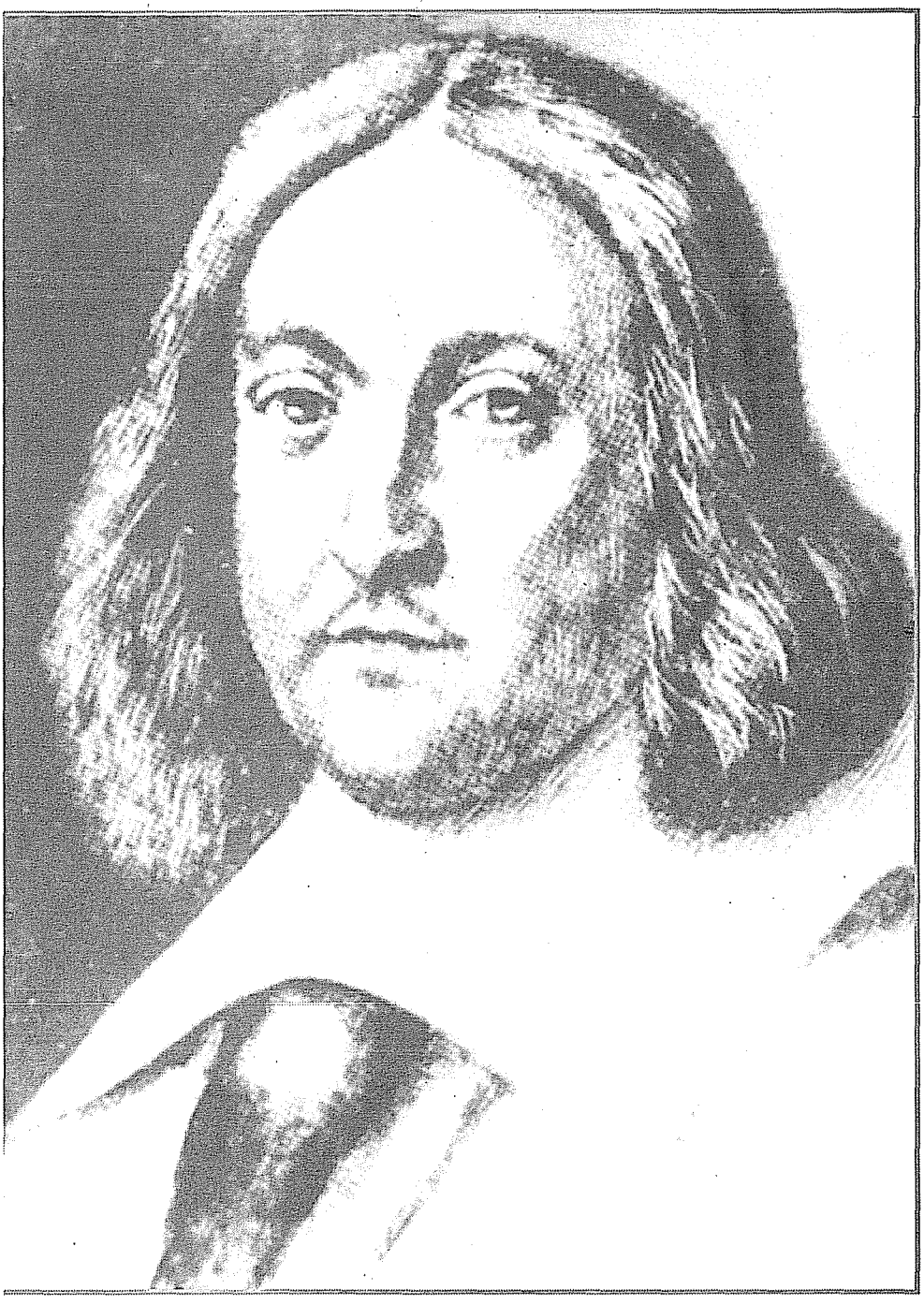
Vol. 32 #1, SEPTEMBER 1993

Gazette

Fermat's Last Theorem: Proved At Last?

See Page 6



Inside:

Snapshots by Paul Sherk of the OAME '93 Annual Conference

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

About the Gazette

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Submission of Articles

Articles for the Gazette should be submitted to Jack Weiner at the above address, while those for the Abacus should be submitted to Amy Parker Deadlines for information to be included in the Gazette or the Abacus are: June 15 for the September issue, October 15 for the December issue, January 15 for the April issue, and April 15 for the June issue.

Notice to Authors

Your articles should be typewritten, word-processed or photocopied from typewritten or word-processed copy double-spaced on 8.5" x 11" paper with 1.5" margins and single sided. Diagrams and figures should be in camera-ready form. Please submit three copies to the editor and keep one as protection against loss. We have all submissions blind-reviewed, so please send your full address and affiliation on a page separate from the manuscript.

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▲ Editorial

September, 1993—Destreaming! Standardized Exams! Social Contract! Wage Freeze! Wage Cuts?!?!!!

Issues! Controversy! Infighting! Out and out fighting!

I think I'd like to start my September by remembering why I love math and why I love teaching.

So let's see. How about Mr. Beardsley, my grade 9 math teacher, who let me come visit him at my former school, when I needed help the next year with my grade 10 math? And Mr. Edwards, who got physically excited, as I did and do, about a particularly neat proof in geometry. Of course, there's Paul Stanbury, an outstanding math teacher and human being. (Note the first name. By grade 13, I was taking liberties.)

At university, Ian McGee, John Wainwright, Peter Hoffman, and of course the incomparable Ken Fryer, played important roles in nurturing my love of math and teaching, as did Hugh Allen and Bill Higginson at MacArthur Teacher's College.

Then there are the people with whom I've taught whose example and experience influence me every day. Here at U. of G., I greatly admire math professors Joe Cunsolo and Joe Mokanski, both multiple teaching award winners. At Parkside High School in Dundas, the whole math department and in particular, Neil Reid, Jim Mattice, and, in the science department, Don Buntain. OAME's Executive Directors Don and Carol Attridge, and current president Ron Lancaster. Math Consultant Gary Flewelling. Hamilton teacher Trevor Thomson, who encouraged me to share my ideas in the first place.*

Most important, and truly too numerous to mention individually, those students, grades 9 through fourth year university, I have been privileged to teach. Every semester I have taught at university and every term I taught in my years at Parkside, there were days, fortunately rare, when everything would go wrong—lessons that didn't work, discipline problems, broken equipment. Most days, though, I would come away smiling. And start the next day smiling.

This is all very personal, and since most of you know few of the people I've mentioned, this may all seem irrelevant. Well, not really, because I believe each of you could have written this editorial, inserting the appropriate names from your experience. And in remembering, you'd be smiling as you wrote.

Sure, these are challenging and in some ways, exasperating times. But I wouldn't trade my job for anything. I have much to be grateful for and many to be grateful to.

September, 1993. A fresh start in every classroom. Have a terrific year.

Sincerely,



** Not enough women in these lists! But I suspect that would not be the case for someone reflecting back on high school and university in the 1990's rather than the 1960's and 70's.*

▲ Contents

Columns

- ▲ Editorial / **1**
  Jack Weiner
- ▲ Letter to the Editor / **2**
  Peter Taylor
- ▲ Council Comments / **2**
  Bob Robinson
- ▲ President's Message / **4**
  Ron Lancaster
- ▲ Math and the World Around Us:
  Fermat's Last Theorem / **6**
  Ron Lancaster
- ▲ Have Your Say / **10**
  Ed Barbeau

Articles

- ▲ Problem Solving in Mathematics:
  A Fresh Perspective
  for Future Teachers / **13**
  Rheta N. Rubenstein
- ▲ Did Pythagoras
  Really Say "$a^2 + b^2 = c^2$"? / **17**
  Geoffrey Roulet
- ▲ That's Sum Triangle / **19**
  Helene J. Sherman and Karen L. Torretta
- ▲ Proof by Induction:
  Understanding at Last? / **24**
  Tom J. Griffiths
- ▲ "Am I Really Crazy?"
  Selected Issues
  Concerning Destreaming / **26**
  James M. Haines
- ▲ Designing a Mathematics Office / **30**
  Mike Zichowski

Notices

- ▲ 21st Annual
  OAME/AOEM Conference / **5**
- ▲ 1993 OAME/AOEM
  Leadership Conference / **9**
- ▲ Nominations for
  OAME/AOEM Council / **31**
- ▲ Call for Award Nominations / **33**

▲ LETTER TO THE EDITOR

REPLY TO GORDON NICHOLLS' ARTICLE ON ENTRANCE EXAMS.

I agree with pretty well everything Gord said about the problems our students face in proceeding to university or college, and the desirability of some form of entrance exam. One of the most positive impressions that remain from my own Grade XIII preparation at Lisgar (1960-61) for the "Departmentals," was the feeling that the teacher was not adversary but ally in a joint campaign against the "system." That was fun and enlivening.

I'd like to comment on his Proposal #1, which concerns university admission. I accept his proposed list of possible examinations, but I suggest that these examinations not be based on the most recent work of the students (ie. OAC) but on a somewhat lower level of material (eg. grade 11-12). This would have a number of advantages, which I shall discuss in the context of the mathematics exam.

1) The material in the OAC math courses (calculus and linear algebra) is new in kind for the student. It is difficult to prepare such material quickly to an examination level for any syllabus except one based on technical skills and type problems.

2) Less sophisticated material would allow for somewhat more discriminating types of questions, and we could test those skills that are really needed by a student to succeed in first-year university. These skills are outlined by Gord in his "second problem" - the ability to review a standard body of material, synthesize information and concepts and write well-reasoned answers. It is often said that what causes a student to fail first-year math is lack of mastery at the grade 11-12 level.

3) It would set the OAC teacher free to provide a real capstone course which ties together a lot of high school material, and allows time for open ended problems, class explorations, and essay writing. Again, this experience would serve the university-bound student well.

Of course, I realize that we will not avoid a certain amount of "teaching to an exam"; but that can work well. As the time of the exam approaches, schools can set up special sessions which play with past exams. If these are good problems, this could be a valuable experience for student and teacher.

I offer one alteration, and that is to require three exams for university entrance instead of two, with one of English and French being compulsory. For example, at Queen's, I expect our Engineering students would be required to offer English or French, Math, and Science. This would also reduce the variance in score due to an unfortunate (perhaps for the student) set of questions or a bad day.

Peter Taylor
Department of Mathematics and Statistics
Queen's University
Kingston, Ontario

▲ COUNCIL COMMENTS

BOB ROBINSON, GOLDEN SECTION CHAPTER REPRESENTATIVE

These notes were taken at the OAME Council Meeting held June 5, 1993 at the Ramada Inn-Airport West in Mississauga. Send any correspondence directly to the editor.

INTRODUCTIONS

Ron Lancaster, our new President, introduced the new councillors.

CORRESPONDENCE

The International Mathematics Olympiad will be held in Ontario in 1995.

A letter has been sent to Directors of Education throughout the province telling them what OAME is all about and specifically mentioning the *Focus on Renewal* document; the *Gazette* and *Abacus;* our Leadership Seminar "Changes in Transition"; and our Annual Conference.

FOCUS ON THE RENEWAL OF MATHEMATICS EDUCATION

The document was released at the Windsor Meeting at a press conference and was included with the June Gazette. This was a joint project of OAME and OMCA. Please look at

the names of those responsible for its development, in particular Judy Crompton and Karen Allan as project managers and Don Attridge for his "behind the scenes" assistance. This timely publication was designed to provide clear guiding statements to curriculum planners about content and methodology for mathematics education from Junior Kindergarten through the end of Grade 9.

VISIONS and ONGOING PROJECTS

Ron Lancaster has directed the Council to address the following issues:

1. Membership
2. Assisting Parents
3. Media Involvement
4. Communication with Universities and Colleges
5. Equity Issues regarding Mathematics Contests

PROFESSIONAL DEVELOPMENT LEADERSHIP'93

"Changes in Transition" is the timely theme and another sellout is anticipated...so enrol early. The seminar will be held on November 10-12, 1993 at the Ramada-Airport West. For more information contact Don or Carol Attridge at 1 Southdale Dr., Markham, ON L3P 1J6 or phone (416) 294-3511.

CONFERENCE'93

Todd Romiens did an outstanding job along with Don Gray as co-chairs of our annual conference in Windsor. Over 1000 attended this highly successful "Annual". A lot of personal touches were evident...Thanks Todd and Don!

CONFERENCE'94

Planning is well underway and we are all looking for another successful and satisfying annual conference in Kingston on May 12-14, 1994. Brian Gallagher and Margo Johnston are the conference co-chairs. David Handley is the chair of publicity.

PUBLICATIONS

Advertising rates and schedules have been established and sent out to publishers and educational supply companies. Advertisements will be appearing regularly in issues of the *Gazette.* Further information is available from Bob Robinson, Advertising Manager. (See the front inside cover for details.)

Councillors are very pleased with the quality of our publications and very appreciative of the many hours volunteered by our editors Jack Weiner and Amy Parker, and their assistants.

Gary Flewelling was commended for his many hours of work in the preparation of the special "Transition Years" issue.

MEMBERSHIP/PUBLICITY

A new membership brochure has been prepared and it will be continually updated.

The committee is investigating ways to increase our membership and publicize our association. The process will involve both maintaining existing memberships and obtaining new memberships.

EARLY YEARS/FORMATIVE YEARS

We invite you to submit appropriate articles for our two publications. For the *Abacus,* we are particularly seeking packages pertaining to Design and Technology, Data Management, Probability and Chance, and Estimation.

TRANSITION YEARS

A project of the committee this year is to produce a support document pertaining to Patterning and Algebra.

SPECIALIZATION YEARS

OAME and OMCA will work together to complete the *Focus on Renewal* by addressing the Specialization Years.

The committee will attempt to produce a Grade 10 document to assist teachers to teach mathematics to students who have come from a destreamed Grade 9.

Another publication will be directed to professional development suggestions for teachers using new technology.

EQUITY

Membership on this committee is open to any member of Council. The annual Math Equity breakfast was very successful with over 110 attending in Windsor. Planning has begun for next year's breakfast which will be held on Friday, May 13, 1994. A meeting has been set up with the University of Waterloo to discuss equity issues as they pertain to mathematics contests.

A personal note, I was honoured and deeply touched when I was notified that I was the recipient of an honourary lifetime membership in OAME. Thank you so much for this honour!

▲ PRESIDENT'S MESSAGE

RON LANCASTER



Change. Can you think of any other word that stirs up as much emotion or concern as this word does? Change is everywhere around us and in my first message to members of the OAME I would like to take some time to look at what I see as the two types of change.

One type of change is external. While this type of change comes from outside of our control, we do have some say in whether we resist the change or work with it. Take graphics calculators for example. There has been an incredible change in the capabilities of these calculators even over the past year. Graphics calculators are now delightfully easy to program, two of them can actually talk with each other and they can even be hooked up to a computer. Whether we like it or not these calculators are here to stay and they are getting cheaper by the hour. What we do have control over is what we do with them. We can either resist the technology or we can embrace it and make changes in what we teach and how we teach it. Some topics and algorithms can now be done in much easier ways by using a graphics calculator and we now have a chance to take a close look at what is being taught to students. Computers along with these graphics calculators are changing how people do and use mathematics in industry and elsewhere. Again we have no control over this change. What we can change is how we prepare our students for the world that they will enter. For many teachers this world is quite different from the one that they were part of when they were in school so it is not surprising that some teachers are having trouble coping with these changes. If you are teaching high school mathematics and you are not using graphics calculators, you owe it to your students to start using them. You will be surprised and pleased with the changes that you will see in your students.

There are many other external changes that we are confronted with on a daily basis and most of these are difficult to deal with. It appears to me (I just turned 40 and these things are sometimes hard to judge as we get older) that students have changed a great deal from when I was in school. They watch TV more, more of them come from less than ideal home situations and school is not the only thing in their lives anymore. Today the biggest source of external change is from the Ministry and there are certainly teachers all over Ontario who are very frustrated with the daily changes in direction and the general lack of support. The OAME is working hard to help teachers with these changes. The June issue of the *Gazette* dealt exclusively with The Transition Years and the *Focus on Renewal* document is full of ideas, thoughts and new directions for teachers.

The other type of change is internal and it is the most important one. Herein lies the real secret for coping with the external changes around us. By internal change I mean things that an individual can do to alter their surroundings and environment. I have experienced a great deal of internal change over the past few years and it all started with a major change that I made several years ago. I am generally quite shy and private, but perhaps I can share with you the changes that I made in the hopes that it may help some individuals to see that internal change can be great. The 1980s was a very special decade for me. I spent those years at Oakville-Trafalgar High School and I was fortunate enough to have had a great deal of recognition for my work in the classroom. I won two National Teaching Awards from The Hilroy Foundation along with two other national awards and a number of other local awards. Well over fifty newspaper and magazine articles were written about my teaching techniques and I had a special relationship with the community of Oakville. I am not one to be boastful and that is not what I am trying to do here. The point that I am attempting to make is that I was very happy and I could have stayed there and been very comfortable for another decade. As the decade ended however I decided that a change would be good for me and I resigned from my position and was subsequently hired by The Peel Board of Education. I went from being a mathematics teacher in a school that knew me to be an acting-head in a school where I was an unknown. The school itself was so different from anything that I had ever been in. I had students from over 65 countries in my classes and with the Gulf war dominating the news, the tension within the school was high at times. The school was non-semestered, yet another big change for me. Within the first three weeks I had 114 timetable changes to cope with. I am not making this up! Something had gone wrong with the timetabling that year and dozens of students came and went from my classes oftentimes on a daily basis. I was having trouble just getting used to dealing with names of students from around the world. On top of all of this, I had a horrendous drive from Hamilton to Mississauga and it was not unusual for me to be on the road for 3 hours or more. My feelings about the change that I had made sank to an all-time low in about November when

the Board announced the termination of a number of consultant positions. These positions were one of the reasons why I had made the big change in the first place. Near the end of the year I realized that I would have to make yet another big change and I took a new position at St. Mildred's-Lightbourn School, an all-girls school in Oakville.

In reading this your first reaction might be to never make any sort of internal change for fear of having the kind of experience that I had. I am pleased to tell you though that the story has a happy ending. This decade is turning out to be even better than the 1980s. St. Mildred's-Lightbourn is a JK-OAC school and for the past two years I have taught Grade 7 and 8 mathematics classes and a Calculus class. Had I not made my original change I never would have experienced such a unique and special timetable. The change that I made also led directly to a new relationship that has blossomed into a permanent one. This relationship is warm and egalitarian and it certainly has been the best change of all.

I would encourage all of you to consider making some internal changes in your life over the next few years. Have you been in the same school for a long time? Would you benefit from changing where you teach? Are you using the same notes and exams year after year? Would there not be an opportunity for some good PD by making up or finding some new problems, activities or projects for your students? The process of making internal changes will help make it easier for you to cope with the external changes that you must deal with in and out of the classroom.

Have a great year and I hope to meet many of you at either a Chapter conference or at OAME'94 in Kingston.

— Ron Lancaster

21ST Annual O.A.M.E. Conference

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Queen's University
Kingston, Ontario
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HOSTED BY QUINTE ST. LAWRENCE MATHEMATICS ASSOCIATION IN COOPERATION WITH O.A.M.E.

▲ MATHEMATICS AND THE WORLD AROUND US

RON LANCASTER, ST. MILDRED'S-LIGHTBOURN SCHOOL

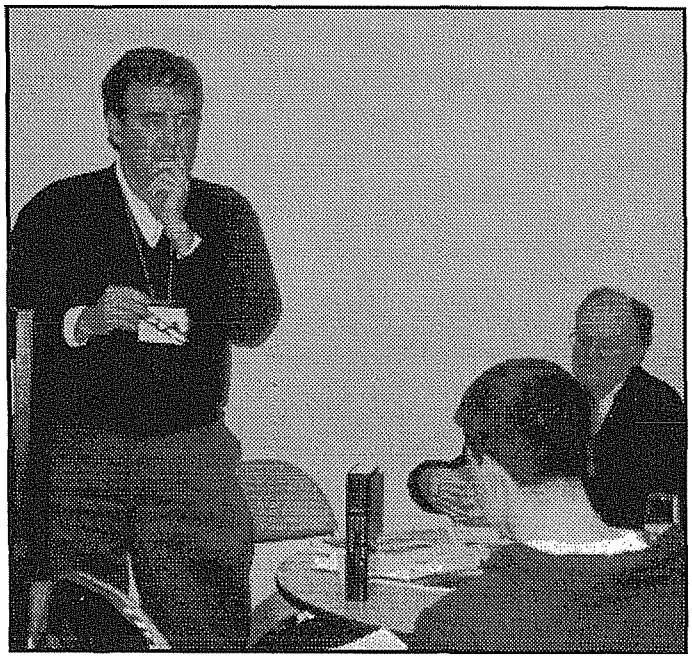
Readers are invited to submit items from newspapers, magazines, books or other sources that contain information that can be related to any area of Mathematics. Sufficient information about the source of the item should be given so that other readers can obtain additional details. Teachers are encouraged to use the contents of this regular feature of *The Gazette* as an aid to helping their students see the connection between Mathematics and the world around us. The content may not fit in with what you are currently teaching to your students, but many of the items can be used to begin your classes in an interesting and engaging manner.

This year I was again able to teach a course at The Phillips Exeter Academy Mathematics and Technology Conference in Exeter, New Hampshire. This conference is by far the best PD experience that I have ever had. It is a week long and participants take two 10 hour courses of their choice. The variety in the courses that are offered is exceptionally good and there is also a software library that is open for people to use at the conference. The conference is usually held during the last week of June and you can obtain more information by calling (603) 772-4311, extension 381.

On the Thursday morning of the conference I got up early and went for a run through the town of Exeter. At about 6:15 I stopped at a convenience store to buy a copy of the New York Times. The truck had just come in and I had to wait a few minutes to buy a copy of the paper. As I started to run again I looked at the front page of the paper and was startled to read that Fermat's Last Theorem had been proven. I literally stopped in my tracks and was spellbound as I read the article. I was amazed and delighted to see how this news had made it to the front page of The New York Times. It was a great moment to be a mathematics teacher. I ran into other mathematics teachers from the conference on the way back to my room and I shared the news with them. During the day other people bought the paper too and the news spread very quickly. I would imagine that by the end of the day everyone at the conference would have heard the news. I wish that I somehow could have conducted a poll throughout the day because the data could have been used as a classic example for individuals who study the dispersion of information.

When I arrived back home I was able to track down a number of other articles that dealt with this news item and there is no question that The New York Times article was by far the best one. The explanations given in the other articles were hard to follow and they did little to promote mathematics. The worst article came in the form of an editorial in The Toronto Star in which the editors made the comment that "many readers will wonder how grown men can waste their time trying to solve a mystery lingering from the 17th century". To the editors credit, they go on to explain the value of such work. But it is hard to imagine how anyone could be so out of touch around gender issues and the editorial did little to make readers feel good about the subject of mathematics.

This is important news for the mathematical community and I encourage you to share this story with your students. Since you probably missed the article from The New York Times we have been given permission to reprint the article in its entirety.



* "At Last, Shout of 'Eureka!' In Age-Old Math Mystery"

by Gina Kolata, The New York Times, June 24, 1993

More than 350 years ago, a French mathematician wrote a deceptively simple theorem in the margins of a book, adding that he had discovered a marvelous proof of it but lacked space to include it in the margin. He died without ever offering his proof, and mathematicians have been trying ever since to supply it. Now, after thousands of claims of success that proved untrue, mathematicians say the daunting challenge, perhaps the most famous of unsolved mathematical problems, has at last been surmounted.

The problem is known as Fermat's last theorem, and its apparent conqueror is Dr. Andrew Wiles, a 40-year-old English mathematician who works at Princeton University. Dr. Wiles announced the result today at the last of three lectures given over three days at Cambridge University in England.

Within a few minutes of the conclusion of his final lecture, computer mail messages were winging around the world as mathematicians alerted each other to the startling and almost wholly unexpected result.

Dr. Leonard Adelman of the University of Southern California said he received a message about an hour after Dr. Wiles' announcement. The frenzy is justified, he said. "It's the most exciting thing that's happened in - geez - maybe ever, in mathematics."

Mathematicians present at the lecture said they felt "an elation," said Dr. Kenneth Ribet of the University of California at Berkeley, in a telephone interview from Cambridge.

Impossible Is Possible

The theorem, an overarching statement about what solutions are possible for certain simple equations, was stated in 1637 by Pierre de Fermat, a 17th century French mathematician and physicist. Many of the brightest minds in mathematics have struggled to find the proof ever since, and many have concluded that Fermat, contrary to his tantalizing claim, had probably failed to develop one despite his considerable mathematical ability.

With Dr. Wiles' result, Dr. Ribet said, "the mathematical landscape has changed." He explained: "You discover that things that seemed completely impossible are more of a reality. This changes the way you approach problems, what you think is possible."

Dr. Barry Mazur, a Harvard University mathematician, also reached by telephone in Cambridge, said: "A lot more is proved than Fermat's last theorem. One could envision a proof of a problem, no matter how celebrated, that had no implications. But this is just the reverse. This is the emergence of a technique that is visibly powerful. It's going to prove a lot more."

Remember Pythagoras?

Fermat's last theorem has to do with equations of the form $x^n + y^n = z^n$. The case where n is 2 is familiar as the Pythagorean theorem that the squares of the lengths of two sides of a right-angled triangle equal the square of the length of the hypotenuse. One such equation is $3^2 + 4^2 = 5^2$, since $9 + 16 = 25$.

Fermat's last theorem states that there are no solutions to such equations when n is a whole number greater than 2. This means, for instance, that it would be impossible to find any whole numbers x, y and z such that $x^3 + y^3 = z^3$. Thus $3^3 + 4^3$ (27 + 64) = 91, which is not the cube of any whole number.

Mathematicians in the United States said that the stature of Dr. Wiles and the imprimatur of the experts who heard his lectures, especially Dr. Ribet and Dr. Mazur, convinced them that the new proof was very likely to be right. In addition, they said, the logic of the proof is persuasive because it is built on a carefully developed edifice of mathematics that goes back more

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than 30 years and is well accepted by researchers.

Experts cautioned that Dr. Wiles could of course have made some subtle misstep. Famous and not-so-famous mathematicians have claimed proofs in the past, only to be tripped up by errors. Dr. Harold M. Edwards, a mathematician at the Courant Institute of Mathematical Sciences in New York, said that until the proof was published in a mathematical journal which could take a year, and until it is checked many times, there is always a chance it is wrong. The author of a book on Fermat's last theorem, Dr. Edwards noted that "even good mathematicians have had false proofs."

Luring the World's "Cranks"

But even he said that Dr. Wiles' proof sounds like the real thing and "has to be taken very seriously."

Despite the apparent simplicity of the theorem, proving it was so hard that in 1815 and in again 1860, the French Academy of Sciences offered a gold medal and 300 francs to anyone who could solve it. In 1908, the Germany Academy of Sciences offered a prize of 100,000 marks for a proof that the theorem was correct. The prize, which still stands though has been reduced to 7,500 marks, about $4,385, has attracted the world's "cranks", Dr. Edwards said. When the Germans said the proof had to be published, "the cranks began publishing their solutions in the vanity press," he said, yielding thousands of booklets. The Germans told him they would even award the prize for a proof that the theorem was not true, Dr. Edwards added, saying that they "would be so overjoyed that they wouldn't have to read through these submissions."

But it was not just amateurs whose imagination was captured by the enigmatic problem. Famous mathematicians, too, spent years of their lives on it. Others chose never to get near it for fear of being sucked into a quagmire. One mathematical genius, David Hilbert, said in 1920 that he would not work on it because, "before beginning I should put in three years of intensive study, and I haven't that much time to spend on a probable failure."

Mathematicians armed with computers have shown that Fermat's theorem holds true up to very high numbers. But that falls well short of a general proof.

Tortuous Path to Proof

Dr. Ribet said that 20th century work on the problem began to grow ever more divorced from Fermat's equations. "Over the last 60 years, people in number theory have forged an incredible number of tools to deal with simple problems like this," he said. "As the tools became more complicated, they took on a life of their own. People lost day-to-day contact with the old problems and were preoccupied with the objects they created."

Dr. Wiles' proof draws on many of these mathematical tools but also "completes a chain of ideas," said Dr. Nicholas Katz of Princeton University. The work leading to the proof began in 1954, when the late Japanese mathematician Yutaka Taniyama made a conjecture about mathematical objects called elliptical curves. That conjecture was refined by Dr. Goro Shimura of Princeton University a few years later. But, Dr. Katz said, mathematicians had no perception through the 1950's to 70's that this had any relationship to Fermat's last theorem. "They seemed to be on different planets," he said.

In the mid-80's, Dr. Gerhard Frey of the University of the Saarland in Germany "came up with a very strange, very simple connection between the Taniyama conjecture and Fermat's last theorem," Dr. Katz said. "It gave a sort of rough idea that if you knew Taniyama's conjecture you would in fact know Fermat's last theorem," he explained. In 1987, Dr. Ribet proved the connection. Now Dr. Wiles has shown that a form of the Taniyama conjecture is true and that this implies that Fermat's last theorem must be true.

"One of the things that's most remarkable about the fact that Fermat's last theorem is proven is the incredibly roundabout path that led to it," Dr. Katz said.

Arcane to the Arcane

Another remarkable aspect is that such a seemingly simple problem would require such sophisticated and highly specialized mathematics for its proof. Dr. Ribet estimated that a tenth of one percent of mathematicians could understand Dr. Wiles' work because the mathematics is so technical. "You have to know a lot about modular forms and algebraic geometry," he said. "You have to have followed the subject very closely."

The general idea behind Dr. Wiles' proof was to associate an elliptic curve, which is a mathematical object that looks something like the surface of a doughnut, with an equation of Fermat's theorem. If the theorem were false and there were

indeed solutions to the Fermat equations, a peculiar curve would result. The proof hinged on showing that such a curve could not exist.

Dr. Wiles, who has told mathematicians that he is reluctant to speak to the press, could not be reached today. Dr. Ribet, who said Dr. Wiles was shy, said he was asked to speak for him.

Dr. Ribet said it took Dr. Wiles seven years to solve the problem. He had a solution for a special case of the conjecture two years ago, Dr. Ribet said, but told no one. "It didn't give him enough and he felt very discouraged by it," he said.

Dr. Wiles presented his results this week at a small conference in Cambridge, England, his birthplace, on "Padic Galois Representations, Iwasawa Theory and the Tamagawa Numbers of Motives." He gave a lecture a day on Monday, Tuesday and Wednesday with the title "Modular Forms, Elliptic Curves and Galois Representations." There was no hint in the title that Fermat's last theorem would be discussed, Dr. Ribet said.

"As Wiles began his lecture, there was more and more speculation about what it was going to be," Dr. Ribet said. The audience of specialists in these arcane fields swelled from about 40 on the first day to 60 today. Finally, at the end of his third lecture, Dr. Wiles concluded that he had proved a general case of the Tatiyama conjecture. Then, seemingly as an afterthought, he noted that that meant that Fermat's last theorem was true. Q.E.D.

People raised their cameras and snapped pictures of this historic moment, Dr. Ribet said. Then "there was a warm round of applause, followed by a couple of questions and another warm round of applause," he added.

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▲ Have Your Say:

Perspective on Evaluation and Achievement

ED BARBEAU, DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TORONTO

** Editor's Note: Todd Romiens, President-Elect of OAME, will respond to Ed Barbeau's column in the December "Have Your Say".*

There is a consensus among my colleagues at University that, while a few students are well-prepared for tertiary level mathematics, the general run of students seem less able to cope with this level of work than used to be the case. I have no reason to believe that they have not been taught much of the required background, or that their teachers are on the whole less conscientious than they were a generation ago.

Rather I think it is because more students have not had the experience of taking ownership of knowledge and have no understanding of what is required to achieve mastery. Much of the blame can be put at the door of a curriculum which touches topics superficially and repetitively and a lack of rigour towards the whole issue of evaluation.

In order to proceed effectively in a course of study, students must internalize the required background. A process of review, summation and integration must occur; what has been learned must be put into some sort of framework and a context established. Furthermore, the student should not be deceived about his mastery of prerequisite material, but be able to have it reliably at his disposal. For most people, this does not come easily or naturally, and often requires some sort of goal and deadline.

Readers who play a musical instrument or engage in a sport will understand precisely what is at issue here. If my golf game is languishing, then its improvement is best achieved by exposing myself to criticism and informed instruction and by setting for myself some goal. If I wish to acquit myself well against an opponent of superior skill, then I cannot be content with relying on the occasional luck shot but will rather analyze my game, practise my weak techniques and try to ensure that when faced with a 300 yard fairway I can call upon a 300 yard drive in the right direction. Similarly, anyone who has taken part in a performance will recall how the whole play or piece of music seems to come together at the last minute.

Out of a wrong-headed solicitousness, much of what goes on in schools seems designed to prevent students from experiencing the challenge and joy of real achievement. It is no wonder that many students consistently assign a lower priority to their academic study than to working part-time and engaging in other activities. They seem convinced that no real consequences will follow from a failure to engage their studies and that the system will somehow pick up after them.

Our students must be encouraged to take their goals seriously. If these goals involve academic study, at whatever level, then we must provide a regime which forces them to give such study absolute priority at certain critical stages. For the study of mathematics, a discipline in which one's ability to proceed depends on mastery of previous material, it is hard to think of a substitute for a few well designed examinations at certain points in the career of a student.

By well designed, I mean an instrument which forces students to integrate and relate ideas, to use their knowledge strategically and to communicate their understanding coherently and at length. The instrument should have the express purpose of advising students about their preparedness to proceed and ensuring the teachers at the next level that their students will be qualified. It should not be used as a means of evaluating teachers, schools or boards or of making comparisons across jurisdictions; if it is desirable to do these things, then other instruments must be found.

I am not talking about a general standardized test to determine whether a student should get a school-leaving diploma. I wish also to distinguish this sort of examination from regular class tests which have a much more localized function of review and diagnosis. Similarly, while multiple choice instruments are well adapted to certain tasks such as efficiently checking basic facts and techniques, I do not see them as playing a role in checking broad mastery.

Specifically, I am proposing two main sets of examinations separated from the regular curriculum and not required for a school-leaving certificate, but required for entry into the next level of mathematics. The first examination would be available to students between Grades 6 and 8, and would be based on arithmetic, ratio and proportion, basic geometry, mensuration and basic combinatorics. Students would be faced with applied

situations (such as might face a citizen in daily life) which would require the use of mathematical judgment and techniques; they would have access to pocket calculators, tables and other reference material. Such examinations would be offered at a level higher than the individual school, probably the local Board of Education, occur twice a year and permit a student as many attempts as are required. There would be no ranking of students, nor numerical grades; rather a student would receive one of the following designations:

A Solid mastery of and flexible control over ideas;

B General competence, with the student highly likely to succeed at the next higher level with the same amount of effort;

C Reasonable grasp of material with a few gaps which can be dealt with at the next higher level;

D Poor understanding of ideas and lack of technical proficiency; student should not proceed unless he/she is particularly well motivated and a remedial program with individual attention is available;

E Should not proceed under any circumstances.

No student would be permitted into high school mathematics without having cleared this examination.

The second examination would be available to students in Grades 11 and 12 and focus on algebra, synthetic and analytic geometry and trigonometry. The problems would be straightforward applications of concepts and techniques, with emphasis on clear and complete exposition. These would be offered on the same basis as the earlier examination, and graded in the same fashion. No student will be admitted to a university course involving mathematics or science without having cleared this examination. Teachers of OAC classes (which after all are directed to the university-bound students) might find it useful to use this examination as a qualifier for this level.

If these examinations were brought into existence, they would have the salutary effect of signifying to students that the system really does care whether they are qualified to pursue more advanced work and that it is serious about enforcing standards to mastery. More importantly, they would fill two important *pedagogical* functions for which there is scant provision at present: including a review and integration of knowledge and understanding achieved over a longer period, and requiring the student to assess how well they have learned a topic and to take corrective action.

Two criticisms are often levelled at such a proposal as this. First, it is complained that teachers will teach for the examination. Secondly, it is complained that students will be burdened with the stigma of failure.

If teaching for the examination means that teachers have to address with their students clarity of style, provide sufficient technical practice to ensure mastery so complete that students can attend to higher order aspects of mathematical problems, and coach students in how to summarize ideas and learn efficiently, then this is not a bad thing. If preparing for an examination means that the student and teacher enter into an alliance to meet a common challenge, is this to be deplored?

The question of failing students is a serious one. First, it should be pointed out that the other side of failure is success. In a regime where no one fails, no one also experiences a feeling of achievement, of a challenge surmounted and mastery attained, of confidence in one's ability to proceed further. There is no doubt that children will fail. Allowing students to repeat an examination will help reduce the sting of failure, as will not using the examination to hold students back from any endeavour except those for which the student has been definitively graded as unqualified. But, most important, failure will occur at the point where it can be acted upon and redressed.

There are a number of reasons that a student may fail. If it is due to lack of diligence, then the remedy lies with the student. If it is due to specific weaknesses, then these should be identified and dealt with in a timely fashion. If the student turns out to lack aptitude, then alternatives should be found. In any case, there is a great deal of flexibility in our current system to allow a student to take a less demanding load or re-enter school with the onset of greater motivation or maturity. It is not failure that should concern us, but failure that cannot be handled productively.

Compare this to what happens to many students at the present time. Much of their school grade is based on superficial tests, which may involve filling in blanks or multiple-choice answers. Buoyed by high grades and promotion from year to year, they have no reason to believe that their grasp of mathematics is in any way deficient, or that mathematics is any more than sterile and trivial procedures. Then at some stage, perhaps in high school, perhaps at college or perhaps at university, they suddenly find that they cannot cope. The techniques for getting high grades that served so well in the past just don't work anymore. The tutor they turn to for help finds that they have fundamental misunderstandings about basic ideas, no control of the most elementary techniques and no sense of any context. In short, the student realizes that he has no intrinsic feel for the subject and that all the years of school have been in vain. If this is not failure of a most devastating sort, then I do not know what is. It is not ethical to allow students to proceed to this level of desperation, where the only realistic option is to drop out and abandon one's

ambitions.

For the past seven years, I have been conducting a modest experiment with a high school correspondence course on polynomials (or theory of equations), followed by an optional examination. This course demands great commitment from the participants, and usually about a half dozen assign themselves the goal of taking the examination. There is no credit given for success in the examination, but I believe that certain benefits accrue to those that pass: they have had the experience of working on their own at material which is probably as difficult as any they will encounter in university, and so should be able to proceed with more confidence; they will have covered material which deals with the algorithmic, technical and conceptual side of mathematics and obtained a broader perception of what a body of mathematics involves; they will have covered specific topics which will help them put into better perspective later work in calculus, algebra, numerical analysis and complex variable theory. Sometimes, I can see a student who may not seem that strong to begin with, struggle with and take command over some reasonably difficult mathematics.

This should be an exciting time for students to go on in science and mathematics. The progress in these areas has been phenomenal in the past two decades, both in depth and breadth. We owe it to our students to encourage them into these fields, to reflect them authentically in our teaching, and, above all, to see to it that they are adequately prepared to take up studies both challenging and rewarding. For this, we must grasp the issue of evaluation resolutely without ideological preconditions. Examinations should be pertinent to specific goals of the students, and be well focused in their intent. The time for action is now.



▲ Problem Solving in Mathematics: A Fresh Perspective for Future Teachers

RHETA N. RUBENSTEIN, FACULTY OF EDUCATION, UNIVERSITY OF WINDSOR



Rheta N. Rubenstein has been teaching preservice and inservice courses in mathematics education at the University of Windsor since 1989. Previously Rheta was a secondary school teacher in Detroit, Michigan.

In the fall of 1991 I was presented with a challenge: in under two hours, give future teachers what may be their only experience in mathematics education. Both my students and I learned a great deal from this experience which I would like to share. My story has implications for activities with preservice teachers. I also invite classroom teachers to consider the implications for work with students in the transition years.

In the University of Windsor Faculty of Education we offer a General Methodology course for all Intermediate/Senior division preservice students. The course includes some exposure to each of the disciplines in a grade 7 or 8 homeroom, as well as a detailed study of a few of them. As the mathematics educator in this team-taught course I needed to plan a two hour workshop for the mathematics overview. The workshop was to be offered to over 100 I/S students in three sections of 20-40 students each.

I chose to focus my session on problem solving in the spirit of the NCTM Curriculum and Evaluation Standards for School Mathematics. I had many goals to accomplish in a short time. I wanted to give students a positive experience by doing non-trivial mathematics, in a way that would build their confidence both in what they could learn and in what youngsters can do mathematically. I wanted to debunk some myths, e.g., that there is one right or best way to solve a problem and that the authority for the correctness of an answer belongs to teachers or answer books. I also wanted to demonstrate that we can make mathematics more accessible to more students by starting with concrete situations. Finally, I wanted to highlight the four overarching Standards: problem solving, communication, reasoning, and connections. As an added twist, the activity had to speak to the full range of students, from the sweaty-palmed mathematics-anxious to the handful of mathematics majors.

The shortness of time was my key. Here was a chance to apply the "less is more" philosophy. I chose one problem, a trusty chestnut: the handshake problem.

Launching the Challenge

I began by asking the students to complete the phrase "Mathematics is..." There were generally two types of responses: attempts at definitions and emotional outpourings (e.g., "my enemy"). We put these on the back burner and returned to them for discussion after the activity.

I told the students that since this was a new year we would consider a problem related to meeting people. "Suppose the forty people in the room each introduced himself or herself to each of the others. Imagine at each meeting that people shake hands. How many handshakes would this be?" (If you are not familiar with this problem, please try it now. If you are familiar, think of multiple solution approaches.)

To get started, we acted out the handshakes of two, three, then four people. We made a table of the results:

number of people	2	3	4
number of handshakes	1	3	6

I also showed them how a diagram might represent these situations, with points for people and segments for handshakes.

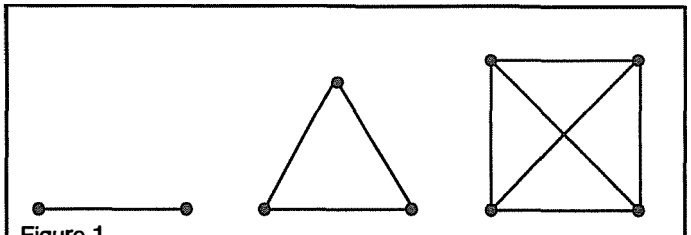


Figure 1

I summarized that we had used four strategies: using smaller numbers, acting out the problem, making a table, and making a diagram.

Then I presented the challenge to their working groups: a) find at least one solution which each member of their group could explain, and b) find as many other ways of solving the problem as they could.

Student Responses

Before I had given the final direction, some students were already buzzing. "There must be a pattern." "I'm sure I learned this in grade 12, but I don't remember it." However, when they heard the two part challenge, there was an audible pause. The nature of the task changed. They were not just looking for a single answer or a solution that made sense to only one of them.

I gave them about 20 minutes and circulated to probe, question, and, as one of my students had described it, eavesdrop. The first time I used this lesson, I had noted in advance a few solutions I had anticipated. Over the three sessions, I was more amazed each time by the creativity of my students and the number and diversity of approaches.

Occasionally, I saw some papers with sigma notation, subscripted variables, and recursion formulas. These students were taken aback when I reminded them that we were supposed to operate at the grade 7 or 8 level. When they recouped and found simpler approaches, they learned something important - what is studied in upper level courses is not necessarily inaccessible to naive learners.

After interviewing groups, I selected representatives to explain group solutions at the board. As others listened, they were usually surprised at the number of ways of thinking about this one problem. I tried to sequence the presentations in a way that connected related strategies and moved from more direct solutions to more insightful ones. Following are most of the solutions that appeared, not all of them, of course, in one session.

Student Solutions

Solution 1

We extended the table a bit and found a pattern.

# of people	# of handshakes	
2	1	
		+2
3	3	
		+3
4	6	
		+4
5	10	
		+5
6	15	

We saw that the last number added, like the five, is one less than the number you are on, six. We know that for 40 people you have to add the numbers from 1 to 39. We don't know why it works, but it's a very clear pattern. [This was the most common pattern found.]

Solution 2

Suppose there are just two people in the room. There is one handshake. Now another person enters. He shakes hands with the two people already there. Now there are $1+2$ handshakes. The fourth person, now, must shake hands with 3 people, so there are $1+2+3$ handshakes and so on. When the 40th person enters, there are $1+2+3+...+39$ handshakes. We didn't have a calculator, so we didn't do it, but we know how to find the answer. [At this point, groups with the first solution saw why theirs worked.]

Solution 3

We thought about starting with the whole group. Suppose I shake hands with everyone; that's 39 handshakes. I'm done. I leave the room. The next person shakes hands with 38 people and leaves the room. This goes on until two people are left. They make one handshake and one of them leaves the room. The person remaining has shaken hands with everyone. So you need to add the numbers from 39 down to 1.

Solution 4

We found a way to add the numbers from 1 to 39:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

39 38 37 36 35 34 33 32 31 30 29 28 27 26 25 24 23 22 21

40 40 40.....40 40 20

There are nineteen 40s and 20 more. That makes 780.

[Groups who had laboured to find the sum were impressed.]

Solution 5

We also started with the full group, but ours looks different. Here is my diagram. You see me ("I") and 39 others.

0000000000000000000000000000000000000000I

I shake hands with 39 people. But each of us could think of ourselves as "I". So each of 40 people makes 39 handshakes. 40 x 39 = 1560. But each handshake has two outstretched arms, we've really counted each one twice. So there are half as many handshakes or 1560/2 = 780.

[Now we have a multiplication and division approach, even more efficient than the clever addition above!]

Solution 6

Suppose there are five people. Call them A, B, C, D, and E. Make a chart of their handshakes. First, we put dashes in where you can't shake hands with yourself. Now let H represent a handshake.

	A	B	C	D	E
A	-	H	H	H	H
B		-	H	H	H
C			-	H	H
D				-	H
E					-

In the lower left there are no handshakes because these people already shook hands. Counting by rows you see that the number of handshakes is 4 + 3 + 2 + 1 = 10. This gives the same pattern of adding the numbers from one less than the number of people down to one. [Many students were surprised by this two dimensional representation which looked and felt different from the others, but held the same information.]

Solution 7

We made a similar chart but left off the last row because there was nothing in it.

	A	B	C	D	E
A	-	X	X	X	X
B	-	-	X	X	X
C	-	-	-	X	X
D	-	-	-	-	X

We saw a rectangle cut in half making a triangle. To see this, notice how the dashes and the x's make the same shape. The area of the triangle is half of 5 x 4 or ten. For 40 people, there would be 40 columns and 39 rows. The area is half of 40 x 39 or 780. That's the number of handshakes.

Solution 8

We continued the table up to ten people and got 45 handshakes. We figured we could divide the 40 people into four groups of ten each. Each group makes 45 handshakes at their own table. Now, suppose group A shakes with group B. There are 100 handshakes. Why? Imagine you are one person in group A. You shake with ten people in group B. Now the next person in A shakes with ten and so on. All ten people in A shake ten times or 10 x 10 = 100. Now the same thing goes for the other pairs.

group A with group B - 100 handshakes

group A with group C - 100 handshakes

group A with group D - 100 handshakes

Now group A is completely done. They have shaken hands with everyone in the room. Now group B members need to shake with each of the other groups.

group B with group C - 100 handshakes

group B with group D - 100 handshakes

Now group B is done. Groups C and D shake hands.

group C with group D - 100 handshakes

Group C is done. But so is group D, because everyone else came to them. The total is 4(45) + 6(100) = 780.

[This was utterly new to me. It was serendipitous that we had a "nice" number of people. A good spin-off, for the mathematically sophisticated, is to demonstrate that this sort of partitioning is always equivalent to the general solution. See the appendix for details.]

Solution 9

Our group also extended the table and found multiplication facts for the even numbers of people.

# of people	# of handshakes
2	1 = 1 x 1
3	3
4	6 = 3 x 2
5	10
6	15 = 5 x 3
7	21
8	28 = 7 x 4

In the multiplication, the first number is one less than the number of people. The second number is half the number of people. For 40 people, we would have 39 x 20 or 780. [Some groups saw the same pattern, but expressed it differently: "one number goes up by twos and the other goes up by ones." By following the pattern, they were able to get to 780.]

Solution 10

Our group found a pattern for the odd numbers of people.

# of people	# of handshakes
2	1
3	3 = 1 x 3
4	6
5	10 = 2 x 5
6	15
7	21 = 3 x 7
8	28
9	36 = 4 x 9

We figured that at 39 the pattern would be 19 x 39 or 741. Then we knew we would have to add 39 which gives 780. [This was harder for students to generalize. The second factor equals the number of people. To find the first factor,

you must subtract one from the number of people, then take half.]

Solution 11

We took ratios of the two columns.

number of people	number of handshakes	handshakes/ people
2	1	1/2 = .05
3	3	3/3 = 1
4	6	6/4 = 1.5
5	10	10/5 = 2
6	15	15/6 = 2.5
7	21	21/7 = 3

We saw the ratios growing by halves. Counting up by halves we found that at 40 we would be at 19.5. Then we multiplied back to get 780 handshakes. [This pattern explained both of the previous two, as well as solution 4 which counted 19 half 40s.]

Wrap Up

Of course several groups, through my insistence on multiple methods and their own motivation for a general solution, determined that n(n-1)/2 was "the formula". I specifically chose to ignore this during the general discussion. Introducing it, I felt, would undermine the larger goals mentioned earlier. Nevertheless, there was a strong need for closure. As one student put it, "The problem is begging for a formula." So at the end of the board presentations I let those who had seen this solution share it. Usually, their explanation built from an image like the one in Solution 5. There are n people in the room. You don't shake hands with yourself, so you shake with n-1 people. That gives n(n-1). But each person has counted the handshake, so you have double what you want. The result is n(n-1)/2. The division by two was difficult for some students. To help we referred back to the diagrams and the two endpoints of each segment or we shook hands and thought about the handshake from the point of view of each person involved. I pointed out that being concrete and visual as well as verbal and symbolic offered the chance for more students to understand.

I then asked the students to reread their earlier writing ("Mathematics is...") and reflect on changes they might make as a result of the preceding activity. From the sharing I know they made some significant steps in realizing that mathematics can be much more than numbers, symbols, and operations of arithmetic.

Finally, we looked at the Ontario Ministry of Education document *Curriculum Guideline, Mathematics, Intermediate and Senior Division, Part Two*. The introductory materials here speak to many of the goals I was targeting: concern for the math anxious student (p. 13), individual differences among learners (pp. 13-14), language as a vehicle for both understanding and expressing mathematics (pp.17-18), experiential approaches (p. 18), and, of course, problem solving (pp. 19-20). These students could now read this resource with some experience to connect with it.

What did we accomplish? I have only my observations to report. Students were actively engaged. They looked for patterns, sought generalizations, raised questions, debated and defended their thinking. There was an excitement in their activity, satisfaction in their accomplishments, and surprise in the diversity of approaches. My goal was to build some appreciation of the spirit of the *Standards:* problem solving, communication, reasoning, making connections. We touched on each of these.

While I know that I had a limited cognitive effect on their understanding of mathematics and its teaching, I think I had a significant affective effect. I believe that those students who are asked to teach mathematics have a positive basis from which to grow. I also hope that some of the fundamental principles they experienced, adopting a problem centred focus, recognizing the roles of language, and appreciating students' diverse ways of thinking could transfer to teaching in other disciplines.

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Ontario Ministry of Education, (1985). *Curriculum Guideline: Mathematics, Intermediate and Senior Divisions, Part Two.*

Appendix

Proof that Solution 8 Generalizes

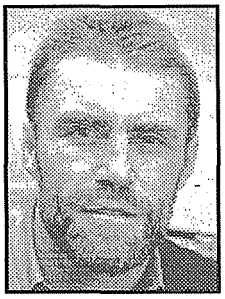
In solution 8 students imagined the 40 persons divided into four groups of ten. The number of handshakes needed was 4(45) + 100(6). Using the notation of combinations, four is the number of groups, 45 = C(10,2), 100 is the square of the number of people in each group, and 6 = C(4,2). In general, the solution to the handshake problem is C(n,2). Suppose a group of n is partitioned so that $n = ab$, forming a groups of size b. We must show that $C(n,2) = aC(b,2) + b^2C(a,2)$.

Proof

$$
\begin{aligned}
aC(b,2) + b^2C(a,2) &= a[b(b\text{-}1)/2] + b^2[a(a\text{-}1)/2] \\
&= 1/2[ab^2 - ab + a^2b^2 - ab^2] \\
&= 1/2[a^2b^2 - ab] \\
&= 1/2[n^2 - n] \\
&= 1/2(n(n\text{-}1)) \\
&= C(n,2)
\end{aligned}
$$

▲ Did Pythagoras Really Say "$A^2 + B^2 = C^2$" ?

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After 14 years as a secondary school mathematics teacher and consultant in Timmins and 3 years as an Education Officer in Sudbury, Geoffrey Roulet is an Assistant Professor at Queen's Faculty of Education.

Practice teaching sessions over the past year and a half have given me a number of opportunities to observe student teachers and secondary school pupils applying the Theorem of Pythagoras. The scene seems to follow a standard script. A class is working on a topic that generates the need to determine the length of a side of a right-angled triangle, (the distance between two points on the Cartesian plane, solving a triangle in trigonometry). The teacher reminds the students that there is a connection between the sides of the figure and possibly invokes the name of Pythagoras in asking for the relationship. Classes always appear to have an answer satisfactory to the teacher as invariably someone responds with, "$a^2 + b^2 = c^2$ ", and this is recorded on the board. Then the problems begin. Which side is "c" and where do the "a" and "b" go? Eventually, after some discussion concerning the longest side or hypotenuse, the triangle is labelled and the missing dimension calculated.

Kepler is quoted as saying (Huntley, 1970):

"Geometry has two great treasures: one is the Theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel."

How did this "measure of gold" come to be reduced to "$a^2 + b^2 = c^2$ "?

The situation is not much different when my students at the Faculty of Education are the pupils. During Intermediate-Senior Mathematics peer teaching sessions the Pythagorean relationship is always given in the algebraic form but fortunately here, there are no problems correctly identifying the sides. With the Junior-Intermediate core mathematics class there is more confusion. As illustration, I present one conversation;

Student (teacher candidate) - "I have to teach square roots and that Pythagorean rule during practice teaching. Do you have any suggestions?"

Instructor (Roulet) - "I'm not sure. What is this Pythagorean rule?"

Student - "You know, $a^2 + b^2 = c^2$ ".

Instructor - "That is interesting. You take the first two letters of the alphabet, square and add them, and you get the third letter squared."

Student (with puzzled expression) - "No, these are not letters. They are really numbers."

"Instructor - "Oh! If I take three numbers, square two and add them together I get the square of the third."

Student (hesitantly) - "I'm not sure. I don't think it works for all numbers."

Instructor - "Then what numbers does it work for?"

Student - "I'm not certain, but I think it has something to do with triangles."

With some work we get the relationship pinned down and agree that if we measured the lengths of the three sides of a right-angled triangle the square of the length of the hypotenuse (longest side) would equal the sum of the squares of the other two values.

We have no historical record of Pythagoras' own statement and proof of the theorem that bears his name but Euclid's presentation in the Elements of 300 B.C. has been preserved in the work of Theon. The English translation of *Book I Proposition 47* is (Apostol, 1988):

"In any right-angled triangle the square which is described on the side subtending the right angle is equal to the squares described on the sides which contain the right angle."

Euclid goes on in *Book VI, Proposition 31*, to expand the original theorem to assert the same area equality whenever similar figures are constructed on the sides of the right-triangle (for an illustration of this see *The Theorem of Pythagoras, Apostol, 1988*). This image of the Pythagorean relationship as dealing with the areas of geometric figures was preserved for 22 centuries. *A School Geometry* by Hall and Stevens (1904), a text used in Canada at the beginning of this century, states the theorem using a translation of Euclid and highlights no algebraic summary of the form "$a^2+b^2=c^2$".

In the 1960's, Ontario students were introduced to the

Pythagorean relationship during the Grade 11 Geometry course. While the language of *Deductive Geometry and Introduction to Trigonometry* (Petrie, Baker, Darbyshire, Levitt & MacLean, 1957), a popular text of the time, was less formal than that of Euclid, the theorem was still strongly linked to the areas of squares constructed on the sides of right-angled triangles. Have the new guidelines led to a suppression of the geometric interpretation? One would think, with the recent emphasis on the equality of the arithmetic, measurement, and geometry strands of the Primary and Junior Division mathematics programs, that student preparation for a geometric approach has increased. The curriculum guideline, *Mathematics: Intermediate and Senior Divisions* (Ontario Ministry of Education, 1985), calls for a three step development of the Pythagorean Theorem through Grades 8, 9 and 10. The suggested sequence is: investigating and applying (Grade 8), stating (Grade 9) and proving (Grade 10). This would appear to be a natural progression and reflects the historical development, with the Babylonians and Egyptians (Hogben, 1960) employing the relationship without possessing a formal proof. Although early application may be of value in providing motivation, it should not treat the relation as a formula into which students just plug numbers.

When we look at how the approved textbooks approach this topic we see that there is a general rush from a brief geometric introduction to an algebraic conclusion. Only Alexander and Klassen in *Mathquest Eight* (1988) refrain from giving an "a, b, c" algebraic summary. This text fully develops the area relation and tackles applications without a need for the "$a^2 + b^2 = c^2$" equation.

It would appear that there is general agreement that the best way for students to remember this mathematical gem is to memorize "$a^2 + b^2 = c^2$". And we are successful. Many students remember the equation. But how much potential excitement, beauty and understanding is lost?

Is any of this important? It certainly would help if students could assign the a, b and c correctly to the sides of a right-angled triangle and some appreciation for the history of mathematics would be valuable. But in the long run the use of "$a^2 + b^2 = c^2$" is probably not a major disaster. The problem is, I believe, that the highlighting of this equation is symptomatic of a rather common and limiting view of mathematics, the view that mathematics must use symbols and can not be talked about in plain language or explored through diagrams.

The existence of this image of our subject was vividly illustrated in my Junior-Intermediate pre-service course. During a demonstration of learning centres, one student presented a Grade 8 activity investigating the Pythagorean relationship. She did a very good job, leading us through the construction of squares on the sides of the triangle and then establishing the equality of areas by sub-dividing the smaller squares and assembling the pieces on top of the largest (see Alexander & Klassen, 1988, p. 395). From this she had us write in our own words a summary statement concerning the areas. But then a further series of questions led the class to label the triangle's sides with a, b and c, express the areas in terms of these letters and finish with "$a^2 + b^2 = c^2$". When I asked the student why she had included the last set of instructions she replied, "To get the mathematical statement of the theorem. I thought that was what we wanted." It would appear that the popular view is that doing "real" mathematics demands the use of x and y or a, b and c.

This tendency to "algebratize" is much too common. Witness the practice of many teachers to require a "Let x be ..." statement as an initial line of every problem's solution. We need to retain plain English, pictures and concrete materials in our mathematics. The development of the Pythagorean Theorem is certainly one place where this can be done.

I would like to thank Hugh Allen of the Faculty of Education, Queen's University, for his helpful comments on previous versions of this paper.

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▲ THAT'S SUM TRIANGLE

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A popular math activity used to develop problem-solving skills and to simultaneously provide addition practice is the Sum Triangle Game (Porter, 1983). Given the arrangement in Figure 1, players must fill in the circles with the numerals from 1-9, using each numeral only once. The sums of the four numerals on each side of the triangle must be equal, but these totals are initially unknown to the players, who may use any problem-solving strategy to arrive at a solution. Often individuals use the guess-and-check method to complete the puzzle. The purpose of this article is to describe a technique that will lead to a pattern for solving the above problem and, in fact, generalize to any similar situation which involves three arithmetic sequences with a common difference.

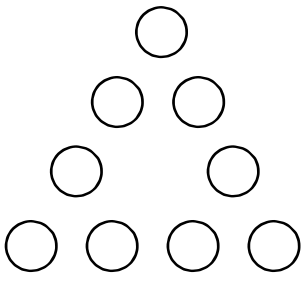


Figure 1

LOWEST POSSIBLE SUM PATTERN

To achieve the lowest possible sum per side, 1, 2, and 3 must be placed in the corners. These numbers will be added in twice once for each of the two sides in which they are located. The partial sums which are found by adding the corner numbers represent consecutive numbers, as shown in Figure 2.

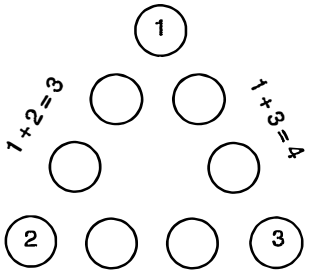


Figure 2 $2 + 3 = 5$

These sums must be added to other sums (the middle numbers on each side) to obtain a common total on each side. To balance the totals of all three sides, the sums of each middle pair must also be consecutive numbers. The unused numbers available for the middle locations are 4, 5, 6, 7, 8, and 9. These numbers, which total 39, must be paired in some way so as to produce a set of consecutive numbers. Since consecutive numbers can be written as $n - 1$, n, and $n + 1$ (McKeague, 1982), the following relationship exists:

$$\begin{aligned} (n-1) + n + (n+1) &= 39 \\ 3n &= 39 \\ n &= 13 \\ n-1 &= 12 \\ n+1 &= 14 \end{aligned}$$

The remaining (unused) numbers can be listed in ascending order and then paired, as shown in Figure 3, to arrive at consecutive sums 12, 13, and 14.

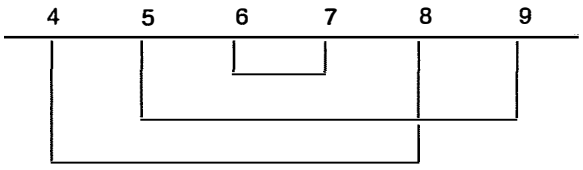


Figure 3

The puzzle will yield the identical sum on each of its three sides when the corner pair having least value (1 and 2) is matched with the middle pair having greatest value (9 and 5), the corner pair having greatest value (2 and 3) is matched with the middle pair having least value (8 and 4) and lastly the comer paid having intermediate value (1 and 3) is matched with the middle pair (6 and 7). In general, to

▲ "Am I Really Crazy?": *Selected Issues Concerning Destreaming*

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After 15 years of teaching mathematics, James M. Haines assumed a position at Lakehead University's School of Education in 1988. Areas of interest include: Preservice Teacher Education, Mathematics Education and Classroom Management.

During the past year a secondary school mathematics teacher, reflecting on the current efforts in Ontario to destream grade 9, posed the following question: "Am I really crazy?" The tone of doubt in the query referred not to the teacher's mental state but a disagreement with plans to destream grade 9 mathematics. The purpose of this paper is to present issues not commonly raised by the proponents of destreaming. Issues which will be addressed are: equity for all, the dropout issue, students as peer models, current trends in math revisions, and guidelines for streaming.

Opposing views regarding educational approaches are not uncommon. For example, there are differing philosophies of cognitive instructional models. Discovery learning, "an instructional approach in which students learn from their own active explorations of concepts and principles" (Slavin, 1991a, p. 192), was advocated by Jerome Bruner. David Ausubel, a critic of discovery learning because not all students always know what is important or relevant, sees value in reception learning, "learning that occurs when students are presented information in organized, teacher-structured lessons" (Slavin, 1991a, p. 194).

Equity for All

Similarly, the Ministry of Education's goal in pursuing equality of education and opportunity for all students is a philosophical issue. The reality is that some parents have always and will continue to provide experiences which will ensure that their children are advantaged. Parents who are concerned about their children's academic preparation are accessing private-sector educational entrepreneurs in growing numbers (Lewington, 1992). In this same article Harvey Weiner of the Canadian Teachers' Federation is quoted as saying that the emergence of commercial enterprises is "an equity concern that these products are not going to be accessible to everyone". Lareau (1989) has noted that "(social) class influences the resources parents have available to circumvent and challenge social institutions" (p.183). Useem's (1992) finding that "districts with high enrollments of students on the fast track tended to be those with high average parental levels of education" (p. 347), adds support to the notion that the home is the major factor in equity. This reality is acknowledged by Conrath (1992) who offers a thought-provoking definition of 'equity':

"... different treatment for different kids with different needs, not the same 'standard' for all kids in spite of multiple intelligences, home and family, and past experience in school" (p. 140).

Assuming equity is an attainable goal, it should receive intense attention during the grades JK to 6 (Dreikurs & Cassell, 1972). If during the first eight years of schooling students aren't "made equal", then the task would become even more difficult in the later grades. For those students with weak academic skills, remediation in those first eight years of school would do much to promote equity. Without changing the home, which Finn (1987) and Dreikurs and Cassell (1990) have stressed, attempts through the school to cure social and economic injustice appear questionable. As Paquette and Allison (1988) state, "If Radwanski should prevail, the school will once again be issued a mandate to cure social and economic injustice" (p. 39).

The Dropout Issue

A major focus of Radwanski's (1987) report was to address the dropout issue in Ontario high schools. However, Hahn's (1987) components of a preventative dropout program at the high school level fails to list destreaming as an issue. The lack of correlation between streaming and dropouts is addressed by Games (1991).

None of this research reveals if such differences are caused by tracking. Students differ in their attitudes toward school before coming to high school. Perhaps these differences would still exist even without tracking.

The only clear way in which cause and effect results may be established would be if some courageous school system randomly assigned some middle schools to a tracking system, and prohibited tracking in other middle schools and in all elementary schools. (p. 292)

Furthermore, Finn (1987) found no "fully satisfactory 'causal' research relating [dropping out] to school factors ... the symptom is not likely to be eradicated by school-based remedies" (pp. 14-15). Finn (1987) adds:

To be sure, the most frequently cited reasons for leaving were associated with school itself. ... But being school-related does not mean that they are school-caused. They may have their origins in the individual, his family, or social class. (p. 15)

Radwanski (1987) concludes, with support from studies, that the dropout rate is between 31 and 33 per cent. His concern is justified. However, The Halton Board of Education (1992) raises serious doubts as to the accuracy of this figure for their county with a reported dropout rate of 4.92% in a study ending in December 1991. The Halton Director of Education, Bob Williams, concluded in the media release: "We have consistently argued that this mythical 30% dropout figure we hear so often, simply doesn't apply to our Board. This retention study validates our belief and the results are testament to the hard work of our staff, students and community."

In light of such radical findings, it would seem reasonable that before Radwanski's "tinkering with the secondary schools" (Coulter, 1991) is implemented that the Halton system be carefully analyzed. Whether Halton is defining dropouts differently, or are actually meeting their needs more effectively, should be clarified. It would appear unlikely that destreaming efforts would reduce the dropout rate to less than 5%.

Students as Peer Models

Another argument that is provided in favour of destreamed classes is that higher achieving students serve as role models. Schunk (1987) has pointed out that fast learners do not serve as models for students of low and average ability. Regarding peer learning, Kanfer and Goldstein (cited in Martin 1981) state: "A model who is too dissimilar to the observer, or whose behavior is far superior (or more sophisticated) than that presently possible for the observer will generally produce poor acquisition effects" (p. 59). Allan (1991) cautions that "children typically model the behaviour of other children of similar ability who are coping well with school" (p. 64). This raises some doubt regarding the modelling value within heterogeneous destreamed classes.

Current Trends in Math Revisions

The efforts to destream grade 9 mathematics run counter to the University of Chicago School Mathematics Project which began in 1983. "The aim of the UCSMP project is to develop and test K-12 mathematics curriculum that is meaningful for average students" (Thomas, 1992, p. 90). "The UCSMP plan anticipates that about half of all eighth-graders would be ready for first-year algebra after studying Transition Mathematics in the seventh grade. ... This would leave most students free to take advanced courses such as calculus, statistics, and computer science courses in the eleventh and twelfth grades" (Thomas, 1992, p. 91). Comparison studies appear to reflect favourably on these efforts.

Slavin (1991a; 1991b) appears to be well aware that mathematics is a subject which should receive special consideration. "There is probably some justification for acceleration programs, such as offering Algebra I to mathematically talented seventh graders, or offering advanced placement in high school" (Slavin, 1991a, p. 284). Becker (1990) notes that "most middle-grade schools (63%) offer algebra to at least some of their seventh- and (more usually) eighth-graders" (p. 451). Algebra I is generally equivalent to Ontario's advanced stream grade 9 mathematics. These beliefs add support to the UCSMP findings.

Even Radwanski (1987), possibly unwittingly, offers support for moving the Ontario's current grade 9 course into grade 8 to better enable students to study calculus. In his report, he states that "unless a person understands the basics of arithmetic, algebra, and geometry, even calculus, he or she won't know what operation to perform with a calculator to solve a given problem" (pp. 44-45).

At a time when evidence is coming forth to challenge students and combat boredom, Ontario's destreaming efforts in mathematics appear to move in the opposite direction.

Guidelines for Streaming

The Radwanski (1987) report's blanket approach to destreaming and its failure to offer guidelines may reflect a lack of awareness of necessity to consider the differences between subjects. Whether Radwanski would have considered Slavin's summary and conclusions (below) for between-class ability grouping, had it been published prior to 1987, is speculation.

... ability grouping plans are beneficial for student achievement when they incorporate the following features.

1. Students remain in heterogeneous classes most of the day and are regrouped by performance level only in such subjects as reading and mathematics in which reducing heterogeneity is particularly important.

2. The grouping plan reduces heterogeneity in the specific skill being taught.

3. Group assignments are flexible and are frequently reassessed.

4. Teachers adapt their level and pace of instruction In regrouped classes to accommodate students' levels of readiness and learning rates.

(p. 71)

Although Slavin doesn't provide specific grades for homogeneous classes in reading and mathematics, it may be safe to assume that he is referring to the elementary school streaming, as reading is not a secondary school subject.

In discussing alternatives to rigid tracking, Braddock (1990) clarifies the issue at both the elementary and middle grades.

If used at all, between-class tracking should be limited to the basic academic courses in which the students' different levels of prior preparation are clearly the major reasons for grouping for instruction. In the elementary grades, research has indicated that grouping plans are most beneficial when students remain in heterogeneous classes for most of the day and are regrouped only in mathematics and reading on the basis of their current level of skill. For the middle grades, it is reasonable to predict that tracking would serve best if it were similarly limited or perhaps even further restricted to subjects (e.g., mathematics) that have specific prerequisites at each step of learning.

(p. 448)

An analysis of these points indicates that the Ontario educational system adheres to most of these policies and mathematics should be exempt from destreaming.

Concluding Remarks

From the literature listed above, the following points could be argued:

- ▲ Higher achieving peers may not serve as role models for those academically weaker students.
- ▲ The impact of factors outside of the school's influence (i.e., the family) have been ignored by destreaming advocates.
- ▲ No cause and effect relationship between streaming/ tracking and dropouts has been established.
- ▲ Guidelines regarding ability grouping have been ignored in Ontario.
- ▲ Not only is support provided that mathematics be exempt from the destreaming movement, but also there is evidence supporting a revised mathematics curriculum, introducing the current grade 9 algebra material in grades 7 and 8.
- ▲ For those teachers of subjects which do not lead readily to streamed classes, the freedom of choice should be given. Flexibility of structure should be encouraged.

Hargreaves and Macmillan (1992) view high schools as "inflexible". They fail to acknowledge that secondary mathematics teachers have adapted to changes, such as the "new math" in the late sixties and the OAC mathematics courses, on more than one occasion. In the case of resistance to destreaming, it may be that they are not opposed to change, but rather change that is not based on sound research findings. Teachers may be opposed to what they perceive as irrational change. It is critical that they resist such change, otherwise every smooth-talking theorist, politician, or report writer will jerk the educational system off on another uncharted course.

The feelings of many Ontario secondary mathematics teachers are well summarized by Christine Walton (1992), mathematics teacher at Toronto's Riverdale Collegiate:

Every student must be helped to fulfil his or her own potential. Unfortunately that cannot be done in a de-streamed environment. Believe me. I've been there. ... I dread the thought of teaching in Toronto or Ontario for the next couple of years. (p. A15)

Sikes' (1992) conclusion that "the imposition of change can lead to low morale, dissatisfaction and reduced commitment" (p. 49) appears to be in clear of Walton's statement.

In answer to the initial question, "Am I really crazy?", the answer is: "No, you're caught in one of education's philosophical debates, but a debate where only one side of the literature is discussed". Hopefully, the Ontario Ministry of Education will consider "other literature", particularly for mathematics.

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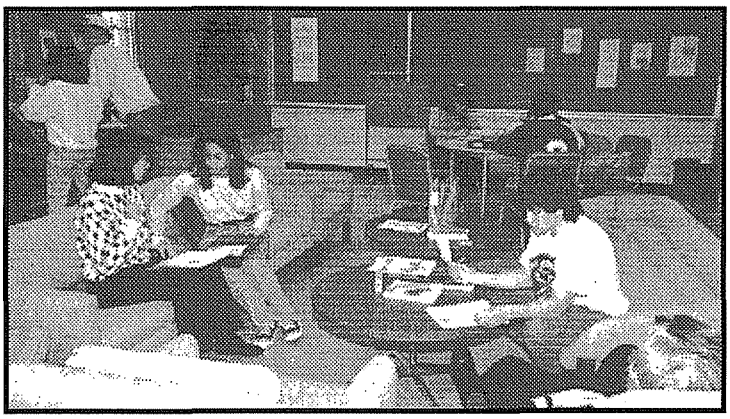
Winner of the OAME'94 Kingston Conference Registration and $200.00 expenses

Ron Scoins U. of Waterloo

★★★★★★★★★★★

▲ DESIGNING A MATHEMATICS OFFICE

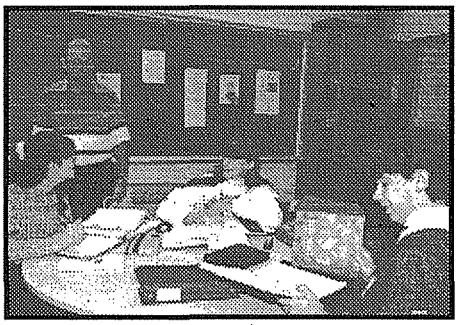
YORK MILLS C.I. MATH OFFICE, BY MIKE ZICHOWSKI, NORTH YORK BOARD OF EDUCATION
PHOTOS BY BARRY SHAINBAUM

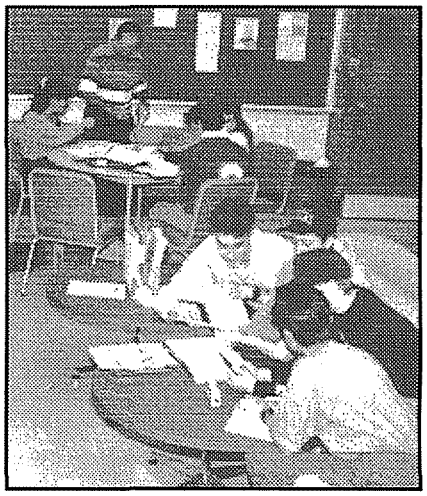


The York Mills C.I. math department is unique in that it operates out of a former designated classroom with absolutely no distinction or barrier between the "math department office" and any math classroom in the school. The key to the design of this office is the sincere belief that the department should function as a team, and that the mathematics department office should be accessible (just like any classroom) by students from 8:15 a.m. to 4:00 p.m.



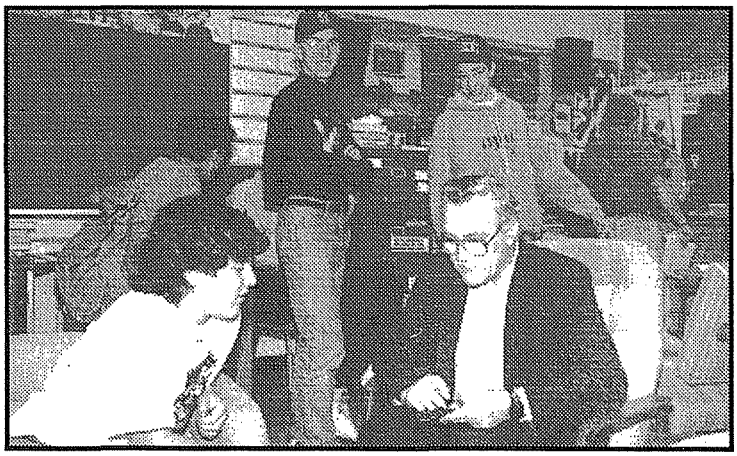
Teachers' desks are arranged around the perimeter of the room, and the central region of the room with its "hand-me-down" sofas and circular dinner and coffee tables remains the primary help station for students with conceptual or problem-solving difficulties. This relaxing and cosy central oasis affords the perfect conditions to tackle those difficult homework problems, offer calculator instruction, review for upcoming tests, allow counselling and guidance, or to have a friendly chat between teacher and student. No appointment is ever required to use this office; the door remains open at all times provided that a department member is present or the school is not participating in another firedrill. Only monthly department or examination meetings necessitate the closing of the office to students and other staff members as well. On all other occasions the room resembles the daily activity of a bus terminal with students and math staff coming and going. The open door policy has created an informal yet productive living room where math anxiety can be effectively managed or adequately suppressed. Mathematics help can be found by simply visiting the "friendliest room in the school" as one student remarked.





The results of this open-door policy are impressive as parents and the guidance department have complimented the mathematics team on bringing down the barriers between the student and teacher or subject; and the students have quickly sensed that this former classroom is now "their problem-solving central" where their efforts coupled with a teacher's patience can produce the required results. It is a room that they indeed trust.

With the addition of light pop music playing on the radio in the background, a coffee machine, small refrigerators, microwave, photocopier, and comfortable chairs, this room is complete to cordially greet any visitors from administrators to students to math colleagues from other schools and school boards. We had succeeded in sweeping the stuffiness out of a closet type, "out of bounds" math office and program, and the benefits have yet to cease.



NOMINATIONS FOR OAME/AOEM COUNCIL 1994-95

Each year, elections are held for President-Elect, 3 Directors and 7-10 Councillors for the Council of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 31, 1994, to:

Jeri Lunney
Past-President, OAME

Include the nominee's qualifications with the Nomination Form. The Nomination Committee will make a final selection of those nominees who will stand for election. Their names and qualifications will be published in March, 1994. Voting will be by mail by members of the OAME/AOEM. The results of these elections will be made known at the Annual Meeting of the OAME/AOEM in Kingston in May, 1994.

NOMINATION FORM

I nominate: _____

Address: _____

_____

For the position(s) of: _____ **President-Elect**

_____ **Director**

_____ **Councillor**

_____ **Nominator**

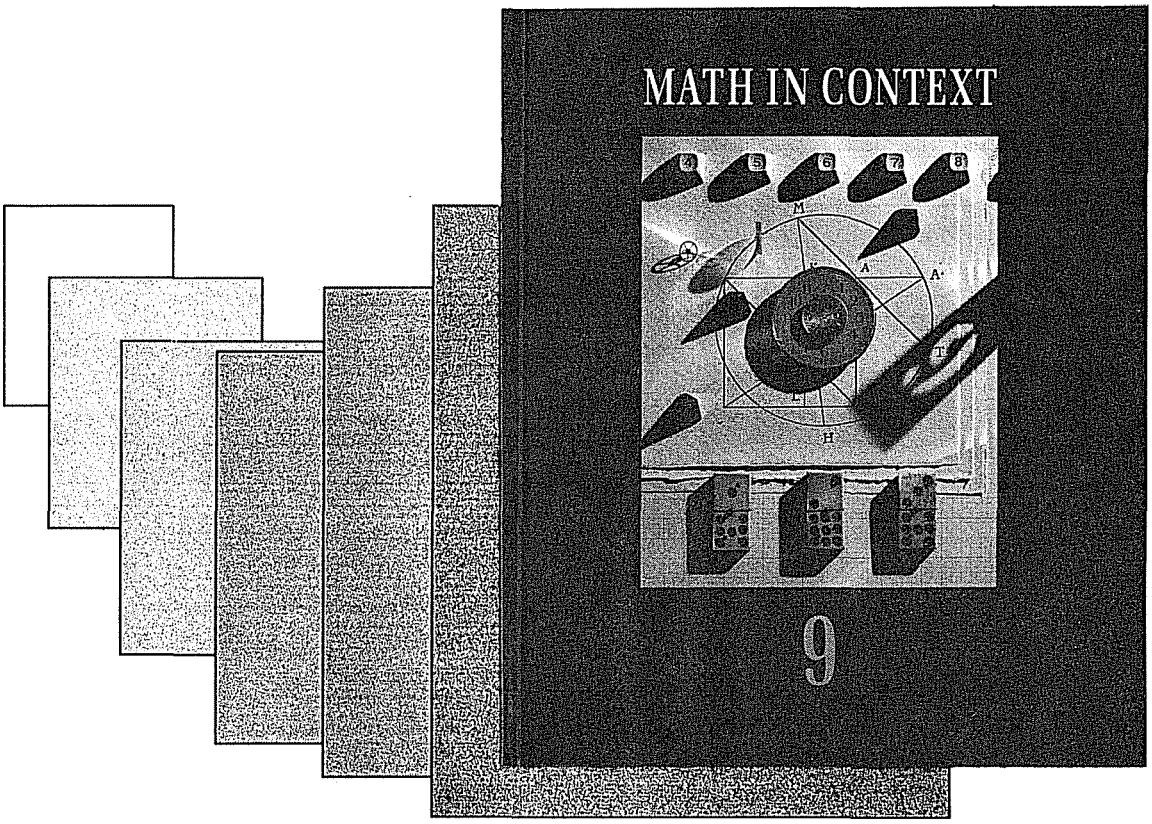
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CALL FOR AWARD NOMINATIONS

LIFE MEMBERSHIP

To qualify for Life Membership in OAME/AOEM a candidate shall have:

- 10 or more accumulated years of membership in OAME/AOEM
- demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular

Deadline for nominations: January 31, 1994.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award, to be presented annually to a person who does not necessarily meet the criteria for the Life Membership Award (i.e., not necessarily an educator or an OAME/AOEM member.) The award is aimed at recognizing people who have supported OAME/AOEM, such as publishers, educational suppliers, hotel personnel, etc.

At this time OAME/AOEM is requesting nominations for consideration for this year's award, to be presented at the Annual Conference.

Deadline for nominations: January 31, 1994.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM has established the Morley MacGregor Award, to be presented annually to an elementary school teacher of mathematics who has demonstrated excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME/AOEM is requesting nominations for consideration for this year's award, to be presented at the Annual Conference.

Deadline for nominations: January 31, 1994.

KENNETH D. FRYER AWARD

In Memory of the late Dr. K.D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, has established the Kenneth D. Fryer Award for Excellence, to be presented annually to a secondary school mathematics department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME is requesting nominations for consideration by the committee for this year's award - to be presented at the Annual Conference.

Deadline for nominations: January 31, 1994

AWARD NOMINATION FORM

Date of Nomination: _____

Award (Check One)

❑ **Life Membership** ❑ **Mona MacGregor Award** ❑ **Morley MacGregor Award** ❑ **Kenneth D. Fryer Award**

I/We nominate (Name of Nominee) _____

School, Organization or Business of Nominee _____

School, Organization, or Business Address of Nominee _____

Principal of Nominee (for the Kenneth D. Fryer Award) _____

Mathematics Department Head of Nominee (for the Kenneth D. Fryer Award) _____

Nominated by (Name, Address & Phone) _____

Please attach a separate sheet to the completed nomination form, outlining the reasons for your nomination and send to Don & Carol Attridge, Executive Directors OAME/AOEM, 1 Southdale Drive, Markham, Ontario L3P 1J6 Fax (416) 472-3272

IMAGE
Algebra
and
Geometry

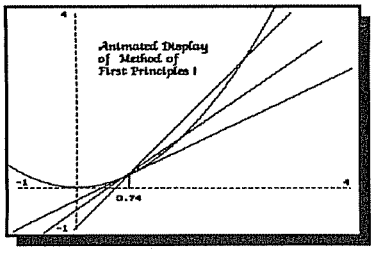
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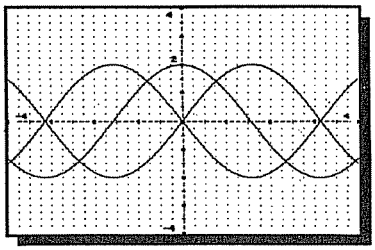
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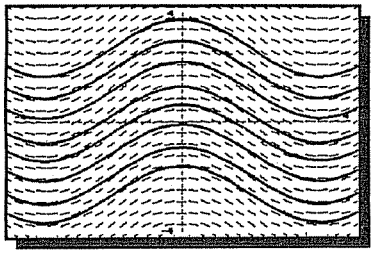


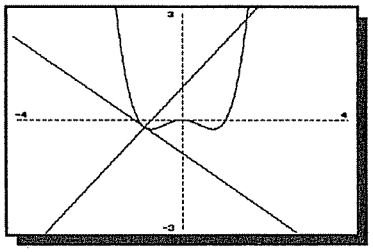
IMAGE-*Calculus* 2.1

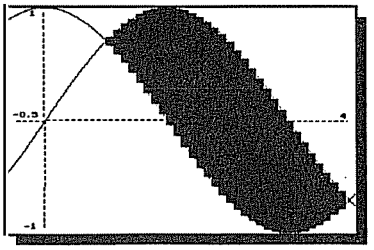
"...we have found this software to be an excellent problem solving tool. It has had a major impact on the way our math classes are taught."

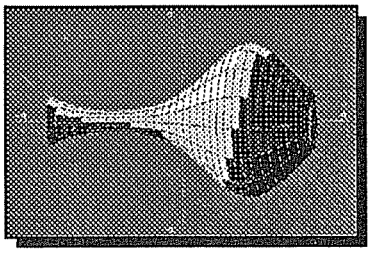
- Department Head (Muskoka)

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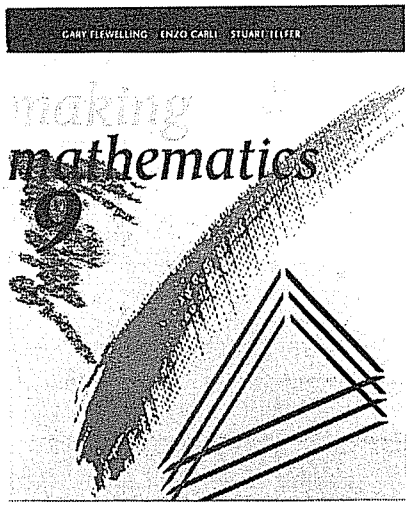
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