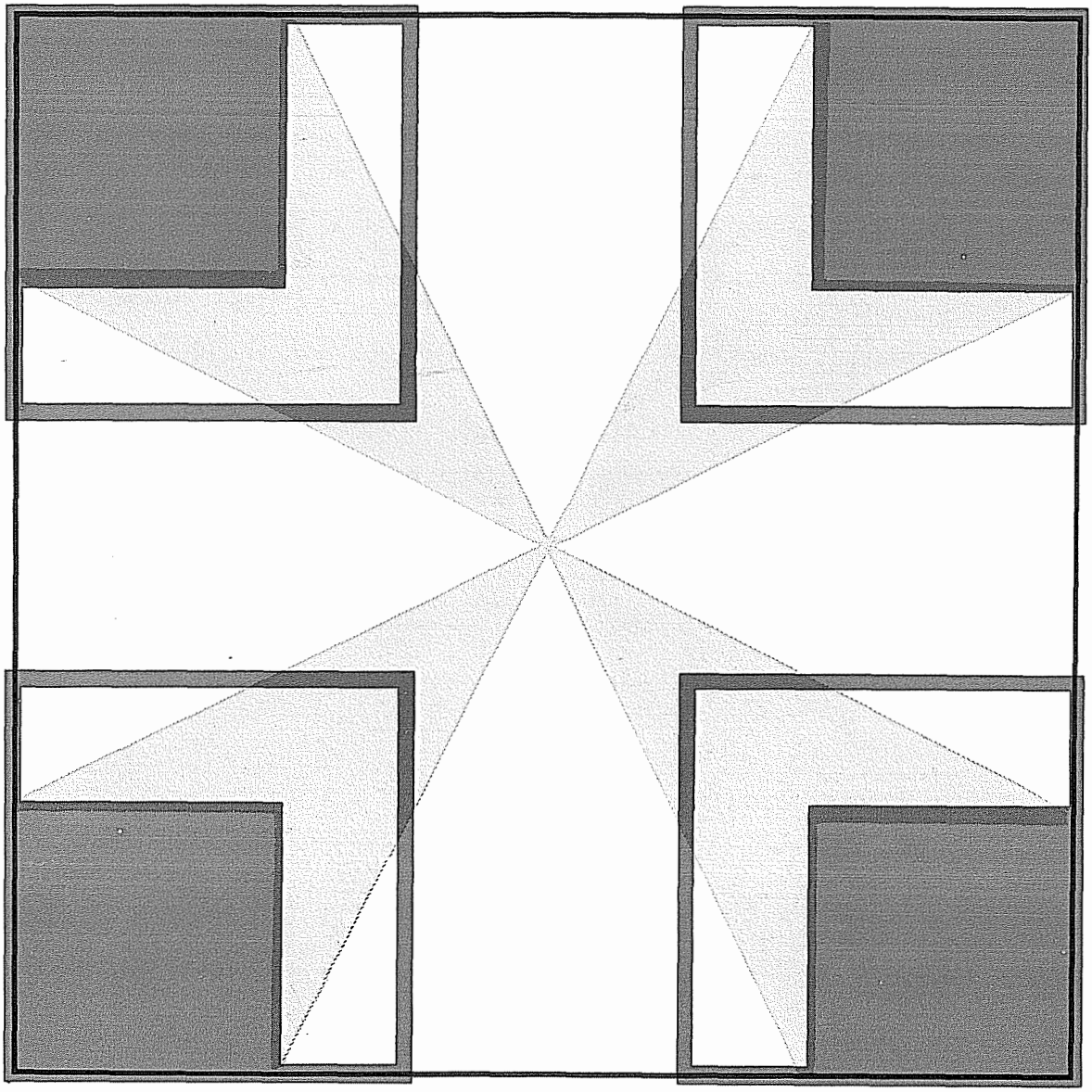
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Volume 30 Number 3 April 1992



THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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June 15 for the September issue
October 15 for the December issue
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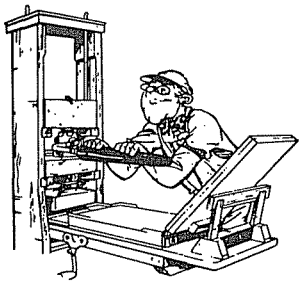
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Barry Onslow & Eric Wood are
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In many ways it is hard to believe that it has already been four years since we assumed the positions of associate editors of the Gazette, working along with Bob Smith as editor. In these first two years, and in the following two when we took over full responsibility for the journal as the co-editors, we have enjoyed a most interesting, pleasant and professionally stimulating experience. It is therefore with a mixture of sadness and satisfaction that we write this, our final, editorial. Sadness because of the many helpful and friendly people we have met and worked with and whose comradeship we shall miss. Satisfaction because we believe that we have made a positive contribution to the quality of our professional journal and we now look forward to other challenges.

There are many people who have helped to produce a high quality journal. First of all we wish to thank our authors for their industrious efforts in producing high calibre material to enrich mathematics education.

Our early decision to send out manuscripts for two blind reviews necessitated an increase in the number of reviewers that were needed and we have appreciated the willingness with which people have agreed to act in this capacity. Without their conscientious reading of our submitted manuscripts and their constructive advice to authors in helping to produce better articles it would have been impossible to maintain the quality of this publication.

To Carol Onslow, for her tireless efforts drawing and re-drawing diagrams; rearranging script and mathematical equations so that everything would "fit"; and, changing the layout "just one more time"; we express our sincere thanks for a job well done.

The cheerful and professional assistance of our typist, Sandra Neubauer has helped us maintain deadlines efficiently, and is very much appreciated.

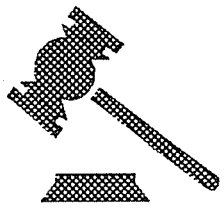
We also appreciate the support for our role given by the Faculty of Education of the University of Western Ontario as we embarked on this undertaking.

Finally we would like to acknowledge the OAME Council members, especially the publications committee and Don and Carol Attridge, for their unstinting encouragement and support of our work. We sincerely hope that the friendships, both personal and professional, that we have made through our work with the Gazette can be maintained as we move on to other things.

We are pleased to welcome our successors. Together they bring a great deal of knowledge, expertise and enthusiasm to the position. We can all look forward to their excellent guidance during the coming years when there will undoubtedly be new mountains to climb. We wish them every success and look forward to reading their first edition.

Barry Onslow and Eric Wood

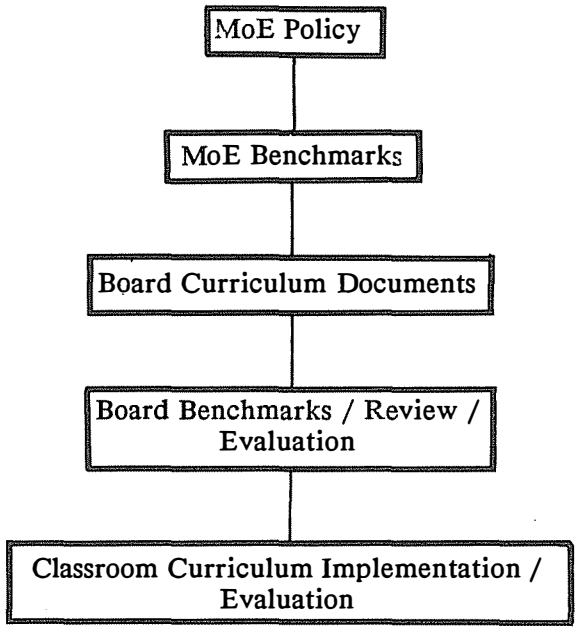
PRESIDENT'S MESSAGE



The O.A.M.E. has been very actively involved in the restructuring of education in Ontario. The Focus on Renewal Committee, the Early/ Formative Years Committee and the Transition /Specialization Committee have responded to the Ministry consultation process and we have been directly involved in reviewing and redrafting Ministry of Education Policy on Formative and Transition Years and, from this document, with the writing of MOE Benchmarks at grades 3, 6, and 9. Each of these initiatives has been done in cooperation with the Ontario Mathematics Coordinators Association (OMCA).

The basis for all of this work has been *Focus on Renewal of Mathematics Education* prepared jointly by OAME/OMCA and the views of our members and others who responded to this consultation paper. We received over 100 responses. These were sorted by topic and concern and the *Focus* document is currently being re-written in direct cooperation with the Ministry of Education Policy statements and Benchmarks. We are involved, your views are being heard by the Ministry of Education through OAME.

It is important that members are aware of the current thinking and policy development of the Ministry of Education. The Ministry is preparing a policy statement for Formative and Transition Years. From this policy, Benchmarks will be written. The Ministry of Education has stated that they will not be preparing "Guidelines" of detailed curriculum but the policy will be *learning outcomes* from which *benchmarks* will be written. From the learning outcomes boards will be expected to prepare curriculum implementation documents, board benchmarks and other materials. This is a very significant change which *downloads* implementation directly to boards and ultimately the classroom:



This new direction leaves much of the responsibility to the classroom teacher. The OAME is currently working on strategies to assist the classroom teachers and boards of education. We will continue to monitor the Ministry of Education policy development and will continue to insist that mathematics educators be directly involved in policy development, benchmarks, implementation strategies and timelines.

This is the OAME's first order of business now and throughout this decade of change.

M. Ann Jones
President

LETTERS TO THE EDITOR



Faculty of Education
Queen's University
Kingston, Ontario. K7L 3N6

Sunday, November 10, 1991

Dear Editors,

Mike, my running partner, is rather eclectic in his interests. The wide ranging discussions arising from his reading help make the Sunday morning marathon preparation kilometres pass quickly. Still, I was surprised by his announce-ment that he was working his way through some of Benoit Mandelbrot's writing on Fractal Geometry. Not an easy task for anyone and certainly a significant undertaking for someone who has not formally studied mathematics for close to twenty years. After all, when I had enquired a week earlier in my Intermediate-Senior Mathematics teacher pre-service course only three of the twenty-nine students had heard of Fractal Geometry.

Mike, like I, is in the 40+ age group of runners known as Masters so it has beeen a few years since he completed his CAAT course in art and design. His job as co-owner of a small construction company requires some mathematics when he design and costs projects. But this is not Fractal Geometry. Mike does not require any more credits in mathematics. he is just ionterested in Fractal Geometry.

"So, Mike, what sparked your interest in this topic?"
"I read a short artical in an art journal and it reminded me of a book that a high school math teacher once lent me and some questions that she posed. I began by locating and rereading as copy of that book, *The Divine Proportion*, and then I just worked from there."
"And who was the high school mathematics teacher who planted this seed?"
"Elaine Harvey at Frontenac Secondary School.

Thank you Mike, & thank you Elaine, OAME Councillor and long time OAME contributor, for making me feel good about being a teacher.

Yours sincerely,

Geofrey Roulet

Huntley, H. E. (1970). *The divine proportion: a study in mathematical beauty.* New York, NY: Dover.

CALCULATOR USE IN MATHEMATICS INSTRUCTION

The issue of calculator use at all grade levels is one that has received much recent attention since the NCTM Standards were released in March, 1989. Although a variety of studies done in the last ten years regarding calculator use in classrooms have shown that student performance is not adversely affected by calculator use, there is disagreement about how the calculator can be most effectively used by teachers and students in the teaching and learning of mathematics.

Current research questions include: 1) For what grades and student ability levels is the calculator appropriate? 2) For what mathematical topics should the calculator be used? 3) How can calculators impact the area of problem solving in mathematics? 4) In what ways will the use of calculators improve student enjoyment of and attitudes toward mathematics? 5) How do the current mathematics curricula need to be changed to enhance student learning with calculators?

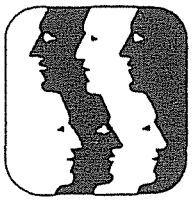
It is important for math educators to realize that calculators are a necessary tool in today's world and instruction in calculator use is needed as part of today's mathematics curriculum. Some suggestions for math educators include: 1) Become knowledge-able about the research on and the benefits of calculator use. 2) Develop school calculator policies for equal access and their use in instruction and testing. 3) Realize the importance of using calcu-lators for exploration, discovery activities, and alternative problem solutions. 4) Incorporate realistic situational problems requiring appropriate calculator use. 5) Promote revised standardized achievement tests with sections designed for calculator use. 6) Utilize the NCTM recommendations for curricular and instructional reforms.

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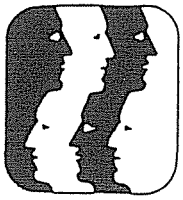
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Counterpoint

IN SUPPORT OF DESTREAMING GRADE NINE



by
Donald Louma
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Mattawa, Ontario

The Ministry of Education's decision to destream grade nine programs in Ontario Secondary Schools has received modest attention in the literature even though the Radwanski report (Radwanski, 1987) and its successor (Ontario Ministry of Education [OME], 1988) have been in circulation for almost four years. A notable exception to this silence is provided by Eric Wood's article, "Radwanski and Streaming: Where is the Evidence?" in the *Gazette* (Sept. 1990). In it, the government's decision to proceed with the Radwanski destreaming recommendation is challenged from a variety of perspectives. Understandably, Wood selectively chooses evidence to support his case while overlooking some other considerations.

Wood seems caught between criticizing the Radwanski Report and the subsequent government policy. The two are quite different. Most importantly, Radwanski suggests destreaming the entire program in secondary school (p. 163) while the government is proceeding only with a destreamed grade nine, (with schools having the option to extend it further, (OME, 1988, p.10)). As well, the government has not adopted the recommendation that there be a virtually common curriculum throughout. I will confine my comments to what is going to happen, namely the destreaming of grade nine and a new philosophy of pedagogy for the "transition years". With this emphasis, it is clear that my line of reasoning will be somewhat different than Wood's.

The first argument presented is that Radwanski has "made the classic error of confusing causation and correlation", by assuming that "this streaming process somehow causes the perpetuation of the under privileged position of these children in society" (Wood, p.7). Although Radwanski (1987) claims that social factors are greatly responsible for children's being placed or placing themselves in the lower streams (pre-school disadvantages, p. 123; immobility between ability groups in elementary school, p. 132; impact of cognitive and developmental psychology, p. 135), these arguments alone would constitute no real evidence of causality. It might simply be that socially disadvantaged children are less intelligent than others and that this correlation, however unfortunate in some respects, is a "fact of life". However, further examination of the evidence does not support such a contention. Although the correlations between ability and stream and socio-economic status and stream are both positive, the relationship is much stronger for the latter than for the former (Porter, Porter, & Blishen, pp. 189-193; Radwanski, pp. 72-73). While these types of statistics cannot "prove" a case in a causal sense, it becomes clear that there is a significant number of academically capable lower socio-economic students in the general and basic streams. The question of exactly how they got there becomes moot in the face of the consequence of a limited educational future once they are there. So schools, despite their avowed purpose of equalizing education opportunity, seem to be structuring inequality.

Wood then introduces an argument surrounding a newspaper article which claims there are well-paying jobs in the technological occupations for which our general level programs are preparing students. Again, the evidence tends not to substantiate this contention. Of the student who begin grade nine taking primarily general level subjects, about 38% graduate (King, p. 18). Further, employers routinely use the Secondary School Graduation Diploma as a filtering device for sorting through candidates (Radwanski, p. 87). The majority of these students, then, become ineligible for

such jobs by entering secondary school at the general level. No doubt there are some jobs for which secondary school graduation is not needed. However, those jobs are unlikely to be of the $50 000 per year variety described in the newspaper article. Further, the assumption seems to be that those students who are not academically gifted automatically will be suited to technological training programs. The argument that those lacking abstract academic skills must necessarily have competence in technical areas continues to be durable despite lack of supporting evidence.

Much of the Wood article centres on the assumed distinction between the educational systems in Canada and the United States, claiming that such disparity denies meaningful comparison and invalidates the American research for Canadian application. While the dearth of Canadian evidence makes this section somewhat difficult to refute, there is evidence, some provided by Radwanski, which tends to establish similarity. His chapter on *Understanding the Issue of Dropouts* draws almost exclusively on Canadian evidence, with Oakes' (1985) study merely providing corroboration.

The contention is that Canadian teachers, being better paid than their American counterparts, and taking "their professional responsibilities to teach seriously" (Wood, p. 8), provide a different learning environment for our students. However, many Ontario drop-outs report that teacher reaction to their decision to discontinue school, was that "the teachers didn't care" (Radwanski, p. 90). Further, some of those leaving school indicated that teacher reaction was "somewhat favourable" (ibid). Given that the vast majority of drop-outs come from the lower streams, it is difficult to sustain the argument that teacher attitude towards these students is substantively different here than in the United States.

Funding formulas in Ontario serve to narrow the huge disparities in educational funding evident in the U.S. between the rich and poor geographic areas. While it is no doubt true that province-wide financial support creates a more equitable situation, there has been considerable erosion in the level of support in recent years. Provincial support has declined from 61.3% of board spending in 1975 to 47.6% in 1984 (Atkinson & Sussman, p. 154). Further, there is indication from recent budgets and the deteriorating financial condition of the province that this trend is likely to continue. These are gross statistics which do not describe the full impact of these funding declines on a rural board with a small commercial tax base. While it remains true that the financial circumstances are better for boards of education here than in the United States, the gap is closing.

Wood notes the difficulties teachers have in dealing with the resistance offered by students in "lower" streams. The superior teachers are reported to "heroically *cajole, badger, threaten*, and persuade their charges to work at academic tasks -- even in the lower tracks" (p. 9) [italics added]. The "heroic" posture of these teachers is seen as contrasting with the attitude of some in the United States who strike a bargain with their students -- "the students behave as long as they are not pushed too hard" (p. 8). Surely neither alternative describes a good learning situation for students.

Delinquency, ("difficult students who are reluctant learners" (Wood, p. 8)), is viewed as a structural problem, another "given" with which we must deal. But Radwanski and others contend that these recalcitrant students are rather *produced* by a system which they see as irrelevant and disempowering, and not some group that necessarily must exist (Radwanski, p. 94).

Then, there is the matter of the content of mathematics curricula. It is suggested that the advanced curriculum, "designed to be suitable for the top 30% of all mathematics students" (Wood, p. 8), will be taught to all students. No such statement exists in the Ministry reports. Rather, it is suggested that the grade nine program be revised to include core areas with a focus on integration of subject matter in an interdisciplinary way (Hargreaves & Earl, p. 116; OME, 1988, p. 5). Radwanski is relatively silent on the specifics of subject content, recommending simply the elimination of the three streams, "replaced by provision of a single and undifferentiated *high quality* educational stream for all students (Radwanski, p. 163) [italics added]. Only in the narrowest

interpretation could this "high quality" be assumed to mean the existing advanced level curriculum. Open is the question of whether the current abstract nature of the advanced program is suitable even for the 30% for whom it was intended. The assumption that the docile bodies found in advanced programs indicate a high degree of contentment and sense of achievement within such programs is likely unwarranted. Rather, it may be evidence of the successful conditioning and the lack of resistance of this favoured group.

Examples of the "academically defensible" mathematics contained in general level programs, "income tax, mortgages, municipal taxation, personal finance, vehicle costs, and so on" are provided (p. 10). No doubt, these are valuable units. However, the topics indicated are all contained within the grade 11 and 12 general level guidelines (OME, 1885b) and will remain there under the proposed changes. To focus on the actual case, we should look at the grade nine curriculum in the advanced and general streams.

GRADE 9 - GENERAL LEVEL

NUMERICAL METHODS

1. Applications of Whole Numbers, Decimals, Fractions, and Integers
2. Powers and Square Roots
3. Applications of Ratio and Rate
4. Statistics

ALGEBRA

1. Variables, formulas, and polynomials
2. Equations
3. Mathematics and the Computer (optional)

MEASUREMENT AND GEOMETRY

1. Measurement and its Applications
2. Constructions
3. Enlargements and Reductions

GRADE 9 - ADVANCED LEVEL

NUMERICAL METHODS

1. Applications of Whole Numbers, Decimals, Integers
2. Powers and Square Roots
3. Fractions and Whole Numbers
4. Applications of Ratio, Rate, and Percent
5. Statistics

ALGEBRA

1. Variables and Polynomials
2. Formulas and Equations
3. Relations
4. Probability (optional)
5. Mathematics and the Computer (optional)

MEASUREMENT AND GEOMETRY

1. Measurement and its Applications
2. Constructions
3. Properties of Plane Figures
4. Coordinate Geometry and Congruence Trans
5. Dilatation
6. Geometry and Navigation

(OME, 1985b, p. 41; OME, 1985a, p. 42)

While the time spent on these topics varies from one stream to another, the argument that the two programs are *philosophically* different in their orientation, and that one seems intended for an entirely different type of learner than the other, is suspect. The notion that a difference in teaching strategies will be enough to make the mathematics learning experiences substantively different in the two streams is unsustainable in the face of this almost identical content. In fact, the only difference appears to be one of the quantity of material taken rather than one of a different "kind" of mathematics.

It is argued that "it is simply not true that a student intending to pursue a career in business needs to know the same mathematics as one who wants to be an electrical engineer" (Wood, p. 10). This contention seems valid but is peripheral to the proposed grade nine destreaming. Most university business schools routinely require calculus for admission, for example. Currently, we teach the same mathematics for all our academically elite students regardless of vocational aspirations. There is a serious question here, but one that has nothing to do with destreaming our grade nine classes.

In Ontario secondary schools, we have the situation where our grade nine mathematics courses are virtually identical in content, one simply being a "watered-down" version of the other. It can be predicted that 62% of the students who begin secondary school in the general stream will drop out of school before graduation having failed mathematics more than any other subject before they do (Radwanski, p. 79). Further, only 4% of them will ever graduate from a post-secondary program (King, p. 18). It becomes difficult to maintain an argument which suggests that grade nine students are placed in the general level "for their own good" in the face of all of this evidence to the contrary.

The destreaming of grade nine is a fait accompli. There will be difficulties with its implementation. Mathematics teachers are accustomed to dealing, in a teacher-centred manner, with what they assume are relatively homogeneous groups (Hargreaves & Earl, p. 43). In recommending the destreaming of grade nine, the government is suggesting a modest beginning for the transformation of secondary education, bringing it closer to its avowed egalitarian purposes. Teacher conservatism and resistance may well consign this initiative to the status of an historical curiosity, but it is my hope that teachers will embrace this opportunity to create a greater equality of opportunity for all their students.

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SEVENTH INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION

The 7th International Congress on Mathematical Education (ICME-7) will be held in the city of Québec (Canada) from August 16 to 23, 1992. It will be the seventh in a series of congresses of the International Commission on Mathematical Instruction (ICMI), following those of Lyons (1969), Exeter (1972), Karlsruhe (1976), Berkeley (1980), Adelaide (1984) and Budapest (1988).

In an effort to meet the diverse needs and interests of the 3000-3500 expected participants, the program will cover all of the major areas of mathematics education at the elementary, secondary and post-secondary levels. Activities will include lectures, working groups, topic groups, workshops, short communications, posters, project presentations, and films, as well as exhibitions of textbooks, software, and other types of materials. Here are a few examples of themes that will be discussed during the congress:

Improving students' attitudes and motivation
Mathematics for early school leavers
Innovative assessment of students in mathematics
Students' misconceptions and inconsistencies of thought
The impact of calculators on the elementary school curriculum
The role of geometry in general education
Probability and statistiics for the future citizen
Modeling activities in the classroom
Students' difficulties in calculus
Undergraduate mathematics for different groups of students
Pre-service and in-service teacher education
Methodologies for research in mathematics education

Founded in 1608 by Samuel de Champlain, the city of Québec, capital of the Canadian province of the same name, is the cradle of French civilization in North America. Because of its unique charm, its historical past and its exceptional location dominating the majestic Saint-Laurent river, Québec is a privileged place which attracts tourists from all over the world. The ICME-7 congress will take place on the campus of Université Laval, which offers facilities and services making it a most convenient place to hold such an international event.

The Second Annoncement of the congress will be published in1991 and will contain detailed information about the program, as well as forms for registration, accommodation, and submission of short communications or posters. In order to receive it, please write to: **Congrès ICME-7 Congress, Université Laval**



MIXING MIXTURES: A GRAPHICAL SOLUTION

by
Bonnie H. Litwiller and David R. Duncan
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

Algebra students are sometimes confused by the abstractness of symbolic solutions to problems. A great deal of benefit can be derived from using graphs to depict relationships. We shall explore an example of this approach, using the often troublesome topic of mixture problems.

Suppose that Lois has two barrels of acid solutions. The first barrel contains a solution of 20% acid (by volume) while the second barrel contains a solution of 40% acid. How many litres of each solution must be mixed together to yield 40 litres of a 35% acid solution?

This problem may be solved graphically as shown in Figure 1.

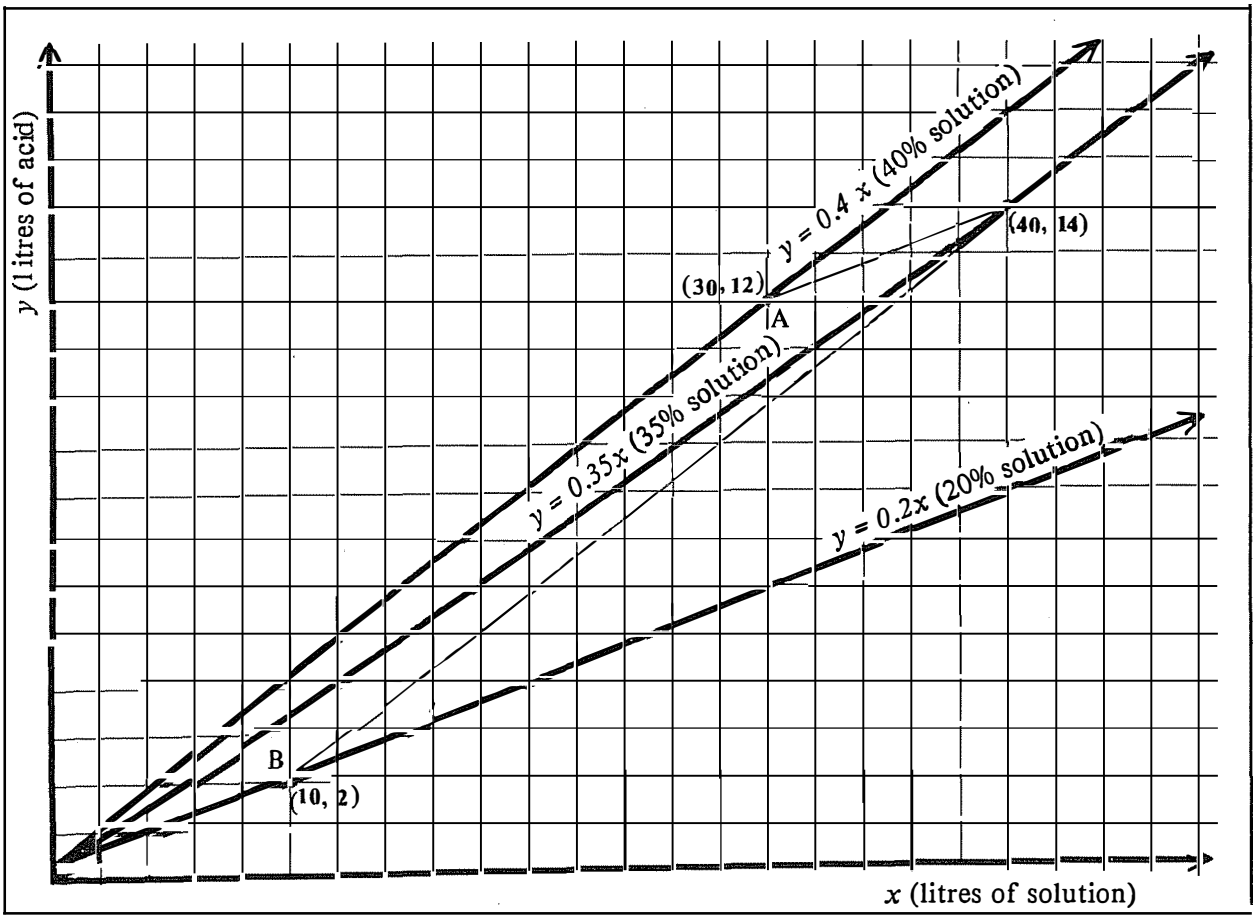


Figure 1. Graphical solution to the mixture problem.

PROCEDURE:

1. Draw the graphs of the three lines.

$$\left.\begin{array}{l} y = 0.2x \\ y = 0.4x \end{array}\right\} \text{ - The original solutions}$$

$y = 0.35x$ - The resulting solution

2. On the line $y = 0.35x$, locate the point (40, 14). This point represents 40 litres of solution and 14 litres (35% of 40) of acid.

3. Through the point (40, 14) draw:
   A. A line parallel to $y = 0.2x$. Let A be the point of intersection between this line and the line $y = 0.4x$.
   B. A line parallel to $y = 0.4x$. Let B be the point of intersection between this line and the line $y = 0.2x$.

4. Determine the equation of the line connecting (40, 14) and A. This equation is:
   $y - 14 = 0.2(x - 40)$
   $y - 14 = 0.2x - 8$
   $y = 0.2x + 6$

5. Determine the equation of the line connecting (40, 14) and B. This equation is:
   $y - 14 = 0.4(x - 40)$
   $y - 14 = 0.4x - 16$
   $y = 0.4x - 2$

6. To find the co-ordinates of A, solve the following system:
$$\begin{cases} y = 0.2x + 6 \\ y = 0.4x \end{cases}$$
   $0.4x = 0.2x + 6$
   $0.2x = 6$
   $x = 30$
   $y = 0.4(30) = 12$ Thus $A(30, 12)$.

7. To find the coordinates of B, solve the following system
$$\begin{cases} y = 0.4x - 2 \\ y = 0.2x \end{cases}$$
   $0.4x - 2 = 0.2x$
   $0.2x = 2$
   $x = 10$
   $y = 0.2(10) = 2$ Thus $B(10, 2)$.

8. Write the solution.
   - From the 40% solution, use 30 litres of liquid containing 12 litres of acid.
   - From the 20% solution, use 10 litres of liquid containing 2 litres of acid.
   - When mixed together these two components would yield 40 litres (30 + 10) of liquid containing 14 litres (12 + 2) of acid. The problem has been solved graphically.

Note that steps 5 and 7 are unnecessary. Steps 4 and 6 establish that 30 litres of the 40% solution, containing 12 litres of acid, were needed. To achieve a final solution of 40 litres of liquid containing 14 litres of acid, the 20% solution must therefore contribute 10 litres of liquid containing 2 litres of acid.

An adroit grapher may be able to draw and measure so carefully that the coordinates can be determined by inspection. The computer should be an aid in this process.

Let us establish algebraically that this method yields the correct solution for a general problem. Let solution 1 contain A% acid while solution 2 contains B% acid. (Also, let $A > B$.)

How many litres of each solution must be combined to produce c litres of solution containing d litres of acid?

$$\text{Let } a = \frac{A}{100} \qquad b = \frac{B}{100}$$

Figure 2 displays the lines $y = ax$ and $y = bx$, representing solutions 1 and 2, while

$$y = \frac{d}{c}x$$

[connecting the origin to (c, d)] represents the final solution.

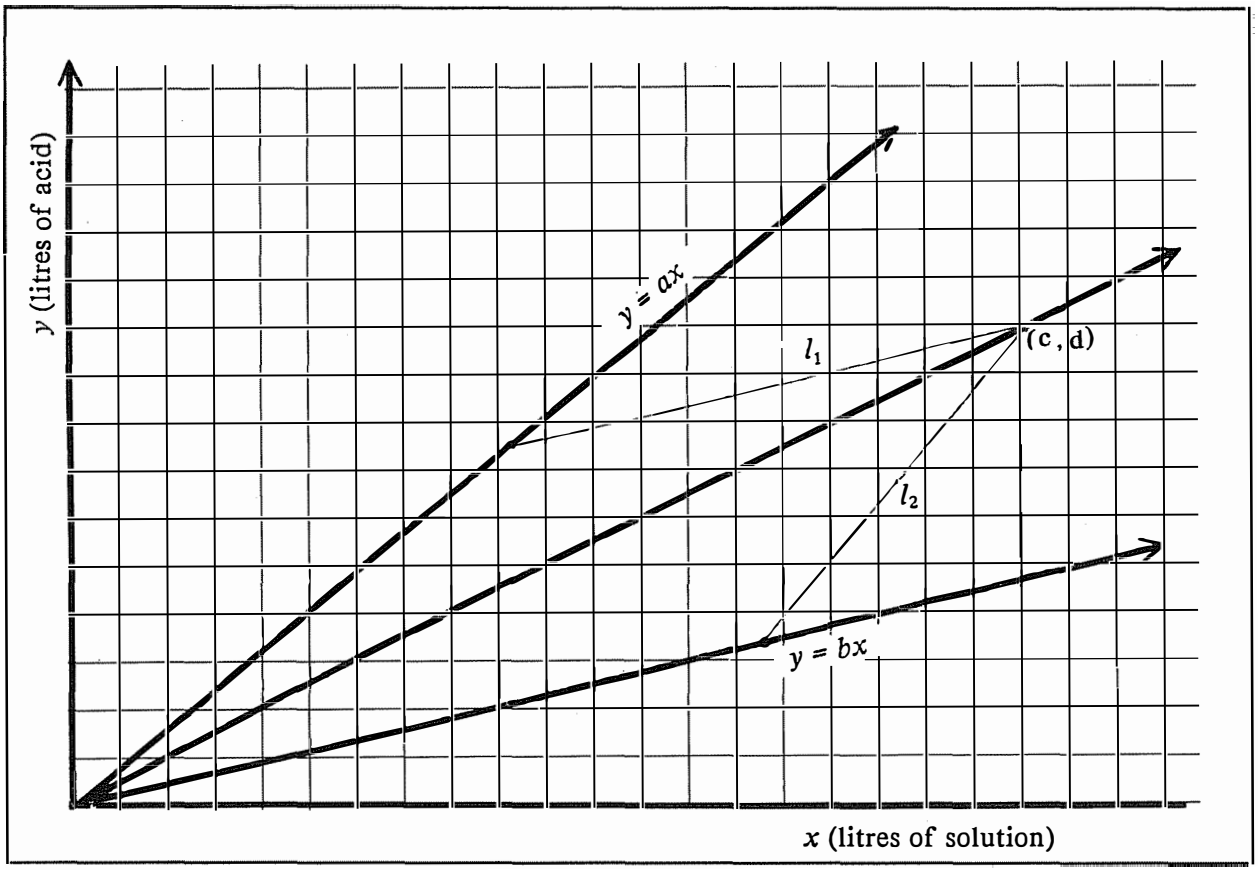


Figure 2. Solution to the general mixture problem.

Let l_1 be the line through (c, d) parallel to $y = bx$ and let l_2 be the line through (c, d) parallel to $y = ax$. The equations of l_1 and l_2 are $(y - d) = b(x-c)$ and $(y - d) = a(x - c)$ respectively.

Solve l_1 and $y = ax$ simultaneously:

$$\begin{cases} y = bx - bc + d \\ y = ax \end{cases}$$

$$ax = bx - bc + d$$
$$ax - bx = d - bc$$
$$(a - b)x = d - bc$$

$$x = \frac{d-bc}{a-b} \quad \text{(litres of liquid from solution 1)}$$

$$y = \frac{a(d-bc)}{a-b} \quad \text{(litres of acid from solution 1)}$$

Solve l_2 and $y = bx$ simultaneously:

$$\begin{cases} y = ax - ac + d \\ y = bx \end{cases}$$

$$bx = ax - ac + d$$
$$bx - ax = d - ac$$
$$(b - a)x = d - ac$$

$$x = \frac{d-ac}{b-a} = \frac{ac-d}{a-b} \quad \text{(litres of liquid from solution 2)}$$

$$y = \frac{b(ac-d)}{a-b} \quad \text{(litres of acid from solution 2)}$$

Verify that the amounts just expressed symbolically yield the correct number of litres in the solution:

1. Total litres of liquid

$$= \frac{d-bc}{a-b} + \frac{ac-d}{a-b}$$

$$= \frac{ac-bc}{a-b} = \frac{(a-b).c}{a-b} = c$$

2. Total litres of acid

$$= \frac{a(d-bc)}{a-b} + \frac{b(ac-d)}{a-b}$$

$$= \frac{ad-abc}{a-b} + \frac{abc-bd}{a-b}$$

$$= \frac{ad-bd}{a-b} = \frac{(a-b)d}{a-b} = d$$

The correct numbers of litres resulted.

Further challenges:

1. How could the method of this article be adjusted to solve the following problem?

Lois has 10 litres of a solution containing 20% acid. How many litres of a 40% acid solution must be added to the original 10 litres to achieve a 35% solution?

2. This graphical method could be thought of as involving the composition of vectors.

Are there other problems (besides the familiar force and velocity compositions) in which composition of vectors can be used?

3. Are there any other graphical strategies that could be used in the problem of this article?

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LA YUPANA - AN INCA COUNTING DEVICE FOR TEACHING YOUNG CHILDREN

by
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Faculty of Education
McGill University
Montreal, Quebec

Much of our present-day practical mathematics stems from the needs of primitive people to decide the time for sowing and harvesting, the need to trade, or to divide up the land. Some of the mathematical tools used by primitive people may be used as simple teaching devices for modern classrooms. In the Inca culture of Peru, there was a complex accounting system which allowed the ruler, or Inca, great control over the administration of the people. Although there are no written records of the Inca culture, drawings and artifacts have survived which allow us to build a picture of Inca mathematics. There is evidence of the use of accounting for census purposes, for keeping records of crops in storage and, equally familiar to us in the modern era, of taxes owed and paid. Figure 1 shows a sixteenth century drawing of an accountant and treasurer holding a tallying device called a *quipu*. The *quipu* was a record keeping device made of string, which was knotted to represent groups of ten and powers of ten, and examples found by archaeologists are on display in Peruvian museums.

One of the intriguing devices used by the Inca's mathematicians for counting and performing the four basic operations was a form of abacus, called the *yupana*. This is shown in the bottom left hand corner of Figure 1. The original *yupana* was probably made of wood and was marked out into five rows and three columns. Each square formed by this grid contained a pattern of depressions; five in the lowest, then three, then two and one in the top square. There is some disagreement about the precise way that the *yupana* was used: was it a base-ten system or a base-five system (Ansion, 1983; Burns, 1981)? There is no doubt that it was a concrete way of tallying and calculating, similar to the abacus used in other cultures. It seems likely that each column represented a place in our familiar decimal system, so that the accountants could reckon in units, tens, hundreds, thousands and tens of thousands, probably using small pebbles or kernels of corn as tokens.



Contador y Tesorero

Figure 1

A simplified version of the *yupana*, made of cardboard and with only two columns for tens and ones, is gaining popularity as a modern teaching tool in Peruvian schools. Recent exposure to this device in Peru was convincing evidence that it is an inexpensive addition to the toolbox for teaching place value to primary school children in any continent. It is an easily and cheaply constructed

piece of equipment that involves the children in learning by touching and moving objects. It can be used by individuals, pairs of children or co-operatively in groups, it adds variety to a mathematics lesson, and introduces our children to another culture. Whether or not the *yupana* was used for base ten or base five computation, it is a useful variation on the "place-value mat" so common in the primary classroom.

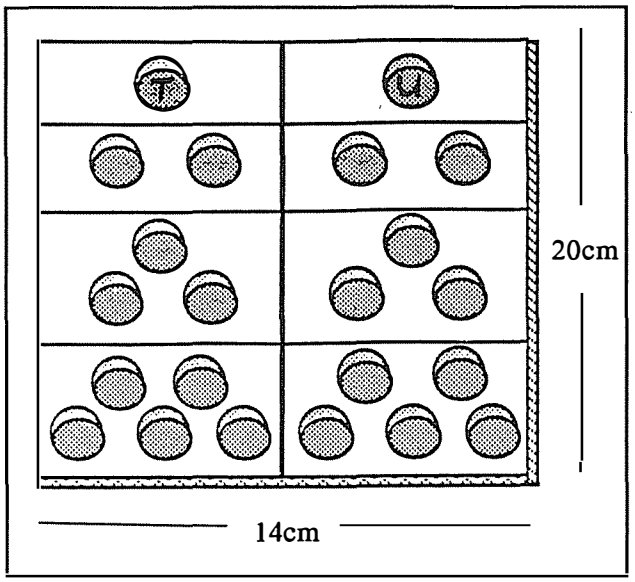


Figure 2

CONSTRUCTING THE YUPANA

Use a piece of corrugated cardboard 14 cm by 20 cm, marked as shown in Figure 2, with 22 circles about 1.5 cm in diameter.

The top (single) circle may be cut larger than the others to act as a reservoir for tokens. There is no need to be too precise with the measurements. Ask for help from parent volunteers or older children in the school to cut out the circles with a craft knife. The circles may either be cut part way through the cardboard, or, to make a stronger *yupana*, you may cut out the circles completely and then stick another piece of 14 cm by 20 cm cardboard at the back of the cut piece.

For each *yupana*, put about 50 tokens such as fava beans or lima beans (appropriate for a device of Peruvian origin!) into a plastic tub with a lid, and you are ready to start. If you feel uncomfortable using foods for manipulatives, small pebbles or medium sized buttons are equally useful. The only criteria are that there should be about fifty tokens for each child and that when working with young children, counters should not be so small that they fit easily into ears or noses. It helps keep things organized if you can put both *yupana* and tokens into a self-sealing lunch bag.

PLACE VALUE

Figure 3 shows the sequence in which counters are successively added to the *yupana* to fill the depressions if the objective is to introduce

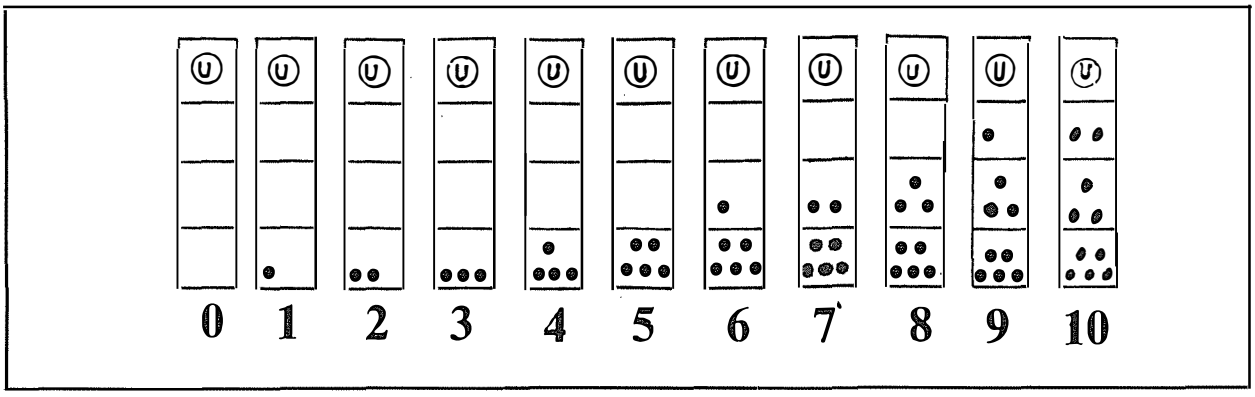


Figure 3

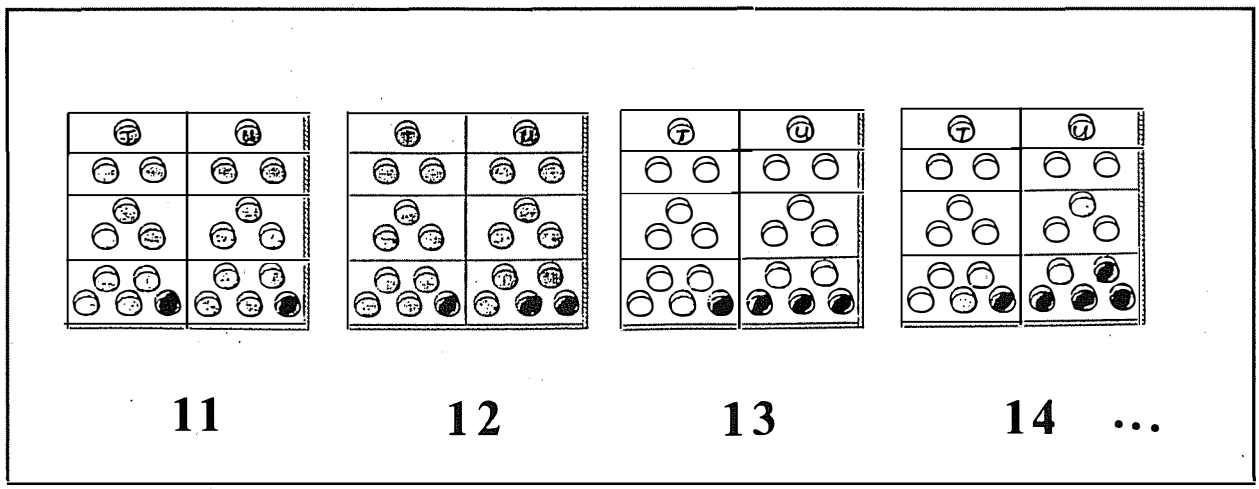


Figure 4

place value in base ten. As children add tokens, they should be encouraged to count them aloud, and later to draw each arrangement and to write the numeral underneath. This helps the child to make the links among the quantity, the sound of the word and the shape and form of the symbol.

When the tenth hollow is filled, and there are ten tokens in the unit column, children may be introduced to the *rule of ten*. As soon as the unit column is filled, all the tokens are removed and replaced by one token in the tens column. You may want to have the columns labelled *T* for *tens* and *U* for *units*, to help the children remember that the lowest numbers are on the right. It is easy to see that quantities between 10 and 19 can be developed by placing one token in the tens column and successively adding tokens to the units column, as shown n Figure 4.

Each time a bean is added, children should be encouraged to touch the columns and say aloud: "One ten and one unit; one ten and two units..." and so on. The act of "touching and saying" will again help to reinforce the links between concrete and abstract. As a final step,

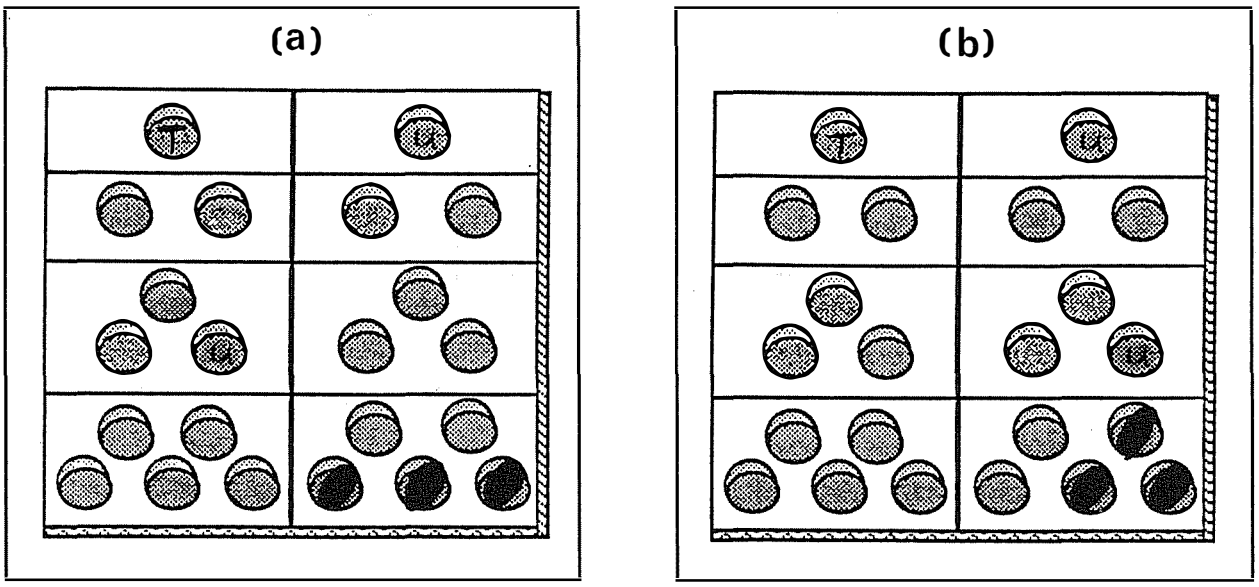


Figure 5

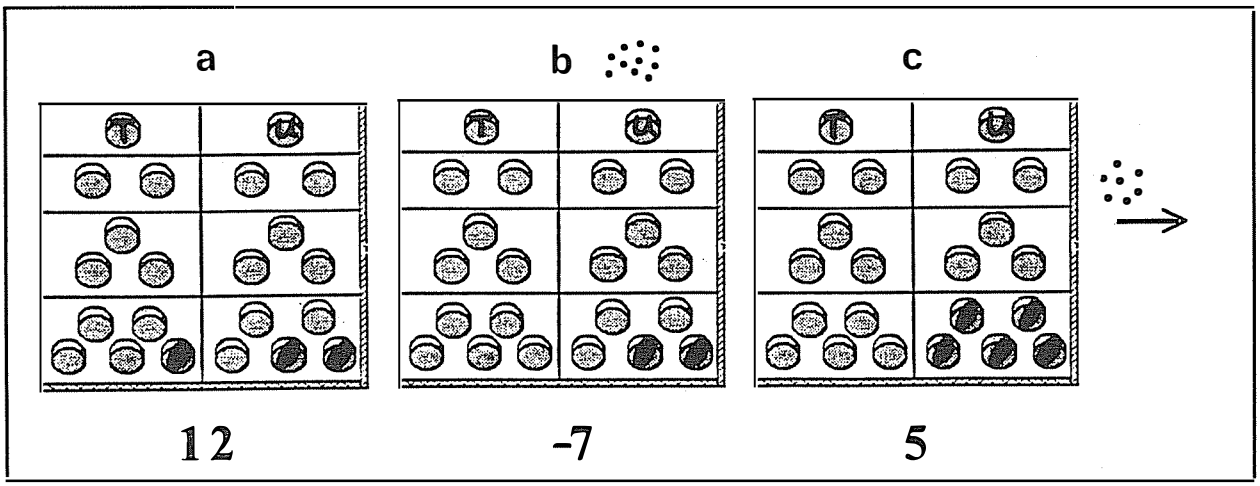


Figure 6

children should be asked to draw the arrangement and finally to write the numeral.

CONSERVATION OF NUMBER

To help children's development of conservation of number, they should be encouraged to see that the arrangement in Figure 5a is equivalent to that in Figure 5b; that the placement of three tokens in a line rather than a triangle does not change the number of tokens, simply their pattern or arrangement. They may then work in pairs and, using the same number of tokens as their partner, make a different pattern.

THE CONCEPT OF ZERO

By placing a number of tokens in the depressions of the *yupana* and removing them one at a time, the children can say aloud the number remaining, to help them learn to count backwards. As a final step, when the last token has been removed, the children would say "zero". Having actually removed all the tokens, the child has a more tangible expression of a difficult concept.

GREATER THAN, LESS THAN, EQUAL TO

Two children, each with a *yupana*, can each place a different number of tokens to show that *5 is more than 3*, or *2 is less than 4*, or *3 equals 3* and should be given plenty of opportunity to verbalize this, using correct vocabulary, before being asked to represent it symbolically.

ADDITION AND SUBTRACTION

When you feel comfortable with the children's grasp of place value, the *yupana* is a useful tool for introducing addition and subtraction. Calculations of the form 3 + 4, or 3 + 2 + 3, or 7 - 2, which do not involve regrouping, may be introduced early in the process, even before the tens column is introduced.

When regrouping is needed, for operations such as 7 + 5, or 12 - 7, the idea of the exchange between ten tokens and one token becomes useful. If children are asked to show 12 on the *yupana* (Figure 6a) and then to subtract 7, they would have to exchange the one ten for 10 units, which can be kept in the units reservoir (Figure 6b). It is then easy for the child to subtract 7 beans, leaving 5 (Figure 6c).

SUMMARY

This tool may be used for exploratory activities in math, for teaching activities for large or small group instruction, or as remediation for a child who has been experiencing difficulty with other teaching methods. Teachers of older students might want to make this an

enrichment project for a group to explore and then demonstrate. The suggestions given in this article may be extended in a variety of ways. For example, the *yupana* may be used to teach higher powers of ten by building *yupanas* with three or more columns. You would add about 8 cm to the width of the cardboard for every new place value you wanted to incorporate. You might decide to paper over any columns that were not yet needed.

If you decide that the children are ready to move beyond the exploratory phase, they can Carry out the activities on their *yupanas* and then draw sketches of them, shading circles to represent tokens. In this case, grid paper is useful for young children if you do not choose to prepare master sheets for them to fill in. Remember the need to spend plenty of time on the bridge between concrete and abstract. A rule of thumb is to allow the children themselves to ask to be allowed to work without the manipulative. They will know when they are ready to move on. In any case, the links among concrete, symbolic and abstract are not likely to be made by most children unless they are given plenty of opportunity for exploration with manipulatives.

The best manipulatives are those simple enough to stand alone. That is, the child can use the manipulative to learn mathematics, without having to spend a lot of time learning how to use the manipulative. They should also be simple enough for the teacher to construct without too much time, effort or money. The *yupana* is one tool that fits the bill on these criteria.

My thanks to Susana Vargas Davila, Martha Villavicencio Ubillus and Alberto Patino of the Catholic University of Peru, Lima, Peru, for introducing me to La yupana.

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MATHEMATICS, SCIENCE & TECHNOLOGY EDUCATION NEWSLETTER

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FIVE STEPS TO TEACHING PROOF IN THE SENIOR MATH CURRICULUM:

A Cooperative Learning Approach
Part II

by
Brendan Kelly
University of Toronto

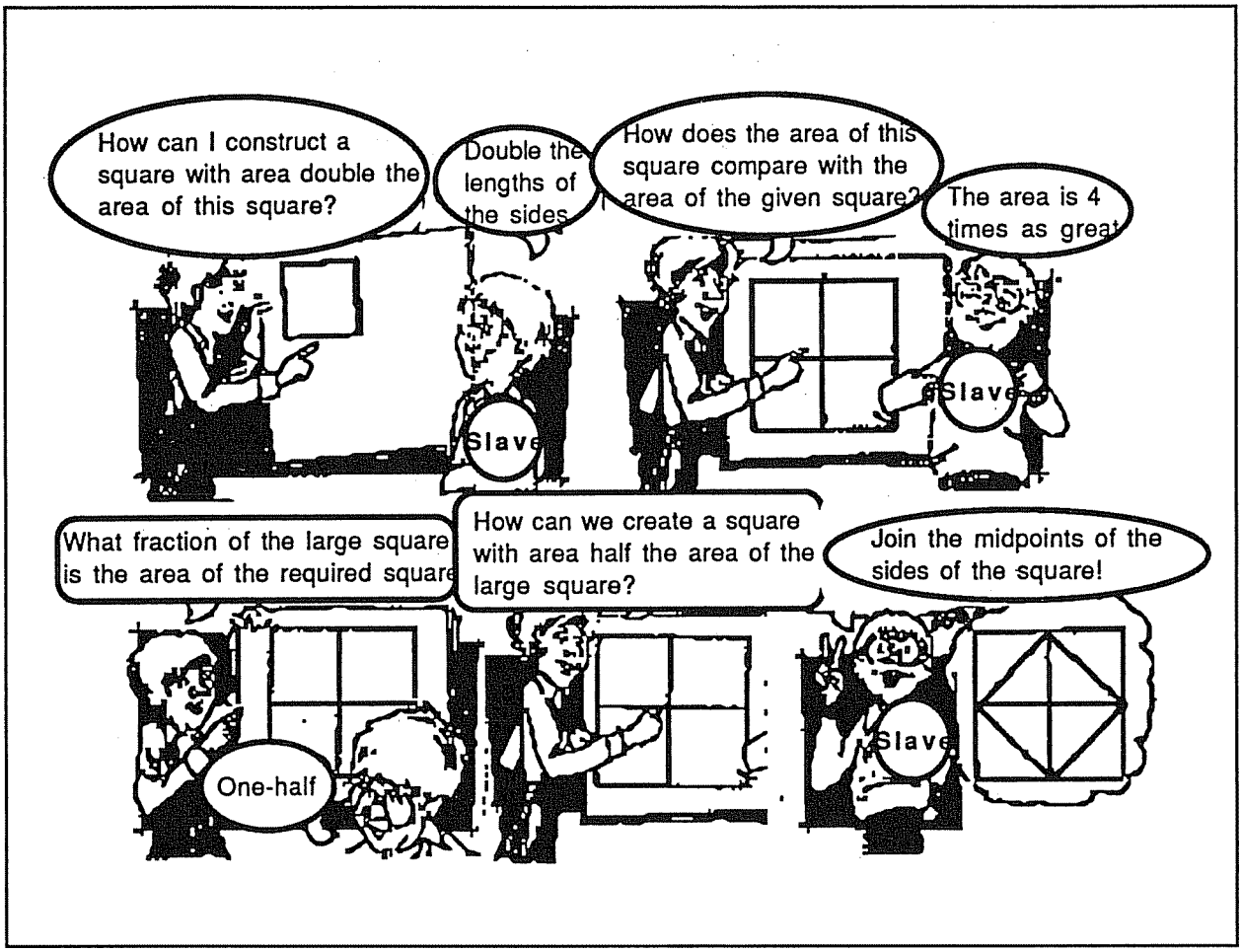
INTRODUCTION

This is part II of a five-part article spanning consecutive issues of the Gazette. In part I, we described briefly the central role of proof in the senior mathematics curriculum and the reasons why we might regard a study of proof as one of the invariants in a changing mathematics curriculum. That article also noted the trend toward alternative instructional methods and, in particular, cooperative learning. The intent of this five-part series is to provide a model to demonstrate how the concept of proof and its evolution might be explored with senior level students in a series of five lessons structured in the cooperative learning format. The intent is to cast the development of proof in its historical context and to show how the quest for certainty led through a cascading sequence of complexities to a point where it was recognized that absolute certainty is an unattainable goal. This development presents the evolution of proof in five independent but sequential lessons corresponding to the five parts into which this article is partitioned.

The lesson in part I of this article, titled The Dawn of Mathematical Proof: Inductive Thinking and Conjecture, referred briefly to the inductive thinking of the ancient Egyptians and Babylonians. It prompted students to conjecture formulas in specific cases and attempted to help students discover why proof is needed to support conjecture. The lesson presented here is designed to take the students through the next step in the evolution of proof in which the ancient Greeks attempted to underpin conjecture with logical argument based on common premises or axioms. This lesson presents a simplified version of the dialogue between Socrates and Meno to stimulate large group dialogue and demonstrate the statement-reason format for proofs. This is followed by two teaching examples which might be used to evoke further discussion. A set of investigation exercises is embedded in the lesson for students to explore and discuss in groups. It is intended that each group will report one or more of its explorations to the class to allow for some important follow-up discussion. Additional investigation exercises are provided as homework to ensure that all students are involved in the learning. We hope that you will find parts or all of it useful and that this might encourage you to contribute further examples of mathematics topics which are amenable to this instructional format.

Lesson 2: The Development of Direct Proof and Generalization

By 600 B.C. the ancient civilizations in Babylonia and Egypt were in decline. Meanwhile in ancient Greece, a bold new society committed to philosophical investigation and deductive thinking was emerging. During the next few centuries, this civilization would contribute more to mathematics and mathematical proof than all the preceding civilizations as well as any to follow in the next 1500 years! The following dialogue (called the Meno dialogue) between the famous Greek philosopher, Socrates and a Greek slave exemplifies the step-by-step series of questions and answers (the so-called Socratic method) which was used over 2000 years ago to teach deductive thinking.



Socrates helped the slave discover how to construct a square with double the area of a given square. To verify or prove that the new square is twice the area of the given square, we reconstruct the dialogue in the following tabular form.

STATEMENT	REASON
1. The large square has 4 times the area of the given square.	It is composed of 4 copies of the given square.
2. The inscribed square consists of 4 triangles. Each is half the area of the given square.	The diagonal of a square divides it into 2 triangles of equal area.
3. Therefore the area of the inscribed square is twice the area of the given square.	Each of the 4 triangles is half the area of the given square.

Table 1. Reconstruction of dialogue.

To make the foregoing proof more convincing, or rigourous, we would have to prove that the quadrilateral formed by joining the midpoints of the sides of a square is itself a square. If this were not the case, the entire proof would collapse. For this reason, it is important that a proof be free from hidden assumptions.

The "statement-reason" format for tabulating deductive proofs facilitates checking an argument for hidden assumptions. This deductive method pioneered by the Greeks over 2000 years ago remains the cornerstone of modern mathematical proof.

TEACHING EXAMPLE 1

Prove that S(n), the sum of the natural numbers from 1 to n is given by the formula

$$S(n) = \frac{n^2+n}{2}$$

Solution

Statement	Reason
1. $S(n) = 1 + 2 + 3 + \ldots + n$	Definition of S(n).
2. $S(n) = n + (n-1) + (n-2) + \ldots + 1$	Commutative law for addition.
3. $2S(n) = (n+1) + (n+1) + \ldots + (n+1)$ $= n(n+1)$	Adding vertically adjacent pairs of numbers in equations (1) and (2).
4. $S(n) = \frac{n(n+1)}{2}$	Dividing both sides of equation (3) by 2.

The following example provides a series of facts which we must use to deduce a logical conclusion.

TEACHING EXAMPLE 2

Mr. Abel, Mr. Bessel and Mr. Cayley each have a grown daughter. One of the daughters is a teacher, one a writer and one a lawyer. Use the facts below to determine whose daughter is the lawyer.

Fact 1 The lawyer often plays tennis with her father.
Fact 2 Mr. Bessel's daughter is friends with the teacher.
Fact 3 The teacher's father plays golf every Sunday.
Fact 4 Mr. Abel does not like sports of any kind.

Solution

Statement	Reason
1. The teacher is not the daughter of Mr. Abel.	Facts 3 & 4
2. The teacher is not the daughter of Mr. Bessel.	Fact 2
3. The teacher is the daughter of Mr. Cayley.	Statements 1 and 2
4. The lawyer is not the daughter of Mr. Abel.	Facts 1 and 4
5. The lawyer is not the daughter of Mr. Cayley.	Statement 3
6. Therefore, the lawyer is Mr. Bessel's daughter.	Statements 4 and 5

INVESTIGATIONS

EXPLORE AND DISCUSS THESE QUESTIONS IN YOUR GROUP.

Use logical argument or algebraic manipulation to prove each statement by the direct method.

1. The product of two consecutive integers is even.
2. The product of three consecutive integers is a multiple of 6.
3. The product of 2 consecutive odd numbers is one less than the square of an even number.
4. Any common factor of two given numbers is a factor of their sum and their difference.
5. The product of the highest common factor and least common multiple of any two numbers is equal to the product of those two numbers.

COMPLETE EXERCISES 6 AND 7 FOR HOMEWORK.

Use the tabular format shown in example 2 to solve the following logical problems.

6. The teacher knew that one of Aaron, Brad, Carol or Dana had cracked the code to the school computer data bank. Each of these students, in turn, made a statement, but only one of those four statements was true.

   Aaron said "I didn't do it."
   Brad said "Aaron is lying."
   Carol said "Brad is lying."
   Dana said "Brad did it."

   Which of the four students cracked the computer code?

7. A mathematician in search of Certainty City arrived in the unfamiliar town of St. Arbitrary. Each inhabitant of St. Arbitrary belonged to one of two religions; the Truthtellers (who always told the truth) or the Liars (who never told the truth). He arrived at a point in the road where it branched in two directions. To determine which of the branches lead to Certainty City, he asked an inhabitant, "If I asked a member of the other religion which road leads to Certainty City, what would he answer?". The mathematician heard the inhabitant's response and then took the opposite road.

   Prove that the mathematician took the correct road to Certainty City.

In part III of this article, to appear in the next issue, we present the third lesson and an important phase in the evolution of mathematical proof, under the heading Indirect Proof and Proof by Counter-example.

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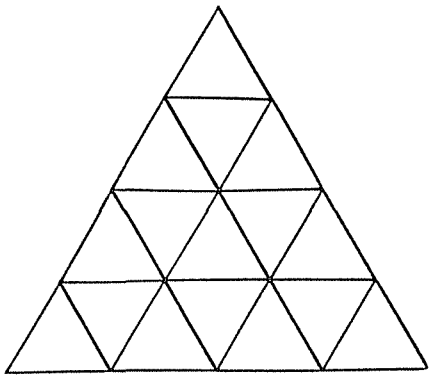
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HOW MANY TRIANGLES?

by
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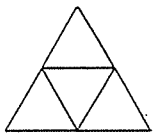


Given an equilateral triangle ABC with each side length equal to 4 units; divide each side into 4 equal lengths. Join the points of section as shown. How many equilateral triangles are formed?

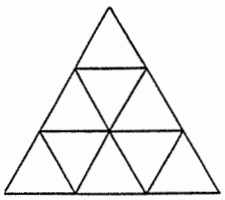
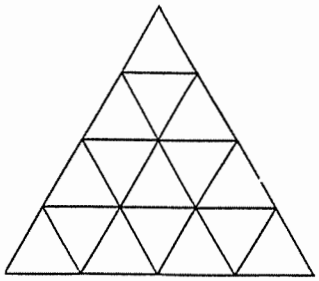
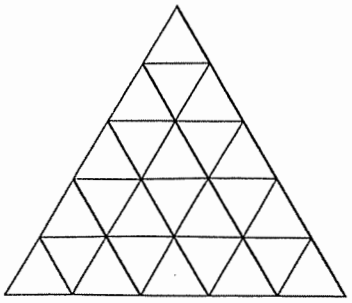
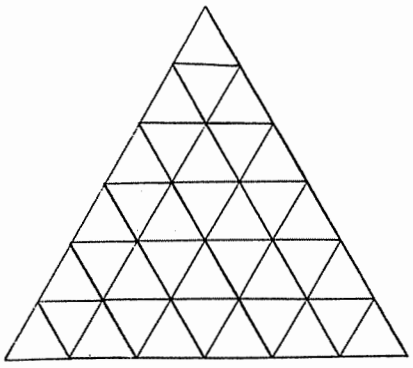
The solution (27) can be found by simply counting. Extend the problem to an equilateral triangle whose sides are n equal parts. Join the points of section as before. How many equilateral triangles can be formed?

The answer is not readily obvious. Mathematical inquiry leads to the discovery of patterns and formulations of summations of a unique series. The devised method is very appropriate for use with the computer.

Let $f(n)$ denote the number of the equilateral triangles so obtained. We shall investigate the cases where n = 1 to 6.

n = (size of each side)	Diagram	$f(n)$ = Number of Equilateral Triangles
$n = 1$		$\therefore f(1) = 1$
$n = 2$		Length = 1: (1+2) + 1 Length = 2: 1 $\therefore f(2)=[1 + (1+2)] + 1$

Note: The number of inverted triangles is 1

n = (size of each side)	Diagram	$f(n)$ = Number of Equilateral Triangles
$n = 3$		Length = 1: (1+2+3) + (1+2) Length = 2: 1+2 Length = 3: 1 $\therefore f(3) = [1 + (1+2) + (1+2+3)] + (1+2)$
$n = 4$		Length = 1: (1+2+3+4)+(1+2+3) Length = 2: (1+2+3)+1 Length = 3: 1+2 Length = 4: 1 $\therefore f(4) =$ [1+(1+2)+(1+2+3)+(1+2+3+4)]+ [1+(1+2+3)]
$n = 5$		Length = 1: (1+2+3+4+5)+(1+2+3+4) Length = 2: (1+2+3+4)+(1+2) Length = 3: 1+2+3 Length = 4: 1+2 Length = 5: 1 $\therefore f(5) =$ [1+(1+2)+(1+2+3)+(1+2+3+4)+ (1+2+3+4+5)]+[(1+2)+(1+2+3+4)]
$n = 6$		Length = 1: (1+2+3+4+5+6)+(1+2+3+4+5) Length = 2: (1+2+3+4+5)+(1+2+3) Length = 3: (1+2+3+4)+1 Length = 4: 1+2+3 Length = 5: 1+2 Length = 6: 1 $\therefore f(6) =$ [1+(1+2)+(1+2+3)+(1+2+3+4)+ (1+2+3+4+5)+(1+2+3+4+5+6)]+ [1+(1+2+3)+(1+2+3+4+5)]

Note: The last part of the sum denotes the number of inverted triangles.

Following this pattern, we can easily write out

$f(7)$ = [1+(1+2)+(1+2+3)+(1+2+3+4)+...+
1 + 2 + 3 + 4 + 5 + 6 + 7)] + [(1 + 2) +
(1+2+3+4)+(1+2+3+4+5+6)]

and

$f(8)$ = [1+(1+2)+(1+2+3)+(1+2+3+4)+...+
+(1+2+3+4+5+6+7+8)]
+[1(1+2+3)+(1+2+3+4+5)
+(1+2+3+4+5+6+7)]

In general, we obtain

$f(n)$ =
[1+(1+2)+(1+2+3)+...+(1+2+3+...+n)]+
[1+(1+2+3)+(1+2+3+4+5)+...+
(1+2+3+...+(n-1))], when n is even,

(Note that there are $n/2$ terms in the last part of the above sum) and

$f(n)$ =
[1+(1+2)+(1+2+3)+...+(1+2+3+...+n)]+
[(1+2)+(1+2+3+4)+(1+2+3+4+5+6)+...+
(1+2+...+(n-1))], when n is odd,

(Note that there are (n-1)/2 terms in the last part of the above sum.)

From this general formula alone, one can naturally write a computer program. Find the total number of equilateral triangles when the size of each side, n, is input.

The following program will execute the task.

```
10 INPUT "ENTER A POSITIVE
   INTEGER N ";N
15 IF N=1 THEN PRINT "ANS. ";N:END
20 T=1
30 FOR X=1 TO N :S=S+T:T=T+X+1:NEXT X
40 IF INT(N/2)<>N/2 THEN 100
50 A=1
60 FOR X=1 TO N/2 : W=W+A:
   A=A+2*X+2*X+1:NEXT X
70 PRINT "ANS"=";W+S
90 END
100 B=1+2
110 FOR X=1 TO (N-1)/2
120 K=K+B : B=B+2*X+1+2*X+2
130 NEXT X
140 PRINT "ANS". ";K+S
150 END
```

Run the above program. The results are:

When n = 2 ans = 5
When n = 3 ans = 13
When n = 4 ans = 27
When n = 5 ans = 48
When n = 6 ans = 78
When n = 7 ans = 118
When n = 8 ans = 170
When n = 9 ans = 235
When n = 10 ans = 315
When n = 20 ans = 2255
When n = 100 ans = 256275
When n = 500 ans = 31406375
When n = 499 ans = 31218625

We now proceed to find the generating function and the recursive formula for $f(n)$.

When n is even

$$f(n) = \sum_{k=1}^{n} t_k + \sum_{k=1}^{n/2} u_k$$

where t_k and u_k can be defined by a recursive formula below:

$$\begin{cases} t_1=1 \\ t_k=t_{k-1}+k, k>1 \end{cases}$$

and

$$\begin{cases} u_1=1 \\ u_k=u_{k-1}+(2k-2)+(2k-1), k>1 \end{cases}$$

When n is odd

$$f(n) = \sum_{k=1}^{n} t_k + \sum_{k=1}^{n/2} u_k$$

where t_k and u_k are given in the form of the recursive formula.

$$\begin{cases} t_1=1 \\ t_k=t_{k-1}+k, k>1 \end{cases}$$

and

$$\begin{cases} u_1=1+2 \\ u_k=u_{k-1}+(2k-1)+(2k), k>1 \end{cases}$$

Finally, we find the generating function for $f(n)$.

1. $n = 2m$ case.

$f(n)=[1+(1+2)+(1+2+3)+...+(1+2+3+...+n)]$ (n terms)
$+[1+(1+2+3)+...+(1+2+3+...+(n-1))]$ ($n/2$ terms)

Let $S_n = 1+(1+2)+(1+2+3)+...+(1+2+3+...+ n)$

$$=\sum_{k=1}^{n} \frac{k(k+1)}{2}$$

$$=\sum_{k=1}^{n} \frac{k^2}{2}+\sum_{k=1}^{n} \frac{k}{2}$$

$$=\frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}\right]+\frac{1}{2}\left[\frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{12}[2n+1+3]$$

$$= \frac{n(n+1)(n+2)}{6}$$

Let $S^1_{n/2}$
$=1+(1+2+3)+(1+2+3+4+5)+...+(1+2+3+...+(n-1))$

$$=1+\frac{3}{2}(1+3)+\frac{5}{2}(1+5)+...+\frac{n-1}{2}(1+(n-1))$$

$$=\left(\frac{1}{2}+\frac{1^2}{2}\right)+\left(\frac{3}{2}+\frac{3^2}{2}\right)+\left(\frac{5}{2}+\frac{5^2}{2}\right)+...+\left(\frac{n-1}{2}+\frac{(n-1)^2}{2}\right)$$

$$=\frac{1}{2}[1+3+5+...+(n-1)]+\frac{1}{2}[1^2+3^2+5^2+...+(n-1)^2]$$

$$\frac{1}{2}[1+3+5+...+(2m-1)]+\frac{1}{2}[1^2+3^2+5^2+...+(2m-1)^2]$$

$$=\frac{1}{2}\left[\frac{m(2m)}{2}\right]+\frac{1}{2}\left[\frac{m(2m-1)(2m+1)}{3}\right]$$

$$=\frac{1}{2}m^2+\frac{1}{6}m(2m-1)(2m+1)$$

$$\therefore f(n)=f(2m)$$
$$=S_{2m} + S^1_m$$
$$=\frac{2m(2m+1)(2m+2)}{6}+\frac{m^2}{2}+\frac{m(2m-1)(2m+1)}{6}$$
$$=\frac{m(4m+1)(m+1)}{2}$$

Now we may use this generating function to evaluate

$f(500)=250(4x250+1)(250+1)/2 = 31406375$

this concurs in the result from computer program.

2. $n=2m-1$ case

$$f(n)=[1+(1+2)+(1+2+3)...+(1+2+3+...+n)] \quad (n \text{ terms})$$
$$+[(1+2)+(1+2+3+4)+...+(1+2+3+4+...+(n-1))] \quad (n-1)/2 \text{ terms}$$

The first part of the above sum is the same as case 1.

We only have to find the sum of the second part.

$$S^1_{\frac{n-1}{2}} = (1+2)+(1+2+3+4)+(1+2+3+4+5+6)+...$$
$$+(1+2+3+4+...+(n-1))$$

$$=\frac{2}{2}(1+2)+\frac{4}{2}(1+4)+\frac{6}{2}(1+6)+...+\frac{n-1}{2}(1+(n-1))$$

$$=\left(\frac{2}{2}+\frac{2^2}{2}\right)+\left(\frac{4}{2}+\frac{4^2}{2}\right)+\left(\frac{6}{2}+\frac{6^2}{2}\right)+...+\left(\frac{n-1}{2}+\frac{(n-1)^2}{2}\right)$$

$$=\frac{1}{2}[2+4+6+...+(n-1)]+\frac{1}{2}[2^2+4^2+6^2+...+(n-1)^2]$$

$$=\frac{1}{2}[2+4+6+...+2(m-1)]+\frac{1}{2}[2^2+4^2+6^2+...+(2(m-1))^2]$$

$$=[1+2+3+...+(m-1)]+2\ [1^2+2^2+3^2+...+(m-1)^2]$$

$$=\frac{(m-1)m}{2}+2\left[\frac{(m-1)m(2(m-1)+1)}{6}\right]$$

$$=\frac{m(m-1)}{2}+\frac{(m-1)m(2m-1)}{3}$$

$$\therefore f(n)=f(2m-1)=S_{2m-1}+S^1_{m-1}$$

$$=\frac{(2m-1)(2m)(2m-1+2)}{6}+\frac{m(m-1)}{2}+\frac{(m-1)m(2m-1)}{3}$$

$$=\frac{m(2m-1)(2m+1)}{3}+\frac{m(m-1)}{2}+\frac{(m-1)m(2m-1)}{3}$$

Simplify this, we have

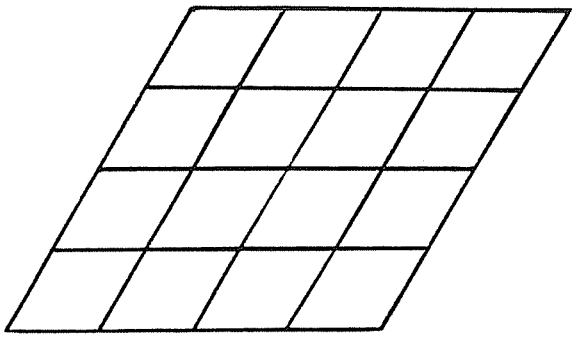
$$\therefore f(n)=\frac{m}{2}(4m^2-m-1)$$

To find $f(499)$, we apply the above formula and have $f(499) = 250(4\text{x}250^2-250-1)/2$
$=31\ 218\ 625$

This is also the answer run by computer.

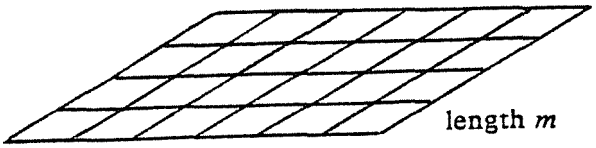
We now conclude this paper by posing the following questions.

1)



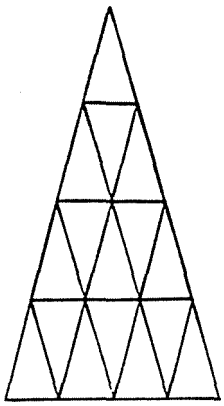
Let the size of each side of a rhombus be n units. Divide each side into n equal parts. Join the division points, as shown. Let $f(n)$ denote the number of rhombuses so formed. Find this generating function of $f(n)$ and write a computer program to evaluate $f(n)$ when n is input.

2)



Let the lengths of two adjacent sides of a parallelogram be n,m respectively where $n>m$. Divide the side with length n into n equal parts and the side with length m into m equal parts. Join the division points as shown. Let $f(n,m)$ denote the number of rhombuses so formed. Find the generating function of $f(n,m)$ and write a computer program to evaluate $f(n,m)$ when n, m are input.

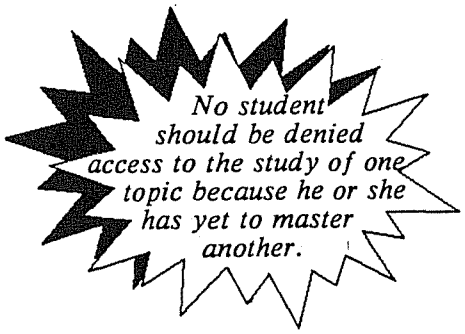
3)



The length of each leg of an isosceles triangle is equal to n units. Divide each side into n equal parts. Join the division points as shown. Let $f(n)$ denote the number of isosceles triangles so formed. Does $f(n)$ have the same generating function as $f(n)$ in the equilateral case?

INTERPRETING THE STANDARDS

(This feature is designed to help contextualize a number of recommendations from the NCTM curriculum and evaluation standards for school mathematics - we invite your comments.)



All students, including students with identified special needs, should have access to a balanced mathematics program. This program should include arithmetic, measurement, geometry and problem solving.

Traditionally, there has been a tendency to focus on basic arithmetic skills involving addition, subtraction, multiplication and division. These operations in the elementary school appeared with whole numbers, fractions and decimals. Emphasis on these topics often resulted in students getting little or no experience, particularly with geometry.

Many teachers insisted upon mastery of discrete arithmetic and drill and practice was the order of the day. We did fractions for 3 months and the students are still struggling.

Students deserve better. Make computation a tool. Introduce technology! Now all students can solve real problems, explore real data, and use mathematics from their own environment. The student struggling to master discrete pieces of unrelated information might blossom in an art unity on symmetry.

Quotes interpreted by Dr. Alex Norrie
Peel Board of education

THINKING THROUGH GEOMETRY WITH HANDS-ON EXPERIENCES

by
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Dept. of Elementary Education
University of St. Louis

Most teachers would probably agree that geometry has not been taught with the same emphasis given to the "basic skills" of number operations. The difficulty many students experience (Allendoefer, C.B., 1969; Fletcher, T.J., 1985) indicates that they need to become involved with confidence-building activities in this interesting and challenging branch of mathematics.

According to guidelines found in *Curriculum and Evaluation Standards for School Mathematics*, published by the National Council of Teachers of Mathematics (N.C.T.M., 1989), the fundamental goal of geometry instruction in the elementary school should be the development of intuition about geometric figures and their properties. Activities which directly involve pupils in exploring, describing, constructing, drawing, measuring, visualizing, comparing, and classifying geometric figures rather than memorizing properties and definitions will present meaningful opportunities as students learn to interpret the spatial world. The National Council of Teachers of Mathematics encourages guiding students to a strong sense of spatial relationships which in turn serve to better prepare the learners for understanding other mathematical topics such as measurement, critical thinking and reasoning skills. The study of geometry offers many advantages to the mathematical program.

The opportunity geometry presents for providing a wide variety of learning experiences has been reinforced by Pierre van Hiele and his wife, the late Dina van Hiele-Geldof (van Hiele, P., 1957; van Hiele-Geldof, D., 1958). The van Hieles delineated characteristics of the thinking process involved in learning geometry in five conceptual levels. Specifically, they are labelled "visualization, analysis, informal deduction, formal deduction and rigor" (Shaughnessy and Burger, 1995, p.420). When accompanied by appropriate instructional experiences, the learning model, which has received increased attention in the United States (Wirszup, 1979), asserts that the learner will progress sequentially from the initial level, visualization, in which space is simply observed, to the highest level, rigor, which focuses on formal abstract aspects of deduction. Thus, just as students progress through levels of increasingly more abstract thought when learning whole and rational number computation, (Gagne, R., 1982; Heddens, J. & Speer, H., 1988) the learners move from simple to more intricate levels of geometric understanding in the van Hiele sequence.

Implicit in the van Hieles' writing and the NCTM's *Standards* is the theme that the learning of geometry is more dependent on the type of instruction received than on the grade level or chronological age of the learner. Students should be presented with geometry experiences which directly involve observing and manipulating materials as well as communicating ideas represented by those materials. The following activities, which are suitable for pupils in grades three and above, provide hands-on opportunities for creating and analyzing geometric figures to develop conceptual understanding.

ACTIVITIES FOR THREE DIMENSIONAL FIGURES

A. Guess My Name Game: Students are provided three dimensional handmade geometric models constructed during previous lessons, three dimensional commercial materials and/or everyday objects such as shoe boxes, balls, tin cans and party hats or ice cream cones. The teacher or group leader than announces a certain number of faces, edges, or other

physical characteristics of one of the solid figures. Working individually or in small groups, students examine the models and guess which one is being described. For example, the teacher or group leader lists one of the following characteristics and the students try to match them to one of the figures. Possible attributes and answers, in parentheses, are:

1. no edges (sphere or ball)
2. curved edges (cylinder or tin can)
3. 6 equal sized faces (cube or sugar cube)
4. 6 edges, 4 faces (triangular pyramid)
5. sides are different shape from the base (square pyramid; rectangular, pyramid or shoe box; triangular pyramid.

B) Odd Shape Out Game: Students, working in small groups, pick 3 models and decide among themselves which model does not "belong" with the group of three. A first group then shows the three models to the class. The other classmates must guess which model the presenting group decided did not belong and why it was chosen as "odd shape out." Whichever group of students is correct is chosen to be the next presenting group. For example, for the models in Figure 1, the answer may be that the first model was chosen because it has 4 vertices, or the third because it has congruent bases.

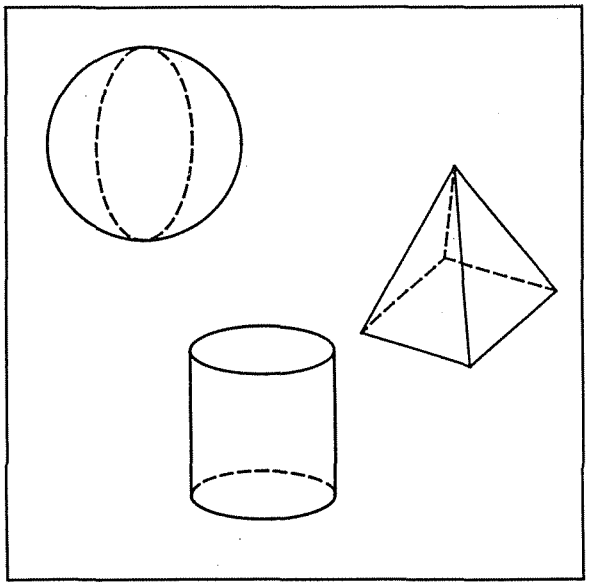


Figure 1

C. Name That Shape Game: Students are first asked to predict which three dimensional models would result from cutting and constructing the shapes outlined in Figure 2.

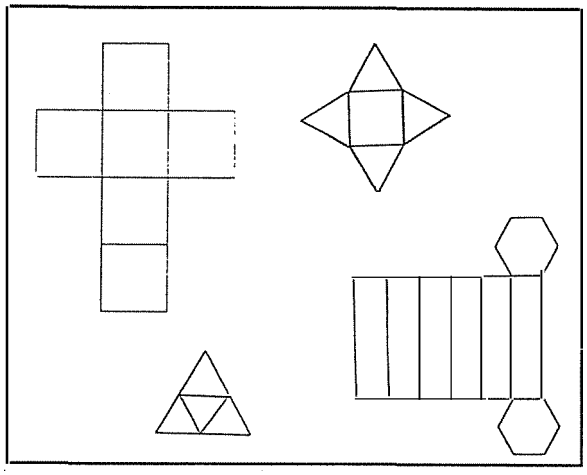


Figure 2

Working in small groups, students make their predictions and then cut and fold to test their judgements. Class discussion should encourage students to state reasons for their choices before they cut and fold shapes because the brainstorming is helpful to those in doubt. Conclude the activity by tracing one or more of the faces of the constructed shapes on paper. It becomes evident to students, for example, that the sides of a cube provide the outline of a square; thus, the two dimensional figures are seen as "parts", or a subset of three dimensional shapes. If several three dimensional figures are traced in various positions, an interesting art object can emerge from this activity.

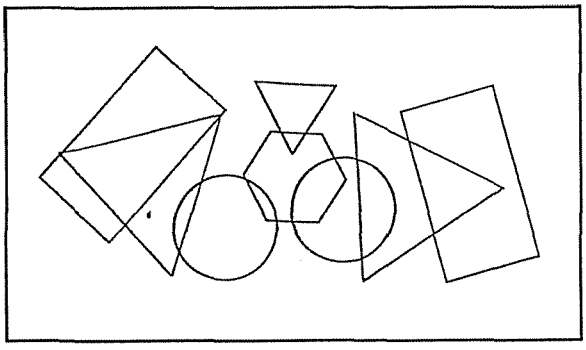


Figure 3

ACTIVITIES FOR TWO DIMENSIONAL FIGURES

Symmetry is a concept that can be taught in many ways. The idea of matching two parts of a figure can first be explored in the real world. For example, students can observe the property of symmetry in objects which contain parts that match in size and shape such as the front and back covers of their books (while indicating that the designs on them do not have to match) or the wheels on a toy car. Students' own bodies can be analyzed as learners notice symmetry in their feet and hands. Next, students can advance to paper models and build shapes on geoboards as described in the following activities.

A. Wax Paper Activity: Wax paper is a useful material for demonstrating symmetry. The matching sides of a figure are easy to see because the fold lines appear darker than the rest of the wax paper. The children enjoy this "change of pace" paper and it is relatively inexpensive to provide. The teacher should cut out and distribute equilateral triangles, regular pentagons, regular hexagons, squares, rectangles, and irregular shapes. Students try to fold the paper shapes so that one part completely covers the other part so that the pupils discover whether the lines they fold along are, or are not, lines of symmetry. If students fold too many lines so that the paper shape is confusing to explore or the creases can not be distinguished, children should begin again. If students make an incorrect fold they will then discover that the sides will not match when, perhaps, they should have. After such explorations of folding and then identifying lines of symmetry, students can tabulate their results as indicated in Table 1. Through organizing the results, students should develop the generalization that there are the same number of lines of symmetry as there are vertices or sides in each of the regular figures. The generalization also acts as a check while students match their folded figures to the results of the table.

Symmetry Table

Figure	# of Sides	# of Vertices	# of lines of Symmetry
Square	4	4	4
Equilateral triangle	3	3	3
Regular Pentagon	5	5	5
Regular Hexagon	6	6	6

Table 1.

The wax paper may be cut along lines of symmetry and traced onto construction paper to create interesting art projects. Students may explain their results during class discussions as they discover which shapes are and are not symmetrical.

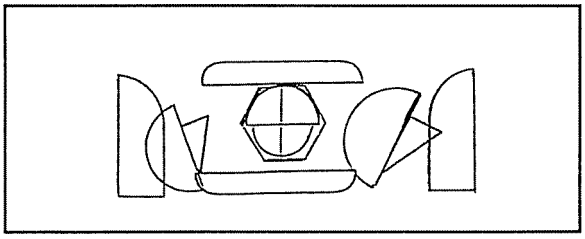


Figure 4

B. Same But Different: Reflection, as a topic in geometry in the elementary school curriculum, is yet another application of the idea of symmetry. Students enjoy using a geoboard, a board with pegs arranged in a 5×5 or 6×6 array. Rubber bands can be stretched over the pegs to form two dimensional figures. A lesson in reflections might begin with the student stretching a rubber band from upper left to lower right to form a diagonal "line". This diagonal line segment is the line of reflection or line of symmetry. One student, working in groups of two, could form a design on one side of the diagonal. The other student would then construct a reflection, or mirror image, on the other side of the diagonal, as indicated in Figure 5. Students must form shapes so that each peg of the original design is the same distance from the diagonal as is its mirror image on the other side of the diagonal.

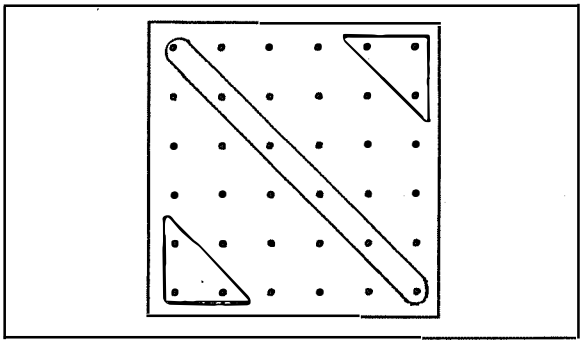


Figure 5

Dot paper could be substituted for the geoboards if none are available. When students work in pairs, they can check each other's work to see that the drawings are, indeed, symmetrical. The matching dots must be the same distance from the line of reflection, also called the line of symmetry. If working independently, pupils might draw or make the figures and submit them to the teacher for verification. Students may want to find other lines of symmetry, such as vertical or horizontal lines down the middle of the geoboard.

ACTIVITIES FOR ONE DIMENSIONAL FIGURES

The concept of a line segment can be presented in a concrete manner so that its properties can be explored. Rather than having students memorize abstract definitions from a text, the geoboard or dot paper can again be used to study this geometric idea. Line segments on a geoboard can be represented by pairs of pegs along with the rubber bands that connect them or by the drawn line connections from dot to dot on the dot paper. Finally, children can learn to identify the defining pegs (points) of a line segment as the endpoints of the segment.

The abstract concept of point can also be made real to the students as they manipulate rubber bands around pegs on a geoboard, each peg representing a point. If dot paper is used instead of the boards, the dots can be thought of as points.

Dot's Right: Using a geoboard, students pick a peg, identify it as *A* and answer the following questions either individually or in small groups.

1. First, connect line segments, each having Peg *A* as an endpoint, to another peg on the board. Each line segment should be connected in a different direction. How many different line segments could be formed, each having *A* as an endpoint? (The students are to discover that infinitely many line segments can have the same endpoint. Lines do not have endpoints and so the teacher needs to make the distinction that the figure being formed is, in fact, a line segment).

2. Identify another peg as *B*. How many line segments can be formed connecting peg *A* to peg *B*?

3. Pick three different pegs that are not in a row. How many line segments are needed to connect the pegs in every possible way? What happens if the pegs are all in a row? Repeat this activity with 4 pegs not in a row and then in a row.

By working with these interesting and stimulating activities as supplementary or entire lessons, teachers will be greatly encouraged by students' grasp of geometric concepts. As youngsters work with the models and think through the related tasks the students will experience the variety of reasoning activities which are encouraged in the NCTM document and in the research for appropriate concept development. Teachers are encouraged to vary the activities according to their own experience and students' abilities so that geometry will become an active, integral and motivating subject in school mathematics.

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by
Ron Lancaster
St. Mildred's-Lightbourn School
1080 Linbrook Road, Oakville, L6J 2L1

For the first time in my career I am teaching students in Grades 7 and 8. I am having one of the best years that I have ever had and I am working hard at putting together interesting assignments and activities for these students. The school that I am at is St. Mildred's-Lightbourn, an all-girls school in Oakville, Ontario. There are about 600 students from JK to OAC and I am learning a great deal from being around these students. This month I thought that readers might enjoy the following problems that I gave to my Grade 8 students on a recent test.

Grade 8 Mathematics
Test 3

Over the past few weeks there have been many people in Southern Ontario who have had the flu. In fact many schools have been half-empty because of the high number of students who have been sick.

Suppose that starting today we were to have a major outbreak of flu here at St. Mildred's-Lightbourn School amongst the 120 Grade 7 and 8 students. Let N be the total number of people who will have at some point been sick t days from today. For the questions given below, please use the following formula.

$$N = (120 \times t^2) \div (t^2 + 25)$$

(a) Find the value of N for every value of t in the table below. Round all of your answers to the nearest whole number. Show all of your work for the first 5 entries only.

$$N = (120 \times t^2) \div (t^2 + 25)$$

t	N
0	
2	
4	
6	
8	
10	
12	
14	
16	
18	
20	
22	
24	
26	
28	
30	

(b) Using your results from part (a), draw a broken-line graph. Be sure to label your axes and include a title for your graph.

(c) Using the formula, find the value of t for which N is equal to 100.

HINT: Look at your answers to part (a) and obtain an approximate answer to this question. Then use a trial and error procedure to zero in on the correct value of t.

(d) Using the formula, is it possible to find a value for t for which N is equal to 140? Why or why not? If it is not possible, how could you modify the formula so that it can be done.

SOLUTIONS AND COMMENTARY

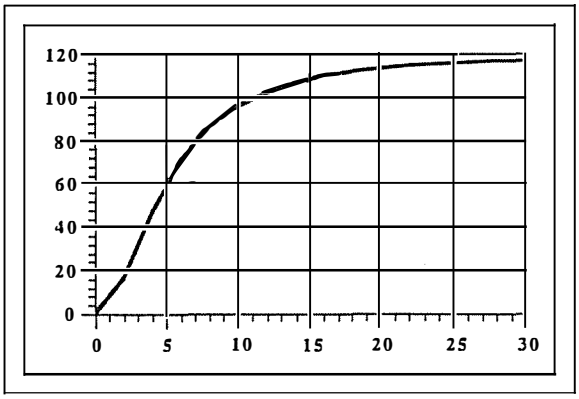
(a)

t	N
0	0
2	17
4	47
6	71
8	86
10	96
12	102
14	106
16	109
18	111
20	113
22	114
24	115
26	116
28	116
30	117

In addition to being an excellent calculator exercise for my students, I also used this question to help parents understand why calculators are needed in today's mathematics classrooms.

My students were quite interested in this question because of the practical nature of the problem and because it was related to some thing that was happening at our school and elsewhere. Their interest would have died a quick death had they done all the calculations by hand. All across Ontario, far too many Grade 7 and 8 students are bored to tears by the current curriculum and by the way it is presented. This type of practical problem would certainly help to revive their interest and passion for learning.

(b)



(c) From the results for part (a), my students realized that the value of t was between 10 and 12. Prior to doing this question, we had spent a great deal of time using the trial and error method for solving equations. With some patience many of them were able to find that t was approximately equal to 11.18. Some of them got carried away and found that t was approximately equal to 11.18033989. I realize that from the point of view of significant digits (which we talked about) that this answer is not reasonable, but the point of the exercise was to use the method of trial and error. After the test several students asked me if their was a way of finding the value of t without using a trial and error approach. I took some time to do it algebraically (which we were just starting to do with much simpler equations) and we found that t was equal to $\sqrt{125}$.

(d) It was interesting to see how my students approached this problem. At first many of them simply continued the chart from part (a). They kept increasing the value of t and found that N was never above 120 and so they concluded that N was never equal to 140.

I thought that this was quite good for a Grade 8 student. There were about 5 of the students though that realized that this really did not prove that N would never be 140. With some initial direction from me they took a look at what happens when you divide t^2 by $t^2 + 25$. Once they saw that this was always below 1, it became clear to them that N would never be greater than 120 and so in particular it would never be equal to 140. Most of the class was able to see that by changing the 120 in the formula to something above 140, it would ther be possible for N to be equal to 140

Teaching Tips

MAKING LINKS

It is important that teachers make students aware of the interconnectedness of mathematical principles. One such illustration of its significance comes from a recent experience.

In teaching a pre-calculus course, the topic of inequalities is discussed in great detail. A typical problem reduces to an inequality such equations as $x^2 - 6x + 8 > 0$. Factoring produces $(x - 2)(x - 4) > 0$. The students recognize that $x = 2$ or $x = 4$ would produce equality with 0. They are encouraged to create a chart using these so-called *breakpoints* to divide the real number line into intervals. Then the signs are examined by testing values in the separate intervals. The resulting chart is shown in Table 1.

	$x-2$	$x-4$	$(x-2)(x-4)$
$x<2$	-	-	+
$2<x<4$	+	-	-
$x>4$	+	+	+

Table 1.

Since we want to solve the inequality: $x^2 - 6x + 8 > 0$, we select the values of x that produce a positive product. The solution is given by $x < 2$ or $x > 4$. In calculus, this is commonly represented as $(-\infty, 2) \cup (4, \infty)$.

Unfortunately, a number of students struggle with this approach. Perhaps they are visual learners or algebraically unfit to feel confident with the process. How can these students be reached? Is there something in their "tool chest" that will allow them to handle such inequalities? As the teacher, one must try to create links with their prior experience.

The majority of students in this course have a sound knowledge of quadratic functions and parabolas. They could accurately graph the function. However, this may be tedious and it doesn't encourage elegance. Rather than drawing an accurate graph, it is sufficient that they note two key facts about the parabola:

1. The graph opens upwards.
2. The vertex lies between the zero s (break points) because of the symmetric features of a parabola.

Without identifying the exact coordinates of the vertex, a rough sketch can be drawn (Figure 1). Accuracy is unimportant provided that we can identify where the graph is above/below the x-axis. It can be seen that $x^2 - 6x + 8 > 0$ when $x < 2$ or $x > 4$.

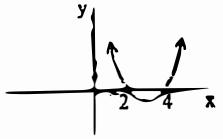


Figure 1.

I know that some students found this graphing method much easier. Others that were comfortable with the chart approach nodded confidently as they saw the graphical analogy. It reinforced their own understanding. Often I must step back and ask the question: "What other approach could be used to illustrate the concept being discussed?" Give the students more options and draw on their past experiences; the results are worth it.

by John Grant McLoughlin

MONTREAL MATH

Connecting the classroom to the REAL world
NCTM CANADIAN REGIONAL CONFERENCE
Montreal, 23025 August, 1992
Bonaventure Hilton Hotel

Why would you consider spending the last few days of your summer holiday at a conference? We asked David Letterman and he gave us the following list of ten reasons.

10. No substitutes to hire, no preps to leave.

9. There is no other NCTM conference in Canada between now and May 1993.

8. Bring the family! There's something for everyone. Cruise on the ST. Lawrence, sun bathe at Ile Notre Dame beach right in the centre of the Expo islands, or jet-boat on the Lachine Rapids (the latter is recommended for the strong of heart only).

7. Enjoy the ambience of over 5 000 restaurants.

6. Take part in the celebration of Montreal's 350th birthday.

5. Visit the Olympic Complex and (hopefully) take in a Montreal Expos game.

4. If you arrive early enough you can participate in the Comedy of Jazz Festivals. If coming during the week of the conference only, you may see a movie star or two as the Movie Festival is in full swing.

3. Join in some of our numerous social events, like Casino Night which will help kick off our conference on Sunday night.

2. See top Mathematics educators from across the continent: Steven Leinwand, Don Frazer, Lee Yunker, Stanley Bezuszka, Frank Ebos, Jim Rubillo, Jill Britton, Frank Demana, and Ron Lancaster to mention just a few. In addition many local speakers are anxious to meet and exchange ideas with you.

1. Live Montreal's unofficial motto: "La vie est belle"; which roughly translated means "Everything is open until 3:00 A.M.".

Complete the form below and return it to Michael McNally, Publicity Chairman, Lower Canada College, 4090 Royal Ave., Montreal, Qc., H4A 2M5.
We will send you specific information on your level(s) of interest and a programme booklet if you desire.

==========

NAME: _____

ADDRESS: _____

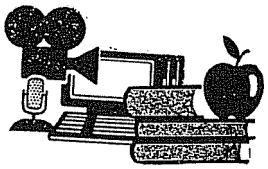
CITY: _____ Postal Code: _____

Level of interest. Circle one or more: K-3 4-6 7-9 10-12 13-above

Would you like a programme booklet? (NCTM members need not request one as you will automatically receive one next May or June). _____

REVIEWS

Editor:
George Gadinidis



UNDERSTANDING FRACTIONS V1.0
L. Gordon, M. Payne, and R. Neufeld

This tutorial was written with the Authorware Professional authoring system, an authoring language. It can be used in grades 5-9. Students can work through the tutorial by themselves, after perhaps an introductory demonstration by the teacher. It can be used by students for review. If enough computers are available (one for each student in a class), students can learn what they need to know about fractions with this tutorial without traditional lessons.

The software can be run on the following computers: Macintosh Plus, Classic, SE, SE II, or II family of computers with System 6.0 software or later and at least 1 MB RAM. The menu has buttons showing which topics have been worked on. The coverage of work with fractions is quite complete:

Meaning of fractions
Equivalent fractions
Adding fractions
Subtracting fractions
Multiplying fractions
Dividing fractions
Order of operations
Improper - mixed fractions
Fractions & decimals

Each topic consists of a Concept Part and a Practice Question Part. Some of the outstanding features are:

Very clear explanations
Well chosen examples
Excellent help
Dynamic visual illustrations
Many practice problems with interaction
Instructions for further study before leaving each topic

As an example of the visual effects, consider explaining the equivalence of 1/2 and 2/4 (see Figure 1). The fraction 1/2 is formed using the figure on the left as follows. The shaded rectangle is moved across the screen to the area where the fraction will be formed. When it arrives there, it changes to a 1 and becomes the numerator, indicating

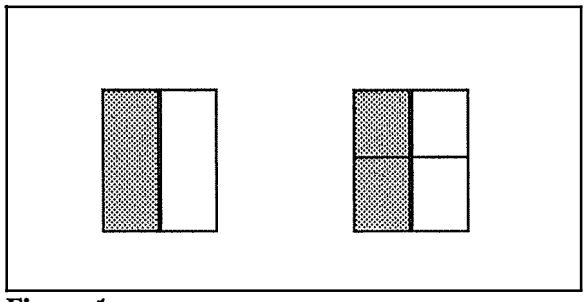


Figure 1

that one rectangle was moved. The two rectangles together are also moved; they change to a 2, and become the denominator. To form the fraction 2/4, a horizontal line is moved in from the side of the screen to make the figure on the right. Then the procedure described above is repeated for forming the new fraction. Since the shaded areas in both figures are equal, the fractions formed are equivalent.

A disadvantage of the dynamic aspect of the software is that it sometimes is a bit slow. Also, the Concept Part of Multiplying Fractions contains an extensive treatment of cancelling common factors. In the practice questions the numerators and denominators still are to be multiplied out separately, and the resulting fraction then simplified. It would also be desirable to have a facility for keeping track of student progress with the program.

Understanding Fractions is available from Logo Publications. Single copies are priced at $49.95, Lab Packs at $99.95, and a site licence is available for $199.95.

Reviewed by: John van Aalst, who is a PEMSTEP student at Althouse College.

24 -- The "I've Got It" Game.

The game of twenty-four consists of a deck of 96 double-sided cards, each side containing four numbers between 1 and 9 (digits may be repeated). The object of the game is to use the numbers, each exactly once, and the four operations (addition, subtraction, multiplication, division) to obtain an answer of 24 (brackets are allowed). There are three levels of difficulty which are indicated by dots in the corners of the cards. The price of the game is $29.00. There is a tournament package at a price of $15.00 and a pocket edition for $9.00.

I first played the game with a group of education students, all eventual teachers of mathematics. We enjoyed the game and found that some of the highest level cards gave us a challenge. We wanted to try the game in a classroom situation so we took the game to our mathematics methods class. We were surprised to find that in a class of math graduates there were feelings of math anxiety created when some of the students were not able to solve the cards as fast as other members in the group. Based on this experience we decided that the use of groups may be useful in reducing such anxiety -- letting all students play while reducing pressure on individual students to come up with answers.

I decided to try the game with my grade ten general level mathematics students. Our course had started with order of operations with whole numbers, so I decided to play this game for part of the review period. I let students pick partners that they would work with in the game. I found that students were very interested in the game. Some students used pen and paper and calculators to try to get an edge on the competition. Other students gave up and let the others solve the problems. When asked about their thoughts on the game, there were responses ranging from "frustrating" to "fun" and "interesting". It may have been a more positive experience if I had picked the teams but I did not know them well since it was the beginning of the year.

I also tried the game with my grade eleven general math class. Again, I let them pick partners to form the teams. We had four games of about three pairs of students so that everyone could play at the same time. One student initially thought it would be a waste of time and another was afraid of not being able to solve any of the problems. After the game was over they both said that they enjoyed the game. Responses to the game included "It was fun and a challenge", "It makes you want to think quickly", and "It gives you a headache"! It was great to see some of the weaker students saying "I've got it!".

This game can be a confidence builder for some of the students that don't often experience success in mathematics. Teams create less pressure on individuals and can lead to positive results. The game could be used at any time to break the routine in a math classroom. It involves practice in basic mathematical operations and all grade levels can benefit from practising the basics. As for reviewing the order of operations for a test, it may be more effective if the students were responsible for writing down their solutions. Every student could be given the same number of cards and be expected to write down the solutions for each of the cards.

I will use this game in my classes again and I encourage you to use it in any mathematics class you may teach. The supplier is:

Suntex International, Inc.
118 N. Third Street

Reviewed by:
Paul Allore
Lindsay CVI
Victoria County Bd. of Ed.

OAME RESPONDS TO THE TRANSITION YEAR CONSULTATIVE PAPER

1991/92 OAME Transition Years Committee.

Excerpts of letter and submission to Gerry Connelly -- chairperson of the Transition Years Work Team.

Attached is a response to the Transition Years Consultative Paper from L'Association Ontarienne Pour L'Enseignement des Mathematiques/Ontario Association for Mathematics Education (OAME). The OAME is pleased to be a participant in the Ministry of Education consultative process. We are responding to the document with a view to the future - the year 2000 and beyond. There are many stages for change and many bridges to be built from the present to the future. The OAME would like to be actively involved in this renewal process.

The mathematics teachers of Ontario are pleased that the Ministry of Education is developing a partnership with educators to insure growth, renewal and continued excellence in mathematics education.

Sincerely,

M. Ann Jones, President
Susan Stuart, Chair
Transition Years Committee

ISSUES

CORE CURRICULUM

A How to describe the essential knowledge, values and skills.

We recommend Option 3.1(e).
The provincial core curriculum policy should be described in terms of essential competencies without specified reference to subject disciplines. Expected outcomes in each of these should be specific at the end of the Transition Years. Essential competencies could (SHOULD) include development of knowledge and skills in the following areas:

- Communication
- NUMERACY
- CRITICAL AND CREATIVE THINKING
- AWARENESS & USE OF TECHNOLOGY
- Independent learning
- Decision-making
- Interpersonal and social relationships
- Appreciation of forms of artistic expression

This option is consistent with the goal of developing broad and integrated competencies. It calls for a revolutionary reorganization of the school curriculum away from the focus on the traditional disciplines. This would require a significant change to the departmental organization of most secondary schools, many elementary intermediate programmes and board of education programme/curriculum departments.

While it represents the alternative most closely aligned to the general goals of the renewal of mathematics education, it also represents the alternative which will be most difficult to implement. It is however, a necessary and essential step towards a realistic mathematics curriculum.

If this option is implemented, mathematics education must be a fundamental building block for the integration of other subject areas. It must be a platform for learning.

A great deal of support, centrally and locally, will be needed over a long period of time to properly interpret the mathematics component of the core curriculum. This option has tremendous ramifications regarding the entire school experience for students, teachers and the general public. This option mandates dramatic realignments of teaching strategies, evaluation, reporting, expectations for achievement and the understanding of the purpose of school mathematics held by the public.

Option 3.1(e) might best be interpreted as an ideal vision of core curriculum for the next century. Option 3.1(g) may be more viable as an attainable intermediate stage. We strongly recommend ten years as a reasonable time period for the full implementation of renewed core curriculum and appropriate teaching methodologies. An "evolutionary" approach to the implementation of the core curriculum is required to insure that the content as outlined in *Focus on Renewal of Mathematics Education, Appendix A*, is fully understood as an integral part of the core curriculum.

B What kind of curriculum documents should the Ministry of Education develop?

...The new core curriculum in the Transition Years will require the development of an unprecedented number of teacher support materials. These materials must be in place prior to deadlines for implementation of the core curriculum and must form the basis for pre-service and in-service programmes. Curriculum materials should be written by practising classroom teachers for practising classroom teachers. Perhaps the best example of this kind of material was the Geometry Resource Package developed for the junior division...

C. How should the curriculum documents be developed?

... that OAME/OMCA be active participants in the process including the development of curriculum policy guidelines, curriculum materials, teacher training and curriculum review. Professional organizations are ideally positioned to communicate with and receive input from practising classroom teachers, consultants and coordinators. Through these organizations the process of curriculum and guideline development will be consultative. The Ministry of Education should allow adequate time for reflective and detailed analysis of any draft documents by all concerned groups and individuals. The deadline for implementation of any policy should be realistic to ensure that all relevant support materials and in-service are in place for a staged, planned implementation.

D. How should technological education be included in the Transition Years core curriculum?

It is necessary that computers, calculators, symbolic manipulators and other relevant technologies be integrates across the core curriculum to support and enhance the deliver of mathematics.

Monies should be made available to implement the above. Hardware and appropriate software are not currently available in the requisite quantities to provide the regular access that the above implies...

E. Criteria for promotion to the Specialization Years.

...Promotion to the Specialization Years should depend on a consideration of many factors other than achievement within a narrowly defined subject disciple. Recommendations should be made jointly by all the teachers of the students in Grade 9, as a team. General learning, organizational thinking and communications skills need to be considered...

F. Assessment Strategies.

As outlined in *Focus on Renewal of Mathematics Education, Appendix A*, we outline the following:

Programme Indicators

1. *Assessment and Evaluation*
- Assessment should be based on a knowledge of how students learn and the needs of the student.
- Assessment should include how well a student learns mathematics independently
- Assessment involves collecting and organizing data.
- Evaluation involves interpreting data and making judgements based on the assessment
- Evaluation and assessment techniques should mirror programme objectives and content as well as instructional approaches.
- Techniques of self- and peer-evaluation should be used.
- A balance of diagnostic, formative and summative evaluation should be maintained.
- Summative evaluation should be an indication of the students' readiness for further mathematics.

2. *Collecting Data for Assessment.*
- Multiple assessment techniques including written models, projects, demonstrations, and problem solving should be used.
- Calculators, computers and manipulative materials should be used by students.
- Methods and instructions should be determined by the type of information needed, the use of the information, and the needs or development level of the students.

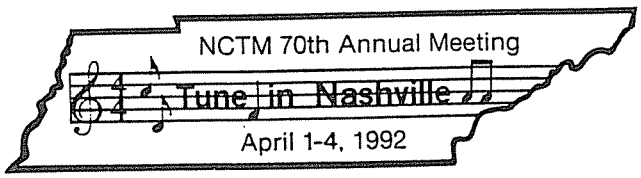
3. *Students' role in Evaluation*
- Students should demonstrate formulation of problems, strategies for problem solving, solutions for problems, verification of results and generalization of solutions.
- Students should understand, interpret, and evaluate mathematical ideas presented in written, oral or visual form.
- Students should demonstrate a willingness to persevere in mathematical tasks.

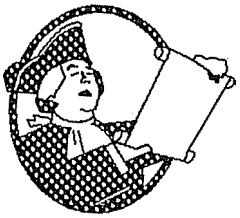
4. *Programme Evaluation*
- Information on programme outcomes, curriculum implementation and instruction should be collected systematically.
- The design of assessment instruments should reflect the relative emphasis of topics within a unit or course.

5. *Role of Examinations/Tests.*
- Standardized tests are only one indicator of programme outcomes.
- Formal examinations should play a minor role in General level, and no role in Basic level mathematics courses.
- Evaluation in Basic and General level courses should be an ongoing process, not an event.

6. *Teacher Pre- and In-service*
- There is a need for both elementary and secondary teachers to meet and have special training to focus on instructional strategies particularly suited for grades 7, 8 and 9 students.

A copy of *Focus on Renewal of Mathematics Education* may be obtained by writing to:
Harry Traint (Secretary/Treasurer OMCA)
c/o Hamilton Board of Education
PO Box 2558
Hamilton ON L8N 3L1





COUNCIL COMMENT

by
Bob Robinson

The following notes were taken at the OAME Council Meeting held January 18, '92. Your comments are welcome and should be sent to the editor.

Conferences

Leadership '91
Although numbers were down, this conference was, once again of high quality. Participants felt the sessions were excellent, as illustrated by comments made on the evaluation forms. Thanks are extended to Mickey Sandblom who co-ordinated the conference.

Leadership '92
Jeri Lunney will co-ordinate the '92 conference which will take place November 11-13, 1992, at the Ramada Inn-Airport West in Mississauga. Jeri reports that there will be no increase in fee and the theme will be "Focus on Renewal". Look for more announcements in future issues of the *Gazette*.

1992 Annual Conference
Jan Scully informed Council that 25 chairs and 85 committee members are working cooperatively to put an outstanding program together for the attendees. The program booklets are ready and are being prepared for mailing. The registration fee will be $120 for members and $155 for non-members -- quite a savings for members. Tours and other social activities are being arranged. The conference will be May 7-9, 1992, at the Sheraton Parkway Toronto North Hotel, located at Hwy.7 and Hwy.404.

1993 Annual Conference (Windsor)
Plans are proceeding well and the theme will be "Connections".
1994 Annual Conference will be in Kingston.

Committee Activities

Publications
Amy Parker will be the editor of the *Abacus* while Jack Weiner and Gary Spoar will become the interim editors for the *Gazette* commencing with the September 1992, issue. A search will continue for a new editor of the *Gazette*. Chapter contributions to the *Gazette* are encouraged. The committee is investigating ways to give the *Gazette* a new look. Council approved and is excited about our publication, to be produced four times a year, probably September, December, March, and June.

Membership/Publicity
The committee is investigating ways to market OAME and our activities. Our membership is now over 2100. The committee goals are being reviewed as well.

Mathematics Equity
There will be a Mathematics Equity breakfast at our annual meeting in May, on Friday at 7:30 a.m. The guest speaker will be Glenda Simms, President of the National Action Committee for the Status of Women. The committee is planning a Mathematics Equity conference. Watch for announcements.

Formative Years/Early Years
A draft of the association's Early Years response to the ministry was distributed. Comments will be received by the committee from council and will be discussed at the March council meeting.

Transition Years
Excerpts from the OAME response to the Transition Years Consultation paper were proposed for insertion in this issue of the *Gazette*. An excellent annotated bibliography, of use to those educators concerned about Transition Years' students, is being prepared and will be presented at a Transition Years session at our May annual Conference.

Specialization Years
The revised Calculus-TIP handbook (draft form) was discussed to provide input to the ministry. The Specialization Years consultation paper had been released just a few days before our meeting. A process has been put into place so that our response will be received by the due date in early July.

Focus on Renewal
Representatives of OAME and OMCA are examining all of the responses to the consultation paper. A vision is being prepared on what mathematics education should be. The next meeting of this group is scheduled for February. Paul Zolis was commended for his assistance and support in this project as was Alex Norrie who hosted the first meeting.

Chapter Reports

CHAMP

The fall mini-conference, MATHFEST, was held on Oct. 24th. Over 130 people enjoyed a delicious dinner, prepared by students at E. C. Drury High School in Milton. During October, the fifth annual CHAMP Mathematics Contests for students grades 8 - 11 were conducted. Once again, these contests proved to be great vehicles for encouraging students to become involved in mathematics activities. Over 5500 students, from 166 schools across four boards, participated. Congratulations are extended to the following schools and the members of their winning teams:

Grade	School
8	New Central P. S. (Halton)
9	Martingrove C.I. (Etobicoke)
10	Woodlands School (Peel)
11	Oakville Trafalgar H.S. (Halton)

Members of these teams along with other individual award winners will be formally recognized at an awards presentation as part of the chapter's Spring Event on April 9, 1992. This after-school activity is unusual in that it combines a wine & cheese social with workshops plus the special awards presentation for our math contest winners and their parents. CHAMP has completed the drafting of a constitution.

COMA

1991/1992 started auspiciously with a very successful conference, jointly sponsored by QSLMA at the Opinicon Lodge near Kingston. Once again, the conference was well attended. COMA has decided to sponsor a delegate to "Leadership 92" but has not yet decided how to choose the recipient. COMA is investigating ways to encourage more participation in their activities from people at the University of Ottawa, Carleton University, and Algonquin College. The annual January mini-conference for 1992 will be held on February 3rd this year. The topic chosen for this year's conference is "Divergent Paths: Different approaches to teaching Mathematics".

Golden Section

The fall conference was highly successful and, in fact, over-subscribed. The next meeting of the Niagara-Brock Mathematics Interface Group will be held on Feb. 10, 1992, at Brock University. The Intermediate/ Senior Golden Section mini-conference's date, time, program and location will be determined at this meeting. Judy Crompton compiled and forwarded Golden Section's response to the *Ontario Focus on Renewal of Mathematics* consultation paper.

NOMA

Many students from faculties of education are starting to attend the conferences. NOMA is planning to host a mini-conference in April in North Bay at Nipissing College.

NWOAME

The third annual mini-conference took place in September in Thunder Bay. "Leading Mathematics Education into the 21st Century" workshops were presented, generating much discussion and leaving many wanting to know more. As of the September AGM meeting, NWOAME has a new executive. Bill Otto takes over the reins from Marilyn Hurrell, who has served as the Chapter's first Past Chair -- hats off for a job well done, Marilyn. Wilf Durham (from Program Council East), who hails from Marathon, becomes the new Vice-Chair, Doug Martin remains on as Treasurer and Carolyn Carlson assumes the duties of Recording Secretary. The Executive of NWOAME is attempting to tie its annual fall conference to the Lakehead Board of Ed's major P.D. Day next October. NWOAME again plans to participate as one of the sponsors of the Lakehead University and NWOAME Math Awards and Student Recognition Day held at Lakehead University each spring.

SAME

SAME's mini-conference was held in October with about 200 in attendance. Their Spring Math Fest (Elementary and Secondary) is scheduled for April 23, 1992, from 4:00 p.m. to 6:00 p.m. The program includes publisher displays, teacher store fronts, a trading post, and table talks of 20 minutes each dealing with: E.S.L. concerns; Transitions; Manipulatives; math equity; and new computer programs.

Other Reports

NCTM

The *Mathematics Teacher* has a new look. Barry Scully reminded Council of the NCTM Annual conference in Nashville in April. More "addenda" publications are coming out.

FINAL AUDITED O.A.M.E. FINANCIAL REPORT FOR MAY 1/90-AUG.31/91				
RECEIPTS			EXPENDITURES	
BALANCE ON HAND	$8782.39		MEETINGS	$23568.70
MEMBERSHIPS	$92834.69		GAZETTE	$18472.53
LEADERSHIP '90	$21600.00		LEADERSHIP	$15649.65
ANNUAL CONFERENCE	$13266.00		CHAPTER REBATE	$11983.36
21ST CENTURY CONF.	$13113.50		21ST CENTURY CONF	$11872.89
CHAPTER LOANS	$4500.00		INVESTMENT	$10000.00
BANK/INVEST INTEREST	$4379.34		POSTAGE	$7784.71
BUTTON SALES	$2400.20		HONORARIA (2 YR)	$6200.00
DONATION	$1500.00		BUTTONS/BAGS	$5382.60
MCGREGOR FUND	$1403.00		CONFERENCE	$4282.65
LEADERSHIP '91	$800.00		SECT.EXPENSES	$2451.64
N.C.T.M. SALES	$475.00		OFFICE SUPPLIES	$2335.12
OTHER	$1412.07		CHAPTER LOANS	$2000.00
			COMMITTEES	$1135.96
			N.C.T.M.	$768.46
			GIFTS/AWARDS	$625.05
			OTHER	$426.55
TOTAL RECEIPTS	**$166466.19**		**TOTAL EXPENDITURE**	**$124941.07**
RECEIPTS LESS EXPENSES	$41525.12			
OUTSTANDING CHEQUES	$231.15			
NET	**$41756.27**			
BANK BALANCE	**$41756.27**			
SAVINGS	**$6365.78**			
INVESTMENTS	**$21000.00**			
Note 1: These OAME books were audited by Bob & Kim McRoberts, Aurora.				
Note 2: These accounts are for 15 months due to change in OAME fiscal year.				

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