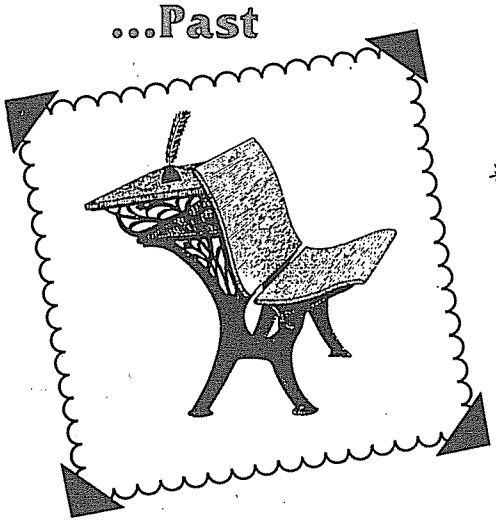
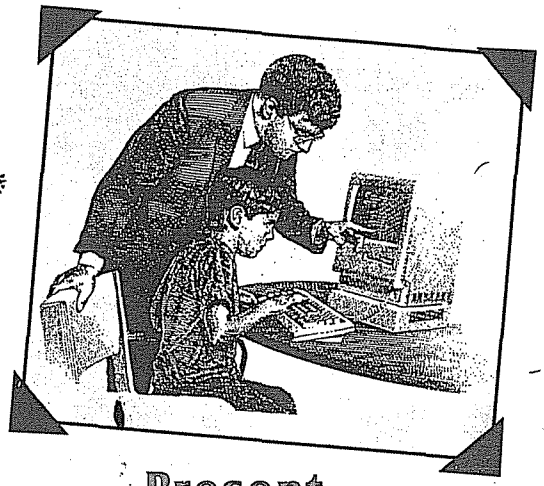
Vol. 33 #2, December 1994

Ontario Mathematics Gazette

Ghosts of OAME....

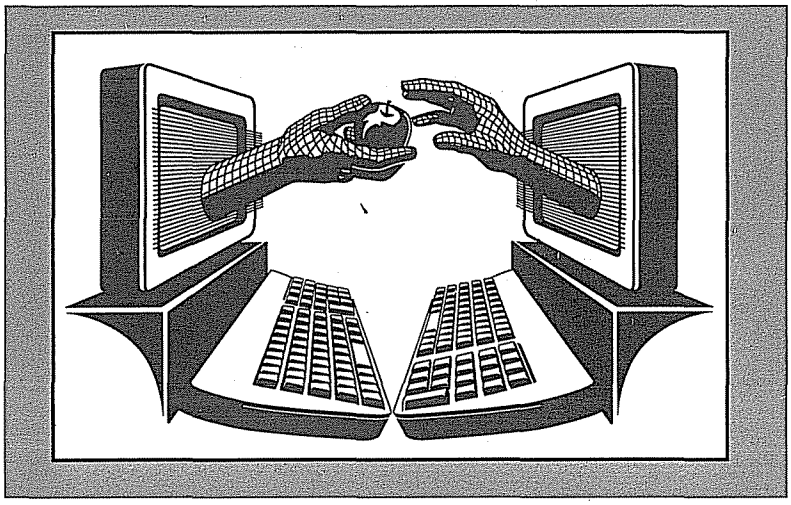
...Past





...Present

...Future



As we reminisce about the past... •Don & Carol •Council Comments •Hazing
cope with the present... •Assessment •Creative Writing •Problem Solving •Destreaming
and look to the future... •Calculator Graphics •Inquiry •Have Your Say •Bonnie & Dave

OAME wishes you a *Happy Holiday Season!*

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

About the Gazette

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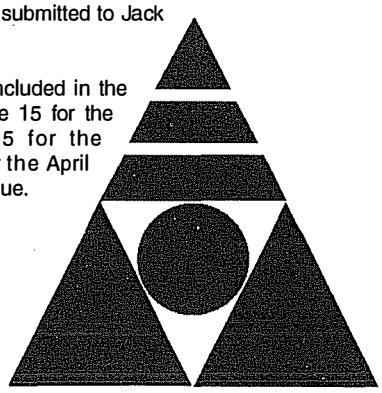
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Submission of Articles

Articles for the Gazette should be submitted to Jack Weiner at the above address.

Deadlines for information to be included in the Gazette or the Abacus are: June 15 for the September issue, October 15 for the December issue, January 15 for the April issue, and April 15 for the June issue.



Notice to Authors

Your articles should be typewritten, word-processed or photocopied from typewritten or word-processed copy double-spaced on 8.5" x 11" paper with 1.5" margins and single sided. Diagrams and figures should be in camera-ready form. Please submit three copies to the editor and keep one as protection against loss. We have all submissions blind-reviewed, so please send your full address and affiliation on a page separate from the manuscript.

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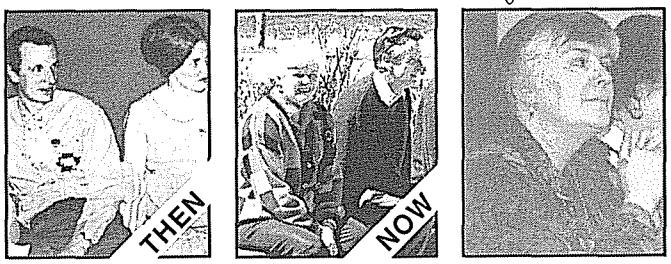
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▲ Editorial

I wish you all a happy and healthy holiday. And now, here's my friend Jeri Lunney.

Jack



A Tribute to Don and Carol

BY JERI LUNNEY

How well I remember my first year as a Director of OAME. There were several new faces around the table, including Don and Carol Attridge. As we learned our roles and got to know each other, I remember wondering how we would survive without Morley and Mona MacGregor to guide us.

Let me reminisce for a few moments about the five years that I worked so closely with the Attridge team as I progressed from Director to President and Past-President. They are indeed a team, complementing each other both personally and professionally. Don worried over budgets as Carol kept us on track at meetings. Carol's warm smile and caring attitude made us all feel a part of the OAME family. Her expertise in organizing conferences and writing terms of reference stand out as examples of her professionalism. They both played the role of host at meetings and conferences, welcoming everyone with their warmth. I think of Don's dedication to having OAME enter the computer age, his worries over deficit budget and his amazement when we made a profit on every venture. How well I remember when Don announced, with his usual flair, that OAME now had over 2000 members. But their garage was always filled with our latest project. OAME invaded their personal lives in every way.

In these years, we saw many changes in Council: four Council meetings instead of three; regular Friday night committee meetings; four issues of the *Gazette;* significant publications such as the *Focus on Renewal;* and active participation in a variety of curriculum activities. OAME played a leading role in NCTM Leadership conferences and had waiting lists for almost every Leadership Seminar. As the Ministry cut back in personnel, OAME became the group that the teachers in our province counted on for leadership. Through all the changes, Don and Carol worked diligently at our sides.

We shared successes and failures, happy times and disappointments, personal and professional tears of joy and frustration. I am glad to be able to thank them, on your behalf, for their dedication in their roles as Executive Directors. Farewell, Don and Carol. Keep in touch.

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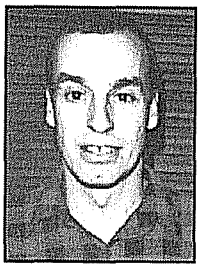
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▲ Have Your Say

STEVE THORPE
MANITOULIN SECONDARY SCHOOL
WEST BAY, ONTARIO



Steve Thorpe has taught Math, Physics and Computer Science for twenty years. At present he is Mathematics Department head at Manitoulin Secondary School. Previously Steve taught in British Columbia and in Malawi, Africa.

As a high school Math teacher for twenty years I have witnessed the evolution of calculators from an expensive toy of the rich to an affordable tool of the masses. A calculator that will last a student through their high school years costs less than a baseball bat.

Have our mathematics courses and our mathematics teaching kept up with the technological changes? I don't think they have. In secondary school we still expect students to add, subtract, multiply, and divide fractions and mixed numbers on paper. Most new cheap scientific calculators have fraction keys (ab/c keys) that allow students to perform these operations with a few keystrokes. We should teach the students how to do this. Another area that should be revised is the work with exponents, especially negative and fractional exponents. The better students will remember how to evaluate 8^{-3}, even weeks after we teach it. Many of the others will give you a negative number as an answer. Students who have learned how to use their calculator effectively will be more likely to give the correct answer.

In the advanced level we spend a lot of time and effort on factoring quadratic trinomials. The students need to learn how to factor so that they can solve quadratic equations. This skill may be important but so are many others for which we don't have time. Calculators costing less than $25 can give the students the roots in a flash.

I'm not advocating that we throw out the encouragement of basic skills, nor should we stop teaching factoring and operations with fractions. But we shouldn't beat our heads against the wall by insisting that all our students become as proficient as we were in "paper calculation". The students have the calculators; let's show them how to use them effectively. Perhaps they will come to enjoy mathematics more if we remove some of the drudgery.

Editor's Note: Hugh Allen's article (page 4) is very closely related to Steve's comments.

▲ President's Message

TODD ROMIENS



Welcome to the real world. When students perform poorly on math and English reviews, we teachers are criticized. When they perform well, we are still criticized, ie., standards are too low. Such was the comment made by our Premier on the results of the latest English review. Mr. Rae also commented, "Failure is not something to be avoided. It is a necessary part of learning."

What ramifications do these statements have for our classrooms if they are carried out? What will be the pronunciation of the Royal Commission on Learning? Perhaps answers to these and other questions will be forthcoming when the Ministry releases its findings sometime before the new year (scheduled date). At the same time the revised Common Curriculum document and Mathematics Standards will also be released.

I have had the opportunity to be on the working team revising the Standards document. I think you'll like the new format. It is cleaner and things you need to learn will be easier to find.

While I'm talking about the Standards document, here is an update on our Special Projects Committee's work on the exemplar items. A successful two day meeting was held in October. Guidelines were discussed on writing exemplar items for each strand and working teams are busy preparing some of them now. As you know, this is a very large and onerous task. The total package cannot be produced in a year, but we have started well. My thanks go to all the members of OAME and OMCA who are doing a wonderful job. We look forward to seeing and presenting the results.

At Council's request, a revision of a draft letter was sent to Education Minister, Dave Cooke, expressing our desire to be kept in the forefront of the Ministry's agenda when it comes to proposals for writing exemplar assessment items. OAME and OMCA have volunteered on your behalf, to oversee and inservice teams from across the province in writing these items. To date, I have not received a reply.

On a sadder note, you all are aware that Don and Carol Attridge have resigned their duties as Executive Directors of OAME. Don and Carol have brought a wealth of expertise and organization to our group that has both inspired us and moved us forward. It has been through their guidance and leadership that we have grown and prospered over the last several years. On behalf of OAME, I would like to thank them for all the time and effort they have put in for us, and wish them a long and healthy retirement. Don and Carol, OAME will miss you both.

I also would like to welcome our new executive directors, Dave and Bonnie Alexander. They are no strangers, as you know, to the math community. They bring a wealth of experience in math education to our organization and I know they will be a tremendous asset.

These are changing times, and exciting times in mathematics education. Have confidence that our association will help you implement the changes. It is easy to focus on the negatives, but keep your chin up, and continue to look forward to Monday morning (not retirement).

▲ Presenting Dave & Bonnie Alexander

DON FRASER
FACULTY OF EDUCATION
UNIVERSITY OF TORONTO



Editor's Note: Most of you know by now that Dave and Bonnie Alexander are OAME's new Executive Directors. I asked Dave's University of Toronto colleague, Don Fraser, to reminisce about his longtime friend.



At a September staff meeting at the Faculty of Education, University of Toronto a few years ago, Dave stood up and eagerly described his experience at a Co-operative Learning Institute he had attended in August. Two years from retirement, after 30 years of teaching, why would he bother giving up part of his summer for that?

The answer is part of what makes Dave tick.

First, Dave is a conscientious, hard worker. Given a job, he gives it 100%. At the Faculty of Education, the various Deans learned to give the tough committee jobs to Dave. Secondly, Dave is a lifelong learner. No big ego here, he's always been willing to explore the latest approaches in teaching, always willing to listen and learn from others. Thirdly, since Dave is such a team player, co-operative learning struck a responsive chord with him.

Dave would be the last person to speak of his accomplishments, but let's look at a few highlights.

- Author of the Council of Ministers of Education, Canada Report on Mathematics Assessment 1993
- Local Organizer, Working Group 18, Methods of Implementing Curriculum Change, ICME 1993
- Curriculum Leader, Provincial Review Mathematics-Grade 10 General, Grade 12 Advanced 1989-1992
- Project Leader and Writer of the Provincial Mathematics Guidelines 1985
- Chairman of Local Arrangements for the Annual NCTM meeting held in Toronto 1982
- Convention Chairman for the regional NCTM meeting held in Toronto 1973

While all this was going on, Dave earned an Ed. D. degree from SUNY in Buffalo in 1977, was Chairman of the Math Education Department at the University of Toronto 1976-1981, and WAS THE FIRST PRESIDENT OF OAME 1973/74.

No wonder that the University of Waterloo in 1982 awarded Dave a Descartes Medal for signal service to Mathematics in the schools of Ontario and no wonder that in 1986 OAME made Dave a Life Member.

Dave is not a one-dimensional person. For years he has been a keen sailor and curler. Even after moving to Lake Rosseau last year, Dave would drive down to Toronto for his Wednesday evening curling. His curling buddy Elgin Wolfe says that Dave "loves the game, is a good strategist, and is competitive but a real gentleman." That's the Dave I know too.

The Alexanders, Bonnie and Dave, are a team. When the NCTM Conference was in Toronto, Bonnie headed up the Hospitality committee. Bonnie found more things for the visitors to do in Toronto than I knew existed and at a reasonable price.

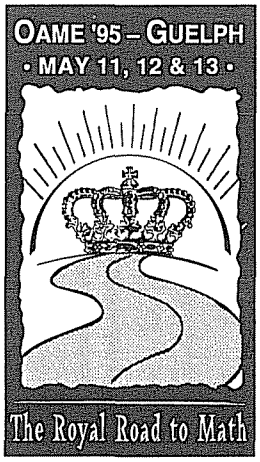
Bonnie and Dave don't live in the past. Creating a new home on Lake Rosseau over the last two years is evidence of that. They bring to the job of Executive Directors competence acquired through the years and an energy and a joie de vivre that defies the years.

But until you've seen Bonnie and Dave, in the wee hours of the morning, swirling around the dance floor, you ain't seen nothing yet.

Welcome home to OAME, Bonnie and Dave.

22nd Annual OAME Conference

le 22e Congrès Annuel de l'AOEM



University of Guelph, Guelph, Ontario

May 11,12 &13, 1995

HOSTED BY: **The Grand Valley Mathematics Association**

▲ Mathematics in Schools: A Form of Intellectual Hazing?

HUGH ALLEN
QUEEN'S UNIVERSITY
KINGSTON, ONTARIO



Hugh Allen has taught mathematics and mathematics education in high schools and universities for over thirty years. Recently retired, Hugh will continue to pursue his interests in music and outdoor activities in retirement.

Editor's note: Hugh includes, with I hope at least a little satisfaction, your current editor as one of his admiring graduates.

Before reading on try to figure out what is being determined in the computation to the right.

```
                    40 + 6 + .8 + .03
                  | 102 712.000 000
1600              |  64 000
                  |  38 712.000 000
4800              |
 720              |
  36              |
5556              |  33 336
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6348              |
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6459.04           |   5 167.232
                  |     209.768 000
6570.72           |
   4.212          |
   0.0009         |
6574.9329         |     197.247 987
                        12.520 013
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Some years ago, you might have seen students in a high school mathematics class doing several computations similar to these.

Not many teachers today would recognize the formal method for finding the cube root of 102 712 correct to two decimal places, but it was common practice - at least according to the starred exercises in the book my father used - in high school about 1913 or 1914. Fortunately, we don't put students through this kind of exercise today. Yet, until about twenty years ago, the formal method for *square* root was a regular staple in the mathematical diet of high school students.

The introduction of calculators meant that students could determine square roots to any reasonable degree of accuracy with a couple of key strokes. Even if the calculator did not have a square root key, students could use a simple, meaningful, iterative process to determine the required square root quickly. And yet, the square root algorithm continued to be taught for many years after calculators were available to be used in schools. Why was this the case? And why does it continue to be the case that students are required to learn algorithms and skills that are no longer necessary given the intent of the curriculum and the technology readily available to students?

It is my contention that the main reasons these topics continue to be taught are that teachers know the topics, are comfortable with them and like to teach them. They make for a nice, neat, skill-type lesson. The teacher is able to do an example or two and then the class can be set to work practising the skill for hours, days or, as in the case of my first example of such an unnecessary algorithm, for months.

An Elementary School Topic

I'm referring to the paper and pencil algorithm for long division. Hands up all those who filled in their own income tax form last year and *chose* to do the calculations involved by means of the standard paper and pencil algorithms rather than by a calculator. In fact, when was the last time you did a long division question of any complexity (say, three-nonzero digits into a five or six digit number)? For most of us, it was either in school when we studied the long division algorithm, or just before we got our first calculator. Yet we continue to see pupils in grades six, seven, and eight practising the long division algorithm. (In some "progressive" classes they're allowed to check their answers by using their calculators.) Do children need to know this algorithm? If, as I believe, they do not, then why is it still being taught?

As I suggested above, one reason is that teachers are comfortable with the algorithm and find it friendly to teach. That does not mean that it's an easy technique for students to master - many fail to do so and, in failing, experience tremendous frustration. If you think about it, you'll realize that the challenging task is to get students to know *when* to divide one number by another and to *estimate the result* before undertaking the division. Getting the answer is really not very difficult - punch, punch, punch and there it is. But does the answer make sense in the context of the problem we're trying to answer? Students should estimate to get a rough idea of what the answer ought to be, and then use a calculator to get the answer to the degree of accuracy appropriate to the question. Now that's not easy to teach!

But the main reason why the long division algorithm continues to be taught is that teachers learned it themselves

when they were in school. And look at us. We're good people and we did long division problems by the pageful; so, *(and here's the key, although most adults won't admit it)* if you want to be a "good person", then you need to do lots of long divisions too. The use of this utterly illogical "rationale" is not new. It appears implicitly in statements like: "When I was your age, I_____." (Fill in the blank with "walked three miles to school" or "held down two jobs" or whatever event you believe to be responsible for the character you possess and your audience lacks.) I can imagine parents in ancient Babylonian times decrying their children's use of Egyptian papyrus. "When I was your age, I carried around five heavy clay tablets wherever I went. What will you do if it rains and your papyrus tears, or you lose your writing implement?"

It's hard to break with tradition - even if there are good reasons for doing so. Tradition gives us rites of passage we believe to be important. Until recently, it was a tradition that medical interns worked long hours - sometimes twenty-four or more at a stretch - even though the medical profession knew that the quality of care was likely to decline significantly toward the end of an intern's shift. But they (the established order) did it, and what was good enough for them was good enough for the new kids.

The military knows all about tradition and brings a boot-camp mentality to much of what it does. They may argue that the constant hazing of recruits who don't fit in is designed to provide a greater sense of camaraderie, but that's hogwash; there are much better and more constructive ways to generate a spirit of teamwork. Even the universities have abandoned much of their traditional hazing of freshmen in favour of undertaking constructive activities designed to give new students a sense of belonging to the community and the institution.

Sometimes, unfortunately, many of the hazing activities are continued out of sheer vindictiveness - "We had to go through this, and by God, you new chaps will too" - although the ordeals are frequently justified as necessary in order to "maintain the standard of the profession (club, etc.)." Very male behaviour!

Tradition, and history in general, may tell you what you should not continue to do. But the reasons for what you choose to do must be grounded in the present and should take into account current attitudes and values as well as the appropriateness of current available technologies. A mathematics curriculum should not appear to have been written by a dead hand. The content must be appropriate for today's students and must reflect the attitudes and needs of society and the current state of technology. Such a curriculum would probably not include a paper and pencil algorithm for long division as a required topic, although it might be included as an historical or cultural topic such as Gelosia multiplication or Chisanbop.

Secondary School Topics

The secondary school mathematics program still contains many examples of outdated, traditional topics many of which mystify students. One reason for their confusion lies in the fact that most of the algorithms are algebraic in nature. For many students it's difficult enough to make sense of algorithms involving twos, threes and thirty-fives, but when the manipulations involve symbols like $a + b$, x^2 and xy^2z it can be utterly confusing.

A delightful illustration of the problems students have with algebraic manipulation occurs in the book *The World of A.J. Wentworth, B.A.* by H.F. Ellis. Wentworth is a mathematics master in an English public (i.e., private) school. One day he had kept the class in because they had become "rather silly and over-excited about taking out some brackets" when converting a product of the form $(a + b)(a - b)$ to a difference of squares $a^2 - b^2$. One of the brighter students complained that in the previous class they had expressed $a^2 - b^2$ in the form $(a + b)(a - b)$.

"Yes, Mason?" I said, not quite understanding what he was driving at.

"Well, sir, now we have started with $(a + b)(a - b)$ and got it to $a^2 - b^2$."

"Exactly. That shows we were right yesterday, doesn't it?"

"Yes, sir, but there doesn't seem to be any end to it. When it's $a^2 - b^2$ we have to work away and get it to $(a + b)(a - b)$ and when it is $(a + b)(a - b)$ you're still not satisfied, sir, and we have all the trouble of making it into $a^2 - b^2$ again, sir. Couldn't we jolly well make up our minds which is best and leave it alone, sir?"

Another boy shouted, "It's not fair, sir," and in a moment the whole set was talking at once, urging me to 'make up my mind' and 'leave well enough alone', and a lot more extravagant nonsense. One boy even had the impertinence to offer odds of three to one on $a^2 - b^2$. [1]

The problem underlying this fictitious student's concern is shared by a great number of real students today. Algebra seems to be a game in which you manipulate symbols for no apparently good reason. Wentworth has failed to teach Mason and the others *when* you might want to express $a^2 - b^2$ in the form $(a + b)(a - b)$ and when you want to leave it as it is. This is the challenging part of teaching. When do you want to think of thirty-five as $30 + 5$ and when do you want to think of it as 5×7? The tough part is knowing when to manipulate an expression into an equivalent form. The manipulation itself is not the challenge. Indeed, many of the manipulations are unnecessary.

For example, here is a favourite topic of teachers that could be left out of the program: "completing the square". I can practically hear the sudden inhalations of breath, the sound of palms struck against chests and claims that, "You have to do 'completing the square' so that students can put a function like $3x^2 - 2x+1$ in the form $a(x - p)^2 + q$ in order to graph it." **WRONG!** There are at least two ways to accomplish what you want without teaching the completing the square technique. I

know it makes a lovely lesson, but that's not the point. The methods below (particularly the second method) build on what the student already knows without requiring new techniques to be learned.

Students' understanding of the graphs of quadratic functions develops through successively graphing functions *of the form* x^2, ax^2, $ax^2 + q$ and $a(x - p)^2 + q$ (one hopes through the use of computer graphing facilities or graphing calculators). In this way, they come to see the algebraic expression of a vertical stretch, af(x); a vertical shift, f(x) + q; and a horizontal shift, f(x - p) on the graph of a given function, f. Students can describe the graph of a quadratic function (axis of symmetry, vertex, etc.) if the function is expressed in one of the forms above. The technique of completing the square usually arises when students need to express a quadratic function given in the form $ax^2 + bx + c$ in the form $a(x - p)^2 + q$.

Example 1 - Method 1

Suppose, to take a specific case, that we want to express the function $3x^2 + 2x + 1$ in the form $a(x - p)^2 + q$.

That is, we want

$$3x^2 + 2x + 1 = a(x - p)^2 + q$$

Expanding the right side gives

$$3x^2 + 2x + 1 = ax^2 - 2apx + (ap^2 + q).$$

Comparing coefficients on both sides of this identity gives

$$a = 3,\ -2ap = 2 \text{ and } ap^2 + q = 1.$$

Solving successively for a, p and q gives

$$a = 3,\ p = -\frac{1}{3} \text{ and } q = \frac{2}{3}.$$

Thus $3x^2 + 2x + 1 = 3(x + \frac{1}{3})^2 + \frac{2}{3}$ and the properties of the graph such as vertex, axis of symmetry, etc., are known to the student as a result of earlier work.

That wasn't so tough, although it may have involved more algebra than you might have wished. If so, you'll love

Example 1 - Method 2

The problem is to graph and describe the function f where $f(x) = 3x^2 + 2x + 1$. The graph of f is identical to and one unit above the graph of g where

$$g(x) = 3x^2 + 2x.$$

The student's knowledge of quadratic functions says that the graphs of f and g have the same axis of symmetry and the vertex of the graph of f is one unit above the vertex of the graph of g.

The axis of symmetry of g is a vertical line lying mid-way between the zeros of g. The zeros of g satisfy the equation $g(x) = 0$; that is, $3x^2 + 2x = 0$.

Solving $3x^2 + 2x = 0$, or equivalently, $x(3x + 2) = 0$, gives

$$x = 0 \text{ and } -\frac{2}{3}.$$

Thus the zeros of g are 0 and $-\frac{2}{3}$ and therefore the axis of symmetry of g (and hence also of f) has the equation $x = -\frac{1}{3}$.

The vertex of the graph of f lies at the intersection of the axis of symmetry and the curve and hence has abscissa equal to $-\frac{1}{3}$ and ordinate equal to $f(-\frac{1}{3}) = \frac{2}{3}$.

Method 2 has the advantage that it introduces no new techniques, but rather, it builds upon and strengthens concepts the student has already learned. Completing the square is an unnecessary technique. The student can do everything based upon their existing understanding of the behaviour of graphs "Okay," you say, "but you still need to know how to complete the square in order to develop the quadratic formula." **WRONG!** There are at least three different ways to develop the quadratic formula without using any completing the square technique. (In the examples below, specific functions are used although the method is completely general and can be applied to the general quadratic equation $ax^2 + bx + c = 0$. Also, it is assumed that the student can solve simple quadratics that "factor".)

Example 2 - Method 1

This method uses the student's understanding of quadratic functions and their graphs. Suppose the problem is to solve the equation $x^2 + 2x - 5 = 0$. The left side doesn't factor (rationally).

The problem is equivalent to finding the zeros of the function f where

$$f(x) = x^2 + 2x - 5.$$

The zeros of f are equally spaced on either side of the axis of symmetry, and the axis of symmetry of f is the same as the axis of symmetry of g where

$$g(x) = x^2 + 2x.$$

The zeros of g are 0 and -2, and hence the axis of symmetry of g (and also of f) has equation x = -1. (Students would likely be able to identify the equation of the axis of symmetry of the graph of f fairly quickly from their work with problems of the type presented in Example 1 - Method 2.)

The zeros of f must be of the form -1 + k and -1 - k (because they're equally spaced about the axis of symmetry).

Hence, f(-1 + k) = 0 and f(-1 - k) = 0.

Substituting gives:

$$(-1 + k)^2 + 2(-1 + k) - 5 = 0$$

$$\text{and } (-1 - k)^2 + 2(-1 - k) - 5 = 0.$$

Expanding and *adding* the two equations gives

$$2 + 2k^2 - 4 - 10 = 0,$$

from which $k^2 = 6$ and hence, $k = \pm\sqrt{6}$.

The method is perfectly general and when applied to $ax^2 + bx + c = 0$ gives the zeros of the corresponding function to be of the form $-\frac{b}{2a} \pm k$ where k turns out to be $\pm\frac{\sqrt{b^2 - 4ac}}{2a}$.

Example 2 - Method 2

Like the previous method, this one uses the fact that the axis of symmetry of the graph of the corresponding function $f(x) = x^2 + 2x - 5$ has equation $x = -1$. Now shift the graph of f one unit to the right so that its axis of symmetry is the vertical axis.

The resulting function, h, satisfies $h(x) = f(x - 1)$ and the zeros of h are exactly one unit to the right of the zeros of f.

From $h(x) = f(x-1) = (x - 1)^2 + 2(x - 1) - 5 = x^2 - 6$, the zeros of h can be determined to be $\pm\sqrt{6}$ and hence the zeros of f (and the roots of the given equation) are found by shifting these zeros back one unit to the left, giving $-1 \pm \sqrt{6}$. Again the method is perfectly general and can be applied to give the familiar quadratic formula.

Example 2 - Method 3

This approach is based upon the student's understanding of quadratic functions. Example 1 illustrates two ways to convert a function of the form $ax^2 + bx + c$ to the form $a(x - p)^2 + q$. Thus, the equation $ax^2 + bx + c = 0$ can be rewritten in the form $a(x - p)^2 + q = 0$, and this last equation can be solved directly.

No doubt there are many other ways to generate the quadratic formula without completing the square. So there appears to be no good reason to spend time teaching this technique. (Other uses of the technique - such as identifying conics in non-standard position - can also be handled by methods similar to that used in Example 1 - Method 1.) Of course, university teachers of calculus might object that "completing the square" is needed in order to handle integrals of rational functions having quadratic denominators.

A Traditional University Topic

Agreed, but this begs the question of whether or not so much time should be spent on techniques of integration such as integration by trigonometric substitution, partial fractions, integration by parts, etc. Perhaps this topic, too, survives largely because of tradition. After all, the purpose of learning all these techniques of integration is usually to be able to evaluate the definite integrals in later applications.

The only problem with this line of reasoning is that the software that is (or should be) available to students of the calculus can be used to evaluate a definite integral to any reasonable degree of accuracy. All calculus teachers know that the challenge for the student is recognizing *when and how* to express a quantity as a definite integral. The calculation of the definite integral is most easily and efficiently handled by a machine.

Note the striking similarity between the "problem" of the elementary school student's use of calculators to perform divisions and the calculus student's use of a computer to evaluate a definite integral. In both cases, the difficult part is understanding what the process (division or forming an integral) means and when it should be used; the actual computation is easily accomplished. There is also a similarity between the response of the traditionalist to the two situations. Cries of "What will you do if your batteries run down or you forget your calculator?" and "You won't be allowed to use it on the exam," are replaced with "What will you do if there's a power failure or there isn't a computer free at the time you want to use it?" and "You won't be allowed to use it on the exam."

Arithmetic and mathematics today are done with calculators and computers. Instruction that fails to recognize this fact is out of step with current thinking. Exams that do not permit the use of tools normally available to a practitioner simply don't make sense. (All right, Dr. Intern, your test will be to see if you can remove that patient's kidney without the use of anaesthetic or surgical implements.)

Tradition can keep us mired in the past if we're not very careful and constantly reassessing our needs and resources. Current practices should reflect current contexts.

Other "Traditional" Secondary Topics

An examination of the high school curriculum reveals several other traditional topics whose time may be at an end. Much of the manipulation of radicals, while in radical form, and particularly the technique of "rationalizing the denominator" seems to be unnecessary given the use of calculators. It is important that students know that there is a number which when multiplied by itself gives the number six, and that the number in question can be written $\sqrt{6}$, but from that point on the value 2.449 can be used instead. Problems requiring students to obtain closed-form simplifications for expressions such as

$$\frac{\sqrt{8} + \sqrt{50} - \sqrt{18}}{6\sqrt{3} - 2\sqrt{5}}$$

seem pointless given the availability of calculators. Traditionally, we've manipulated radicals and required students to be able to convert entire radicals to mixed radicals in order to obtain an answer in closed form. Is there still a good reason for continuing to require this? (The writers of recent textbooks have made a concerted effort to downplay manipulations of this type.)

Another topic for re-examination is long division of one polynomial by another. Grade 12 teachers claim that they need to teach long division of polynomials (sometimes even including the synthetic form) in order to get on with the subsequent work on the factor theorem. **WRONG!** In fact, you don't need long division in order to obtain the quotient and remainder when $2x^4 - 5x^3 - 3x^2 - 6x + 7$ is divided by $x - 2$ (or even by $x^2 - 3x + 1$, for that matter). There is a very quick and

simple approach that avoids the use of the long division algorithm and that reinforces and builds upon the student's understanding of the meaning of division and its relationship to multiplication. The long division algorithm is not necessary in order to allow students to use the Factor Theorem. In fact, given the availability of symbolic manipulation programs, the traditional inclusion of the Factor Theorem itself might be in need of re-assessment.

No doubt, a close scrutiny of the high school mathematics program would reveal several other topics whose presence in the curriculum would be difficult to justify on anything other than a traditional basis.

Contests and Competitions

What about the very best mathematics students who enjoy taking part in mathematical competitions and contests? It could be legitimately argued that they need to know techniques such as completing the square and be familiar with a wide variety of algebraic and numerical manipulations. And the reason they do is that many of the questions posed on these competitions favour students with these skills. Most contest papers seem to value the content and approaches destined to be de-emphasized in school mathematics - at least according to the NCTM *Standards.* It could be argued that these contests are thinly-veiled attempts to justify the retention of certain traditional techniques in the school curriculum. As long as success in contests requires candidates to know traditional algorithmic procedures, these algorithms will prevail - although I doubt that we'll return to teaching obscure factoring types such as $a^3 + b^3 + c^3 - 3abc$. However, the contests and competitions are designed for our best students and they seem able to handle whatever is sent their way.

Summary

In some ways, the decision as to what constitutes appropriate content in a mathematics curriculum comes down to what you mean by "mathematics". There was a time when arithmetic was a university subject. The expansion of mathematical ideas, combined with changes in technology, have resulted in several metamorphoses of the elementary and high school mathematics curriculum in the past 200 years. What it means to "do mathematics" has changed - witness the lack of acceptance of recent computer proofs in some parts of the mathematical community. Mathematics may still be a creation of human minds, but the resources available to those minds are vastly different than they were even fifty years ago. In the light of current technology, quaint algorithms such as paper-and-pencil long division and the formal methods for extracting cube roots and square roots, as well as the need to rationalize denominators and perform other algebraic and arithmetic manipulations may be of historical interest only.

We need to ensure that the topics we teach are not simply reflections of the topics we were taught in school, and that the kind of mathematics we ask students to do is not chosen solely because we did that kind of mathematics and are familiar with it. The program must make sense in light of the demands of today's society and the availability of current technology. This is the direction in which the NCTM *Standards* is leading us. The standards described in this NCTM document, along with their recommended changes in emphasis in content and instructional practice make school mathematics today very different from the mathematics done in schools as recently as twenty years ago. Reports such as the *Premier's Council Report* make clear that the algorithmically-based curriculum we experienced as students will not give us graduates with the problem-solving and interpersonal skills needed for the future. For us to ignore the directions indicated by documents like the *Standards* and the *Premier's Council Report* in favour of teaching outdated topics and traditional, "comfortable" material amounts to little more than a form of intellectual hazing. It may be that many of our favourite mathematics topics and lessons of the past no longer serve the best interest of the students. The "paces" through which students should be put must lead them to the mathematics for *their* time.

Footnote

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The Carleton-Ottawa Mathematics Association

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Chapters will collaborate with the schools and boards within their areas to select the teams. COMA provided each Chapter with a comprehensive registration package, including sample activities, at the September Council meeting.

CONTACT PERSON:

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▲ Creative Writing Experiences in the Mathematics Classroom

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Jennie Lynne Lamb was a high school student in Ms. Ryan's class. She is currently working on a degree in Broadcast Journalism at the University of Texas at Austin.

Reading, writing, and mathematics can be effectively integrated through creative writing experiences. The major requirement should be to relate the writing experience to pertinent mathematical concepts. Teachers and students can have fun in the classroom while exchanging information and ideas.

Standards for School Mathematics

Mathematics as communication in the *Curriculum and Evaluation Standards* promotes students' clarification and reflection about mathematical ideas. The *Standards* also emphasize reading, writing, discussing, and listening in the mathematics classroom. These play important roles in learning and using mathematics.

Mathematics as communication can be achieved through creative writing assignments. Students are given opportunities to communicate, first through the discussion of ideas, then through their written expression of the ideas. When they "talk mathematics" they construct knowledge, connect ideas, and develop a deeper understanding about mathematics (*NCTM*, 1989).

Classroom discussion, before writing begins, can aid in the understanding and connection of concepts (Hoyles 1985). As students explain thoughts and ideas, they modify each others' ideas as they modify their own. As they write their stories, they continue to clarify and develop a better understanding of concepts (Lappan and Schram 1989). This paper addresses how students and teachers can work together, through language and writing activities, and how that work pertains to the *Professional Standards for Teaching Mathematics* (1991).

Standards for Teaching Mathematics

According to the *Professional Standards,* a change in current teaching practices is needed. Teachers should provide students with a variety of opportunities through individual, small-group, and whole-class work and lead them to question and answer a variety of problems. Students will not only gain new knowledge in the process, but will also make connections with previously learned information.

Creative writing can be assigned at any grade level. The assignment can be expanded beyond the writing experience. The writing can take on the form of a journal entry. Students can be assigned to write a research paper, dramatize a story, or make a variety of videos such as situation comedies, music videos, commercials, or news reports (Lamb, Lamb, and Ryan, 1990).

Assigning a creative writing project in isolation will not be enough to interest or encourage students. Teachers should use the *Standards, as a guide,* and let teaching take on new meaning. With practice, teachers can comfortably and happily include writing activities as they supplement mathematics instruction.

Second-Grade Explanations of Regrouping

Jenny had been teaching subtraction with regrouping for about one week. The students were given many opportunities to verbally exchange ideas and Jenny, after one week, was ready to assess their understanding. She wrote two subtraction problems and the following statement on the chalkboard:

33	33
-12	-26

I need to trade one ten for ten ones when writing the problem _____ because.....

This problem was designed to be the students' journal entry for the day. The students were to choose one of the two problems, then explain why regrouping would be necessary.

This was not only a good way to get the students to reflect on subtraction with regrouping but also a way for Jenny to determine their level of understanding. In an interview Jenny said, "I wanted them to not only be able to show, but to verbalize and communicate understanding through written form." She provided copies of some of the journal entries.

Jerry:

When doing 33-26 = 7 because when you have this problem 33-12 = 21 you don't have to brak apart and 33 - 26 you do and also you trade because thar is not 6 to take away so you need to take apart a ten.

Faye:

I need to trade becaus when you can't answer 3 - 6. Why becaus if you have 3 apples you can't subtract 6 apples you half to trade the other number like 26 - 38 you can't do that. I mean you need to brake apart to get a higher number than 6 so you can subtract blanck minus 6 =

Kathy:

I need to trade 1 ten for 10 ones when doing the problem 33 - 26 = 7. Because you cannot take away 33 - 26 = You need to break apart a ten in order to get enough ones to do the problem. You will take away the whole from the part and get the other part.

As Jenny reflected on the students' journal entries, she became quite enthusiastic; she was happy to see them write about their thinking. She believed some students showed their understanding but also found that some were just going through the motions. Some students had memorized the procedure relating to the subtraction algorithm but were unable to explain the reasoning behind the process. She knew she had some additional work to do before they came to a real understanding of subtraction with regrouping.

A Fourth-Grade Sample

Sandra often searched for projects that would interest her fourth-grade students. She enjoyed integrating media and mathematics. On one occasion, she proposed a creative writing assignment. She left the assignment open-ended, wondering about the possible stories that would evolve. Sandra was quite impressed by Jennie's story:

All is well in the world thanks to Multipliable Girl. She knows the multiplication facts better than anyone and her knowledge keeps everyone safe.

Everything was going well at school until the mathematics teacher started giving division for months at a time. One student was very happy in the class; he could do division without scrap paper and soon everyone realized he was the Dreaded Divisor. He lived in fear of Multipliable Girl because she was the quickest with her multiplication facts. He decided to do away with her and demanded a showdown. There would be no killing, instead, they were going to outsmart each other.

The showdown began. Multipliable Girl posed a division problem and the Dreaded Divisor answered. Then he posed a multiplication problem and she answered. Multipliable Girl had a plan ... she would let him take the lead, then she would say a few zero problems and the Dreaded Divisor would be defeated.

When the Dreaded Divisor was in the lead, Multipliable Girl asked, "What is zero divided by seven?" The Dreaded Divisor began to get nervous, he didn't like to think about zero, but he correctly answered, "zero." He asked, "What is nine times nine?" Multipliable Girl quickly answered, "Eight-one." Then she asked, "What is six divided by zero?" The Dreaded Divisor knew he had been tricked and ran screaming from the room.

The Dreaded Divisor never suspected that anyone knew that dividing by zero was not possible. Multipliable Girl's plan worked, she was able to outsmart the Dreaded Divisor. Once again, all is well. Multipliable Girl's work is done but when she is needed, she is always ready.

Application to the *Professional Standards*

The journal entries and the story of Multipliable Girl are examples of what students can do when given the opportunity to write. Teachers can use the *Standards* as a guide to encourage and foster mathematical knowledge and growth. By using the *Standards,* the teacher can easily cover all important aspects of a mathematics lesson.

In order to incorporate the *Standards* into instruction, teachers will need to: a) develop worthwhile mathematical tasks: b) determine their role in the classroom discourse; c) define the student's role in discourse; d) construct tools for enhancing discourse; e) develop a secure learning environment; and f) become skilful in their analysis of teaching and learning. The *Curriculum and Evaluation Standards* and the *Professional Standards* describe what teachers should know and be able to do in the mathematics classroom. The remainder of this article will address the application of these standards to classroom situations.

a) *Worthwhile Mathematical Tasks.* Before the lesson can begin, the teacher should be knowledgeable of students' backgrounds and interests. An informal survey can be done the first day of school or each time a new mathematical topic is introduced. Teachers must encourage students to make connections and should use interesting and motivating tasks which promote the understanding of concepts.

"Good tasks are ones that do not separate mathematical thinking from mathematical concepts or skills, that capture students' curiosity, and that invite them to speculate and to pursue their hunches" (***NCTM,*** 1991). Offering the creative writing experience is just one example of how teachers can extend the students' range of thinking and curiosity in mathematics. This will be an invaluable experience for students, but teachers will also benefit. Teachers will learn what motivates their students, what their interests are, and gain insight into their thinking.

b) *The Teacher's Role in Discourse.* Assigning a writing experience without discussion and participation could be counterproductive. The teacher needs to involve students in discussion, listen to their ideas, and challenge their thinking. A creative writing lesson could begin with the teacher reading a story like Multipliable Girl to the class. The class would discuss the story and its implications to mathematics. If the goal were to assign a creative writing story, the next step would be to discuss other heroes and villains, legends, and science fiction characters. How might students solve some mathematical dilemma using a different character? The teacher's priorities would be to promote thinking and provoke reasoning.

Discussion is the beginning of a creative writing process. As students call out names of heroes, the teacher writes the names on the board. The teachers asks, "What might this hero do?" Then, the students are guided through a discussion of the latest mathematical concept taught. The teacher continues to question, "What problems have you had in this area and what problems might your characters have? How can they be helped?" "How could you have been helped?" All of this is written on the board which will refresh the students' memory as they begin to reflect on the discussion. At this point the assignment could be to work with a partner to write a story or write a journal entry that reflects a new question, a dilemma, any concerns, or what was learned.

c) *Student's Role in Discourse.* The teacher's job is to provide the environment; the student's job is to listen, respond, make connections, and solve problems. Once the classroom discussion has taken place, students should be given a chance to take some of the ideas discussed and talk them over with partners. This will not only help them formulate ideas but also synthesize ideas and concepts.

After talking over ideas with partners, students begin writing rough drafts. As they write, they continue to reflect and connect ideas and concepts. With the completion of the rough draft, students should be given a chance to meet with partners again. They can then read each others' papers and offer suggestions. Once corrections have been made, students write their final copy and prepare for class presentations and discussions.

If writing reflections in journals is the given assignment, a rough draft won't be necessary. They may begin writing as soon as an assignment is given. They may also exchange viewpoints with a partner before beginning the writing process.

d) *Tools for Enhancing Discourse.* A variety of tools to stimulate students' thinking should be easily accessible. The teacher should act as facilitator as the students write and discuss ideas. Students may use cubes to describe the shape of a hero or a city, or they may use graph paper as they reflect on ways to solve a geometric problem. If students are experiencing difficulty, the teacher might suggest using a different tool. Variety is the key...whether cubes, blocks, beans, colour markers, graph paper, calculators, or computers...as long as there are choices, the students will be stimulated.

There will be many opportunities to make mathematical connections when students take turns presenting their stories. After each story, discussions dealing with the story (characters and content) can revolve around various approaches to reasoning. Not only will students see ideas presented in many ways, the writer will also have a chance to reflect upon the interpretations given.

e) *Learning environment.* Crucial to the success of this as well as any other activity in the classroom, is the learning environment. The teacher must first provide a safe, secure environment, one in which students are willing to take risks. The teacher and students must respect each others' ideas. The teacher must also make it known that mistakes are acceptable. Only then will students take the risk to expose their creativity.

These are not one-lesson activities. Students will need sufficient time to make sense of the content and the activity. Special folders can be provided for "ongoing work". The teacher should be available and should plan to move around the room in order to listen, converse, and guide. A necessary part of individual attention is the probing and connecting of ideas whenever possible. The *Professional Standards* describe the learning environment of the classroom as a community filled with people collaborating to make sense of

mathematical ideas.

f) *Analysis of Teaching and Learning.* Creative writing experiences open the door for ongoing analysis. The teacher, as facilitator, constantly observes as students work, question, and respond. The teacher can begin evaluation as journal entries are read or as the class discusses Multipliable Girl. Constant interpretation and reflection are required. The teacher should be wondering about the connections students are making and making mental notes as discussion and observation progresses. The teacher might consider questions such as:

- How did students react when they found division by zero was impossible?
- Does anyone still think it's possible?
- Can you think of other problems that might have similar results?
- What about feedback on their "writing skills" grammar, spelling, etc.?
- Can you write, in your journal, other examples or problems that require regrouping?
- Draw a picture of how you'd subtract with regrouping.
- Can you give me an example or a problem that would require two regroupings?

As students write, the teacher can observe and evaluate each student's thoughts and ideas. During the observations the teacher can guide, challenge, and extend students' ideas and help to develop their reasoning abilities. Through this process, the teacher will know if a student is understanding concepts or developing misconceptions.

Conclusions

The creative writing experience can be considered a worthwhile mathematical task. Once a teacher has an idea, the *Professional Standards* can be used as a guide. The teacher should know if the assignment challenges and interests the students. Will the assignment engage their intellect and challenge them to make connections? The teacher should remember to listen to students' ideas and decide when and how to connect mathematical ideas and language to those ideas. The teacher must provide a secure environment, one in which students can question, exchange ideas, and read their stories without fear of humiliation.

The teacher should constantly gather information about his/her students. Are students connecting previously learned information to new information? Are they learning sound and significant mathematics? Are they developing positive attitudes towards mathematics? What will the next step be? What more do they need to know?

Teaching means questioning, listening, and observing. By fitting a variety of activities into the *Professional Standards,* teachers may have an easier time assessing the worth of their efforts. With enough use, not only will the ideas become internalized, but also their practice.

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As most of you know, Canada is hosting the International Mathematics Olympiad in July, 1995 at York University.

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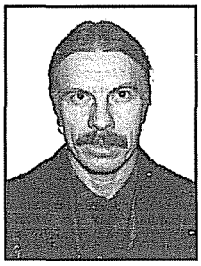
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▲ PICTURE THIS!

BY RAY NOWAK
PEEL REGION BOARD OF EDUCATION



Ray Nowak is currently Head of Mathematics at Bramalea Secondary School in the Peel Board of Education. His Math & Music presentations have been seen at many conferences including NCTM (annual) in Nashville and ICTCM (International) in Chicago. Ray's classroom explorations for the TI-82 will be published by Brendan Kelly Publishing in 1994.

Visualization is a very powerful right-brain investigative and creative tool in mathematics. Graphing, for example, can be used to "view" the behaviour of algebraic functions, to "see" the solution for non-trivial equations, and to "picture" real or artificial shapes and symmetries. Numerical techniques can "supercharge" this visualization process. For instance, simple algorithms can make approximate "eye-ball" solutions very precise, plot "curves" that cannot be expressed as explicit functions, and generate "images" requiring pixel by pixel calculations.

The graphics calculator gives students personal access to this synergistic mathematical approach. The following procedures and programs were used by high school students to explore the applications above. The examples are not developed as self-contained teaching units, but are designed as utilities that can be executed to investigate the impact of this technology. The programs are listed in the Appendix and are written for the Texas Instruments TI-81 calculator, but can be modified for other graphics calculators.

Equations & Inequations

1. The obvious use of the graphics calculator is to graph functions. Equations can easily be solved by writing them as functions and finding the zeros (x-intercepts). For example, the equation

$$\sin^2 x + x = 2$$

can be solved by graphing the corresponding function entered into the calculator as

$$Y_1 = (\sin X)^2 + X - 2$$

and locating the zero(s) using the TRACE and ZOOM features.

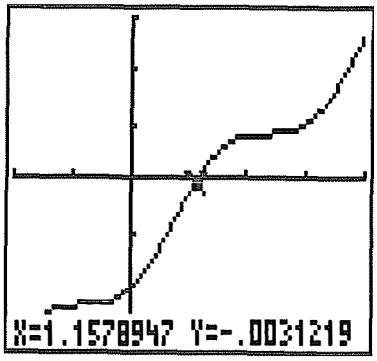


Figure 1.

Figure 1 shows the graph of Y_1 and its zero $x \approx 1.16$ after "tracing" the function using viewing window RANGE values

$$-2 \le X \le 4 \quad \text{and} \quad -3 \le Y \le 3$$

and calculator in radian MODE. Using "Zoom In" several times displays a more exact solution $x = 1.1596953$.

2. Another related application involves solving inequations by graphing. Messy absolute value inequalities such as the following are usually solved by factoring and using the case method:

$$|x^2 - 3x - 10| < |x^2 - 6x + 8|.$$

The calculator solution requires three simple steps:

a) rearrange the inequation to obtain zero on the right side;

$$|x^2 - 3x - 10| - |x^2 - 6x + 8| < 0$$

b) enter the left side of the inequation into the calculator as a function;

$$Y_1 = \text{abs}\,(X^2 - 3X - 10) - \text{abs}\,(X^2 - 6X + 8)$$

c) graph Y_1 and determine the intervals (x values) for which the function is negative.

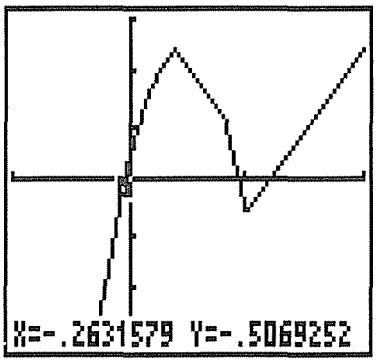


Figure 2.

Figure 2 shows the TRACE feature being used to approach the right end point of the first interval in the solution. The RANGE values for this viewing screen are

$$-5 \le X \le 10 \text{ and } -15 \le Y \le 15.$$

3. A simple numerical program NSOLVE (prgm 1) employing the Newton-Raphson method can be used to find more precise zeros for function Y_1 as follows:

a) enter the PRGM menu and select EDIT screen to key in the program (see TI-81 Manual),

b) GRAPH the function and use TRACE to move the cursor to the desired zero,

c) enter PRGM menu and EXECute the NSOLVE program.

The three "numerical" zeros of the last function were found and used to define the following intervals for the inequality solution:

$$(-\infty, -0.212) \text{ and } (4.712, 6.000).$$

Students can use these routines to visualize and solve any equations or inequations involving combinations of built-in functions on the calculator.

Parametric Equations

1. The parametric MODE of the TI-81 can be used to obtain quite intricate graphs using simple "polar" functions. A neat way to define the function (so as to key it in only once) is to enter it as parametric equation X_{2T} and to use X_{1T} and Y_{1T} to convert it into rectangular coordinates. For example, the constant function r(T) = 1 is entered as:

$$X_{1T} = X_{2T} * \cos T$$

$$Y_{1T} = X_{2T} * \sin T$$

$$X_{2T} = 1 .$$

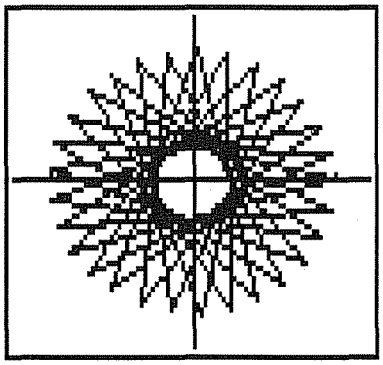


Figure 3.

This generates the SUN in Figure 3 by connecting 29 (4321÷149) points on a polar circle using RANGE values

$$0^\circ \le T \le 4321^\circ \text{ , Tstep} = 149^\circ$$

$$-1.25 \le X \le 1.25 \text{ , } -1.25 \le Y \le 1.25$$

and "Connected Line" graphing MODE.

2. Functions X_{1T} and Y_{1T}, and RANGE were modified as follows to transform the 9-petal polar rose curve defined in X_{2T} into the BUMBLEBEE in Figure 4:

$$X_{1T} = X_{2T} * \cos 4T$$

$$Y_{1T} = X_{2T} * \sin 7T$$

$$X_{2T} = \sin 9T$$

$$\text{with } 0 \le T \le 360^\circ \text{ and Tstep} = 2^\circ.$$

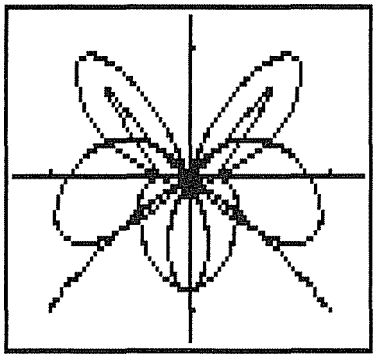


Figure 4

Students can alter constants 4, 7, and 9 in the trig functions to produce other wild looking graphs. Parametric equations can also be used to generate many curves involving physical motion such as cycloids, conics and spirals (Yates 1974).

Calculus Graphs

1. Another application allows us to approach the derivative with a new slant. The built-in TI-81 MATH function NDeriv was used to plot the numerical derivative or "slope function" Y_2 of the defined function Y_1 (in radian MODE) by entering:

$$Y_1 = \sin (2\sin X)$$

$$Y_2 = \text{NDeriv}(Y_1, 0.001).$$

Both functions were "turned on" and plotted together on the GRAPH screen shown in Figure 5 for RANGE values:

$$0 \le X \le 6.283, \quad -3 \le Y \le 3, \quad \text{Xscl} = 1.57.$$

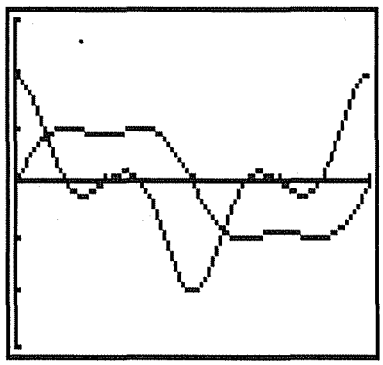


Figure 5

Calculus students can find the zeros of graph Y_2 (with Prgm 1) if Y_1 is changed to Y_2 in line 4 and use them to calculate the local "max-min" values for Y_1. They can also "take the derivative of Y_1", enter it into Y_3, and check their answer with

the graph of Y_2. For this example,

$$Y_3 = 2 * \cos (2\sin X) * \cos X.$$

2. A short numerical program INTPLOT (prgm 2) employing Simpson's Rule can be used to plot the integral or "area function" for Y_1. Figure 6 shows previous graph Y_1 and its integral.

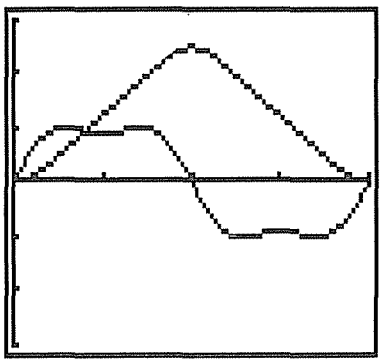


Figure 6

Some observations are necessary:

a) the original function must be defined in Y_1 (unless changed in the program);

b) in order to see the original function also, "turn on" function Y_1;

c) the numerical value of the definite integral (AREA) of Y_1 from Xmin to Xmax can be obtained after running INTPLOT by pressing ENTER (and this accuracy can be increased by decreasing half-step size H in line 3);

d) INTPLOT can be used to check answers to "find the integral of Y_1" questions by comparing graphs;

e) functions that are tough or impossible to integrate symbolically (such as Y_1) can be plotted and analyzed;

f) you cannot ZOOM or TRACE a graph DRAWN from a program (on the TI-81).

Implicit Functions

1. Graphing implicit functions f(x,y) = 0 presents an interesting exploration. RELPLOT (prgm 3) can be used to plot any relation in x and y, such as

$$x^3 + y^3 = 3xy.$$

In order to graph this equation using the program, three steps are required:

a) rearrange the equation to obtain zero on the right side;

$$x^3 + y^3 - 3xy = 0$$

b) replace y in the equation with v (since variable y gives TI-81 error message 17) and enter the left side as a function;

$$Y_1 = X^3 + V^3 - 3XV$$

c) enter PRGM menu and EXECute RELPLOT.

The program takes a few minutes to complete the graph since it systematically checks for a "sign" change in Y_1 using (x,y) values of every pixel on the screen.

A better visual approach is to plot the corresponding inequation f(x,y) < 0 by changing the 12th program line from

"If FZ < 0" to "If Z < 0".

This produces Figure 7 in which the "border" of the filled-in region represents the graph of the original FOLIUM OF DESCARTES equation for window values

$$-2 \le X \le 2 \text{ and } -2 \le Y \le 2.$$

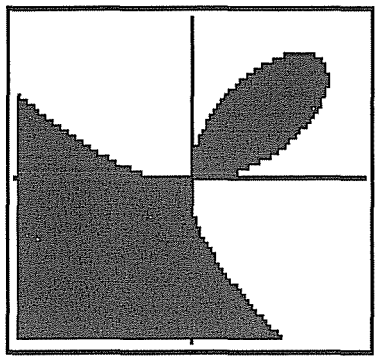


Figure 7

There are many other famous curves that can be "sketched" using this program (Yates 1974, 199).

2. The SNOW ON THE WINDOW PANES in Figure 8 is the plot of the extraordinary relation

$$y - [y] < x - [x]$$

with $-3 \le X \le 3$, $-3 \le Y \le 3$, and where [ ] represents the greatest integer function (TI-81 MATH function "Int").

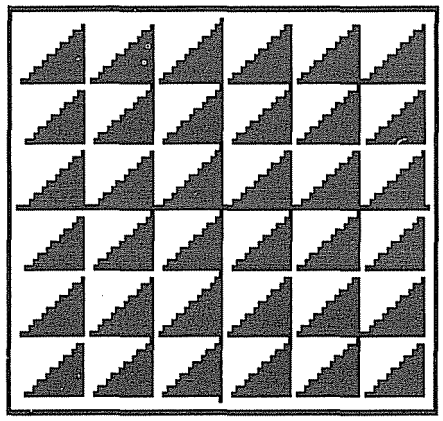


Figure 8

Fractal Plots

In the final application, the graphics calculator was programmed to generate FRACTAL or "self-similar" shapes.

1. IFSPLOT (Prgm6) creates fractal images using the "chaos game" algorithm (Barnsley 1988, 91). This iterated function system (IFS) "randomly" selects affine transformations from a set of four pre-determined transformations that can be represented by the matrix equation

$$T_i \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_i & b_i \\ c_i & d_i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix}$$

where i = 1,2,3,4. The first transformation chosen is applied to any point (x,y) in the plane, the second to the image point of the first, the third to the second image point, and so on. Image points are plotted after the tenth iteration to exclude spurious points. The screen RANGE values for the FERN in Figure 9 are:

$$-3 \le X \le 3, \quad 0 \le Y \le 11, \quad Xscl = 0, \quad Yscl = 0.$$

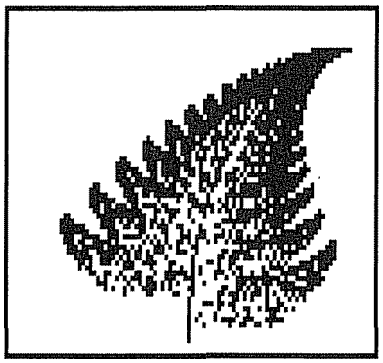


Figure 9

The first four rows of calculator matrix A are used to store the six coefficients of each transformation such that row i contains coefficients a,b,c,d,e,f for transformation T_i (see Appendix). The 5th row of matrix A is used to store data that is used by the program so that the probability of transformation T_i being applied in any iteration is given by P_i where:

$$P_1 = 0.85, \quad P_2 = 0.07, \quad P_3 = 0.07, \quad P_4 = 0.01.$$

This helps produce a fairly "good" rendering of the image with only 5000 iterations. Students can create other fractal forgeries by changing the elements of matrix A (Barnsley 1988, 87).

2. JULSET (Prgm 4) plots a class of fractal shapes known as Julia sets. An iterative process uses a complex transformation such as

$$T(z) = z^2 + c$$

to plot points in the plane. The transformation in the program is rewritten in terms of its real and imaginary parts so that

$$T(x,y) = (x^2 - y^2 + a, 2xy + b)$$

where z = x + yi and c = a + bi. The value of the constant c for the SEAHORSE VALLEY in Figure 10 is defined in program lines 3 and 4 where

$$a = -0.74543, \quad b = 0.11301.$$

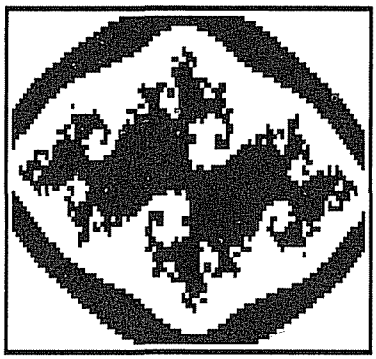


Figure 10

Other exotic fractals can be produced by changing the values of a and b (Peitgen and Richter 1986, 193). Each pixel on the screen is given (x,y)coordinates such that

$$-1.5 \le X \le 1.5 \text{ and } -1 \le Y \le 1.$$

Each point (x,y) has transformation T applied to it, and then reapplied to the resulting image point, and then reapplied again to the image point resulting from that, and so on. If the image points do not escape a circular region of radius $\sqrt{10}$ from the origin after 40 iterations, then the original pixel becomes a member of the set of seahorses' pixels. The ring surrounding the seahorses is formed by pixels for which an image point escaped the boundary radius in exactly 3 iterations. The running time of the program is cut in half using the origin symmetry of the graph. Using computers, this simple procedure can produce stunning, colorful, complex fractal art (Peitgen and Richter 1986).

3. MANDSET (Prgm5) uses the same complex transformation T defined in prgm 4 to graph the famous Mandelbrot set which is the "blueprint" for Julia sets.

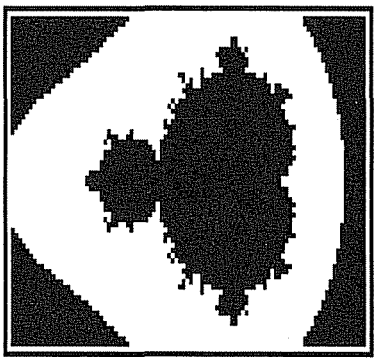


Figure 11

Each pixel on the screen is given coordinates (a,b)such that

$$-2 \le a \le 1 \text{ and } -1 \le b \le 1$$

where c = a + bi and the transformation T varies as c changes for each pixel. Prgm 5 uses exactly the same iterative procedure as Prgm 4 except that the initial starting point for transformation T is always z=(0,0). The MANDELBROT SET in Figure 11 was produced using only 30 iterations. The program exploits the x-symmetry of the set to reduce graphing time by factor two.

Conclusion

The graphics calculator offers a stimulating visual approach to the study, application, and exploration of mathematics that will soon be economically accessible to most students. It has the potential of making many traditional skills, such as factoring to solve equations, almost obsolete, while rendering many new topics, such as fractals, quite relevant to the curriculum. These are truly interesting times!

Endnotes

CLRAXES (Prgm 7) was written as a subroutine for the "fractal" programs (4,5 and 6) to get rid of the axes. Variables D and E represent the dimensions ΔX and ΔY for each pixel on the screen and are global. All figures were produced and stored as files using the Texas Instruments PC-81 emulation software.

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Barnsley, Michael, *Fractals Everywhere,* Boston: Harcourt Brace Jovanovich, 1988.

Peitgen, H.O., and P.H. Richter, *The Beauty of Fractals,* New York: Springer-Verlag, 1986.

Yates, Robert C., *Curves and Their Properties,* N.C.T.M., 1974.

Appendix

Prgm1:NSOLVE

```
:X→Z
:Lbl 1
:Z→X
:X-Y1/NDeriv(Y1,0.001)→Z
:If abs (X-Z)>abs (X/1E10)
:Goto 1
:Disp "X="
:Disp Z
:End
```

Prgm2:INTPLOT

```
:ClrDraw
:DispGraph
:(Xmax-Xmin)/200→H
:0→S
:Xmin→X
:Y1→L
:Lbl 1
:X→U
:S→V
:X+H→X
:Y1→M
:X+H→X
:Y1→R
:H(L+4M+R)/3→A
:S+A→S
:Line(U,V,X,S)
:R→L
:If X<Xmax
:Goto 1
:Pause
:Disp "AREA="
:Disp S
```

Prgm3:RELPLOT

```
:All-Off
:ClrDraw
:PT-On(0,0)
:(Xmax-Xmin)/95→D
:(Ymax-Ymin)/63→E
:Ymin→V
:Lbl 1
:Xmin→X
:0→F
:Lbl 2
:Y1→Z
:If FZ<0
:PT-On(X,V)
:Z→F
:X+D→X
:If X≤Xmax
:Goto 2
:V+E→V
:If V≤Ymax
:Goto 1
:End
```

Prgm4:JULSET

```
:All-Off
:Prgm7
:-0.74543→A
:0.11301→B
:40→N
:10→R
:0→J
:Lbl 1
:Ymin+JE→Q
:0→I
:Lbl 2
:Xmin+ID→P
:PT-On(P,Q)
:PT-On(-P,-Q)
:P→X
:Q→Y
:1→K
:Lbl 3
:X2-Y2+A→Z
:2XY+B→Y
:Z→X
:If X2+Y2>R
:Goto 4
:IS>(K,N)
:Goto 3
:Goto 5
:Lbl 4
:If K=3
:Goto 5
:PT-Off(P,Q)
:PT-Off(-P,-Q)
:Lbl 5
:IS>(I,95)
:Goto 2
:IS>(J,31)
:Goto 1
:End
```

Prgm5:MANDSET

```
:All-Off
:Prgm7
:30→N
:10→R
:0→J
:Lbl 1
:Ymin+JE→B
:0→I
:Lbl 2
:Xmin+ID→A
:PT-On(A,B)
:PT-On(A,-B)
:0→X
:0→Y
:1→K
:Lbl 3
:X2-Y2+A→Z
:2XY+B→Y
:Z→X
:If X2+Y2>R
:Goto 4
:IS>(K,N)
:Goto 3
:Goto 5
:Lbl 4
:If K=3
:Goto 5
:PT-Off(A,B)
:PT-Off(A,-B)
:Lbl 5
:IS>(I,95)
:Goto 2
:IS>(J,31)
:Goto 1
:End
```

Prgm6:IFSPLOT

```
:All-Off
:Prgm7
:5000→M
:0→X
:0→Y
:1→N
:Lbl 1
:Rand→R
:0→K
:Lbl 2
:K+1→K
:If R>[A](5,K)
:Goto 2
:[A](K,1)X+[A](K,2)Y+[A](K,5)→Z
:[A](K,3)X+[A](K,4)Y+[A](K,6)→Y
:Z→X
:If N>10
:PT-On(X,Y)
:IS>(N,M)
:Goto 1
:End
```

Prgm7:CLRAXES

```
:ClrDraw
:(Xmax-Xmin)/95→D
:(Ymax-Ymin)/63→E
:0→I
:0→J
:Lbl 1
:Xmin+ID→X
:Ymin+JE→Y
:PT-Off(X,0)
:IS>(J,64)
:PT-Off(0,Y)
:IS>(I,95)
:Goto 1
:End
```

Matrix [A] Elements:

0.85	0.04	-0.04	0.85	0.00	1.6
0.2	-0.26	0.23	0.22	0.00	1.6
-0.15	0.28	0.26	0.24	0.00	0.44
0.00	0.00	0.00	0.16	0.00	0.00
0.85	0.92	0.99	1.00	0.00	0.00

▲ AM I REALLY CRAZY? NO, HOWEVER....

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THUNDER BAY, ONTARIO



Patrick Brady is a Sessional Lecturer, Pre-Service Teacher Education Program in the Faculty of Education, Lakehead University, Thunder Bay, Ontario. He taught in a First Nations community in Northwestern Ontario during the 1980's. Prior to that he taught in Nigeria, West Africa. He has a particular interest in the problems and issues involved with minority education in general and Native education in particular.

Abstract

Few issues have raised as much controversy among educators as the Ontario Ministry of Education's destreaming initiative. Haines (1993), in a recent edition of the Ontario Mathematics Gazette, raised a number of questions concerning this initiative which he believes have been ignored by the proponents of destreaming. This article examines the issues raised by Haines and attempts to provide an alternative point of view.

Haines (OMG, 1993) in his commentary regarding the current debate surrounding the destreaming of secondary education in Ontario, identified four areas which he stated were not "commonly raised by proponents of destreaming."[1] These were the questions of: (a) providing equity for all, (b) addressing the dropout issue, (c) the function of students as peer models and (d) the potential negative effect that destreaming might have on mathematics instruction. The purpose of this article will be to examine each of these issues and to provide an alternative point of view to Haines' position.

Systemic Barriers

Central to Haines' position was his argument questioning the use of destreaming as a vehicle for the promotion of "equality of education and opportunity for all students."[2] This stance ignores the issue of how the practice of streaming functions as a systemic barrier to the success of a significant number of secondary school students, particularly disadvantaged or minority students.

Mackay and Myles, Wilson and Danziger have all documented the negative effect streaming has on First Nations students. As Mackay and Myles stated:

> *Our data also indicate that native students are consistently 'placed' willingly and unwillingly in general and basic levels. While this may be appropriate for some, others are being underestimated or underestimating themselves and are having their poor self-image confirmed by school staff who are either biased in their assessment of Native students' abilities, untrained, or overworked. [3]*

Given the fact that dropout rates are significantly higher among basic and general level students when compared to their advanced level counterparts (King & Hughes, 1985) the negative effects of streaming on First Nations students (dropout rates approach 90% in some areas) becomes readily apparent to even the most casual observer. As Wilson notes:

> *The well-being and academic excellence displayed by students in elementary school on the reserve could not withstand the onslaught of negativism as they attempted to move into the mainstream system. Students had clear perceptions about the cause of their lack of success in high school. They perceived that they were isolated in the school - isolated from the system, from the white students, and from the teachers. [4]*

This isolation was caused, in large part, by their being streamed into low track streams which lowers what Byrne refers to as "academic SC (self-concept)".[5]

While there is no guarantee that destreaming will ensure equity in educational outcome for all students, it may serve to break down some of the systemic barriers currently confronting disadvantaged and minority students in Ontario secondary schools.

The Dropout Issue Re-visited

One of the more interesting aspects of Haines' article was his use of the Halton Board of Education's dropout statistics as supporting data for his claim that it would be "unlikely that destreaming efforts would reduce the dropout rate to less than 5%."[6] The selection of the Halton Board of Education was interesting in that this particular jurisdiction encompasses an area that is scarcely representative of the rest of the province. An examination of the 1991 Census documents (Statistics Canada, 1991) revealed that the median family income for the federal electoral district of Oakville-Milton (located in the heart of the Halton Board of Education's territory) was $68,770, and 4.8% of the population were considered to be low income families. These figures contrast strongly with province-wide statistics which found the median family income to be $50,046 with a low income family rate of 10.9%. In other words, the Halton board draws many of its students from a socioeconomic environment that is significantly more advantaged than the general provincial population. Given this factor, and studies by

Radwanski and especially Hull, which indicate a strong correlation between socioeconomic status and the propensity to drop out, it is of little wonder that the Halton Board enjoys such a high rate of student retention.

As Radwanski stated:

> What appears abundantly clear is that instead of providing socioeconomically disadvantaged children with genuine equality of opportunity ... our school system has the unintended effect of relegating large numbers of these children to lower tiers of education...from which they are statistically unlikely to graduate... [7]

While the correlation with socioeconomic status is not perfect, it is sufficiently strong to suggest that streaming, at best, serves to reinforce the existing social structure and, at its worst, severely inhibits the educational opportunities of the disadvantaged.

Streaming as a Reinforcer of Negative Peer Models

Haines further raises doubts as to whether or not superior students could serve as role models for their less academically endowed peers in a destreamed class. Again Haines has neglected to look at the other side of the coin: how streaming contributes to the creation of a negative school environment for many low track students. Gamoran and Barends, for example, have enumerated the different manner in which teachers interact with high and low track students. Oakes found that:

> Upper track teachers tend to be more enthusiastic, to vary their methods of presentation, and use more constructive criticism than teachers in lower track classes. [8]

Similarly, Vanfossen, Jones and Spade have commented on how the practice of streaming produces negative peer models as low track students are "segregated in the classroom with peers who have lower aspirations".[9] These and other factors result in the widening of what Byrne refers to as the "academic SC (self-concept) differential between low-and high-track students."[10]

Perhaps the negative effects of streaming are best summarized in the following commentary provided by McDermott (1989):

> Our schools divide people into halves: those who can and those who cannot...There are thousands of students every day who are insured success simply because the dropouts have disappeared from the competitive roles. Where would the successful be without dropouts? [11]

Overall, while destreaming may not guarantee the provision of positive role models for academically weak students, the inverse practice certainly guarantees a supply of negative role models, peer models who scarcely contribute to a positive learning atmosphere in lower track classrooms.

Streaming as a Nimby Issue

Finally, while Haines argued that mathematics should exempted from the destreaming initiative, it is an argument that exhibits one serious flaw: the tendency to view all issues from a tunnel vision perspective. Simply stated, mathematics is only one component of the secondary school curriculum and therefore, is not more or less important that any other component. To claim that mathematics should be given special consideration is to approach the destreaming initiative from the NIMBY (Not In My Backyard) perspective, an approach that is becoming increasingly prevalent in Canadian society as a whole.

Haines was correct, however, when he stated that the destreaming controversy is one of "education's philosophical debates." [12] It is a debate that examines the role of education in society in general, and the function of the secondary school within that system in particular. In its most rudimentary form the debate requires society to address a basic question: is the educational system to be a vehicle for inclusion or a barrier of exclusion? In other words, do we continue to marginalize low-track students within the school system and wider community or do we destream and remove the systemic barriers that prevent disadvantaged and minority students from reaching their full potential? It is a question that readers will have to answer for themselves.

Footnotes

1. J. Haines, *Am I Really Crazy? Selected Issues Concerning Destreaming,* Ontario Mathematics Gazette, September 1993, p. 26.
2. IBID, p. 26.
3. R. Mackay and L. Myles, *Native Student Dropouts in Ontario Schools,* Queen's Printer for Ontario, Toronto, 1989, p. 56.
4. P. Wilson, *Trauma In Transition,* Canadian Journal of Native Education, 1992, 19(1) p. 50.
5. B. Byrne, *Self-Concept And Academic Achievement: Investigating Their Importance As Discriminators Of Academic Track Membership In High School,* Canadian Journal of Education, 1990, 15(2).
6. J. Haines, *Am I Really Crazy? Selected Issues Concerning Destreaming,* Ontario Mathematics Gazette, September, 1993, p. 27.
7. G. Radwanski, *Ontario Study Of The Relevance Of Education And The Issue Of Dropouts In Ontario Schools,* Ontario Ministry of Education, 1987, p. 78.
8. J. Oakes, M. Lipton, *Detracking Schools: Early Lessons From The Field,* Phi Delta Kappan, 1992, 73(6), p. 424.
9. B. Vanfosse, J.D. Jones and J.Z. Spade, *Curriculum Tracking and Status Maintenance, Sociology and Education,* 1987, 60, p. 116.

(Continued on page 36)

▲ Teachers, Parents, Students: A Dialogue on Transition

The group of teachers listed below are trying to put together a flyer for mathematics teachers of Ontario to give to parents of Transition Years students. The brochure's purpose is to inform parents just what is happening in a typical Grade 9 Transition Years Mathematics classroom *if the teacher has bought into the OAME's Focus on Renewal.*

As part of a field test, some parents of Grade 9 students were given a draft copy of this document. These parents seemed to like many things about the document. They appreciated the explanation of the six strands that the students are to work in. In addition, they felt that they had a better grasp on how their children were to be evaluated.

Our interpretation of teacher responses to this document is different. They are afraid that the program reflective of Transition Years policies will underserve their students now and in the future. They also feel a lack of direction from the Ministry of Education, especially a lack of clearly defined mathematics curriculum. There is also trepidation in being faced with teaching strategies which were not a part of their own teacher training.

Therefore, we present the following dialogue. It is designed to help teachers visualize what they and parents can expect from a Transition Years Mathematics Program which is

- in tune with the OAME/OMCA *"Focus on Renewal"* document and
- in the spirit of the Ministry of Education Transition Years and Common Core Curriculum documents.

We want you to act and/or react. Write the editor (see the masthead page for the address, fax number, or email address) to let us know what you think. Is this document of use to you? How would you change it? What do you think? Give copies of this to some colleagues and get them to respond. This is your chance to have input!

Arnie Carefoote Don Gray

Barry Scully David Zimmer

The Ontario Association for Mathematics Education Transition Years Flyer

DRAFT COPY

Today is a time of change. Education must meet the needs of students living in the INFORMATION AGE, not the INDUSTRIAL ERA. Learners still need many of the old basics, but will need more and different skills. Tomorrow's citizens will need to be effective communicators, self-directed, team players, creative problem solvers, productive workers, and curious about the world around them.

An important focus of the educational community has been the early adolescent years (Grades 7-9) now called the...

Transition Years

Transition in Student's lives:

- A time of great change in the learner (physical, emotional, intellectual)
- A time of transition, from elementary to secondary school
- A time for variety of teaching/learning strategies/assessments

We are now modifying the traditional high school program in the following ways:

- We are restructuring the curriculum to reflect the needs of adolescents. This curriculum development also emphasizes the development of learning skills of those adolescents more than the previous curriculum.
- We are encouraging the home to become partners with the school in the development and delivery of programs and in the education of children.
- We are trying to keep parents informed about these changes and the implementation of new educational strategies, techniques and programs.

The following is what the Ontario Association for Mathematics Education Transition Years Committee thinks that you need to hear while having . . .

A Conversation with Your Child's Math Teacher

P(parent): *What is my child studying in this math class?*

T: The *basic skills,* as you know them are still being taught. But your child is also learning estimation and *communication skills,* as well as problem solving strategies.

We are trying to connect math, its ideas and applications to other subject areas and are moving away from math as a body of isolated concepts and procedures.

Our math program is organised around six strands:

- **problem solving**
- **number sense**
- **data management**
- **patterns and algebra**
- **measurement**
- **spatial sense & geometry.**

P: You've mentioned a number of different, unfamiliar topics. Could you explain what some of them mean? Let's start with problem solving.

T: *Problem solving is* the process of knowing what to do when you don't know what to do. Students are encouraged to develop skills and procedures in order to solve real-life problems. In doing so, they learn how topics are connected to each other and to other subjects.

The students also develop skills in speaking/writing with their classmates and others. Their creative, curious minds are challenged to seek alternate solutions and to create their own problems.

Problem solving skills extend beyond mathematics but math offers many opportunities for students to practise and develop these skills.

P: Why are the skills in the *data management* section important?

T: The information age is demanding that our informed citizens be able to understand and manipulate data, be able to critically analyze that data and then make informed decisions.

A glance at any newspaper reveals that much information and misinformation is conveyed through charts and graphs. At the same time, many decisions are based on statistical survey data. I want your child to make decisions and to base those decisions on the best information available.

P: When you talk about the manipulation of information, don't students need to have some *sense of numbers* and some skill in *using the existing technology* (calculators and computers)?

T: To help students acquire this number sense, we use many estimation and drill activities, games and also calculators. The issue of using calculators in the classroom has been controversial and, to date, we have not used this technology to the best advantage.

All students should be encouraged to learn to judge whether a calculator is needed or whether mental math should be used. Calculators take away the burden of multiple paper and pencil activities and provide opportunities for exploring mathematical concepts. Students must still be able to predict the appropriateness of the calculator answer. "Is it approximately correct?" With the use of calculators, time can be spent on

- **deciding which operations to use and in which order**
- **deciding when a solution makes sense (approximation)**
- **testing to see if the solution fits the problem.**

P: Do you mean that my child *will be taught the basics,* but will also make use of *calculators and/or computers* to explore/develop problem solving skills?

T: That's it! Do you have any other questions?

P: Looking at my child's report, I see that *portfolios, journal writing, group work, and self-evaluation* are all used to determine my child's "mark". *Do tests or exams count* any longer?

T: Yes they do. Tests and exams are still very much a part of every math program. However, *any single form of assessment is too limiting* to fully describe a student's knowledge of math. Other techniques of assessment include

- **performance on a given task**
- **student peer evaluation**
- **presentations**
- **observation check list**

As well, we have an obligation to use these diagnostic tools to determine areas of strength or weakness. This could then help determine remedial or enrichment activities to customize each student's program.

P: Please explain why *cooperative group learning* is used in the classroom.

T: Statistics show that when students share their knowledge and perceptions by working in groups, *achievement and interpersonal relationships* are affected in a positive way - motivation is higher, anxiety level is lower and, as a result, more learning takes place.

Students must be active, to talk, to write, to listen about what they are learning. This communication is often (though not always) best done in group situations where each member of the group has responsibilities for, and contributes to, the goal of the group.

Many of the skills developed in this approach have been identified by the Conference Board of Canada as *critical skills required* for the Canadian work force.

P: How can I help my child with *homework?*

T: The fact that we are now talking is the first step. The communication between home and school needs to be open and honest. Your child will be more motivated to complete assignments if you show an interest in the work, offer your support, provide a quiet workplace, and provide rewards (not necessarily bribes) for successfully meeting your (realistic) expectations.

One specific suggestion would be in encouraging your child to be organized. Encouraging the use of a personal calendar to track tests, projects and assignments as well as outside activities would help develop necessary learning and decision making skills.

P: I like the idea of working with the teacher. It makes me feel comfortable to know you and what you are asking my child to do. It is also comforting to talk about my child's progress. Tell me about this *Grade 9 destreaming.*

T: In the past, students entering Grade 9 were required to select their mathematics program at one of three levels - basic, general, or advanced. This selection has been postponed until Grade 10, making the Grade 9 program much more similar to the Grade 7 and 8 classes. The mix of student ability levels is more like that found in Grade 7 and 8 classes. This has been done for a variety of reasons:

- **to reduce stress/anxiety of moving from elementary to secondary school**
- **to provide all secondary school students with access to the entire math program, regardless of their previous experiences**
- **to delay (for one year) the selection of a particular math program**

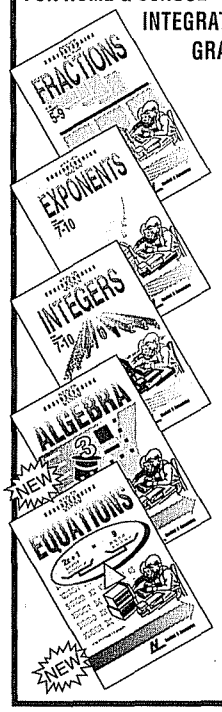
P: Our time is up. It was nice to meet you and I thank you for all the useful information.

T: Please take this brochure which will explain the changes which we are trying to implement. I hope that we meet again when you or I have any comments or questions about your child's progress.

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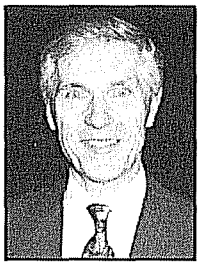
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▲ Inquiry - How Does it Conflict With Current Beliefs and Practices?

JOHN L. CLARK
TORONTO, ONTARIO



John Clark is a retired coordinator of mathematics for the Toronto Board of Education. Recently he completed research into teaching through inquiry at grades 7 and 8. He is interested in issues related to curriculum and teacher development.

[This is the first of two articles dealing with the concept of inquiry. This article explores the meaning of inquiry. The next will deal with inquiry in the classroom.]

Using loops of string, grade 7/8 students are trying to make triangles according to certain conditions. One student says "It is impossible to make a right equilateral triangle." Another student asks "Why not?"

Inquiry is a powerful concept not just in teaching mathematics but across the curriculum because underlying it are fundamental beliefs about knowledge, learning, and the roles of teacher and student which conflict with how many subjects are presently taught. In order to foster mathematical inquiry faithfully in the classroom, we need to examine fundamental beliefs about education and find ways to resolve what are sure to be major conflicts. We need to examine our beliefs about mathematics itself, what we hope students will learn, and how we can effectively teach these things. In this article, I inquire into the concept of mathematical inquiry to clarify its meaning and implications for learning mathematics, and examine potential conflicts between teaching which takes inquiry seriously and other approaches.

Critical Thinking

As educators in a democracy one of our primary goals should be to develop critical thinking in students. Critical thinkers do not see things as either black or white and right or wrong. They usually accept that most situations are ambiguous, strive to see both sides of a question, and make generalizations tentatively. They believe that knowledge is relative and contextual, reasonable people can disagree, and

some interpretations can be better than others. But with critical thinkers, the locus of control for determining meaning and understanding is with the individual, not external authority. Can mathematics teachers teach students to think critically? Yes we can, but it requires concentrated effort. We also need to think more critically about mathematics itself.

The Nature of Mathematics

"How can you inquire into mathematics - it's either right or wrong!" This comment made by many adults and students reflects a belief in the absolute certainty of mathematical knowledge. There is no ambiguity; all there is to know is known and usually found in a textbook. There is no place for a student's naive question or fresh insight into familiar concepts and relationships. As the teacher, I "know", and as the student your job is to get "it" from me. But isn't mathematics evolving like other fields of knowledge?

Why does mathematics among all subjects in the curriculum enjoy this perception of certainty? A poem is open to interpretation, the causes of the Persian Gulf War can be debated, and the problems of the environment are complex. Of course, other subjects can be taught with absolute certainty; for example, in a history class, students may be taught five of the causes of World War II. But mathematics is also open to interpretation. Do all mathematics teachers share the same understanding of notions such as proof or rational numbers?

Resnick (1988) discusses how different perceptions of the discipline of mathematics lead to corresponding views of learning mathematics as a "well-structured discipline" and the corresponding emphasis on symbol manipulation and rules without questioning or discussion by students. She notes research in other fields of learning which "all tend to treat learning as a process of interpretation, justification, and meaning construction" (p. 33). She states that "All of this suggests that we urgently need to begin investigating possibilities for teaching mathematics as if it were an ill-structured discipline" (p. 33).

Given that most teachers view mathematics as unambiguous and as a well-structured discipline, attempts to foster inquiry in classrooms will be difficult unless we address basic questions concerning the nature of mathematics itself.

Learning Mathematics

Polya states that for "students to gain a sense of the mathematical enterprise, their experience with mathematics should be consistent with the way mathematics is done" (Schoenfeld, 1992; p. 339). Thus Polya believes that in order for students to know what mathematics is about, they should learn how to do mathematics the way mathematicians do. Many authors believe that the process of making and testing conjectures is at the heart of how mathematicians work (Ernest, 1991; Kleiner, 1991; Polya, 1945). This means that by studying mathematics, students should learn to make and test conjectures. But can they learn this? How can teachers foster inquiry and still teach a course of study?

A Grade 7 boy says, "The bigger the denominator the smaller the fraction." Another student asks, "Does this work for two-thirds and two-fifths?"

The above illustrates how students make and test conjectures. We all do this from a very early age because it is an integral part of our language development and understanding of various concepts. Today researchers agree that effective learning must be accompanied by understanding (Resnick, 1988). Generally, when we inquire into a question or problem, we seek meaning and try to understand what makes it tick. We do this by questioning, gathering data, and making conjectures. Then new questions arise which lead to further data collection and refining of conjectures. In my experience, when students are given the freedom to explore and generate their own questions, they conjecture what they believe is true, and through discussion clarify their understanding. Students are willing and able to inquire in order to learn what we feel we must teach including such things as multiplication, fractions, equations, or functions.

I have observed, however, there are few opportunities for students to inquire. The dominant teaching mode presumes that mathematical knowledge can be passed from teacher to student. Teachers (and students) over the years have come to expect that good teaching comprises explaining, telling, and teachers asking questions about known facts. This pattern of instruction, often called "transmission", is in conflict with teaching through inquiry.

Student Questioning

Teachers (secondary more than elementary) frequently say they teach using the Socratic method and by this mean a series of carefully constructed questions which they as to lead students to new knowledge. Teachers pose questions to a group of students and one answers on behalf of others. Educational jargon refers to teachers' high order questions designed to elicit students' higher order thought. This form of Socratic questioning is based on many assumptions which should be challenged.

Saul (1992; p. 117) maintains that Socrates did not use questions the way many teachers do today; he used questions to facilitate dialogue. Today every question is intended to produce an answer but Socrates assumed that every answer raises a question. Thus questioning should stimulate shared exploration and openness not closure.

There are good reasons to be skeptical of the belief that teacher questions elicit higher level student thought (Carlsen, 1991; Cazden, 1986; Dillon, 1982b). Dillon (1988) who has studied teacher and student questioning at some depth believes the person who benefits from higher order questions is more likely the questioner (Dillon, 1982a). In other words, the formulation of a probing question is preceded by some hard thinking. He notes that most questions teachers ask are not genuine or inquiry questions because "they do not bear on information doubted, desired, or sought, but on information known and taught" (Dillon, 1982b; p. 160). He believes that

constant teacher questioning can in fact make students more passive, reactive, and overly dependent on the teacher. In my experience, students in most classes do not ask questions of any substance. They do ask, however, questions such as: Is this right? Is this what you want? What do I do next? Certainly, with inquiry, teachers need to ask far fewer questions and students many more.

A Grade 8 girl asks "Do all shapes have a half? Does this one?", then goes to the blackboard and draws an irregular quadrilateral. The class begins to buzz. Some students focus on the diagram at the board and others have a discussion using rapid drawings on paper. A student goes to the board with a ruler and tries to measure the sides of the quadrilateral. Another student says, "It depends on what you mean by half."

A fertile question which student inquirers often ask spontaneously is the "What if...?" question (Brown and Walter; 1983). In the anecdote above, the students had been investigating fractions with rectangular paper strips. The girl is asking "What if it wasn't a rectangle?" Because the teacher had developed students' respect for one another's questions, the girl's questioning stimulated further inquiry among other students. Inquiry was also facilitated by the fact that the teacher was not perceived as the authority but someone who would listen and join in student discussion. Teachers can teach students to ask questions like the "What if...?" and encourage them to question themselves and others in their investigations.

In inquiry, it is those conducting an inquiry who ask the important questions. The above classroom anecdotes illustrate that when students are given the opportunity to inquire, they ask probing questions. Fostering mathematical inquiry in the classroom, therefore, requires a shift from teacher questioning to student questioning and new roles for both teacher and student in classroom communication. But to achieve this students need to gain greater ownership and responsibility for their learning. New teacher and student roles resulting from a shift to inquiry are in conflict with many accepted practices of teaching mathematics.

Problem Solving and Inquiry

Since the 1980s the focus in the mathematics curriculum has been problem solving. How does it fit in with inquiry? I believe that there are at least three major differences between inquiry and many current practices involving problem solving. First, with problem solving, it is the teacher (or textbook) who usually poses and formulates problems. With inquiry, however, students also question and generate problems which become the focus of daily classroom work. Second, with problem solving, the emphasis is usually on getting a right answer or solution. With inquiry, however, problems do not just lead to answers and solutions but to new questions and problems producing a sustained investigation. Third, problem solving is often not used to teach the core curriculum but is practised after major skills and concepts have been taught usually by transmission. With inquiry, however, questioning, problem formulating and solving are integrated into the cutting edge of learning what are considered to be the most important content and processes defined by the curriculum. Inquiry, therefore, includes problem solving but is more.

Summary

Inquiry as an approach to learning mathematics serves two functions. First, it is a process of actually doing mathematics, that is, generating new mathematics and solving problems. This is mathematics as inquiry. Second, it is a process of learning mathematics, that is, acquiring key concepts, skills, and relationships. This is mathematics by inquiry. The view expressed by many authors is that these two functions are inseparable; principled mathematical knowledge and power are gained through the process of inquiry which is a combination of questions, investigating, discovering, and problem posing, formulating, and solving.

In current practice, there are at least these two very different approaches to teaching mathematics, transmission and inquiry. But they are incompatible because underlying each are a number of assumptions which lead to conflicting pedagogical beliefs: mathematics as well-structured versus ill-structured, teacher imposition versus negotiation with students, teaching through telling versus dialogue, teacher control of knowledge versus shared exploration, and student as object versus student as subject and autonomous learner. Transmission and inquiry are endpoints of a continuum between which most classroom practices lie.

A recent Ministry curriculum working document for mathematics makes frequent reference to inquiry (OMET, 1993). The vision for mathematics education advocated in current reforms such as the NCTM Standards (1989) takes inquiry seriously. What is being advocated is a dramatic shift from transmission toward inquiry and therefore a paradigm shift of values. What is being called into question are fundamental beliefs about education, knowledge, mathematics, and learning. The implications for curriculum design, professional development, assessment, and accountability are profound and far reaching. Taking inquiry seriously means examining critically strongly held beliefs and practices.

In a subsequent article I will explore the implications of an inquiry approach for classroom practice including issues of time, depth and breadth, classroom communication, roles of the teacher and student, lesson preparation, and assessment.

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▲ ASSESSMENT

JENNIFER WEINER
GUELPH, ONTARIO

Welcome to the inaugural edition of the Assessment column. The purpose of this column is to provide teachers with a practical, useful assessment tool in each issue of the Gazette. Although it is recognized that there are many ways of assessing students (and readers are welcome to submit examples of alternate forms), the final exam continues to be a reality in secondary schools today. We are not advocating for or against this as an appropriate method for evaluating students' learning; we are simply describing the situation as it exists.

In this column we have provided a grade nine destreamed exam. Teachers can use this column as they wish. Here are some suggestions: 1) Use the exam for your own information, e.g., to compare what you are emphasizing in your course with the sample exam's emphasis or to compare your style of questioning with that of others. 2) Take interesting questions from the sample exam and tailor them to suit your needs. 3) Use the sample exam as a practice exercise for your students.

Please note: This exam was not prepared specifically for this column. It was an exam actually used last June in an Ontario high school. We present this, not as an OAME creation, but simply as a sample of what's "out there".

If you have an exam or other assessment item that you feel would be appropriate for this column, please submit it to: Jennifer Weiner, c/o Dept. of Mathematics and Statistics, University of Guelph, Guelph, Ontario, N1G 2W1.

Note: All references to individual boards, schools or teachers will be removed.

CALL FOR AWARD NOMINATIONS

Deadline for nominations: January 31, 1995.

LIFE MEMBERSHIP

To qualify for Life Membership in OAME/AOEM a candidate shall have:

- ▲ 10 or more accumulated years of membership in OAME/AOEM
- ▲ demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award, to be presented annually to a person who does not necessarily meet the criteria for the Life Membership Award (i.e., not necessarily an educator or an OAME/AOEM member.) The award is aimed at recognizing people who have supported OAME/AOEM, such as publishers, educational suppliers, hotel personnel, etc.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM has established the Morley MacGregor Award, to be presented annually to an elementary school teacher of mathematics who has demonstrated excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

KENNETH D. FRYER AWARD

In Memory of the late Dr. K.D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, has established the Kenneth D. Fryer Award for Excellence, to be presented annually to a secondary school mathematics department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME is requesting nominations for consideration by the committee for this year's awards - to be presented at the Annual Conference.

AWARD NOMINATION FORM

Date of Nomination: _____

Award (Check One)

☐ **Life Membership** ☐ **Mona MacGregor Award**
☐ **Morley MacGregor Award** ☐ **Kenneth D. Fryer Award**

I/We nominate (Name of Nominee) _____

School, Organization or Business of Nominee

_____

School, Organization, or Business Address of Nominee

_____

(for the Kenneth D. Fryer Award)

Principal of Nominee _____

Mathematics Department Head of Nominee _____

Nominated by (Name, Address & Phone) _____

Please attach a separate sheet to the completed nomination fonn, outlining the reasons for your nomination and send to Ron Lancaster, Past President OAME

MAT 1W Mathematics Examination June 1994 Time: 1.5 hours

My name is _____

My teacher's name is _____

Complete neatly–organized solutions must be developed in the spaces provided.

Calculators may be used but not shared.

Decimal answers must be correct to one decimal place.

Use your time efficiently.

The following formulae may be useful.

$$C = \pi d, \quad A = bh, \quad A = \pi r^2, \quad A = 0.5bh, \quad A = \frac{h(a+b)}{2};$$

$$A = 4\pi r^2, \quad V = Bh, \quad V = \frac{\pi r^2 h}{3}, \quad V = \frac{4\pi r^3}{3}$$

Approx. Marks

(8) **1.** The rainfall for each month in a year in Ourtown is listed in the chart.

Determine the:

(a) mean monthly rainfall.

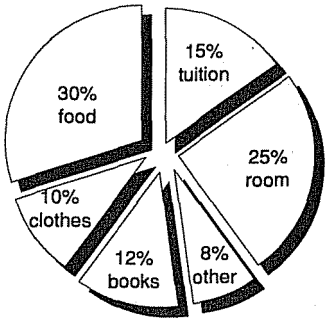
(b) mode.

(c) median.

(d) percentage of the annual rainfall received in September.

Month	Rainfall in mm
January	11
February	12
March	27
April	59
May	66
June	73
July	66
August	88
September	80
October	70
November	58
December	26

(3) **2.** The graph at the right shows how university expenses are distributed for a typical student. The total amount spent in one year is $9000. How many dollars are spent on food?



(4) **3.** During a "20% off" sale, the price of a pair of running shoes was $59.95. Determine the regular price correct to the nearest cent.

(10) **4.** Correctly complete the following chart.

Common fraction in **Lowest Terms**	Decimal Fraction	Percent
	0.75	
$1\frac{2}{5}$		
		8%
	0.625	
$2\frac{1}{20}$		

(28) **5. Without using your calculator,** determine the **exact** value of each of the following.

(a) $\frac{2}{3}+\frac{3}{7}$

(b) $12\frac{1}{2}-9\frac{7}{8}$

(c) $2\frac{2}{3}\times\frac{9}{32}$

(d) $1\frac{4}{5}\div 4\frac{1}{6}$

(e) $4\frac{7}{8}-2\frac{5}{8}\times 1\frac{1}{3}$

(f) $-14-(+20)+(-15)-(-18)$

(g) $-42\div 7-8(-9)$

(h) $-\frac{2}{3}+5\frac{5}{8}-1\frac{3}{4}$

(i) $3^{100}\ (3^2)\div 3^{98}$

(j) $5^0+5(2^3)^2$

(16) **6.** Simplify each of the following.

(a) $2x - 3y - 6x + 2y$

(b) $5x^3 - 3x^2 + 2x^3 - x^2 - x^3$

(c) $3(5x) - 24x \div (-8)$

(d) $-6xyz \div 3x - 3y(4z)$

(e) $2x^3(-3x^2) \div 6x$

(f) $(2y^4)^3$

(g) $-3x^2y^4(16yx^3) \div (-4x^2y)$

(15) **7.** Solve each of the following equations for x.

(a) $3x - 5 = 10$

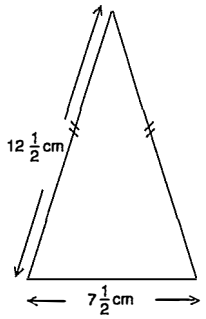
(b) $2x + 9 = 7x - 11$

(c) $\frac{2x}{3} - 3 = 7 + \frac{3x}{2}$

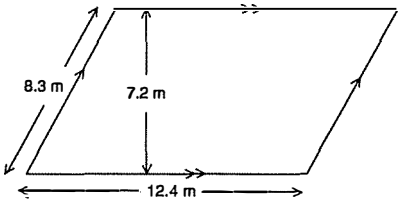
(d) $3 : 9 = x : 41$

(e) $3 - \frac{2 - x}{4} = x + 5$

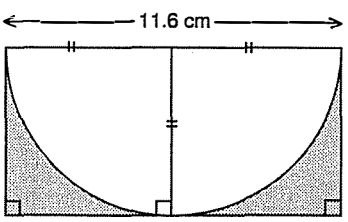
(3) **8.** Determine the perimeter of the diagram at the right.



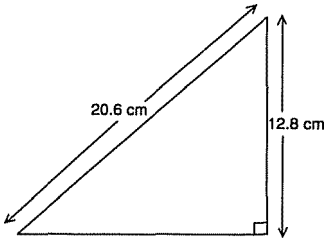
(3) **9.** Determine the area of the diagram at the right.



(5) **10.** Determine the area of the **shaded** region at the right.



(4) **11.** Determine the length of the unknown side in the diagram at the right.



(4) **12.** Determine the volume of a cone whose height is 8.4 cm if its base has a diameter of 10.2 cm.

(5) **13.** In the blank provided, print an algebraic expression to represent the:

(a) number which is one more than twice x. _____

(b) value (in cents) of y dimes. _____

(c) first odd number smaller than x if x is odd. _____

(d) number which is 4 less than half of x. _____

(e) sum of $5 - y$ and $2y + 4$. _____

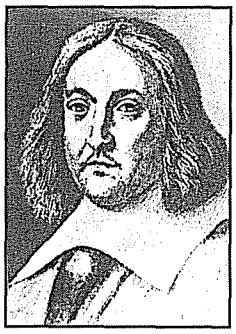
(3) **14.** After examining his video collection, Rob realizes that the ratio of his number of rock videos to his number of science fiction movies is 4:3. If Rob has a total of 91 videos, determine how many of these are science fiction movies.

(5) **15.** One number is 14 less than four times another number. If their sum is 541, determine the numbers.

(6) **16.** Chad has been saving dimes and quarters. He has 16 more quarters than twice the number of dimes. If their value is $20.80, determine the number of each type of coin.

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Did you **double check** all solutions and are they as neatly organized as possible? (Remember that marks will be deducted for poor quality solutions.) Leave your finished examination unfolded. Have a safe summer!



Pierre de Fermat
1601 – 1665

Update on Fermat

Editor's Note: This arrived by email on October 25. Karl Rubin is a Professor of Mathematics at Ohio State University.

As of this morning, two manuscripts have been released

- Modular Elliptic Curves and Fermat's Last Theorem, by Andrew Wiles
- Ring Theoretic Properties of Certain Hecke Algebras, by Richard Taylor and Andrew Wiles.

The first one (long) announces a proof of, among other things, **Fermat's Last Theorem,** relying on the second one (short) for one crucial step.

As most of you know, the argument described by Wiles in his Cambridge lectures turned out to have a serious gap, namely the construction of an Euler system. After trying unsuccessfully to repair that construction, Wiles went back to a different approach, which he had tried earlier but abandoned in favour of the Euler system idea. He was able to complete his proof under the hypothesis that certain Hecke algebras are local complete intersections. This and the rest of the ideas described in Wiles' Cambridge lectures are written up in the first manuscript. Jointly, Taylor and Wiles establish the necessary property of the Hecke algebras in the second paper.

The overall outline of the argument is similar to the one Wiles described in Cambridge. The new approach turns out to be significantly simpler and shorter than the original one because of the removal of the Euler system. (In fact, after seeing these manuscripts Faltings has apparently come up with a further significant simplification of that part of the argument.)

Versions of these manuscripts have been in the hands of a small number of people for (in some cases) a few weeks. While it is wise to be cautious for a little while longer, there is certainly reason for optimism.

Karl Rubin

▲ READERSHIP SURVEY REPORT

OAME PUBLICATIONS COMMITTEE

In the April issue, the OAME Publication Committee for the *Gazette,* published a Readership Survey in the form of a questionnaire. The purposes of this article are to report the results of the survey and to respond to the many constructive criticisms and comments.

First, we would like to thank the approximately 150 readers who responded to the survey. We appreciate the time, effort and thought that was put into answering the questions and writing the comments. Of course, with a readership over 2000, we were hoping for lots and lots of responses. On the other hand, since most of the collating of data was done in the summer by John Kersley of the committee, perhaps we should be thankful that 2000 people did not respond.

The Readership Survey consisted of a series of questions and a chance to comment on the various sections of the *Gazette* including the *Abacus*, on style and format, on what our readers expect from the *Gazette* and the *Abacus* and on possible improvements.

Here are the results.

65% of the respondents said they read the *Gazette* within the first week of receiving it; 7% said the journal sits on the shelf, collecting dust. We were pleased to note most read it soon after receiving it.

Note: The following per cents are based on a rating of 6 or more out of 10. (1 was of little value; 10, of great value.)

Column / Section	Significant Value
Editorial column	63%
Letters to the Editor	63%
President's Message	59%
Have Your Say	78%
About Problem Solving	87%
Council Comments	48%
Math Mania	73%
ABACUS	78%
Articles	92%
Notices	60%
Advertisements	49%

It is obvious that one cannot publish a journal that caters to every need. Thus, the results are not surprising. One respondent wrote, "Each reader picks and chooses relevant or pertinent information according to one's specific needs." A few people commented on the use of advertisements, a recent addition to the *Gazette.* A couple said they did not think we should advertise at all. Another said, "I would like to see a limit on advertising." One person commented, "I realize these probably have value to OAME as an income source." They do. At the same time, the OAME does not endorse the advertised products.

Overall, 89% of the respondents rated the journal 6 or more out of 10. "With fiscal restraint and less money for professional development, it's a wonderful way to keep on top of new ideas." "...a fine magazine. Please continue." "Every mathematics teacher should read it." Then there were these comments: " Seems to becoming of less value as time goes on". "Of the publications I receive, *Math Teacher, SSM Gazette, MAA*, it is the least useful." "It is disappointingly secondary ...would like a broader focus and more balanced sections."

We were pleased that 83% of those responding said they liked the style and format of the publication. Quite a few commented that the new style has greatly improved the appearance of the journal. One person suggested the addition of cartoons for a light humorous touch. Another person suggested that a glossy cover would look great. Others have a different opinion, "too formal, needs to be more user friendly."

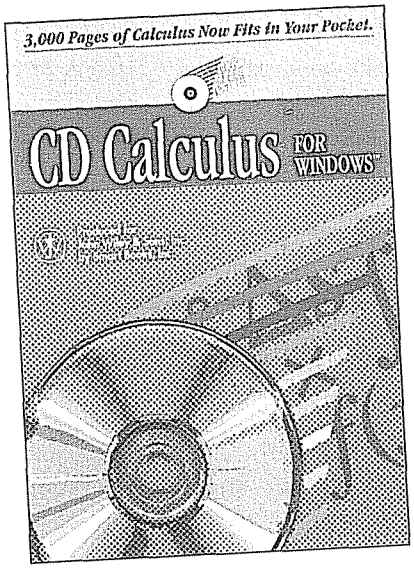
About 75% said they see the journal as a helpful teaching resource. From a department head, "Absolutely. I keep the *Gazette* on file in the department as a resource." "Without fail, I use at least one thing out of the *Gazette* the week I receive it." If we are able to act on most of the suggestions that we received from this survey, the journal would definitely be a more helpful teaching resource. About 72% said they discuss or share ideas from the *Gazette.* Finally, 79% said they recommend the journal. "It is good value for the money."

About the *Abacus.* 63% regarded it as a helpful teaching resource; 68% liked its style and format; 53% share or discuss it and 63% recommend it. "The pull out quality of the *Abacus* makes it quite easy to share with others." The general impression is that most who responded like what the *Abacus* had to offer, but two people had these comments: "The *Abacus* is often practical regarding activities, but issues are rarely treated," and "I would appreciate it, if the 'photocopiable' material were complemented by thought-provoking articles."

We appreciated all the comments and suggestions for improvement. They were very helpful in informing us how the *Gazette* is perceived by our readers and what they see as the purposes of the journal. The following are a summary of the main suggestions from the questionnaires:

1. The *Gazette* should be practical. "The articles should be relevant to the actual teaching topics. It just seems to emphasize the vast gap between the wonderful theory of research and the harsh reality of the classroom! In these

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times of change, something on 'Teaching Tips' might be valuable. Something teachers can try in their classrooms right away." "I need more articles that are specific to classroom use. Sometimes you have it; most of the time you don't."

2. Our readers want reviews of the latest products on the market, such as videos, software, books, and other resources. They also want to know how to obtain any interesting resources, i.e., names and addresses. We agree; this is very important. The *Gazette* is currently looking for someone to be a Resource Review Editor. Anyone interested should contact Jack Weiner, at the address listed on the inside front cover.
3. Readers want articles on how these latest products are being used in the classrooms directly connected to what they are teaching. One person suggested a feature, "Around the Province" to identify and describe how the products are used in the classrooms. "more emphasis on technology i.e., graphing calculators and computers." "More computer related articles."
4. People are interested "in how teachers are successfully adapting to the latest education policies such as the *Provincial Standards* and *The Common Curriculum*."
5. People want the *Gazette* to be an open forum, a place for the discussion of educational policy as it pertains to mathematics and of controversial issues in mathematics. They want both sides presented and invite readers to respond.
6. The biggest complaint is the lack of articles for elementary and primary teachers. The *Abacus* helps to fill this need but, clearly, it is not enough. "Being an elementary teacher, I find most aspects of the *Gazette* beyond my needs. I'd like to see a better balance between elementary and secondary articles." "Devoting a significant portion of the *Gazette* to early years and primary divisions is suggested." We agree with these comments. We are always seeking articles for the early years.

The committee is investigating many of these ideas for possible implementation in future issues of the *Gazette* and the *Abacus*.

In order for the *Gazette* to improve and meet the needs of the mathematics teachers of Ontario, we need more people to submit articles for publication. The following is a list of more specific topics that readers have requested:

1. Use of computer networks for students and teachers.
2. Sample topic tests, one elementary, one intermediate and one secondary per issue.
3. Integration with other subjects, e.g., history, French, English, geography, music as well as science and technological applications.
4. Samples of student work, especially projects or in-depth problem solving.
5. Articles geared to the basic and general level programmes.
6. Fractals.
7. Math contests, samples of contests from other countries, articles on how students have done on the various math competitions, publishing elegant student solutions from these contests.
8. History of mathematics, psychology of mathematics.
9. Alternative assessment and evaluation.
10. Practical games, puzzles, etc.

There were many other constructive suggestions concerning layout, design and content. These ideas have been passed on to Jack. Again, we would like to thank all those who responded to the survey and/or submitted comments. The committee always welcomes readership comments and suggestions through the chair, Bill Otto.

Finally, to Tom, who wrote, "It would be nice to see my trig article in print." Well, we try to please.

THE OAME PUBLICATIONS COMMITTEE

Editor's Note: The Publications Committee and the OAME Council extend thanks to John Kersley for his many hours of work compiling the data for this report.

NOMINATIONS FOR OAME/AOEM COUNCIL 1995-96

Each year, elections are held for President-Elect, 3 Directors and 7-10 Councillors for the Council of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 31, 1995, to:

Ron Lancaster, Past-President, OAME

Include the nominee's qualifications with the Nomination Form. The Nomination Committee will make a final selection of those nominees who will stand for election. Their names and qualifications will be published in March, 1995. Voting will be by mail by members of the OAME/AOEM. The results of these elections will be made known at the Annual Meeting of the OAME/AOEM in Guelph in May, 1995.

NOMINATION FORM

I nominate: _____

Address: _____

_____

For the position(s) of:

☐ **President-Elect** ☐ **Director** ☐ **Councillor**

Nominator _____

Address _____

_____

▲ Council Comments

PETER SAARIMAKI
COORDINATOR OF MATHEMATICS
TORONTO BOARD OF EDUCATION



Introduction

It is with a sense of trepidation that I have taken over this column from Bob Robinson. He has set some very high standards. (Sorry about the creeping provincialism already.) Bob has indicated that he would be very happy to hear from one and all. Bob can be reached at 12303 Harbour Pointe Blvd., T-103, Mukilteo, WA 98275; home phone: (206) 513-9600, FAX: (206) 349-1728.

I see my purpose here as providing you, OAME members, a snapshot of something very few get to see - namely the more human side of Council meetings.

The readership survey taken in *The Gazette* last spring had some very good news, including the fact that over half of you read Council Comments. Now what can we do for those who don't. As a start, each of you could share your *Gazette* with one colleague. Let me know what they have to say.

These notes were taken at the OAME Council meeting held Saturday October 1, 1994. Comments about Council and its activities should be directed to the President. Comments about my comments can be sent to the editor.

General

After an intensive search and lengthy interviews, Dave and Bonnie Alexander were appointed as the new Executive Directors. Don and Carol Attridge will help with the transition and their contributions will be more fully recognized at next Council meeting. They have asked that there be no fanfare, but their *fans,* who are legion, I'm sure will have some *fair* ideas of their own.

OAME '94 Conference

Margo Johnson reported from QSLMA that the annual OAME '94 conference in Kingston was very successful, especially considering the implications of the social contract. Over $36,000 has been forwarded to the OAME to further the support of mathematics in Ontario.

Resources

As usual, Ron Lancaster brought a collection of samples, freebies and handouts. How else would we ever learn about the many resources available and see the variety of newspaper clippings? For example, have you heard about the circular protractor, with three concentric rings, measures angles in radians - available from RADE? For more info FAX (514) 349-3317.

I did mention that I wanted to share some snapshots of the human side of Council. Chairman Todd ran the meeting wearing a Mickey Mouse sweatshirt. It has a group picture of Mickey's friends, whom Todd indicated were really members of Council. Who do you think Goofy represented? (There is no winner for this question.)

Special Projects

OAME and OMCA are embarking on a project to provide support materials to the *Provincial Standards.* At a two day retreat in October, OAME members of the Special Projects committee will join consultants and coordinators from the OMCA to start the year-long process. The materials will start with the Transition Years and should be ready for field-testing in the new year. If you have any questions or wish to offer your class for piloting, contact Judy Crompton (info on inside back cover of this Gazette).

Committees

Early/Formative Years

This energetic group continues to focus on providing teachers with classroom usable activities, for the *Abacus.* The next materials will deal with Volume and Capacity. (For kids and their music, that's how loud it is and how long you can stand it.)

Transition Years

What does a Transition Years classroom look like in the light of the *Focus on Renewal?* The committee is producing a pamphlet to send home to parents to let them know what life is like in Grade 7, 8 or 9 classrooms. In this *Gazette,* you will find a draft for your reaction.

Specialization Years

The many responses to the draft discussion paper last spring has encouraged the OAME committee, with members of the OMCA, to refine their proposals for a future discussion paper. The committee spent a whole day before Council on the next draft and plan to have it out in the schools and education circles early next year.

Publicity and Membership

The Faculties of Education are being targeted this year with copies of the Focus, articles from the *Gazette* and the *Abacus* and a letter outlining the advantages of OAME membership so prospective teachers are aware of us before they graduate. Local chapters will follow up with personal contacts with their Faculties.

Publications

The Search for an advertising manager was successful as you will note in the masthead. Highlights of the results of the readership survey were presented to Council and are printed elsewhere in this issue.

Miscellaneous

There were various business items, but you can talk to your chapter reps about anything of interest to you.

At what other subject group would you expect a member of the executive table to stand up part way through the meeting and ask "Is there a calculator in the house?" Seems Don needed one and was short!

e-mail

Email and the *Internet* are hot issues for teachers, and especially for math types. Dave Zimmer reminded Council members of this FREE resource. More information on joining OTF's *Creating a Culture of Changes* is available from the CCC Coordinators at (416) 966-3424, or

(800) 268-7061 or FAX (416) 966-5450. Remember, it's free to Ontario teachers.

Chapters

One regret I have is that most of the wonderful conferences planned by chapters this fall, shared with Council at the Oct. 1 meeting, will be over by the time you read this in the December issue. Each chapter puts on some marvellous Professional Development activities. Keep in touch with your local chapter to be aware of plans, *before* they happen.

COMA

This enterprising chapter is hosting the first annual Ontario Mathematics Olympics next June 3, 1995. They are inviting *each chapter* to select a team of students by running its own mini-Olympics, and to provide the necessary support to bring its team to Ottawa for the competition and socialization.

CHAMP

The 1994 CHAMP math contests for Grades 7 to 11 were scheduled for Nov. 2 with the mini-conference, MATHFEST '94, following the next day.

GOLDEN SECTION

Golden's new rep. Bob Saari (no, that *is* his *full* name) described this chapter's fall conference, scheduled for October 20 at the Embassy Suites in St. Catharines. It was an exciting program.

GVMA

Their very popular series of resource books will have a new addition next spring, with the publication of a new set of Finite exams. The fall conference on Nov. 10 highlights Dr. Larry Smith, Professor of Economics, U of Waterloo.

ISOMA

David Pratt from Queen's University will be the key-noter at the fall conference on October 27, talking on conventional versus mastery models of curriculum planning.

NOMA

Pat Rogers, Todd Romiens, Nancy Moore and Judy Crompton headline a strong line-up for *Making Connections* on Oct. 14 and 15. Also in the list was Michel Lyons from Quebec, talking of his program built around chess and math.

NWOAME

The fall conference November 25-26, will use different lures for *Fishing the D-Stream* and will feature Marg Warren as guest speaker. (Will she be working for scale, fishing for participation, casting for suggestions or just alluring?)

SAME

The Nov. 3 conference will *Focus on Geometry*. SAME will definitely be involved in the first international AMTNYS-OAME summer workshop '95, being held at Scarborough College. More information on this bringing together of mathematicians from Ontario and New York State, Transition and Specialization Years focus, is available from your chapter rep. or John Kersley at (905) 389-1367, ext. 207.

TEY^{2}MS

Teachers will be *Expanding the Power* on Oct. 27 at the fall conference. They will have two opportunities to benefit from twenty presenters and win prizes with key-noter Dr. Beth Savan, Director of Environmental Programs at the University of Toronto.

Y^{4}MA

November 15 will see *Powerful Mathematics* workshops and a "Pep Rally" for the two Olympic events this year. Members are involved in the AMTNYS-OAME summer workshop '95.

An historic event occurred. Council adjourned at 2:40 p.m., well ahead of it's normal 4:00 plus time. Thanks are extended to Todd for his efficiency in running the meeting and his dedication to the tasks.

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For more info contact: John Kersley



The Quadratic Function

Did you know that Al Fleming, Principal of the University of Toronto Schools, is retiring?

Al has been a math educator in Ontario for more than thirty years, both of high school students and math teachers (through the Faculty of Education, University of Toronto). In 1970, several mathematics teachers, including Al Fleming, gathered together at Thornlea Secondary School to mark the first set of Euclid mathematics contest papers, sponsored by the Canadian Mathematics Competition from the University of Waterloo. This began a long association with math contests in Ontario.

In 1985, Al was presented with the René Descartes Award for Signal Service to the Teaching of Mathematics in the schools of Ontario by the Faculty of Mathematics, University of Waterloo.

To honour Al's achievements, a spring celebration, The Quadratic Function, will be held in the quadrangle of Hart House, U of T, on May 17, 1995.

For further information, contact
Margaret Rieger or write The Quadratic Function

About Problem Solving: Misconceptions as Fodder for Mathematical Thinking

JOHN GRANT MCLOUGHLIN
EDUCATION PROGRAM
OKANAGAN UNIVERSITY COLLEGE



Consider the following problem posed by Mary Grace Kantowski [1]:

Suppose I have two six-sided cubes. Each cube has a square on three of its faces, a triangle on two of its faces and a circle on one face. If the cubes are rolled 1000 times, what combination of shapes will occur most often?

Instinctively many will feel that this is straightforward. Obviously the most common combination will be made up of two squares because the square is the most common figure on each cube. Say it isn't so. Many are surprised to realize that the likelihood of getting a triangle and a square is considerably greater. In fact, it is expected that this combination would occur about 83 more times than a pair of squares. Although an experiment involving a fixed number of tosses does not guarantee a particular result, it is likely that the mathematical expectations will not be too far off when the pair of cubes are rolled 1000 times.

Why is the likelihood of a square-triangle combination greater than that of the square-square combination? Imagine the pair of dice spiralling through the air. One die lands. It's a square. Now the square-square combination is most likely by a 3:2 ratio over the square-triangle combination. But the first die should land on a triangle one-third of the time, thus eliminating the possibility of getting a pair of squares in these cases. Meanwhile, the likelihood of getting a triangle and a square would increase significantly in such a situation. The underlying fact is that only a circle on the first die would rule out the possibility of getting a square-triangle combination, whereas, a circle or a triangle would make it impossible to obtain a pair of squares. If you're not convinced by this logic, carry out the experiment yourself. Constructing a tree diagram may also help you to better grasp the essence of this problem.

Counterintuitive results provoke thought. Upon realizing that one's hunches are producing incorrect conclusions, it is likely that there exists an increased level of intrinsic motivation to address the problem. The motivation stems from a desire to figure out the flaw in one's own reasoning process. Problem solving is not meant to be easy; rather it is intended to be a struggle. Therefore, such problems offer potential for rich problem solving experiences.

A wonderful article by Shmuel Avital and Ed Barbeau [2] provides thirteen examples of problems in which the intuitive approach seems to lead many to invalid conclusions. They identify five sources that may lead to such results: lack of analysis, unbalanced perception, improper analogy, improper generalization, and false symmetrical reasoning. The following two examples from their article provide insight into two of these sources.

A railroad track 24 metres long, whose endpoints A and B are fixed, expands by 2 centimetres when heated by the sun. Assuming that it buckles to create an isosceles triangle ABC with the ground as base, estimate how high the midpoint M will rise to position C.

Although the expansion seems minimal in terms of the length of the bridge, it may surprise you to verify that the midpoint, M, rises about half a metre. Many expect much less of a rise due to the tendency to improperly generalize from life experience. That is, minimal changes are generally associated with minimal effect.

The result may be obtained by considering an isosceles triangle with a base, AB, of 2400 centimetres. The lengths of the equal sides, AC and BC, would be 2402/2 or 1201 centimetres. The equal sides would be the respective hypotenuses of congruent right triangles, AMC and BMC, each having bases of length 1200 centimetres and a common altitude MC. The length of MC can be calculated using the Pythagorean Theorem. You may wish to verify that this length is slightly less than 50 centimetres or half a metre.

Consider a second example from the same article. This time the reader is encouraged to examine a probability problem and see how the result may differ from commonly held intuitions. The source of the misconception exhibited in this example deals with an improper application of symmetry.

A bag contains a certain number of black and white balls. Two balls are drawn at random. It is known that the probability that one is black and the other is white is 1/2. What can be said about the number of balls of the two colours in the bag?

Many suspect that there must be an equal number of black and white balls in the bag. Suppose that was the case. Revealing the colour of one of the balls selected would not affect the probability of getting one ball of each colour. Instead the question could be restated as a conditional probability problem: "What is the probability that the second ball selected is black (white) given that the first ball selected was white

(black)?"

In order to get the desired probability of 1/2, one would ideally like to have an equal distribution of colours following the initial selection. Therefore, starting out with an equal number of black and white balls does not produce this situation, as desired. In fact, the required distribution of B black balls and W white balls must satisfy the following equation:

$$\frac{\binom{B}{1}\binom{W}{1}}{\binom{B+W}{2}} = \frac{1}{2}$$

Readers are challenged to find the relationship between B and W that satisfies this equation. The result is rooted in an intriguing mathematical relationship.

More fundamental concepts may also give rise to such curious solutions. The notion of an average speed is often misunderstood. Here are two problems dealing with averages that you may wish to share with your students:

Suppose that you drive up a hill at a constant speed of 30 km/hr. At what speed would you need to drive down the same hill so that your overall average speed on the hill would be 60 km/hr?

A person drives from home to school at an average speed of 60 km/hr. The average speed travelling along the identical route from school back to home is 40 km/hr. What is the average speed for the trip from home to school and back again?

It is intriguing to observe people when they discover that seemingly simple questions have placed them in uncomfortable situations. Grappling with such misconceptions provides potential for meaningful mathematical discourse and reasoning to take place. If you have any favourites that you would like to share with the Gazette readers, I would be pleased to publish them in a future issue. Send along your ideas.

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Acknowledgment

I would like to thank Peter Brouwer for his editorial assistance in the preparation of this issue's column.

(Continued from page 19)

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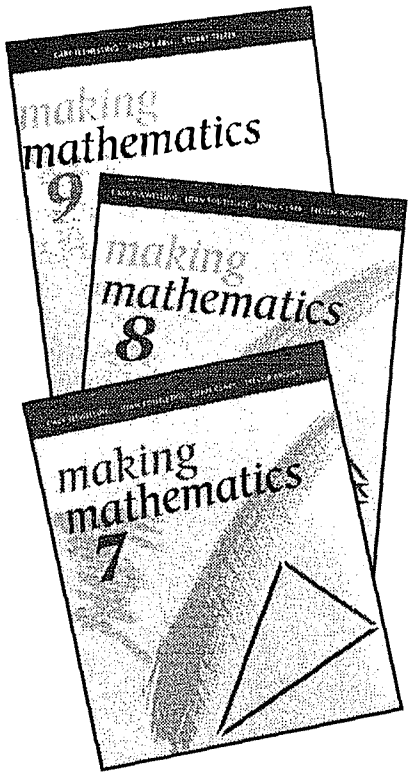
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