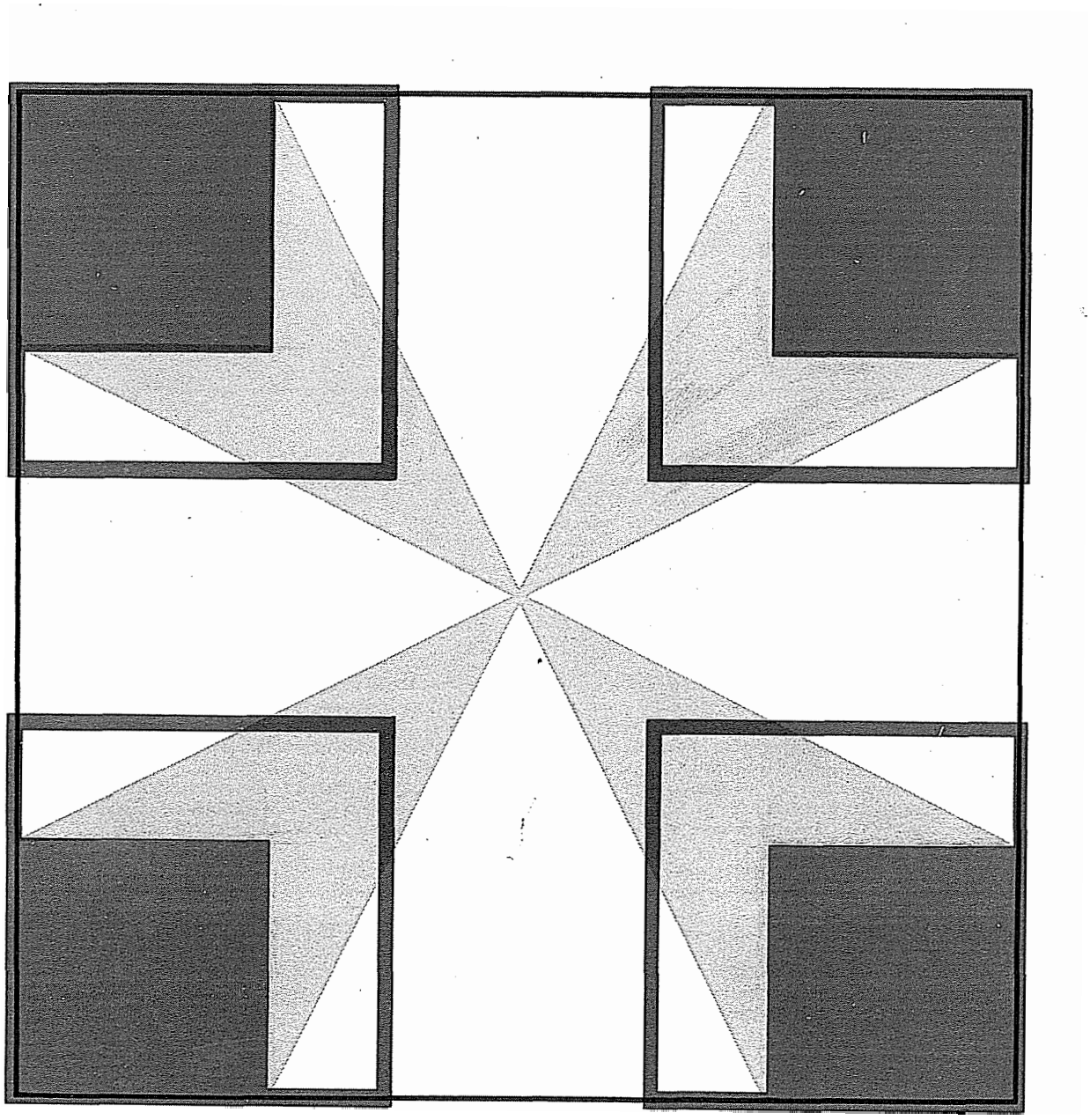
ontario mathematics gazette

VOLUME 28 NUMBER 2 DECEMBER 1989



THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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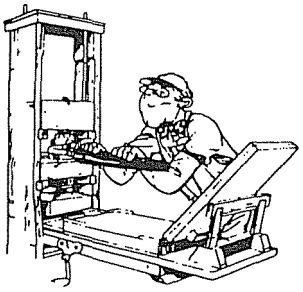
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by
Barry Onslow

December's issue of the Gazette focuses on the teaching of **Statistics and Probability** which has, until recently, been a much neglected area of mathematics. The NCTM *Standards* document has brought about a resurgence of interest in statistics and probability, but one wonders how we can fit this topic into an already over-crowded curriculum. We have to decide what is important for the futures of our youth. No longer do we need to train students to behave like inexpensive calculators. The future rests on educating our students to be numerate, creative and analytic - to be able to adapt to an ever-changing and complex society.

Torturous weeks answering long division questions or solving quadratic equations could be better spent using technology to perform the repetitive calculations while freeing the students' minds for the more appropriate tasks of understanding the processes and applying their knowledge to solve real-world problems. Statistics and probability offers an opportunity to study real-world problems which are useful, interesting and challenging. However, as Alan Sontrop points out in his article *Frozen Mannequins*, text book examples are often not real-world problems.

A quote from the NCTM *Standards* document succinctly illustrates the importance of the topic:

> *Collecting, organizing, describing, displaying and interpreting data, as well as making decisions and predictions on the basis of that information, are skills that are increasingly important in a society based on technology and communication (p.54).*

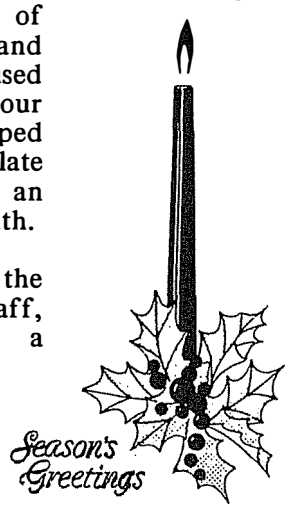
University students already have as much use for statistics and probability as the more traditional OAC components of calculus or algebra and geometry. Moreover, statistics and probability is probably more important to the everyday functioning of the average person than almost any other single aspect of school mathematics. If we want a population capable of processing information accurately and relevantly, then the study of statistics and probability in our schools is essential.

In the early grades it is important for students to build an intuitive sense of probability through experimentation and debate. From such a foundation the theoretical aspects studied in later grades can be linked to real world experiences and students will have access to a *bridge* which allows abstract concepts to be explored within a context which makes sense. Within the real world context, aspects of conjecture, inquiry, analysis, debate and proof, can be aptly demonstrated using statistics and probability. Conjecturing is a powerful pedagogical strategy for examining concepts. When students make predictions, perform investigations and discover conflicting (or confirming) results they become active participants in the learning process rather than passive receivers of knowledge.

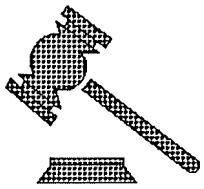
Being able to discern between the truths and fallacies within statistics and probability is yet another aspect which cannot be forgotten. Benjamin Disraeli has been credited with stating that there are three kinds of lies - lies, damned lies and statistics! Although this condemnation is excessive, students do need to understand that statistical analysis does not provide evidence of unequivocal truth, but it does provide evidence that is generally better than hearsay or opinion.

The feature articles in this issue provide excellent examples of how statistics and probability can be used beneficially in our schools. It is hoped that they stimulate further interest in an exciting area of math.

On behalf of all the editors and staff, I wish you all a peaceful Christmas and the very best of wishes for the New Year.



PRESIDENT'S MESSAGE



We've had the Speech from the throne but in ten years there will be little difference.

Things will be as they were. More will drop out, more will fail and no child will get what he or she deserves.

Things will not be more individualized, more equitable, not when all are painted with the same brush.

These are words heard throughout staff rooms in this province. We seem to be facing another round of political change with little effort planned to help the teachers.

The emotions expressed now are different from those of ten years ago when OS:IS was *in uterero.* I hear a whisper of hope expressed in comments such as:

> If only the government would allow time for teachers to be involved in the emergence of these changes, then we would have something worth the battle.
>
> If they would allow us to be involved at the developmental stages, we could really effect good and necessary changes. But they continue to involve us, the professionals in the field, only at the reaction stage. This is, again, change imposed from above. Real change can happen only if we, the stakeholders, have a share in painting the picture. Then we can take ownership and feel commitment to implement the plan.
>
> We want to be proactive, not reactive.

The Action Plan 1989-94 is a draft document on how the senior administrators at Queen's Park see themselves implementing the initiatives demanded by the Minister of Education. It is floating around to different groups in the province. Its sub-title is:

"Restructuring the Education System.
- The Foundation Years (Junior and Senior Kindergarten)
- The Formative Years (Gr. 1 to 6)
- The Transition Years (Gr. 7 to 9)
- The Specialization Years (Gr. 10 to OAC)
- Technological Education
- Teacher Education"

The rumours I have heard so far are about the dates for input from various groups and the number of intended changes at some levels. Let me tell you about the activities of the councillors and executive members of OAME. At our last meeting, the council instructed me to write a friendly letter asking the senior person overseeing the restructuring project to allow us to appoint one person to sit on each of four committees: The Formative Years, The Transition Years, The Specialization Years and Teacher Education. I am a member of the Ontario Mathematics Coordinators Association as an OAME representative. I have joined a sub-committee working on the restructuring issue as well. Approaches similar to those outlined below are being undertaken by members of that committee. So the Ministry will hear from us from two directions. The voices are in synch. Maybe we will be heard.

The Primary/Junior committee members have been following the development in the Renewal of the Formative Years project. They have a meeting planned in February to prepare a response to this document. The big question, of course, is "how will this renewal document stand up in view of the new plans?" The Action Plan simply states that the Ministry initiative for this level will be "to revitalize the curriculum from grades one to six by focusing on the development of literacy, analytical and communication skills." From what we can see, the dates for the renewal document and the Action Plan paper are in conflict. We have lots of questions to ask.

The Intermediate/Senior committee is also sending a letter to the people in charge of the Transition Years suggesting many questions that teachers will need the Ministry to answer before curriculum changes can even be addressed.

1. What will a destreamed classroom look like?
2. How will exceptional students receive the special treatment to which they are entitled?
3. What measures will be taken to be sure that students going on to The Specialization Years will have the choices necessary to accommodate their needs now that they have come from destreamed grade 9?
4. What resources will be provided to help implement the changes?
5. What teacher in-service will be provided?
6. How will evaluation reports to students and parents change to reflect the new philosophy of the Transition Years?
7. Can we assume that mathematics specialists will still be teaching mathematics?

These are just a few of the questions to be posed. Their letter will offer the services of committee members as people with answers to these questions. The letter from this committee will support my general request for representation on the planning committees. If they are unsuccessful in their attempt to have

a representative on the development team, then they will be ready to react when the time for Reaction Groups is called. There is a common thread through the thoughts expressed by math teachers brainstorming on how they see implementation plans. People are saying:

- The content has to go.
- The focus must be on getting the students to use mathematics as a tool in focusing, clarifying and expressing their ideas.
- We can not have tests with fixed information and questions that have only one answer.
- There has to be some focus on research and analysis skills.
- The old basics just won't cut it ten years from now.

A similar approach should be taken in contacting The Specialization Years group. My great fear is that the Ministry plans little or no change at this level. They will simply redefine the total credits issue and be done. Because people in the mathematics community are very busy responding to the de-streaming issue and to those changes being developed for the Transition Years, we may be neglecting issues that need to be addressed to The Specialization Years group. I know a statement needs to be made about evaluation changes for these years but more importantly, the pure mathematics in the advanced level program can not be delivered to the 80%-90% of the students who may very well end up in MAT 2A. Our existing MAT 2A, 3A , 4A, Calculus and Algebra stream should be called "Mathematics for Nerds" or "Mathematics for Researchers" and at most 12% of the students should go that way. A new mathematics curriculum with the changes outlined in the NCTM Standards document needs to be created. New methods of delivery must be ensured. And resources must be allocated to provide the computers, graphics calculators, symbolic manipulators, and software necessary to deliver mathematics to the majority of students. The new course must be perceived as an academic course with high expectations and sound goals. It might be called "Mathematics for Success", Math for Managers and Executives", or "Math For Those Who Will Make This Province Great". Parents, students and post secondary people must approve of the course and hold it in high regard. There must be an OAC attached to the stream. The majority of students will be in a core grade 10 program. At the grade 11 level, the families decide that future requirements will be met by the MTT or MTB or by having the students in the OAC Applied Mathematics Program.[1]

Teacher Education, both pre-service and in-service, needs attention. If the change is not going to be given time to grow from within and emerge as plans the teachers can feel ownership of, then large amounts of money need to be allocated for work required to get the teachers on-board and confident enough to deliver the dreams. We all need to learn how to teach using cooperative learning models. We all need to learn to develop individualized research projects for students to learn to use mathematics. We need to embrace the question of what it takes to teach people to mathematize. My hope is that in the end we will have math programs that give the power of freedom inherent in the ability to use mathematical techniques to problem solve and to do critical thinking. That is what mathematical literacy is. We also need to learn to use the new tools - hardware, software, graphics calculators and symbolic manipulators - so students can learn to use these in helping them address the issues of the next century. With the commitment to real change, we will be able to feel pride in our delivery of skills for the year 2000. We will be giving mathematical power to everyone - not to the few who just manipulate algebra well enough to get to the end of the problem.

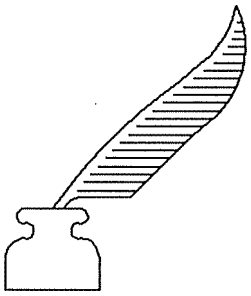
We have not asked for representation on the Technological Education group but feel very strongly that we need to lend our support to its revitalization. This province needs good technicians and technologists. We need to insist on a giant advertising campaign that will tell parents that letting their children enter technological fields will not result in their being dead-ended in jobs. If anything, these people will be more ready for the future than the academics. I think the campaign should be as large as the "Anti-drinking and Driving" campaign. Tech education is necessary and should be considered a respectable path for students of any ability, including those who would never have strayed from advanced level in the past.

The part in all this that excites me is that mathematics teachers are involved with positive, creative, and extremely important ideas they are willing to share with the senior planners at the Mowat Block. I hope the planners will listen, The promise comes from a statement made in the Action Plan. "Key to the success of the plan of action is that consultation on policy must mean a discussion of valid policy options rather that participation in the refinement of a particular ministry position." Sounds like they want to hear from us. Let's speak in unison!

Mary Lou Kestell
December 1989

[1] The opinions expressed in this paragraph are those of your President and do not necessarily reflect the views of the council, any other individual or group.

LETTERS TO THE EDITOR



3 October 1989
Faculty of Ed.
The University of
Western Ontario

Dear Barry:

This letter is a comment on the article by Dave Hunter "WHERE ARE WE AND WHERE DO WE GO FROM HERE" that appeared in the September issue of the Gazette. He has clearly outlined the problems that affect the teaching of mathematics in our schools and his arguments/observations mirror exactly with my experiences.

His long term solution of having the Ministry set external exams will give a more reliable standard on which to judge students' achievement. I have some problems with external exams, but my point is that these exams will not necessarily change the way math is being taught. My experience tells me that teachers will teach the material the way it is presented in the textbooks. I think the Ministry must take leadership and get off its "duff" - not by introducing external exams, but by approving textbooks that reflect the approaches and philosophy that are laid out in the guidelines. The state of California recently rejected "all mathematics textbook series submitted for authorized adoption" [1] because it wanted to stress that "mathematics textbooks must be designed to develop student capabilities to address and solve complex, subtle, and unpredictable problems". Will our Ministry assume such responsibility and take some action?

Thanks Dave for a well written, concise, precise viewpoint article.

Ron Reesor

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[1] National Research Council. (1989). Everybody Counts. A Report to the Nation on the Future of Mathematics. Washington, D.C: National Academy Press.

Lester B. Pearson
Catholic High School
Gloucester, Ontario
K1J 8M5

3 Oct. 1989

Dear Editor,

Dave Hunter's observations and criticisms of the present state of mathematics education across the province deserve much attention and hopefully will create some positive reaction.

Two apparently divergent issues raised by Mr. Hunter - the need for more problem solving experiences and the need for more rigorous and objective standards - might be the most crucial in increasing the quality of the learning of mathematics and the prestige of the teaching of mathematics in our province.

As Mr. Hunter implies, the ultimate goal of education in general and mathematics in particular is the creation of independent, confident problem solvers. In mathematics, students must be challenged to solve a variety of problems and appreciate the variety of problem solving strategies. The problem folder referred to by Mr. Hunter enables the student to enlarge his portfolio of problems and strategies and provides an opportunity to see relations among the various branches of mathematics. A range of problems could be created to challenge students who need to appreciate the practical daily applications of mathematics as well as mathematically mature students who can enjoy the abstractions involved in a theoretical problem.

Mr. Hunter also recommends the creation of provincial standards through compulsory provincial examinations. I feel that this form of assistance from the Ministry will provide teachers with the support necessary to maintain the integrity and rigour of the advanced level courses. Such examinations should reduce the temptation to inflate the marks of university-bound students and act as

a filter to select those students for whom the advanced level courses were designed. Provincial examinations, I believe, would also precipitate the re-examination of our mathematics curriculum which, as this article suggests, is crucial.

The success of the changes recommended by Mr. Hunter depends on the agent of these changes. The classroom teachers and board consultants who are attuned to the difficulties encountered by adolescents in the mathematics classroom must be the key players in the revisions Mr. Hunter suggests.

Yours sincerely,

Bill Murphy (V.P.)

3 Kingscross Square
Etobicoke, Ontario
M8Z 4M4
(416) 232-1937

June 8, 1989

Dear Editor,

Readers of the *Gazette* who are involved in teaching enrichment level mathematics, or who have a bored student in their class who is having an easy time with the regular curriculum, might be interested in *Mathematical Mayhem*. *Mayhem* is a problem-solving journal for high school and undergraduate students *by* students. Last summer after the International Mathematical Olympiad in Australia, Patrick Surry and Ravi Vakil, two Canadian Team members, hatched a plan to communicate the excitement of mathematics to other students. A generous grant from the Samuel Beatty Foundation helped turn this dream into reality, allowing *Mayhem* to be sent out free to all interested during its first year (although a nominal fee will be charged after this summer). Since then, five issues have been released and the staff has swollen to eight.

Each issue of *Mayhem* contains articles on a variety of topics as well as several problem sections ranging in difficulty from elementary high school to undergraduate. The problems are proposed by readers, and solutions to the problems are invited from subscribers and printed in later issues. Also, most articles are written by students. Thus *Mayhem* depends on an active and interested readership.

If you are interested, or if you have a potentially interested student, contact Ravi Vakil at the above address to receive your complimentary copy of issue 4.

Yours Sincerely,

Ravi Vakil.

Conestoga College
August 8th 1989

Dear Editor,

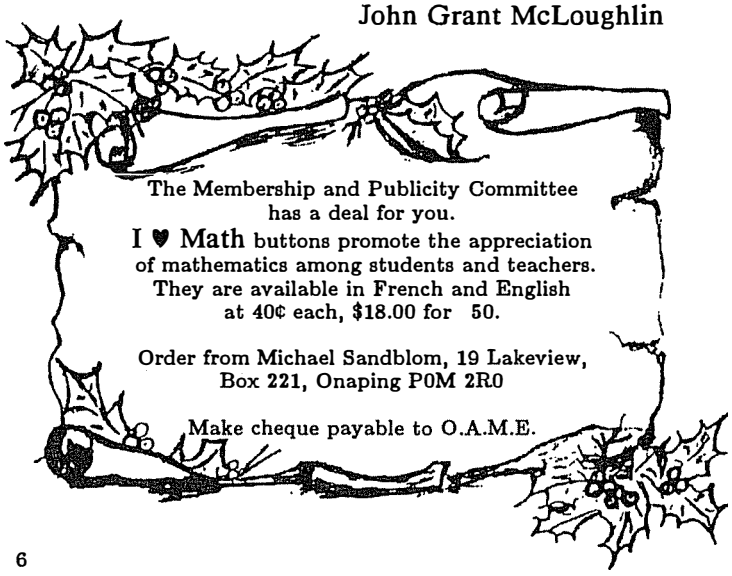
Since your forthcoming issue will be focussed on statistics and probability I thought your readers might enjoy the following problem

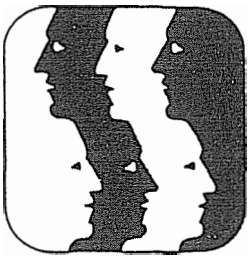
In a room darkened by a blackout, a drawer contains 8 blue socks, 10 black socks, and 6 grey socks, how many socks must be takenout of the drawer to ensure that there will be:
a) a pair of socks of the same colour?
b) a pair of black socks?

In both circumstances, students have to consider the worst possible scenarios to arrive at the correct conclusions.

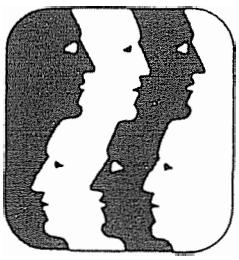
Answers: a) 4 socks b) 16 socks

John Grant McLoughlin





Viewpoint



WHY PROBABILTY AND STATISTICS MUST BE TAUGHT IN THE SCHOOL MATHEMATICS CURRICULUM

by
James E. Schultz
Department of Mathematics
The Ohio State University
Columbus, Ohio 43210

This Viewpoint is based on the keynote address given by the author at the "Mathematics for the 1990s" conference held at the Faculty of Education, University of Western Ontario on October 28th 1989.

Though there is a wide range of opinions, odds are the average teacher under estimates the value of probability and statistics in the school mathematics curriculum. Yet these topics are so relevant and natural that we might not even be aware that three terms from probability and statistics - *range, odds* and *average* - appeared in just the first half of the first sentence!

Having myself taught a semester course in probability and statistics several times in a Wisconsin high school over twenty years ago, I can say with confidence that some textbooks have been available in the secondary schools for quite a long time. Probability and statistics have also long been included as topics in elementary school texts. This is not to say these topics have often been taught. This Viewpoint cites some reasons why we should teach probabilty and statistics and addressess some of the possible barriers.

Probability and statistics are exceedingly important in our everday lives and are becoming increasingly more visible. For example, it would be virtually impossible to read any major newspaper any day of the year and not encounter statistics. (Note that this could hardly be said for long division, multiplying mixed numbers, or factoring polynomials!) As early as 1975 the National Advisory Committee on Mathematical Edcuation (NACOME) recommended "integration of statistical ideas throughout the curriculum at all levels." In 1976 the National Council of Supervisors of Mathematics identified "reading, interpreting, and contructing tables, charts and graphs" and "using mathematics to predict" as two of ten basic skill areas. The National Council of Teachers of Mathematics (NCTM) called for the exploration of statistics in its *Agenda for Action of 1980* and in the *Standards...* of 1989.

Support for the teaching of probabilty and statistics can also be found in other sources. The Preface of the *Teaching of Probability and Statistics* Yearbook of the NCTM gives several reasons for the inclusion of probability and statistics in the school curriculum, including (1) they provide meaningful applications of mathematics at all levels; (2) they provide methods for dealing with uncertainty; (3) they give us some understanding of the statistical arguments, good and bad, with which we are continually bombarded; and (4) they are inherently interesting, exciting, and motivating topics for most students.

In *School Mathematics for the 1990s*, prepared by the International Commission on Mathematical Instruction, the position is given that a sound mathematical education will enable people to handle the mass of data with which they are constantly bombarded in this information age. Writing in *The Elementary School Journal* (1984) Mary Lindquist said "There is no doubt that the skills of organizing and interpreting data are crucial in a technological and numerical world."

Some teachers may feel that they lack the knowledge necessary to teach probability and statistics, since these subjects may not have been emphasized in their own coursework. While sound preparation is of course important (as it is with any subjects you teach) there is now more and more support for teachers. For example, the joint committee of the American

Statistical Association and the National Council of Teachers of Mathematics has generated many helpful suggestions. Their recommendations include statements like "real data should be used whenever possible" and "the emphasis should be on good examples and building intuition" and not on using statistics to deceive. Moreover, text materials for students and teachers are rapidly improving to make the job easier. Appreciating probability and statistics as a dynamic and exciting way of dealing with uncertainty rather than as a body of complex formulas can go a long way to becoming comfortable with these topics. While there is no substitute for intensive study, teachers can begin to make the much needed improvements in their own backgrounds to remove this barrier.

While there has not been an abundance of good text materials, this is happily changing. More and more excellent materials are becoming available through projects such as the Quantitative Literacy Project, the Middle Grades Mathematics Project, and the University of Chicago School Mathematics Project, as well as through commercial sources. Many of these materials are highly accessible to most students. Lack of suitable materials is no longer a barrier.

Some teachers feel that they do not have time to teach probability and statistics because students have not yet mastered fundamental arithmetic skills. Such a stance is quite ironic, for statistics and probability provide relevant, interesting settings to develop necessary computation skills. In fact, in a world becoming more and more decimalized due to technology and the use of the Metric System, probabilty provides one of the strongest remaining arguments that students need computational skills with fractions at all.

In studying statistics, students should experience gathering, summarizing, and using data, statistical measures, and sampling, with applications to areas such as weather, geography, consumerism, social studies, recreation, physical fitness, and science. Specific topics should include representations using stem and leaf plots, box plots, bar and line graphs, and scattergrams. In the study of probability, students should experience experimental probability, the use of area models, and techniques of simulation. Neither statistics nor probabiity should be mere manipulation of formulas.

As a final comment, note that the title of this Viewpoint refers to why probability and statistics should be taught. These topics are far too important to be chapters at the end of the book which get skipped for lack of time. If anything is certain, it is that we should teach probability and statistics.

O.A.M.E. / A.O.E.M. LIFE MEMBERSHIP

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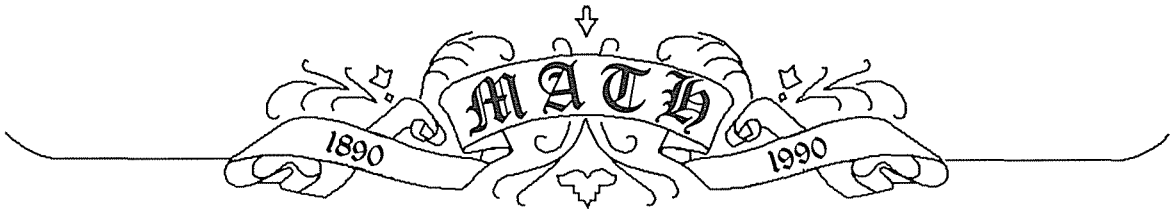
(a) 10 or more accumulated years of membership in OAME/AOEM, and

(b) demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular.

Nominations for Life Membership in the O.A.M.E. / A.O.E.M are to be submitted to:

Paul Lessard

Nominations with supporting information should be sent to the above address on or before **January 12, 1990.**



ARE YOU AN "OLD" MATHEMATICS TEACHER?
(OR A DESCENDENT OF ONE)

Joan Routledge (416-727-4167) and Jack LeSage (705-728-9958) have been asked by the Ontario Association for Mathematics Education to produce a History of Mathematics Associations in Ontario to celebrate 100 years.

If you have any information, convention programmes, pictures, minutes, vignettes, etc involving any past Mathematics organizations in Ontario (math sections of OEA, OATMP, OATM, OMC, OCI, etc) please write:

Jack LeSage



ON CHERCHE DES MANUSCRITS

Nous avons besoin de vos idées!
On encourage les lecteurs à soumettre des manuscrits qui examinent les techniques dans la classe ou des questions de l'enseignement de mathématiques (écrits en français) pour la critique.

Les instructions pour les auteurs peuvent être trouvées au début de la revue.

ON CHERCHE DES CRITIQUES

Afin de faciliter la politique de la gazette à propos de la publication d'articles en anglais et en français, il nous faut des critiques pour les articles français. Veuillez envoyer votre nom à:

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c/o University of Western Ontario
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BUILDING STATISTICAL CONCEPTS THROUGH VISUALIZATION AND VERBALIZATION

by
Rheta N. Rubenstein
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The recent document, *Curriculum and Evaluation Standards for School Mathematics* (Commission on Standards, 1989), challenges mathematics educators to involve students more actively in the construction of mathematical knowledge. Following are some activities designed for junior and intermediate students that address this challenge in the area of statistics. Each activity involves a visualization component in which students use concrete materials to see directly a statistical measure. This is followed by a verbalization component in which students are asked to explain, apply, or interpret what they see. The discussion step helps students make the transition from concrete examples to abstract concepts.

Activity 1: STAND TALL

OBJECTIVE: Develop vocabulary of data, mode, median, maximum, minimum, and range.

Students in a junior classroom arrange themselves by height around the periphery of their classroom. The concepts of less than, greater than, and equal to, are applied in a very direct way. The heights of the students are **data**: pieces of information. Students are then asked to discuss various attributes of their heights. For example, students of equal height. This height, the most common in the set of data, is called the **mode**. Students find the middlemost student in the ordered line; his/her height is called the **median**. If there are an even number of students there are two in the middle. In this case, a height halfway between those two is the median. (In general, in a set with an even number of elements, the median is the average or mean of the two middlemost values. However, the term mean is introduced in a later activity so it is not used here).

The tallest and shortest students stand back-to-back. The difference between their heights, which can be observed directly, is the **range** of heights in the class. The heights of those two students are **minimum** (least) and **maximum** (greatest) heights of the students in the class. If possible, the line of students is photographed and the new terms identified in the picture. Alternatively, strips of masking tape can be place on one wall to represent the height of each student in the class with labels placed on minimum, maximum, median, and mode heights. In either case the students gain a direct, visual understanding of some basic statistical terms.

To begin to build interpretive powers, students are asked to talk about situations in which these measures might be used. For example, suppose you were designing doorways for a school. Which measure is most important? (maximum) Suppose you were ordering graduation gowns for the class in various lengths. Which length would you order in greatest quantity? (mode) Suppose you were designing a bookcase so that all students could reach the top shelf. Which measure do you need? (minimum)

After students have an opportunity to collect and analyze other similar data such as heights of books on a library shelf, ask them to stretch their thinking with questions like the following. Is the difference between the minimum and the median equal to the difference between the maximum and the median? (Not generally.) How different would they expect the median or modal heights to be for another class of the same grade level? (Little.) Could they imagine another class of the same grade level with a much greater range of heights? (Yes, with very tall or very short students.) Could a class of students three years older have the same median height? (Very unlikely.) the same range? (Possibly.)

Activity 2: PENCIL PARADE

OBJECTIVE: Draw and interpret box plots.

Students work in groups of four. They pool pencils, crayons, or pens to have 15 writing

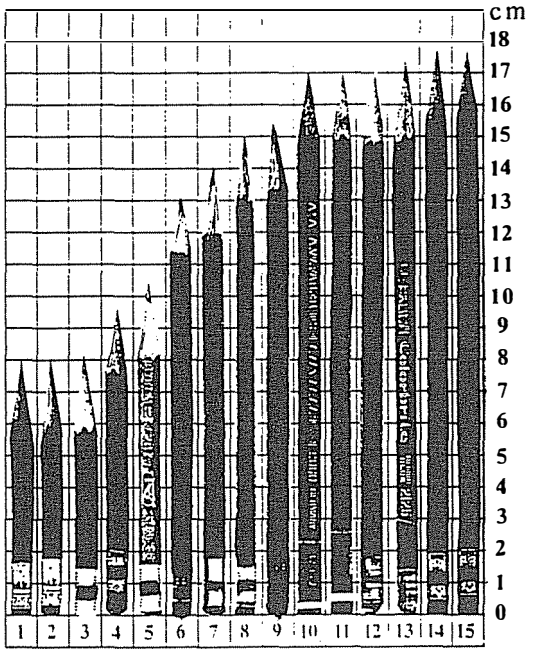


Figure 1.

implements of various lengths. They arrange the items in order by length on centimetre graph paper placing each one in its own column (see Figure 1). The teacher can do the same on the overhead projector with a transparency with a centimetre grid.

After reviewing the concepts from the previous activity, a new display of data is introduced, a **box plot**. The box plot uses five measures. Three of them are familiar: minimum, maximum, and median. These three measures are located in the heights of the writing implements. The other two are the median of the lower half of the date, called the **lower quartile** and the median of the upper half of the data, called the **upper quartile**. Using 15 data values, allow each of these five values to correspond to a specific value of the set; the minimum is the height of the first item, the maximum is the height of the 15th, the median is the height of the 8th, the lower quartile is the height of the 4th, the upper quartile is the height of the 12th. In general, when finding the lower and upper quartiles in a set of data with an odd number of values, the median is excluded from each half when finding the median of that set. Thus using 15 (or any value of the form 4n + 3) simplifies this introductory activity.

Figure 2 shows how the heights of the items can be directly copied across the page to create a box plot. The "box" represents the middle half of the data. The extensions, sometimes called "whiskers", represent the lower and upper quarters of the data. The example in Figure 2 has a longer whisker at the bottom of the box than at the top. Students are asked to connect these attributes to the pencil lengths themselves. After some discussion they should determine that the pencils in the top quarter vary less in their heights than do those in the bottom quarter. Similarly, the students should see that the upper portion of the box (for pencils 15-17cm) is shorter than the lower portion (for pencils 9.5-15cm). This reflects the fact that pencils in the lower group vary more in their heights than do those in the upper group. Again, discussion is the vehicle to build connections between the original heights and their more abstract representation in the box plot.

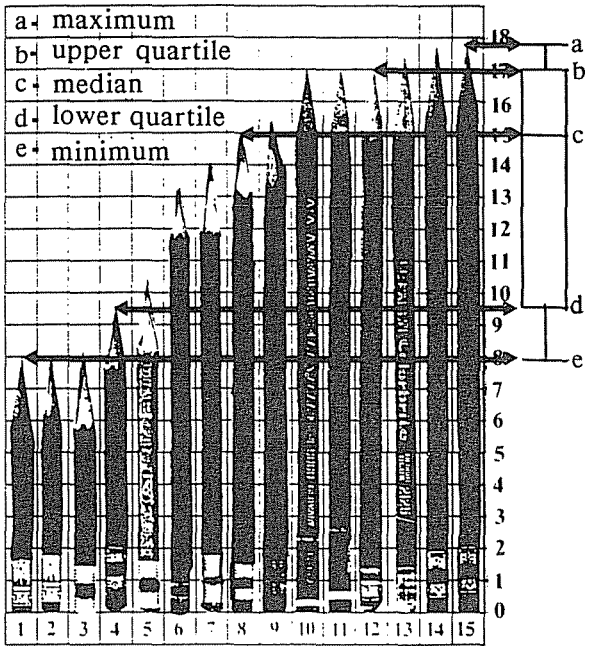


Figure 2.

Activity 3: **LET'S SHARE**

OBJECTIVE: Develop concept of mean as a value derived from sharing a total equally.

To begin, students make stacks of blocks of heights four, two, and six. Then they rearrange the blocks to form equal sized stacks (see Figure 3). At this point there should be some discussion about sharing the blocks equally among the three positions. Repeat this stacking and rearranging for several sets such as {9, 2, 1}, {7, 4, 4}, {7, 9, 5, 3}. Then, ask students to discover a way to find the mean without actually rearranging the blocks. If necessary, ask whether the total stays the same

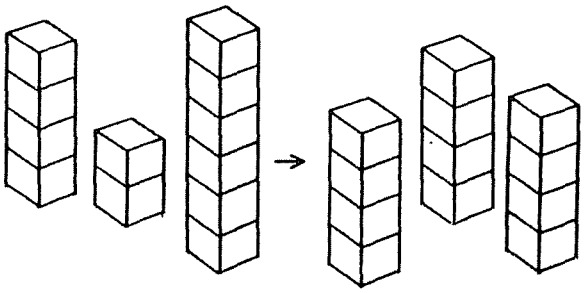
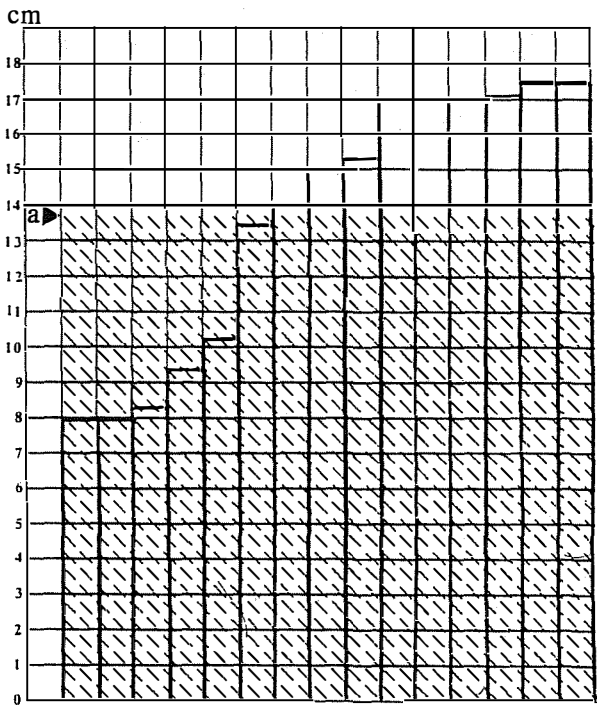


Figure 3.

and how this, together with the number of stacks, can determine the mean. Ultimately they should see that they must divide the total number of blocks by the number of stacks.

Next, return to the pencil lengths of the previous activity. Have students make a display of the heights of their pencils on the centimetre graph paper. Have them include a scale or simply place a ruler along the edge of the graph paper. Then, discuss what the mean of the heights represents. Although the pencils cannot be taken apart like the blocks, the mean is the constant height all the pencils would have if the same amount of wood and lead were evenly *shared*. How is this found? Find the total number of millimetres and divide them equally among the number of *stacks*, which in this case is the number of pencils. Students may use calculators to do so. Another visual display of the meaning of the mean uses the overhead projector. Use coloured acetate and make a rectangle with height equal to the mean and width equal to the width of the 15 bars. This is overlaid on the display of original heights (see Figure 4). Ask students to talk about what they see. They should recognize the total length by which short bars are increased is equal to the total length by which the tall bars are shortened. They may count squares and fractions of squares to verify. Again, the idea is that the total length is constant.

As an extension, ask students to discuss some special sets. In each case have them compare the median and mean of a new set with those of a set with which they are already familiar. For example, suppose all their pencils were short golf pencils except for one ordinary unsharpened new pencil. Or, suppose ordinary pencils were used and one giant pencil from a carnival. Students begin to see that while an



a: mean = 13.6 cm

Figure 4.

extreme value has little effect on the median of a data set, it can have a dramatic effect on the mean. (Parts of Activity 3 are adapted from Bohan and Moreland, 1981, who have tips for other activities as well.)

Activity 4: **COUNT AND BALANCE**

OBJECTIVE: Create and interpret a frequency distribution

Each student is asked to report to the nearest centimetre the length of his longest pencil. In this case there will be many of the same length. Students and teachers together record this information as it is being reported. In quite a natural way a **frequency distribution** develops as an x is placed in each appropriate column (see Figure 5). This display shows the lengths of the pencils on the horizontal axis and the number of times each occurs (frequency) on the vertical axis. Students are asked to identify the mode in this display (value with tallest column, in this case 16cm)

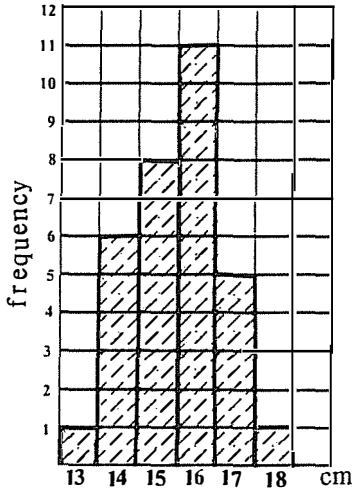


Figure 5.

and the median (length corresponding with the middle value when counting the number of x's from bottom to top while moving left to right; in this case also 16cm). For a greater challenge, students are asked to find the mean without having to add together all the values. Some should suggest that they can use multiplications derived from the graph. See Table 1.

The mean is 496/32 or 15.5 cm. This, too, has a physical meaning in terms of the graph. Note that the point associated with each length is the centre of the square labelled with that

Frequency x (height of bar)		Value of bar (length of pencils represented)	=	total cm represented by that bar
1	x	13	=	13
6	x	14	=	84
8	x	15	=	120
11	x	16	=	176
5	x	17	=	85
1	x	18	=	18
32 pencils				496 cm

Table 1.

number. Then 15.5 is on the line dividing the columns for 15 and 16. With a paperclip, punch a very small hole near the horizontal axis at that point. Carefully cut out the graph. Suspend the graph by the paperclip through the hole (see Figure 6). It should balance from that point. That is, the bars should fall vertically. Students should compare this to a seesaw. On a seesaw a heavy person can be balanced by a light person if the light one sits farther from the pivot. Similarly, a bar near the mean ("pivot") of a frequency distribution can be balanced by a shorter bar farther from the mean. The students should take another look at the numbers involved to confirm this relationship.

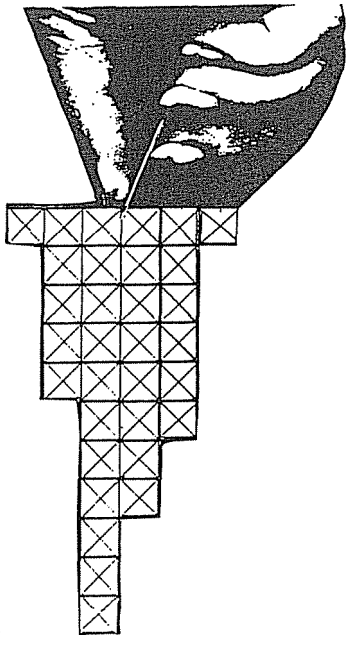


Figure 6.

Activity 5: SLIDES AND STRETCHES OF DATA

OBJECTIVE: Recognize effects of transformations on a data set.

Recall, as in Activity 2, some data for a set of pencils. Ask students to place an eraser tip on the end of each pencil. Thus the length of each pencil is increased by one centimetre. Have students answer questions like the following. What is the effect on the mode, median, mean? [each is increased by 1] What is the effect on the range? [none] Other similar events are discussed. For example, suppose we have data on the weights of all the students in the class. Now every one puts on a 1 kg set of jogging anklets. How are the statistical measures affected? Suppose the manager of the local fast food restaurant has statistics on the pay for a certain week in December for all the teens working there. Now they each receive a $20.00 bonus for New Year's Day. How are the statistical measures affected? Students enjoy creating their own such examples.

Similarly, have students consider multiplication factors. Suppose all the pencils in the data set are cut in half. What affect does this have on the mean, median, mode and the range? [each is halved] One way of visualizing this is by finding the centre of each pencil and aligning these locations on the edge of the graph paper and covering the lower portions. Again, analogous questions can be raised. For example, suppose the teens working at the fast food shop all get a 15% raise. How does this affect the statistical measures of the original salaries? [mean, median, mode and range are each multiplied by 1.15]

Activity 6: THE LONG AND SHORT OF IT

OBJECTIVE: Interpret pairs of box plots.

[These activities are designed for older intermediate students and are derived from Maher (1981)]. First, have students review some data with a box plot and interpret long and short whiskers and portions of boxes as in Activity 3. Then have them find their heights to the closest centimetre. Make two box plots, one for boys and one for girls. Consider the results in Figure 7.

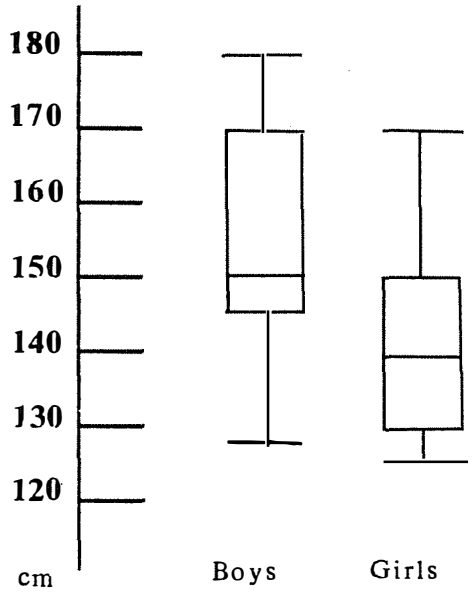


Figure 7.

Ask students to discuss differences and similarities between the box plots and the meanings of these comparisons. Clearly, the range for the boys is greater; their heights are widely disparate. The majority of the girls (those in the lower 75%) are closer in height than the lower 75% of the boys. This group of girls is more alike in height than the same proportion of the boys. The median for the boys is about equal to the upper quartile for the girls meaning that half the boys are taller than three quarters of the girls. The heights of the middle 50% of the girls are evenly distributed about the median. This suggests that the distribution of heights among the middle 50% of the girls is fairly even. The top whisker for the girls is long. This means there are a few girls much taller than the majority. The boys have a wide spread in heights between the median and the upper quartile. This suggests that the third quarter of the boys have very different heights. In contrast, the boys in the second quarter are rather consistent in their heights.

Learning to interpret box plots is challenging. Again, having the students physically arrange themselves in order by height and identify those at median and quartile positions assists

everyone in visualizing the relationship between the display and the data.

Extension: To provide opportunities to do more interpretation the class may hold a Measurement Fair. Committees of students take charge of various events. These include measurement of physical as well as action attributes. Physical attributes include length of handspan, armspan, stride, vertical reach, and cubit (elbow to longest fingertip). Action attributes include distances for throwing, long jump, and standing high jump. For other possibilities see Pagni (1979). Each committee then organizes, analyzes, displays, and reports its findings. Comparison box plots are used to find differences or similarities by gender, athletic involvement (participates in sports or not), or age (e.g., over or under 14). Bulletin board displays of the findings will be of interest for several weeks following the Fair.

Summary

This article has illustrated some activities using modest materials emphasizing visualization and discussion to help junior and intermediate students acquire, understand, use, and interpret key statistical ideas. The reader is encouraged to try them or to use them as models for similar activity-based instruction.

REFERENCES

Bohan, Harry & Moreland, E. J. (1981). Developing some statistical concepts in the elementary school. In A. P. Shulte & J. R. Smart (Eds.), Teaching Statistics and Probability, Yearbook. (pp. 60 -63). Reston, VA: The National Council of Teachers of Mathematics.

Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics. (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: The National Council of Teachers of Mathematics.

Maher, C. A. (1981). Simple graphical techniques for examining data generated by classroom activities. In A. P. Shulte & J. R. Smart (Eds.), Teaching Statistics and Probability, Yearbook. (pp. 109-117). Reston, VA: The National Council of Teachers of Mathematics.

Pagni, D. L. (1979) Applications in school mathematics: Human variability. In S. Sharron & R. E. Reys, Applications in School Mathematics, Yearbook. (pp. 43-58). Reston, VA: The National Council of Teachers of Mathematics.

1989 McCartney Scholarship Winner

The Ontario Association for Mathematics Education continued to sponsor the McCartney Scholarship, financed by Science Research Associates, again in 1989.

The judges this year were William Renoy, Prince Edward Collegiate Institute, Picton and Janet Richards of Iroquois Falls Secondary School.

The winner was Philip Jong of Earl Haig Secondary School, North York Ontario. Philip was a pupil of "Bill" Bissett a former councillor and long time member of OAME. Philip was born in Hong Kong and came to Canada with his parents in 1984. His father, Paul Jong is in the real estate business. His mother, Helen is an accountant. He has two brothers.

Philip received his elementary education at Rosedale Elementary School, Toronto. He attended Zion Heights Junior High School, North York before entering Earl Haig. To further his education, Philip will be attending the University of Toronto and hopes to specialize in a field of mathematics and/or the medical sciences.

Philip has won various mathematical awards culminating in the winning this year of a silver medal at the International Mathematical Olympiad.

Mona and Morley were very pleased to reward Philip with this scholarship.

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LOTTERY WINNING PROBABILITIES

by
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Thunder Bay, Ontario

Lotto 649 provides an example of a situation whose sample space contains millions of 7 points. Consequently the calculation of the probability of various outcomes will require a method of counting large numbers of points. This problem and its analysis will illustrate some aspects of the combinatorics and probability segments of the Ontario Academic Course in Finite Mathematics.

A person "plays" Lotto 649 by choosing six of the numbers from 1 to 49 and pays $1.00 for a ticket bearing the six numbers chosen. On the day of the draw, six "winning" numbers are generated at random (without replacement). Then a seventh number, the bonus number, is generated. If a player's set of numbers matches the six winning numbers, that player wins a jackpot prize of at least one million dollars. The actual amount of prize money depends on the number of tickets purchased. For instance, the draw held on Wednesday, February 15, 1989 had a jackpot prize of $2 485 211.00. It is possible that more than one player will be eligible to win the jackpot prize, in which case the prize is shared. Subsidiary prizes are awarded as follows:

a. The player wins second prize ($187 771.50 on February 15) if five of the six numbers on the player's ticket match winning numbers and the remaining number matches the bonus number.

b. The player wins a third prize ($4 248.20 on February 15) if five of the six numbers on the player's ticket match winning numbers and the remaining number does not match the bonus number.

c. The player wins fourth prize ($143.20 on February 15) if four of the six numbers on the player's ticket match winning numbers.

d. The player wins fifth prize (always $10.00) if three of the six numbers on the player's ticket match winning numbers.

In order to count the number of points in the Lotto 649 sample space and in the events specified by (a), (b), (c), and (d), the reader should recall that the number of ways of choosing x items without replacement from a list of n items is

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

where $n! = n(n-1)(n-2) \ldots (3)(2)(1)$. Consequently, the number of ways of choosing six numbers from the numbers 1 to 49 is

$$\binom{49}{6} = 13\ 983\ 816$$

So the probability of the player winning a jackpot prize is

$$\frac{1}{13\ 983\ 816} = 0.0000000715$$

To win a second prize, five of the player's numbers must be from the list of six winning numbers and the player's remaining number must match the bonus number.

There are $\binom{6}{5}$ ways of choosing 5 of 6 winning numbers and $\binom{1}{1}$ way of choosing the bonus number.
Consequently there are $\binom{6}{5}\binom{1}{1} = 6$ points in the event described by (a).

To win third prize, five of the player's numbers must have been chosen from the list of six winning numbers and the player's remaining number must not match the bonus number but, instead, must be one of the 42 numbers that were not drawn. Therefore the event in the sample space described by (b) contains

$$\binom{6}{5}\binom{42}{1} = 6(42) = 252 \text{ points.}$$

To win a fourth prize, four of the player's numbers must have been chosen from the list

of six winning numbers and the player's remaining numbers must be from the list of 43 other numbers (the bonus number has no special importance). Therefore the event is the sample space described by (c) contains

$$\binom{6}{4}\binom{43}{2} = 15(903) = 13\ 545 \text{ points}$$

Similarly, the event in the sample space described by (d) contains

$$\binom{6}{3}\binom{43}{3} = 20(12\ 341) = 246\ 820 \text{ points.}$$

As a result, the event of not winning any prize contains

$$13\ 983\ 816 - (1 + 6 + 252 + 13\ 545 + 246\ 840) = 13\ 723\ 192 \text{ points}$$

Since the probability of an event equals the number of points in the event divided by the number of points in the sample space, the probability of winning a particular prize can be found by dividing the number of points in the event associated with that prize by 13 983 816. The probabilities of winning the various prizes are summarized in the following table.

JACKPOT PRIZE $\frac{1}{13\ 983\ 816}$ = 0.0000000715	**SECOND PRIZE** $\frac{6}{13\ 983\ 816}$ = 0.000000429
THIRD PRIZE $\frac{252}{13\ 983\ 816}$ = 0.0000184	**FOURTH PRIZE** $\frac{13\ 545}{13\ 983\ 816}$ = 0.000969
FIFTH PRIZE $\frac{246\ 820}{13983816}$ = 0.0177	**NO PRIZE** $\frac{13\ 723\ 192}{13\ 983\ 816}$ = 0.981

One can also compare the number of prize winners in the February 15, 1989 draw with the anticipated number of winners for that draw if all 13 983 816 possible tickets had been purchased and no ticket was duplicated.

NUMBER OF WINNERS

PRIZE	FEB. 15, 1989	ANTICIPATED
JACKPOT	0	1
SECOND	5	6
THIRD	169	252
FOURTH	9 640	13 545
FIFTH	199 971	246 820

The above table shows that the actual number of winners on February 15 was smaller than what might have been anticipated, an indication that perhaps fewer than 13 983 816 (different) tickets might have been purchased for the February 15, 1989 draw. Based on the above information, a crude estimate of the number of different tickets purchased for the February 15, 1989 draw could be calculated using:

$$\frac{\text{actual number of winners}}{\text{anticipated number of winners}}(13\ 983\ 816)$$

$$= \frac{0 + 5 + 169 + 9\ 640 + 199\ 971}{1 + 6 + 252 + 13\ 545 + 246\ 820}(13\ 983\ 816)$$

$$= \frac{209\ 785}{260\ 624}(13\ 983\ 816)$$

$$= 11\ 256\ 043$$

The above estimate makes the assumption that the proportion of "actual number of winners" to "anticipated number of winners" also reflects the fraction of the 13 983 816 different tickets that was actually sold. Because of the possibility of duplicate winning tickets, it is likely that the above type of estimate will err on the high side.

FROZEN MANNEQUINS

by
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There is a type of problem which at first glance appears to be *applied* math but turns out to have little to do with practical reality. These problems have to do with arrangements of objects in circular patterns and they still appear in current math texts where they continue to baffle more than a few students. The absurd assumptions needed to obtain *correct* answers to such problems supports the perception that you need to be somewhat perverse or obtuse to enjoy this type of problem solving.

Here are three related examples based on anonymous current sources, that students might encounter in the study of probability.

(1) How many ways can 5 people be seated at a round table (Answer: 24)

(2) How many ways can ten basketball players form in a circular huddle around their coach? (Answer: 362880)

(3) A necklace consists of a particular clasp, seven identical beads and one unique bead. How many ways are there to arrange the necklace? (Answer: 4)

Any creative photographer knows the answer to the first two questions is infinity. And any parent will tell you that when the necklace of example 3 adorns an adolescent neck, there are many more than 4 ways to locate the clasp or the unique bead with respect to a left earring, with respect to a trademark alligator or polo rider sewn on a sweater, or with respect to a streak of blonde hair near the right temple.

Jefferson Hayseed III, a typical better-than-average math student, believed his seventeen years of hands-on familiarity with tables, which included more than two years as a server at local restaurants, was more than adequate to justify his answer of infinity to the first problem. The *correct* answer in the back of the book astonished him, so he cogitated some more and recalled that he had ecountered five-seaters in fast-food outlets where the tables and seats ere rigidly fixed to the floor. Such five-seater tables, he concluded, could support no less than 120 different arrangements of frozen mannequins. In disbelief he rechecked the answer in the back of the book, but no way could he imagine a five-seater which supported only 24 seating arrangements. Poor Jefferson was devastated, a victim of a limited and overly realistic frame of reference.

Listen Jeff, you know that empty silo - that approximation of a right circular upright cylinder - which your dad abandoned when free trade made Wisconsin cheese potentially more attractive than Ontario cheese? Imagine spending your inheritance transforming it into a group meditation chamber for originators of perverse math riddles. We'll clean it out and brick up the entrance. Then we'll hire the best tradesmen in Eastern Ontario, to re-plaster the inside. Every defect will be covered so meticulously, we'll end up with a near-perfect cylindrical room. We'll get them to level the floor and install a retractable conical ceiling. Precisely in the apex of the cone, we'll have installed a solitary light fixture. As you'll see, this fixture will have to have an infinity of vertical planes of symmetry. Then we'll get them to make a perfectly flat, round table and fix it precisely in the centre of the new floor with five identical seats rigidly fastened so that the distance between them is equal and constant. No picture or other ornament such as a ventilation grill may adorn the chamber. It will be bright, bare and clean - but because of mathematical constraints, the light switch will have to be installed outside the chamber and the air inside may be stale at times. A final inspection will reveal that everything is absolutely ideal - right down to the hue, stiffness and texture of the table and seats. Not even a fly speck will be visible anywhere inside the renovated chamber - everything will be uniformly constant as far as human perception is concerned.

Now we are ready to seat the first of our five authors. Start with the one who originated the problems involving round tables. We'll blindfold him with an edible, blue cheese flavoured bandana - a current lo-tech favourite. With a strong necklace clasp - the very rare kind used in arrangement problems which presuppose a plane of symmetry perpendicular to the plane of the necklace, along a diameter which goes through the centre of the clasp - we'll buckle him in a

basket at the business end of a crane boom. When the basket is suspended we'll spin it wildly - we'll use a vertical axis of rotation - and hoist the dizzy author through the open top of the silo cylinder. By a minor miracle, an event which has low probability, he'll dock in one of the five seats, totally disoriented. We'll dislodge him by remote control, quickly retract the basket and slam the roof shut.

Eventually, he'll remove his blindfold, and munching it, will contemplate his ideal surroundings. "Which chair am I in?" he'll muse. Assuming his new environment, he'll soon realize the absurdity of his question. He will exclaim "This room has no reference point and spinning in that basket has disoriented me! I don't know north from west, so all five seats are equivalent and mathematically - there is only one way for me to be seated here!"

Let's leave him there to contemplate his fate for a while. Maybe he'll get the blues and become "cheezed off" like so many students who got little encouragement from cryptic answers to idiotic arrangement problems. When he finally dozes off we'll retract the hatch and lower the creator of the basketball problem. And when the slumbering first author awakens, he'll see the rival author standing near him. "Have a seat", he'll say. "Using me as your reference, you can choose any of four places to sit!"

I see by your look Jeff, that you are anticipating the rest of this fantasy . . . You're right! The next guy lowered will be the necklace guy . . . and yes, right again . . . he'll have three seats to choose his from. Which makes a total of - one times four times three - twelve ways of seating the first three meditators. Great! I believe you'll be able to finish this. And good luck getting the *correct* answers to the rest of those arrangement problems.

Teaching Tips

On the first day of classes we distribute textbooks to our students and expect them to be immediate experts on how to use these books. Too often the text becomes just a source for problems. Therefore it makes sense to spend some time with our students on how to use the text. One could have the class examine:

a) the table of contents
b) the index at the back - use specific examples
c) the structure of the chapters
d) the structure of the exercises - easy to difficult
e) the structure of the review - chapter/cumulative
f) the answers to the questions
g) how the author has highlighted the important results - use of boxes, different colours
h) the extras that are included - historical notes, puzzles

With a better understanding of the structure of the textbook, students may even find it useful in their study of mathematics.

Suggested by:
Ron Reesor
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PROBLEMS

by
Ronald Lancaster

1. This year (1989) there were two Fridays that landed on the thirteenth of the month. (January 13th and October 13th)

   a) When was the last time that we had two "Friday the thirteenths" in one year? When will the next time be?

   b) Is it possible to have three "Friday the thirteenths" in one year? Why or why not? Is it possible to have none? (or one? or four? or five?)

   c) Why is "Friday the thirteenth" considered to be unlucky?

2. Using a calculator find the value of $(3 + \sqrt{8})^n$ for $n = 1,2,3,\ldots 10$. Include as many decimal places as your calculator allows and discuss your results.

3. Find the smallest positive value of x for which $\sin(x^\circ) = \sin x$. (i.e. the sine of x degrees is equal to the sine of x radians)

4. Let $f(x) = kx^r$. Find the values of k and r for which $f'(x) = f^{-1}(x)$.

Solutions and Commentary

1. Last October, Marion Lamoureux, a colleague of mine, came and asked me if I had any "Friday the thirteenth" problems that she could use on a test that she was giving on October 13. I showed her a few things that I had done several years ago and she put together the question as it is given here. She used it as a bonus question and allowed her students to hand their work in on the following day. I spoke with five or six of her students and they really liked her idea.

This is a great problem because there are so many ways of approaching it. It is also something that students at the elementary level can work on too! One way of getting started is to realize that the entire calendar for any year is determined by two things:

1) The day of the week that January 1 lands on. There are seven possibilities of course.

2) The type of year that it is. There are two types, a regular year with 365 days and a leap year with 366 days. Leap years occur every 4 years. An exception to the four year cycle occurs in a century year (1500, 1600, 1700...). Century years are leap years only if they are divisible by 400.

From the above it follows that there are only 14 different calendars. They can be categorized as shown below.

January 1	Type of Year
Monday	Regular
Monday	Leap
Tuesday	Regular
Tuesday	Leap
...etc.	

All you need to do now is look at each of the 14 cases and see where the thirteenth day of each month lands. You will find that it is possible to have only 1, 2, or 0 "Friday the

thirteenths". You might also discover that our calendars repeat every 28 years.

2. Were you surprised by what you found? Did you notice that as n got larger, $(3 + \sqrt{8})^n$ approached being integer valued. In fact, on my calculator I found that

$$(3 + \sqrt{8})^{10} = 45\ 239\ 074$$

Clearly something is wrong here, because $(3 + \sqrt{8})^{10}$ is an irrational number. Why is it so close to being an integer? Why is $(3 + \sqrt{8})^{11}$ even closer to being integer valued? [and $(3 + \sqrt{8})^{12}$ etc.]. Play with this for awhile and I'll make some comments about this in the next issue. You might also want to look at $(4 + \sqrt{15})^n$, $(5 + \sqrt{24})^n$, etc.

3. $\sin(x^0) = \sin x$

$\sin x - \sin(x^0) = 0$

$$\sin x - \sin\left(\frac{\pi x}{180}\right) = 0$$

$$2 \sin \tfrac{1}{2}\left(x - \frac{\pi x}{180}\right) \cos \tfrac{1}{2}\left(x + \frac{\pi x}{180}\right) = 0$$

The above equation has an infinite number of solutions. The smallest positive solution is given below (the details are left for you).

$$x = \frac{180\,\pi}{180 + \pi}$$

$$\simeq 3.087702086$$

I have been giving this question to my top senior math students for quite a few years (I cannot remember where I first saw it). I would encourage you to use it too! One thing you might do is ask your students to enter 3.1 on their calculators and compute the sine in either mode. They should find that the two results are almost the same. Ask them to find a number for which the two results are even closer to each other. If they keep going, most of them will be able to come up with something close to 3.087702086. Challenge your better students to find the exact value. This might also be an appropriate time to graph $y = \sin(x^0)$ and $y = \sin x$ on the *same* set of axes. In fact, all calculus students should be shown this because it makes it very clear that the derivative of $\sin x$ is $\cos x$ only if x is in radians.

4. You should have found that

$$r = \frac{1 + \sqrt{5}}{2} \qquad \text{(golden ratio!)}$$

and $k = r^{1-r}$

Thus $f(x) \simeq .74274294462469x^{1.61803398874929}$

Here is a suggestion for you. On one of your calculus tests give your students this function and ask them to find the inverse (you could do this with Grade 11 or 12 too). Later, on another test, give them the function again and ask them to differentiate it. When you take up the test, ask them to compare and discuss the two questions and their answers.

Can you find a similar function for which $f''(x) = f^{-1}(x)$? or
$f'''(x) = f^{-1}(x)$ etc.

Dr. Tom Davison from McMaster University recently sent me a beautiful way of solving Q1 from the last issue (go back and take a look at the problem and how I solved it). He let the roots of the quartic polynomial be b-3e, b-e, b+e, b+3e and solved the problem in several lines. This is a most unusual way of writing four consecutive numbers and I doubt that most people would come up with this. Dr. Davison is well known for his unique way of solving problems and is often able to see what other miss.

I have not yet received any solutions to the geometry problem from the last issue (Q2, September 1989). If you come up with something send it along to me. In the next issue, I will run solutions to both this problem **and** the geometry problem from the April 1989 issue (Q3). So far I have received five different solutions to that one!



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SO, YOU HAVE TO TEACH MULTIPLYING



by
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As educators we are faced with the problem of differentiating the curriculum for the gifted to provide appropriate learning experiences. In the field of Mathematics the normal route to go is acceleration, however, this does not totally fill the Ministry mandate nor is it always appropriate to the students' needs. From this basis I would like to suggest an approach to mathematics for the gifted that will help deal with this issue.

Too often we look at mathematics courses as arithmetic: if the student can demonstrate a mastery of the skills, then it is assumed that they know the concept. Mathematics is often interpreted as product learning only, instead of process learning as well.

To add meaning to these comments one must first deal with some terms. According to Mary Baratta-Lorton (1976) arithmetic is mastery of skills, while mathematics is concept development. Arithmetic can be seen as mastery of the basic facts and the use of them to obtain correct answers. These students usually have difficulty with problem solving because the concept of addition or multiplication or whatever is not mastered. They know that 5 + 3 = 8 but they do not understand the concept of 'eight' or the concept of addition.

I would like to suggest that mathematics be viewed as process learning with the product a necessary part of the process.

Programming for the gifted should be based on the development of mathematical concepts and an understanding of the processes in mathematics. Gifted students can master the facts and in that way show mastery of the product but as the information base expands, the product will not be memorizable so the student must understand the process involved. In order to enrich the math program, we must look at the process of math and be able to develop the concepts.

To this end I have been working with a group of grade 7 students from an enrichment program. We worked on the basic facts, but even such a topic can be broadened to increase the students' understanding and help them appreciate the patterns and relationships in the number system.

MULTIPLYING TABLE

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Figure 1

First we looked at the simple multiplying matrix (see figure 1). After discussing patterns and making observations such as the digit used most, least, and row and column similarities, we wrote the table in mod 9.

I have a challenge for you. Here are some numbers and then the number written in mod 9. Can you finish the pattern?

Base 10	12	43	63	71	35	23	87	432	65
Mod 9	3	7	9	8	8	5	?	?	?

A multiplication table, when written mod 9, is formed by taking the remainder when an element in the original table is divided by 9. For example, 8 x 7 = 2 (mod 9) because 8 x 7 = 56 and 9 divides into 56 6 times with a remainder of 2. Since the remainder when

dividing by 9 can only be 0, 1, 2, ..., 7, 8, a mod 9 system has only 9 elements. If the original system is infinite then these 9 elements will repeat themselves over and over.

Now the students made the multiplying table in mod 9 and started making observations about it (see figure 2). Students began to see in a concrete way, the similarities between the 3, 6 and 9 times tables. One student said "Why do we have to learn the tables in chronological order. It would be easier to learn the tables as families."

VEDIC SQUARE

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

Figure 2

We spent several lessons looking for patterns on the multiplying table. Students coloured each number a different colour to emphasize the symmetry of the chart. We also observed that each diagonal was a palindromic number. In all, we listed more than 30 different patterns in the chart.

It was my intention to help students see patterns in the number system and to learn their multiplying facts. We made circle graphs of each of the multiplying tables (see figure 3). To make the circle graphs we divided a circle into 9 equal sections and labelled them with the numbers from 1 to 9. Students chose the table they wanted to graph and joined the points on the circle in the order of the mod 9 chart. eg. For the 7 table they started at 7 joined to 5, the 5 to 3, the 3 to 1 and so on.

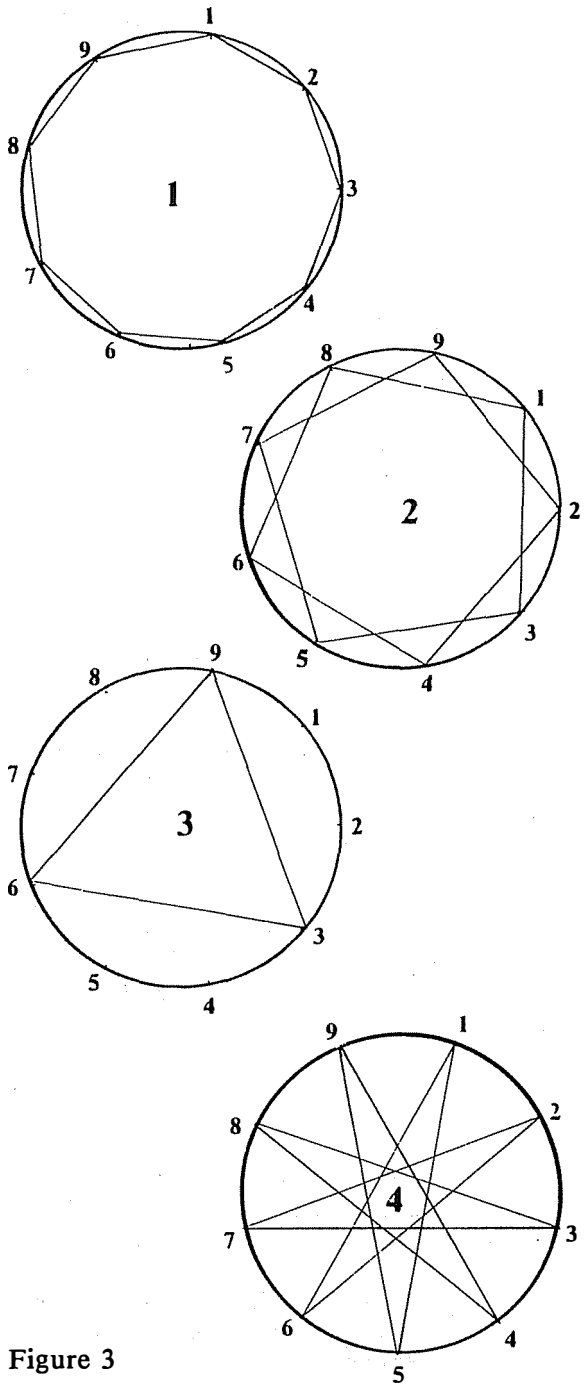
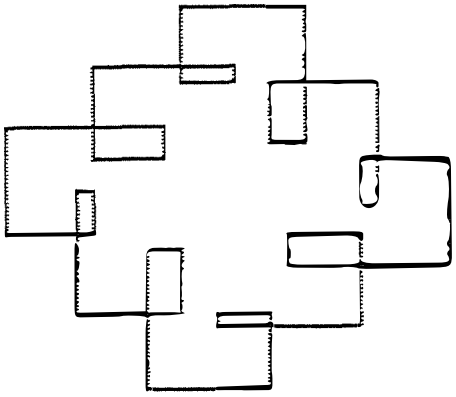


Figure 3

The similarities of the tables now became evident visually. As the students worked with these graphs, interesting observations came up in discussion. Alexis observed, "On a circle graph, if you take 2 numbers that if added equal 9, they will form the same pattern. "Having graphed the tables we started working with Logo on the computer to represent them

pictorially. We used one table and wrote it in mod 9. We multiplied each number by 5 (if we had used the numbers as they were, the Logo graph would have appeared too small.

Seven Times	7	14	21	28	35	42	49	56	63
Mod 9	7	5	3	1	8	6	4	2	9
X 5	35	25	15	5	40	30	20	10	45



```
TO TWO
  FD 10 RT 90
  FD 20 RT 90
  FD 30 RT 90
  FD 40 RT 90
  FD 5 RT 90
  FD 15 RT 90
  FD 25 RT 90
  FD 35 RT 90
  FD 45 RT 90
END

TO TWOALL
  REPEAT 4 [TWO]
END
```

Figure 4.

These numbers were used in order as the FORWARD distance and in between each forward command we made a right turn of 90 (see figure 4).

Students were given time to experiment with this work. They used different angles in their programs, or changed the angles by making each angle increase by 1 degree through the program.

Students now had several experiences working with the multiplying table. Each time we did a new activity I would get the students to start with the multiplying table and make the Vedic Square. The table written in mod 9 is called the Vedic Square. I was introduced to this idea by Lowell Hovis of Seattle, Washington. However, I have not been able to get any information on the Vedic. I do not know if it is the name of a Mathematician or a made-up title for the table. If you can come up with any information about Vedic I would appreciate hearing from you.

By the time I had worked with a group of students for a year with the Vedic Square and other activities like it, the students knew their multiplying facts just as well as when we had spent the year memorizing them.

We also worked on other patterns with the multiplying facts. I introduced them to the Ethiopian way to multiply. The Ethiopians did not know how to multiply but they could get the product of any two numbers. They knew no more mathematics than how to add, double, or half a number. Yet with this little knowledge they could correctly find the product of any two numbers. This is how they did it:

13 X 15
~~6~~ ~~30~~
3 60
1 120
195

1. They put 13 in a column at the left and 15 in a column at the right.

2. They divided 13 by 2; that is, they halved 13. Of course the answer is 6 1/2. But they knew nothing about fractions; so they dropped the 1/2 and wrote 6 below 13.

3. They doubled the number in the right column and wrote 30 below 15.

4. They halved 6 and doubled 30.

5. They kept this up until the last number in the left column was 1.

6. They did not like even numbers and believed they should be destroyed. All even numbers in the left column were crossed out

along with their partners. So, they crossed out the 6 along with its partner, 30.

7. The numbers in the right column were added. The sum is 195, and that is the right answer.

Activities such as these provide students with another method of getting the correct answer but even better let them examine the process involved. Why does this method work? That is the question that is always asked, but I do not always want to provide the answer.[1] Students now ask that question of themselves without me saying it. In so doing the students are looking at the process of mathematics.

Other activities which achieve a similar result are The Napier's Bones, Finger Multiplying and Lattice Math. Why do we multiply the way we do? When looking at this question and at other methods of obtaining the same result, we help students look at the process of multiplying. The answer to this question is not as important as the fact that we look at the question and explore other methods of multiplying.

Another activity that generates patterns is the exploration of palindromic numbers. These are numbers that are read the same backward or forward. Students started experimenting with the Fibonacci sequence. They discovered that the sequence was palindromic if written in Mod 9. However, the sequence had to be written to beyond 60 numbers before the pattern was evident.

All these activities on their own are just more exercises but the patterns of mathematics are shown through them and some of the mystery of mathematics is revealed. You can design a program for these students that is not acceleration. It means taking the topic beyond the fringe and providing for them a diversion of thought and idea.

[1] *We leave the proof as a challenge. Correct solutions will be published in the next issue of the Gazette.*

BIBLIOGRAPHY

Charles, Randall I. (1986). Fact Strategies: Bridging the Way to Better Math Performance. Learning 14(5), pp. 54, 56-60.

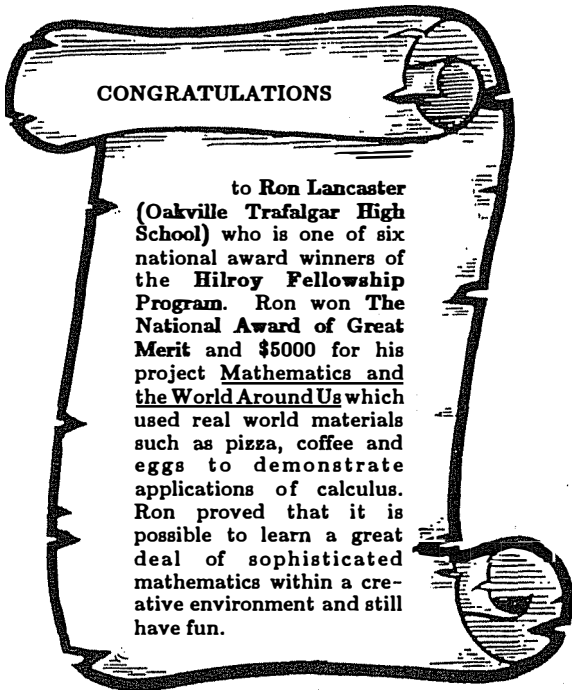
Glenn, William H. and Johnson, Donovan, A. (1960). Invitation to Mathematics. Garden City, :Doubleday.

Kelly, B., Carlow, C., Neufeld, R. V., Symington, J. & Worth, J. (1986). Mathquest Grades 1-8, Don Mills, Ontario: Addison-Wesley.

Kennedy, Leonard M. (1970). Guiding Children to Mathematical Discovery. Belmont, CA: Wadsworth.

Pattison, Sylvia J. & Phillips, Jo. (1975). Math Activities Bonus Book.Portland, MA: J. Weston Walch.

NCTM, (1979). Applications in School Mathematics. National Council of Teachers of Mathematics. Yearbook.





WITCHES, TWO HEADED COINS, AND RAFFLES

Some Intriguing Probability Models

by
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Probability is one of the core topics in the OAC Finite Mathematics course. Having taught probability (as a component of the business statistics course) at Conestoga College, I have become increasingly aware of the inherently complex nature of this field of mathematics. As with many areas of mathematics, proper selection and application of formulae may be used to solve most problems. However, no other secondary school topic can be handled so well with common sense and logical thinking. In spite of this, mention of the word *probability* tends to raise the anxiety level of most students.

Why is probability threatening to students? Mathematics is presented as a subject with right answers, for which students are rewarded. Generally, effort is rewarded by crediting incomplete solutions with part marks. This pattern of evaluation is observed throughout secondary school. Probability does not readily fit this mold. Complex problems may lend themselves to solutions of one or two lines. A correct answer usually will earn full marks. Conversely, an incorrect solution tends to perhaps receive less part marks than students deserve because of the minimal amount of written work. A good teacher can often identify the error by considering how the numbers may have been derived. For example, if a student writes "The probability of rolling three consecutive sixes with a die equals 3/6 or 1/2", the teacher will likely assume that the student added 1/6 + 1/6 + 1/6. If I am unable to make sense out of an incorrect answer, my tendency is to award no marks and indicate that a clear explanation of the process may result in some marks. This has proven satisfactory. It is difficult to pry the thought processes of students given a few lines of work. One may argue that the student deserves no marks in such instances. Often that becomes apparent.

However, our educational system continues to struggle in its efforts to produce creativity and independent thinking. To simply discount wrong (alternative) answers would serve to reinforce this flaw in the system. Instead, we must open ourselves to divergent ways of thinking. I maintain that unorthodox or ingenious solutions, whether correct or incorrect, provide a basis for meaningful discussion. Through such discussion, students will benefit from recognizing that more than one approach is valid. Also, incorrect solutions may illustrate pitfalls that are commonplace and can be avoided in the future. This part of learning is too often neglected. In my opinion, an excellent teacher can take a wrong answer and turn it into a tool for learning.

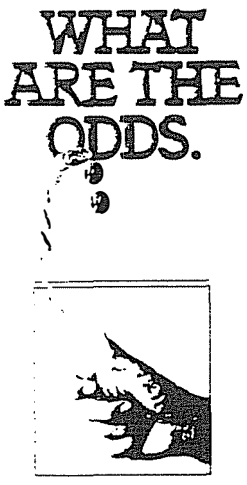
Don't be mistaken. Teachers will also benefit. Three immediate benefits are realized, namely:

- a better understanding of the thought processes employed by students.

- an opportunity to observe the pedagogical value of selected problems and thus, assess their merit for future use.

- a less threatening atmosphere for mathematical learning, one in which insights and differences are valued.

Probability lends itself to the ideas outlined above. I have selected three of my favourite problems to discuss in greater detail. These problems can be solved using a variety of models. They have been thoroughly examined in my classes at Conestoga College as well as in David Laurenson's finite math class at University of Toronto Schools. It is hoped that at least one of these problems will prove to be useful in your own classroom experience.



TWO HEADED COIN PROBLEM

Two coins, one fair and the other two headed, are available. One of the coins is randomly selected and tossed twice with heads resulting

both times. What is the probability that the fair coin was chosen?

(Source: Chris Springer's Stat 230 class at Waterloo)

This problem deals with conditional probability. Therefore, a tree diagram could be employed to solve the problem. This approach permits the student to define and visualize all possible outcomes prior to selecting those that are favourable from those that are feasible. In this situation, the feasible outcomes include all cases where heads appeared on both tosses. Note that to be considered favourable, the fair coin must have been used.

Alternatively, the problem could have been solved using a formula. This could be applied in conjunction with the tree diagram or separately. The formula for conditional probability could be applied. Michael Falkenheim, a student at University of Toronto Schools, solved the problem quickly by simply writing

$$\frac{\frac{1}{4}}{\frac{1}{4}+1}$$

Any of these solutions are indeed valid. However, if I was required to explain this problem to a class, a tabular representation would probably be the best method to use. Assume that the fair coin can land heads (H) or tails (T) and the two headed coin can land heads (H1) or heads (H2). All possible outcomes can be represented as follows:

Fair Coin			Two Headed Coin		
H	H	*	H1	H2	*
H	T		H1	H1	*
T	H		H2	H1	*
T	T		H2	H2	*

All eight possible outcomes are equally likely. Since both coins are equally likely to be selected, the probabilities of each of the outcomes listed above is 1/8. All of the outcomes that produce two heads are identified with an asterisk. It is apparent that the probability that the fair coin was selected equals 1/5.

THE RAFFLE PROBLEM

Only 100 tickets were sold for a raffle. You and four friends each have single tickets. If two tickets are drawn, what is the probability that exactly one of you will win a prize? (Note: Tickets drawn are not replaced.)

Choose the expression that gives the desired probability.

a) (5/100) (95/99) + (95/100) (5/99)

b) $$1 - \frac{\binom{95}{2} \div \binom{5}{2}}{\binom{100}{2}}$$

c) $$\frac{\binom{5}{1}\binom{95}{1}}{\binom{100}{2}}$$

Which answer did you select? All of the expressions above equal $0.0\overline{95}$. That probably surprises you. How are these expressions derived? They are all based upon sound logical reasoning.

The first answer depends upon mutually exclusive events being defined. Either the first draw is won by one of the five and the second one lost, or vice-versa. The respective probabilities are then added.

The second expression is easily explained if the definition of a complement is understood. The complement of the favourable outcome is represented by all of the unfavourable outcomes. In this case, the unfavourable outcomes include either 2 wins or 2 losses for the group. The probability of exactly one win is then found by subtracting the probability of the complement from 1.

The simplest way to understand the problem is reflected in the third expression. Simply stated, 2 of 100 tickets will be drawn and we require that they include 1 of the 5 held by this group and 1 of the remaining 95 tickets. The simplicity of this explanation is appealing.

THE WITCHES PROBLEM

Two witches enjoy meeting each other over a cauldron of tea. Both witches have serious shortcomings, however. First, each witch is poorly organized and arrives at the meeting place randomly between midnight and 1:00 a.m. Second, each is notoriously evil-tempered and becomes outraged on having to wait 15 minutes or longer for her companion. Thus, the following temper-saving arrangement has been agreed on: when either witch has waited fifteen minutes - or when one o'clock arrives and she is still alone - she disappears at once, not returning until the next night. Here is our problem: On a given night, what is the probability that the two witches meet?

(Source: Mathematical Challenge II Plus Six, NCTM)

A variety of models can be used to solve this problem. For a more complete discussion of this problem, refer to the source in which you will find an article entitled *Probability, Geometry, and Witches* by J. Philip Smith.

Two solutions to the problem will be discussed here. The first is based upon the arrival time of Witch A. Let us define two mutually exclusive events, namely:

A: Witch A arrives between 12:15 & 12:45 a.m.

B: Witch A arrives between 12:00 & 12:15 or 12:45 and 1:00 a.m.

Note that P(A) = P(B) = $\frac{1}{2}$. Also, note that if event A occurs, Witch B can meet Witch A provided that she arrives in the 15 minute period immediately preceding or following the arrival of witch A. If event B occurs, either the time preceding or after the arrival of Witch A is reduced to 7.5 minutes (1/8 hour) on average. Thus, the probability of meeting is reduced to 3/8 in these cases. Overall, the probability that the witches meet equals

$$(1/2)\,(1/2) + (1/2)\,(3/8) \text{ or } 7/16.$$

Most student would not easily follow the solution outlined above. However, an elegant solution to this problem exists. It is unusual because it involves geometry. Begin by constructing a unit square in the first quadrant of a standard Cartesian plane. Let the arrival times (fraction of an hour after midnight) of witches A and B be represented by x and y respectively. Then the region defined by $x - y < 1/4$ will represent all of the possible meeting times. Since the unit square has an area of 1 square unit, the area of the shaded region represents the desired probability. This equals 7/16 or .4375 (See Figure 1).

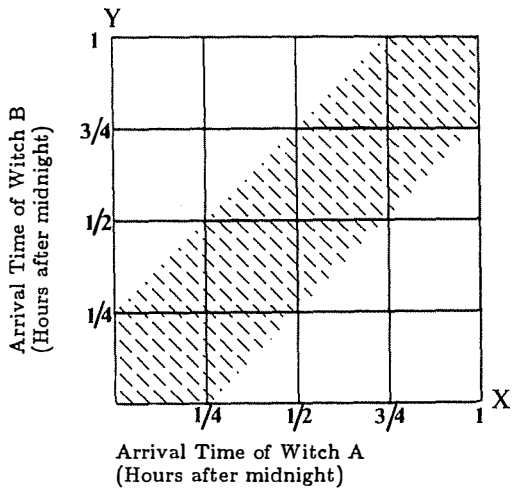


Figure 1.

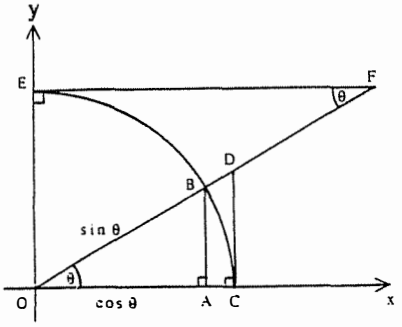
Students and teachers are most impressed by this elegant approach. It illustrates another application of coordinate geometry in addition to introducing the concept of geometric probability.

CONCLUSION

It is important to encourage the development of alternative models. Furthermore, when we as teachers become aware of innovative solutions or problems with a little more spice, we should be encouraged to share these with appropriate student audiences and our own professional colleagues.

Let me close by sharing an experience from a college class. The problem concerned a deck of 52 cards and the probability of being dealt 5 cards belonging to the same suit. Steve Dickie, a student in the class, asked me what would happen if two jokers were added as wildcards. I agree to solve the problem before the next class. He did the same. Our answers were identical but our methods differed. He constructed a tree diagram, whereas, my solution was based on combinations. Both of us were enlightened by the apparent insight of the other. Such experiences add to the satisfaction of being a teacher or student of mathematics.

The second diagram suggested by Mr. Todd Romiens on page 21 of the April (1989) edition of the *Ontario Mathematics Gazette* brings to mind two other diagrams. The first, which is simply an extension of that on page 21, illustrates diagramatically some basic trigonometric relations. Some students find it helpful to see trigonometric ratios used as numerals (which they really are) to represent something visual.



OCE is a sector of a unit circle.

CD and EF are tangents

$\triangle OAB \sim \triangle OCD \sim \triangle FEO$

$$\frac{\sin\theta}{\cos\theta} = \frac{CD}{1} = \frac{1}{FE} \Rightarrow CD = \tan\theta \quad \& \quad FE = \cot\theta$$

$$\frac{\sin\theta}{1} = \quad = \frac{1}{FO} \Rightarrow FO = \operatorname{cosec}\theta$$

$$\frac{\cos\theta}{1} = \frac{1}{OD} \Rightarrow OD = \sec\theta$$

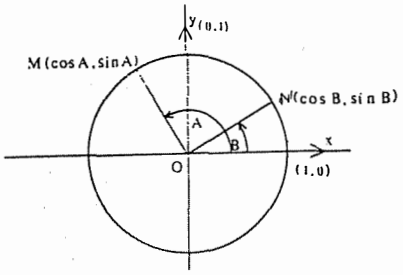
Since the three triangles are right-angled,

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

Of course, this diagram illustrates only general relations for the restricted cases of acute angles.

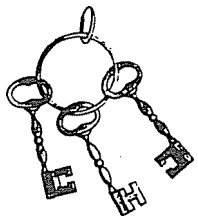
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The second diagram illustrates that the expansion formulae are, in fact, dot products.



$$\overrightarrow{OM} \cdot \overrightarrow{ON} = |\overrightarrow{OM}|\ |\overrightarrow{ON}|$$
$$\cos(A - B) = (\cos A, \sin A) \cdot (\cos B, \sin B)$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

Submitted by:
Geoffrey G. Steel
136 Parkview Drive
Hamilton, Ont. L8S 3Y3



MATCHING CAR KEYS AND DOORS: A REAL WORLDPROBABILITY PROBLEM

by
Bonnie H. Litwiller and David R. Duncan
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa



Probability is an increasingly important topic in mathematics education. To teach probability effectively and meaningfully, real world problems are needed. Consequently, probability teachers are always on the lookout for examples that they can use in their classes. We shall describe a real world probabilistic situation.

People may wonder if their car keys might open the doors of other cars of the same model. To investigate the likelihood of this occurring we need information about the construction of car keys. According to the manufacturer, the key for one popular model (call it Q) of car has five "notches". Each of these notches can be cut to six different depths. The lock itself is configured similarly to be opened by exactly one key.

How many different keys can be constructed this way? By the Fundamental Principle of Counting (FPC), there are

$$6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \text{ or } 6^5 \text{ or } 7776$$

different notch-depth arrangements for a key. Suppose that all of these 7776 arrangements are equally likely. How likely is it that the key of one car owner would open the car door of another car of this model Q?

This problem can be thought of in two ways:

1) In a group of model Q car owners, what is the probability of at least one "match"; that is, of at least two people having identical key arrangements, enabling them to open each other's car doors?

2) A specific person, Alex, purchases a car of model Q. What is the probability that he can open another car door of the same model?

These problems will be discussed separately.

Problem 1

Let n be the number of people, each owning one car of model Q. If $n > 7776$, at least two of them must have the same key arrangement. Therefore, suppose $n \leq 7776$. Visualize these n people receiving their keys, one by one. Each person has a choice of 7776 possible arrangements. Again using the FPC, there are 7776^n different ways in which these n people can receive their keys.

It would be very difficult to analyze all possible ways in which various numbers of matches could occur. Instead, first consider the case in which there are not matches. For this to occur:

Person 1: all 7776 arrangements are possible.

Person 2: 7775 arrangements are possible since Person 2 must avoid Person 1's arrangement.

Person 3: 7774 arrangements are possible since Person 3 must avoid the arrangements of both Person 1 and Person 2.

Person 4: 7773 arrangements are possible.

Person 5: 7772 arrangements are possible.

⋮ ⋮

Person n: $7776 - (n-1)$ or $7777 - n$ arrangements are possible.

Next compute the probability that there are no matches. Recall that the arrangements are random. The probability is thus:

$$\frac{7776 \cdot 7775 \cdot 7774 \cdot 7773 \cdot \cdots (7777-n)}{7776^n}$$

Then, the probability that at least one match occurs is:

$$1-\frac{7776 \cdot 7775 \cdot 7774 \cdot 7773 \cdot \cdots (7777-n)}{7776^n}$$

Table I displays the probability of at least one match for various values of n.

Table 1

n	Probability of at least one match	n	Probability of at least one match
1	0	35	0.0738
2	0.0001	36	0.0779
3	0.0004	37	0.0822
4	0.0008	38	0.0866
5	0.0013	39	0.0910
6	0.0019	40	0.0956
7	0.0027	41	0.1002
8	0.0036	42	0.1050
9	0.0046	43	0.1098
10	0.0058	44	0.1147
11	0.0071	45	0.1198
12	0.0085	46	0.1249
13	0.0100	47	0.1300
14	0.0116	48	0.1353
15	0.0134	49	0.1406
16	0.0153	50	0.1460
17	0.0173	51	0.1515
18	0.0195	52	0.1571
19	0.0218	53	0.1627
20	0.0242	54	0.1684
21	0.0267	55	0.1742
22	0.0293	56	0.1801
23	0.0320	57	0.1860
24	0.0349	58	0.1919
25	0.0379	59	0.1980
26	0.0410	60	0.2040
27	0.0442	61	0.2102
28	0.0475	62	0.2164
29	0.0509	63	0.2226
30	0.0545	64	0.2289
31	0.0581	65	0.2353
32	0.0619	66	0.2417
33	0.0657	67	0.2481
34	0.0697	68	0.2546
69	0.2611	88	0.3899
70	0.2677	89	0.3968
71	0.2742	90	0.4037
72	0.2809	91	0.4106
73	0.2875	92	0.4175
74	0.2942	93	0.4244
75	0.3009	94	0.4313
76	0.3077	95	0.4382
77	0.3144	96	0.4450
78	0.3212	97	0.4519
79	0.3280	98	0.4587
80	0.3349	99	0.4655
81	0.3417	100	0.4723
82	0.3486	101	0.4791
83	0.3554	102	0.4859
84	0.3623	103	0.4926
85	0.3692	104	0.4994
86	0.3761	105	0.5061
87	0.3830		

continued...

It is surprising how rapidly the probabilities mount. Only 105 keys were sufficient to produce a probability greater than 1/2 that at least one match would occur.

Problem 2

Problem 2 differs from Problem 1 in that a specific person (Alex) is identified to do the "matching." We are not concerned whether other keys match, only whether Alex matches someone else.

Suppose that there are n keys besides Alex's. Again, first compute the probability that Alex does not match any of these other n keys. These n keys can be arranged in 7776^n distinct ways. For each of these keys one possible setting matches Alex's, but 7775 are different from Alex's. Thus, the number of distinct ways in which these n keys could be arranged differently from Alex's is 7775^n. The probability that all n keys are arranged differently from Alex's is $\frac{7775^n}{7776^n}$ or $\frac{7775}{7776}$.

The probability that at least one of these keys matches Alex's is then $1 - \left(\frac{7775}{7776}\right)^n$

Table II reports probabilities for selected values of n.

Table II

n	Probability of at least one person matching Alex's key
1	0.00013
2	0.00026
3	0.00039
4	0.00051
5	0.00064
⋮	⋮
100	0.01278
⋮	⋮
105	0.01341
⋮	⋮
1000	0.12068
⋮	⋮
2000	0.22680
⋮	⋮
3000	0.32011
⋮	⋮
4000	0.40216
⋮	⋮
5000	0.47431
⋮	⋮
5389	0.49996
5390	0.50002

The results of problem 2 are greatly different from those of problem 1. To many students, these problems seem essentially the same. They are decidedly not so. Problem 1 yields much larger probabilities of matches than many people would expect, while the probabilities of problem 2 increase very slowly.

In problem 1, only 105 keys were necessary in order that the probability of at least one match exceeded $\frac{1}{2}$. In problem 2, 105 keys, besides the key of Alex, yielded a probability of only .01341 that Alex would match someone. A total of 5390 keys were necessary to give Alex a 50% probability of matching someone.

Logarithms could be used in problem 2 to determine the number of keys that would yield various probabilities. For example, to find the number of keys needed so that the probability of Alex's matching at least one other key would exceed $\frac{1}{2}$, proceed as follows:

$$1-\left(\frac{7775}{7776}\right)^n > \frac{1}{2}$$

$$\left(\frac{7775}{7776}\right)^n < \frac{1}{2}$$

$$n \log_{10}\left(\frac{7775}{7776}\right) < \log_{10}\left(\tfrac{1}{2}\right)$$

$$n > \frac{\log_{10}\left(\frac{1}{2}\right)}{\log_{10}\left(\frac{7775}{7776}\right)}$$

$$n > \frac{-0.30103}{0.000055854}$$

$$n > 5389.6$$

Since n must be an integer, the smallest possible value of n is 5390 (table II).

Challenges for the reader:

1) The alert reader will recognize that these two car key problems are isomorphic to the familiar *birthday problems*. These are:

a) In a group of n randomly selected people, what is the probability that at least two people share the same birthday anniversary?

b) If Alex comes into a room of n randomly selected people, what is the probability that he has the same birthday anniversary as at least one of the people in the room?

2) Look for other situations in which these *matching* questions arise in real world situations.

END-OF-YEAR SCAVENGER HUNT

This is a fun and challenging way to review work taught during the year. Make up a quantity of questions covering different topics. I usually add some interesting, non-routine problems to solve as well. I separate the questions, according to difficulty and time required to answer, into 3 groups - 2 marks, 5 marks, 8 marks. The questions are put on dittos: the 2 mark questions first, and the 8 mark questions last. In a narrow margin on the right I write the marks each question is worth.

The student make their own groups - 4 is best but 3 is also allowed. Two math periods are given to answer the questions. In order to make marking easier only 1 question set per group is submitted after the first period. Each question which has been done is marked. If the answer is correct, the number in the right-hand margin is circled. If the answer is incorrect, I stroke through the mark in the right-hand margin and this questions may not be attempted again. The students are told before the hunt begins that there are no part marks. Only correct answers count. The group is encouraged to check each other's work carefully before submission. The questions are marked after the first math period and the leaders at that time are announced. After the second math period prizes are given to the winning group.

e.g. (Grade 7)

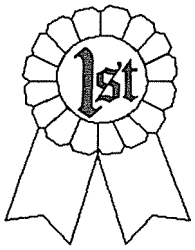
		MARKS
1.	There is a shape labelled *Question #1* on the bulletin board. Its area is _____.	2
2.	Answer all of these correctly: a) $1/2 + 2/3$ c) $2/3 \times 4/5$ b) $3/4 - 1/2$ d) $3/4 \div 1/2$	5
3.	George has red, blue and brown pants. He has a yellow shirt and a red shirt. He has a blue tie and a green tie. How many different outfits can he wear when he chooses pants, shirt and a tie? Draw and colour all his different possible outfits.	8

Suggested by:
Barb Kaiser
Princess Elizabeth P.S.
London, Ontario

AWARDS FOR THE TEACHING OF PROBLEM SOLVING

The award for the teaching of problem solving is sponsored by Chester Carlow, in cooperation with OAME, to recognise innovative and stimulating methods of teaching problem solving in the elementary school.

In the search for Excellence in Elementary Mathematics Education, **Jeanne Pherand**, Chalmesford, Ontario won an honourable mention for her submission.



First Prize of $300 is awarded to **Martha Milne** Ottawa, Ontario. Here is the submission which earned her first prize...

This past year, while taking Special Education Part II, Gifted Option, I chose to study the phenomenon of metacognition and how it affects the problem solving process. My interest in this topic was triggered by the thoughts outlined below.

The major focus of the education system at the beginning of this century was the acquisition of knowledge. The products of thinking were deemed of greater importance than the process of thinking. Today, this has changed. The bounds of knowledge now far exceed the capacity of a single human mind. The ever-accelerating rate of scientific and societal change makes it necessary for us to emphasize the process of acquiring and using knowledge effectively.

The computer has entered this scenario as a helpmate. As computers increasingly pick-up the task of storing and manipulating information, human minds are set free to engage in activities that call for unique human talents. Problem-solving is one such activity, and metacognition is one such human talent.

Metacognition is the awareness, regulation and evaluation of one's thinking. Metacognition acts like a quality control device, allowing a thinker to independently gauge the value of his or her own ideas.

For my study of metacognition and problem-solving, I set up four, forty-minute sessions with two gifted children. My intent was three-fold. Firstly, I wished to gain an understanding of how these students arrived at their answers. Secondly, I wanted to know if they were aware of how they had solved the problems. Thirdly, I wanted to see how the students' performance would alter after they were offered problem-solving models to use.

The participants in my study were ten and eleven years of age and part of a self-contained gifted class. When working on the problems, they used Lochhead and Whimbey's "Talk Aloud" approach. The solver uttered her thoughts out loud while the other acted as listener and prompter.

Each of the girls attempted to solve a different math problem. The problems came from Ruth Douglas's book *Tools for Problems Solvers*. One of the problems used is as follows:

Your mum put a chocolate bar in your lunch every four days, and a bag of chips every five days. If you had both treats in your lunch today, how long will it be before you again get this double treat?

After arriving at a solution with which they were satisfied, the students then studied a Math Problem-Solving Model and solved another problem using the model.

A similar set-up was used for the non-math problems, but for these the students attacked the problems as a team. I made up the non-math problems, ensuring that the problems suited the children's age and situation. Both of the students are members of the school choir. One problem they dealt with was: The quality of your choir's singing is going to suffer if members don't start to attend practises more consistently. The model used for this

problem type was Treffinger and Isaksen's Creative Problem-Solving Model.

The usefulness of the models I offered to the students depended on the type of problem on which they were working. Both found that the math model was of little use. One girl said that the step-by-step approach constrained her thinking.

The Creative Problem-Solving Model, on the other hand, was appreciated by both students. They felt that the model helped them organize their thoughts and generate more ideas. I noticed that the girls employed their metacognitive skills more frequently when following the model. They were more open to each other's ideas and monitored and evaluated their thinking more readily.

Observing these two students has led me to conclude that more attention should be paid to the development of metacognitive skills. In order to succeed with their problems, my two students has to be aware of what solution strategies they had at their disposal and how and when to use them. They had to be aware of when to use their creative thinking abilities. They needed to know how to evaluate their progress and when to be satisfied with a result.

Problem-solving models can be very useful, but they are mere tools. It is a child's metacognitive skills which will determine how useful these tools will be. Hopefully, future investigation will further illuminate the nature of metacognition, and how to recognize it in action.

The students I chose for my study were gifted. All children, though, need to develop metacognitive skills. Global and individual problems continue to become more urgent and more complex. Students who become aware, flexible, self-evaluating thinkers will be best equipped to deal effectively and creatively with these problems.

Bibliography

Douglas, R. (1989). Tools for Problem Solvers. Ottawa Board of Education, Ottawa.

Forrest-Pressly, D. et al. (Eds.), (1985). Metacognition, Cognition and Human Behaviour. (Vols. 1-2). Orlando, FL: Academic Press.

Ginsburg, H. P. (1983). The Development of Mathematical Thinking. Orlando, FL: Academic Press.

Isaksen, S. & Treffinger, D. (1985). Creative Problem Solving: A Basic Course. Buffalo, NY: Bearly.

Lochhead, J. & Whimbey, A. (1980). Problem Solving and Comprehension. Philadelphia, : Franklin Institute Press.

ATTENTION ELEMENTARY SCHOOL TEACHERS

If you are an elementary school teacher who had developed "neat" and effective ways and means of improving the teaching of problem solving in your classroom, school, or district, then this announcement is for you.
Chester Carlow, in cooperation with OAME, is sponsoring awards for the improvement of teaching problem-solving in the elementary school: $300 for first place, $200 for second place and $100 for third place.

WHAT IS REQUIRED:
Teachers are asked to submit a report (5 copies) which details the innovative intructional strategy(ies) which were used, and the results relative to the imporvement in student's abilities to solve problems.
Teachers should select an instructional strategy which looks promising in terms of previous theory and research on problem solving. This will promote steady progress in the area.

Complete information about the structure of the reports can be obtained from:

J. Symington
50 Copeland Rd.,
Brampton, Ont. L6Y 2S7

Mary Lou Kestell
228 Manitou Way
Ancaster, Ont. L9G 1Y2

Chester Carlow
87 Risebrough Ave.
Willowdale, Ont. M2M 2E2

Your completed entry (5-11 pages) should be sent to Chester Carlow before April 1, 1990.

DON'T KEEP THOSE EXCITING IDEAS A SECRET ANY LONGER,

LET US HEAR FROM YOU!



A statistician is a person who draws a mathematically precise line from an unwarranted assumption to a forgone conclusion. Anon.

Definition of a statistician: someone who's happy with his head in the oven and his feet in the fridge, because he thinks he's pretty comfortable "on average". Anon.

If at first you don't succeed, you're just about average!

Bill Cosby.



Teaching Tips

Three of the teaching ideas and techniques I have found to be most useful additions to my repertoire in the last few years are:

☞ Give a short test or quiz after every exercise, classwork or homework. Mark it as the students complete it. Make sure the faster ones have more work to entertain them. Return the test and take it up immediately. Record the marks, and count them as part of the term mark. This gives students immediate feed-back on their knowledge and understanding. It prepares them for the kind of questions to be asked on a major test to be given later on the same topic. It gives them a chance to remedy deficiencies in their knowledge before such a test. It gives the teacher lots of marks for assessment.

☞ Record marks on a computer. The difference in the computation time preparing for reports is incredible. The flexibility of weighting different aspects of the evaluation techniques is a delight.

☞ In order to prepare for anecdotal reports, I keep a record of my reactions to my students work as I mark the two tests prior to the report. This gives me much more confidence that my comment on the report makes sense, especially earlier in the year when I do not know the students well.

Suggested by:
Tom Griffiths
A. B. Lucas Secondary School
London, Ontario

INTERVIEW

Taped Talks with Bob Smith

...Dr. Peter Hilton

R.S: It is the fifteenth annual convention of O.A.M.E. I am seated comfortably in the hospitality room in the Ramada Inn, Niagara Falls. Dr. Peter Hilton is my guest for this interview. My first interview for O.M.G. was with Dr. Hilton and our interview then centered around math anxiety. He seems to have struck a note at that time because we had many letters from people who were doing research on this topic. Before we begin the interview, where are you located now, Peter?

P.H: Since 1982 I've been at the State University of New York, the Binghampton Campus, they were nice enough to give me the title of Distinguished Professor which I like to say sanctions my rather eccentric interpretation of my duties.

R.S: I think it was well earned.

P.H: Thank you very much. I'm very happy there and will stay there for the remainder of my career in University teaching. I continue the same mix of activities as when we first spoke, I am still a research mathematician doing my undergraduate and graduate teaching but also taking an intense interest in problems in mathematics education at all levels and in all the countries I have visited. I have been fortunate enough to be able to visit a number of different countries and I can compare their experiences and their attempted solutions with ours and see where the differences and similarities lie.

R.S: Specifically the last time I had the opportunity to hear you was at the San Francisco conference of NCTM and we both had the good fortune to listen once again to Dr. Polya. As I recall you did some play acting on stage, to set up an introduction of Dr. Polya.

P.H: Yes, that's right, yes.

R.S: Dr. Polya has since passed on. His work is still a key part in problem solving. But you have been bothered by the emphasis on problem-solving!

P.H: Yes, I would say that I have been bothered. I'm bothered by the emphasis on *solving* problems. I would like to see an equal emphasis on identifying problems which can be tackled with the resources we currently have available at our disposal. That means, for example, problems which are appropriate to students at various levels. It also means, of course, problems which are appropriate to mathematicians at the present level of knowledge of mathematics. I'm very keen to emphasize, first of all, that the natural direction of mathematics is from answer to question not from question to answer. If students understand what are reasonable questions to ask in the presence of their existing knowledge and understanding then they are thinking mathematically. That is one of the things that bother me this emphasis always on problem-solving. The second difficulty about a lot of the work being done in problem-solving is that the reason why problem-solving is difficult is because by hypothesis you don't know in advance the method you are going to use. It virtually follows that you don't even know in advance that you are going to be able to solve the problem. Problem-solving is nevertheless almost always done in a context in which you are supposed to know you can solve the problem if only you find the right method. Moreover there is supposed almost to be an algorithm for finding the right method. I believe a false picture is being created.

You can give hints as you know. You can say that you should look at some special cases, or you should simplify the problem a bit first. Exercises at the end of a chapter are often phrased in the form of a word-problem, but

you know with absolute certainty, first, that what you have just learned is very acutely relevant to the solution of this problem, and, second, that you have been given precisely the data you need to solve the problem. So I don't like to give the impression that you can, as I say, make the solving of problems algorithmic. There has to be the element of inspiration, there's got to be the informed selection from your repertoire of knowledge and techniques of what is likely to work and there has got to be the judicious selection of problems as well. Mathematics has been incredibly successful, but it's been successful because mathematicians know when to ask *what* questions. If we were, for example, to take our questions from some non-mathematician who would select them on the basis of their importance and some moral or social sense, our success rate would plummet catastrophically. I think that there is a glibness, there's a simplification of the real difficulties involved in all this talk of problem-solving.

I say more seriously than jokingly that all the discussion I've seen of problem solving omits one of the most important of all human attributes. That trait is knowing when to give up, to recognize that it isn't a good problem, to know it's not the appropriate time. A machine doesn't do that and we can. We can say this is not a good use of our time. I think that we should be inculcating more of this as a general sort of good practice with regard to the doing and applying of mathematics.

R.S: A developer of good judgement is what I hear coming through.

P.H: That's right, you've heard very well.

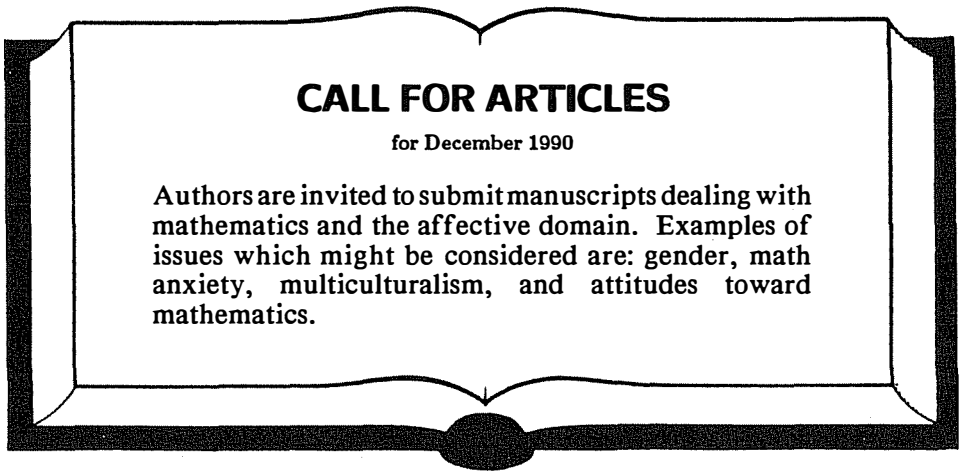
R.S: That of course is one of the more difficult aspects and yet, just because it is difficult it must not be ignored.

P.H: Let me tell you a bit about a series called *Real Math* with which I am involved. It is a K-8 programme, but I would like to refer to how it deals with very young children. We have as a thread going through their education what we call thinking stories. These thinking stories usually use mathematical technique that the children already have, but by no means necessarily the thing they just learned. But they may not require mathematical technique. They may appear to be in a form in which they look like familiar problems, for example, the following "If it takes one dog 10 minutes to swim across a stream, how long does it take 3 dogs?" It looks like a problem of a kind that they know how to solve but in fact, if they think about it, they realize it's not quite of that kind. There are other problems where they can say, "we don't have enough information", and there are other problems where the answer is very much in the form of a decision. For example, it wasn't reasonable for Johnny's father to ask Johnny to get all that done in 30 minutes. Answers are of the kind that really occur in life where we rarely are simply satisfied to get a number. We really have to take a decision on the basis of the number we've got. Some of the problems ridicule excessive accuracy. eg: Is it sensible for one boy to say to the other, "I walked farther than you walked yesterday because you walked 185/243 miles and I walked 187/243 miles"? Is that a really sensible conversation? At the very elementary level, you can at least develop in the children the capacity to select from their repertoire, you can encourage them to believe that that's what they have to do when solving problems. The traditional way in fact makes it very difficult for them to do that, because they always believe they know how they are meant to tackle problems.

R.S: Your statements place you in line with David Wheeler's main point in his address. He feels mathematics must be of the world, it has to make sense in the real world, the world that the children are going to live in, and that we must try to give some of our time in classrooms to developing this aspect of judgement as opposed to the mechanical skills and technical problem-solving techniques.

P.H: Yes, the only thing that really upsets me is that David should have decided it was necessary to say it to as enlightened group as this one. If he is right that it is necessary to say it, then the situation in Ontario isn't quite as good as I imagined it. Of course what he says is true but who could possibly doubt it? It's so true today because we have machines to achieve the accuracy we need. It's simply unnecessary to inflict on human beings the painful duty of carrying out the slow hand algorithms to get accurate answers; understanding why the hand algorithms work, that's mathematics, but being able to execute them fast and accurately has nothing to do with mathematics at all.

R.S: Agreed. It was good to chat with you once again Peter. Thank you.



CALL FOR ARTICLES

for December 1990

Authors are invited to submit manuscripts dealing with mathematics and the affective domain. Examples of issues which might be considered are: gender, math anxiety, multiculturalism, and attitudes toward mathematics.

KENNETH D. FRYER AWARD

In Memory of the late Dr. K. D. Fryer, OAME in conjunction with the Grand Valley Mathematics Association established the Kenneth D. Fryer Award for Excellence, to be presented annually to a Mathematics Department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME is requesting nominations for consideration by the committee for this year's award. It will be presented at the annual conference in Ottawa.

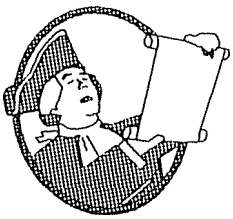
Deadline for nomination submission: March 1, 1990

KENNETH D. FRYER AWARD
NOMINATION FORM
1989-1990

I/WE nominate School: _____
Address: _____
Principal: _____
Math Dept. Head: _____
Nominated By: _____

Please attach a separate sheet to the completed nomination form (or copy) outlining the reasons for your nomination, and forward them to:

Don and Carol Attridge
Sec/Treas. OAME
1 Southdale Road
Markham, Ontario L3P 1J6



COUNCIL COMMENT

by
Bob Robinson



These comments are based on the Council Meeting held at the Ramada Inn, Airport West, on October 14, 1989.

NEW REPRESENTATIVES

The following new representatives were welcomed to council:

Neal Reid (GVMA)
Peter Richardson (SWOAME)

CONFERENCES

OAME '89 - OTTAWA

The OAME '89 financial report was presented and accepted. Well done... Ottawa!

OAME '90 - Hamilton (May 9-12, 1990)

The program is in place with over 150 presentations planned and a banquet. The speakers are keen, the publishers are enthusiastic and the international flavour of this particular annual is exciting as it will serve as the Canadian NCTM regional meeting into the bargain. Truly, all roads are leading into Hamilton in May where we will experience "Mathematics: A Free Trade of Ideas." See you there! Contact George Knill or Bob Robinson for information.

OAME '91 - ETOBICOKE

The Credit Humber Association for Mathematics Promotion will host this Annual in 1991. General planning sessions have taken place already and a major promotional campaign is planned for May at the Hamilton Conference. A leadership strand for administrators and trustees is being planned for the Thursday morning sessions. For information contact Jeff Martin, Bob Jones (Conference Co-Chair) or Lis Henry (CHAMP representative).

NCTM LEADERSHIP CONFERENCE

Barry Scully reported on the NCTM leadership conference held in Toronto July 14-16, 1989. The sharing of ideas and getting a national view of education were very beneficial to all who attended. Barry made several recommendations to council which will be addressed by the executive. OAME's representatives were Jeri Lunney, Jan and Barry Scully. Jan and Barry are also the NCTM representatives to council. Contact them for the procedure to order NCTM materials. Ordering through our representatives helps OAME as we get a percentage of any sales made.

OMCA (Ontario Mathematics Coordinators Association)

President Mary Lou Kestell described an interesting retreat she attended as our representative to the OMCA. The overall theme was "Evaluation." May Lou also described other initiatives of this important mathematics association.

Global Education

A most interesting and informative presentation on Global Education and possible roles for mathematics educators in this project was delivered. For more information on the worthwhile project, contact Thomas G. Lyons, Project Co-ordinator, OTF/CIDA Global Education Project, Ontario Teachers' Federation

Hilroy Fellowship

Ron Lancaster is the most recent recipient of the Hilroy Foundation Fellowship Award. Well done, Ron! Ron told councillors ways they can get involved with this foundation. For more information contact Ron Lancaster

CHAPTER REPORTS

CHAMP (Credit Humber Assoc. For Math. Promotion)

New CHAMP committees for this year to help deal with the growing membership and activity level are: Publicity, Fund-Raising, Contest, Mathfest and Spring Event.

MATHFEST '89 will be held October 19 at Port Credit S.S. with 14 publishers displaying materials. This event is used as a membership drive.

Peel Board is again running a Math Day on November 22 for participants of all ages and levels of ability. A four day workshop on Active Learning was held at the end of August and one of the strands included Math educators. Neil Davidson will be conducting a P.D. session on cooperative learning in mathematics on November 17 in Peel.

The constitution has yet to be finalized.

COMA (Carleton Ottawa Mathematics Association)

The joint COMA-QSLMA Fall Conference "Towards 2001" on October 3-4, 1989 was a tremendous success. Over 100 participants attended the various workshops at the Opinicon Lodge in Chaffey's Locks. Presenters from the political arena, as well as chairpeople from two school boards and a number of ministry people ensured a lively and informative exchange of information and ideas, particularly on the subject of destreaming and its effects on the mathematics curriculum. Congratulations to the organizers for a job well done.

COMA plans to host a workshop-dinner activity this coming January. A second activity is a workshop devoted to looking at the future of COMA. This event will be in response to the OAME survey form that was circulated last spring to all the chapters.

Golden Section (Hamilton-Niagara Region)

No activities are planned for this winter. The annual Mini-Conference will be held in the spring of 1990.

GVMA (Grand Valley Mathematics Association)

The next meeting of GVMA is being held in Nov. 1989. The history of this long standing chapter of OAME is being written. Sample OAC exams for the three courses are ready for sale at the Hamilton Conference in May.

NOMA (Northern Ontario Mathematics Association)

The 1989 annual conference was held on Thursday, October 26 and Friday, October 27, 1989 at La Salle Secondary School in Sudbury. The theme is "Evaluation In The Mathematics Program." A report on this activity will be in the next Council Comment.

NWOAME (Northwestern Ontario Assoc. for Mathematics Education)

NWOAME '89, the First Annual Fall Conference, was a great success. Over 100 teachers attended. Area Councillors voted to hold a similar Conference next Fall. It is our hope to establish the end of September as NWOAME meeting time, and have teachers plan early to attend.

Membership in NWOAME is growing.

Pine Ridge Mathematics Assocassociation (Oshawa-Peterborough Region)

PRMA members are presenting at the Board-Wide P.D. Day on November 17th. Ten different mathematics workshops are offered.

A new format for the annual Mathematics Awards Night is being considered. The objective is to increase the involvement of individual schools since currently, a small number of schools are represented. Feedback from other districts is requested.

QSLMA (Quinte St. Lawrence Mathematics Association)

QSLMA and COMA joined forces to organize the recent fall conference at Chaffey's Locks. See COMA's report for details. A spring conference is being planned for April, 1990 to be held at RMC in Kingston.

SAME (Scarborough Association for Mathematics Education)

The Ninth Annual Mini-conference took place on Tuesday, October 17 at Maplewood H.S. from 3:45-9:00 p.m. Look for a report on this event in the next Council Comment.

They are planning another affair on April 4, 1990. Stay tuned for more information.

SWOAME (Southern Ontario Association for Mathematics Education)

The fall mathematics conference is scheduled for Friday, December 1, and Saturday, December 2, 1989. The Friday evening dinner will be held at the Teutonia Club and the elementary and secondary sessions will be at Holy Names High School. This year's theme is "Using the Calculator and Computer In the Mathematics Classroom."

Y^4MA (York University, North York Board, York Region Board, York Region RCSS Board)

Mini-conference will be at St. Robert's Catholic High School. The theme is "Mathematical Investigations."

The chapter plans to write a constitution and a newsletter and conduct a membership drive, among other things.

COMMITTEE REPORTS

Primary/Junior - Sue Stuart, Roland Morrow.

The committee is planning to provide input to the renewal of the Formative Years project of the Ministry. OAME has representatives on the various committees associated with the provincial reviews in mathematics at the grade 6 and grade 8 levels.

Intermediate/Senior - Anne Jones, Jeri Lunney.

A survey on the number of qualified mathematics teachers has been distributed and is to be returned by October 31, 1989. A full report is expected in February. The committee will request of the ministry that OAME be consulted as a reaction group to the elimination of streaming issue in the Transition Years. General specific questions have also been directed to the ministry for their response.

Membership - Lis Henry, Mickey Sandblom.

The committee has designed a new brochure but are still exploring ways to make this material more appealing to the eye. Special stickers will be available at the Hamilton Conference. Be sure to look for the membership form in the NCTM/OAME program booklet for Hamilton and elsewhere in this publication. Any person wishing to volunteer to serve on the membership table at the Hamilton Meeting should contact Don Attridge.

Should we have a new logo? Do we need a new logo? Contact Lis Henry.

Publications - Ron Lancaster, Jack Weiner.

Jack Weiner introduced Amy Parker, the potential new 1991 Abacus Editor to Council. Focus issues of the Gazette are being discussed, as are different ways in which the Abacus can be presented. New features such as "Mathematics In the News" with Ron Lancaster acting as a "clearing house" is one of the new features we can look forward to in upcoming issues of the Gazette.

Futures - Peter Lenczewski, J. Symington.

The "survival binder" project appears to have been tabled permanently. A professional liaison with industry and the "real-life" application of mathematics will be pursued.

Women In Mathematics - Lorna Wiggin, Mary Lou Kestell.

The annual Women In Mathematics Committee Breakfast is being planned. This highly successful breakfast is on Friday, May 11, in the Webster Room of the Hamilton Convention Centre. Funding is being investigated and admission to this event is by ticket only. Contact Lorna Wiggan for more information. The committee is also developing a comprehensive speakers list for the use of members who are planning for conferences or seminars.

Nominations - J. Symington

This committee chaired by J. Symington has Jan Scully and Ron Sauer as its members. Persons wishing to be nominated or who wish to nominate individuals to the position of President-Elect or councillor should contact members of the committee.



Next Council Meeting:
Saturday, February 10, 1990.
Ramada Inn,
Airport West.

NOMINATIONS FOR OAME/AOEM COUNCIL

Each year elections are held for President-Elect, 3 Directors, and 7-10 Councillors for the Council of OAME/AOEM.

Nominations for these positions are to be submitted to:

J. Symington

Please include the nominee(s)' qualifications with the Nomination Form below to be received by J. Symington **no later than 4:00 p.m. January 12, 1990.** The nomination committee will make a final selection of those nominees who will stand for election. Their names and qualifications will be published in March, 1990. Voting will be by mail by members of OAME/AOEM.

At the Annual Meeting of OAME/AOEM in Hamilton in May 1990, the results of these elections will be made.

>>

NOMINATION FORM

I Nominate

Address _____

_____

_____

for the position(s) of: _____President-Elect

_____Director

_____Councillor

_____Nominator

_____Address

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

1989-90 EXECUTIVE & COUNCILLORS

Past Pres.	Symington, J
Pres.	Kestell, Mary Lou
Pres-Elect	Sandblom, Michael
Director	Lunney, Jeri
Director	Morrow, Roland
Director	Weiner, Jack

MEMBERSHIP DUES

Subsciptions to the *Gazette* alone are no longer available; however, members of O.A.M.E./A.O.E.M. receive the *Gazette* and *Abacus* free. Membership dues for 1 year are $25.00 in Canada, $27.00 in United States and $30 outside Canada and United States Membership dues for 5 years are $112.00 in Canada, $122.00 in United States, $140.00 outside of Canada and United States. Orders for memberships should be sent to:

Secretary-Treasurers

Don and Carol Attridge, O.A.M.E.

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COUNCILLORS TERM EXPIRING 1990

Fullerton, Olive

Handley, David

Jefferies, Mary Lynn

Jones, Ann

Lessard, Paul

McNabb, George

Rogers, Dr. Pat

Wiggan, Lorna

COUNCILLORS TERM EXPIRING 1991

Collins, Eileen

Cornwall, Jan

Davidson, Dave

Griffiths, Tom J

Harvey, M. Elaine

Mitchell, Janet

Lancaster, Ronald

Robinson, Robert

COUNCILLORS TERM EXPIRING 1992

Carefoote, Arnold

Knill, George

Romiens, Todd

Sauer, Ronald

Scully, Barry

Stuart, Susan

Warburton, Wendy

CHAPTERS AND REPRESENTATIVES

CHAMP(PAME)	Henry, Lis
COMA	Lenczewski, Peter
GOLDEN	Khan, Leslie
GVMA	Telfer, Stuart
NOMA	Maloney, Jerry
NWOAME	Hurrell, Marilyn
PINE RIDGE	Ennis, Ron
QSLMA	Evans, Douglas
SAME	Haehnel, William
SWOAME	Richardson, Peter
Y^4MA	Scully, Janet