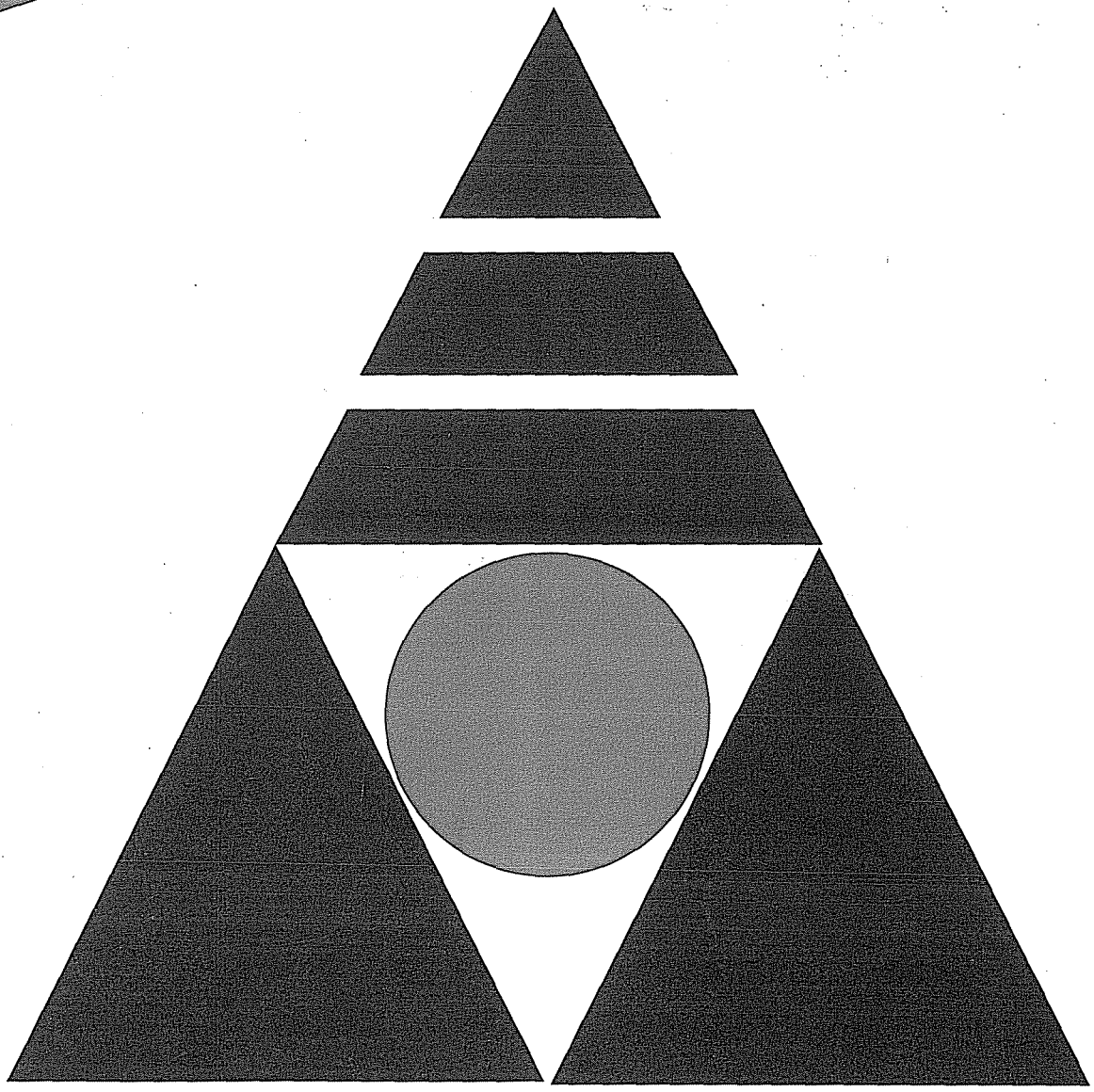
Vol. 31 #3, April 1993

Ontario Mathematics Gazette

Out with the old, in with the new!
See Page 1



ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

About the Gazette

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Submission of Articles

Articles for the Gazette should be submitted to Jack Weiner at the above address, while those for the Abacus should be submitted to Amy Parker Deadlines for information to be included in the Gazette or the Abacus are: June 15 for the September issue, October 15 for the December issue, January 15 for the April issue, and April 15 for the June issue.

Notice to Authors

Your articles should be typewritten, word-processed or photocopied from typewritten or word-processed copy double-spaced on 8.5" x 11" paper with 1.5" margins and single sided. Diagrams and figures should be in camera-ready form. Please submit three copies to the editor and keep one as protection against loss. We have all submissions blind-reviewed, so please send your full address and affiliation on a page separate from the manuscript.

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▲ Editorial

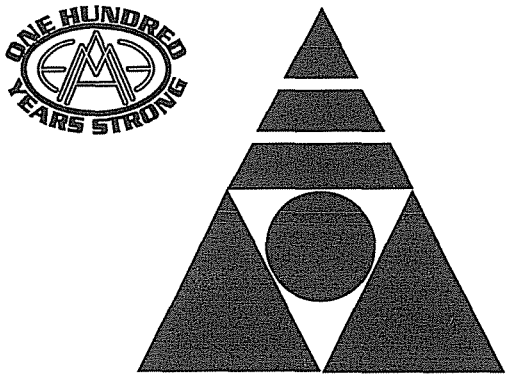
BY JACK WEINER

I am composing this editorial while monitoring (with the help of some graduate students) my 220 honours calculus students. They are writing the dreaded integration test. Actually, they are much more worried than they need to be. These students are bright, enthusiastic, and they work hard! I know they will do well. University admission averages have risen substantially recently and the caliber of students I see in my first year classes reflects this. I don't believe they are necessarily smarter than previous classes. But they are more dedicated. And they're fun to teach. And they laugh at my (often pitiful) jokes.

Gord Nicholls has contributed our inaugural OMG opinion column, Have Your Say. His comments concern the almost astronomical university admission standards. We invite your response to Gordon, as well as to any item in the Gazette. If you have an idea for a future Have Your Say, please submit a draft, or call me to discuss the possibility.

I'm told spring is inevitable, so let's keep smiling in spite of the slush still outside this (as I write) mid-March. And look forward to your very first June issue of the Ontario Mathematics Gazette, subtitled, The Transition Years: A Teacher's Survival Guide.

All the best, Jack Weiner



Front Cover:

We proudly present the new OAME/AOEM logo, prepared by the Publicity and Membership Committee and adopted by Council on January 23, 1993. For comparison, we've placed our retiring logo in the bottom left corner.

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▲ GUEST EDITORIAL: THE INTEGRATION ISSUE

BY GARY FLEWELLING

Imagine a Grade 7 classroom. The theme of the week is deserts. On each student's desk rests an 8.5 by 11 sheet of paper with the drawing of a palm tree on it. On the palm tree are placed 40 arithmetic drill questions. Math has just been (badly) integrated into the theme of deserts.

Integration is all about making connections, making connections within a discipline, making connections between disciplines and making connections to situations, issues, resources or skills that are interesting, comprehensible, relevent, useful or important to the student.

Integration is a significant issue in the Transition Years. If integration is handled badly, mathematics education runs a real risk of being trivialized. If integration is handled well, mathematics education will be enriched.

Integration will likely be handled badly if mathematics is integrated away as an identifiable subject, if the perception of mathematics a narrow one or if there is ignorance of the direction in which mathematics education is moving. Under one or more of these conditions, people are all too likely to accept mindless drill sheets on palm trees as appropriate mathematics education.

In the Dec. 92 issue of the Gazette, in an article entitled *Sept. 93: Yang or Yin?*, I expressed my concern over proposals or policies that would see schools opt for a style of integration that would cause mathematics to disappear as an identifiable subject. Since publication of that article, Judy Crompton, the math consultant from the Lincoln board, and I had an opportunity to meet for a morning with Jack Miller of OISE and share with him our concerns about what we see as a problematic form of integration. We wanted to meet with Jack because of his outstanding reputation in the area of the integrated curriculum and holistic learning. The meeting turned out to be quite interesting and worth sharing. We reached general consensus on the following ten points. (Consider sharing these points with your Transition Years committee colleagues.)

1. Organizing program by subjects is the most reasonable way to get started in Sept. 93.
2. Ministry policy for the Transition Years allows Grades 7, 8 and 9 to be treated in similar fashion.
3. Start from subjects and build connections. Teachers should make connections in their math class in such a way that, "...the interrelationships among subjects and topics (are) evident and meaningful to the students." (Transition Years Policy, Section 1.7) Teachers who make these connections in the math classroom will naturally and increasingly reach out to other teachers to cooperatively plan student activities. An incremental grass roots approach is needed here.
4. Sept. 93 math courses in Grade 9 should not be watered-down advanced or beefed-up general courses. The Grade 7, 8 and 9 math courses should be significantly different in content and approach from those presently being taught.
5. All subjects should incorporate the ten cross-curricular processes listed in the Common Curriculum draft policy. Though each subject will put its own filter on the student's universe, each subject should integrate and make connections from within and across disciplines.
6. Math, Science and Technology is an arbitrary and limiting grouping of subjects. Mathematics needs to be connected to all disciplines.
7. Holistic learning can take place within a subject structure. The positive examples of holistic learning that we shared at the meeting all came from subject-based settings.
8. The transmission (traditional), transactional (inquiry) and transformational (holistic) approaches to teaching all have their place in the classroom.
9. A comprehensive set of mutually agreed upon standards of achievement and a corresponding set of quality test items in the form of Ministry Benchmarks can very positively link curriculum and assessment. The use of standardized math tests should not be encouraged.
10. Change is coming too fast.

What does math taste like?

Math tastes like bubble gum. At first it is hard to chew but gradually it gets easier to chew and then you can blow bubbles. POP! I figured out the math problem. Eventually after working on it for a long time, it gets stale and you spit it out.

– A grade 9 Math Student
at St. Paul High School, Nepean, Ontario

Submitted by Jeri Lunney

▲ President's Message:

BY JERI LUNNEY

Being President of Ontario Association for Mathematics Education this year has been both exciting and rewarding. The position gave me, a classroom teacher, unique access to information and people. I have particularly enjoyed having the opportunity to work on Council with a group of professionals committed to excellence in mathematics in our province.

As we get close to Sept. 93, many secondary teachers are more and more concerned about issues such as destreaming and further restructuring of high school credits. This year, OAME committees have been involved in a variety of activities which will be of benefit to all of the mathematics teachers in Ontario. We have worked on projects relating to Benchmarks, the School Achievement Indicators Program (SAIP) and curriculum renewal. OAME members have played key roles in a number of initiatives taken by the Ministry of Education including Benchmarks and the School Achievement Indicators Program. We have worked on committees, written letters and attended meetings.

- Barry Scully and the Transition Years Committee have been preparing curriculum materials to assist teachers as they prepare for major changes in Sept. 1993.
- Peter Saarimaki and the Early and Formative Years Committee are preparing activities related to the Benchmarks for the Abacus.
- Jerry Maloney and the Specialization Years Committee are addressing curriculum issues at grades ten to OAC.
- Judy Crompton, as Chair of the Focus on Renewal Committee, coordinated OAME's response to the Ontario Provincial Mathematics Benchmarks, in addition to developing a revised version of the "Focus on Renewal of Mathematics Education."
- Marilyn Hurrell and the Publicity and Membership Committee presented a new logo at the January Council.
- The Publications Committee, chaired by Peter Lenczewski, is very proud of the Gazette's new look. Editor Jack Weiner and his staff are to be congratulated for their dedication and enthusiasm in producing this professional journal.
- Barry and Jan Scully, our NCTM representatives, serve as liaisons with our parent organization, in addition to bringing revenue into the organization through the sale of NCTM materials.
- The Leadership Seminar, held at the Ramada Airport West in Mississauga in November, was a fabulous success.
- I have travelled to several Chapter meetings to give greetings from OAME. All thirteen Chapters are actively promoting mathematics in their local regions and beyond. OAME's unique chapter structure has been key to the success of our organization in serving the needs of our vast province.

As this special year comes to a close, I would like to pay a special tribute to the Executive of the OAME who work long hours in the service of our organization. Ann Jones, Ron Lancaster, Todd Romiens, Ron Sauer and Susan Stuart: you are the best! Thank you; it has been a privilege to work with you. Carol and Don Attridge, Executive Directors of OAME, give many hours to this organization. Carol, you are the heart and Don the conscience of Council. Your leadership and dedication are appreciated. I appreciate your hard work and your friendship.

Thank you for a wonderful year. I look forward to seeing many of you at the annual conference in Windsor. Todd Romiens and his committees have planned an excellent program. See you in Windsor in May!

More Notes from the Desk of the President

In November the newspapers were filled with the news that grade ten would be destreamed and OAC's would be dropped. The following letter was sent to the Minister of Education on November 20, 1992.

Dear Mr. Silipo:

The Ontario Association for Mathematics Education (OAME) is concerned about the recent press reports and other indications that further restructuring of education in the Transition and Specialization Years is being considered by your Ministry. In particular, we would like to address the destreaming of grade ten and the further restructuring of the Ontario Academic Courses. We realize that these press reports and rumours are just that. However, through our conversations with Ministry officials and because of our knowledge of information flowing to Directors and Trustees organizations, we cannot dismiss these reports as merely rumours. It is our belief that the changes being considered are not only educational issues, but are linked with educational funding models.

As an association, we have actively supported the necessary reform to the Transition Years and have

participated in the Benchmarks evaluation process. We would like to be directly involved with the discussions on the possible destreaming of grade ten and the changes to the Ontario Academic Courses. The renewal at the Transition Years is strongly based on research related to the needs of the adolescent learner (Hargreaves) and the positive educational outcomes of a destreamed environment (Slavin, Oaks). We are concerned, however, that the changes proposed for grade ten and the OAC's are being driven more by funding models than by educational outcomes.

The process of consultation, and more importantly, collaboration, has been the reason for many of the positive activities around the Transition Years. We would caution that changes put into place without collaboration and for other than well-documented educational reasons, will lead to resistance from educators, parents and the community. The Ontario Association for Mathematics Education would welcome opportunities to collaborate with the Ministry of Education on these issues. We represent mathematics educators in Ontario who care about the process of learning and the best educational outcomes for our students.

Yours truly

Jeri Lunney, President

I also promised in the December Gazette to bring you up to date on our involvement with the Ministry on the topic of National Indicators (SAIP). The following letter was written in November after Ann Jones and Lorna Morrow met with Ministry personnel. Now that the new education minister, David Cooke, appears to favour more testing, it is important that OAME remain involved with the Ministry so that the interpretation of student results is both realistic and fair.

Dear Mr. Silipo:

Thank you for responding to our letter of June 10, 1992 by arranging a meeting with Morris Liebowitz, Michael Koslow and Burle Summers to discuss our concerns about the National Indicators Project for mathematics education.

As we indicated during our discussions, the Ontario Association for Mathematics Education does not support this model for evaluation. A short answer or multiple choice summative test that a student completes in a single sitting does not reflect the full range of activities that are involved in learning mathematics. Students should be exploring, predicting, conjecturing, justifying, discussing and writing mathematics. Ontario teachers are attempting to implement these procedures in their classrooms, considering them important aspects of a student's development.

Teachers are also investigating alternative means of assessment, such as interviews, portfolios and performance-based tests using manipulatives, to evaluate the processes of mathematics as well as the products. Given the nature of a multiple choice/short-answer test, few, if any, of these aspects of a good mathematics program will be assessed by such methods. At best, such a test can only be a snapshot in time of students' recall and simple application of basic facts and concepts.

While we realize that few, if any, changes can be made at this point to the National Indicators Project, we would hope that if Ontario takes part in future large scale evaluation projects, a process form of examination will be seriously considered. Currently the Learning Research and Development Centre and the National Centre on Evaluation in the United States are piloting a test that is based on performance tasks, projects and portfolios. Such tests do not treat evaluation as an event, but as part of the process of learning. We attach an article which provides information about these evaluations.

The Ontario Association for Mathematics Education would be pleased to collaborate with the Ministry of Education on an evaluation process that is more congruent with current evaluation strategies. It is our belief that such a process would be more appropriate to the needs of our students and to the need to be accountable to parents and the public.

Jeri Lunney, President
Ann Jones, Past-President

Shirley Zanin wrote to me after seeing the December Gazette. Shirley, a wonderful mathematics teacher who retired from teaching last June, was my teacher at Almonte High School and taught two of my children at Nepean High School. She wrote:

"How about Mattel's Barbie? It's probably the best publicity math has had in eons. If the comment had been 'Math is boring', I'd sue, but 'math is hard' is right on! Why shouldn't it be? The mistake is that Ken should also be saying it...math is being complimented...I suggest that you get in touch with Mattel and have them award Barbie an Honours Math degree. Seems the only honourable thing to do and it's probably the only thing she doesn't already have!"

Correction

In Jeff Irvine's article, "Game Theory, the Mathematics of Competition" (OMG Dec. 92), the following table should have appeared as Figure 8.

		Pitcher Throws HI	MD	LO
Batter Guesses	HI	.4	.2	.0
	MD	.3	.4	.2
	LO	.0	.3	.4

▲ More Pricing Games

BY L. KEITH ROY, DEPARTMENT OF MATHEMATICAL SCIENCES
LAKEHEAD UNIVERSITY, 955 OLIVER ROAD, THUNDER BAY, ONTARIO P7B 5E1



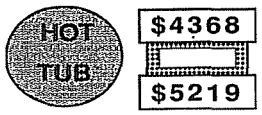
Educated at Queen's, Keith teaches statistics at Lakehead University. A frequent contributor to the Gazette, he has given many invited talks and workshops for groups of students and teachers.

My recent article (OMG, Dec. 91) featured four pricing games from the popular television game show "The Price Is Right" and their analysis using concepts presented in the OAC Finite Mathematics course. Here are several more such games.

Short Games

Some of the games featured on the show take very little time to complete. Most such games can be analyzed quickly and easily in terms of finding the probability of the contestant winning the game.

Unless otherwise stated, it will be assumed that all choices made by the contestant are random choices.



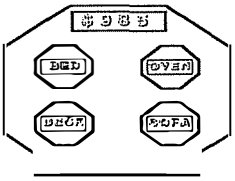
Double Prices

The contestant is presented with two choices for the price of the hot tub and will win that prize by choosing the correct price. The contestant has probability 1/2 of being correct.



Most Expensive

If the contestant can guess which of the three prizes is the most expensive, she/he will win all three prizes. Using a random choice, the probability of winning is 1/3. Of course, some knowledge of relative price sizes on the part of the contestant could eliminate some of the risk and increase the probability of winning.



Danger Price

To win all four prizes, the contestant must choose, one at a time, the three prizes which are not priced $985. The probability of being successful is 1/4 since the contestant need simply choose which of the four prizes to avoid.



Squeeze Play

The price of the trip begins with a 2 and ends with a 9. One of the other digits is to be eliminated. That is, the price of the trip is either $2469 or $2479 or $2679. Once the contestant chooses which digit to eliminate, the other digits squeeze together. Suppose the 4 was eliminated. The result is



Faced with a random choice of three possible digits to eliminate, the contestant has probability 1/3 of winning the trip.



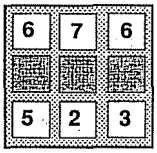
Pick a Pair

For each of the products on display there is another in the display with the same price. That is, there are three sets of pairs. The contestant picks two of the products and their prices are revealed. If the prices match, the contestant wins the featured prize, in this case, living room furniture.

There are C(6,2) = 15 ways of choosing two of the six products so that the probability is 1/15 of randomly choosing the two lowest priced products, 1/15 of finding the pair with the highest price, and 1/15 of finding the other pair, for a total of 1/5.

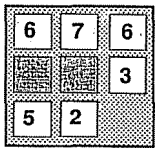
An alternative method notes that, after a first choice has been made, only one of the five remaining products has a price that matches.

If the prices of the contestant's two choices fail to match she/he is given a second chance to match the price of whichever of the two she/he chooses. Since four products remain, the probability of success on the second chance is 1/4. The probability of having to use the second chance and being successful is (4/5) (1/4) = 1/5, so the overall probability of winning is 2/5.

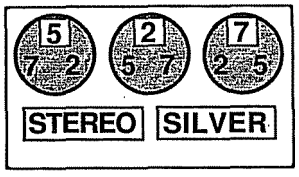


Two For the Price of One

In order to win two featured prizes, a television and bedroom furniture, the contestant must correctly guess the price of the television. The first digit in the price is either 6 or 5, the second digit is either 7 or 2, and the third digit is either 6 or 3. To aid the contestant, one of the digits, whichever the contestant chooses, will be revealed. Suppose the contestant picks the third digit.



The contestant has probability 1/2 of correctly choosing the first digit and probability 1/2 of correctly choosing the second digit. Consequently, the probability of winning the game using random choices is 1/4.



Safe Crackers

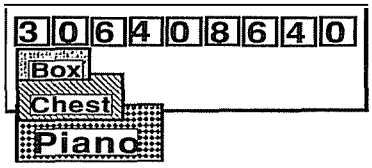
The safe contains two prizes: a stereo and a silver tea service. The price of the silver tea service is also the combination to the safe.

The three digits in the price of the silver tea service can be ordered 3! = 6 ways, so that the probability of success using simply a random choice is 1/6. However, the 2 is most likely too small to be the first digit. So the combination is one of 5-2-7, 5-7-2, 7-2-5, or 7-5-2 which gives a probability of success equal to 1/4.

This game sometimes features choices like 0, 5, 6 where, most often, the 0 is the last digit. So the combination is either 5-6-0 or 6-5-0 which gives a probability of success of 1/2.

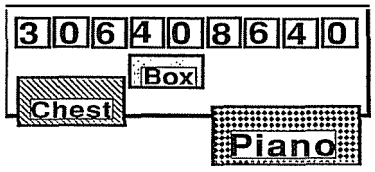
Longer Games

Some of the other games featured on the show take a bit longer to complete. Again, unless otherwise stated, it will be assumed that all choices made by the contestant are random choices.



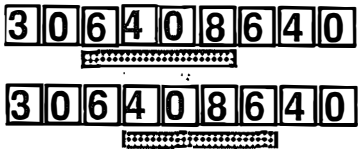
Make Your Move

In order to win all three prizes the contestant must slide the prize labels horizontally and position them below the proper prices. The labels must not overlap. One possibility is



There are 3! = 6 ways of arranging the labels, so the probability of being correct is 1/6. However, other factors enter into consideration.

Since the initial digit in the price of the piano cannot be 0, the only possibilities for the price of the piano are 3064, 6408, 4086, and 8640. Consider the following cases.



The above two cases uniquely determine the positions of the other labels. The following case presents two possible ways of arranging the other two labels.



The case:



is highly unlikely for two reasons: firstly, 3064 is a rather

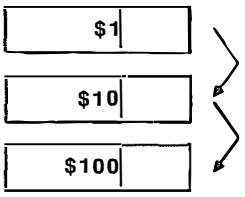
small price to pay for a piano, and secondly, the price of either the box or the chest is unlikely to begin with a zero.

Consequently, the probability of success can be increased to 1/4 if the contestant gives some thought to the situation.

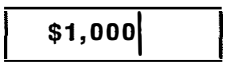
The Grand Game

There are six featured products, four of which are priced under $2.60. The contestant starts with one dollar and a zero is added to that amount each time a product priced under $2.60 is chosen. So the amount of money won by the contestant increases to $10, then $100, then $1000, then $10,000 as long as she/he avoids choosing one of the products priced over $2.60. If the contestant reaches $1000, she/he must choose whether to quit at that point, or to risk losing the $1000 by continuing.

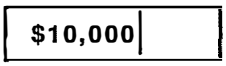
Since there are C(6,4) = 15 ways of randomly choosing 4 items from 6 items, a contestant with no idea about the price of any of the six products would have probability 1/15 of winning the $10,000. However there always seems to be two obvious choices, so almost every contestant reaches the $100 mark.



At this point two products of the remaining four are priced under $2.60. Thus the probability is 2/4 of reaching the $1000 mark.



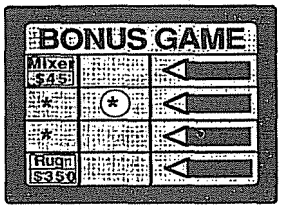
The probability of winning the $10,000, given that the contestant has reached the $1000 mark and chooses to go on is 1/3.



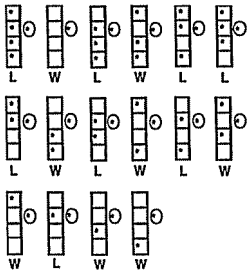
So a contestant who reaches the 100 dollar mark has probability (2/4)(1/3)=1/6 of winning 10,000 dollars.

The Bonus Game

Signs indicating the wrong price of four featured products are revealed one at a time. The contestant is asked to indicate whether the real price is higher or lower than the price on display. The contestant wins each of the products for which a correct guess was made. If the contestant is incorrect, the sign is moved to the right and hidden under an arrow, leaving a * at the left of the display. The contestant also has a chance to win a bonus prize (usually furniture or a trip) if, at the end of the game, the middle column is illuminated to reveal a * next to a prize sign in the column at the left.



For a fixed position (chosen before the game begins) of the * in the middle column, there are sixteen possible results.



In the above, W indicates a situation where the contestant wins the bonus prize, and L indicates the loss of that prize.

If the contestant randomly guesses "higher" or "lower" as each product is revealed, she/he will have probability 1/2 of being wrong, resulting in a * in the left hand column, and probability 1/2 of being correct (no *). Consequently all of the sixteen possibilities listed above will have probability

$$\left(\frac{1}{4}\right)^4 = \frac{1}{16}$$

Since eight of the above possibilities result in a win of the bonus prize, the probability of winning that prize is 8(1/16) = 1/2.

However, suppose the contestant uses her/his prior knowledge concerning the four featured products. Assuming that the contestant has probability 2/3 of being correct about any of the featured products and 1/3 of being incorrect, the sixteen possibilities listed above will no longer be equally likely. The probabilities for the possibilities resulting in a W are respectively

$(2/3)^4 = 16/81$,

$(2/3)(1/3)^3 = 2/81$,

$(2/3)^2(1/3)^2 = 4/81$ (three times) and

$(2/3)^3(1/3) = 8/81$ (three times).

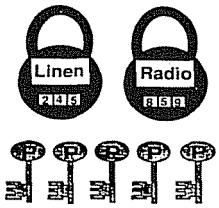
So, in this case, the probability of the contestant winning the bonus prize is

$16/81 + 2/81 + 3(4/81) + 3(8/81) = 2/3$.

The wary reader might have predicted the above result using a line of reasoning similar to the following. If the contestant is twice as likely of being correct about each featured product, as opposed to being incorrect, then the eight possibilities resulting in a W will together be twice as likely as the group of possibilities resulting in an L.

Master Key

The contestant is confronted with two featured products: the linen and the radio. The price of the linen is either $24 or $45 while the price of the radio is either $85 or $59.



The contestant will win a choice of one of five keys if she/he guesses the correct price of the linen and another choice if she/he guesses the correct price of the radio. One of the five keys unlocks the car, one unlocks the freezer, one unlocks the desk, one unlocks none of the three, and one, the master key, unlocks all three.



If the contestant has probability 1/2 of correctly guessing the price of each of the featured prizes, then the probability that she/he wins two keys is (1/2)(1/2) = 1/4, while the probability of not winning any keys is also (1/2)(1/2) = 1/4. So the probability of winning only one key is 1 – 1/4 – 1/4 = 1/2.

If the contestant wins only one key, the probability that the key will unlock the car is 2/5 since both the master key and car key will unlock the car.

If the contestant wins two keys, the probability that both keys will unlock the car if

$$\frac{2 \cdot 1}{5 \cdot 4} = \frac{1}{10}$$

the probability that only the first key chosen will unlock the car is

$$\frac{2 \cdot 3}{5 \cdot 4} = \frac{3}{10}$$

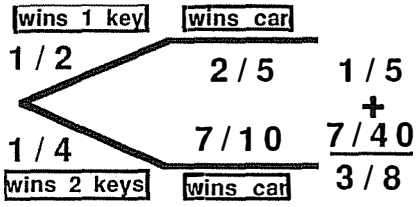
and the probability that only the second key chosen will unlock the car is

$$\frac{3 \cdot 2}{5 \cdot 4} = \frac{3}{10}$$

So, if the contestant wins two keys, the probability of winning the car is

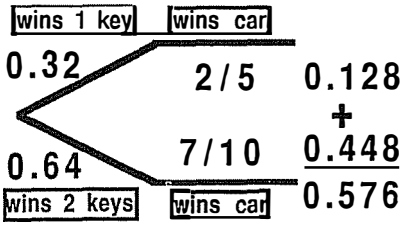
$$\frac{1}{10} + \frac{3}{10} + \frac{3}{10} = \frac{7}{10}$$

The above information is used in the following tree diagram.



Consequently, if the contestant has probability 1/2 of correctly guessing the price of each of the featured prizes, the probability that she/he will win the car is 3/8.

Suppose the contestant has probability 0.8 of correctly guessing the price of each of the featured prizes. Then she/he has probability (0.8)(0.8) = 0.64 of winning two keys, probability (0.2)(0.2) - 0.04 of winning no keys, and probability 1-0.64-0.04 = 0.32 of winning just one key. The following tree diagram results.



Consequently, if the contestant has probability 0.8 of correctly guessing the price of each of the featured prizes, the probability that she/he will win the car is 0.576.

ANNUAL CONFERENCE

The Ontario Educational Research Council Annual Conference will be held Friday and Saturday, December 3 & 4, 1993, Ontario Institute for Studies in Education.

For information contact:
Dr. John Bramwell,
London Board of Education

DECEMBER 3 & 4 • 1993

(See *Call for Papers,* pg. 9 this issue.)

REVIEW:
NOTES ON A SUMMER CONFERENCE

LORNA MORROW
TORONTO, ONTARIO

August in Quebec City saw a number of OAME members participating in the 7th international Congress for Mathematical Education (ICME-7), many in leadership roles.

To describe the conference, I turn to the words of Eric Muller, a very busy trouble-shooting member of the National Committee. For the final Newsletter of the Congress, Eric wrote:

> *I wonder how you will remember ICME-7?*
>
> *For me it was first and foremost a people conference...It was also...a conference that relied on technology and tried to address its implications...Technology extended through the Miniconference to very many other sessions throughout the Congress...Hundreds of computers, some with projection facilities, were used...This technology, for all its imperfections, is changing our world, including the world of ICME...and is certainly here to stay. Do we know enough yet about how to prepare our students to function effectively and efficiently in an ever-expanding technological world? Mathematics educators must surely have something to say.*
>
> *Last, but not least, it was a Congress trying to reach out to a concern for others - disadvantaged groups, groups who find mathematics particularly difficult, minority groups, individuals with handicaps, etc. etc.*

OAME can be proud of its members who contributed to the success of this international gathering by acting as speakers, Local Organizers, or committee members. Some of these were Dave Alexander, Ed Barbeau, John Clark, John Del Grande, John Egsgard, Gila Hanna, Bill Higginson, George Knill, Erika Kundiger, Ron Lancaster, Ron Lewis, Lorna Morrow, and Barry Onslow.

There were 383 Canadians in attendance, a turnout exceeded only by the US with 787. All told, 88 countries were represented, some of the developing countries by virtue of ICME's policy of underwriting expenses for such delegates. The opportunity to meet and talk with mathematics educators from such a range of countries was a unique experience, and one OAME members took full advantage of.

Those of us who participated came away with new friends, new ideas, and renewed enthusiasm for mathematics education. For those of you who were not able to attend, ICME-8 will be held in the summer of 1996 in Seville, Spain.

Proceedings of the Congress will take different forms. There will be an official Proceedings book published and sent to participants. This will have brief descriptions of the issues discussed in the Working Groups, Theme Groups, and other sessions. In addition, some of the Working Groups will be producing a more detailed collection of papers which will be advertised in professional journals when they are completed, late this fall or early next year.

Call For Papers

The Ontario Educational Research Council invites proposals for the presentation of research papers at its annual conference, December 1993.

Submit a brief summary including purpose, method, findings and implications along with the title of your paper to the Program Chair or President. All presenters will be expected to register at a special reduced rate for the conference. Detailed information will be available at the time the paper is accepted.

Dr. John Bramwell,
Conference Chair
London Board
of Education

Dr. Mary Hookey
President, O.E.R.C.
Nipissing University
North Bay

▲ Specialization Years Committee Comments

BY JERRY MALONEY

In the spring of 1992, as part of its mandate from OAME, the Specialization Years committee prepared a response to the Ministry of Education's Specialization Years Consultation Paper. In its response the Committee addressed the following issues.

- What are the most important educational needs of students at this stage in school?
- What should being a secondary school graduate mean?
- How should the role of the school and the role of the family be related?
- Overcrowded Curriculum
- Subject Specialization in Teacher Education
- Teacher Education
- Gender Equity
- Racial and Ethno-cultural Equity
- Gender and Education/Career Choices
- Curriculum Organization
- Credits for External Courses
- Curriculum Renewal
- Assessment and Evaluation
- Educational Technology
- Needs of Students New to Canada
- Integrated Learning Activities
- The Guidance Program
- Reporting Student Achievement
- Additional Issue: Funding

The major issues centred around teacher education, curriculum renewal and the use of technology. Following are excerpts from the document submitted addressing these issues.

Subject Specialization and Teacher Education

Subject specialization alone is not enough; there is a need to provide pre-service and in-service training for teachers on the variety of learning processes, new technologies, critical thinking-skills, co-operative learning, adapting to a variety of learning styles, independent learning, and so on. Prospective teachers must be taught in a manner similar to how they are to teach - exploring, conjecturing, communicating, reasoning, and so on. There is no evidence that generally the current pre-service training of mathematics teachers reflects the directions of mathematics education in Ontario as it is evolving to prepare students for the 21st century.

Calculators, computers, courseware, and manipulative materials are necessary for good mathematics instruction; the teacher can no longer rely solely on chalk, chalkboard, paper, pencils, and texts. Simply providing teachers with these materials will not produce a new program. Teachers must also know how to integrate this technology into a quality mathematics program. It is of paramount importance that adequate in-service training be provided to impress upon current teachers the value and the need for new, improved curricula and methodologies. Without regular, on-going staff development programs, there will be little effective change.

Part of the solution could be found on closer connections between faculties of education and subject teacher groups such as OAME and OMCA. More than this, the Ministry of Education must encourage partnerships between faculties of education and local school boards. This would ensure flexibility and co-operation in sharing staff. Successful classroom teachers could then be seconded part-time or full-time to faculties of education. Likewise, all faculty of education staff must be required to teach at least one full year out of every five in a school at the appropriate level. Placement of practice teachers must be based on the skills of the associate teacher rather than on who is available or who requests student teachers. Practice teaching must be for longer periods of time and must involve some mentoring on a long term basis.

There must be standards across the province for faculties of education. Most appear at this time to be on their own with little accountability to anyone.

Because of the high turnover of mathematics staff in the next few years and an expected shortage of students in teachers' colleges with mathematics background, there needs to be a thrust in the faculties of education to recruit

students with the appropriate background to replace math and science educators.

Curriculum Renewal

There is a need in each of the courses to revise the curriculum and ensure that what we are teaching better meets the needs for students in the 21st century, as well as takes advantage of the technology that is available. The material needs to involve realistic applications. Rather than continuously reteaching skills that students will need later, the material must be taught when needed. In each of the courses there must be opportunities for optional sections that will meet individual needs rather than attempt to have courses that try to meet all needs.

With the impending elimination of credits in grade nine, the whole issue of required credits for a graduation diploma will need to be revised. We suggest that in the specialization years there is a need to continue the credit system, including grade 10, and that students should have the equivalent of three compulsory credits in mathematics in these years. These credits could be broken into smaller modules or fractional credits so that individual needs of students can be better met. The first half credit could be common to all students taking a particular course but the next two quarter credits could be chosen from a variety of quarter credits that better meet the needs of the individual student.

Educational Technology

The use of technologies in mathematics education has a profound effect on the curriculum we can teach and need to teach. It also affects the methods we use and will require additional in-service training and a focus on pre-service training for future mathematics teachers.

Presently the committee is dealing with some very contentious issues. Since the Ministry has not yet given Ontario teachers some clear indications of where we are going in the specialization years the committee is attempting to look at many of the broader issues. Our task at present is to assist in the preparation of the Specialization Years part of the OAME/OMCA Focus on Renewal Document. The Committee is struggling with issues such as destreaming of grade 10, the place of Math for Business and Technology and generally the suitability of our present advanced level curriculum in preparing our students for the 21st century. If you have any concerns or suggestions please contact Jerry Maloney, Chair of the committee.*

**See the inside back cover of this issue for Jerry Maloney's address and business number.*



The Publication's Committee is pleased to announce the appointment of Bob Robinson to the position of Advertising Manager of the Gazette.

Bob brings to the position an extensive background in mathematics education. He was a teacher and mathematics consultant with the Board of Education for the City of Hamilton. During those years Bob served as an education officer in the Curriculum Policy Branch of the Ontario Ministry of Education, Director and Councillor with OAME, Executive Director and Director of NCTM where he was chair or member of three major committees, and member of OMCA. He is an active member of NCSM and an associate member of ASSM.

Bob has chaired or coordinated many mathematics curriculum committees and mathematics education projects, and has had a direct role in staff development and in-service training. He is aware of the needs and interests of the mathematics educators that make up the readership of the Gazette; hence, he is well qualified to solicit advertisements that will be of interest and help to the readership while generating revenue that will be used to improve and expand future issues of the Gazette.

We are honoured to have Bob in this position and look forward to working with him.

– Peter Lenczewski
Chair, Publications

Editor's note: For more information, contact Bob Robinson, Advertising Manager, OMG, Box 63062, University Plaza, Dundas, Ontario L9H 6Y3

▲ HAVE YOUR SAY:
ENTRANCE EXAMS- THE TIME HAS COME

BY GORDON NICHOLLS
HEAD OF MATHEMATICS
PRESTON HIGH SCHOOL

As the twentieth century comes to a close, many educators in Ontario are assessing the achievements of the past and are also trying to determine future directions. I think that we, the mathematics teachers of Ontario, must also look at the past and search for answers to the many problems that surround us in the schools. The following questions require thought and the quest for solutions:

1. In the 700 or so high schools in Ontario are we evaluating students in a consistent way?
2. Do students have a realistic view of their own abilities when they leave high school?
3. Are Ontario students being admitted to post secondary institutions in a fair and consistent manner?
4. Why has the percentage of Ontario Scholars increased from 16% in the 1970's to over 30% in 1987 and yet the success rate of Ontario students in International Mathematics studies has declined?
5. Are the criteria for awarding Canada Scholarships satisfactory when the retention rate in first year of university has been only about 55%?

Do these questions disturb you? They certainly have caused me to question whether or not we have appropriate methods for evaluating students and whether the admission system to our colleges and universities is fair. As I searched for answers I took a historical look at where we were 25 years ago, what we did to improve the environment and instruction in our schools and where we are today. The conclusions I have drawn are in part determined by the historical developments of the Seventies and Eighties. I am writing this article with the view that in the Seventies many mistakes were made in Ontario education. In the last 12 years much has been done to reverse the excesses of the Seventies, yet still maintain a degree of local autonomy and preserve the creative spirit required for excellent teaching and learning. The list of compulsory courses has returned to 16. The 1985 guidelines attempted to standardize mathematics curriculae and still allow for some local options. The Seventies concept of the self-directed highly motivated learner is still with us even though large numbers of students do not display these attributes. Unfortunately our theoretical "highly motivated learner" is a very human being whose motivation is affected by peer pressure, home environment, societal attitudes to work ethic and despair over a sinking economy. In many cases self-directed learning is replaced with virtually no learning.

As we begin the last decade of the century I think we have to find solutions for *several serious problems.* **The first problem** is the diversity of preparation among our first year university students. The abilities to write clearly and correctly, to reason logically, to execute algorithms correctly and to approach problem solving with a variety of approaches are a critical part of the education for all students who wish to attend a university. The feedback we are getting from vehicles like the Descartes competition and the International Math Study is that too large a percentage of our students cannot demonstrate these abilities to the level we would like. We also know that if we elect to use the inexpensive multiple choice type measuring tool, then we will not be promoting writing, organization and clarity of expression. **The second problem** we need to address is how to improve our students' ability to review a standard body of material, synthesize information and concepts, and to write well reasoned answers to questions based on this material. We know that for a small group of creative students it is possible to achieve these goals in other ways; however, for the vast majority of students, the removal of external measuring instruments has led to an unwillingness to master the material to the standards required. **The third problem** is the ability of our students to perform under pressure. One form of pressure performance is meeting a boss' deadline; a second type of pressure is to prepare an essay, problem set or lab report and then submit the well written result on time; and a third form of pressure is to write well constructed answers under the time constraints of an exam. Writing exams is a skill that most students can acquire with practice. Today's two term and semestered schools give students only 50% of the practice they acquired a decade ago. We must look for ways to give students more opportunities to prepare for and write exams. **The fourth problem** we have to deal with is fairness of admission to the universities and the colleges. In the past few years tight financial constraints have caused universities to limit enrolment. To do this in as fair a way as possible, we need to have some criteria that are more broadly based than the marks from the individual school. The 1966 Committee on the Grade 13 Year addressed this issue. Unfortunately the Committee's recommendations seem to have been forgotten by most parties.

In my opinion the best solution to the problems listed above is to return to a form of external examination that requires the students to write well reasoned essay style solutions. To achieve this reform I would suggest the following:

Proposal #1

The first component is to introduce a system of university entrance exams. These exams would be required of all students and would count for 45% of the entrance mark. The next 45% would come from the general average of the best six grade OAC subjects. The final 10% would be determined by a list of factors such as the number of times the exams were tried, extra curricular record, confidential recommendation, previous awards, oral skill in languages etc. To help the universities identify special talents, one confidential form should be prepared for each student and submitted to the University Application Centre in Guelph. *All students applying for university would be required to write two university entrance exams.* Each exam would be three hours in length and essay style in format. The two exams required from a student would be determined by the choice of faculty in the university. Each faculty would specify the two exams required. These exams would be written twice a year – either in December or in early June. Preparation of these exams would be done by a mixed committee of experienced high school and university teachers. These committees would be organized so that they would work 6 to 10 months in advance of the examination date. The exams offered would cover specified topics from the curriculum. The questions would be designed to give a performance evaluation of the students at a variety of difficulty levels. In each exam, except English and French, students would be required to answer two out of three sections. The following are possible examinations that could be offered:

1. Mathematics–based on Calculus, Algebra and Geometry and Finite math.
2. Science–based on Biology, Chemistry, and Physics.
3. Humanities–based on Geography, History and Sociology.
4. Business–based on Accounting, Economics and Law.
5. Languages–based on French, German, Spanish and Latin.
6. English–based on grade 13 English. Possible components could be content, analysis of a sight passage and précis writing.
7. French – similar structure to the English paper.

An entrance exam system will cost money. In my opinion the proposed system of University admission would have to be financially self-supporting. Students would have to pay a fee for each paper they wished to write. Without a detailed cost analysis it is difficult to be precise, but I would estimate that a fee of $50 to $75 per exam would cover all marking and administrative costs. Marking would be done at all universities in the province using a team of high school teachers and university people. Papers would be assigned to university centres based on a student's first choice of university. After marking, all results would be put on a master data file and shared with all universities in the province. A tight deadline of one week should be allowed for the marking and creation of the data files. The papers of all students who apply for scholarship would be handed over to the scholarship committee at the university of first choice. These committees could remark any or all papers to help them in their decision making.

Although the idea of writing only two entrance exams is unusual it has a cost and time efficiency that is desirable and lends itself to the selection of groupings that would fit the entrance requirements for most faculties. The final decision about which exams should be selected would be made by each university. The following suggestions are offered so the reader can see how the system might work.

- Engineering–Math and Science Exams
- Mathematics and/or Computer Science–Math and Science Exams
- Modern Languages–English and Humanities exams
- Business–Business and either the Math or English exams
- Science–Science and either the Math or English exams.

It is important to note that a student could graduate from grade 13 (O.A.C. if you wish) and not write any of the above exams. These exams would only be required if a student wished to apply for University entrance. In order to facilitate the excellent work that has been done with mature students over the last 25 years it would be important for staff who have worked in the mature student program to develop a special set of admission criteria for these students.

Proposal #2

The situation at the community colleges has been altered radically in the last few years. Programs that used to be easily accessible are now over-subscribed and many students do not get their first choice. To introduce an element of fairness and also to raise the prestige level for entrance, I propose that a set of common entrance exams be written by applicants to the C.A.A.T. These exams would be two hours in length and could be written in either December or early June. The exams would be based on the

content of general level programs and would only be written by those students who wished to attend a community college. Initially I think it would be advisable to require all applying students to write either an English or Mathematics exam. Mathematics would have to be written by those students wishing to enter the technical and business areas that require the mathematics of grade 11 and 12. The second exam would be determined by the college department a student wanted to enter. A fairly wide variety of exams would have to be offered. In addition to a written exam I think it would be important to allow the colleges to invite selected border-line students to come to the college to do a practical exam. Financial independence would be required of this exam system and the students would be required to pay a fee of approximately $50 per paper. The marking of the papers would take place at the colleges using combined teams of high school and college teachers. All exam results would be entered into a data file and shared with all colleges in Ontario.

In closing, I would suggest that we need to introduce forms of testing that go beyond the individual school level. We need to do this in order to assure both parents and ourselves that certain basic standards are being met and that fairness based on merit is being used to allocate the admissions to our post secondary institutions. These tests should be prepared and marked by teachers of the students, but the marking schemes and the required standards to be attained must have a broader scope than that of an individual school. I think these proposals would give a common denominator factor for entry into these institutions that will guarantee a greater element of fairness. Hopefully the students will rise to these challenges of these tests and work in conjunction with their teachers to achieve success. This form of motivation can be viewed and used as a very positive factor for educational development, or it can be viewed as being both negative and destructive. I have had the personal good fortune to both study as a student and to work as a young teacher in environments where the positive approach to these challenges was stressed. The students in those learning environments were able to achieve both success and a feeling of tremendous accomplishment.

I have written this article with the hope that it will stimulate debate and reaction. The time for action is now because Ontario is on the brink of major educational change. If the problems I have identified do indeed exist and the proposals I have suggested are to work, then you, the community of mathematics teachers, must take the initiative and work for the changes suggested. All the usual methods for implementing change are available to you, but you must make the effort to see that the ideas are discussed. To act is to exert some control over your teaching career—the time is now!

THE 20TH ANNUAL OAME CONFERENCE

LE 20EME CONGRÉS DE AOEM ANNUEL

Hilton International Hotel & Cleary Convention Centre

Windsor, Ontario • May 13-15, 1993

Sponsored by: SouthWestern Ontario Association of Mathematics Educators in cooperation with OAME/AOEM

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Full Conference	$99cdn/$85us	$134cdn/$110us
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Saturday Only	$30cdn/$25us	$45cdn/$38us



▲ NINE POINT CIRCLE (REVISITED)

DR. GURCHARAN SINGH BHALLA
BRONX COMMUNITY COLLEGE OF THE CITY
UNIVERSITY OF NEW YORK, BRONX, NY 10453

The Nine-point Circle Theorem is a remarkable and prominent work of the modern elementary geometry of a triangle. The theorem states that in a triangle, the circle passing through the midpoints of the sides of the triangle passes through the feet of its three altitudes, and the midpoints of the three segments joining the orthocentre with the vertices of the triangle. Thus in triangle ABC with Ø as orthocentre (the intersection of the three altitudes), D, E, F, the midpoints of sides AB, BC and CA respectively, H, I, J, the feet of three altitudes from A, B and C, respectively, and K, L, M, the midpoints of segments ØA, ØB and ØC, respectively, all lie on a circle known as the nine-point circle.

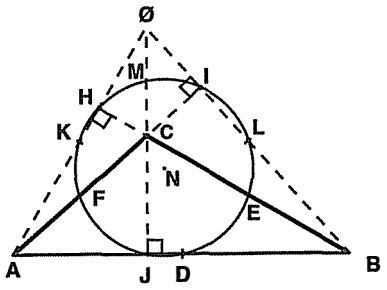


Figure 1

The Nine - Point Circle

In the early nineteenth century a German scholar, Karl Wilhelm Feuerbach (1800-1834), published a pamphlet including the nine-point circle and his theorem known as Feuerbach's Theorem. The nine-point circle was known as Feuerbach's circle in Germany. The circle was also erroneously called Euler's circle. The theorem was also independently discovered by Benjamin Bevan, John Butterworth and John Whitely.

In this article we will justify the Nine-point Circle Theorem by using the methods of analytical geometry. We will also prove two corollaries.

i. The center N of the nine-point circle (the point of the intersection of perpencidular bisectors of the line segments DE, EF and FD) bisects the line segment joining the orthocentre Ø and Q, the center of circle passing through vertices A, B, and C of Δ ABC.

ii. The centroid G of the triangle ABC divides the segment ØQ in the ration 2:1.

Let the vertices of triangle ABC be A = (0,0), B = (a,0), and C = (b,c). Then the vertices of midpoints D, E, F are

$$D = \left(\frac{a}{2}, 0\right),\ E = \left(\frac{a+b}{2}, \frac{c}{2}\right),\ F = \left(\frac{b}{2}, \frac{c}{2}\right)$$

I. We will find the coordinates of N, radius ND and the equation of the circle.

Let N be the centre of the circle through D, E, and F. The midpoint and slope of DE, respectively, are

$$\left(\frac{a+b}{2}, \frac{c}{2}\right) \text{ and } -\frac{c}{b}$$

The slope of the perpendicular bisector is b/c.

The equation of the perpendicular bisector of DE is

$$(1)\quad y - \frac{c}{4} = -\frac{b}{c}\left(x - \frac{2a+b}{4}\right)$$

Similarly, the equation of the perpendicular bisector DF is

$$(2)\quad y - \frac{c}{4} = \left(\frac{a-b}{c}\right)\left(x - \frac{a+b}{4}\right)$$

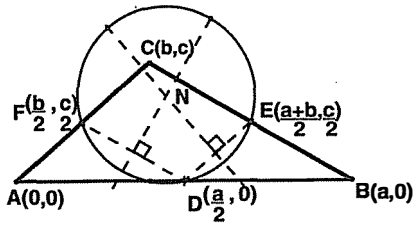


Figure 2

Finding the Centre N of the Nine-Point Circle

From (1) and (2), we find

$$N = \left(\frac{a+2b}{4}, \frac{c^2+ab-b^2}{4c}\right)$$

The coordinates of D are (a/2,0).

Therefore,

$$ND^2 = \left(\frac{2b-a}{4}\right)^2 + \left(\frac{c^2+ab-b^2}{4c}\right)^2$$

The equation of the circle through D, E, and F is

$$(3)\quad x^2 + y^2 - \frac{a+2b}{2}x - \frac{c^2+ab-b^2}{2c}y = -\frac{ab}{2}$$

II. We will show that the circle (3) passes through points H, I, and J, the feet of the three altitudes from vertices

A, B, and C of ΔABC. H is the intersection of the altitude from A to BC.

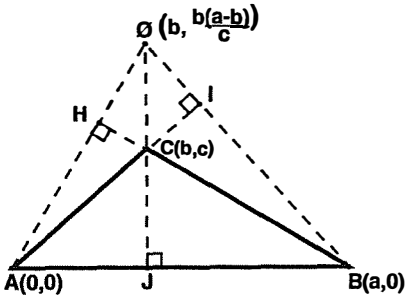


Figure 3

Locating the Orthocentre Ø

The equations of BC and AH respectively are

$$(4)\quad y = \left(\frac{c}{b-a}\right)(x-a)$$

$$(5)\quad y = \left(\frac{a-b}{c}\right)x$$

From (4) and (5), we find

$$H = \left(\frac{ac^2}{(a-b)^2+c^2}, \frac{ac(a-b)}{(a-b)^2+c^2}\right)$$

H lies on the circle, since its coordinates satisfy (3) as indicated below:

$$\frac{a^2c^4}{\left((a-b)^2+c^2\right)^2} + \frac{a^2c^2(a-b)^2}{\left((a-b)^2+c^2\right)^2} - \frac{a+2b}{2}\left(\frac{ac^2}{(a-b)^2+c^2}\right)$$
$$-\left(\frac{c^2-b^2+ab}{2c}\right)\left(\frac{ac(a-b)}{(a-b)^2+c^2}\right)$$
$$= -\frac{a^2c^2}{(a-b)^2+c^2}$$
$$-\frac{a^2c^2+2abc^2+ac^2(a-b)-ab^2(a-b)+a^2b(a-b)}{2\left((a-b)^2+c^2\right)}$$
$$= -\frac{abc^2+ab(a-b)^2}{(a-b)^2+c^2} = -\frac{ab}{2}$$

Similarly we can find the coordinates of the feet I and J of the altitudes from B and C and show that they satisfy (3).

III. To show that points K, L, M also lie on the circle we find their coordinates and then verify that they satisfy (3).

Points K, L, M are the midpoints of segments ØA, ØB, ØC.

Ø, the orthocentre of the triangle ABC, is the intersection of altitudes AH, BI and CJ.

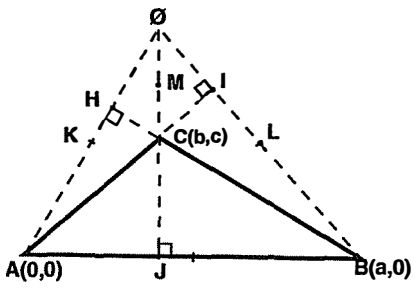


Figure 4

Locating Points K, L and M

From the equation of AH,

$$y = \left(\frac{a-b}{c}\right)x$$

and the equation of BI,

$$y = -\frac{b}{c}(x-a)$$

we find coordinates of Ø to be

$$\left(b, \frac{b(a-b)}{c}\right)$$

and therefore, the coordinates of K, L, and M are

$$K = \left(\frac{b}{2}, \frac{b(a-b)}{2c}\right)$$

$$L = \left(\frac{a+b}{2}, \frac{b(a-b)}{2c}\right)$$

$$M = \left(b, \frac{b(a-b)+c^2}{2c}\right)$$

The point K lies on the circle (3) since its coordinates satisfy the equation of the circle as indicated below:

$$\frac{b^2}{4} + \frac{b^2(a-b)^2}{4c^2} - \frac{a+2b}{2}\cdot\frac{b}{2} - \frac{c^2+ab-b^2}{2c}\cdot\frac{b(a-b)}{2c}$$
$$= \frac{b^2-ab-2b^2}{4} + \frac{b(a-b)\left(b(a-b)-c^2-ab+b^2\right)}{4c^2}$$
$$= \frac{b^2-ab-b(a-b)}{4} = -\frac{ab}{2}$$

Similarly, we can prove that the points L and M all lie on the circle. Hence, from I, II and III we conclude that points

D, E, F, H, I, J, K, L, and M lie on the circle, known as the nine-point circle.

IV. We will now establish the first corollary of the Nine-point Circle Theorem: that N, the centre of the nine-point circle, bisects the line segment joining the orthocentre Ø and the centre Q of the circle through A, B, and C. Q is the intersection of the perpendicular bisectors of AB and AC whose equations are, respectively

(6) $x = \frac{a}{2}$ and

(7) $y - \frac{c}{2} = -\frac{b}{c}\left(x - \frac{b}{2}\right)$

From (6) and (7), we find the intersection

$$Q = \left(\frac{a}{2}, \frac{c^2 - ab + b^2}{2c}\right)$$

Also, recall that the orthocentre Ø of ΔABC is

$$Ø = \left(b, \frac{b(a-b)}{c}\right)$$

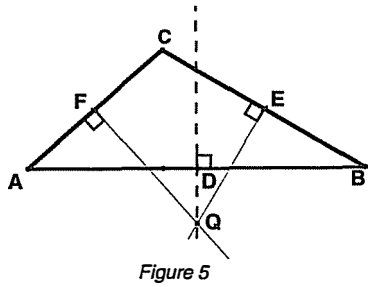


Figure 5

Finding Q, the Centre of Triangle ABC

By the midpoint formula, the midpoint of ØQ is

$$\left(\frac{a+2b}{4}, \frac{ab-b^2+c^2}{4c}\right)$$

which are the coordinates of N.(Refer to part I.)

(V) To prove the second corollary we will show that the point T dividing the line segment ØQ in the ratio 2:1 has coordinates equal to the coordinates of the centroid G of the triangle ABC.

Figure 6 below illustrates the formula for the point of division of a line segment into a given ratio (here, 2:1).



Figure 6

Point of Division of a Line Segment into a Given Ratio Format.

Coordinates of T

$$= \frac{n}{m+n} \text{ coordinates of } Ø + \frac{n}{m+n} \text{ coordinates of } Q$$

Therefore, the coordinates of T are

$$\frac{1}{3}\left(b, \frac{b(a-b)}{c}\right) + \frac{2}{3}\left(\frac{a}{2}, \frac{c^2-ab+b^2}{2c}\right)$$

$$= \left(\frac{a+b}{3}, \frac{ab-b^2+c^2-ab+b^2}{3c}\right) = \left(\frac{a+b}{3}, \frac{c}{3}\right)$$

Also, using median AE, we find the coordinates of G to be

$$\frac{1}{3} \text{ coordinates of } A + \frac{2}{3} \text{ coordinates of } E$$

$$= \frac{1}{3}(0,0) + \frac{2}{3}\left(\frac{a+2}{2}, \frac{c}{2}\right)$$

$$= \left(\frac{a+b}{3}, \frac{c}{3}\right)$$

This proves the second corollary.

In this article we have used the methods of analytical geometry to prove the Nine-Point Circle Theorem along with two corollaries. Incorporated in the high school mathematics curriculum, these methods not only improve students' knowledge of analytical geometry but also enrich and enhance their learning of plane geometry.

▲ DESTREAMING: WILL IT LEAVE US UP THE CREEK?

PETER SAARIMAKI,
COORDINATOR OF MATHEMATICS,
TORONTO BOARD OF EDUCATION

A mathematics perspective of Rosedale Heights Secondary School in Toronto

1. The top view

Rosedale Heights came into being in September 1990 with over 100 students entering the non-streamed grade 9 program. The school, in what was formerly Castle Frank High School, was brought to life by a committee of the Toronto Board of Education to provide an educational model unique in North America. In its commitment to a common non-streamed curriculum, the model rests on values of mutuality, the development of autonomous learning, and the highest standards of professionalism in curriculum development and teaching.

The school established several clear goals, specifically to:

- ▲ provide a program to meet the needs of students with a variety of abilities, interests and backgrounds in preparation for a wide range of post-secondary futures;
- ▲ maximize each student's potential development through
  - non-streamed classes with a non-limiting curriculum
  - access to technical, business and academic options to accommodate different skills and aptitudes;
- ▲ maintain high standards of achievement for all its students;
- ▲ retain a high percentage of students to graduation;
- ▲ involve parents/guardians, teachers and students in a meaningful participatory role in the school; and
- ▲ build links to industry, community agencies and colleges and universities.

When these goals were applied to a philosophy of curriculum, certain beliefs about learning became evident.

Education is defined as a learning experience for everyone who is involved - students, staff, family, community.

Learning occurs best in an atmosphere that challenges people to extend their intellectual, attitudinal and skills boundaries.

Every student deserves and is able to take advantage of enriched learning materials and processes.

Learning is most useful when it is integrated with both theoretical and practical components and when it is connected to a coherent whole, incorporating:

- ▲ content integrated into progressive course units;
- ▲ theory from academic inquiry combined with applications-based information and experience; and
- ▲ the world of issues and behaviours in which we live.

The essential features of the curriculum at Rosedale Heights include:

- ▲ a common curriculum in each subject and in every grade that offers courses common for all students regardless of ability, with a progressive continuum of development that allows students to participate at a self-regulated pace;
- ▲ a blended non-streamed curriculum in which required subject content, skills objectives and time allotments have been restructured to produce a common curriculum, often combining objectives from more than one subject area;
- ▲ a non-limiting curriculum with full access to OAC prerequisites;
- ▲ a holistic, integrated curriculum that fosters appreciation for the inter-relationships of content from different subject areas, enabling students to identify connections and patterns of experience and knowledge;
- ▲ an emphasis on the development of critical thinking skills and attitudes which promote the ability to select and interpret knowledge;
- ▲ descriptors, such as inclusive, systematic, useful, issues-based, real, open, which identify the curriculum as dynamic and relevant to student needs.

2. The front view

At Rosedale Heights the common blended curriculum requires strategies and processes that emphasize

- ▲ teaching defined as a learning process for both the teacher and the student;

- individualized, student-paced programs taught with student centred, cooperative strategies;
- whole group activities that provide students with cohesive experiences to balance individualized learning and bring students together as part of the school community;
- active learning through projects, independent research and hands-on learning, small group learning, peer tutoring and other strategies that encourage creative and independent thinking skills;
- team teaching that builds on the cross-curricular integration of courses;
- evaluation defined and implemented as a process of learning, with a range of evaluation strategies to support and extend student understanding of progress:
  - self/peer/group/teacher (to include the full range of perspectives of those involved)
  - diagnostic (to ascertain needs and develop criteria for appropriate instructional strategies)
  - formative (to promote learning in process)
  - summative (to determine achievement);
- technology driven by educational need, with multi-media learning tools and audio-visual and computer facilities (a business lab, a cross-curricular lab, individual classroom use);
- the student as instructor, through informal and formal peer tutoring and through trained and supervised teaching assistantships.

The implications of these strategies led to the realization that the classroom becomes more of a student-centred resource centre and the teacher takes on the roles of organizer, coordinator and facilitator over a variety of concurrent activities.

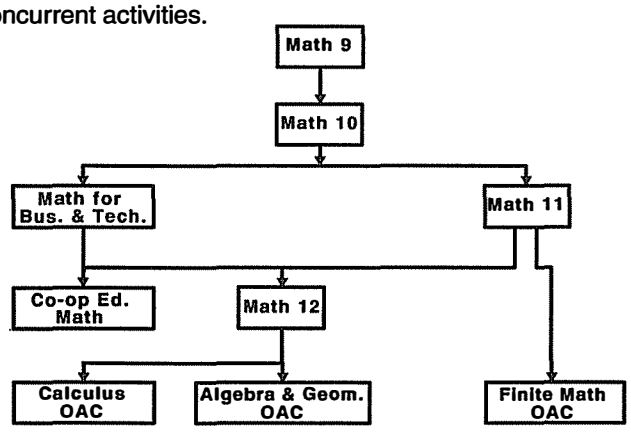


Figure 1

The grade 9 non-streamed course leads into grade 10 (non-streamed), which in turn leads into a grade 11 transition course with areas common to the advanced and general level courses (see flow chart in figure 1).

At each stage, students are exposed to all the topics while the curriculum and teaching strategies are adapted to address each student's individual needs. This recognizes that students' interest, goals and achievements are not static, and ensures students are not trapped in one level.

3. The side view

The mathematics program was designed so that the content in each topic (e.g. grade 9 topics are shown in figure 2) is accessible to all students. This means that the range of content topics is open to all, not that all students will experience the same coverage of each topic. Whenever possible, the depth of exploration is related to the level of abstraction at which the students are capable of operating. Higher levels are available to all, not required of all, but encouraged of all.

Sample Grade 9
Mathematics Course Outline
Rosedale Heights Secondary School

A. Numerical Methods
- Application of Whole Numbers
- Decimals and Integers
- Powers and Square Roots
- Fractions and Rational Numbers
- Applications of Ratio, Rate and Percent

B. Algebra
- Variables and Polynomials
- Formulas and Equations
- Relations

C. Measurement and Geometry
- Measurements and its Application
- Construction
- Properties of Plane Figures
- Coordinate geometry and Congruence
- Transformation

D. Statistics

E. Math and the Computer

Figure 2

Each unit is introduced by a motivational activity, such as a video about shark population sampling and census estimates as a lead into statistics. Using activities such as these underscores the value placed on students' learning guided by the search to answer questions, first at an intuitive, empirical level, then by generalizing, and finally by justifying or proving.

Knowledge emerges from the students' experiences

with these problems and the students recognize the need for particular skills, concepts or procedures. This active, constructive view of the learning process is reflected in the way the various topics are taught. As recommended in the NCTM Standards[1] and the Cockcroft report[2], instructional techniques are varied and include opportunities for:

- ▲ group and individual assignments;
- ▲ appropriate project work;
- ▲ discussion between teacher and student, and among students;
- ▲ practice in mathematical methods;
- ▲ exposition by the teacher.

Mathematics is approached as a whole where concepts, procedures and intellectual processes are interrelated. Through specific instructional activities, the common curriculum makes deliberate attempts to connect ideas and procedures both among different mathematical topics and with other content areas.

The overall goal is to help students become mathematically literate, to empower them, while still providing them with equal access and opportunity, allowing for individual flexibility, so that their options are kept open. Specifically, each student should:

- ▲ learn to value mathematics;
- ▲ become confident in their own mathematical ability;
- ▲ become a mathematical problem solver;
- ▲ learn to communicate mathematically;
- ▲ learn to reason mathematically[3].

4. The inside view

Students are encouraged to complete all objectives in each unit of study. Some are considered compulsory for which students must show a demonstrable level of understanding. The other objectives are negotiable, are designed to extend the competencies of all students, and are exposed to the students.

Integration with other subjects occurs naturally in each unit, so that Numerical Numbers, for example, involves exercises related to Home, Banking and Business. Concepts are introduced and reinforced with manipulatives in each unit. Well used materials include algebra tiles, integer cubes and various mathematical games.

Individualization comes through enrichment activities, independent study, and peer tutoring. Enrichment activities grow out of and beyond the objective and are planned by the students, with or without the teacher's direct involvement based on the theme of the unit in question.

Student evaluation is continuous, varied, flexible and growth oriented. Term work includes such traditional tools as tests, quizzes and homework assignments, counting for 50% of the overall mark. Examinations contribute 30% with compulsory objectives having the greatest weight.

Attitude and work habits account for the remaining 20%. Students are assessed holistically on a range from 0 to 3 for such attributes as working cooperatively, up-to-date notebooks, homework, effort, use of time, respect for others, and having appropriate materials. This scheme emphasizes the importance given to work skills and the affective domain.

Evaluation is done by the teacher, the student, peers and groups. The assessment process is continuous and is part of the overall policy of reporting to parents; in writing about eight times in the year (in each subject) and through four formal parent-teacher-student interview nights.

5. The bottom view

The bottom line, of course, is "does it work?". The first year has had many successes, some challenges, and some revisions. The course outline was manageable. The expectations for students were reasonable. The extra work required, as for any new program, was tiring yet exhilarating because, ... at the bottom line: students took hold of their learning and felt good about it.

Even as the first year unfolded, the second and following years were being planned for. As a pilot program, monitoring is ongoing and, as data becomes available, as techniques become tried (and true!), as longitudinal studies are evaluated ... then we will get a clearer picture of where it all leads.

6. The global view

As far as "destreaming" goes, we are already in the creek. Many teachers in "regular" grade 9 classes are dealing with a wide range of student abilities, a disparate variety of backgrounds, different learning styles and lack of motivation. A non-streamed population already exists in many classrooms, with its attendant problems and challenges, even before any Ministry mandate. Projects such as Rosedale Heights can provide all teachers with strategies and solutions to practical real-life situations.

As with any challenge in education, solutions often lie in the flow of ideas and practices from the classroom, from programs that have been designed with the student as the central focus. So, yes we're in the creek, but I think we're going in the right direction.

1. Curriculum and Evaluation Standards for School Mathematics; NCTM; 1989; page 10

2. Mathematics Counts: The Cockcroft Report; HMSO; 1982, para 243
3. NCTM Standards; page 5

"Destreaming: Will it Leave Us Up The Creek?" – *an update as of Jan. 1993.*

Rosedale Heights now has destreamed both grade 9 and grade 10. The common curriculum is open to all students and each course is written so that it is composed of a continuum of skills and content to challenge all students. At the grade 11 and 12 level, students are taking P or N courses, depending on whether the course is a Prerequisite or Not for OAC courses. Students experience varied program methods, including various cooperative learning strategies, independent study, cross-curricular initiatives, peer tutoring and interdisciplinary work.

Students entering grade 9 next September will also have a choice of two integrated programs: Integrated Creative Arts or Business and Innovative Technology. In addition the school has implemented a teacher-advisor program and developed a unique student record of achievement that is used in conjunction with an anecdotal interim report.

Mathematics in the Transition Years

A Summer Institute for Teachers
Aug. 16 – 20, 1993, Queen's University

The Mathematics, Science and Technology Education Group in the Queen's Faculty of Education is sponsoring a limited enrollment Summer Institute for teachers entitled, Mathematics in the Transition Years: Strategies and Content for Non-Streamed Classes, from the 16th to the 20th of August, 1993. Further information is available from:

MSTE Group Summer Mathematics Institute

▲ **5 Steps** *continued from pg. 23*

subject in which analytical thinking is all pervasive, but it is also the subject which applies these processes to discover its own scope and limitations! As an added bonus, they may look back upon these classroom discussions as one of the most stimulating and enjoyable experiences in their study of mathematics. Give it a try and then share your experiences with us. Good luck!*

**We might pause to question whether there is a rigorous definition of luck.*

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▲ Five Steps to Teaching Proof in the Senior Math Curriculum - A Cooperative Learning Approach

BY BRENDAN KELLY

Introduction

This is part V of a five-part article spanning consecutive issues of the Gazette. In part I, we described briefly the central role of proof in the senior mathematics curriculum and the reasons why we might regard a study of proof as one of the invariants in a changing mathematics curriculum. That article also noted the trend toward alternative instructional methods and, in particular, cooperative learning. The intent of this five-part series is to provide a model to demonstrate how the concept of proof and its evolution might be explored with senior level students in a series of five lessons structured in the cooperative learning format. The intent is to cast the development of proof in its historical context and to show how the quest for certainty led through a cascading sequence in complexities to a point where it was recognized that absolute certainty is an unattainable goal. This development presents the evolution of proof in five independent but sequential lessons corresponding to the five parts into which this article is partitioned.

The lesson in part I of this article, titled *The Dawn of Mathematical Proof: Inductive Thinking and Conjecture,* referred briefly to the inductive thinking of the ancient Egyptians and Babylonians. It prompted students to conjecture formulas in specific cases and attempted to help students discover why proof is needed to support conjecture. The lesson presented in part II, titled *The Development of Direct Proof and Generalization,* was designed to take the students through the next step in the evolution of proof in which the ancient Greeks attempted to underpin conjecture with logical argument based on common premises or axioms. The lesson presented in part III, titled *Indirect Proof and Proof by Counterexample,* presented three examples of indirect proof, two of which were invented by the ancient Greeks. In lesson IV titled *The Quest for Ultimate Rigour,* we introduced students to one of the Zeno paradoxes from the 4th century BC, to explain the motivation for the relentless pursuit of rigour that reached a peak intensity at the beginning of this century. However, it is in this final part, titled, Crisis in the Search for Absolute Proof, that students learn of the unhappy yet intriguing climax to a two thousand year struggle for absolute truth. Rather than attempting to explain the detailed implications of Gödel's theorem, we introduce the students to some of the set theoretic paradoxes which lie at the heart of the completeness-consistency dilemma. Two investigation exercises are provided in this lesson for students to explore and discuss in groups. It is intended that each group will report one or more of its explorations to the class to allow for some important follow-up discussion. An additional investigation is provided as homework to ensure that each student participates in the learning.

We hope that you will find this lesson useful and that this might encourage you to contribute further examples of mathematics topics which are amenable to this instructional format.

Crisis in the Search for Absolute Proof

By the end of the 19th century it was generally believed that mathematical proof had finally reached the ultimate level of rigour and that all mathematical theorems could be deduced from a small carefully chosen set of axioms. However, in 1902, Bertrand Russell enunciated the paradox that now bears his name; a paradox that pulled the rug from under the mathematical framework established by those who believed that all mathematics could be logically deduced from a carefully chosen set of axioms. Gottlob Frege, a German mathematician, who had just completed the major part of his life's work wrote the following comments at the end of this publication dated 1903.

> *"A scientist can hardly encounter anything more undesirable than to have the foundation collapse just as the work is completed. A letter from Mr. Bertrand Russell put me in this position just as my work was all but through the press."*

The Russell paradox was only one of several discoveries which revolutionized the general conception of mathematical proof.

Teaching example 1

We are told that a male barber in a certain village

shaves all those men and only those men who do not shave themselves. Does the barber shave himself?

Solution:
Either the barber shaves himself or he does not.

Statement	Reason
1. Suppose the barber shaves himself.	*This is a conjecture.*
2. Then he is shaved by the barber.	*Statement 1.*
3. But the barber shaves only those who do not shave themselves.	*Given condition.*
4. Therefore the barber does not shave himself.	*Statements 2 and 3.*

Statement 4 contradicts the conjecture, so the conjecture that the barber shaves himself is false. Therefore, the barber must not shave himself. But this contradicts the condition that the barber shaves everyone who does not shave himself. Clearly the given condition is inconsistent with either possibility.

The foregoing example should be used to demonstrate to students that we must be extremely careful in applying the method of indirect proof. In some cases the condition given in a problem may not allow either a statement or its negation to be true!

Teaching example 2

A man who was about to be executed was given an opportunity to make one last statement. If his last statement were true the man would be beheaded. If his last statement were false he would be hanged. The man said "I will be hanged." Explain why the executioner could not perform his duty and the man was allowed to go free.

Solution:
Either the man was to be beheaded or hanged.

Statement	Reason
1. Suppose the man is to be beheaded.	This is a conjecture.
2. The man's last statement is false.	Statement 1.
3. The man must be hanged.	Statement 2 and condition.

Statement 3 contradicts the conjecture that the man is to be beheaded. Therefore the man is to be hanged. However, if the man is to be hanged, his statement is true and the executioner must behead him, contradicting that he is to be hanged. Since either supposition leads to a contradiction, the man could be neither hanged nor beheaded and was set free.

The quest to prove that all mathematical theorems could be deduced from an appropriate set of axioms was ended abruptly in 1931 when a young Austrian mathematician, Kurt Gôdel, enunciated his famous "incompleteness theorem." This theorem asserted in effect that in an axiomatic system of sufficient complexity (such as that defined by the Peano axioms) some theorems would be undecidable. That is, some theorems could be neither proved nor disproved using only those axioms and the principles of logic!

Investigations

Explore and discuss these two paradoxes in your group.

1. According to legend Epimenides, a famous Cretan poet in the sixth century BC, made the statement "All Cretans are liars." Prove that this statement cannot be true.

2a. The statement "Every rule has an exception," is a familiar adage. Explain why the statement in quotes cannot be true.

2b. The statement in "part a" is amended to read, "Every rule except this one has an exception." Is the amended statement self-contradictory? Explain your answer.

Complete the following exercise for homework.

3. Mr. Protagoras, the senior partner in the law firm Punitive, Damage and Fault trained a new young lawyer, Mr. Novice, on courtroom procedures. They agreed that Novice would pay Protagoras for the instruction out of monies received on his first successful trial. After the instructions, Novice set up his practice but no clients arrived. Protagoras threatened to sue Novice for payment, reasoning as follows:

   "Either I will win this lawsuit or you will win it. If I win, you will pay me in accordance with the court judgement. If you win, you must pay me in accordance with our agreement. Either way, I get paid."

   "Not so", responded Novice. "If I win, then in accordance with the court decision I need not pay you. If you win, then I have yet to win my first trial and by our agreement, need not pay you."

   Whose argument is correct? Explain your answer.

If this sequence of five lessons is successfully presented and received, your students should emerge from the experience realizing that mathematics is not only the

▲ *Continued on page 21*

▲ COMPUTERIZING MATRICES: A FERTILE CONNECTION

BY TIM MCNAMARA

Tim McNamara begins his 16th year of classroom teaching this fall, the last six of which were at his high school alma mater, The Nichols School of Buffalo (New York). His other duties at Nichols include Coordinator of Upper School Mathematics, Faculty Editor of the soon-to-be released international math student journal, The Nth Degree, and Head Coach of the school's varsity baseball team. He has spoken at the 1990 Hamilton NCTM Canadian Regional Meeting and 1991 OAME Meeting in Etobicoke and has five articles published in four different state math teacher journals (NY, CT, PA, and NJ). Tim's work has led to his two-time secondary level selection (1990 and 1991) as a finalist in New York State for the Presidential Award for Excellence in Mathematics Teaching.

"Students who are able to apply and translate among different representations of the same problem situation or of the same mathematical concept will have at once a powerful, flexible set of tools for solving problems and a deeper appreciation of the consistency and beauty of mathematics." <NCTM Standards, p. 146>

In reality, however, such opportunities rarely exist for our students, primarily because most secondary math courses are textbook-driven with lessons presented in increments of difficulty rather than in an interrelated fashion. The instructor mindful of this curricular circumstance nevertheless finds solace in the collection of fresh insights and creative applications available in a wealth of math periodicals. Introducing one of these at the appropriate time and possibly adding a personal variation on that theme encourages our students to begin to develop the desired "big picture" mentality implicit in connectivity.

A case in point begins with an interesting article appearing in the April 1985 edition of the NCTM's Mathematics Teacher, written by Daniel C. Alexander and entitled "A Matrix Application Technique for Secondary Level Mathematics." Mr. Alexander shows us how to take N given coplanar points and construct an (N - 1) th-degree function through those points by means of manipulating an equation of representative matrices. For example, let's examine his connection of algebra and matrices by considering how to proceed from any three given points (x_1,y_1), (x_2,y_2), and (x_3,y_3) to a quadratic function of the form

$$y = c + bx + ax^2 \text{ or } y = cx^0 + bx^1 + ax^2$$

The student first needs to understand that, in order to properly evaluate "c", "b", and "a", he or she must set up and solve a like-numbered system of simultaneous equations (which many second-year algebra and discrete math textbooks use as a prelude to demonstrating methods of solution such as Cramer's Rule and finite differences, respectively). Combining the above:

$$y_1 = cx_1^0 + bx_1^1 + ax_1^2$$

$$y_2 = cx_2^0 + bx_2^1 + ax_2^2$$

$$y_2 = cx_3^0 + bx_3^1 + ax_3^2$$

Mr. Alexander then transforms this system into a matrix equation. He labels the column of y's

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

the (3x3) array of x's

$$A = \begin{pmatrix} x_1^0 & x_1^1 & x_1^2 \\ x_2^0 & x_2^1 & x_2^2 \\ x_3^0 & x_3^1 & x_3^2 \end{pmatrix}$$

and those sought after coefficients "c", "b", and "a"

$$C = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

He thus obtains the matrix equation Y = AC.

Solving for C from the equation Y = AC involves A^{-1}, the multiplicative inverse:

$A^{-1} Y = A^{-1} (AC) = (A^{-1} A)C = I_3 C = C$

Textbooks generally discuss an algorithm which begins with the (3 x 6) matrix $[A: I_3]$ and, through a series of elementary row operations, transforms the left half into I_3 and the right half into A^{-1}. From the teaching and evaluation perspectives, however, this method presents too many variations and, in fact, may be omitted entirely by an instructor with that curricular option. Some calculators are equipped to find A^{-1} quickly, but let's consider staying the course Mr. Alexander charted for us and discovering some

new avenues for learning mathematics connectively. Specifically:

1. research a more mechanical approach to finding A^{-1},
2. write a computer program to evaluate A^{-1}, and
3. incorporate the program into another program which, using three points, will identify the quadratic function itself.

1. Some books available on the subject of matrix algebra discuss a method of finding A^{-1} by means of cofactors (or minors). There are four steps to the process, best demonstrated by an example. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Step #1: Find the determinant of A. In this case, Det (A) = 3.

Step #2: Identify the cofactor for each element in A, and create a new cofactor matrix with those numbers. To show how to find one, we target $12 = a_{33}$. Temporarily "blocking out" both the 3rd row and 3rd column of matrix A, we are left with the (2 x 2) matrix:

$$\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$$

whose determinent is $1 \bullet 3 - 1 \bullet 2 = 1$. Repeating this for each member of matrix A, we obtain the matrix

$$\begin{pmatrix} 11 & 7 & 2 \\ 9 & 9 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$

Step #3: Negate every other element of this new matrix, that is, a_{12}, a_{21}, a_{23}, and a_{32} (those elements for whom the sum of the subscript digits is odd) and then divide each by the determinant. This yields

$$\begin{pmatrix} \frac{11}{3} & \frac{-7}{3} & \frac{2}{3} \\ -3 & 3 & -1 \\ \frac{1}{3} & \frac{-2}{3} & \frac{1}{3} \end{pmatrix}$$

Step # 4 Transpose the above matrix (rows become columns, and columns become rows), and the process is complete:

$$A^{-1} = \begin{pmatrix} \frac{11}{3} & -3 & \frac{1}{2} \\ \frac{-7}{3} & 3 & \frac{-2}{3} \\ \frac{2}{3} & -1 & \frac{1}{3} \end{pmatrix}$$

The reader should verify that $A \bullet A^{-1} = I_3$.

2. Besides being fairly routine to follow, the step-by-step nature of the cofactor method for finding A^{-1} translates well into the computer program "Three by Three" (Figure 1). Pay particular attention to step 100 (a rounding function for answers to three decimal places), steps 300 and 320 which find the determinant for

$$\begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix}$$

and steps 400-480 which yield the inverse

$$\begin{pmatrix} R & S & T \\ U & V & W \\ X & Y & Z \end{pmatrix}$$

All of these essentially represent the four steps outlined previously. Encouraging our students to make the connection between the algorithm in part 1 and the steps in the program (as opposed to using some "canned" calculator or software package) fosters a deeper understanding between the two.

3. "Given the three points (-4,3), (-2,-5), and (1,13), find the equation of the parabola whose graph passes through all three."

Let us coordinate what we now know about the "order 3" inverse matrix and develop a second program, "Parabola 3 Points" (Figure 2), which will take us to quadratic functions and beyond. For

$$y = Lx^0 + Kx^1 + Jx^2$$

We have the three equations

$$3 = L(1) + K(-4) + J(16)$$

$$-5 = L(1) + K(-2) + J(4)$$

$$31 = L(1) + K(1) + J(1)$$

Therefore, we need to solve

$$\begin{pmatrix} 1 & -4 & 16 \\ 1 & -2 & 4 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} L \\ K \\ J \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \\ 13 \end{pmatrix}$$

Steps 400 to 580 compute the required inverse matrix and then steps 600 to 620 compute the required coefficients

▲ *Continued on page 32*

▲ BOOK REVIEW: RELEARNING MATHEMATICS

JEFF CATANIA
FACULTY OF EDUCATON, CLASS OF 1992
UNIVERSITY OF WESTERN ONTARIO

Relearning Mathematics
A Different Third R – Radical Maths
Marilyn Frankenstein
Free Association Books:
London, UK. 1989 ISBN 1-85343-091-9

"Friends, comrades, gentlefolk – ARISE! To the oppressed and neglected of our society – gather round! To those wallowing in mathematics anxiety and ignorance, failed by your schools and left to flounder in a sea of innumeracy – band together! ARISE, I say! Sum your fractions, multiply your numbers and integrate your factors into one great power! For united we stand, divided we fall! Take up the mighty arithmetic, algebra and calculus. With them we shall solve the great inequality and form a new math! A fairer math! A radical math!!!"

Strong words for a meek and mild subject like mathematics? One might agree, but not after Relearning Mathematics. It presents the informed but oft-forgotten notion that mathematics cannot be separated from our non-neutral society or from other ways of knowing about the world, such as reading and writing. Marilyn Frankenstein agrees with Anthony Petrosky that all domains:

> *"...are aspects of understanding, and theories that attempt to account for them outside their interactions with each other run the risk of building reductive models to human understanding."*

These aspects of understanding are blended elegantly in the text, which presents basic mathematics learning from a strong socio-political platform, and relates this learning to personal empowerment at every stage. The result is a powerful formula to help those with math anxiety overcome their fears and gain math literacy.

One might expect such a work from Frankenstein, who has in her varied background taught mathematics to both children and adults. This experience lends her a strong affinity for students with mathematics anxiety, which shows through in the caring, sensitive nature of the text. She somehow balances an urgency for using mathematics to enact change with sound, comforting methods to demystify the meaning of numbers and variables. Her earnest yet caring tone throughout is a balm to the anxious student that carries them from idea to idea with curiosity and confidence.

Part One of the text exemplifies this sensitivity. In it, the student/reader identifies their math anxiety (if any) and is given a myriad of opportunities to explore mathematics in a non-threatening way. Students then read, write and think their way through a do-it-yourself math therapy. A section devoted to mathematics anger follows which will show, even to the most analytical among us, that with all its connections to politics, art and recreation, mathematics is both vital and exhilarating.

Supporting these resources is a strong methodological backbone of effective strategies for success in mathematics: sections on calculator use, journal writing, group work, and self-evaluation precede any numerical work or theory. From this base, one is brought to the language of mathematics slowly and painlessly, encountering a vast array of ideas and activities along the way. Frankenstein has created a lush resource of strategies and problems for mathematics teachers everywhere to draw from: articles, charts, truths, misconceptions, conversations, stories, art, cartoons, journals, excerpts, statistics, poetry, graphs, photos, essays, history, fables, error-analysis, and yes, even straight mathematical drill. Far from appearing contrived, these exercises are thoughtful and relevant, demanding more than just fact recollection and algorithmic skills to solve. Examples of such exercises and excerpts from the text show this quality.

> *"... a Hindu saying: Ten men live in such a way that they allow one to take precedence. Without this one they have as little meaning as zeros unless they are preceeded by a 'one'."*

Write briefly about what this saying means:

- *a. literally, that is, in terms of the use of the zero in the place value system;*
- *b. figuratively, that is, how ... the use of zero in the place-value system relates to people organizing themselves in a group;*
- *c. do you agree or disagree with this saying, and why?"*

Read the following ... [and] write about:

- *a. which information was most interesting to you and why;*
- *b. which information was confusing, and what*

questions do you need answered before you can understand that information.

"... Our base 10 system is certainly a consequence of this, while the French words for twenty, eighty and ninety-vingt, quatre-vingts and quatre-vingt-dix - suggest an older base 20 system. The Mayas ... also used a base 20 system 1,500 years ago to create calendars more accurate than the Gregorian one we use today. Even the ancient Babylonian-Sumerian base 60 system, which survives in our measurement of time, angles and geographical position, was probably derived from finger counting. ...

"[from a fable] ... Numbers have revealed themselves to only two or three geniuses in human history, but geniuses die and all numbers remain alive. One living genius is 9-year-old Elizabeth Smith, who is about to give up studying numbers because other people think she is crazy for doing it.

"In a dream one night (or some dreamlike out-of-body experience), 6 is translated from Numberland to Elizabeth's room. He meets her, talks to her, convinces her to keep studying numbers and is wafted back to Numberland a changed digit. He is convinced now that numbers and people can meet, and he has a nagging doubt about the immortality of numbers ..."

Full, clear solutions to the exercises complement every exercise. And somehow with each cognitive advancement there is mathematics education lurking in the wings, prompting the reader to regard mathematics not as a barrier to be vaulted but as a 'tool for empowerment'.

This prompting is vigorous and continual. The problems are divided into categories by their cognitive nature, but the content rarely leaves the arena of social action. The following are excerpts from problems in the signed numbers and mathematics anger chapters respectively:

"... Reagan's first year is showing itself ... in a shift of dollars away from states and cities and toward a military buildup financed largely by cuts in domestic programs. Discretionary spending for the military was up $30.3 billion and discretionary non-military domestic spending was cut $25.4 billion."

"In Mexico, where 80 percent of children in rural areas are undernourished, livestock consume more grain than the entire population – livestock that is then exported to the USA to make hamburgers."

For adults, these issues are a welcome application of the material, but to younger students they might be a distraction, especially when each new problem offers another thought-provoking issue to attend to. Furthermore, some statistical infirmities and errors in the solutions may require a teacher to administer the text to a younger class carefully, especially to ensure that the reading level of any particular problem is appropriate.

However, the text is a wonderful resource for mature children and adults in any educational environment, especially for those experiencing mathematics anxiety. It has the flexibility to be used as an independent, remedial resource and offers abundant excursions for advanced students – indeed it is a treasure-trove of ideas for math educators. In addition there is patient, thorough instruction in mathematical principles with logically presented examples, drill, problems and solutions. All of the actual mathematical instruction is presented at the start of each chapter in a clear, organized way, never dominating the text but supporting the more relevant issues around which mathematics revolves. Clearly, the range and flexibility of the text as a tool for learning is its greatest strength. Due to this flexibility however (and a high density of ideas) only a minimum of content is covered – an amount approximately equal to one-half of the grade nine general-level course. A second textbook is to be published which would continue where the first leaves off. Nonetheless, Frankenstein has created in her first work a remedial mathematics teaching instrument that synthesizes a lifetime of study and reflection. Her passion for social change and empowerment of the oppressed, although by no means novel, is unique in the realm of mathematics education and provides a refreshing, human departure from the sterile curricula of today. How many students of these curricula reject mathematics literacy as unimportant or unattainable? Relearning Mathematics knows their struggle and rallies them for a revolution. What are the odds of finding educational artistry such as this in the non-political textbooks used in our schools, and what is the percentage chance that teachers will be able to freely use such a work in their classrooms? The answer, perhaps, is determined by each individual's determination to enrich their teaching and to question the nature of mathematics education today.

"The odds of finding a pearl in a pearl oyster are 1 to 12,000. What is the percentage chance of finding a pearl in a pearl oyster?"

COUNCIL COMMENT

BOB ROBINSON
GOLDEN SECTION CHAPTER REPRESENTATIVE



These notes were taken at the OAME Council Meeting held January 23, 1993 at the Ramada Inn-Airport West in Mississauga.

Correspondence

If a Mathematics Advisory Group (MAG) is formed, the representative for OAME will be Arnie Carefoote.

A grant has been awarded to OAME by the NCTM to assist us with the "Focus on the Renewal of Mathematics Education" project.

Correspondence is ongoing with the Ministry of Education regarding our concerns about the National Indicator's Project. Other letters have also been sent regarding recent press releases.

Math Equity Breakfast

The Annual Breakfast is scheduled for Friday, May 14 at the Windsor Annual Meeting.

Nominations

Nominations have been accepted and the membership is encouraged to exercise its opportunity to vote.

Publications

The June Issue of the Gazette will be a focus issue pertaining to the "Transition Years". Advertising opportunities are available for the September issue. For more information contact: Bob Robinson, Advertising Manager, Box 63062, University Plaza, Dundas, ON, L9H 6Y3, Phone (416) 628-5563, Fax (416) 628-9250

Membership

Council has adopted a new logo. See the front cover of this issue.

Early Years/Formative Years

The committee is dealing with: "What is it that the classroom teacher needs to implement the new curriculum?" The committee will arrange for articles to appear in forthcoming issues of the Gazette. Ron Sauer will represent OAME on the Provincial Primary/Junior Curriculum Committee of the Ministry. The Early Years/Formative Years committee chair is Peter Saarimaki.

Transition Years

Continuing discussions about suggested curriculum revisions will result in draft materials being available for discussion at the March Council Meeting. The committee chair is Barry Scully.

Specialization Years

This committee is working very hard, with little input coming from the Ministry. Draft documents are being produced. The committee chair is Jerry Maloney. (See Jerry's report on page 10.)

Focus on the Renewal of Mathematics Education

The draft version (Jan. 1993) was presented for discussion by Judy Crompton. Responses and comments to the Early Years and Formative Years portions were to be sent to Judy by February 26.

Benchmarks

OAME's response to the September, 1992 draft of the Ontario Provincial Mathematics Benchmarks was presented and discussed by Council.

NCTM

Jan Scully presented materials for the Chapter reps and announced new periodicals to be released by the council and also new fees.

OAME'93

Todd Romiens is excited about the upcoming Annual in Windsor. Although there is emphasis on the Transition Years, there will be offerings for all levels from K through OAC's.

OAME'94

Kingston will be the host for OAME'94. The site will be Queen's University with Brian Gallagher and Margo Johnston serving as chairs.

Leadership'92

Very positive and supportive evaluation responses were distributed by Jeri Lunney. Well done, Jeri!!

Leadership'93

We are looking forward to Leadership'93 which will be chaired by Ron Lancaster who along with his committee is putting together an exciting program. The focus will be on the Transition Years and will deal with issues such as

assessment, enrichment, integration, teaching styles, classroom management, and technology. This event will take place on November 11 through November 13.

Chapter Reports

COMA

Over 150 registrants from various locations throughout the valley were expected to attend their January mini-conference. COMA executive has been frustrated regarding having access to the Leadership seminars organized by OAME.

CHAMP

MATHFEST was held on November 5th in Brampton. Over 175 people enjoyed the productive work sessions and delicious dinner provided by the students of North Peel Secondary School. The sixth annual CHAMP Mathematics Contests for students in grades 8-11 took place in November. This year approximately 5700 students from 180 schools across four boards participated. The winning schools were:

Grade 8: New Central P.S. (Halton)

Grade 9: Martingrove C.I. (Etobicoke)

Grade 10: The Woodlands School (Peel)

Grade 11: Martingrove C.I. (Etobicoke)

GOLDEN SECTION

The Niagara-Brock Interface Group met in November to consider a response to the Benchmark document. In general, the group was not in agreement with the content of the document. Their annual Intermediate/Senior teacher's meeting is scheduled for Thursday, April 22, 1993 at Brock. The focus is the Transition Years.

GVMA

The Spring Conference will be April 15 in the Davis Centre at the University of Waterloo. The new set of Calculus exams is still in the development stage but will be ready for June. Jack Weiner's (et al) new publication will be entitled "Technology in the Math Classroom". There will be chapters on Symbolic Math Processors, Logo in your Classroom, Graphing Calculators, and Using Spreadsheets in the Classroom.

ISOMA

The ISOMA Executive met at Upper Canada College in November and decided that there were many informal issues which should be brought before the membership. A newsletter has been produced to help address these issues. The annual fall conference has been scheduled for Thursday, October 18, 1993.

NOMA

NOMA is proceeding with plans to hold their fall conference on October 1 and 2, 1993. They will again offer Math Certificates to all schools served by NOMA. Math awards will be given to the winners of their Math contest at the Intermediate level. NOMA would like to receive any draft or final curriculum guidelines re destreaming.

OSLMA

Plans for a Spring Conference are nearly complete. The location will be the Royal Military College at Kingston. Speakers will present information on a destreamed, student centred, learning program for grade 9 math at Perth Collegiate, cooperative learning, evaluation, cross-curricular activities and "Grass Roots UP".

SAME

SAME has plans underway for their spring conference in mid-April which will appeal to elementary teachers. In the fall they experimented with a single-topic, after school workshop designed to appeal to teachers of the senior levels. The event was very successful and they plan to use this format more often. This spring SAME will be initiating an annual SAME Classroom Materials Development Awards. Teachers will be asked to submit proposals for developing their own classroom materials required to present a unit in mathematics.

SWOAME

Over 110 people registered for their fall conference which was a success. Their efforts are now concentrated on providing an outstanding annual conference in May.

WOMA

A Spring Conference will be held in April in Tillsonburg. One of WOMA's projects is to publish a more "formal" newsletter.

Y^{4}MA

A very successful conference was held last fall.

▲ MATH IN THE NEWS:
MATHEMATICS AND THE WORLD AROUND US – APRIL 1993

RON LANCASTER, ST. MILDRED'S LIGHTBOURN SCHOOL,
1080 LINEBROOK ROAD OAKVILLE, ONT. L6J 2L1
Leave all messages with Donna Cossitt – (416) 845-2386 – or FAX: (416) 845-4799

Readers are invited to submit items from newspapers, magazines, books or other sources that contain information that can be related to any area of mathematics. Sufficient information about the source of the item should be given so that other readers can obtain additional details. Teachers are encouraged to use the contents of this regular feature of The Gazette as an aid to helping their students see the connection between mathematics and the world around us. The content may not fit in with what you are currently teaching to your students, but many of the items can be used to begin your classes in an interesting and engaging manner.

▲ ▲ ▲

What a Difference Five Years Makes in the First Language of Residents in the Greater Metro Toronto Area, *The Toronto Star,* September 16, 1992, pg. A23.

This article discussed in detail how the face of Toronto has changed over the past five years. The chart given to the right was included in the article.

Language	1986	1991	% Change
English	2 395 150	2 584 050	7.8%
Italian	193 205	189 265	-2%
Chinese	94 210	175 075	86%
Portuguese	78 035	95 305	22%
Polish	37 965	58 540	54%
Spanish	29 805	55 725	87%
French	43 845	49 800	27%
German	52 760	48 110	-9%
Greek	45 960	46 875	2%
Tapalog (Philipino)	13 315	36 905	177%
Punjabi	12 535	31 965	136%
Ukrainian	29 130	24 605	-15%
Hungarian	19 575	24 915	7%
Arabic	8 955	19 560	118%
Tamil	2 055	19 235	836%
Vietnamese	8 875	18 985	114%
Persian (Farsi)	3 875	16 889	336%

Students from all parts of Ontario will probably be fascinated with the numbers from this article and the information should help to acquaint them with how the face of Toronto has been changing. Teachers might want to use the following questions as part of an assignment that integrates mathematics with topics from the social sciences.

Questions

a. Assuming that the first two columns are correct, are there any errors in the figures given in the % change column? Correct all mistakes that you find.

b. Assuming that no other languages are involved, find the population of Metro Toronto in 1986 and 1991. Find the percent change in the population of Metro Toronto from 1986 to 1991.

c. Find the average of the numbers in the % change column. Do this in the usual way by adding up the 17 numbers and then dividing this total by 17. Compare this answer to your answer to part b. Should they be the same?

d. Assuming that each language group continues to grow at the same rate and that no other languages are involved, find the population for all 17 language groups for the next 50 years. Use a constant interval of 5 years and for each time period rank the language groups from largest to smallest.

e. After how many years will the English language group account for 50% of the total population? After how many years will the Tamil group be the largest?

Comments

In the next issue I will include answers to the above questions. I will be using a spreadsheet to work out all of the answers and I would encourage you to do so too. These questions are suitable for students of all levels from Grade 7 to OAC. If these questions are used as part of a multi-cultural unit, then you might have your students research each language and learn at least one expression from each one.

▲ ▲ ▲

It wasn't mathematics that elevated Stronach, Letter to The Editor from David White of Toronto, *The Toronto Star,* November 8, 1992

"Frank Stronach, Chairman of Magna International, complains that his annual pay of $5 million is inadequate for the hours he's worked over the years. He is quoted as saying, "*Look, if I counted all the hours I spent since Day 1, I don't think I'd be getting minimum wage.*"

▲ ▲ ▲

Questions

a. At the time of this article Stronach was 60 years old. If the $5 million is payment for every hour that Stronach has been alive, what would his hourly pay be?

b. How long would it take a person earning minimum wage $6.35 per hour and working 40 hours per week to earn a total of $5 million?

Comments and Answers

a. In his letter to the Editor, David White worked out the answer to this question and he came up with a rate of about $9.50 for every waking and sleeping hour since birth.

b . In a second letter to the Toronto Star on the same topic and day, Libby Routh of Toronto proposed and solved this question. She determined that it would take more than 378 years to earn $5 million.

▲ ▲ ▲

Education, Just Testing, *The Globe and Mail,* February 1, 1993.

"Given a regular pyramid V with a square base, there is a ball with its center on the bottom of the pyramid and tangent to all edges. If each edge of the pyramid base is of length a, find the following quantities:

1. the height of V;

2. the volume of the portion common to the ball and the pyramid."

Comments and Answer

The article mentions that this is a problem from an entrance paper for Humanities students at Tokyo University. The following answers appeared at the end of the article and I would be interested in hearing from readers of this column who are able to verify these answers.

a. The height of V is

$$\frac{\sqrt{2}a}{2}.$$

I wonder if students would have been given credit had they given the answer

$$\frac{a}{\sqrt{2}}$$

b. The required volume is

$$\left(\frac{7\sqrt{6}}{54} - \frac{1}{4}\right)\pi a^3$$

This is a nice problem, but I do wonder why the Humanity students are being asked this type of question. Like the tetrahedron, I guess they really must be on the ball.

Other Recommended Articles

By the Numbers, by Kleinwort Grieveson

CEO John Brew likes nothing better than unraveling a good math puzzle or two. *The Institutional Investor,* August 1986, pg. 21. You might want to share the following quote from this article with your students. Kleinwort Grieveson's Brew: The key to his love of mathematics he says is laziness:

"Those who are mathematically inclined are always on the lookout for an easier way to solve problems."

▲

Mathematics of Coin Tossing predicts No, *The Toronto Star,* October 22, 1992, pg. A25.

▲

Should Tracking Be Derailed? Mix, Don't Match, Selecting the Smart Set, *The New York Times,* November 1, 1992, Education Section.

Three articles with different viewpoints about destreaming.

▲

Family Math: Helping to cure fears of the world's Barbies, Louise Brown, *The Toronto Star,* November 3, 1992, pg. B2.

Louise Brown has a column in The Toronto Star every Tuesday and she often writes about educational issues. Her columns are well worth reading.

▲

Who Owns 'X'?, *The New York Times,* November 8, 1992, Section 9.

An interesting article about the controversy surrounding the licensing of goods related to the movie Malcolm X. The article fails to point out 'X' is in fact owned by the mathematical community!

▲

Suspected Toilet Vandal Arrested, *The Friday Ryersonian,* November 20, 1992.

"Esclante says that when he was asked why he was vandalizing the toilet, the suspect replied that he had failed a math test"

Many thanks to Allan Gillis of Ryerson for this item.

▲ Math In the News:

On the Trail of the Meaning of Life, *The Toronto Star,* November 28, 1992.

This is an excellent article about Ann Druyan, activist and writer from New York City. The excerpt from the article given below shows how easy it is for a mathematics teacher to tum a student away from the subject.

"She was derailed from her interest in science and math when a junior high teacher ridiculed her over her excitement of pi. "I raised my hand and said 'You mean this applies to every circle in the universe?' and the teacher told me not to ask stupid questions," she recalls. "And there I was having a religious experience, and she made me feel like such a fool."

▲ Matrices: *continued from page 25*

$$\begin{pmatrix} R & S & T \\ U & V & W \\ X & Y & Z \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -5 \\ 13 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 2 \end{pmatrix}$$

We now have the final equation, $y = 3 + 8x + 2x^2$, from which we can choose to graph its parabola, find its zeroes with methods ranging from the quadratic formula to Newton's successive derivatives, identify highlights of the parabola such as the vertex and the focus, all with the possibility of adding commands to the basic "Parabola 3 Points" program. Remarkable, then, are the consequences of combining ideas for both ourselves and our students, as if putting pieces of a math puzzle together in different ways gives fresh insights and a renewed appreciation. Perhaps that's what the true message of the classroom really is:

"A little learning is a dangerous thing;
Drink deep, or taste not the Pierian spring;
There shallow draughts intoxicate the brain.
And drinking largely sobers us again."

– Alexander Pope
Essay on Criticism

Golden, Neal, **Computer Programming in the BASIC Language,** Harcourt Brace Jovanovich, Inc., New York, 1981.

Pettofrezzo, Anthony J., **Matrices and Transformations,** New York: Dover Publications, Inc., New York, 1966.

School Mathematics Study Group, **Introduction to Matrix Algebra,** A.C. Vroman, Inc., Pasadena, 1965.

Working Groups of the Commission on Standards for School Mathematics, **Curriculum and Evaluation Standards for School Mathematics,** National Council of Teachers of Mathematics, Reston, VA, 1989.

The Program

```
]LOAD THREE BY THREE
]LIST
10  REM THIS PROGRAM USES
    THE COFACTOR METHOD
20  REM OF FINDING THE
    INVERSE OF A 3 X 3 MATRIX
50  HOME
100 DEF FN A(X) = .001 * INT
    (1000 * X + .5)
110 PRINT "THIS PROGRAM WILL
    COMPUTE THE"
120 PRINT "INVERSE OF A 3 X 3
    MATRIX."
130 PRINT
150 PRINT
**************************
160 PRINT
170PRINT "READING ACROSS
    EACH ROW, PLEASE"
180 PRINT "ENTER THE
    ELEMENTS OF MATRIX A,"
190 PRINT "SEPARATING EACH
    NUMBER BY COMMAS:"
200 INPUT A,B,C,D,E,F,G,H,I
300 M = A * E * I + B * F * G + C *
    D * H - (C * E * G + A * F * H +
    B * D * I)
320 IF M = 0 THEN PRINT : PRINT
    : PRINT "SINCE MATRIX A
    HAS ITS DETERMINANT = 0."
    : PRINT "IT, THEREFORE,
    CAN NOT HAVE AN
    INVERSE." : GOTO 999
400 R = FN A((E * I - F * H) / M)
410 S = FN A((C * H - B * I) / M)
420 T = FN A((B * F - C * E) / M)
430 U = FN A((F * G - D * I) / M)
440 V = FN A((A * I - C * G) / M)
450 W = FN A((C * D - A * F) / M)
460 X = FN A((D * H - E * G) / M)
470 Y = FN A((B * G - A * H) / M)
480 Z = FN A((A * E - B * D) / M)
500 PRINT
510 PRINT " THE INVERSE OF
    MATRIX A IS:"
520 PRINT
530 PRINT, R,S,T
540 PRINT
550 PRINT
560 PRINT, U,V,W
570 PRINT
580 PRINT
590 PRINT X,Y,Z
999 END
]RUN          (FIGURE 1)
THIS PROGRAM WILL COMPUTE
    THE INVERSE OF A 3 X 3
    MATRIX.
**************************
READING ACROSS EACH ROW,
    PLEASE ENTER THE
    ELEMENTS OF MATRIX A,
    SEPARATING EACH
    NUMBER OF COMMAS:
?1,2,3,1,3,5,1,5,12
THE INVERSE OF MATRIX A IS:
3.667      -3       .333
-2.333     3        -.667
.667 -1    .333
]LOAD PARABOLA 3 POINTS
]LIST
10  REM THIS PROGRAM USES
    THE COFACTOR METHOD
20  REM OF FINDING THE
    INVERSE OF A 3 X 3 MATRIX
30  REM TO ASSIST IN
    IDENTIFYING THE EQUATION
    OF
40  REM THE PARABOLA
    THROUGH THREE GIVEN
    POINTS.
100 HOME
110 PRINT "THIS PROGRAM
    FINDS THE EQUATION OF
    THE"
120 PRINT "PARABOLA
    THROUGH THREE GIVEN
    POINTS."
130 PRINT
140 PRINT
**************************
200 PRINT
210 PRINT "PLEASE ENTER THE
    COORDINATES OF THE"
220 PRINT "FIRST POINT,
    SEPARATED BY COMMAS:"
230 INPUT B,N
240 PRINT
250 PRINT, "THE SECOND
    POINT:"
260 INPUT, E,P
270 PRINT
280 PRINT "THE THIRD POINT:"
290 INPUT, H,Q
300 A = 1
310 D = 1
320 G = 1
330 C = B ^ 2
340 F = E ^ 2
350 I = H ^ 2
400 M = A * E * I + B * G + C * D * H
    - (C * E * G + A * F * H + B * D *
    I)
410 IF M = 0 THEN PRINT : PRINT
    : PRINT "SINCE MATRIX A
    HAS ITS DETERMINANT = 0."
    : PRINT "IT, THEREFORE,
    CAN NOT HAVE AN
    INVERSE.": GOTO 999
500 R = (E * I - F * H) / M
510 S = (C * H - B * I) / M
520 T = (B * F - C * E) / M
530 U = (F * G - D * I) / M
540 V = (A * I - C * G) / M
550 W = (C * D - A * F) / M
560 X = (D * H - E * G) / M
570 Y = (B * G - A * H) / M
580 Z = (A * E - B + D) / M
600 L = R * N + S * P + T * Q
610 K = U * N + V * P + W * Q
620 J = X * N + Y * P + Z * Q
700 PRINT
710 PRINT
**************************
720 PRINT
730 PRINT "THE EQUATION OF
    THE PARABOLA IS:"
740 PRINT
750 PRINT "Y = ";J;"X^2 + ";K;"X +
    ";L
999 END
]
RUN
THIS PROGRAM FINDS THE
EQUATION OF THEPARABOLA
THROUGH THREE GIVEN
POINTS.
**************************
PLEASE ENTER THE
COORDINATES OF THE FIRST
POINT, SEPARATED BY COMMAS:
?-4,3
THE SECOND POINT:
?-2,-5
THE THIRD POINT:
    (FIGURE 2)
?1,13
**************************
THE EQUATION OF THE
PARABOLA IS:
Y = 2X^2 + BX + 3
```

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Ann Pearce

Barry Scully

Ontario Association for Mathematics Education/Balance Sheet

Sept. 1, 1991 to Aug. 31, 1992

Assets

Current Assets:	
Cash	$ 54,309.42
Short Term Investments	21,000.00
Prepaid Exp.-Conference '93	4,000.00
	$ 79,309.42

Liabilities and Surplus

Current Liabilities:	
Accounts Payable	$ 0.00
Membership fees Paid In Advance	50,000.00
Leadership '92 fees Paid in Advance	4,475.00
Total Current Liabilities	$ 54,575.00
Surplus:	
Balance Sept. 1, 1991	$ 18,890.00
Operating Surplus for the Year	5,843.52
Balance Aug. 31, 1992	$ 24,734.42
	$ 79,309.42

Statement of Operations
Sept. 1, 1991 to Aug. 31, 1992

Revenue	
Membership Fees	$ 62,339.45
Conference Fees	47,117.00
Interest Earned	3,275.94
Leadership '91 (net)	1,982.44
Other	2,904.10
	$117,618.93
Expenses	
Meetings Expense	$ 31,552.72
Gazette	26,203.76
Chapter Rebates	12,574.95
Committees	11,672.50
Honoraria	8,560.00
Ontario Standards	4,532.40
Election Expenses	2,397.03
N.C.T.M.	2,004.95
Centennial History Project	2,000.00
Conference Expenses	1,164.14
Secretarial Expenses	986.46
Postage	846.44
Gifts and Awards	828.78
Telephone and Fax	828.78
Other	2,084.36
	$ 111,775.41
Operating Surplus for the Year	**$ 5,843.52**

Your 1992-93 Council in Action







