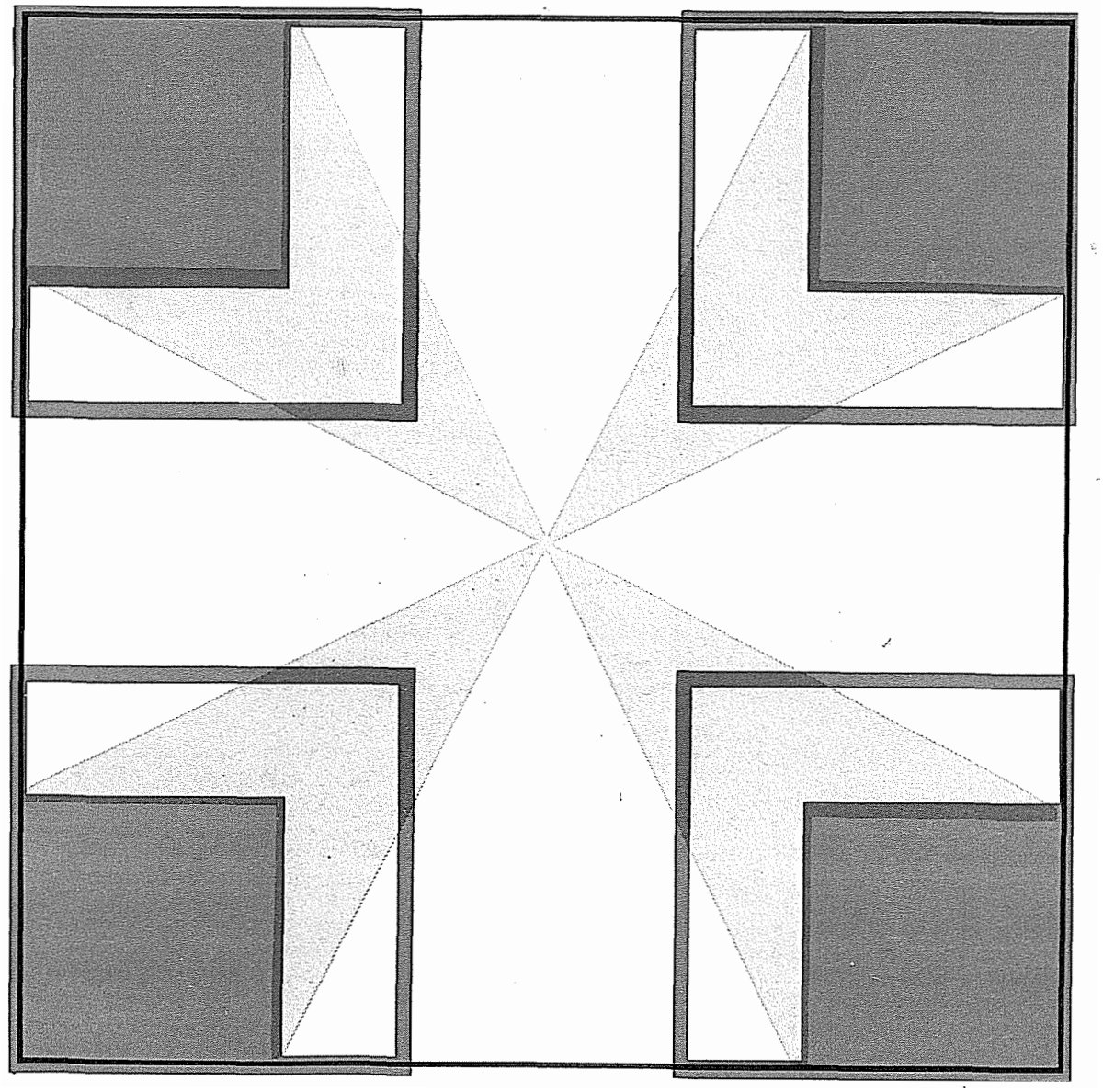
ontario mathematics gazette

Volume 30 Number 2 December 1991



THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

EDITORS

Barry Onslow
Faculty of Education,
The University of Western Ontario

Eric Wood
Faculty of Education,
The University of Western Ontario

Reviewing Board

D. Baron	D. Gilchrist	R. Lancaster	P. Rogers	J. Symington
E. Bridson	T. Griffith	J. Grant M^{c}Loughlin	T. Romiens	S. Telfer
E. Carter	D. Handley	E. Muller	K. Roy	J. Weiner
K. Cooper	E. Harvey	J. Palmer	I. Schnapp	P. Weygang
J. Cornwall	R. Isenegger	A. Parker	J. Scully	L. Wiggan
S. Crossman	A. Jones	J. Pitman	W. Schofield	N. Wilson
O. Fullerton	B. Kaiser	R. Reesor	R. Smith	A. Worrall
G. Gadanidis	I. Kleiner	G. Reid		

NOTICE TO AUTHORS

Articles submitted for publication should be typewritten or reproduced from type written copy, double-spaced on 8½" X 11" paper with 1½" margins, and one side of the paper. Diagrams and figures should be in copy ready form. THREE copies should be submitted to the editor and one kept by the author as protection against loss. Please send the author's full address and affiliation on a page SEPARATE from the manuscript as a policy of blind reviews has been adopted. If possible please submit a black and white, 5" X 7" portrait photograph suitable for publication. Any photographs of equipment should be a minimum of 5" X 7" in black and white.

AVIS AUX AUTEURS

Les articles qui sont soumis pour publication doivent être dactylographiés ou une photocopie de l'originale, écrit à double interligne sur papier 8½" X 11" avec une marge de 1½" et écrit seulement sur un côté de la feuille. Les diagrammes et les figures doivent être prêts pour imprimer. TROIS copies doivent être remises à l'éditeur et l'auteur devrait garder une copie en cas de perte. L'auteur devrait aussi inclure son adresse au complet et son affiliation en éducation sur une page SEPAREE du manuscrit. Les articles sone révisés par un comité qui ne serait pas informé du nom de l'auteur pour maintenir l'anonymat. Si possible veuillez soumettre une photo portrait noire et blanche, 5" sur 7" en grandeur, convenable à la publication. Les photographies de l'équipment devraient être au moins 5" sur 7" en grandeur et de couleur noire et blanche.

The *Gazette* is indexed in *Canadian Education Index*.

DEADLINES for submission of articles or information to be included in O.M.G. are as follow:
October 15 for the December issue
January 15 for the April issue
June 15 for the September issue

Unless otherwise indicated in the article, permission to photocopy material from the *Ontario Mathematics Gazette* is granted to classroom teachers for instructional use, to authors of scholarly papers and to librarians who wish to place a limited number of copies of articles on reserve. In all such instances due credit should be given to the author(s) and OAME. Permission must be sought for any commercial use of the content from this journal. Other uses of OAME journal material should be reported to the OAME.

Cover design:	Lois Armstrong, Media Production Centre, Faculty of Education, U.W.O.
Typing:	Sandra Neubauer, Faculty of Education, U.W.O.
Lay Out:	Carol Onslow, U.W.O.

V. Cairns

December 1991

ISSN 0030-3211

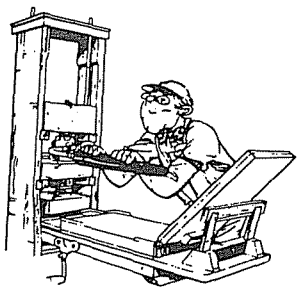
TABLE OF CONTENTS

EDITORIAL 2
PRESIDENT'S MESSAGE 3
LETTERS TO THE EDITOR 4

VIEWPOINT:
Tom Griffith 7
Four Pricing Games
L. Keith Roy 10
Non-Standard Word Problems
Jim O'Keefe 16
A Survey of Pre-Calculus Skills
Lucreda A. Hutton & Jeffrey X. Watt 18
Computational Pizazz
K. Allen Neufeld 23
Computer Tests Can Change Evaluation
Lawrence Sher & Patricia Wilkinson 25
History of QSLMA
M. Elaine Harvey 28
If This is Science, Why Are we Doing Math?
Kate Le Maistre 32
Making Sequences Easy with a Scientific Calculator
Richard Dewey 36
Five Steps to Teaching *Proof* in the Senior Mathematics Curriculum. Part 1.
Brendan Kelly 38
Reviews
George Gadanidis 43
Council Comments
Bob Robinson 46

Barry Onslow & Eric Wood are
co-editors of the Gazette for 1991/92.

Articles should be submitted to
Eric Wood
Faculty of Education, U.W.O.
1137 Western Road, London, Ontario, N6G 1G7



The *Ontario Mathematics Gazette* is printed by:
POLE PRINTING
Box 69 Forest, Ontario.
519 786-5112





As a child it seemed to me that the time between one Christmas and the next was just a few seconds short of eternity. It is a clear indicator that old age is catching up with me when I perceive that I just finished writing a Christmas editorial and that I can't be writing another one so soon!

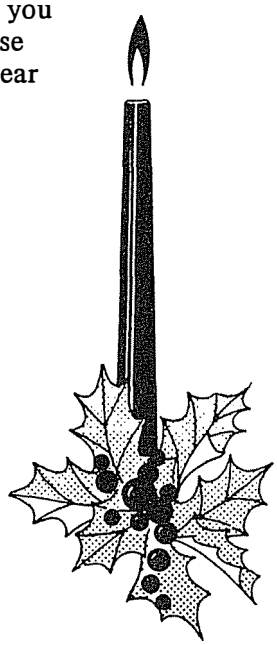
In spite of this apparently meteoric passage of time, much has happened in the year since I last wrote about the Christmas issue of the *Gazette*. Many OAME committees are examining the implications of various Ministry initiatives in the mathematics education field at all levels and this issue once again tries to put the new reform documents and ideas into a form that can be helpful to the classroom teacher.

For those teaching senior grades, Keith Roy's article on various pricing games is sure to generate interest in probability while the findings of a research study on pre-calculus skills should help us to examine the assumptions that we make about what calculus students do and should know. Although Jim O'Keefe's article does not contain 4 calling birds, 3 french hens, 2 turtle doves or a partridge in a pear tree, it does discuss a budgie problem -- the best we could do in a recession! For elementary teachers, Kate LeMaistre has provided an interesting way of putting mathematics and science together in the classroom. Allen Neufeld's ideas will add a spark of interest (often needed around Christmas time) to computational tasks.

Technology has continued to become more important in mathematics teaching and two articles discuss the use of both calculators and computers in the secondary classroom.

Finally, in this issue we introduce the first in a five-part series that looks at proof, a major strand in the senior curriculum, and the way that it can be discussed in a mathematically defensible and yet interesting way.

In closing, let me join with Barry in wishing you and all those near and dear a happy and safe holiday season.



PRESIDENT'S MESSAGE

Season's Greetings

Leading mathematics into the 21st century
What is the new vision for math education? It is a very confused scenario with three basic questions: What should we teach? How should we teach? How should we evaluate?

In this message I would like to address the first question and will be dealing with the other two in subsequent issues.

What should we teach?
If you ask people in business or industry if they think that education should prepare students for the workplace, they will unanimously say YES!

In you ask these same people what should be included in our mathematics curriculum to prepare students they will list statistics, logic, probability, measurement and integration of mathematics skills into other subject areas. Business people will also include "interpersonal skills". By this they really mean how to get along in the office, "problem solving" individually and as a member of a team.

Are the mathematics skills addressing these needs or are we currently teaching the mathematics skills that people have who are now being laid off? We talk about life-long education, "upgrading" but are we start them off at the same point, with the same mathematics skills that those 30 and 40 year olds who are now having to re-train?

Our society needs people who can reason, work together, and have a background which allows them to integrate what they have learned and be able to move that knowledge to the jobs, to their personal interests and hobbies.

In 1990, Statistics Canada, on behalf of the National Literacy Secretariat conducted a survey to assess the functional reading, writing and numeracy skills of adult Canadians, ages 16 to 69. While the literacy statistics and levels were widely reported, the numeracy skills are equally important. Many numeracy tasks were evaluated that involved either a single operation or a series of numerical operations. The numeracy skills measured reflect not only the ability to perform numeric operations, but also the ability to achieve them within the context of everyday tasks requiring the use of documents and forms. From this information they found three levels of numeracy:

Level 1

14% - Canadians at this level have very limited numeracy abilities which enable them to, at most, locate and recognize numbers in isolation or in a short text.

Level 2

24% - Canadians at this level can deal with material requiring them to perform a simple numerical operation such as an addition or subtraction.

Level 3

62% - Canadians at this level can deal with material requiring them to perform simple sequences of numerical operations which enable them to meet most everyday demands.

What does this tell us about our schools? As you would expect, the more schooling, the higher the percentage in Level 3. But, surprising, only 64% of Canadians who graduated high school were in Level 3 and only 83% of those who attended university were in Level 3.

Were they asking the wrong questions or were they trying to evaluate numeracy tasks that are not taught, especially to students working at the advanced level? We believe that students transfer our formal mathematics curriculum to real world applications. Some do, many don't.

Another interesting finding relates to age.

Age	Level 1	Level 2	Level 3
16-24	14%	30%	56%
25-34	9%	22%	69%

Does this mean that our recent students are less numerate that those who were in the system in the last 10 years? This could be one conclusion but more likely, in their working life they re-trained themselves to be able to handle "real life" numerical problems because they were exposed to them and because they had to.

It is necessary to redefine the mathematics curriculum, the scope and sequence of topics if we are to truly make our mathematics education more authentic and realistic to the needs of the 21st Century.

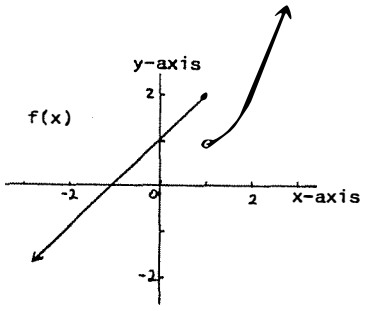
LETTERS TO THE EDITOR



In the Mathematics Department of Dunbarton High School, several colleagues and I arrived at a standstill in a discussion of the Calculus concepts *Does not exist*, *No limit* and $+\infty$ *or* $-\infty$.

Both the mathematical definitions and their uses are ambiguous and hence are not clear. I will attempt to present our argument.
Consider the following four diagrams.

a)



$\lim_{x \to 1^+} f(x) = 1$

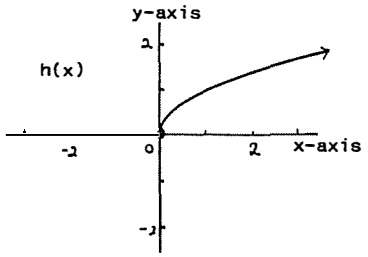
$\lim_{x \to 1^-} f(x) = 2$

$\lim_{x \to 1} f(x)$ Does not exist

$\lim_{x \to +\infty} f(x) = +\infty$ or No limit

$\lim_{x \to -\infty} f(x) = -\infty$ or No limit

c)



$\lim_{x \to 0^+} h(x) = 0$

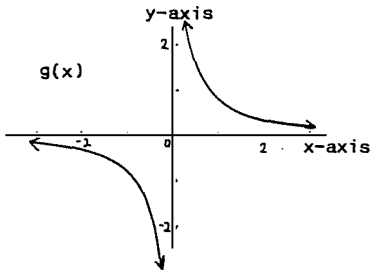
$\lim_{x \to 0^-} h(x)$ Does not exist

$\lim_{x \to 0} h(x)$ Does not exist

$\lim_{x \to +\infty} h(x) = +\infty$ or No limit

$\lim_{x \to -\infty} h(x)$ Does not exist

b)



$\lim_{x \to 0^+} g(x) = +\infty$ or No limit

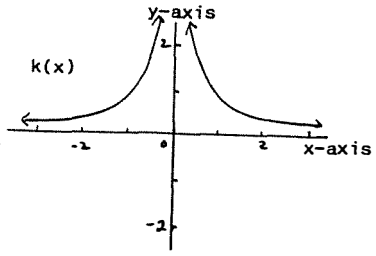
$\lim_{x \to 0^-} g(x) = -\infty$ or No limit

$\lim_{x \to 0} g(x)$ Does not exist

$\lim_{x \to +\infty} g(x) = 0^+$

$\lim_{x \to -\infty} g(x) = 0^-$

d)



$\lim_{x \to 0^+} k(x) = +\infty$ or No limit

$\lim_{x \to 0^-} k(x) = -\infty$ or No limit

$\lim_{x \to 0} k(x) = +\infty$ or No limit

$\lim_{x \to +\infty} k(x) = 0^+$

$\lim_{x \to -\infty} k(x) = 0^+$

NOTE:

The use of $+\infty$ or $-\infty$is used to represent a directional (positive or negative) approach of an extremely large value, part of the extended real number set.

ie. as $x \to 0$ then $k(x) \to +\infty$ can be expressed as $\lim_{x \to 0} k(x) = +\infty$

After further consideration of the diagram, we collectively concluded that there may be a distinction between *Does not exist, No limit and* $+\infty$ *or* $-\infty$.

One interpretation is as follows:

Given: A function $M(x)$ is discontinuous at 'a' in the following way:

IF: $\lim_{x \to a^+} M(x) = L$ and $\lim_{x \to a^-} M(x) = K$
where $L \neq K$ and L and $K \in R$

THEN: $\lim_{x \to a} M(x)$ Does not exist

OR

IF: $\lim_{x \to a^+} M(x) = +\infty$ and $\lim_{x \to a^-} M(x) = -\infty$
or NO limit or NO limit

THEN: $\lim_{x \to a} M(x)$ Does not exist
Using the extended real numbers.

Furthermore, the distinction that the term *No limit* is to $+\infty$ *or* $-\infty$, is as the term *speed* is to *velocity*, where *No limit* and *speed* have a quantitative quality and $+\infty$ *or* $-\infty$ and *velocity* have a directional quality. We collectively conclude that $+\infty$ *or* $-\infty$ is a more accurate description than *No limit*. *Does not exist* can be interpreted as no value or directional approach as shown in examples a) through c). In other words *Does not exist* is not equivalent to $+\infty$ *or* $-\infty$ or *No limit*.

There are many calculus texts and mathematics dictionaries which support our view. However, I have found no references which clearly define the three terms presented in this discussion. It is my intention to ask both memberships to consider this view for discussion and if possible clarify the definitions of *Does not exist*, *No limit* and $+\infty$ *or* $-\infty$.

Sincerely Yours,

A. Vagenas F. Vitale
M. Ricciuto C. Hunt
M. Melino W. Mullen
Dunbarton High School

Dear Editor:

Recently, an individual in Sometown, Ontario, requested my help in validating her claim of second prize in a Honda contest. Part of her letter and the ballot follows:

> Professor Weiner: Here is a (replica) of the Honda Ballot with the "skill testing" question.

TO WIN A HONDA NIGHTHAWK

1st Prize: 1991 Honda Nighthawk 250cc and Canada Safety Council course (approx. value $3,650.00)
2nd Prize: 10 Compact Discs (Approx. value $200.00) See reverse side for rules)

Name:_____

Address:_____

_____

Winners must correctly answer the following skill testing question:

$22 + 14 \div 3 \times 4 - 3 =$ ___



Honda has stated that 45 and 37 2/3 are *both* correct answers. Would you please answer the question and explain how (why?) you arrive at your answer. Would you also state your opinion on whether both 45 *and* 372/3 are correct answers. Thank you very much for you time. Your assistance with this is greatly appreciated.

I asked Gary Flewelling to join me in writing a response because I thought the combined influence of a university mathematics teacher and a county board of education mathematics consultant might convince Honda to grant the lady her legitimate second prize. I know this sounds pompous, but gee whiz, there's $200 in compact discs at stake here. Enjoy!.

Jack Weiner.



Well, There are Two Right Answers
or
"How I Learned to Ignore BEDMAS and Become Richer For It!"

Dear Terry:

Thank you for your fax regarding the Honda contest skill testing question.

As you know, you were required to evaluate the following expression in order to claim your prize: 22 + 14 ÷ 3 x 4 - 3

Your answer of 37 2/3 is correct. You have applied rules regarding order of operations, which most of us refer to with the acronym, "BEDMAS".

The set of rules which BEDMAS imposes is not arbitrary. In fact, these rules force us to evaluate such an expression as your test question in a manner consistent with common sense. We will elaborate on this a little later*.

But first, you mention that the Honda representative to whom you spoke said that "a correct way" of working this out is to execute each operation in the order in which it appears. It is almost certainly the case that this individual has confused the symbolic expression

22 + 14 ÷ 3 x 4 - 3

with the verbal instructions,
"twenty-two add fourteen *(pause)*, divide by three *(pause)*, multiply by four *(pause)*, subtract three."

The "(pause)" is significant in these verbal instructions, for each pause cues you to execute an arithmetic operation. This is how Honda has determined an answer of 45. For the Honda answer to be correct, the skill testing question would have to read something like, [(22 + 14) ÷ 3] x 4 - 3

As promised, we would like to emphasize that the order or of operation rules do lead to "reasonable" answers, while Honda's method can lead to ridiculous results. Here's a simple example.

Terry, you have $100 and you plan to save $10 per week for the next twenty weeks. Clearly, you will, in just under five months, have 100 + 10 x 20 dollars saved. Now you know, and we know, and almost everyone else will agree that you will have saved $300.

However, our friend at Honda would maintain, following the "Honda method", that your account now has
100 + 10 x 20 = 110 x 20 = 2200, that is $2200. We wonder whether Honda products are priced using this method! Both of us hope that we have clarified not only why you are correct, but also why the Honda representative is not.

Yes, Terry, there really is exactly one correct answer.

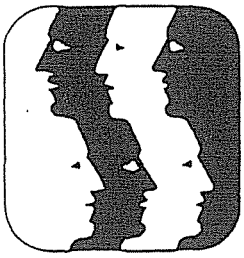
Yours sincerely,

Gary Flewelling
Mathematics Consultant
Wellington County Board of Education.

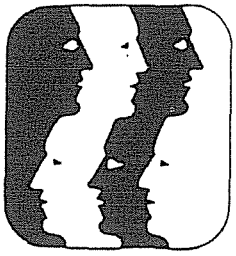
Jack Wiener
Assistant Professor
Department of Mathematics and Statistics
University of Guelph

P.S. Our consultation fee is 1000 - 500 x 2 dollars. What did you say? The cheque is in the mail?

* For the mathematicians in the audience, BEDMAS is consistent with the field properties of the real number system.



Viewpoint



BAD LANGUAGE IN MATHEMATICS

by
Tom Griffiths
A.B. Lucas Secondary School
London, Ontario

Many of our students are experiencing progressively more problems in mathematics as they pass through high school. This is due to several reasons. One reason is that they attempt to learn mathematics as a series of unrelated and virtually nonsensical processes. Another is that they try to learn and remember these processes by catch words or phrases which are not related to the concept or algorithm to be learned.

Mathematics is a language whose words and sentences are presented and communicated in the form of codes or ciphers such as expressions, identities or equations. Our students do not initially understand or communicate in this language, and hence we have to use their day to day language to describe and explain the mathematical processes and concepts. This is difficult enough for many of them, without the language being in the form of meaningless jargon. If such jargon is used, the learner is involved in yet a second level of translation and interpretation. It is, therefore, extremely important that we use words very carefully in our teaching. These words must be in the student's vocabulary if they are being used to describe a logic chain or mathematical concept. New words should not be used to describe new ideas. The words must also accurately describe the concept or process.

An example of a frequently misused word is "cancel" to describe the process of simplifying fractions. As teachers we have been using this word in this context for so long that we do not think of its meaning. However, there have been attempts to get rid of this word in this context for a long time. I recall articles in this journal in the 1960s. According to the dictionary, the word cancel means to cross out, annul, or to make without value, or in other words, change to zero. If the expression $3 + 2x - 3$ is simplified to $2x$, then the 3's have been cancelled or eliminated. This I consider a correct use of the word. However, if the fraction 6/9 is simplified to 2/3, nothing has been cancelled. Dividing the factors of 3 in both the numerator and denominator by the common factor of 3 gives quotients of 1, not 0. If $(x+2)/(x+3)$ is changed to 2/3, then the x's have been cancelled. We have all seen this interpretation from our students, and if the word cancel is used in this way, it is a correct application of the word, but not the mathematical concept. Further, as well as being incorrect in this context, the word cancel does not assist the learner in understanding the process of simplifying fractions in any way. If the phrase "divide numerator and denominator by a common factor" is used, the learner can at least think through the instruction. He or she can then work on understanding the words, such as "factor", one at a time, and gradually understand the process and apply it correctly. The word "cancel" gives the teacher a quick and easy "mathemagical" word to use as jargon, but it does not help the learner, who is left to learn by repeatedly observing a process. The learner must then attempt to copy the process correctly, not only without the necessary understanding, but also without understandable instruction. the word "cancel" does not convey the correct logic or algorithm. This is not a good learning situation.

In good mathematics the concepts should be clearly and simply defined. This should be

done through the use of words which also explain the concept or algorithm, or at least a named reference to a mathematical rule. In simplifying fractions, either a detailed phrase should be used, or perhaps a reference to the "Golden Rule of fractions", but not cancelling. The phrase "Golden Rule" may be the mathematicians name or jargon for the process of simplifying fractions, but at least it is not a misnomer.

Another example of a similar misuse that I find causes my students progressively more difficulty as they go through high school, comes from the solution of equations. What mathematical significance or logic does "take it over to the other side and change the sign" have? There are a great many students who never understand that the logic of solving equations depends on keeping both sides equal by applying the same process to both sides.

Sadly, as they progress through school, many of our students learn an increasing number of these jargon words and phrases, but without any understanding of their meaning or the logic behind them. The result is that they find themselves attempting to apply the same increasing number of nonsense processes. They do get surprisingly good at being able to produce the correct answer. However, when it comes to putting the ideas together in such endeavours as learning Calculus, the students are lacking the logic, concepts and skills, to help them make the links between the ideas. This causes many students to drop mathematics.

We need to make sure that the language we use has meaning; that it describes the process correctly; and that the words we use do explain, or give links to the explanation of the logic and the associated algorithms. We need to rethink our use of words. We should not take for granted that the words we learned in school, and have used all our lives, actually make sense. This is not easy for us, but very important for our students.

Let us mathematics teachers make a determined attempt to help our students understand mathematics by eliminating "bad language". Let us pay very careful attention to using only words which convey precise meanings, and that can be understood by our students.

O.A.M.E. / A.O.E.M. LIFE MEMBERSHIP

To qualify for **Life Membership in O.A.M.E./A.O.E.M.** a candidate shall have:

(a) 10 or more accumulated years of membership in OAME/AOEM, and

(b) demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular.

Nominations for Life Membership in the O.A.M.E. / A.O.E.M are to be submitted to:

Don & Carol Attridge
Secretary-Treasurer O.A.M.E.

Nominations with supporting information should be sent to the above address on or before **January 13, 1992.**

MATHEMATICS, SCIENCE & TECHNOLOGY EDUCATION NEWSLETTER

MSTE will report on math, science, technology and computer education research and teaching practices, as well as advertise programmes, upcoming seminars and workshops, and the availability of publications and resource materials of interest to educators.

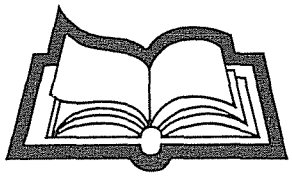
Aim: to promote greater awareness of the links that can be made between mathematics, science and technology, and to investigate how these links can result in better teaching.

Information about workshops, conferences or resources sponsored by your organization and of interest to MST educators can be sent to the Editor. (Publication dates: December and April)

NEW

To be added to mailing list educators are invited to contact:
The Editor, *MSTE NEWS*
Mathematics, Science and Technology in Education Group
Faculty of Education
Queen's University
Kingston, Ontario K7L 3N6

NEW PUBLICATION



MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former O.A.M.E. Co-Secretary-Treasurer, O.A.M.E. established the Mona MacGregor Award, to be presented annually to a person who has made a significant contribution to O.A.M.E. but who does not necessarily meet the criteria for the Life Membership Award (i.e. **not necessarily an educator or an O.A.M.E. member.**) The award is aimed at recognizing people who have supported O.A.M.E. such as publishers, educational suppliers, hotel personnel, etc.
At this time O.A.M.E. is requesting nominations for consideration for this year's award. It will be presented at the Annual Conference. Nominations are to be submitted to:

Don & Carol Attridge
Secretary-Treasurer O.A.M.E.

Deadline for nominations is January 13, 1992.

FOUR PRICING GAMES

by
L. Keith Roy
Department of Mathematical Sciences
Lakehead University

GAME 1

One of the games faced by contestants on the CBS Network's popular game show "The Price Is Right" is "Spelling Bee". In order to win a car, the contestant must spell the word *CAR* by making selections of cards from a display of thirty cards of which eleven bear the letter *C*, another eleven bear the letter *A*, another six cards bear the letter *R*, and the remaining two cards have the whole word *CAR* printed on them. The actual letter or word is printed on the backs of the cards, while the front of each card has "$500" printed on it. The contestant is given two free choices of cards and is given the opportunity to win up to three more choices by correctly guessing the prices (within ten dollars above or ten dollars below the actual retail prices) of three featured consumer products. As well, if any of the contestants are lucky enough to correctly guess the exact price of any of the three products, they automatically win all three extra choices. The letter or word printed on the backs of the chosen cards is revealed one card at a time, and, at each step, the contestant is offered the amount of five hundred dollars times the number of still unrevealed cards to withdraw from the game. Most contestants choose to remain in the game until all the letters or words printed on the backs of the chosen cards have been revealed.

The analysis of the above game, in the likely case where the contestant chooses to remain in the game until the very end in hope of winning a car, makes use of techniques presented in both the Combinatorics section and the Probability and Applications section of the Ontario Academic Course in Finite Mathematics.

The least complicated approach is to find the probability of a contestant failing to win the car, and then to subtract this number from one in order to find the probability that the contestant will win the car. There are several cases to be considered.

1) If the contestant is lucky enough to win three additional choices, for a total of five choices, he/she will fail to win the car if neither of the cards containing the word CAR have been chosen and one (or more) of the three letters C, A, R, was not chosen. The number of ways of choosing five cards from a display of thirty cards is

$$\binom{30}{5} = 142\ 506.$$

The number of ways of choosing neither of the two CAR cards, none of the eleven C cards, and five cards from the seventeen A cards and R cards is

$$\binom{2}{0}\binom{11}{0}\binom{17}{5} = 1(1)(6\,188) = 6\,188.$$

Consequently, the probability of failing to win the car with five card choices because only A's and/or R's were chosen is

$$\frac{6\,188}{142\,506}.$$

Similarly, the probability of failing to win the car with five card choices because only C's and/or R's were chosen is also

$$\frac{6\,188}{142\,506}.$$

The number of ways of choosing neither of the two CAR cards, none of the six R cards, and five cards from the twenty-two C cards and A cards is

$$\binom{2}{0}\binom{6}{0}\binom{22}{5} = 26334,$$

so that the probability of failing to win the car with five card choices because only C's and/or A's were chosen is

$$\frac{26334}{142506}.$$

As a result, the probability of failing to win the car with five card choices is

$$\frac{6188}{142506}+\frac{6188}{142506}+\frac{26334}{142506}+\frac{38710}{142506}.$$

2) Suppose the contestant wins two additional choices, for a total of four choices.

The probability of failing to win the car with four card choices because only A's and/or R's were chosen is

$$\frac{\binom{2}{0}\binom{11}{0}\binom{17}{4}}{\binom{30}{4}} = \frac{2380}{27405}.$$

This is also the probability of failing to win the car with four card choices because only C's and/or R's were chosen.

The probability of failing to win the car with four card choices because only C's and/or A's were chosen is

$$\frac{\binom{2}{0}\binom{6}{0}\binom{22}{4}}{27405} = \frac{7315}{27405}$$

so that the probability of failing to win the car with four card choices is

$$\frac{2380}{27405}+\frac{2380}{27405}+\frac{7315}{27405} = \frac{12075}{27405}.$$

3) Suppose the contestant wins only one additional choice of cards, for a total of three choices.

The probability of failing to win the car with three card choices because only A's and/or R's were chosen is

$$\frac{\binom{2}{0}\binom{11}{0}\binom{17}{3}}{\binom{30}{3}} = \frac{680}{4060}.$$

This is also the probability of failing to win the car with three card choices because only C's and/or R's were chosen.

The probability of failing to win the car with three card choices because only C's and/or A's were chosen is

$$\frac{\binom{2}{0}\binom{6}{0}\binom{22}{3}}{4060} = \frac{1540}{4060}$$

so that the probability of failing to win the car with three card choices is

$$\frac{680}{4060}+\frac{680}{4060}+\frac{1540}{4060} = \frac{2900}{4060}.$$

4) Suppose the contestant wins no additional card choices, for a total of only two choices.

The contestant will fail to win the car if neither of the cards she/he chooses contain the word CAR. The probability of this happening is

$$\frac{\binom{2}{0}\binom{28}{2}}{\binom{30}{2}} = \frac{378}{435}.$$

The problem of how many additional card choices the contestant will win still remains.

Suppose that, because of her/his experience as a shopper, a contestant has a better than 50% chance (let's say 60%) of guessing a price within the plus or minus ten dollar range for any particular product, and that the contestant's success or failure concerning the price of a particular item is independent of her/his success or failure with respect to the price of any other items. Further, suppose that the contestant will not be able to guess the exact price of any of the three products. This then would depict a binomial situation, that is, the probability of x successes and consequently x additional card choices, for $x = 0, 1, 2$, or 3, is

$$\binom{3}{x}(0.6)^x(0.4)^{3-x}$$

The tree diagram (Figure 1) puts all of the above information together.

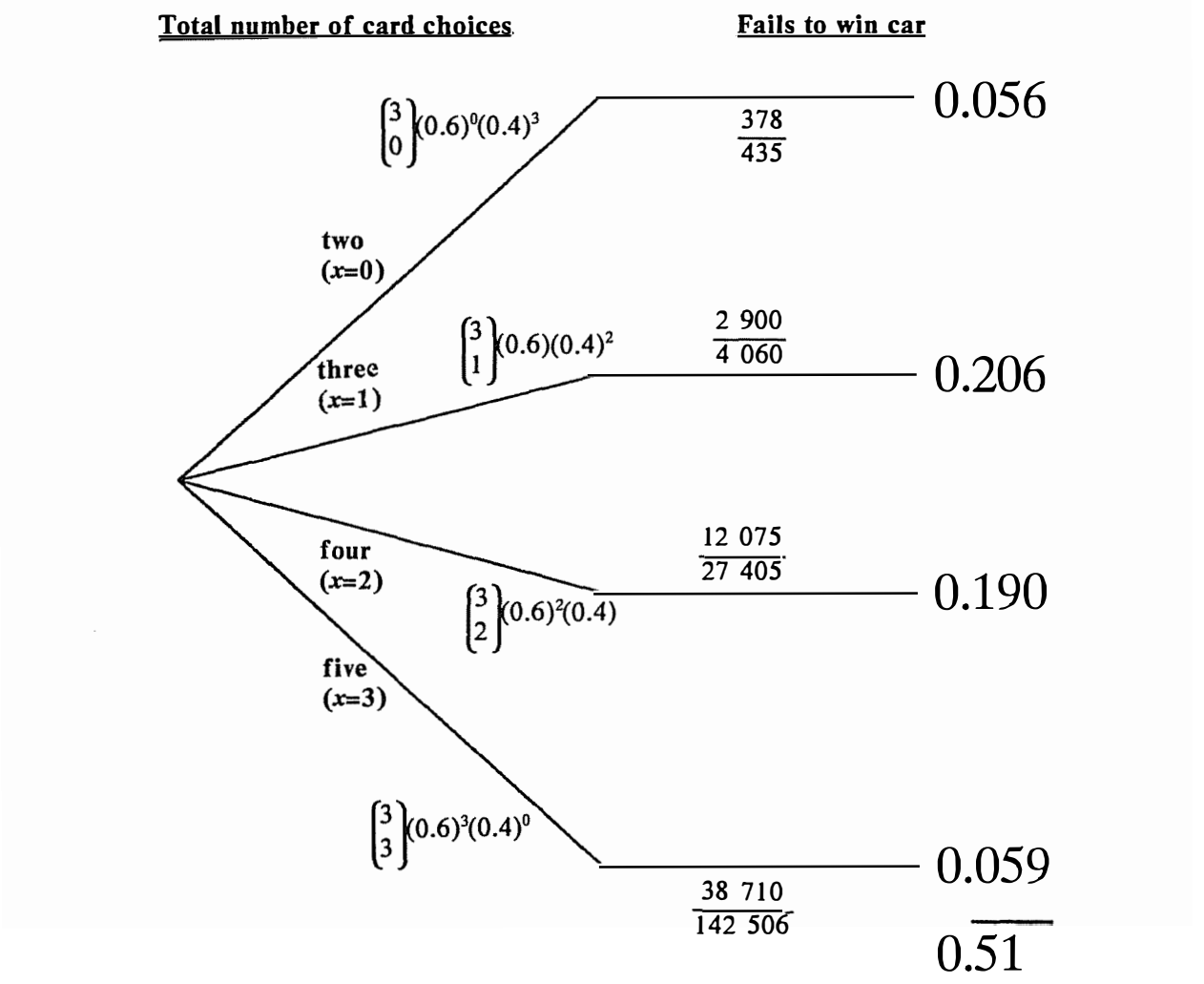


Figure 1

In Figure 1, the second set of branches depicts the event that a contestant will have three card choices and fail to win the car. The probability of this event is

$$\left[\binom{3}{1} (0.6)(0.4)^2 \right] \left[\frac{2900}{4060} \right] = 0.206$$

Thus the probability that a contestant will fail to win the car is

$$0.056 + 0.206 + 0.190 + 0.059 = 0.51$$

so that the probability of a contestant actually winning the car, assuming the binomial probability of 0.6 discussed earlier, is

$$\mathbf{1 - 0.51 = 0.49.}$$

GAME 2

Another interesting game often featured on the same program is the "Penny Ante Game". In order to win a fabulous prize, the contestant must guess the prices of two featured products. The price of the first product is somewhere in a list of four possible prices, and the price of the second product is somewhere in a second list of four possible prices. The contestant is given three "Price Is Right" pennies and must surrender a penny to the game show host each time a wrong guess is made. If, at the end of the game, the contestant has at least one penny in her/his possession, she/he wins the prize.

The contestant first tries to guess the price of the first product by choosing one of the prices from the first list, surrenders a penny if that guess is wrong, and proceeds to guess prices from the first list until either she/he runs out of pennies or the correct price for the first product is chosen. If the contestant still has at least one penny in her/his possession and has guessed the price of the first product, she/he tries to guess the price of the second product by choosing from the second list of prices. If the contestant guesses the price of the second product before running out of pennies she/he wins the prize.

Consider the case of a contestant who has no idea about the proper price of either product and simply proceeds by randomly choosing prices from the first list, and, if she/he still has at least one penny remaining, randomly choosing prices from the second list.

As the game is played, sequences such as [s][ffs] will result, where f and s refer respectively to failure and success in a particular attempt to guess the price of a product, and the square brackets are used to separate guesses for the two products. The sequence [s][ffs] represents the situation where the contestant is lucky enough to guess the price of the first product on the first try, fails twice to guess the price of the second product and therefore loses two pennies, and then guesses the proper price of the second product. On the other hand, sequences like [ffs][f] would result in the contestant losing the game.

Suppose the contestant chooses prices at random. There are four mutually exclusive sequences which would result in the contestant losing the game:

[fff] which has probability

$$\left[\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right) \right] = \frac{1}{4}$$

of occurring,

[ffs][f] which has probability

$$\left[\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right) \right] \left[\frac{3}{4} \right] = \frac{3}{16}$$

of occurring,

[fs][ff] which has probability

$$\left[\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) \right] \left[\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) \right] = \frac{1}{8}$$

of occurring,

and [s][fff] which has probability

$$\left[\frac{1}{4} \right] \left[\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right) \right] = \frac{1}{16}$$

of occurring.

So the probability that a contestant will fail to win the game if prices are chosen at random is

$$\frac{1}{4} + \frac{3}{16} + \frac{1}{8} + \frac{1}{16} = \frac{5}{8}.$$

Contestants who have some idea about the retail prices of the two products will, of course, have a smaller probability of losing.

GAME 3

A third interesting game on "The Price Is Right" is "The One Away Game". The contestant is presented with a five digit price for a car and is informed that all digits are incorrect but each is only one integer away from the correct digit. For instance, when confronted with 23717, the contestant can replace the 2 with either a 1 or a 3, the 3 with either a 2 or a 4, the 7 with either a 6 or an 8, the 1 with either a 2 or a 0, and the 8 with either a 9 or a 7. The contestant is then told how many digits of the resulting price are correct. If one or more are correct, the contestant is allowed to change the appropriate number of digits to the other alternative. Of course, the wrong digits could be selected. However, if the resulting price is correct, the contestant wins the car.

It will be assumed that a contestant, when faced, for instance, with a first digit of 2, is always able to correctly judge whether the car that is featured has a price somewhere between \$10 000 and \$20 000, in which case the contestant will change the first digit to a 1, or whether the price is somewhere between \$30 000 and \$40 000, in which case the contestant will change the first digit to a 3. So the probability of the contestant winning the car will depend only on what happens with the last four digits. It will also be assumed that the contestant will make a random choice of the two alternatives for each of the last four digits.

There are five mutually exclusive ways that a contestant can win the car. For x=0, 1, 2, 3, or 4, the contestant will win the car if x of the last four digits are correct, and all of the 4-x incorrect digits are randomly selected from the last four digits and are changed. The probability of x of the last four digits being correct comes from the binomial distribution

$$\binom{4}{x}(0.5)^x\,(0.5)^{4-x} = \frac{\binom{4}{x}}{16}.$$

The probability that the proper 4-x incorrect digits are selected from the last four digits is

$$\frac{1}{\binom{4}{4-x}}$$

for x = 0, 1, 2, 3, or 4, since there are

$$\binom{4}{4-x}$$

ways of choosing 4-x items from 4 items.

Consequently, the probability of winning the car by selecting the right choices for all four last digits, and not having to make any changes, is

$$\frac{\binom{4}{4}}{16} = \frac{1}{16},$$

while the probability of winning the car by having three correct digits out of four and successfully choosing to change the proper digit is, using x = 3,

$$\frac{\binom{4}{3}}{16} \times \frac{1}{\binom{4}{1}} = \frac{1}{16}.$$

Indeed, the probabilities for x = 2, 1, and 0 are all 1/16. So the probability of the contestant winning the car under the assumptions outlined above is 5(1/16) = 5/16.

GAME 4

One of the oldest games featured on the program is "The Any Number Game". The contestant will win one of three prizes: an automobile, a secondary prize, or the amount of money, in dollars and cents, in the piggy bank. The digits 0 through 9 appear once and only once on the game board which consists of spaces for the last four digits in the price of

the car, the three digits in the price of the secondary prize, and the three digits representing the amount in the piggy bank. The contestant chooses digits one at a time and these are displayed on the game board in their proper places. The contestant will win the prize which has its price completed first.

Assuming that the contestant chooses the digits in random order, there are five mutually exclusive ways of winning the car.

(i) Suppose the contestant chooses the last four digits in the price of the car without choosing any of the six digits associated with the other prizes. The probability of this happening is

$$\frac{\binom{4}{4}\binom{6}{0}}{\binom{10}{4}} = \frac{1}{210}.$$

(ii) The contestant might choose three of the digits in the price of the car as well as one of the six digits associated with the other prizes during her/his first four choices, then choose the remaining digit in the price of the car on her/his fifth choice. The probability of this happening is

$$\frac{\binom{4}{3}\binom{6}{1}}{\binom{10}{4}} \times \frac{1}{6} = \frac{1}{52}.$$

Notice that when the contestant is about to make her/his fifth choice, there are still six digits remaining, only one of which belongs in the price of the car.

(iii) The contestant might choose three of the digits in the price of the car as well as two of the six digits associated with the other prizes during her/his first five choices, then choose the remaining digit in the price of the car on her/his sixth choice. The probability of this happening is

$$\frac{\binom{4}{3}\binom{6}{2}}{\binom{10}{5}} \times \frac{1}{5} = \frac{1}{21}.$$

(iv) The contestant might choose three of the digits in the price of the car as well as three of the six digits associated with the other prizes during her/his first six choices, without winning any prize, then choose the remaining digit in the price of the car on her/his seventh choice. The probability of this happening is:

$$\frac{\binom{4}{3}\binom{3}{2}\binom{3}{1}}{\binom{10}{6}} \times \frac{1}{4} + \frac{\binom{4}{3}\binom{3}{1}\binom{3}{2}}{\binom{10}{6}} \times \frac{1}{4} = \frac{3}{35}.$$

Notice in the above expression, that there are two ways of picking three digits associated with the two subsidiary prizes without winning either prize: either one or two of these digits must be associated with the piggy bank.

(v) Finally, the contestant might choose three of the digits in the price of the car as well as four of the six digits associated with the other prizes during her/his first seven choices, without winning any prize, then choose the remaining digit in the price of the car on her/his eighth choice. The probability of this happening is

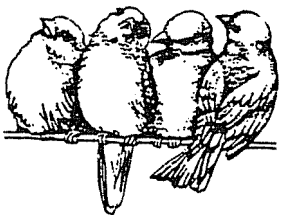
$$\frac{\binom{4}{3}\binom{3}{2}\binom{3}{2}}{\binom{10}{7}} \times \frac{1}{3} = \frac{1}{10}.$$

Obviously, if no prize has been won by the time seven digits have been chosen, there remains only one blank space on the board in the price of each prize, and the contestant will win whichever prize is associated with the next digit chosen. So it is impossible for nine or ten digits to be chosen.

Consequently, the probability of winning the car in "The Any Number Game" by choosing digits at random is:

$$\frac{1}{210} + \frac{1}{52} + \frac{1}{21} + \frac{3}{35} + \frac{1}{10} = 0.26.$$

NON-STANDARD WORD PROBLEMS



by
Jim O'Keefe
Head of Computer Education

The following word problem is a little different than most encountered in high school. It can't be solved using the standard methods one might employ simply because the number of unknowns the information produces is greater than the number of equations.

It can be solved by using a combination of standard methods and a little "mathematical intuition".

PROBLEM

A bird collector has $100.00 to spend. A pet store is selling off a special shipment of budgies at incredibly low prices.

Blue budgies are $10.00 each, green budgies are $3.00 each, yellow budgies are $.50 each. Tax is included in the prices.

The bird collector wants to buy exactly 100 budgies and wants to spend exactly $100. He also wants to purchase at least one budgie of each colour.

How many blue, green, and yellow budgies should he buy?

SOLUTION

Let x represent number of blue budgies.
Let y represent number of green budgies.
Let z represent number of yellow budgies.

(1) $x + y + z = 100$
(total number of birds must be 100)

(2) $10x + 3y + 0.5z = 100$
(total cost of the birds must be $100.00)

As can be seen, there are two equations and three unknowns. This system of equations can't be solved using the standard methods taught in high school.

Let us take the equations a little farther and see what we can shake out of them.

(3) $20x + 6y + z = 200$ $\quad 2 \times (2)$

(4) $19x + 5y = 100$ $\quad (3) - (1)$

Solving (4) for y gives:

(5) $y = \dfrac{100 - 19x}{5}$

We know that the collector is buying live birds which means that he can't purchase a fraction of a bird. Therefore, the three unknowns must be positive integers.

For y to be a positive integer, the following must be true:

a) 5 must divide evenly into $100 - 19x$
b) In order for a) to be true, $100 - 19x$ must have as its last digit a five or a zero
c) In order for b) to be true, $19x$ must have as its last digit a five or a zero
d) In order for y to be positive, $19x$ must be less that 100

Looking at the multiples of 19, there is only one that satisfies all of a), b), c), and d). That number is 95.

Therefore:	$19x = 95$
	$x = 5$
From (5)	$y = 1$
From (1)	$z = 94$

Substituting the above values into equation (2) will verify the solution. Therefore, the bird collector should purchase 5 blue budgies, 1 green budgie, and 94 yellow budgies.

KENNETH D. FRYER AWARD

In Memory of the late Dr. K. D. Fryer, OAME, in conjunction with the Grand Valley Mathematics Association, established the **Kenneth D. Fryer Award for Excellence,** to be presented annually to a **Secondary School Mathematics Department** demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.
At this time OAME is requesting nominations for consideration by the committee for this year's award. It will be presented at the annual conference in Toronto.

Deadline for nomination submission: March 2, 1992

KENNETH D. FRYER AWARD NOMINATION FORM 1991-1992

I/WE nominate
SCHOOL: _____
ADDRESS: _____
Principal: _____
Mathematics Dept. Head: _____
Nominated By: _____
_____

Please attach a separate sheet to the completed nomination form (or copy) outlining the reasons for your nomination, and forward them to:

Don and Carol Attridge
Secretary/Treasurers O.A.M.E.

WE NEED TO HEAR FROM YOU!



Help O.A.M.E. respond to the Ministry of Education's consultation paper on The Specialization Years (now in draft form). Issues include:

* over crowded curriculum
* equity of access to mathematics (gender, culture)
* curriculum renewal - relevance, levels of instruction, types of courses, cross-curricular integration, ...
* teacher education - new teachers and in-service

Send your opinions before January 15 to:
Jerry Maloney, Chair
OAME Specialization Years Committee

A SURVEY OF PRE-CALCULUS SKILLS

A Survey To Identify Those Pre-Calculus Skills That Are Weak or Missing Among Students Who Do Not Succeed in Calculus

by
Lucreda A. Hutton & Jeffrey X. Watt
Department of Mathematical Sciences
Indiana University Purdue University at Indiana

INTRODUCTION

This study surveyed college mathematics instructors teaching first semester calculus in the United States. The vast majority of the students in these courses are being introduced to calculus for the first time. Because of the many differences in the learning environment between high school and college, the results of this study might vary in those areas where calculus is taught at the high school level. This would warrant a similar survey to be conducted with high school calculus teachers in Ontario where an introduction to calculus is taught at this level.

A growing concern of many college mathematics educators is the low success rate of their calculus students. Calculus is a required subject in several academic disciplines and affects 31% of all undergraduate students in the United States (Young, 1988). Calculus has become one of the most important courses offered at the collegiate level, and it is among the top five courses in annual enrollment (Steen, 1988). Currently, there are 700 000 students enrolled in calculus courses at the post-secondary level, of these, 100 000 are enrolled in two year and junior colleges (Anderson & Loftsgaarden, 1987).

Despite the growing number of students required to complete the study of calculus, a large number of them do not succeed. A study conducted by the Mathematical Association of America (MAA) in 1987, reported that a wide range of failure rates existed among various colleges and universities. It was found that between 35 percent of the students enrolled in smaller schools and 60 percent of the students enrolled at larger schools will fail their calculus course.

The combination of the large numbers of students required to complete calculus and the high failure rates of students in calculus warrant the investigation of the prerequisite mathematics skills of these students. Thus, the purpose of this study was to survey mathematics instructors' perceptions of which pre-calculus skills are lacking among their students who fail. The results would then be used to help identify areas in the pre-calculus and calculus curriculum that might be used as variables for investigation in future research projects.

METHOD

SURVEY DESIGN

The instrument used to collect the data was a one page paper-and-pencil survey of 26 items. These items are listed in Figure 1. The respondents were asked to rate each of the items on a five point scale as to that item's perceived significance in impeding student achievement in calculus.

The items were constructed from the results of interviews conducted with eight instructors of mathematics who regularly teach the calculus sequence. The instructors were asked to generate items that might be stumbling blocks

for students' success in calculus. These items were then grouped into five subject areas. They were Algebra, Logarithms, Trigonometry, Geometry, and General. For example, the subject area, General, included such items as study habits, attendance, problem solving, and amount of material covered.

I ALGEBRA (ALG)

1. Generally weak algebra skills.
2. Errors with signed numbers.
3. Rules of exponents.
4. Rules of inequalities.
5. Complex fractions.
6. Solutions of equations.
7. Variable denominators.
8. Basic understanding of functions.
9. Basic understanding of inverse functions.
10. Basic graphing skill.
11. Lack of experience with limit concepts.
12. Lack of experience with sequences.

II LOGARITHMS (LOG)

1. Relationship between logs and exponents.
2. Using properties of logarithms.

III TRIGONOMETRY (TRIG)

1. Arc functions.
2. Knowing trig identities.
3. Making basic identity substitutions.
4. Solving trig equations.

IV GEOMETRY (GEOM)

1. Lacks 3-D experience.
2. Cannot make meaningful sketches.
3. Cannot generalize volume concept to complex 3-D figures.

V GENERAL (GEN)

1. Poor study habits.
2. Poor class attendance.
3. Word/application problems.
4. Lacks general problem solving skills.
5. Too much material in course.

Table 1 Survey Items on instrument

SUBJECTS

The subjects selected to participate in the survey were 137 volunteers from across the nation who teach college calculus. The sample was significantly weighted in favour of two States. One third of the volunteers were from New York, one third from Indiana, and the remaining one third from a variety of other States. The reason for this weighting was because the results of the study would be used by mathematics educators in New York and Indiana for possible curriculum and instruction changes.

RESULTS

Figure 1 lists the 26 items surveyed in the order that they appeared on the survey. In Table 1, the different items are identified by a group and item number. The five groups are ALG, LOG, TRIG, GEOM, and GEN. An item referred to as GEOM1 would be "lacks 3-D experience," and GEOM2 would be "cannot make meaningful sketches."

The subjects were asked to score each of the items using a scale from 0 to 4. The score of 0 represented the response "not noticed as a problem", and the score of 4 represented the response "very often a problem."

The mean score for each item was used to rank items from 1 (the highest mean score as very often a problem) to 26 (the lowest mean score). The top five ranked items are listed in Table 1.

DISCUSSION

The ranking of the top five items was not statistically significantly different between the various geographical areas from which the subjects were selected. Thus, the data was not further analyzed by groups in this study. This finding demonstrates that there is strong agreement nationally about which pre-calculus skills are considered the most serious problems in calculus.

RANK	ITEM	MEAN	STATEMENT
1	GEN3	3.25	Word/Application Problems
2	ALG1	3.08	Generally Weak Algebra Skills
3	GEN4	2.99	Lacks Problem Solving Skills
4	ALG9	2.85	Understanding Inverse Functions
5	GEOM1	2.78	Lacks 3-D Experience

0.00 = not a problem
4.00 = very often a problem

Table 2. Five highest ranked items.

Historically, it has been true that "word" problems or applications have been one of the stumbling blocks in mathematics. These data indicate that they are still considered a major problem for calculus students. This item, GEN3, was ranked as the number one stumbling block. It is important to note that item GEN4, "lacks general problem solving skills" was ranked third. This result indicates that these two items were clearly seen as related problems.

During the past decade, the focus of mathematics education has shifted towards problem solving. The results from this study support the need for this shift. These data indicate a continuing weakness in problem solving performance despite the current focus on problem solving in the mathematics curriculum.

Item, ALG1, "generally weak algebra skills", was ranked second. This concern was no surprise as good algebra skills are essential for success in calculus. However, the fact that most calculus students have had two or three years of algebra indicates a large scale problem in remediation of these students. It is interesting to note that the ranking of this item was slightly higher than "problem solving" which indicates that teachers perceive the lack of algebra skills as serious of a problem as the ability to problem solve.

The authors believe that weak algebra skills contribute to the poor problem solving performance of students. In turn, these two items may have an additive affect on calculus achievement. However, this study did not address the question of whether weak algebra skills contributes to a breakdown in George Polya's third or fourth step of problem solving. Certainly, algebra skills play an important role in step three: carrying out the plan.

ALG9, "understanding inverse functions", was ranked fourth. The concept of inverse functions is considered fundamental to the study of calculus.

GEOM1, "lacks 3-D experience", was ranked fifth. The mean value of this item was 2.78 which is considered a moderate to frequent problem. Spatial ability appears to be important to success in calculus even during the first semester. Perhaps some of the difficulties with application problems stem from a lack of spatial experience or ability.

When students enter primary school their world is three-dimensional. Our paper-and-pencil curriculum soon conditions them into two-dimensional thinking. For example, the ball becomes a two-dimensional picture. Then, in higher mathematics these students find it difficult to think about three-dimensional problems.

CONCLUSION

The authors and the other calculus teachers represented in this study are in agreement that application problems, weak algebra skills, poor problem solving skills, and lack of spatial ability are the major contributing factors to students' failure in first year calculus. The spatial ability of students needs to be nurtured throughout the curriculum. Likewise, meaningful application problems and problem solving exercises should be emphasized in all mathematics courses.

Further investigation into the identification of the specific weak algebra skills is warranted. Also, the study of the relationship between spatial ability and applications as they related to the study of calculus seems to be indicated. Do all pre-calculus curriculums include three-dimensional work? To what extent are students encouraged to think about spatial problems? It appears that mathematics curriculum should include and encourage spatial thinking.

It is not so obvious how to improve the algebra curriculum. Perhaps it is time to determine what can be removed from the pre-calculus curriculum in order to focus more on those algebra skills meaningful to calculus. Calculators and computers should be helping to make some of these decisions apparent.

REFERENCES

Anderson, R. D., & Loftsgaarden, D. O. (1987). A Special Calculus Survey: Preliminary Results. *Calculus for a new century*, Mathematical Association of America.

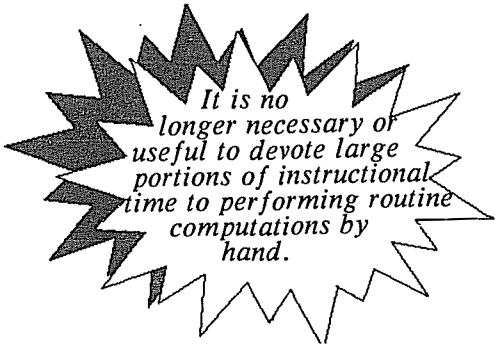
Steen, L. A. (1988). Calculus Today. *Calculus for a new century: a pump, not a filter*, (MAA Notes No.8), 10-13.

Young, G. S. (1988). Present Problems and Future Prospects. *Calculus for a new century: a pump, not a filter*, (MAA Notes No. 8), 172-175.

The authors are indebted to Elaine V. Alton who offered insightful guidance and support on this study.

INTERPRETING THE STANDARDS

(This feature is designed to help contextualize a number of recommendations from the NCTM curriculum and evaluation standards for school mathematics - we invite your comments.)



For school students in grades 5 to 8 many mathematical experiences other than computation should be the focus of the program. Computation is an important part of mathematical communication and should be combined with the power of calculators and computers. Students require more experience with evaluating and interpreting the results of computations done by machine.

Computations should support meaningful experiences in geometry, measurement and other areas of mathematics. Wide varieties of computation and estimation techniques including mental computation should be applied in different mathematical settings. Computation should arise from a need to know not simply for its own sake.

Emphasis should be placed on appropriate methods fro performing computation. Should you use mental arithmetic, written algorithms, a calculator or a computer? Students should translate results from various computational scenarios into solutions. Computation should be viewed not as a goal in itself, but as a multifaceted tool for knowing and doing.

Quotes interpreted by Alex Norrie
Peel Board of Education



SEVENTH INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION

The 7th International Congress on Mathematical Education (ICME-7) will be held in the city of Québec (Canada) from August 16 to 23, 1992. It will be the seventh in a series of congresses of the International Commission on Mathematical Instruction (ICMI), following those of Lyons (1969), Exeter (1972), Karlsruhe (1976), Berkeley (1980), Adelaide (1984) and Budapest (1988).

In an effort to meet the diverse needs and interests of the 3000-3500 expected participants, the program will cover all of the major areas of mathematics education at the elementary, secondary and post-secondary levels. Activities will include lectures, working groups, topic groups, workshops, short communications, posters, project presentations, and films, as well as exhibitions of textbooks, software, and other types of materials. Here are a few examples of themes that will be discussed during the congress:

Improving students' attitudes and motivation
Mathematics for early school leavers
Innovative assessment of students in mathematics
Students' misconceptions and inconsistencies of thought
The impact of calculators on the elementary school curriculum
The role of geometry in general education
Probability and statistiics for the future citizen
Modeling activities in the classroom
Students' difficulties in calculus
Undergraduate mathematics for different groups of students
Pre-service and in-service teacher education
Methodologies for research in mathematics education

Founded in 1608 by Samuel de Champlain, the city of Québec, capital of the Canadian province of the same name, is the cradle of French civilization in North America. Because of its unique charm, its historical past and its exceptional location dominating the majestic Saint-Laurent river, Québec is a privileged place which attracts tourists from all over the world. The ICME-7 congress will take place on the campus of Université Laval, which offers facilities and services making it a most convenient place to hold such an international event.

The Second Annoncement of the congress will be published in1991 and will contain detailed information about the program, as well as forms for registration, accommodation, and submission of short communications or posters. In order to receive it, please write to: **Congrès ICME-7 Congress, Université Laval**

COMPUTATIONAL PIZAZZ:
TEACH YOUR STUDENTS TO CREATE PUZZLES FOR THEIR PEERS.
MAGIC CROSS-OUT

by
K. Allen Neufeld
Faculty of Education
University of Alberta
Edmonton, Alberta.

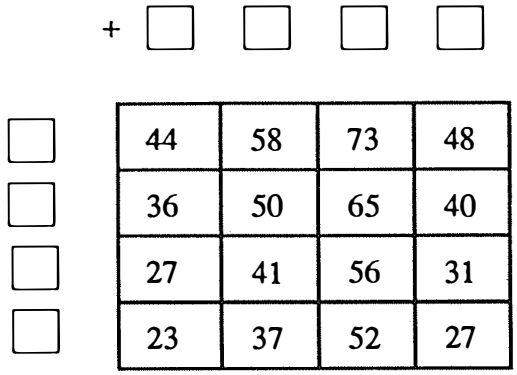
There aren't many children, or adults for that matter who do not enjoy the challenge of a puzzle. The purpose of this article is to provide teachers with a procedure for creating mathematical puzzles based on a selected computational skill. Once a teacher is familiar with the procedure, it is not difficult to encourage students to create puzzles for one another. Some children may not be able to create puzzles of their own, but they can still benefit by solving puzzles created by a classmate.

The 100 basic facts for a particular operation like addition (or multiplication) are often consolidated into a 10 by 10 array which is commonly called a table. Because the addends (or factors) are presented in order from 0 to 9 to the left and at the top of the table, the sums (or products) within the table exhibit fairly obvious patterns. The commutative property is illustrated in that the set of sums (or products) below the diagonal is symmetric to the set above the diagonal. The identity property for addition is illustrated by the first column and row of the addition table, while the identity property for multiplication is indicated by the second column and row of the multiplication table.

The puzzle activity that follows is based on the sums of a randomly chosen set of addends not necessarily restricted to the basic digits of 0-9. The activity is presented in four parts. The first part is based on a set of quotients which are also the addends for a previously designed addition table. In the second part, a procedure is presented which results in a surprising relationship between the addends and a selected set of sums within the table. The third part consists of a set of directions to assist teachers and students to design their own puzzles. Finally, the fourth part includes a set of questions to lead the reader to discover the rationale for the procedure.

Part 1 - Divide and Find the Missing Addends

Copy the chart into your exercise book.



Place the eight quotients from the following division problems into the correct boxes at the top and left of the chart. Each quotient is a missing *addend*. For example, which two quotients *add* to make 44?

a. $72 \div 3 =$ b. $96 \div 3 =$

c. $80 \div 5 =$ d. $55 \div 5 =$

e. $78 \div 3 =$ f. $84 \div 7 =$

g. $90 \div 6 =$ h. $82 \div 2 =$

Part 2 - Play "Magic Cross-Out" With Your Chart

a. Find the sum of the eight quotients which you placed in the boxes at the top and left of the chart.

b. Choose any number in the chart, circle it, and cross out all other numbers in the same row and column (Example - suppose you chose 41; your chart would now look like the one below).

44	58	73	48
36	50	65	40
~~27~~	(41)	~~56~~	~~31~~
23	37	52	27

c. Choose another number which has not been crossed out, circle it and cross out all other numbers in the same row and column.

d. Choose a third number, circle it and cross out as before.

e. Circle the one remaining number.

f. Find the sum of the circled numbers. Is it the same as the sum from part a?

Part 3 - Make a Magic Cross-Out Game for Your Friend

a. Draw a 4 by 4 chart of 16 squares.

b. Choose any eight numbers and lightly pencil one to the left of each row and on top of each column.

c. Find the magic sum of these eight numbers; write the magic sum on a slip of paper.

d. Fill in the sixteen squares by adding the appropriate rows and columns.

e. Erase the eight lightly pencilled numbers.

f. Ask your friend to play "Magic Cross-Out" by circling numbers and crossing out rows and columns.

g. Show your friend that the sum of the circled numbers is the same as your magic sum.

Part 4 - Why Does This Work?

a. Do you ever circle more than one number in any row or column?

b. Do you have to cross out the uncircled numbers?

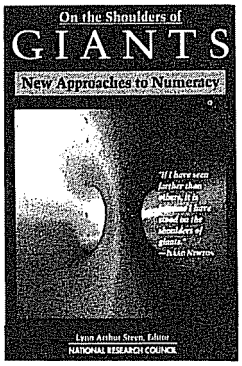
c. Could you circle the four numbers on the diagonal?

d. Is it sensible that the sum of the four numbers on the diagonal is the same as the eight "outside" numbers (addends)?

e. Would a 3×3 or a 5×5 chart work?

f. Would a 2×3 chart work?

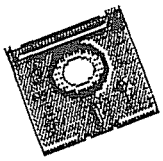
New from the Mathematical Sciences Education Board...



What mathematics should be learned by today's young people and tomorrow's workforce? **On the Shoulders of Giants: New Approaches to Numeracy** is a vision of the richness of mathematics expressed in essays on dimension, quantity, uncertainty, shape, and change.

ISBN 0-309-04234-8; 1990, 250 pages, 6 x 9, index, hardbound with jacket, $17.95. To order using VISA/MasterCard/American Express, call toll-free 1-800-624-6242, Monday-Friday, 8:30-5:00 EST. Call (202) 334-3313 in the Washington metropolitan area. Send a check or money order to National Academy Press, 2101 Constitution Avenue, NW, Washington, DC 20418. Please add $2.00 for postage.

COMPUTER TESTS CAN CHANGE EVALUATION: GREEN GLOBS, MONEY MANAGER, MASTERY AND COLLABORATIVE TESTING



by
Lawrence Sher & Patricia Wilkinson
Professors of Mathematics
Manhattan Community College
New York, N.Y.



The technology of computer games can provide a testing procedure that supplies direct motivation for students to spend more time on their mathematics and other subjects.

For example, in a Pre-calculus course, we have used a computer game called *Green Globs* (Dugdale & Kibbey, 1985) as a test. In this game, 13 globs of 1/2 unit diameter are randomly distributed on a graph of the plane. The students are then asked to find equations that, when graphed, hit as many globs as possible. The more globs hit with one equation, the higher the score. Students may start the game over and try another curve, if their equation barely misses a glob. We have found that our students will spend hours to improve their scores without much encouragement and without complaining. One of the authors was so captivated that he stayed up three nights trying to improve his score. The students simply don't realize that they are working hard to improve their learning.

This kind of learning is so different from our present form of testing -- one attempt, pass or fail, a school technique that is not often mirrored in society. The constant opportunity to fail can be destructive to students' confidence. We will show some computer non-destructive testing procedures that may motivate students to maximize their existing abilities in academic situations. A major method used will be a mastery procedure in which the test is not intended to be taken only one time, but to be taken many times to allow students to improve their scores. The random generator in computers allows a sufficient number of tests to be easily created so that multiple attempts become feasible.

Any computer mathematics program with a scoring system or for which a scoring system can be created can be used in mastery testing. *Green Globs* has the advantage that it can be used several times throughout a pre-calculus course. When the students knew only the equations of straight lines, a score of 30 was sufficient to achieve credit for mastery. By the time the students learned the equations for all the conic sections, the score needed to show mastery can be increased to 100.

We first introduced mastery learning techniques into mathematics classes at our large urban community college when the City University of New York began Open Admissions. Faced with large numbers of students under-prepared in arithmetic and elementary algebra, we instituted mastery learning and testing in remedial mathematics. We first divided the arithmetic course into modules, and then created multiple forms of an exit test for each module and several forms of the final examination. Mastery testing allows students to fail tests without penalty and to retake these tests, but requires that they pass <u>all</u> tests, with a close to perfect score and not just have a passing average. Our success with this program caused us to realize that the mastery learning strategy which worked so well for us with developmental courses could and should be extended to our upper level courses.

We discovered that the *Linear and Quadratic Graphs* program on the *Green Globs* disk provides just such computerized mastery tests for a pre-calculus course. This software shows a parabola on a grid. It asks the student to type the equation of the curve into the computer. The student's equation is then graphed on the same axes as the parabola, and students can see if they are correct or if not, they see how close their attempt came. If the students are correct, they move to a higher level and are given a more difficult parabola to

recognize. When the highest level parabola has successfully been identified, the student has mastered the parabola. The nice part of this program is that the student's attempt is graphed, and most importantly mistakes can be learning opportunities. *Linear and Quadratic Graphs* also has similar mastery tests for lines, circles, ellipses, and hyperbolas. Similarly, Kelly's *Discovery Learning in Trigonometry* provides mastery tests for trigonometric functions using the same method. The graphing section of *Calculus* (Broderbund Software), reverses this process by starting with an equation. Students are then asked to use a mouse to graph the equation. The correct graph is shown and students can compare this answer with the correct one. Because there is no scoring method in this software, the teacher must create print materials to use it as a test.

Collaborative learning, also called cooperative learning, is a very successful educational technique and is best known as a method through which minority students are able to achieve success in the mathematical sciences. Collaborative learning refers to a set of instructional methods in which students work in small, mixed-ability learning groups. Students are responsible not only for learning the material, but also for helping their groupmates to learn. When students have to organize their thoughts to verbalize and communicate their ideas to their teammates, they must engage in a process that greatly enhances their own understanding of content (Slavin, 1987). The collaborative classroom has a helpful, non threatening environment in which one can try out ideas and ask for assistance. After finding a disproportionate failure rate for well-prepared Black and Hispanic students in Calculus at the University of California Berkeley, Uri Treisman organized collaborative honours workshops for these students. Groups of four to seven students met to work out the answers to honours calculus problems. Students learned that there are various ways to look at problems and that you don't have to find them all by yourself. The collaborative learning strategy coupled with expectations of excellence dramatically increased minority retention in the calculus course, and led to success in later courses. This technique is now being used successfully at varied institutions ranging from prestigious four-year colleges to historically African-American colleges, to two-year college settings.

Surprisingly, in most successful collaborative learning programs, neither collaborative testing nor any team testing is a part of the student's evaluation. The Borough of Manhattan Community College Mathematics Department has received a National Science Foundation Calculus Reform grant to adapt the collabor-ative learning techniques of Treisman and others to minority urban community college calculus students. One of our successful strategies is collaborative testing.

Two serious questions must be addressed before we adopt collaborative testing. 1) Do the best students in the group do all the work while the others do nothing? 2) Do the weakest students not contribute even if most of the others work? We found some answers by videotaping a Manhattan Community College pre-calculus class using *Green Globs* as a collaborative exercise.

We found by observing the videotape that it was clear that almost all students contributed to finding high scoring solutions, and we did not find any student who was uninvolved. While this may not hold true for all collaborative exercises, we can say that at least in this collaborative exercise the better students did not dominate.

We have used *Green Globs* as both a mastery and competitive test. A pre-calculus class was divided into teams, and was told that any team which achieved a score above a given mastery level would get credit for a perfect score on a quiz. Also, the team that achieved the highest score would receive bonus points. In *Green Globs*, the students can get a new set of globs by pressing a key. Interestingly, a skill not usually evaluated is tested - whether students can determine if the arrangement of globs allows a good scoring opportunity, and whether they can find a better game board if there is a poor scoring arrangement of globs. We have not yet found a name for this skill and are open to suggestions.

Collaborative testing, (which a critical colleague recently reminded us is presently called cheating), combined with individual tests may prove a better indicator of success in a working environment than the present solo tests. In the business and science positions to which our students aspire, team work is considered a plus.

Money Manager (Chap & Sidewater, 1983) and *Estimation Skill Builder* (Rasmussen, 1983) are software packages that can be used as mastery or collaborative tests in an arithmetic course. *Money Manager's* problem is to plan a budget for eight weeks, and end up with the most money at the end. Since accidents are randomly distributed in the program, such decisions as whether or not to buy insurance prove critical. In *Estimation Skill Builder*, the student can get bonus points by either estimating quickly or getting an exact answer. The decision whether to estimate or to compute the actual result is the constant challenge.

A colleague at a sister institution of our university recently experimented with collaborative testing (Muir, 1989). Calling her method tandem testing she had students who were repeating a basic mathematics course study and take preliminary tests in pairs. Her success in increasing the number of students who passed the University's Freshman Skills Assessment Program (FSAP) Mathematics Test from 52% to 71% indicate the value of this testing strategy.

The importance of students being able to communicate mathematical ideas in writing cannot be over emphasized. If we expand the nature of this paper to consider evaluative instruments other than tests, some other interesting software can be considered. *Master Grapher* (Demana & Waits, 1989) can be used in either collaborative or individual student papers to find similarities and differences between functions. Any number of statistical computational packages can change the emphasis of papers in statistics courses from computation to analysis of data in a writing assignment.

We believe that improving education does not stop with improving teaching methods, but must include changing testing. Computer tests make mastery testing simple. While individual tests should be the main measure of a student's score collaborative testing used as a part of the student's grade will show teamwork skills that are not evaluated today.

An unexpected pleasure in our experience of using computer tests is the fact that so many strategies come together in a way that maximizes student motivation, involvement and success.

REFERENCES

Broderbund Software. (1987). *Calculus* [Computer program]. Sensei Software.

Chap, A. R. & Sidewater, D. A. (1983). *Money Manager* [Computer program]. Computer Age Education Inc.

Demana, F. & Waits, B. K. (1989). *Master Grapher* [Computer program]. Columbus Ohio.

Dugdale, S. & Dibbey, D. (1985). *Graphing Equations* [Computer program]. Pleasantville, NY: Sunburst Communications.

Dugdale, S. & Dibbey, D. (1985). *Green Globs.* [Computer program]. Pleasantville, NY: Sunburst Communications.

Kelly, J. (1982). *Discovery Learning in Trigonometry* [Computer program]. Conduit.

Muir, B. (1989). My Students Want To Take Math Tests! Tandem Testing With A Basic Skills Class. *Mathematics In College*, A Journal of the CUNY Mathematics Discussion Group.

Rasmussen, S. (1983). *Estimation Skill Builder* [Computer program]. Berkeley, CA: Edusoft.

Slavin, R. E. (1987). Cooperative learning and the cooperative school. *Educational Leadership 45*(November), 7-13.

QUINTE-ST.LAWRENCE MATHEMATICS ASSOCIATION

History

by
M. Elaine Harvey

Reaching back into the cobwebs of history for the roots of an organization is a strange and fascinating task. One may never know when conception actually took place or when the birthing pains of labour began. In the case of the Quinte-St.Lawrence Mathematics Association, the actual date of birth can be identified as November 6, 1974, but we must remember that there were mathematics educators of European origin in the Quinte-St.Lawrence area for close to two hundred years before that. As well the mathematics of the aboriginal peoples has not yet been identified or integrated into the curriculum by the descendants of the incoming settlers.

Perhaps the St.Lawrence College/OATM Conference held on September 26, 1970, was a catalyst for the conception of the QSLMA. (The Ontario Association for the Teachers of Mathematics was the precursor of OAME). Don Douglas of St.Lawrence College welcomed guests to this full-day conference. Bill Cruden, then principal of the Kingston campus and later president of the three-campus College, delivered the luncheon address entitled, "Possible Ways St.Lawrence College Can Aid Area Schools in their Mathematics Program."

The following year

> David Wing and Hugh Allen invited teachers of mathematics to an evening which they had organized at Duncan McArthur Hall (on November 25, 1971). The meeting was enlivened by Dr. Kenneth Fryer (whose topic was "A Mathematical Pot-pourri"). A hundred people were counted at the dinner, and afterwards three presentations were given by Alan Duffield, Donald Orr, and Hugh Allen. The evening was a great success.
>
> Dr. Fryer, during his talk, made several references to the nonexistent "Eastern Lake Ontario Mathematics Association". Apparently these remarks were too provocative to resist, for...a meeting (was soon) organized by Jim Boyd at Loyalist Collegiate (Kingston) of the new Association.

Prior to Jim Boyd's meeting, Dr. John Coleman provided the impetus for a "Meeting for Area Mathematics Teachers and Queen's Mathematics Department" which was held on February 13, 1974, with representatives from Belleville, Brockville, Campbellford, Elgin, Elginburg, Ernestown, Gananoque, Kingston, Napanee, Perth, Prescott, Smiths Falls, Sydenham and Trenton.

Then in May of that year, the area hosted the highly successful first annual conference of the reorganized provincial association--OAME--under the capable co-chairs, Hugh Allen and Bill Anglin. Jim Boyd headed the program committee which included Bill Higginson and Bruce Kirby. When asked to make a report to OAME Council about program plans, Jim said, "I have only one word to say--help!" The registration fee for members was $6.00, and, in spite of Jim's initial concern, excellent workshops were held all day Saturday and Sunday morning. The program booklet noted that this was the "83rd Convention of the Ontario Association for Mathematics Educators".

In the fall of that year Bill Higginson wrote

> Mark Twain once claimed that reports of his death had been "greatly exaggerated". In a similar vein rumours of the demise of our fledgling math group have also been false. Following a recent meeting of the planning group ([Hugh] Allen, [Gaye] Beckwith, [Jim] Boyd, [Terry] Hicks, [Bill] Higginson, [Bruce] McColl, [Bill] Woodside) there are plans for a number of activities to be held during the 1974-75 academic year.

The letter went on to announce a meeting to be held in November:

> Part of the plenary session on 6 November will be the presentation of a proposal for affiliation of our group with OAME. Ideas for names of the organization (LALGFMA would not seem to be an acronymist's delight but use your imagination) will be gratefully received.

The parental struggle to name the baby was influenced by all relatives and friends. LALGFMA (Lennox and Addington, Leeds and Grenville, Frontenac Mathematics Association) was, not surprisingly, rejected. A further letter from Bill gives more suggestions:

> It has been suggested that a possible name for this organization be the Kingston Area Mathematical Association of KAMA...Quinte-St.Lawrence Mathematical Association is another possibility. OAME

councillors, Jim Boyd, Hugh Allen, David Wing and Doug Evans will lead a writing team which will prepare a proposal for presentation at the meeting." A roast beef dinner will be available at LCVI for $2.75.

This inaugural meeting held at Loyalist C.V.I. in Kingston began with coffee at 4:00pm on November 6, 1974. Leo Jonker gave a session on "An Extension of Euler's Relation for Convex Polyhedra to Toruses andpheres". After-dinner sessions were conducted by a panel chaired by Bruce McColl on CAAT programmes; by Hugh Allen on "What I'm Doing with My 240's [MAT 2G0's]; by Jim Tennant on the "Involvement of Elementary School Teachers in KAMA"; and by Jim Boyd on "Acceleration along a Straight Line: some problems".

Then at 8:00 pm the session entitled "Formation of KAMA: An OAME Chapter--Reactions and Resolutions" resulted in the birth of the Quinte-St.Lawrence Mathematics Association. David Handley later wrote,

As one of the 80 or so present at the inaugural meeting, I was not in favour of the name "QSLMA" as it was difficult to pronounce. I favoured the acronym "Quasimodo" which I felt had a certain 'ring' to it. It stood for "Quinte and Seaway Institute for Mathematical Operators Development Organisation". Unfortunately, the vote went against the literary title!!

It was probably Jim Boyd, the first president, who coined the nickname, "Quizzelma", which is used by some with affection to this day. Other founding officers were Jim Tennant--vice president, Hugh Allen--secretary-treasurer, and councillors David Handley, Heather Breck, Dale Shannon and Steve Wight.

While choosing a name had been a difficult task, writing a constitution was easy. It wasn't done! QSLMA founders decided to follow the principle of commonlaw and dispense with a constitution. In the early 1980's Norah Wilson and Elaine Harvey were asked to draw up a non-constitution, but even that idea was given up and the constitution question was again discarded as being unnecessary and restrictive.

In these early years dinner meetings were held three times a year after work. The location was generally a school in the central area of the region although meetings have been held in Trenton and Prescott. At the February 1975 meeting at Duncan McArthur Hall, John Bates and Aina Villa from Davenport School, Toronto, spoke on "The Teaching of Number from K-6". John is now director of the Frontenac County Board. In December, "A Forum on Improving Student Attitudes Towards Mathematics" was held at Jeffrey Hall, Queen's University.

Jim Tennant gave a session at the March 1976 meeting entitled "Problem Solving is a Problem!" The October meeting at Napanee Secondary School was accompanied by dinner at the Campbell House. "You pay $2.00, QSLMA pays $2.00 for your meal."

A memorable meeting was held in March, 1977, at Sydenham High School when George Scroggie, Ministry of Education, led a general session for all before dinner entitled "The Intermediate Revision - A Progress Report". The church basement dinner was excellent but no doubt George will not forget the roasting he got from the members who were not very happy with the ministry plans for the intermediate division.

The April meeting that year was held at Gananoque Secondary School with dinner at "The Inn" overlooking the St.Lawrence River. A survey at that time determined that members continued to support holding three meetings per year.

Hugh Allen's suggestion for a QSLMA logo was adopted during the fall of 1977 and has been used ever since. It is thought to be related to the ancient symbol for the Society of Pythagoras. At the November meeting at Napanee S.S., Peter Battey-Pratt spoke on Spin Theory. He noted, "There is a surprising new mode of rotation that you probably won't believe even when you see it!" He was right! We didn't! But we should have! Peter actually demonstrated it.

At the February 1978 meeting at Queen's University, Jim Whitley speaking on "Man and Measurement" defined an acre as a garden plot equal to the amount of ground you can plough in one day with a team of oxen. Hugh Allen discussed "Splitting the Triangle" in Gems for Seniors, and Elaine Harvey spoke on "Equation Solving with a Difference". Bob Gunter, as secretary, wrote to the members soon after,

As the meal at the last meeting cost only $3.25 and you paid $5.00, you have a credit of $1.75. You could either (circle one) donate . . . , refund . . . , deduct from cost of April meeting, or donate it to the Hugh Allen Hockey Memorial Fund.

Dr. John Coleman wrote, "I have heard that the meeting went extremely well and the supper was superb at quite a reasonable cost."

The April meeting that year at Gananoque Secondary School included the session by Margaret Buskard and Linda Connor, "Are you having problems sleeping at night wondering how to (a) follow the New Math guidelines, and (b) use activities meaningfully, and (c) provide for enrichment, and (d) individualize your instruction?" Peter Taylor's session, "Why Johnny Can't Loop the Loop", actually began with a "once-upon-a-time" fairy tale. He advised teachers, "Organize our lives so that our principal activity is not teaching but learning. Find the right problems at the right time. They come out of my learning." Then he launched into some exciting stimulating problems.

Sessions in 1979 included Mary Jane Ellison's and Bill Higginson's discussion, "Frames: A New Approach to the Teaching of Number" based on a method for introducing basic number concepts developed by John Bates of the Toronto Board, and Hugh Allen spoke on "How to get a lot from a little".

The newsletter for a meeting in 1979 read, "See you at Sydenham on November 8 for a little math and an old time church supper!" One speaker was the late Peter Ullrich, math consultant with the Ottawa Board of Education, who spoke on the Magic Square. Dr. John Coleman, head of Mathematics at Queen's arrived late for the church dinner and ate in the kitchen with the women. They seemed to enjoy it as much as he did.

A 1981 newsletter curiously stated, "Because OAME have run out of back copies of the Gazette and Abacus, we cannot accept new enrollments in OAME for the 1980/81 year. However, new members will be welcome to QSLMA." At the April meeting that year Bill Higginson spoke on "Stephen Leacock and Stress Analysis of a Strapless Evening Gown" and Dick Willmott spoke on "What and Why are Fuzzy Sets?" noting that the set of tall men is fuzzy.

There was a very small attendance at the Feb. 1982 meeting at Duncan MacArthur Hall in spite of the "pay-if-you-eat" cafeteria facilities. This plan had been instituted to avoid the problem of the previous year when several people paid for dinner, but were unable to attend the meeting because of a blizzard that evening. Ann Jones from the MOE in Ottawa spoke on the effect of microcomputers on the Ontario mathematics program, and Elaine Harvey gave the following introduction to one of the speakers:

> **Who graduated in math and psychology from UBC? Who was hired to teach math at Sharbot Lake High School by Bill Higginson's father? Who taught Bill Grade 13 math? Who later taught at Nepean in Ottawa and then George Henry in North York? Who was math chairman at Woodbine Junior High School? Who was educational supervisor in Math for TVOntario? Who has been a freelance mathematician for the past six years? Who co-authored the Aha! Box and the Paradox Box with Martin Gardiner (who ran a popular series on recreational mathematics in the Scientific American)? Aha! It's Bob Tappay.**

Concerned with falling attendance at meetings, in 1983 president Norah Wilson and secretary Beth Thompson instituted changes that revitalized the organization. The number of meetings was reduced from three to two per year and they were not longer located in the high schools. In April the Royal Military College hosted a meeting for the first time with an elegant banquet in the officers' mess arranged by Professor Lyman Smith. It was such a success with a hundred and fifty participants that quite a number of dinner meetings have been held there since that time.

The next notice of meeting read, "The splendid R.M.C. banquet is a tough act to follow but after some hard and devious negotiations we can offer a superb ROAST BEEF DINNER with all the trimmings at the Queen's Faculty Club." Members were encouraged to attend by asking them to guess the number of hockey sticks in the pickle jar on the registration table and win a free registration to OAME '84 in Kingston. At this meeting Hugh Allen commented in a talk,

> **It's exciting to have discovered something. It's somewhat exciting to hear of someone else's discovery, but it's much more exciting to discover it yourself. But there is a need when you discover something to talk about your discovery.**

And then he introduced a new congruence postulate:

> **When two sides and a non-included obtuse angle from one triangle are respectively equal to two sides and a non-included obtuse angle from another triangle, the triangles are congruent.**

QSLMA hosted the highly successful and well attended 1984 OAME conference at Kingston. Elaine Harvey was conference coordinator and Brian Gallagher was program chair. For the

first time the conference ran from Thursday to Saturday instead of Friday and Sunday. Now mother and father mathematicians could celebrate Mother's Day at home with their children.

The program booklet mentioned,

> You may have noticed the sequence of numbers on your OAME '84 pen . . . This sequence comes to you courtesy of Peter Battey-Pratt (who probably stole it from somebody else). Anyway, we are having another contest! If you can figure out how the sequence continues, and why, you may win a prize.

The sequence was:
0, 1, 10, 2, 100, 11, 1000, 3, 20, 101, . . .

Unfortunately, Ken Fryer was fatally ill and was unable to attend the conference. His endearing dry humour was evident in the session he had planned to offer: "Some Mathematical Musings: The title says it all. Don't come if you're expecting to be enlightened or uplifted mathematically." Flora MacDonald, MP for Kingston and the Islands, remarked during her presentation at this conference, "Real reform can't be brought about in a year or two."

At the annual banquet Elaine Harvey recognized the recent death of Grace Miller, mathematician in her own right and wife of Dr. Norman Miller, who was then 95 years old. Elaine noted that he "had a distinguished career in mathematics and is regarded by many of his students as the best teacher they ever had. "And there is a reason." he said to me once. "Teaching is an art. Don't ever forget it." Dr. Miller was a life member of OATM.

A newsletter of 1986 read,

> Our Fall extravaganza on Oct. 17 is to be held in the picturesque setting of the Hotel Kenney on the shores of the Rideau Canal at Jones Falls.... We hope you will be able to attend. In fact, why not relax and stay over night at the Hotel?

This was followed in the Spring by another successful RMC affair. The announcement for the next meeting began:

> Our Fall extravaganza on Oct. 27 (1987) is to be held in the picturesque setting of the Carriage House Motel on the shores of the St. Lawrence River at Gananoque. (Does this have a familiar ring? It seems QSLMA members are fussy about the setting!) Here we will celebrate the 300th anniversary of the publishing of Newton's "PRINCIPIA".

Joan Routledge, past president of OAME and mother of math-head, Doug Routledge of QSLMA, spoke on the topic "Key in the Brain" about the need for K-6 students "to develop a mental agility with numbers and operations" now that "calculators and computers have taken over most of the tedious number crunching required in real-life situations".

Dave Handley as president in 1987 cautioned members going to the annual conference, "To those of you going to Sudbury - enjoy yourself (but remember you are representing QSLMA!!)."

Another successful meeting was held at RMC in April 1989 with 132 teachers attending. A report to Council stated, "90 people enlisted in the Canadian Navy for the evening so that they could enjoy a roast beef dinner in the Senior Officers' Mess."

In October of 1989 QSLMA joined COMA to offer a two-day conference at the Opinicon Lodge, Chaffey's Lock. The keynote address by Maurice Liebovitz, education officer, MOE was entitled, "Towards 2001". This was the first annual joint sponsorship of this conference which was first initiated by COMA in 1986.

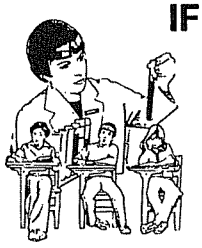
Ever interested in the social as well as the mathematical, a 1990 QSLMA invitation read, "You are invited to bring your spouse to the gala banquet, followed by the toe tapping music of the Limestone City Jazz Band. Math sessions first. Meet your partner in the Officers' Lounge at RMC." Do you think the spouses know the quotation from August de Morgan, "It is easier to square the circle than to get round a mathematician?"

Year	President
1974-76	Jim Boyd
1976-77	Jim Tennant
1977-78	Hugh Allen
1978-80	Elaine Harvey
1980-81	David Borrowman
1981-82	Peter Battey-Pratt
1982-84	Norah Wilson
1984-86	David Handley
1986-88	Doug Evans
1988-89	David Wing
1989-91	Brian Gallagher

Editors' note:
This historical account from QSLMA was inadvertently omitted from the Centennial Issue, April '91. Our apologies to QSLMA and our readers.

Benchmarks III and Ron Lancaster's Problems Column will appear in the April issue.

IF THIS IS SCIENCE, WHY ARE WE DOING MATH?



by
Kate Le Maistre
Faculty of Education
McGill University
Montreal, Quebec

For three consecutive years, my son's October science program consisted of collecting leaves. One year, the class researched the names of the leaves and classified them as well, but for the other two years, they just collected them. At about the same time, I realised that my son was experiencing the common Grade 5 confusion between perimeter and area, so I took one of the maple leaves and asked casually: "I wonder how many ways we can think of to find the perimeter of this leaf?" His answers helped him to clarify his ideas and gave me the idea of using more open-ended questions in mathematics class. At the same time, I realized that such problem-solving often occurs naturally when science activities are involved. Since then, I have asked such open-ended questions of a variety of classes at the Grades 4 - 8 level, and taught some math concepts by presenting science problems.

One of the common questions in math class is: "Why do we need to do this?" Teachers of children in the upper elementary grades soon realise that their students need justifications. Just as important, our curricula are so densely packed that we sometimes ask ourselves that same question: "Why are some topics included in the math program?" Are they there just to solve the word problems on the right hand page of the two page spread of the textbook? Or is there a practical rather than a contrived - "textbook" - reason for learning the math content?

It is certainly true that abstract skills become more meaningful to children when the skills are used in a realistic context. Putting this another way, it is easier to learn a mathematical process when it is needed to resolve a problem situation, and this is true of many other real-life situations; when did *you* learn to program the record feature of your VCR? When the salesman showed you in the store? Or when there was a late movie you wanted to tape for viewing at a more convenient time?

Teachers report that "less able" students often perform better in a problem-solving situation where the problem is real-life, and needs the application of real-life skills as well as mathematical skills. This is possibly because our math programs expect a high level of abstraction of our students. Even when the textbook talks about visits to the supermarket, or planning for camping trips, unless the students are *really* going to the supermarket, or *really* planning a camping trip for themselves, it is difficult for the situation to move out from the pages of the textbook for all but the best students in your class. Try grouping these practically-oriented students with the computational experts in the class and watch the success of both groups improve.

We know that nothing succeeds like success. Because it is easier for most students to solve problems based on real situations, their success rate improves and correspondingly, so does their confidence level. How many adults do you know who say "I could never do math", and when prodded, recollect that it was between Grade 6 and Grade 8 that their math skill deteriorated? In these grades, textbooks introduce few startlingly new math concepts - same content, bigger numbers is the usual analysis - but the situations often become more removed from the students' experience. This is compounded in our classrooms if students have limited experiences of the world, and part of our time must be spent in broadening their horizons - as well as teaching math.

One of the best ways of interesting students in events outside their immediate world is for

teachers to share their own experiences. Part of my background is in the physical sciences, so it's natural for me to use questions from science when I want to give some purpose to teaching mathematical concepts. Here are some challenges that are appropriate for upper elementary or junior high school students. All of them involve finding the answer to a question. All of them involve the manipulation of simple and cheap materials. All involve creativity, resourcefulness and enjoyment and all are appropriate for either the math slot or the science slot in your timetable.

Can you ... find the perimeter of a maple leaf?

(This activity will identify the students who are confusing area and perimeter. It helps teach the concepts of perimeter and the use of indirect measurement. It will amaze you with the creativity that students demonstrate.)

Can you ... devise a method to measure the thickness of a piece of paper?
... the volume of a raindrop?
... the mass of one crystal of table salt?
... the area of the head of a pin?
... the diameter of a hair from your head?

(All of these activities involve indirect measurement; the management of small numbers; the use of calculators; measuring instruments; metric units and conversion between units. They also show children that sometimes, an exact answer is impossible with the instruments available to us, but that an approximation is useful. Here's an example of the use of estimation techniques where the student can't solve first, then round off to give an "estimate".)

Can you ... build a bridge out of drinking straws which will span 0.5 m and support a mass of 100 g?

(The strength of a triangle is a fundamental concept to civil engineers, and, it seems to me, one of the few practical reasons for teaching students about triangles. This activity can be made more challenging for junior high school students by increasing the span and mass required, or varying the materials allowed, or by asking them what is the greatest mass that their bridge can support. In other formats, it is a common challenge for undergraduate engineers.)

Can you ... build a glider that flies farther than anyone else's?

Can you ... build a parachute that floats more slowly than anyone else's?

(Both these activities involve the scientific principle of controlling variables, better called a "fair test" with young students. If you want to get into such depth, you can also use these activities to teach some basic principles of aerodynamics. If not, your students will still have plenty of opportunity to measure lengths and calculate areas.)

Can you ... design a pendulum to measure seconds?

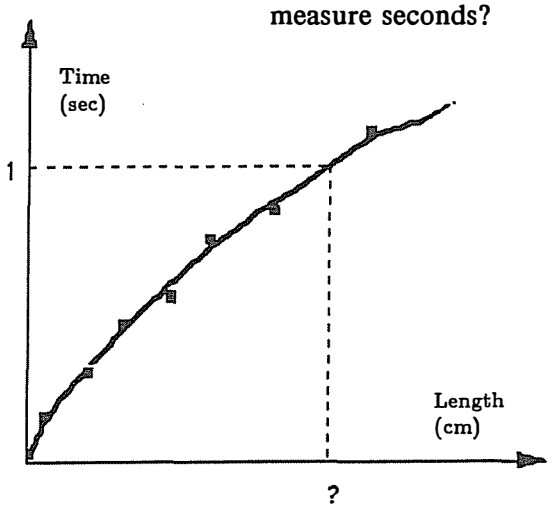


Figure 1: Time vs Length for a Pendulum

(Here is another opportunity to practise measuring length, since the period of a pendulum depends only on the length of the string. Most students will not know this, and

will assume that the mass on the end of the string makes a difference too. Allowing plenty of time for free exploration of these factors is an excellent application of a constructivist teaching approach. You should encourage very careful measurements, as students tend to bend their results to fit their hypothesis. This is an activity which allows interpolation from a line graph, and could be used with students ready to draw and interpret line graphs. Younger children can solve the problem effectively by a trial and error approach.)

Can you ... find the height of a tree or a tall building?

(Some people are familiar with a method for finding the height of inaccessible objects using a clinometer. Since this involves fairly careful measurement of angles, it is difficult for many elementary school children. If they do not measure the angles accurately, the results are so inaccurate that children are discouraged. Also, the application of the formula for the calculation is meaningless for most upper elementary children.

There are other methods involving ratios and the measurement of the shadow cast by the tall object compared with the shadow cast by a child. The following method, which does not depend on a sunny day, is based on similar triangles, but involves only two measurements of length, and no measurement of angles. The only requirement is that you can stand back from the tree a distance equal to the height of the tree. This method does not usually appear in textbooks, so it will be described in more detail.

Note how much mathematics and how little equipment is involved in this simple activity, and how the activity will develop mathematical vocabulary in a casual way. Students will work with isosceles right triangles, similar triangles, measurement of length and ratios. They will also learn how to fold a piece of rectangular paper to make a square.)

Find the Height of a Tree

1. Fold a piece of paper on a diagonal. Fold up the rectangular strip at the bottom. What is the name of the figure you have made? (Look at the lengths of the sides. What can you say about two of the lengths?) Hold your paper so that the extra rectangular strip is parallel with the ground.

2. Hold the triangle to your eye and sight along the folded edge. Have your partner stand alongside you to make sure the bottom edge of the paper is parallel to the ground.

3. Move backward or forward until you can see the top of the tree at the tip of your triangle.

4. Measure the distance, d, from you to the object, and the distance, h from your eye to the ground. Explain why (d + h) is the height of the tree.

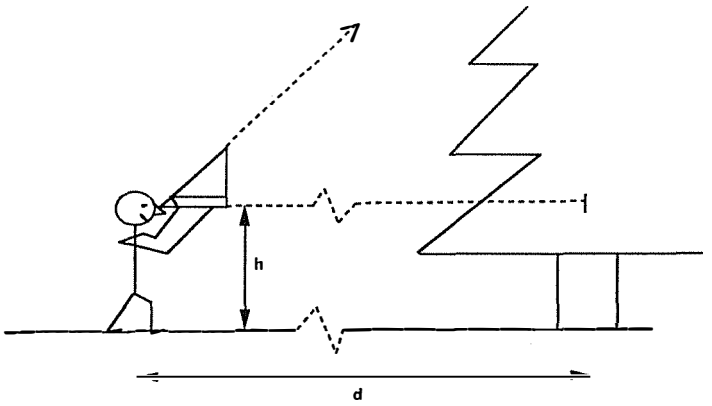


Figure 2: How Tall is the Tree?

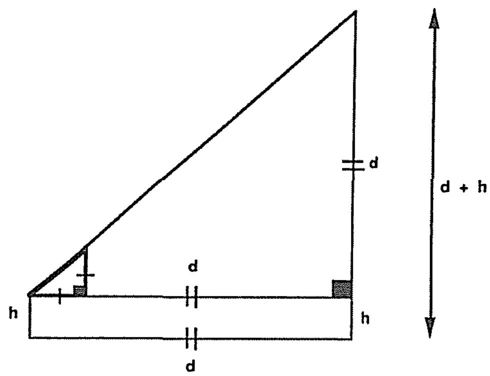


Figure 3: Calculating the Height of the Tree

All these activities are appropriate for children around middle school age and all of them employ objectives from the math and science curricula of most authorities. They have the advantages of being cheap and easy to perform and - most important - they are fun for students. I hope some of them will fit into your classroom needs or that they will act as triggers for you to think of other activities.

NOMINATIONS FOR OAME/AOEM COUNCIL

Each year elections are held for President-Elect, 3 Directors, and 7-10 Councillors for the Council of OAME/AOEM. Nominations for these positions are submitted to:

Michael Sandblom
Past President O.A.M.E./A.O.E.M.

Please include the nominee(s)' qualifications with the Nomination Form below to be received by M. Sandblom **no later than 4:00 p.m. January 13, 1992.** The nomination committee will make a final selection of those nominees who will stand for election. Their names and qualifications will be published in March, 1992. Voting will be by mail by members of OAME/AOEM. The results of these elections will be made known at the Annual Meeting of OAME/AOEM in May 1992.

>>

NOMINATION FORM

I Nominate

Address ______________________________

____

____

for the position(s) of: _______President-Elect
_______Director
_______Councillor

______________________________Nominator

______________________________Address

____

MAKING SEQUENCES EASY WITH A SCIENTIFIC CALCULATOR

by
Richard Dewey

The following is an extension of work in the O.A.C. Algebra geometry course both in the areas of induction and the matrix solution of linear systems.

A question often found is one like:

> Conjecture the formula for the sum of the first n terms of the sequence: 3, 3, 6, 12, 21, ...

The resource book *Finite Differences* (Seymour & Shedd, 1973) gives an interesting approach to problems of this type. The following is a variation to their solution that utilizes matrix algebra. In addition, we will show how calculators like the Texas Instrument TI-81 and the Sharp EL-9000 can be used to easily produce a formula for either the nth term or the sum of the first n terms of any sequence whose general term is a polynomial.

Let us consider the above sequence:

t_1	t_2	t_3	t_4	t_5	...
3	3	6	12	21	...

We will now calculate the differences between the terms and list them below the original sequence

t_1	t_2	t_3	t_4	t_5	...
3	3	6	12	21	...
	0	3	6	9	

and continue the process of taking differences until we have reached all zeros

t_1	t_2	t_3	t_4	t_5	...	
3	3	6	12	21	...	
	0	3	6	9	...	
(1)		3	3	3	...	steady state of 3s.
			0	0	...	

The number of steps required to produce this steady state of differences is the same as the degree of the polynomial representing the general term t_n. In the above case the expression would be:

$$an^2 + bn + c$$

There are a number of ways to solve for the values of a, b, and c.

First, one could create a function: $f(n) = an^2 + bn + c$ and substitute values 1, 2, 3 for n.

$$\begin{array}{ll} f(1) = a + b + c & = t_1 \\ (2) \quad f(2) = 4a + 2b + c & = t_2 \\ f(3) = 9a + 3b + c & = t_3 \end{array}$$

Producing the linear system

$$\begin{array}{ll} a + b + c & = 3 \\ 4a + 2b + c & = 3 \\ 9a + 3b + c & = 6 \end{array}$$

Solving this system results in the values

$$(a,b,c) = (\frac{3}{2}, -\frac{9}{2}, 6)$$

$$\therefore t_n = \frac{3n^2 - 9n + 12}{2}$$

An alternative shorter solution is provided in *Finite Differences*. Replacing t_1, t_2, t_3 by $f(1)$, $f(2)$, $f(3)$ gives

$a + b + c$,	$4a + 2b + c$,	$9a + 3b + c$, ...
	$3a + b$	$5a + b$
		$2a$

By comparing this to (1) we get:

$2a = 3$, $3a+b = 0$, and $a + b + c = 3$ and, therefore,

$$(a, b, c) = \left(\frac{3}{2}, -\frac{9}{2}, 6\right)$$

The following solution stems from a matrix solution to the system of equations produced in (2). The matrix equation would appear as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} t_1 & t_2 & t_3 \end{bmatrix}$$

or $A\ X = C$

To solve this equation one must first find the inverse of the coefficient matrix A

$$A^{-1} = \begin{bmatrix} 0.5 & -1 & 0.5 \\ -2.5 & 4 & -1.5 \\ 3 & -3 & 1 \end{bmatrix}$$

Since $A^{-1}AX = A^{-1}C$
$X = A^{-1}C$

What is interesting about this approach is: *for any sequence requiring two steps to reach a steady state* the matrix A will be the same as the one above and, therefore, *the inverse matrix will always be the same.*

If one has a scientific calculator which has the capability of processing matrix arrays, not only can one define A and calculate A^{-1} but one can make a *permanent* record of A^{-1} for future use. It then becomes a simple matter to define the matrix C and find the product $A^{-1}C$. The resulting matrix will provide the coefficients of the general term t_n.

This process extends to matrices of m dimensions. Therefore, depending on the storage capabilities of the calculator used, one can easily find a statement of the general term of a sequence or an expression of the sum of the terms for any sequence which reaches a steady state after n differences. (Note that the expression for the sum of n terms will have a degree one greater than the expression for the nth term.)

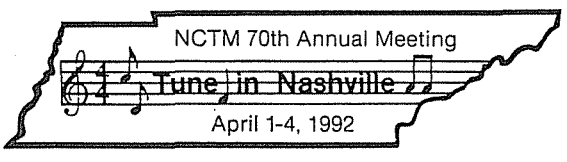
In general:

$$A = \begin{bmatrix} 1^n & 1^{n-1} & 1^{n-2} & \dots & 1^0 \\ 2^n & 2^{n-1} & 2^{n-2} & \dots & 2^0 \\ 3^n & 3^{n-1} & 3^{n-2} & \dots & 3^0 \\ .. & .. & .. & - & .. \\ .. & .. & .. & - & .. \\ .. & .. & .. & - & .. \\ n^n & n^{n-1} & n^{n-2} & \dots & n^0 \\ (n+1)^n & (n+1)^{n-1} & (n+1)^{n-2} & \dots & (n+1)^0 \end{bmatrix}$$

Each of the inverses of these matrices could be stored for future use, making the tedious task of *conjecturing* the formula for the nth term or the sum of the first n terms an interesting application of matrix algebra.

BIBLIOGRAPHY

Seymour, D. and Shedd, M. (1973). *Finite differences: A pattern-discovery approach to problem solving*. Palo Alto, CA: Creative Publications.



FIVE STEPS TO TEACHING PROOF IN THE SENIOR MATH CURRICULUM: A COOPERATIVE LEARNING APPROACH
Part I

by
Brendan Kelly
University of Toronto

INTRODUCTION

THE ROLE OF PROOF IN THE SENIOR CURRICULUM

As the computer and the graphing calculator become standard tools in the mathematics classroom, mathematics curricula will undergo dramatic changes. Reading and interpreting graphs from a visual display will emerge as an important mathematical skill, obviating the need to sketch functions by laboriously creating a table of values. (This gives a new meaning to the term functional literacy!) Systems of linear equations will be solved by computing the inverse matrix of the coefficients, if it exists, or by an iterative method rather than employing tedious computational methods of elimination. Many traditional topics in mathematics will be eclipsed by topics from contemporary mathematics, such as fractals and the mathematics of chaos, which rely on iteration and recursion. As we move toward the year 2000, we will participate in a complete renovation of the high school mathematics curriculum more far-reaching than in any previous era!

During this new era of change, mathematicians and mathematics educators will be re-evaluating the rationale for mathematics as part of the high school curriculum and deciding what is important and what must be discarded. Any search for what is fundamental to mathematics will ultimately yield the concept of proof as a prime invariant, pervading all topics and transcending all technological advances. Without proof, we have no theorems, only conjectures. Even problem solving in mathematics may be described as a process beginning with exploration followed by conjecture and culminating in proof.

Unfortunately, too many of our students graduate from high school perceiving mathematics as a collection of skills or recipes to be applied to the solution of problems in the way that the arithmetic operations are applied to the solution of trivial numerical computations. They regard proof as a redundant abstract formalism which they rarely encounter except in Euclidean geometry. Students are often heard to ask, "Why are proofs offered in the study of such things as the remainder theorem and the binomial theorem when in fact, tests and examinations require only that the student know how to apply them?".

A strong case could be made for the assertion that one of the fundamental objectives of all high school mathematics programs is to help students learn to formulate rational arguments as well as analyse the validity of arguments presented to them. The Ontario curriculum guideline, Mathematics: Intermediate and Senior Divisions: Part III (1985), includes proof as one of the process components to be incorporated in mathematics instruction at all grade levels from 9 through O.A.C.. On page 18 of that document we find the following statement:

> ...the teacher should remember that it [an understanding of proof] is an out-growth of a mixed set of skills, habits and attitudes. These include being aware of assumptions, listening to and evaluating arguments, recognizing patterns (including cases where patterns cannot be extended without proof), and being willing and able to think hypothetically.

In their blueprint for mathematics curricula of the 1990's titled Curriculum and Evaluation Standards for School Mathematics (1989), the

N.C.T.M. (National Council of Teachers of Mathematics) presented fourteen standards for high school mathematics. In the discussion of the third standard called Mathematics and Reasoning, we find the following:

> Inductive and deductive reasoning are required individually and in concert in all areas of mathematics. A mathematician or a student who is doing mathematics often makes a conjecture by generalizing from a pattern of observations made in particular cases (inductive reasoning) and then tests the conjecture by constructing either a logical verification or a counter-example (deductive reasoning) .... In addition, this standard proposes that college-intending students should learn the more formal methods of proof required for college-level mathematics. ...A third goal, also a departure from the existing curriculum, is to give increased attention to proof by mathematical induction, the most prominent proof technique in discrete mathematics.

COOPERATIVE LEARNING

Concurrent with the revolution in the content of school mathematics curricula there is a wide-spread challenge to the traditional ways of teaching mathematics. The trend toward incorporating peer tutoring and group learning in the classroom has led to calls for the use of cooperative learning as one of several teaching strategies in mathematics. While large group instruction may continue to be the predominant instructional format in mathematics, it would seem that there are several opportunities in the curriculum to employ cooperative learning techniques and in so doing enhance both teaching and learning. To this end, we believe that it is useful to share with colleagues some examples of mathematics topics which we have taught successfully using the cooperative learning format.

In this article, we attempt to provide a model to demonstrate how the concept of proof and its evolution might be explored with senior level students in a series of five lessons structured in the cooperative learning format. The intent is to cast the development of proof in its historical context and to show how the quest for certainty led through a cascading sequence of complexities to a point where it was recognized that absolute certainty is an unattainable goal. This development presents the evolution of proof in five independent but sequential lessons. Each contains a brief presentation followed by one or more teaching examples which might be used to evoke discussion. A set of investigation exercises is embedded in each lesson for students to explore and discuss in groups. It is intended that each group will report one or more of its explorations to the class to allow for some important follow-up discussion. Additional investigation exercises are provided as homework to ensure that all students are involved in the learning. Since these five lessons span too many pages for a single issue of the *Gazette*, we are presenting this article in five parts of which this article is part I. We hope that you will find parts or all of it useful and that this might encourage you to contribute further examples of mathematics topics amenable to this instructional format.

LESSON 1.

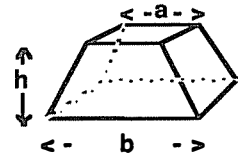
THE DAWN OF MATHEMATICAL PROOF: INDUCTIVE THINKING AND CONJECTURE

Historians of mathematics do not know when mathematics first emerged in its earliest forms though they agree that place value, and the four arithmetic operations may have been in use as early as 5700 B.C. in Mesopotamia. (Of course, the notation and the algorithms we use today were not developed until the last few centuries.) Furthermore the ancient Egyptian civilization between 4000 B.C. and 1800 B.C. used mathematics in measurements related to the design and construction of impressive structures such as the great pyramids.

One of the most remarkable mathematical achievements of that civilization was discovered in 1930 when the translation of an ancient Egyptian document called the Moscow papyrus (c 1850 B.C.) revealed a procedure for calculating the volume of the frustrum of a particular truncated square-based pyramid! Though the ancient Egyptians had no algebraic notation for expressing this procedure as a formula, they described verbally a rule for computing the volume which would be expressed today in the general case using the formula:

$$V = \frac{1}{3}h(a^2 + ab + b^2)$$

where h denotes the height of the frustrum and a and b, the sides of the top and bottom faces.



How this procedure was discovered is one of the great enigmas in the history of mathematics. Most historians believe that it was neither derived deductively nor proved in the general case. Prior to the ancient Greek civilization in the first millennium B.C., mathematics was relatively unstructured, containing specific, albeit ingenious techniques related to the solution of particular problems. There is no evidence in any of the ancient documents of proofs of the kind we have today. It is believed however, that impressive discoveries of the kind described above were happened upon by what is called inductive thinking. That is, long tables of measured volumes corresponding to particular values of a and b were compiled and patterns were sought relating the values of a, b and V. Clearly the discovery of a formula by inductive thinking does not constitute a proof, but rather an educated guess which mathematicians call a conjecture. The following example shows how we could use inductive thinking to conjecture a formula for the sum of the natural numbers from 1 to n.

TEACHING EXAMPLE

Use the table below to help you conjecture a formula in terms of n for S(n), the sum of the natural numbers from 1 to n.

n	1	2	3	4	5	6	7	...
$S(n)$	1	3	4	10	15	21	28	...

Solution: If we plot the values of S(n) for these values of n we obtain the graph in Figure 1.

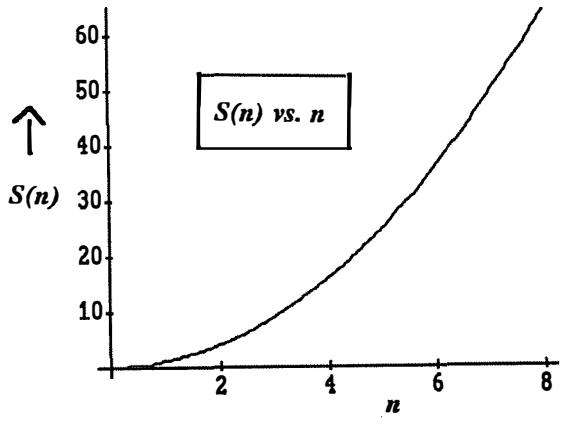


Figure 1

n	1	2	3	4	5	...
n^2	1	4	9	16	25	...
S(n)	1	3	6	10	15	...

We observe in the table that S(n) is less than n^2. However, closer study of the table reveals that $n^2 + n$ is double the value of S(n) for all values of n up to $n = 5$. That is:

$$n^2+n=2S(n) \text{ or } S(n)=\frac{(n^2+n)}{2}$$

We note that this is only a conjecture. We have not proved the formula for all values of n and therefore the formula may fail for a value of n not checked. It is important to remind students that a conjecture is not a theorem and may therefore be false.

In 1640, the great mathematician, Pierre Fermat, conjectured that all numbers F_n defined by:

$$F_n = 2^{2^n} + 1$$

are prime for all n. For example, substituting $n = 0$, 1, 2, 3 and 4 into the formula for F_n, we obtain:

$$\begin{aligned} F_0 &= 2^{2^0} + 1 \\ &= 2^1 + 1 \\ &= 3 \end{aligned}$$

Since the graph is not linear in n, we conjecture that S(n) may be quadratic in n.

To test this hypothesis, we make a table of n, n^2 and S(n) and look for a pattern which relates n, n^2. and S(n).

$$F_1 = 2^{2^1} + 1$$
$$= 2^2 + 1$$
$$= 5$$

$$F_2 = 2^{2^2} + 1$$
$$= 2^4 + 1$$
$$= 17$$

$$F_3 = 2^{2^3} + 1$$
$$= 2^8 + 1$$
$$= 257$$

$$F_4 = 2^{2^4} + 1$$
$$= 2^{16} + 1$$
$$= 65\,537$$

A century later Leonard Euler disproved the Fermat conjecture by showing that $F_5 = 641 \times 6\,700\,417$ and is therefore not prime! The uncertainty of inductive thinking had led the ancient Greeks to pursue a more systematic method for developing and verifying conjectures. The end result was deductive geometry!

INVESTIGATIONS

Explore and discuss these questions in your group.

1. For each of the following sequences write: (i) the next 3 terms (ii) the *n*th term

a) 1, 8, 15, 22, ...
b) 1, 4, 9, 16, ...
c) 1, 2, 4, 8, 16, ...
d) 1, 4, 10, 20, 35, ...

2. Let D_n denote the last digit of the number 7^n when written in base 10.

   Write D_n for $n = 0$ through 9.

a) What is the last digit of 7^2? 7^3? 7^4? 7^5?
b) If you know the value of n how can you determine the last digit of 7^n?
c) What is the last digit of 7^{143}?

3. Let R_n denote the number of regions into which is circle is divided by n diameters.

a) Write the values of R_n for $n = 1, 2, 3$ and 4.
b) Conjecture a formula for R_n.

Complete exercises 4 and 5 for homework

4. Let H_n denote the number of handshakes which take place at a political rally involving n people when each person shakes hands with every other person exactly once.

a) Write the values of H_n for $n = 2, 3, 4, 5$ and 6.
b) Conjecture a formula for H_n.

5. Make a table listing five pairs of numbers, their highest common factor and their least common multiple. Then conjecture a relationship between the highest common factor and the least common multiple of any two numbers.

In part II of this article, to appear in the next issue, we present the second lesson and the next phase in the evolution of mathematical proof under the heading The Development of Direct Proof and Generalization.

REFERENCES

Bell E. T. (1945). *The Development of Mathematics*. N.Y.: McGraw-Hill.

Boyer C. B. (1968). *A History of Mathematics*. N.Y.: John Wiley & Sons.

Chace A. B. (1979). *The Rhind Mathematical Papyrus*. Reston, VA: National Council of Teachers of Mathematics.

Devaney R. L. (1990). *Chaos, Fractals and Dynamics*. Reading, MA: Addison-Wesley.

Gleick J. (1987). *Chaos: Making a New Science*. N.Y.: Penguin.

Hofmann J. E. (1967). *The History of Mathematics to 1800*. Totowa, N.J.: Littlefield Adams.

Math Appetite Begins at Home

(NU) - For many children, math is about as appetizing as broccoli. But it need not be. Parents concerned about their children's learning can make a big difference, according to the National Council of Teachers of Mathematics.

Mom and dad need not be math wizards to talk numbers with their youngsters, say the professionals at NCTM, the largest organization of its kind in the world.

All it takes is patience, a little creativity and a healthy dose of caring.

Teachers say it is imperative that children master math basics early, so they can move confidently to higher levels later.

Teachers Care Too

And while teachers are natural allies for parents in this learning, confidence-building process, experience shows that often a little extra time spent at home with mom and dad can pay big dividends.

The NCTM has prepared two brochures, available in English and Spanish, that can provide parents with some useful pointers.

One brochure, "Help Your Child Learn Math," outlines the essentials needed by a child to learn mathematics and also provides some examples parents can use with children of all ages. Here's one: "It takes mother 45 minutes to get to work. When should she leave the house to get to the office by 9 ... (and) get home by 6:30?"

The second, "Using Calculators to Improve Your Child's Math Skills," gives parents a chance to work with children on a variety of problems and activities.

Single copies of the brochures are free on request. Send a self-addressed, stamped, business-size envelope to the National Council of Teachers of Mathematics, Dept. NU, 1906 Association Drive, Reston, VA 22091.

National Council of Teachers of Mathematics

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former Secretary-Treasurer of O.A.M.E., O.A.M.E. established the ***Morley MacGregor Award****, to be presented annually to an* ***elementary school teacher of mathematics*** *who has demonstrated excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.*

At this time O.A.M.E. is requesting nominations for consideration for this year's award. It will be presented at the Annual Conference in May.

Deadline for Nomination Submission: March 2. 1992

MORLEY MACGREGOR AWARD
Nomination Form 1991-1992

I/We nominate:
NAME:_____

SCHOOL: _____

BOARD: _____

Nominate by: _____

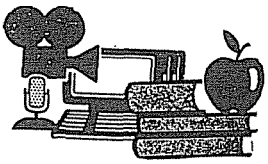
_____

_____

Please attach a separate sheet to the completed nomination form (or Copy) outlining the reasons for your nomination, and forward them to:
Don & Carol Attridge
Secretary-Treasurers

REVIEWS

Editor:
George Gadanidis



EXPLORING GEOMETRY:
Geometric preSupposer

One of the key ideas embedded in the NCTM *Standards* is that *"knowing" mathematics is "doing" mathematics*, which involves much more than the traditional approach of a socratic lesson followed by related seatwork. "Doing" mathematics implies that learning is *guided by the search to answer questions -- first at the intuitive, empirical level; then by generalizing; and finally by justifying (proving).*

If you are inclined towards facilitating your students to become "doers" of mathematics, then you would probably be interested in taking a closer look at the *Geometric preSupposer*. This software package enables students to easily make geometric constructions and measurements that would normally be done more slowly and less accurately by hand. Students can investigate the patterns that emerge from the constructions and measurements and make conjectures (generalizations) which can be tested by further constructions and measurements.

Described are two activities which can be carried out using the *Geometric preSupposer*:

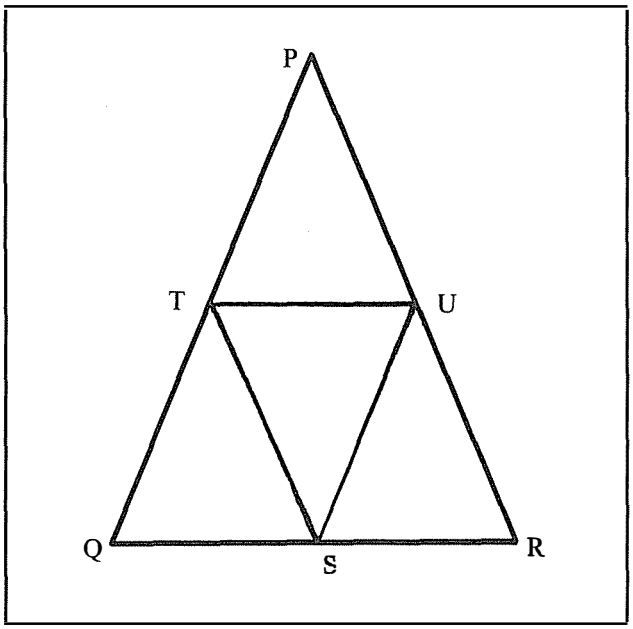
ACTIVITY ONE:

Draw triangle **PQR** (acute, scalene).
Draw line segments joining the midpoints of the three sides.
Measure the angles in the construction.
Measure the areas of the various figures in the construction.
Measure the lengths of the various line segments in the construction.

What <u>patterns</u> do you notice in your measurements?
What <u>conjectures</u> can you make based on your measurements?

Repeat the above instructions for a different triangle.

Do the patterns in the measurements of the second construction support your conjectures?



ACTIVITY TWO:

Draw a regular polygon (3 to 18 sides).
Measure the angles.
Draw and count the diagonals that can be drawn.

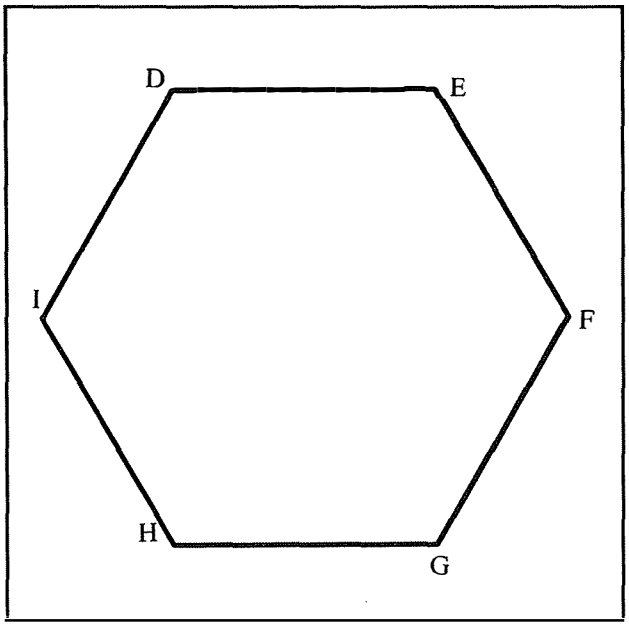
Repeat for four other regular polygons.

What patterns do you notice?
What conjectures can you make?

Use your conjectures to make predictions about a regular polygon not yet constructed.
Construct the polygon and make appropriate measurements to test your prediction.

Predict which of your conjectures would not hold true for irregular polygons?

Check your prediction with a construction.



We noticed these features/characteristics of the *Geometric preSupposer*:

- it is fun; motivates reluctant learners
- it promotes an active approach to learning geometry
- it encourages conjecturing (a mathematical activity often overlooked in our classrooms)
- it promotes the mathematical language related to geometry
- it allows a study of geometry that starts at the empirical, hands-on level
- it is fairly easy to use
- it come with a teacher's guide
- constructions and measurements can be saved for future reference

This software package can be used in the classroom in a variety of ways:

- as an activity centre (if you have computers available for use in your classroom)
- as a whole class activity (if you have access to a computer lab)
- as part of a teacher demonstration (if you have access to an LCD tablet for the overhead projector)
- as an enrichment activity

The Geometric preSupposer: Points and Lines
Grades: 5-adult
For: IBM or Tandy
Price: one disk, backup, teacher's guide, quick reference card

3.5/5.25 Disk	$132
3.5/5.25 Lab Pack (10)	$396
3.5/5.25 IBM Network	$528

From: Sunburst Communications, Scarborough, ON

Also available in this series:

Geometry problems and Projects for use with the Geometric preSupposer ($73)
The Geometric Supposer : Triangles
The Geometric Supposer : Quadrilaterals
The Geometric Supposer : Circles
(for grades 8 and up; prices are the same as those for *preSupposer*)

Reviewed by:
Jane Franklin
Consultant, Duram Bd. of Ed.
and
George Gadanidis
Faculty of Education.
University of Western Ontario

LEADING MATHEMATICS INTO THE 21ST CENTURY CANVAS TOTE BAGS

The O.A.M.E. Membership
and
Publicity Committee has a deal for you!

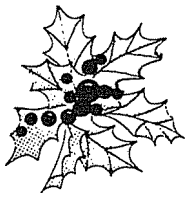
The **"Leading Mathematics Into the 21st Century" Canvas Tote Bags**
promote the appreciation of mathematics among students and teachers.
They are white, large, and extremely durable and available at $15.00 each.

Make cheque payable to O.A.M.E. and order from:
Don & Carol Attridge

The Membership and Publicity Committee has a deal for you.

I ♥ Math buttons promote the appreciation of mathematics among students and teachers. They are available in French or English at 75¢ each or $25.00 for 50.

Make cheque payable to O.A.M.E. and order from:
Don & Carol Attridge



COUNCIL COMMENT

by
Bob Robinson



The following notes were taken at the OAME Council Meeting held September 21, 1991. Your comments are welcome and should be sent to the editor.

Committee Structure
The standing committees of council, their chairs (C) and executive (E) liaisons are: Publications - Peter Lenczewski (C), Ron Lancaster (E)); Publicity/Membership - Marilyn Hurrell (C), Mickey Sandblom (E); Focus for Renewal - Dave Zimmer (C), Ann Jones (E); Formative Years - Peter Saarimaki (C), Sue Stuart (E); Transition Years - Barry Scully (C), Sue Stuart (E); Specialization Years - Jerry Maloney (C), Jeri Lunney (E); Mathematics Equity - Jan Cornwall (C), Ann Jones (E); Terms of Reference - Jeri Lunney (C), Sue Stuart (E)).

Committee Activities

Publications
The committee commended Jack LeSage and Michael Tabor for the outstanding Anniversary Issue released April, 1991. The committee, along with the editors, are going to investigate ways to give our publication a "new look".

Membership/Publicity
Each chapter is being encouraged to have a membership/publicity representative. Publicity activities utilizing newspapers, videos, and other forms of media will be discussed.

Focus for Renewal
The *Ontario Forces on Renewal of Mathematics: A Consultation Paper* has been released. The response form (enclosed with the consultation paper) is to be returned to Paul Zolis (Mathematics Department, Scarborough Board of Education) or Mickey Sandblom by November 30, 1991. All those interested in the future of mathematics education in the province should respond to all or part of this paper. An unsolicited proposal has been sent to Marion Boyd, Minister of Education, regarding OAME involvement in mathematics education initiatives. Although a polite response to our proposal has been received from the minister, we will seek further information from the ministry regarding the status of our proposal.

Formative Years
This committee will prepare a response to the Early Years consultation paper for discussion by the Council at their next meeting.

Transition Years
The membership will be informed about the OAME's response to the consultation paper through excerpts which will appear in an issue of the *Gazette*. The committee is to gather a list of appropriate resources including a list of "transition years" speakers. A resource kit for new members is also under consideration.

Specialization Years
We are anxiously awaiting the release of the *Specialization Years Consultation Paper*. In anticipation of its release the committee is discussing the following concerns: the overcrowded curriculum; equity of access; curriculum renewal; cross curriculum learning; and teacher education.

Mathematics Equity
Reference was made to the special publication *Girls in Science: Discovering Their Choices* published by the Canadian Congress for Learning Opportunities for Women. The following articles were distributed: The Quest for Equity. *FWTAO Newsletter*, June, 1988; Women's Ways of Knowing: Implications for Teaching Mathematics. *Women and Mathematics Education Newsletter*, Spring, 1991; Mathematics for a Multicultural Society, *Mathematics, Teachers, and Children*. London: 1988.

Chapter Reports

CHAMP
Congratulations to CHAMP for an outstanding OAME '91. The program, commercial

exhibits, and social functions were all well received by those who were fortunate to attend this conference. Now CHAMP will concentrate on other exciting mathematics activities which they have conducted in the past.

COMA
The fall conference in collaboration with QSLMA is scheduled to be held at the Opinicon Lodge, north of Kingston, on October 9 and 10. Del Smith who provided great leadership in COMA has stepped down from the chairmanship and is replaced by Shirley Reid. The membership is growing.

GOLDEN SECTION
The Golden Section is hosting a set of Primary, Junior, and Intermediate workshops on Wed. October 23, 1991, at the Parkway Suites in St. Catherines. The chapter is planning a similar set of workshops in the Hamilton area early in 1992 and its annual Intermediate/Senior meeting in April, 1992, at Brock University.

GVMA
The Fall Conference will be on November 7, from 3:50 p.m. to 8:15 p.m. Each member will receive "Problems from the Waterloo County Math League". The Spring Conference is planned for April 2, 1992. Jack Werner is setting up three new committees to make up a new set of OAC mathematics exams to be ready by the summer of '92. The publications committee is investigating the possibility of a publication dealing with the application of mathematics computer software programs across the mathematics curriculum.

NOMA
The Fall Conference was Friday evening (Sept. 20) and Saturday morning (Sept. 21) at Nickel District Secondary School in Sudbury. Registrants came from Wawa to Moosonee to Parry Sound. Nineteen sessions were presented along with commercial displays and a wine and cheese social. Evening workshops on destreaming and strategies to make it work were held last April. The evening was free to NOMA members and was over-subscribed. NOMA's "Award for Excellence" certificates were distributed to elementary schools in June.

NWOAME
NWOAME was one of five sponsors of the Lakehead University/NWOAME Math Awards and Student Recognition Day held last spring. As many as 20 students were invited from each secondary school in the region. The NWOAME '91 conference is September 27-28, in Thunder Bay. During this event elections will be conducted. Marilyn Howell, the first and only chair of NWOAME, is stepping down to concentrate on her recent election to the position of councillor with OAME. The new representative on council is Bill Otto.

Pine Ridge
Mathematics workshops are being organized for the Durham PA day on November 11, 1991. The annual mathematics mini-conference workshops, commercial displays, and dinner is to be offered on April 9, 1992. The chapter's annual awards night will be June 4, 1992. A Pine Ridge Mathematics Contest is being explored.

QSLMA
Jim Whitley, known to many of us, has accepted a position in Saudi Arabia to establish a new education system. A very successful mini-conference took place last April at the Royal Military College. The joint COMA/ QSLMA fall conference is being held at Chaffey's Locks on October 9-10, 1991.

SAME
SAME has several goals for the 1991-92 year. Among them are:
- to establish a peer mentoring network to facilitate the number of teachers who can conduct mathematics inservices.
- to develop strategies to develop confidence, success and a love of mathematics in all students, including E.S.L. and special needs students, and girls.
- to increase memberships in SAME and OAME in order to develop increased awareness and knowledge of the NCTM standards.

SWOAME
The fall conference will be on Friday evening and Saturday November 1-2, 1991, at the Royal Marquis Hotel in Windsor. Much of the activity of SWOAME is directed toward the hosting of the OAME '93 conference.

WOMA
On October 26, 1991, WOMA is presenting its first annual conference. Sessions and panel discussions for all levels from K-OAC will be provided. Publishers and educational suppliers will be there as well. The theme is School Mathematics - in Transition? and the keynote

ONTARIO ASSOCIATION for MATHEMATICS EDUCATION
ASSOCIATION ONTARIENNE pour L'ENSEIGNEMENT DES MATHEMATIQUES

Join the over 1900 mathematics educators, kindergarten through college and university, who enhance mathematics education in Ontario. Your membership:

INCLUDES OAME/AOEM Publication - *Gazette* and *Abacus*
SUPPORTS professional Development through the Annual Provincial Conference, the Leadership Seminar, and Regional Mini-Conferences
PROVIDES opportunity for local input with Chapter Affiliation
PROMOTES Mathematics through Excellence in Mathematics Certificates, I Love Math Buttons, and OAME/AOEM Awards,
ENCOURAGES networking and information sharing
STIMULATES Problem-Solving at all grade levels

O.A.M.E./A.O.E.M. MEMBERSHIP APPLICATION FORM

NAME_____
(first name(s)) **(surname)**

HOME ADDRESS _____
(street)

_____
(town/city) **(province)(postal code)** **(telephone)**

EDUCATIONAL INSTITUTION _____

ADDRESS _____
(board)

_____
(postal code) **(telephone)**

Elementary _____ **Secondary** _____ **CAAT** _____ **University** _____ **Other** _____

Chapter Affiliation (Check one):

CHAMP ○ COMA ○ GOLDEN ○ GVMA ○ NOMA ○ NWOAME ○
PINE RIDGE ○ QSLMA ○ SAME ○ SWOAME ○ WOMA ○ Y^{4}MA ○

O.A.M.E./A.O.E.M. Membership: New __ Renewal __

Membership Fees:
__ one year $30 ($35 outside Canada)
__ five years $135 ($165 outside Canada)
__ retired and student members $20

Send form and cheque, payable to O.A.M.E., to:

Don Attridge

O.A.M.E. 1991-1992 COUNCIL

PRESIDENT
Jones, M. Ann

PAST PRESIDENT
Sandblom, Michael

PRESIDENT-ELECT
Lunney, Jeri

DIRECTORS
Lancaster, Ron
Sauer, Ron
Stuart, Susan

CHAPTERS AND REPRESENTATIVES

CHAMP	Rodger, John
COMA	Payne, Robert
GOLDEN	Robinson, Bob
GVMA	McLennan, Murray
NOMA	Maloney, Jerry
NWOAME	Otto, William C.
PINE RIDGE	Firth, Ron
QSLMA	Evans, Douglas
SAME	Cornwall, Jan
SWOAME	Robinson, John G.
W.O.M.A.	Webster, Wally
Y4MA	Feldman, Phil

COUNCILLORS - TERM EXPIRING 1992
Anderson, Larry
Carefoote, Arnold
Knill, George
Morrow, Roland A.
Romiens, Todd
Scully, Barry
Warburton, Wendy

COUNCILLORS - TERM EXPIRING 1993
Handley, David M.
Lenczewski, Peter
McNabb, George A.
Morrow, Lorna J.
Scully, Janet
Weiner, Jack
Wiggan, Lorna
Zimmer, David

COUNCILLORS - TERM EXPIRING 1994
Crompton, Judy
Harvey, M. Elaine
Hurrell, Marilyn
Reesor, Ron
Saarimaki, Peter
Trainl, Harry F.
Warren, Margaret

GAZETTE EDITORS
Onslow, Barry
Wood, Eric

SECRETARY-TREASURERS
Attridge, Don & Carol