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Ontario Mathematics Gazette

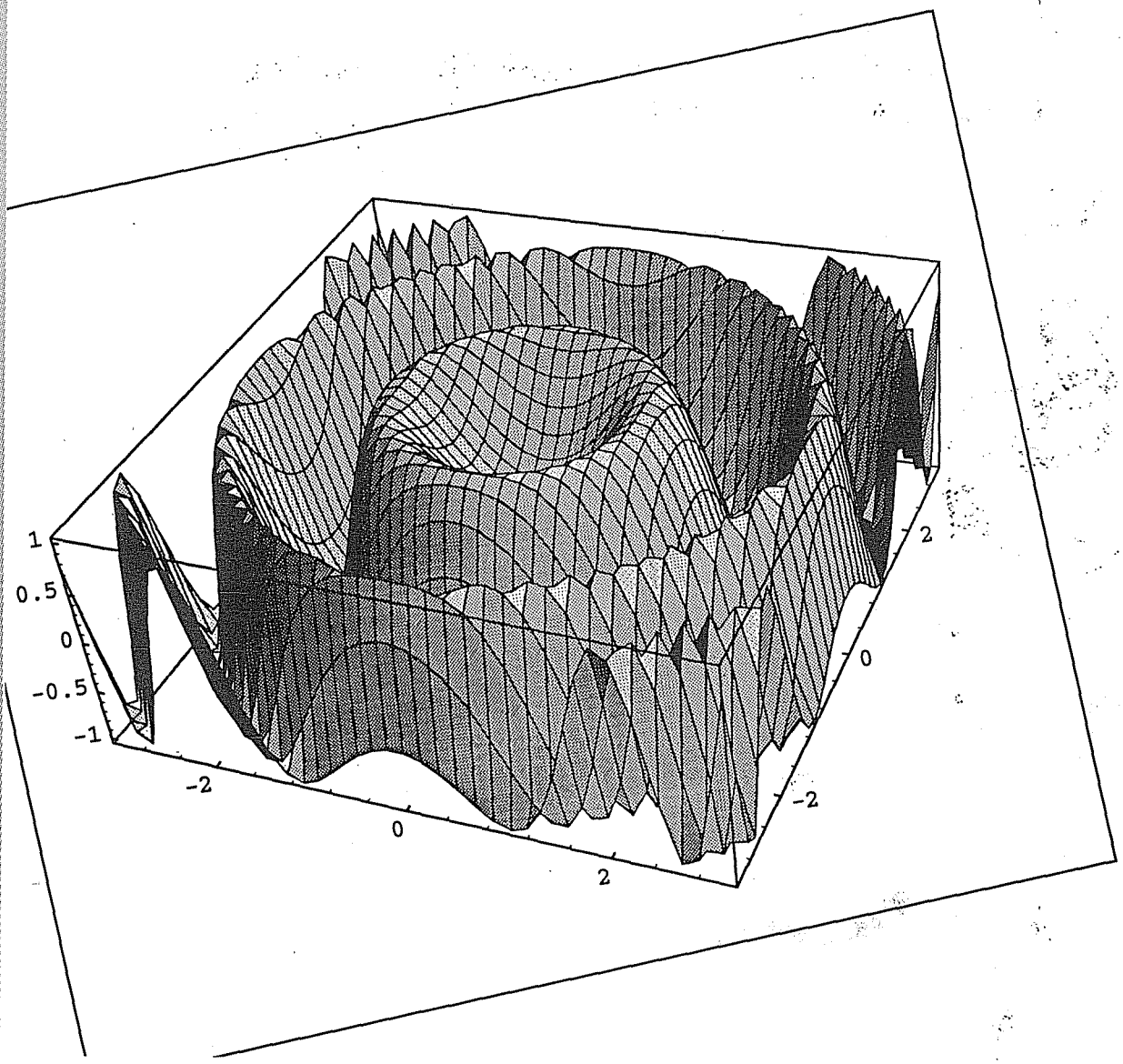
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Articles for the Gazette or the Abacus should be submitted to Jack Weiner at the above address.

Deadlines for information to be included in the Gazette or the Abacus are: June 15 for the September issue, October 15 for the December issue, January 15 for the April issue, and April 15 for the June issue.

Notice to Authors

Your articles should be typewritten, word-processed or photocopied from typewritten or word-processed copy double-spaced on 8.5" x 11" paper with 1.5" margins and single sided. Diagrams and figures should be in camera-ready form. Please submit three copies to the editor and keep one as protection against loss. We have all submissions blind-reviewed, so please send your full address and affiliation on a page separate from the manuscript.

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▲ EDITORIAL



Spring is definitely in the air. Actually, it's in the puddles that soaked me and my companions through our lunchtime run. I hope you had a refreshing March Break and are happily back in the class for the home run stretch to June.

I also hope you can and will attend the OAME Annual in Kingston. Every member of OAME was sent a program and registration form. If you didn't receive yours or want extra copies for your colleagues, contact

Brian Gallagher or Margo Johnston

Watch for "Focus on Featured Speakers", scattered throughout this issue of the Gazette. These contain mini-biographies of many conference speakers.

One of the interesting articles you will read in this issue is "Strings of Successes", by David Duncan and Bonnie Litwiller. They examine the probability of strings of heads in successive tosses of a coin. One of the original reviewers of this article suggested that the problem posed needed to be motivated. Why would you want to know the probability of four heads in succession, for example? In their revised submission, David and Bonnie presented the problem in a perfectly natural context.

This may seem a minor detail to some of you, but it was important to me. First, the article was much improved for having the problem motivated. Second, the reviewing process itself worked beautifully. My reviewer did her job. She read the article thoroughly, evaluated its relevance and merit for the Gazette. When she decided it should be published with modifications, she checked carefully both the mathematics and grammar and made excellent constructive suggestions. In response, Bonnie and David acted upon her suggestions and produced a revision that improved an already good article. Of course, an author might act on some suggestions and have good reason not to respond to others. But understand that the reviewer is a partner, not an adversary.

This is a backhanded editorial way of thanking those reviewers (in the majority) who do their job conscientiously, and thanking the contributors (in the majority) who take constructive criticism in the positive way it is intended.

This issue marks the end of Amy Parker's tenure as editor of the Abacus. Amy has contributed to the Abacus for many years in many ways. Before her involvement as editor, she submitted activities, usually in French. (Amy is a French immersion teacher.) Later, she learned the ropes of editing while helping previous Abacus editors Anna and Colin Garnham. Three years ago she assumed the editorial mantle. The usual term is two years and she has gone beyond the call of duty in taking on a third year. Bonne chance, Amy.

Jack

MATH MANIA

By Paul Khanna



Editor's Note: Paul Khanna is a second year Engineering student at the University of Guelph.

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▲ LETTERS TO THE EDITOR

Editors Note:

Ed Barbeau contributed the September, 1993 "Have Your Say" and Todd Romiens responded in December, 1993. Here is Ed's response to Todd's column:

Dear Editor

I am quite happy to enter into a discussion of my article, but not on the basis of points that I did not make and hope that readers did not draw from the article. I do not propose examinations as a "quick fix" nor solely out of self-interest, although I must confess to being at a loss why one might seek out unprepared students to teach or witness unresponding the anguish of students who suddenly realize how deficient their backgrounds are.

I do not believe that one can conclude from my article that "mathematics is linear", but it would be fair to attribute to me the perception that the subject is cumulative to the extent that a lack of proficiency with earlier material can sabotage the learning of more advanced material. I would be interested to hear from any teacher whose experience does not accord with this.

Nor do I see from the observation that learning at times requires some effort the requirement of a "regime that tries to force learning." It is my experience that while children can be curious, astute, ingenious, perceptive and divergent, most of them do not spontaneously subject themselves to the discipline necessary to secure proficiency, tie up loose ends, clarify ideas and express themselves cogently.

A final comment: I am not ignorant of the situation in either elementary or secondary schools, having spent a fair bit of time in institutions at both levels and talked to teachers there. There are many complex situations that I know teachers have to face, and which to some extent are shared by the university. But at the end of the day, the issue is really that either a kid wants to learn mathematics or he doesn't; if he does, then we as professionals have to make it clear what is necessary.

Yours sincerely,

Ed Barbeau

Dear Editor

A couple of months ago I read a newspaper article describing the report of the Provincial Auditor about the education levels of Senior Public School mathematics teachers. The article claimed that 60 percent of Ontario's grade seven and eight math teachers have not taken university level mathematics courses.

I find this very disturbing. How can students in grade seven learn to appreciate the beauty of mathematics if their teachers are themselves blind to it?

I am currently a third year student in the Honours Mathematics program at McMaster University. This past year, my professors have been emphasizing the aesthetics of mathematics. With every course I take I become more aware of deeper aspects of math, aspects that were not stressed in my introductory calculus and linear algebra courses. Aspects I share now when I discuss mathematics.

I worry that as a result of the inadequate preparation of their teachers, elementary school students will never experience the aesthetic side of mathematics. It is my experience that many students, early on in their academic lives, learn to dislike math, and cringe at the thought of studying it beyond high school. They find it difficult or boring, but take it because they need it to get a good job.

Well, a strong grounding in math will help a student find employment, but we must emphasize the beauty of the subject as well. We must make students aware of the aesthetics of the subject along with its technical content. I believe doing so will inevitably increase student interest.

For example, don't simply show how to use Pythagoras' Theorem. Include a discussion of its history. Show some of its implications in number theory. I don't expect all elementary students to delve deeply into number theory, but a taste of such mathematics can go a long way towards nurturing positive feelings for the subject.

My personal goal is to become a mathematics teacher. I hope to enrich the lives of my students through the medium of mathematics. I cannot imagine doing this with only OAC Calculus. I think it is time to insist on minimum university qualifications for teachers of mathematics, regardless of the grades they are teaching.

Regards,

Chris K. Brady

▲ PRESIDENT'S MESSAGE

RON LANCASTER



Recently, OAME and OMCA submitted a written report to the Royal Commission on Learning.

The writing team included: Chuck Lemenchuk, Mathematics Program Consultant for the Halton Roman Catholic Secondary School Board; Peter Saarimaki, Coordinator of Mathematics for the Toronto Board of Education; and Barry Scully, Mathematics Program Consultant for the York Region Board of Education.

Here is the report:

TO THE ROYAL COMMISSION ON LEARNING FROM THE ONTARIO MATHEMATICS COORDINATORS ASSOCIATION AND THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

January 31, 1994

The OMCA and OAME were pleased to make a joint oral presentation to the Commission on November 4, 1993 in St. Catharines. Our two presidents, Ron Lancaster (OAME) and Peter Saarimaki (OMCA), were able to share their vision of mathematics education in Ontario by distributing the landmark document, Focus on Renewal of Mathematics Education.

The Purpose and Direction of Our School System

Literacy, both linguistic and mathematical, is the top priority of education. This priority must be reflected in all policy documents and decisions. This includes compulsory math courses in the Specialization Years and also must include the development of new math courses to meet the needs of more aspects of the business community.

While the Common Curriculum sets out general directions, the Ministry must support, encourage and coordinate the necessary resource documents that teachers need to implement the proposed philosophy as intended. The Common Curriculum, while it does give us statements about the principles of learning, teaching and curriculum, is not as succinct or pithy as the Image of the Learner nor as visionary as the Goals of Education we have lived with for so long.

Therefore we recommend that the Ministry of Education:

- make two math courses compulsory in the Specialization Years
- make allowances for alternative math courses to meet the needs of the local community
- provide support to local curriculum developments through coordination of projects, finances, and sharing of expertise
- summarize the vision of education in a brief page of prose

The Program in Our Schools

"Just as our vision of the twenty-first century differs dramatically from that of the twentieth century, so the vision of mathematics education ... differs significantly from the traditional vision of the subject."[1] As we look to specific recommendations for programming, we can do no better than refer you to the Focus on Renewal documents. We are proud of the generous use of quotations from this production in the Ministry's Provincial Standards in Mathematics. This is but one example of collaboration between the Ministry and subject organizations that we must continue in these times of fiscal restraint.

Teachers need more direction and support than currently available. The promised resource packages for alternate assessment tools and portfolios are needed now. If the Ministry were to act as a broker to facilitate the development and distribution of resources, teachers could be assured of quality materials, in tune with provincial directions, and financed in time to be of use.

Teachers benefit when they can see a whole division's expectations at a glance, before they review their program. The complaints of a crowded curriculum implies a need for a reasonable space for mathematics in the overall program. New techniques are needed and teachers need time and inservice to become familiar with them and have the opportunity to implement them.

Integration is one technique that can be very powerful and yet can be misused, or misunderstood. There have to be very strong subjects on which to build integration. Integration by itself will not and cannot make mathematics disappear.

With all these changes, teachers and teachers-to-be need well planned, consistent and long-range inservice. Teacher education should be a four year concurrent program instead of one year after a B.A., a year just touching on many aspects of teaching, limiting mathematics to only six weeks.

Therefore we recommend that the Ministry of Education:

- make teacher education a four-year concurrent program
- include more specialist courses in the B.Ed. program
- ensure that teacher training programs need up-to-date content, theory and information on assessment and evaluation based on current research in each year of the program
- make inservice a mandatory expectation for all teachers

Inservice must be on-going and beneficial. It must also be made worthwhile. Teachers cannot be expected to do extra work without "rewards". Inservice should be equated with credits towards a further degree, as is the case in the USA. Inservice could also result in increased qualifications towards upgrading. Business does not expect employees to give up their own time for the employers' desires. Boards as well must see the benefits of teacher inservice.

Teachers in training make use of practising teachers, usually termed associate teachers. The selection process needs to be looked at more closely so that mentoring can be included as an expectation. Reward associates with Ontario awards similar to the Prime Minister's MST Awards. Those so selected could be given reduced teaching loads as part of the

1 Focus on Renewal of Mathematics Education, OAME/OMCA, page 2

award, allowing them to be functional associates.

Therefore we recommend that the Ministry of Education:

- set up Premier awards to recognize excellence in teaching, with the recipients being given reduced teaching loads and mentoring opportunities
- financially support the production of videos with best-teaching practices for inservice (as collaborative ventures between MET, TVO, the teaching federations and subject associations)

Accountability in Education

The Provincial Standards performance indicators need exemplars on the laser-disc for inter-active use, or at the least videos. Only in this way will teachers know what the expected level of performance will look like. We must recognize that while values and attitudes are very important, they are difficult to measure objectively at any significant level.

We continue to support the Mathematics Education Forum, a loose collection of eight autonomous bodies, and realize that it is still up to the bodies themselves to disseminate and promote the products of the Forum. The Forum provides a very useful venue for dialogue among the various levels of education that doesn't exist otherwise. Local versions of the Forum would be very useful for establishing or strengthening the educational continuum throughout the province. We need to ensure that secondary standards and post-secondary expectations are consistent, allowing students to be prepared as best as possible for their future. If we can agree on the final outcomes and significant signposts, then we can help students over the various transitions.

We see the recently announced mathematics action symposium as the start to a long-term relationship among the stakeholders in mathematics education.

Therefore we recommend that the Ministry of Education:

- promote the mathematics action symposium as the start of a long term forum for the stakeholders in mathematics education
- reaffirm it's support, including funding, to the previously announced initiatives including an assessment package and a portfolio resource
- continue to promote the use of alternative assessment tools throughout the curriculum and that this be reflected in programs such as Provincial Reviews

The Organization of Our School System

In Outcome Based Education, time is not a factor. Program and organizational components must mirror this. Having a teacher stay with a group of students over a three year period would certainly help put the focus on divisional expectations. Even if this grouping were just for a mentor/advisor program, it would provide a form of continuity that is often missing now.

At the secondary level, blocking of classes is necessary so that there is the flexibility and opportunity to integrate. Students need to have access to larger time slots to "get into" more holistic topics and teachers need the common time for discussion and planning.

Therefore we recommend that the Ministry of Education:

- encourage and promote long term pairings of teachers and student groups
- facilitate blocking of classes in secondary schools and encourage timetabling that allows for common planning time for teachers

Conclusions

A new era is dawning, and in many classrooms, has arrived. Educators, parents and the community in general are spending a lot of time discussing where education should be going, and how we are going to get there, and how are we going to know when we've arrived. The Common Curriculum is a start but the Ministry must support and encourage further discussions and resource materials.

We are all working towards education helping our young people:

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- pursue ethical and responsible personal growth

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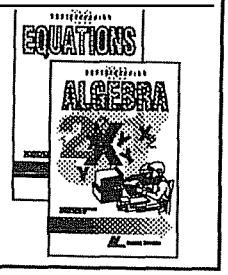
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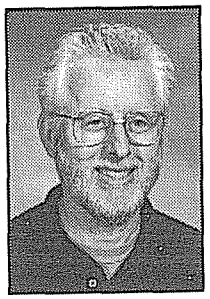


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▲ Have Your Say: Exam Writing in the Schools

EDWARD W. MCFARLANE



Edward W. McFarlane was born and educated in Toronto. He received a B.A. in mathematics and a M.A. in theoretical physics, both from University of Toronto. He has taught mathematics and physics for the past 32 years, mainly in Scarborough, and has co-authored three publications commissioned by the Ontario Mathematics Commission and published by O.I.S.E. He retired December 31, 1993.

Once again we have a proposal for external examinations for entrance into university - see the Gazette, April 1993, "Entrance Exams - The Time Has Come", by Gord Nicholls. As is often the case, these proposals stem from a wistful look at the past and a longing for good old fashioned discipline. But the argument usually turns on the decrease in abilities of the students along with the supposed fairness of an external exam.

The feedback from the Descartes competitions and the International Math Study are taken as evidence that "too large a percentage of our students cannot demonstrate abilities to the level we would like." But the reliability of the IMS is suspect due to reports of "cheating" by many of the top rated countries. As well, the multiple choice nature of their questions leads to testing which overemphasizes memorized information and one step problems. Even more problematic is the use of the Descartes competitions as a basis of evidence: first, the competitions are written by only a small percentage of our students; second, the vast majority of the students who write the Descartes have come up through the system of the Canadian Mathematics Competitions which appeal to those students who love competitions and especially those who are able to respond quickly based on considerable memorized information.

The only other evidence offered about the abilities of our students is that "the retention rate in first year university has been only about 55%." Here again we have problematic evidence. To what extent is this a problem of the universities? Stuart Smith has just completed a study of teaching at the universities and found the universities wanting in many areas. At the very least, we all know how their first year class sizes have mushroomed in the past thirty years. Furthermore, their increased emphasis on research over teaching has inevitably led to a decrease in the opportunities to develop and nurture the students they receive and has hence allowed teaching to be more dominated by the passing on of memorized information.

We are also urged to accept the inherent fairness of an external exam. Only in this way can parents and teachers be assured that, "certain basic standards are being met and that fairness based on merit is being used to allocate the admissions to our post secondary institutions." But everyone knows that these so-called basic standards are very debatable and hence will be politically decided upon. This has always meant in the past that certain topics must be covered and this list of topics soon grows. Standard texts and standard teaching inevitably ensues. These outcomes result from the weak assumption that fairness is an attainable quantity. We will always be faced with different schools and different teachers. Surely diversity and the encouragement of innovation are qualities about which we should think twice before limiting them. Standard exams will without doubt do this.

Does the proposal for external exams stem from a wistful look at the past and a longing for good old fashioned discipline? We are asked to "look at where we were 25 years ago" and to regard the "developments of the Seventies and Eighties" as key to understanding the "many problems that surround us in the schools." It is implied that, among other things, the external exams before 1966 were more successful in placing the right students in the right programs at university. But in my years in Mathematics, Physics and Chemistry at the University of Toronto, 70% of the students who began the program did not complete it. "Standardization" has never given evidence that it will improve the preparation of our students. It is advanced that "for the vast majority of students, the removal of external measuring instruments has led to an unwillingness to master the material to the standards required." But this lament as well ignores the experience of Ontario mathematics teachers in the past 27 years. Before the exams were removed teachers complained about the imposition of the 80-20 rule. As soon as the exams were removed the boards were pressing grade 13 teachers to pass a high percentage of their students. (A 91.2% pass rate was asked for in Scarborough.) This resulted in a mushrooming of the number of Ontario Scholars (from an average of 3 or 4 per year at my school to an average of

over 25). Then came the credit system which encouraged large numbers of students to obtain credits at summer school. Many teachers have, I think justifiable, reservations about the quality of summer school credits. These pressures gradually led teachers to set exams which rarely covered more than one term. The idea of an exam that covers the year's work and is written by all students has virtually vanished. All this has culminated in courses and exams which overemphasize memorized information. Little wonder that some students shop for teachers and schools and show an "unwillingness to master the material."

You may conclude that "The Time Has Come" but surely there are better alternatives than the quick fix external exam. Teachers must find means of mutual support in order to resist the trendy "educators" who influence our Ministry and Boards into swallowing one bandwagon after another. Teachers need to develop curriculum that can pass the test of peer review. Mathematics teachers need to improve their networking so that the best among us, such as Gord Nicholls, can be heard by all. This will involve improving the appeal of the Gazette and targeting the role of OAME and its local chapters. The textbooks must be revamped to reflect the needs of students to acquire a sufficient number of basic tools that will allow them to develop and solve significant problems. Then perhaps our students will have the opportunity to master some mathematics rather than just memorizing information. Can teachers in this way gain some control over their teaching careers? I believe so and I also agree that "the time is now."

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Dennis Nykoliation

(Friday Morning at 0900)

President and General Manager, Black & Decker Canada Inc. Mr. Nykoliation was born and educated in Manitoba. He is a graduate of University of Western Ontario's Business School Executive Program. Dennis worked for NCR Canada and General Electric prior to joining Black & Decker in 1984. He was appointed Vice-President Sales Distribution and Customer Service in 1988. He is actively involved with several business associations including the Canadian Management Centre and the Business Council on National Issues. Mr. Nykoliation is well qualified to enlighten us on Quality Control - Everyone's Concern.



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▲ Strings of Successes: A Probability Connection



DAVID R. DUNCAN
AND BONNIE H. LITWILLER
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AND COMPUTER SCIENCE
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The NCTM *Curriculum and Evaluation Standards for School Mathematics* emphasizes the importance of connections between mathematics and the "real world". Probability situations provide a fertile ground in which to develop such connections. Teachers are consequently interested in situations which lend themselves to probability discussions. We shall present one such situation involving flipping coins.

Suppose that a mathematics teacher, Phyllis, wishes to construct a true-false examination for a contest. In order to avoid inadvertently introducing a pattern to the correct true-false responses, she flips a coin before composing each question. If the coin shows a "head" she composes a question for which the correct answer is "true"; if the coin shows a "tail" she composes a question for which the correct answer is "false".

At some point during the test construction process, Phyllis notices that she has composed several consecutive "true" questions resulting from a string of consecutive heads on her coin; she wonders whether this is an unusual occurrence which might lead a student to surmise that a predictable pattern of responses was present.

This question of likelihood can be answered in at least two ways. One interpretation of likelihood would involve focusing on the specific flips that yielded heads and computing the probability of that precise outcome. For instance, the probability that a specific sequence of five flips yields all heads is

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\left(\frac{1}{2}\right)^5=\frac{1}{32}$$

But, suppose that Phyllis is interested in the probability that a string of heads of a specific length occurs **sometime** in the flipping process; she does not restrict her attention to a predetermined location for this string. This problem would be very difficult to approach by direct calculations of probabilities. For example, to determine the probability of getting a string of exactly three heads in exactly 10 flips would require a consideration of all possible placements of this string, together with attention to the preceding and succeeding flips (if appropriate) to be sure that they are tails.

This is the type of problem that may be best solved by constructing a model. Consequently, we constructed lists of all possible outcomes when a fair coin is flipped each of 2, 3, 4, ..., 12 times. (A computer can be an aid in such a process.) For a given number of flips (n), each of the 2^n possible head/tail sequences of length n are equally likely The task is then to count the number of head/tail sequences that contain strings of consecutive heads of various lengths.

To conform to the usual convention in such a problem, we will designate the occurrence of one coin face as "success" and the other as "failure". We will arbitrarily designate a head as a success, and a tail as a failure.

For example, if a fair coin is flipped three times, the following eight equally likely sequences are possible:

HHH	THH
HHT	THT
HTH	TTH
HTT	TTT

How many strings of heads of various lengths are there?

- In two of these sequences (HHT and THH), there is a string of exactly two heads (successes). The probability of such an occurrence is then $\frac{2}{8}$ or $\frac{1}{4}$
- In one of these sequences (HHH), there is a string of exactly three successes. The probability of such an occurrence is $\frac{1}{8}$
- Altogether the probability that a string of at least two successes occurs is $\frac{3}{8}$

Let us consider as another example the situation in which Phyllis flips a fair coin four times. The following 16 equally likely sequences are possible:

HHHH(4)	HTHH(2)	THHH(3)	TTHH(2)
HHHT(3)	HTHT	THHT(2)	TTHT
HHTH(2)	HTTH	THTH	TTTH
HHTT(2)	HTTT	THTT	TTTT

Beside each sequence the length of the longest string (greater than or equal to 2) of consecutive heads contained in the sequence is indicated in parentheses.

- ▲ The probability of getting a string of exactly two heads is then $\frac{5}{16}$
- ▲ The probability of getting a string of exactly three heads is $\frac{2}{16}$
- ▲ The probability of getting a string of exactly four heads is $\frac{1}{16}$
- ▲ The probability of getting a string of at least two heads is $\frac{5+2+1}{16} = \frac{8}{16}$ or $\frac{1}{2}$
- ▲ The probability of getting a string of at least three heads is $\frac{2+1}{16} = \frac{3}{16}$

Table I and Table II report results of n flips for n = 2, 3, 4, ..., 12. Table I reports the number of outcomes containing success strings of lengths exactly 2, 3, ..., 12, while Table II reports the instances where success strings of lengths at least 2, 3, ..., 12 occur.

Table I

No. of Flips (n)	No. of Possible Head/Tail Sequences	No. of Success Strings of Length Exactly:											Cumulative Totals
		2	3	4	5	6	7	8	9	10	11	12	
2	4	1											1
3	8	2	1										3
4	16	5	2	1									8
5	32	11	5	2	1								19
6	64	23	12	5	2	1							43
7	128	47	27	12	5	2	1						94
8	256	94	59	28	12	5	2	1					201
9	512	185	127	63	28	12	5	2	1				423
10	1024	360	269	139	64	28	12	5	2	1			880
11	2048	694	563	303	143	64	28	12	5	2	1		1815
12	4096	1328	1167	653	315	144	64	28	12	5	2	1	3719

Table II

No. of Flips (n)	No. of Possible Head/Tail Sequences	No. of Success Strings of Length at Least:										
		2	3	4	5	6	7	8	9	10	11	12
2	4	1										
3	8	3	1									
4	16	8	3	1								
5	32	19	8	3	1							
6	64	43	20	8	3	1						
7	128	94	47	20	8	3	1					
8	256	201	107	48	20	8	3	1				
9	512	423	238	111	48	20	8	3	1			
10	1024	880	520	251	112	48	20	8	3	1		
11	2048	1815	1121	558	255	112	48	20	8	3	1	
12	4096	3719	2391	1224	571	256	112	48	20	8	3	1

In each case in either table, the probability of the outcome in question is the number of times it can occur divided by the number of possible head/tail sequences. For instance, when a fair coin is flipped 10 times, the probability that a string of exactly 4 consecutive heads will result is

$$\frac{139}{1024} = 24.5\%$$

Suppose that Phyllis wishes to focus on success strings of length **exactly** 2. What is the probability that she achieves such a string when she flips the coin different numbers of times? Table III reports the results which are achieved using entries from Table I.

TABLE III

No. of Flips n	Probability of Getting a String of Exactly Two Successes		
2	$\frac{1}{4}$	=	0.2500
3	$\frac{2}{8}$	=	0.2500
4	$\frac{5}{16}$	=	0.3125
5	$\frac{11}{32}$	=	0.3438
6	$\frac{23}{64}$	=	0.3594
7	$\frac{47}{128}$	=	0.3672
8	$\frac{94}{256}$	=	0.3672
9	$\frac{185}{512}$	=	0.3613
10	$\frac{360}{1024}$	=	0.3516
11	$\frac{694}{2048}$	=	0.3389
12	$\frac{1328}{4096}$	=	0.3242

Note that the probabilities increase from n = 3 to n = 7 and then decrease from n = 8 to n = 12.

Suppose that Phyllis wishes to focus on success strings of length **at least** 2. What is the probability that she achieves such a string when she flips the coin different numbers of times? Table IV reports the results which are achieved using entries from Table II.

TABLE IV

No. of Flips n	Probability of Getting a String of At Least Two Successes		
2	$\frac{1}{4}$	=	0.2500
3	$\frac{3}{8}$	=	0.3750
4	$\frac{8}{16}$	=	0.5000
5	$\frac{19}{32}$	=	0.5938
6	$\frac{43}{64}$	=	0.6719
7	$\frac{94}{128}$	=	0.7344
8	$\frac{201}{256}$	=	0.7852
9	$\frac{423}{512}$	=	0.8262
10	$\frac{880}{1024}$	=	0.8594
11	$\frac{1815}{2048}$	=	0.8862
12	$\frac{3719}{4096}$	=	0.9080

Note that these probabilities are increasing throughout the table.

Challenges for the reader:

1. Continue Tables I and II for greater values of n.
2. Extend Tables III and IV using the extended Tables I and II. Do the probabilities of these extended tables appear to be approaching limits?
3. Build new Tables III and IV for strings of lengths other than 2.
4. Find other real world connections for which the results of this article would apply. For example suppose that a basketball player has a 50% chance of success in each foul shot attempt.
5. The calculations in this article assumed that P(head) = P(tail) for each flip of the coin. What would happen if the coin were "weighted" so that this assumption no longer applied? For instance, suppose that P(H) = 0.60 while P(T) = 0.40 for each flip of the coin.
6. Suppose that Phyllis wishes to calculate the probability of sequences of either heads or tails of certain lengths. How can this be done?
7. Suppose that Phyllis places correct foils in a multiple-choice examination by repeatedly rolling a fair die. What is the probability that she gets strings of any specific die face?

▲ A Survey To Identify the Importance Assigned to the NCTM's General Curriculum Standards

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The NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) reflect what should be of value or of importance in the mathematics curriculum.

The NCTM's Professional Standards for Teaching Mathematics (1991) envisions teachers as agents for the implementation of these curriculum standards. While the relationship between teachers' conceptions of mathematics and their instructional behavior when teaching mathematics is complex, studies (Bush, 1982; Thompson, 1982, 1985) have uncovered a consistent connection between teachers' beliefs, values, and preferences, and their teaching practice. The present study investigated the importance assigned to each of the general standards by both preservice and current teachers.

There are 13 K-4 Curriculum Standards: 4 general standards which convey the spirit and type of mathematical experiences teachers are to implement and 9 content standards reflecting specific content areas that are to be addressed. The general standards articulate what is important and also form a common basis for the entire K-12 mathematics curriculum. These four general standards are: Mathematics as Problem Solving, Mathematics as Communication, Mathematics as Reasoning, and Mathematical Connections.

The degree of importance assigned to each of the general standards by current and future teachers will, in part, influence their implementation. For preservice teachers and also for current teachers, some of the standards may be valued and assigned greater importance; this would make their implementation more likely. On the other hand, other standards may be viewed as less valuable and assigned lower importance which would reduce the likelihood of their implementation.

The vision offered by the curriculum standards for school mathematics must be endorsed both by teachers and those students preparing to become teachers, if implementation is to proceed. Any standard that is viewed by students as having low importance should receive greater emphasis and attention in teacher preparation, since the standards should form an integral part of the mathematics curriculum for students preparing to become teachers. If preservice and current teachers have similar value structures, these areas assigned low importance should be emphasized during in-service activities. If not, appropriate adjustments in emphasis can be made for in-service activities to encourage implementation by current teachers.

METHOD

Subjects

The sample consisted of 68 preservice teachers and 56 current teachers. The preservice teachers included all junior level elementary and early childhood education majors at a state college in the Eastern U.S. They were surveyed on the first day of class during the Spring 1991 semester. The current teacher group included elementary school teachers, whose subject responsibilities included mathematics, from six New Jersey public schools. They were surveyed during the Spring 1991 semester by a volunteer teacher from each school who distributed and collected the surveys during a regularly scheduled school faculty meeting. These six volunteer teachers did not complete the survey.

Materials

Each of the general curriculum standards is elaborated in a list of student activities followed by a Focus section and a Discussion section with example problems. From the elaboration of each of the 4 general standards, 3 representative statements were selected. As an example, from the elaboration on problem solving the following three statements were selected: to use problems involving everyday situations, to show how to solve problems in more than one way, and to have students create their own

problems from real world activities. The statements were put in random order and a 12-item paper and pencil survey was constructed.

The instructions for the survey indicated its purpose was to improve preservice mathematics education. No mention of, nor connection to, the curriculum standards was made to the subjects. The respondents were required to rate each statement on a five point Likert-Scale: 1) of no importance or not applicable, 2) of low importance, 3) of medium importance, 4) of high importance, 5) of extremely high importance.

RESULTS

A series of Analyses of Variance using the importance rating given each of the statements as the independent variable yielded no significant differences among the responses of the current teachers from the six different schools. Consequently, the six teacher groups were combined and treated as a single group.

Group I (preservice teachers) and Group II (current teachers) were compared using T-tests for each of the 12 items. Group Means, Standard Deviations, and T-Values for each are presented in Table 1.

Current teachers' ratings were significantly higher than preservice teachers for four items. These four significant differences were equally distributed over the four general standards. Current teachers' ratings were higher than those of preservice teachers on all items except for one, that of relating physical materials, pictures, and diagrams to mathematical ideas, but this margin was not significant.

To rank the importance ratings given each of the four general standards, the ratings for each of the three corresponding statements were totaled and averaged to yield a score between 1 and 5. Groups I and II were then compared using T-tests with the rating of each general standard as the dependent variable. Group Means, Standard Deviations, and T-Values for each are presented in Table 2.

Importance ratings were higher for current teachers than for preservice teachers for all of the general standards; in two cases, the differences were statistically significant and in the other two cases the differences approached significance at the .05 level. Also, both groups ratings of importance of the four general standards resulted in the same hierarchy.

DISCUSSION

Analysis of the surveys indicated that current teachers assigned a higher degree of importance than preservice teachers to the general standards and to statements representing them, and more specifically, that Mathematics as Communication was assigned the lowest importance rating by both preservice and current teachers.

Four statements were rated significantly higher in importance by current teachers than by preservice teachers: to show how to solve problems in more than one way, to have students write or talk about mathematics, to use patterns and relationships to analyze mathematical situations, and to relate or connect two different topics in mathematics.

The significantly higher importance ratings given these statements could suggest that by virtue of being teachers and attempting to convey mathematics to their students these statements or strategies become more important. If students are having difficulty understanding a topic, teachers may realize the value of alternate methods of solving problems, student discussions, or connections to previous topics in conveying material. While current teachers assigned higher value to these statements, preservice teachers may have yet to recognize their importance.

The statement rated the lowest in importance of the twelve statements by both groups was to read and discuss literature with students concerning mathematics. This result was echoed by Paulos (1992) when he attests to the myth that mathematics and narrative are disparative activities. If mathematics is devoid of such experiences, it fosters the unfortunate sterilization and compartmentalization of mathematics. Lively and creative dialogue can give spirit to mathematics and is indicative of the type of experiences teachers are to implement. Both preservice and in-service education needs to affirm the importance of reading and discussing mathematics.

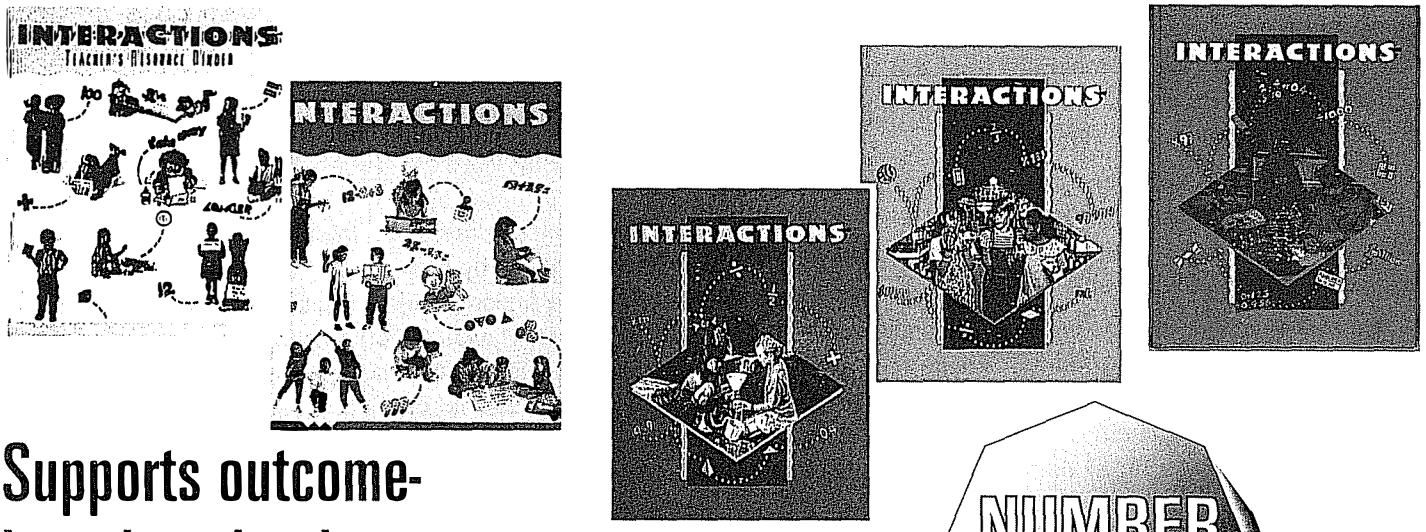
When the ratings of statements corresponding to the general standards were combined, Mathematics as Communication was rated by both preservice and current teachers as having the lowest importance. Statements corresponding to communication involved relating physical materials, pictures, and diagrams to mathematics along with reading, writing, and discussing mathematics. These activities are a marked departure from past practices which emphasized algorithms for their own sake, use of only hand computation, and single correct answers. Opportunities for students to experience Mathematics as Communication should be encouraged and implemented by teachers.

Overall, the general standards were recognized as valuable and rated as important by many preservice and current teachers. However, for others the areas rated as lower in importance require greater emphasis and continued attention in both preservice and in-service education, if the vision of implementation is to proceed.

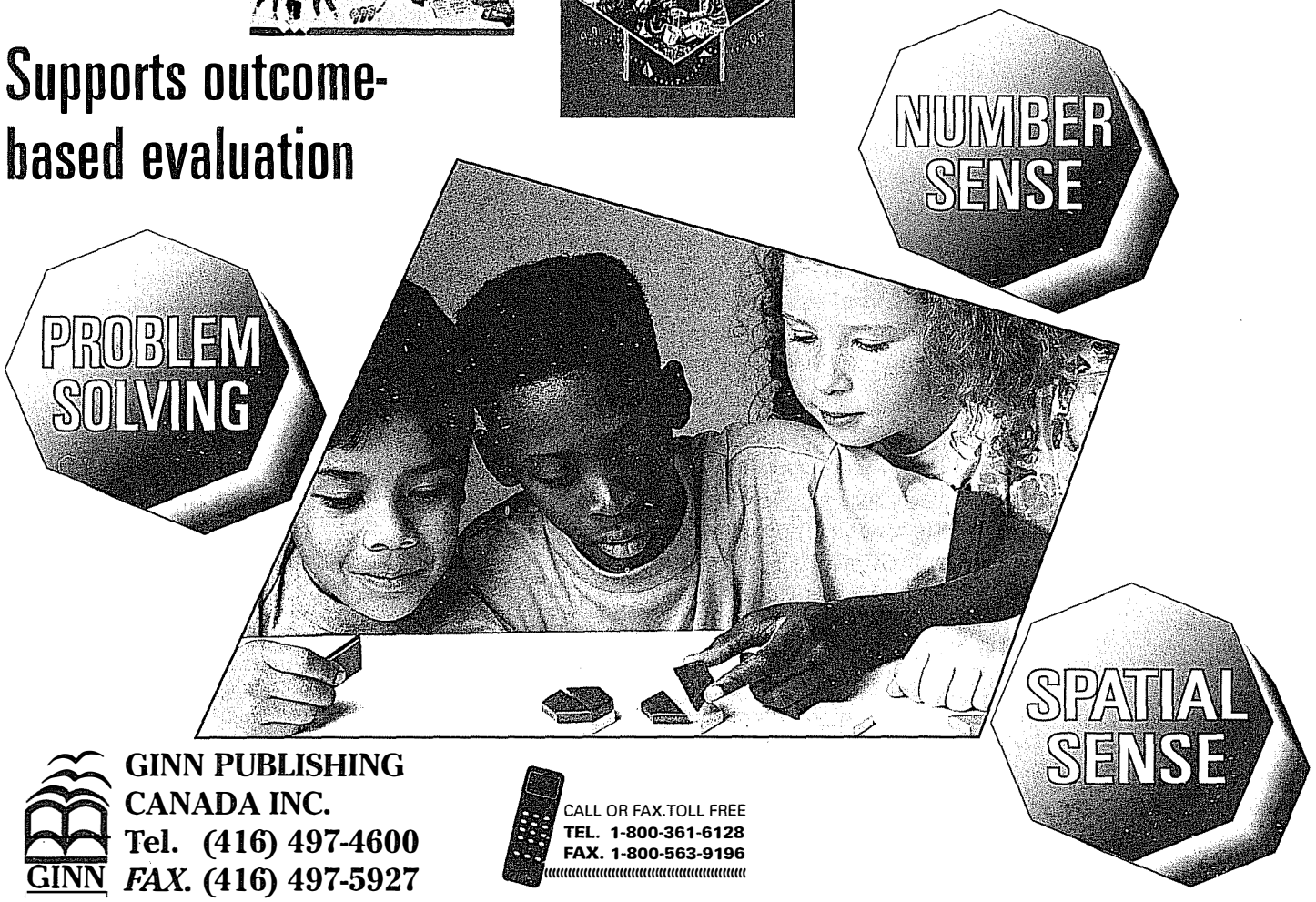
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▲ ELEMENTARY! MY DEAR WATSON

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ERIC WOOD is an associate professor. He teaches mathematics methods to prospective teachers of secondary school mathematics and is interested primarily in better ways to prepare and professionally develop teachers.

Pupils in our high school classrooms have difficulty connecting real world representations with mathematical symbolism. Yet to understand a mathematical concept, it is crucial that our pupils connect abstract symbolism with concrete representations whenever this is possible (NCTM, 1989). A question which often provides some indication as to how a person has learned mathematics is 10 ÷ (1/2). When this question has been presented to prospective elementary and middle school teachers, a few have contributed the answer 5 (their answer being sensible for the model they are using - 10 shares in half), but most give the correct answer 20. When asked to explain why the answer is 20, however, the vast majority are unable to do so and resort to rules which have been memorized during their school years. As one student teacher wryly explained:

"Ours was not to reason why.
Just invert and multiply."

When these same individuals are asked, "How many people can attend a party if there are 10 pizzas and everyone received half a pizza?", the correct answer of 20 is always quickly contributed, but the student teachers do not necessarily recognize the connection between the verbal question and its equivalent mathematical representation of 10 ÷ (1/2). If this lack of understanding is so prevalent among such an elite group, what sort of mathematical understanding do most of our high school students possess? Implicit understanding between reality and symbolism does not occur automatically for a great many of our pupils. Concrete representations must not be restricted to elementary school classrooms.

Admittedly, not all mathematical symbolism can be modelled in a real world context. Yet when students do connect symbolism to concrete representations they are able to construct their own mathematical understanding and recognize mathematics as an integrated body of knowledge rather than a collection of isolated facts. It is the responsibility of educators to provide opportunities which enable pupils to travel comfortably, in either direction, between symbolism and concrete representations.

The activities which follow are aimed at helping students develop representations for mathematical symbolism. They are intended to illustrate examples of bridges which can make such journeys possible and show how strategies often advocated for the elementary school can be applied effectively in high school settings. It is our belief that pupils have the potential to understand mathematics if they are given opportunities to develop connections between symbolic mathematics and appropriate models. If we do not assist in facilitating such connections we might well be developing a future generation skilled in symbolic manipulation (a skill which is becoming less and less

important), rather than a population empowered with an understanding of mathematics and able to appreciate the importance of mathematics for solving real world problems.

The first example presented in this article illustrates how concrete materials can be used to help pupils build representations to assist their understanding of calculus. The second example, on factoring, shows how a geometric model of an arithmetic concept may be extended to algebra. The third example offers a strategy to help broaden a pupil's schema of triangle, and the conditions under which triangles are congruent.

Area Under A Curve - Three Ways!

Consider the case of finding the area under a curve y = f(x). Traditionally this would be accomplished by integrating the function between the appropriate limits. But it is also possible to first get a good approximation to the answer using an experimental approach and concrete materials. All that is required is a sheet of Bristol board, an overhead projector, a paper cutter and an equal arm balance.

First draw an accurate graph of the function using metric graph paper with 1 cm by 1 cm squares further subdivided into square millimetres. (Assume that 1 cm is 1 unit.) Also carefully outline 1 square centimetre of the graph paper somewhere away from the curve. Use this graph to make a transparency. The Bristol board is taped to the blackboard and this transparency is projected onto it so that you can trace the curve (and the square centimetre that was outlined) onto the Bristol board. If the required area is now cut out and placed on the equal arm balance, the mass of the cutout piece can be measured as can the mass of the unit square that has been cut out. Since the area of the square is 1 square unit, a simple ratio of these two masses will allow you to calculate the area in question regardless of how curved the boundaries are!

Now that you know reasonably accurately what the area is, you can take the area that has been cut out and use the paper cutter to shear it into strips. These need not all be the same width and, in fact, it is preferable to do this cutting a bit arbitrarily so that the students won't think that there is anything special about the width of the strips. Divide your class into groups and have each group calculate, for their strips, the area of a rectangle with length equal to the long side of the strip and the area if they had a rectangle with length equal to the short side of their strip of cardboard. These two areas are averaged and this becomes the approximation for the area of that particular strip in square cm. Summing these values for all groups gives a measure of the area under the curve in square cm. The final step is to divide this value by the number of square centimetres in one square unit which can be found by calculating the area in square cm of the unit square. This step produces an answer for the area in square units which should agree very closely with the value produced by finding the mass. Finally, use the calculus method to calculate the exact value for the area and compare it to the two others that the students have already calculated.

Factoring

Let's consider a second example to illustrate concrete material use. In the elementary grades children are often asked to find factors of given numbers. A standard practice is to relate finding factors to area and an associated rectangular geometric model. For example, with a number like 12, children may be given 12 tiles and asked to prepare all the different rectangles that they can find. Variations of this activity would include asking students to illustrate all such rectangles on a geoboard. Children might then be expected to record their findings using graph paper.

Applications of this arithmetic to geometric model can likewise be made to multiplication activities in algebra. Consider the illustrations in Figures 1 and 2 and notice how they may relate to their arithmetical counterparts of, say, 3 x 17 or 3 x 10 + 3 x 7, and 35 x 27 or 30 x 20 + 30 x 7 + 5 x 20 + 5 x 7.

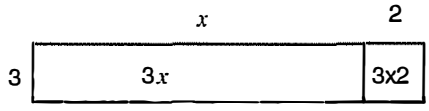


Figure 1. Rectangular Model for 3(x + 2).

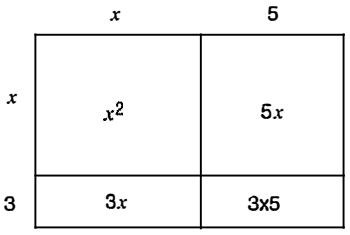


Figure 2. Rectangular Model for (x + 3) (x + 5).

Now for the factoring! Once students have had the multiplication activity experiences as outlined above, they should be ready to tackle the reverse process.

For these activities students will need a set of tiles made out of Bristol board or some other strong paper. The tiles required are large squares of dimension x by x (say 9.8 cm by 9.8 cm), strips x by 1 (9.8 cm by 1 cm) and small squares 1 by 1 (1 cm by 1cm) (see Figure 3). The 9.8 cm length is somewhat arbitrary. The idea is to select a value that is not a multiple of the unit (see Bidwell, 1972).

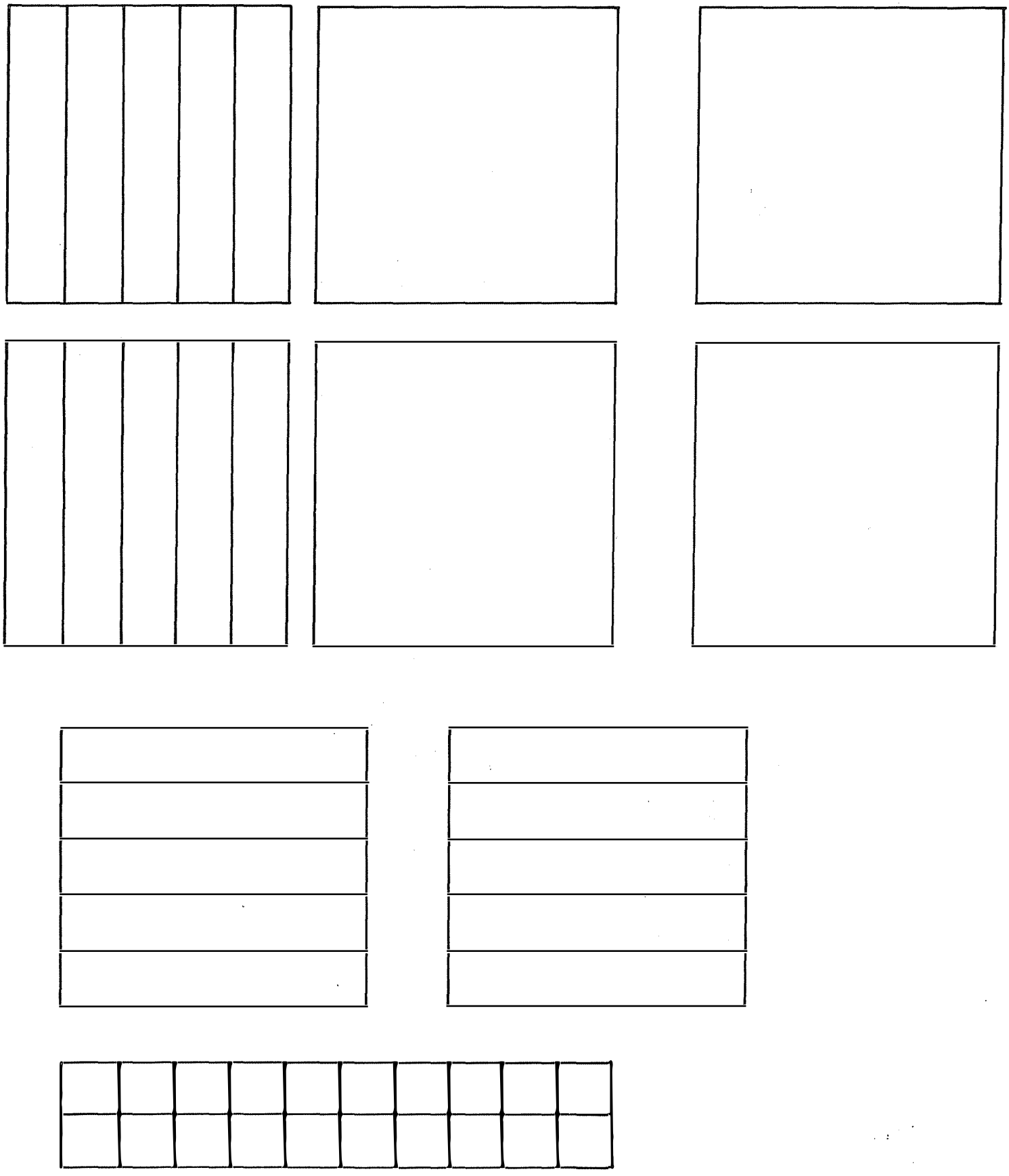


Figure 3. Blackline Masters for the Tiles.

To factor an example such as $x^2 + 5x + 4$, where the coefficient of the x^2 term is 1, students will have to assemble one x^2 tile, five x tiles and four unit tiles. After placing the x^2 tile, the next problem becomes how to arrange the five x and the four unit tiles to help form the rectangle. If we concentrate on the four unit tiles, ignoring the number of x tiles, then two possibilities exist as four can be factored as 2 by 2 or 1 by 4. Notice from the diagrams in Figure 4, however, that the 2 by 2 option requires four rather than five x tiles. Thus, only the 1 by 4 option satisfies the original condition of the problem. The factors of the trinomial may then be read from the model demonstrating the 1 by 4 option.

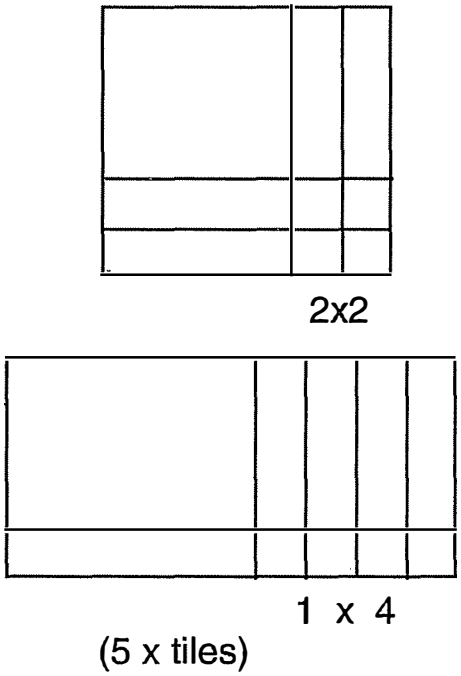


Figure 4. Options for factoring the trinomial $x^2 + 5x + 4$.

Notice how the thinking processes used in creating the model parallel those used in the traditional factoring algorithm. Students first factored the x^2 (that is, they first placed the x^2 tile), then considered the possible factor pairs of the 4, the 1 by 4 and 2 by 2. They finally selected, after finding the option that required five of x, the 1 by 4 option as meeting the condition of the problem; that is, they formed a rectangle, and hence were able to name the factors as $(x + 1)$ and $(x + 4)$. Extensions using this model may be found in Howden (1985).

Congruent Triangles

The third example relates to the meaning of triangles. It is very likely that if you ask your students to draw a "triangle", most of them will draw ones similar to Figure 5. It is also very likely that many of them would describe the triangle in Figure 6 as an "upside-down triangle".

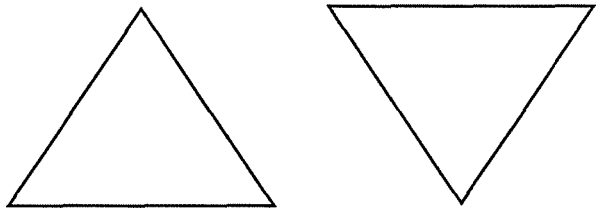


Figure 5 *Figure 6*

For a significant number of students, this is a symptom of a narrow understanding of the term "triangle", which prevents them from accepting that the two triangles may be congruent. Some students incorrectly assume that congruent triangles are oriented identically. They also have difficulty with pairs of corresponding sides in similar triangles that don't have identical orientations. These problems are also symptoms of the narrowness of the experience with triangles that we provide for our students in secondary school classrooms. One way to broaden student experience with triangles, and hence broaden their understanding, is to involve students in physically constructing and manipulating various triangles.

To discover the properties that relate directly to the concept of triangle congruence, ask students to physically construct triangles using the conditions SSS, SAS, and ASA, as well as SSA, AAS, and AAA, as outlined in Activity 1. Notice that students should discover which of these conditions describe unique triangles. They should also discover that the orientation of triangles is irrelevant when we are considering congruency. This activity will build a solid foundation on which to develop a more formal and more abstract approach to the concept of congruent triangles. It is suggested that, as an introduction to this activity, the teacher demonstrates one of the constructions to familiarize students with the construction process and the materials to be used.

Students should be encouraged to describe and defend their findings. Student constructions may be displayed under the headings of SSS, SAS, ASA, AAS, SSA, and AAA, to emphasize that in each of the first four categories, the triangles constructed are always congruent. The SAA case should be discussed in terms of its relationship to the ASA and AAS cases, leading to the conclusion that all three cases fall in the same category of conditions for congruence; that is, one of SAA, ASA, or AAS, necessarily implies the other two. Note that the concept of "similarity" may also be developed by creating a parallel set of activities.

CONSTRUCTING AND COMPARING TRIANGLES

MATERIALS NEEDED:

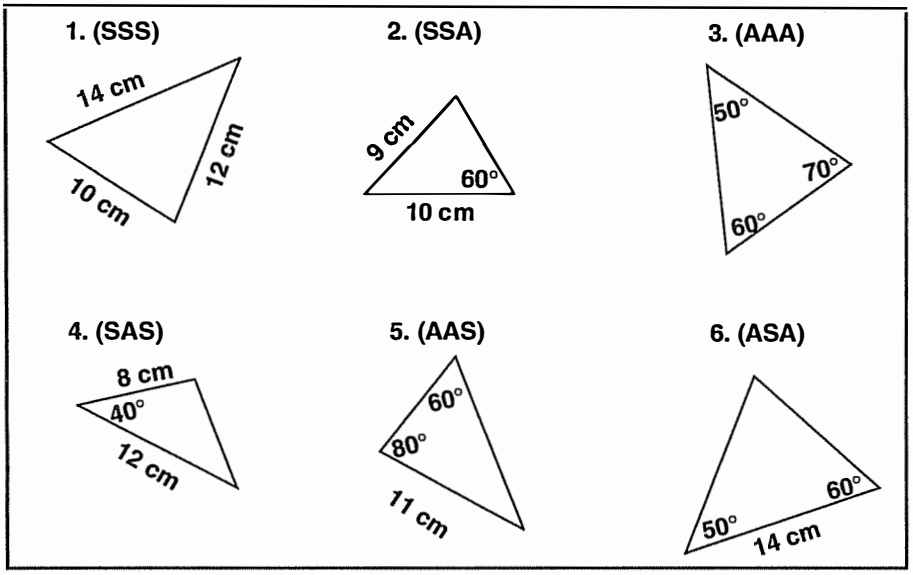
straws (20 cm), pipecleaners (6 cm), scissors, rulers, protractors

ACTIVITY:

Construct the six triangles described below by cutting the straws to the required sizes and joining them using the flexible pipecleaners; use a protractor to measure the required angles.

In each case, if it is possible, construct another triangle which is different from the first.

(S = side; A = angle)



Circle the conditions that describe constructions which are unique (that is, the ones that create only one type of triangle).

SSS SSA AAA SAS
AAS ASA

Activity 1. Triangle congruency.

Editor's Note: These triangles were drawn accurately to scale and then reduced.

Conclusion

The purpose of this article has been to provide the rationale for, and examples of, activities that use concrete materials with students in both junior and senior mathematics classes. We are convinced that judicious use of concrete materials can enhance the mathematical experiences of all students. We hope that this article serves as a framework to help teachers seek out other topics where concrete materials may be appropriately used.

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(Continued from page 13)

A Survey To Identify the Importance Assigned to the NCTM's General Curriculum Standards

Table 1. Means, Standard Deviations, and T-Values

General Standards and Corresponding Statements	Group I (n = 68)	Group II (n = 56)	T
1) Mathematics as Problem Solving			
To use problems involving everyday situations	4.59 (0.60)	4.63 (0.56)	0.35
To show how to solve problems in more than one way	4.18 (0.67)	4.45 (0.74)	2.14*
To have students create their own problems from real world activities	3.97 (0.73)	4.20 (0.82)	1.62
2) Mathematics as Communication			
To relate physical materials, pictures, & diagrams to mathematical ideas	4.25 (0.63)	4.23 (0.71)	-0.15
To have students write or talk about mathematics	3.60 (0.81)	4.05 (0.77)	3.14**
To read & discuss literature with students concerning mathematics	3.13 (0.93)	3.38 (1.04)	1.37
3) Mathematics as Reasoning			
To have students justify and explain their thinking	4.46 (0.66)	4.61 (0.53)	1.39
To relate that the method of solution of a problem is as important as the answer	4.29 (0.75)	4.56 (0.66)	1.88
To use patterns and relationships to analyze mathematical situations	3.87 (0.73)	4.16 (0.83)	2.09*
4) Mathematical Connections			
To have student link conceptual and procedural knowledge	4.07 (0.68)	4.23 (0.63)	1.35
To relate or connect two different topics in math	3.76 (0.74)	4.13 (0.66)	2.84**
To use math in other curriculum areas besides math	4.01 (0.76)	4.23 (0.71)	1.63

* probability <.05 **probability <.01

Table 2. Means, Standard Deviations, and T-Values

General Standards	Group I (n = 68)	Group II (n = 56)	T
1) Mathematics as Communication	3.66 (0.59)	3.89 (0.69)	1.95*
2) Mathematical Connections	3.95 (0.58)	4.20 (0.52)	2.44**
3) Mathematics as Reasoning	4.21 (0.52)	4.43 (0.54)	2.41**
4) Mathematics as Problem Solving	4.25 (0.51)	4.44 (0.54)	1.88*

* probability <.07 ** probability <.02

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FOCUS ON FEATURED SPEAKERS

Bernard Shapiro

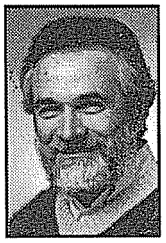
(Thursday Morning at 0900)

A native of Montreal, Dr. Bernard Shapiro is a graduate of McGill and Harvard Universities. A versatile and widely experienced scholar, he has been Professor and Associate Dean of Education at Boston University, Dean of Education and Vice-President (Academic) and Provost of the University Institute for Studies in Education. A former member of the executive committee of the International Association for the Evaluation of Educational Achievement, he has also served as Chairman of the governing board of the Centre for Educational Research and Innovation in Paris. From 1986 to 1993 he played a number of key roles in the Ontario Civil Service, including Deputy Minister of Education, Deputy Minister and Secretary of Management Board, and Deputy Minister of Colleges and Universities. A Professor of Education at the University of Toronto since February 1993, he was named in December of 1993 as Principal-Designate of McGill University.

▲ Geometry and Four Maximum Problems



Harris S. Schultz is a Professor of Mathematics at California State University, Fullerton. In 1989 he was named Outstanding Professor of the nineteen campuses of The California State University.



Mario Martelli teaches Mathematics at California State University, Fullerton. He works on Dynamical Systems, Bifurcation Theory and Differential Equations. He recently completed a book on Discrete Dynamical Systems and Chaos for undergraduates (Logman, 1992).

The following four maximum problems are frequently included in Calculus text books. In each one of them the quantity p is a fixed positive real number.

Problem 1. Among all rectangles ABCD (Fig. 1) with perimeter 2p, find the one with largest area.

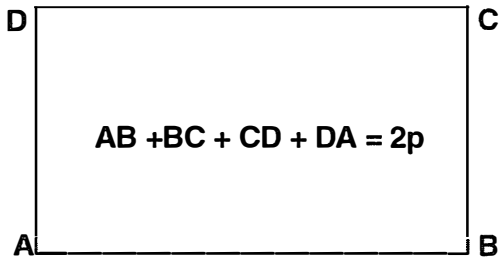


Fig. 1 A rectangle with perimeter 2p. The unit used to measure it is irrelevant to the problem.

Problem 2. Among all isosceles triangles ABC (Fig. 2) with perimeter 2p, find the one with largest area.

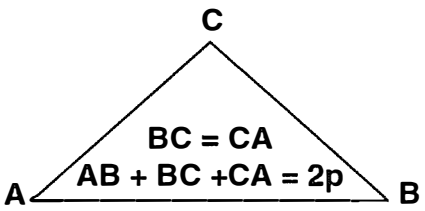


Fig. 2 An isosceles triangle with perimeter 2p. We assume BC = CA.

Problem 3. Among all rectangles ABCD (Fig. 3) such that BC + CD + DA = 2p, find the one with largest area.

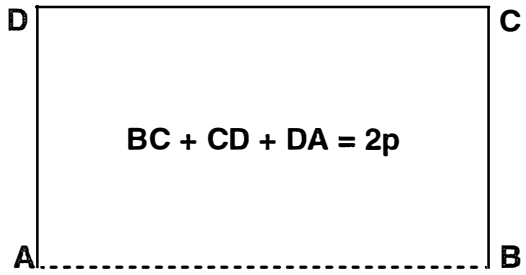


Fig. 3 The side AB is not included in 2p.

Problem 4. Among all isosceles triangles ABC (Fig. 4) such that BC = CA = p (and consequently, BC + CA = 2p), find the one with the largest area.

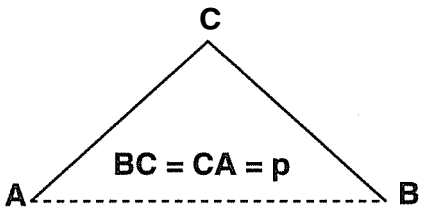


Fig. 4. The side AB is not included in 2p.

The solutions can be found with Calculus. In the first problem, the rectangle with maximal area is the square with AB = BC = CD = DA = p/2. In the second problem the isosceles triangle with maximal area is equilateral, with AB = BC = CA = 2p/3.

At this point, most students in a typical Calculus class are probably expecting that the solutions to problems 3 and 4 are the same as for problems 1 and 2 respectively. They will be a little surprised to find that the solution to problem 3 is the rectangle for which AD + BC = CD = p and that the solution to problem 4 is the isosceles right triangle. One of our Calculus students, Steven Beauchemin, was indeed troubled by these different answers. Discussions with him pointed out that there is a hidden consistency among all four answers. It also led to our discovering that each solution can be easily derived from one simple inequality, based on a geometric argument (for a related result see [2]).

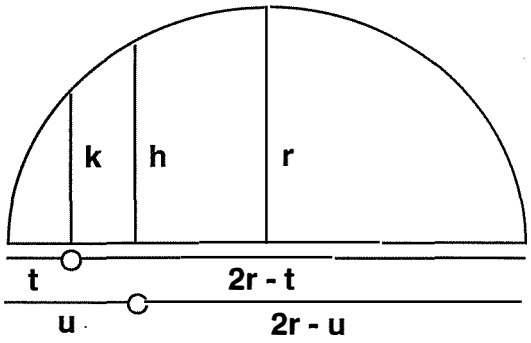


Fig. 5 Recall that u/h = h/(2r - u).

From Figure 5 we derive $u(2r - u) = h^2$, $t(2r - t) = k^2$, whenever $t, r \in [0,2r]$. Therefore, the inequality $|r - u| \le |r - t|$ implies

(1) $t(2r - t) \le u(2r - u) \le r^2$

Remark. Notice that $u(2r - u) = r^2$ if and only if $u = r$.

This last property can be rephrased as follows. Given a positive real number c, the largest product among all pairs of numbers, a, b such that $a+b = c$ is obtained by choosing $a = b = c/2$ and it is equal to $c^2/4$

Below we solve all four problems using (1). The same inequality can be used to solve two additional and more challenging problems:

(a) among all triangles with perimeter 2p, find the one with largest area;

(b) among all quadrilaterals with perimeter 2p, find the one with the largest area.

The solution to problem (a) is the equilateral triangle, and the one to problem (b) is the square. These two solutions provide the hidden consistency among the answers to the four problems mentioned previously. In fact, problems 3 and 4 can be considered as special cases of problem (b) with a reflection argument illustrated by Figure 6.

Problem 3 is solved by finding the rectangle with largest area among the ones with perimeter 4p. Problem 4 is solved by finding the rhombus with largest area among the ones with perimeter 4p. Hence, Problems 1, 3 and 4 are particular cases of the more general question posed in (b). Accordingly, their common answer is the square. We also notice the consistency between the answers to (a) and (b). In both cases the solution is provided by the regular polygon of the respective family.

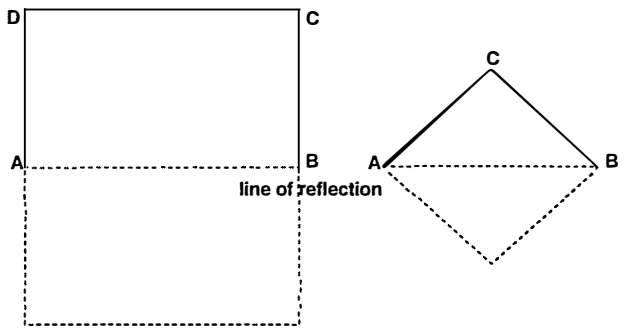


Fig. 6 Double the two figures along the line of reflection.

We now show how inequality (1) can be used to solve all four maximum problems, as well as problems (a) and (b).

In Problem 1 we want to maximize $x(p-x)$, where $x = AB$. Thus, we must choose $x = p/2$.

In Problem 3 we want to maximize $x(2p - 2x) = 2x(p - x)$, where $x = BC$. Thus, we must choose $x = p/2$.

Problem 4 can be solved by maximizing the square of the area of the triangle. This is given by (see Fig. 7) $\text{Area}^2 = p^4\sin^2\gamma(1-\sin^2\gamma)$. Hence we must choose $\sin^2\gamma = 1/2$, i.e., $\gamma = 45°$.

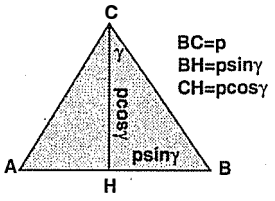


Fig. 7 The area of the triangle ABC is $p^2\sin\gamma\cos\gamma$

In Problem 2, we can use Heron's formula to obtain

(2) $\text{Area}^2 = p(p-x)(p-x)(p-y)$

where $x = BC = CA$, $y = AB$, and p is the semiperimeter $\left(\text{that is } p = \frac{2x + y}{2}\right)$. Letting $t = 2x-1$, $u = \frac{1}{3}$, and $r = \frac{x}{2}$ we have

$$|r - u| = \left|\frac{1}{2}x - \frac{1}{3}\right| \le 3\left|\frac{1}{2}x - \frac{1}{3}\right| = \left|\frac{3}{2}x - 1\right|$$
$$= \left|(2x - 1) - \frac{1}{2}x\right| = |t - r| = |r - t|$$

Without loss of generality we can assume $p = 1$. Then $2x + y = 2$, and we obtain

(3) $\text{Area}^2 = (1-x)(1-x)(2x-1)$ for $x \in [\frac{1}{2}, 1]$

By inequality (1), we obtain

$$(2x - 1)(1 - x) = t(2r - t) \le u(2r - u) = \frac{1}{3}\left(x - \frac{1}{3}\right)$$

It follows that

$$(1-x)(1-x)(2x-1) \le \frac{1}{3}(1-x)\left(\frac{2}{3}-(1-x)\right)$$

By the remark we made after (1), the right hand side of this inequality is largest when

$1-x=\frac{1}{3}$, that is, when $x = 2/3$.

From $2x + y = 2$, we derive $y = 2/3$. Hence the triangle is equilateral.

We now apply the same procedure to establish that among all triangles (not necessarily isosceles) with perimeter 2p, the equilateral is the one with largest area. We assume again, without loss of generality, that the semiperimeter is 1. Denoting the three sides by a,b,c and using Heron's formula we have

$\text{Area}^2 = (1-a)(1-b)(1-c) = (1-a)(1-b)(a+b-1)$ since $a+b+c = 2$.

From (1),

$$(1-b)(a+b-1) = (1-b)(a-(1-b)) \le \frac{a^2}{4}$$

Hence

(4)

$$\text{Area}^2 = (1-a)(1-b)(a+b-1) \le (1-a)\frac{a}{2}\frac{a}{2}$$

Letting $t = 1-a$, $u = \frac{1}{3}$, and $r = \frac{1}{2}-\frac{a}{4}$, we have

$$\begin{aligned} |r-u| &= \left|\frac{a}{4}-\frac{1}{6}\right| \\ &\le 3\left|\frac{a}{4}-\frac{1}{6}\right| \\ &= \left|\frac{3a}{4}-\frac{1}{2}\right| \\ &= \left|(a-1)+\left(\frac{1}{2}-\frac{a}{4}\right)\right| \\ &= |-t+r|, \\ &= |r-t|, \end{aligned}$$

so that, by (1)

$$(1-a)\frac{a}{2} = t(2r-t) \le u(2r-u) = \frac{1}{3}\left(\frac{2}{3}-\frac{a}{2}\right).$$

From (4) we obtain

(5) $\text{Area}^2 = (1-a)(1-b)(a+b-1) \le (1-a)\frac{a}{2}\frac{a}{2} \le \frac{1}{3}\left(\frac{2}{3}-\frac{a}{2}\right)\frac{a}{2}$

The product on the right hand side is largest when

$$\frac{a}{2}=\frac{1}{3}, \text{ i.e., } a=\frac{2}{3}$$

For obvious reasons of symmetry, $a = b = c = 2/3$ and the triangle is equilateral. This result cannot be obtained using only Calculus of one variable.

The area of a quadrilateral with sides, a,b,c,d and perimeter 2p can be found from the equality (see [1])

(6) $\text{Area}^2 = p(p-a)(p-c))(p-d) - \frac{1}{2}abcd\,[1+\cos(\alpha+\gamma)]$

where α and γ are the angles of two opposite vertices (see Fig. 8).

We can assume $p = 1$. Moreover, since we are searching for the maximum value of the area we can also assume $\alpha + \gamma = 180°$ (thus minimizing $\cos(\alpha + \gamma)$). In fact, the quadrilateral can be considered to be made of four rigid bars free to rotate around the points A,B,C and D. Without changing the perimeter, we see that $\alpha + \gamma$ can be made smaller than 180° and larger than 180° with suitable rotations of the sides. By continuity, there will be a position when $\alpha + \gamma = 180°$.

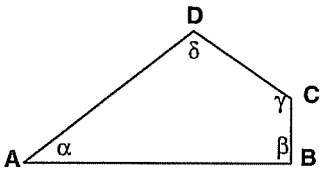


Fig. 8 The angles α and γ can be replaced by β and δ since $(\alpha + \gamma) + (\beta + \delta) = 360°$.

With these conditions equality (6) becomes

$\text{Area}^2 = (1-a)(1-b)(1-c)(1-d)$

$= (1-a)(1-b)(1-c)(a+b+c-1)$.

From (1),

$$(1-c)(a+b+c-1) = (1-c)(a+b-(1-c)) \le \frac{(a+b)^2}{4}$$

Hence,

$$(1-a)(1-b)(1-c)(a+b+c-1) \le (1-a)\frac{a+b}{2}(1-b)\frac{a+b}{2}$$

Now,

$$(1-a)\frac{a+b}{2} = (1-a)\left(1-\frac{a-b}{2}-(1-a)\right) \le \frac{1}{4}\left(1-\frac{a-b}{2}\right)^2$$

Similarly,

$$(1-b)\frac{a+b}{2} \le \frac{1}{4}\left(1-\frac{b-a}{2}\right)^2$$

Therefore,

$$\begin{aligned} &(1-a)\frac{a+b}{2}(1-b)\frac{a+b}{2} \\ &\le \frac{1}{16}\left(1-\frac{a-b}{2}\right)^2\left(1+\frac{a-b}{2}\right)^2 \\ &= \frac{1}{16}\left(1-\frac{(a-b)^2}{4}\right)^2 \end{aligned}$$

Since $|a-b| \le 2$, the quantity on the right reaches its maximum value when $a = b$. For reasons of symmetry we conclude that $a = b = c = d = 1/2$, that is, the quadrilateral is a rhombus. From the equality $\alpha + \gamma = \beta + \delta = 180°$ it follows that the rhombus is a square.

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▲ WRITING MATHEMATICS

MARC GARNEAU C.I.



Ed Bernard is a Vice Principal at Marc Garneau Collegiate Institute in East York. He will present a workshop on "Writing Mathematics" at the OAME Annual in Kingston on Friday, May 13, from 10:15 to 12:30.

All young children, except the most severely handicapped, possess a language, and can repeat stories sequentially and in detail after hearing them a surprisingly few times. People retain these stories for their whole lives. Can techniques from language experiences assist in the learning and assessment of mathematics?

In order to attempt an answer this question, I visited several mathematics classes and told the students that I was going to give them an unusual mathematics assignment. The instructions I gave the students were as follows.

> " First, I'll give you a non-mathematical example of the assignment and let you try it. We'll discuss and take up aspects of this assignment. Second, I'll give you the mathematical assignment. Third, I'll return in a few days, take up the assignment, explain to you what I'm trying to accomplish, and solicit your views.
>
> "I'm going to put two non-mathematical expressions on the board. When you see the two expressions, I want you to tell me the content *(the story)* that you associate with these two expressions.
>
> "Here are the two expressions: glass slipper and pumpkin.
>
> "Now, tell me the content (the story) that you associate with these two expressions." *(Teachers may have to use other examples depending upon the cultural backgrounds of their students.)*

Most students said these expressions referred to the story of Cinderella. When asked to begin telling the story, most students began "Once upon a time", and were able to tell the story. I pointed out that although they had likely not heard this story for a long time, they had an amazing amount of correct detail in their stories and the content of their stories was well sequenced.

> "Now for the mathematics assignment.
>
> "This time I will put a title on the board, and you write the story." *(The classes I visited had recently completed their study of fractions, and I had the students' names and scores on their fraction tests.)*

The differences between adding and multiplying fractions.

> "Remember the Writing Process you use in your English classes. List the appropriate content, that is, what you know about the title, sequence the content, imagine a story into which to put your content, and write out a full account of the story. Give a lot of detail. Use only words."

When I read the stories, many students employed introductory and concluding sentences. For example, "There are a few differences between the addition and multiplication of fractions. ... These are the differences between the addition and multiplication of fractions".

In a few cases, students incorporated their content into a traditional type of story. For instance, "Once upon a time there was a boy named Bob who didn't know how to do fractions. His friend helped him after school one day. ... Bob finally understood how to do fractions, and passed his fraction test." In another version, the conclusion stated, "and the boy not only passed his next fraction test, years later he became a math teacher." And in another case, "Adding and Multiplying were talking to each other one day. Adding asked Multiplying to tell how he is different than Adding regarding fractions. ... Well, although we both deal with numbers, we are different in a few ways."

I had prepared a marking scheme prior to administering these assignments. I awarded marks for the title, an introductory sentence with a bonus for a creative introduction, a conclusion, and for spelling, grammar, and writing style. In marking the content of the addition of fractions, I was looking for some discussion of finding a common denominator, of adding the numerators, and placing the sum of the numerators over the common denominator. In marking the multiplication of fractions, I was looking for a description of multiplying the numerators together, and placing this product over the product of the denominators. I also awarded marks for a reference to reducing answers to their lowest terms, of dividing in the multiplication of fractions to reduce the size of the numbers, a reference to the order of operations, a comment on signed numbers, and a discussion of dealing with mixed numbers.

As many as forty percent of the students' marks increased on this assignment over their marks on traditional

fraction tests, and many substantially. Some of the students received marks similar to their marks on the traditional fraction tests. As many as forty percent of the students' marks decreased. Most of the students whose marks decreased were New Canadians; they were unable to describe or write about mathematical operations albeit some of them scored the highest on the traditional types of tests. This may imply a lack of demand for language usage in many mathematics classrooms.

I believe that 'writing mathematics' offers opportunities for many students to exhibit knowledge not demanded by traditional mathematics tests. It is important in all subject areas to employ a variety of types of questions, and a variety of types of evaluation instruments in order to meet the various talents of each of our students, and to allow students to be successful in some area of each course. For instance, in visual arts courses students are not evaluated in only one medium. Various media are explored, for instance, sketching, watercolours, oil, charcoal, clay, etc., as well as art history, and the greater the variety, the more likely we will match an aspect of the course with a talent of a student. Consequently, in mathematics, we should employ the traditional types of testing as well as assignments of the type I am suggesting in this paper in order to meet the variety of talents that students possess.

When people get up in the morning and get dressed for work, they are not going out to do mathematics or geography. People are going out to do an integrated task. It is important that they can communicate what they are doing to fellow workers, clients, supervisors, and so on. The business world and post-secondary schools want graduates who can communicate clearly and creatively. They appear to want young people who can describe as well as apply what they know. In the world of work a mathematics major does not just do mathematics. A top mathematics student, then, should be able to describe theory and applications as well as solve problems and perform calculations. For example, he or she should be able to divide fractions ***and*** describe the algorithm.

Schools need a variety of delivery approaches and assessment instruments in mathematics in order to meet the talents and learning styles of different students.

In universities most courses in mathematics are not offered to mathematics majors, but to people in other disciplines who require a variety of mathematical skills in their fields of study, for example, engineering, computer studies, science, business, the social sciences, and so on. Consequently, being able to communicate about mathematics is imperative for all people involved in this context, especially mathematics professors.

If writing mathematics is an important aspect in understanding and reporting mathematical concepts, then I hope this paper may point to a way of commencing this activity in our classrooms.

When I returned to the classes in order to take up the assignments, to explain what I was attempting to do, and to solicit the students' views, I discussed how very young children, except the most severely handicapped, have learned a language and stories. In fact, it is amazing how a young child can repeat a story such as, Cinderella, after hearing it a surprisingly few times. I pointed out that the students have heard the story of fractions many more times than they have heard the story of Cinderella, and they have heard about fractions more recently than Cinderella. They should be able to describe operations involving fractions. Some students said, but there was no pressure or expectation to learn Cinderella at our mothers' knees, and at the time, Cinderella was interesting to us. The study of fractions (or mathematics in general) is not interesting to many of us. These students have pointed out the very basis for commencing this type of exercise (writing mathematics) at a much earlier stage in the educational process. I explained to them that I was seeking additional methods for assessing students in mathematics. Some students thought that seeking additional assessment methods in mathematics was a good idea. Others, of course, did not like this idea. Some students recalled that they had been asked in the past to write about how they felt after writing a math test, and some said that they had been asked to write poetry concerning their experiences in mathematics.

In order to implement 'Writing Mathematics', I believe we need some applications in learning and assessing mathematics which arise from learning language and stories.

- In learning language and stories, we give countless examples of how words are used. In mathematics, we need to give countless examples of how numbers are used. Further, we need to give examples of applications , for instance, from science, technology, computer studies, sports, geography, business, and so on.
- I suggest that topics in mathematics need to be presented in an overview rather than in isolated pieces. Language and stories are learned holistically. We did not learn Cinderella by hearing bits of the story now and then, and out of sequence. By learning (or seeing) the whole story, association and sequence are present. Association and sequence are two aspects of what is meant by the material having 'meaning', of not being like nonsense syllables. There is a lot of research on the amount of time and effort required to learn nonsense syllables as opposed to learning meaningful

material. Nonsense syllables are learned more slowly (research suggests that four times the effort is required), and are quickly forgotten. Does mathematics come across as nonsense syllables to some children?

In mathematics, we can go back later and deal with difficult points in isolation, rather than teaching the whole topic in a series of isolated points. I suggest that we need to present the 'whole fraction story': what are fractions, examples of what fractions look like, how fractions are used, pictures of pieces of pie, the parts of a fraction named and labelled, the four operations as they apply to fractions noting the similarities and differences among the operations, examples of the four operations, and so on. As well as association and sequence being present in this methodology, students would have the whole story of fractions in their notes in one place for study purposes.

- Part of the testing in mathematics, then, should include the natural questioning which is used in evaluating language and stories as well as the traditional means of testing.
- Assignments and assessment in mathematics need to involve great variety, and not just textbook-like questions. For instance, in introductory geometry, assignments and assessment need to include problem-solving, research, writing, conceptual knowledge, aesthetics, and so on. These attributes can be met through learning the shapes, learning the parts of each shape, labelling, learning perimeter, area, and volume, applying the formulae to perimeter, area, and volume, creating mobiles of shapes, researching these shapes as they appear in nature, writing the story of, say, 'Area', the use of computers in generating these shapes, estimation of widths of rivers and heights of trees, and so on.
- The common elements of 'problem-solving' need to be discussed noting that there are often more ways than one to solve problems. The learning style of students may mean that each student prefers one way over another.
- One very important method of learning in the humanities involves a student asking questions prior to commencing a topic, and then having his own questions answered during his readings. We remember the answers to our own questions better than we remember the answers to someone else's (the teacher's) questions. For instance, if we were about to study the American Civil War, we might ask what are the dates of the war, who were the leaders of the two sides, what were the two sides, how did the war start, what were the causes, where was the first battle, what were the major battles, what was the last battle, who were the generals, who won, reasons for the outcome, what was going on in the world at the time of this war, does anyone really win a war, what are the effects of the war immediately after, and today, are there any feelings about the war still present in America, and so on.

As a person reads about a topic, new questions will arise. As a person has more experience asking his own questions, he will get better at asking questions prior to commencing a topic, and thus, will get better at remembering significant details about a topic.

In the case of introducing fractions one might ask: what are fractions, what does the word fraction mean, can you add, subtract, multiply, and divide fractions, what are the rules for these operations with fractions, what are some applications of fractions, is there a relationship between fractions and decimals, and fractions and percent, and so on.

Since writing this article in April, 1993 I have conducted a conference on "Writing Mathematics" for OCLEA at OISE in November, 1993 and will again on April 12, 1994. I will also present a workshop at OAME's annual conference on May 12, 1994 in Kingston, Ontario. As you might imagine, from the first presentation and subsequent planning, I have greatly expanded my ideas on the topic. The presentation now includes:

- Reading, Writing, Listening, and Talking Mathematics.
- Application exercises in the Destreamed Classroom.
- Integration of mathematics in other subject areas.
- Assessment in mathematics.

If there is a germ of an idea in this paper, I suspect each reader will be able to add many other applications. I would be very glad to hear from you concerning new applications that you can make to this idea.

Further readings in alternative approaches in mathematics:

Marilyn Burns, *"Helping Your Students Make Sense Out of Math,"* Learning 88, 16 (1988): 31-6.

Grace Burton, *"Writing as a Way of Knowing in a Mathematics Education Class,"* Arithmetic Teacher, 33 (1985): 40-5.

David M. Davison and Daniel L. Pearce, *"Using Writing Activities to Reinforce Mathematics Instruction,"* Arithmetic Teacher, 35 (1988): 42-5.

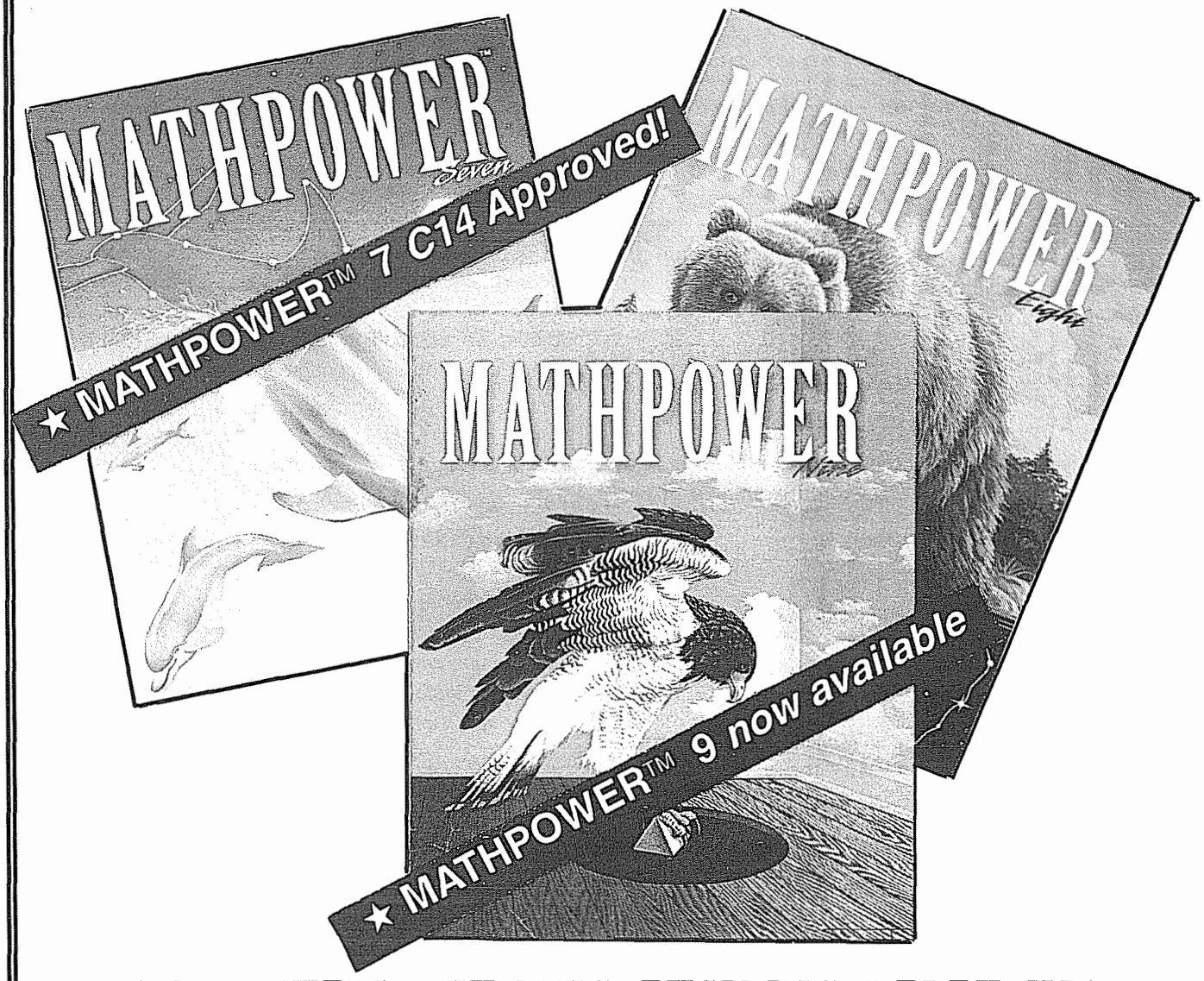
Christine S. Evans, *"Writing to Learn in Math,"* Language Arts, 61 (1984): 828-35.

Bill Kennedy, *"Writing Letters to Learn Math,"* Learning, 13 (1985): 59-61.

(continued on page 32)

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▲ FACTORING THE TRINOMIAL...AN ALTERNATIVE APPROACH

CAROL HARE
THE WOODLANDS SCHOOL
PEEL BOARD OF EDUCATION



Carol (Gugins) Hare was born in Toronto and raised in Calgary. She earned a B.Sc. and B.Ed. at the University in Calgary. She has taught for the Calgary, Regina, Carelton and Peel Boards of Education. Carol is currently enjoying teaching math and computer science at The Woodlands in Mississauga.

One of the best things about teaching is that you get paid for learning. One of my favourite examples of this occurred when Nima Arkai-Hamed, a senior student at Lorne Park Secondary School in 1989, favoured me with the following method for factoring trinomials. The best thing about this approach is that it is strictly mechanical and does not rely on "finding the factors that...." My students call it the "Nima Approach".

When factoring a trinomial of the form $ax^2 + bx + c$ one of the methods we teach is decomposition. The method is developed by looking at the way the product of the binomials is obtained.

Consider the following example:

$(3x + 4)(2x - 5)$
$= 6x^2 - 15x + 8x - 20$
$= 6x^2 - 7x - 20$

The integer –7 is the sum of the coefficients –15 and +8 which in turn are the factors whose product equals 6(–20), or in general form, 'ac'. With such an observation we generate the method known as decomposition.

Faced with the problem "Factor completely $6x^2 - 7x - 20$" a student must 'find' two numbers that sum to –7 and have a product of –120. After finding these numbers to be –15 and 8 the student continues:

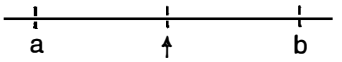
$6x^2 - 7x - 20$
$= 6x^2 - 15x + 8x - 20$
$= 3x(2x - 5) + 4(2x - 5)$
$= (2x - 5)(3x + 4)$

The most difficult step in the decomposition method is "finding the factors of_____whose sum is_____" *That is the problem!*

Here is Nima's method for finding such factors.

Consider the trinomial $x^2 + 42x + 437$. We are looking for the factors of 437 that sum to 42.

If we assign the values a and b to the two factors and place them on a number line it is clear that the median of a and b is (a + b)/2.



$$\text{median} = \frac{a+b}{2}$$

We know the value of a + b is 42; therefore the value of (a + b)/2 will be the median 21. It follows that one of the numbers we are looking for is less than 21 and the other number is the same amount greater than 21.

We could then represent these specific factors as (21 + k) and (21 – k), with k > 0, we know the product of these factors is 437.

$(21 - k)(21+k) = 437$
$441 - k^2 = 437$
$-k^2 = -4$
$k^2 = 4$
$k = 2$

This value can now be added and subtracted from 21 to provide the factors 23 and 19. From this point we would continue with the decomposition method.

$x^2 + 42x + 437$
$= x^2 + 23x + 19x + 437$
$= x(x + 23) + 19(x + 23)$
$= (x + 23)(x + 19)$

Using a similar approach we can factor $4x^2 + 60x + 221$. The median of the factors of the product of 4(221) is 30. It follows therefore that

$(30 + k)(30 - k) = 4(221)$
$900 - k^2 = 884$
$-k^2 = -16$
$k = 4$

The desired factors are 30 + 4 = 34 and 30 - 4 = 26. Continuing using decomposition the final result would be

$6x^2 + 60x + 221 = (2x + 17) + (2x + 13)$

Similarly $6x^2 + 5x - 221$ can be factored as follows. The median of the factors of 6(–221) is 2.5; therefore we can

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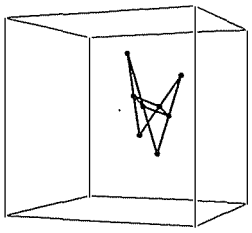
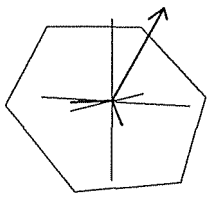
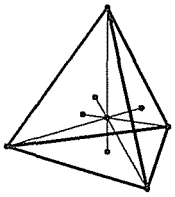
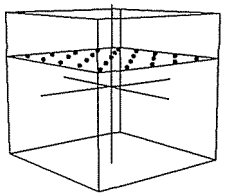


IMAGE-*Algebra & Geometry* 2.1

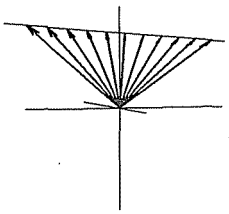


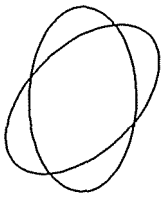
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For a first hand look at **IMAGE-*Algebra and Geometry***, **IMAGE-*Calculus*** and **IMAGE-*Calculator*** plan to attend a two-hour demonstration at OAME '94 in Kingston, May 12-14. Alternatively, to obtain further information contact:
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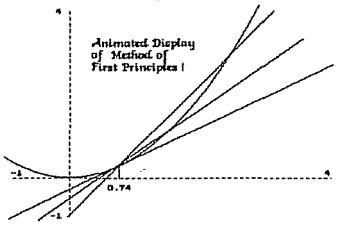
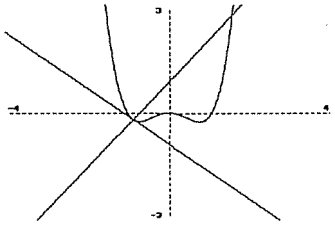
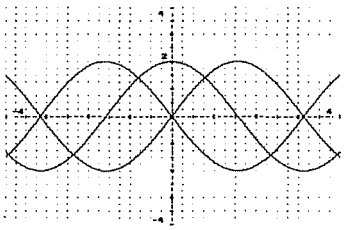


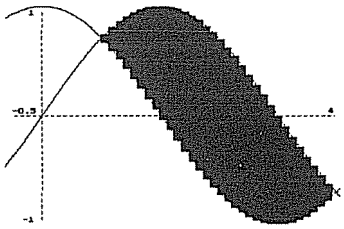
IMAGE-*Calculus* 2.1

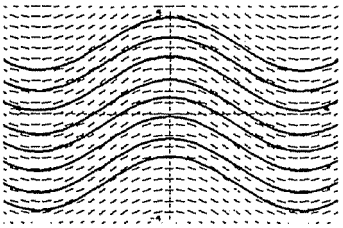


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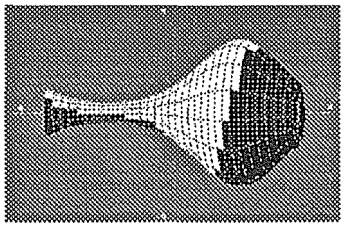


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"...the many advanced features like areas between curves and polar graphing let me <u>see</u> and appreciate my work for the first time."
- OAC student (Oxford County)



▲ COUNCIL COMMENTS

BOB ROBINSON
GOLDEN SECTION CHAPTER REPRESENTATIVE



These notes were taken at the OAME Council Meeting held January 22, 1994 at the Ramada Inn-Airport West in Mississauga. Please send any correspondence directly to the editor.

HONOURARY MEMBERSHIP

Don Attridge was awarded an honourary membership in the OMCA by Peter Saarimaki.

AWARDS COMMITTEE

Nomination announcements for awards appeared in the December issue of the Gazette and some nominations were received. Nominations were closed on January 31. Announcements of award recipients will be made at the annual meeting or banquet in Kingston.

MATH EQUITY

Another outstanding breakfast is being planned for our Annual Meeting in Kingston. The committee is preparing a handout for the occasion. A questionnaire was distributed to Council which addressed mathematics contests and equity issues. The committee is discussing multicultural issues as well as the issue of destreaming. Elaine Harvey shared with us the video entitled "Story of Android" which is designed to encourage students, especially girls, to become interested in science and mathematics. It is aimed at Grades 4-6. For more information on the work of the equity committee, please contact Elaine Harvey.

NOMINATIONS COMMITTEE

Nominations deadline for positions of Councillor, Director, and President was January 31, 1994. By the time you read this you should have received your ballot. We trust that you returned it. Successful candidates will be announced at the annual meeting.

INTERNATIONAL MATHEMATICAL OLYMPIAD (IMO)

The annual IMO is a mathematical competition which currently involves approximately seventy-five countries. Each country selects a team of up to six secondary school students to send to the competition. The Canadian Mathematical Society (CMS) selects Canada's team.

The competitors set a nine hour contest in two four and one-half hour sessions held on successive mornings. The contest consists of six questions, three of which are presented each day.

The participating countries are responsible for getting their teams to the host country and back home again. While attending the competition, the host country provides the needs of all visiting teams.

In 1995, Canada will host the Olympiad. On July 13, 1995, the jury will meet in Waterloo to set the paper. The students will arrive on July 16 and stay at York University, where they will write the competition on July 19 and 20. On the 20th, the jury will move to York for the marking, which will be completed on July 22. The awards ceremony will take place on July 24, and the teams will depart Canada on July 25.

If you would like more information about the competition or would like to become involved in the organization, contact:

- Dr. Graham Wright
  Secretary of IMO 1995 Department of Mathematics
  University of Ottawa

- Prof. Richard Brown
  Chair of York Site Committee
  Department of Mathematics and Statistics
  York University

- Mr. Tom Griffiths
  Member of York Site Committee (OAME Liaison)
  A.B. Lucas Secondary School

MATHEMATICS EDUCATION FORUM

Steve Halperin, chair of the "Forum", addressed the Council and provided background on this recently established group. Workgroups have been created to develop materials for Geometry, Data Analysis and OAC Calculus.

PUBLICATIONS COMMITTEE

The committee is discussing the merits of compiling a

collection of activities from past issues of the Abacus. The committee will coordinate the creation, printing and distribution of new publications. The committee is also seeking a Book Review Editor for the Gazette. The committee developed a readership survey which is included with this issue. They hope to ascertain the value of our two publications as perceived by the membership. We welcome your input.

OAME'94

Everything is well-in-hand for our annual in Kingston. It's going to be a great conference on May 12, 13, 14, 1994 at Queen's. There will be no sessions over the noon hour, so you can visit the exhibits and meet with friends. See you there!

LEADERSHIP'94

Leadership'93 was a sellout and a huge success. Todd Romiens is coordinating another exciting Leadership. Watch for announcements and plan to attend this event, set for November 9 and 10, 1994.

CONSTITUTION AND BY-LAWS

These are being up-dated by Elaine Harvey, Lorna Morrow and Todd Romiens.

MEMBERSHIP/PUBLICITY

A new-membership package is being designed. It will be sent to Faculties of Education for distribution to students. In this regard, a letter will be sent to instructors asking if they would like to have an OAME Councillor come to talk about the merits of joining the OAME.

EARLY YEARS/FORMATIVE YEARS

The committee has developed a unit of activities for Grades 2-6 on the topic of Data Management, Chance, and Probability. This appears in this issue of the Abacus. Their next project will pertain to volume.

TRANSITION YEARS

The Transition Years: Parent Brochure will be in draft form for discussion by the committee at its March Meeting.

SPECIALIZATION YEARS

The committee is continuing to work on the last portion of the Focus on Renewal Document. A joint meeting with OMCA is being planned to discuss this venture.

CHAPTER REPORTS

Note: Because many of the spring conferences will have taken place by the time you receive this issue of the Gazette, I have not included them in my "comments".

CHAMP

During early November, CHAMP conducted their seventh annual CHAMP Mathematics Contests for students in Grades 8-11. Once again, these contests proved to be great vehicles for encouraging students to become involved in mathematics activities. Over 5600 students from 171 schools across four boards participated.

Beginning in 1994, CHAMP will sponsor one member of its executive to attend the OAME Annual conference. In addition, they decided that the chapter will pay the registration fees for four CHAMP members (one from each of their four boards) to the annual conference. The lucky members will be chosen by random draw from those who apply. The draw will take place at their annual Springfest.

GOLDEN SECTION

GOLDEN discussed the OAC Calculus document produced by the Mathematics Education Forum workgroup. Plans have been made for a meeting of Intermediate/Senior teachers in April.

Their mailing to those members of OAME from the Niagara-Hamilton region who had not designated a chapter affiliation was 100% successful. Signed forms were turned to the chapter representative who gave them to the Executive-Director for chapter rebate.

Their mailing to those few former members of OAME from the Niagara-Hamilton region who had not renewed membership was moderately successful as well.

GVMA

Sample Grade 9 exams are going to be published in time for the Kingston Conference. No exam will be identified as to its origin. If you have a sample you wish to share, send it directly to Jack Weiner. See the Gazette masthead for Jack's mailing address.

ISOMA

The chapter has passed its constitution which is essentially the same in structure as that of the OAME.

Over 60 members attended their fall mini-conference at Upper Canada College last October. Next year's fall mini-conference will be held at the Country Day School.

The chapter is going to investigate the rather low

elementary school level percentage of their membership.

The annual general meeting for ISOMA will be held as usual at the OAME May Conference in Kingston on Friday afternoon.

NOMA

NOMA's new president is Dan Charbonneau. Jerry Maloney is the secretary. About 150 people attended their fall mini-conference last October. Next fall's conference will be on September 30 and October 1.

NWOAME

Joe Hall has volunteered to revive the Chapter Newsletter. Marilyn Hurrell is running for Director of OAME and Bill Otto is running for Councillor of OAME. Joe Hall will also be running as Chapter Chair in September when Bill Otto's term expires. Information will be forthcoming regarding Lakehead University NWOAME Math Awards Day.

PRMA

The PRMA and the Durham Board of Education will partner to co-sponsor the annual Durham Mathematics Competition which will be held in early May. The competition uses a "Reach for the Top" format with oral, overhead and relay questions. Most Durham secondary schools enter a team composed of Grade 9, 10 and 11 students. The PRMA is attempting to initiate a similar competition for Grade 6, 7 and 8 students.

Three boards have produced resource documents which are based upon the Common Curriculum and contain strategies for teaching mathematics to Grade 7, 8 and 9 students. Contact the PRMA representative Rick White for details.

QSLMA

Needless to say QSLMA has been very busy coordinating the "Annual". We are all looking forward to it.

SWOAME

Watch for details for the May mini-conference.

WOMA

WOMA members are pleased to see the revamped version of the WOMA newsletter, THE SINE POST. Under the leadership of Ed Duffield, they have put together a professional-looking newsletter filled with items that no math teacher can do without.

Plans are well under way for OAME'96. Ron Reesor and Kaye Appleby recently took the committee for a tour of the new convention centre in London. It is a terrific new facility with state-of-the-art technologies and many modern innovations.

Y^{4}MA

Almost 200 participants attended their annual fall conference. The Y^{4}MA newsletter "THE EXPONENT" was distributed to members in late October. The feature article was "Writing in Mathematics". The next issue is due in April, 1994.

Editor's Note: Two Chapter Reports, SAME and COMA, highlighted Math Teaching Award Winners. Watch for John Roger's article featuring these teachers and others form across Ontario in the June Gazette.

(continued from page 26)

Writing Mathematics

Coreen L. Mett, *"Writing as a Learning Device in Calculus,"* Mathematics Teacher, 80 (1987): 534-37.

M. K. Paik and E. M. Norris, *"Writing in Mathematics Education,"* International Journal of Mathematical Education in Science and Technology, 15 (1984): 245-52

Diane Vukovich, *"Ideas in Practice: Integrating Math and Writing Through the Math Journal,"* Journal of Developmental Education 9, (1985): 19-20.

(continued from page 28)

Factoring the Trinomial ...An Alternative Approach

represent the factors as (2.5 + k) and (2.5 – k).

$(2.5 + k)(2.5 - k) = 6(-221)$
$6.25 - k^2 = -1326$
$k^2 = 1332.25$
$k = 36.5$

The factors are 2.5 + 36.5 = 39 and 2.5 - 36.5 = –34. Decomposition from this point produces the result

$6x^2 + 5x + -221 = (3x - 17)(2x + 13)$

I have been told that this method was illustrated in a journal about 10 years ago but to date I have not found the reference.

▲ About Problem Solving: Creating Problems

JOHN GRANT MCLOUGHLIN, DEPARTMENT OF MATHEMATICS, MEMORIAL UNIVERSITY, CORNERBROOK, NEWFOUNDLAND.

Today a letter arrived from a friend. She has recently reentered the mathematical arena after taking a sabbatical over the past few years. One of the things that strikes her is the cleverness involved in some mathematical problem creations. She writes:

"Creating math problems is a whole area of mathematics which I've never explored. Have you thought about how younger students work at creating problems? Have you done any? It seems terribly difficult but somebody somewhere must be doing it."

Indeed some people have devoted considerable time and energy towards the creation of problems. However, I suspect that relatively few people have actively engaged themselves in creating novel problems that go beyond changing the numbers from familiar exercises in textbooks. Problem creation is an enjoyable and challenging pursuit for others. The efforts of the Canadian Mathematics Competition crew and those affiliated with them exemplify the spirit. People ought to be encouraged to become involved in such efforts. The timely arrival of my friend's letter has challenged me to reflect upon my experiences in this area.

What makes a good problem? This is a subjective question that one needs to address personally. It is not the purpose of this article to explore the question. Instead I would like to share some ideas and experiences that may spark your interest.

Mathematical misconceptions may serve as a stimulus for the creation of intriguing problems. For example, $\frac{16}{64}$ equals $\frac{1}{4}$

Cancelling the 6's produces a correct result. What other fractions share this property? Investigating more open-ended questions is likely to develop an increased level of conceptual understanding. Queen's University Mathematics Professor, Peter Taylor [1] suggests that we take a few problems such as these to form the basis of a problems-based curriculum in high school algebra. The curious mathematical result

$2\sqrt{\frac{2}{3}} = \sqrt{2\frac{2}{3}}$ is posed as an example in his article.

Another practical suggestion is to start by taking two seemingly unrelated topics, and then try to create a problem that links these topics. Let me share three such examples. At a professional development day, Ron Dunkley, from the University of Waterloo, presented the following problem:

> The sides of a triangle have lengths 3, 7, and 8 cm respectively. Show that the measures of the interior angles form an arithmetic sequence.

Another interesting problem connects geometry and number properties in a succinct but untraditional manner. This problem appears in an article by Tony Gardiner [2]:

How many triangles have all three angles perfect squares (in degrees)?

A third example was shared by Bob Alexander at a workshop he offered to his math education students at the University of Toronto.

> Select three numbers (a, b, c) that form an arithmetic sequence. Then select another three numbers (d, e, f) that form an arithmetic sequence. Find the intersection point of the lines $ax + by = c$ and $dx + ey = f$

One need not look far for sources of problems. Brown and Walter [3] provide numerous examples of problem posing activities that are designed to generate mathematical learning from familiar content knowledge or physical structures. Suppose that we consider the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, ... One could use it as a starting point to generate a list of so-called attributes. These attributes can then become the source of mathematical investigations.

Although I cannot do justice to the work of Brown and Walter, I will suggest a few directions that one could move. Perhaps it is observed that the square of any term in the sequence is one less than the product of its adjacent terms. Can we prove this mathematically? It is evident that the first two terms are 1's and subsequent terms are found by adding consecutive terms. What would happen if we changed the first two terms and/or the operation involved in creating the sequence? The authors use the term "What-if-not" to encourage readers to challenge assumptions, thus, to investigate uncharted territories.

Students will become more involved in the learning process if they play an active role in the direction it takes. "The Art of Problem Posing" is a book that deserves to be

read by mathematics educators. It contains a diverse collection of activities and challenges to engage the reader. The interplay between problem posing and problem solving is also discussed. Solving problems requires us to rethink plans of action and pose problems. Of course, a problem must also be posed prior to the solving process being initiated.

The way that mathematics is presented affects the learning experiences of our students. The following type of exercise may be found in numerous textbooks;

> Given that the perimeter of a rectangle is 44 cm and its area is 112 cm^2, find the length and width of the rectangle.

Despite the routine appearance of such an exercise, it may also play a role in the development of problem solving abilities. Instead of focusing on the answer, suppose that we shift our focus to the process. The exercise can be rephrased to invite students to solve the exercise in as many ways as possible. Kimberley Sherman and myself [4] outline six ways of proceeding to solve this particular exercise. Although that article was primarily concerned with implications for evaluation, it points to another issue. It is probable that most mundane exercises may be reframed to provide greater educational value.

Most of us have little or no formal training in creating mathematical problems. In order to develop such abilities, I feel it is important to forge ties with others who are interested in creating problems. You may volunteer to assist with the preparation of a local math contest. Perhaps colleagues or students would join with you to form a problem solving group. The group may attempt to solve problems from publications such as School Science and Mathematics, Journal of Recreational Mathematics, or mathematics contests. Writing for mathematics' leagues and contests has been the single most valuable source of my personal development in this area. It is the sharing of ideas, perspectives, mathematical strengths and weaknesses that is so valuable.

It would be wonderful to hear from readers and problem solvers. Perhaps you would like to share a problem solving experience from your classroom. Maybe you have a problem that you would like to pose to other readers. Has one of your students created a challenging problem? The problems in the last issue remain open for comments. I hope to hear from some of you before saying anything about them in the next issue.

I would like to close by sharing a timely problem that can be adapted to almost any grade level. To celebrate a new year, 1994, the challenge is to use each of the digits once to make expressions equivalent to 1, 2, 3 and so on. There are no restrictions to be placed on the use of mathematical symbols, parentheses, or operations. For example, factorials and square roots may be used freely. Did you observe that all of the digits are perfect squares and so is 49? Some of you may wish to turn this challenge over to your students. Note that all four digits must be used in each expression. For example, $22 = 9 + 9 + 4$ is not allowed. However, $22 = 9 + 9 + 4 \times 1$ is fine. Digits may be combined to make expressions such as $42 = 91 - 49$

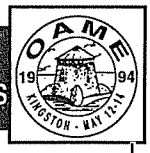
Have fun. It's a wonderful year in terms of mathematical potential. The year 2000 won't be nearly as exciting!

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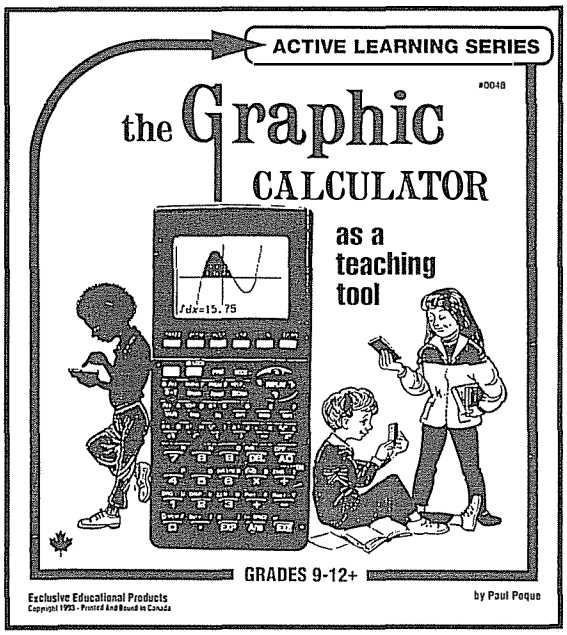
Don Fraser

(Saturday Morning at 11:30)

Don Fraser has been Professor of Education in the Faculty of Education at the University of Toronto since 1969. A widely-admired teacher, he was Head of the Department of Mathematics at Emery Collegiate Institute in North York before coming to FEUT. In recent years he has taught at Winfield Junior High School full-time for the 1986-87 academic year, University of Toronto School (a Grade 7 class from 1987-1990), and at Lawrence Heights Middle School (Math Fun Breakfast Club, 1993). His publications include the popular Newspaper Math and Sports Math, published by Dale Seymour, the Toronto Star's Go Jays! and Go Leafs! and Mathemagic published by Dale Seymour and Addison-Wesley. A much sought after speaker at professional meetings of teachers of mathematics, he was a Keynote Speaker at the 1993 Regional Meeting of the National Council of Teachers of Mathematics in New York City, and Banquet Speaker at the NCTM's 1993 Annual Meeting in Seattle. In 1992 Professor Fraser was selected as "Teacher of the Year" by students at the Faculty of Education of the University of Toronto.

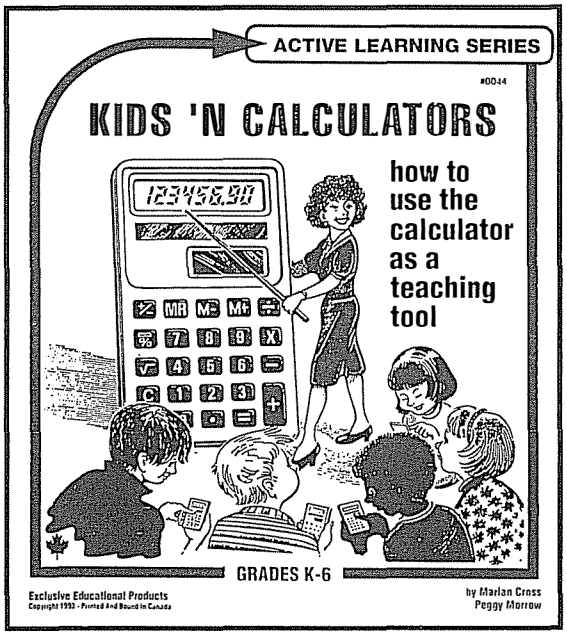
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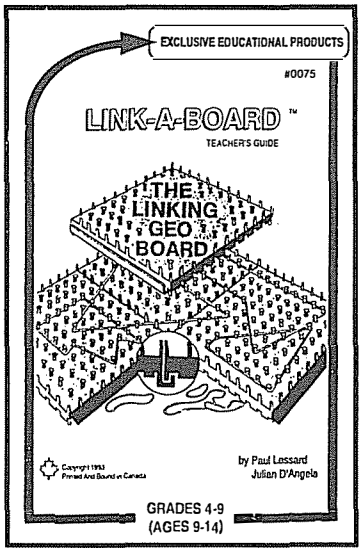
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WATCH FOR OUR 1994 CATALOGUE

▲ MATHEMATICS AND THE WORLD AROUND US



RON LANCASTER
ST. MILDRED'S-LIGHTBOURN SCHOOL

Readers are invited to submit items from newspapers, magazines, books or other sources that contain information that can be related to any area of mathematics. Sufficient information about the source of the item should be given so that other readers can obtain additional details. Teachers are encouraged to use the contents of this regular feature of The Gazette as an aid to helping their students see the connection between mathematics and the world around us. The content may not fit in with what you are currently teaching to your students, but many of the items can be used to begin your classes in an interesting and engaging manner.

1. **Hey kids, pass up the chips - and become a millionaire,** The Toronto Star, August 29, 1993

"Next time you're about to plunk a loonie on the counter for a pop or bag of chips, consider this: An 18-year-old who sets aside $1 a day for 47 years, at 12.5 per cent compound interest, will have a million dollar nest-egg by age 65."

(a) Verify the claim made in the above excerpt.

(b) How would the size of the nest-egg change if the rate of interest were changed to half of what it was given to be in the article? Would the size of the nest-egg be half of a million dollars?

(c) How would the size of the nest-egg change if the 18-year-old were to set $1 aside for only half of the time given in the article? Would the size of the nest-egg be half of a million dollars?

(d) How would the size of the nest-egg change if the interest rate were double what it was given to be in the article and if the student were to put aside $1 for only half of the number of years given in the article. Would the size of the nest-egg stay at one million dollars?

2. **"Chrysler MagicWagon Sweepstakes",** Shoppers Drug Mart, September 1993

The skill-testing question given below appeared on the entry form for this contest.

Multiply 482 x 351
Add 818
Divide by 500
Subtract 157

(a) Find the answer to the above skill-testing question.

(b) In part (a) you should have found that the answer came out to be a whole number. Keep all of the numbers in the skill-testing question the same except for the number 482 in the first line. Replace the 482 with other numbers and see if you can find some other cases for which the final answer continues to be a whole number.

(c) Make up a question of your own similar to the one given in part (b).

3. **"Exports to Asia",** The New York Times, November 11, 1993

In this article the following figures were given for the total amount of yearly exports from the United States to Australia, Brunei, China, Hong Kong, South Korea, Malaysia, New Zealand, the Philippines, Singapore, Taiwan and Thailand.

year	yearly exports (in billions)
1988	92
1989	107
1990	115
1991	122
1992	129

The differential calculus can be used to find the equation of the line that best fits this data. Let the equation of the line be $y = mx + b$, where y represents the total number of exports (in billions of dollars) x years after 1988.

Note that the data given in the above chart consists of the following 5 points:

(0,92) (1,107) (2,115) (3,122) (4,129)

Let D be the sum of the squares of the vertical distances from these five points to the line $y = mx + b$. Note that

$$D = (b-92)^2 + (m+b-107)^2 + (2m+b-115)^2 + (3m+b-122)^2 + (4m+b-129)^2$$

The line of best fit is the one for which D is as small as possible. The values of m and b can be determined by

finding $\frac{dD}{dm}$ and $\frac{dD}{db}$ and setting both of these derivatives equal to 0 and solving the resulting system for m and b.

(a) Find m and b by following the above procedure.

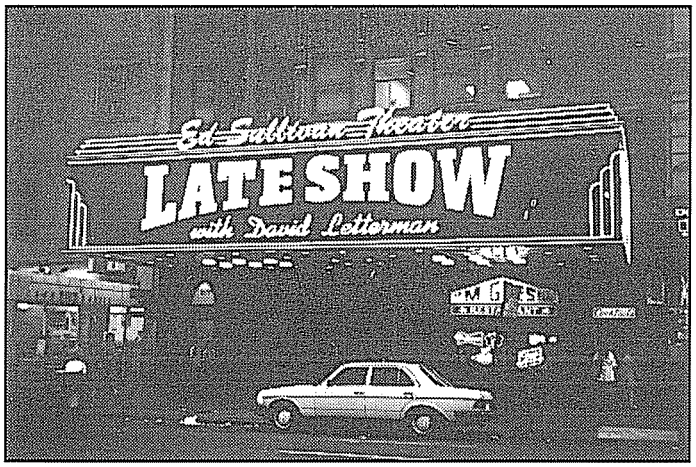
In finding $\frac{dD}{dm}$ treat b as a constant and in finding $\frac{dD}{db}$ treat m as a constant.

(b) Use the equation of the line of the best fit from part (a) to predict the total number of exports for the year 2000.

4. **"The Beatles Appear on Ed Sullivan, Pop Culture is Born",** The Toronto Star, February 9, 1994

"Ed Sullivan was so desperate to get the Beatles' North American live TV debut, having been scooped by his arch-rival Steve Allen, he offered them an incredible $100000 fee for three performances, two live and one taped."

"Recalling 5 Days That Shook The World", The New York Times, February 6, 1994



"In addition to the crowd lined up outside the theatre, an estimated 73 million viewers tuned in to "The Ed Sullivan Show" on February 9, 1964.

(a) What day of the week was February 9, 1964?

(b) Find the next year after 1994 when February 9 will land on the same day that it did in 1964.

(c) Suppose that from 1964 to 1994 the average rate of inflation was about 5% per year. Take the fee paid to the Beatles in 1964 and keep increasing it by 5% per year and find out what it would grow to by the year 1994. How does this compare to what rock groups receive for a concert today?

(d) How does the number of people who watched the Beatles' first show on March 9, 1964 compare to the number of viewers who watched the Harding-Kerrigan competition during the Winter Olympics last month?

Comments and answers

Due to space limitations, answers for all of these questions will appear in the June 1994 issue of the Gazette. If you use Question 4 with your students you might want to rent **"The Beatles: The First U.S. Visit",** an Apple-authorized video released in 1990. On February 9 of this year I showed my Grade 7 students the video and then I gave them an entire assignment that involved the Beatles first visit to America. Most of my students knew the words to every song in the video, an amazing testament to the longevity of this group and they really enjoyed working on the questions. I showed some slides that I took of the Ed Sullivan Theater in New York, one of which is given below and because it is now the new home of the David Letterman Show they were quite familiar with the building and its present use.

FOCUS ON
FEATURED SPEAKERS

Veronica Lacey (Thursday Morning at 11:30)

The charismatic Veronica Lacey is a recognized visionary and innovative leader in education. She is the recipient of numerous awards of distinction including OISE's Educator of the Year Award and YWCA's Woman of the Year Award and has served as a member of the federal government's Prosperity Steering Committee. Veronica is currently the Director of Education for the North York Board. She is committed to ensuring the success of public education by implementing a program with clearly defined student outcomes and system wide assessment of reading, writing and mathematics. She regards public accountability as a high priority in fulfilling her system's mission of "empowering all students to learn, to achieve success, and to participate responsibly in a global, pluralistic society."

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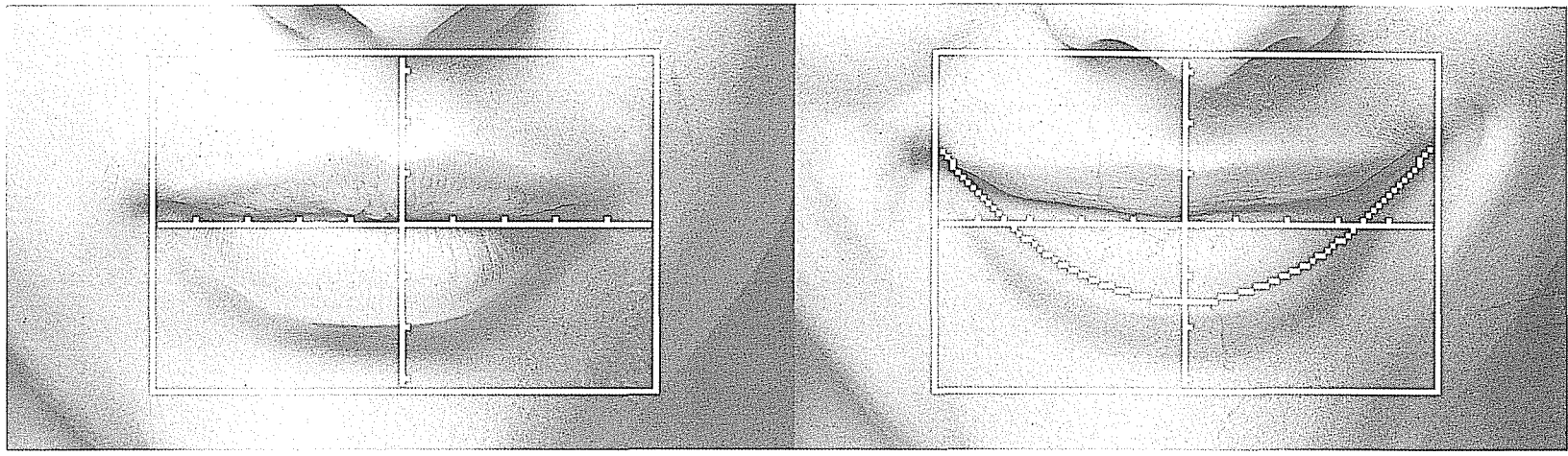
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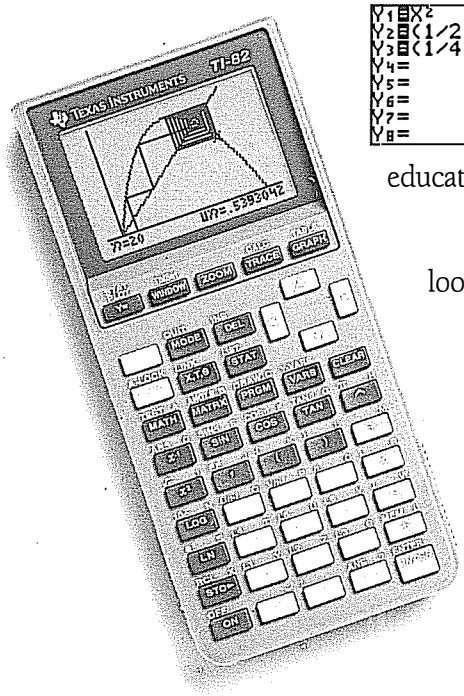
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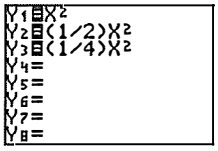


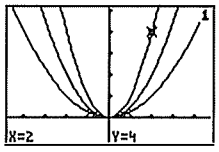
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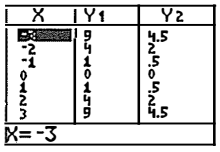
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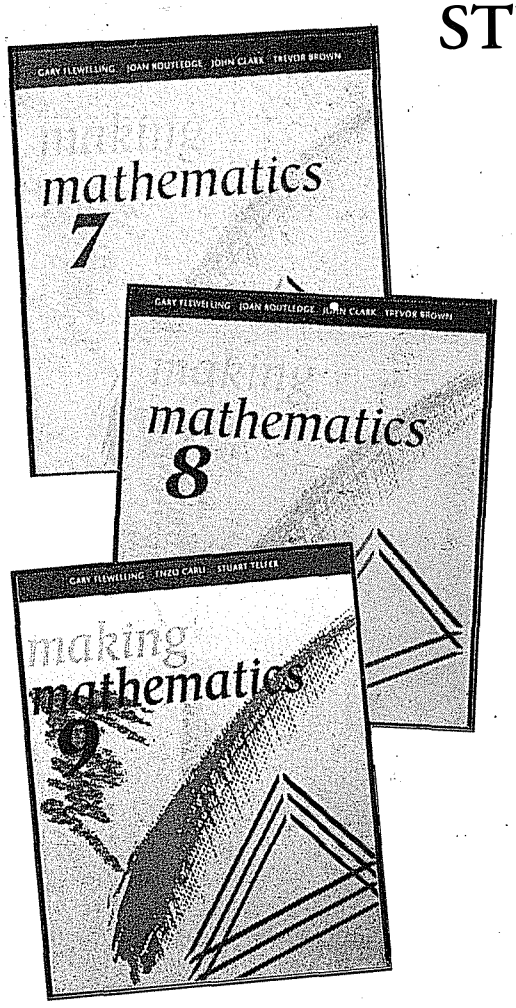
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