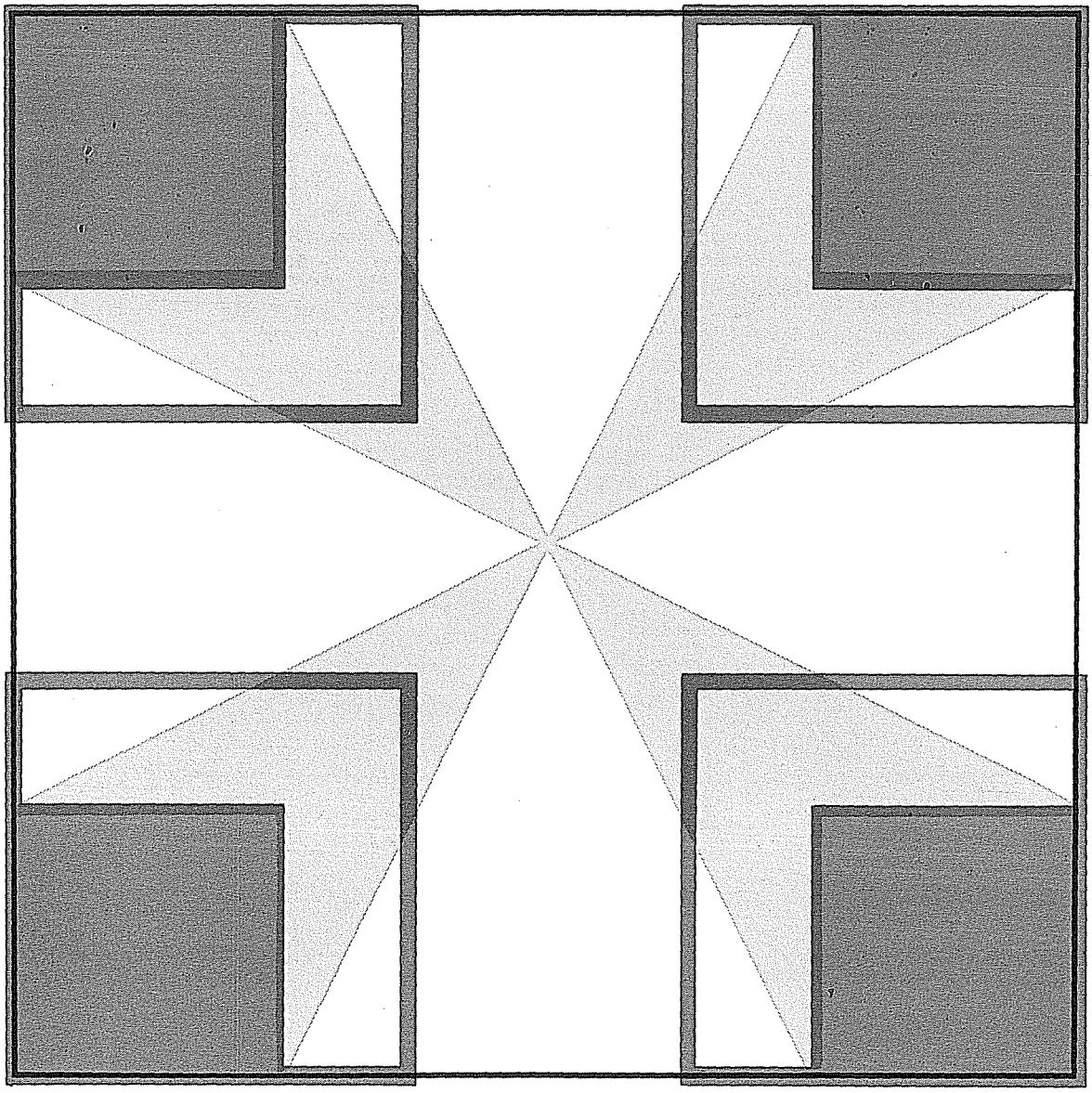
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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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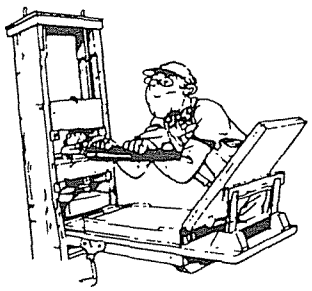
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by
Eric Wood

The Christmas season has always been one of my favourite times of the year -- in spite of the somewhat chaotic pace of life that comes with it. Teachers' lives too are chaotic from time to time, and, as mathematics teachers we can now take solace in the fact that chaos theory has become a study in its own right! A discussion about chaos theory is just one of the many interesting mathematical nuggets presented in Sylvia Gold's Viewpoint column.

Readers are encouraged to react to our Viewpoint columns by writing to the editor -- whether you agree or disagree is not important, just write to express your view. Furthermore, I know from my discussions with teachers and other educators around the province that many of my colleagues have interesting perspectives and opinions on the state of education in general and mathematics education in particular. Please take this opportunity to put your thoughts on paper and send them to us. If you wish your submission to be considered for the Viewpoint column, please indicate this intention because although the manuscript will be blind reviewed as usual, the process that we use is slightly different for this opinion piece.

The NCTM *Standards* have now been widely promulgated and in this issue a number of articles are devoted to ways in which the classroom teacher can address some of the issues that the *Standards* raise. For example, The Toronto Benchmarks Program is introduced in the first of a three-part series to illustrate the way in which one school board is approaching the issue of student evaluation and assessment.

Another significant idea in both the NCTM *Standards* and Ontario Ministry guidelines is the focus on group learning. Putting this kind of pedagogy into practice in the mathematics classroom is made easier by the practical and interesting suggestions in Rheta Rubenstein's article on mathematics learning groups. Keith Roy has provided a game which will stimulate students at the secondary level to think about and investigate some ideas related to probability theory, another important strand advocated in the *Standards*.

Concrete materials are often advocated as important for elementary school students but as Robert Gerver illustrates in his article on related rates, they are just as appropriate in higher level mathematics courses such as calculus. Finally, Rena Upitis shows how mathematics and music are closely linked through a study of Bach Preludes.

Ron Lancaster's problems this month take on a somewhat different flavour and will be a perfect way to spend some time in front of a Christmas fire while sipping eggnog!

We wish all our readers and their families a safe and happy holiday and look forward to yet another year of mathematical sharing in 1991!



PRESIDENT'S MESSAGE

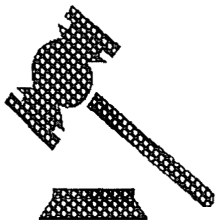
Here it is December and already I am in the second half of my term as president of OAME. As 1991 nears and our celebration of 100 years of mathematics' organization in Ontario is set to begin, it is noteworthy that the history preparation is well under way and will be printed soon. At the same time as we look at the past, we are looking toward the future.

Committees dealing with various divisions of the school program are busily preparing our points of view for input into various Ministry of Education committees. Particularly, we are interested in putting together ideas for the Foundation and Formative Years which require input to the Ministry by the end of this month. Although there is more time available, the Transition Years are being studied carefully with a view to being able to put forward our position when it is required. The Specialization Years are not being viewed as closely by the Ministry at this time, but we are preparing ourselves for the time when this information is needed.

OAME is also looking to the future in its conferences and planning. Mary Lou Kestell organized a very successful Leadership Conference in November that dealt with the future relationships and needs between industry, business and education and the role of mathematics in growth and development.

With the future in mind, all councillors will be attending a training session in January on Leadership in Mathematics for the Twenty-First Century. With the information gleaned at these sessions to be led by Alex Norrie, Bob Robinson and Mary Lou Kestell, councillors will be able to leave and hopefully go out into the province to acquaint other teachers as well as administrators and other school officials about the directions we should be heading as we near the year 2000.

Allow me to spend some time dealing with chapters and their relationship to the parent body, OAME At this time there are twelve chapters covering all areas of our province. They vary in size frcm large in numbers and strength like the Grand Valley Mathematics Association (GVMA) and the Scarborough Association for Mathematics Education (SAME) to large in territory such as the Northwestern Ontario Association for Mathematics Education (NWOAME) and the Northern Ontario Mathematics Association (NOMA). Some have been part of OAME since it began in its present format in 1973 while others such as the Western Ontario Mathematics Association (WOMA) are taking their first steps. Regardless of age or size, each chapter is an integral part of the total organization. Chapter representatives become council members first and our elections are then held to fill the rest of the positions on council for an annual operating group of forty. Chapter reps are full-fledged council members and are expected to take a full part in all Association activities as well as looking after the interests of their organizations back home.



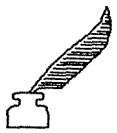
Chapters are also given serious consideration when setting OAME membership fees. At present $9.00 out of each $30.00 fee is slated for return to your chapter. Our task of making these refunds will be made much easier if each of you indicate your chapter affiliation at the time you remit your fees. These refunds will help support chapter activities while the remainder of the fee will allow the provincial body to work at making our voice heard at all levels of government, business and industry. We look forward to each of you continuing to support both your local chapter and the provincial organization as we move into the nineties and beyond.

As the festive times approach, may I wish all of you a most happy season and good health and happiness as we move into our centennial year. All the best to all of you.

Mickey Sandblom,
President



LETTERS TO THE EDITOR



St. George's School of Montreal

Dear Editor:

Professor Cohen's September presentation of aspects of cryptology as an application of number theory prompts me to share with interested fellow-members a "lead" that seems not to be well known. The American Cryptogram Association, now sixty years young, has a wonderfully challenging journal (six a year) featuring sixty or more systems of recreational cryptography, questions posed being well graduated as to difficulty, and all kinds of self-paced, amiable competition being supported. The Association has a particular welcome for student members (computer-oriented or paper-and-pencil competitors), and I find my McGill University gifted/talented summer 'teens often thrive on this kind of opportunity. The group met in Montreal in August, holding its Annual Meeting at St. George's. The 1991 meeting is in Buffalo. Teachers and students can obtain particulars and sample journals from the A.C.A., 18789 West Hickory Street, Mundelein, Illinois 60060.

Harold Don Allen, Ed.D

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Dear Editor,

Thank you for the excellent article by Eric Wood in the September 1990 issue of the *Gazette*. *Radwanski and Destreaming* is must reading for all mathematics teachers. In fact, I have suggested to department heads in our board to send copies to their principal and superintendent.

Rejection of the Radwanski report or concerns over destreaming should not cloud discussion by mathematics teachers of the issues in grades 7-9. The Ministry's focus on the Transition Years offers us an opportunity to re-think our programs, our evaluation practices, out instructional strategies, and our school structures to better meet the academic and *social* needs of adolescent learners. The hope is that strong, Ontario-based research will justify and direct any structural change. As well, teachers must continue to evaluate and modify programs to reflect the mathematics skills required by early adolescents, both in and our of the classroom.

Stu Telfer.

CORRIGENDUM

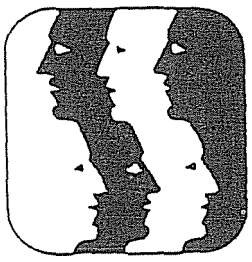
The article **"Practical problems arising out of number theory"** by E. L. Cohen, which appeared in the September 1990 issue of the *Ontario Mathematics Gazette* contained several misprints.

Page 24, right column:
bottom blank line after "but are more necessary than"
should read "ever, for example, because of communications"

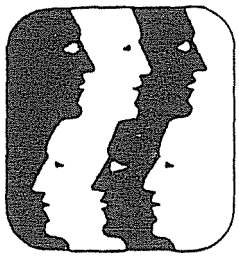
Page 26, left column:
line 5 of EXAMPLE should read "24 12 10 20 17 12 20 -- 15 19 4 5"
instead of "-- 24 12 10 20 17 12 20 20 15 19 4 5"

Page 26, left column:
4 lines from bottom, should read: "4a + b ≡ 20 (mod 26)"
instead of "4a + b ≡ 15 (mod 26)"

Our apologies to the author and readers.



Viewpoint



WOMEN AND MATHEMATICS FROM ALCHEMY TO CHAOS

by
Sylvia Gold
Director of Research and Evaluation
Royal Commission on New Reproductive Technologies.

Editor's Note: This article was originally presented at the Women and Mathematics Breakfast at the NCTM/OAME Regional Conference, Hamilton, May 1990.

All educators are concerned that, despite their best efforts, the enrollment and retention figures in mathematics and science classes is disturbingly low. It is especially low for girls. In the past we tried to fix the students, especially the girls, treating them for math anxiety or setting up separate classes. That didn't work. Now we are being pushed by economic factors to increase our mathematics and science graduates.

ECONOMIC IMPLICATIONS

There is a growing concern that Canada will be faced with a shortage of mathematicians and scientists in the decades to come. The Prime Minister has established a scholarship program to encourage young people to study in this area. Fifty percent of the scholarships are awarded to girls. But we have a major shortage of mathematics graduates on our hands. As Anita Ross, IBM Vice President of Personnel noted recently, of all students who enter grade nine, only 18 percent complete university, while 19 percent complete community college.

In a standard international science examination involving 17 countries, Canada ranked 12th. Ross went on to point out that by the year 2000, 30 percent of new jobs will require new skills in mathematics, science and language that come from university degrees or higher. Yet in 1987 there were only 300 Ph.D. graduates in engineering and applied sciences in Canada. After subtracting those who leave for other countries, the supply for Canadian business, institutions of learning and government is very limited.

Some U.S. statistics are rather alarming, particularly if they foretell the future Canadian situation. The U.S. expected the shortfall of bachelor degree graduates in science and engineering in 1989 to be about 40 000; the projected deficit in the year 2006 is 640 000 (*Wall Street Journal*, February 22, 1990, P.A1, A8).

Where is the next generation of mathematicians and scientists going to come from? Demographic studies tell us they will come from untapped sources: women, aboriginal people, and racial minorities. We must make students from these groups welcome in our mathematics and science programs. Perhaps

the new emerging concepts and language of mathematics, growing out of chaos theory, will make that connection.

MAKING CONNECTIONS: WOMEN'S PERSPECTIVES AND NEW MATHEMATICAL CONCEPTS.

Having presented a rather bleak picture of the deficits in enrolments in mathematics and their consequences, and suggested that we must find better ways of encouraging students, especially girls, to feel a connection and attraction to mathematical studies, let me ask you to reflect upon the following question:

Try to connect some of what we know about women's ways of conceptualizing the world and the vocabulary of chaos theory. My thesis is that chaos vocabulary and concepts are more congruent with women's ways of knowing the world than machine age language and therefore this truly new mathematics should be very attractive to all students but especially females. By integrating chaos thinking into our mathematics programs we should encourage higher enrolments and retention. Let me attempt to demonstrate why this is more than a leap of faith.

My inspiration for this position stems from several books on women's ways of knowing and chaos theory. It also comes from my experience and the struggle we all face, as teachers, in imparting knowledge and motivating learning.

Chaos vocabulary and concepts are more congruent with women's ways of knowing the world.

First, what is mathematics anyway, and if it is so important how come I can't answer that question? Second, can a look between the covers of books on chaos theory help me understand how mathematicians think, what they think about, and what their relevance is to society? My next question is, what insights does this reading and thinking give us in order to understand the antipathy of students, particularly girls, to science and mathematics?

Back to the first question. What is a mathematician? Perhaps a mathematician is someone who enjoys getting up at the crack of dawn to watch the sun rise and greet the day with the birds. Or someone who sees life as a series of Venn diagrams - overlapping, connected, interactive.

Let me offer you a description by a young university graduate now working at CCMD. She says,

> "A mathematician is someone who pushes the limits of mathematics. Given that mathematics is defined as a tool by which scientists can study the phenomena of nature, a mathematician attempts to refine these tools. He (sic) does this through equations because equations allow him to study in the abstract and produce generalities which the scientists can then use to study the more concrete aspects of nature."

Then again, perhaps mathematicians are abstract thinkers who translate events into formulas and equations that ordinary mortals cannot figure out, no matter how elegantly they (the abstractions, not the mathematicians) are constructed. Legend has it that a prize winning doctoral thesis in mathematics was only one page long.

Sometimes mathematicians are trapped into trying to force people to fit into numbers, percentages, graphs and statistics. These are called demographics, polls and projections. But in saying this I might be moving away from mathematicians to professions which use mathematics in their cause, such as economists and statisticians.

Truly, the lay person has difficulty understanding what mathematicians do, what

practical value they bring to society, or how mathematical thinking contributes to human endeavour. A facile response points to consumers' needs to understand mortgage rates, discounts, area and budgets. Basic measurements and control over our income and expenditure are where it's at. Joanne, the young woman I quoted earlier, sees mathematics in a broader context and argues that mathematics should be compulsory throughout high school, so that all the options for further study remain open. She says, "Educators should not allow students to drop a class as important as mathematics simply because the students find it boring. That does not solve the problem. The solution is to make mathematics interesting and demystify it so that students can see how important and practical mathematics can be. . . because it teaches them how to think and how to come up with solutions. Getting the answer isn't the important thing. The important thing is the steps a student takes in getting those answers."

The dictionary helps expand our understanding a little. A mathematician is one who is versed in mathematics, or, an astrologer. Astrologer??

Mathematics is the collective name for geometry, arithmetic and certain sciences involving geometric reasoning, as astronomy and optics. Let's reflect back to the curious reference to astrology.

The dictionary tells us that Astrology is practical astronomy - the application of astronomy to the prediction of events natural and moral. It was of two kinds.

A) Natural astrology, the calculation and foretelling of natural phenomenon as tides and eclipses, obsolescent since the 17th century.

B) Judicial astrology, the art of judging of the occult influences of stars upon human affairs, the only meaning since the 17th century.

Speaking of astrology, recently astrologers expressed their outrage at newspaper editors who caution their readers against accepting horoscopes as scientific fact. "They are nothing but entertainment," say the editors. Sydney Omarr, former journalist and now a professional astrologer finds the disclaimer trend dangerous. He says, "Once you permit this kind of thought control and prejudice, it will go over into other areas. . . tomorrow it'll be the bridge column, then the financial column." Joan Quigley, former astrologer to Nancy Reagan, says, "Astrology is scientific in this sense: it is based on astronomy, the same way a doctor bases his diagnosis on lab reports and the economist bases his predictions on computer-gathered statistics. Only I think astrology is more reliable than those two." *(Wall Street Journal,* April 27, 1990).

This is science of the past, of alchemy and harmony, and perhaps confusion between the real and the supernatural, of scientific truth and mythical and artistic interpretation.

All this is to say that lay people have little understanding of what mathematicians do beyond clocks, checking grocery bills and calculating interest rates. The challenge, mysteries, problems and applications of mathematics are not understood or appreciated. Should we therefore be surprised when students drop mathematics when it becomes abstract? Is it a surprise that it remains mysterious and arcane to many? In this information age where there is no longer a need for arithmetic if you can afford a simple calculator why move on to calculus?

The emerging scientific paradigm bears a stronger relationship to women's ways of knowing.

I suggest Joanne is on the rights track. There is a need to demystify mathematics, to make it fun and interesting and more than an

intellectual exercise. The giants of mathematics may never become household words, like Madonna or Wayne Gretzky, but I cannot name one modern day mathematician - except for some I have been reading about in books on chaos. Here I find the excitement of discovery, the art and science of mathematics, the rigour and experimentation and play that we have to make real and enticing to our students.

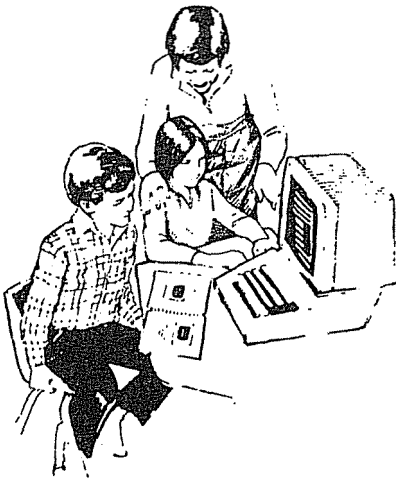
FROM THE HISTORY OF SCIENCE

A reading of the history of science makes it clear that mathematical and scientific thinkers are, like the rest of us, part of the social, intellectual, and economic contexts of our time. The philosopher Alfred Whitehead tells us that "the seventeenth century had finally produced a scheme of scientific thought framed by mathematicians, for use of mathematicians. The great characteristic of the mathematical mind is its capacity for dealing with abstractions; and for eliciting from them clear-cut demonstrative trains of reasoning, entirely satisfactory so long as it is those abstractions which you want to think about." (Toffler, 1988, p. XII-XIII)

The classical science of the seventeenth and eighteenth centuries pictured a world of precision in which chance played no part, in which all the pieces came together like cogs in a cosmic machine. Coincidentally, this was the age of the rise of the factory civilization and the age of the machine. This was the on-set of factory-based industrialization, of manufacturing machines and heroic engineering breakthroughs, railways, textiles, and finally cars. But today we see the limitations of the machine model of reality. We see a new paradigm (Toffler, 1988, p. XII-XIII).

Traditional science in the machine age tended to emphasize stability, order, uniformity and equilibrium. But today's shift from heavy inputs of energy, capital and labour to information and communication signals a new scientific world model. And here we begin to pick up chaos theory. Contrasted to stability and equilibrium, chaos talks about fluctuating sub-systems within systems, where a single fluctuation or combination of them may become so powerful as a result of positive feedback that it shatters pre-existing organization. Out of this disorder and chaos, through a process of self-organization, order and organization arise. Parallels in social realities would be an economic crash, technological upheaval, or a paradigm shift (Toffler, 1988, pp. XI-XVII).

Chaos thinkers are looking for the whole, with an eye towards pattern, a taste for randomness, for jagged edges and sudden leaps. The American physicist Shaw said, "You don't see something until you have the right metaphor to let you perceive it." Chaos theory talks about fractal geometry, strange attractors, turbulence and patterns. It is very much an articulation of our information age since none of these concepts would have been made explicit without the computer.



As a young student Benoit Mandelbrot was frustrated by the highly abstract geometry he was taught at school and became fascinated by the non-geometric irregularity of the world around him. His observations and mathematical visualization led him to see repeating patterns or self-similarity, a powerful way of generating shape. The concepts I am trying to understand that come from fractal geometry and mathematicians' conception of chaos theory are concepts such as patterns in

turbulence, iterations, repetition of parts, and the image of the whole in each part, as in a hologram. I find this exciting because, just as the mathematics of the machine age was congruent with the rationality of social and economic thinking, perhaps the thinking within chaos theory - of the relationship between the whole and its iterative parts, fluctuations and change, chaos in regularity and stability and order born out of turbulence and chance is reflective of our era and helps us understand it.

How different these concepts are from the concepts of the machine age - concepts of order, sequence, control and power.

WOMEN'S WAYS OF KNOWING

How can this thinking change matters so that more students, girls and boys, are loving mathematics and staying with it?



I find hope in these new mathematics concepts for enticing girls and women into mathematics and science in greater numbers. Let me explain why. In the book *Women's Ways of Knowing*, (Belenky et al, 1986) the authors, psychologists, interviewed women from many walks of life who were undergoing changes in their lives. The authors were interested in a number of aspects of these women's lives, including the influence of schools in promoting or hindering their development.

The authors find that once knowers assume relationships in knowledge rather than linear sequence in knowledge, and a more holistic frame of reference, they feel responsible for questioning and examining the systems they will use for constructing knowledge. They feel more confident to engage in the learning process, and often point out, "You are asking the wrong questions" or "Your question is out of context".

The authors conclude that educators can connect with female students if they emphasize connection over separation, understanding over assessment, and collaboration over debate.

Let me make the point another way, by referring to the field of law to illustrate the need for educators to recognize gender differences. Justice Bertha Wilson of the Supreme Court of Canada has made this point well. In an address to the Fourth Annual Barbara Betcherman Memorial Lecture at Osgoode Hall last February, Justice Wilson discussed the impartiality of judges. She urges judges and legislators to make room for female values in their work.

She notes that feminist scholars make these points. One, women view the world and what goes on in it from different perspectives than men; and two, women judges, by bringing to bear that perspective on the cases on which they sit, can play a major role in introducing judicial neutrality and impartiality in the justice system.

Justice Wilson refers to the work of Harvard education Professor Carole Gilligan whose research has found that women see themselves connected to others and as members of society, where men see themselves as autonomous and independent. Justice Wilson concludes by saying that "If women lawyers and women judges through their differing perspectives on life can bring a new humanity to bear on the decision-making process, perhaps they will

make a difference. Perhaps they will succeed in infusing the law with an understanding of what it means to be fully human". To be fully human means to fully recognize the cognitive powers and perspectives of women as well as men, to value connectedness as well as independence.

Having said this, I think that if we were to draw two circles, one containing a description of the way in which we believe males see the world and another containing a description of the way in which women see the world we would have overlapping circles. I contend that the emerging scientific paradigm bears a stronger relationship to women's ways of knowing - patterns, relationships, wholeness, calm within turbulence, and change followed by new patterns and calm. Obviously many men are comfortable with such analysis as well.

The point I wish to emphasize is that as teachers of mathematics and science we have a golden opportunity to attract women to mathematics and science as these areas of study open up visions that are congruent with women's views of the world. Just as the rationality and scientific model of the past were the dominant themes in schools and in society, so the emerging themes from chaos theory and the information age must influence curriculum. I believe that, given the opportunity, female students will tune into the exciting concepts that are emerging.

CONCLUSION

The following suggestions hopefully serve as an appropriate conclusion to my remarks.

- Let us fix the way we teach mathematical understanding and language so that it is attractive to female and male students alike, if for no other reason than the fact that the domestic and international economic health of Canada depends on graduates in this field.

- Let us build on the knowledge we have about the ways women perceive the world and the link to learning, so that, in Justice Wilson's words, our classrooms and our humanity are whole.

- Let us support the efforts of voluntary organizations which are encouraging youngsters to undertake mathematical and scientific studies. For example in January 1990 the Girl Guides of Canada launched a nation-wide Girl Guide Water Project in cooperation with Environment Canada. Among the activities in which the guides participated were checking on the type of, chemical constituents of, and the polluting characteristics of the kinds of liquid soaps and cleaners used in the home and school and to promote the use of environmentally safe materials. Other programs focused on safe drinking water, heritage river preservation, fish habitats, shoreline erosion, and water use conflicts. An overall objective of this program was to encourage girls to prepare themselves for a future in environmental sciences and related careers.

- A final suggestion. To make mathematics attractive to students, particularly female students, we should try to help them understand the relevance of mathematical thinking to the larger social context; that mathematicians and scientists interact with their societies and that they all share in certain values and ways of looking at the world. The alchemists believed in the harmony of science and nature; perhaps they were true ecologists.

Francis Bacon viewed science as the conquering of nature, as power over nature - a view that predisposed Western society to elevate scientific thinking and led to, among other manifestations, scientific management, which we can still recognize in our schools today. The manifestations are time, sequence, control, order and linear thinking.

In the same way mathematical thinking today is paralleling social thinking. Chaos theory talks about patterns, ambiguity, calm in the midst of turbulence, choice, iteration, aesthetics, non-linearity. These are the new words of organizations, schools included. Managing diversity, dealing with ambiguity, thinking in patterns of relationship and

connectedness - all these values bear an affinity to chaos theory.

Our young people are living with ambiguity. How aptly mathematical thinking of this new age can be in helping them understand our physical and social world! Young women will relate much more easily to the connectedness, the ambiguity, turbulence and calm of chaos.

To end on my beginning point - to encourage enrollment and retention in math, we should not attempt to fix the student. We should fix the way we teach mathematics.

REFERENCES

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Teaching Tips

ANGLES WITH GARBAGE TIES

The introduction of geometry at the Grade 9 level involves defining and measuring different types of angles. To help students have a visual image of these angles each one should be given a garbage tie and the following instructions:

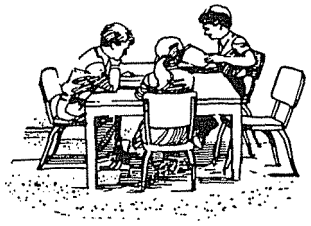
a) Form an acute angle - estimate its measure - trace it onto a sheet of paper - measure it with a protractor.
b) Repeat a) several times.
c) Form an angle which you estimate to be 45° - trace it - check the estimate with a protractor.
d) Repeat c) for 30°, 60°, and 90°.
e) Repeat a) through d) for obtuse angles.

These instructions could be teacher/student directed by using an overhead projector.

Note: 1. The same type on instructions could also be used when classifying triangles.
2. The use of garbage ties gives a hands-on experience when studying perimeter.

Submitted by Ron Reesor
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London, Ontario.

MATHEMATICAL SORTING ACTIVITIES FOR FORMING LEARNING GROUPS



by
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The National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* encourages teachers to have students work in groups for appropriate segments of the learning program (Commission on Standards, 1989, p.19). Similarly, the Ontario Ministry of Education supports these arrangements in its promotion of language across the curriculum: "...schools should provide an environment in which students are encouraged to use language to explore concepts, solve problems, organize information, share discoveries, formulate hypotheses, and explain personal ideas. Students need frequent opportunities to interact in small-group discussions..." (1984, p.7).

Many teachers who strive to achieve these goals use learning groups in their classrooms. They find that it is beneficial to put students into new teams intermittently during the year. In this way, they benefit from communicating about mathematics with a wider range of pupils as well as from acquiring needed social skills. If classes are going to reorganize on a regular basis, then the teacher needs some strategies for sorting students into new groups. Randomization is one strategy. Use a standard deck of cards and select "four of a kind" for each group to be formed. Shuffle and distribute. Students find the others with the same value and form a team. But why not be more imaginative? Following are some other sorting ideas which have the advantage of using mathematical concepts in the process.

Following a unit on geometry, for example, prepare a deck of cards as shown in Figure 1 designed for nine groups of four students each. After the cards are shuffled and distributed, students look for others whose cards make a category with theirs. Of course, the students do not know in advance what the categories are. While other students are still sorting themselves out, those in groups can be asked to identify the distinguishing properties in their category and other figures that could be members of their group. When everyone is "sorted" each group should share the results of these discussions with the entire class. Difficulties or alternative sorting arrangements may also be discussed.

The deck can be adapted to different arrangements. For smaller groups (two or three students is sometimes preferred), reduce the size of each category. For more or fewer groups, change the number of categories. When designing the deck of cards the teacher must be sure that items fall into unique categories. For example, Figure 1 does not include a square because it could be classified as a regular polygon or as a parallelogram. To provide a double-check for students, draw each item within a category in a different colour. Then, before groups are finalized, students may check that each member has a differently coloured card.

Other ideas for decks of cards may be even easier. Figure 2 shows categories for integers, again carefully avoiding numbers that could be categorized in two different ways. For fractions, simply use sets of equivalent fractions for the categories. Cards can include models of the fractions to reinforce the meaning, as well as the names of the fractions. Using this same concept of equivalence, categories can be expressions that all simplify to the same value. A classic of this type is sets

one dimensional figures

right angled triangles

parallelograms

obtuse angled triangles

regular polygons

isosceles triangles

prisms

pyramids

non-polyhedra 3-D figures

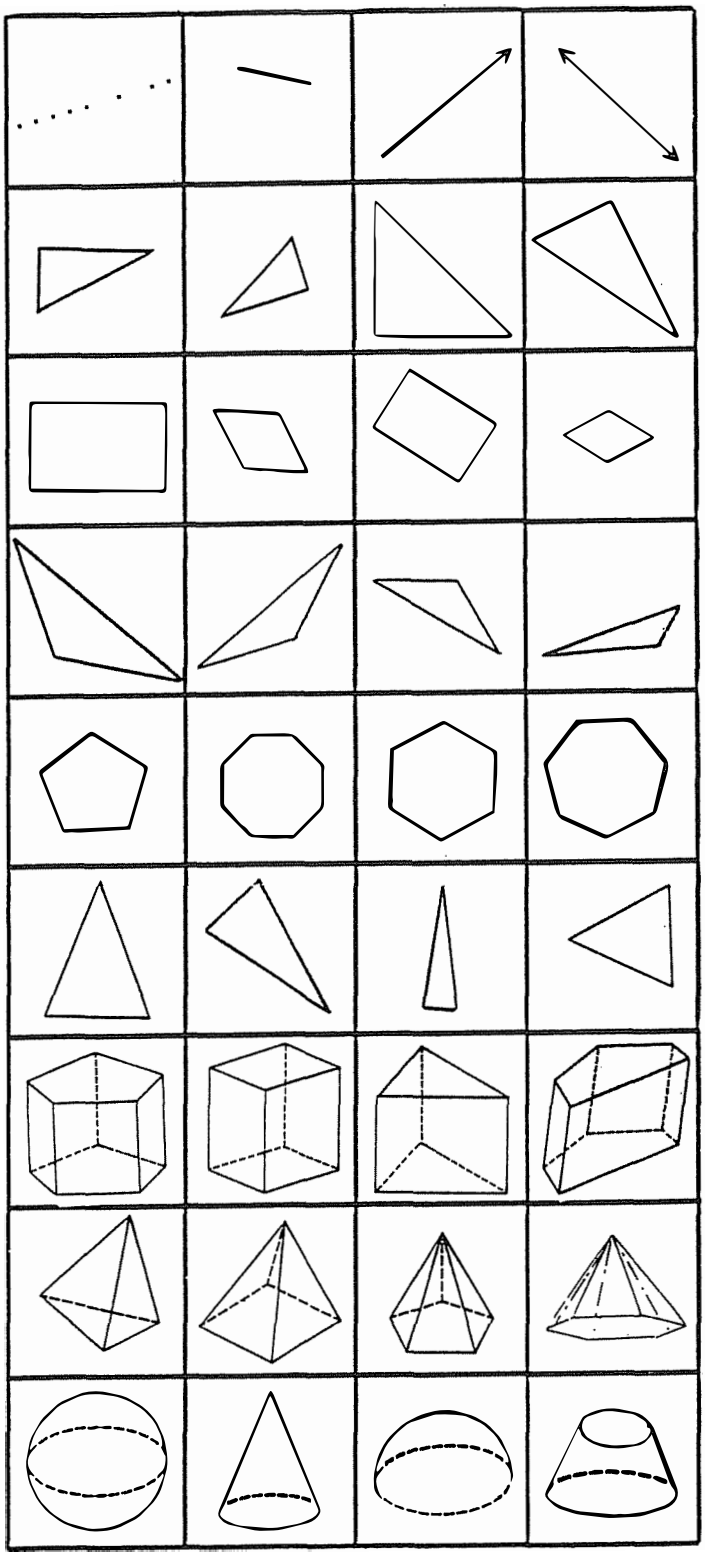


Figure 1

multiples of 3	6	18	15	24
primes	19	31	53	73
squares	4	16	25	121
cubes	8	27	125	512
multiples of 7	14	28	42	84
palindromes	8888	262	404	110011
negatives	-2	-19	-46	-11
numbers in scientific notation	$4X10^3$	$4.91X10^5$	$1.09X10^3$	$8X10^5$

Figure 2

of equivalent fractions, decimals, and percents. Another is expressions using order of operations that simplify to the same value. These can be numerical expressions, or, after students have begun algebra, variable expressions that simplify to the same value or expression.

For measurement, consider a set of cards as shown in Figure 3 in which students sort measurement units into categories by the attribute being measured. After students have had experience with statistics they can sort themselves into the following categories: bar graphs, line graphs, circle graphs, box plots,

length	mm	cm	km	m
area	mm^2	m^2	km^2	hectare
volume	mm^3	m^3	km^3	cm^3
capacity	L	mL	kL	cL
mass	g	kg	cg	tonne
speed	cm/sec	m/min	m/hr	km/da
unit price	¢/oz	$/kg	¢/L	$/L
time	sec	min	hr	da
money	francs	dollars	pesos	lire

Figure 3

Maria and Kim are at a 40% off sale.	Maria buys a jacket originally marked $59.	Kim buys a sweater originally marked $28.	How much money did Kim & Maria save?

Figure 4

centres (mean, median, mode, and their formulas or descriptions), and spreads (range, variance, standard deviation, and their formulas or descriptions).

A more elaborate set of categories can be built from problems. Figure 4 shows one "category" which is a problem with four pieces of information. The names, Maria and Kim, become the links which students use to identify their fellow group members. When groups have been formed the students work together to answer the final question. Alternatively, the question card can be omitted (for groups of three) and the groups formed can be asked to write and solve at least three questions using their given information.

While these examples focus at the junior and intermediate levels, this concept is easily adapted both up and down the curriculum. Figure 5 illustrates a set of cards for primary students which is based simply on representations of the same numerical value. Another idea for this level is to use numbers in the same decade (20s, 30s, 40s, etc.) as the categories. Or students can be told that their groups will be made of four consecutive numbers. For example, 9, 10, 11, 12 is one category; 98, 99, 100, 101 is another. At the

one	1	•	4 - 3
two	2	••	5 - 3
three	3	•••	2 + 1
four	4	••••	2 + 2
five	5	•••••	4 + 1
six	6	• •••••	3 + 3
seven	7	•• •••••	3 + 4
eight	8	••• •••••	5 + 3
nine	9	•••• •••••	10 - 1
ten	10	••••• •••••	7 + 3

Figure 5

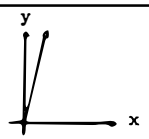
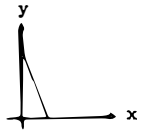
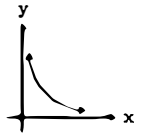
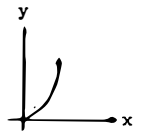
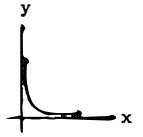
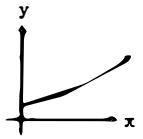
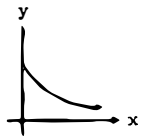
direct variation	$y = kx$	Each cm on this map represents 1 km.	
linear, but not direct variation	$y = mx + b$	A cylindrical beaker filled with 100 cc of water is draining at the rate of 5 cc per sec.	
inverse variation	$y = \frac{k}{x}$	The length of a rectangle with area 24 cm varies as the width changes.	
square variation	$y = kx^2$	The area of a circle depends on the radius.	
inverse square variation	$y = \frac{k}{x^2}$	The intensity of the light from a bulb depends on the square of the distance from the source.	
exponential growth	$y = ab^x$	An investment in a bank at 10% interest compounded continuously grows with time.	
exponential decay	$y = ab^{-x}$	The value of a car decreases by 20% each year from its previous year's value.	

Figure 6

senior level, the topic of functions and models offers opportunities for similar categorizing as illustrated in Figure 6. Here one card names a type of function, one card provides a situation which is modeled by that type of function, one card provides a graph, and one provides an equation. For a simpler activity, use just one function, for example, linear. Then each category can have a specific situation modeled by a linear equation, the specific graph, and the specific equation.

After students have used this activity a few times, they begin to appreciate and recognize more easily the idea of equivalence which is a strong theme in many mathematical topics. As a teacher, you know you have made progress when they ask to generate their own such decks for future class rearrangements.

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B|E|N|C|H|M|A|R|K|S IN MATHEMATICS

The Members of the Toronto Board of Education Benchmark Team are Freda Appleyard, Peter Billingsley, John Clark, Margaret Evans, Dr. Sylvia Larter, Phoebe Rankin, and Donald Rutledge

Editors' Note: Benchmarks in Mathematics is the first of an invited series of articles discussing an alternative assessment instrument developed at the Toronto Board of Education.

BACKGROUND

In May 1987, the Toronto Board of Education mandated the development of standards for student achievement in mathematics and language at the end of grades 3, 6 and 8 to be used by teachers in evaluating students and reporting to parents. The resulting standards are now known as Benchmarks. The Benchmarks for Grades 3 and 6 were delivered to the schools in the spring of 1990, and grade 8 Benchmarks will be delivered in late fall of 1990. It is also the Board's intention to develop Benchmarks for the secondary panel, probably at the end of grade 10.

It was the opinion of the team who had the responsibility for developing the Benchmarks, that there was an enormous gap between current practices in large scale evaluation systems, and the philosophy of Ministry and Board guidelines. Ministry guidelines stress experiential learning and higher order thinking skills. The Toronto Board guidelines place great emphasis on teacher observation as a method for evaluating child development, and on the importance of language in learning. These process elements are not generally reflected in large scale evaluation systems which are more oriented to objectives which can be assessed through paper and pencil methods. The team also believed that the content and methods used in a system-wide evaluation make a strong statement to teachers about what is important in the learning experiences they provide for their students. Therefore, in developing the Benchmarks an attempt was made to bridge the gap between testing and learning by establishing standards and methods of evaluation which were consistent with the broadest goals of the expected curriculum.

DEVELOPMENT

A Benchmark Team has had the responsibility for the content and methods used in the program. The Team consists of a curriculum superintendent, and representatives from the Research, Language and Mathematics Departments. The philosophy and methods for language and mathematics have been generally the same, and the Team believes that by working together, the quality of the Benchmarks has been greatly enhanced.

The Benchmark Team was assisted by a Mathematics Advisory Committee of teachers, vice-principals, principals and consultants. This committee helped to shape the overall approach and methods which were used in the assessment. Objectives were selected from Ministry and Board guidelines and the committee worked hard to design tasks which encompassed not only the content objectives, but also the use of manipulatives, problem solving and student language. The goal was to design tasks which were, as far as possible, similar to good classroom activities which teachers now use in their classrooms. The idea of videotaping student performances grew out of the desire to be able to observe how students think and solve problems, and to have a record of this for all teachers to use. Detailed scripts were written for each task and all of the tasks were field-tested before being administered.

INTERVIEWING THE STUDENTS

A ten percent representative sample of the total student population in each of Grades 3, 6 and 8 was drawn randomly. Included were students from French Immersion, Special Education and ESL classes. Each student was interviewed for about five hours and about two hours were recorded on videotape. The interviewers were recently retired teachers who understood children and our school system. The interviewers received one week of training and the interviews took place in the spring of 1988 for Grades 3 and 6, and 1989 for Grade 8.

ANALYZING THE DATA

Most of the scoring of the students' performances was done by classroom teachers during the summer months. The teachers were paid and worked under the leadership of members of the Benchmark Team. A majority of the videotaped performances were scored holistically. Analytic scoring was also used, mostly with the students' non-videotaped performances and written work. Briefly, here is what is meant by holistic and analytic scoring.

Holistic scoring is much like a sorting activity. In the beginning as you observe each student's performance or written work, you decide that it is very good, average or poor. All of the performances gradually get sorted into groups, perhaps three initially, or more as the number of performances increases. The observations which are used as the basis for placing each student are called criteria. The criteria are drawn out of the students' performances and not determined in advance. In the Toronto Benchmarks, five holistic levels were used. The final holistic data for a particular activity include the percentage of students rated at each of the five levels of performance. There was no attempt to bell-curve the holistic data. The graphs for the data vary according to the activity and the students' performances.

Analytic scoring is more like conventional scoring where the number of students giving a particular response is documented. In Benchmarks, as well as determining the percentages of students giving correct responses, significant other responses or errors were also documented.

Much of the literature on holistic scoring supports the view that it is as reliable as analytic scoring and it is more efficient. In Benchmarks, the combination of holistic scoring and observations of students' videotaped performances provided a powerful tool for evaluating higher order thinking skills.

THE BENCHMARKS

What emerged from the process described above are three libraries of videotape and print Benchmarks.

Grade 3 Library (M3) - 16 videotapes and 10 print Benchmarks

Grade 6 Library (M6) - 12 videotapes and 15 print Benchmarks

Grade 8 Library (M8) - 12 videotapes and 14 print Benchmarks

The Benchmarks Libraries include a broad range of activities in arithmetic, measurement and geometry, and problem solving is integrated throughout. In the majority of Benchmarks, students were required to perform tasks using manipulatives and explain their responses to various questions. The videotapes provide a unique opportunity to observe how students think about important concepts, and how they solve problems.

Here are samples of the tasks which you will see on the videotapes in each Library:

Grade 3

- place 9 balls in two baskets in as many ways as possible
- add and subtract two numbers with base ten blocks
- estimate and measure the length of a curve
- sort a set of geometric solids

Grade 6

- estimate answers to real problems involving multiplication
- compare two fractions using fraction pies
- carry out an experiment involving a pair of dice
- estimate and then use water to measure the capacity of a bowl

Grade 8

- estimate and then measure the number of beans in a jar
- find the time needed to walk the distance between two points on a map
- investigate the patterns which result from adding two-digit numbers and their reversals
- make a circle graph for a set of data

The Benchmarks are not tests. They are demonstrations of what Toronto Board students can do on activities which most teachers now use in their classrooms. The majority of the Benchmarks contain the following components: a set of objectives, a task, a scoring system, demonstrations of student performances at different levels, and a set of data. Some of the print Benchmarks do not have demonstrations of student performances.

IMPLEMENTATION

The Board is looking at the next few years as the period of implementation. The process will be school-based and led by the principal with the support of a school team of teachers and consultants. All vice-principals, principals and support staff have received intensive in-service on the content of the Benchmarks and the roles of various staff leading implementation.

The use of Benchmarks is not prescriptive. They are to be seen as information and it is expected teachers will shape their use both in evaluating students and reporting to parents. The use of Benchmarks, however, is mandatory and all teachers will be involved in their implementation. Teachers will be able to integrate Benchmarks gradually into their ongoing program. A system-wide Benchmark Implementation Committee will monitor implementation by acting as a clearing-house for good ideas and revising procedures for reporting to parents.

SUMMARY

We believe the Toronto Board of Education Benchmarks have the following strengths:

1. They have high content validity because they align evaluation with ongoing classroom practice and they evaluate a wide range of objectives.

2. They provide a holistic approach to evaluating students' higher order thinking skills in activities which are a part of the regular program.

3. They provide a resource for discussing how students learn and what classroom activities promote their growth.

4. They provide a wide base of potential information for evaluating students and reporting to parents.

5. They respect teachers' professionalism by not making judgements of their students for them.

We in the Toronto Board of Education feel very proud and privileged to have been a part of such an exciting and worthwhile project.

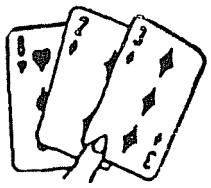
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Make cheque payable to O.A.M.E.

FAIR GAME OR FOUL



by
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Lakehead University
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A few years ago, a group of teachers and parents hosted a "Casino Night" for students from grades seven and eight. The students used play money and, at the end of the evening, used their accumulated wealth to bid on various prizes donated by local merchants.

A number of different games, of varying levels of interest and popularity were featured. One particular table, where the games to be played should have resulted in a student losing slightly more often than winning, became quite popular later on in the evening because some students seemed to win more money at that table than they lost.

The poster in Figure 1 was used to describe the games that could be played at that particular table.

INSTRUCTIONS

There are 4 kings and 2 aces face down on the table.
You must choose which of the games you wish to play.

GAME 1



Pay $1.00 to play and choose 2 cards.
If both cards are kings, you will receive $2.00.

GAME 2

Pay $1.00 to play and choose 2 cards.
If both cards are aces, you will receive $10.00.

Figure 1 Game instructions

The probability of a player winning $10.00 in game 2 is the product of *[the probability that the first card chosen is an ace]* with *[the probability that the second card chosen is an ace, given that the first card was]*, that is (2/6) (1/5) = 1/15. Notice that the player should receive $15.00 instead of $10.00 each time he/she wins in order for this game to be fair.

The two games listed above are among the many con games and other examples of faulty statistical reasoning contained in *Flaws and Fallacies in Statistical Thinking*, by Stephen

The probability of the player winning $2.00 in game 1 is the product of *[the probability that the first card chosen is a king]* with *[the probability that the second card chosen is a king, given that the first card was]*, that is (4/6) (3/5) = 2/5. Notice that, for five plays of this games, a player will pay a total of $5.00 but can expect to win only twice, receiving a total of $4.00. Consequently, the player should be entitled to receive $2.50 instead of only $2.00 each time she/he wins in order for this game to be fair.

Campbell, (Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1974). This book (only 200 pages) should prove an interesting source of examples for teachers of statistics or probability at both the elementary and secondary school level.

Since both of the games described above are unfair, what happened at the table in question to produce unanticipated increases in a player's accumulated winnings?

It was discovered some time later that the person operating the games at that particular table didn't quite understand the instructions and allowed some players to proceed as follows:

A player would select game 1, pay $1.00 and select one card. If that card turned out to be a king, the player would select a second card and would receive $2.00 if that second card was also a king. (So far, there has been no violation of the rules contained in the instructions.) However, if the first card chosen turned out to be an ace, the player would say something like; "Obviously I have lost game 1. I'll try for two aces.", then would pay an additional $1.00 before selecting the second card. If the second card was also an ace, the player would receive $10.00. (This method of playing game 2 certainly is a violation of the rules since the player has already found one ace and needs only to find the second ace among the remaining five cards, with a probability of 1/5 as opposed to a probability of 1/15 of finding <u>both</u> aces in the real game 2.)

Figure 2 will help in the analysis of the new game invented inadvertently by the players.

Following the branches in figure 2, a player will either:

(a) pay $1.00 to play, select a king then another king, and receive $2.00 with probability (2/3)(3/5) = 6/15 (a net increase of $1.00),
(b) pay $1.00 to play, select a king then an ace, and consequently lose with probability (2/3)(2/5) = 4/15 (a net decrease of $1.00),
(c) pay $1.00 to play, select an ace, pay an additional $1.00 then select a king, and consequently lose with probability (1/3)(4/5) = 4/15 (a net decrease of $2.00), or
(d) pay $1.00 to play, select an ace, pay an additional $1.00 then select the other ace, and receive $10.00 with probability (1/3)(1/5) = 1/15 (a net increase of $8.00).

The player's expected increased bankroll after one play of this new game is

$$(\$1.00)(\frac{6}{15}) - (\$1.00)(\frac{4}{15}) - (\$2.00)(\frac{4}{15}) + (\$8.00)(\frac{1}{15}) = \frac{2}{15}\ \textit{dollars}$$

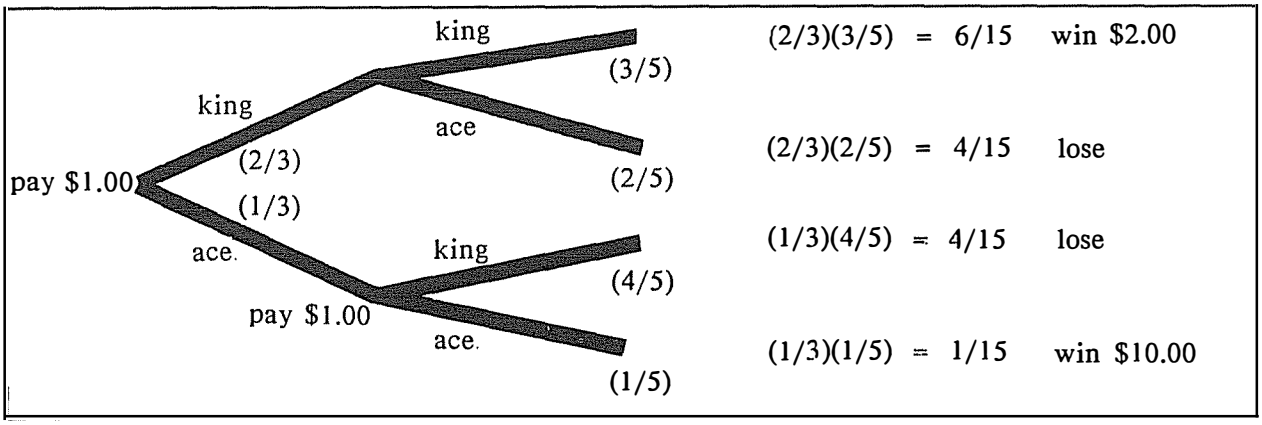


Figure 2 Analysis of new game

NCTM NEW PUBLICATIONS

Algebra for Everyone, published by the Mathematics Education Trust (MET), is available now to order. The book, edited by Edgar Edwards, Jr., is a set of essays written by top mathematics education professionals. Each essay pertains to a specific aspect of teaching the fundamentals of algebra to the entire population. *Algebra for Everyone* is written for supervisors and teachers who influence the teaching of algebra and who can and will address the problems facing underachieving populations. You can order this publication from NCTM.
1990, 89pp., #030N4, $7.00.

Mathematical Challenges for the Middle Grades, compiled and edited by William D. Jamski. This collection of puzzles and problems from 15 years of the *Arithmetic Teacher* is grouped by categories (Numbers, Special Counting, Computation, Oldies but Goodies, Offbeat and Unusual, Geometry, and Probability) and is certain to challenge and amuse students and teachers in Grades 5-8.
1990, 48pp., #425N4, $6.25.

The Story of Pi, created and written by Tom Apostol. This product is a computer-animated videotape produced at the California Institute of Technology that uses motion, colour, and sound to enhance textbook instruction. Pi is defined, computed, and examined in terms of history and its uses in various fields. The tape comes with a program guide/workbook that includes exercises, applications and projects.
1989, 24 min (VHS only),
#470N4 (videotape and guide), $25.00;
#471N4 (program guide only), $4.00
10 or more guides, $2.00 each.

How to Use Cooperative Learning in the Mathematics Class, by Alice F. Artzt and Claire M. Newman. The authors discuss the reasons for using cooperative learning in the mathematics classroom, the benefits for all students, and ways teachers can begin a cooperative learning program and make it work. Graded sample activities (elementary through high school) and an extensive bibliography are included.
1990, 63pp., #424N4, $6.95.

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USING MANIPULATIVES AND DIAGRAMS TO HELP STUDENTS RELATE TO RELATED RATES IN CALCULUS

by
Dr. Robert Gerver
North Shore High School

Often our calculus students are the best students our school has to offer. They are responsible, conscientious and concerned for their academic futures. They are extremely successful in school mathematics - but how much do they enjoy it? Can they really *do* mathematics?

Because these students are intrinsically motivated, textbooks and teachers often deprive them of logical, interesting presentations with manipulatives that provoke thought. New topics are presented via lectures and students jump right into solving problems. Can a student gain any mathematics appreciation after a 180-day diet of lectures and homework assignments?

North Shore High School's honours mathematics classes begin calculus in the 11th grade. Students take the Advanced Placement Calculus Exam at the end of the 12th grade. This two-year program allows teachers to develop topics with understanding. There doesn't seem to be much value to cramming material at the expense of understanding. Ask yourself:

- How many of your students really understand ϵ and δ limit proofs?
- How many of your students can handle relative extrema problems that are not mimicks of previous problems?
- How many of your students can apply calculus to economics, biology and engineering in addition to the traditional physics?
- Did you ever spend time allowing students to verify areas found by integration by counting the squares under the curve on graph paper?
- How many of your students can intelligently discuss the interrelationships of $y = e^x$, $y = lnx$ and $y = \frac{1}{x}$?
- When a student solves a related rates problem, can he/she interpret the answer in a complete English sentence?

This article will present a complete introductory, one-day lesson that is devoid of assignments. It is full of visuals and thinking questions. This type of lesson should be created to introduce every topic so students gain an appreciation and understanding of calculus. The topic presented here will be an introduction to related rates problems.

WHAT ARE RELATED RATES?

First, let's examine the terms. By their nature, a problem using a derivative is using a rate. By this point in the semester, the students have dealt with one rate of change per problem. So the plural, rates, connotes a significant departure from previous work.

To understand *related* rates, it's smart to introduce *unrelated* rates. For instance, the rate at which you drive home from school and the rate at which your students read are unrelated rates. One can change without affecting the other; they are independent. This two-minute explanation primes the student for related rates.

FORMULAS

The previous night's homework should have students review these formulas, which will be used in related rates problems:

- area of rectangle
- area of circle
- volume of right circular cylinder
- volume of cone
- volume of rectangular parallelopiped
- surface area of rectangular parallelopiped
- volume of sphere
- surface area of sphere
- pythagorean theorem
- similar triangle formulas

We don't want to get hung-up on "forgotten formulas" during our introductory lesson.

USING MANIPULATIVES AND DIAGRAMS

EXAMPLE 1 We are now ready to discuss related rates. Pose a problem about a 10m ladder slipping down a wall. Draw a diagram, as shown in Figure 1.

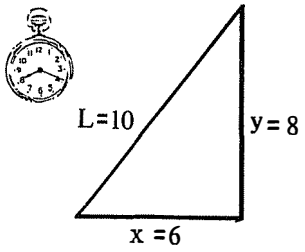


Figure 1. Ladder leaning against wall

Include a clock in any diagram that involves changes over time. Imagine the ladder slipping one meter along the ground, as in Figure 2. How many of your students think that it also slipped one meter down the wall?!

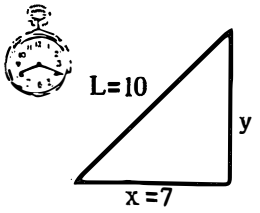


Figure 2. Do your students think y = 7?

It is clear that 7m is not correct, from the Pythagorean Theorem, but it is surprising how many students don't realize how the lengths and rates interact. To visually display this, make the following manipulative. You will need a ruler, cardboard and a marker. Follow steps **a** through **g**:

a) Cut the cardboard into a 40cm × 50cm rectangle. The bottom is the 40cm side.
b) Cut off a 8cm × 8cm square from the lower right corner (see Figure 3).

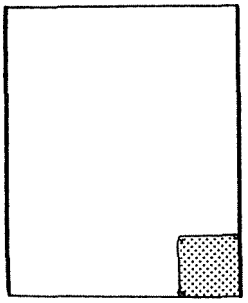


Figure 3.

c) Represent the ground and the wall by drawing in lines as shown in Figure 4.

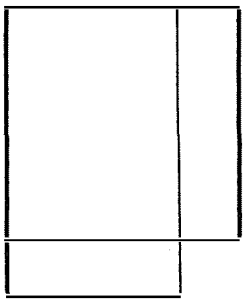


Figure 4.

d) Mark off 3cm increments on the ground. Label these 1-10, as shown in Figure 5.

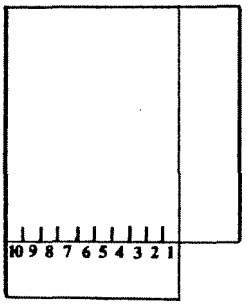


Figure 5.

e) Draw a clock in the corner. Label the wall and ground y and x, respectively (see Figure 6).

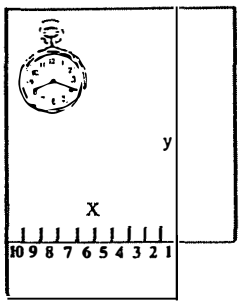


Figure 6.

f) Score the x and y sides, then fold forward as shown in Figure 7.

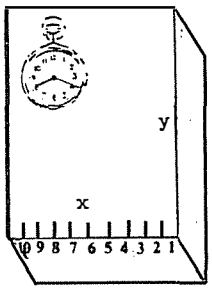


Figure 7.

g) Use marker and the back of a wooden ruler to make your "ladder". The finished Slip-o-Meter is shown below:

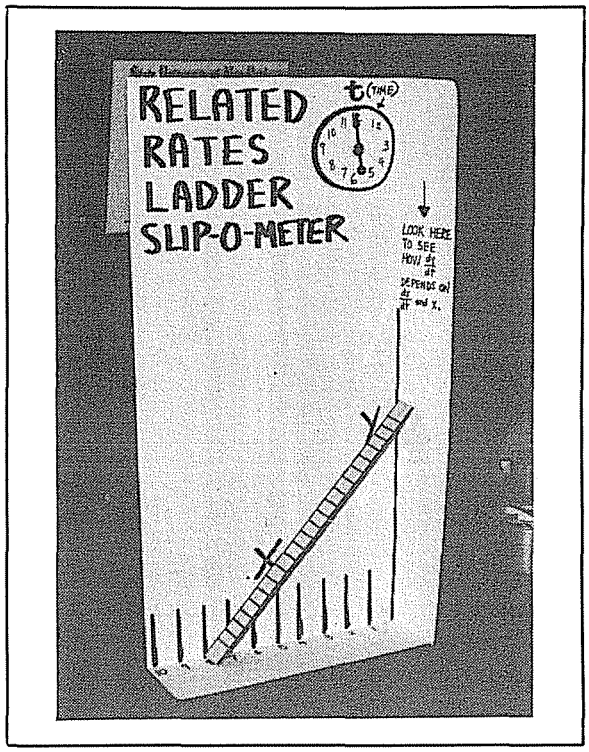
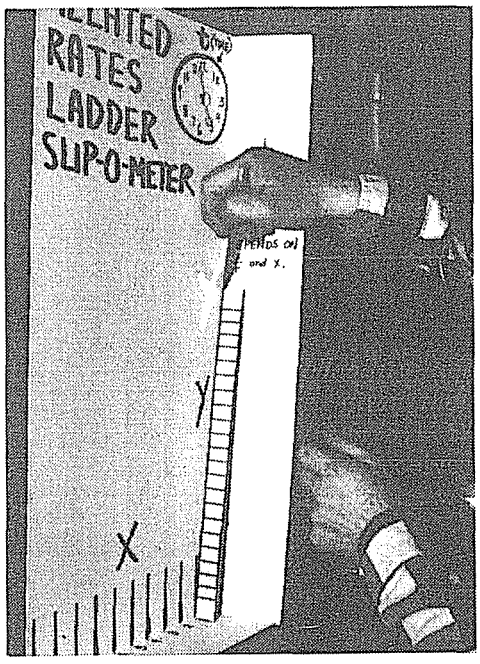
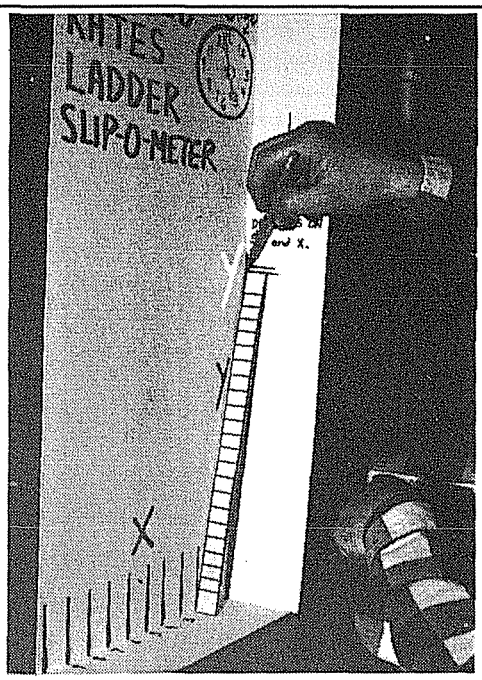


Figure 8.

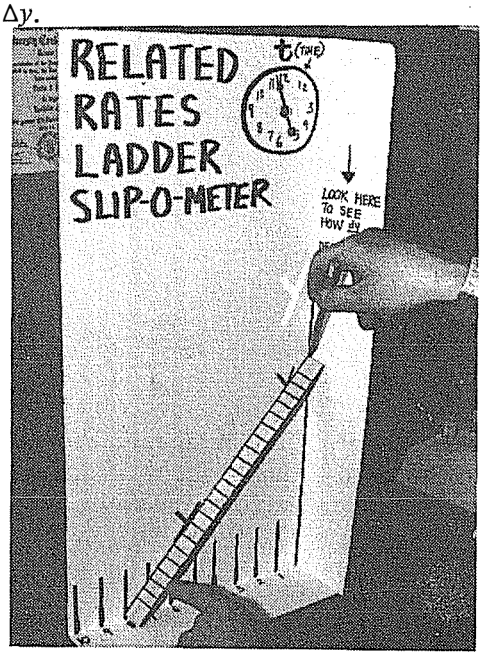


1. Place the ladder's foot at the line labelled 1. Lean the ladder against the wall and mark where the ladder top hits the wall.

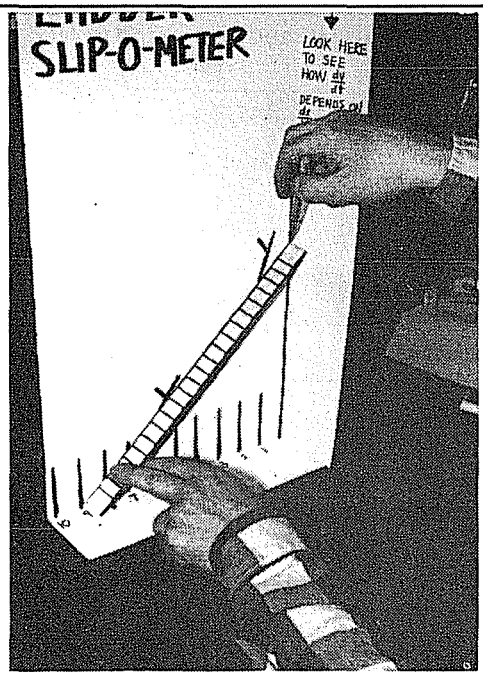
Figure 9. *Continued on p. 26*



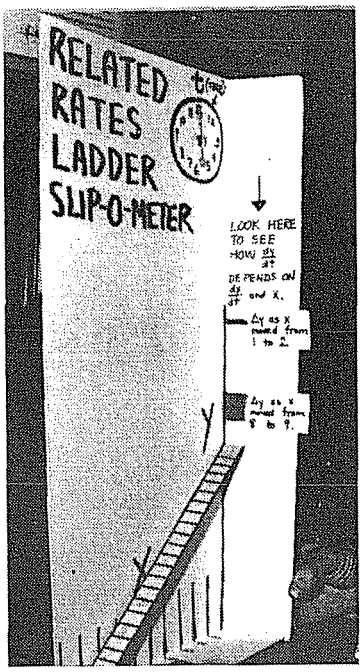
2. Move the ladder's foot to the line labelled 2. Lean the ladder against the wall and mark where the ladder top his the wall. Show students the distance represented by Δy.



3. Now, place the ladder's foot at the line labelled 8. Lean the ladder against the wall and mark where the ladder top hits the wall.



4. Move the ladder's foot to the line labelled 9. Lean the ladder against the wall and mark where the ladder top hits the wall. Show the students the distance represented by Δy.



5. Point out the fact that Δy depends on x and Δx. Also, elicit the fact that dy/dt depends on x, Δx and dx/dt.

Figure 9. *Continued from p. 25*

Figure 9 shows a picture sequence that can be used to show the interdependence of the rates. Move the foot of your ladder (ruler) in 3cm increments, marking off the changes in y. Students clearly see that dy/dt depends on x and dx/dt. They also can understand the difference between dy/dt and dy/dx. Too often our students can find derivatives, but they really can't interpret them.

EXAMPLE 2. Most students think the rear end of a car consists of two wheels bolted to one axle. They don't realize that if the car is making a turn, the inner wheel spins at a different rate than the outer wheel. Hence, there cannot be one axle - there are two axles connected to a *differential* which allows the coexistence of related rates (see Figure 10).

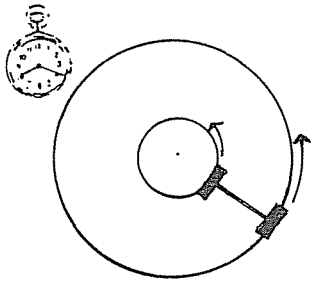


Figure 10. Diagram of car's turning track (aerial view).

A Tinker-Toy rod and two wheels tightly pressed on can be passed around the class. Students playing with it will realize that the axle would break if it didn't have a differential (see Figure 11).

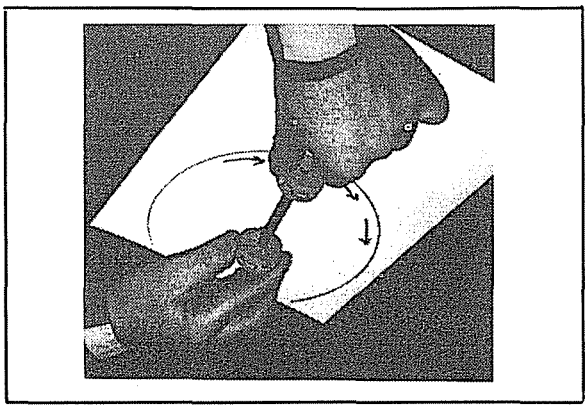


Figure 11. Using a Tinker-Toy.

EXAMPLE 3. Typical related rates problems include balloons and kites flying away from people. A geoboard can be used to show students what variables are changing dependent upon the problem's specific scenario.

EXAMPLE 4. A funnel can be used to show students that adding water to a conical reservoir will change the depth and radius of the water.

EXAMPLE 5. A cylinder can be filled with water to contrast the changing radius in the conical reservoir problem. Students can also see that the rate of fill (dV/dt) affects the rate at which the water level rises (dh/dt).

EXAMPLE 6. Blow-up a balloon, discussing the rates of change in volume and surface area.

EXAMPLE 7. Related rates problems sometimes feature a person walking away from a lightpole, monitoring his speed and shadow length. Turn out the classroom lights and use a flashlight to show students how the shadow's length and the shadow tip's speed are related to the person's height and walking speed.

The physical examples are light, interesting and very thought-provoking. The lesson can end with a series of thinking questions about each example:

1. In a cylinder being filled with water, if dV/dt is constant, must dh/dt be constant? Must dr/dt be constant? Must dr/dt be positive? (See Figure 12.)

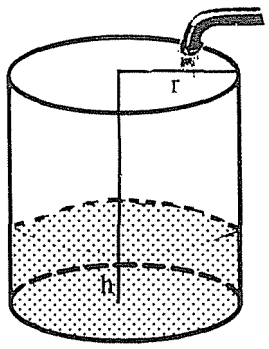


Figure 12.

2. In a cone being filled with water, if V changes must h change? Must r change? Must dV/dt change? Must dr/dt change? (See Figure 13.)

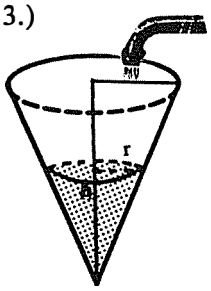


Figure 13.

3. In a rectangular trough being filled with water, if $V = L \times W \times H$, if V changes, must L change? Can L change? (See Figure 14.)

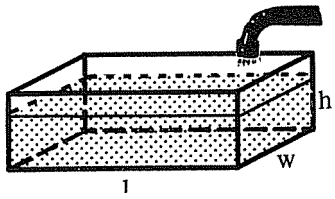


Figure 14.

4. A ladder L slips down a certain wall. Does dL/dt depend on dx/dt? What must dL/dt equal? (See Figure 15.)

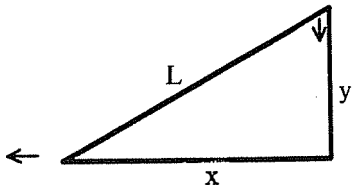


Figure 15.

5. A leaky conical reservoir is being filled at a constant rate, dV/dt. The water leaks out at a variable rate, dL/dt. The change in volume is represented by $dV/dt - dL/dt$. If $dV/dt > dL/dt$, then is dh/dt positive or negative? (See Figure 16.)

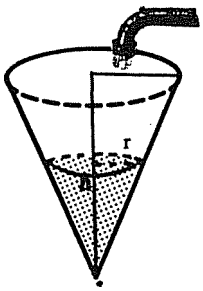


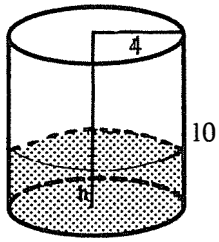
Figure 16.

As you can see, questions of this type will reveal whether or not the students have optimized their chances of understanding the math problems that will follow.

Students must be able to interpret the answers to these problems. Although students obviously see that the statement "I weigh 111 gallons" is ridiculous, they lose sight of the units in related rates problems. Carrying the units throughout the problem could aid understanding. Notice how the following example's answer neatly has the correct units inherent in its answer.

"A cylindrical tank has height 10m and radius 4m. It is being filled with water at a constant rate of $2m^3$ per second. How fast is the water level rising when the water is 6m deep?

SOLUTION:



$$V = \pi r^2 h$$
$$V = \pi(4^2)h$$
$$(m^3) = \pi(m^2)(m) \quad \leftarrow \text{The units check}$$
$$\frac{dv}{dt} = 16\pi\frac{dh}{dt}$$
$$\left(\frac{m^3}{\text{sec}}\right) = (m^2)\pi\left(\frac{m}{\text{sec}}\right) \quad \leftarrow \text{The units check}$$
$$2 = 16\pi\frac{dh}{dt}$$
$$\frac{2}{16\pi} = \frac{dh}{dt}$$
$$\frac{m\,\not{m}^3}{(\not{m}^2)(\text{sec})} = \frac{m}{(\text{sec})} \quad \leftarrow \text{The units check}$$

This inspiring treatment allows the students to understand, appreciate and actually like math. It provides a nice break in a pressure-packed curriculum. If every topic was given a similar introductory treatment, we'd be graduating *more* "math-philes" and *fewer* robots. *

MATH, MUSIC AND COMPUTERS: LINKING BACH PRELUDES AND ALGEBRAIC FUNCTIONS

by
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Faculty of Education
Queen's University



Examples abound from centuries past of mathematicians who were also amateur musicians, and musicians who dabbled in mathematics. Einstein was not only a mathematician, but also a violinist. Mozart took segments of music motifs, classified them in musical ways, and then used random numbers to create pieces, apparently to the delight and amusement of onlookers. Schoenberg, a 20th century composer, was "compel[led] to compose mathematically, . . . bas[ing] his writing on a twelve-tone system instead of the conventional scale." (Kaufmann, 1960, p. 174).

That one person might be both a mathematician and a musician speaks of the possibility of core similarities within the domains themselves. The deep and undeniable relationships amongst the mathematics of Godel, the art of Escher and the music of Bach, were explored at length by Hofstadter (1979). He writes:

Mathematicians were among the first admirers of Escher's drawings, and this is understandable because they often are based on mathematical principles of symmetry or pattern. . .the Strange Loop is one of the most recurrent themes in Escher's work. [C]ompare its six-step endlessly falling loop with the six-step endlessly rising loop of [Bach's] Canon per Tonos. The similarity of vision is remarkable - Bach and Escher are playing one single theme in two different "keys": music and art (Hofstadter, 1979, p. 11-13).

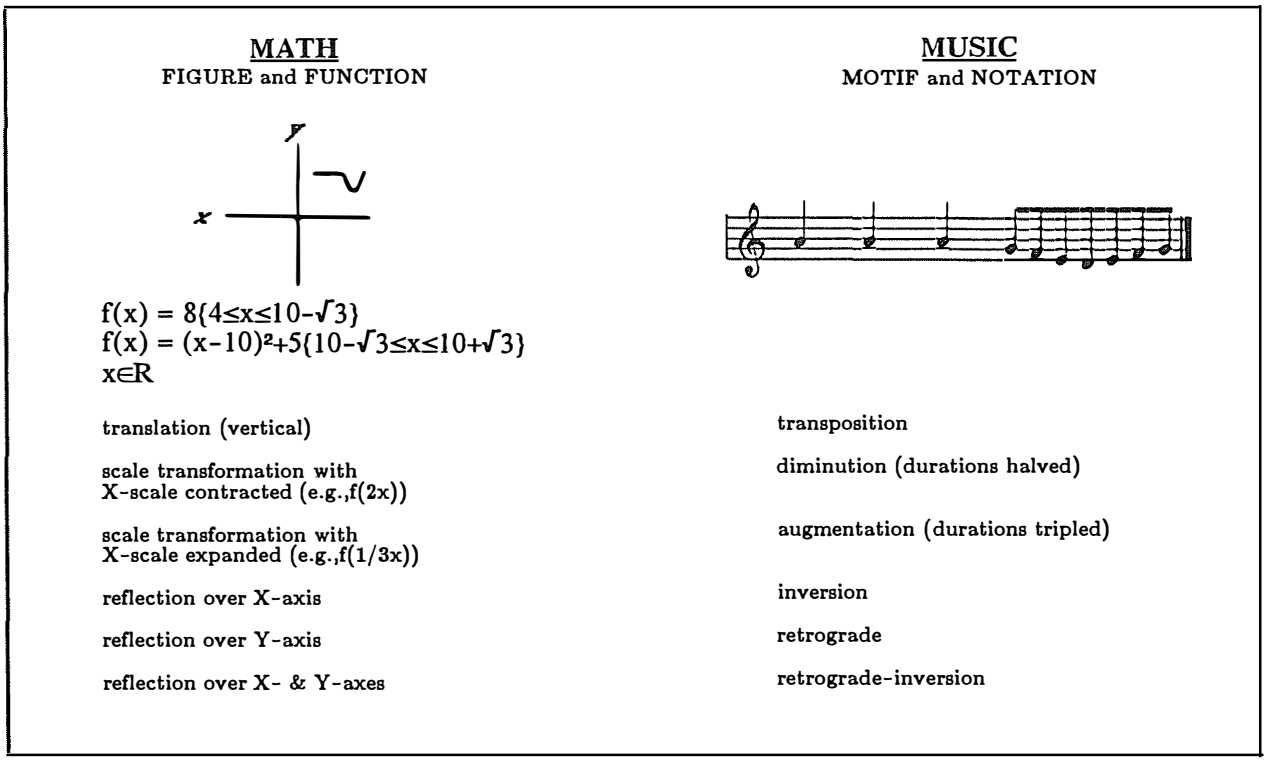


Figure 1. Corresponding Math and Music Transformations

And further:

Godel's discovery involves the translation of an ancient paradox in philosophy into mathematical terms. . . [a] paradox [that] is a one-step Strange Loop, like Escher's Print Gallery. . . (Hofstadter, 1979, p.17).

And finally:

Godel and Escher and Bach were only shadows cast in different directions by some central solid essence." (Hofstadter, 1979, p.28).

Despite these special relationships between mathematics and music, and other domains as well, a different kind of relationship often exists between students and their feelings towards high school mathematics. Many of us have heard students ask their secondary mathematics teachers questions of the following nature: "Why are we learning this stuff? What is the practical value of what we're learning here?" Teachers might answer with a statement like this: "What you are learning now will be important when you study other areas of math later, such as differential calculus, and apply that to practical problems in a professional field." (This answer rarely satisfies students -- after all, if they are looking for relevance **now**, they are hardly going to be pleased with a response which tells them to wait even longer.) Alternatively, teachers may say something to the effect: "It'sworth learning for its own sake -- these are really exciting concepts."

(Students tend to look rather dubious at this show of love for mathematics, believing their teacher perhaps, but wondering all the while where the excitement lies.) While the example developed in this paper is by no means a solution to the perennial problem of student motivation, it provides an interesting way of showing that mathematics is, indeed, linked to other disciplines in an intriguing and beautiful way.

When I was a teenager, I had the pleasure of learning to play the piano with a remarkable teacher. Unlike other music teachers with whom I had studied in the past, this teacher heard and saw and felt beauty in the most mundane musical expressions -- including the dreaded scales and chord exercises that we were expected to practice. I have come to believe that her love for scales and the like was due in part to her ability to see relationships in music in a highly patterned, indeed, mathematical way. Further, she could make those relationships apparent to even her most stubborn students (of which I was one). One way of doing this was to ask students to colour their Bach Preludes and Fugues, showing the various manipulations of the theme or motif in different colours. Thus, the original motif might be coloured red, the transformed motif played a fifth higher in green, the upside-down or inverted version in yellow, and so on. By the time the entire piece was coloured, the relationships amongst the manipulations of the motif were abundantly clear. At the time, I remember thinking this was pleasantly

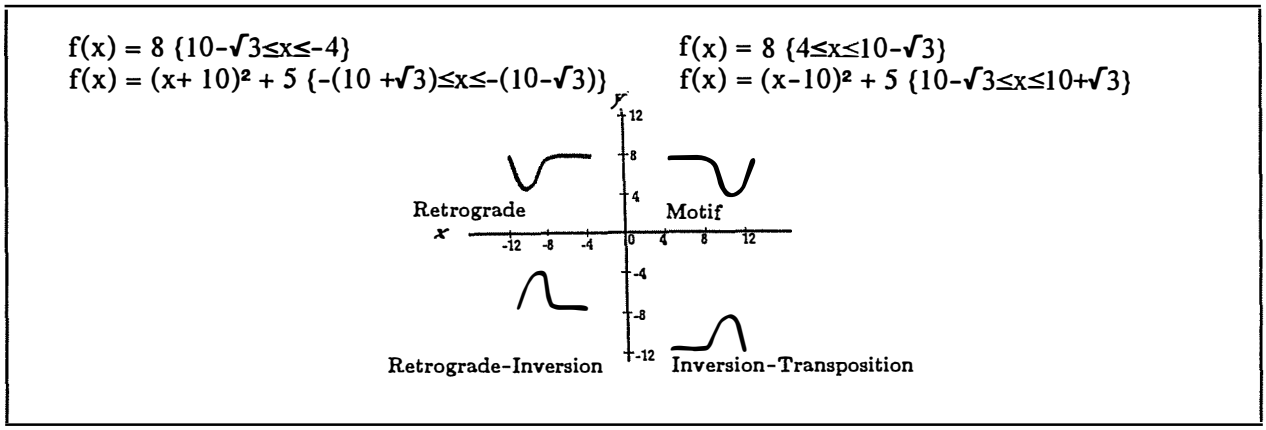


Figure 2. Math and Corresponding Music Transformations on the Cartesian Coordinate System
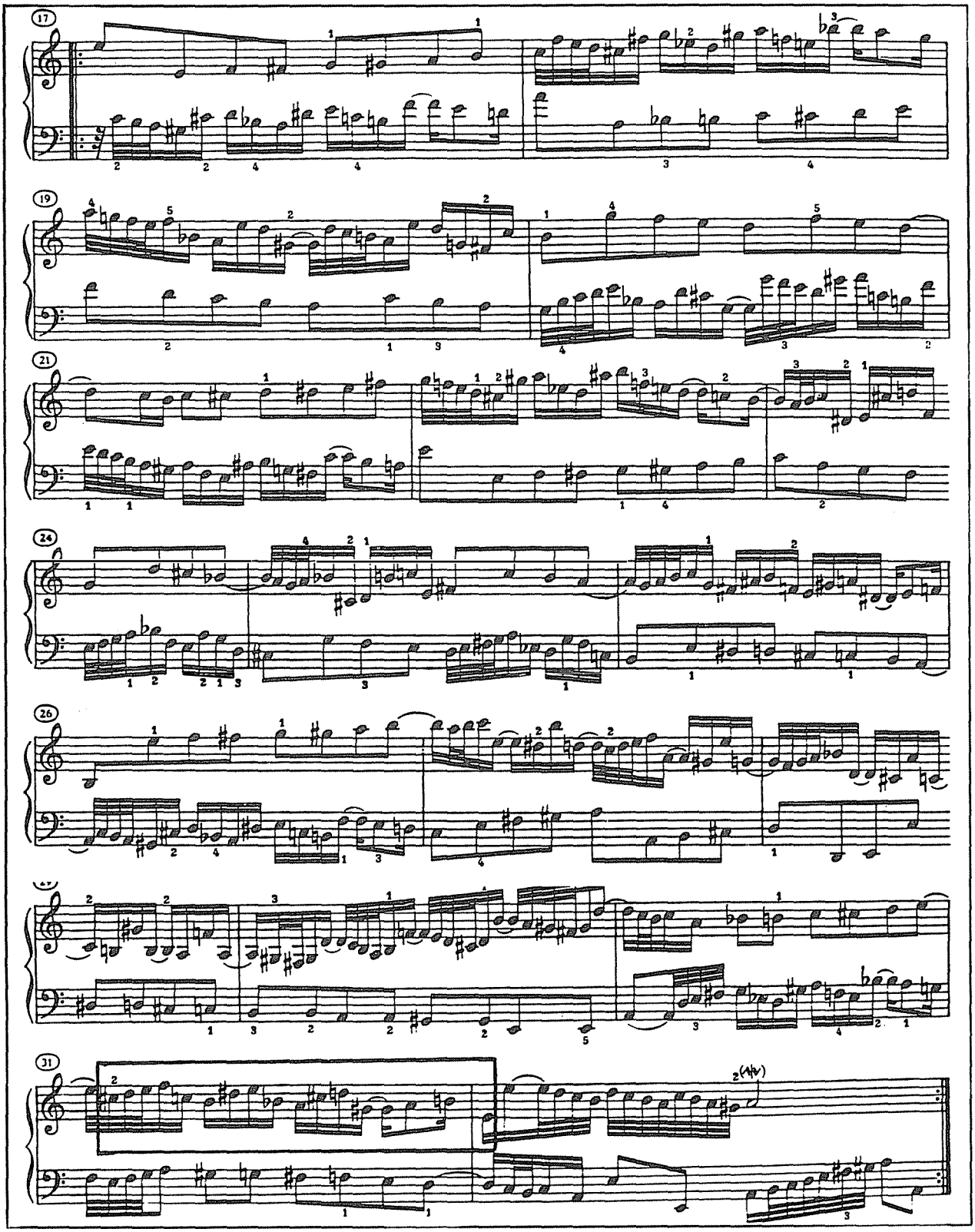


Figure 3. Excerpt from Prelude XX, Book II, *The Well-Tempered Clavier*, J.S. Bach

unusual, and that it vaguely reminded me of the transformations we were learning about in algebra. But beyond that, the experience was shelved.

Some years later, when I found myself teaching both music and mathematics, I remembered colouring the Bach Preludes and Fugues. Could this experience be brought into the classroom in some way? I was sure that it could. In the remainder of the present paper, I describe how this might be done, both by mathematics teachers with music background, and by those without.

CORRESPONDING TRANSFORMATIONS

To begin with, there are several transformations in mathematics and music which can be seen as corresponding to one another. If a figure is placed on the Cartesian Coordinate System, and then moved up or down along the Y-axis a fixed amount (say, 3 units up), we say that there has been a vertical translation, and the new figure can then be described in terms of a new function (see Figure 1). This is the same idea as taking a musical motif and moving it up or down a fixed interval (say, up a fifth), resulting in the transposition of the original motif, which can be described in terms of a new notations (see Figure 1). Similarly, scale transformations and reflections have musical equivalents. A scale transformation where the X-scale is contracted (e.g., f(2x)) is similar to diminution (shortening all of the durations of each musical event by a fixed factor (X) while keeping the pitches the same (Y)). A scale transformation where the X-scale is expanded (e.g., f(1/3x)) is akin to augmentation (changing all of the durations so that each event is longer, but the relative durations remain the same). Reflection over the X-axis is the same as inversion (playing the motif upside-down), and reflection over the Y-axis is the same as retrograde (playing the motif backwards). The combined reflection over the X- and Y-axes is the same a retrograde-inversion. Of course, other combinations are also possible -- translating a figure and reflecting it over the X-axis would find its musical equivalent in a transposed and inverted motif.

One way of demonstrating these relationships to students is to take a small excerpt of music such as the one given in Figure 1, copy it onto a small piece of transparency film, and show the transformations using the overhead projector. The relationships shown in Figure 2 will then emerge.

AN EXTENDED EXAMPLE: *THE WELL-TEMPERED CLAVIER*, PRELUDE XX, BOOK II, J.S. BACH

After the corresponding relationships between mathematical figures and musical motifs are described as above, a complex musical example can be shown and played. I have reproduced a portion of one of Bach's Preludes for this purpose. (Perhaps a student in the class can play the piece. If not, there are numerous recordings of this Prelude available.) More simple examples can be found in the Two and Three Part Inventions, and in fact, from the work of many other composers as well.

Figure 3 shows the second page of Prelude XX. The original motif appears in Measure 31, on the last line. It way typical of Bach to play the motif only once at the beginning, transform it in many ways, and then repeat the original motif near the very end of the piece, bringing closure to the work. In Figure 4, all of the blank spaces represent the places where the original motif has somehow been transformed. These transformations include inversion, transposition, inversion-transpositions, and partial repetitions of the motif, as indicated. It is usually a surprise to students to see how much of the piece can be "explained" by these transformations. In fact, if one were to take the secondary figure and perform the same analysis, removing the portions which were transformations of the secondary figure, there would be hardly any notes left from the excerpt.

A STUDENT EXERCISE

In my experience, by this point, many students are at least intrigued by a less than conventional math class, and are eager to try their hand at composition through transformation. I provide each student with a small piece of transparency film which has a staff marked on it, and overhead pens and instruments for students to share (synthesizers, pianos, and xylophones work particularly well, but the instruments used will depend on availability and student/teacher expertise). Students are given these instructions:
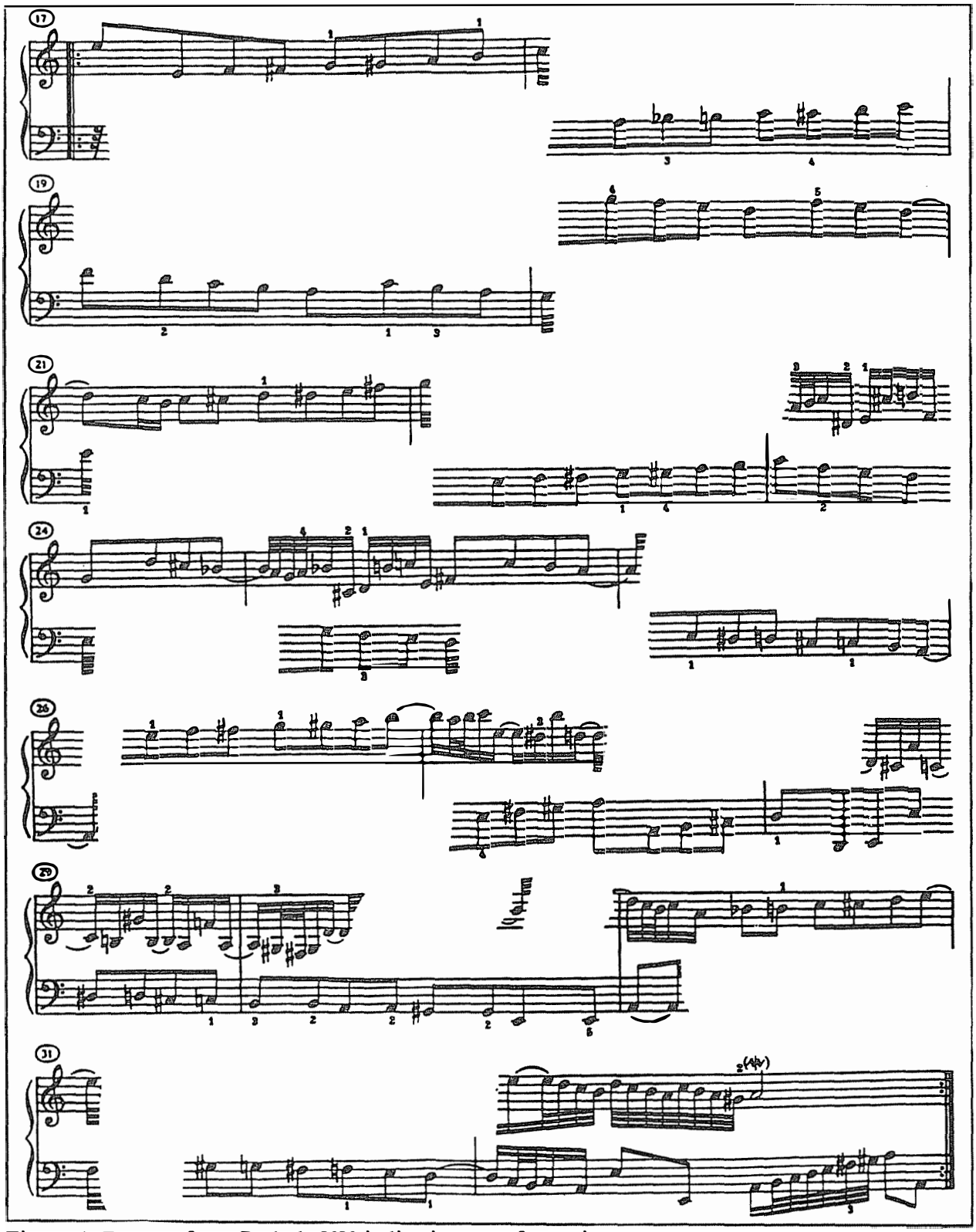


Figure 4. Excerpt from Prelude XX indicating transformations

This except is the same as the one appearing in Figure 3. However, now there are blank spaces where the motif is either transformed or repeated in its original form.

Play around with the instrument until you find a 5 or 6 note motif that you like, making sure that the motif has at least two different durations (i.e. slow and fast notes). Then write it, or have someone help you write it on the transparency. Once this is done, experiment with the transformations by rotating the transparency, identifying the transformations you find most pleasing.

An example of one student's work, based on the above directions, is given in Figure 5.

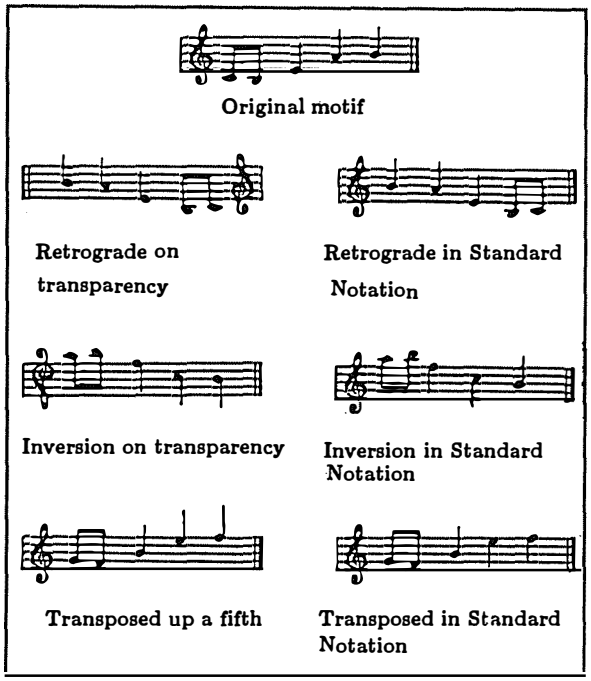


Figure 5. Transforming a Motif

(from Upitis, R. (1990). *This Too is Music.* Portsmouth: N.H.: Heinemann Education Books)

Once the students have identified their favourite transformations, the next step is to combine the transformations into a pleasing pattern to give their composition an overall form and coherence. In one case, when a student had found the transformations she liked but was having trouble combining them (transposed an octave, retrograde, and transposed-inversion), I suggested she follow the pattern of her sweater. The sweater was striped, with pink as the predominant colour. She used the original motif for pink, and the other colours for the other transformations. The resulting piece is shown in Figure 6. The composition is remarkably lovely -- and the student, Veronica, had no formal music training.

COMPUTER PROGRAMS

There are also ways of using computer music programs to tease out and augment students' understanding of these relationships. Some students may prefer working with a composition program over using acoustic instruments, finding the manipulations easier to make with the computer input device rather than having to master an instrument. Others may use the two types of media in tandem.

Any number of computer composition programs could be used in conjunction with the application I have described thus far. I use Musicland as an example, an Ontario Ministry of Education program available on the ICON.

One component of the Musicland program allows the user to "draw" a motif through gesture, using the trackball on the ICON. This gesture is then represented as a graphic music notation, similar to standard notation in terms of pitch, but using rectangles to show duration. The sound durations are directly proportional to the length of the rectangle, that is, a rectangle of four units length would play in real time twice as long as a rectangle of two units length (see Figure 7).

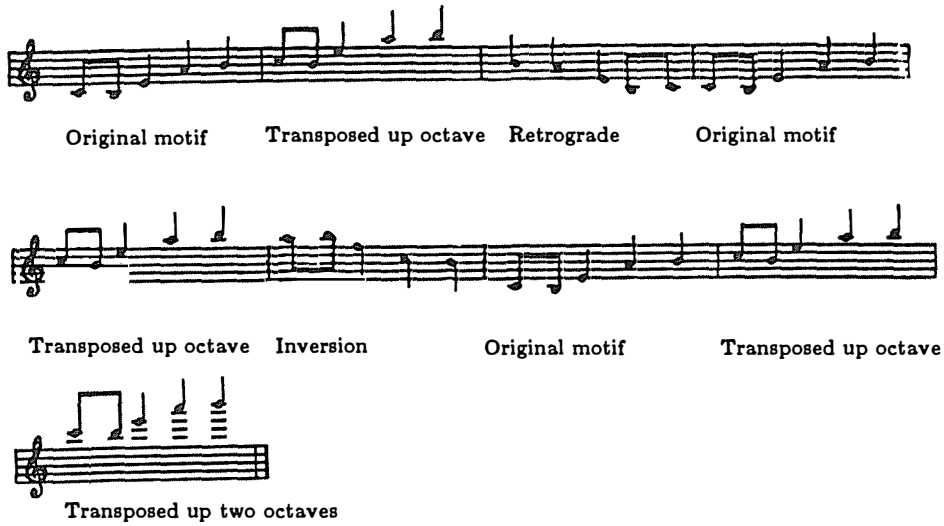
Once the motif has been drawn and represented by the music notation, it can then be transformed in all of the ways previously outlined -- transposition, diminution, augmentation, inversion, retrograde, and combinations of all of the transformations. Each of the transformations is represented by a different colour -- reminiscent of the days when I once coloured the music of Bach. The use of colour in the Musicland program, not surprisingly, is for the same effect as that intended by my piano teacher. Through colour, the various transformations can be viewed separately as well as in relation to one another.

The most obvious advantage to using this program over the transparency film method is that one can hear each transformation without having to play the results on an acoustic

instrument. Further, the composition can be assembled directly on the computer, and heard at any time as it is being created. Thus, for a piece like Veronica's Pink Sweater, Veronica, using the computer, might well have heard her piece as she went along, rather than writing the whole thing, all the while imagining the sounds, and then hearing it played later on an acoustic instrument by someone else. Finally, with a computer program, the work can be saved and manipulated again at a later date.

It is important, however, to realize that even without this kind of program, the same results can be achieved with the simple materials described earlier -- transparency film, pens, and an acoustic instrument. In a similar vein, when using computer music programs in this

The piece as it was first transcribed from the transformations to the motif on the transparency film:



The piece in standard music notation, with some minor modifications to the rhythm, and a simple accompaniment based on three notes of the motif:
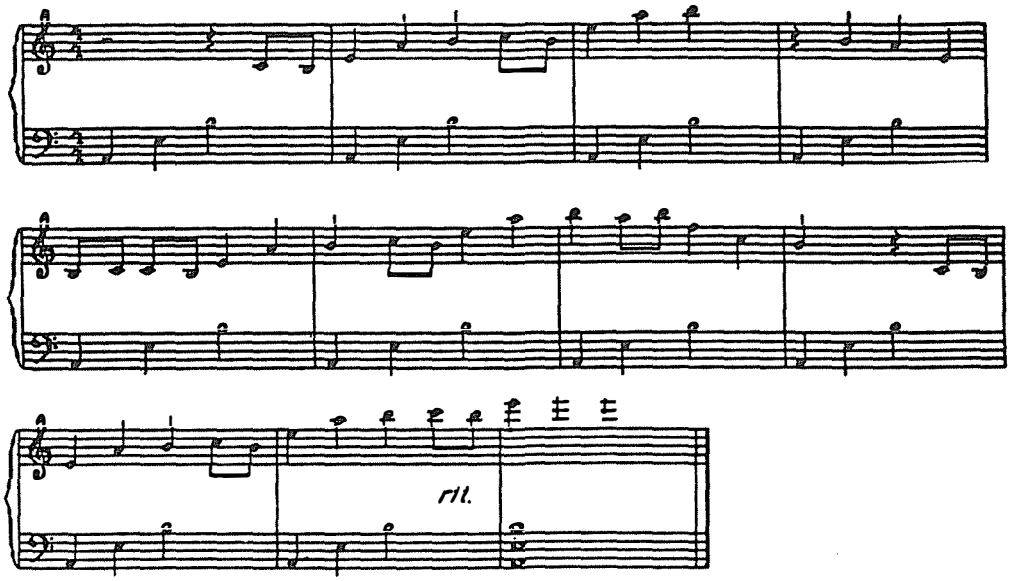


Figure 6. Veronica's Pink Sweater
(from Upitis, R. (1990). *This Too is Music.* Portsmouth: N.H.: Heinemann Educational Books, Inc.)

kind of an application, there are a number of programs which could be adapted for this purpose, even those that are much simpler than Musicland.

Recently, when I described the work of Veronica and showed the Musicland program to a secondary school mathematics teacher, he grew very excited at the prospect of trying this in his classroom. What is crucial though, is that he immediately seized on the possibility of using a very simple Radio Shack computer music program for another extension of the transformation theme. The computer music program he described took lists of numbers, corresponding to different pitches, and then played them through the built-in speaker. He suggested that students might enjoy taking a series of points from a function, such as the ones pictured in Figure 2, defining those points as a list, and then hearing them played. Further, he suggested that students could even perform the mathematical transformations on the list, predicting the musical results, and then, as did Veronica, combine various forms of the motif into a piece of music. He concluded our conversation with the important observation that it didn't really matter which computer program was used, as long as the student could somehow enter a musical or mathematical figure, and then make transformations of the original. Similarly, this idea could be adapted for students of different ages and abilities. I have used the Bach Prelude as a starting point for Grade 10 students through to Grade 13 (in Ontario, the O.A.C. level) students.

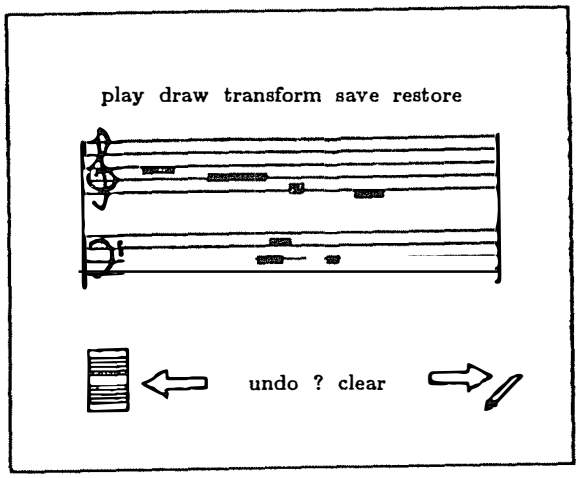


Figure 7. A motif represented by the graphic notation of *Musicland*

CONCLUDING REMARKS

Earlier I stated that an exercise such as this one could serve as a way of motivating students to look more closely at the mathematics they are required to learn. This is far less significant, however, than another possible outcome of having experimented with this kind of work. Asking students to compose music, whether through mathematics or other means, is a creative, generative undertaking. Students have little difficulty in identifying composition as a creative endeavour, and music as a medium for personal expression. However, fewer students see mathematics in the same light -- as something which can provide them with tools for expression, as a way of generating new ideas, and ultimately, as a domain for growth and learning. One hopes that by making these kinds of links between domains, students will come to see the nature of mathematics in a new way. Bach himself was captivated by the beauty of mathematical manipulations, and was reportedly quite aware of the mathematical flavour to his work -- its recursive nature, and more generally, the importance of repetition, symmetry, and overall pattern. How fitting, then that his compositions could be used to loop back and link music and mathematics once again.

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REPORT ON THE FIRST MEETING OF THE WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITION

by
Tom Griffiths
Lucas Secondary School
London, Ontario.

The first meeting of the World Federation of National Mathematics Competitions was held this year at Waterloo University from August 16-21. Approximately 50 delegates were in attendance representing 16 nations from all parts of the world.

The conference included sessions devoted to selecting from previously submitted problems and improving on them. The delegates worked on this in teams of five or six every morning. In the afternoon, 20-minute, prepared papers were presented. The highlights of these exceptionally high quality presentations were the papers presented by Dr. Anthony Gardiner of England, Nikolai Konstantinov of the USSR, given in Russian with simultaneous translation, and Professor Alexander Soifer from the USA. Needless to say most of the rest of the time was taken up by socializing and exchanging ideas.

The hospitality presented by the University of Waterloo was exceptional, and I am sure that all of the delegates left with a very positive impression of Canada, and a few extra pounds.

On the final day of the conference a meeting was held to discuss topics of mutual interest and concern. The following proposed revision of the goals of the federation was passed.

"The Federation provides a focal point for those interested in, and concerned with, conducting national mathematics competitions for the purpose of stimulating the learning of mathematics. Its objectives include:

a) serving as a resource for the exchange of information and ideas on mathematics competitions through publications and conferences;

b) assisting with the development and improvement of mathematics competitions;

c) increasing public awareness of the role of mathematics competitions in the education of all the students and assuring that the importance of that role is properly recognized in academic circles;

d) creating and enhancing professional links between mathematicians involved in competitions around the world."

As a delegate, one of the strongest impressions the conference left me with was pride in Canada. We have the largest percentage participation of students of any country. We also have the largest involvement of non-university teachers of any country. The only other country which had a considerable involvement of these teachers is the United States. Most of the university delegates from the other countries expressed the opinion that high school teachers could not handle the level of work required. I hope that the strong high school Canadian delegation modified this opinion. Furthermore the non-university Canadian teachers present not only exhibited a high level of ability in problem solving, but submitted questions which gained considerable acclaim.

In all, the conference was a great success and will be followed up by a second in Bulgaria in 1992. There will also be a meeting of the organization in Quebec City in 1992 at ICME (International Conference of Mathematics Education).



PROBLEMS

by
Ron Lancaster
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945 Bloor St. E.
Mississauga, ONT. L4Y 2M8

In order to give readers of this column a change of pace, I have decided to run a large collection of puzzles in this issue, rather than the usual set of problems. These puzzles are commonly called alphametics, a term coined by J.A.H. Hunter many years ago. I created all of these alphametics in the late 1970s and early 1980s. The first thirteen alphametics were published in the *Journal of Recreational Mathematics* and the next two appeared in the syndicated column "Fun with Figures". The last one was accepted years ago by the *Journal of Recreational Mathematics*, but it has not yet been published.

In all alphametics, each letter represents a different digit. Two letters cannot represent the same digit and no word can begin with the digit 0. The goal is to determine what digit is represented by each letter. While some trial and error is generally required, you should try to apply some logic in order to minimize the number of combinations that need to be considered.

Quite a few of my students enjoy solving alphametics and I would encourage you to give your students a copy of these. If they create one of their own, send it to the *Journal of Recreational Mathematics* for possible publication.

1. Immodesty

This alphametic does not imply that other journals are not fun, but that **JRM** offers the most **FUN**.

```
  J R M
  T H E
  F U N
-------
  O N E
```

2. Final reward

Since we all want to minimize our time in **HELL**, it makes sense that the number of summands in the following alphametic is to be as small as possible.

```
    H E L L
    H E L L
    H E L L
      . .
      . .
      . .
    H E L L
    H E L L
    H E L L
-----------
H E A V E N
```

3. Non-smoker's lament

Since lung cancer is primarily caused by smoking, it only makes sense that in the following alphametic **SMOKE** is prime.

```
      O K
      N O
  J O K E
      D O
    N O T
---------
S M O K E
```

4. Non-smoker's lament revisited

As was the case in #3, **SMOKE** is also prime in the following alphametic.

```
  S M O K E
  M A K E S
        M E
-----------
C H O K E
```

5. Drag racer's lament

The digit 5 may not be used again.

```
  H A T E
      T O
      D O
      5 5
  N E E D
      T O
-------------
  S P E E D
```

6. Speciality of the house

```
        A
    T R E A T
  C A R T E R
  P E A N U T
-------------
  B U T T E R
```

7. Count on it

```
    E A S Y
      A S
    O N E
    T W O
-------------
  T H R E E
```

8. New morality

```
    N U D E
      N O T
    R U D E
      N O R
-------------
  C R U D E
```

In fact, it's the least **CRUDE**.

9. Mastermind

In a recent game of Mastermind between two cunning opponents, the code-breaker broke the code in five logical moves, as shown below. interestingly enough, the code pegs form an alphametic which has a unique solution. Can you find it?

C O D E	P E G S
	W W G R
	W W O O
	Y Y B B
	W O B B
O W B B	

K E Y	P E G S
	B
	W B
	B B
B B	W W
B B	B B

10. An invitation!

A prime **BALL** here!

```
  C O M E
    O N E
  C O M E
    A L L
  H A V E
          A
-------------
  B A L L
```

11. Pre-election

```
      M E E T
            A
  P E A N U T
  F A R M E R
          M R
-------------
  C A R T E R
```

12. Inflationary alphametic

Keeping in tune with the times, make the value of **DOLLARS** as small as possible.

```
        $ $
        $ $
        $ $
        . .
        . .
        . .
        $ $
        $ $
        $ $
-------------
  D O L L A R S
```

NOMINATIONS FOR OAME/AOEM COUNCIL

Each year elections are held for President-Elect, 3 Directors, and 7-10 Councillors for the Council of OAME/AOEM. Nominations for these positions are to be submitted to:

Mary-Lou Kestell

Please include the nominee(s)' qualifications with the Nomination Form below to be received by M.L. Kestell no later than **4:00 p.m. January 12, 1991.** The nomination committee will make a final selection of those nominees who will stand for election. Their names and qualifications will be published in March, 1991. Voting will be by mail by members of OAME/AOEM. The results of these elections will be made known at the Annual Meeting of OAME/AOEM in Toronto in May 1991.

>>

NOMINATION FORM

I Nominate ____

Address ____

____

for the position(s) of: ___________President-Elect

___________Director

___________Councillor

____Nominator

____Address

KENNETH D. FRYER AWARD

In Memory of the late Dr. K. D. Fryer, OAME, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award for Excellence, to be presented annually to a Mathematics Department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME is requesting nominations for consideration by the committee for this year's award. It will be presented at the annual conference in Toronto.

Deadline for nomination submission: March 1, 1991

KENNETH D. FRYER AWARD
NOMINATION FORM
1990-1991

I/WE nominate School: ______________________________
Address: ______________________________
Principal: ______________________________
Mathematics Dept. Head: ______________________________
Nominated By: ______________________________

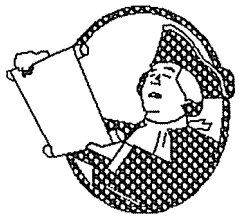
Please attach a separate sheet to the completed nomination form (or copy) outlining the reasons for your nomination, and forward them to:

Don and Carol Attridge
Secretary/Treasurers O.A.M.E.



COUNCIL COMMENT

by
Bob Robinson



The following are based on the OAME Council Meeting held on September 22, 1990. Your comments are welcome and should be sent to the editor.

In Memory

Members of council stood in silence in memory of Morley MacGregor (past secretary-treasurer) who passed away in June. A letter was received from Mona MacGregor thanking OAME for flowers and the many letters of condolence.

Committee Structure

The following standing committees have been established: Futures, Formative Years, Transition Years, Specialization Years, Women and Mathematics, Publications, and Membership.

Leadership Seminar

This important annual event is to be held November 14-16, 1990 at the Ramada Inn, Airport West. Two nights accommodation as well as 5 meals and 2 receptions are included in the reasonable fee of $265. There have been some changes in the program this year and over 15 publishers/suppliers will be there.

NCTM Regional Meeting - Hamilton

This was a highly successful annual meeting of the NCTM/OAME regional meeting. Over 2 200 people attended to make this the largest NCTM regional meeting held in Canada. A full report will be presented to Council at its January, 1991 meeting.

CHAMP 1991

Preparations are well under way for our annual meeting in Etobicoke during the second week in May, 1991. There are plenty of exciting meetings planned. There will be many extra curricular activities as well. Presiders for sessions are still required.

International Conference on Mathematics Contests

Tom Griffiths reports that a most successful conference took place at the University of Waterloo this past summer.

International Mathematics Olympiad

This international event will be at York University in 1995. It is hoped that OAME will become actively involved.

Leading Mathematics Into the 21st Century

Mary Lou Kestell, Alex Norrie and Bob Robinson attended a "Leading Mathematics Into The 21st Century" workshop recently and received some excellent material to assist in the presentation of workshops pertaining to the reform of mathematics education. These individuals are prepared to educate others as a part of this project. For further information contact Mary Lou Kestell or Alex Norrie.

CHAPTER REPORTS

CHAMP

CHAMP has a new logo and is preparing a new constitution. Their mathematics contest for Grades 8-11 was written on October 24, 1990. Mathfest '90 took place on October 25, 1990. A great deal of time and effort is directed toward making their annual meeting in May, 1991 a huge success.

COMA

A publishers' display and their general meeting was conducted last April. The annual QSLMA/COMA fall conference took place on October 3-4, 1990.

GOLDEN SECTION

Their annual seminar was presented at Brock University last April and planning is taking place for the 1991 seminar.

GVMA

This chapter is writing a short history of GVMA and is planning a fall conference for November 8, 1990. They are finishing the resource book Teaching Mathematics in the Transition Years.

NOMA

The fall conference was held on September 28-29, 1990. Student achievement is rewarded with a NOMA certificate of excellence and cash awards are presented to zone champions in the mathematics competition.

NWOAME

NWOAME '89 was a success and was co-sponsored by Lakehead University. Cash prizes were awarded on Mathematics Awards Day. NWOAME '90 was held

September 28-29, 1990. The chapter is also writing a brief history of its organization.

PINE RIDGE

The chapter is considering a new format for their Mathematics Awards Night. Members of Pine Ridge are presenting 10 workshops on their P.D. day, November 17.

QSLMA

The spring conference last April was a success, as was the joint COMA/QSLMA fall conference held in October.

SAME

The annual fall conference was held on October 15 and plans are proceeding for Springfest in April.

SWOAME

The fall conference is planned for November 9-10. A review of the chapter's constitution is in process.

WOMA

An inaugural dinner took place on November 13. Their spring conference will be on April 4, 1991. The chapter membership is a nominal $10.00.

Y^4MA

The fall conference will take place on November 13. The chapter is planning a membership drive, the production of a newspaper and special speaker events.

COMMITTEE REPORTS

Women and Mathematics

Over 110 teachers attended the excellent breakfast at the Hamilton NCTM/OAME meeting. Sylvia Gold's speech entitled "From Alchemy to Chaos" is published as the Viewpoint, page 5, of this issue of the Gazette. The committee is planning to compile a portfolio of resources related to women and mathematics. Capable women are encouraged to seek nomination to the OAME Council.

Membership

Faculties of education, consultants and department heads will be contacted to promote membership in OAME. Plans are being made to advertise in the Forum and OTF journals. There may be a special centennial "I Love Math" button produced. We'll keep you posted.

History of OAME

The authors of this special publication are Jack LeSage and Joan Routledge with Michael Tabor as desktop publisher. The publication date is planned for April, 1991.

Publications

Anyone wishing to review articles submitted for publication in the Gazette should submit their names to the editors. Anyone wishing to be the editor for the Abacus or the Gazette should contact any member of council. The terms would be from September, 1991 to April, 1993.

Specialization Years

This committee will monitor MOE development regarding the Specialization Years and will serve on such projects as provincial reviews, Calculus TIP, OAIP, etc. They will work for relevance in the secondary mathematics curriculum particularly with respect to new technology. An OAME speaker's bureau will be established.

Transition Years

Results of the Availability/Need for Mathematics Teacher Report were discussed. The committee is anxiously awaiting the release of the consultation paper hopefully this fall.

Formative Years

The group will initiate a response to the Formative Years Consultation paper and the Issues accompanying the working paper.

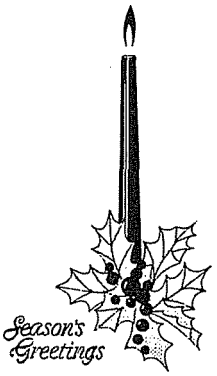
NCTM

Janet and Barry Scully were commended for the fine job they are doing as NCTM representatives. Anyone wishing to review manuscripts submitted to NCTM should contact the president of OAME.

TVOntario

It was noted that it is TVO's 20th birthday and that new mathematics programming is available.

Next Council Meeting:
Saturday, January 12, 1991
Ramada Inn
Airport West



ONTARIO ASSOCIATION for MATHEMATICS EDUCATION
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Join the over 1900 mathematics educators, kindergarten through college and university, who enhance mathematics education in Ontario. Your membership:

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ENCOURAGES networking and information sharing
STIMULATES Problem-Solving at all grade levels

O.A.M.E./A.O.E.M. MEMBERSHIP APPLICATION FORM

NAME _____
(first name(s)) **(surname)**

HOME ADDRESS _____
(street)

_____
(town/city) **(province)(postal code)** **(telephone)**

EDUCATIONAL INSTITUTION _____

ADDRESS _____
(board)

_____
(postal code) **(telephone)**

Elementary _____ **Secondary** _____ **CAAT** _____ **University** _____ **Other** _____

Chapter Affiliation (Check one):

CHAMP ○ COMA ○ GOLDEN ○ GVMA ○ NOMA ○ NWOAME ○
PINE RIDGE ○ QSLMA ○ SAME ○ SWOAME ○ WOMA ○ Y^4MA ○

O.A.M.E./A.O.E.M. Membership: New __ Renewal __

Membership Fees (effective May 1,1990 to April 30, 1991)
__ one year **$30** ($35 outside Canada)
__ five years **$135** ($165 outside Canada)
__ retired and student members **$20**

Send form and cheque, payable to O.A.M.E., to:
Secretary/Treasurer
Don Attridge

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NOMA	Maloney, Jerry.
NWOAME	Hurrell, Marilyn.
PINE RIDGE	Ennis, Ron.
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SAME	Haehnel, William.
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COUNCILLORS TERM EXPIRING 1991
Collins, Eileen.
Cornwall, Jan.
Davidson, Dave.
Fullerton, Olive.
Griffiths, Tom J.
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Mitchell, Janet.
Robinson, Robert.

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Romiens, Todd.
Sauer, Ronald.
Scully, Barry.
Stuart, Susan.
Warburton, Wendy.

COUNCILLORS TERM EXPIRING 1993
Handley, David.
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Wiggan, Lorna.
Zimmer, David.

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