematics, Science and Technology) symposium was held recently and plans are under way to publish the proceedings.

It is possible that a statement on equity will be prepared in the near future.

Mention was made to an excellent mathematics education article which was published in the Toronto Star in March.

Publications - Ron Lancaster (Chair), Jack Weiner (Executive Liaison).

The editors of our fine publication, Bob Smith, Barry Onslow and Eric Wood would like to receive lots of letters and reactions to the articles and problem solving sections.

The purpose of the Gazette will be examined and how to address the needs of student teachers as well.

FUTURE MEETING DATES

Executive	Council
September 8, 1989	October 14, 1989
November 17, 1989	February 10, 1990
April 1, 1990	

MATHEMATICS EDUCATION TRUST

The Axial Curve Rotator, a reaching unit by Walter M. Hunter dealing with the construction of a device that mechanically produces the three-dimensional shape resulting from the rotation of any algebraic line or curve around either axis on the coordinate plane. 24 pp., $4.

Beyond Numbers: The Mathematics Literature Connection, an activity resource developed by Roslynn Seidenstein and John P. Madison for grades K–6. Combines the seemingly disparate areas of mathematics and children's literature to introduce, reinforce, and broaden mathematics skills and concepts. 32 pp., $5.

Econographs for Graphacons, a reaching unit for high school juniors or seniors, is a one-week study of graphs used in economics. The title come from the students of Benilde–Saint Margaret's High School (Saint Louis Park, Minnesota) on whom Mary Joe Aiken field-tested her unit. 26 pp., $3.

Liquefied Natural Gas: A Potential for an Abundant Energy Supply or a Potential for Danger, developed for high school by Joseph Fishman (College of Staten Island, City University of New York), is a one-week reaching unit applying mathematics to societal issues. 45 pp., $5.

Mathematics and Your Horse, an activity-filled teaching unit by Cathy Anne Barkley geared toward middle school, is designed to help students care for a real or an imaginary horse properly while reviewing basic math skills. 40 pp., $5.

Mathematics Curriculum Outline and Sample Activities for Gifted Students, Grades 2–8, is a set of materials resulting from a project conducted by Alexander Tobin, director of mathematics education for the Philadelphia school district. Contributing authors are Alan Barson, Robert Murphey, and Donald Scheuer. 36 pp., $3.

Model Rocket Math, prepared by Laura Reeves (Rio Grande High School, Albuquerque), outlines a two-week course for 11th- and 12th-graders that includes constructing and launching rockets, calculating burnout statistics, and other aspects of rocket flight. 28 pp., $4.

Probability: Actual Trials, Computer Simulations, and Mathematical Solutions, by Karen Doyle Walton and J. Doyle Walton, appropriate for classes from seventh grade through beginning college level, in which actual trials are performed, results are recorded, computer simulation of the problem is run several times, and the mathematical solution of the problem is presented. Diskette written for Apple IIe/11c and supplies included. 42 pp., $6.

Order from the
Mathematics Education Trust
1906 Association Drive Reston VA 22091
Tel (703) 620-9840.

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

1989-90 EXECUTIVE & COUNCILLORS

Past Pres.	Symington, J
Pres.	Kestell, Mary Lou
Pres-Elect	Sandblom, Michael
Director	Lunney, Jeri
Director	Morrow, Roland
Director	Weiner, Jack

MEMBERSHIP DUES

Subsciptions to the *Gazette* alone are no longer available; however, members of O.A.M.E./A.O.E.M. receive the *Gazette* and *Abacus* free. Membership dues for 1 year are $25.00 in Canada, $27.00 in United States and $30 outside Canada and United States Membership dues for 5 years are $112.00 in Canada, $122.00 in United States, $140.00 outside of Canada and United States. Orders for memberships should be sent to:

Secretary-Treasurers

Don and Carol Attridge, O.A.M.E.

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COUNCILLORS TERM EXPIRING 1990

Fullerton, Olive

Handley, David

Jefferies, Mary Lynn

Jones, Ann

Lessard, Paul

McNabb, George

Rogers, Dr. Pat

Wiggan, Lorna

COUNCILLORS TERM EXPIRING 1991

Collins, Eileen

Cornwall, Jan

Davidson, Dave

Griffiths, Tom J

Harvey, M. Elaine

Mitchell, Janet

Lancaster, Ronald

Robinson, Robert

COUNCILLORS TERM EXPIRING 1992

Carefoote, Arnold

Knill, George

Romiens, Todd

Sauer, Ronald

Scully, Barry

Stuart, Susan

Warburton, Wendy

CHAPTERS AND REPRESENTATIVES

CHAMP(PAME)	Henry, Liz
COMA	Lenczewski, Peter
GOLDEN	Khan, Leslie
GVMA	Telfer, Stuart
NOMA	Maloney, Jerry
NWOAME	Hurrell, Marilyn
PINE RIDGE	Ennis, Ron
QSLM	Evans, Douglas
SAME	Haehnel, William
SWOAME	Richardson, Peter
Y4MA	Scully, Janet