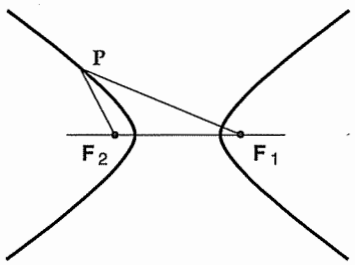
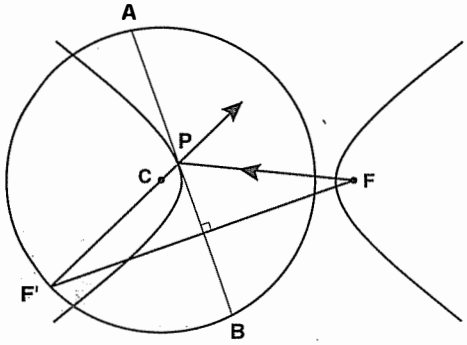
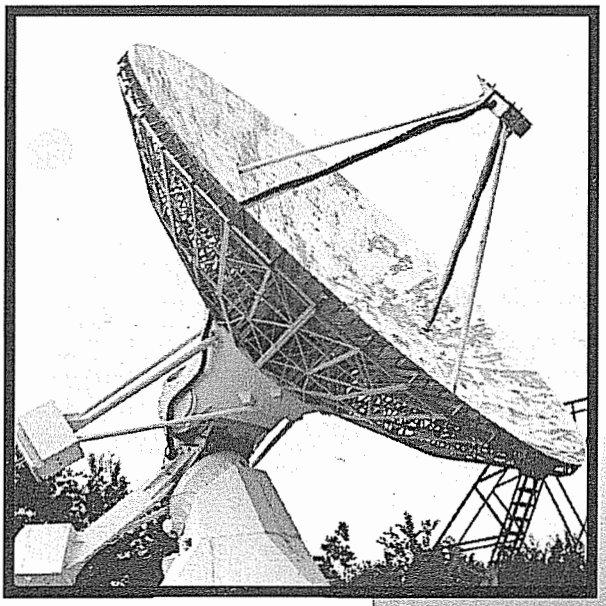
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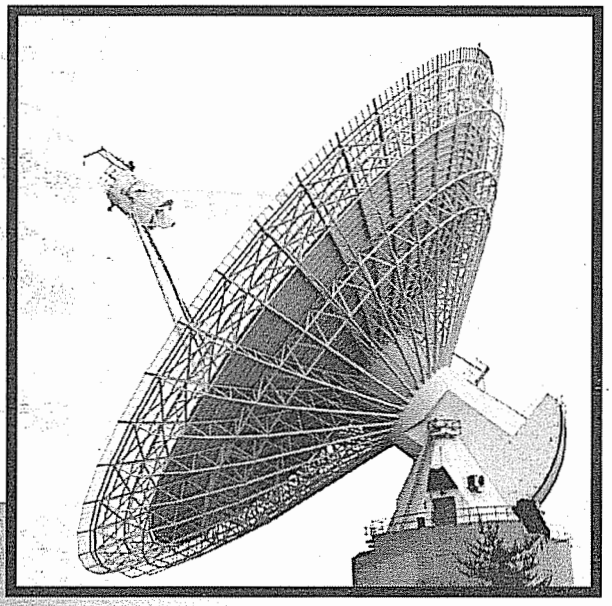
Gazette

VOLUME 1 NUMBER 31 SEPTEMBER 1992











SEE PAGE 28 FOR "REFLECTOR PROPERTIES OF THE CONICS", JOHN DEL GRANDE'S NEW ARTICLE!

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

ASSOCIATION ONTARIENNE POUR L'ENSEIGNMENT DES MATHÉMATIQUES

About the Gazette

The Gazette—ISSN 0030-3211—is indexed in the Canadian Education Index and is published four times a year. Its publication Mail Registration Number is 10264.

EDITOR

- Jack Weiner
  Department of Mathematics and Statistics
  University of Guelph
  Guelph, Ontario

ASSOCIATE EDITORS

- Gary Spoar
  Department of Mathematics and
  Statistics University of Guelph
  Guelph, Ontario

- Mary Lou Kestell
  Head of Mathematics
  Highland Secondary School

INPUTTING

- Lynn Hamilton—University of Guelph, Guelph, Ontario

TECHNICAL ART & COVER PHOTOS

- Tom Olien—Humber College, Toronto, Ontario

DESIGN & DESKTOP PUBLISHING

- Michael Tabor—MARK III Inc., Toronto, Ontario

PRINTING & BINDING

- Pole Printing, Forest, Ontario

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SUBMISSION OF ARTICLES

Articles for the Gazette should be submitted to Jack Weiner at the above address, while those for the Abacus should be submitted to Amy Parker

Deadlines for information to be included in the Gazette or the Abacus are: June 15 for the September issue, October 15 for the December issue, January 15 for the April issue, and April 15 for the June issue.

NOTICE TO AUTHORS

Your articles should be typewritten, word-processed or photocopied from typewritten or word-processed copy double-spaced on 8.5" x 11" paper with 1.5" margins and single sided. Diagrams and figures should be in camera-ready form. Please submit three copies to the editor and keep one as protection against loss. We have all submissions blind-reviewed, so please send the your full address and affiliation on a page separate from the manuscript. Please include a photograph of yourself—in black and white if at all possible.

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EDITORIAL

JACK WEINER

Welcome to Volume 31 of the Ontario Mathematics Gazette. I am honoured to be your new editor and look forward to the challenge of producing a professional journal that you will find stimulating, useful, enjoyable and relevant.

I am grateful to outgoing editors Barry Onslow and Eric Wood for easing me into the job gently. They have set a high standard over their four-year tenure and I will do my best to maintain it. They assure me they are only a phone call away for advice. Eric and Barry, thank you, but you may live to regret the offer!

Mary Lou Kestell and Gary Spoar have been appointed associate editors, thank goodness. Mary Lou, a past president of OAME, is currently Head of Mathematics at Highland Secondary School in Dundas, and is in constant demand as a speaker on mathematical issues. Gary is an Associate Professor of Mathematics at the University of Guelph. He has often given enrichment talks to talented high school students and is committed to keeping open the lines of communication among teachers of mathematics at all levels. You will hear their distinctive voices as we share (fight for!) the editor's page.

Amy Parker continues as Editor of the Abacus. Also she graciously helped proofread this issue and I will certainly impose upon her for help in the future.

You will notice a new look to the Ontario Mathematics Gazette. Michael Tabor, of MARK III Inc., has been contracted as our Desktop Publisher. Michael was responsible for the widely praised layout in the OAME Centennial Gazette.

There are a few changes in the structure of the Gazette. We have gone from three issues per volume to four, reducing the size from 64 pages per issue to 48. There are many reasons for this, but the most important, in my view is immediacy. We can communicate about the many important issues, such as destreaming and standardized testing, as they arise.

Finally, the eternal plea of the editor. Each of us has something to share. I encourage, cajole, beg you. Submit articles! Respond to columns! Comment on articles! Actively help create and improve our journal!

Best wishes for a terrific school year.

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President's Message

Jeri Lunney

It is a privilege for me to introduce myself as your President for 1992-93. For over a quarter of a century I have worked in a variety of roles in education as teacher, consultant and Vice-Principal. Currently, I am Department Head of mathematics at St. Paul's High School, a 7-OAC school in Nepean. Since 1984, I have been active in OAME activities, as Chapter Representative for COMA, Councillor, Director and President-Elect. I look forward to working with the talented group of people on this year's Council.

OAME has enjoyed the close relationship that we have had with Ontario Mathematics Coordinators' Association (OMCA) over the past year. A joint committee has studied and summarized the reactions to the Ontario Focus on Renewal of Mathematics: A Consultation Paper. Your responses were greatly appreciated and are the basis for the revision of the document.

In the past year, OAME has worked in concert with the Ministry and OMCA to articulate learning outcomes and benchmarks for mathematics at grades 3, 6 and 9. The timelines have been tense; money has been in short supply; and a large number of teachers have devoted their time to meet the frantic deadlines set by the Ministry. Now that the draft copies are in your hands, it is critical that you become involved. Get together with teachers in your region and respond to the draft version before it becomes final.

Once again, we face the challenge of renewing mathematics curriculum. Funds are limited, yet the Ministry and our Boards expect even more from us. Consultants have been sent back to the classroom, there have been labour disruptions and our teachers are tired. At the first Council meeting in June, I asked councillors to work together with me to support the teachers of the province as they face these issues. OAME will listen to your ideas and concerns and is ready to give practical assistance. This year's committees have been formed and are already at work. This is a year for action.

In the past few weeks I have been speaking with a large number of people: elementary and secondary teachers, university professors and parents. The issues of the day concern us all. As discussions and plans for the renewal of mathematics education proceed, it seems easy for some to focus on the negative aspects. Teacher-bashing has become the norm in our country as we hear of the results of provincial and international reviews in mathematics. Even within our own ranks, there is dissention regarding the role of mathematics contests, issues involving equity, back to basics versus process in mathematics. We are bashing each other instead of working together to find solutions. Let's get to work and find solutions!

Things to Watch for in 1992-93

The Gazette will have four editions this year. Watch for the new look as the Publications Committee puts its plans into action.

CONFERENCES

- The Leadership Seminar, "Focus on Renewal", will be held at the Ramada Inn on November 11-13. This conference features keynote speakers Myrna Ingalls, Burle Summers, Jack Weiner and David Zimmer. The comprehensive program includes sessions of interest to teachers of mathematics from Kindergarten to OAC and is open to the first 70 registrants.
- A new conference to be held on February 25-26 has been organized by the Equity Committee and will focus on gender issues, ESL and ESD as well as special needs students.
- The annual OAME conference will be held in Windsor on May 13-15, 1993. (Notice that this is not Mother's Day weekend.) Let's get those bus, plane and train loads organized and meet in Windsor for OAME '93.

Call for Members

Use the application form on page 34 to join the over 2200 mathematics educators, kindergarten through college and university, who enhance mathematics education in Ontario. Your membership:

Includes OAME/AOEM Publications—Gazette and Abacus ▲ Supports Professional Development through the Annual Provincial Conference, the Leadership Seminar, and Regional Mini-Conferences ▲ Provides opportunity for local input with chapter affiliation ▲ Promotes mathematics through "Excellence in Mathematics" Certificates, "I Love Math" Buttons, and several OAME/AOEM Awards ▲ Encourages networking and information sharing ▲ Stimulates Problem-Solving at all grade levels

Concrete Algebra

Michelle Helson-LeBreux—Peterborough, Ontario

Using your hands to solve equations can open up equation solving to many students who do not have the intellectual tools to work in traditional ways.

In my experience teaching grade 9 and 10 general and basic level math, equations have proved to be the most difficult and frustrating topic both to teach and to learn. If students master equations in the form $x + a = b$, they cannot seem to make the jump to $ax = b$. If they do, then combining the two types of equations is another stumbling block. Even in cases where I've felt I successfully taught the equations unit, I've found that the students can't apply those skills to problems encountered later in the course.

Now I've worked out a new method to teach solving equations. I hope that a more concrete approach to the problem will:

- eliminate some of the tension and frustration in the classroom
- help the students retain their skills for use in other contexts.

This method can be done with algebra tiles but I've chosen to use two colours of construction paper with triangles and squares in each colour. For the purposes of this article, I will refer to "shaded" and "unshaded" shapes.

Solving $x + a = b$, where a and b Are Integral

As a first example, arrange the shapes in the following way:

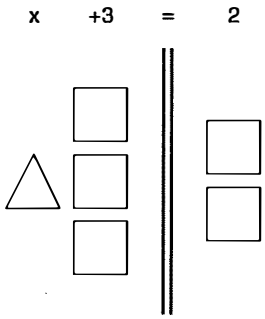


Figure 1

Now let us challenge ourselves to get one unshaded triangle by itself on the left side of the double line, using an ordered procedure governed by a small set of rules. The most important rule is that any changes made on one side of the double line must be repeated on the other side. That does not help us much with the above scenario but the next rule can be applied immediately: a shaded and an unshaded square on the same side of the double line cancel each other out. Since we would like to get rid of 3 unshaded squares on the left side of the double line, we should add 3 shaded squares to both sides. The arrangement in figure 1 becomes:

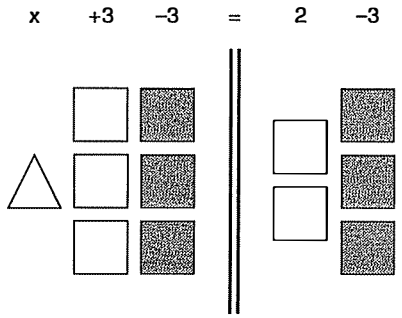


Figure 2

Now cancel shaded and unshaded squares in pairs.

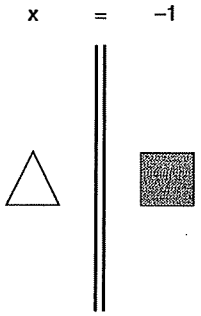


Figure 3

The challenge has been met: there is one unshaded triangle alone on the left side of the double line. What has been accomplished? Well, if one makes the following assignments:

- unshaded triangle represents x
- unshaded square represents 1
- shaded square represents –1
- double line represents =

then one will realize that we have solved the equation $x + 3 = 2$.

SOLVING $x + a = b$, WHERE a AND b ARE RATIONAL

The method works with fractions too, especially those that are easy to recognize visually, like $\frac{1}{2}$ and $\frac{1}{4}$.

The following sequence of diagrams represents the process of solving $x - 2\frac{1}{2} = -5$.

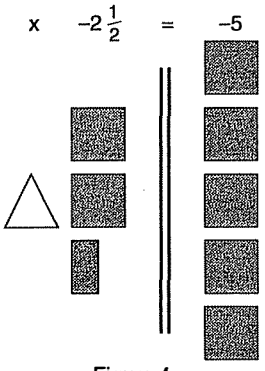


Figure 4

To get the triangle by itself add $2\frac{1}{2}$ unshaded squares to both sides.

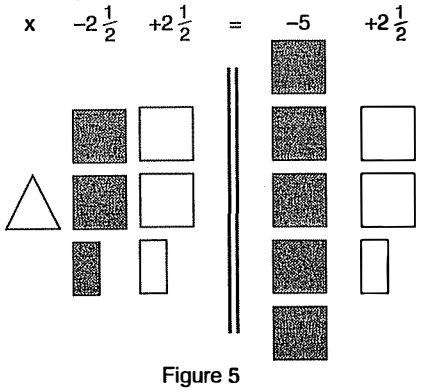


Figure 5

At this point we are ready to eliminate shaded and unshaded pairs of squares.

Two pairs can be eliminated easily—leaving half an unshaded square and three shaded squares on the right side of the double line.

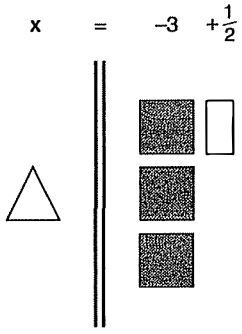


Figure 6

What we can do now is cut one of the shaded squares in half and eliminate one half with the half unshaded square, leaving the following to represent the solution to the equation:

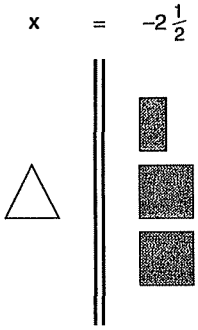


Figure 7

SOLVING $ax = b$, WHERE a IS A POSITIVE INTEGER GREATER THAN 1

Our first example of an equation of the form $ax = b$ will be $2x = 4$, represented:

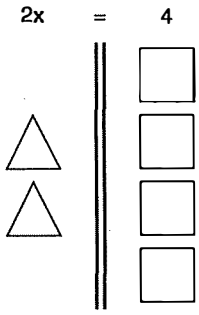


Figure 8

Students should be able to see the difference in structure between this example and the previous ones. Put simply, this one has more than one triangle on the left side of the double line. Therefore, the need for a different procedure should not be a surprise. The goal of getting one unshaded triangle by itself on the left side of the double line does not change.

What we will do is divide the squares on the right side into groups, one group for each triangle on the left. Each group must have the same number of squares.

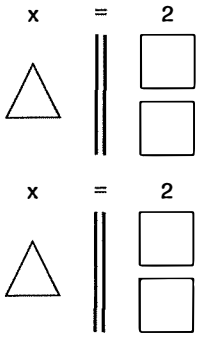


Figure 9

The solution to the equation is represented by any one group containing one triangle and some squares. In this case, the solution is $x = 2$.

The following diagrams demonstrate the entire process to solve $2x = -5$.

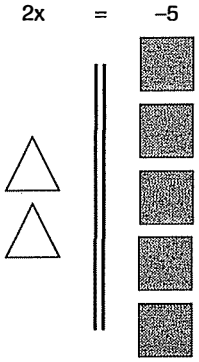


Figure 10

Regroup the shaded squares.

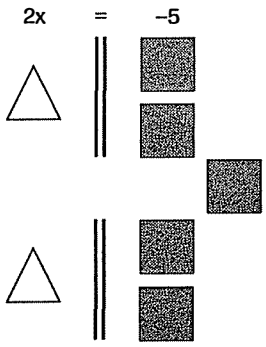


Figure 11

The one remaining black square cannot be added to either group because the groups must have the same number of squares. It must be cut in half and one half must be added to each group.

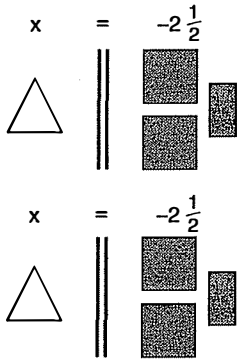


Figure 12

The solution, represented by either group, reads $x = -2\frac{1}{2}$.

Solving $ax = b$ where a is a negative integer

The method can be extended so that the coefficient of x can be a negative number. We will let a shaded triangle represent $-x$. Therefore, the equation $-3x = 8$ would be represented as follows:

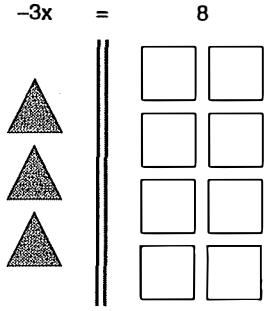


Figure 13

The grouping procedure does not change.

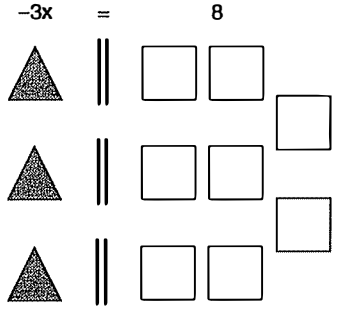


Figure 14

This time there are two outstanding unshaded squares and three groups to fit them into. The simplest way to proceed is to cut each square into thirds and place two thirds with each group as follows:

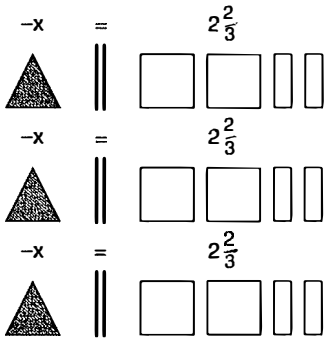


Figure 15

But this equation is still unsolved. In every previous example, we have concluded with one unshaded triangle by itself on the left side of the double line. Here we have a shaded triangle. We introduce a new rule: One may reverse the shading of a triangle only if one reverses the shading of every other shape on both sides of the double line.

We are left with:

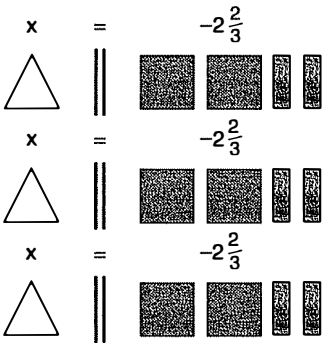


Figure 16

SOLVING *ax = b*, WHERE *a* IS A RATIONAL NUMBER

There remains to do an example of the form $ax = b$ where a is a fraction such as $\frac{1}{2}$.

We will find that it is better to use the form $\frac{1}{2}x$ than $\frac{x}{2}$.

We represent the equation $\frac{1}{2}x = 3$ as follows:

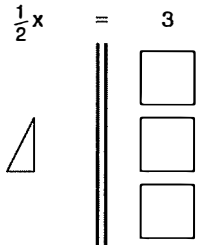


Figure 17

The half triangle represents $\frac{1}{2}x$.

The procedure, when dealing with portions of triangles (whether shaded or unshaded) will be to repeat what is given as many times as are necessary to make a full triangle. In this case, only one copy is needed. The next step would appear as follows:

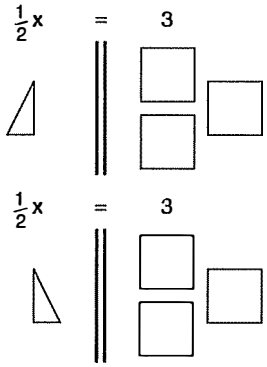


Figure 18

Now, combine the two sets of shapes as follows. This is the final step in the example, representing the solution $x = 6$.

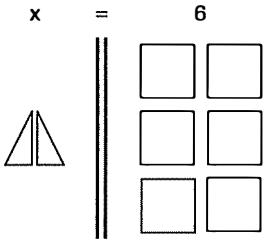


Figure 19

Precise measurements are not necessary. The half triangle used to represent $\frac{1}{2}x$ does not have to be exactly half the area of the full triangle. What is important is that the students realize that the shape has been cut into two parts. An assumption that the parts are of equal size is adequate.

A Final Example

Our final example demonstrates the procedure to solve an equation of the form $ax + b = c$. In particular we will solve $3x + 4 = -3$.

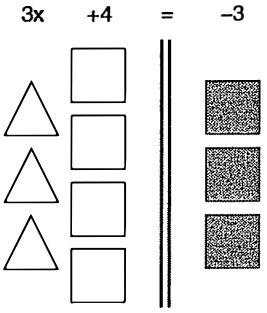


Figure 20

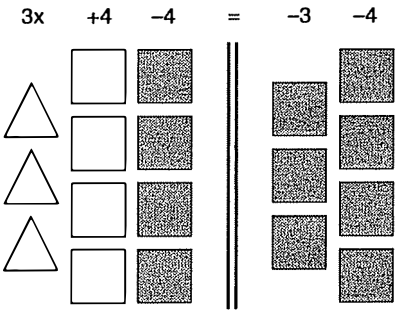


Figure 21

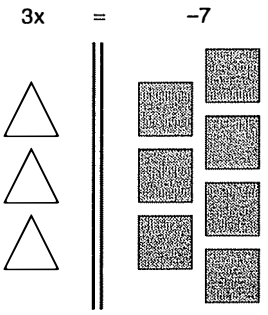


Figure 22

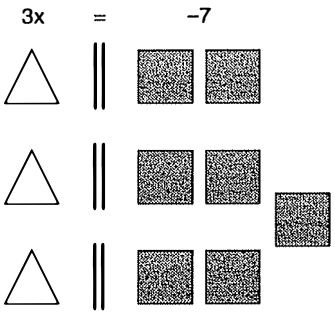


Figure 23

The solution is $x = -2\frac{1}{3}$

Summary

A summary of all the rules follows:

1. The object is always to isolate one unshaded triangle on the left side of the double line.
2. Whatever changes are made on one side of the double line must also be made on the other.
3. A shaded and an unshaded square on the same side of the double line cancel each other out.
4. The shading of any triangle may be reversed if the shading of every other shape is reversed.

In the past, I've felt like I've drowned my students in cases, processes, and rules. Not only are there too many procedures but there is no concrete way of determining which one is called for. A good abstract thinker who is confronted with $2x = -9$ may be able to identify the need to divide both sides by 2 but there is nothing in $2x = -9$ to point a more concrete thinker in that direction. My method of solving equations provides visual cues to the learner at the start of the problem and throughout the solution.

While I, personally, don't find visual cues helpful, I realize what a disservice I've been doing my students by assuming the same is true for all of them. I do not intend this method to be considered strictly as a lead into the algebraic form of solving equations although it can function in that capacity. I believe it can stand on its own in that a student who wishes to use it exclusively can do so.

Of course it is too cumbersome to solve equations with large constants or minute fractions by moving around scores of tiny shapes but I would hope that students who become proficient through practice will eventually be able to visualize the processes without having to be involved in them physically. This visualization is still quite different from the traditional algebraic solutions of equations—even though it is more theoretical than concrete manipulations. It is different because it relies on very simple symbolic representations (colours and shapes) and it uses no arithmetic operations.

In short, I believe that I have exhausted every traditional method of explaining equations. I have found that many students do not have the intellectual tools to access the information the way I have been presenting it, but they have other tools. It is incumbent on the teacher to make use of them. This concrete method of solving equations is my attempt at that.

Council Comment

Bob Robinson—Golden Section Chapter Representative

The following notes were taken at the OAME Council Meeting held June 6, 1992, at the Ramada Inn-Airport West in Mississauga. Your comments are welcome and should be sent to the editor.

Welcome

Jeri Lunney, the incoming President, welcomed the councillors to the first meeting of their new term. Jeri outlined the exciting challenges which lie ahead and our involvement in various projects. A red business binder was presented to each of the council members.

National Indicators

Lorna Morrow and Arnie Carefoote represented OAME at a May meeting where testing procedures for students were discussed. There are several issues which are of concern to OAME such as the length of the assessment, the emphasis on multiple choice instruments, the assessment of only a small part of what mathematics we are teaching our students, and the timing of the assessment (May, 1993). Concern was also expessed that the Ministry should make a statement indicating its awareness of the limitations of this assessment. Plans are underway to send a letter of our concerns to the Ministry of Education.

Annual General Meeting (1992)

Lorna Morrow and John Del Grande were appointed new Life Members of OAME. Don and Carol Attridge became Executive-Directors of the association and ISOMA (Independent Schools of Ontario Mathematics Association) became a new chapter.

Focus on Renewal

Past-President, Ann Jones, reviewed the accomplishments of the "renewal" project and commented on the excellent relationship and continued liaison with the Ontario Mathematics Coordinators' Association (OMCA) and the Ministry of Education. OAME continues to play a significant role in developments within the province including the following: responses to the Early Years, Formative Years, Transition Years, and Specialization Years consultation papers; Benchmarks Project; writing of Learning Outcomes and our continued involvement with the OAIP projects. We must ensure that OAME is a force within the province. Mathematics and mathematics education is highly complex and in a state of flux.

New Visions

President Jeri Lunney praised Ann for her leadership and other members of council for continuing and fostering further positive relationships with the Ministry of Education. Ron Sauer will act as our liaison with the OMCA. We will continue our "Focus on Renewal". There is much excitement about the work ahead and the number of council meetings this year will be increased from three to four to better accomplish our goals.

Mathematics/Art Scholarship

Jeff Martin, as a guest to the council meeting, explained that Sheila Greenspan, a keynote speaker at the Annual Conference, wished to have her honoraria applied to fund three Mathematics/Art Scholarships for grade 12 and OAC students. The successful applicants would attend an advanced studio course at the Art Gallery of Ontario for 30 Saturdays with the intention of granting an OAC independent study credit in visual arts. Questions arose regarding the selection committee and availability to all areas of the province. Since the scholarship would be ongoing, it was agreed that this scholarship would be adopted for one year only and that it be reviewed by the Awards committee for the future.

NCTM

Our representatives to NCTM will continue to be Barry and Jan Scully. They attended the NCTM Provincial Representatives Leadership Conference held in Calgary in July.

Two new journals will be published by NCTM and members were advised to read the News Bulletins for new products and developments from NCTM. It was also suggested that we promote ourselves better by recommending ourselves and fellow colleagues as speakers for regional and annual conferences.

Committee Reports

PUBLICATIONS

There will be a new format for the *Gazette* and there will be four issues. Jack Weiner, the editor, reported that future issues will include photos and a "Have Your Say" column dedicated to issues in mathematics and mathematics education.

Council approved recommendations regarding paid advertisements in the *Gazette*. Among these recommendations were the following: that OAME accept paid advertisements; that a disclaimer regarding such ads appear in the *Gazette*; that ads be accepted at the discretion of the editor; that advertising rates be established; that multiple ads receive a discount; that ads be in camera-ready form and placed at the discretion of the editor; that a schedule of advertisement submissions be established; that a guideline for the amount of advertising be established; and that every effort be made to have the first advertising appear in the April, 1993, issue. The Publications committee was directed to appoint an Advertising Manager for the *Gazette* for one year.

EARLY YEARS/FORMATIVE YEARS

The focus of this committee will be to provide practical and useful classroom activities for the Benchmarks and make regular submissions to the *Abacus*. The Early Years and Formative Years encompass over half of a student's elementary school experiences as well as two-thirds of the learning outcomes in the provincial Benchmark project.

TRANSITION YEARS

The committee plans to begin work on a recommended curriculum for Transition Years mathematics which would include knowledge, skills, and attitudes. This curriculum would be based on the five dimensions of mathematics and learning outcomes presently being developed by the Ministry.

SPECIALIZATION YEARS

The committee is preparing a summary of last year's OAME response to the Ministry's consultation paper. This summary appears elsewhere in this issue. The committee is developing a questionnaire for circulation regarding innovative schools and their teaching strategies, assessment and evaluation techniques, and the names of contact persons. It is planned to create a directory of those schools with innovative programs and a speakers' list.

MEMBERSHIP/PUBLICITY

The committee will seek out a new logo design which will be presented to the Executive and Council for approval at the October meetings. Michael Tabor, as a guest of Council, presented some fine suggestions for OAME promotions. Decisions on these will be made in the fall along with budget considerations.

FOCUS ON RENEWAL

The "Focus on Renewal" document will be revised in partnership with OMCA. A response to the Ministry's Benchmarks will be prepared through the involvement of OAME chapters and council. Plans are being made to continue our involvement with the Benchmark Project and the National Indicators Project. We will monitor and respond, as appropriate, to each of these projects and publish appropriate articles in the *Gazette*. Discussions are taking place regarding the production of a communication and information kit dealing with "renewal" and assessment. The kit would be for the general public, parents, trustees, school board adminstrators, teachers, faculties of education, student teachers, post-secondary educators, business/industry and the media. A press conference is also being planned.

Conferences

OAME CONFERENCE 1992

Jan Scully reported that over 1200 registrants experienced this highly successful annual conference. She complimented the assistance by the numerous teacher and student volunteers who assisted in so many ways. A set of recommendations and up-dated conference guidelines was presented to Todd Romiens, the chair of our next annual conference in Windsor. Jan was commended for her leadership in coordinating this very fine and successful "annual".

OAME CONFERENCE 1993

Todd Romiens indicated that plans were in order for Windsor in May, 1993. Recommendations for speakers should be sent directly to Todd. We are anticipating a great conference with outstanding facilities in the "south-west". We also hope to make this an international event by having participation by our border neighbours.

LEADERSHIP 1992

Jeri Lunney and her committee have put together a wonderful program for this event. Chapters are encouraged to have representatives from their areas attend. This is a super-value seminar. The registration is limited and the all-inclusive fee of $295 includes 3 full days of workshops, 2 nights accommodation, meals, banquet, reception, and an "intimate" publishers' display.

MATHEMATICS EQUITY 1993

Plans are well underway for the Math Equity conference to be held February 25-26, 1993, at OISE. Suggestions for speakers to address gender equity issues, ESL/ESD issues, and special needs students in the Early Years, Formative Years, and Transition Years as they pertain to mathematics education should be sent to Shirley Fairfield at the Scarborough Board of Education.

CHAPTER REPORTS

GENERAL

The Chapter Representatives met on Friday after dinner with the Executive and Committee Chairs at the Ramada Hotel. Bill Otto chaired this meeting and noted that these representatives are VIP's in the association and their liaison with teachers in the chapters is appreciated by OAME. President Jeri Lunney welcomed the group and shared her experiences as a "chapter rep". She encouraged the sharing of ideas and told them how they are key people because they represent the "grass-roots" of the organization.

CHAMP

Mathfest 1992 will be held October 15, 1992. This afternoon/evening event will include two workshop sessions, dinner, publisher display and a great opportunity to share ideas.

The CHAMP mathematics contests will be on Wednesday, November 4, 1992. The contests are currently for Grades 8 through 11. Discussions are underway to include Grade 7 for next year. Also under consideration is a plan to have the top Grade 8 students create problems for the Grade 7 contest.

GOLDEN SECTION

The mini-conference last April had 120 particpants. Future workshops are being planned.

GVMA

Two new publications will be available in November, 1992:

1 *Sample Calculus Exams,* Volume 2

2 *Technology for Teaching Mathematics* (identifies where computer technology—such as graphing calculators and spreadsheets—can be used in the mathematics curriculum)

NOMA

The fall conference will be held in Sudbury on Sept. 25-26, 1992. The displays and Friday evening social plan to be stimulating. For the first time at a NOMA conference, a "trading post" will be established.

New award certificates for mathematics have been created in both French and English. A new logo contest is being conducted involving art and graphics departments at local high schools. The Sudbury Board has made a good start regarding their Math Olympics Project. There are hopes to expand this next year.

NWOAME

On May 29, 1992, NWOAME was a sponsor at the Lakehead University/NWOAME Mathematics Awards Luncheon. This annual event honours the top math students from northwestern Ontario high schools. About 120 students attended with over 80 awards presented.

NWOAME is cooperatively planning with the PD committee of the Lakehead Board of Education an annual fall conference for Oct. 23. (NWOAME 1992) The theme for the day is: "A Day of Mathematics with NWOAME: Strategies for Transition".

PRMA

Over 250 teachers attended "Mathematics: A Social Event" mini-conference held April 9, 1992. PRMA organized a mathematics competition for schools in the Durham Region. The format was similar to "Reach for the Top". The 9th Annual Mathematics Awards night was held on June 4, 1992. Medallions were given to the top student in each school and plaques awarded to the top two schools in Durham. A fall professional activity day is being organized.

QSLMA

A very joyous "Spring-Fling" was conducted April 9, 1992, at RMC. A fall conference is being planned in cooperation with COMA at Opinicon Lodge.

SAME

"Table Talk" math was successful last spring. The Annual Fall Conference is being planned. They are planning to bring in teachers from across panels to exchange ideas. A "door-knocking" approach will be implemented to try to bring new members into the chapter.

SWOAME

The 4th annual "dinnerlude" meeting was successful. Prior to this "dinnerlude", a consortium meeting was conducted to discuss local educational issues. Another meeting similar to this is envisioned for September. Represented at the consortium were St. Clair College, University of Windsor, elementary schools, and secondary schools.

WOMA

The April 9, 1992, conference was well attended and a fall conference for October 24, 1992, is planned. OAME 1996 will be held in London or Sarnia.

Y⁴MA

The chapter is getting "back-on-track" after a highly rewarding and outstanding hosting of the " 1992 annual".

OTHER NEWS

CONGRATULATIONS!

- **Barbara Beckley**, Gateway PS—Morley MacGregor Award
- **John Champ**, President of Houghton Mifflin—Mona MacGregor Award
- **John Del Grande**—OAME Life Member Award
- **Ann Jones**—Ottawa Business Women Education Award
- **Jeri Lunney**—Descartes Award, University of Waterloo
- **Lorna Morrow**—OAME Life Member Award
- **Unionville High School**—K.D. Fryer Award
- **Jack Weiner**—Distinguished Professor of the Year Award, University of Guelph

OAME/AOEM

FOCUS ON RENEWAL

1992 LEADERSHIP CONFERENCE

NOVEMBER 11 - 13, 1992—RAMADA INN AIRPORT WEST

5444 DIXIE ROAD, MISSISSAUGA, ONTARIO 416-624-1144

ACTIVITY-BASED WORKSHOPS (JK-OAC) ON THREE THEMES:

- Mathematics as Communication
- Mathematics as Problem Solving
- Mathematical Connections

LEADERSHIP STRAND—FOUR KEYNOTE SPEAKERS:

- Myrna Ingalls
- Burle Summers
- Jack Weiner
- Dave Zimmer

ALL INCLUSIVE REGISTRATION FEE: $295—INCLUDES 3 DAYS OF WORKSHOPS, 2 NIGHTS ACCOMMODATION, MEALS, BANQUET, RECEPTIONS, PUBLISHERS DISPLAYS

FOR MORE INFORMATION CONTACT:

Don and Carol Attridge
OAME Executive Directors

CREATIVITY COUNTS!

Attention: Math (& Art) Students!

INTRODUCING OAME MATH/ART SCHOLARSHIP

The Ontario Association for Mathematics Education recognizes the importance of creative thinking in fields associated with mathematics. In fact, it is innovation not manufacturing that drives the economy in today's rapidly changing world.

In recognition of the link between creativity and problem solving in mathematics, the OAME joins the Art Gallery of Ontario to offer scholarships to mathematics students (Grade 10-12) who display creative problem solving in both Mathematics and the Visual Arts.

SCHOLARSHIP AWARD

The OAME Math / Art Scholarship consists of three free one year enrolments in the Advanced Studio course at the Anne Tanenbaum Gallery School at the Art Gallery of Ontario (Value $250 each). While the Gallery School operates independent of the Ministry of Education, credit for independent study through the Advanced Studio course may be granted towards the OAC in Visual Arts.

TO APPLY

Submit two (2) examples of YOUR creative solution to mathematical problems and two examples of YOUR creative efforts in the Visual Arts. (Letters of recommendation from your Mathematics and Visual Arts teachers may be included but are not Mandatory.) Include a completed copy of the OAME Mathematics/Art Scholarship Application form—shown below—and mail your submission to:

Steven Bowie
Head—Anne Tanenbaum
Gallery School
Art Gallery of Ontario
317 Dundas Street West
Toronto ON M5T 1G4

DEADLINE: OCTOBER 15, 1992

OAME MATHEMATICS/ART SCHOLARSHIP APPLICATION

☐Mr ☐Ms Name		*Birthdate*
Address & City		Postal Code
Home Telephone		
Name of School		
Address & City		Postal Code
Name of Teacher		Telephone
Please enclose:	☐ 2 examples of creative math	☐ 2 examples of art work (any 2D media, slides or photos for 3D work)
Signature of Applicant		Date
Office Use: Date Received	Interview	Notification Date

Joseph Solomon Delmedigo— Mathematician and Teacher

Sandra M. Pulver—Pace University, New York NY

This paper explores a small facet of the works of Joseph Solomon Delmedigo which heretofore has not been adequately analyzed, and will provide an additional body of knowledge with regard to original contributions of Jewish scholars in the history of mathematics and science.

Interest in mathematics per se is greatly increased if historical remarks and facts are given within the context of any article. Knowledge of the origins and background of a subject can give added meaning to its study [Cajori, p. 2]. A study of the history and nature of things deepen the mathematical insight, and hence leads to better understanding of mathematical concepts today [Miller, p. 407].

Even teachers in the early seventeenth century were trying—with great difficulty, and not too much success—to interest their students in mathematics by inserting within their texts practical problems intended to be of use by merchants, bankers and tradesmen; problems with which the ordinary people would be confronted in their day to day business dealings.

Joseph Solomon Delmedigo (1591-1655), who wrote exclusively in Hebrew, presents several examples of practical arithmetic to both excite and teach his students. Delmedigo develops, in *Elim*, pp. 367-370, written in 1629, a series of puzzles or brain teasers, much the same as those found in modern day texts on commercial mathematics. He first presents his problems and incorrect answers with tongue in cheek, and then provides the correct solutions for his students.

These kinds of puzzles were not uncommon in Delmedigo's time, and often appeared in books on commercial mathematics. Delmedigo presents a repertoire of practical examples of buying and selling items such as eggs, tobacco, nuts, rolls, etc., using monetary units common at that time and in each case, he discusses the fallacy inherent in the particular series of commercial transactions. He writes that each of these transactions appears to render a profit or a loss, but shows that each, in turn actually constitutes, paradoxically, the other situation.

As evidenced in *Elim*, Delmedigo is not the best of teachers. Each of his examples could have been explained in a simpler, more lucid manner using the mathematics available at the time. From our stand point today, he was repetitious, didactic, wordy, and unexciting. However, these practical arithmetic problems do provide valuable insight into the mathematics used by the community of businessmen at that time, and the writings of their educators who tried to teach it.

As in Cocker's *Arithmetic*, written in 1677, most units must be supplied by the reader of *Elim*, regardless of whether a linear measure, such as length, is meant, or whether a Delmedigo is referring to a square measure, such as area, making much of his writing unclear to students of mathematics of his day (Yeldman, p. 87).

Grammatical and typesetting errors abound in his Hebrew dissertation. During the time of Delmedigo, type was set by hand. Typesetting fractions, especially, was a difficult and complicated procedure and it is therefore understandable that errors would result in their printing.

Following now are some of the commercial mathematical problems presented by Delmedigo, in Hebrew, to his coreligionists in order to teach them to enter the secular world and thereby integrate them more completely into the communities in which they lived.

Delmedigo's lesson begins:

PROBLEM I

How do 2 persons buy at the same price and sell at the same price, and the same quantity, and yet one profits and the second incurs a loss?

Let us assume that 2 merchants bought tobacco. Each one bought 81 litres for 72 florins (a former gold coin of Florence, first issued in 1252), so that each of the 9 litres cost 8 florins.

The first merchant sold 40 litres at one time and received 6 florins for each group 6 litres he sold, and at another time, sold the remaining 41 litres, each 3 litres for 2 florins. It would seem that he sold each 9 litres for 8 florins. Yet, when one performs the calculations correctly, this first merchant only receives 67 $\frac{1}{3}$ florins.

Therefore, a part of his original principal has been lost.

The second merchant sold 40 litres at 3 florins for each 2 litres; and the remaining 41 litres were sold at 5 florins for each 7 litres, so that it would seem that he sold all 9 litres for 8 florins. Reasoning correctly, however, one will find that this merchant profited, since he actually receives 89 $\frac{2}{7}$ florins.

Delmedigo asks his students, "Why is the initial solution incorrect?" He then continues: "This fallacy is clearly explained:

The merchant sold 6 litres for 6 florins

and 3 litres for 2 florins

or incorrectly 9 litres for 8 florins

It seems, therefore, that the merchant should earn what he spent, but does not.

Correctly, the first merchant earns:

$$\frac{40 \text{ litres}}{6 \text{ litres}} \times 6 \text{ florins} = 40 \text{ florins}$$

and $$\frac{41 \text{ litres}}{3 \text{ litres}} \times 2 \text{ florins} = 27\frac{1}{3} \text{ florins}$$

for a total of $67\frac{1}{3}$ florins.

The second merchant sold 2 litres for 3 florins

and 7 litres for 5 florins

or, incorrectly 9 litres for 8 florins.

It seems, therefore, that the merchant should earn what he spent.

Correctly, the second merchant earns:

$$\frac{40 \text{ litres}}{2 \text{ litres}} \times 3 \text{ florins} = 60 \text{ florins}$$

and $$\frac{41 \text{ litres}}{7 \text{ litres}} \times 5 \text{ florins} = 29\frac{2}{7} \text{ florins}$$

for a total of $89\frac{2}{7}$ florins.

It is therefore evident that the first merchant incurs a loss while the second makes a profit."

PROBLEM II

Someone buys 12 eggs for 6 ksitot, that is, 2 eggs for each ksita, and another 12 eggs at 6 for a ksita. And in this manner, it can be viewed that 8 were bought for 2 ksitot. According to this calculation, he ought to sell all of them for 6 ksitot in order to realize a return of the principal, but he would actually lose ksitot.

"Please explain," Delmedigo asks of his pupils.

Delmedigo continues: "The specious argument given here is as follows:

In the first group, 2 eggs were bought for 1 ksita

In the second group, 6 eggs were bought for 1 ksita

or incorrectly, 8 eggs were bought for 2 ksitot.

The merchant incorrectly reasons that since he bought a total of 24 eggs at 8 eggs for 2 ksitot, or 4 eggs per ksita, he must sell all 24 of them for 6 ksitot.

Reasoning correctly however, he actually paid

$$12 \text{ eggs} \times \frac{6 \text{ ksitot}}{12 \text{ eggs}} = 6 \text{ ksitot}$$

and $$12 \text{ eggs} \times \frac{1 \text{ ksitot}}{6 \text{ eggs}} = 2 \text{ ksitot}$$

or 24 eggs for a total of 8 ksitot.

He would therefore lose 2 ksitot if he sold the 24 eggs for 6 ksitot."

PROBLEM III

One hundred nuts were bought at 3 for a pashut, and another 100 at 4 for a pashut. Therefore, one may conclude that 200 nuts were bought at 7 nuts for 2 peshutim. But if the merchant sells them in this manner, he will lose money.

"How can we explain his mistake?" Delmedigo asks his pupils. Continuing, Delmedigo writes, "The error is explained in this manner.

The merchant purchased 200 nuts:

$\frac{100 \text{ nuts}}{3 \text{ nuts}}$ for 1 pashut = $33\frac{1}{3}$ peshutim

$\frac{100 \text{ nuts}}{4 \text{ nuts}}$ for 1 pashut = 25 peshutim

for a total cost of 200 at $58\frac{1}{3}$ peshutim.

If he sells 7 nuts for 2 peshutim, he will earn only

$\frac{200}{7} \times 2$ peshutim = $54\frac{1}{7}$ peshutim

His loss will be $1\frac{4}{21}$ peshutim."

PROBLEM IV

If one bought 100 litres of falafel for 100 ksitot, and sold 50 litres at $\frac{5}{4}$ litre for 1 ksita and the remaining 50 litres at $\frac{3}{4}$ litre for one ksita, it apparently seems that he had no profit and no loss, (for he sold a total of 100 litres for 100 ksitot). But it is not so, for he did make a profit.

"Can you explain this?", Delmedigo asks his students.

Continuing Delmedigo states: "The merchant bought 100 litres of falafel for 100 ksitot.

They were then sold for

$\frac{50 \text{ litres}}{\frac{5}{4} \text{ litres}}$ for 1 ksita = 40 ksitot

and $\frac{50 \text{ litres}}{\frac{3}{4} \text{ litres}}$ for 1 ksita = $\frac{200}{3}$ ksitot

for a total payment of $106\frac{2}{3}$ ksitot. His profit, therefore, was

$6\frac{2}{3}$ ksitot."

Delmedigo shows that students of mathematics make these errors because they are using an additive rather than a multiplicative structure, and not using weighted averages.

The following refers to a modern day problem in which pupils have difficulties similar to those of Delmedigo's.

If we drive 300 km at 100 km/h to a location and return the same 300 km at 60 km/h, what is the average speed?

Most students will choose 80 km/h. They want to add the speeds and divide by 2, rather than find the correct answer of 75 km/h.

The correct method of approach is difficult for many students to understand.

It is necessary for us to use rate = $\frac{\text{total distance}}{\text{total time}}$

but we must calculate the total distance and total time first.

total distance = 300 + 300 = 600 km

$$\text{total time} = \frac{300 \text{ km}}{100 \frac{\text{km}}{\text{h}}} + \frac{300 \text{ km}}{60 \frac{\text{km}}{\text{h}}} = 3 \text{ h} + 5 \text{ h} = 8 \text{ h}$$

Therefore using average rate = $\frac{\text{total distance}}{\text{total time}}$,

$\frac{600}{8} = 75 \frac{\text{km}}{\text{h}}$.

Therefore the solution is not 80 km/h but 75 km/h!

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Five Steps to Teaching Proof in the Senior Math Curriculum— A Cooperative Learning Approach: Part III

Brendan Kelly—Faculty of Education, University of Toronto

This is part III of a five-part article spanning consecutive issues of the Gazette.

Introduction

In part I, we described the central role of proof in the senior mathematics curriculum and reasons why we might regard a study of proof as one of the invariants in a changing mathematics curriculum. The article also noted the trend toward alternative instructional methods such as cooperative learning. The intent of this five-part series is to provide a model to demonstrate how the concept of proof and its evolution might be explored with senior level students in a series of five lessons structured in the cooperative learning format. The intent is to cast the development of proof in its historical context and to show how the quest for certainty let through a cascading sequence of complexities to a point where it was recognized that absolute certainty is unattainable. This development presents the evolution of proof in five independent but sequential lessons corresponding to the five parts into which this article is partitioned.

The lesson in part I of this article, titled "The Dawn of Mathematical Proof: Inductive Thinking and Conjecture", referred briefly to the inductive thinking of the ancient Egyptians and Babylonians. It prompted students to conjecture formulas in specific cases and attempted to help students discover why proof is needed to support conjecture. The lesson presented in part II, titled "The Development of Direct Proof and Generalization", was designed to take the students through the next step in the evolution of proof in which the ancient Greeks attempted to underpin conjecture with logical argument based on common premises or axioms. The lesson presented in this article, titled "Indirect Proof and Proof by Counterexample", is the third lesson in the sequence. It pauses in the historical development of proof to present three examples of indirect proof. A set of investigation exercises is provided for students to explore and discuss in groups. It is intended that each group will report one or more of its explorations to the class to allow for some important follow-up discussion. Additional investigation exercises are provided as homework to ensure that all students are involved in the learning. We hope you will find parts or all of it useful and that this might encourage you to contribute other mathematics topics amenable to this instructional format.

Lesson 3—Indirect Proof and Proof by Counterexample

In lesson 1, we observed that Euler proved Fermat's Conjecture false by showing that F_n (the nth Fermat prime) is not, in general, a prime number. By providing a single exception to a general rule or theorem, we disprove the theorem itself. Such an exception is called a counterexample.

It is also possible to prove a conjecture false by showing that the assumption that the conjecture is true leads to a contradiction. This method of proof is called the indirect method. The following example demonstrates the use of the indirect method of proof.

TEACHING EXAMPLE 1

To prevent Al, Betty and Cory from dominating the High Rollers Social Club, the membership of the executive committee of that club was restricted by these conditions.

1 If Al is a member, then Betty is not a member.

2 If Al is not a member, then Cory is a member.

3 If Betty is a member, then Cory is not a member.

Prove that Betty is not a member of the executive.

SOLUTION

Either Betty is a member of the executive or she is not.

STATEMENT

1 Suppose Betty is a member of the executive.

2 Then Al is not a member of the executive.

3 Cory is a member of the executive.

4 Betty is not a member of the executive.

REASON

1 This is a conjecture.

2 Condition 1 and statement 1.

3 Condition 2 and statement 2.

4 Condition 3 and statement 3!

Statement 4 contradicts the conjecture in statement 1, so the conjecture must be false. That is, Betty is not a member of the executive.

The indirect method of proof was used over 2000 years ago by the ancient Greeks who applied it in the proof of some of the most powerful and beautiful theorems in mathematics. The great 20th century mathematician, G.H. Hardy once wrote

The method of indirect proof is a far finer gambit than any chess gambit: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game!

The next two examples of indirect proof are theorems which first appeared in Euclid's Elements (about 300 BC).

TEACHING EXAMPLE 2

Prove that there are infinitely many prime numbers.

SOLUTION

Either the set of prime numbers is finite or it is infinite.

STATEMENT

1 Suppose the set of prime numbers is finite.

2 Then there must be some greatest prime, p.

3 Let q be the integer which is 1 greater than the product of all the prime numbers. That is, $q = 2 \times 3 \times 5 \times 7 \times 11 \times \ldots \times p + 1$.

4 q does not have any prime number as a factor.

5 q is a prime number.

6 q is a prime number greater than p.

REASON

1 This is a conjecture.

2 Statement 1.

4 From statement 3, when q is divided by a prime, the remainder is 1.

5 Statement 4.

6 Statements 3 and 5.

Statement 6 contradicts statement 2, so the conjecture must be false. That is, there is no largest prime and the number of prime numbers is infinite.

TEACHING EXAMPLE 3

Prove that $\sqrt{2}$ is an irrational number.

SOLUTION

Either $\sqrt{2}$ is rational or it is irrational.

STATEMENT

1 Suppose $\sqrt{2}$ is a rational number.

2 There exist integers m and n with no common factor such that $\frac{m}{n} = \sqrt{2}$.

3 $m = \sqrt{2}\, n$

4 $m^2 = 2n^2$

5 m^2 is even.

6 m is even.

7 4 is a factor of m^2.

8 4 is a factor of $2n^2$.

9 n is even.

10 2 is a common factor of m and n.

REASON

1 This is a conjecture.

2 Definition of a rational number.

3 Multiplication of equation 2 by n.

4 Squaring both sides of equation 3.

5 From equation 4, m^2 has 2 as a factor.

6 Otherwise m^2 is odd, contradicting 4.

7 The square of an even number has a factor 4.

8 From equation 4.

9 If n is odd, 4 is not a factor of $2n^2$.

10 Statements 6 and 9.

Statement 10 contradicts statement 2, so the conjecture must be false. That is, $\sqrt{2}$ must be an irrational number.

INVESTIGATIONS

Explore and discuss these questions in your group.

Give a counterexample to disprove each statement.

1 All prime numbers are odd.

2 If $a^2 = b^2$, then $a = b$.

3 $\frac{m}{n} < \frac{m+1}{n+1}$ for any positive integers "m" and "n" where $n \neq 0$

4 $\frac{x}{y} < \frac{2x}{y}$ for any real "x" and "y" not equal to zero.

5 There is a prime number between any pair of consecutive multiples of 5.

Use the indirect method to prove each statement.

6 There is no largest integer.

7 $\sqrt{3}$ is an irrational number.

Complete exercises 8 through 11 for homework.

Use the indirect method to prove each statement.

8 $\sqrt{5}$ is an irrational number.

9 The product of two even numbers is even.

10 The linear equation $ax + b = 0$ has exactly one solution in x for fixed values of a and b when a is not zero.

11 If two given lines in the same plane are perpendicular to a third line in that plane then the two given lines are parallel to each other.

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Loomis, E.S., **The Pythagorean Proposition**, NCTM, Reston VA, 1972

NCTM, **Historical Topics for the Mathematics Classroom**, NCTM, Reston VA, 1969

Steen, L.A., **On the Shoulders of Giants: New Approaches to Numeracy**, National Academy Press, Washington DC, 1990

In part IV of this article, to appear in the next issue, we present the fourth lesson and perhaps the most significant phase in the evolution of mathematical proof under the heading, "The Quest for Ultimate Rigour".

MATH EQUITY COMMITTEE CONFERENCE

WHERE—OISE

WHEN—FEBRUARY 25 & 26, 1993

WHAT—EQUITY ISSUES:

- Gender
- ESL/ESD
- Special Needs Students

CONTACTS

- Shirley Fairfield
- Susan Stuart

MORE DETAILS TO FOLLOW

The Transition Years– An Annotated Bibliography

Ron Firth, George McNabb, Lorna Morrow, Ron Reesor, Bob Robinson, Todd Romiens, Barry Scully–Chair of the Transition Years Committee– Janet Scully, Sue Stuart, Lorna Wiggan

This bibliography is divided into three sections: General Reading, Mathematics—General Reading, and Mathematics—Activities.

General Reading

- philosophy and organization of the middle school
- pedagogy, discipline, homework, curriculum development
- the young adolescent
- parental involvement
- equity
- evaluation
- cooperative learning strategies
- self-esteem
- dealing with at-risk students

Addy, R., Henderson, C., Knox, W., **The General Store: Meeting the Needs of the General Level Student**, Professional Development Committee, OSSTF, 1980

A resource that looks at the needs of the general level student and approaches to deal with them effectively. The articles within the resource point to a highly personal, democratic, holistic, and humanist approach and corresponding strategies to support these approaches.

Alexander, W.M., George, P.S., **The Exemplary Middle School**, Holt, Rinehart and Winston, New York, 1981

The book "defines the middle school concept, describes in detail the characteristics of exemplary middle schools and illustrates the implementation of these characteristics in practice."

Berla, N., Henderson, A.T., Kerewsky, W., **The Middle School Years: A Parent's Handbook**, National Committee for Citizens in Education, Columbia, MD. 1989

"This handbook has been written to suggest ways every parent can help improve the achievement of the middle school-age children." Topics include behaviour, organization of middle schools, middle school policies.

Blyth, D.A., Karnes, E.L., **Philosophy, Policies, and Programs for Early Adolescent Education: An Annotated Bibliography**, Greenwood Press, Westport CT, 1981

This book includes sections on "Prescriptions for and Descriptions of Middle Schools", "Internal Organization of Schools for Early Adolescents: Team Teaching, Departmentalization, and Related Concepts", "Discipline and Problem Behaviour in Schools for Early Adolescents".

Carnegie Council on Adolescent Development, **Turning Points**, Carnegie Council on Adolescent Development Press, Washington, DC, 1989

A ground breaking report that highlights compelling reasons why the teaching and curriculum delivered to middle grade students must change. The authors recommend eight strategies for transforming middle grade schools. Well written and an easy read.

Clarke, J., Wideman, R., Eadie, S., **Together We Learn**, Prentice-Hall, Scarborough ON, 1990

A practical resource to help teachers implement small group learning strategies in their classrooms. It also contains relevant information and forms on how to evaluate student progress in small group situations.

Cornfield, R.J., Coyle, K., et al., **Making the Grade: Evaluating Student Progress**, Prentice-Hall, Scarborough ON, 1987

A resource to aid teachers in refining their evaluation skills and to encourage them to augment traditional methods with other ones. It provides teacher-prepared activities for effective evaluation of a student's progress within the student's program.

George, P.S., Oldaker, L.L., **Evidence for the Middle School**, National Middle School Association, Columbus OH, 1985

Part I represents "Research in Middle Level Education" (a review of research activities, research strategy, and recent research findings). Part II, "A Survey of the Effectiveness of Middle Level Schools", presents the design of the study, representations of the exemplary middle school, and a summary and conclusions.

Glasser, W., **The Quality School: Managing Students Without Coercion**, Harper and Row, New York NY, 1990

The author shows that the traditional coercive management in schools is the root of the problem and suggests that we replace the "bossing" that turns students and staff into adversaries with a system of management that brings them together.

Hargreaves, A., Earl, L., **Rights of Passage: A Review of Selected Research About Schooling in the Transition Years**, Queen's Printer, Toronto ON, 1990

"The main areas of focus for the review were identified as: The key characteristics of elementary schools and secondary schools and their impact on student learning and development;...The transition process itself;...the curriculum in the Transition Years;...[and] innovative strategies of assessment and evaluation which support and are integrated into the learning process itself during the Transition Years."

Johnson, M. (Ed.), **Toward Adolescence: The Middle School Years–Seventy-ninth Yearbook of the National Society for the Study of Education–Part I**, University of Chicago Press, Chicago IL, 1980

Authors include Joan Lipsitz, Conrad Toepfer, David Elkind, etc. Topics of the papers include: "The Age Group", "The Family", "The Mass Media and Popular Culture", "Promoting Cognitive and Psycholinguistic Development", "School-Based Research", etc.

Johnston, J.H., Merkle, G.C., **What Research Says to the Middle Level Practitioner**, National Middle School Association, Columbus OH, 1986

This book presents a selection of columns from "What Research Says to the Practitioner", first published in issues of the Middle School Journal. Topic include Middle Grade Organization, Effective Middle Level Principals, Teacher Stress, Classroom Management, Ability Grouping, Critical Thinking, Self-Concept.

King, A.J.C., **The Adolescent Experience**, OSSTF Research Committee, Toronto ON, 1986

This book reports the findings of a two year study of adolescents across the province. Of particular interest are the sections of self-esteem, participation in extra-curricular activities, gender relations, effective schools, and student achievement.

Lee, E., **Letters to Marcia: A teacher's guide to anti-racist education**, Cross Cultural Communication Centre, Toronto ON, 1991

A practical book which deals with concerns of the thoughtful teacher and suggests activities to promote an anti-racist approach to education.

Lounsbury, J.H., **As I See It**, National Middle School Association, Columbus OH, 1991

This book is comprised of articles which appeared in the Middle School Journal after 1984. topics include homework, interdisciplinary teaming, values.

Merenbloom, E.Y., **The Team Process in the Middle School: A Handbook for Teachers, 2nd ed.**, National Middle School Association, Columbus OH, 1986

The author provides middle level teachers with "thorough, practical, reality based, and field tested" strategies and methods to implement a team approach to organization in the middle school. Chapters include "Role and Function of a Team", "Appropriate Teaching Strategies", "Effective Leadership at the Team Level."

McPartland, J., Slavin, R., **Increasing Achievement of At-Risk Students at Each Grade Level**, *[No publisher available at press time, ed.]*, 1990

This examines the status of currently proposed or implemented programs aimed at increasing achievement of at-risk students from elementary schools to high schools. Identifies effective programs and practices that improve the achievement of these students.

National Study of School Evaluation, **Middle School/ Junior High School Evaluative Criteria: A Guide for School Improvement**, NSSE, Falls Church VA, 1979

This book is comprised of twelve sections, which include: School and Community, Design of Curriculum, Learning Areas, Student Activities Program.

Oppenheimer, J., **Getting it Right: Meeting the Needs of the Early Adolescent Leamer**, FWTAO, Toronto ON, 1990

Research findings in this book report on the following: Effective/Successful Schools, Evaluation of Successful Schools, Meeting Basic and Development Needs, School Climate, School Organization, Discipline, Pre-Service and In-Service for Intermediate Teachers.

Osborne, K., **Middle Years Sourcebook: Some Suggestions for the Education of Early Adolescents**, Manitoba Dept. of Education, 1984

"The topics dealt with in this book were identified by the Middle Years Program Review Committee" and include the following: Characteristics of the Middle Years Youngster, Teaching in the Middle Years, Evaluation of Students, Discipline.

Schniedewind, N., Davidson, E., **Open minds to equality: A sourcebook of learning activities to promote race, sex, class, and age equity**, Prentice Hall, Englewood Cliffs NJ, 1983

The Title speaks for itself.

Strahan, D.B. (Ed.), **Middle School Research: Selected Studies 1986**, National Middle School Association Research Committee, Columbus OH, 1986

The seven papers in this volume "address a range of concerns expressed by middle school educations: Self-concept as it relates to family structure; strategies for extending concepts in science, social studies, and problem-solving; perspectives on art curricula; organizational structures of effective middle schools; and problem-solving strategies used by middle grades principals."

MATHEMATICS—GENERAL

Many of the same topics as above are dealt with, but with specific reference to mathematics:

- cooperative learning
- problem solving
- assessment
- use of manipulatives
- mainstreaming
- mathematics curricula and reasons for change

Artzt, A.F., Newman, C.M., **How to Use Cooperative Leaming in the Mathematics Class**, NCTM, Reston VA, 1990

This small volume briefly outlines different models of cooperative learning; suggests ways of implementing them, and provides a variety of problems for use in classrooms from primary to secondary level.

Burns, M., **A collection of math lessons: From grades 6 through 8**, The Math Solution Publications, 1990

This book describes how the authors introduced and conducted the investigations with students. The activities involve the students as active problem solvers, using hands-on materials in cooperative small groups and writing and discussing mathematics.

California State Department of Education, **A Question of Thinking: A First Look at Students' Performance on Open-ended Questions in Mathematics**, Sacramento CA, 1989

A study is described in which students were given open-ended questions in mathematics to solve. An analysis of actual student responses suggests ways of using such responses for assessment. Although the study used grade 12 students, the conclusions are of interest for all levels.

Charles, R., Lester, F., O'Daffer, P., **How to Evaluate Progress in Problem Solving**, NCTM, Reston VA, 1987

The book outlines evaluation techniques such as observation, self-assessment, holistic scoring and multiple-choice tests. Ways to organize for use of any or all of these are given, along with ways to use evaluation results. Examples of student work help illustrate and explain the ideas.

National Research Council, **Everybody Counts: A Report to the Nation on the Future of Mathematics Education**, National Academy Press, Washington DC, 1989

This report examines mathematics education as one system, from kindergarten through graduate school. It deals with all major components of the system, from curricula, teaching, and assessment to human resources and national needs. It also charts a general course for the future of mathematics education.

National Research Council, **Reshaping School Mathematics: A Philosophy and Framework for Curriculum**, National Academy Press, Washington DC, 1990

The report proposes a framework for reform of school mathematics. The steps essential to achieving this reform are outlined as reshaping the curriculum, improving teacher pre-service, developing better means of assessing student performance, and preparing more effective and appropriate instructional materials.

Silvey, L. (Ed.), **Mathematics for the Middle Grades (5-9)**, Yearbook of NCTM, Reston VA, 1982

This collection of essays includes such diverse topics as help for mainstreamed learning disabled students, developing mental arithmetic, mathematics out-of-doors, using games effectively, developing computer literacy, statistics, and problem-solving. Discussions are linked to research to give a theoretical background to the practices being suggested.

Steen, L. A. (Ed.), **On the Shoulders of Giants: New Approaches to Numeracy**, National Academy Press, Washington DC, 1990

"This volume—is part of a national dialogue on mathematics education", part of which includes Everybody Counts and reshaping School Mathematics mentioned above. The "numeracy" of the title is broadly interpreted to include pattern, dimension, uncertainty (probability, data analysis), and shape. Each of six chapters deals with one of these major topics, providing theoretical backgrounds as well as useful suggestions that can be easily incorporated into the classroom. Examples of applications from several other disciplines suggest ways to integrate mathematics with other subjects.

Stenmark, J. K., **Assessment Alternatives in Mathematics: An overview of assessment techniques that promote learning**, Lawrence Hall of Science, Berkeley CA, 1989

Why we assess, what we assess, and how we might do so are all dealt with in this 40-page booklet. Among the useful topics are samples of student work, questionnaires for self-assessment, descriptions of portfolios, a scoring rubric for open-ended questions and an extensive bibliography on assessment.

Stenmark, J. K. (Ed.), **Mathematics assessment: Myths, models, good questions, and practical suggestions**, NCTM, Reston VA, 1992

This book documents some of the changes in assessment practices in mathematics and includes a broad collection of techniques and practices that emphasize student thinking.

Willoughby, S.S., **Mathematics Education for a Changing World**, ASCD, Alexandria VA, 1990

This small, easy-to-read book discusses reasons why change is necessary, what different professional groups are saying and doing to encourage change, how to implement change, and the problems encountered when doing so. Two specific topics dealt with in more detail are problem-solving and the use of technology.

MATHEMATICS—ACTIVITIES

These books include activities, lessons plans, ready-to-use ideas, etc.

- data collections and interpretation
- patterns and functions
- problems for cooperative groups
- more problems

Canadian Mathematics Competition, **Problems, Problems, Problems, Volume 4**, University of Waterloo, Waterloo ON, 1991

The fourth in this series contains problems from Gauss and Pascal contests (Grades 7, 8, 9). Problems are classified under topics such as decimals, percentages, averages, ratios, two-dimensional geometry, counting and logic, and so on.

Curcio, F.R. **Developing Graph Comprehension: Elementary and Middle School Activities**, NCTM, Reston VA, 1989

The book provides a selection of activities, black-line masters, and suggestions for making teacher aids to help students develop facility with several kinds of graphs including stem-and-leaf plots. Suggestions for computer use are included.

Erickson, T., **Get It Together: Math Problems–Grades 4-12**, Lawrence Hall of Science, Berkeley CA, 1989

This book gives math teachers materials to introduce and foster cooperative problem solving in their classrooms. It is a collection of over 100 math problems for groups of 2-6 students in grades 4 and beyond. The problems cover wide range of subject matter and difficulty. The book also includes advice on management and assessment.

Flewelling, G., Weiner, J. (Eds.), **Teaching Math in the Transition Years**, Grand Valley Math Association, Waterloo ON, 1990

A collective of 38 activities for grades, 7, 8 and 9. Suggestions for starting cooperative activities and sample lessons are included.

Jamski, Wm.D. (Ed.), **Mathematics Challenges for the Middle Grades from the Arithmetic Teacher**, NCTM, Reston VA, 1990

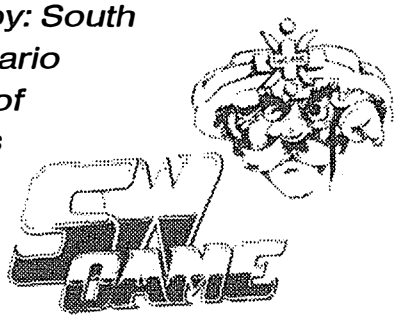
"This book is...for...those middle-grade mathematics teachers who are constantly searching for a wide variety of appropriately challenging and motivating problems." Jamski has grouped these problems from the last fifteen years of the Arithmetic Teacher under such headings as Numbers, Computation, Special Counting, Geometry, Probability. The answer section gives the reference(s) for each problems. The teacher who pursues these references will find another treasure trove of material for his/her classroom.

THE 20TH ANNUAL OAME CONFERENCE/ LE 20 EME CONGRES DE AOEM ANNUEL

Hilton International Hotel & Cleary Convention Centre

Windsor, Ontario May 13-15, 1993

Sponsored by: South Western Ontario Association of Mathematics Educators in Cooperation with OAME



Phillips, E. et al., **Patterns and functions: Addenda series, Grades 5-8**, NCTM, Reston VA, 1992

A collection of activities in which the unifying themes of NCTM's Curriculum and Evaluation Standards—problem solving, communication, reasoning, and mathematical connections—are exemplified. Many of the activities are also open-ended, involve the use of concrete materials and could be effectively used in preparing students for algebra and functions. (Readers are encouraged to examine other books in the Addenda series.)

School Achievement Indicators Program

Lorna Morrow

On June 1 and 2, two representatives from OAME/AOEM and two from OMCA were invited to a meeting with Morris Liebowitz, Ministry of Education, to examine items proposed for the Council of Ministers of Education, Canada (AMEC), School Achievement Indicators Program, popularly called National Indicators.

OAME/AOEM representatives were Arnold Carefoote (Scarborough) and Lorna Morrow (North York). OMCA representatives were Peter Saarimaki (Toronto) and Ron Sauer (Waterloo).

The aim of the meeting was to identify matches or mismatches between test items and Ontario curricula. This report described the nature of the test, the task we had, and recommendations and concerns that we presented on behalf of the mathematics education community of Ontario.

The Nature of the Test

The test will be given to a sampling of 13 and 16 year olds across Canada in May, 1993. between three and four thousand students at each level in each province will be tested, in both French and English.

Both Content and Problem Solving tests will be given. No student, however, will be tested in both areas. Thus, the sample size for each component will be between 1500 and 2000.

The strands chosen to measure students' skills in mathematical content are designed to evaluate levels attained on:

- numbers and operations
- algebra and functions
- measurement and geometry
- data management and statistics

"The strands chosen to measure students' skills in problem solving are designed to evaluate levels attained on:

- a range of problems and solutions
- the use of numbers and symbols
- the ability to reason and to construct proofs
- proving information and making inferences from databases
- pursuing evaluation strategies
- demonstrating communication skills"

(Mathematics Assessment Criteria: Draft, July 4/91)

Criteria for five levels of achievement in each content strand and each problem solving strand have been written and used to develop content test booklets at five different levels. There is just one problem solving booklet, containing items at all five levels.

The Content test will be in two parts. The first part is a thirty-minute teacher-scored Placement Test of 12 items that are multiple-choice and short answer in form. A students' success on this test determines which two of the five "level" booklets that student will then be given to complete. Each booklet will contain 25 items (again multiple-choice and short answer). Students will be allowed two hours (or more, if necessary) to complete them.

The Problem Solving booklet will contain 10 multi-step problems of a variety of types. Full solutions will be expected. Students will be given two hours to complete the test.

The use of calculators and appropriate manipulatives is encouraged for both Content and Problem Solving tests. As well, a sheet of common formulae (e.g. area, volume, slope) will be provided for each student.

All items will have been field tested. Data from the field tests are being analyzed to determine which items to retain for the 1993 testing.

The Placement Test is being validated by having a sample of students write both the Placement Test and a selection of items from all 5 "level" booklets.

The purpose of the June 1 and 2 meeting was, as stated earlier, to identify items that match Ontario curricula for 13 and 16 year olds. However, mismatches will still occur on the final forms since a topic such as probability, while optional in Ontario, is core in the majority of provinces and territories.

The group also contributed editorial comments (suggestions for changes in wording, in format, etc.). These will be compared with comments from similar groups in the other provinces and the territories. Changes will be made by a majority agreement.

Concerns and Recommendations

We had a number of concerns about both format and administration of the test. Some of these are outlined below.

A major concern was that the items test only a narrow range of content and they do not reflect a wide range of assessment techniques — at best they will give a snapshot of student achievement. We asked that a statement reflecting this be part of early (i.e. pre-test) publicity. Such a statement following the publication of test results will not be effective.

Another concern was the length of the test. Our students are not accustomed to two hour tests. Moreover, this type of testing contradicts the Intermediate/Senior Ontario Mathematics Guideline's position that:

Written tests or examinations, no matter how well constructed, do not provide all of the information needed for a valid evaluation.... Whatever form of assessment is used, care should be taken to ensure that it reflects the instruction given.... Most of the teacher's assessments.... should be formative. (page 23)

Thirdly, testing in mid-May raises concerns about motivation for students, particularly 16 year olds. Students who are not motivated to do well will not present an accurate picture of mathematics education in Ontario.

Fourthly, giving the same Problem Solving booklet to 13 and 16 year olds must, of necessity, mean presenting 13 year olds with problems they are not expected to solve. This may have a negative effect on the self-esteem and self-confidence of many students. Even for those students who will not be affected this way, the frustration level can be expected to exceed acceptable levels, particularly since ESL and LD students are included in the sample.

Some of these concerns have been addressed in a letter to the Minister of Education from Jeri Lunney, President, OAME/AOEM on behalf of Council and members of the organization. OMCA is planning a similar action. This is part of a continuing effort on the part of both groups to monitor and have some impact on mathematics education across the province.

If you wish further information, contact your Chapter representative or any other member of Council.

Curriculum & Evaluation Standards for School Mathematics Quotes

Interpreted by Dr. Alex Norrie

Standards Quote #6 Primary Junior—*"Work with fractions should draw on children's experiences in sharing."*

In every result of testing, evaluation, or assessment ever published fraction objectives are always badly done by the majority of students. This applies across the elementary grades. Why?

Fraction concept development is poorly done in the nations classrooms. We do not spend enough time in concept development. Operations are introduced at too early a level. Concept development should dominate probably until grade 8.

Teachers themselves do not clearly understand fraction concepts or fraction operations. This can lead to rote teaching and from this students emerge with less than adequate concept development.

Sharing, sharing, sharing! That is a practical application of fraction concepts. Pre-schoolers do it! School kids do it! Adults do it! Why not use it as a model for teaching fraction concepts?

Reflector Properties Of the Conics

John Del Grande

Paper folding offers secondary school students an excellent opportunity to use materials in a hands-on activity to develop mathematical concepts.

A student's first encounter with the conics usually occurs in the study of quadratic functions. The reflector properties of these curves are often cited as applications in the design of parabolic, elliptic and hyperbolic reflecting surfaces. The proof of such properties is usually found in analytic geometry after a study of tangents and their properties. The attempt to prove these properties in a semi-intuitive way involving Euclidean geometry and the results of a well known paper folding exercise motivated the development of this paper.

Paper folding, described in this paper, offers secondary school students an excellent opportunity to use materials in a hands-on activity to develop mathematical concepts — an opportunity these students seldom get. As the paper is folded the creases outline a curve called an envelope where an envelope is defined as a curve that touches every member of the system of lines.

The reflector properties of a reflecting surface are known from physics. When a ray of light is reflected from a flat surface, the angle of incidence equals the angle of reflection. Note that the angles of incidence and reflection are defined from the normal or perpendicular to the surface. Of course, the complementary angles are also equal.

In the diagrams that follow either of the pairs of equal angles may be used.

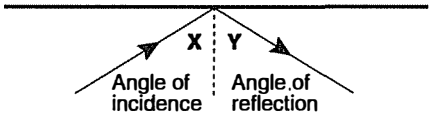


Figure 1

When the surface is curved, the curved surface can be considered to be flat at the point at which the ray strikes it. The flat spot has the same direction as the tangent at the point. The surface reflects "locally" as if it were the tangent to the curve.

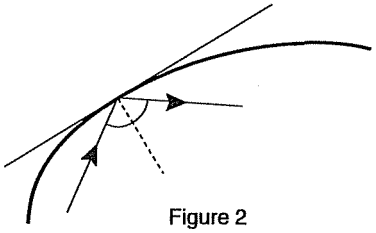


Figure 2

For a circle, the tangent is perpendicular to the radius at the point of contact. Thus a light beam emanating from the centre of a circular mirror would be reflected back through the centre.

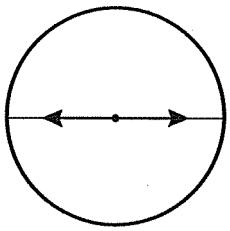


Figure 3

The definitions of the conics used in this development follow.

A parabola is a planar curve consisting of the set of all points that are equidistant from a fixed straight line called a directrix and a fixed point called a focus. For example, for a focus F and a line l in a plane the set of all points, P, for which $PF = PA$ forms a parabola.

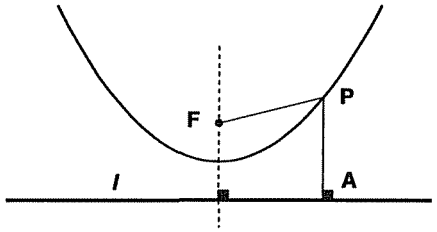


Figure 4

An ellipse is a plane curve consisting of the set of all points the sum of whose distances from two fixed points F_1 and F_2 the set of all points, P, for which $F_1P + F_2P = c$ (a constant) forms an ellipse.

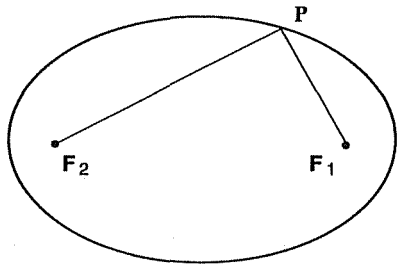


Figure 5

A hyperbola is a plane curve consisting of the set of all points the difference of whose distances from two fixed points, called foci, is a constant. For example, for two fixed points F_1 and F_2 the set of all points, P, for which

$|F_1P - F_2P| = c$ (a constant) forms a hyperbola.

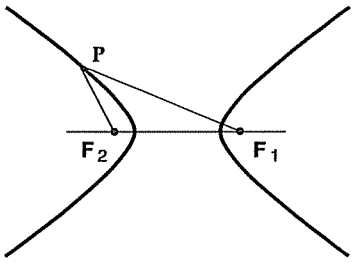


Figure 6

THE PARABOLA

- Use a sheet of waxed paper with a line *l* drawn on it.
- Select any point, *F*, on the paper but not on the line.
- Fold the paper so that *F* falls on the line.
- Crease the paper.
- Repeat the folding and creasing until a curve is outlined,

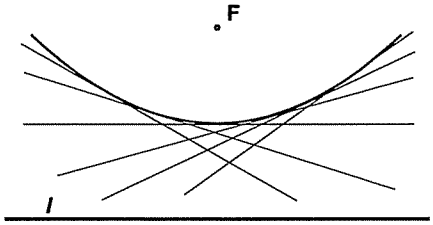


Figure 7

A red plexiglass mirror or Mira may be used to map *F* onto the line *l* with the mirror line replacing the creases. The more creases that are made the smoother the curve.

(If waxed paper is used the creases and the envelope can be projected on a screen using an overhead projector.)

The curve outlined, or the envelope, seems to be a parabola with focus *F* and directrix the line *l*.

To show that the curve outlined is indeed the parabola with focus *F* and directrix *l* we first prove that there exists a point, *P*, on each crease line such that $PF = PF'$ where PF' is the perpendicular distance from *P* to *l*. This point P is common to the parabola and the crease line. Then we show that this point is the only point common to the crease line and the parabola — that is, the crease line is tangent to the parabola.

If the point *F* falls on the point *F'* on line *l* after a fold then the crease, *AB*, is the perpendicular bisector of the segment *FF'*. That is, *F'* is the image of *F* upon reflection in the crease line.

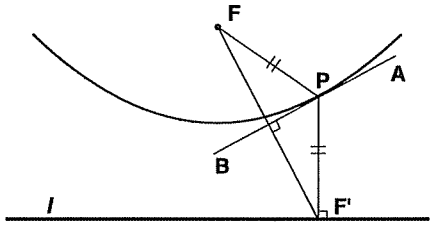


Figure 8

If there is a point, *P*, common to the crease line and the parabola then, $PF = PF'$ (where *P* is on the crease line) and *PF* equals the perpendicular distance from *P* to *l* (where *P* is on the parabola).

Thus, the perpendicular distance from *P* to *l* is PF'.

That is $\overline{PT'} \perp l$.

To find *P* draw a perpendicular to *l* at *F'* to intersect the crease line at *P* as in Figure 8.

$\therefore$ *P* is a point common to the crease line and the parabola.

To show that the crease line is tangent to this parabola we must show that no other point on the crease line can lie on this parabola.

Select any other point *P'* on the crease.

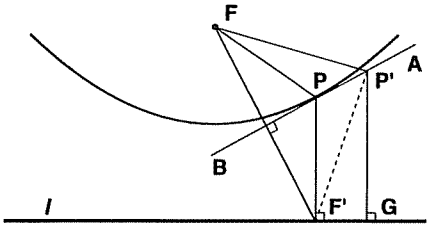


Figure 9

Drop a perpendicular *P'G* from *P'* to the line *l*.

Since *P'* is on *AB*, $P'F = P'F'$.

In triangle *P'F'G* we have $P'F' > P'G$.

$\therefore P'F > P'G$. For *P'* to be on the parabola, $P'F = P'G$, which is not true.

Therefore *P'* cannot lie on the parabola.

Since the parabola and the crease line have one and only one point in common, the crease line is tangent to the parabola.

Thus all the crease lines are tangent to a parabola whose focus is *F* and directrix the line *l*.

The envelope of the crease lines is a parabola.

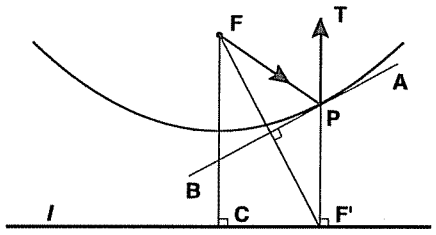


Figure 10

Figure 10 shows the waxed paper when a point *F* is mapped onto the point *F'* in the line *l*. The parabola envelope is sketched where *AB* is tangent to the parabola at *P*.

Extend *F'P* to *T*.

Drop a perpendicular from *F* to the line *l*. *FC* is the axis of symmetry of the parabola.

Triangle *FPF'* is isosceles.

Since *AB* is the crease line

$AB \perp FF'$

$\therefore \angle FPB = \angle F'PB$

But $\angle F'PB = \angle APT$

$\therefore \angle FPB = \angle APT$

A light ray emanating from *F* striking the parabola at *P* is reflected from the tangent at *P*, so that the angle of

incidence, $\angle FPB$, equals the angle of reflection, $\angle APT$.

$\angle FPB = \angle APT$

$\therefore$ The ray is reflected along PT.

But $\overline{PT} \parallel \overline{FC}$.

$\therefore$ The reflected ray is parallel to the axis of the parabola.

A light beam emanating from the focus of a silvered parabolic surface, such as the reflector of a car headlight or of a flashlight, will upon reflection be parallel to the axis of the parabola.

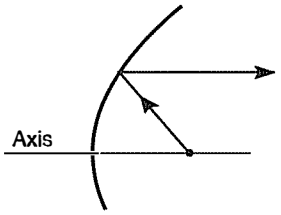


Figure 11

THE ELLIPSE

- Draw a large circle, with centre C, on a sheet of waxed paper and mark a point F inside the circle.
- Fold the paper so that F falls on the circle and crease the paper.
- Continue folding and creasing so that F falls onto the circle each time.
- The crease lines outline a curve which appears to be an ellipse with foci at F and C.

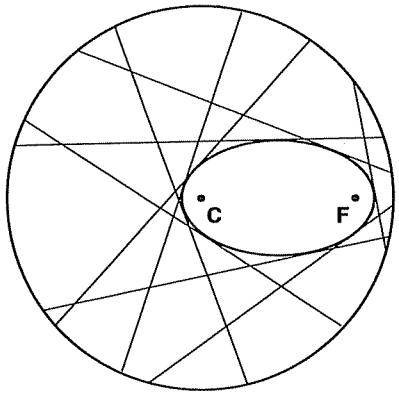


Figure 12

The curve is an envelope of the straight lines because each line touches the curve.

To show that the curve outlined is indeed an ellipse we prove that there exists a point on each crease line that is on an ellipse and that this point is the only point common to the crease line and the ellipse — that is, the crease line is tangent to the ellipse.

If F falls on F' after a fold, then the crease, AB, is the perpendicular bisector of FF'. Join F' to C, cutting AB at P.

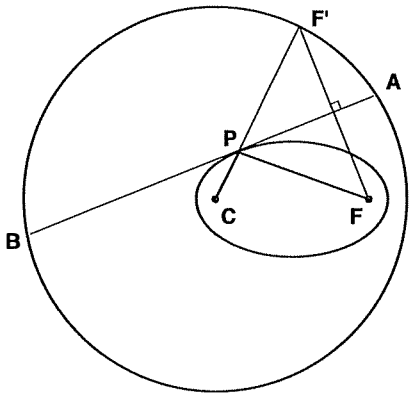


Figure 13

Now $FP = F'P$.

Notice that $F'P + PC = r$ (radius of the circle)

$\therefore FP + PC = r$ (a constant)

$\therefore P$ lies on an ellipse with foci F and C.

For any other point P' on AB,

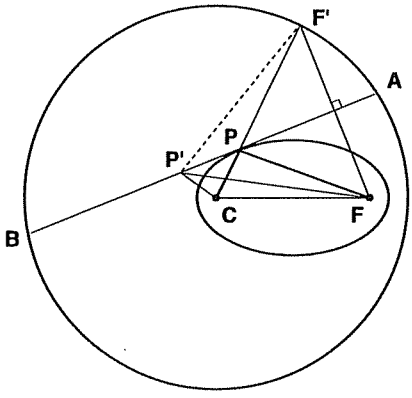


Figure 14

we have triangle $F'P'C$ and $F'P' + P'C > F'C$.

$\therefore FP' + P'C > F'C$ $(F'P' = FP')$

$\therefore FP' + P'C > r$ $(r = F'C)$

$\therefore P'$ is not on the ellipse.

Thus the ellipse and each line have one and only one point in common. That is, the crease line AB is tangent to the ellipse. Thus, each crease line in Figure 12 is tangent to an ellipse with foci F and C, and the crease lines outline an ellipse.

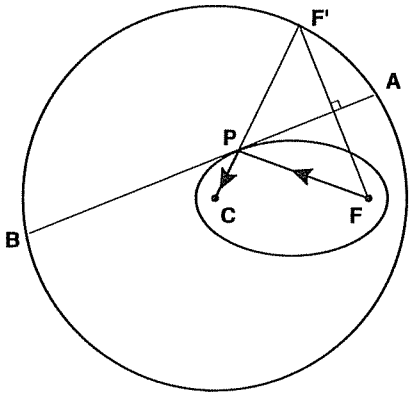


Figure 15

In Figure 15 the point F is folded onto the point F' on the circle with centre C, and the line AB is the crease line. The ellipse envelope is sketched. The ellipse has foci F and C and AB is tangent to the ellipse at P.

AB is the perpendicular bisector of FF'.

Triangle F'PF is isosceles.

$\overline{AF} \perp \overline{F'F}$

$\therefore \angle F'PA = \angle FPA$

But $\angle FPA = \angle BPC$.

$\therefore \angle FPA = \angle BPC$

A ray of light emanating from F along FP striking the ellipse at P is reflected from the tangent line AB so that the angle of incidence, $\angle FPA$, equals the angle of reflection, $\angle BPC$.

That is, $\angle FPA = \angle BPC$.

$\therefore$ the reflected ray lies along the line PC and passes through the focus C.

$\therefore$ a ray of light emanating from one focus is reflected from the elliptic surface and passes through the second focus.

If we have an elliptical reflector then a light beam emanating from one focus, F_1, upon reflection from the elliptical surface will pass through the second focus, F_2.

Also for an elliptical pool table with a perfectly resilient perimeter, a ball passing over one focus will upon rebound from the perimeter pass through the second focus.

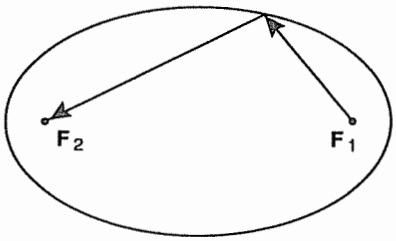


Figure 16

THE HYPERBOLA

- Draw a large circle, with centre C, on a sheet of waxed paper and mark a point F on the outside of the circle.

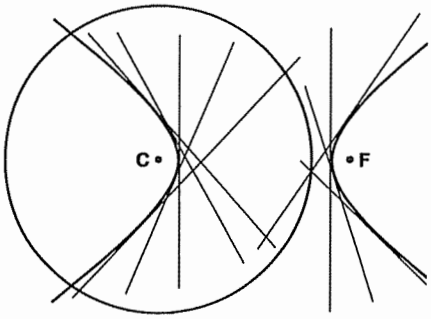


Figure 17

- Fold the paper so that F falls on the circle and crease the paper.
- Continue folding and creasing so that F falls onto the circle each time.
- The crease lines outline a curve which appears to be a hyperbola with foci at F and C.
- The curve is an envelope of the straight lines because each line touches the curve.
- To show that the curve outlined is indeed a hyperbola we prove that there exists a point on each crease line that is on a hyperbola and that this point is the only point common to the crease lines and the hyperbola — that is, the crease line is tangent to the hyperbola.
- If F falls on F′ after a fold, then the crease, AB, is the perpendicular bisector of FF′.
- Join F′ to C cutting AB at P.

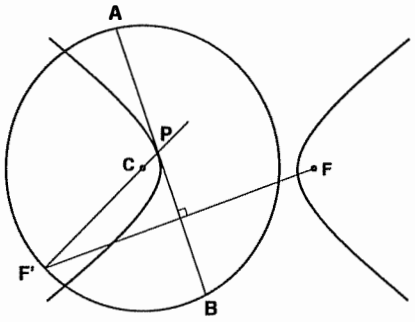


Figure 18

Now FP = F′P.

Notice that F′P - PC = CF′ = r (radius of the circle)

Since F′P = FP

$\therefore$ FP - PC = r (a constant).

$\therefore$ P lies on a hyperbola with foci F and C.

Let P′ be any other point on AB.

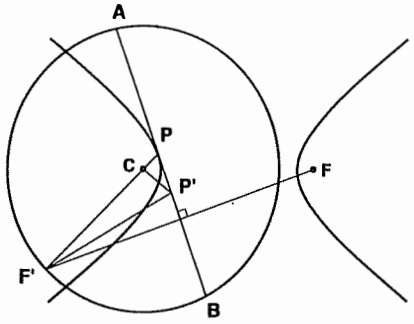


Figure 19

In triangle F′CP′ we have

F′C + P′C > F′P′

$\therefore$ F′C > F′P′ – P′C or F′P′ – P′C < F′C

But FP′ = F′P

$\therefore$ FP′ - P′C > F′C (where F′C = r)

If P′ is on the hyperbola then FP′ – P′C = F′C, which is not true.

$\therefore$ P′ is not on the hyperbola.

Thus the hyperbola and each crease line have one and only one point in common. That is, the crease line AB is tangent to the hyperbolas. Thus, each crease line in Figure 17 is tangent to a hyperbola with foci F and C and all the crease lines outline a hyperbola.

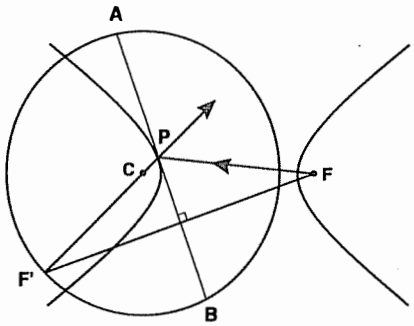


Figure 20

In Figure 20 the point F is mapped onto the point F′ which lies on a circle with centre C and the line AB is the crease line. The hyperbola is sketched showing AB as a tangent to it at P.

Extend F′P to T. AB is the perpendicular bisector of FF′.

Triangle PFF′ is isosceles

$\angle FPB = \angle F'PB$

But $\angle F'PB = \angle APT$

$\therefore \angle FPB = \angle APT$

If *FB* is the line of a ray of light emanating from *F* and reflected from the tangent line at *P*, then *PT* is the line of the reflected ray and *PT* passes through *C*.

$\therefore$ a ray of light emanating from one focus is reflected from the hyperbolic

surface and passes through the second focus when extended.

A light beam emanating from the focus of a silvered hyperbolic surface will upon reflection from the hyperbolic surface move on a line which when extended will pass through the second focus.

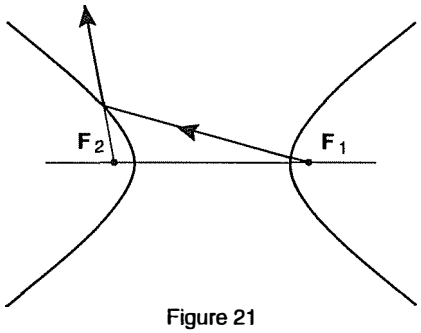


Figure 21

APPLICATIONS

Most headlights and searchlights have an intense light source at the focus of a parabolic reflecting mirror. The light rays that strike the parabolic mirror are reflected along rays parallel to the axis of the parabola. Thus most of the light generated at the source is concentrated in a beam of parallel rays of light.

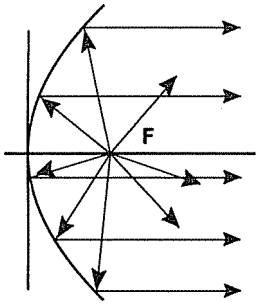


Figure 22

However light that is not directed at the mirror will scatter in different directions. For greater efficiency a reflecting hemisphere is so located that the light source is also at the centre of the hemisphere.

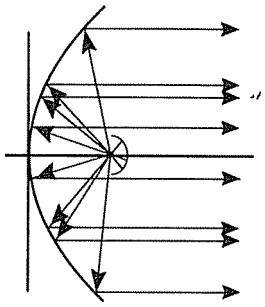


Figure 23

Light rays striking the hemispherical surface are reflected back through its centre, which is also the focus of the parabola, onto the parabolic reflector and are finally reflected in a set of parallel rays. Thus all the light emanating from the source can be concentrated into a beam of parallel rays.

Light from a distant source, such as light from the sun or the stars, is seen in parallel rays. If these rays strike a parabolic reflector from a direction parallel to the axis, the rays will be reflected through the focus of the parabola. This is the principle of the telescope and solar furnaces. This also explains the origin of the word 'focus' which is Latin for fireplace or burning point.

A famous landmark in Old London is the whispering gallery in the Tower. This gallery is partly elliptic in shape with foci a considerable distant apart. A whisper at one focus can be heard distinctly at the other because sound waves emanating from one focus of the elliptic walls will upon reflection converge at the second focus.

The story is told of the Ratskeller in Bremen, the walls of which are shaped somewhat like an ellipse, that the city fathers were remarkably well informed about the feelings and views of the citizens of the city. The former drank their wine at a table that was situated at a focus, and thus could hear distinctly the conversation of a distant table, which stood at the other focus and about which the Berger congregated.

Hyperbolic and parabolic mirrors are used in large telescopes. A Cassegrain telescope has a main parabolic mirror, *AA*, with focus F_1 and a convex hyperbolic mirror, *BB*, with foci F_1 and F_2.

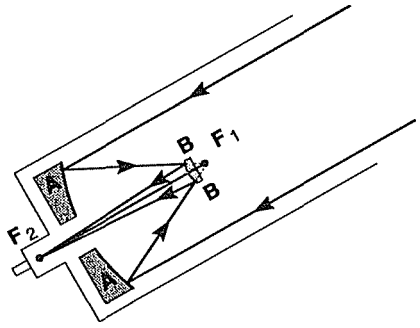


Figure 24

Parallel rays from a distant star strike the parabolic surface and are reflected towards the point F_1. Since F_1 is the focus of the hyperbolic reflector then all reflected rays will converge at F_2 the second focus of the hyperbolic reflector. In this way a large quantity of very faint light from the distant star is concentrated at F_2 and can be detected with photographic equipment or in some cases by the naked eye. Other variations of this mirror combination can be used to further enhance the light from stars.

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Response to The Early Years Consultation Paper

OAME Early Years/Formative Years Committee—
Ann Jones, President,
Peter Saarimaki, Committee Chair

*Following are **excerpts** (Issue 2, Issue 7, etc.) from the response sent to Paulette Laforest-Norman, team leader of the Ministry's Early Years Work Team. The response was prepared by The Early Years/Formative Years Committee and refined through consultation at Council.*

General Comments

The Ontario Association for Mathematics Education/ L'Association Ontarienne pour l'Education's is very appreciative of the Ministry of Education's continuing efforts to involve the full educational community in the consultative process to restructure education in Ontario. There are several comments of an overarching nature that are presented here, before specifically addressing each issue.

These comments on the Early Years are offered in the belief that all documents and policies from the Ministry should constantly and consistently reaffirm the K-OAC continuum. They should all derive from a common, connected view of the learner and learning. The Image of the Learner and the learning environment can serve as sign posts while we explore the development of skills and attitude towards our common goal.

The important first years of schooling, from Junior Kindergarten to grade 6, must have special attention as children develop attitudes, confidence and skills.

Throughout all levels of schooling, we must provide a balanced and broadly based curriculum that

a) promotes the spiritual, moral, cultural, mental and physical development of students in school and in society, and

b) prepared students for the opportunities, responsibilities, and experiences of adult life.

Mathematics has a vital role to play in all aspects of the education of children and educators must be clear aboutthe role, philosophy and aims of mathematics.

We need to realize, and help others realize that,

- mathematics is not simply "numerical skills", as it has been identified in the areas of growth
- mathematics is an essential element of communication
- mathematics is a search for pattern
- mathematics is imagination, initiative and flexibility
- mathematics is problem solving...and problem posing
- mathematics is a process, a creative activity

In the consultation paper (page 14), we would suggest that two sub-headings be used under "Growth Related to Ideas" to parallel those in language, namely: numeracy and mathematical skills.

Numeracy would include an awareness of the concept of number, uses of measurement, and communication of facts and ideas.

Mathematical skills would include sorting, classifying, counting, shape recognition, and representation.

These support the philosophy of the following general goals in mathematics for students—that they:

- learn to value mathematics
- become confident in their ability to do mathematics
- become mathematical problem solvers
- learn to communicate mathematically
- learn to reason mathematically

As well, in identifying specific skills for mathematics, they should be consistent with the view of mathematics as being made up of three strands (arithmetic, measurement and geometry) with estimation and problem solving interwoven throughout.

The recognition of patterns through concrete experiences and explorations from everyday life is central. Children must have the opportunity to work in a systematic way, to work independently, to work cooperatively, and to work at some things in depth.

The OAME/AOEM is willing, and eager, to work with the Ministry to develop a true picture of mathematics, one that supports the Image of the Learner through the eyes of mathematics educators and helps prepare our students of today to be responsible citizens for tomorrow.

ISSUE 2

Should provincial policy describe learning in Junior Kindergarten and Kindergarten with regard to the following?

a) Role/play _____ ☑ YES

b) Role/language _____ ☑ YES

c) Role/adults _____ ☑ YES

d) Areas of growth _____ ☑ YES

Comment

a) Policy statements about the role of play must include descriptions of the attributes of play, concrete examples, and purposes of play. These policies must truly reflect current educational research. The policies must also include descriptions of the characteristics of the learners at each age. They must describe rather than prescribe.

Free exploration has a valued place in the child's learning but there is also a time for direction—a time to know when to move on, and where. The role of play must reflect the social, emotional, physical and cognitive needs of the child for we are concerned with the education of the whole child.

b) The most effective learning takes place when there is social interaction. Learning is a matter of communication. There is a strong interdependency between learning and language acquisition. Communication goes beyond the written word to encompass such things as role-playing, modelling, as well as concrete, pictorial, graphic, and symbolic representations. The valuable role that mathematics plays in helping children learn and communicate must be recognized.

c) The role of adults must deal with professionals, paraprofessionals, and all who have contact with or impact on the child. The role of the teacher is changing and evolving to become a multi-dimensional leader, listener, co-learner, facilitator, observer, recorder, reporter, etc. The role and expectations of other significant adults must also be clarified.

d) The role of problem solving and problem posing should be expanded to encompass all aspects of learning. The desired outcomes should be supported by examples of specific, observable behaviours.

ISSUE 7

Should provincial policy provide direction to school boards with regard to staffing in Junior Kindergarten and Kindergarten as it relates to the following?

a) Class size _____ ☑ YES

b) Pupil-adult ratio _____ ☑ YES

c) Teacher aides (special needs) ___ ☑ YES

Comment

Staffing in Kindergarten classes should be consistent with relevant current Ministry policies. With school board class sizes in grades 1 and 2 targeted for a maximum of 20 on average (P/P Memo 107), the Early Years class sizes should be appropriately smaller, in line with the Ministry's stated rationale that this is one way to improve the quality of a child's education in the early years.

a) We believe that smaller Kindergarten classes will benefit the children, parents and teachers. With this rationale as a guide we propose that class sizes in the Kindergartens should be, on average, 18 with a maximum of 22. Special consideration should be given to mixed Junior Kindergarten/Kindergarten classes to either reduce the maximum class size or reduce the pupil-adult ratio.

b) Pupil-adult ratios should be in the neighbourhood of 9:1, with the adults comprised of licensed teachers and paid (not volunteer) educational assistants (EAs). The EAs may be interested adults from the local school community or paraprofessionals but must receive ongoing training relevant to their position.

c) Teacher aides for children with special needs still need to be provided, above any and all staffing identified in (a) and (b).

d) Funding must be consistent with other Ministry objectives on the Formative Years. As well, Kindergarten teachers

need specific on-going training in child development, learning theories and other relevant topics. Serious consideration should also be given to physical layouts of classrooms and minimum floor space, with special emphasis on consistency with regulations concerning the same children in daycare facilities.

With due regard to these statements concerning staffing and EAs, schools must still encourage parent volunteers. Their assistance can enhance the quality of education children receive in these crucial years by increasing their opportunities to interact with an adult.

ISSUE 8

Should provincial policy provide direction to school boards with regard to procedures for assessment, evaluation and reporting of children's individual patterns of development and learning?

1 Assessment

a) Teachers' observations _____ ☑ YES

b) Parents' observations _____ ☑ YES

c) Child care observations _____ ☑ YES

Comment

The Collection of information concerning children's growth, development and progress needs to be obtained with a variety of tools. No one assessment tool can find all the information we need.

Assessment should be based on teachers' observations as the cornerstone, but supported by parents' observations, EA's observations, conferencing, student demonstrations, portfolios of work, peer comments and self appraisals. The collection of information over time provides insights into stages of development as well as growth patterns. Collections of students' work in portfolios should be built up collaboratively between the child and the teacher with each contributing significant samples.

2 Evaluation

a) Early Identification _____ ☑ YES

b) Promotion _____ ☒ NO

c) Retention _____ ☒ NO

Comment

a) Both assessment and evaluation must be related to the objectives of the program. In practice, assessment and evaluation are part of a blended package whereby we collect data and make inferences about children's progress and program delivery.

Identification of children's learning abilities and needs is a constant requirement in education. Obviously the more we know and the earlier we know it, the better able we will be to help children learn to their best ability and to provide suitable learning experiences.

b) c) Programs should be developmentally appropriate and "promotion or retention" is not the issue. Evaluation should be formative rather than summative and should focus on the progress that the child has made, not on comparisons with peers.

As children proceed in the Primary division, we agree in principle with recommendation #19 in The First Report on the Select Committee on Education (1988) that "age appropriate promotion with required remedial and preventative programs be the norm." Research has consistently been against grade retention as a general policy. Any and all decisions about a child's progress through school must be joint decisions among the various partners, made with the child's interests, strengths and background at heart, and with due regard to the child's self-esteem and confidence.

3 Reporting

a) Written reports _____ ☑ YES

b) Parent-teacher conferences _____ ☑ YES

Comment

There should be regular opportunities for parental classroom visitations and parent-teacher conferences. Written reports should represent the ongoing dialogue about the child's development between the teacher and the parents.

This dialogue should start in the spring preceding full-time attendance as a part of the Early Identification Process. This period of registration should include home visits, time in the classroom for the child, and interviews with the child and parents. Part of the program should also include orientation courses for parents to help them understand how young children learn.

All of the preceding comments are predicated on the need for more teacher training and inservice. Teachers must have access to ongoing professional development dealing with the expectations, objectives and programs in the Early Years. They should learn how to assess and evaluate—with a variety of tools—programs and children.

ISSUE 13

Should additional training components on Junior Kindergarten, Kindergarten and child care co-ordination be provided in principal and supervisory officer courses?

a) Principals _____ ☑ YES

b) Supervisory officers _____ ☑ YES

Comment

Recognizing the role of principal as curriculum leader and the role of supervisory officer as a leader of leaders, people in both these positions must be knowledgeable about all aspects of child development, learning theories and all levels of program. Decision makers who are aware of the value of the Early Years especially, will be better able to appropriately allocate space, staff and funding.

Realizing that the majority of leaders and decision makers have already taken their preparation courses, the Ministry, Universities, and School Boards should provide ongoing inservice for current leaders (Supervisory Officers, Principals and curriculum resource personnel).

ISSUE 16

Should the Ministry of Education offer professional development to teachers in schools where French is the language of instruction to enhance their knowledge, awareness and understanding of the cultural and linguistic needs of French-language children in Ontario?

1 Teachers trained in Ontario

a) Orientation sessions _____ ☒ NO

b) Regional workshops _____ ☒ NO

2 Teachers trained outside Ontario

a) Orientation Sessions _____ ☒ NO

b) Regional workshops _____ ☒ NO

Comment

All teachers in Ontario need to receive professional development concerning the cultural and linguistic needs of children in Ontario. These must reflect the multiracial, multicultural communities in which we live and therefore inservice must be relevant to the local level and not some provincial average. This will encompass the growing portion of our population for whom the language of instruction in school is not their first language. We need to adapt all programs to reflect the local, regional, national and global communities in which we live.

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Readers are invited to submit items from newspapers, magazines, books or other sources that contain information that can be related to any area of Mathematics. Sufficient information about the source of the item should be given so that other readers can obtain additional details. Teachers are encouraged to use the contents of this regular feature of The Gazette as an aid to helping their students see the connection between Mathematics and the world around us. The content may not fit in with what you are currently teaching to your students, but many of the items can be used to begin your classes in an interesting and engaging manner.

Where the Sky Is the Limit

The Toronto Star, Monday, July 20, 1992

Artist's concept is given for a 220-storey skyscraper in a proposed urban development plan by the Mitsui Construction Company in Tokyo. The 1321-metre high building is designed to house one million people.

QUESTIONS

a) How many people will live on each floor?

b) How high is each floor?

c) How do your answers for parts a and b compare with apartment buildings in your area?

d) Will this building be taller than the CN Tower in Toronto?

e) Do you think that most tenants will own a car? If so do you think that enough underground parking spaces could be provided?

COMMENTS AND ANSWERS

a) It seems incredulous that there will be about 4545 (1 000 000 ÷ 220) people on each floor.

b) The height of each floor will be about 6 m (1321 ÷ 220).

c) The answer to part b compares favourably with most apartment buildings in North America. The answer to part a is huge. Can you imagine having over 4500 people living on the same floor as you?

d) The building will be much taller than the CN Tower.

e) It is hard to even imagine how many floors of underground space would be required for the tenants of this building. It is a good thing that many people in Tokyo do not own a car.

I just finished reading the July 27, 1992 issue of *The New Yorker* Magazine and I noticed an interesting piece of information related to this item in an article on Page 68. Many years ago Frank Lloyd Wright designed a mile-high skyscraper project to be built in Chicago. He designed a garage capable of holding fifteen thousand cars, but forgot to consider how long it would take to empty the garage at the peak of rush hour. The article gives other examples of where Wright brushed aside the actual needs of a client in favour of his own intuitive and sometimes outrageous appraisals of those needs.

Town lost in thought at prospect of maharishi taking over base

USA Today, July 28, 1992

"Along with improving personal well-being, transcendental meditation practitioners believe the whole world enjoys the benefits—such as reduced crime and increased business activity—if enough people meditate at the same time. The optimum number for group meditation is 7000, Maharishi International University Dean Robert Wynne says, because that number is the square root of 1% of the world's population"

QUESTIONS

a) Using the information given in the above excerpt, find the population of the world.

b) If the population of the world was quadrupled, how would this effect the optimum number for group meditation?

COMMENTS AND ANSWERS

a) Let the population of the world be P. The value of P can be found by solving the equation given below.

$$7000 = \sqrt{0.01P}$$

You should find that P comes out to be 4.9 billion.

b) If the population goes from P to $4P$, the optimum number for group meditation will become twice what it was because of the presence of the square root in the formula.

StarPhone logs 8 million calls

The Toronto Star, August 1, 1992

"StarPhone, The Star's telephone, news and information service logged its 8 millionth call yesterday, making it the leader among audiotex information systems in North America. ... StarPhone, which went into business last September 14, answers more than 30 000 calls a day for sports scores, horoscopes, weather, lottery numbers and dozens of other types of information"

QUESTIONS

a) Is the headline consistent with the information given in the article?

COMMENTS AND ANSWERS

a) There are 322 days from September 1, 1991 to July 13, 1992 inclusive. If your students end up with 321 days, it may be because they forgot that 1992 was a leap year. At an average rate of 30 000 calls per day this would mean that they should have received 9 666 000 phone calls during this period of time. Therefore the headline is reasonably consistent with the information given in the article.

Rents to rise 4.9 per cent in 1993

The Toronto Star, August 2, 1992

"Next year's legal rent increase has been pegged at 4.9 per cent. Coming on the heels of this year's 6 per cent increase and 5.4 in '91, it means rents will have gone up 16.3 per cent in three years."

QUESTIONS

a) Is the statement that rents will have gone up by 16.3 per cent correct or incorrect?

COMMENTS AND ANSWERS

a) The statement is incorrect. I have just finished writing a letter to The Editors of The Star and I hope that the letter is selected for publication. If I were in school right now I would have had my students send in some letters too. To see that the statement is not correct, take any amount of money, say $100 and increase it by 5.4%, then 6% and finally by 4.9%. You should end up with an answer of $117.20 (rounded up) which corresponds to an overall increase of about 17.2%.

Drabble Cartoon

The Globe and Mail, August 3, 1992

First frame shows Norman stroking his pet duck and saying "1342, 1343, 1344,..."

Next frame shows his Mother looking quite puzzled and asking him "What on earth are you doing?"

Following frame shows Norman responding to his Mother with "Oh, Hi Mom! Dad says that if I stroke a duck one million times, I'll absorb enough electricity to light this light bulb!"

Final frame shows Norman's father privately saying to his mother "Hey, it keeps him out of my hair for a couple of hours!"

QUESTIONS

a) If Norman is able to stroke his duck once every second, will this "keep him busy for a couple of hours"?

COMMENTS AND ANSWERS

a) It will take Norman over 277 hours to stroke his duck one million times. His father was way off!

Which derivatives do CFOs really use?

The Institutional Investor, Page 141

"Overall, derivatives remain an acquired taste: Many chief financial officers swear by them, but others shun them. In this survey a number of CFOs, some from quite large companies, indicated that they don't use derivatives regularly and don't intend to. The majority do use them, however, and a small contingent has begun delving into the more arcane instruments."

COMMENTS

A huge grin appeared on my face when I came across this article and I am looking forward to sharing it with my Calculus class this fall. I will either post it on the bulletin board or run a copy of it as a filler item on their derivative test.

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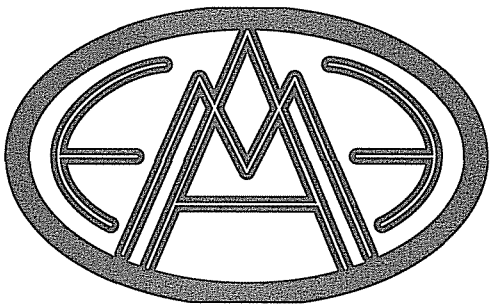
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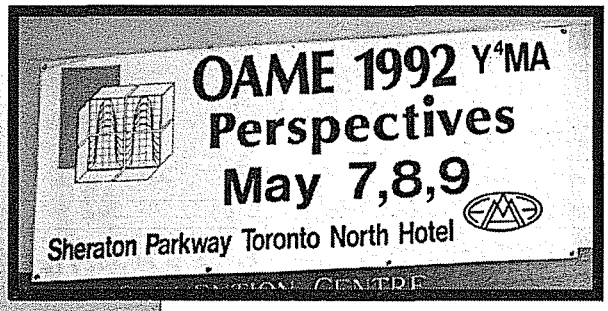
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AT THE CLOSING SESSION OF OAME '92 RON LANCASTER LET FLY AN AEROBIE. WHAT ARE THE CHANCES THAT IT WOULD SAIL TWENTY-FIVE METRES ACROSS THE BALLROOM AND LAND ON THE DOOR HANDLE? WELL IT DID. MATH MAGIC, RON!

The telescope photographs on the cover of this month's Gazette were taken by Tom Olien. Shown is the 46-m radio telescope at the Algonquin Space Complex in Northeastern Algonquin Park—one of several Canadian research instruments operating at this site.

This summer saw the opening of the Algonquin Space Campus, managed by ISTS—the Institute for Space and Terrestrial Science. The Algonquin Space Campus is a summer camp program for teens that combines the water and forest of Algonquin Park with active exploration of space science and technology using the facilities of the Algonquin Space Complex.

For further information contact Chris Coggan, Director, Algonquin Space Campus, ISTS, (416) 675-3311.