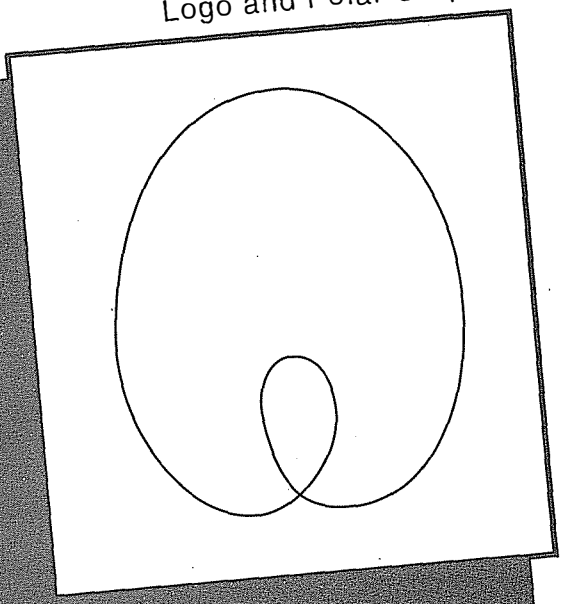
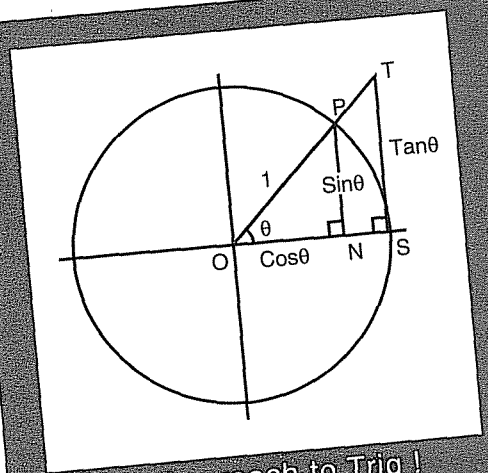
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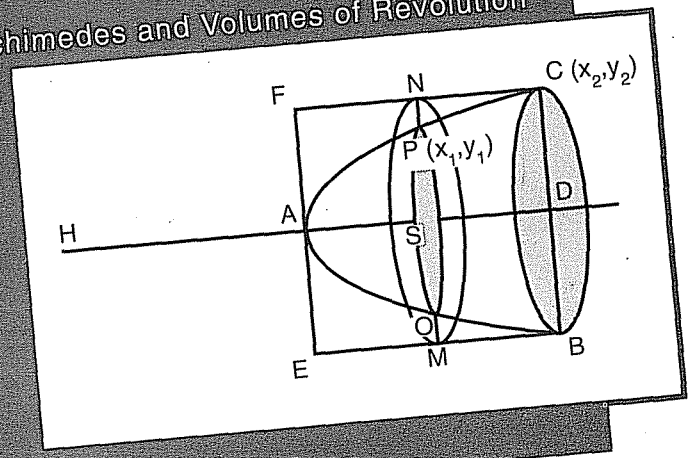
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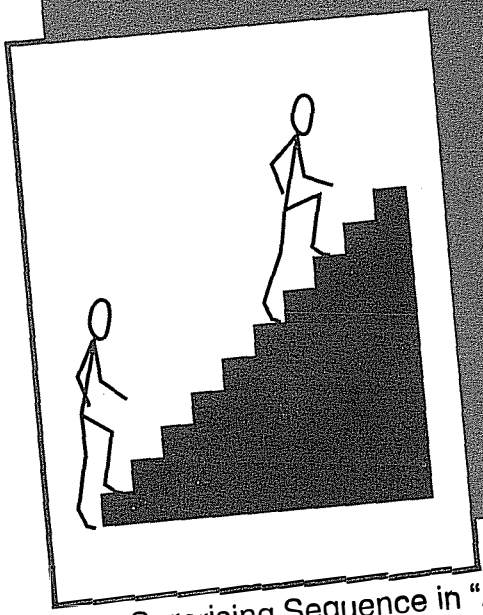




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Articles for the Gazette should be submitted to Jack Weiner at the above address.

Deadlines for information to be included in the Gazette or the Abacus are: June 15 for the September issue, October 15 for the December issue, January 15 for the April issue, and April 15 for the June issue.

Notice to Authors

Your articles should be typewritten, word-processed or photocopied from typewritten or word-processed copy double-spaced on 8.5" x 11" paper with 1.5" margins and single sided. Diagrams and figures should be in camera-ready form. Please submit three copies to the editor and keep one as protection against loss. We have all submissions blind-reviewed, so please send your full address and affiliation on a page separate from the manuscript.

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Editorial



Sometimes, I get a little confused as to what my role is as editor.

Certainly, there is the administrative side. When articles are submitted, I write a letter acknowledging the submission. Then, I parcel the article out to three reviewers, thanking them in advance for their time and effort and giving them a date by which to return the article with comments, constructive criticisms and recommendations.

Next comes the easy or hard part, depending...

"It is my pleasure to inform you that your article has been accepted for publication ...";

"It is the opinion of the reviewers that your article needs revision before it can be considered for publication ...";

"It is the consensus of the reviewers and editors that your article is unsuitable for publication ..."

There is the matter of putting an issue together. I try to select a balance of articles from the accepted bin which spans the grades and touches upon a variety of areas. With my secretary and associate editors, I prepare the files and work with my graphics experts to effectively present the material. We try to remain true to the authors' preferences and to the journal's style. And we try to minimize the errors.

That's a lot of work, but it's also the reasonably straightforward part of being an editor.

Here's my editorial quandary. Occasionally in this issue of the Gazette, an author has stated in a single sentence or paragraph a rather controversial opinion, but without substantiation. In a column such as "Have Your Say" (and note there was no submission for this column in this issue), the author would state the opinion and argue why he or she feels this way. You, the reader, might disagree and you might write a reasoned response to air your opposing viewpoint.

But as I said, these controversial statements appeared as isolated paragraphs within articles and without argumentative backup. I am not going to tell you where they are. I hope you will find them yourselves.

Now, as editor, should I have said to the author that either this opinion should be backed up or deleted? Or should I do as I am doing? Run the article as written. The opinion is part of the author's overall message. It's up to us, the readers, to respond, in writing, if we feel differently.

So find these statements. Write if you disagree. Write if you disagree with me. Write.

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Editor's Note: You will find several OAME/95 Conference photos in this issue, courtesy of perpetual OAME chronicler, Paul Sherk.

▲ LETTER TO THE EDITOR

I'd like to comment on two of the articles in the April issue of the Gazette.

(1) Factoring the Trinomial.. An Alternative Approach

I would like to suggest a method I used in my classes some 50 years ago! The method used in the article bears some relationship to the one I used.

In this article it is suggested that factoring trinomials such as $6x^2 - 7x - 20$ reduces to:

finding the factors of -120 where the sum is -7.

The process can be made much easier, especially if the factors are hard to find, if the statement is turned around.

Find two numbers whose sum is -7 and whose product is -120.

	Addends	sum	product
First trial	-8,1	-7	-8
Second trial	-10,3	-7	-30
Third trial	-20,13	-7	-360
Fourth trial	-15,8	-7	-120

Finding addends is a very simple arithmetic operation.

The author's second method of finding an average comes naturally when one finds "trial" addends. For example,

Find two numbers whose sum is 60 and product is 221 x 4 = 884

	Addends	sum	product
First trial	30,30	60	900
Second trial	28,32	60	896
Third trial	36,34	60	884

After a little experience the first and second trial could have been eliminated immediately as one can predict the last digit in the product. If a calculator is handy it helps to ascertain how close you are to the correct pair of numbers.

Find two numbers whose sum is 42 and product is 437

	Addends	sum	product
First trial	21,21	42	XXX(ends in 1)
Second trial	20,22	42	XXX(ends in 0)
Third trial	19,23	42	437

Find two numbers whose sum is 5 and product is -1326

	Addends	sum	product
First trial	45,-40	5	XXX(ends in 0)
Second trial	40,-35	5	XXX(ends in 0)
Third trial	39,-34	5	-1326

The method I'm suggesting required "number sense" - a facility that students should be encouraged to acquire.

(2) Elementary My Dear Watson

(a) The method for factoring using "strips and squares" was used extensively by W.W. Sawyer. I watched Sawyer lead about 70 Grade 6 students to solve such factoring problems with strips and squares at Don Mills Collegiate - well over 25 years ago. Norman Gillespie did a number of workshops using this method in the good old days. A complete development can be found in a 20 year-old grade 9 Ontario text, "Mathematics Insights and Applications", by Del Grande, Jones & Morrow, Gage Publishing, 1975.

(b) Could it be that students have some misconceptions about congruency because teachers inadvertently impose some false notions on students? Basic ideas of transformation of figures should be introduced and used throughout the grades starting in Kindergarten. Seeing congruent figures in different orientations is a skill that is acquired by children after using a great number of transformation activities. I refer interested readers to the following publications.

John J. Del Grande, *Spatial Perception and Primary Geometry, Learning and Teaching Geometry*, 1987 Yearbook of the National Council of Teachers of Mathematics.

Curriculum and Evaluation Standards: Addenda Series, Geometry and Spatial Sense,
-Kindergarten, pages 6,9
-Grade 1, pages 18-20

Sincerely yours,
50 years and counting,

John Del Grande

ANNOUNCING...

The First Annual

The Carleton-Ottawa Mathematics Association
in association with
The Ontario Association for Mathematics Education
presents...

THE FIRST ANNUAL

ONTARIO MATHEMATICS OLYMPICS FOR GRADE 7 AND 8 STUDENTS

LOCATION: OTTAWA ***DATE: JUNE 3, 1995***

Two teams will be chosen from each of the 14 Chapters of OAME to compete in this activity. Each team will consist of four students (two boys, two girls; two from grade 7, two from grade 8).

Chapters will collaborate with the schools and boards within their areas to select the teams. COMA will provide each Chapter with a comprehensive registration package, including sample activities, at the September Council meeting.

CONTACT PERSON:

Jeri Lunney, Box 1564, 157 Elgin Street, Almonte, Ontario K0A 1A0 Phone and Fax (613) 256-0687

▲ President's Message

TODD ROMIENS



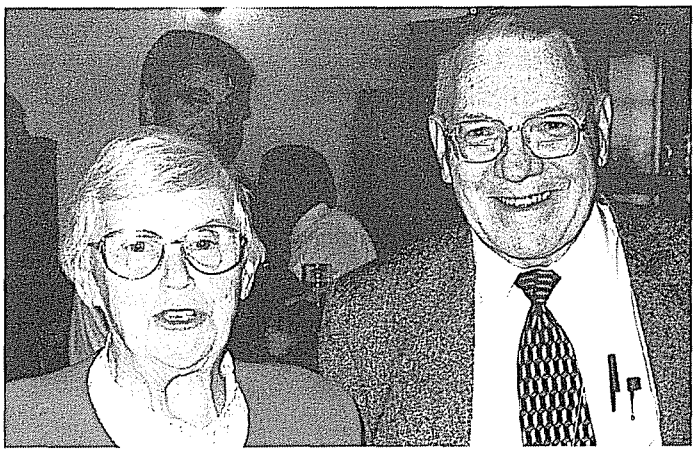
Welcome back from what I hope was a restful and exciting summer break. By now you are back to your teaching duties and I hope your kids are energetic, enthusiastic and motivated to do mathematics for another year. This year promises to be an outstanding one for math educators across the province. The winds of change have been blowing for a few years now and I think are finally making inroads in many areas. These include the use of portfolios, journal writing and alternate assessment methods. Many teachers are implementing these changes with much success. There has also been an increase in the use of current technology, including graphics calculators and computers and teachers are seeing the positive impact that these have on how we teach curriculum. OAME has and will continue to support these initiatives.

This will also be the year that the Commission on education will be making its report. I expect little in it that will affect mathematics education directly. However, if there is a restructuring to eliminate OAC's, it will have a significant impact on the curriculum we teach.

The Ministry has been dragging its feet in many areas that are of concern to us all, including the Transition Years curriculum, the role and form of the Specialization Years, inservice and implementation of the Common Curriculum document and the writing of exemplar items for the Standards document. It is my belief that OAME can be proactive and not reactive in these areas, and has already taken some steps to be so.

We have formed a Special Projects Committee whose role will be to identify issues in mathematics education that may require resources that would benefit OAME members and to coordinate the preparation of these resource materials. The first project to be undertaken by the committee is outlined below.

OAME Conference 94



OAME Special Projects Committee Assessment Project

Outcomes

- a set of field-tested alternative assessment activities for Transition Years to support the Provincial Standards.
- increased understanding for committee members and field testers regarding alternative assessment and the Provincial Standards

Plan

Work jointly with OMCA

October 17/94

- participate in the OMCA Retreat (description attached)
- clarify understanding of open-ended activities or "activities that invite students to explain their reasoning"
- generate an initial set of activities
- identify a chair for each strand
- after the retreat, the strand chairs will complete the work begun in their groups: refine activities, identify/create others are necessary

Selecting Teachers for Field-Testing

- project leader will coordinate the selection of teachers to field-test items
- from OAME Chapters: 3 teachers from each chapter, one for each of Grade 7, 8, 9
- from OMCA members: recommended teachers

December 11, 12/94 (Sunday, Monday)

- strand leaders meet
- selection of items to be field-tested
- distribution to pilot teachers

January 95-March 95

- field testing
- student solutions to be returned to project leader by March 10/95
- include a questionnaire for teacher — comments on the process and success of the activity

March 24, 26/95 (Friday, Sunday)

- meet to pull together results of piloting
- establish contents of final document

April 95-May 95

- design and proofing of final document
- printing for release at OAME Annual Conference

It is our intention to get the job done and to be a greater benefit to teachers. Other projects will be identified in the near future.

Two other new committees have been formed as well. The "Promotions" committee will liaise with other committees and make sure that OAME materials, resources and membership applications will be available at all chapter, leadership and annual meetings. The "Future Conferences" committee will among other things liaise with the committee running the current OAME annual conference, to offer advice and suggestions and to further enhance the communication between executive and planning committees.

There are many other happenings that are of great interest to members, but I'll leave them to future issues. If you are an OAME member, I encourage you to get involved and get others to join. If you are new to our Gazette, join up - the benefits are worthwhile.

P.S. Don't forget the Leadership Conference on Nov. 9-11. Sign up today - space is limited.

▲ CALCULUS, A STUDENT-CENTRED APPROACH:

RICHARD DEWEY
CLARKSON S. S.
PEEL REGION BOARD OF EDUCATION



Richard Dewey has been teaching with the Peel Board of Education since 1971 and is currently teaching math at Clarkson Secondary School. Richard has been a speaker at many conferences throughout Ontario and has spoken at OAME conferences on several occasions. He has had articles published in both The Mathematics Teacher and The Ontario Mathematics Gazette. His greatest ambition is to make mathematics fun for all students.

Calculus, properly presented, can be one of the most useful and rewarding math courses offered at the secondary school level. The purpose of this article is to provide the calculus teacher with some new ideas, approaches, and photocopyable work sheets that can be used in the Calculus classroom.

The motivation for this article came from two sources. First, a conversation with Dr. Peter Taylor of Queen's University at the 1991 OAME Conference; secondly, from trying new ways of making my own Calculus class last year student-centred, rather than teacher-centred. One of the projects we accomplished was an investigation into how Calculus is used in the work place. The students were asked to write a short essay based upon an interview with a practising professional such as an engineer or scientist. The only requirement was that the person not be in education. The results were rather startling: most students informed me that their prospective subjects explained that they *never used Calculus*.

This couldn't be true! I suggested that they try again. When the subjects were asked again, they explained that they were referring to *the Calculus techniques they had learned in school*. The data used by practising engineers is often analyzed using computer programs and they, as one engineer said, "haven't evaluated a limit or taken the derivative of a polynomial function since I was in school." When I looked at our last Calculus exam I found that nearly half the exam was based upon the evaluation of limits and the derivatives of polynomial and radical functions. And to make matters worse, we didn't even allow the use of programmable graphics calculators (not to mention computer programs). It was clear that we had to bridge the gap between our existing curriculum and those Calculus based activities used by mathematicians and engineers in the field.

I remembered that Peter Taylor, in the preface to his distinctive text *Calculus, The Analysis of Functions,* refers to the differences in his approach as an attempt "to be a process-based curriculum; that is, the material and problems are primarily chosen with an eye to engaging the student in the process of doing mathematics through such things as classroom activity, small group work, project work, problem-solving, reading, and writing." This seemed to be just what I needed. The approach I decided to choose for my Calculus class was that of problem-solving through cooperative learning. A necessary condition for problem-solving is that the problem be one that the student *has not previously experienced*.

This approach, for many students, can be all too novel and thereby troublesome. I dealt with this by remembering Neil Davidson's article, "Small-Group Cooperative Learning in Mathematics". In his discussion of "math avoidance" and "math anxiety", he refers to small group learning as "a social support mechanism for the learning of mathematics." He adds, "Small groups provide a forum in which students ask questions, discuss ideas, make mistakes, learn to listen to others' ideas, offer constructive criticism, and summarize their discoveries in writing." The size of the groups used in my class was kept between two and four. The groups were either heterogeneous or homogeneous in their ability although I found that heterogeneous groupings of two were more productive. As one of the parents contributed during a parent's night interview, "the smart farmer, in the days of horses and wagons, would hitch the more aggressive horse with the slower horse. That way the stronger horse would always set the faster pace and carry the weaker horse when it tired." I concentrated on cooperative pairs rather than larger groupings because I also use the McMaster University observer-coach, six step strategy for problem-solving. This technique is especially useful in the solution of "word problems" found in related rates, optimization and differential equations sections. Details of the method and its use will be the subject of a future article.

The problem found in the work sheets was taken from Peter Taylor's Calculus text (p. 9). The objective of the lesson is to have the students gain an understanding of the

Bennett, and Stevahn's book, *Cooperative Learning, Where Heart Meets Mind.* In this method the students do the exercise in pairs, think about their response individually for a period of time, then share their ideas with their partner in an attempt to reach a solution to the problem. For a cooperative lesson to be successful it is very important that a high degree of structure be provided. Before handing out the work sheets we discussed the process; how the students were to work together; and the objectives, both in terms of learning to work cooperatively and the mathematical concepts. Rather than present a statement of facts and requirements I designed a series of steps that the students would work through. What follows is a *photocopyable copy* of that exercise. When you use this in your classroom it is necessary that you stress the importance of discussion within the group, both what is happening and why.

importance of slopes, curvature and secant lines in the interpolation of data. The students perform the exercise using the Think-Pair-Share method as outlined in Kevin Crouse's article "Cooperative Learning" and in Bennett,

The members of the group must be aware of their role in the collaborative process. Each member must:

a. Be responsible for his or her own work and behavior
b. Be willing to assist the other member(s) of the group
c. Accept an active role in the solution of the problem
d. Seek assistance from the teacher only after he or she has exhausted every possibility within the group.

Materials: One set of activity sheets for each student, rulers, calculators and a sharp pencil.

Directions: Divide the class into pairs. This can be done by ranking the students by their current class average and pairing students; lowest with highest, second from lowest with second from highest, etc. An alternative method is to pair the students randomly. Take a deck of playing cards and pair value and colours (red two's, black two's, red three's, etc.) for each member of the class; then randomly distribute the cards, face down, on a table or desk. Each student chooses a card then finds his/her partner by locating the card with the same colour and face value.

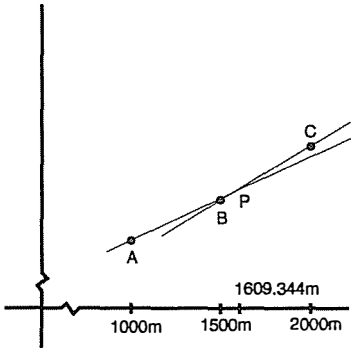
After the students have seated themselves next to their partner, distribute the first page of the activity sheets, two sheets per team. Instruct the students to read the data and instructions carefully and answer *individually,* in pencil, question number one. Allow the students about three minutes to answer the questions on their own. At the end of this time instruct the students to stop and explain what they did to their respective partner. This should take approximately two minutes. Then allow another few minutes for the pair to work together to correct any mistakes and finish answering the questions. During this time you should circulate among the students to get an idea of how well they have understood the task. You might then either elect to have a class discussion of the first sheet or go on to the remainder of the activity sheets.

Before distributing sheets two and three check that each person has a ruler for the graph. Caution the students that the points **must be graphed with great accuracy** and that the accuracy of the graph is **essential** for answering the question about curvature. You may also want to review the meanings of concavity and secant line, the method for determining slope and the equation of a line.

The process is the same for the second two sheets as for the first. Give the students five to ten minutes to work on the sheets individually, then let them check, discuss and finish the sheets with their partner. It is very important that you **do not** give them enough time to finish the sheets on their own. While the students are working together you should circulate and help them **ask questions** which will guide their partners to the correct solution. At the end of the exercise you can have a representative of each group report on their findings or have a class discussion of the unit. This **follow-up discussion** is a **vital element** of the cooperative learning process. During the discussion period the students should be given the opportunity to summarize their work. They should discuss the meaning of an estimated result, the significance of the graph and each type of approximation.

In the first type of approximation it is assumed that the average speed for 1500 metres is faster than that for the mile. Using this speed produces a lower limit. The upper limit is found by assuming that the average speed for 2000 metres is slower than that for the mile.

In the second type of approximation, the fact that the curve is concave upward is used to produce a better approximation of the time. The fact that the average time between 1000 and 2000 metres is 212.65 seconds and that the time for 1500 metres is 212.2 seconds indicates a slight concavity upwards.



The line through 1000 metres, A, and 1500 metres, B, will produce a secant line that will pass below the value for the mile and will thereby give a lower bound. The line through B and 2000 metres, C, will pass above the mile point and give an upper bound. Because the rate of change is so gradual, these bounds will be very close together.

Depending on the students' past experience with cooperative learning, the unit together with the ensuing discussion should take from 40 to 70 minutes to complete. I find these discussions particularly fruitful as it gives me a chance to learn my students' strengths and weaknesses. Occasionally there is a

bonus when a student tells me something that I hadn't previously thought of, making it a learning experience for me too.

Answers:

1 a. $v_1 = 7.0688$ m/s

b. $v_2 = 6.8634$ m/s

c. $t_l = 227.6685$ s

d. $t_u = 234.4814$ s

Therefore the time for the mile must lie between 227.7 and 234.5 seconds.

2 a. concave upwards

b. answers will vary, eg., since the slope of AB = .1566 and the slope of C = .1584 it must be concave upwards

d. $y = .1584x - 25.4$

e. $t_u = 229.52$ s

f. $y = .1566x - 22.7$

g. $t_l = 229.32$ s

Therefore the time for the mile must lie between 229.3 and 229.5 seconds.

Additional questions: Here is a question that I used in a quiz to evaluate their understanding of the unit. Jack T. Colton and Joan Wilder* were sitting in a hot sultry tavern in Cartagena, Columbia one Valentine's Day reminiscing over their past experiences in Columbia when the waiter delivered another cool Martini. The temperature of the drink upon arrival was 5°C. Five minutes later the temperature of the drink had risen to 12°C. After a total of 10 minutes the temperature was 18°C. The drink was then allowed to stand until it had reached the room temperature of 34°C. This process took a total of 25 minutes.

1. On a piece of graph paper accurately sketch the points described, putting the time on the horizontal axis and the temperature on the vertical axis.
2. Describe the type of curvature.
3. Use the best method possible to determine what the temperature of the drink was 15 minutes after it was served.

BONUS (one point): What was the object of Jack and Joan's original adventure in Columbia? (Hint: not all hearts are red.)

Further questions can be found in Peter Taylor's Calculus text, pages 13 and 14.

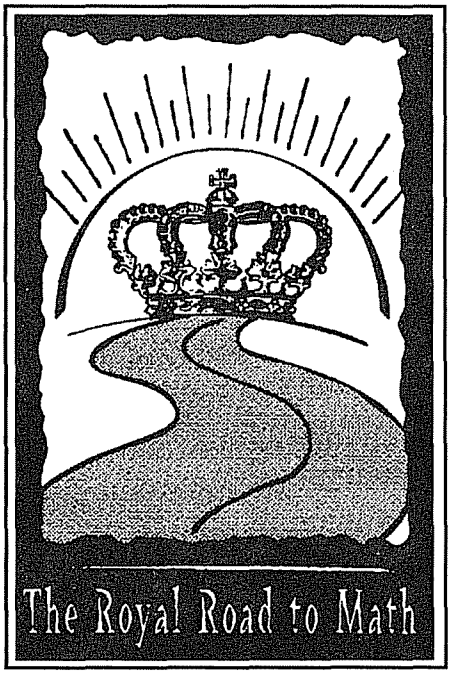
*The hero and heroine in the film, "Romancing the Stone".

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22nd Annual OAME Conference

le 22e Congrès Annuel de l'AOEM



University of Guelph
Guelph, Ontario

May 11,12 &13, 1995

HOSTED BY:
The Grand Valley Mathematics Association

ESTIMATING THE MILE RECORD

At the right is a table of the men's metric world track records as of August 1976 (taken from the Guiness Book of World Records). At that time, the one mile was still a major track event, contested by the top runners. Based on the metric data, provide the best possible estimate of what the mile record should have been in August 1976.

Note that 1 mile equals 1609.344 m.

Metric World Records

x metres	t seconds	Year Set
100	9.96	68
200	19.81	71
400	43.9	68
800	103.5	76
1000	133.9	74
1500	212.2	74
2000	291.4	76
3000	455.2	74
5000	793.0	72
10000	1650.8	73
20000	3444.2	76
25000	4456.8	75
30000	5490.4	70

1. To calculate the average speed of an object that starts from rest and finishes at some time t (seconds) over a given distance x (metres), simply divide x by t to get velocity v in m/s.

   a. Calculate the velocity for a measured distance just under the mile.

   v_1 =

   b. Calculate the velocity for a measured distance just over the mile.

   v_2 =

   c. To get a *lower limit* for the time for the mile divide the number of metres for the mile by v_1.

   t_ℓ =

   d. To get an *upper limit* for the time for the mile, divide 1 609.344 m by v_2.

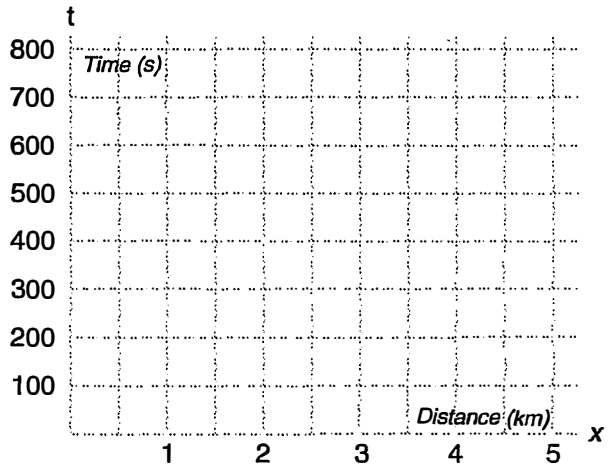
   t_u =

   Therefore the time for the mile must lie between:

   _____ and _____ seconds.



2. A better estimate comes from an investigation of the graph obtained by plotting the values on an axis. Put the distance, x, on the horizontal axis and the time t on the vertical axis. Since we are only interested in the mile we will only use distances up to 5 km.



a. Is your graph straight, slightly concave up, or slightly concave down? (Use a ruler to see.)

b. Explain how your data reflects this property. *(Hint: compare successive slopes between the intervals of distance.)*

c. Mark the points on your graph for 1000 m, 1500 m, 1609.344 m, and 2000 m with the letters A, B, P and C respectively.

d. Find the equation of the secant line between the points B and C.

e. This secant line represents an *upper bound* for the mile time. Solve for the upper bound by substituting the distance in metres for the mile. Your new t_u is:

t_u =

f. To get a better lower bound, we will use the secant line between the points for 1000 m and 1500 m, ie., A and B. Find the equation of the secant line through the points A and B.

g. This secant line represents a *lower bound* for the mile time. Solve for the lower bound by substituting the distance in metres for the mile. Your new t_ℓ is:

t_ℓ =

We have now produced a new and much more accurate range for the mile time.

Therefore the time for the mile must lie between:

_____ and _____ seconds.

Your results should have produced an interval within two tenths of a second! That's a great improvement over what we had before. Whatever the "concave-upness" of the graph means physically, it is evidently a powerful property.

What, by the way, was the mile record? Would you bet that it lay inside your interval? Well, it should! As of August, 1976, the mile record, set by John Walker of New Zealand at Gothenburg, Sweden, on August 12, 1975, was 3 minutes, 49.4 seconds, which comes out to be 229.4 seconds.

Eight Years of Polynomials

EDWARD J. BARBEAU
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Ed Barbeau has been teaching mathematics at the University of Toronto for 27 years. He is keenly interested in keeping the lines of communication open between the high school and university mathematics communities and between mathematicians and the general public. He has often been a guest on CBC's Quirks and Quarks and welcomes invitations to visit high schools.

During the past eight years, the Faculty of Arts and Science at the University of Toronto has offered a correspondence course for high school students on the mathematical topic of polynomials, with an optional examination each May. I would like to comment on the origins of this course and report on its subsequent development.

In the early 1970's, I was dismayed at the fragmentation of the high school mathematics curriculum in response to the so-called "spiral approach". While this approach does recognize that mathematics is mastered only over time through a process of recalling and reinforcing past work, its effectiveness depends on covering a sufficient amount of work at each turn of the spiral for students to see the significance of what they are doing and obtain a level of skill. In a given year, there must be an appropriate focus for the area covered.

Accordingly, I wrote an essay to suggest one unifying thread - the theory of equations - which would involve algebraic and geometric components, secure high school skills and provide a firm basis for later work. As the 1970's progressed, I drafted a set of notes and tried out the material over five meetings during one year of the Metro Mathematics Club. (I took over this club from W.W. Sawyer when he retired to Cambridge.) The students seemed to enjoy the sessions.

The course at the University of Toronto was catapulted into existence by two main factors. The first was the desire of many students who, while not having completed their full high school program, had completed OAC mathematics and approached the university for acceleration into tertiary courses for academic credit. Such requests were occasionally granted, but the results were not always satisfactory. First, there was the problem of combining high school and university schedules. Secondly, many such students were responding to pressure either from themselves to obtain lots of credits or from teachers or parents egging them on, rather than from an intrinsic interest in and love for mathematics. Thirdly, many had obtained high school marks with little effort and were not prepared to deal with mathematics that was not immediately comprehensible and that required a certain amount of effort to negotiate. Fourthly, their interest in mathematics was not deeply rooted; it was merely the subject in which their grades were highest. Some students taking university courses prematurely succeeded at first, but fell apart academically later on; at least one fell prey to the drug culture.

Of course, there have been successes, but their incidences are higher among students who can be persuaded to defer university work until after high school graduation, and who have in the meantime broadened their mathematical foundation in informal ways. We have given such students access to a mentor in the department and free run of the mathematics library.

Thus, the polynomials course was designed to be a resource for students who had completed all their high school mathematics. It would extend the curriculum in ways relevant to university work without covering material in standard university courses.

The second factor leading to the polynomials course was the desire of the Faculty of Arts and Science to provide a program for high school students in various disciplines. This would involve Saturday seminars, visits to the university and mentorships. The Faculty agreed to sponsor and administer the polynomials course, partly as a model for similar ventures in other fields. It now offers a correspondence problems course as well for students planning to enter the Physics and the Chemistry Olympiads. A fee is charged to cover costs.

In many ways, the study of polynomials or the theory of equations is ideal for students wishing to continue in mathematics. It combines down-to-earth material on algorithms and practical computation with deeper, more theoretical material, and incidentally fosters the acquisition of skills in manipulation useful for any science student. It is a unifying locus for a variety of mathematics: analysis, algebra, combinatorics, geometry, number theory, numerical analysis and complex variables; in addition, the area has a long and rich history. While the student may never see much of the specific material in a university course, the experience of work-

ing with polynomials will lead to a better understanding of more general and abstract areas of mathematics.

Apart from the subject matter, the polynomials course offers experience in independent study and provides a mathematical experience more intense than the standard high school fare that will ease the transition to a university level of performance.

The polynomials course was created not only for students, but also for teachers. It provides an additional resource they can use with some of their students, gives them access to interesting mathematics for their own professional development, and suggests directions that may be taken in future curriculum revisions. Indeed, the mathematical and pedagogical dimensions were explored with two groups of teachers in our graduate Master of Science in Teaching program, and some useful advice from them was incorporated into the program.

In the autumn of 1985, we signed up our first class of students. We were expecting about twenty takers; over one hundred signed up. The participants were sent a set of notes and problem sets to be submitted regularly for marking and critical comment. Monthly lectures on campus provided a general perspective and dealt with examples. At the end of the year, an optional examination was given. As expected, the initial enrolment was too large and half the students never submitted even one set of solutions. The effective participation fell to about twenty and four signed up for the final examination, three passed. It was notable that the students who remained tended not to be those in their final school year (as intended), but students in Grades 11 and 12 who had yet to complete school mathematics. The seniors did not want to be diverted from optimizing their school grades for university admission and scholarships.

In the second year of the program, short videotaped lectures were sold for personal use. Despite injunctions to would-be participants that a great deal of commitment would be necessary, the initial enrolment was again over one hundred, with the same pattern of drop off. In the third year, responding to an apparent need for some sort of extracurricular experience, we offered three courses - polynomials for the truly committed, combinatorics for those not willing to work quite so hard, and recreational mathematics for those not prepared to make a substantial effort. The combinatorics course consisted of a number of units, each produced by a different professor. The enrolment pattern for the polynomials course remained unchanged, while the other courses each attracted only about thirty students. So we went back to offering only the polynomials course. In 1989, the notes and problem sets became the basis for a textbook, *Polynomials,* by E.J. Barbeau, published by Springer-Verlag in New York.

Students often ask what they *should* be doing in the course or complain that the amount of material covered is excessive. This is beside the point. The course is not offered for any kind of credit, deliberately so. However, one high school took advantage of a Ministry of Education regulation that allowed it to count the course as half a high school credit for its students. Performance in the course is also considered by campus scholarship committees. If a student elects to take the examination, then there is a recommended schedule of deadlines that should be respected. Otherwise, the material is there for students and teachers to handle as they wish. They can set their own goals, and suit themselves about the pacing and amount of material covered.

It is currently fashionable to object to formal examinations. However, they can have a valuable pedagogical effect. They can induce students to review and integrate work taken over a long period of time and, in preparation, make sure that their grasp of the material is robust. It is my experience that this does not otherwise occur, with the result that students do not have at their disposal in mathematics courses material they may have been taught at an earlier stage. Those students who do pass the examination can be sure that they are working at university level and that the standards of the examination are rigorously maintained. It is hoped that the traditional culture shock faced by students entering university will be considerably reduced. Indeed, the successful candidates on the examination worked steadily and hard during the year. While some of the failing students did so for lack of preparation, others did in fact produce some good work - there just was not enough of it.

We should not skirt around the necessity for students continuing in mathematics and science to be fluent in algebra, geometry and trigonometry. This takes thought and a sustained effort, something that is difficult to muster for students with lack of motivation, heavy job or social commitments and a philosophy of doing the minimum for a grade. However, students with the appropriate skills can take thirty seconds instead of fifteen minutes to complete a standard calculation, avoid inappropriate and needlessly complex manipulations and generally improve their chances with higher level material. The route to mastery of a musical instrument or a sport is valid also for mathematics. As teachers, we can help students learn more efficiently and devise assignments to smooth out difficulties, but we do no one a service - not students, not parents, not teachers, not taxpayers - by giving false advice to students through inflated grades and shielding them from their proper responsibility to engage their studies.

Where do we go from here? First of all the polynomials course will continue. Some teachers have indicated to me that some of the material, such as Horner's method for evaluating polynomials, has been incorporated successfully into their regular courses. Material on factorization, quadratic equations, numerical solutions of polynomial equations and interpolation could be profitably given to a wide range of students. While almost all the students who have participated in the course have been from Ontario, I would be willing to accept into the fold students from other parts of Canada as well as from other countries.

There are other courses that I would also like to see offered, with the dual purpose of extending experiences of the better students and doing a dry run on material that might find its way into the standard curriculum. Possibilities are:

(1) **Mathematical physics:** Calculus would be taught explicitly for application in physical problems; participants would progress towards a derivation of Kepler's Laws from the basic postulates of Newton.

(2) **Applied mathematics:** This would cover some accessible modern applications of mathematics in such areas as biological modelling, experimental design, optimization and economics. It would draw on recursions, linear algebra and matrix theory, probability and combinatorics.

(3) **Dynamical systems:** This would treat quadratic recursions, chaotic behaviour and fractals.

(4) **Geometry:** This course would offer an eclectic approach to the solution of geometric problems by means of synthetic and analytic methods, transformations, vectors and complex numbers.

(5) **Combinatorics:** This course would proceed from basic results to more sophisticated methods of enumeration and derivation of identities, such as generating functions. Also included would be some historical problems, applications and graph theory.

(6) **Trigonometry:** There would be substantial treatment including the use of complex numbers, with applications and an extension to spherical trigonometry.

(7) **History of mathematics:** Apart from giving a survey of important developments, ideas and people, modules in this course would give examples of mathematical research in various eras. There is a lot of elementary material available in the literature.

These courses involve new material and perspectives that have emerged in mathematics over the past three decades. These deserve a place in the school curriculum. All of them would be demanding, but certainly accessible to the student who wants to continue in mathematics and science. They could serve as laboratories to determine which topics and treatments could be incorporated into the regular curriculum.

It would be desirable to have a constellation of bodies which would make these courses available on a volunteer basis - university departments, school boards and individual teachers and professors. The present curriculum, at least in Ontario, is patently unsatisfactory and cannot be allowed to persist for long. Let us prepare now so that we can build soundly for the new millennium.

Appendix 1

Report on the examination for the University of Toronto Polynomials course for the period 1986-1993.

The number of candidates for each year were

1986	4	1987	2
1988	3	1989	5
1990	6	1991	6
1992	6	1993	6

The successful candidates were

1986
- Ravi Vakil, Martingove, Cl., Etobicoke, ON
- Bryan Feir, St. Michael's University School, Victoria, BC
- Michael Glaum, Thomas A. Blakelock H.S., Oakville, ON

1987
- Ron Bodkin, Nepean H.S., Ottawa, ON
- Miki Narui, White Oaks S.S., Oakville, ON

1988
- Clifford Lam, Markham D.H.S., Markham, ON

1989
- Jeff Higham, Stephen Leacock C.I., Scarborough, ON
- Dale Woolridge, Agincourt C.I., Scarborough, ON
- Kevin Chiu, Appleby College, Oakville, ON

1990
- Naoki Sato, University of Toronto Schools, Toronto, ON
- Peter Milley, C.W. Jeffrey S.S., North York, ON
- Lleweilun Smith, Woodstock C.I., Woodstock, ON
- Ricky Wai-Ki Ho, Leaside H.S., East York, ON

1991
- Kevin King-Hin Cheung, Agincourt C.I., Scarborough, ON
- Philip Chong, W.L. MacKenzie C.I., North York, ON
- Gregory Smith, Collingwood C.I., Collingwood, ON
- Hong Liang Chang, Riverdale, C.I., Toronto, ON
- Sara B. Feyler, Lampton Central C.V.I., Petrolia, ON
- Sze King Tsoi, Port Credit S.S., Mississauga, ON

1992
- Edward Leung, University of Toronto Schools, Toronto, ON
- Hamid Mahvidi, Sir Sandford Fleming S.S., North York, ON
- Sindi Sambourin, Cornwall C.&V.S., Cornwall, ON
- Clement Yip, Port Credit S.S., Mississauga, ON

1993
- Adrian Chan, Upper Canada College, Toronto, ON
- Joe Callaghan, De La Salle College, Toronto, ON
- Alexander Yong, Del La Salle College, Toronto, ON

Appendix 2

University of Toronto
Faculty of Arts and Science
May, 1990
Polynomials
Examiner E.J. Barbeau

Time: 3 hours

No notes nor books are permitted.
A pocket calculator may be used.

Answer any five questions.
Each question is worth 20 marks.

1. Consider the polynomial

$$p(t) = 12t^5 - 22t^4 - 17t^3 + 10t^2 + 2t - 21.$$

(a) Determine an upper bound for the absolute value of the zeros of p(t) (real and complex, non-real).

(b) Determine all rational zeros of p(t).

(c) Determine how many and locate approximately real non-rational zeros of p(t).

2. (a) By means of the substitution $z = x + yi$, render the complex equation

$$z^3 - z + 1 = 0$$

as a system of two real equations in x and y.

(b) On the same axes, sketch the graphs of the two real questions in (a) and thence solve the equation $z^3 - z + 1 = 0$ graphically.

3. Let k be a fixed real number. Define the following functions:

$$g_1(x) = \frac{1}{k - x}$$

$$g_n(x) = g_1(g_{n-1}(x)) \qquad \text{for } n \geq 2.$$

$$f_n(x) = (g_n(x)-x)\,(k-g_{n-1}(x))(k-g_{n-2}(x))\ldots(k-g_1(x))(k-x)$$

for $n \geq 1$.

(a) Verify that

(i) $$g_2(x) = \frac{k - x}{k^2 - kx - 1}$$

(ii) $$f_1(x) = x^2 - kx + 1$$

(iii) $$f_2(x) = k(x^2 - kx + 1)$$

(b) Prove that, for $n \geq 1$,

$$f_{n+2}(x) = kf_{n+1}(x) - f_n(x).$$

(c) Deduce from (b) that, for each positive integer n

$$f_n(x) = (x^2 - kx + 1)u_n,$$

where $u_1 = 1$, $u_2 = k$ and $u_{n+2} = ku_{n+1} - u_n$.

4. (a) Determine necessary and sufficient conditions on the coefficients a, b, c of the cubic

$$t^3 + at^2 + bt + c$$

that its zeros, u, v, w, satisfy both of the conditions

$$u^2 + v^2 + w^2 = 1 \quad \text{and} \quad \left(\frac{1}{u}\right)^2 + \left(\frac{1}{v}\right)^2 + \left(\frac{1}{v}\right)^2 = 1$$

(b) Deduce that, for any cubic satisfying the condition of (a),

$$(a + c)^2 = (b + 1)^2.$$

(c) Prove that any cubic satisfying the condition of (a) cannot have all of its zeros real, but that one of its zeros is +1 or −1.

5. Factor each of the following as a product of irreducible polynomials over Q:

(a) $x^4 + 4$

(b) $3x^3 + 22x^2 + 45x + 28$

(c) $2x^3 + x^2y^2 - 2xy - y^3$

(d) $x^5 - 5x^4 + 5x$

(e) $x^2 + xy + xz - yz - 2y^2$.

6. Solve for integers x and y the diophantine equation

$$x^3 - y^3 = 3xy + 1$$

7. (a) Explain why there is at most one polynomial f(t) of degree less than 5 for which:

$$f(0) = -3,\ f(1) = 7,\ f(2) = 5,\ f(3) = -2,\ f(4) = 1.$$

(b) Express the polynomial described in (a) in the form

$$f(t) = a_0 + a_1(t + 4) + a_2(t + 4)^2 + a_3(t + 4)^3 + a_4(t + 4)^4,$$

for suitable real numbers a_i.

8. Sketch the graph of the following equation, marking significant points on the graph:

$$y = 12x^5 - 200x^3 + 1500x - 1784.$$

9. Let $f(x) = |x|$ for $-1 \leq x \leq 1$.
Justify or refute the following statement:

the polynomial of degree not exceeding 3 which minimizes

$$\max \{|f(x) - g(x)| : -1 \leq x \leq 1\}$$

has the form $g(x) = a + bx^2$ for some positive reals a and b.

Report on the 1990 May Polynomials examination

There were six candidates, of whom four were successful.

Question 1 (attempted by 3 candidates): The maximum marks awarded for the parts were (a) 6; (b) 6; (c) 8. Two students provided bounds for the roots (not optimal) and two successfully found the unique rational zero. However, one did not give any justification for his assertion that there were no other rational zeros. For part (c) Fourier-Budan was the method of choice, but no more detailed analysis was tried to deal with the ambiguous cases.

Question 2 (attempted by 4 candidates): Part (a) and the hyperbola arising in (b) were fairly well handled, although one student misread the transverse and conjugate axes and had the hyperbola passing through the origin. One student with a reasonable graph failed to sketch in the asymptotes.

Question 3 (attempted by 4 candidates): One of the failing candidates did not understand how to deal with the notation. Another was able to complete only part (a). The remaining two handled the question quite well, although one made no attempt on the relatively easy part (c).

Question 4 (attempted by 5 candidates): (a), (b) and the identification of 1 or -1 as a zero in (c) were reasonably well done. No one attempted the remainder of (c).

Question 5 (attempted by 4 candidates): Two completed all the factorizations, although neither indicated why the quartic factor in (d) was irreducible. However, generally candidates should improve their factoring skills with more practice, since the approach was pedestrian even where successful in the end.

Question 6 (attempted by 1 candidate): The solver guessed the solution (x,y) = (t+1, t), then tried (x,y) = (t+k, t). This would have been a successful line of attack had he not misread the resulting equation as an identity in t rather than a conditional equation in t and k.

Question 7 (attempted by 5 candidates): The methods used were the method of undetermined coefficients (leading to five linear equations in five unknowns) and the Lagrange polynomial (theoretically neat but difficult to execute in practice). One succeeded in arriving at the correct answer for (a). No one had an effective algorithmic approach to (b) and attempts got irretrievably bogged down.

Question 8 (attempted by 5 candidates): The solutions while generally correct were hardly fluent. Students are not adept at manipulating expressions in order to yield information, and not everybody was able to resist the temptation to set everything equal to zero, even when there was no reason to do so. The graph sketches were generally poor; critical points were not identified and the significance of the inflection points as separators of concave and convex parts of the graph was not indicated.

Question 9 (attempted by 1 candidate): The one student, although failing over all, made a creditable stab at probably the most subtle question on the paper.

While there was some nice work done on the paper, even by the failing candidates, every candidate with the same amount of knowledge, but with more care and manipulative skill, could have gained at least five marks. For those going on in mathematics (and they all should consider it), more practice in developing this skill would yield a good dividend. An old-fashioned course in trigonometry would provide an excellent background for university-level work.

Appendix:

Marking scheme

1. (a) 6 (b) 6 (c) 8
2. (a) 5 (b) 4 for imaginary part, 6 for real part
   5 for indication of graphical solutions
3. (a) 9 (3 for each) (b) 7 (c) 4
4. (a) 8 (4 for each condition)
   (b) 4 (c) 8 (4 for each part)
5. 4 for each factorization
6. 10 for the generic solution
   10 for showing absence of other solution
7. (a) 4 (b) 16
8. 10 for handling function, first and second derivatives,
   10 for graph
9. holistically marked

Computer Assisted Instruction: A Necessity in the Elementary Classroom

SARAH FROST
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Sarah Frost is a French and English Communications teacher at Wedgewood Junior School in Etobicoke. She has presented various computer workshops for ECCO and is a professional development workshop leader for FWTAO. Sarah began her M. Ed. thesis on computer studies, through Brock University, in January of 1994.

I envision a world where personal lap-top computers will be the "slates and chalk" in the elementary classroom of the future. Computers will be individually programmed to fit the needs of each child, with all children learning via their own terminals in one classroom community. Physically and mentally disabled children will work side by side with non-handicapped children, and foreign translation will be done at the push of a button. Telecommunication networks in the near future will permit students to better address global issues through direct inter-action with "real-world" participants, including other schools around the globe.

As educators, we have a major responsibility to ensure that our students are prepared for their technologically-oriented future, because a new proficiency will be required for the "Information Age". The educational environment must provide the opportunity to master skills necessary for success.

This paper will outline the rationale which supports my vision of the role of the computer in the school of the near future. It will establish why elementary educators must integrate effective computer-assisted instruction (CAI) into their regular programming, and conclude with recommendations to meet the challenge.

Computer technology has had an enormous impact on society. For the education system to be relevant and effective, this influence must be reflected in curriculum. As it is essential that school prepare students for the future, this places a tremendous burden of responsibility on education. Through education, children must become independent thinkers able to take charge of their learning. Students need to be comfortable with technology and feel confident that they have the ability necessary to cope with the changes taking place around them. It is the educators' responsibility to provide a non-threatening environment where risks can be taken and skills can be developed. Within this environment, individual differences in learning can be accommodated, students become active participants in their own education and are motivated to seek knowledge. Through the ability to learn, youth can acquire and maintain the skills they will require in such a rapidly changing world.

Children learn and interact with their environment in a very inquisitive and playful manner. When computers are taught at an early age as an extension of their learning, a positive relationship develops between the learner and technology. This creates an environment where children see computers as a regular part of their learning process. The more positive exposure a child has to computer experiences, the more constructive and meaningful the relationship becomes. It should be remembered that "one acquires computer skills much as one learns how to use the pencil or drive a car: not for the sake of mastering them but for the sake of doing something worthwhile with them."[1]

Given that a child must be able to use technology on a daily basis in his/her future, this relationship with technology must be initiated as early as Kindergarten. Learning is a cumulative process. Children begin to better understand new concepts and new experiences by building on what they have already experienced. It is acknowledged by Swick that "the critical factor in children's development of concepts is their active involvement in acquiring additional experiences so that they can construct more complete and meaningful concepts."[2] A child's ability to relate to technology in a positive and rewarding fashion provides the foundation essential for continued learning. Accordingly, children's opinions of the value and purpose of computers will be highly influenced by the kind of programs they are exposed to at a very early age. When students are exposed to programs which require decision-making skills and problem solving, they tend to develop more "interactive" ideas about the value and uses of computers. When some children were exposed to passive computer games, their impression of the computer was that the computer had many answers in it. However, those exposed to more "responsive contexts" commented about all the things you can do with the computer: write, paint, count and even

develop your own games.[3] In Logo, "the child programs the computer. And in teaching the computer how to think, children embark on an exploration about how they themselves think."[4]

We must establish an environment where children form positive attitudes about the technology around them. "With this in hand, they will model the 'interactive' use of the technology in other learning efforts."[5] The skills learned while working with the computer can be easily applied to other learning environments and form the basis of a skill-set which will be as necessary in the 1990's as writing and reading.

One of the most powerful influences computers have on learning is the opportunity for students to improve their self-esteem by working independently at their own level in a non-threatening environment. When students make choices and decisions about their learning, they are motivated to continue the process. They are accountable and responsible for their own learning and make decisions that reflect upon themselves directly. With many types of software, learners can be encouraged to be better risk takers, thus stimulating their imagination and decision-making skills. Immediate, non-threatening feedback is offered, encouraging the testing of alternative solutions to a given problem. It allows them to take responsibility for their learning and become more autonomous learners, who are comfortable with their naturally inquisitive interests.

Within this positive learning environment is the opportunity for teachers to reinforce the skills necessary for students to work cooperatively. Cooperative learning can further promote the development of computer skills because it permits students to become both teacher and learner as they encourage and assist each other with their learning. "Group work allows students to observe, imitate and learn from each other, keep each other on task, and share a sense of accomplishment."[6] Establishing rules and routines before specific assignments are initiated, provides the structure necessary for this type of learning. Each group will evaluate its own performance after the task is complete. This will encourage the modification of techniques and behaviours that have proven unsuccessful. With regular exposure to working in cooperative environments students will prepare themselves for the life skill of learning how to work with others.

A major hindrance to the establishment of an effective school environment is some teachers' fear of and lack of experience with computers. Clearly, "if teachers do not see the computer as an integral component of schooling that occupies a natural place in their instructional approach, then some of the predictions concerning technology's failures may be realized."[7] Many teachers did not grow up using computers, and are not comfortable using them in their classrooms. It can be very intimidating trying to keep up with the changing technology of our society. As many teachers have not had adequate instruction in the area of computer-assisted instruction, it is difficult, if not impossible, for many of them to implement an effective computer program for their students. Furthermore, it is very demanding to expect teachers to provide an integrated curriculum which offers equal access to the computer for all students when there is only one computer in the classroom. This only adds to the educator's anxiety about technology.

This issue can only be addressed by budgeting for more computer resources in the classroom and assisting teachers on a regular basis with regular instruction on how to effectively use computers in the classroom. Teachers should be encouraged to take professional courses which keep them up to date on current methodology and software. It is also the responsibility of the school board to keep its staff informed about the constantly changing computer technology through the provision of mandatory teacher inservice on a regular basis. If computers are to become the "slates and chalk" of the classroom, it is essential that all educators feel confident and at ease with computers.

CAI across the curriculum is becoming a natural method of teaching students and should be part of the regular learning environment for all students. When integrated into regular routines, computers become a normal and non-threatening part of the education process.

CAI has applications in all subject areas. With theme-based and child-centered learning, various software programs can be used to enhance the core learning objective. However, students need to understand that the computer is a learning tool, a resource useful in all areas of the curriculum, not simply a machine that provides right and wrong answers in math. Computers should be utilized as another medium through which educators can teach the standard curriculum. "In a properly integrated approach, basic concepts of problem solving, information handling, and the social issues will be reinforced and applied in many different ways throughout the curriculum in science, math, society studies, and the language arts."[8] The more exposure they have to the different skills they can acquire with the aid of a computer, the more comfortable they will be about learning, and transferring this acquired knowledge to other problems and situations.

A CAI program is more effective when each teacher integrates the use of computers across all curriculum areas directly in the classroom. Computer labs are currently popular and useful in elementary schools because most have

enough terminals for an entire class to work on the same project at the same time. Some teachers prefer working with computers in a lab environment because they are not comfortable with children working on different activities at the same time. They are also effective environments for teaching specific skills or instructions on the use of a specific piece of software/hardware application. Having the entire class writing stories or learning a specific math concept, at the same time, seems easier to manage and schedule. The use of a computer lab does not, however, accommodate the different learning styles of students. Some children may prefer to do mathematics in an afternoon when the computer lab has been slotted for the morning. Having the lab assigned as a specific "period" does not reinforce the use of technology throughout the entire curriculum. For a computer lab to work effectively, there must be enough flexibility to permit students to use the lab whenever they wish. This is very difficult to timetable and regulate, as one computer lab must be accessible, at least part of the day, to all classes. With the provision of sufficient computers in each classroom, there will be a limited need for computer labs. Children will perform the required classroom activities at the time of the day when they feel most comfortable, rather than when they are timetabled to work in the lab. They will be provided with enough hardware and software to work independently on their own computers whenever they are ready to complete an assignment. At present, this is a difficult task to accomplish. It is almost impossible for all students to have equal access to computers in the classroom because often only one or two computers are available.

These limited resources make it very difficult for teachers to integrate computer learning on an individual level across the curriculum. Therefore part of the education system's responsibility to the school is to ensure that schools are provided with the best and most applicable computer systems available. Sufficient computers, printers, and video equipment must be available in each school so that all children have equal access throughout the learning process. It is also imperative that the hardware is properly serviced and maintained. Adequate funds must be budgeted to keep the software and hardware current. Without adequate funds it is impossible to properly maintain computer equipment systems in such a rapidly changing world. Adequate funds must also be provided for teacher education. This obviously needs the support and approval of both the taxpayer and trustees.

Additional support for computer-assisted instruction is also found in the Ontario Ministry of Education's June 1991 Draft Policy entitled "Computers Across the Curriculum". Very specific aims, goals, objectives and responsibilities have been proposed. After careful examination and feedback from educators and the Ministry of Education, certain aspects of this document are most likely to be made law. While it is not relevant at this time to present specifics from the draft policy, it is important to note that the Ministry of Education has recognized the necessity of instruction in computers as part of an integrated school curriculum to prepare students for their future. Recognition that "it is the responsibility of our educational system to prepare learners for this world of change by integrating the use of computer technology across the curriculum,"[9] will require resource commitments at the board level.

This prompts the question about external influences on a child's education. When effective learning environments have been established in the schools, children will develop a natural and positive relationship with computers and technology. Children will be comfortable with the use of computers inside and outside of the classroom. Because some parents, however, will not have computers in the home, it is ultimately in the schools where students will best learn about the purpose of computers. It is essential that parents try to learn about how technology affects their children's lives, and take an interest in the computer assisted instruction in the schools. Through sharing and learning together with adults, Papert found that children's interest in mastering computer skills is heightened, their understanding of the technology is broadened, and their actual usage of computers increases.[10] Papert and Swick's research confirms that "supportive, continuing child-adult interactions in computer environments strengthen both children's and adult's construction of viable computer constructs."[11] Swick identified a growth-oriented schema suggesting the early introduction of computer skills is appropriate. It is a fact that "access to an explorative use of the computer certainly increases the likelihood of their integrating it into a life-long learning perspective."[12] While the need for computing resources external to the school environment (e.g., home), is recognized at higher levels of education, it is certainly progressive to promote this at the elementary level.

School curriculums are determined by societal values and norms. Both the private and public sectors promote and fund technological change. Just as it is the educators' responsibility to provide a supportive learning environment, it is the private sector's responsibility to support the school systems. The private sector must continue to promote the advances in technology by funding more school programs.

The range of software available permits many applications in an educational setting. Nonetheless, children "do not learn from software or courseware: they learn from the goal-directed intellectual activities that good software affords and invites."[13] When properly integrated, these types of programs accommodate students' different learning

styles and abilities. With the use of word processing programs, for example, students no longer need to worry about the mechanical aspects of writing. The editing process encourages each child to feel comfortable using their imagination to create a first draft. They are more inclined to be creative when the demand for correct spelling and grammar is absent from the first draft. They learn through the process of editing, that corrections are made when, and only when, they are ready. Just as students do not create at the same rate or in the same manner, not all students are ready to have drafts corrected at the same time. Students who cannot write will dictate their stories to an adult who will type them into the computer. Those who write but with difficulty may not be ready to have their work edited. Their first draft should be left as is. Some students who feel uncomfortable with the keyboard should be encouraged to write their corrected and final draft on the computer. They must be allowed to work at their own speed and not feel pressured. This promotes the different levels of ability of our students. "The computer is not simply a writing tool, but an environment which offers unique opportunities for collaboration and playful stimulation of the child's thinking and writing."[14]

Through the use of graphic programs, children develop spatial conceptualization skills and a sense of direction. With Logo, for example, students learn about sequencing, and how to use angles and degrees in their own programming. It is an excellent medium to teach students about how to problem solve. Other graphic programs offer students the opportunity to draw maps, create graphs, make posters and certificates, and draw their own coloured pictures. With many desktop publishing programs children can accompany their stories with graphics using the same program.

Databases are important tools that teach children how to discover differences and similarities among groups of things. It offers students a very effective way to learn about how to apply their classification skills. They learn how to analyze trends and patterns, and how to report what they have discovered. Hypotheses are tested and modified. With assistance from their teacher even children in Grade 2 can learn how to use a simple database. Organizing and classifying information is a skill that must be learned by all students if they wish to survive in such a technologically demanding world. Students should be introduced to spreadsheets and how to use them to predict outcomes, beginning in Grades 5 or 6. It is essential that our students know how to read and write maps, charts, and graphs if they wish to succeed in the future. Through this natural progression of CAI, the essential elements of word, picture, and numbers, all assist in developing an approach to learning.

As change will be the only constant in our world, it is essential that students' preparation for "The Information Age" begins as at young an age as possible. The education system is accountable for providing our students with life skills training. The school system must provide the environment where adequate learning of such skills can occur. If students do not develop computer literacy skills during their years at school, they will encounter difficulties coping with the world around them. Computer illiterate adults will face the strong possibility of unemployment.

Success is dependent on a cooperative approach which draws upon the resources available in the private and public sectors. If students are to have the best opportunity available to learn about computers and their effective application, the following recommendations must be implemented.

Teachers must be properly trained about computers on the use and application of computers through mandatory core courses while attending their Bachelor of Education program. School boards must keep their staff up to date on current software methodology, by providing regular mandatory computer inservice programs during school hours. School administration teams must support the positive environment teachers are trying to establish for effective CAI, and assist in the development of computer based curriculum programs.

Continued private and public sector funding will provide students with the best and most current hardware and software programs. With the support of taxpayers, trustees, and school administrations, continued funding for computers will keep the computer hardware/software current and properly maintained. Funding to upgrade teachers' skills in addition to inservice seminars must also be accounted for.

The far reaching implications of these recommendations are no less unsettling than the thought of an education system incapable of meeting the challenges of life beyond the classroom. My vision is a world in which computer technology plays a central role in all facets of the education system. In order to prepare our students and the school system for a time when computers are the "slates and chalk" of learning, we must act now. Computer-assisted instruction is the essential process that will prepare our students for the world in the future. It is clear that "thinking about the computer's role in education does not mean thinking about computers, it means thinking about education."[15]

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FOOTNOTES

1 **Gavrial Salomon**, *The Computer Lab: A Bad Idea Now Sanctified,* Educational Technology, October, 1990, p. 51.

2 **Kevin J. Swick**, *Designing Computer Environments for Young Children: Toward an Extension of Their Growth,* Educational Technology, December, 1989, p. 12.

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4 **Seymour Papers**, *Mindstorms, Children, Computers and Powerful Ideas,* Basic Books, Inc., New York, 1980, p. 19.

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6 MECC Centre for the Study of Educational Technology, Centre Points: *Cooperative Leaming in Computer Based Instruction: Key to Success is Structure of Environment,* Educational Technology, January, 1991, p. 60.

7 **Larry Miller, Dale Burnett,** *Using Computers as an Integral Aspect of Elementary Language Arts Instruction:* Paradoses, Problems, and Promise Junior Language Arts OAIP, p. 66.

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9 Ontario Ministry of Education, *Computers Across the Curriculum* (Consultation Document), June, 1991, p. 4.

10 **Kevin J. Swick,** *Designing Computer Environments For Young Children: Toward an Extension of Their Growth,* Educational Technology, December, 1989, p. 12.

11 **Kevin J. Swick**, Ibid., p. 12.

12 **Kevin J. Swick**, Ibid., p. 12.

13 **Gavrial Solomon,** *The Computer Lab; A Bad Idea Now Sanctified,* Educational Technology, October, 1990, p. 51.

14 **Catherine C. Morocco, Susan B. Newman,** *Word Processors and The Acquisition of Writing Strategies* Journal of Leaming Disabilities, Vol. 19, Number 4, April, 1986, p. 244.

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ARCHIMEDES AND VOLUMES OF REVOLUTION

JOHN DEL GRANDE, RICHMOND HILL



John Del Grande has been a major contributor to mathematics education in Ontario and beyond for over fifty years. He is the author of several textbooks and has published articles in many journals, including the Ontario Mathematics Gazette and the Mathematics Teacher. In 1992, he was elected a Life Member of OAME.

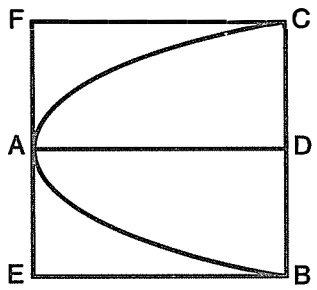
Archimedes, born in 287 B.C., was Syracuse's most famous son. He has often been declared as the greatest mathematician of all times. Students frequently see or hear the name of Archimedes mentioned in their studies of science, engineering, and mathematics but rarely do they have an opportunity to get a glimpse of the sheer genius of the man. There is no question that Archimedes' work on the volume of a sphere and of a paraboloid of revolution were both original and exciting. He did not restrict himself to the artificial restrictions of Plato whose proofs involved only results that could be obtained by straight edge and compasses. Archimedes utilized the principle of the lever in an ingenious way to solve this problem. He was a true applied mathematician.

The works of Archimedes have survived throughout the centuries in a few manuscripts written by others. In a copy of a letter written to his friend Eratosthenes, called the *METHOD,* he outlines how he made use of his knowledge of mechanics in order to discover solutions that enabled him to give rigorous mathematical proofs for problems on area and volume.

This manuscript brought to light Archimedes' knowledge on the method often used in calculus in the topic of integration. The *METHOD* contained the novel way in which Archimedes found the volume of solids. He imagined cutting up a desired solid into a large number of thin parallel slices or discs and suspending them at one end of an imaginary lever so that they balanced another solid whose volume and centre of gravity were known. The method used by Archimedes to find the volume of a sphere was published in the March 1993 issue of "The Mathematics Teacher". In Proposition 4 of the *METHOD* Archimedes shows that the volume of a paraboloid of revolution is one-half the volume of the circumscribing cylinder. This paraboloid has many modem day applications as it is found in the headlight of cars, searchlights, radar and television antenna, and in telescopic mirrors. His argument, in this paper, is written in modem terms to be understood by students with a knowledge of the principle of the lever and the centre of gravity properties of a cylinder.

THE VOLUME OF A PARABOLOID

The fundamental idea of Archimedes' approach (known as his method of equilibrium) was to cut up a required area or volume into a number of thin parallel slices. He would then design a level system to balance these slices. Knowing the "mass" and centroid of these slices he could by a summation process discover the required formulas. The following illustrates Archimedes' method for finding the volume of a paraboloid of revolution.



Draw a parabola CAB with axis AD

Enclose the parabola with a rectangle FEBC with axis AD.

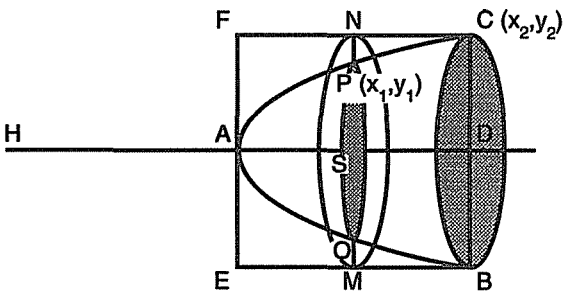
Rotate the parabola about AD to get a paraboloid of revolution.

Rotate the rectangle about AD to get a cylinder.

Produce DA to H so that HA = AD.

Take a cross section of the paraboloid and the circumscribing cylinder as shown in the diagram.

Think of HAD as the arm of a balance with fulcrum at A.



Let HD and FE be x and y-axes respectively.

If the parabola is in standard position its equation is of the form $y^2 = 4px$.

Slice the paraboloid along MN at right angles to AD.

The sections of the paraboloid and cylinders are circles of areas C_1 and C_2 with radii SN and SP respectively.

Let the coordinates of P be (x_1,y_1) and of C be (x_2,y_2)

Therefore $y_1^2 = 4px_1$

and $y_2^2 = 4px_2$

Therefore $\frac{y_1^2}{y_2^2} = \frac{4px_1}{4px_2} = \frac{x_1}{x_2}$

The area C_1 is πy_1^2.

Since $SN = DC = y_2$, the area C_2 is πy_2^2.

Therefore $\frac{x_1}{x_2} = \frac{\pi y_1^2}{\pi y_2^2} = \frac{\text{area of circle in parabola}}{\text{area of circle in cylinder}} = \frac{C_1}{C_2}$

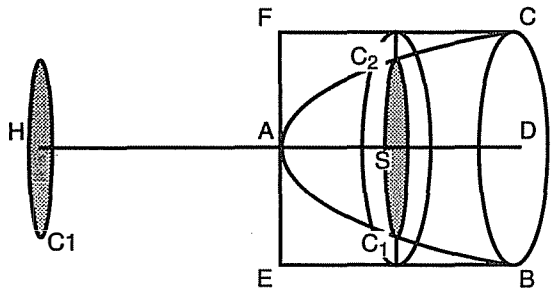
Therefore x_2 • (circle in parabola) = x_1 • (circle in cylinder)

Written in another form we have $C_1 \bullet x_2 = C_2 \bullet x_1$

But $AH = AD = x_2$

Now if the section C_1 is placed at H we get from the theory of the lever that the Section C_1 at H balances the section C_2 at S.

Therefore AH• (circle in parabola) = x_1• (circle in cylinder)
= AS• (circle in cylinder)



If each section is given a small thickness and the mass of each section is proportional to its volume made up the area of section times the small thickness, then the section of the cylinder, remaining where it is, balances the corresponding section of the paraboloid when placed at H.

Now proceed to slice up the cylinder and paraboloid with a series of cuts starting at A and moving across to D. Continue placing the sections of the parabola at H and we eventually will have the whole paraboloid at H, that is, the mass of the paraboloid will be concentrated at H. The sections of the cylinder move from A to D and will make up the whole cylinder with axis AD. Thus, if the paraboloid itself is placed so that its centre of gravity is at H then its mass will balance the mass of the cylinder whose cross section is FEBC.

The centre of gravity of the cylinder lies along AD at the midpoint, K, of AD.

where AK = KD.

Therefore we have (mass of paraboloid) • AH
= (mass of cylinder) • AK.

But mass is proportional to volume.

(volume of paraboloid) • (AH)
= (volume of cylinder) • (AK)

But AH = 2 AK

Therefore, 2 • (volume of paraboloid)
= volume of the circumscribing cylinder.

The volume of the paraboloid of revolution is one-half that of the cylinder that circumscribes it.

Archimedes' method using a large number of very small slices can be made perfectly rigorous using the modern theory of limits. His method essentially becomes the same as integration where the procedure is a picturesque and effective illustration of the concept of an integral as the limit of a sum.

This novel and famous work of Archimedes gives teachers the opportunity to digress and enrich their mathematics classes with important historical features of the subjects. The preceding presentation, if carefully developed, lies within the capabilities of most Grade 11 and Grade 12 students.

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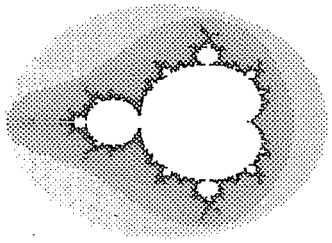
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LOGO AND POLAR GRAPHS

JIM O'KEEFE, ST. PETER'S S. S. , PETERBOROUGH



Jim O'Keefe has been teaching for 8 years, 5 as head of computers at St. Peter's Secondary. He has a particular fondness for the 'C' programming language, pizza and The Beatles.

Secondary and post-secondary teachers often consider Logo to be a children's programming language. This attitude likely stems from the fact that most of the well known Logo commands are only two characters in length. There may also be a connection between this attitude and the idea that the lines drawn in Logo are created by a turtle carrying a pen across the screen.

Although children can easily use Logo with its simple commands and can enjoy its immediate graphical feedback, Logo is much more than a programming language for children. Logo is a derivative of the LISP language which is best known for being used in artificial intelligence experiments. Logo can do some very sophisticated list processing, but it is best known for its ability to draw graphs with simple commands. Its simple graphing commands combined with its ability to handle trigonometric functions makes it ideally suited for teaching the concepts involved with creating polar graphs. Logo is also readily accessible to most people who have access to a computer.

If your main objective is just to have polar graphs drawn, then a graphing calculator would be the most convenient tool to achieve your goal. If you want to teach students how to plot polar graphs and to understand why they look the way they do, then programming in Logo on a computer is the ideal tool. There are two reasons for this. First of all, in order to write a computer program, the programmer must have a clear and concise understanding of the problem being solved. If he/she does not have this understanding when the project is begun, then through trial and error on the computer (and perhaps some guidance from a teacher), an understanding can be achieved by the time the problem is solved and the computer program is completed. Secondly, Logo is the ideal language for the specific example being discussed here, because its graph drawing commands are indeed simple enough for most people to master quickly. The student is not frustrated by the syntax of the computer language and can focus on the mathematics involved. As well, one cannot overlook the fact that any time a computer is involved in a lesson, the interest of most students is immediately increased even before the lesson has begun.

I have written the following program for graphing linear polar equations (Figure 1). This program should run on most computers using a Logo interpreter with little or no modifications. After it is typed into the interpreter it is run by typing: **trig**

This is a fairly straight forward program but does require some explanation.

The line:

```
make "r ( 4 * sin (3 * :an ) )
```

represents the equation for the graph being drawn. Notice that the equal sign is removed and that the right side is bracketed. The angle of each polar co-ordinate is represented by the variable :an.

The next two lines:

```
make "x ( :r * cos :an ) * :scale
make "y ( :r * sin :an ) * :scale
```

are the standard formulae for converting from polar to Cartesian co-ordinates, which is what Logo most easily uses. The variable :scale is given a value of 20 and is used to increase the size of the graph on the screen.

The line:

```
if :an = -90 [pu]
```

is simply used to lift the pen up while the turtle is moved from the centre of the screen to the first (x,y) co-ordinates. The variable :an is given a value of -90 when the program begins to signal that this is the first call to "to draw" and that the pen should be lifted to prevent a line being drawn from the centre of the screen to (x,y).

At first glance, this program appears to have no looping structure. This is because it uses a programming technique known as recursion. The function "to draw" calls itself.

The line:

```
if :an < 270 [draw :an +5]
```

is the line in "to draw" which makes the call to itself. This continues until the variable :an becomes equal to 270 and then the recursion "unwinds" itself and the program ends. For those interested in programming, recursion (a function making a call to itself) is a technique which can be employed in most languages, but is not very useful for most

applications. Logo does however lend itself to extensive use of recursion.

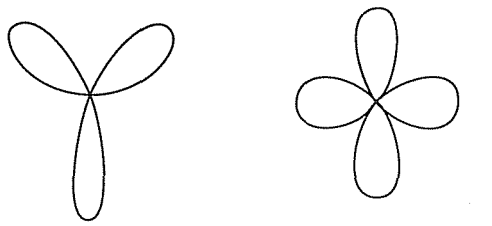
The accompanying graphs appear elongated in a verticle direction because the number of pixels (picture elements) on a computer screen is not the same for the vertical and horizontal axes. Trying to compensate for this difference would somewhat complicate the program.

Although this program does not display the more sophisticated aspects of Logo (with the exception perhaps of the use of recursion), it does allow students with limited computer programming skills to use a programming language to achieve a goal. Students can make changes to both the program and equations to explore and better understand polar graphs.

In the bibliography, I have listed two books which have excellent sections explaining polar graphs.

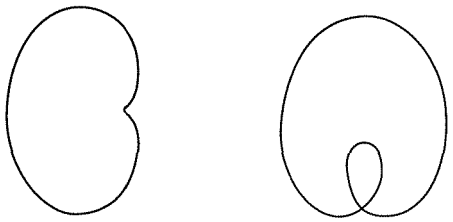
```
to trig
   local ":an
   cs
   make "an -90
   draw :an
end
to draw :an
   ht
   local ":r
   local ":x
   local ":y
   local ":scale
   make "scale 20
   make "r ( 4  * sin (3 * :an ) )
   make "x ( :r * cos :an ) * :scale
   make "y ( :r * sin :an ) * :scale
   if :an = -90 [pu]
   setpos se :x :y
   pd
   if :an < 270 [draw :an + 5]
end
```

FIGURE 1.



r = 4 * sin(3 * :an)
Three-Leaved Rose

r = 4 * cos(2 * :an)
Four-Leaved Rose



r = 2 * (1 - cos(:an))
Cardioid

r = 2 + 4 * sin(:an)
Limaçon

FIGURE 2

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Introducing the Trigonometric Functions

TOM J. GRIFFITHS
A.B.LUCAS SECONDARY SCHOOL, LONDON



Tom J. Griffiths was a member of the OAME council for twenty-four years. Tom is a frequent contributor to the Gazette and has served as a reviewer for many years.

About forty years ago trigonometry was initially introduced to students, including myself, as a year of studying the analysis of triangles. Hence the trig functions were introduced as ratios of the sides of a right angled triangle. Every possible combination of right angled, and non-right angled triangle in two and three dimensional situations was studied and numerically analyzed. Of course, since the magic electronic guessing boxes were not yet invented, we had the added delight of performing all calculations using log tables, or, in more permissive situations, slide rules. The following year we then went on to study a totally different topic called trigonometry, in which there were sine functions to graph and differentiate. These functions were no longer tied to triangles, and they took values for angles over 180° as well as for mysterious things called radians. And so trigonometry was studied as two totally unrelated subjects. Unfortunately judging from the text books we use today, written in the late 1980's, we have still not discovered that the two approaches to trig can be easily integrated.

First it needs to be reiterated that trigonometry is very useful in the study of triangles, vectors, complex numbers and polar co-ordinates, all of which use the trigonometry functions based on the study of the right angled triangle. Also trigonometry functions are needed as functions which can be graphed, transformed, have an associated algebra with equations and identities, and which can be analyzed using calculus. In order to have functions which can be used in all these roles I have found it necessary to totally rethink my definitions of the trigonometric, or circular functions.

I have not been the only one to be doing this. In the mid-sixties the authors of the textbooks of the time were playing with such periodic functions as the "Winding" function. This attempt, although very valid as an approach to trigonometric functions through their periodic nature, was sadly reduced to ridicule by being termed as the "Wing-ding" function, and was thus unfairly put beyond serious consideration. However, in the seventies I was fortunate enough to come across teachers who were trying to come up with better mousetraps in this area, and most were using the idea inherent in the winding function. That was that the trigonometric functions are in fact based on the circle and are periodic. This is why they are known as circular functions. The major step forward here was to associate the functions with a point on a circle of radius r. The functions are related mappings from the angle subtended at the centre to the x and y values of the point **P** in the diagram.

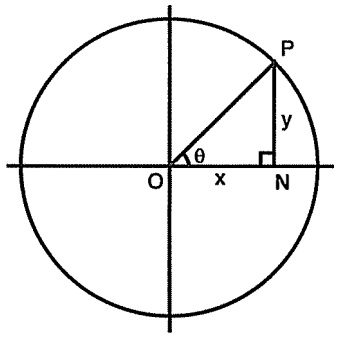


Figure 1

$$\text{Cos}\theta = \frac{x}{r} \text{ and } \text{Sin}\theta = \frac{y}{r}$$

For angles between 0° and 90°,

these are directly equivalent to (A)

$$\text{Cos}\theta = \frac{\text{adj}}{\text{hyp}} \text{ and } \text{Sin}\theta = \frac{\text{opp}}{\text{hyp}}$$

and give $x = r\text{Cos}\theta$ and $y = r\text{Sin}\theta$

By this definition the right angled triangle imbedded in the circle can be removed, and the sides given the names we old codgers were brought up with, i.e., opposite, adjacent and hypotenuse, and we can use those good old standby's **SOHCAHTOA** and **CAST.** Hence triangles can be solved in the traditional way.

I also found that if the idea of the point on the circle was followed through, not only could the study of right angled triangles be followed, but the idea of the ratios or functions having values for obtuse angles became simple. Also the idea of angles over 180° or even those obtained by multiple revolutions or revolutions in the negative direction made sense. Anywhere on the circle gave values for the trigonometric functions. Hence the functions could be graphed fairly easily.

Also the identity

$$\text{Cos}^2\theta + \text{Sin}^2\theta = 1$$

became more obvious as a direct result of the sides of the right angled triangle in the circle since

$$x^2 + y^2 = r^2 \text{ , and so from (A),}$$

$$(r\text{Cos}\theta)^2 + (r\text{Sin}\theta)^2 = r^2$$

Therefore $\text{Cos}^2\theta + \text{Sin}^2\theta = 1$

The **CAST** rule is now redundant as the sign of the ordered pair (x,y) dictates the sign of the trigonometric functions, since r is always positive.

This approach helped many of us in the seventies to give a better introduction for trigonometry. Unfortunately we had to do this with little help from most of the available texts, which seem to be totally dominated by the opp/adj/hyp definitions. It is interesting to speculate as to why the authors of the texts were not using the reasonably well known simpler approach to help us and our students understand the topic. Could it be that the texts are dedicated to sales and not leadership? Will teachers not buy new approaches? Will we buy only what we are comfortable with? I do not believe so. I have seen reticence to tackle a new idea change to complete attachment to it, in the idea of dealing with graphs by transformations. This approach was not used at all in the 60's. It was imbedded in the curriculum of 1969, and hence had to be in the text books. It is now the standard approach. Textbooks can lead the way, but perhaps it does need the Ministry to lead first in dictating methodology in some instances.

After using the circle radius r approach for a while, I then realised that I was still making the story unnecessarily complicated. Why not use a circle of radius one, the unit circle. With the sine and cosine functions defined on this circle, the actual lengths of x and y are respectively the cosine and sine functions. This is the approach I have found to be the most compelling for myself, and the simplest to help me introduce trigonometry in all its facets to my students.

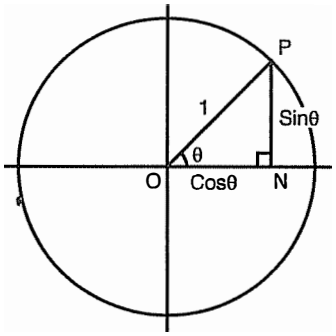


Figure 2

It has many advantages. The graph of the sine and cosine functions can be graphed as a direct 1:1 correspondence of the lengths NP and ON as θ varies. This is now my introductory lesson to the circular functions, and the students actually construct these graphs a point at a time by physical measurement, while simultaneously obtaining the same values on their calculators. Other trigonometric graphs can be obtained by transformation procedures.

The acronym **SOHCAHTOA** now becomes redundant with the idea that cosine and sine are x and y respectively. Note that the c and s of cosine and sine are in alphabetical order, as are x and y. This helps students remember which is where.

For the study of the right angled triangle, the triangle **OPN** can be removed from the circle, and magnified by r. This gives the same processes for the study of triangles as described earlier.

It can also be noted that the identity $\cos^2\theta + \sin^2\theta = 1$ is directly obtainable from the diagram.

The tangent function is also obtainable as a direct measurement as is shown in Figure 3. This comes from the definition of

$$\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$$

and the concept that triangles ONP and OST are similar, so

$$\frac{PN}{ON} = \frac{\text{Sin}\theta}{\text{Cos}\theta} = \frac{TS}{OS} = \frac{TS}{1}$$

Hence the length TS is Tanθ.

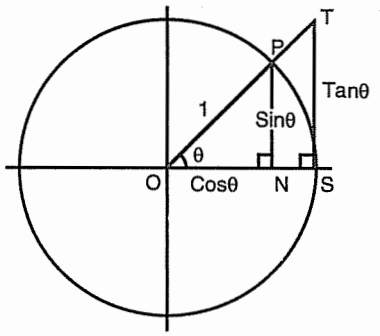


Figure 3

The introduction of radian measure is now relatively simple as a mapping of arc length to angle. The circumference of the unit circle is also the arc length for one complete revolution and is 2π.

In order to solve trigonometric equations, the same unit circle can be easily used for reference. For example the equation

$$\text{Cos}\theta = -\frac{1}{2}$$

can be solved using the circle or cosine graph in Figure 4.

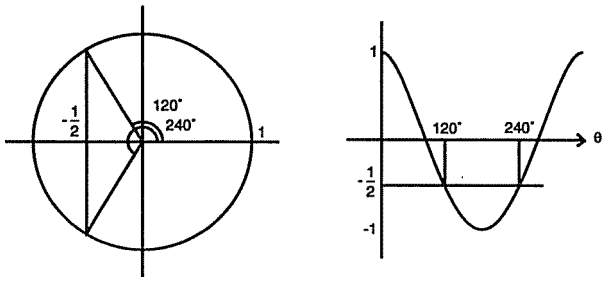


Figure 4

In order to obtain the expansions of Sin(A+B) and Cos(A+B) the circle can again be used. If the point (CosB, SinB) on the circle is rotated by the rotation matrix for angle A by matrix multiplication,

$$\begin{bmatrix} CosA & -SinA \\ SinA & CosA \end{bmatrix}$$

then the following calculation gives the required expansions.

$$\begin{bmatrix} Cos(A+B) \\ Sin(A+B) \end{bmatrix} = \begin{bmatrix} CosA & -SinA \\ SinA & CosA \end{bmatrix} x \begin{bmatrix} CosB \\ SinB \end{bmatrix}$$

$$= \begin{bmatrix} CosA\ CosB - SinA\ SinB \\ SinA\ CosB + CosA\ SinB \end{bmatrix}$$

I include a non-transformation development of this expansion of sin(A+B) for $0^\circ < A < 90^\circ$, $0^\circ < B < 90^\circ$ as an appendix.

I have found that this introduction to trigonometry has given my students, and hence myself, a far more integrated and satisfying understanding of the concepts. I also find that the unit circle is an excellent reference for correcting and clarifying misunderstandings for my students.

I am sure that I will continue to change my approach in this and many other topics, but I find that this is what keeps mathematics teaching fresh and interesting for me, and hopefully for my students.

Appendix

Lemma

The area of triangle $ABC = \frac{1}{2}ab\,SinC$

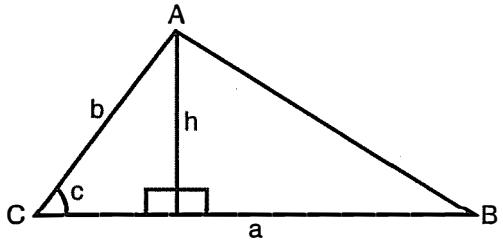


Figure 5

Area of triangle $ABC = \frac{1}{2}ah$

But $h = bSinC$

Therefore area of triangle $ABC = \frac{1}{2}ab\,SinC$

Proof that $Sin(A+B) = SinACosB + CosASinB$, for $0^\circ < A < 90^\circ$, $0^\circ < B < 90^\circ$

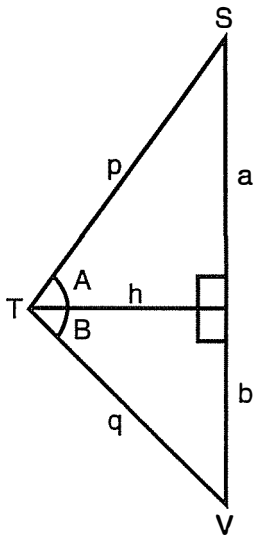


Figure 6

Area of triangle STV from lemma and $\frac{1}{2}$ (base) x (height)

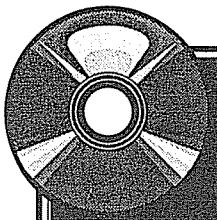
$$\frac{1}{2}pqSin(A+B) = \frac{1}{2}(a+b)h$$

Therefore,

$$Sin(A+B) = \frac{ah}{pq} + \frac{bh}{pq}$$

$$= \frac{a}{p} x \frac{h}{q} + \frac{b}{q} x \frac{h}{p}$$

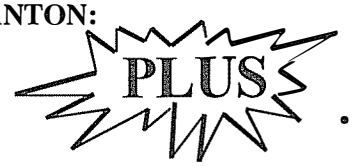
$$= SinACosB + CosASinB$$



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Halving the Hexagon: A Geometric Counting Problem

DAVID R. DUNCAN AND BONNIE H. LITWILLER
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
UNIVERSITY OF NORTHERN IOWA



The NCTM *Standards* encourage teachers to emphasize the connections between branches of mathematics. Geometry has often been treated as an isolated branch of mathematics. However, there are some interesting links which can be forged between geometry and, for example, combinatorics. One such connection arises from the dissection of a regular hexagon into six congruent triangles as shown in Figure 1.

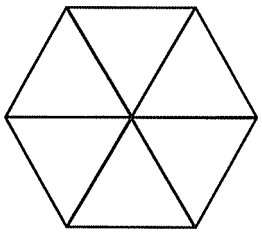


Figure 1

Question 1:

If the hexagon is assumed to have one unit of area, in how many ways can "one-half" be illustrated using a single colour? Assume that the hexagon may not be rotated or reflected.

To illustrate one-half, three of the six congruent triangles must be selected.

These can be selected in $\binom{6}{3}$

or $$\frac{6!}{3!\,(6-3)!} = 20 \text{ ways.}$$

Three of these ways are shown in Figure 2.

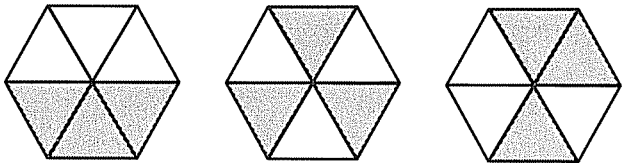


Figure 2

Question 2:

Question 1 required that all three chosen triangles be the same colour. What happens if more than one colour is permitted in identifying the three triangles which compose one-half?

Case 1:

Three colours (red, yellow, and blue) with each colour being used only once. The problem then involves a succession of two tasks.

Task 1: Choose the three triangles in

$$\binom{6}{3} = 20 \text{ ways.}$$

Task 2: Assign the three colours to these three triangles. This can be done in

$$\binom{3}{1,\ 1,\ 1} = \frac{3!}{1!\ 1!\ 1!} = 6 \text{ ways.}$$

Then, by using the Fundamental Principle of Counting, the number of ways that these two tasks can be consecutively performed is 20 **x** 6 = 120 ways. To illustrate, observe that the following six colour arrangements (using red, yellow, blue) are possible for the first arrangement of Figure 2.

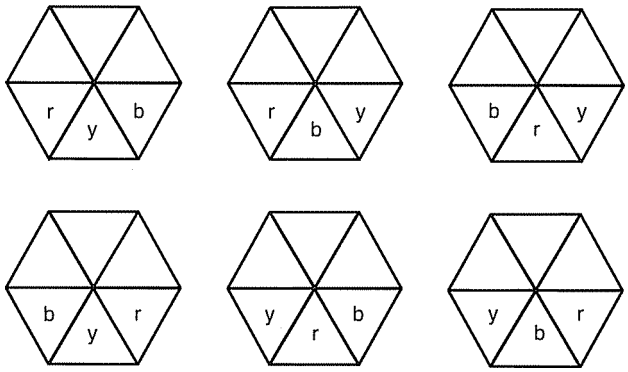


Figure 3

The same thing could be done for each of the 20 initial

representations of one-half. The students are encouraged to complete these cases as well.

Case 2:

Two colours (red and yellow) with each colour to be used a minimum of once, and a maximum of twice. There are three tasks needed to accomplish this assignment.

Task 1: Choose the three triangles. Again there are

$$\binom{6}{3} = 20 \text{ ways to do this.}$$

Task 2: Since two colours are to be used, one colour must be used twice and the other colour once. Thus we next select the two triangles which will have the "majority colour". This can be done in

$$\binom{3}{2} = 3 \text{ ways.}$$

Task 3: Assign the colours to these three triangles which have been designated in tasks 1 and 2. this can be done in

$$\binom{2}{1,1} = 2 \text{ ways.}$$

Again using the Fundamental Principle of Counting, the number of ways that these three tasks can be performed consecutively is 20 **x** 3 **x** 2 = 120 ways.

Figure 4 illustrates the two ways in which the actual colours can be assigned after the three triangles have been identified and a particular assignment of the two triangles to receive the majority colour have been specified. Let M represent the majority colour while m represents the minority colour in this hexagon. Students are encouraged to organize the unpictured cases as well.

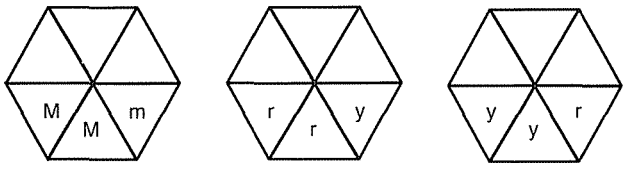


Figure 4

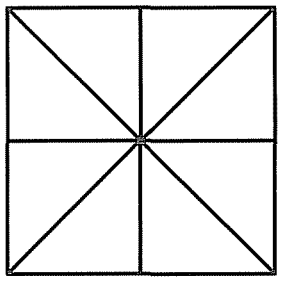
The grand total of all the ways in which the three triangles can be chosen and coloured to illustrate one-half is:

using one colour:	20
using two colours:	120
using three colours:	120
	260

Would your class have predicted that there would have been as many as 260 ways of choosing and colouring three triangles to illustrate one-half?

Challenges for the reader and his/her students:

1. Each of the methods described in this article assumes that the colours to be used are already determined. What would happen if the one, two or three colours were selected from a colour box of 64 colours?
2. It was initially stated that the original regular hexagon was fixed and could not be rotated or reflected. What would happen if rotations and reflections were allowed? Would more or fewer colouring possibilities result?
3. Suppose a square were divided into eight congruent triangles as shown. In how many ways could you select and colour triangles to illustrate one-half?



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▲ Book Review

PETER LENCZEWSKI
ADULT HIGH SCHOOL
O.B.E., OTTAWA, ONTARIO

ACADEMIA'S ADVANTAGE STUDY GUIDES - CALCULUS

ACADEMIA (1993) Inc., Toronto, 1993

Academia is a new company trying to find a niche in the mathematics textbook market that has been neglected until now. The Advantage Study Guides are tools designed to help students maximize their grades. They are specifically custom tailored for Ontario high school students, and are based on the Ministry of Education Curriculum Guidelines - Mathematics - Intermediate and Senior Division 1985 - Part Three. The Guides are **not** textbooks. They are **not** designed to replace neither the text nor the classes. What they really are is a consumable product (space is left in the book to work out most of the questions) to be used to review and prepare for tests and exams.

The first Guide to be published was the OAC Calculus. It comprises over 300 pages and covers the Ministry Guidelines very thoroughly. It is instructive to look at the format of the Calculus Guide which consists of four major components.

1. **NOTE SUMMARIES** - Very brief summaries of the main ideas. For this reason, students still need to attend classes and use their issued textbook (the Guide points this out). This section may contain some very basic illustrative examples. When important, the Guide develops theory from basic principles (e.g., the sum rule for derivatives).
2. **QUIZZES** - At the end of each section. Space is provided for working out the questions on the page, and *complete and detailed* solutions then follow. This is the section where most of the material is explained.
3. **PROBLEMS** - At the end of each chapter. The approach taken is to divide the page down the middle: the questions and space for solutions are on the left side of the page, while detailed solutions are provided on the right side. The student covers up the solution and works through the question. If a "sticking" point is encountered, the worked solution can be checked, but just enough to get past this point. A good variety of problems covers all the major types with very thorough, detailed and easy to follow solutions provided.
4. **EXAMS** - At the end of the Guide. A major resource, a set of three slightly different exams is provided. These try to represent typical exams that students may encounter. Again, full solutions are provided. Unlike the Quizzes and Tests, no space is provided for scratch work.

The first exam has an interesting mix of questions. The questions stress problem solving rather than skill or recall. It covers all major topics and is quite challenging.

The second exam, while similar to the first, contains more calculus application problems. I prefer it to the first, except for the lack of graphing questions. More interesting questions but not as balanced. It also is fairly challenging. The third exam is more conventional and tends to be more skill oriented than the other two. I found it to be the best balanced of the three.

In reading through the Guide, I had a sense of "deja-vu". Much of the work had a strong resemblance to my teaching notes, even to the point of highlighting the "tricks" or "sticking" points a student might encounter. The material was very easy to follow and very detailed without being overly verbose. Seldom did the Guide assume a step - everything was explained. This is the type of help students often need but cannot get in their texts. The Guide takes a very "common sense" approach, making it very user-friendly.

Within the constraints of its size, all major topics are covered in detail. The Guide delivers what it promises; it helps to fill in and prepare the student for tests and exams. It is particularly useful to the student who misses a class or needs to see a problem explained in detail.

Is it perfect? Close.

A few weaknesses were present, none of them critical.

TYPOS - Most have been corrected, but a few still got through. They are very obvious, and due to the clarity of presentation will be easily picked up by the student.

GRAPHS - Used when needed. However, I would like to see more graphs when solving certain questions. Though not required, the graphs make the explanations easier to follow.

STEPS - A *few* clarifying steps in the solutions to certain questions in the logs and exponents section were left out. These are almost trivial, but my experience has been that students really struggle with this section, and *nothing* is trivial to them!

ANTIDERIVATIVES - This section, the last one, seems to be a bit hurried. Nothing critical, just a sense of urgency that is similar to the one I get towards the end of the course as I struggle to finish everything before the exam. Maybe this is not a bad idea - it mirrors real life!

I wish to reiterate that the weaknesses were minor and detract little from an excellent student (and teacher) resource. At the reasonable price of $24.95, the Guide is almost indispensable for the students needing extra help in preparing for tests and exams.

I highly recommend it.

CALL FOR AWARD NOMINATIONS

LIFE MEMBERSHIP

To qualify for Life Membership in OAME/AOEM a candidate shall have:

- 10 or more accumulated years of membership in OAME/AOEM
- demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular

Deadline for nominations: January 31, 1995.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award, to be presented annually to a person who does not necessarily meet the criteria for the Life Membership Award (i.e., not necessarily an educator or an OAME/AOEM member.) The award is aimed at recognizing people who have supported OAME/AOEM, such as publishers, educational suppliers, hotel personnel, etc.

At this time OAME/AOEM is requesting nominations for consideration for this year's award, to be presented at the Annual Conference.

Deadline for nominations: January 31, 1995.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM has established the Morley MacGregor Award, to be presented annually to an elementary school teacher of mathematics who has demonstrated excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME/AOEM is requesting nominations for consideration for this year's award, to be presented at the Annual Conference.

Deadline for nominations: January 31, 1995.

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In Memory of the late Dr. K.D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, has established the Kenneth D. Fryer Award for Excellence, to be presented annually to a secondary school mathematics department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME is requesting nominations for consideration by the committee for this year's award - to be presented at the Annual Conference.

Deadline for nominations: January 31, 1995.

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Award (Check One)

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Please attach a separate sheet to the completed nomination form, outlining the reasons for your nomination and send to
Ron Lancaster, Past President OAME

▲ BOOK REPORT

PETER SAARIMAKI
MATHEMATICS CONSULTANT
TORONTO BOARD OF EDUCATION

Curriculum Frameworks for Mathematics and Science - TIMSS Monograph No. 1

by David Robitaille et al
Pacific Educational Press, 1993
102 pages

The Third International Mathematics and Science Study (TIMSS) will be administered by the International Association for the Evaluation of Educational Achievement (IEA - don't ask!) to students from over 50 educational systems in 1994-1995. As Canada, and Ontario, will be included, this little book (three chapters and four appendices) setting out to identify the basics of the review and inform the reader about curriculum comparisons, is a necessity for educators.

"To understand educational systems and to be able to draw valid comparisons among them, information about curriculum and about instructional practices must be available along with data on student outcomes." To accomplish this, the three major foci of TIMSS are described in one third of the chapters as : "an in-depth analysis of mathematics and science curricula, and investigation of instructional practices based mainly on teacher self-report data, and an assessment of students' mastery of the curriculum as well as their attitudes and opinions."

The conceptual framework for TIMSS, as is was for SIMS and SISS (math and science were previously tackled separately), involved studying three curricula:

- the intended (system) - the society at large
- the implemented (classroom) - local community
- the attained (student) - personal background

Data will be collected from a variety of levels and over a long period of time, but the main study will involve schools, teachers and students in the second quarter of 1995 (in the Northern hemisphere). Out of this TIMSS will report on such items as: opportunity to learn, students' achievement, use of technology, participation rates (particularly with regard to gender-based differences), tracking and streaming, the role of textbooks, and instructional practices.

Three populations of students will be studied centering around 9-year-olds, 13-year-olds and those in their last year of secondary school, both generally and for those specializing to some degree in mathematics or physics.

"The main intention of TIMSS is to provide detailed descriptions of student achievement and to explain variations in such achievement through description of national education systems and their intended and implemented curricula."

The last chapter goes into the framework details for coding math and science, including:

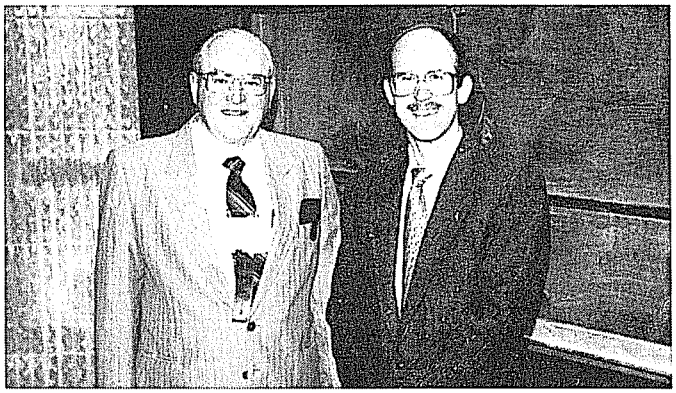
- content-numbers, geometry, functions, etc.
- performance expectations-know, reason, solve,...
- perspectives-attitudes, careers, interest,...

As well there is the textbook analysis process, with examples from various countries, and a sample curriculum guide analysis, using the NCTM Standards. The complete coding with detailed categories is given in the appendices (eg., percentages are 1.1.2.4 and 2.4.5 represents justifying and proving). Any assessment task would then have at least three codes to identify and compare it.

"Research (on curriculum) contributes by informing the decisions faced in practice, informing them not only with specific facts but also with ways of perceiving curricular situations, thinking about them, and acting upon them." We know from previous international (SIMS, IAEP) national (SAIP), and provincial reviews, that parents, the public and the media are always very interested in the results. By being more knowledgeable ourselves about the "tests" and how they work, we will be able to put the results in their proper perspective and work with other interested stake holders to improve mathematics programs.

A *little* book but definitely a good read and a worthwhile resource for the mathematics teacher, the science teacher, curriculum writers, administrators, and those with an interest in international comparisons in education.

OAME Conference 94



▲ OAME SPECIAL PROJECTS COMMITTEE ASSESSMENT PROJECT

"Bringing the Provincial Standards into the classroom: Activities that invite students to explain their reasoning"

The Ontario Provincial Standards for Mathematics provide a "metre stick" against which the performance of individual students can be compared. They also provide a framework within which teachers may begin to use alternative assessment strategies in mathematics. To achieve either of these possibilities, however, it is essential that teachers have available a source of exemplary assessment activities. The Ministry of Education and Training is in the process of planning to involve boards in the generation of activities. Unfortunately, the timeline for this project is unclear while the need for activities is immediate. Accordingly, the OAME Special Projects Committee, in conjunction with the Ontario Mathematics Coordinators Association, have undertaken the task of writing and field testing activities for the Transition Years. The activities will be published in a document for sale across the Province.

The project is planned to follow the timetable below:

October 94: 2-day writing meeting, during which initial activities will be generated and assessment planned
October-November 94: completion and refinement of sample activities
December 94: selection of activities for field testing; distribution to field test teachers
January-March 95: field testing of sample activities in Grades 7, 8 and 9
March 95: submission of field test results, including sample student solutions; review of results; put together final document
April-May 95: design of final document; printing
May-June 95: release of final document

The sample assessment activities will be written so that there is a focus on reasoning and so that students are invited to explain their reasoning.

Teachers who are interested in field testing the activities may contact:

Judy Crompton
P.O. Box 29017, 125 Carlton Street
St. Catharines, Ontario, L2R 7P9
Telephone (905) 685-7884

NOMINATIONS FOR OAME/AOEM COUNCIL 1995-96

Each year, elections are held for President-Elect, 3 Directors and 7-10 Councillors for the Council of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 31, 1995, to:

Ron Lancaster
Past-President, OAME

Include the nominee's qualifications with the Nomination Form. The Nomination Committee will make a final selection of those nominees who will stand for election. Their names and qualifications will be published in March, 1995. Voting will be by mail by members of the OAME/AOEM. The results of these elections will be made known at the Annual Meeting of the OAME/AOEM in Guelph in May, 1995.

NOMINATION FORM

I nominate: _____

Address: _____

_____

For the position(s) of: _____ **President-Elect**

_____ **Director**

_____ **Councillor**

_____ **Nominator**

_____ **Address**

_____

▲ About Problem Solving

JOHN GRANT MCLOUGHLIN
EDUCATION PROGRAM
OKANAGAN UNIVERSITY COLLEGE



Sources of problems vary from teacher to teacher. Some create their own problems to be solved while others may refer to books. Although the term "problem" seems to conjure widely di ferent images for math teachers, it is likely that most could benefit from enhancing their own awareness of potential sources. This article features one problem from a source that is familiar to many of you. However, it is unlikely that you have used the source in the way that I'd like to share.

While teaching a first year (Grade 9 level math) class at University of Toronto Schools, I required some time off to attend the OAME conference - a fine reason - in Ottawa. The students needed some assigned work. It seemed like an excellent opportunity to let them tinker with a few problems from different mathematical areas. Curiously enough, that month's NCTM Mathematics Teacher calendar contained a wide range of such material. Only three or four of the questions involved trigonometry or such topics beyond the knowledge base of these students. The assignment was straight forward: "Do any ten of the calendar questions." Upon my return, it was evident that the students had enjoyed this challenge. Many had attempted as many as 25 questions. I was surprised to find several correct answers that did not appear in the calendar solutions. We learned a lot discussing the questions.

The remainder of the article focuses attention on the discussion of one of the problems. Few opted to hand in a solution to this question. None of the solutions were correct but that was unimportant. We learned more because of that fact.

The question on May 26 read as follows:

A staircase has ten steps. You can take one or two steps at a time; you can take them in any order. In how many different ways can you go up the staircase?

Feel free to try the question before reading further.

The question posed difficulties because students were inclined to list all the ways and the list became too long. It is unlikely that one would correctly list all of the possibilities accurately. Such a situation suggests that it may be worth considering a simpler problem - shorter staircases.

Suppose that the staircase had only one step. Then, only one way would exist to reach the top. What would happen with two steps? You could walk up two steps initially or take two single steps. Hence, two possible ways would exist. Let us represent these ways as (2) and (1,1), thus, indicating the order and length of the respective steps. One may wish to construct a table in an effort to observe a pattern. The following table summarized the variety of ways to reach the top of staircases of various lengths.

# of Steps	Possible Ways	Total
1	(1)	1
2	(2),(1,1)	2
3	(2,1),(1,2),(1,1,1)	3
4	(2,2),(2,1,1), (1,2,1),(1,1,2),(1,1,1,1)	5
5	(2,2,1),(2,1,2),(1,2,2), (2,1,1,1),(1,2,1,1),(1,1,2,1), (1,1,1,2),(1,1,1,1,1)	8

The pattern emerged at this stage. Many noticed the Fibonacci sequence (a topic previously studied) appearing as the total number of ways. Hence, it was conjectured that 89 possible ways existed for climbing a staircase with ten steps. Note that this pattern was discovered through class discussion. Two more questions merited our attention.

Why does the Fibonacci sequence arise in this situation? Consider the final step taken. One can reach the top in either of two ways: a double step from two steps down or a single step from one step down. Note that these two ways are always distinct (mutually exclusive). Hence, if we know the number of ways to reach these two steps, we need only add the number of ways together to obtain the total number of ways of ascending the entire staircase. This is based on the same principle that underlies the Fibonacci sequence. Any term after the second one is equal to the sum of the two terms immediately preceding it. From a problem solving perspective, the discussion demonstrated the value of working backwards as a viable strategy.

Secondly, we wanted to find out if anyone could list or explain a process for finding the 89 ways. The easiest approach appears to involve counting the number of double steps taken along the way. Only one ways exists with no double steps and similarly with five double steps. If one double step is included, eight single steps are required. Hence, the double step can be taken at nine possible points on the ascent. If two double steps are taken, six single steps must also be included. This makes for eight steps, two of which are doubles. These may be taken in (8x7)/2 or 28 ways. Continuing in this matter, we find that there could be three, four or five double steps. These produce 35, 15 and 1 way respectively. Altogether the total number of ways is indeed 1+9+28+35+15+1 = 89.

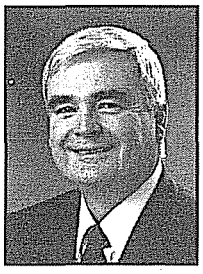
In closing, I would like to offer one question that appeared as the May 2 entry on the same calendar. It has numerous solutions. Try to generate an extensive list as a class. You are encouraged to send along solutions or any comments about problem solving.

Use eight different digits to form this product.



Council Comment

BOB ROBINSON
GOLDEN SECTION CHAPTER REPRESENTATIVE



PREAMBLE: It is with sadness and many, many fond memories of my consecutive years (about 23) on Council that I inform you that I will not be writing the "Council Comments" any longer. By the time you read this column I trust that all the "administrative" procedures for my move to Washington State with my wife Jo Anne and son Christopher, will be in place. I hope you, the reader, have found the column to be informative and it has provided you with an update of the activities of your Council. Thank you for your support and the knowledge I have gained from so many of you, my friends and esteemed colleagues.

My successor as the writer of Council Comments is Peter Saarimaki. Look for his "Comments" in forthcoming issues. Good luck, Peter. I know you will enjoy this assignment as much as I have.

These notes were taken at OAME Council Meeting held June 4, 1994 at the Ramada Inn-Airport West in Mississauga. Please send any correspondence directly to the editor.

GENERAL

A warm welcome was given to Sandy Woodcock, the representative of our newest chapter "TEY^{2}MES" pronounced "teams". Our new chapter consists of educators from Toronto, East York, City of York, and Metro Separate School Boards. Welcome Sandy!

Congratulations to Jack! Yes, our own Jack Weiner has received an Outstanding Teacher Award from OCUFA (Ontario Confederation of University Faculty Associations). Wonderful...well deserved!

SUMMER INSTITUTE

Look for an announcement in the near future about a joint ATMNYS/OAME summer institute for the summer of 1995. Council is in the discussion stage of hosting this annual institute in partnership with the Association of Teachers of Mathematics in New York State.

VISIONS

Todd Romiens, our New President, outlined the following issues and activities which OAME will have to address over the coming years.

- Exemplar items for the Standards Document
- How will the Specialization Years blend with the Transition Years?

Need to hear about the success stories with respect to the Transition Years. There still needs to be a lot of work done at this level. Need to hear about the transition to Grade 10.

- How to implement the "common curriculum" for Grades 1-9.

Need to hear about the successes with integration. How should an integrated unit look?

- Outcome based education: what is OAME's role?
- Look at creative ways to increase membership.
- Continue our partnership with the Ministry.
- Improve our liaison with the Faculties of Education.
- Continue to work with the Ministry of Education and Training.
- Ways to inform the media about the good things happening in mathematics education.

COMMITTEES

Early Years/Formative Years

The committee is planning to prepare a volume and capacity unit for the Abacus. An exemplar resource package is also being prepared.

Transition Years

The Communicating with Parents brochure will be completed this term. An annotated bibliography of non-commercial resources for teachers of the formative years will be compiled by the committee. If you wish to submit any materials, please contact Barry Scully in the Mathematics Department at the York Region Board of Education.

Specialization Years

Doug Evans and his committee are revising further the Specialization Years section of the Focus on Renewal document.

PUBLICITY AND MEMBERSHIP

John Rodger's committee is putting the final touches to the joint OAME/NCTM joint membership form. "Post-it" type notes are now available. There will be a focus on Faculty of Education students this term. Letters and packages will be sent out.

PUBLICATIONS

We are pleased to announce that Jennifer Weiner is the new Abacus editor. Good luck..best wishes, Jennifer! The committee is looking into including an assessment column, featuring for example, sample exams in future issues. Look for results of our readership survey in an upcoming issue.

FUTURE CONFERENCES

This small committee is in the process of recommending sites to the Executive for our future Annuals. A conference manual is being developed.

MATH PROMOTION

This committee is concerned about our connecting and communicating with the public. They are exploring several "avenues". If you have any ideas, please contact Ron Lancaster.

SPECIAL PROJECTS

One project is to generate exemplary assessment instruments to support our Provincial Standards. We will work jointly with OMCA. They will also explore alternate assessment strategies for the Transition Years.

MATH EQUITY

The May breakfast in Kingston was a huge success. The speech by Anne Jones was excellent and it has become such a popular event that some people could not get in.

The committee will continue to offer "breakfasts" at our Annuals. Other issues to be addressed by the committee are:

- Focus on multicultural aspects on equity
- Mathematics representation from minority groups
- Physical and verbal assault issues
- Address representation by elementary teachers on the Council and in the Association

NCTM REPRESENTATIVE

Our "new" NCTM rep. is Mickey Sandbloom. Welcome back Mickey!!

Kingston Conference

Over 900 attended this wonderful conference. Well done, all you hardworking folks in the Kingston area.

Guelph Conference

Plans are heating up and progressing well for the conference at the University of Guelph. If you have any questions contact Jack Weiner, Department of Mathematics and Statistics at the University of Guelph.

LEADERSHIP

The Leadership Conference will focus on the Primary to Transition Years. The theme is: CONNECTIONS: PRIMARY THROUGH TRANSITION. The "Leadership" is always successful, but the registration is limited to 72 people; so...don't be disappointed and get your request in early.

CHAPTERS

COMA

The first annual Ontario Mathematics Olympics will be hosted by COMA on Saturday, June 3, 1995 in Ottawa. Each chapter will receive: sample activities, suggestions for organizing a local competition, and further details about accommodations, registration costs, etc. Two teams of four will represent each chapter. Each team will consist of two girls and two boys, two from Grade 7 and two from Grade 8.

GOLDEN SECTION

A response to the Specialization Years questionnaire has been sent via Eric Muller to the committee. Plans are underway for our October mini-conference for Primary and Junior teachers. The chapter will arrange a visit to Niagara for IMO'95 participants. Any student who can speak the language of any of the visitors from the various countries and would like to assist should contact Eric Muller in the Mathematics Department at Brock University.

GVMA

Plans are proceeding well for the 1995 Conference at the University of Guelph. Grand Valley's publications sold well at Kingston. The Grade 9 destreamed exam package sold out. More will be produced.

NOMA

The executive is finalizing its schedule for its fall conference which will be in Sudbury on Friday, October 14 and Saturday, October 15. The theme is "Making Connections".

NWOAME

Almost 200 Canadian Mathematics Competition participants from Zone 61 (roughly from Hornepayne to Kenora) were awarded Honour Roll certificates at the Lakehead U./NWOAME Awards Day. NWOAME'94 is scheduled for September 23-24.

PINE RIDGE

Congratulations to Ryan O'Donnell for his achievement of a gold medal in the 1994 Fermat Contest. Ryan is a student at O'Neill Collegiate in Oshawa. O'Neill Collegiate was also the winner of the "Reach for the Top" Durham Mathematics Competition held in May.

SAME

The Fencott Award for Remarkable Progress in Mathematics was awarded to a student who took math at the general level until grade 12 and is now finishing her OAC's.

TEY^{2}MS

During the past year math educators from Toronto, East York, the City of York, and the Metropolitan Separate School Board came together to establish this new chapter. They are currently meeting in committees to plan the mini-conference for Oct. 27 which will be held at St. Patrick Secondary School in the MSSB.

Y^{4}MA

The fall conference will be held November 15, 1994. They will be working on the International Math Olympiad that will take place at York University in July, 1995.

This is "Ten-Four" my good buddies!

Bob

O.A.M.E. 1994-1995 Council

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To Be Announced

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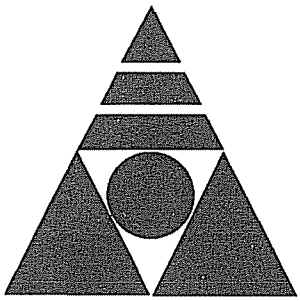
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