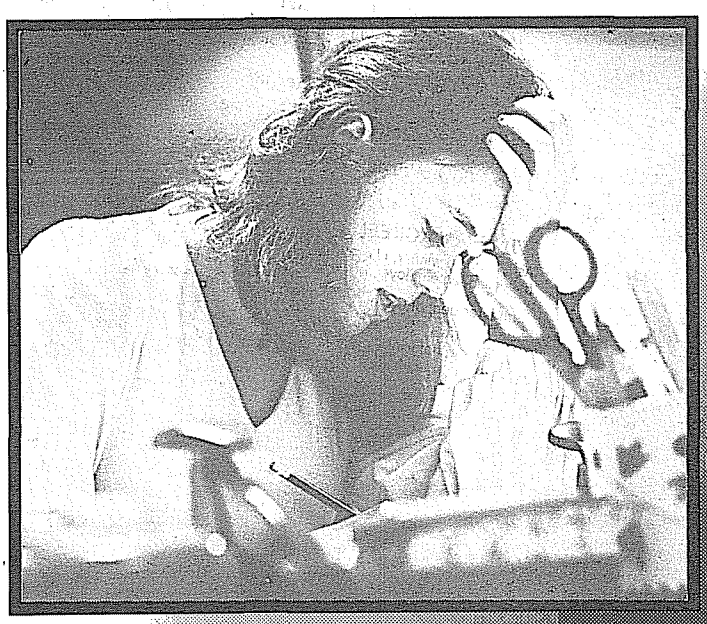
THE ONTARIO MATHEMATICS

Gazette

VOLUME 31 NUMBER 2 DECEMBER 1992



"Math can be hard sometimes ..." Barbie dolls are much in the news these days after Mattel's recent "math class is hard" gaffe. For commentary see Lorna Morrow's letter on page 3 & Shelly Beauchamp's article on page 15. And take a look at page 36.



"... but much more often it's fun—and we're good at it!" Right: Julia McAllister, Michelle McKinnon & Jennifer Vincent—at the Northern Summer School of Science & Technology, a program for high school students from Northwestern and Northcentral Ontario.

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

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Articles for the Gazette should be submitted to Jack Weiner at the above address, while those for the Abacus should be submitted to Amy Parker

Deadlines for information to be included in the Gazette or the Abacus are: June 15 for the September issue, October 15 for the December issue, January 15 for the April issue, and April 15 for the June issue.

NOTICE TO AUTHORS

Your articles should be typewritten, word-processed or photocopied from typewritten or word-processed copy double-spaced on 8.5" x 11" paper with 1.5" margins and single sided. Diagrams and figures should be in camera-ready form. Please submit three copies to the editor and keep one as protection against loss. We have all submissions blind-reviewed, so please send your full address and affiliation on a page separate from the manuscript. Please include a photograph of yourself—in black and white if at all possible.

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Editorial

Mary Lou Kestell

Welcome to the second Gazette of the 1992-93 school year. I am very excited about what we have to offer. Our President, Jeri Lunney (page 2) points out some concerns about the National Indicator's Test and reviews our work with the Ministry on Benchmarks. Berle Summers, as a Leadership speaker, indicated that the Ministry Benchmarks Program has received very positive feedback from such groups as business, engineering and accounting professionals.

Our articles support teachers' search for effective ways to teach current topics using computer software (page 8), game theory (page 16) and courses such as Calculus (page 26) from an investigative, inquiry-based approach. The York Mills Collegiate people offer an fabulous idea for setting up ways to have students prepare videos to teach each other how to prepare for exams (page 30). "The Graph of Life" offers interesting analysis opportunities for students (page 13).

Advertisements for OAME/AOEM's Annual Conference invite you to Windsor in '93 (page 3). The Equity Conference–a first for OAME/AOEM–will happen at OISE in February (page 13). Teachers and publishers all across North America will tell you that OAME/AOEM Conferences are the best and could easily compete with NCTM Regionals almost anywhere.

I could not write this editorial without referring to the Barbie controversy. We include a short article (page 15) and a letter to the editor (page 4) regarding the dolls. I have been realizing over time, how socially acceptable it is in North America to have hated mathematics. Witness Mattel's defence, "Well math **is** hard." As mathematics educators, we do have to face our self-image problem straight on and start to change it.

In attending the Leadership Conference I heard more than one group of people saying, "It is a very exciting time to be involved in Mathematics Education. This is where things are happening. The spotlight, that 10 years ago shone on language arts people, is now directed our way!" We have a chance to shine.

Speaking of shining, have a very happy holiday season and let your light shine in the hearts of many. Remember to celebrate Newton's Birthday on December 25th along with your family's traditional ceremonies. Happy New Year!

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President's Message

JERI LUNNEY

Congratulations to the Publications Committee, to the Editors and Michael Tabor, for the Gazette's new look. The diagrams and photographs are professionally done; the articles are excellent; and there are many activities suitable to be used in the classroom.

The response to this year's Leadership '92 seminar, "Focus on Renewal", held at the Ramada Hotel, Airport West, Mississauga on November 11-13 was overwhelming. The committee regrets that not everyone on the waiting list could be accommodated. The activity-based workshops were based on three themes: Mathematics as Communication, Mathematics as Problem Solving and Mathematical Connections. Keynote speakers included Myrna Ingalls, Burle Summers, Jack Weiner, and David Zimmer. As Chair of the conference, I would like to take this opportunity to thank the members of the organizing committee (Don and Carol Attridge, Judy Crompton and Ron Sauer) and the speakers who gave their time to make this seminar such a huge success.

The response to the Leadership Seminar is indicative of the concern teachers across the province have for the changes currently taking place in education; benchmarks, learning outcomes, de-streaming, national testing. OAME will continue to play an integral role in assisting teachers across the province. The mini-conferences hosted by local Chapters and the annual conference in Windsor provide excellent professional development. Support your local Chapter. Get involved:

- Share your favourite activity with others.
- Make a presentation at a staff meeting or at a professional development day.
- Attend a mini-conference.
- Write an article for the Gazette.
- Come to the annual OAME conference in Windsor.

OAME has worked closely with the Ministry of Education as the provincial benchmarks and learning outcomes were developed. Burle Summers, Director of the Learning Assessment Branch and Gail Rappolt, Coordinator of Field Development, Provincial Benchmarks Project have involved OAME members on committees and have kept us well informed. The first provincial benchmarks for student achievement in mathematics at the end of grades 3, 6 and 9 have been developed and distributed to Boards.

The six strands, (inquiry and problem solving, number sense and numeration, spatial sense and geometry, patterning and algebra, data management and probability, estimation and measurement) are indicative of the changes which have taken place in the past few decades. What we once considered "basic skills" are no longer sufficient. Number crunching alone is inadequate in this age of technology.

The following quotation from a memorandum written to Directors of Education may help to clear up some of the misconceptions about benchmarks.

"Provincial Benchmarks are broad standards for the province against which to assess students' learning. They set worthwhile goals for all, by describing expected provincial levels of achievement. Their primary focus is to make a clear statement to teachers, parents and students about expected results. The current provincial mathematics benchmarks indicate a provincial standard for the end of each division (grades 3, 6 and 9).

How will they be used?

- Provincial benchmarks (standards) will provide a concrete framework for both student and program assessment.
- Provincial benchmarks will act as references for teachers when assessing individual students and reporting to parents.
- Provincial benchmarks will provide information for parents and guardians about what they can expect their students to be learning.
- Provincial benchmarks will provide a concrete basis for school boards to develop board-wide assessments.
- Provincial benchmarks will also be used to inform the auditing of province-wide achievement.

It is clear that the Learning Assessment Branch expects teachers to have the responsibility for assessing the student's progress, evaluating individual students, planning instruction and reporting to parents on the student's progress. Benchmarks are not instruments used to fail students; rather they allow teachers to make appropriate assessments and provide more focused assistance to enable more students to achieve success. It is not intended that teachers simply use the sample evaluation questions as test items. Student evaluation tools such as

portfolios, written work by students, individual and group projects, teacher and observation, student evaluation and written tests should be used. I believe that this message agree with OAME's vision of evaluation.

An external examination, whether it be a school board's examination, a provincial review or a national test is not sufficient to assess a student's progress. We must make the public aware that such tests assess only a part of the learning that has taken place. The following quotation from my letter to the Minister of Education, Tony Silipo, expresses OAME's concern about the process being used in the National Indicators Test for thirteen-year-olds and sixteen-year-olds.

> *It is clear that multiple choice and short answer items are the most time-efficient for large-scale testing. However, these items provide only a snapshot of a student's achievement. Therefore, we recommend that, before the administration of the test, the Ministry, in its communications with members of the public, media, and educators, stress that the results measure only a very small, albeit important, part of a student's experience in a well-balanced mathematics program. In addition, it is critical that the Ministry clearly articulate the relationships between the results of the testing and their use in the refinement of curricula.*
>
> *...The fact that the National Indicators Test requires thirteen and sixteen year olds to sit for a two hour summative test is not only inconsistent with what is known about assessment but is counter-productive. Students are being encouraged to learn in a cooperative mode and to become productive members of our technological society. Teachers are being encouraged to use teaching and assessment methods which prepare students for the twenty-first century. Yet the assessment methods to be used on the National Indicators Test will be merely summative test items, a small part of a comprehensive evaluation of a student.*

As a result of this letter, I have been invited to meet with Mr. Silipo and Ministry of Education Officer, Moe Liebowitz, to discuss the issues. I will keep you informed. For more information on the School Achievement Indicators Program, read Lorna Morrow's article in the September Gazette.

As we enter the holiday season, I wish each and every one a happy holiday season and a peaceful and prosperous new year.

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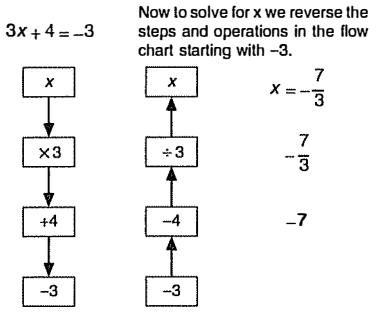
LETTERS

SKYSCRAPERS & PROBLEM-SOLVING WITH FLOWCHARTS

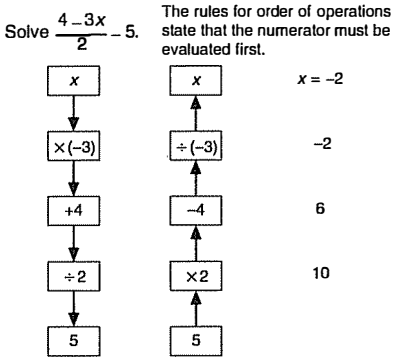
John J. Del Grande, Newmarket

In Ron Lancaster's article on the proposed Japanese skyscraper he might have wondered how one could move the occupants of the building in elevators. It has been conjectured that in such buildings people might never leave them in a life time.

I would also like to follow up on the article by Michelle Helson-LeBreux. Solving first degree equations in one variable can be illustrated using flow charts and order of operations. For example, in the equation $3x + 4 = -3$ start with x and evaluate the expression $3x + 4$ using order of operations — which can be shown in a flow chart.



This looks after all cases in the article. Another example:



TEEN TALK BARBIE

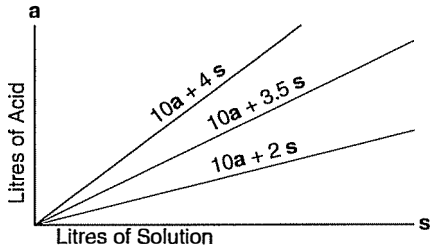
Lorna Morrow, Toronto

While those of us in mathematics education are trying to ensure that all students have access to good math programs and that girls as well as boys are encouraged to continue with math and science courses, Mattel is doing its best to undermine our efforts. According to a message recently on University of California email, Mattel is selling a product called "Teen Talk Barbie" which says four phrases, among which is "math class is hard". Mattel justifies this by replying that they are not saying the other classes are easy, and that they have a doll available that says "I want to be a doctor". Just how they expect Barbie to succeed in this aim without mathematics is not made clear. Since Barbie dolls are popular with very young children (ages 4 and up), this negative message about math could be influencing children even before they get to school. We should all be making our concerns clear to Mattel, and to the Canadian Toy Council that endorses Barbie dolls as a 'good' toy for children.

MIXING MIXTURES

Peter Weygang, St. Thomas

The article by Litwiller and Duncan, "Mixing Mixtures: A Graphical Solution" (Volume 30, #3), was interesting. However, to my taste, it was somewhat contrived. The usual hierarchy of problem solving methods would place a graphical solution somewhat lower than an algebraic solution. Indeed, in many instances, algebraic relationships are models for information which first appeared in a graphical form. A vector method presents an alternative solution which reaches towards a higher level of sophistication.



With reference to the diagram we have $m(10\mathbf{a} + 4\mathbf{s}) + n(10\mathbf{a} + 2\mathbf{s}) = 10\mathbf{a} + 3.5\mathbf{s}$

Equating vector **a**'s coefficients: (1) $m + n = 1$.

Equating vector **s**'s coefficients: (2) $4m + 2n = 3.5$.

Substituting n from (1) into (2):

(3) $4m + 2(m - 1) = 3.5$, and so $m = 0.75$ and $n = 0.25$.

Thus the ratios $\frac{\text{Stronger solution}}{\text{Weaker solution}} = \frac{m}{n} = \frac{0.75}{0.25} = \frac{3}{1}$.

This ratio can be used to determine the composition of any given volume of mixture. Specifically we have:

30 litres of 40% + 10 litres of 20% = 40 litres of 35%.

Five Steps to Teaching Proof in the Senior Math Curriculum: A Cooperative Learning Approach– Part IV

Brendan Kelly, University of Toronto

Introduction

This is part IV of a five-part article spanning consecutive issues of the Gazette. In part I, we described briefly the central role of *proof* in the senior mathematics curriculum and the reasons why we might regard a study of proof as one of the invariants in a changing mathematics curriculum. That article also noted the trend toward alternative instructional methods and, in particular, cooperative learning. The intent of this five-part series is to provide a model to demonstrate how the concept of proof and its evolution might be explored with senior level students in a series of five lessons structured in the cooperative learning format. The intent is to cast the development of proof in its historical context and to show how the quest for certainty led through a cascading sequence of complexities to a point where it was recognized that absolute certainty is an unattainable goal. This development presents the evolution of proof in five independent but sequential lessons corresponding to the five parts into which this article is partitioned.

The lesson in part I of this article, titled, "The Dawn of Mathematical Proof: Inductive Thinking and Conjecture", referred briefly to the inductive thinking of the ancient Egyptians and Babylonians. It prompted students to conjecture formulas in specific cases and attempted to help students discover why proof is needed to support conjecture.

The lesson presented in part II, titled, "The Development of Direct Proof and Generalization", was designed to take the students through the next step in the evolution of proof in which the ancient Greeks attempted to underpin conjecture with logical argument based on common premises or axioms.

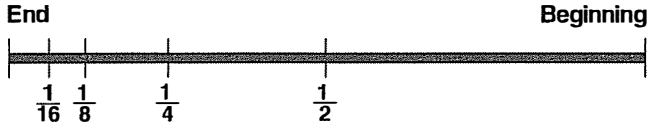
The lesson presented in part III, titled, "Indirect Proof and Proof by Counterexample", presented three examples of indirect proof, two of which were invented by the ancient Greeks.

In this article, we present lesson 4, titled, "The Quest for Ultimate Rigour". Here, we introduce students to one of the earliest paradoxes in mathematical thinking, discovered by Zeno in the 4th century BC. This paradox reflects the difficulties experienced by the ancient Greeks in coming to grips with the limit concept and provides a strong rationale for the pursuit of rigour. An investigation exercise with several parts is provided in this lesson for students to explore and discuss in groups. It is intended that each group will report at least one of its explorations to the class to allow for some important follow-up discussion. An additional investigation exercise is provided as homework to ensure that all students are involved in the learning. We hope that you will find parts or all of it useful and that this might encourage you to contribute further examples of mathematics topics which are amenable to this instructional format.

Lesson 4–The Quest for Ultimate Rigour

Whenever mathematicians have begun to think that certainty was within grasp, a paradox has surfaced which has shaken the foundations of mathematical thinking. So it was in the 4th century BC when the Greek philosopher, Zeno of Elea made the assertion known as Zeno's racecourse paradox:

A runner can never reach the end of a racecourse in a finite time.



STATEMENT

1 In order to finish a race, a runner must first pass the point $\frac{1}{2}$, located half the way between the beginning of the course and the finish line.

2 It will take a finite time to reach the point $\frac{1}{2}$.

3 After the point $\frac{1}{2}$, there is another half way point marked $\frac{1}{4}$.

4 There are an infinite number of points which the runner must reach and each will take a finite time.

5 The total time for the race is infinite.

REASON

1 $\frac{1}{2}$ is between the runner and the finish line.

2 It is a finite distance from 1 to $\frac{1}{2}$

3 The remaining interval is divided in half.

4 Statements 1, 2 and 3 can be repeated an infinite number of times.

5 The sum of an infinite number of finite times is infinite.

But we know from experience that the runner can reach the finish line. Where is the flaw in Zeno's argument? The following example may help your students discover where Zeno's argument breaks down, particularly if you model the process using a cardboard metre stick which you cut at each stage.

TEACHING EXAMPLE

A metre stick is cut in half. One piece is placed in a basket. The remaining piece is cut in half and one of those pieces is placed in the basket. Again the remaining piece is cut in half and one of the pieces is placed in the basket. This process is repeated *n* times.

a) Write as a sum of fractions the total length of the first 6 pieces placed in the basket.

b) Write the length of the *nth* piece to be placed in the basket and the length of the remaining piece.

c) Write as a sum of fractions the total length of the first *n* pieces placed in the basket.

d) Write an algebraic expression for the total length of the first *n* pieces in the basket.

e) Suppose this process could be continued an infinite number of times. Would the length of the stick ever exceed one metre?

SOLUTION

a) The total length of the first 6 pieces of the metre stick in metres is $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}$.

b) The length of the n^{th} piece in metres is $\frac{1}{2^n}$. This is also the length of the remaining piece.

c) The total length of the first *n* pieces inside the basket is $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots+\frac{1}{2^n}$

d) The total length of all the pieces is 1 m.

Therefore, the total length of the pieces inside the basket is $1-\frac{1}{2^n}$ metres.

That is, $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$

e) Since we started with a stick one metre long, it is clear that the sum of the lengths of the pieces can never exceed 1 m no matter how many times the stick is divided. That is, the sum of the infinite number of fractions $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$ can never exceed 1. Furthermore, the sum must approach arbitrarily close to 1 as we toss more and more of the fragments into the basket. To verify this mathematically, we observe from part d, above, that the sum of the series up to and including the *nth* term is

$1-\frac{1}{2^n}$

and this approaches 1 as *n* becomes infinitely large.

This example shows that the sum of an infinite number of finite quantities may be finite. This calls into question the reason supporting the fifth statement in Zeno's argument above. Paradoxes such as this prompted mathematicians to put mathematics on firmer foundations by specifying a basic set of "self- evident" truths or axioms from which all mathematical theorems could be logically and unambiguously derived. This would (presumably) expose all hidden assumptions and remove paradoxes.

INVESTIGATIONS

Explore and discuss this exercise in your group.

1 A bicycle was traveling 10 m/s toward a brick wall. When the bicycle was 60 m from the wall, Fearless Photon left the front tire of the bicycle and flew straight toward the wall at a speed of 15 m/s. Upon reaching the wall, Photon turned and flew toward the bicycle at 15 m/s. Fearless continued to fly from the wall to the bicycle tire and back until becoming trapped between the bicycle tire and the wall.

a) How far was Photon's first trip from the bicycle tire to the wall?

b) How far was Photon's first trip from the wall to the bicycle tire?

c) Assuming that Photon changes direction instantaneously, how long did it take Photon to fly from the tire to the wall and back?

d) How long did it take Photon to fly the first two return trips to the wall and back?

e) How many trips did Photon make before becoming trapped? Was each trip a finite distance?

f) How long did it take Photon to make the infinite number of trips?

g) Use Zeno's reasoning to explain why Photon could not make an infinite number of trips.

h) Explain the flaw in the argument you provided in "g".

2 Complete Exercise 2 for homework.

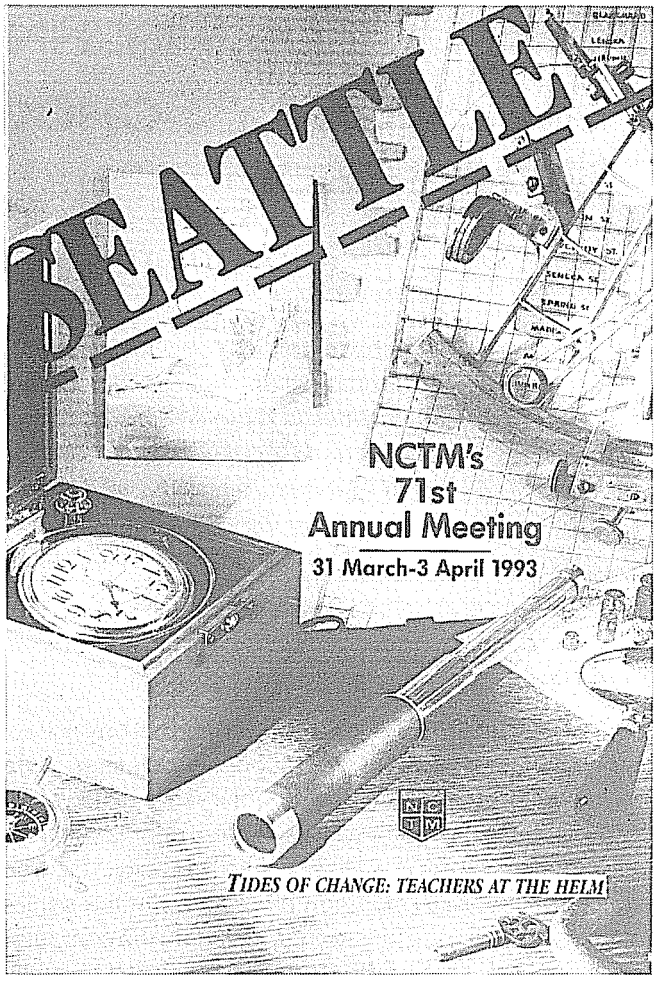
a) What is the value of the following series when $x=1$? $1-x+x^2-x^3+x^4-x^5\ldots$

b) It is easily verified by multiplication that

$$1-x+x^2-x^3+x^4-x^5\ldots=\frac{1}{1+x}.$$

What is the value of the right side of this equation when $x = 1$? Explain how the right and left hand sides of this equation can have different values.

The fifth and last part of this article will appear in the next issue of the Gazette. In lesson 5, titled, "Crisis in the Search for Absolute Proof", we will lead students from the relatively easily resolved paradox of Zeno into the irresolvable paradoxes of the twentieth century.



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The Use of Technology in One Secondary School Mathematics Classroom

C.R. D'Arcy, Royal St. George's College, Toronto

It has become quite clear that technology can and is changing the way we teach Mathematics and specifically, Calculus. However the integration of these new capabilities into the classroom should be done carefully and responsibly. Each educator must identify what advantages he or she hopes to gain for their students and how best to reap those benefits. One particular blend of curriculum and technology in the classroom has led to improvements in the volume of material investigated and the depth of its understanding by the students.

Introduction

Royal St. George's College is an independent day school for boys in downtown Toronto offering classes from Grades 4 through OAC. As a university preparatory school, it draws the majority of its student population from upper middle class families within the city. Course enrollments average eighteen per class with the senior classes slightly less. Classrooms, including the computer laboratory, are small and modestly equipped. Slightly more than half of the senior mathematics students have access to computers at home.

In 1985, the Ontario Ministry of Education introduced a new set of guidelines for an Ontario Academic Credit in Calculus (ref 1). There was a noticeable emphasis on the role of technology throughout its curriculum:

"In the planning and development of courses, consideration should be given to relevant ways of incorporating knowledge, skills and attitudes related to the computer. Alternative strategies for teaching and learning mathematics using the computer should be explored."

Textbooks organized to satisfy the new requirements were not available in time for commencement of the 1986-1987 academic year and no specific suggestions for supporting technology were offered in the document. This set of circumstances provided the motivation to develop a new Calculus environment for the students of St. George's.

The goal was to produce an integrated learning environment consisting of a central text supported by highly graphic software and accompanying laboratory exercises. Four years later these components are in place and subject to continual scrutiny. Graduates of the College continue to derive benefit from these materials in their first year university courses.

The Textbook

Technical word processing software packages were readily available in 1986, one of which was purchased to write the primary text. Since then, all assignments, tests, exams, software documentation and laboratory exercises have enjoyed the high quality of printed output that these packages offer. In retrospect, it should not have been so surprising that students were quick to respond with a higher quality of presentation in their own submissions. Some students purchased copies of the technical software for their personal use. A three-ring binder format was chosen for the textbook to permit students to merge their homework with the text. This ability reinforces the personal nature of the environment. Appendices in the text include Answers to Exercises, Tables of Derivatives and Integrals, Proofs, Tests and Exams, Laboratory Exercises and the IMAGE-Calculus™ User's Guide. (See reference 2.)

THE SOFTWARE

Relatively few algebraic skills lack a corresponding geometric model. The mathematician who coined the phrase, "a picture is worth a thousand symbols", was of course referring to graphic software.

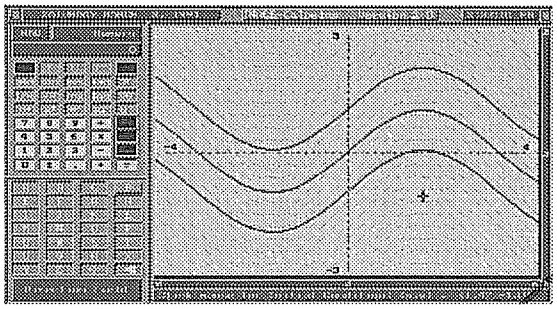


Figure 1

A number of software packages for Calculus existed in 1986, however they proved to be inappropriate for use at the College for a variety of reasons. Most packages were designed to showcase their algebraic power with little or no concern for the supporting geometry and their price precluded the purchase by students. Finally, the organization of the topics were certainly not tailored to the guidelines of the Ministry. IMAGE-Calculus™ was crafted with the needs of St. George's College students in mind as a tool for the rapid display and investigation of key concepts. Indeed, a personal copy for each student is included with their textbook for use as required at home. The software package, written for the DOS platform, also runs on the ICON under QDOS.

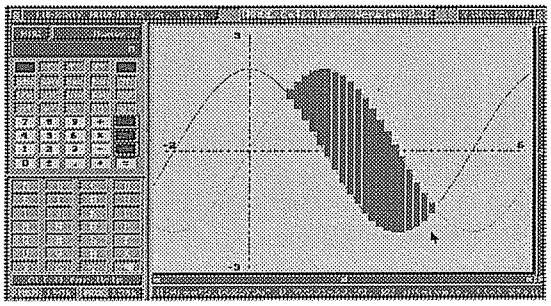


Figure 2

As a tool for the single variable Calculus course, core topics include the symbolic entry of functions, graphing with overlay capabilities, limit analysis, qualitative analysis, tangent lines, numerical integration with applications of area, arc length and solids of revolution, exponential growth, and Newton's method. Enrichment topics, some of which form the basis for laboratory exercises, include Maclaurin's series, parametric and polar equations, conics, surfaces of two variables and an introduction to chaos and fractal geometry to name a few. Rapid save, load and print utilities, as well as a built-in scientific calculator complete the software environment.

LABORATORY EXERCISES

A number of factors have combined to drive the need for independent laboratory exercises. To adequately prepare our graduates for university it is necessary to exceed the curriculum stated by the Ministry. This is due mostly to a wide variety of expectations in first year university mathematics courses. In addition, as teachers we are called upon to measure student performance using methods beyond those of formal tests and exams. Furthermore, much of the beauty of mathematics, perhaps those topics that can capture the imagination of its students, never seem to make it into the core curriculum but nevertheless are based on some wonderful (and simple) mathematics. Finally, beyond simply using technology in the classroom, we must provide an incentive for students to incorporate these tools into their own work.

The laboratory exercises were devised to satisfy the rationale provided above. Topics include applied maximum and minimum problems, arc length, Maclaurin's series, Newton's method, area between curves, volume and surface area of solids of revolution, antiderivatives and derivative fields, truncation error of approximate integration, parametric and polar equations, conics, functions of two variables and asymptotes. Each exercise requires the support of the IMAGE-Calculus™ software and leads students through a number of investigations aimed at broadening their understanding of the concept. For their final submission, a word processing software package with graphic import capabilities is required to merge the body of their text with the screen dumps from IMAGE-Calculus™. The evaluation of each lab is based on the quality of their submission and a short oral interview to discover their level of comprehension.

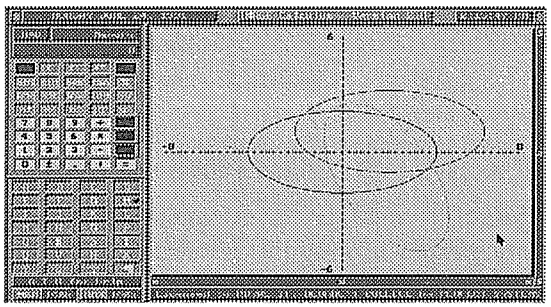


Figure 3

THE CLASSROOM

In addition to the curriculum environment, the physical environment imposed by the classroom has experienced change as well. A movable cart supporting a computer, an overhead projector and a display tablet is positioned at a different student's desk at the commencement of each class. White marker boards installed around the classroom serve as a perfect vehicle for both the display and colour analysis of the live output of the software. Having the students control the action of the software during the class quickly reduces any phobia they might have to using technical software and also frees the instructor to steer the analysis at the board.

The course is conducted in the mathematics classroom. Any requirements the students may have of the computer laboratory are satisfied on their own time. The scheduled forty minute periods are for the instruction of Calculus; the use of technology is for the support of classroom discussion. Finally, the technicultural society, of which the students are a product, has come to place value on rapid visual display. Advertising, computer games, videos and movies often capitalize on the quick graphic dissemination of information to drive home their message usually at the expense of eroding the attention span of the audience. This phenomenon has put pressure on traditional teaching styles. A mathematics concept demonstrated with the assistance of a computer graphic display, seems to carry an implicit advantage over blackboard geometry drawn by hand. The use of animation further enhances its value and acceptance.

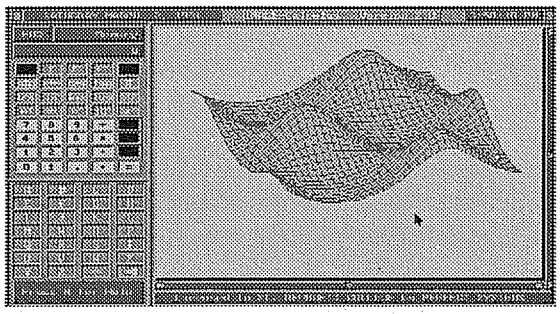


Figure 4

SAMPLE INVESTIGATIONS

Three examples will serve to demonstrate how algebraic fundamentals can be brought to life with graphic support.

1 With its zoom capability IMAGE-Calculus™ can highlight the convergence of $y = \sin x$ and $y = x$ as x nears 0 at the origin, reaffirming the result.

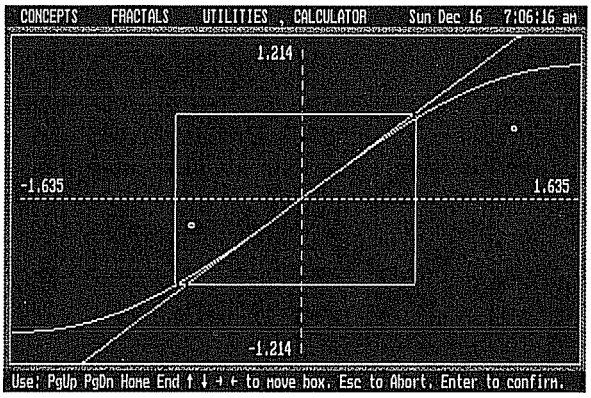


Figure 5

2 The change of base formula for logarithms permits the evaluation of logarithmic expressions for bases other than those found on the standard calculator. Explicit use of the formula can permit the definition and subsequent graphing of a family of logarithmic functions, as shown below.

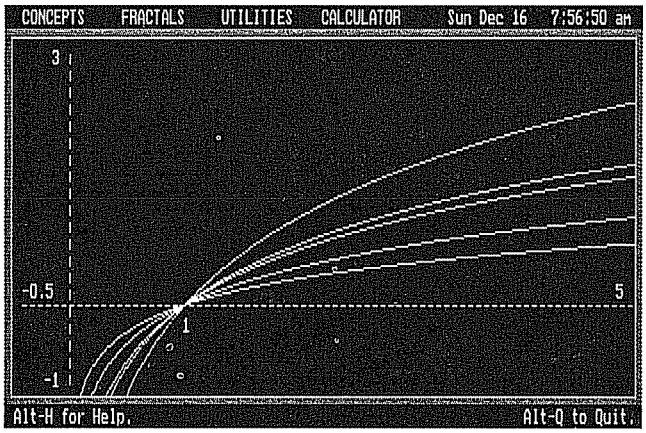


Figure 6

3 Many functions are defined algebraically for the express purpose of testing students' ability to correctly apply the rules for differentiation. The ability to visualize unusual functions, such as the famous exponent to exponent to exponent function below, removes another level of abstraction.

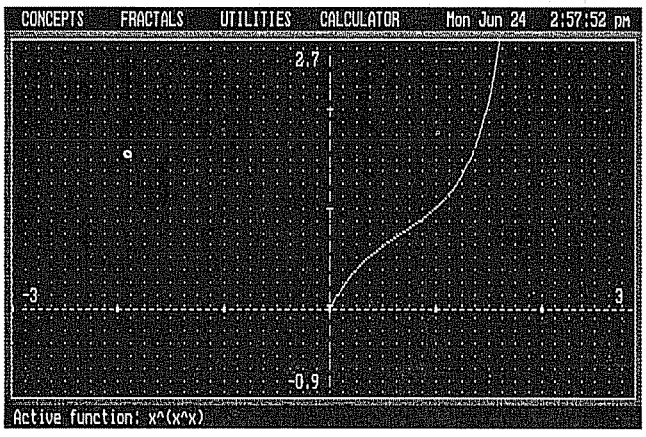


Figure 7

Summary

This paper is by no means intended to be a prescription for dovetailing technology and curriculum but rather as a reference for proven techniques that are having a positive impact on the instruction and learning of OAC Calculus.

References

1 **Curriculum Guideline, Mathematics**, Ontario Ministry of Education, 1985

2 D'Arcy, C. R., **IMAGE-Calculus** (Interactive MAthematics/Graphics Environment for Calculus), 1991

3 D'Arcy, C. R., **O.A.C. Calculus**, 1991

Readers interested in further information regarding the availability of the either the IMAGE-Calculus™ software or the corresponding Laboratory Exercises can write to:

Chris D'Arcy
Head of Mathematics
Royal St. George's College
120 Howland Avenue
Toronto, Ontario
M5R 3B5

Call for Contributions!

The June, 1993 issue of the Ontario Mathematics Gazette will have a special focus:

A GRADE 9 MATH TEACHER'S DESTREAMING SURVIVAL GUIDE

Like it or not, the grade 9 classroom is to be destreamed in September, 1993. So we are putting together an issue that will feature:

- an introductory overview of destreamed grade 9 math
- several lesson plans based on the early part of the course
- a discussion of evaluation techniques for the destreamed classroom

Please submit your contributions to Jack Weiner by January 31, 1993. See the inside front cover for submission instructions.

Review of Mary Cope's Game, "Dr. Math"

Brenda Malvern, Brantford CI&VS

Math can be fun--and our students can learn something, too! These were the main goals for Mary Cope of Brantford Collegiate Institute & Vocational School as she developed an interesting new game, especially for Intermediate Level math students. DR. MATH-EQUATIONS has been an idea developing for Mary who wanted to introduce more variety in methods and hands-on activity in math classes.

DR. MATH is a user-friendly game which provides drill in equations—a common area of difficulty for many students. The game offers lots of flexibility for teachers and students; for example, teachers can add their own questions.

DR. MATH can be played by an entire class, small groups or individuals. The playing time can vary from 10 minutes to an hour or more--a terrific addition to classrooms for those "Snow Days", "Track Days", etc. The game is equally appealing to General and Advanced students, and the winner can be either the poor math student or the usual achiever. Students may also get involved in administering the game.

Mary has sample-tested DR. MATH with colleagues at BCI&VS and student teachers at Althouse College and received an enthusiastic response from staff and students. The game was also used by a French Immersion teacher at BCI&VS who "...found DR. MATH to be an enjoyable and stimulating way to enhance and reinforce new concepts—the game changed the attitude of a challenging class". DR. MATH was introduced to adults at the Brant Alternative Learning Centre, who found it a welcome change of pace.

In DR. MATH, Mary has developed an excellent variety of questions that cover a wide range of difficulty. Also, it is very easy to add in more questions. If the game continues to be successful, Mary has plans for DR. MATH-FRACTIONS, DR. MATH-EXPONENTS and DR. MATH-INTEGERS. DR. MATH-EQUATIONS, which comes with all supplies and equipment for the game in an attractive box, costs $30 (shipping and tax included).

Readers interested in further information regarding DR. MATH can call or write to:

Mary Cope

NEWS FROM NCTM

ADDENDA SERIES UPDATE

The Addenda Series are teacher-friendly books that help translate the NCTM Standards into classroom practice.

- **Kindergarten Book** #453N3, $9.50
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UPCOMING CONFERENCES

- Regional Conferences: **Winnipeg**, Manitoba May 6-8, 1993; **New York City**, New York May 27-29, 1993
- Annual Conferences: **Seattle**, Washington March 31-April 3, 1993; **Indianapolis**, Indiana April 13-16, 1994

NEW EXCITING TITLES

- **Calendars for the Calculating**, Vol. 2. #496E1, $7.00
- **Fractals for the Classroom** (Part One: Introduction to Fractals and Chaos; Part Two: Complex Systems and The Mandelbrot Set) #440E1 (part 1) $29.00; #507E1 (part 2) $29.00
- **How to Use Children's Literature to Teach Mathematics**, #428E1, $TBA

THE VIDEO LIST EXPANDS

- **Algebra for Everyone**
- **Calculators for Classrooms**
- **The Polished Stones**
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- **Reaching Higher**
- **Similarity**
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2. Include your membership number, expiry date, and address used for membership purposes.
3. Subtotal your order, then deduct 20%. This is one of the many benefits of being an individual member of NCTM.
4. Send your order form and a cheque made payable to NCTM-Ontario in U.S. funds.
5. Send your form and cheque to your Ontario representatives:

Janet and Barry Scully
NCTM-Ontario Representatives
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Richmond Hill, On L4C 6B4

The Graph of Life

William W. Dodd, W.A. Porter Collegiate Institute, Scarborough, Ontario

A Graph of Life can be an interesting, open-ended exercise for mathematics students. Such an exercise can help students develop basic skills in computation, graphing and estimating. Reflecting on the patterns in a Graph of Life can motivate students to investigate time management and personal planning.

The process of creating a Graph of Life is intrinsically interesting to students and can contribute to several instructional objectives such as practising basic computations, developing graphing skills, developing estimation skills, and introducing time management and personal planning. Every moment of our lives is spent doing something. Our time is spent on activities such as eating, sleeping, going to school, watching TV and going on vacation. Surprisingly, a single graph can easily be constructed to illustrate the time spent on major activities by an individual during his/her entire life. I call the result a 'Graph of Life'.

Constructing a Graph of Life is an exercise that takes only one class period and leads to the production of a novel and informative result.

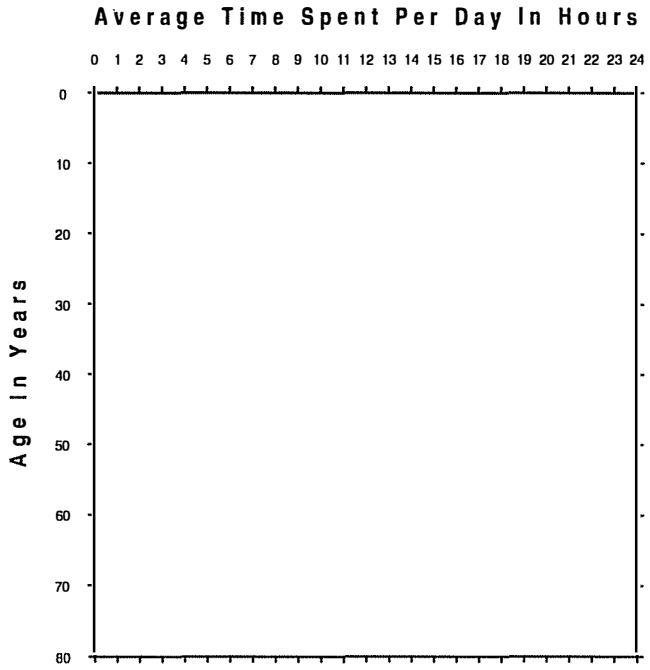


Figure 1—Axes for a Graph of Life

Preparing to Construct a Graph of Life

As shown in Figure #1, the vertical axis is used to represent the years in a person's life and the scale ranges from 0 to 80. (The average Canadian has a life expectancy of approximately 80 years.)

The horizontal axis is used to represent the average number of hours per day spent on an activity during a given year. This axis has 24 divisions, one for each hour in a day. An activity can be shifted horizontally to any position that is graphically convenient.

Math Equity Conference

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Before beginning a graph, it is also necessary to decide whose life is going to be graphed - the student's own life, or the life of a typical student, or the life of a typical urban dweller etc.

Life activities are plotted on the graph not as points, but as regions. As a first estimate, each region is rectangular in shape. The width of a region is the average number of hours per day that are spent on an activity and the height of a region corresponds to the number of years the individual engages in this activity.

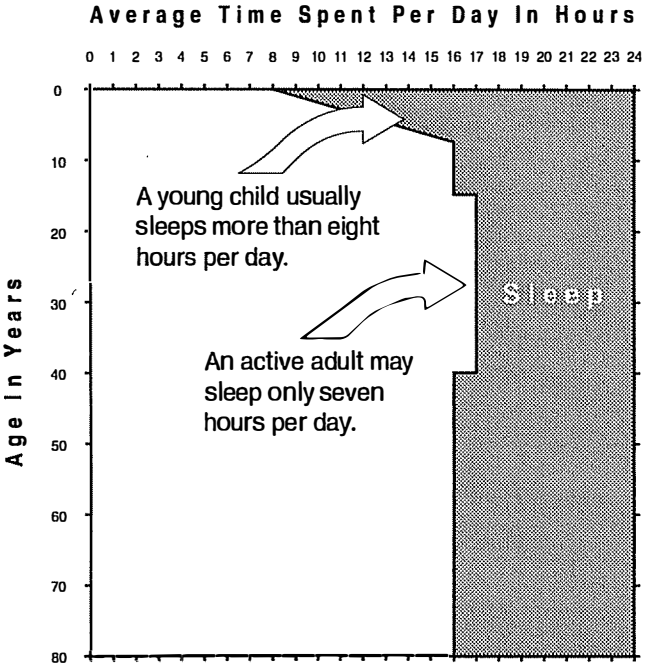


Figure 2—Time Spent on Sleep

Plotting An Activity That Occurs Every Day of the Year

Some activities occur virtually every day of the year - such as sleeping, eating and grooming. Consider sleeping as an example. If a person sleeps an average of 8 hours per day, each day of the year for his/her entire life, then a section representing sleep could be graphed (or carved out of the square) as a rectangle with dimensions 24 hours x 80 years. See Figure #2.

Students may want to reflect on this simple result. One third of this person's life (and of most healthy persons) is spent asleep! In addition, the basic rectangular shape may be modified after a more detailed consideration of sleep patterns. Babies and children often sleep more than 8 hours per day, while active teens and adults often sleep less than 8 hours per day.

Plotting An Activity That Does Not Occur Every Day of the Year

Some activities such as attending school and going to work represent major events in a person's life, but do not occur every day (For example, weekends and holidays may not be included.) Plotting an activity that does not occur every day requires a few more computations.

The general strategy is to estimate the total hours spent on an activity during a complete year then to divide by 365 to obtain a daily average on an annual basis. For example, consider the time a person spends attending elementary school from grade one to grade eight. Along the vertical axis, one can estimate that the normal ages corresponding to elementary school range from 6 years to 14 years. Along the horizontal axis, one has to estimate the total hours spent in school per year and then divide by 365. A typical school day extends from 9:00 A.M. to 3:30 P.M. including a one hour lunch break and two 15-minute recesses.

The net result is that a child actually spends only 5 hours per day in school. In Ontario there are 185 instructional days in a school year. Thus, in one year an elementary student spends and average of 5 x 185 / 365 = 2.5 hours per day attending school. The region, representing time

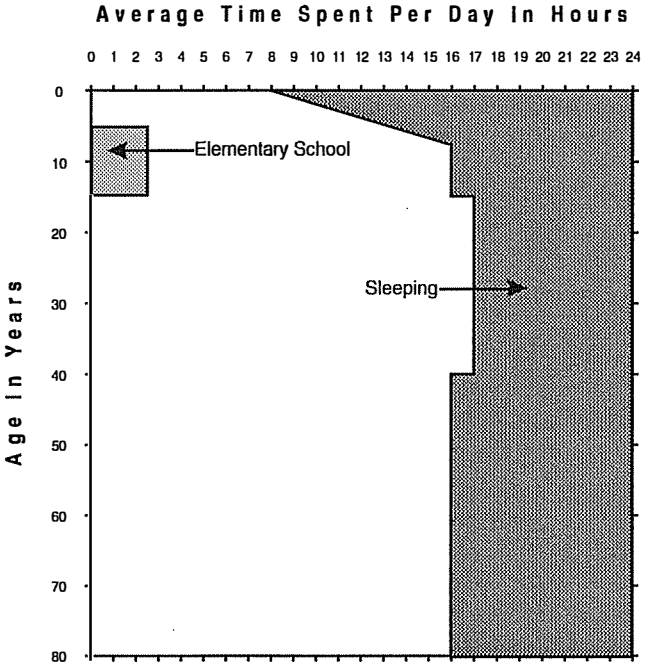


Figure 3—Time Spent At Elementary School

spent at elementary school, can now be plotted on the Graph of Life as a rectangle with dimensions 8 years x 2.5 hours. See Figure #3.

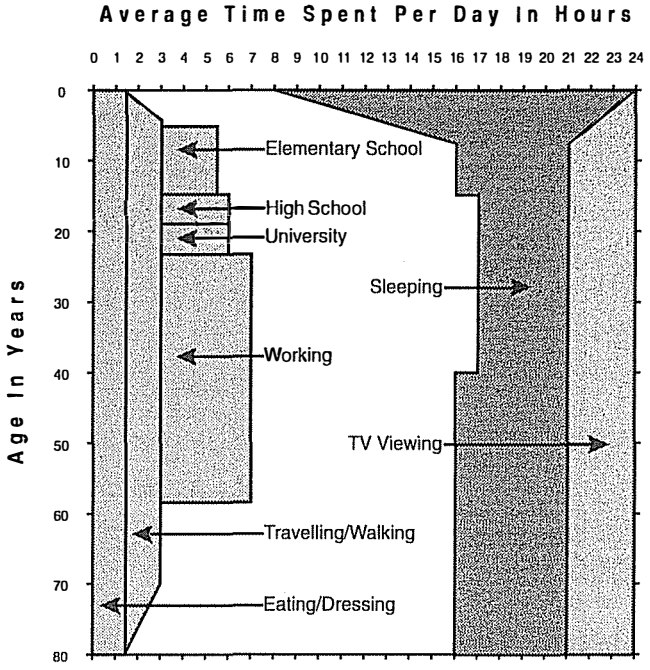


Figure 4—A Graph of Life

Additional Comments

In Figure #4, a Graph of Life with several more activities has been plotted to illustrate the life of a person who has gone to university, and then worked until retirement at age 60. The remaining blank area represents time that has not yet been accounted for.

As well as examining a predesigned graph, it can be instructive for each student to construct a graph that might apply to his/her own past and future. Students can also be asked to determine how they might plot any of a wide variety of life activities on the graph. For example, how could the time spent on child rearing be estimated and graphed? How could the time spent on a part-time job be graphed?

Students are usually surprised by the relatively small portions of the Graph of Life that are covered by school and by work. Student interest in the graph can be used to initiate discussions about the use of leisure time, the importance of maximizing learning opportunities in school and the importance of personal time management.

It can also be an informative exercise to ask students to investigate the effects produced by changing assumptions. For example, how would the Graph of Life change if a person were to reduce television viewing by 50%, or if a person were to retire 10 years earlier than planned?

Aftermath of Barbie

Shelly Beauchamp, Toronto

A talking doll has made "math anxiety" front page news. In October, The Globe and Mail reported that The Mattel Toy Company has been inundated with complaints from individuals and organizations, including The National Council of Teachers of Mathematics, objecting to Teen Talk Barbie's words "Math class is tough." In response to these complaints in Canada and The United States, Mattel has offered to exchange dolls which utter the offensive phrase and promised to delete the sentence from dolls made in the future. Mattel claims that it would be too difficult to recall dolls now on sale, because the phrase is only found in 1 out of every 4000 dolls*.

At the same time that this outcry was occurring, other people such as The Globe and Mail's science columnist were leaping to Barbie's defense. Math is hard the writer contended, so why shouldn't Barbie say so? Such comments I believe speak more of the writer's own dislike of mathematics than to those of the young girls who play with Barbie. In fact it is clear that pre-pubescent girls do better in math classes than their male counterparts. For the most part they do not find math to be tough. So the message delivered is a particularly pernicious one: that being a "teen" means not only having breasts and fancy clothes (one writer in The Globe and Mail pointed out that a real female with Barbie's figure would be in a body cast) but it also means hating math.

The Barbie episode has had some useful fallout. Hopefully it will lead Mattel to re-examine their methods of market research and to get in touch with the realities of young girls and their parents in the 90's. It also offers teachers an opportunity to discuss the issue of girls and math anxiety with their classes. Perhaps students could come up with some alternative phrases for Barbie and send them to Mattel. Maybe they could start with "How come Ken gets more airtime in math class?"

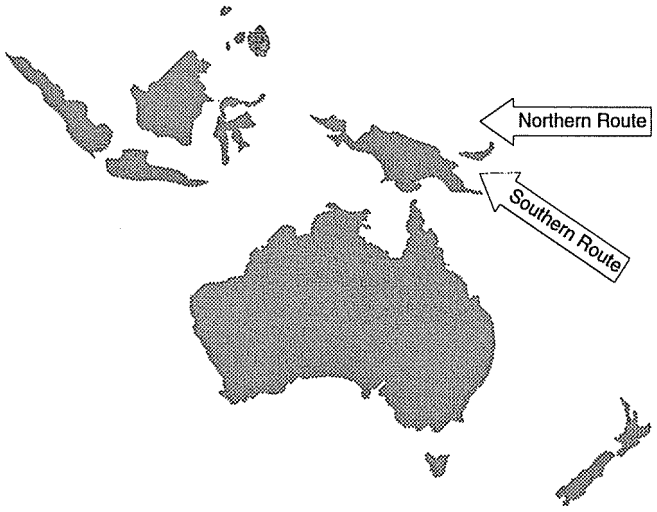
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* In the article Patricia Pape, Mattel's vice-president of marketing, is quoted as saying that there are 64 different looks to Teen Talk Barbie and 80 different voice chips saying 4 different things. She said they would have to search all the dolls and that the chance of getting it is one in 4000. Where does the 4000 come from?

GAME THEORY—THE MATHEMATICS OF COMPETITION

JEFF IRVINE, BRAMPTON CENTENNIAL SECONDARY SCHOOL, BRAMPTON ONTARIO

Game theory presents an interesting and entertaining area of mathematics which I have found to be a great "hook" for Finite Mathematics students. The mathematics can be kept fairly simple, while the problems can be formulated as "real world" situations. An added bonus is that even a simple game theory problem can contain elements from almost every major topic in the Finite OAC.



The Battle of the Bismark Sea

Payoff Matrix for Kenny—Measured in Days of Bombing

Allied Choice	Japanese Choice: North	Japanese Choice: South
North	2	2
South	1	3

Payoff Matrix for Imamura—Measured in Days of Bombing

Allied Choice	Japanese Choice: North	Japanese Choice: South
North	-2	-2
South	-1	-3

Figure 1

In February 1943 Japanese Admiral Imamura was ordered to transport troops across the Bismarck Sea to reinforce the Japanese occupation army in New Guinea. Imamura's task force could sail either north of the island of New Britain (the shorter route, but often with rain and bad weather) or south of New Britain (longer but better weather). General George Kenney, Commander of the Allied Air Force in the Pacific, learned of the Japanese plans, but did not know which route the Japanese would choose. Kenney's problem was how to deploy his airplanes. Should he concentrate his forces along the northern route or the southern route? If he chose the wrong route, he could recall his planes, but the number of days of bombing would be reduced. Kenney's "payoff matrix", measured in days of bombing, is shown in Figure 1. Imamura's payoffs are of course the negatives of Kenney's.

This is an example of a TWO PERSON ZERO SUM GAME (whatever Kenney wins Imamura loses). Neither opponent knows what the other will choose. However, we can assume that both opponents are rational—i.e., they will select a course of action which will give them the best possible results no matter what strategy the other adopts. Examine the payoffs above. Can you find the optimal strategy for the two Admirals? Remember, you cannot control what the other player will choose!

In the actual battle, both admirals chose the Northern route, and the Japanese fleet was subjected to two days of heavy bombing. Is this consistent with your analysis?

Game theory, the branch of mathematics which analyzes situation of completion and co-operation, is generally considered to begin with John von Neumann, a mathematician, and Oskar Morgenstern, an economist, who jointly published *The Theory of Games and Economic Behavior* in 1928. A GAME consists of two or more players who can select among various STRATEGIES, resulting in

PAYOFFS. It is assumed that the players are rational and are able to choose freely from their strategy sets. The simplest kinds of games to analyze are zero sum, where the winner's payoffs are equal to the loser's losses. Elementary game theory can involve logic, probability, matrices and linear programming. A survey of the literature in elementary game theory shows that similar kinds of problems occur in multiple references, often with only slight differences. This article is not intended as a textbook on game theory. But game theory is an interesting and somewhat refreshing application of many senior level topics in mathematics. Here are a few examples which I have used in OAC Finite Mathematics.

POLITICAL CAMPAIGN

There are two days left before the next Federal Election. Chretien and Mulroney have focused on two key SWING ridings, which generally foretell national trends. Neither will know the other's plans until it is too late to make changes to their respective schedules. Each has three available strategies:

Strategy 1—Spend one day in each Riding.

Strategy 2—Spend both days in Riding A.

Strategy 3—Spend both days in Riding B.

There are some advantages in not competing head to head in a riding. Estimates of votes gained or lost from the three strategies (measured in percent) are:

Payoff Matrix for Chretien

Chretien's Strategy	Mulroney's # 1	Mulroney's # 2	Mulroney's # 3
# 1	1	2	4
# 2	1	0	5
# 3	0	1	-1

Figure 2

THE BOX & CIRCLE TECHNIQUE

Many elementary games can be analyzed using the "box and circle" technique: Put a box around the MINIMUM value in each ROW. Now put a circle around the MAXIMUM value in each COLUMN (see Figure 3). If an element is both boxed and circled, it is called a saddle point, and is the rational choice for both opponents every time the game is played. Why? Because a saddle point is the "best of the worst". No matter what the opponent does, the player who chooses a saddle point is guaranteed to win at least this amount (or lose at worst this much). The saddle point is called the value of the game. It follows that a game is fair only if its value is zero. (No player can make a positive long run return if both players use optimal strategy.)

Saddle Points—the Best Choice for Both Opponents

Chretien's Strategy	Mulroney's # 1	Mulroney's # 2	Mulroney's # 3
# 1	[(1)]	(2)	4
# 2	(1)	[0]	(5)
# 3	0	1	[-1]

Figure 3

So both Chretien and Mulroney should follow Strategy #1 (one day in each riding) with a predicted gain of 1% for Chretien (1% loss for Mulroney).

HEADS & TAILS

Not all games have saddle points. Consider the following simple game: Rod and Connie each independently write "Heads" or "Tails" on slips of paper, and then compare the slips. Connie's payoffs are shown in Figure #4. Is this a fair game? Can either Connie or Rod formulate a strategy to maximize their expected returns if the game is played repeatedly?

Payoff Matrix for Connie

Connie Picks	Rod Picks: Heads	Rod Picks: Tails
Heads	- $4.50	$2.00
Tails	$2.00	- $0.50

Or, In Units of Fifty Cents

Connie Picks	Rod Picks: Heads	Rod Picks: Tails
Heads	- 9	4
Tails	4	- 1

Figure 4

There is no "pure strategy" (or saddle point) in the game. If one player consistently picks "Tails" the other player can profit from recognizing the first player's pattern of play. This game requires a MIXED STRATEGY: play a combination of both strategies, but without a pattern in your play!

THE MIXED STRATEGY SOLUTION

Suppose Connie plays Row 1 with probability x (where $0 \le x \le 1$) and Row 2 with probability $1 - x$.

			Rod Picks: Heads	Rod Picks: Tails
Connie Picks	x	Heads	−9	4
	$1-x$	Tails	4	−1

Figure 5

To find the optimal strategy for Connie, compute the EXPECTED VALUE for each possible set of strategies. (Recall that the expected value of a set of outcomes is the sum of the payoff for each outcome times the probability of the outcome occurring.)

The expected value for Connie if Rod plays Column 1 is:

$$E_{c1} = -9x + 4(1-x)$$

The expected value for Connie if Rod plays Column 2 is:

$$E_{c2} = 4x + (-1)(1-x)$$

Connie cannot control what Rod selects. Therefore, she can insure herself a minimum amount by selecting x such that $E_{c1} = E_{c2}$.

This will give Connie the same long run return no matter what Rod does.

$$-9x + 4(1-x) = 4x + (-1)(1-x)$$
$$-9x + 4 - 4x = 4x - 1 + x$$
$$-18x = -5$$
$$x = \frac{5}{18} \text{ (about 28\% of the time)}$$
$$1 - x = \frac{13}{18} \text{ (about 72\% of the time)}$$

The common value of E_{c1} and E_{c2} is shown below.

$$-9\left(\frac{5}{18}\right) + 4\left(\frac{13}{18}\right) = \frac{7}{18}$$

(converting back to dollars, about $0.19 per play)

Note that Connie cannot make her selections with any discernible pattern, since if she does, Rod (being rational) will identify the pattern and profit from it. Connie needs to make her selections in a random manner, with relative odds of 5 to 13. To do this she needs an external randomization device such as the one shown in Figure 6.

Randomization Device
for Connie

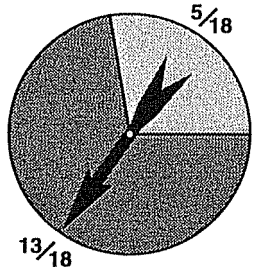


Figure 6

Games can be analyzed from a matrix orientation if desired. The following problem, taken from reference 9, illustrates some of the matrix approach.

WAR

Attacking bombers are attempting to bomb a military installation which is protected by fighters. Each night the bombers will attack HIGH or LOW. The lower bombing runs make the bombing more accurate. Similarly, each night the fighters can look for the bombers either HIGH or LOW. Credit the bombers with six points if they avoid the fighters, and zero if the fighters find them. Also credit the bombers with three extra points for accurate bombing if they fly low.

Payoff Matrix for Bombers

		Fighters: HIGH	Fighters: LOW
Bombers	HIGH	0	6
	LOW	9	3

Figure 7

Determine optimal strategies for the bombers and for the fighters. Also give explicit instructions to the bomber commander and the fighter commander so that by flipping coins they know what to do each night.

FOR BOMBERS: Let x be the probability that they fly HIGH. Then $(1-x)$ is the probability that they fly LOW.

Strategy vector for bombers is [HIGH,LOW] = $[x, 1-x]$

Expected values for bombers:

$E_{c1} = 0x + 9(1-x)$. $E_{c2} = 6x + 3(1-x)$.

Solving $E_{c1} = E_{c2}$ gives $x = \frac{1}{2}$.

Strategy vector for bombers is $B = \left[\frac{1}{2}, \frac{1}{2}\right]$.

The obvious extension of the graphical solution is to use linear programming to locate the minimum or maximum values.

Here is a problem from business (which could also be solved graphically) which illustrates briefly the linear programming approach.

THE GASOLINE PRICE WAR

Two gasoline stations in Georgetown (Petro and Metro) are waging a price war to increase market share. Metro has three strategies which it employs:

- ▶ Sell at the Regular Price (RP)
- ▶ Sell at a Cut Rate Price (CR)
- ▶ Sell Below Cost (BC)

Petro is constrained by its Head Office from using the Below Cost strategy. Expected differences in daily profits (Petro – Metro) are shown below (in hundreds of dollars):

Payoff Matrix for Petro

		Metro's Strategy RP	CR	BC
Petro's Strategy	RP	2	-2	1
	CR	-2	3	-1

Figure 10

Determine Petro's optimal counter strategy.

Suppose Petro charges Regular Price with probability x.

			Metro's Strategy RP	CR	BC
Petro's Strategy	x	RP	2	-2	1
	$1-x$	CR	-2	3	-1

Figure 11

Expected Gains for Petro are as follows:

If Metro charges Regular Price, $E \le 2x + (-2)(1-x)$ [1]
If Metro charges Cut Rate Price, $E \le (-2)x + 3(1-x)$ [2]
If Metro charges Below Cost Price, $E \le 1x + (-1)(1-x)$ [3]

When simplified, these become:

$E \le 4x - 2$ [1']
$E \le -5x + 3$ [2']
$E \le 2x - 1$ [3']

Arbitrarily select the third inequality and write as an equation using a slack variable s:

$E = 2x - s - 1$

Substitute for E in the other inequalities and simplify:

$2x + s - 1 \ge 0$ [1"]
$-7x + s + 4 \ge 0$ [2"]

LINEAR PROGRAMMING PROBLEM

Maximize $E = 2x - s - 1$, Subject to
$2x + s - 1 \ge 0$
$-7x + s + 4 \ge 0$
$0 \le x \le 1$
$s \ge 0$

Solve this problem using the simplex method (appropriate computer software is useful here).

Results are: For Maximum, $x = \frac{4}{7}$

Maximum value of E is $\frac{1}{7}$

(about $14.29 extra profit per day).

So Petro can actually GAIN business if Metro instigates a price war which involves Below Cost sales.

Note that a solution for METRO's optimal strategy would involve three variables, making a linear programming solution a necessity.

Game theory presents an interesting and entertaining area of mathematics which I have found to be a great "hook" for finite mathematics students. The mathematics can be kept fairly simple, while the problems can be formulated as "real world" situations. An added bonus is that even a simple game theory problem can contain elements from almost every major topic in the finite OAC. Game Theory can also be used as an independent study unit topic at the OAC level. Selected areas of the game theory can make excellent enrichment topics at other grade levels.

My favourite references on this topic are 2, 3, 5, and 6. For a treatment which emphasizes the matrix approach, see reference 3. For more on linear programming in game theory, see reference 6. Reference 14 is a fairly high level

text which examines non-zero sum games, co-operation strategies and a lot more. For a source of problems in game theory, see references 1 and 10. I would welcome comments and suggestions, as well as stories of your own experiences should you decide to include some game theory in your programme.

Appendix–Homework Games For You To Try

Instructions: Find a partner/opponent. Play each of the following simple games several times, and record your winnings (or losings). Can you find a winning strategy for each game?

THE INVISIBLE COIN FLIP

Select one of you to be Player A and the other to be Player B. Without showing your opponent, write H or T on a slip of paper. Compare the two papers. If one is an H and the other a T, there is no winner. If both are H's, Player B pays Player A four points. If both are T's Player A pays Player B four points. Repeat this game at least ten times.

A MATRIX GAME

Select a Player A and Player B as before. Simultaneously, Player A selects a ROW (r) of matrix M and Player B selects a COLUMN (c) of matrix M. Player B pays to Player A the number of points shown at entry m_{rc}. (If the entry is negative, Player A pays Player B).

$$M = \begin{bmatrix} 2 & -100 \\ 3 & 50 \end{bmatrix}$$

Repeat this game at least ten times.

FINGER FIGHT

Select Players A and B as before. On the count of three, each player reveals either ONE or TWO fingers. If the TOTAL is EVEN, Player B pays Player A that many points. If the TOTAL is ODD Player A pays Player B the points. Repeat this game at least ten times.

LOOK FOR WINNING STRATEGIES IN EACH GAME!

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CALL FOR AWARD NOMINATIONS

LIFE MEMBERSHIP

To qualify for Life Membership in OAME/AOEM a candidate shall have:

- 10 or more accumulated years of membership in OAME/AOEM
- demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular

Deadline for nominations: **January 31, 1993**

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award, to be presented annually to a person who has made a significant contribution to OAME/AOEM, but who does not necessarily meet the criteria for the Life Membership Award (i.e. not necessarily an educator or an OAME/AOEM member.) The award is aimed at recognizing people who have supported OAME/AOEM, such as publishers, educational suppliers, hotel personnel, etc.

At this time OAME/AOEM is requesting nominations for consideration for this year's award—to be presented at the Annual Conference.

Deadline for nominations: **January 31, 1993**

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former Secretary-Treasurer, OAME/AOEM has established the Morley MacGregor Award, to be presented annually to an elementary school teacher of mathematics who has demonstrated excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME/AOEM is requesting nominations for consideration for this year's award—to be presented at the Annual Conference.

Deadline for nominations: **March 1, 1993**

KENNETH D. FRYER AWARD

In Memory of the late Dr. K. D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, has established the Kenneth D. Fryer Award for Excellence, to be presented annually to a Secondary School Mathematics Department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME is requesting nominations for consideration by the committee for this year's award—to be presented at the Annual Conference.

Deadline for nominations: **March 1, 1993**

AWARD NOMINATION FORM

Date of Nomination:_____

Award (Check One) ☐ Life Membership ☐ Mona MacGregor Award ☐ Morley MacGregor Award ☐ Kenneth D. Fryer Award

I/WE nominate (Name of Nominee)

School, Organization, or Business of Nominee

School, Organization, or Business Address of Nominee

Principal of Nominee (for the Kenneth D. Fryer Award)

Mathematics Department Head of Nominee (for the Kenneth D. Fryer Award)

Nominated By (Name, Address & Phone)

Please attach a separate sheet to the completed nomination form, outlining the reasons for your nomination and send to: Don & Carol Attridge

Creative Problem Solving Experiences for Prospective Elementary School Teachers

Robert Svochik, University of Akron

Whether it be the ancient Greek mathematician Archimedes taking a bath to discover the amount of gold contained in a crown or, more recently, a group of Teenage Mutant Ninja Turtles marketing everything from mugs to T-shirts, creativity has paid dividends for a long time. In our information-based, global community, creative problem solving is, indeed, a valued skill.

The NCTM Standards (1989) contain many suggestions for school mathematics, especially the importance of teaching problem solving. Standard one for grades K-8, "Mathematics as Problem Solving" emphasizes that students should have numerous and varied experiences developing strategies, generalizing solutions, verifying and formulating problems. Similarly, the mathematical connections standard for grades 5-8 states the following:

> *"If students are to become mathematically powerful, they must be flexible enough to approach situations in a variety of ways and recognize the relationship among different points of view." (NCTM, 1989, p. 84)*

Children should have problem solving experiences that emphasize convergent and divergent thinking experiences.

In elementary mathematics methods classes, college students need to experiment and solve problems which require elements of creativity. It is assumed that experiences with creative problem solving will transfer to later teaching experiences with children. Thus, the examples in this paper have been developed and used with elementary education majors so that both a value and skill for creative problem solving may be explored.

Researchers such as Guilford (1962) and Torrance (1984) have indicated that creative ability is independent, after a certain point, from intelligence tests measuring convergent thinking. Thus, intelligence tests do not, in themselves, measure creative thinking. Convergent thinking involves linear, logical, and sequential processes. On the other hand, creativity seems to involve divergent thinking which, in turn, requires flexibility, uniqueness, fluency, as well as the ability to elaborate upon a response. Thus, a creative person when confronted with a problem, tends to emit a large number of responses, shifts categories easily, provides answers which are unique, and can defend and explain findings.

Examples where creativity plays an important role are abundant in business and industry. Listing of alternatives, combining them, seeking relationships, and an openness to irregularities are often observed. For example, Edison was working on telephone transmission and incandescent lamps at the same time. He was stumped on both tasks, having apparently exhausted alternatives. On his table was a mixture of tar and lampblack which he had been testing for his telephone transmitter. He absent-mindedly played with this material and rolled it into a string. Suddenly, an inspiration came to him. He felt that the carbon material filament could solve the problem of the electric bulb. Subsequent experimentation proved him right.

More recently, 3M Company was working on alternative formulas for an extremely strong adhesive. Several tests yielded weak adhesives that did not meet the quality control requirements. Someone at the lab discovered that this adhesive had a valuable use. The weak adhesive could be pressed on paper and easily removed. Thus, the "post it" stickers were discovered. The role of creative play, accidents, and even apparent failures, all contribute to the creative process of the aware person.

Suppose this question is posed: how many ways can you use a barometer to measure the height of a building? (Note that the barometer is incidental; we could have used a thermometer just as well.) Often, students who are first exposed to this problem tend to fixate on a "school" solution; that is, they try to remember what a barometer does. Some elementary education majors seem to be stuck at this level and since they cannot specifically recall

what a barometer does, they generate few alternatives. The element of prejudging results narrows the creative field and vastly reduces the creative flow.

Elementary mathematics students need to have "brainstorming" experiences, where they list several alternatives for problems without prejudging their results. These alternatives can be considered within a planning stage of problem solving. Experiences such as this can promote creative responses to problem situations.

Consider the following samples of solution possibilities uninhibited by judgmental constraints:

1. Use the barometer as a measuring device, then convert to common units.
2. Drop the barometer, time its fall, and plug the result into a formula.
3. Set up a similar triangle situation. Use proportional reasoning to find the height of the building.
4. Use the barometer as a weight attached to a piece of string. Measure the string length.
5. The barometer way. Since barometers measure air pressure, and air pressure varies inversely with height, we can look up a simple physics formula to find the height.
6. Why waste time? Management consultants earn big bucks by helping organizations save time. So, call the building architect or engineer and offer to trade a nice, new barometer for the building's height. After all, we live in an information-based society!

Let's look at another example of a problem requiring a unique approach. Suppose you have the following items: a curved drinking goblet or glass able to hold at least three or four golf balls, another glass able to hold at least one golf ball, and, of course, one golf ball. The glasses and ball are arranged as shown in Figure 1.

The problem is to use only the goblet, leaving its orientation downward (it may be lifted as long as the top points downward) to "pick up" the golf ball and place it in the other glass. The second glass, top pointed upward, cannot be moved.

Solving this problem requires a unique response. You could try lifting and sliding the goblet with the golf ball underneath only to find that you are stuck because the other glass cannot be moved. Focusing on the relationship between the curvature of the goblet and the ball might induce the idea of an orbit and now a solution is born.

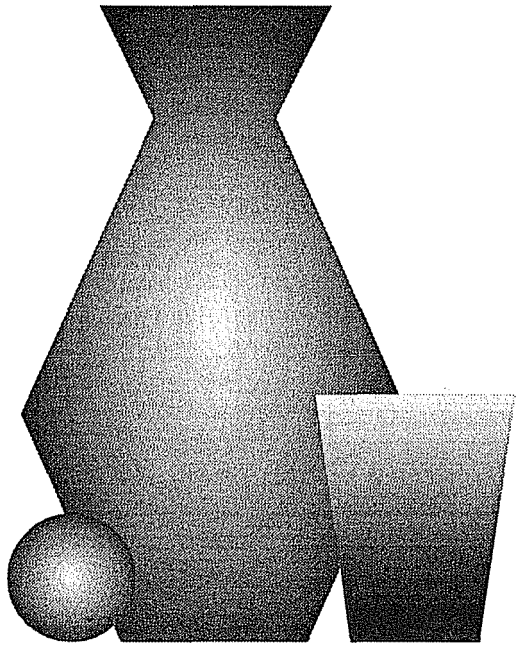


Figure 1

Lifting the goblet and placing it on the table over the ball, then rotating the goblet, causes the ball to bounce against the goblet and spin. The ball will stay in orbit as you lift the goblet over the other glass and drop the ball in. (By the way, this requires a little practice. However, with a little skill a creative solution can occur.)

Another nice problem which causes us to shift categories and to exhibit flexibility is the following: A man was found dead in the desert with a pack on his back. By asking questions which can be answered in a yes or no fashion, determine the cause of death. Here are some sample questions and answers.

Did the man have water in his pack? No.

Did the man starve to death? No.

Was the pack heavy? No.

Other students focus on the pack and think of it as a backpack. An erroneous assumption to be sure. In fact, the pack is a crucial bit of information. However, one must be flexible enough to shift categories and think of other kinds of packs. For example, suppose we think of the pack as a parachute pack. Aha, the man jumped out of an airplane and his parachute did not open! This problem involves raising a great number of questions (fluency) as well as flexibility.

Another problem involves a woman who takes four small, linked chain pieces to her jeweler. The four chains are shown in Figure 2.

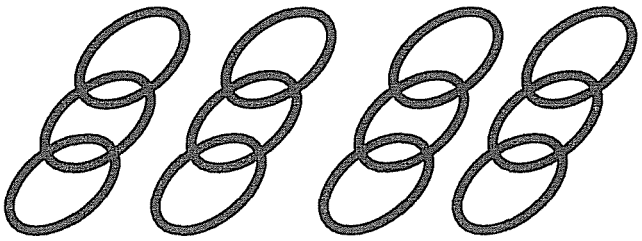


Figure 2

The lady wants the jeweler to make one big necklace out of these pieces. In order to reduce the cost, the lady wants to pay for only three links to be opened and closed in the connection process. How can the jeweler do this? Thinking of the pieces of chain as four segments would yield a solution by opening an end point then closing a connection. Repeating this three more times yields a completed necklace. But the request calls for only three openings and closings. A perceptual shift helps here. If we think of the chains as a set of individual links we can take one of the chains apart and obtain three open links. Further, viewing the remaining segments as a triangle shape rather than an immutable square or circular shape, we can close the three individual links as connectors near the vertex points of the triangle. This solution, shown in Figure 3, satisfies the conditions of the problem. By visualizing the problem in a unique way, a creative solution is found.

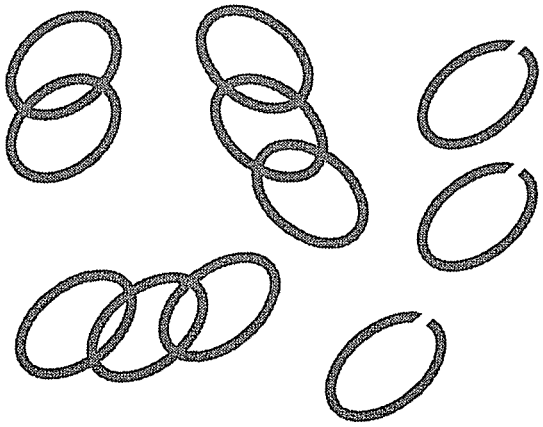


Figure 3

Although the problems presented here are suggested for elementary education majors, many of the same problems can be used with children in grades 5-8. A creativity corner of the mathematics classroom can contain several puzzles, problems, and humorous riddles. Some sample activities are presented below.

Sample Activities

1. How can you put your left hand completely in your right-hand pants pocket and your right hand completely in your left-hand pants pocket. Both at the same time? (You are wearing the pants.)
2. How many ways can you express 100 using the fundamental operations and the number four?
3. How many uses for the numeral 2715 can you find?
4. Visualize three playing cards adjacent to one another. A four is just to the right of a three. Another four is just to the left of a four. There is a diamond just to the left of a heart, and a diamond just to the right of a diamond. Can you name the three cards?
5. A man is racing for home as he is chased by a man in a uniform. As he approaches home, he sees a man wearing a mask and holding a dreaded object. He turns and runs back in the direction in which he came. What familiar event is occurring?
6. List all the different uses that you can think of for a ruler.

Selected Solutions

1. Wear them backwards
2. $\frac{444-44}{4}=100;\ 44+44+4+4+4$
3. House address; flower garden decorations; answer to a math problem; numerical jungle gym; etc.
4. 3 of diamonds; 4 of diamonds; 4 of hearts
5. Baseball game rundown
6. Hammer; measuring device; drawing device; cricket stick; garden marker, etc.

Editor's Note: Watch for Rheta Rubenstein's article, "Problem Solving In Mathematics—A Fresh Perspective for Future Teachers", in the next volume of the Gazette, for a practical application of Professor Svochik's theme.

A Proposal for OAC Calculus

Peter D. Taylor, Department of Mathematics and Statistics, Queen's University

I'm not happy with the OAC Mathematics courses, especially Calculus and Algebra and Geometry. I don't think they are interesting enough to really engage many of our students, I don't think they allow the teachers enough scope and flexibility and freedom to be real professionals, and finally I don't think they prepare the students particularly well for the study of math and science at university. First I'll give a few details about these concerns, and then I'll present some examples of the sort of problems I think might work well in OAC Calculus.

Problems with the OAC Math Curriculum

THE CURRICULUM IS NOT SUFFICIENTLY ACTIVITY-ORIENTED.

Math is something you do, not a body of facts and techniques to learn. Of course, the facts and techniques are important, but only if they can be used to support real activity. I don't think our students have enough experience of mathematics as an elusive prize, to be tracked down and tamed. An OAC Math class should be full of arguments and discussion, students coming to the board to present their proposals for the answer to the current question, students huddled in small groups plotting new strategies, etc. Not all problems engage the students in this way; not all curricula support such activity very well. It is the duty of the curriculum designer and the text-book author to foster where possible this style of learning.

THE CURRICULUM IS NOT SUFFICIENTLY RELATED TO THE LIFE OF THE STUDENT.

This is not the same as saying that there should be more real-world applications; a lot of what is called the real world is not of great interest to most of my students. Nevertheless, the comings and goings of their everyday lives can be a rich source of mathematical problems. They also respond to beauty and elegance, and to paradox, contradiction, and the unexpected—and to pictures...

THERE SHOULD BE MORE PICTURES.

Students need much more practice in working with graphs and pictures, knowing how to use them to guide their thinking and improve their understanding. Of particular importance is the ability to move back and forth between the pictures and the symbols, between geometry and algebra. This is an essential skill for the graduate students I work with in biology and economics. The functions they encounter cannot be derived from the simple elegant laws of physics (which have shaped so much of our modern mathematical curriculum); instead they must base their analysis on more general properties such as the shape and concavity of the graph. A good example is the optimal foraging problem (#6, below). They often find such problems unexpectedly difficult; somehow their early mathematics courses have not served them well. The curriculum is too closely specified, too narrowly technical.

Have you ever looked at the OAC English guidelines? You should—you'll be impressed. They give a number of general ideas about what's to be accomplished in the technical and intellectual growth of the student, something about the place of discussion and independent work, the importance of diversity and balance in the assigned texts, and the importance of oral and written communication, but not so much by way of detail. They give you the impression that the teacher is assumed to be a person of some integrity and wisdom, capable of independent and imaginative thought. On the other hand, the OAC Calculus guidelines could hardly be more different. They are excessively constrained, not only in content, but even in order and timing. They are so plugged with technical skills, that it's difficult to find time to explore a set of ideas with flexibility and grace. It is in fact not so important to the university that its students enter with a uniform set of technical skills.

My First-year Calculus Course at Queen's.

This year David Pollack and I, each in our own course, are using some problems of the type that I present below. We actually have two pedagogical aims in the course. One is focused on classroom dynamics, and the other is designed to encourage the students to take more responsibility for their own learning.

CLASSROOM DYNAMICS

We choose problems which provoke discussion and debate. At intervals the class moves into small groups (of four) to try to pin down the analysis or find the right construction; my tutor and I wander around and ask questions. After that we reconvene as a class and the students wield the ruler and the chalk. There are important skills of cooperation and communication developed here, but it takes time. We can't afford to be in a hurry.

INDEPENDENT LEARNING

I spend no class time on the technical skills of calculus; rather the course is structured so that the students struggle with these on their own or in their working groups, using worked examples and problems, with the possibility of tutorial help two nights a week. When they think they have mastered a certain unit (e.g., the derivative rules) they take a module test which they will only pass if they get 8 or 9 right out of 10. They can take the test as many times as they wish without penalty. We hope this provides a non-intimidating way to obtain the important technical mastery that the course requires. This approach fosters an important mode of learning, and it's also how I manage to make the time to have discussion and exploration in class.

My class (Math 121) has already had OAC Calculus (though they are still a very heterogeneous bunch) and David's (Math 122) has not, and for that reason he meets them for an extra hour a week. So far, the problems are working well for both of us, though we are finding lots of ways in which the students need better preparation in their basic geometric/algebraic skills. The students, on their part, find the problems demanding and often frustrating, but they seem to be enjoying the process and the classes are certainly lively. David has asserted that if the students coming out of OAC knew nothing more than how to solve these problems, they'd be well prepared for university calculus.

Outline of a Proposed OAC Calculus Curriculum

There are five sections. In fact, I'd actually be happy with less than this; it's important not to feel rushed—we need time to play.

1. **Geometric and algebraic analysis with function graphs.** Draws on Grade 12 skills, and motivates concepts of rate of change, slope, and concavity. The problems at the end are examples of this. Good classroom stuff.
2. **The derivative.** Calculation of the first and second derivatives of polynomial, rational, and power functions. Simple applications to rate of change. Should lend itself well to independent self-paced study. (Students need more experience with this.)
3. **The geometry of function graphs.** A collection of geometric problems about intersections of curves and lines, solved with the old algebraic and our new analytical tools. Again this is nice stuff for the classroom.
4. **Exponential, logarithmic, and trigonometric functions.** Most emphasis here on the exponential function—increasingly important for understanding the processes of growth and decay in the biotic world. Important probabilistic applications too. Much of this is also suitable for an independent self-paced approach.
5. **Mathematical modeling.** A return to the problems of chapter 1, with our new analytical tools. Take simple real-life situations, formulate optimization problems, and solve them.

By way of comparison, the current guidelines have four parts: Limits and Derivatives (38 h), Applications of Derivatives (28 h), Anti-Differentiation (19 h), and an allowance for extensions and options (25 h) for a total of 110 hours. The first two parts make up 60% of the course, and are essentially covered in the middle three sections. By omitting anti differentiation (which I am happy enough to do), we are left with 40% of the course for sections 1 and 5.

Finally a word about OAC Algebra and Geometry. I would really like to see an activity-oriented curriculum here as well, but because of the nature of the subject, this seems to be more difficult to provide. Doug Dillon (Athens DHS) wrote his Ph.D. Thesis with me a couple of years ago on this question, and made some progress; many others

have also thought about this. There's more work to be done.

EXAMPLES

I end with a few examples of section 1 problems. Remember that these problems are to be solved by geometric constructions (e.g., using a ruler to draw a line parallel to a given line, or to draw a line from a point tangent to a given curve). I should also say that a lot of class time is spent constructing and talking about the graphs: why do they have the shape they have? You may find these problems difficult at first; that's expected. They'd make a great focus for those PA days!

1. **Minimum cooking time.** We have a large ham at room temperature (20°C) which we want to heat to 80°C as quickly as possible. We have at our disposal a conventional oven, maintained at a constant temperature of 160°C, and a microwave oven. By coincidence, it turns out that the cooking time is the same for both ovens and is exactly 2 hours.

   The temperature-time graph for each of the two ovens is shown below. Explain why they have different shapes.

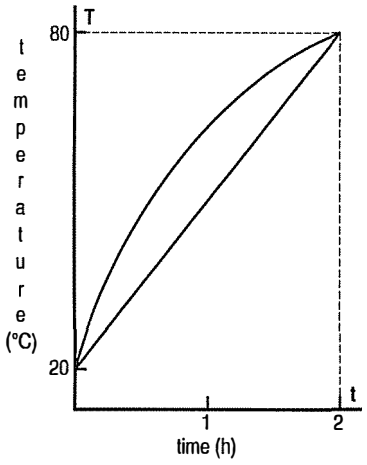


Figure 1

   Show geometrically that by switching from one oven to the other at an appropriate time, we can actually shorten the 2-hour cooking time. On your graph, show clearly how the optimal switch over point is obtained, and how the resulting minimum cooking time is obtained. Draw the resulting $T - t$ graph.

2. **Lining up.** At 8 a.m. the physical education centre opens and people start to line up to book squash courts for the following day. At 8:10 the sheets come out, and the line-up starts to move at the rate of 6 people per minute, that is, it takes each player 10 seconds to sign up for a court.

   Below is a graph of the total number n of people who have arrived to sign up for a court by t minutes past 8. For example, the height of the graph at $t = 30$ is $n = 200$. That means that the 200th person arrives at 8:30.

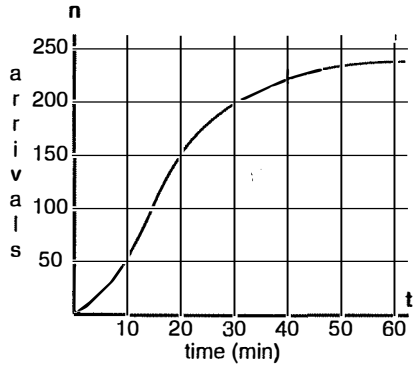


Figure 2

   Show how to interpret the following on the graph: (a) the length of the line at 8:30; (b) the time at which the line-up disappears; (c) the time at which the line is the longest; (d) the total time the 120th person to arrive has to stand in line; (e) the arrival time of the person who stands in line the longest. There are 120 "prime-time" courts and these go to the first 120 people to arrive. Suppose you want to get one of these, but spend the least possible time in the line. At what time should you arrive?

3. **Optimal advertising.** You have a fund-raising concert at the school and you want to know how much to spend on advertising over the local radio station. The more you advertise, the more people you can expect to get out to the concert (up to a point) and the more money you'll make in ticket sales, but also, the higher will be your advertising costs. What should you do to maximize your profit?

   To answer this you certainly need to know how ticket sales will depend on the amount of advertising. Figure 3 shows the graph of expected total ticket sales T against number x of advertising spots. For example, the graph shows that with no advertising, you can expect to sell 100 tickets and with 40 spots you can expect 400.

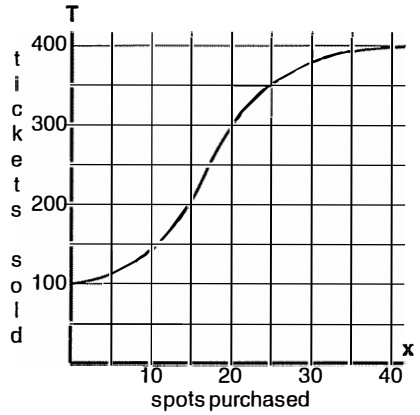


Figure 3

(a) Suppose tickets to the concert sell for \$5 apiece, and advertising costs \$25 per spot. On the same set of axes, plot graphs of your total expected revenue R from ticket sales, and total advertising cost C against number x of advertising spots purchased.

(b) Show geometrically how to find the number x of advertising spots which maximizes your expected profit P, defined as the difference between revenue R and advertising cost C.

(c) Now solve (b) with the same ticket price, but assuming an advertising cost of \$60 per spot.

4. **Optimal cookie size.** You have inherited from your great grandfather a wonderful recipe for oat-bran cookies and you decide to try to make a summer job out of it. Each night you prepare 6 kg of batter, cut it up into cookies, and bake them, and the following morning you take them round to the local juice bars. Your problem is to decide how big to make the cookies.

   The smaller you make them, the more cookies you'll be able to get from your 6 kg of batter, but the lower will be your revenue per cookie.

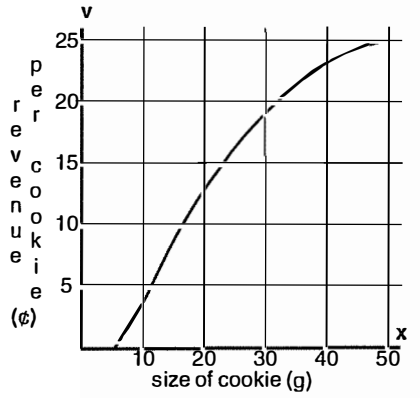


Figure 4

In fact, here is a graph of your revenue v (cents) per cookie against its size x (g). No one is interested in a cookie that is smaller than 5 g, and cookies that are larger than 50 g are unwieldy.*

You want to choose the cookie size which will maximize your daily profit. Your only costs are the ingredients which you have calculated to be \$2 per kg of batter.

(a) Assuming you have no trouble selling all your cookies, find, on the above graph, your optimal cookie size.

(b) Now suppose there is an additional packaging cost. Health regulations require the cookies to be individually wrapped in "self-seal" plastic pouches, and these cost you 5 cents each regardless of the size of the cookie. Does this change the optimum cookie size?

5. **Optimal driving speed.** A friend has offered to drive your car to Vancouver so it will be waiting for you when you get off the plane. She is in no hurry, so is happy to drive at any speed you want, provided that you pay for the gas.

   Below is the graph of the gas consumption z (L/h) of the car against the speed v (km/h) at which it is driven. Explain why the graph has this form. (Gears are a complicating factor here. The graph is drawn assuming the top gear is used.)

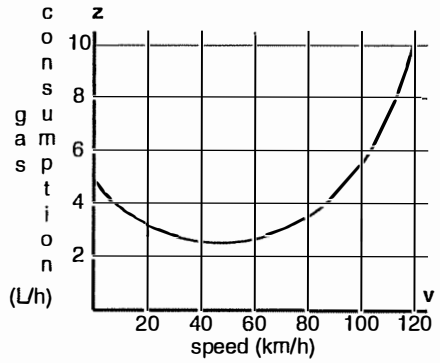


Figure 5

(a) How fast should she drive to use the least gas for the trip?

(b) Now suppose that, in addition to paying for the gas, you have also agreed to pay her a wage of \$3 per hour of driving. If the price of gas is \$0.50 per litre, what is the driving speed which will minimize your total cost?

Editor's Note: ...and possibly unhealthy?

6. **Optimal foraging time.** A bird spends its time foraging among patches of berries. When it finds a patch, it stays there until the patch is fairly well depleted, and then it goes off in search of a new patch. What the bird wants to know is how well it should "pick the patch over." If it stays too long in a patch, it will be spending valuable foraging time looking for the hidden berries that remain, but, on the other hand, when it leaves the patch, it has to spend some time finding another.

The following model of the situation is quite simplistic, but it captures the essence of the bird's problem, and is, for that reason, the basic foraging model found in the ecological literature. What we need to know to solve the bird's problem is the amount of food energy it gets from a patch in terms of the time it spends in that patch.

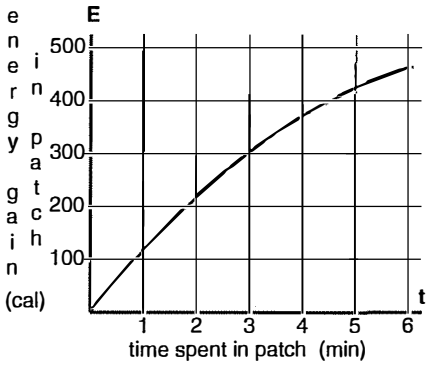


Figure 6—Graph of energy E (cal) acquired from a patch against the time t (minutes) spent foraging in the patch.

For small t the graph is nearly linear, suggesting a constant rate of intake of food. But as t increases, the patch becomes depleted, food items are harder to find, and the rate of food intake decreases. Thus the graph is concave-down.

Another thing we need to know is the amount of time and energy required to find a new patch once the bird decides to leave. Let's suppose that it takes (on average) 2 minutes to locate each new patch. Suppose that the amount of energy expended in this search is (a) negligible (b) 60 calories (mainly in flight). In each case, how long should the bird spend in a single patch to maximize its overall intake of energy over a day of foraging?

Acknowledgments

I am grateful to George Gadanidis, Hugh Allen, Bill Higginson, Geoff Roulet, and two referees, for their encouragement and many valuable comments.

Exam Review—On Video!

Mike Zichowski,
York Mills CI,
Toronto

Math video reviewing is an innovative concept that has been effectively used at York Mills Collegiate Institute to reduce math exam anxiety. There exists a general consensus amongst math educators that a well-prepared math student who has reviewed previous examinations and tests will significantly reduce the level of stress associated with upcoming exams and consequently perform well. This belief has resulted in my math department making available to all students (grades 11 to O.A.C.) a booklet containing previous examinations so that students may familiarize themselves with the various types of questions that have appeared in the past. This package is supplemented by a math video where students of a particular grade level show the viewer methods of solving the problems that appear in the review exam package. Students instruct the viewer (their peers) on methods of approaching and solving problems from previous exams by means of written solutions on a blackboard and in front of two video cameras.

The result is a two hour video filled with math solutions, delivered by students, for students in need of review assistance for the upcoming examination. The math review video is made available on an overnight loan basis for students to use at home with their VCRs, pausing and rewinding sections that need review or reinforcement. Consequently, they alleviate exam anxiety.

Other department members have successfully used the video for classroom instruction purposes. Instead of the teacher solving the problem on the board, the video is played and the video solution is discussed and analysed by the teacher and students for its effectiveness.

Math review videos have now been produced for our grade 11 and 12 advanced mathematics courses and Finite Math. Each year the previous video production is expanded by the addition of new solutions to the most recent exam. This personal form of review has replaced much of the panic associated with math exams and made our role as teachers much more satisfying.

COUNCIL COMMENT

BOB ROBINSON, GOLDEN SECTION CHAPTER REPRESENTATIVE

These notes were taken at the OAME Council Meeting held October 31, 1992, at the Ramada Inn-Airport West in Mississauga. Send your comments to the editor.

MATHEMATICS EQUITY

We are looking forward to a successful February Mathematics Equity Conference to be presented on February 26, 27 and 28, 1993. There will be three strands dealing with 3 issues od Gender, Special Needs Students and ESL/ESD Students.

PUBLICATIONS

The Council is very pleased with the format and presentation of the September Gazette. There is some concern about the lack of elementary content. This will improve over future issues. The editor is anxious to receive any suggestions regarding how our publication can be made better. Plans are in progress to publish a "transitions" theme edition for the June Gazette. (See page 11.) Some Grade 9 topics suggested thus far are: a) What your experiences? What is working? What is not working? b) Content: Suggestions for content including actual lesson plans. c) Assessment and Evaluation. d) How to handle multi-talented groups of students.

Yours truly has been appointed advertising manager to seek out advertisers for upcoming issues of the Gazette. For information regarding ads for the Gazette please contact Bob Robinson, Advertising Manager, Ontario Mathematics Gazette, Box 63062, University Plaza, Dundas, ON L9H 6Y3, Phone: (416) 628-5563, Fax (416) 628-9250.

FOCUS ON RENEWAL

a) Benchmarks This has been a very rushed and the Council has concerns about this rushed process. When reviewing the Benchmarks, teachers should address whether the materials are suitable and how they would be used in the classroom. Questions which need to be addressed are: 1) How will the teachers use the benchmarks? 2) What are the links (connections) between the benchmarks and the common core curriculum being developed by the ministry? Judy Crompton, Lorna Morrow and Ralph Connelly will coordinate OAME's response to the ministry.

b) Common Curriculum The first rough draft of this ministry document apparently: 1) deals with "Perspectives on Learning, Teaching, and the Curriculum;" 2) outlines 10 elements of the common curriculum; 3) describes the components in terms of "Language Arts, Self and Society, and Mathematics, Science, and Technology."

c) Focus on Renewal An OAME/OMCA team worked for two days on a revision for the document. The revision is an ongoing and cooperative venture of these two associations.

CONFERENCES

LEADERSHIP '92

This year's "leadership" was a sellout with a waiting list. A handbook to help future organizers has been created.

OAME '92 ANNUAL

Dick Baron, Finance Chair, presented the final report of the very successful conference hosted by Y4MA. Registration (1250) far exceeded expectations. "Perspectives" was a most fitting description for a conference that brought together such a variety of topics and talented speakers. Jan Scully and Bill Nimigon, co-chairs of the conference, are to be congratulated for putting together such a fine conference on such short notice.

OAME '93 ANNUAL

All committees have been meeting regularly to plan for the Windsor Annual. Preparation is progressing very well. For more about hotel accommodations, registration, and program contact: Todd Romiens, Windsor Board of Education, 451 Port St. W., Windsor, ON N9A 6K1.

Editor's Note: "Council Comments" will get more room next issue. Thanks for your understanding, Bob.

SEPTEMBER 93, YANG OR YIN?

GARY FLEWELLING,
MATH CONSULTANT,
WELLINGTON COUNTY
BOARD OF EDUCATION

Reform in the Transition Years is to begin this coming September. Speaking to this reform is Ministry of Education Policy/ Program Memorandum No. 115A, dated November 2, 1992. It reads in part, "When Grade 9 programs are organized in a manner that integrates the traditional subject areas ...". It is statements such as this one that have motivated me to put in writing some of my hopes and fears for Mathematics Education (and education in general) in Ontario.

September 1993 can be the date when Math Education really takes off or the date when it loses an engine and begins a fatal nose-dive just beyond the runway of educational reform.

September 1993 can be the date when confidence begins to be restored in the minds of parents and business people or it can be the date when the stage begins to be set for a swing back to the basics, the basics of the 1950's.

September 1993 can mark the beginning of a more interesting, relevant, and student-centred math program or it can mark the date when an incomplete and incoherent math program begins.

September 1993 can be the date when the math program becomes a richer experience for the student and when the door to the math class opens wider to let in a world of contexts or it can be the date when the math door closes, math ceases to exist as a subject and the cause of numeracy takes a severe beating.

September 1993 can mark the ushering in of the Yang, the bright and positive, or the Yin, the dark and negative.

I believe that the two differing sets of outcomes are, in large measure, consequences of two differing views of integration.

It is my experience that the best 10%-20% of our teachers recognize the importance of integration and are teaching mathematics in an integrated and holistic fashion now. Many of these teachers are found in our K-6 classrooms. They plan math units that integrate from within and without the math curriculum. They plan in cross-curricular elements, such as, communications, critical thinking and problem-solving. They take into consideration the individual differences of their students. Though the total time allotted to math instruction is fixed, the amount of time they allot to it daily varies depending on the nature of the student activity. They integrate the use of calculators and computers. They often plan into their units elements of science, technology, history, art, music, the environment, and personal health and well-being. They usually connect mathematics to interesting, relevant or significant contexts. Often they embed activities within a theme and often they don't. Often the math lesson is also a science lesson and often it is not. Often the math class looks like a Science lab, often it looks like a Language Arts class and often it looks like neither. Frequently the activities have an investigative flavour and frequently they don't. Their students receive a rich, coherent and balanced math experience. Their students are motivated, knowledgeable and skilled. Their students would measure up very well against Ministry benchmarks. The parents of their students are delighted.

The Ministry does not appear to be promoting this brand of integration. I am deeply concerned that the Ministry of Education and a number of other educators are, inexplicably, putting a different and very dangerous spin on the meaning of integration. The Ministry is putting forward a vision in which Mathematics, Science and Technology go into a melting pot and disappear as individual subjects. They see all the concepts and skills associated with this area being taught in a thematic holistic fashion. Where more than one teacher is involved with the teaching of this area, the teachers involved would participate in a significant amount of co-planning.

What makes the Ministry believe their brand of integration will work? Which Transition Years pilot projects can they point to for evidence to support their proposed brand of integration? Where is the research to support their approach? My curriculum experience tells me that Science and Technology, at the Grades 7-9 level, are insufficiently rich to act as contextual vehicles for a complete and balanced math program. (Art and Language offer at least as many integration opportunities.) Can a teacher deliver Math/Science/Technology through themes? My observations lead me to conclude that themes can be used to deliver only part of a math program. And, where is the evidence that suggests that teachers can plan and work well together on a day-to-day, unit-to-unit and theme- to-theme basis? Our experiences with open-concept classrooms and team-teaching suggest that there are significant (insurmountable?) obstacles here.

Our best teachers are teaching an effective integrated program now. Our focus over the next three years, the implementation period being given to the Transition Years, should be on doubling, tripling or quadrupling the number of these teachers.

I believe our best teachers can implement the Ministry's version of an integrated program or they can deliver a quality mathematics program. They cannot do both. If they can't do both, which one do you think parents will want?

Fortunately, because of many constraints, likely over 90% of schools will be forced to organize themselves for September 1993 in such a way that Mathematics will continue to exist as a subject. The various forms of integration can then be tested and refined over a three year implementation period by real teachers working in real classrooms filled with real students. By September 1996 we should know which forms of integration hold promise. We can use Ministry benchmarks to help us decide.

Review Of Peter Taylor's "Introduction To The Analysis Of Functions"

Garth Fraser & Karen Schindler,
Faculty of Education, UWO

What is "mathematics in the real world"? Peter Taylor's new *calculus* textbook (don't let the title fool you) is the best effort we have seen to link "real-life" problems and mathematics in a textbook. The text is unique in content and style. Emphasis is placed on applications in the natural and social sciences, and graphical function analysis. The problems lead the student to the mathematical theory, while the reverse is true in most other textbooks. Taylor calls this problem-solving approach a "process based curriculum," where the process of *doing* mathematics is just as important than the *content* of mathematics.

Taylor's text is written primarily to fit the first two semesters of a college calculus course, but also with the hope of inspiring a revision of the Ontario High School curriculum. Much of the content parallels the OAC Calculus course as described in the Ministry of Education Curriculum Guideline. However, some topics from the Guideline, such as differential equations, are absent.

Taylor's style is conversational and expository. The formal treatment and technical jargon usually found in mathematical texts is absent. While Taylor avoids recipe algorithms, he does provide extra comments on how to attack difficult problems. Paragraphs of explanation are the norm and not the exception.

Taylor brings much of his personality and experiences into the examples and problems used. The overall effect is to make the book very similar to an individualized student study guide. As Taylor notes:

> *Often I make comments which look like they might better belong in a teacher's manual, but I have felt that students might often find such comments interesting...*
>
> *I prefer not to spend time on a formal proof ...Most students are completely happy with an intuitive feel that the rules are what they ought to be.*
>
> *It's always a nice challenge to see how much information you get with as little calculation as possible.*

Taylor's comments also serve to encourage the student, providing a hint of what can be achieved from mathematical investigations.

> *That's what doing math is all about ...the power that you get from these examples comes from the process of designing and building a coherent structure in which they can be understood.*

Taylor's purpose is clearly to teach for comprehension. This approach does take more time. For high school students not accustomed to solving problems, the exercises will sometimes be very challenging. Students who are used to depending on a formula or a recipe algorithm will not have such aids to fall back on in the text. However, the assumption is that if the mathematics underlying the calculus is understood, retention will be stronger.

The first chapter, "The World: A Graphical Tour" presents a collection of optimization problems drawn from a variety of everyday activities, from "Getting a Squash Court" to "Optimal Cookie Size". As Taylor explains, these problems have the student start with the world, and extract the mathematics from it. Students and teachers alike will find these problems challenging for this very reason. The problems focus on graphical solutions, using the shape of the graph, upper and lower bounds, and maximums and minimums to analyze mathematical problems, without using derivatives. One of the example problems, with the discussion that it would generate, could easily fill a 70 minute class period.

Chapter 2 introduces derivatives, using concepts similar to those found in most calculus text books. However, Taylor typically leads the reader to identify a pattern through investigation, as opposed to presenting the rule first, and justifying the rule through examples. Taylor acknowledges that mathematical rules cannot be avoided; however, they are presented in a positive way:

In the subsequent sections, we will discover further rules. They are to be welcomed: they will permit the rapid, automatic calculation of derivatives, and free us from the difference quotient approach.

With the presentation of the sum, product and quotient rules of differentiation, the author throws an intriguing twist at the reader with the following challenge: "Convince me that the rules are what we expect". He encourages the student to set aside mathematical proofs, and use something intuitive and that's easy to think about (as illustrated by his own "Hot Milk and Coffee Proof of the Sum Rule", and the "Manchurian Coffee Bean Proof of the Product Rule"). Such intuitive proofs are important for students' comprehension and retention of the material, and students should respond favorably to this approach. However, intuitive proofs should not replace formal proofs, which the text almost completely avoids. Formal proof is an important method of demonstrating the consistency of mathematics to which students at least need to be introduced. Still, any formal proof should occur *after* an intuitive approach such as Taylor's.

The chain rule of differentiation provides another specific example of the difference between Taylor's strategy, and that found in the standard calculus textbook. Instead of introducing the chain rule by looking at an arbitrary function, such as $F(x) = (2x^2+3)^{1/2}$, Taylor begins by identifying a need for the chain rule:

> *We grow magic puffballs in the back garden. Their radius r in centimetres is given by the formula $r = 9t/(2+t)$, where t is the time in hours since they have winked into being. But it is their mass that is of the greatest commercial interest, since that is what determines their value. The mass in grams of a puffball of radius r is $m = 2r^3$. What is the growth rate of a puffball in grams per hour at time $t=1$?*

A solution is found through discussion, the author summarizes the mathematical idea that was utilized, and *then* the procedure is given a name.

Chapter 3, which corresponds to chapters on curve sketching in other texts, has again a somewhat backwards approach. The author typically starts an investigation with a picture or geometric observation, and then moves to the equation or algebraic formulation. As Taylor suggests, "this approach has more of a detective flavour to it." The author has high expectations of the students here; the graphing problems are challenging, and stress is placed on the understanding of shapes and patterns. The mathematics student who has a background limited primarily to numerical manipulations may have trouble with some of the material that is presented. However, with practice, this approach should help students develop the skills necessary to formulate a perceived geometric pattern.

The fourth chapter examines the calculus of exponents and logarithms in a manner that emphasizes analytic geometry as opposed to algebra. The number e^x is introduced as the function where the slope of the tangent of the function is equal to the height of the function. A great deal of emphasis is placed on growth and decay problems:

> *I have a large ham at room temperature (20°C) which I want to heat to 80°C as quickly as possible. I have at my disposal a conventional oven maintained at 160°C and a microwave oven. Each by itself will do the job in 2 hours. But I figure that by starting with the conventional oven and then switching over to the microwave, I can get the job done faster. In order to minimize the total cooking time, after how long should I switch ovens, and what is the resulting total cooking time?*

The fifth chapter deals with optimization, again using a graphical approach. Sections are titled the Optimal Selling Price, Optimal Insulation, Optimal Driving Speed, The Effectiveness of Advertising, and Quality Control. Taylor's emphasis on optimization is greater than most other calculus textbooks.

In Chapter 6, the author introduces integration from a graphical perspective, while other texts deal with integration after antiderivatives and differential equations. In Chapter 7, calculus applications to probability are considered. This section is unique to Taylor's text and particularly examines the relationship between probability and areas of regions on a graph.

This text is clearly a departure from the conventional approach to a mathematics textbook. It serves to make mathematics more relevant and more interesting by engaging in situation-based problems of mathematics from a graphical function analysis perspective. It is worthwhile reading for any calculus teacher who is interested in providing a comprehension-based mathematical education.

Taylor, Peter , **An Introduction to the Analysis of Functions**, Wall & Emerson, Toronto, 1992, $29.50. To order a copy, contact Wall & Emerson at 6 O'Connor Dr, Toronto, M4K 2K1, (416) 467-8685 or Fax (416) 696- 2460.)

Editos's Note: See Peter Taylor's Article "A Proposal for OAC Calculus" on page 26.

MATHEMATICS & THE WORLD AROUND US

RON LANCASTER, ST. MILDRED'S-LIGHTBOURN SCHOOL, OAKVILLE

Readers are invited to submit items from newspapers, magazines, books or other sources that contain information that can be related to any area of Mathematics. Sufficient information about the source of the item should be given so that other readers can obtain additional details. Teachers are encouraged to use the contents of this regular feature of The Gazette as an aid to helping their students see the connection between Mathematics and the world around us. The content may not fit in with what you are currently teaching to your students, but many of the items can be used to begin your classes in an interesting and engaging manner.

A CAN CAN KEEP COOL ON ITS OWN

THE TORONTO STAR, 1988

"The fridge soon may be obsolete, at least for keeping your drinks cold. Superior Marketing Research of Salt Lake City, Utah has made what it describes as the first breakthrough in can technology since the ring pull popped up in the early 60's - a self cooling can....Within 90 seconds the temperature of the can has dropped by about 20°C to just above freezing"

QUESTIONS

a) Use the information given in the article to develop an equation that relates the temperature T (in degrees Celsius) of the can and the time t (in seconds), with $t = 0$ corresponding to when the tab is pulled.

HINT: Using Newton's Law of Cooling it follows that the relationship between T and t is of the form

$T = T_0 e^{-kt}$

where T_0 is the initial temperature of the can and k is a constant.

b) The correct answer to part a is $T \approx 21e^{-0.0338t}$

Use this equation to answer each of the following questions.

i) What will the temperature of the can be 30 seconds after the tab is pulled?

ii) When will the temperature of the can be 12°C?

i ii) Sketch the graph of T versus t for $0 \le t \le 120$

COMMENTS AND ANSWERS

Unfortunately I do not know the exact date in 1988 that this article appeared in The Toronto Star. I can tell you though that a similar article appeared in The New York Times on January 21, 1990. If you are teaching the Grades 11 or 12 Mathematics courses, don't pass this question by because of the e that appears in the equation. Just use a base of 10 instead of *e*.

a) When $t = 90$ the temperature of the can is just above freezing. Using $T = 1$ for the temperature at this time (your students might use something else that is close to 1) it follows that $T_0 = 21$.

The value of k can then be found by noting that if $t = 90$ then $T = 1$.

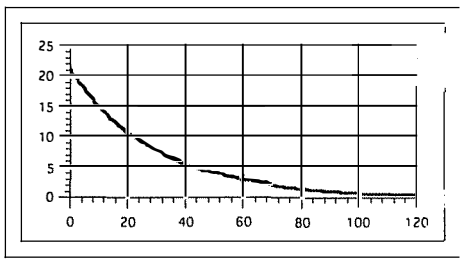
Thus $1 = 21e^{-90k}$ and so $k \approx 0.0338$.

Therefore $T \approx 21e^{-0.0338t}$

b) i) The temperature will be about 7.6°

ii) The temperature will be 12°C after about 16.6 seconds.

iii)



BARBIE SALES BOOMING

THE GLOBE AND MAIL, MONDAY, AUGUST 24, 1992

"Since 1987, sales of Barbie dolls have doubled to $840 million, half of Mattel Inc.'s business overall."

QUESTIONS

a) From the information given in the article, can you conclude that Mattel sold more Barbie dolls in 1992 than in 1987?

b) What additional information would you need to know in order to be able to determine how many Barbie dolls were sold in 1992?

c) Suppose that the total sales of Barbie dolls went up by r% per year from 1987 to 1992. Using the information given in the article, find the value of r.

d) How does the value of r found in part c compare with the average rate of inflation from 1987 to 1992?

COMMENTS AND ANSWERS

a) No! It is possible that fewer dolls were sold in 1992. This could have happened if the price of a Barbie doll skyrocketed from 1987 to 1992. I often see this type of headline in newspapers and it is quite misleading. No conclusions can be made if all we know is that the total sales of an item have increased over a period of time. We need to know how the price of the item has changed over those years.

b) You would need to know the retail price of the Barbie doll. The number of Barbie dolls that were sold could be found by dividing $870 million by the retail price.

c) $870 = 435(1+r)^7$

$r = \sqrt[7]{2} - 1 = 0.1040895137$

The value of r is about 0.10 and so the total sales went up on the average by about 10% per year from 1987 to 1992.

d) It is higher.

STARPHONE RINGS UP ITS 10-MILLIONTH CALL

THE TORONTO STAR, FRIDAY, OCTOBER 16, 1992, PAGE A2

"StarPhone received its 10 millionth call yesterday-just days after marking its first year in service."

QUESTION

What is the average number of calls received by StarPhone

a) per day ?

b) per hour ?

c) per minute?

d) How does your answer to part a compare to the average number of phone calls that a teenager receives in a day?

COMMENTS AND ANSWERS

a) $10\,000\,000 \div 365 = 27397$

On the average StarPhone received 27397 calls per day.

b) 1142 calls per hour

c) 19 calls per minute

d) For many households, the answer to part a is much lower!

OTHER RECOMMENDED ARTICLES

"Intel claims to have world's fastest computer", The Globe and Mail, Saturday, June 1, 1991—News item about a computer manufactured by Intel Corporation that is capable of performing 8 600 000 000 calculations in a second.

"Numbers Are No Problem For Pollsters. Words Are.", The New York Times, Sunday, August 9, 1992—Excellent news article that discusses how the wording used on surveys can dramatically affect the results of a poll.

"Writers should take care interpreting poll results", Letter to the Editor, The Toronto Star, Saturday, September 12, 1992—Excellent letter that describes how the Toronto Star misinterpreted the results of a poll.

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1992 - 1993

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Arnold Carefoote

Ralph Connelly

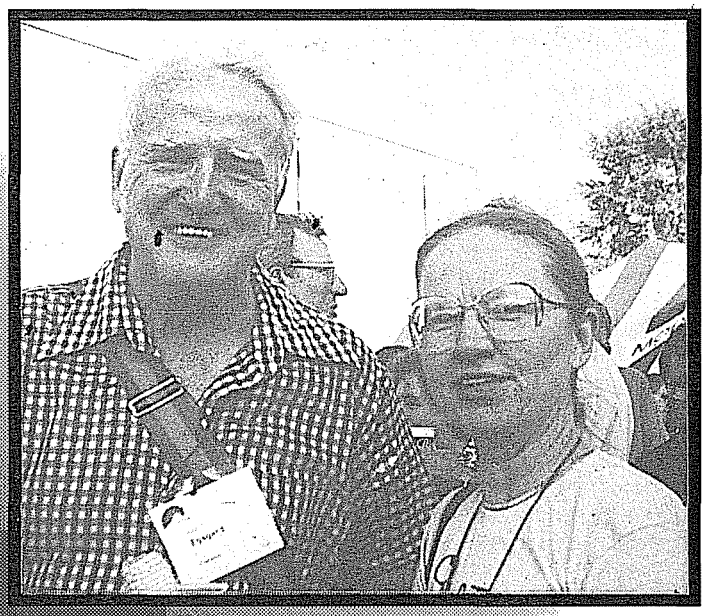
Shirley Fairfield

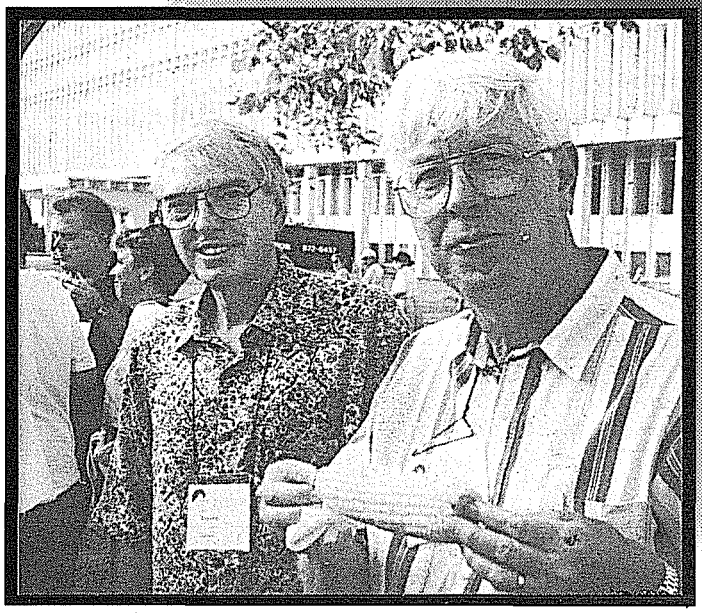
Jerry Maloney

Barry Onslow

Ann Pearce

Barry Scully





Left: Ron Scoins & Ron Dunkley.
Above: Lorna Morrow & John Egsgard
Four of the Ontario mathematics educators enjoying this summer's ICME Conference—the first to come to Canada. See the next issue of the Gazette for reports from Lorna Morrow and Ralph Connelly.

Photo Credits: Back Cover, Courtesy of Lorna Morrow. Front Cover, Left—Courtesy of School Services of Canada. Front Cover Right—Peter Puna, Lakehead University Photography Department. Peter's photo on the front cover shows three students at the Northern Summer School of Science & Technology, a program held in July for high school students from Northwestern Ontario (at Lakehead University) and Northcentral Ontario (at Laurentian University). Ninety students attend a four-week session to study math, biology, chemistry, physics, geology, forestry, and engineering. For more information contact the coordinator—Marilyn Hurrell