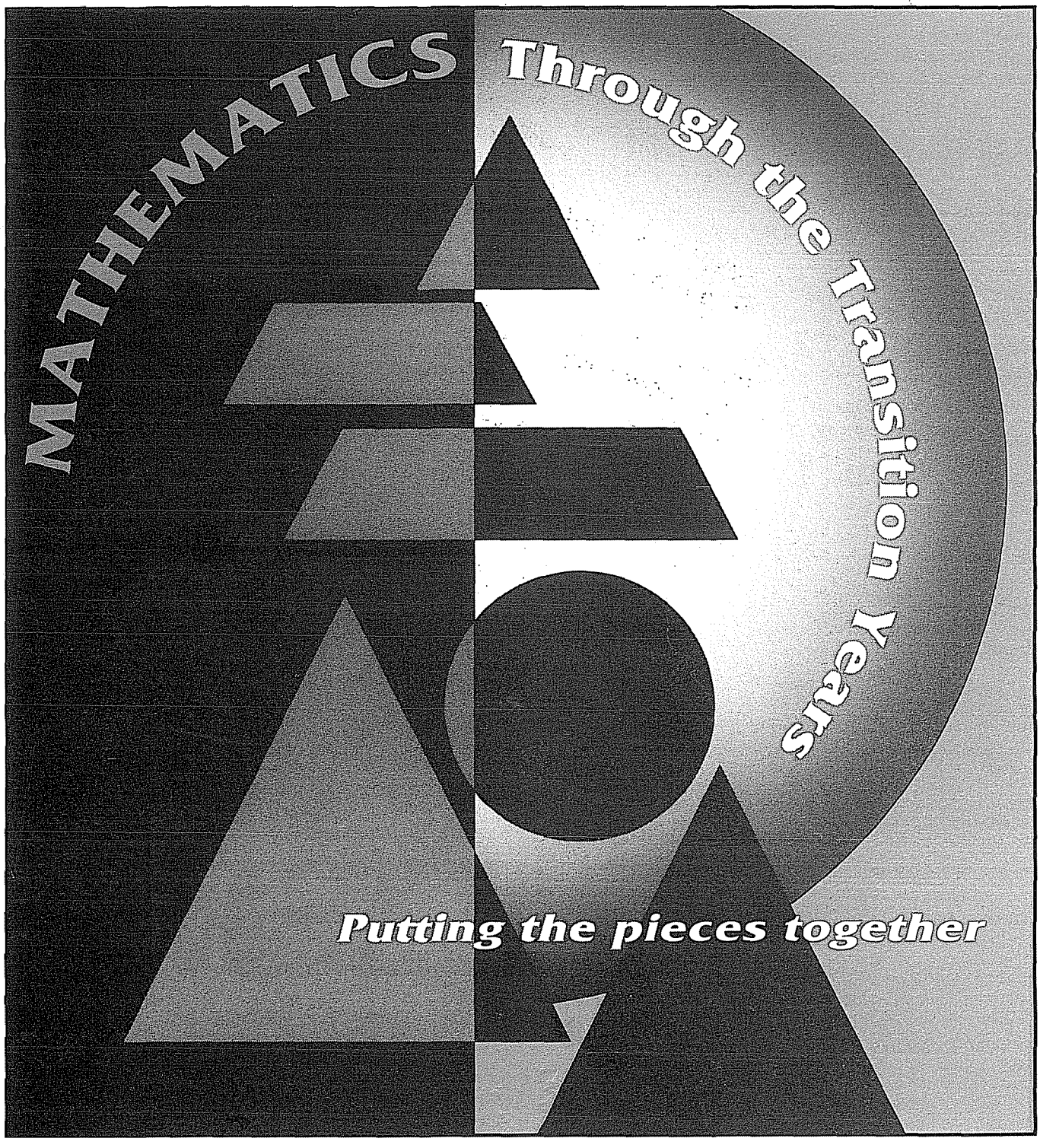
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Deadlines for information to be included in the Gazette or the Abacus are: June 15 for the September issue, October 15 for the December issue, January 15 for the April issue, and April 15 for the June issue.

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Editorial

by Gary Flewelling

The Ontario Ministry of Education has set out to restructure education from K to OAC. Programs from Grades 1-9 are now to be based on the learning outcomes contained in the working paper *The Common Curriculum, Grades 1-9,* February 1993. This document supersedes *The Formative Years,* 1975, and the subject guidelines for Grades 7 to 9, including those developed under *Ontario Schools, Intermediate and Senior Divisions* (OSIS), 1989. Before the mathematics component of the program envisaged in *The Common Curriculum* can become a classroom reality, the learning outcomes will need to be interpreted, expanded upon, and clarified. There also needs to be a clear understanding of the new emphasis in mathematics education and how the new mathematics program should differ from the present one. This inaugural "focus issue" of *The Ontario Mathematics Gazette* has been designed to provide for some of this needed clarification for mathematics in grades 7, 8 and 9, the Transition Years.

Acknowledgements

As editors, we are pleased to acknowledge the contribution of several people in the preparation of this issue of the Gazette.

Of course, we are grateful for the tremendous work of the two teams that prepared ***The Focus on Renewal in Mathematics*** and ***The Transition Years: A Working Document.*** These two documents form the foundation of the entire issue.

Many teachers from across the province contributed articles and activities. Each is identified with his/her article. Amy Parker gathered and edited Abacus material, which we have blended into the activity package. Mary Codrington, of the Wellington County Board of Education and Lynn Hamilton, of the University of Guelph, created the computer files for our desktop publishers, who in turn, did their usual superlative work. Jennifer Kells, a mathematics student at the University of Guelph, provided many hours of invaluable (and free!) help proofing the document.

Finally, we thank OAME for its sponsorship of this very special Focus edition of the Ontario Mathematics Gazette.

Gary Flewelling, *Guest Editor*
Wellington County Board of Education

Jack Weiner, *Editor*
University of Guelph

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MINISTRY EXPECTATIONS

Ministry expectations are set out in the form of learning outcomes in The Common Curriculum (Feb'93) and Ontario Provincial Mathematics Benchmarks (Sept'92 Draft). Learning outcomes describe what students should know, should be able to do, and should value as a result of their learning experiences. They provide the base on which to build school programs. To help clarify the implications for mathematics, Miriam Pearlman, from North York, analyzed the two Ministry documents and identified the learning outcomes related to mathematics. She organized them into four sets: cross-curricular learning outcomes, cross-MST (Math, Science, Technology) learning outcomes, cross-mathematics learning outcomes and those learning outcomes related to a specific mathematics strand. These outcomes are listed on the following two pages. When reading each learning outcome, add the stem, "By the end of Grade 9 students will".

Ten Essential Cross-Curricular Learning Outcomes

i. be able to use language to think, learn, and communicate effectively

ii. be able to employ mathematical knowledge and skills

iii. be able to use scientific methods and apply scientific perspectives

iv. be able to use a wide variety of technologies

v. demonstrate an understanding of history, geography, and cultural forces

vi. demonstrate a commitment to peace, social justice, environment, and global perspective

vii. be able to interact and work effectively with others

viii. value work and learning of all types

ix. be able to exercise aesthetic judgement

x. be motivated to build healthy lifestyles and relationships

Cross-MST (Math, Science, Technology) Learning Outcomes

i. understand the relationships among events, time, distances, and velocity

ii. acquire information by reading mathematical, scientific, and technological journals, manuals, and books and by consulting databases; ask clarifying or extending questions; and apply the information accurately

iii. demonstrate the ability and willingness to work independently and with others, and to explain the importance of collaboration for the advancement of knowledge in MST

iv. conduct an accurate, thorough inquiry with persistence and creativity and explain why these and other qualities are required in investigations using MST

v. examine and objectively evaluate a variety of perspectives before making judgments during inquiries and research

vi. identify, compare, and evaluate objects, processes, and systems drawn from a variety of cultures in order to demonstrate the contribution to MST by people of both genders and from a variety of racial/cultural backgrounds

vii. identify relationships among MST in order to conduct interdisciplinary inquiries at the regional, national, and global levels

viii. use MST skills and knowledge to investigate a variety of issues, and demonstrate an awareness of the benefits and limitations of such knowledge and skills

ix. use statistical information to identify groups of people who may be under-represented in careers in MST and discuss ways of eliminating such inequities

x. use advanced features of word-processing, telecommunications, database-management, and spreadsheet technologies to organize, analyze, interpret, and communicate information in a fashion appropriate to the intended audience

xi. recognize and describe complex patterns and relationships in natural and human-made environments

xii. use computers to develop, explore, and evaluate simulations/models of real or imaginary situations, products, systems, and processes

Cross-Mathematics Learning Outcomes

i. use a variety of methods, technologies, models, checklists; sketches, graphs, and charts; oral, written and pictorial symbols; and technological devices in order to investigate, acquire skills and knowledge, and communicate effectively in MST

ii. value the fact that problems may be solved in different ways and respect and accept other people's solutions

iii. use concrete materials, computers, instruments, and other tools and equipment to construct representations of concepts and understand procedures in MST

iv. understand, select, and use a broad range of mathematical, scientific, and technological concepts and skills in inquiry and design activities

v. understand that learning in MST occurs through the process of overcoming obstacles and solving problems

vi. recognize, compare, and use alternative representations of the same idea

vii. examine their own interests, aptitudes, and abilities in relation to a variety of career and work opportunities and investigate the role of MST in these occupations

viii. establish personal goals and work towards achieving them in order to maximize their learning in MST

ix. ask questions and pursue answers through independent inquiry and research, and communicate the findings to a variety of audiences

x. demonstrate a positive attitude towards MST; confidently engage in independent inquiry, research and the manipulation of concrete materials, and show respect for the successes and contributions of others

xi. use calculators and computers to explore patterns and relationships, and solve problems relating to regional, national, and global settings

xii. use a variety of their own and generally accepted problem-solving strategies in MST

xiii. use a variety of strategies to pose and solve a wide range of everyday problems drawn from relevant experiences across the curriculum and having an MST dimension

Learning Outcomes Related to Specific Mathematics Strands

Inquiry and Problem Solving:

i. use a variety of strategies to pose and solve a wide range of everyday problems drawn from relevant experiences across the curriculum and having an MST dimension

Number Sense and Numeration:

i. use computational estimation and mental calculation to arrive at an approximate value, and judge the validity of reasonableness of answers in a variety of problem-solving contexts

ii. value and use mental calculations and the skills of computational (and measurement) estimation, and be able to apply those skills to new problem-solving tasks

iii. demonstrate in context, with and without the use of a calculator and/or computer, a thorough knowledge of arithmetic facts and accurate computational and mental calculation skills involving whole numbers, decimals, fractions, percents, integers, ratios and rates, powers and square roots

Spatial Sense and Geometry:

i. demonstrate understanding of spatial and geometric relationships by investigating 2-D, 3-D, and transformational geometry

ii. use co-ordinate geometry to describe and represent relationships defined by tables, graphs, and equations or formulas

Patterning and Algebra:

i. represent mathematics symbolically

ii. use and appreciate the value of algebraic notation in mathematics

iii. use algebraic notation to represent the general terms of a pattern and employ algebraic modelling as a strategy in solving relevant real-life problems

iv. solve problems using linear equations with one variable

Data Management and Probability:

i. gather data from a wide variety of sources, analyze and interpret it, make decisions based on it, and communicate the information and decisions in a fashion appropriate to the intended audience

ii. investigate and apply the concepts of probability and statistics in the course of inquiries

Measurement:

i. identify, develop, and apply procedures for measuring the characteristics of a variety of shapes, liquids and solids and for examining the interrelationships between and among mass/weight, volume/density, speed/velocity, distance, displacement and time, using concrete materials and appropriate units

ii. value and use mental calculations and the skills of computational and measurement estimation and be able to apply those skills to new problem-solving tasks

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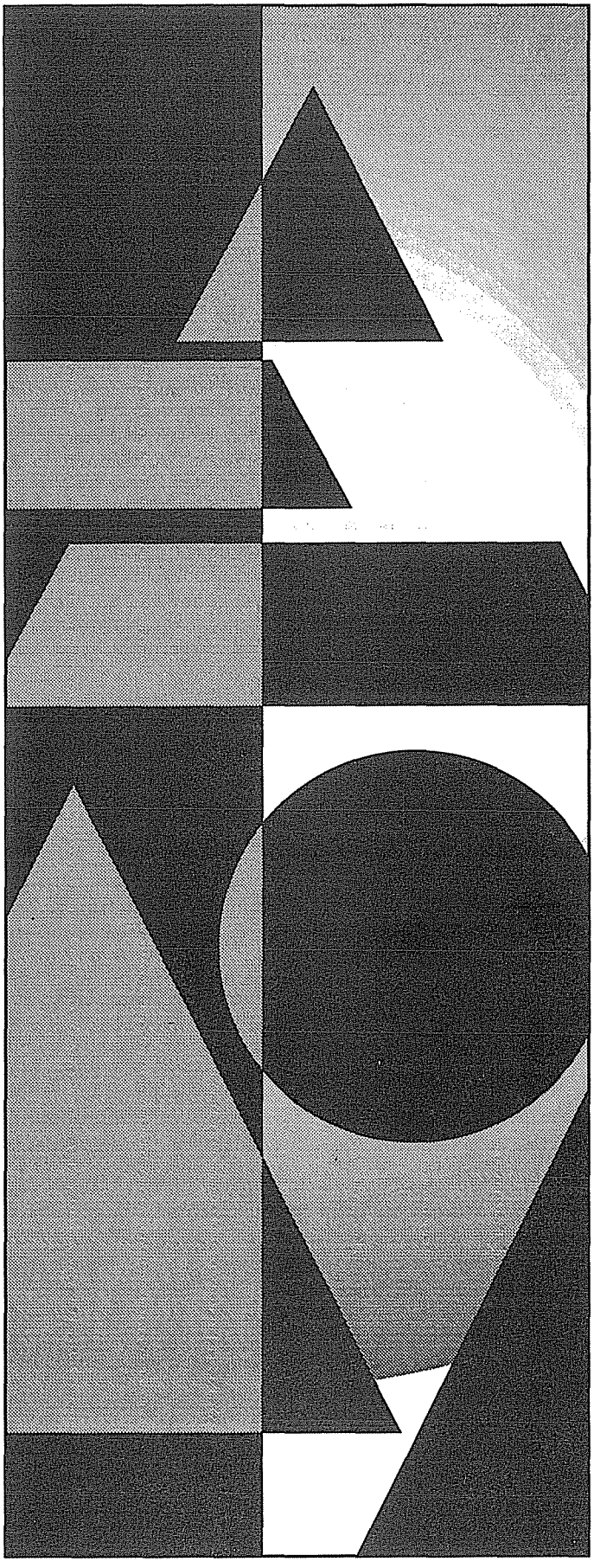
Italicized learning outcomes are found in both The Common Curriculum and Provincial Benchmarks. All other outcomes are from The Common Curriculum.

▲ TRANSITION YEARS MATHEMATICS: THE BASICS

What is defined as the basics of mathematics changes over time to reflect such things as societal priorities, the needs of business, industry and institutions of higher education, changes in current technology and the findings of educational research. Below is a current list of mathematics basics for the Transition Years.

By the end of Grade 9, students will demonstrate:

1. number and operation sense.
2. paper/pencil computational skills.
3. mental computational skills.
4. basic calculator and computer skills.
5. estimation skills in arithmetic and measurement.
6. an ability to judge the reasonableness of an answer.
7. an ability to justify answers and their solution processes.
8. an ability to apply concepts and skills to solving problems related to everyday situations.
9. an ability to collect, organize, display and interpret data and make inferences and convincing arguments based on data analysis.
10. an ability to determine the empirical probability of an event, conduct simple simulation and to apply probability to predicting outcomes.
11. an ability to recognize, describe, create, and generalize patterns and build mathematical models to predict the behaviour of phenomena that exhibit an observed pattern.
12. an ability to pose problems related to a problem situation.
13. an ability to work alone and with others to solve problems.
14. an ability to use a variety of techniques to approach and work on problems.
15. an ability to discuss, write, read and listen to mathematical ideas.
16. an ability to use rates and ratios in problem situations.
17. an understanding of properties and relationships among geometric figures.
18. an ability to visualize, represent, and describe geometric figures and spatial relationships.
19. an ability to create and analyze geometric structures and patterns.
20. an ability to measure, calculate, and construct lengths, perimeters, areas, volumes, angles, capacities, and masses in an application setting.
21. an ability to use a personal set of measurement referents.
22. an understanding of the structure of our measurement system.
23. confidence in their ability to do and use mathematics.
24. an appreciation of the power and pervasiveness of mathematics.
25. an enjoyment of mathematics.



Mathematics in the Transition Years

On the next 14 pages you will find a description of mathematics for grades 7 to 9. The description is written under the following six dimensions:

1. **Problem Solving and Inquiry**
2. **Number Sense and Numeration**
3. **Spatial Sense and Geometry**
4. **Measurement**
5. **Patterning and Algebra**
6. **Data Management and Probability**

Each dimension is described under the headings Emphases, Summary of Outcomes, Learning Outcomes (Grades 7, 8 and 9) and Discussion.

The Emphases, Summary of Outcomes and Discussion sections include descriptions of mathematics in both the Formative Years (Grades 1-6) and the Transition Years (Grades 7-9). This is done in the belief that a good understanding of mathematics in the Transition Years requires an understanding of mathematics in the earlier grades. Only the Learning Outcomes are restricted to Grades 7-9.

The Emphases, Summary of Outcomes and Discussion comments are taken from the document, *Focus on Renewal of Mathematics,* published in May, 1993. This important document is an Ontario interpretation of the NCTM's document entitled *Curriculum and Evaluation Standards for School Mathematics.* The Focus on Renewal document is a joint project of the Ontario Mathematics Co-ordinator's Association (OMCA) and the Ontario Association for Mathematics Education (OAME).

The Grade 7 to 9 Learning Outcomes represent a Wellington County curriculum team's interpretation of the learning outcomes identified by an OAME Transition Years curriculum committee.

You can obtain a copy of *"Focus on Renewal in Mathematics"* document and the OAME Learning Outcomes document by contacting OAME Executive Directors, Don and Carol Attridge. (See the back inside cover for address, phone and Fax information.)

▲ Inquiry and Problem Solving

Problem solving is the primary goal of all mathematics instruction and an integral part of all mathematical activity. ***– NCTM Standards***

Inquiry and Problem Solving: **Emphasis**

Decreased Emphasis on:

- using contrived word "exercises"
- assigning out-of-context "story" problems
- using clue words to determine operation
- giving students problems without first teaching strategies
- recording formalized solutions
- requiring the exclusive use of the approach "Let x represent..."
- solving "type" problems

Increased Emphasis on:

- problem solving derived from the student's world that are of interest to the student
- solving problems that require higher levels of thinking
- recording formats appropriate to the strategy used
- developing a wide variety of problem-solving strategies
- applying problem solving in other subject areas
- representing problem-solving situations orally, concretely, pictorially, numerically, graphically, and algebraically
- judging the reasonableness of results
- using a problem-solving and investigative approach to learning mathematics
- formulating questions from problem situations
- developing a procedure or model for solving problems in mathematics

Inquiry and Problem Solving: **Discussion**

Children arrive in school with open, inquiring minds. They are already familiar with the inquiry process through their experiences with play. Their most important need in the early Formative Years (Grades 1, 2, 3) is to have opportunities to continue their natural inclination towards inquiry learning.

Shared Discovery: Teaching and Learning in the Primary Years (Ministry of Education, 1985)

Inquiry Learning

Inquiry learning is natural to young children. Primary teachers are familiar with and use the seven main stages of the inquiry process (as shown on page 16) to assist students in becoming responsible for their own learning decisions.

Problem Solving Model

Another form of the Inquiry Model is a multi-step process based on George Polya's research (shown on page 7 as a five-step problem-solving model). This model provides problem solvers with a framework within which to tackle problems. It is recommended that this general model for mathematics not be directly introduced to most children before Grade 3. However, this model can be useful as a further mental framework for primary teachers as they plan the classroom environment, take advantage of incidental learning situations, and pose questions to help students to become more successful problem solvers. As students progress through the grades, direct instruction using this model should occur. A poster of the model should be prominent in the classroom for reference during instruction and problem-solving activities.

Problem Solving as a Focus

Students should solve problems on a regular and frequent basis. Problem solving should not be viewed or presented as an isolated unit of study; rather, it should provide the context in which mathematics skills and concepts are learned. In a classroom where problem solving is an integral part of instruction, there is an emphasis on exploring a variety of different kinds of problems (with a special focus on problems where the solution is not

Inquiry and Problem Solving: Summary of Outcomes

Formative Years	Transition Years
• understand and use a variety of problem-solving strategies • employ a process or model for problem solving, such as: – *Understand the problem* – *Make a plan* – *Carry out the plan* – *Look back* – *Communicate the solution* • judge the reasonableness of answers to problems and justify solutions	• solve a variety of routine and non-routine problems • carry out a variety of investigations by following an inquiry model such as: – *Exploring* – *Inquiring* – *Predicting possibilities* – *Planning and collecting* – *Deciding* – *Communicating* – *Evaluating* • pose problems from the student's world
	• pose extensions to solved problems • use the Scientific Method: – *Hypothesis* – *Devise/conduct experiment* – *Draw conclusions from observations* – *Validate/revise hypothesis* • use the Technology Design Procedure: – *Develop a focus* – *Develop a framework* – *Develop a product* – *Implement a plan* – *Reflect on the process and the product* • recognize the links between the problem-solving models for mathematics, science, and technology

immediately evident), and on students sharing their thinking. This is a setting where solving problems in different ways is applauded, where the process is valued as much as the solution, and where students are actively involved in creating problems from real-world activities.

Problem Solving as a Process

Students, both individually and collaboratively, should deal with problems that are open-ended, that involve assumptions and limitations, and for which multiple solutions may be possible. Typical arithmetic problems are restricted — there is usually one correct answer and one way to reach that answer. Such problems are not typical of real mathematics or real problem solving.

Problem solving is a process that may involve starting and stopping, failure and success, consideration and rejection, testing and exploration. It is also important that modelling of the process of problem solving occur in the classroom. Students sometimes have the impression that successful problem solvers just know what to do, and come up with a solution right away. If the teacher models or thinks out loud while solving a problem, and encourages students to do the same, all students may acquire a better appreciation of the process.

Manipulative materials such as pattern blocks and tangrams are excellent for providing opportunities for open-ended problem solving. It is important that the teacher encourage students to share varying solutions, and highlight the process by which they were reached.

▲ Number Sense and Numeration

Children must understand numbers if they are to make sense of the ways numbers are used in their everyday world. ***– NCTM Standards***

Number Sense and Numeration: Emphasis

Decreased emphasis on:

- assigning rote practice with complex algorithms
- memorizing rules
- practising arithmetic skills without a meaningful context

Increased emphasis on:

- investigating numbers and operations in a meaningful context
- teaching number concepts and operations for understanding through patterns and the use of concrete and pictorial models
- developing personal methods for computation
- learning grounded in experiences related to aspects of everyday life that have meaning for students

Number Sense and Numeration: Discussion

Number sense is an intuition about numbers. Students require purposeful classroom experiences which will assist them in:

- *developing number meanings;*
- *exploring number relationships with manipulatives;*
- *understanding the relative magnitudes of numbers;*
- *developing intuitions about the relative effect of operations on numbers, and*
- *developing referents for measures of common objects and situations*

– NCTM Standards, p. 39-40

To encourage these understandings, students need to manipulate physical objects and use their own language to explain their thinking.

Students with good operation sense recognize situations in the real world where a particular operation would be useful. They can also construct appropriate models, and describe the properties of an operation, relationships among operations, and the effects of an operation on a pair of numbers.

Mental Mathematics and Computational Estimation

Students should develop mastery of basic one-digit computational facts by the end of Grade 6. To meet this goal, students require time to identify relationships between numbers and to develop efficient thinking strategies. Mastery is a critical component of fluency with paper-and-pencil and mental computation, as well as estimation.

Techniques of mental mathematics should be introduced gradually along with concepts of place value and paper-and-pencil computations. For example, the mental skill of adding numbers ending in zero, such as 20 + 30 + 60, can be learned by a child who understands place value and can add 2 + 3 + 6.

After students have established a firm base in mental mathematics, the systematic teaching of strategies of computational estimation should occur. Computational estimation should include, not only applications involving whole numbers and decimals, but also those involving fractions and percent. In fact, early experiences in estimating with percent and fractions help to establish related number sense.

It is essential that students develop strong skills in mental mathematics and computational estimation, in order to support the development of the ability to judge the reasonableness of answers produced by a calculator or computer.

Paper-and-pencil Computations:

▲ Whole Numbers and Decimals

Many paper-and-pencil computations, such as long division, operations with large numbers or calculating the square root of a number, have become less used in light of the availability of calculators and computers. Emphasis on estimation in partnership with competence in mental calculation will ensure a student's ability to cope in our technological society.

Students (and adults), however, may continue to need some facility with paper-and-pencil computations. The following guidelines are recommended as being reasonable **maximum** limits for this kind of computation **by the end of Grade 6:**

Addition	three 3-digit numbers
Subtraction	two 3-digit numbers
Multiplication	2-digit by 2-digit numbers
Division	3-digit by 1-digit numbers (including short division)

It is important that the paper-and-pencil computational procedures be introduced through the use of concrete materials, and that skill be developed through practice in context. It is inappropriate to engage students in pages of isolated drill of computational skills.

In the Transition Years, students who have achieved the maximum required levels should be given opportunities to maintain them through practice in context; students who have not achieved the required levels should be given opportunities to extend their skills.

▲ Fractions and Rationals

Concepts and operations with fractions should be introduced using concrete materials. Models like physical materials and diagrams should be used to relate fractions to decimals, to find equivalent fractions, and to explore operations with fractions and decimals. Fraction symbols should be introduced only when students have developed the concepts and oral language to make them meaningful.

In the Formative Years, the focus should be on developing fraction sense, whose components are described in the "Summary of Outcomes". Operations with fractions should be limited to addition and subtraction, and multiplication involving only calculating a fraction of a whole number. Formal procedures should be introduced only after students have had significant opportunities to develop understandings using concrete materials. In operations with whole numbers, there is a recognized multi-stage process which leads students from the concrete to the abstract, or from base 10 materials to the formal algorithms.

Similarly, as students explore fractions with concrete materials such as pattern blocks, the notion of addition and subtraction becomes apparent. For example, where a yellow hexagon equals one whole, a green triangle has a value of

$$\frac{1}{6}$$

Simple counting of two triangles represents

$$\frac{1}{6}+\frac{1}{6}=\frac{2}{6}\ .$$

If four green triangles are counted out to represent

$$\frac{4}{6}\ ,$$

and one is removed, the operation of

$$\frac{4}{6}-\frac{1}{6}=\frac{3}{6}$$

has been performed. These kinds of concrete experiences are critical before attempting formal operations with fraction symbols.

It is recommended that, initially, simple denominators such as 2, 3, 4, 5, and 10 be used. As students gain more experience and skill in working with fractions, denominators such as 6, 8 and 12 can be included.

In the Transition Years, the focus should be on using fractions in the context of ratios, rates, and percent. As well, fraction sense should be extended to include developing skill in operations with fractions, bearing in mind that the most frequent real-life use of fractions involves estimation. Signed fractions (Rational Numbers) can be introduced in Grade 9, but only simple calculations should be pursued.

▲ Calculator Use

In the Formative Years, calculators should be used for investigations, problem-solving experiences, and real-life applications in which the complexity of the numbers is greater than the computational level of the students. In the Transition Years, calculators should be used except when the purpose of an activity is to develop paper-and-pencil computational skills.

When using calculators at all grades, the focus should be on the development of the effective use of a calculator, including judging the reasonableness of the answer produced. As well, students must learn to judge whether a calculator is needed for a particular computation, or whether the computation is appropriate to mental mathematics. The calculator is a powerful aid to computation, but its use must not be allowed to interfere with the intelligent application of good number sense and numeration skills.

▲ Decision Making in Computation

It is critical that students have access to a wide range of computational tools, including concrete materials, mental mathematics, estimation, paper-and-pencil procedures, and calculators. As problem solvers, students will decide which tool is most appropriate under a given set of circumstances.

Number Sense and Numeration: ***Summary of Outcomes***

Formative Years

- develop and apply number sense
- learn place value in relation to operations, mental mathematics, estimation, and magnitude, not in isolation
- apply skills of mental mathematics and computational estimation
- use a calculator effectively for complex calculations and as a tool for learning number concepts and skills
- make decisions in computations (For example, Is a calculation needed? What precision is needed? What computational tool is appropriate? (mental mathematics, computational estimation, paper-and-pencil calculation, calculator)

Transition Years

- master the basic one-digit computational facts
- apply paper-and-pencil computational skills with whole numbers and decimals to a reasonable level of difficulty
- develop fraction sense by:
  - *understanding and applying the concept of fraction*
  - *representing fractions concretely, pictorially, and symbolically*
  - *identifying, generating, and applying equivalent fractions*
  - *estimating with fractions and mixed numbers– ordering fractions and mixed numbers*
  - *understanding and applying various representations of the same quantity (i.e., fraction – decimal fraction)*
- add, subtract, multiply and divide fractions having simple denominators
- calculate a fraction of a whole number
- maintain (or continue to develop) the paper-and-pencil skill levels in whole numbers and decimals reached by the end of the Formative Years
- add and subtract fractions with simple denominators
- apply estimation involving operations on fractions
- apply operations with rational numbers in simple examples
- represent percent using pictorial models and concrete materials
- estimate with percent
- operate with percent
- apply percent in real-life situations, including sales tax, discount, commission, interest
- perform operations and problem solving with ratio and rate
- apply the concepts of ratio and rate in projects or themes that make connections between topics in mathematics and between mathematics and other subjects (For example, the Golden Ratio calculation and interpretation of ratios and rates in science and technology; mathematics and the environment)
- recognize the use of integers in a wide variety of applications
- use concrete materials to develop skills in operating with integers
- simplify expressions involving operations with integers
- operate with factors, powers, and square root (including estimation and computation using a calculator)

Number Sense and Numeration: **Learning Outcomes**

7 8 9	i	identify and use place values in context from millions to thousandths.
7 8 9	ii	identify the effect of an operation on a pair of numbers. (e.g. $5 \div 0.2 > 5$)
7 8 9	iii	apply in applications the order of operations.
7 8 9	iv	perform whole number and decimal computations using paper and pencil. Maximum limits: – addition, three 3-digit; -subtraction, two 3-digit; – multiplication, 2-digit by 2-digit; -division (short), 3-digit by 1-digit
7 8 9	v	recognize and apply the appropriate type of computation for a given context, that is, mental calculation, estimation, paper-pencil computation, or calculator.
7 8 9	vi	approximate sums, differences, products, and quotients of whole numbers and decimals using a variety of strategies.
7 8 9	vii	use a calculator for awkward computations.
7 8 9	viii	judge the reasonableness of answers calculated by paper-and-pencil or calculator.
7 8 9	ix	understand the meaning of integers and identify contexts in which integers occur.
7 8 9	x	add and subtract integers with models.
7 8 9	xi	identify equivalent and approximately equivalent fractions.
7 8 9	xii	add and subtract fractions with and without models.
7 8 9	xiii	multiply and divide fractions with models.
7 8 9	xiv	express hundredths as fractions, decimals, and percents.
7 8 9	xv	estimate and solve problems involving percents and whole percents of numbers (e.g., sales tax, discounts, tips).
7 8 9	xvi	use their number concepts and skills in a wide variety of applications.
8 9	xvii	identify and use place values in problem contexts.
8 9	xviii	use calculators for awkward computations, compacting numbers where necessary.
8 9	xix	solve problems involving addition and subtraction of integers without the use of models.
8 9	xx	solve problems involving the multiplication and addition of fractions.
8 9	xxi	solve problems by using equivalent rates and comparison of rates.
8 9	xxii	use tables and the unit method to solve ratio, rate and percent problems (e.g., if 8% of a number is 50, find 7% of the number).
8 9	xxiii	relate squares and cubes of numbers to the areas of squares and the volumes of cubes.
8 9	xxiv	relate and estimate the square roots and cube roots as the lengths of squares and cubes.
9	xxv	record and interpret numbers in scientific notation.
9	xxvi	approximate the value of expressions using scientific notation or compaction of numbers.
9	xxvii	solve problems involving the addition, subtraction, multiplication and division of integers
9	xxviii	solve problems involving the addition, subtraction, multiplication, or addition with rationals.
9	xxix	solve ratio and percent problems algebraically.
9	xxx	understand powers involving integral exponents.
9	xxxi	use the power laws (in numerical cases only).

▲ Spatial Sense and Geometry

Spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world. — ***NCTM Standards***

Spatial Sense and Geometry: Emphasis

Decreased emphasis on:

- practising isolated skills out of context
- memorizing rules, definitions, and formal vocabulary

Increased emphasis on:

- developing spatial sense and the ability to visualize
- communicating geometric ideas through reading, writing, listening, and speaking using the student's own language, while developing and using the language of geometry
- connecting geometry with other strands in mathematics (e.g., measurement, patterning, data management) and with other subjects (e.g., art, technology, geography)
- using a wide variety of materials and techniques, including computers, for hands-on investigation and exploration
- creating of designs and structures

Spatial Sense and Geometry: Summary of Outcomes

Summary of outcomes in the formative years include:

- construct two-dimensional and three-dimensional models using a wide variety of materials and explore their properties and the interrelationships of the parts
- sketch three-dimensional structures
- recognize and describe symmetry in two-dimensional figures and three-dimensional objects
- create patterns and designs, including tessellations
- apply slides, flips, and turns by using a variety of materials

Summary of outcomes in the transition years include:

- visualize, represent, and describe two-dimensional and three-dimensional geometric figures and spatial relationships
- use a variety of methods and materials to apply the classical ruler-compass constructions
- explore and apply the properties of plane figures (e.g., Opposite Angles Theorem, Isosceles Triangle Theorem)
- use a variety of materials to identify the properties and effects of translations, rotations, reflections, and dilatations
- explore patterns in geometry and relate them to number patterns
- use coordinate geometry to describe and represent relationships defined by tables, graphs, and equations or formulas
- use computers and graphics calculators to develop and apply understandings in coordinate geometry
- identify and apply the mapping rules for transformations in coordinate geometry

Spatial Sense and Geometry: Discussion

Spatial Sense

The student's world is primarily spatial. Spatial sense is the intuitive feel for one's surroundings and the objects in them. Students need a rich environment that contains a variety of geometric objects that they can manipulate in order to discover the connections between them. This will necessarily lead students to see geometry as a dynamic discipline that describes objects and how they move. Students also need to view geometry as being connected to other topics in mathematics as well as to other subjects.

Communication Skills

Students need to develop a complete repertoire of communication skills in geometry. In the Formative Years, the formal language of geometry is not nearly so important as having students explain concepts and observations, orally, using their own language, and in pictures or in print. In the Transition Years, the student's personal understandings can grow into a correct and effective use of the vocabulary of geometry. The teacher's role in language development is important. The teacher should use a blend that includes both the student's own language and the correct mathematical terminology.

Spatial Sense and Geometry: Learning Outcomes

7 8 9	i	construct triangles, quadrilaterals and regular polygons using a variety of tools and strategies.
7 8 9	ii	classify, identify, name and describe triangles, quadrilaterals, pentagons, hexagons, octagons and decagons.
7 8 9	iii	identify properties of simple 2-D shapes, such as, congruent parts, angle sum, and line and rotational symmetry.
7 8 9	iv	identify and apply the properties of parallel and perpendicular lines and congruent figures.
7 8 9	v	classify, identify, name, describe, and construct simple prisms, pyramids and regular polyhedra.
7 8 9	vi	create, analyze, describe and extend linear geometric patterns, identifying the pattern generator and pattern symmetry.
7 8 9	vii	identify figures that will tile the plane and create tessellations of the plane identifying associated symmetries and patterns.
7 8 9	viii	graph ordered pairs in the first quadrant.
8 9	ix	carry out basic constructions using a variety of tools and techniques.
8 9	x	identify basic properties of translations, rotations, reflections and rotations and use these properties to both construct the transformation of simple shapes on dot or plain paper and identify the transformation linking a figure and its image.
8 9	xi	perform basic constructions using a variety of tools and strategies (e.g., paper-folding, transparent mirrors, set square, protractor, compass, computer).
8 9	xii	create and analyze designs using geometric patterns in a variety of real-life contexts and cultures.
8 9	xiii	create similar figures and scale drawings, identifying the properties of similar figures and using these properties to interpret scale drawings (e.g., maps and scale models).
8 9	xiv	sketch 3-D figures on dot paper, construct simple 3-D figures using cubes from their top, side and front views, and identifying views given models.
8 9	xvi	tessellate in 3-D, identifying figures which will fill space (relating to packaging problems).
8 9	xv	graph in all four quadrants.
9	xvii	plot the translation, reflection (in the axes and y=+/-x) and rotation (multiples of 90 degrees) of points in the coordinate plane and identify these transformations from their mapping rules.
9	xviii	identify and apply the angle properties of supplementary and complementary angles, and intersecting and parallel lines.
9	xix	identify the properties of angle and perpendicular bisectors.
9	xx	draw similar figures in the coordinate plane and identify dilatations from their mapping rules.

Visualization

Visualization is also an important skill because students need to be able to draw images in their minds of how shapes look, and to be able to manipulate these images mentally. Geometry is best learned in the active mode. Students discover relationships and develop their spatial sense by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures.

Coordinate Geometry

The study of coordinate geometry in the Transition Years should include both experiences in the context of applications (graphing of data from experiments and formulas) and the formal mathematical treatment of the coordinate geometry on the xy-plane. Students' experiences in graphing and interpreting should include both linear and non-linear relations. In terms of the xy-plane, by the end of Grade 9, students should have an understanding of the concept of a relation, of the relationship between the form of an equation and the shape of the corresponding graph, and they should have particular skills with respect to linear relations, such as graphing from an equation. Computers and graphics calculators are very powerful aids in studying coordinate geometry. All students should have access to computers or graphics calculators.

▲ MEASUREMENT

Measurement is of central importance to the curriculum because of its power to help children see that mathementics is useful in everyday life. –NCTM

Measurement: Emphasis

Decreased emphasis on:

- using conversions out of context
- promoting rote memorization of formulas

Increased emphasis on:

- developing measurement concepts and formulas through the use of concrete materials, pictorial models, and patterning activities using measurement to solve problems and in practical applications

Measurement: Discussion

Importance of Measurement

Understanding measurement is essential for all students. Measurement has practical applications which will enable students to function in today's world. It also helps them to learn other topics in mathematics because many of our teaching models have a measurement base. Measurement concepts are also important in other curriculum areas such as science and environmental studies. Good instruction in measurement should not only teach skills, but should also expand students' ability to learn and understand mathematics.

Measurement Activities

Students must be actively involved in problem-solving situations if they are to learn about measurement. Understanding of measurement, both attributes and process, must be carefully developed through a variety of activities that allow students to compare, order, and then measure. Formulas should grow out of many situations in which students informally explore relationships by manipulating objects and models, and identify generalizations to observed patterns. In the Formative

Measurement: Summary of Outcomes

Formative Years

- measure using non-standard units
- develop a need for estimation and measurement with standard units
- develop and use a personal set of referents or anchors for a variety of measures
- use a wide variety of measurement tools
- select the most appropriate unit in a given situation
- measure, record, and estimate with increasing accuracy
- apply skills to measure temperature, time, distance, length, perimeter, area, capacity/ volume, mass, money, and angles
- develop and apply generalizations for the perimeter and area of a rectangle, square and triangle, and for the volume of a rectangular prism

Transition Years

- continue developing a personal set of referents or anchors for a variety of measures
- estimate, measure and construct length, perimeter, area, volume, angle size, capacity, and mass in problem-solving settings
- develop and apply formulas for the circumference of a circle; the area of a parallelogram, trapezoid and circle; the volume of a prism, pyramid, and cylinder; the surface area of a prism, pyramid and cylinder
- determine the perimeter and area of irregular figures and the volume/ capacity of irregular objects

- understand and apply the relationships between and among commonly used units in the Metric System: (e.g., mm ↔ cm, cm ↔m, m ↔km)
- use measurement concepts and skills to solve problems in practical applications and in familiar situations

Years, formulas should be created by students as the need arises. These formulas should be expressed in the students' own language.

Attributes of Measurement

Types of measurement, such as length, area, capacity and volume, need to be encountered on a continual basis in informal, significant problem-solving settings. Integration with other appropriate topics in mathematics as well as other curriculum areas will provide students with a greater insight into measurement attributes, concepts, and appropriate units.

Estimation

Estimation should be a natural part of all measurement activities, both in mathematics and in other curriculum areas. A variety of estimation strategies should be encountered and students should come to realize that the accuracy of the estimate needed will vary according to the situation. It is of the utmost importance that students become comfortable with estimation.

Measurement: Learning Outcomes

7 8 9	i	use a personal set of references or anchors for a variety of measures.
7 8 9	ii	round measures to appropriate accuracy in all measurement activities.
7 8 9	iii	estimate and measure the perimeters of regular and irregular 2D figures whose boundaries are line segments or portions of curves.
7 8 9	iv	construct regular and irregular figures with given perimeters using a variety of materials.
7 8 9	v	estimate and measure the areas of regular and irregular 2D figures whose boundaries are line segments and/or portions of curves.
7 8 9	vi	construct regular and irregular 2D figures with given area using a variety of materials.
7 8 9	vii	identify the inter-relation between figures with fixed perimeters (area) and their corresponding areas (perimeters).
7 8 9	viii	solve real-life problems involving the use of perimeter and area, using appropriate degree of accuracy.
7 8 9	ix	compare and order a variety of containers by estimating and by measuring their capacities in mL and L.
7 8 9	x	compare and order angles by estimation and measurement of their angles in degrees.
7 8 9	xi	estimate and measure masses of objects in grams and kilograms.
8 9	xii	state and apply the formula for the perimeter of a rectangle (including squares)
8 9	xiii	state and apply the formula for the area of a rectangle (including squares)
8 9	xiv	state and apply formula for the circumference of a circle and estimate the value of pi.
8 9	xv	state and apply the formula for the area of a circle.
8 9	xvi	determine the area of regular and irregular figures using a variety of techniques.
8 9	xvii	state and apply the Pythagorean Theorem.
8 9	xviii	state and apply the formula for the area of a parallelogram, triangle & trapezoid.
8 9	xix	identify the change in perimeter and area of shapes if their dimensions are changed by given factor.
8 9	xx	make scale drawings and models.
8 9	xxi	state and apply the formula for the volume of rectangular prisms and cylinders.
9	xxii	by estimation and measurement determine the surfaces areas of rectangular and triangular prisms and cylinders.
9	xxiii	measure volumes by displacement methods or capacity measures.
9	xxiv	identify the relationship between the volume of a prism (cylinder) and its corresponding pyramids (cones)
9	xxv	solve problems involving surface area, volume and capacity.
9	xxvi	identify the change in volume of shapes when one or more dimensions are changed by a given factor.
9	xxvii	using models, solve practical problems involving irregular 3D figures (e.g., surface area of a vehicle, volume of wood in a tree, capacity of kidney-shaped pool)

▲ PATTERNING AND ALGEBRA

Children who are encouraged to look for patterns and to express them mathematically begin to understand how mathematics applies to the world in which they live. ***– NCTM Standards***

Patterning and Algebra: Emphasis

Decreased emphasis on:

- using variables and numerical expressions in isolation
- manipulating of symbols

Increased emphasis on:

- determining, describing, extending, and creating patterns in mathematics strands, including number, geometry, and measurement
- providing experiences which include the use of concrete and pictorial materials
- creating models to represent problem situations

Patterning and Algebra: Discussion

Patterns

Patterns are everywhere. Patterns are the link students need to make connections between mathematics and the world in which they live. Exploring patterns leads to students developing mathematical power and gaining an appreciation for the beauty of mathematics.

The exploration of patterns by students in the Formative and Transition Years is critical to the development of the concept of variable and the understanding of algebraic thinking.

Patterns in Geometry

In geometry, students need opportunities to build structures and study patterns in tiling, wallpaper, quilts, and furniture arrangements. Students can even form human patterns with their bodies in the gym or outdoors.

Patterns in Measurement

Measurement activities are rich sources of patterns since to measure means to repeat a standard or non-standard unit with a scale or tool, and formulas are nothing more than the general statement of some observed pattern. Students need an environment that includes many opportunities to measure and identify patterns.

Patterns in Number

Number patterns are abundant in the student's world. Nature and our modern technological world provide many numerical patterns. The calculator can be used extensively to explore patterns involving repeated digits and skip counting. Language is another rich source of patterns. Students can examine patterns in sounds, words or stories.

Algebraic Modelling

Algebraic modelling is a powerful method for solving problems at Grade 9, but starts informally much earlier. As a result of patterning investigations or problems solved using a patterning strategy in the earlier grades, students will have had foundation experiences. These experiences must be in context and developmental so that students find they make sense.

Patterning and modelling are not limited to number work. Measurement and other formulas in the later Formative Years should be determined as the general term of an observed pattern. In the Transition Years, the introduction of coordinate geometry and linear relations will provide similar relevant experiences leading to formal solution of linear equations in Grade 9.

It is important that the teacher view algebraic modelling as one strategy amid a wide range of strategies and not leave students feeling that this is memorization work. If a given problem can be solved using algebraic modelling or some other strategy, the teacher should value all reasonable methods.

In Grade 9, formal treatment of algebra begins. It is important that students learn to solve linear equations including the necessary prerequisite manipulation skills. Students' experiences with manipulation of algebraic expressions, however, should be limited to simple examples necessary to solve equations.

Appropriate Example:

Simplify $3(2x - 1) - (5 - 3x) + 5(x - 6)$

Inappropriate Example:

Simplify $2a^3b^2(ab - 4a^5) - b^3(a^4 - ab^2) + 6a^8$

Patterning and Algebra: Learning Outcomes

7 8 9	i	create , describe, analyze, and extend a variety of patterns derived from number and geometric concepts, using a variety of materials.
7 8 9	ii	use concrete and physical representations to establish an understanding of variables.
7 8 9	iii	represent simple patterns with tables, graphs, and rules.
7 8 9	iv	model real-life situations using numerical expressions.
7 8 9	v	create and evaluate numerical expressions with and without brackets using the order of operations.
8 9	vi	create, describe, analyze and extend a wide variety of patterns using a variety of materials.
8 9	vii	predict and verify a pattern beyond the point where physical modelling is practical.
8 9	viii	represent simple patterns with tables, graphs, and rules (use words and algebra).
8 9	ix	describe in words and with variables, rules derived from simple patterns.
8 9	x	interpret algebraic expressions that model real-life situations using word phrases.
8 9	xi	translate word phrases modelled from real life situations into algebraic expressions.
8 9	xii	evaluate algebraic expressions by substitution with and without calculators (whole numbers then integers).
8 9	xiii	solve linear equations informally.
9	xiv	do algebraic manipulation involving addition/subtraction of like terms, addition/subtraction of polynomials, distribution of a constant monomial, with and without the use of physical methods.
9	xv	to model real-life situations using linear equations.
9	xvi	use a balanced situation to model and write equations.
9	xvii	solve equations using formal methods using the algebraic skills listed above.
9	xviii	model real-life situations using inequations.
9	xix	solve inequalities using informal and formal methods.
9	xx	substitute into real-life formulas and then solve the resulting equation.
9	xxi	graph relations using data modelled by real-life information.

Patterning and Algebra: Summary of Outcomes

Outcomes for the formative years include:

- describe, analyze, extend and create a variety of patterns
- express generalizations to patterns in words and develop formulas as the generalization of an observed pattern
- read and interpret data from a variety of sources including graphs, charts, lists, tables, tally charts and text
- use patterns to represent and solve problems

Outcomes for the transition years include:

- describe, analyze, extend and create a variety of patterns
- express generalizations to patterns in words and algebraically, and develop formulas as the generalization of an observed pattern
- use patterns to represent and solve problems
- describe and represent patterns with tables and graphs
- make predictions based on pictorial representations, tables of values, graphs, and equations
- solve problems by using tables, graphs, and equations
- develop the concept of variable, expression and equation
- model problem situations by linear equations and inequations to solve related problems
- solve linear equations by informal and formal methods
- graph relations

▲ DATA MANAGEMENT AND PROBABILITY

Collecting, organizing, describing, displaying and interpreting data, as well as making decisions and predictions on the basis of that information, are skills that are increasingly important in a society based on technology and communication. ***– NCTM Standards***

Data Management and Probability: Summary of Outcomes

Outcomes for the formative years include:

- collect, organize, describe, display and interpret data
- graph using a variety of materials, including manipulatives
- read and interpret data from a variety of sources including graphs, charts, lists, tables, and tally charts
- use computers to draw graphs
- formulate and solve problems based on collected data
- make decisions based on collected data

Outcomes for the transition years include:

- generate questions related to a problem situation; decide on the information needed and methods of collecting the data; collect, organize and display information; interpret the data; make inferences and convincing arguments based on the data analysis
- use a variety of methods to gather data from primary and secondary sources
- calculate and interpret means, medians, and modes
- determine probabilities experimentally
- conduct simple simulations

Data Management and Probability: Discussion

Real World Connections

The revised mathematics program must demonstrate increased attention to data management and probability. Graphing, statistics, and probability are closely linked and provide a strong connection between school mathematics and the real world. They provide students with the opportunity to think, use, understand, and interpret meanings of number – in other words, develop number sense.

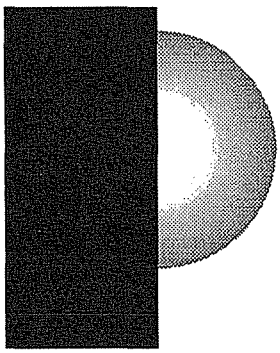
Data management and probability activities provide a meaningful context for computational skills and the setting for representing, interpreting, and discussing information that applies to other mathematics concepts and skills. These activities help develop critical-thinking and problem-solving skills.

Probability

The study of probability in the Formative Years should be kept to an informal/incidental treatment of chance as it relates to games and simple everyday situations. Experimental probability and simulations should be developed in the Transition Years. Theoretical probability should be left to the Specialization Years.

Applications

Graphing, statistics, and probability should not be treated in isolation. They should be used as tools to apply mathematics concepts and skills in a meaningful context throughout the curriculum. Activities should emphasize collection of data, interpreting, formulating hypotheses, and devising procedures to test them. From statistics gathered from projects, concrete graphs, calendars, and weather information in early grades to more complex probability experiments in later grades, activities must be concrete, manipulative, and based on student's experiences.



"Schools must use a variety of assessment and evaluation strategies."

(Transition Years, Grade 7, 8 and 9, pg. 4)

Data Management and Probability: Learning Outcomes

7 8 9	i	collect graphs from various sources and then classify them according to their type.
7 8 9	ii	identify the characteristics of each type of graph.
7 8 9	iii	identify the type of information that is associated with each type of graph (continuous, related or unrelated)
7 8 9	iv	recognize the suitability of different types of graphs for displaying specific data.
7 8 9	v	read and interpret graphs.
7 8 9	vi	construct graphs from given data or data found in other sources.
7 8 9	vii	pose and answer questions about the graphs constructed and those found.
7 8 9	viii	develop intuitive concepts of "chance" (probability) using spinners and other similar activities.
7 8 9	ix	determine and express the "chance (probability)" of events as fractions using the terms "fair/unfair", "likely/unlikely".
8 9	x	determine mean, median, and mode from collected data.
8 9	xi	use mean, median or mode to analyze tabular data.
8 9	xii	identify the various methods used to collect data from a variety of sources and then state the advantages and disadvantages of each method.
8 9	xiii	create a survey based on learner-decided (teacher assisted) problem.
8 9	xiv	analyze questions as to bias/non-bias.
8 9	xv	conduct a survey using good sampling techniques.
8 9	xvi	organize and interpret the information from a survey.
8 9	xvii	display information from a survey using an appropriate graph.
8 9	xviii	interpolate and extrapolate information from graphs.
8 9	xix	identify patterns and trends and make predictions from graphs.
8 9	xx	estimate populations by taking samples of various sizes.
8 9	xxi	explain why actual results do not always match predicted results.
8 9	xxii	predict outcomes by creating a probability experiment or simulation to model a problem.
9	xxiii	create a formula or an equation from data displayed in a chart/table.
9	xxiv	design and conduct an experiment to test an hypothesis.
9	xxv	collect, organize and display the data gathered from an experiment.
9	xxvi	when possible, create an equation to represent data of an experiment. (Note: many problems will not generate an equation.)
9	xxvii	given an equation, display data in the coordinate plane.
9	xxviii	pose and answer questions about the data.
9	xxix	determine probabilities of events from their relative frequencies as obtained from: i) an individual's results, and ii) class results.
9	xxx	explain why actual results do not always match predicted results.
9	xxxi	identify the relationship between sample size and the accuracy of expected results.
9	xxxii	conduct simulations to predict probability of occurrence of complex events.

▲ Assessment and Evaluation

The purpose of assessment is to improve learning. Without assessment the teacher, and the student may never know how well that which was meant to be taught was in fact "caught". The following two pages, taken from the OMCA/OAME Focus on Renewal document, looks at some important aspects of assessment.

Outcomes-based education stresses process. Outcomes are the observable/measurable knowledge, skills, and values that students are expected to have acquired and developed at certain key stages of their schooling. They describe what students should know, be able to do, and value as a result of their learning experiences. Content becomes the tool for attaining outcomes. This shift toward outcomes-based education requires a shift in assessment practices.

Increased attention should be given to a broad range of assessment practices such as:

- assessing what students know and how they think about mathematics;
- focusing on a broad range of mathematical tasks and taking a holistic view of mathematics;
- using multiple assessment techniques, including written, oral, and demonstration formats;
- using calculators, computers, and manipulatives;
- recognizing non-cognitive outcomes of mathematical programs such as attitudes, motivation and appreciation, and assessing process as well as product.

Tests are one way of communicating what students know. But mathematical power involves characteristics such as effective communication skills, problem-solving skills, higher-order thinking, creativity, persistence and curiosity. To give students opportunities to demonstrate these characteristics, a variety of assessment strategies should be used.

The following assessment strategies are recommended in order to supplement the traditional paper-and-pencil testing: performance assessment, open-ended questions, investigations, journals, observations, conferencing/ interviewing, portfolios, and self-assessment.

1. Performance Assessment

Performance assessment in mathematics involves presenting students with a mathematical task, project, or investigation, then observing, interviewing and examining their products to assess what they actually know and can do. A performance task examines the process as well as the product. It provides engaging contexts and enhances motivation as students work on real tasks.

Performance assessment gives information about a student's ability to:

- use mathematical concepts and skills;
- reason soundly and raise questions;
- think flexibly, changing strategies when one does not work;
- use manipulative materials, equipment, calculators and computers;
- work together in groups;
- persist, concentrate and work independently;
- communicate and use mathematical language through discussing, writing, and explaining their ideas and questions;
- observe carefully in order to infer and formulate hypotheses, and
- design and conduct experiments and investigations.

2. Open-ended Questions

A question is open-ended when a student is given a situation and is asked to communicate a response. A variety of successful responses exist.

Open-ended questions assess whether students can:

- recognize the essential points of the problem;
- organize and interpret information;
- make generalizations;
- write for a given audience;
- understand basic concepts;
- use appropriate mathematical language and representation, and demonstrate originality and/or creativity.

3. Investigations

Investigations are explorations of mathematical questions but they may be related to other subject areas. Investigations deal with problem posing as well as problem solving.

Investigations allow students to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect needed information;
- organize information and look for patterns;
- persist, looking for more information if needed, and
- discuss, review, revise and explain results.

4. Journals

A mathematics journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and

respond to open-ended questions.

Journals allow opportunities to:

- formulate, organize, internalize and evaluate concepts;
- clarify thinking;
- identify strengths, weaknesses, and interests, and
- modify programs

5. Observations

Research has consistently shown that the most reliable tool for evaluation is the ongoing, in-class observations of students by teachers. Students should be observed individually and as they work in groups. Through systematic, ongoing observation, the teacher can gather information related to students':

- attitudes toward mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strengths/weaknesses;
- preferred learning styles;
- areas of interest;
- work habits;
- social development, and
- development of mathematics language and concepts.

In order to ensure the observations are focused and systematic, a teacher may use checklists, a set of questions and/or a journal as a guide.

Teachers should develop a realistic plan for observing students. When planning, teachers might consider:

- observing a small number of students each day;
- focusing on one or two aspects of development at a time, and
- tape-recording or videotaping student activities for later analysis.

6. Conferencing/Interviewing

An interview includes a planned sequence of questions, whereas a conference implies discussion, with student and teacher sharing ideas. The main purpose of conferencing or interviewing is to explore the student's mathematical thinking and to assess the student's level of understanding of a particular concept or procedure.

During a conference/interview students can be asked to:

- explain the process used to arrive at a solution;
- justify their thinking;
- suggest an alternate solution strategy;
- discuss their likes and dislikes, and
- discuss their perceived strengths and weaknesses.

A successful conference/interview includes:

- using prepared questions;
- putting students at ease;
- explaining that you are looking for thinking;
- posing a problem;
- taking notes;
- being a good listener;
- being non-judgmental, and
- doing instructional intervention in a separate setting.

Generally, most conferences/interviews should be brief and informal, and occur naturally in the course of everyday mathematical experiences.

7. Portfolios

A portfolio is a showcase for student work, a place where many types of assignments, projects, reports, and writings can be collected. Progress in, attitudes towards, and understanding of mathematics can be seen in a comprehensive way.

Student portfolios can provide:

- evidence of performance:
- a permanent record of student performance, and
- an active role for students in assessing and selecting their own work.

A portfolio may include the following examples of student work:

- written descriptions of investigations
- pictures and dictated reports from younger children
- descriptions and/or analyses of problem situations
- responses to open-ended questions
- homework problems
- photographs of a project
- group reports
- video, audio, or computer-generated examples of student work
- work of a personal choice with reasons for that choice
- excerpts from a student's mathematics journal
- a description of how the student feels about mathematics
- student self-report about what has been learned

8. Student Self-Assessment

Self-assessment promotes metacognition skills, ownership of learning, and independence of thought. Self-assessment can be a questionnaire following a co-operative activity or project, asking how well the group functioned or it can be a daily writing in a journal.

In student self-assessment practices, the teacher should look for:

- signs or change and growth in attitudes, mathematical understanding, and achievement;
- alignment of student's beliefs about his/her performance with their actual performance, and
- a match between student's and teacher's views of expectations and criteria for evaluation.

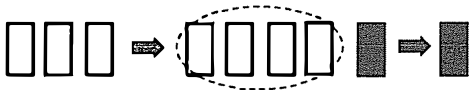
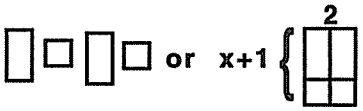
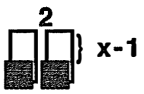
MANIPULATIVES: USING INTEGER CHIPS

Physical models allow students to use concrete things to help them think about and come to an understanding of abstract things. Shown below is a developmental sequence, using coloured chips, to teach students how to carry out operations on integers.

Numerical Expression	Physical Representation	Comments
1	○	a green chip
3	○○○	3 green chips
-1	●	a red chip
-4	●●●●	4 red chips
(any pair of opposites)	○ and ●, or ○○ and ●● etc.	equal numbers of each chip
0	○● (or any pair of opposites)	the important "zero property"
3+(-4)		using the zero property to combine integers
(-2) + (+4) + (-5)		Of which chip are there more? *(gives sign of sum).* How many more? *(gives magnitude of sum)*
3-5		add zeros when necessary
-3 – (-4)		student eventually concludes, "to subtract, add opposite"
3-4+5-6 *[think 3 + (-4) + 5 + (-6)]*	without chips	Of which are there more? (ans. -ve) How many more? (ans. 2)
2x(-3)		use integer addition 2 groups of -3 (=-6)
3 x (-3) = ___ 2 x 6 = ___ 2 x (-3) = ___ 1 x 6 = ___ 1 x (-3) = ___ 0 x 6 = ___ 0 x (-3) = ___ -1 x 6 = ___ -1 x (-3) = ___ -2 x 6 = ___	without chips	use patterns to give meaning to (-ve) x (-ve) and (-ve) x (+ve)
a) 12÷(-2) = ? b) (-12)÷(-3) = ?	without chips	use multiplication knowledge a) ? x (-2) = 12 b) ? x (-3) = -12

Manipulatives: Using Algebra Tiles

Shown below is a developmental sequence, using coloured tiles, to teach (Grade 9) students how to work with algebraic expressions. The emphasis here is to develop algebraic skills necessary for simplifying and solving linear equations. Note how the developmental sequence parallels that of the integer sequence on the previous page.

Algebraic Expression	Physical Representation	Comments
1		a green square with area 1 sq. unit 
x		a green rectangle with area x sq. units 
3x		3 green rectangles
–x		a red rectangle
Any pair of opposites		
0		"zero property"
2x +3		unlike tiles
3x–4x		add zeros if necessary
2x –x – 3x *[think: 2x+(–x)+(-3x)]*		Of which tile are there more? (ans. -ve) How many more? (ans. 2). Therefore answer -2x
2x+1+x–4		combine like terms
(3x-1)–(2x–2)		adding zeros when necessary. student concludes "to subtract, add opposites"
2 (x+1)		multiplication as addition; multiplication as area
2 (x–1)		multiplication as area using zero property

▲ Student Math Journals

In this article M. Elaine Harvey from Kingston shares her experiences with student math journals.

Tell your class that they are going to begin keeping a math journal. Students will have five or ten minutes at the end of math class every Friday to make their journal entries. They will write about something to do with the class, their understanding of the work, or their goals or achievements in the course. Tell the students that you will read what they write and that you will frequently respond in writing, but that you will honour the confidentiality of their journal entries. You will not show their journals or discuss their comments with anyone else. However, give them permission to show comments to anyone they wish. The students who wrote the comments that are quoted in this article have given me written permission to publish them anonymously. Keep the journals locked in the classroom so that there is no security risk or danger of losing them. I have changed any names that appear in their comments. It is important to emphasize to your students that the journals are for communication only and not for evaluation.

I am usually pretty casual about my instructions as to what I want the student to write. Usually at the beginning of the course I ask them to say whether they like math and what kind of achievement they are aiming for in the course.

Student: *Math courses seem to be the stable factor in my mark record.*

When I read those entries, I usually keep a tally and afterwards tell the students that, say, 60% of them like math, 20% hate it and the rest can tolerate it.

After the first week or two of the course I ask them to indicate whether they have any positive or negative comments to make about how the class is run. As I read their journal entries I write my own responses and make notes on general trends that I feel are worth sharing with the class. When I adjust my teaching as a result of their comments, I want them to know that it was their comments that made the difference. I also make a list of those comments that I want to follow up in person such as the following:

Student: *I'm a little bit confused on some of the long division questions but after practising I think I'm getting along a bit better.*

Sometimes I will ask them how they feel about the course so far or how they feel about the test that is coming up or the one that they just got back.

Teacher: *Your unit 1 mark is 86%. I was very pleased with that.*

Student: *Hey not Bad. But Unit 2 is gettin wierd I don't understand.*

Note that I never mark spelling or grammar errors in the students' journals because I use them as a communication tool. As such the reader has just as much obligation to understand as the writer has to express the ideas clearly. Acting in an authoritative manner by marking errors limits the freedom of expression of the student.

Sometimes I don't give any suggestions at all:

Student: *We lost our first football game and I was quite upset. Math is going well.*

Teacher: *Sorry about the loss. I hope the rest of the season goes better. Your Unit 1 math mark is 84%. I'm pleased with that. Are you?*

Student: *Yes, I am pleased! Hopefully it will continue.*

Another student: *Mary (not her real name) and I help each other with our math but I will try not to interrupt the class anymore. Sorry! I'm not really pleased with that mark but I know it's my fault and I will try and bring it up.*

I never write instructions or questions for journal entries on the board because I am trying to break down the barriers to communication that traditionally exist between teacher and student. I want to make it as casual as possible.

I obtained class sets of duotangs for the journals in colour codes—one colour for each class. I have reused these several times. I find the colour coding useful, but if your school can't afford duotangs, surely you or your students can be innovative and create some kind of cover. Just remember that uniformity is an advantage if you are carrying them home to read or handing them out in the classroom.

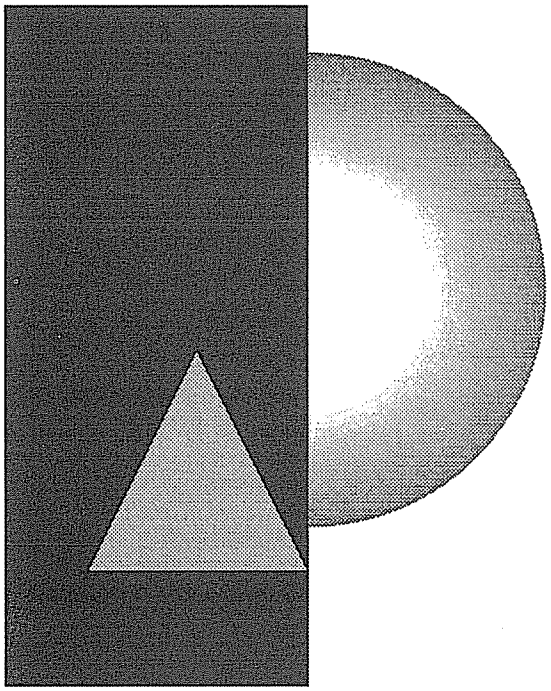
With encouragement students will begin to write notes within their math questions. Perhaps they will explain what method they should use even though they are unable to get started with a question. Or they may note that they recognize that an answer doesn't make sense and give the reason they feel that way. Sometimes one might say, "I'm not sure how to do this but I'll try anyway". This is one way of giving themselves permission to make mistakes without erasing their whole answer.

Soon they will be ready to attempt a written explanation of a particular mathematical technique. It is best to start with a technique that you are sure the students understand. When they become accustomed to doing this, you might be able to ask them to learn a new concept on their own and write a description of it. It would be considered effective if other students could follow it. This may open the door to students instructing each other, or to conducting interviews with other students or adults to research a variety of topics.

▲ Learning Activities

The kind of activities in which students are involved and the way they are involved are two crucial factors in creating meaningful learning experiences. The following pages contain sample activities, each of which exhibits a number of the following characteristics:

- **relevant to students**
- **challenging and engaging**
- **involves the use of manipulatives**
- **integrates problem solving**
- **has an investigative flavour**
- **encourages the student to make generalizations**
- **encourages discussion**
- **involves students working in small groups**
- **open-ended**
- **makes connections to various areas in mathematics, to other subjects and to interesting contexts**
- **encourages reflection**
- **encourages the meaningful involvement of a wide range of students**
- **encourages the use of a variety of assessment techniques.**



"Strategies for using learning materials should include activities that enable students to work co-operatively. In working with others, students encounter different perspectives and approaches that can help them improve their own information-gathering skills and enhance their ability to understand and apply the information they have collected."

(Teaching Destreamed Grade 9 Classes, pg. 5)

▲ The Buffalo Problem

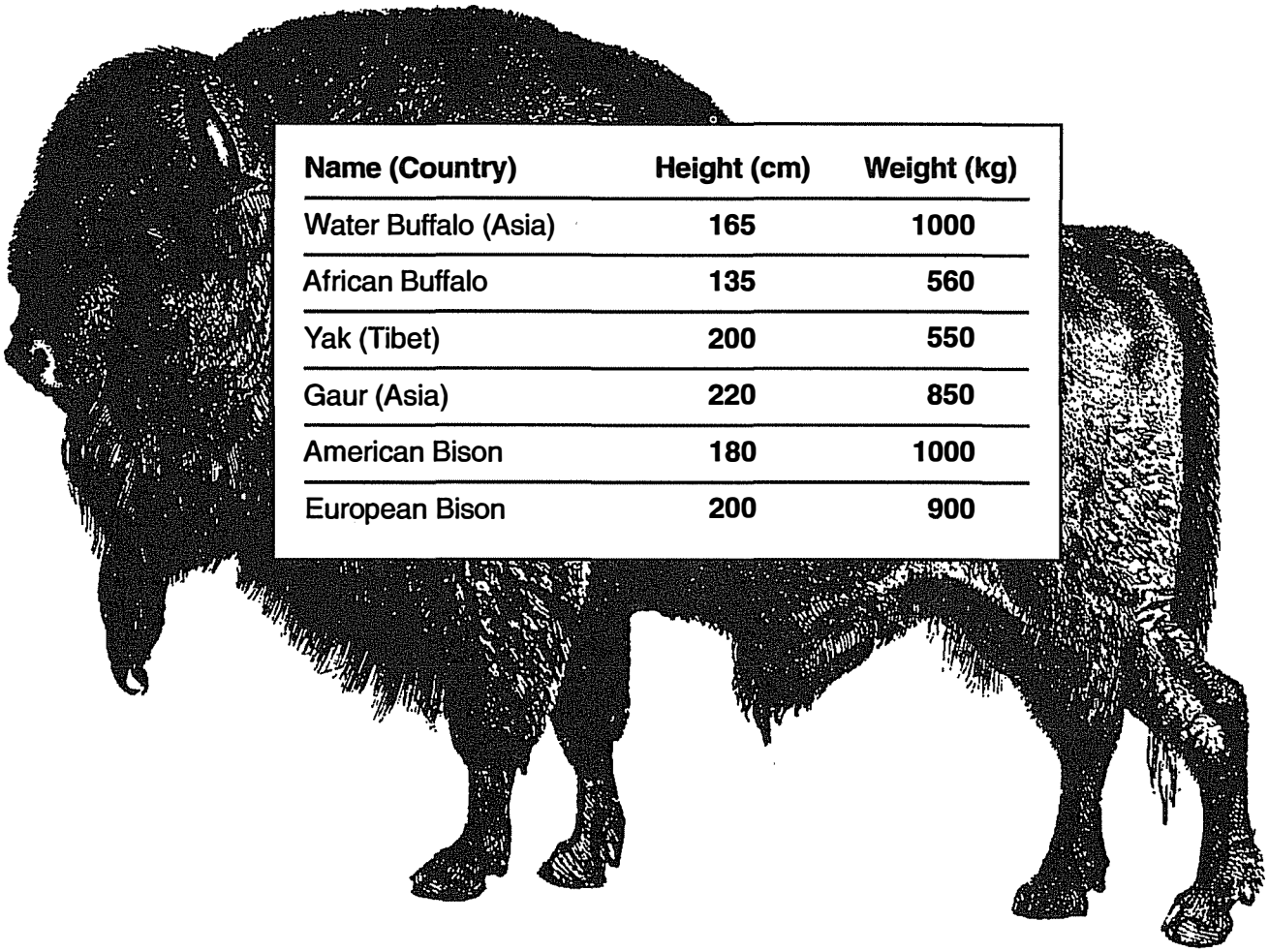
The following question appeared in an insert in a '93 NCTM News Bulletin.
Such open-ended problems are a wonderful source of mathematically rich activity and discussion.

This problem should be given to small groups to work on. Students can be asked to individually write up their responses to the question. The teacher will have to employ a holistic marking strategy here.

A Sample Holistic Scoring Strategy

1. Place student responses in four or five piles from high to low levels of performance.
2. Identify the characteristics that place a response on a given pile (holistic scoring criteria).
3. Write a level or grade on each student's work along with comments justifying the rating. Comments concerning how to improve the rating could also be added.

Which is the Largest Buffalo?



Name (Country)	Height (cm)	Weight (kg)
Water Buffalo (Asia)	165	1000
African Buffalo	135	560
Yak (Tibet)	200	550
Gaur (Asia)	220	850
American Bison	180	1000
European Bison	200	900

▲ Some Number Problems

Instead of giving students a page of computation questions to do, try one of the following problems, from Lorna Morrow of North York, as a group activity and listen to the group discussions. Then have the groups present (and attempt to justify) their solutions.

Write fifteen numbers between 3.4 and 3.5

The mathematics: the knowledge that no matter how close two numbers are, there is always another one between them. This is known as the density of numbers. Students will need to understand both tenths and hundredths to complete this problem. Notice that there is no period at the end of the sentence. This is to avoid confusing children who are really not certain what to do with decimal points or who might have difficulty distinguishing between decimal points and periods.

A number rounded to the nearest tenth is 9.8
Describe all such numbers.

The mathematics: density (as above); place value; rounding. Students may be able to identify several numbers that fit the description given above; it is a more difficult task to describe the set of such numbers.

Write three fractions between 1/4 and 1/3.

The mathematics: common denominators; equivalent fractions; density. It is interesting to observe students as they change the given fractions to twelfths (3/12 and 4/12) only to find that this does not give them the "answer". As a hint, you may wish to ask them why they chose 12 as a denominator or if they could use any other denominator.

Suppose the '8' key on your calculator is broken.
Describe how you could still use your calculator to calculate 357 x 888

The mathematics: the meaning of multiplication; the multiplication algorithm; distributive property. This problem demands number sense rather than the rote application of a memorized procedure. There are several ways to complete the problem: 357 x 1000 - 357 x 112 or 35700 + 35700 + 35700 + 35700 + 35700 + 35700 + 35700 + 35700 + 3570 + 3570 + ... (8 times) ... + 357 + 357 + .. (8 times)

Suppose the decimal point on your calculator is broken.
Describe how you could still use your calculator to calculate 34.25 + 17 + 8.1

The mathematics: place value; addition of decimals. One of the simplest ways to solve the problem is to write each number using hundredths, then add on the calculator without the decimal points (i.e. 3425 + 1700 + 810) and then place the decimal point correctly.

The answer to a division question is '4R2'
Write four different questions that could have this answer.

The mathematics: meaning of division; the division algorithm; division as the inverse of multiplication. This problem points out the meaninglessness of such an answer since 4R2 could mean four and two-sevenths (from 30 ÷ 7) or four and two-fifths (from 22 ÷ 5) or four and two-thirtieths (from 122 ÷ 30). That is, the "R2" has no clear meaning unless we know what the original division question was. Notice that as soon as we suggest a meaning for "R2" we have solved the problem – i.e. four and two sevenths as an improper fraction is thirty sevenths or 30 ÷ 7.

The answer to an addition question is '3 1/2'.
Write four different questions that could have this answer. How many could you write? Explain.

The mathematics: addition of fractions; meaning of fractions; common denominators. The simplest answer is probably 3 + 1/2 which is also the meaning of 3 1/2. Some students may be uncomfortable calling "3" a fraction because of previous experience, but since "3" can be written in a/b form (eg. 6/2, 9/3) it is a fraction. Some students may recognize simple patterns and write 3 + 1/2; 2 1/2 + 1; 2 + 1 1/2; etc. Such students are exhibiting number sense and should be commended.

Why can't 12 x 13 = 135?
Why can't 162 ÷ 9 = 15?
Why can't 37 + 58 + 19 = 97?

The mathematics: number sense; divisibility rules; estimation. For the first problem, students should recognize that the units digits, 2 and 3 cannot give a units digit of 5 in the answer. For the second problem, use similar reasoning. For the third problem, the sum of the units digits must give a units digit of 4, not 7; estimating suggests that the answer must be about 120.

Consider 6394 ÷ 12. Invent a story where the answer is a) 532 b) 533 c) 532 R10 d) 532 5/6 e) 532.8333...

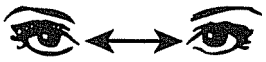
The mathematics: number sense; an understanding of the meaning of division; recognition that some situations demand 'rounding down', some 'rounding up', some a more exact answer than others. In other words, students must recognize the importance of context in assigning meaning to mathematics.

▲ Finding Out About Yourself

Math activities, such as this one from Jan Scully of York Region, can teach students about the world in which they live and about themselves.

Work with a partner:

- My height is _____.
- The distance around my forehead is _____.
- The distance between my eyes is _____.



- The distance around my waist is _____.
- The distance around my fist is _____.
- The distance from my knee to the floor is _____.
- The length of my foot is _____.
- The width of my foot is _____.
- The distance around my neck is _____.



- The distance across my eyes is _____.
- The distance from the base of my nose to the base of my chin is _____.
- The distance from the end of one outstretched hand to the end of the other outstretched hand is _____.
- The distance across my four fingers is _____.
- The distance from my knee to the top of my head is _____.
- The distance from my elbow to my wrist is _____.

Fill in the blanks with the name of the body lengths.

		Estimated ratio
_____	to _____	= 1 to 1
_____	to _____	= 1 to 2
_____	to _____	= 3 to 1
_____	to _____	= 4 to 1
_____	to _____	= 2 to 3

1. Compare your results in the chart with another pair.
2. How are your results alike? different?
3. Write about what you found out about yourself.

▲ Field of Vision

In this activity from Jan Scully, York Region, angle measurement is not carried out in isolation but it is connected to an interesting topic of investigation, field of vision in animals.

Work in a group of four:

In this activity you will perform measurements to determine the greatest angle you can see in a vertical direction. WORK SAFELY!

- To have an accurate angle measure, you must keep your head still while you are being tested. However, you can move your eyes as much as you have to in order to see the object.
- Invite a member of your group to stand close to the chalkboard and look straight ahead.

1. Draw a line on the blackboard outward from their eye parallel to the floor. *(See figures)*
2. Have them look upwards. Hold a pencil above their head.
   - Move the pencil outward until they tell you they can see it.
   - Draw a segment from their eye to that part of the pencil they first saw. (see figure 2)
3. Have them look downwards. Put the pencil beside one of their shoes.
   - Move the pencil outward along the floor until they tell you they can see it.
   - Draw a segment from their eye to that part of the pencil they first saw. (see igure 3)
4. Measure and record the 3 angles made between the segments.
5. Repeat steps 1-4 for each member of your group.

6a. Compare the size of the angles.

- What is the size of the 2 smaller angles?
- the greater angle?

The greater angle is the vertical field of vision.

- What is your vertical field of vision?
- Do boys and girls have a different field of vision?
- Write a report about the above angle. Share your report with other groups.

Figure 1

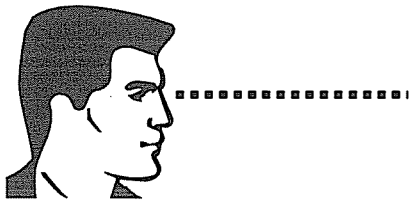


Figure 2

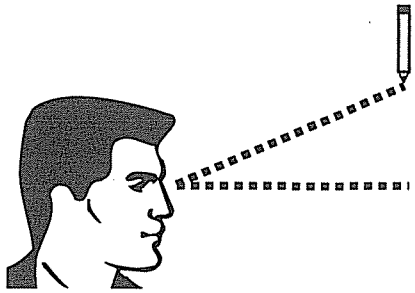
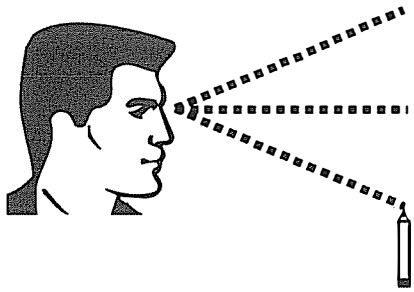


Figure 3



6b. Create an activity that can measure the greatest angle you can see in a horizontal direction. This angle is called the horizontal field of vision.

Research information about the horizontal field of vision for animals that prey upon other animals like wolves, foxes, hawks, and owls. How does the field of vision for the above animals compare to animals that do not prey upon others like budgies and canaries?

▲ Filling an Area: No Gaps and No Overlaps

The following geometry activity, submitted by Barry Scully of York Region, has groups of students investigating which simple 2-dimensional shapes will tile a surface.

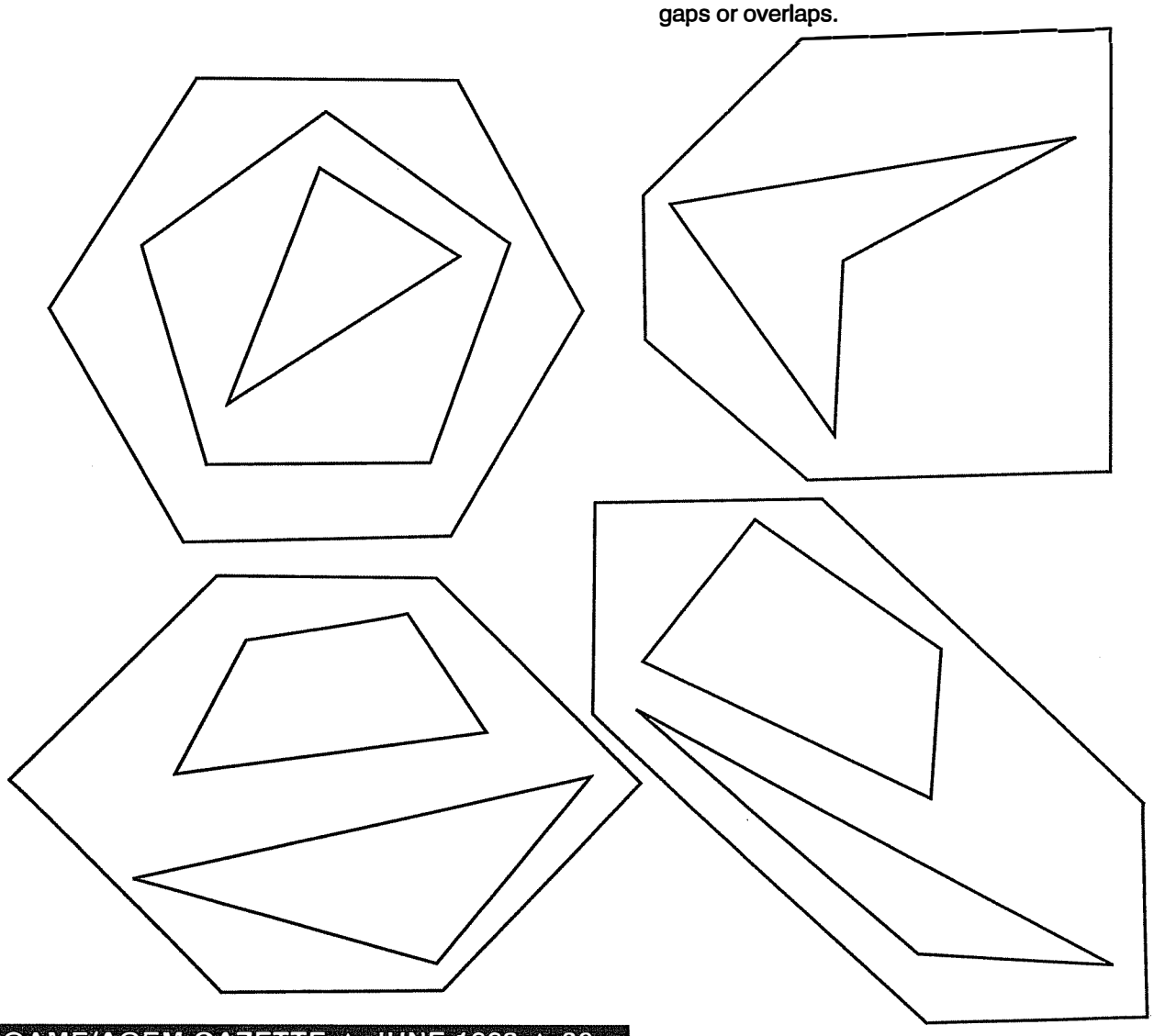
- Work with a partner.
- You will need scissors and construction paper.

Instructions:

- Choose one of the figures below to use as a template.
- Cut about 12 copies of the figure from construction paper.

The problem:

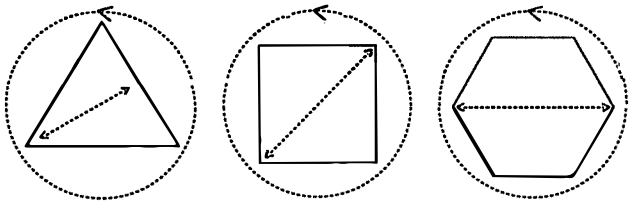
- Arrange the 12 copies to cover an area with no gaps and no overlaps.
- Repeat for other figures.
- Make a list of the figures that can cover an area with no gaps or overlaps.
- Compare your results with other groups.
- Why do you think some figures work and others do not?
- Find or create figures that can cover an area with no gaps or overlaps.



▲ Circumference of a Circle: an Investigative Approach

A change in approach to a familiar topic can often lead to students having a better understanding of the associated concepts and procedures. This is illustrated in the following activity developed by Gary Flewelling.

Have students set up and conduct an experiment that will allow them to answer this question.

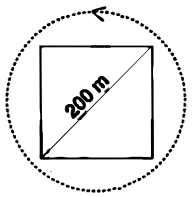


How much farther is it around a regular polygon compared to the distance across it?

Break students into groups of 2 to 5. Each group should select a different family of regular polygons, for example, one could investigate equilateral triangles. Other groups could investigate squares, pentagons, hexagons, octagons, decagons, etc. Each group should find or construct ten different-sized members of their polygon family, share the measurement and calculation workload and average their results.

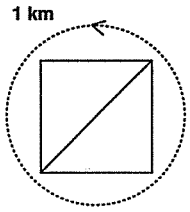
Have the groups pool their results in a table on the blackboard or overhead, and copy it in their notes. Through questioning have the class note that each regular polygon has associated with it its own particular constant. For example, the distance around any square is about 2.8 times as long as the distance across the square. This answer assumes that the distance across a square is defined as the distance from one vertex to its opposite vertex. (The answer is 4 if the distance across is defined as the distance between midpoints of opposite sides.)

At this point, you can pose two types of questions involving the indirect measurement of the perimeter of a regular polygon given its width and the indirect measurement of the width of a polygon given its perimeter.



Example 1

Calculate the perimeter of a square that is 200 metres wide. (answer: about 2.8 x 200 = 560m)



Example 2

Calculate the distance across a square that has a one kilometre perimeter. (answer: about 1000 ÷ 2.8 = 375m)

Have each group make up two problems like the examples above, and place them on the board for the others to solve. Have students write in their journals, with examples, what they learned.

Follow this up the next day with a similar investigation involving circles, with perimeter being referred to as circumference and distance across referred to as diameter. Include a typical set of application questions involving either the indirect measurement of circumference or diameter given the circle's diameter (or radius) or circumference, respectively.

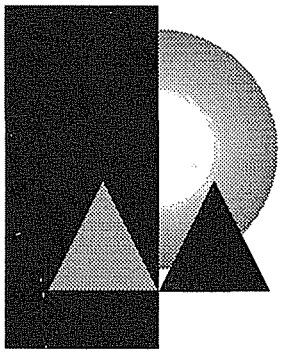
▲ Estimation Problems

The following series of estimation problems, submitted by Loma Morrow of North York, are taken from, adapted from, or triggered by "The I Hate Mathematics! Book" and "Math for Smarty Pants" by Marilyn Burns from Little, Brown and Co.

Students can work with the problems one at a time or they can work at the whole set of problems in pairs or small groups. In the latter case, give students these instructions:

1) Select a problem
2) Estimate an answer
3) Describe a method you could use to check your estimate
4) Carry out your method, if possible.

The value here is in the discussion of the methods and the realization that some quantities will always be estimates since there is no practical way to measure them. Other quantities may be approached through sampling or surveys.



"Schools shall provide for heterogeneous and flexible groupings of students within classes and shall ensure that all students have an opportunity to achieve the learning outcomes specified in The Common Curriculum, Grades 1 to 9."

(Transition Years, Grade 7, 8 and 9, pg. 4)

Problems

1. If you had a piece of string as tall as you are, how many times do you think you could wrap the string around your head?
2. Which is greater—the length of your foot or the distance around your wrist? Your neck?
3. Is there enough paper in a roll of toilet paper to go all the way around this room?
4. Is a large box of cereal taller or farther around?
5. If you were to do a survey on the street in front of the school, do you think you would see more people wearing shoes with shoelaces or more wearing shoes without shoelaces?
6. How many pieces of popped popcorn do you think would fit into a wastebasket?
7. Which do you think would be worth more—a stack of nickels as tall as you or a row of dimes laid edge to edge from your toes to your head as you are lying down?
8. Is a soft drink can farther around than it is tall?
9. Try to draw a rectangle the size of a dollar bill, a circle the size of a quarter, a rectangle the size of a chiclet, a circle the size of a telephone ear piece. Can people do this accurately?
10. About how long a line could you draw with one new pencil before it was worn out?
11. How many cars could be lined up bumper to bumper between here and your city hall?
12. If you asked people to choose their favourite one-digit number, what do you think would be the most commonly chosen number?
13. If you were to try spending $1 000 000.00 by spending $100.00 every hour, day and night, how long would it take you to spend it all?
14. Which would you rather have — your weight in nickels or your weight in quarters?
15. Could you carry $1 000 000.00 in $2.00 bills? in $10 bills? in $1 coins?

▲ Investigating the Surface Area of a Sphere

Jan Scully, from York Region, brings nutrition into the math classroom with this hands on measurement activity. (*At the end of the activity the students eat their oranges.)*

Work in Pairs:

- Measure the circumference of an orange.
- One partner completes the first 3 columns of the chart.
- The second partner cuts the orange peel and finds the area it covers by arranging the peel on centimetre grid paper.
- Partners work together to complete the last three columns of the chart.
- Pairs share their findings by recording them on a common chart.
- What could be a formula we could use to find the surface area of a sphere?

Circumference	Radius	Area of circle	Surface Area of sphere	$\frac{\text{SA sphere}}{\text{A circle}}$	Surface area of sphere
	$r = \frac{c}{2\pi}$				$= \square \times$ Area of circle

▲ A Pythagorean Pyramid

In this `hands on' construction activity submitted by Loma Morrow from North York, students make three congruent pyramids. The pyramids are specially selected to fit together to form a cube.

You will need: scissors, tape or glue, paper, geometry construction instruments

- Using one decimetre as a unit, draw an enlargement of Diagram 1.
- Add tabs as shown in Diagram 2.
- Make two more copies of this figure so that you have three altogether.
- Cut out the figures and fold on the line segments to make pyramids. Use glue on the tabs to help hold the figure together.
- Fit the three pyramids together to construct a cube.

Compare the volume of the cube with the volume of one pyramid. Compare the base area of the cube with the base area of a pyramid. Compare the height of the cube with the height of a pyramid.

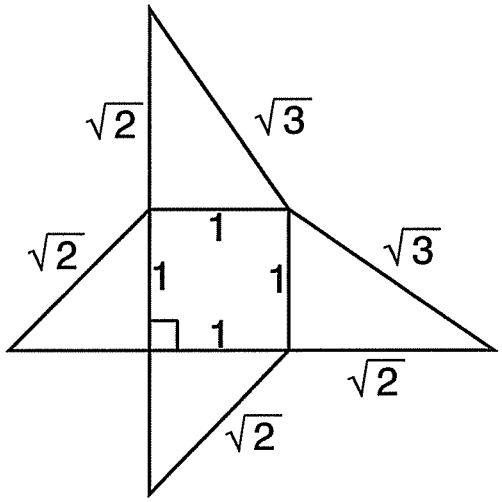


Diagram 1

Teacher Notes:

This activity gives students practice in using construction instruments (compasses, protractors, transparent mirrors, set squares, rulers, etc.)

In order to enlarge the figure, students need to solve the problem of constructing a segment that is √2 units or √3 units long. Students also need some familiarity with scale drawings or dilatations.

Fitting the three pyramids together to construct a cube is a problem in itself demanding spatial abilities.

Students may conclude that the area of a pyramid on the same base and having the same height as a given cube has one-third the volume of the cube. Generalizing to all rectangular prisms is not likely at this stage. However, see the "Prisms and Pyramids" activity for an extension that leads to a more general conclusion.

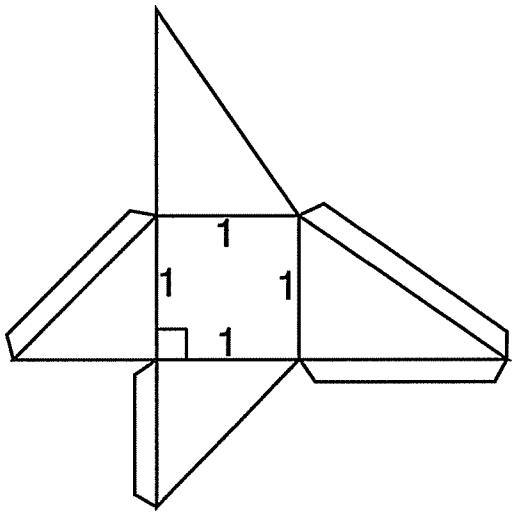
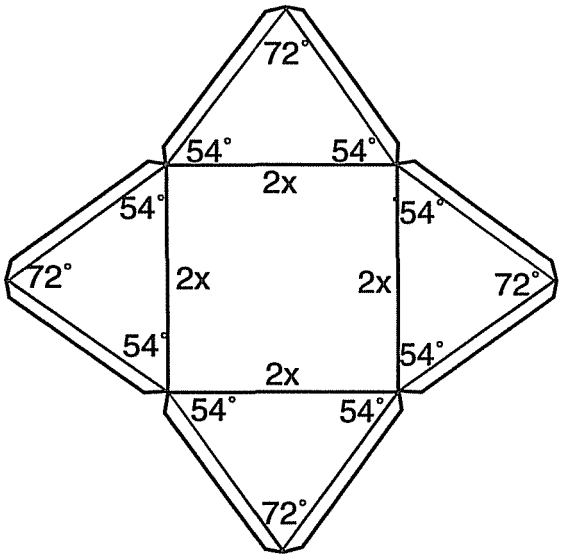


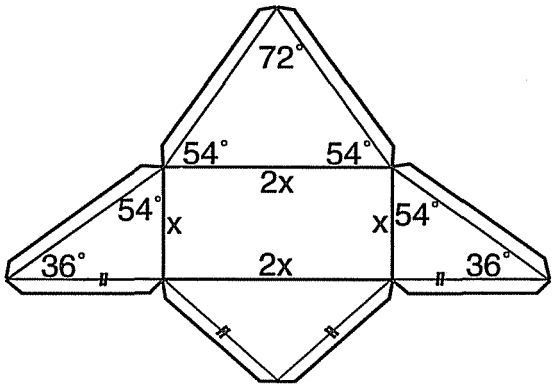
Diagram 2

▲ Prisms and Pyramids

This activity, also submitted by Loma Morrow, is an extension of the previous activity "A Pythagorean Pyramid". The end product is a set of hinged pyramids that fold into a rectangular prism and demonstrate the fact that the volume of a pyramid is 1/3 the volume of its related prism.



Pyramid 1 (Make 1)



Pyramid 2 (Make 4)

You will need: scissors, tape or glue, paper, geometry construction instruments.

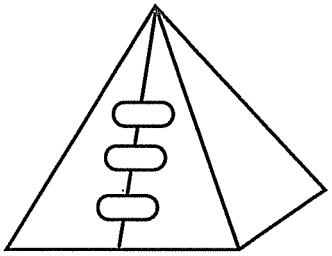
Copy the diagrams above. The side length of the square (2x) should be between 7 and 10 cm to get a model of reasonable size.

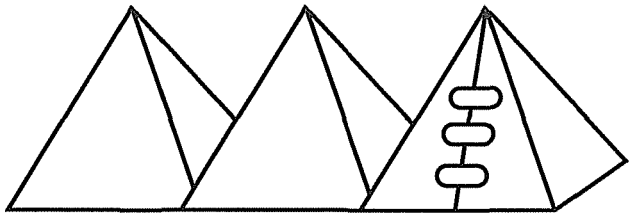
- Cut out the figures, fold, and glue or tape to make 5 pyramids.
- Tape two of Pyramid 2 together to make a hinged copy of Pyramid 1.
- Repeat with the other 2 copies of Pyramid 2.

You should now have 3 copies of Pyramid 1, two of which are hinged on one face.

- Line up the pyramids as shown.
- The hinged pyramids are on the ends with the hinged faces on opposite sides.
- Tape the bases of the pyramids together.
- Fold the model to build a rectangular prism.

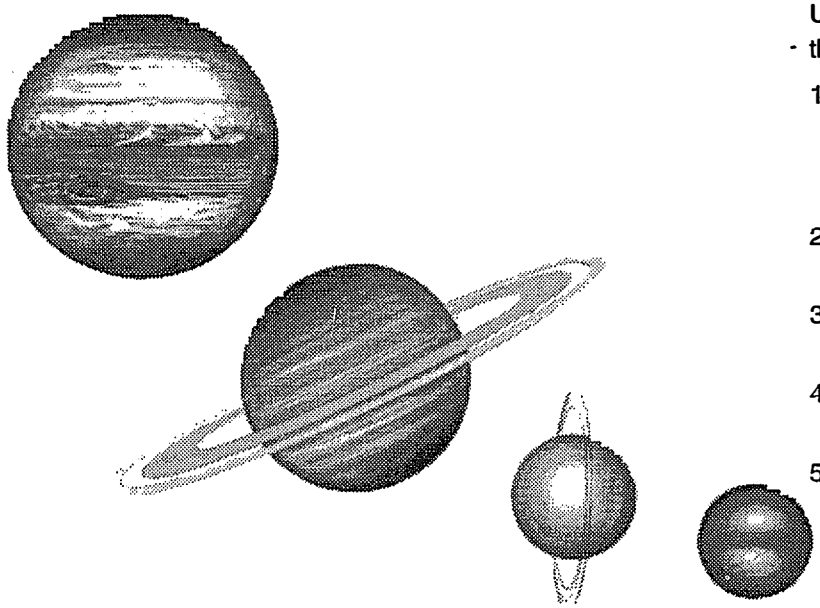
Compare the volume of the prism with the volume of Pyramid 1. Compare the base area of the prism with the base area of Pyramid 1. Compare the height of the prism with the height of Pyramid 1.





▲ The Solar System: Algebra in Context

Modelling situations by linear equations or inequations and connecting mathematics to the world outside of the mathematics classroom are two of the emphases of algebra in the Transition Years. This activity from Gary Flewelling of Wellington County can be done by students working independently or in small groups.



Use the information in the following table to help you answer the following questions:

1. If the sun had a mass of one tonne (about the mass of a car) then the earth would have a mass of about 3 grams. Explain how a person could come up with such a result.
2. If the sun had your mass, calculate the mass of the earth.
3. If the sun had a diameter that allowed it to just fit in your classroom, how big would the earth be?
4. Make a scale model of our solar system that would fit into your classroom.
5. Make up and solve three more problems about our solar system. Share these problems with another group of students.

OUR SOLAR SYSTEM

	Diameter (km)	Distance to sun (10^6km)	Mass ("earths')
SUN	1 392 000		332 830
MERCURY	4878	57.9	0.38
VENUS	12 104	108.2	0.8
EARTH	12 756	149.6	1.0
MARS	6787	227.9	0.1
JUPITER	142 800	778.4	317.8
SATURN	120 000	1423.8	95.2
URANUS	51 200	2868.7	14.5
NEPTUNE	48 860	4492.1	17.2
PLUTO	2300	5926.5	0.002

▲ Discovery with Cubes

Don't include too much detail in your instructions, only enough to give students a starting point to develop their own approaches and strategies. In this activity, a few students will find it helpful to have a set of interlocking cubes available so that they can make and study physical models of the cubes involved in this activity.

Some cubes have been stacked and glued together to form these larger cubes.

If each large cube is dropped into a bucket of paint and completely submerged how many of the smaller cubes will end up:

a) painted on more than three sides?

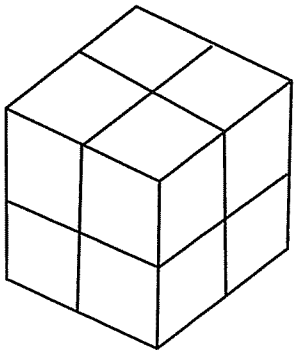
b) painted on three sides?

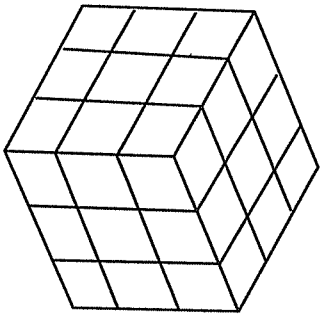
c) painted on two sides?

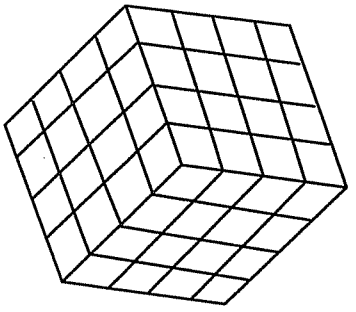
d) painted on one side?

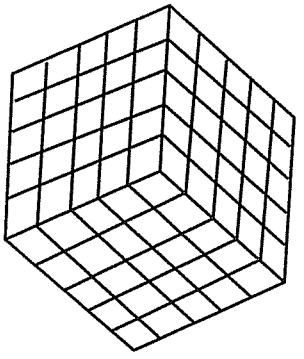
e) not painted?

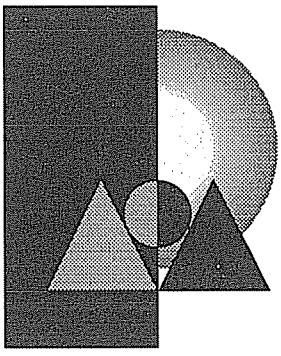
1. Record your results in a table.
2. Add to your table the results for a 6 x 6 x 6 cube.
3. Predict table entries for three other larger cubes. Explain the patterns you used to make these predictions.
4. What would your table entries be for an n x n x n cube?











"A good rule of thumb is to try to include in every learning situation opportunities for direct experience, reflection, instruction and application."

(Teaching Destreamed Grade 9 Classes, pg. 2)

▲ STUDENT BODY GRAPHING

This is a simple yet effective activity developed by Stu Telfer, Waterloo County, that gives concrete meaning to the notion of function and ordered pairs. It offers an exciting change of pace by allowing students to do mathematics outside the classroom.

MATERIALS NEEDED

- length of rope (optional), line markings for grid

OVERVIEW OF ACTIVITY

- A large set of X - Y axes is drawn on an outside playing field. Lines can be drawn very easily using lime line markings on a grass field or using sidewalk chalk in a parking lot. Just using large markers such as orange pylons will also work. If available, a grid can be drawn on a field using Super Stripe (white athletic paint used to line fields).
- The unit markings on the X-axis are approximately 1 metre or 1 student pace apart. The units on the X-axis should be marked, but it is not essential to mark units on the Y-axis.
- Group your students into teams. Teams of 11 are ideal (5 positive, 5 negative and zero) but are not essential.
- Have one group take their position on the X-axis with a student at each unit marking, all facing the first and second quadrant. Students are given a command and each is asked to take that many paces.

Example: *Teacher –*
"Take as many paces as twice the number you are standing on".

The student standing at +3 would take 6 paces forward, the student standing at -2 would take 4 paces backward, etc. Of course, the result will be the students lined up in a "reasonably" straight line of slope 2. If appropriate the teacher could ask the students to state the equation of the straight line (y = 2x) that is the result of the command.

Periodically switch groups. It is possible to show the intersection of two lines by giving one command to a group, telling them to remain in place, then giving a different command to the second group. Places where two students are together are points of intersection (or a discipline problem).

If a length of rope is available, have students hold on to a rope to give a better picture of a straight line.

HINTS AND SUGGESTIONS

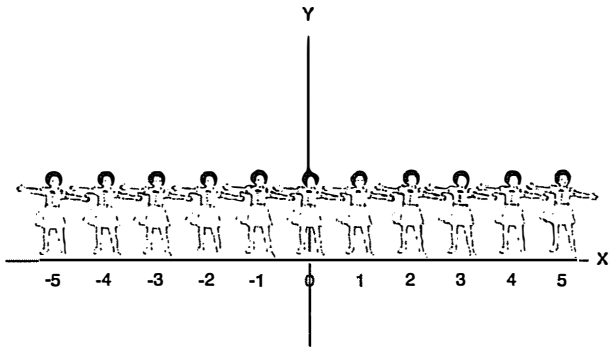
- Have the students "practise pacing" before your initial commands so that they understand what is expected and to allow paces to be fairly equivalent in length. An explanation for students on the negative X-axis is important.
- Keep the first few examples simple. For some you may wish to restrict activity to the first quadrant only (positive X values only).

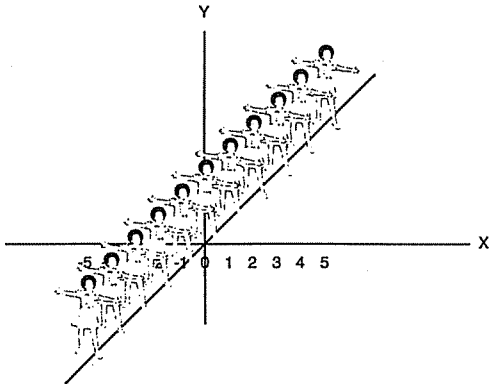
EXAMPLES OF COMMANDS

- "Take as many paces as double the number you are standing on. (Allow students to move.) Now take one more step forward." (y = 2x + 1)
- "Turn around and take as many paces as the value you are standing on!" (y = -x)

EXTENSION

- If students have had previous experience with graphing straight lines, first tell them an equation (eg., y = 2x - 1) and then ask what instruction(s) would generate this line.
- Activities can be created to illustrate properties of parallel lines (eg., one group paces y = 2x and the other y = 2x + 4) and the properties of slope (eg., one group paces y = 2x and the other paces y = 3x or y = -2x).





▲ The Mathematics of the Pendulum

The math classroom should frequently have the look and feel of a science lab with students making conjectures and verifying them through experimentation. The pendulum investigation, submitted by Barry Scully from York Region, is a good example of such an activity.

- Work in a group of three. Answer the following questions before you begin the activity. Answer them individually, then share your thoughts with the group. Try to come to some agreement before beginning the activity.
- A grandfather clock uses a pendulum, gears, and masses to keep proper time. Not all clocks are the same height but they all "tell" the same time. How do you think clock-makers determine the length of the pendulum to make an accurate clock?
- At the end of each pendulum is a mass. How do you think clock-makers determine what size of mass is needed to make an accurate clock?

Now collect the materials you need: string, washers, stopwatch, masking tape, ruler, graph paper

Problem A:

- How long is the pendulum that has a time of swing (a period) of 1 s and a mass of one washer?
- Write about how your group solved this problem. Can you think of any other ways you might have solved the problem? Write about them.

Problem B:

- Change the length of the pendulum and determine the corresponding time of swing for each new length. Make a table:

Length of Pendulum (cm)	Time of Swing (s)

- Record this information on a graph. What type of graph did you choose? Why?
- Make 3 observations from your graph. What effect does the length of the pendulum have on the time of swing?

Problem C:

- Change the mass at the end of the pendulum and determine the corresponding time of swing for each new mass. Make a table:

Number of Washers	Time of Swing

- Record this information on a graph. What type of graph did you choose? Why?
- Compare this graph with the graph from Problem B. Record your observations.
- What effect does the mass at the end of the pendulum have on the time of swing?

Wrap-up:

- Compare the results of Problems A, B, and C with your original theory. Adjust your theory if necessary.

(Note 1. The teacher might wish to add an activity in which students explore the effect of amplitude of swing on the period of the pendulum.

Note 2. Pendulums in grandfather clocks are not simple pendulums. As an extension, students could research the nature of clock pendulums.)

▲ Activities to Stimulate Statistical Concept Development

In this article on Data Management and Probability, Eric Muller, Department of Mathematics, Brock University, illustrates with three sample activities the importance of exposing students in the Transition Years to the concepts of variability and quantity of information.

Data Management and Probability are exciting and important parts of the Transition Years mathematics curriculum. They are exciting because they can absorb and stimulate students at all levels. They are important because of their widespread use to analyze various types of information. This area of applied mathematics is called Statistics. It is worth noting that statistical experiments are controlled by the scientific process, which requires that repetitions of an experiment lead to the same conclusion, within a specified level of error. These conditions will appear, on the surface, contradictory. My advice is not to emphasize the making of decisions but rather to provide experiences which explore alternatives.

Which statistical concepts should our students grasp by the time they complete their transition years? The concept of **variability** is at the top of my list. Students start to explore variability by comparing and pooling their results with those of the other students in the class. Pooling exposes students to a second important statistical concept, namely that of the **quantity of information.** In most situations, one improves one's understanding of the situation by increasing the amount of information. When dealing with large sets of data, avoid repetitive and boring massaging of data; use spreadsheets to do that. I recommend that our activities in data management aim at generating **relative frequency distributions,** as these suggest a bridge to probabilities. In fact they are experimental probability distributions. Our students should leave their transition years with some experience of bias and a desire to question data statements in news reports and advertising. Plan on developing a project with a teaching colleague who has some expertise in communications.

We now outline the types of activities which would enrich the transition years experience. Data gathering activities such as, the number of sports articles in a newspaper, the number of cars passing the school per minute, the weight of candy eaten per student per day, etc., can be good for involving students, raising their awareness of certain issues and developing their data management skills. However, these activities fall short because the underlying mathematical models are complex. In other words, there is a difficult gap between the experiments and the probability models which statisticians would use in these situations. My suggestion is to expose students in their transition years to activities involving manipulatives such as dice and spinners. There the underlying probability model is accessible to the mathematically talented student. The emphasis in these activities should be on the type of information data to record and on what is the most suitable way to display the data in order to meet one's objectives. The purpose of the activities should not be paper and pencil exercises in probability.

In the December 1987 issue of the Gazette I described a game which uses two dice. The advantage of this game is that students can grow with it; it has various levels of play. The game is typical of the activities I would recommend for the transition years. Here are three more activities to expose students to the concepts of variability and quantity of information. (In the first two the probability distributions are known, in the third a comparison is made with published results).

Activity 1: Exploring the distribution of coloured chips in a bag.

A large number of transparent chips, of four of more colours, is counted and placed in a opaque bag. Taking turns each student chooses, without looking, a fixed number of chips (say 10), records the distribution and replaces them in the bag before the next student makes a choice. Based on their information, and given the total number of chips in the bag, each student is asked to make a prediction of the distribution in the bag.

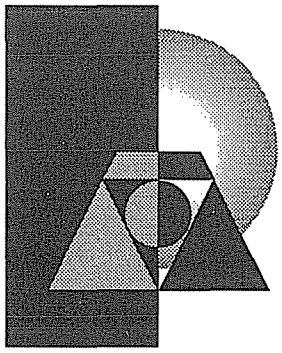
The samples and predictions are compared and their variability discussed. Each student is asked to repeat the sampling this time choosing twice as many chips as before. The change in variability of the samples and predictions compared to the first sampling is discussed. The activity concludes with a comparison of the pooled information with the actual distribution.

Activity 2: Predicting the minimum number of rolls of a die required to reach or exceed a given total.

Before starting the experimentation students are asked to give a worst case (maximum number of rolls) to get 50 (say) and a better case (minimum number of rolls) to get at least 50. Each student then performs the experiment a number of times. The information is now pooled and a suitable display of all the data is produced. What was the worst case - the most rolls? What was the best case - the least rolls? Given this information what would the class use as its best estimate of the number of rolls to get at least 50? Is this information useful to estimate the total we would obtain by rolling the die a fixed number of times (20 say)? The experiment should be repeated with spinners, for example, a spinner with 5 equally probable numbers and a spinner where the angles associated with each number are not equal, i.e., not all numbers are equally likely.

Activity 3: What do we mean when we say that our town had 5.3 cm of rain in September?

This activity involves generating data and comparing class results with those published in the local newspaper. Each student will need a flat bottomed clear plastic container with vertical sides, for example, a flat bottomed plastic bottle could be cut. Using a permanent marker, a vertical centimetre scale is drawn with 0 at the bottom. Each student is asked to place his or her container outside in an open space and to record the depth of water after each rainfall. At the end of the month the individual totals are entered in a suitable table and the total rainfall is estimated and compared with a newspaper report. Discussion should centre on possible causes of the variability of individual totals. Also discuss the location, within the class distribution, of the quantity reported in the newspaper.



"Basing the curriculum on problems seen in context improves learning for all types of learners, including high achievers and students who have difficulty with traditional methods of learning."

(Teaching Destreamed Grade 9 Classes, pg. 2)

▲ PROBLEM-SOLVING BINGO

This problem-solving bingo game is an idea from the TOPS Problem-solving Decks put out by Dale Seymour publications several years ago. Loma Morrow of North York has used it from grade 2 to grade 8, "always with success".

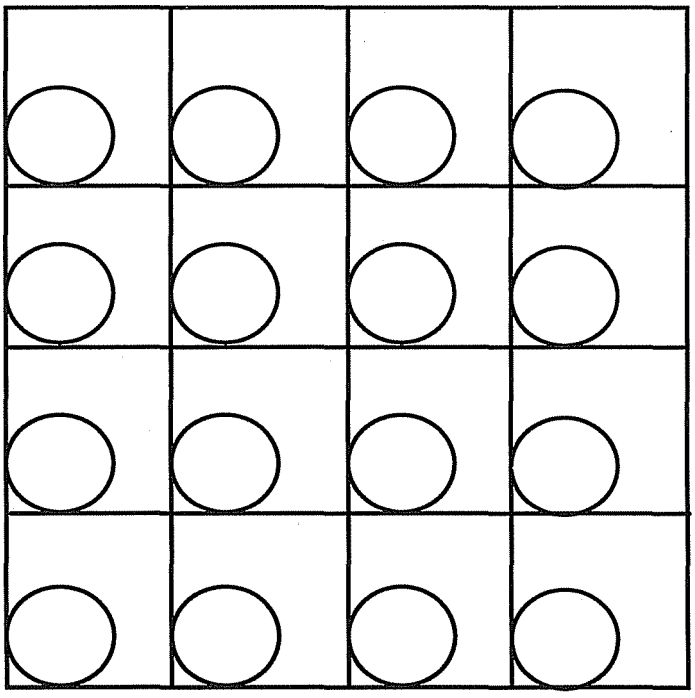
Rules:

1. In each square of the board write one of the numbers from the choices given. Do not look at the problems.
2. Select a problem and solve it. Look for the answer on your bingo board. If the answer is on the board indicate this by writing the number of the problem in the small circle in that square on the bingo board.

   If the answer is not on your bingo board, or if you have already used that answer for another problem, you "lose a turn". That is, you have solved a problem but it does not help you towards bingo.
3. First team to get four in a row marked on the board is the winner.

Choose from

1	6	7	8
10	12	13	18
20	24	30	36
45	54	62	64
96	162	200	450
26 500			



Teacher Notes:

- Students may work individually, but pairs or small groups are very effective.
- Each team of players needs one bingo board and one collection of problems. A convenient way to package the problems is to reproduce a set for each team, cut the problems apart and put them into an envelope.
- Scrap paper, calculators, and manipulatives should be available for students to use as they wish.
- Problems can be taken from any source — old text books, supplementary materials, NCTM publications, etc.

  Two good NCTM sources are:

"Mathematical Challenges for the Middle Grades: From the Arithmetic Teacher" William D. Jamski (Ed.), NCTM, and the companion volume "Mathematical Challenges for the Middle Grades: From the Mathematics Teacher Calendar Problems".

This particular set of problems is taken from Gauss contest papers over the last few years. A good source of such problems is "Problems, Problems, Problems" Volume 4 (and the forthcoming Book 6) from Canadian Mathematics Competition, Faculty of Mathematics, University of Waterloo, Waterloo, Ontario, N2L 3G1.

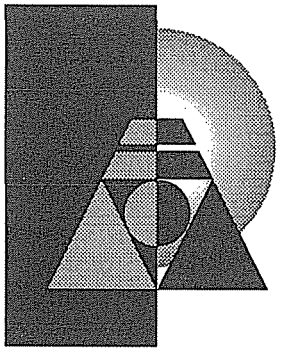
Answers are given here for easy checking of "Bingo". However, if students interpret problems differently and can justify alternative answers, these should be accepted.

The problem number is given first, then the answer.

1-12	2-7	3-12	4-26 500
5-1	6-96	7-200	8-8
9-12	10-24	11-62	12-450
13-13	14-10	15-6	16-18
17-64	18-162	19-54	20-10
21-20	22-45	23-36	24-30

Problems:

1. One mouse said to another, "If you give me one piece of cheese, then we will have an equal number of pieces." The other mouse replied, "If you give me one piece, then I will have double the number that you have." How many pieces did they have together?
2. Each number in a sequence is obtained by adding the two previous numbers. The 6th, 7th and 8th numbers of the sequence are 29, 47, and 76. What is the third number in the sequence?
3. A railway engine is 12 m long. A model is build to 1/100 scale. What must be the length of the model in centimetres?
4. If Bob earns $25,000 a year and Frank earns 6% more than Bob, then how many dollars does Frank earn in a year?
5. If the average of the five numbers 4.9, 3.1, 7.2, k, and 9.3 is 5.1, then what is the value of k?
6. When 16 litres are added to a gas tank that is half-full, then the tank is two-thirds full. What is the capacity of the tank in litres?
7. To make concrete, mix 4 shovels of stone, 2 shovels of sand, and 1 shovel of cement. How many shovels of stone would be needed to make 350 shovels of concrete?
8. When 62 is divided by a certain number, the quotient is 7 and the remainder is 6. What is that certain number?
9. What is the greatest possible product of two whole numbers that have a sum of 7?
10. If 3/4 of a number is 12, then what is 3/2 of the same number?
11. What is the sum of all the even divisors of 32?
12. What is the sum of the digits in the tens places of all two-digit whole numbers?
13. In a set of five numbers, the average of the first two numbers is 10, and the average of the last three is 15. What is the average of all five numbers?
14. A certain number is multiplied by 8. This result is then increased by 8. The final answer is 88. What was the original number?
15. Of 33 students in a class, 18 belong to the mathematics club, 17 belong to the science club, and 4 belong to neither club. How many students belong to both clubs?
16. On a test consisting of 30 questions, Sue had 50% more right answers than she had wrong answers. Each answer was either right or wrong. How many answers did Sue have right?
17. If you travel 48 km in 45 minutes, what is your speed in kilometres per hour?
18. What is the sum of all whole numbers, less than 100, that are multiples of 6 and end in a 4?
19. If 12 345 679 x 9 = 111 111 111, and 12 345 679 x n = 666 666 666, then what is the value of n?
20. Two whole numbers are in the ratio of 2 : 5. The product of the two numbers is 40. What is the greater number?
21. Arin and Jim together have $1.35. Arin has only nickels while Jim has only dimes. Arin has six more coins than Jim. How many coins do the two of them have together?
22. A ball that bounces straight up and down falls, initially, from a height of 16 m. Each time, it bounces up to half the height from which it just fell. When the top of its bounce is 1 m from the floor, what is the total distance, in metres, that it has travelled?
23. If one-third of a number is 6, then what is twice the number?
24. A 70 cm by 84 cm rectangle is entirely covered with congruent square tiles. What is the least number of tiles required?



"The program must be organized in such a way that the interrelationships among subjects and topics are evident and meaningful to the student".

(Transition Years, Grade 7, 8 and 9, pg. 3)

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Harry F. Traini

Margaret Warren

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Arnold Carefoote

Ralph Connelly

Shirley Fairfield

Jerry Maloney

Barry Onslow

Ann Pearce

Barry Scully

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Kay Appleby

Radford De Peiza

Barry Ferguson

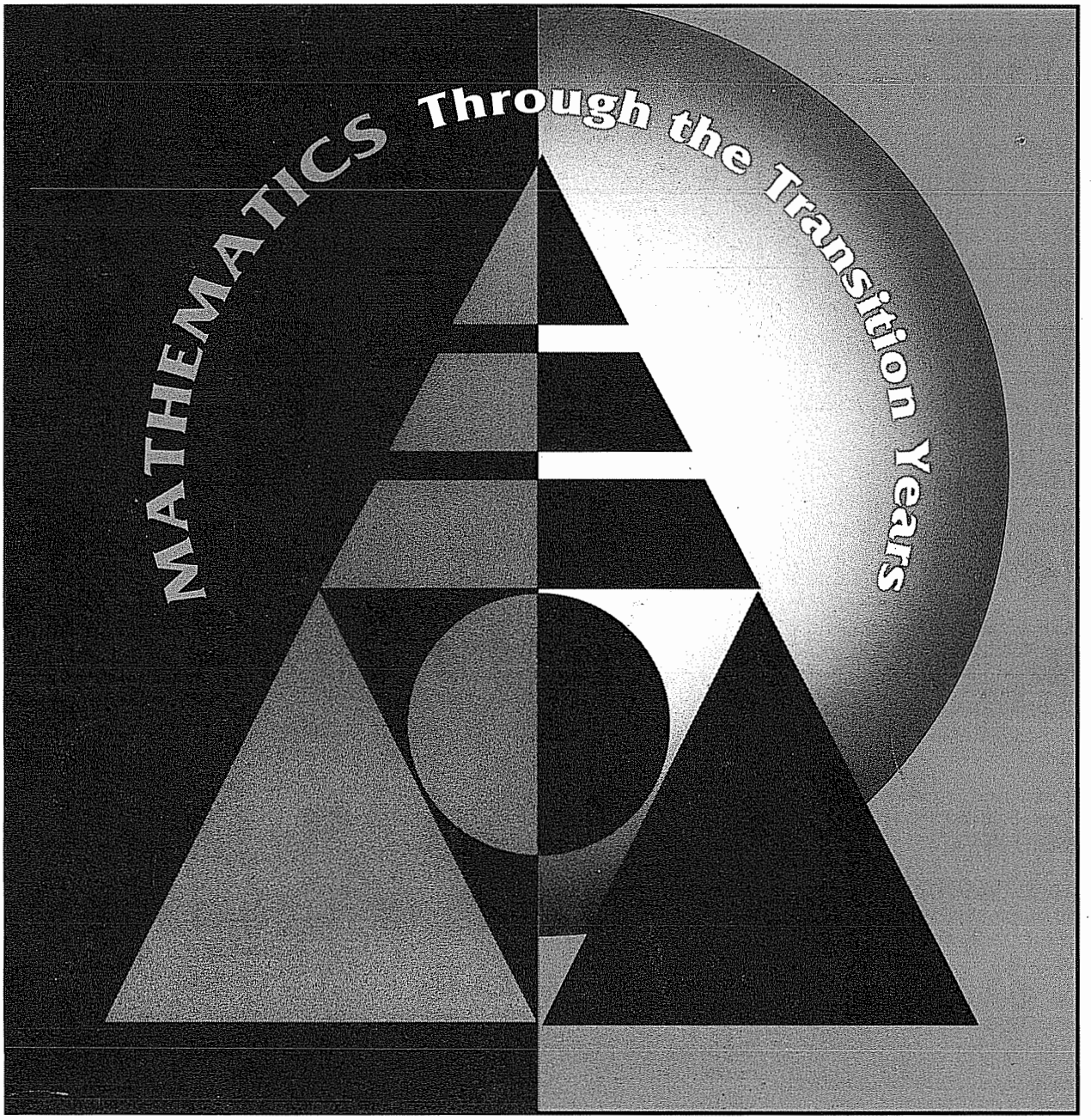
Myrna Ingalls

Peter Lenczewski

Lorna J. Morrow

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