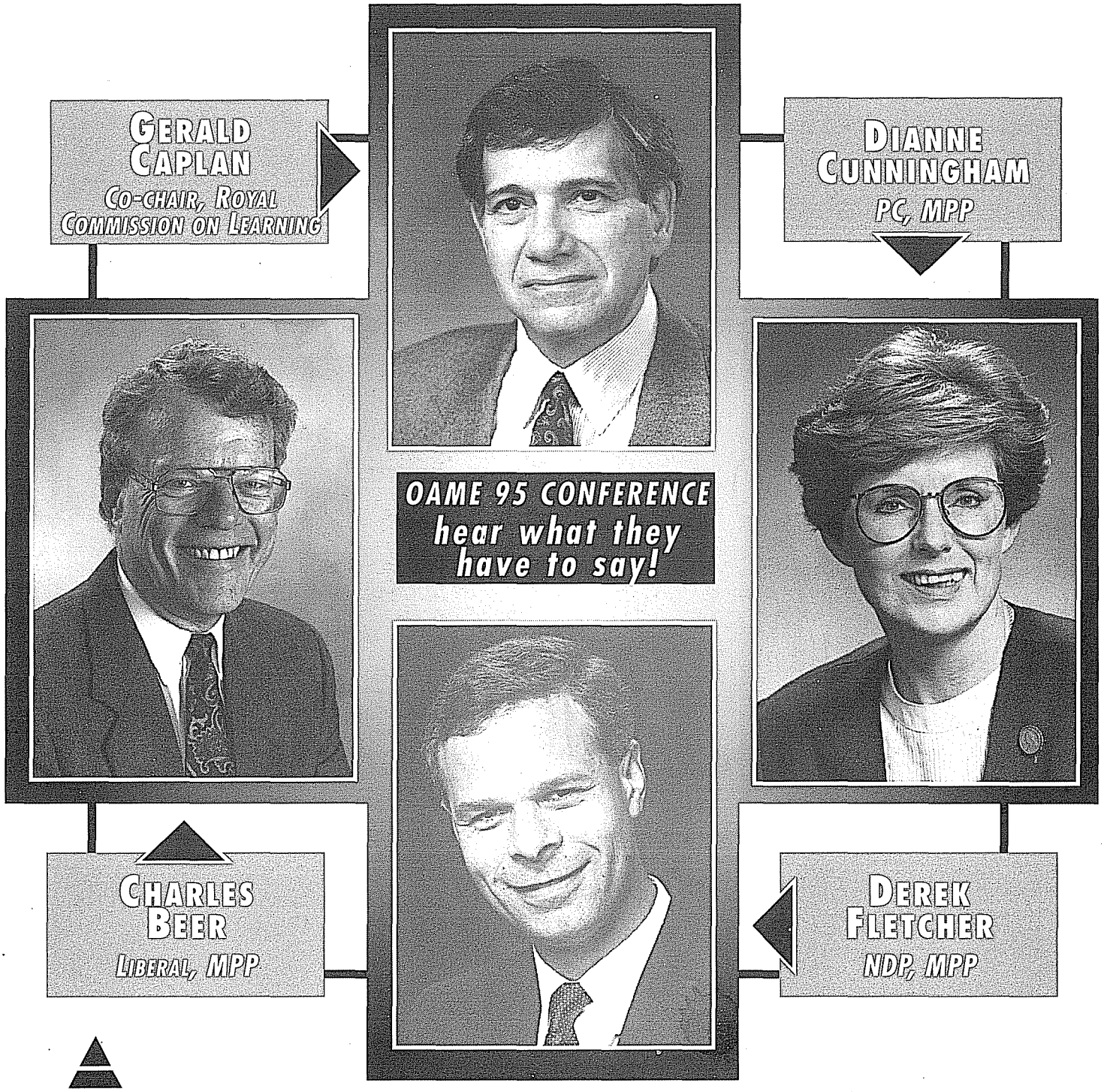
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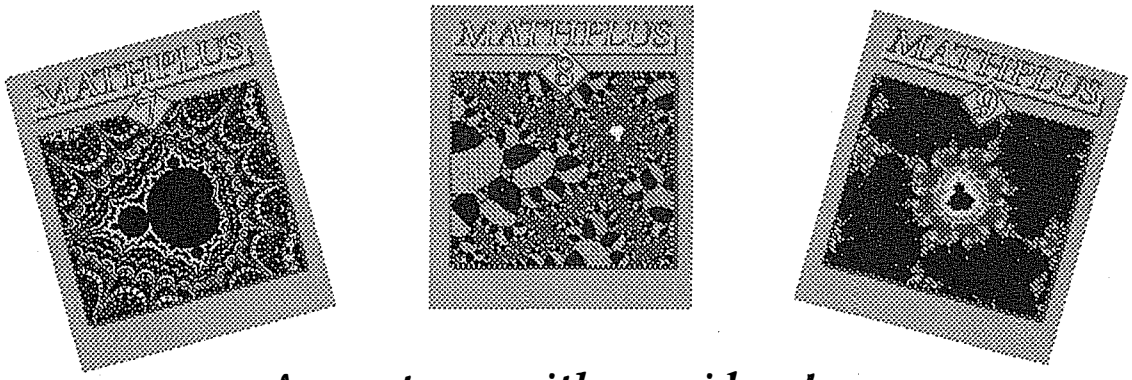
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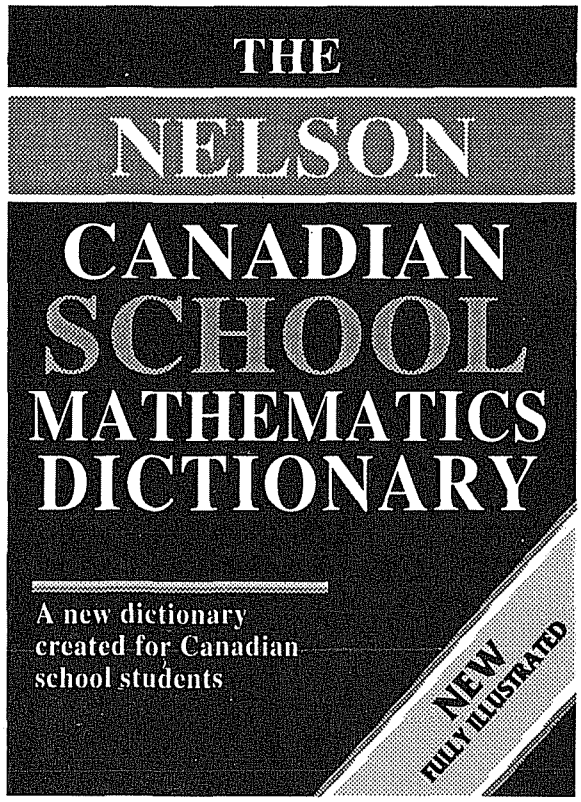
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▲ Editorial



Everyday now for two months, I have listened with nervous anxiety to the morning news. Did Bob call the election? No, please no. Mr. Rae, please wait until Thursday, April 6, at the earliest. If I recall, there must be at least 37 days from the date the election is called to the day we go to the polls. This would take us to Saturday, May 13. Since a weekend election is highly unlikely (is it even legal?), the election would then take place no sooner than Monday, May 15.

Why do I care? Have I got a conference for you. What does the conference have to do with the election date? Read on.

I am truly excited about this year's conference, OAME/95. One, it is occurring at the University of Guelph, my home away from home. Believe me, this place is gorgeous in spring (and summer and fall and winter, but I admit, I'm biased). I can almost guarantee you'll fall in love with both the campus and the city. Two, many dedicated people have put a lot of time and effort into organizing the conference. The resulting program is terrific—well over a hundred and fifty sessions including fifty for the early-formative years. Thursday night, we have famous fractal geometer Robert Devaney. You may have seen his amazing fractal videos on NOVA. The opening keynote address will feature the president of the University of Waterloo, Dr. James Downey. He authored the New Brunswick version of the Royal Commission on Learning.

And that leads me back to the my early morning anxiety. Saturday, May 13, after the OAME General Meeting, we have only one session. But what a session. Ladies and gentlemen, let me introduce to you: Mr. Gerald Caplan, Co-Chair of the Royal Commission on Learning, Ms. Dianne Cunningham, Ontario Conservative Education Critic, and Mr. Charles Beer, the Ontario Liberal Education Critic. We tried to get Minister of Education and Training Dave Cooke, but he had a scheduling conflict. The NDP will be represented by Guelph MPP Derek Fletcher. The topic for discussion: the Royal Commission on Learning and the Future of Education in Ontario. University of Guelph President Mordechai Rozanski will moderate.

This panel discussion couldn't be more timely or more important. Time will be set aside for questions from the audience. Incidentally, for those of you who only plan to attend on Thursday or Friday, your one day registration includes a ticket (if you request one when you preregister) for Saturday.

The Royal Commission is already having a major impact on education in this province. If my morning anxiety is justified and there is an election before May 13, the make-up of the panel may change and the electric potential of this event may be somewhat grounded out. (I checked with my physicist friends and they say this metaphor works.) That will not diminish its importance.

But if the election is still forthcoming, then this Saturday panel promises to be exciting, informative, and may quite possibly give you just what you need to decide where your vote should go.

Come to OAME/95. Come for the fun, for the math, for the friends, for the memories that you will make, and, of course, for the forum on education. See you in May.

Jack

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▲ PRESIDENT'S MESSAGE



TODD ROMIENS

The nice thing about being in mathematics education today is that there is never a dull moment. In fact, things are happening so fast that it is sometimes difficult to keep up. By the time you read this message, some of the information in it will, no doubt, be outdated.

By now you have probably seen the revised copy of The Common Curriculum, the Math and Language Standards, and have received some version of the report of the Royal Commission on Learning. There were 167 recommendations put forward in the report and it seems Mr. Cooke is anxious to act on a number of them. I would like to comment on a few recommendations in light of their impact on the mathematics community.

The first of these is the curriculum issue. Mr. Cooke had promised to "begin implementing a province-wide curriculum and provide classroom-ready teaching materials... This curriculum will pave the way for a standard provincial report card." My question to Mr. Cooke is: "Who is going to write these curricula and teaching materials for the province?" Since their inception, OAME, in conjunction with OMCA, has been providing many materials and ideas for mathematics instruction. Our Special Projects Committee has done a marvellous job providing exemplary assessment ideas across the province. Will our organizations be included in the process of defining curricula at all levels? If not, who will? Some of Mr. Cooke's comments cause me concern. He has said: "Marks will not only mean something, they will mean the same thing throughout the province." Is he suggesting that we halt the forward thinking processes of authentic assessments, wherein we are focusing on process rather than on product, assessing mathematics in different ways beyond paper and pencil tests, and reporting student evaluations in more meaningful ways to parents? Does he honestly intend that a numerical symbol will satisfy parents in their quest to know more about the ability levels of their children, and that this mark will actually represent identical meaning and interpretation throughout the province? Are all teachers expected to be at the same place at the same time in their delivery of curriculum? Are these desirable goals for education?

And what about the students who do not "measure up?" What about those who, for various reasons, are not ready to learn - the ones who disrupt the education of others by causing problems in our classrooms? Is there anything in the report of the Royal Commission that addresses these realities? The answer I have heard is that "remediation should be provided." Can this be possible in a time when resource people are being eliminated, learning disabled students are being integrated, and consultative services are being dismantled? Classroom teachers will again be asked to do more and more with less and less.

Secondly, we have the issue of school councils which "must be established at every school in Ontario by September, 1995." I think this can be a positive step toward helping parents get involved with their child's education. We cannot accomplish shared responsibility and shared accountability without parental help. I see that meaningful dialogue and communication can occur in this forum. My fear, of course, is the fringe element that is crying out for a return to "the old ways" and how much influence they will wield in this forum. If these people have a say in "curriculum and program priorities," how does this relate to the Ministry's stand on providing 90% of the curriculum? There are many other important issues that merit comment. Unfortunately, space does not provide the opportunity to do so.

Since this will be my last message as president of OAME, I would like to thank you, the members, for your support over the past year. In many ways it has been a very exciting tenure. I see our organization strongly influencing the teaching and assessing of mathematics into the twenty-first century. I see an overwhelming desire on the part of our membership to improve mathematics instruction in all schools. I see teachers with great enthusiasm for teaching mathematics and the creative ability to teach it effectively. Truly, these are changing times. Nevertheless, I am confident that collectively, we will weather the current storms of controversy in order to provide a sound mathematics education for Ontario's students.

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CONTACT PERSON:

Jeri Lunney

▲ Letters to the Editor

Dear Editor:

Congratulations on yet another outstanding issue of the *Gazette*. While the latest issue (December, 1994) contained articles that should be of interest to all mathematics educators, the article by Hugh Allen *("Mathematics in Schools: A Form of Intellectual Hazing?")* is something that every secondary mathematics teacher should read. As far back as I can remember I have been uncomfortable with the mathematics that is taught to students. I enjoyed being able to read an article that got right to the heart of why I feel the curriculum is failing to give students a true sense of what mathematics is all about. I think that many of our students view mathematics as a hoop that they need to jump through in order to get to the next step of their education. The thoughts shared by Hugh in his article could go a long way to helping students view the subject as a step towards becoming a better thinker. It would also help them to look at the world around us from a mathematical point of view and to realize that mathematics is useful from both a pure and an applied perspective.

While Hugh's points mainly centred around the process of completing the square, his comments could be equally applied to so much of what we teach. All mathematics teachers should begin the process of rethinking the curriculum and a serious attempt should be made to blend together the content so that students do not see mathematics as a set of terminal skills. This will be a painful process for some educators. It is easy to be convinced that the material on Page 57 is important because it will be used on Page 58, as well as next year. In the long run this approach does little to promote numeracy, nor does it give the majority of students a sense that mathematics is a subject that is worth pursuing.



Ron Lancaster
St. Mildred's Lightbourne School
Oakville

Dear Editor:

I would like to take the opportunity to comment on Hugh Allen's Article *"Mathematics in Schools: A Form of Intellectual Hazing?"*, which appeared in the December, 1994 issue of the *Gazette.*

I was both excited and disappointed by what Hugh had to say in his article. One of the major differences between teaching at a university and at high school (I've taught both) is the amount of time one has available for research and advanced st. . In this article, Hugh explains why "...it continues to be the case that students are required to learn algorithms and skills that are no longer necessary..." by making the statement that "...the main reasons these topics continue to be taught are that teachers know the topics, are comfortable with them and like to teach them."

Secondary school teachers are bound by course outlines written from ministry guidelines which mandate the curriculum. The amount of flexibility in the teacher's choice of content is governed by the department head. The department head's choices, in turn, are made under the direction of the Ministry of Education. The bottom line is that the classroom teacher may not have the right to choose what is being taught or even the method of presentation even though, when the classroom door is shut, **many of us do!**

The second major constraint on the secondary school teacher is the lack of time. Each classroom teacher is required to contribute to the overall running of the school. In addition to the uncountably finite number of quizzes, tests, assignments and an every increasing list of other assessment instruments which must be evaluated and marked, they are probably doing one or more extra-curricular activities. These activities include coaching a sport, being a staff sponsor of a club, yearbook, student council, and being on one or more committees in the school such as exams, commencement, transition years, learning outcomes, to name a few. Teachers are spending their lunches, after school time and even spares designed for lesson preparation assisting the department head with departmental duties, maintaining contact with parents regarding progress and attendance, assisting students with their studies, math contests, and the list goes on. What we are most deficient in is the time, not the desire to develop those innovative, creative lessons that can "... set the class to work practising the skill for hours, days or ...".

Now for the **positive side** of my comments about Hugh's article. When we get the time to read the journals (I read this one during the Christmas break) many of the articles are either about something we already do or are about something we will probably never get a chance to do. I'm not saying they are of no value, but rather, that in comparison I found Hugh's article to contain something that I can, and will, put to use. It was nice to see how he used transformational geometry to solve an algebra problem. Those of us who are using the graphic calculators can use this as an additional tool to connect traditional algebra to technology. I really enjoyed the way he left us with something to think about (how to divide $2x^2-5x^3-3x^2-6x+7$ by x-2 without using the division algorithm.)



The main import of this letter is in the form of a request. **"Please sir could we have a little more...!"** I am sure there are many of us who would like to see a continuation of articles of the same nature (but on a more positive note).

Richard Dewey
Individual Education Centre South
Mississauga, Ontario

(Continued on page 35)

▲ Have Your Say
Reflections on a NCTM Regional Conference



JAMES M. HAINES
ASSOCIATE PROFESSOR
LAKEHEAD UNIVERSITY
THUNDER BAY, ONTARIO

After 15 years of teaching mathematics, James M. Haines assumed a position at Lakehead University's School of Education. His areas of interest include preservice teacher education, mathematics education and classroom management.

Attending the NCTM Southern Regional Conference in Tulsa, Oklahoma, from October 13-15, 1994 has provided many opportunities for reflection and comparison with current issues in Ontario Education.

The Commitment Issue

The first general question raised was: is there more of a commitment toward mathematics education in the Southern United States than in Ontario? (This is not to say that there is.) Three facts form the basis for raising this question. The first observation was the large number of elementary school teachers in attendance. Secondly, there were several references to inservice programs, institutes and professional development centres in the midwest. Finally, two calculator companies had made significant financial and time commitments to train large numbers of teachers in the use of their graphing calculators. One company had reportedly provided hundreds of thousands of dollars in overhead graphing calculators to workshop participants. The two-hour Casio-sponsored workshops were designed to reach every Oklahoma teacher in grades 6 to 12. Texas Instrument provided similar but more lengthy two-day workshops. Although these facts do not guarantee a better quality of education, they do provide the potential for establishing a base.

Calculus Reform

A second question, brought up in a session entitled "The Calculus Reform Movement", involved the future approach to the content of first-year university calculus and possible ramifications for high school. Two revolutionary university calculus texts, the "Harvard" and the "Ostebee-Zorn" have been piloted in the United States under the sponsorship of the National Science Foundation. Both are reported to be de-emphasizing "paper-and-pencil manipulative skill work" approaches to calculus, as advocated in the NCTM Standards (1989). Another goal is increase the numbers of students successfully completing first-year calculus. There is a potential that this movement may have a major impact on the way in which high school mathematics is presented and conceptualized.

The Graphing Calculator

A third issue raised at the Tulsa conference revolved around the issue of what might be considered appropriate use of the graphing calculator. Given that a Curriculum and Evaluation Standards for School Mathematics assumption for grades 9-12 is that "scientific calculators with graphing capabilities will be available to all students at all times" (NCTM, 1989, p. 124), how and for what topics will teachers use graphing calculators? Some insight into this issue was provided in a session, "Algebra Lessons using the Graphing Calculator".

Question #1

The presentation involved the topic of solving the quadratic equation $x^2 - 8x + 12 = 0$ by three methods: by factoring, by graphing, and by the use of the formula. The factoring method was rather straight forward and mathematically sound. However, the adventure into the use of the graphing calculator was quite fascinating.

The reason for the initial concern was that the calculator was limited to graphing functions, and $x^2 - 8x + 12 = 0$ does not represent a function. The presenter miraculously changed the equation to $y = x^2 - 8x + 12$. The resulting graph was a parabolic curve. The x-intercepts were read and routinely placed in the product of two binomials, with the appropriate sign changes. At no time was the point made that the graphs of the initial quadratic equation, two parallel vertical straight lines, and the quadratic function, a parabola, were vastly different.

The use of the quadratic formula, the third method, was a logical progression from the quadratic function which had been created, but, like the second method, removed from the initial quadratic equation.

Question #2

The second question reversed the process involved in question #1. In the first part, the roots of a quadratic, 6 and -6, were provided with the directions to write the factors. In a "slip", $(x-6)(x+6) = 0$ was accepted as the correct response (an equation, not "the factors"). In the second part, participants were to write the quadratic equation, $x^2 - 36 = 0$. The final step was to sketch the graph. Again, miraculously, the equation became $y = x^2 - 36$. When graphed, the initial quadratic expression, which had been converted to a quadratic function, appeared erroneously as a parabola.

Thoughts On Appropriate Use of Graphing Calculator

There is no doubt that "the graphics calculator offers a stimulating visual approach to the study, application, and exploration of mathematics" (Nowak, 1994, p. 13). However, the above examples raise a question: are the graphing calculators being promoted for inappropriate use?

It might be argued that the two questions discussed earlier are examples of inappropriate use of the graphing calculator to artificially solve problems. Graphing non-functions is not within the capabilities of the calculator. To convert equations to functions in order to generate a graph with the appropriate x-intercepts may have little mathematical meaning for the students. If a goal of mathematics is to promote understanding, then such exercises may do little more than promote the calculator to the level of a fancy game involving "rote memorization of facts and procedures" which is supposed to receive decreased attention (NCTM, 1989, p. 129).

Conclusion

The opportunity to reflect on possible future changes and comparisons between regional differences is one of the values of attending conferences. Such times help to prepare for reasoned change and maximize the chances for successful implementation. The Tulsa conference was this and much more.

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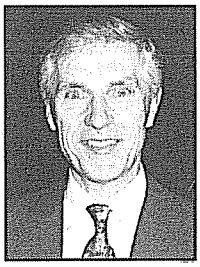
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▲ Moving Towards Inquiry - What are the Implications for Curriculum?

JOHN L. CLARK
TORONTO, ONTARIO



John Clark is a retired coordinator of mathematics for the Toronto Board of Education. Recently he completed research into teaching through inquiry at grades 7 and 8. He is interested in issues related to curriculum and teacher development.

Many current curriculum documents refer to inquiry as a preferred approach to teaching mathematics. In the December issue of Gazette, I described how inquiry conflicts with transmission, a method of instruction which presumes knowledge can be passed from teacher to students, also a method which dominates teaching in many elementary and secondary classrooms. Underlying inquiry and transmission are conflicting beliefs about education, knowledge, mathematics, and learning.

Teaching by inquiry is based on beliefs that education should foster critical thinking and develop autonomous learners. The purpose for knowledge is thoughtfulness, not a storehouse of skills and information. Mathematics is primarily a process of making and testing conjectures and effective mathematics learning results from a search for meaning. Underpinning transmission are a set of beliefs which are in direct conflict with those for inquiry. While many who teach by transmission espouse some of the same inquiry beliefs, their classroom practices demonstrate the opposite.

FOOTNOTE: There was a typographical error in the article on inquiry in the OAME Gazette, December, 1994. On page 23, in the paragraph beginning "Resnick (1988) discusses how...", the paragraph should begin, "Resnick (1988) discusses how different perceptions of the discipline of mathematics lead to corresponding views of learning mathematics. She notes the widely held view of mathematics as a "well-structured discipline" and the corresponding emphasis on symbol manipulation and rules without questioning or discussion by the students."

How we view the discipline of mathematics itself leads to corresponding views of learning mathematics (Resnick, 1988). Thus if we view mathematics as a dynamic ill-defined discipline consisting primarily of making and testing conjectures, this is how we will try to teach it. Viewing mathematics itself as essentially a process of inquiry leads to teaching mathematics through or by inquiry. Teaching by inquiry stresses student questioning, discussion, and problem posing, and greater student responsibility for and ownership of learning. But viewing mathematics as a static rigidly structured and hierarchical discipline leads to a very different set of beliefs and practices for teaching it.

This article and the next are based on a case study of three teachers who moved towards inquiry in their classrooms. In this article, I describe how the conflict of values between transmission and inquiry resulted in difficulties for the teachers in the curriculum they had to teach, and I explore the implications of their experience for curriculum development and implementation.

In the next article, I will describe how moving towards inquiry produced changes in patterns of classroom communication, roles of teacher and student, and student assessment, and I will explore their implications for professional development.

Changes In the Teachers' Practices

In 1991-92, for about 14 months, I worked with three grade 7/8 teachers in a study-group as they attempted to change from teaching mathematics by transmission to a more active exploratory approach which we called inquiry (Clark, 1993).

Previously, the teachers taught mostly from the front chalkboard, and through questioning, led students through solutions to skills-based questions. This was followed by seatwork consisting of similar questions usually drawn from textbook exercises. The teachers assessed students on their ability to solve the same kinds of questions. There was limited use of manipulative materials and little physical activity, and student discussion was mostly about how to do questions and the correctness of answers. To illustrate the change from these practices, here is a summary of an investigation we used to teach fractions.

As an introduction, the teachers asked their classes to discuss in groups what they knew about fractions. Each group made a class presentation using notes on large sheets of chart paper which were taped to the chalkboard for future reference. The students were then instructed to cut out ten strips of paper, each 15 cm by 3 cm. One strip was labelled "1 whole". Another was folded into halves and each section labelled "1/2". Similarly, the other strips were folded and labelled to show quarters, eighths, thirds, sixths, twelfths, fifths, and tenths. The teachers also told their classes to describe in writing the patterns they discovered while making the fraction strips. Students retained their own set of strips in an envelope for future discussions.

Over a period of about three weeks, the investigation continued through small group and whole class discussions. As well as fraction strips, students used pieces of note paper and square-grid paper. The approach from day to day was flexible and each class grew out of the preceding day's discussions. Although much was common to the three teachers' classes, different things happened. The topics investigated were the order relations, equivalent fractions, the four operations, applications, and the meaning of fraction, numerator, and denominator. Here are some highlights of what happened.

- A student said that to compare fractions, the whole has to be the same. This led to the problem of how to share equally two non-congruent rectangular sheets of paper among four students.
- A student asked "Does every shape have a half?"
- While clarifying the meaning of numerator and denominator, a student wrote 6/3 = 2 on the board as an example. This sparked a lively discussion as students made drawings to illustrate.
- Students usually generated their own problems. For example, they were asked to write and then order three fractions. One student wrote on the board {1/3, 1/5, 2/3} and another student said "That is easy!" The teacher used the comment as an entry point to investigate sets of fractions which students believed were easy, harder, or hard to order.
- The preceding activity led to a classification of problems involving sets of fractions in which numerators were each one, denominators were all the same, numerators were all the same, and numerators and denominators were different.
- Students used a variety of problem solving strategies which included approximations and what they called "common sense".
- There was not always time to discuss every student's question or comment, so one teacher wrote some of them on a flip chart under the heading "New Questions", and returned to discuss as many as possible.
- The teachers frequently asked small groups and the whole class "What do you think?", "Is that always true?", and "What is your proof?"
- At the beginning, during, and end of the investigation, students were asked what they knew so far about fractions and to illustrate with examples.
- One student wrote seven pages at home describing his "theories" for the solution to the student generated question, $\frac{5}{12}=\frac{9}{?}$.
- On one occasion, a class of students lined up excitedly for their turn at the board as they wrote about 25 fractions equivalent to 3/5.

The teachers used similar investigations in other topics which they developed from open-ended questions for which there usually were many answers, and tasks in which students could make important discoveries and conjectures, often through observation and discussion with manipulatives. As an investigation continued, students as well as teachers posed problems and questions and the response in one lesson determined what would be attempted in the next. Students discussed and wrote daily about mathematics, not only in mathematical notation, but in everyday language as well. The reasonableness of conclusions and correctness of solutions were determined primarily by discussion and consensus. The teachers assessed student progress holistically by observing daily work, examining write-ups of investigations, and tests in which explanations and justifications for answers were required.

A Different Experience of the Curriculum

Teaching mathematics this way the teachers' experience of the curriculum was different in three ways. First, they found that one problem, comment, or question led to new ones which could not be predicted. The thread of inquiry which unravelled was not a straight line but an unpredictable path through any topic. This is contrary to the dominant hierarchical organization of knowledge found in detailed curriculum guidelines, textbooks, and scope and sequence charts in teacher guides. As a result, a major difficulty the teachers encountered was being able to predict what would happen in the coming week, tomorrow, or even in the next few minutes. They usually waited to see what students said or did, but found this was risky because it often led into unknown territory. Questions arose for which the teachers were not sure of the answers or which involved content not on the grade 7/8 curriculum. This experience of the curriculum was student-directed and, for students, established a continuous thread of connections from one idea to the next. And learning was dynamic, not static. The students demonstrated that there are potentially an endless number of paths through mathematical knowledge. Pollak (1987) calls this "cross-country" mathematics.

Our experience demonstrates that students can, indeed should, have a say in what path to take through mathematical ideas. By providing them this opportunity, teachers promote "intentional learning" (Resnick, 1988) in which learning is firmly rooted in awareness and motivation. Approaching curriculum this way acknowledges that there is not just one reality of mathematics, usually that which is imposed, but multiple realities which are functions of the previous experiences of students and teachers and who they are (Lincoln and Guba, 1985).

The second difference in the experience of the curriculum was the way the teachers planned for classroom instruction. The traditional method is a lesson plan prepared in advance stating precise ends and means. The direction of planning for learning is from an objective or learning outcome to how to teach it. Unfortunately as Brown (1991) discovered, this approach rarely leads to serious mathematical thought. In our experience, however, the teachers effectively reversed the direction of planning. They began with an activity, problem, or question often suggested by students themselves and this gradually generated many concepts and skills, as well as serious mathematical thought. This approach to instructional planning is advocated by Lampert (1990, 1991) who begins instruction with good problems. Our study demonstrates how a question such as 5/12 = 9/? can also motivate students to develop their own theories. Therefore, it raises the question of what makes a good starting point for student learning, and suggests that one characteristic at least should be the potential to engage with powerful mathematical ideas. The fraction investigation, for example, promoted conjecturing, proof, classification, order, and equivalence.

The third difference was in depth of instruction. In the interest of coverage, teaching by transmission often glosses over concepts, but the teachers discovered in inquiry that any concept, problem, or question is potentially the tip of an iceberg. They were surprised how long students were able to dwell on "obvious" ideas, or at least ideas which previously had taken much less time "to teach". The dilemma this produced was one of depth versus breadth of learning. The teachers were torn between developing a powerful knowledge of some things or a general knowledge of many things.

Curriculum Issues

These differences in how the teachers experienced the curriculum reveal two issues. First is the specification of curriculum by grades. In our study, student inquiry led outside the prescribed curriculum for a grade in many directions. It led across to another topic, back to first principles taught in lower grades, and forward to work of higher grades. The student's question, "Do all shapes have a half?", has the potential to generate mathematics which cuts across several grades. Although elementary school teachers may feel the problem more acutely, some students at any grade level, given freedom to inquire, will outpace their teachers. Perhaps being able to withstand feelings of insecurity or inadequacy is a higher teaching qualification than the ability to be an authority or keep a step ahead (A.T.M., 1967). In our experience, the teachers' feelings of insecurity were partially alleviated as they grew more comfortable with their emerging role of co-inquirer rather than authority figure.

These experiences reveal that the traditional curriculum by grades is actually an external structuring of knowledge for the purposes of accountability and management. It assumes that mathematical knowledge is finite, quantifiable, and can be apportioned into small pieces to be learned one at a time.

Alternative views of curriculum are based on different views of knowledge and who should control it. There is a sense in which in authentic inquiry, students and teachers should begin with a *tabula rasa*, a field of knowledge with no strings attached. The extent to which the dictates of a curriculum and

forces of accountability should be allowed to impinge upon authentic inquiry is a difficult question. The dilemma which the teachers faced in our study was reconciling the dictates of the curriculum with the pull of students' questions and observations. What is needed to ease this dilemma is a curriculum in which the priority for inquiry is firmly entrenched, and the importance attached to facts and skills does not diminish this. The Ontario Ministry of Education (1985) guideline for intermediate mathematics presents the philosophy and process components of the curriculum in the first part and then lists mainly content in the second part. In practice, many teachers and department heads either ignore the philosophy and process components and focus primarily on content, or find textbooks which effectively do the same thing. Our study raises the questions of what form a curriculum guideline should take if it is to support mathematical inquiry and still satisfy public accountability, and how much and what content in any grade are absolutely essential.

Our study raises a second curriculum issue, the justification for topics in the curriculum. It is common for teachers of mathematics to have to justify what they teach and they frequently do this unsatisfactorily by saying it will be needed sometime in the future. The teachers questioned why they should have to teach topics such as the order of operations or prime and composite numbers. We eventually agreed that if we taught something, we should be able to demonstrate at least that either the topic was useful to students in everyday life or it promoted mathematical thought. Being able to justify our teaching this way consistently was not easy. Some topics were taught simply because the teachers felt they had to or it was something they had always done.

The Problem of Time and Coverage

The preceding discussion raises the closely related problem of time and coverage. The pressure to cover a course of study within the available time exists for most elementary and secondary teachers of mathematics. In our study, the pressure originated in the teachers' previous experiences, the Board's curriculum guideline, textbooks, external and internal student assessment, and the climate of the school determined by colleagues, principal, and parents. Being grade 7/8 teachers, they also perceived pressure from secondary schools to have students ready for grade 9 with the knowledge of certain concepts and skills.

The teachers frequently expressed concern about the length of time an investigation took because they were worried about the curriculum they had to teach. They reminded me on occasion that they taught more than mathematics and felt stretched because of the work load.

In our discussions, we examined critically the phrase, "covering the course of study." We acknowledged the superficiality of check-lists in which teachers check off items of knowledge as having been done. The teachers recognized that student understanding within and among classes is uneven, and that for many students, there may never be enough time to understand certain concepts adequately.

The goal of developing an inquiry approach produced a new set of pressures which arose out of the desire to accommodate students' questions, interests, and discoveries in daily classroom work. In investigations, students behaved differently. The teachers said the variety of things students did and said was fascinating. They frequently expressed the view that their students needed time to "muck around", to play, to reflect on their learning, and time for ideas to sink in. The teachers often commented how actively engaged their students were, how they stayed on task, and surprisingly, how their interest could be sustained for long periods of time of up to an hour or more. They found that an investigation could continue profitably beyond the anticipated duration, and time spent was not being wasted because many important concepts were emerging and student interest was high. It was difficult to know when to end an investigation. Through inquiry, the teachers were learning to uncover content, not cover it.

But the pressure to cover content was always present. The teachers believed in some investigations that students were not learning certain skills and concepts which the teachers viewed as important, so they looked for opportunities to squeeze them in. For example, during an investigation involving the classification of triangles, the concept of an angle and the use of a protractor were on the teachers' minds. These topics were not coming up naturally in the investigation so the dilemma was whether to impose them, or wait for another investigation. The teachers also felt that they had to "fill in the gaps" in the backgrounds of their students which included not only knowledge of certain content, but also comfort learning with manipulatives.

Hargreaves (1990) notes a difference in the perception of time by administrators and teachers; administrators tend to speed it up and teachers slow it down. Administrators have a technical-rational concept of time which exerts external pressures on teachers. Teacher conception of time, however, emphasizes personal relationships and flexible management of many simultaneous demands. He argues that the solution to the impasse between the two conceptions should include bridging the division between curriculum implementation and development. All teachers should be involved in the development of the curriculum they will have to teach.

Thus there are many priorities for teachers' time and the time-coverage issue has many dimensions. When the teachers said they felt pressure to cover the course of study within the available time, it usually reflected a view of knowledge as commodity and teaching as transmission. When they acknowledged that they were fascinated by the responses of their students, and they needed time to experiment, discuss, and draw their own conclusions, it reflected a view of knowledge as process and learning as inquiry. The teachers were torn between the pressure to cover the course of study and the pull of students' questions and discoveries. It is a struggle between external control of learning and that which

can be achieved by teachers and students working together. In our study, how teachers viewed time and coverage at any stage was an indication of their philosophical position on the continuum between extreme expressions of transmission and inquiry.

Summary

A curriculum based on inquiry differs from one based on transmission in fundamental ways because underlying the two are conflicting views of knowledge and who should control it.

The transmission view of knowledge is that it is a commodity which is quantifiable and can be broken down into small bits. The corresponding curriculum is a highly prescriptive and structured organization of mathematical knowledge. The path through is determined prior to learning and controlled externally. Starting points for learning are objectives or learning outcomes. With inquiry, however, knowledge is viewed as process. Information, facts, and skills are tools for solving problems and making and testing conjectures, not ends in themselves. There are many paths through mathematical knowledge and the path taken can be decided collaboratively by teachers and students. Starting points for learning are questions, often poorly formulated initially, and are posed jointly by students and teachers.

By stressing information, a transmission curriculum leads to shallow learning and assessment procedures, and pressure on teachers to cover a course of study. Teaching facts and skills out of meaningful contexts robs them of their role and educational usefulness. An inquiry curriculum, however, is dynamic, not static, and stresses meaning, mathematical thought, big ideas, and deep exploration.

As the teachers in our study moved towards inquiry, they found themselves in an untenable position buffeted by conflicting pressures and expectations. To address this problem, we must debate the fundamental beliefs about knowledge, mathematics, and learning which underpin the expected curriculum and make a clear choice between transmission and inquiry. The curriculum cannot be both a transmission and inquiry curriculum.

Once we agree on the philosophy of the intended curriculum, then we must deal with how it can be taught in the classroom. Taking inquiry seriously in the classroom poses major challenges for both teachers and students. I will describe these more fully in an article which will appear in the next issue of Gazette.

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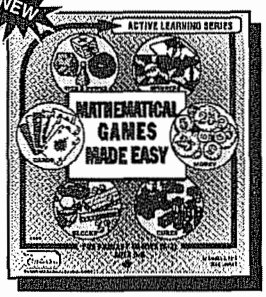
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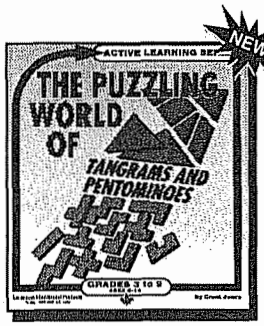
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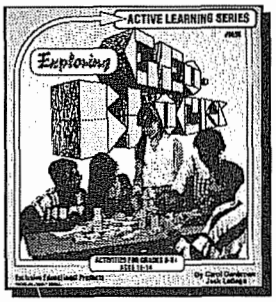


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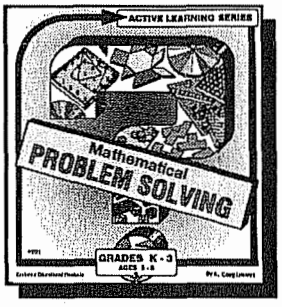
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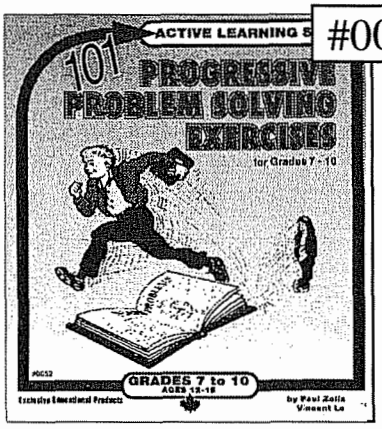
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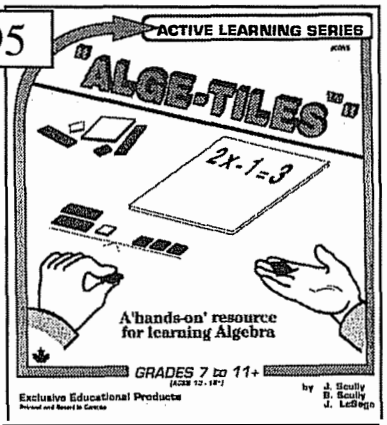
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Addressing the Problems: Preparing Future Teachers for the Mathematics Classroom

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As a teacher of pre-service elementary teachers, I was very interested in the September 1993 issue of the Gazette. Two articles addressed the very real concerns in preparing future teachers for the mathematics classroom.

Geoff Roulet, in his article "Did Pythagoras Really Say '$A^2 + B^2 = C^2$'?" (p17) talks about the pre-service teachers' tendencies to "algebratize" all mathematics solutions and points to their inability to talk about mathematics in plain language (p18). In my experience at a Faculty of Education, I have also discovered that many of them are "algebraic" or "algorithmic" dependent and are rarely able to explain formal procedures or the use of an algorithm in their own words. Often statements such as, "Oh, that's what that means!", or "I know how to do it, but don't ask me why!" are heard in the classroom. The pre-service teachers struggle with various forms of representation, find it difficult to break skills into meaningful, teachable sub-skills, and hesitate when trying to verbalize their thinking about a concept such as multiplication.

Rheta Rubenstein, in her article "Problem Solving in Mathematics: A Fresh Perspective for Future Teachers" (p13) highlighted another serious problem faced by the Faculties of Education in Ontario. She describes to us how pre-service teachers at the University of Windsor are given a two-hour workshop "in what may be their only experience in mathematics education."

Ms. Rubenstein realizes that, in a mere two hours, she will have "limited cognitive effect on (the students') understanding of mathematics and its teaching."(p 16) In fact, we know that it is more likely that the two-hour workshop will have no cognitive effect on these future teachers and that the affective results will be short term. It is likely that there will be no lasting effects on the participants' attitudes towards mathematics teaching and therefore will not make a difference towards the way they decide to teach mathematics in the future. This is not to imply that the workshop was not thought-provoking at the time. Joyce and Showers (1980, 1983), however, among others, have reported extensively on the lack of transfer of new teaching skills from workshop settings. Teaching skills must first be developed and then transferred to the classroom setting. Transference takes place when teachers have the opportunity to practise, see the innovation in action and are coached when using the skill or approach. Simply having heard about an innovation does not guarantee knowledge, skill, and acceptance, and certainly will not lead to transfer.

The Problems

Two serious problems exist. First, many pre-service elementary teachers do not have a solid understanding of mathematical concepts. For example, many express frustration when attempting to find strategies for presenting common algorithms to learners. When asked to explain a concrete model for the subtraction algorithm, many of my students were at a loss for mathematical words and concepts. This is supported in a study conducted by Martin A. Simon (1993). He found that pre-service teachers exhibited serious weaknesses in their understanding of division as a model of situations (p 251).The study showed that knowledge of division was procedural and sparcely connected to conceptual knowledge or real-world situations (p 252). Other studies conducted by Tirosh & Graeber (1989) and by Ball (1990) reveal the same weaknesses in other areas as well. We might therefore assume that these pre-service teachers with weak conceptual backgrounds will be unable to make sound instructional decisions to enhance children's understanding of mathematical concepts and to make assessment decisions based on analysis of childrens' work.

The second problem is a matter of time. The number of hours allotted to mathematics in a pre-service teacher's education varies considerably from faculty to faculty. With any amount - from a mere two hours to a rare forty hours - of formal mathematics methodology instruction as their background, newly qualified teachers in this province are being employed to teach mathematics in all grades from Junior Kindergarten to Grade 10. Past experience tells us that these students are likely to rely on the familiar by employing the teaching strategies that they experienced when they were in school. This will surely not lead to classrooms and schools that are prepared for the 21st century.

Looking For Solutions

The problems of weak mathematical understanding and lack of time allotted to mathematics methodology were the two

problems that were tackled by the faculty at Nipissing University. In order to try to deal with the problems, two solutions were considered and adopted.

Nipissing University offers undergraduate students "Orientation to Teaching", which is a prescribed set of courses taken as part of an undergraduate degree program in preparation for the Faculty of Education. Three years ago, it was decided to add among the required courses "Algebraic Foundations of Arithmetic", a course for those students who might not be able to be successful in other undergraduate mathematics courses and who were intending to become elementary school teachers. The course emphasizes an understanding of our number system and uses applications from elementary mathematics. Since its inception in 1990, this has become the mathematics "course of choice" for a large number of students who arrived at university with few mathematics courses in their secondary school portfolio and for those who are math anxious. Of the 215 Primary/Junior pre-service teachers in the Bachelor of Education program in the 1993-94 school year, 32% had taken a university mathematics course of some type. Approximately 65 are graduates from the Orientation to Teaching and approximately 46% of those had taken the Algebraic Foundations course. Because this is listed as a first year mathematics credit course, a few practising teachers have also enrolled and we hope that more can be encouraged to do the same.

The second proposed solution to the identified problems was to increase the number of hours allotted to Primary/Junior mathematics methodology. A brief was prepared and presented to the Education Faculty highlighting the problems, the number of hours allotted to mathematics in the elementary systems across Canada (approximately 10% of the student hours), and the results of our pre-service teacher surveys from a number of years (in the 1992 spring survey, 76% indicated a need for an increase in hours alloted to math methodology). It was decided that the Primary/Junior pre-service students would receive an extra 12 hours of mathematics methodolgy instruction, increasing the course from 24 to 36 hours.

Evaluating the Results

In order to evaluate the effects of these two structural programme changes, a committee is in the process of creating a formal study to collect data on the effects of the undergraduate mathematics course on dispositions towards mathematics and mathematics teaching and learning. The study will assess two groups of successful candidates of this course: those who continue their studies in the Faculty of Education and those who are presently teaching in elementary classrooms. The second group is important in the study because it is with this group that it will be possible to observe effects, if any, that the course may have on the behaviours of the teacher in a classroom setting.

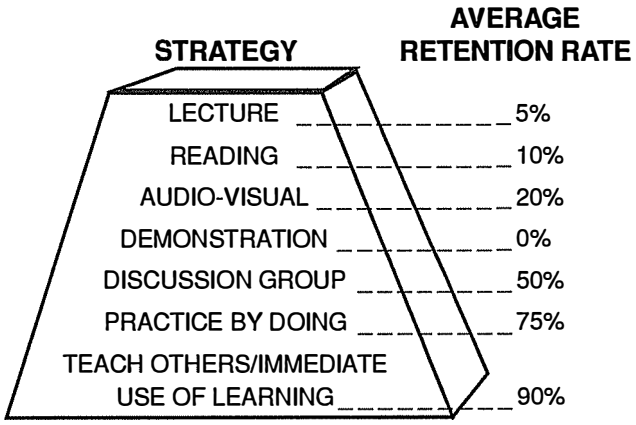
Evaluating the effects of the increase in the number of course hours for mathematics methodology will be more difficult. We may be able to compare the comments on the year end course evaluations and the comments about the program structure, but comparative achievement or attitudinal data is not available.

Continuing Improvements

Changes in the structure of our pre-service mathematics methodology program are expected to produce some positive effects in students' cognitive knowledge. We cannot say, however, that this is the end of the needed improvements to the program. An improved mathematical background and more opportunity to explore mathematics may make a difference, but current writings also point to other changes that are needed to ensure effective mathematics education of future teachers.

The N.C.T.M. *Professional Standards for Teaching Mathematics* describes the needs of pre-service teachers in the "Standards for the Professional Development of Teachers of Mathematics" (p 123-173). The first standard states that mathematics education instructors should "model good mathematics teaching" (p 127) with a refocus from "presenting content in lecture and demonstration to active participation and involvement" (p 127).

The Peel Board of Education's Program Development Department published the "Learning Pyramid" that supports the N.C.T.M. standard :



It was necessary to reform our program in more ways than simply increasing the number of classroom contact hours. What we wished to do was to empower pre-service teachers to become learners of mathematics, to be willing to take risks in their learning, to value different strategies, and to encourage their participation in mathematical discourse (*Standards,* p 127). A program in which the students were actively learning about mathematics, discussing, and teaching others would satisfy these goals and help to ensure retention of learning.

The fundamental goal of our mathematics methodology program is to ensure that the pre-service teachers develop ways of knowing mathematics (p 6) and through that, understand how children know mathematics. Because I believe that learning and teaching are collaborative, I called heavily on my knowledge of cooperative learning to develop the strategies used in the presentation of the program. Students work in base groups, pairs, expert groups and individually to tackle

problems, discover teaching strategies and explore new materials. This year, for the first time, large group assignments were integrated with other subject areas with exciting results.

Pre-service teachers are hesitant at first to take risks in learning, and like many of the children in our schools, look to the instructor for the "right way" and for "the answer". They show frustration at first, and at times exhibit anger that they are required to question, probe, pose questions and generalize. Many say that this is the first time in their post-secondary school career they have been expected to actively take part in the learning. As the groups gain more confidence in themselves as independent learners, it becomes evident that they are enjoying the experience and that their understanding is growing. Peer-teaching reaches a level where the students, in general, feel comfortable enough to question and challenge each other. The level of mathematical language steadily improves. As with many other current studies, we are beginning to notice an increase in the use of active strategies when they are required to teach mathematics in their practicum classrooms. They seem to use more collaborative techniques and begin to plan for learners to be active participants.

In Summary

In order to prepare future teachers who will create mathematical classrooms that embody the visions of the N.C.T.M. Standards, teacher education must provide environments where they can come to understand the power of doing mathematics, the power of talking mathematics and the power of making mathematics. It is imperative that we continue to question the mathematical preparation future teachers are being offered. We must continue to do everything within our power to improve mathematics teaching and learning by valuing and improving teacher education.

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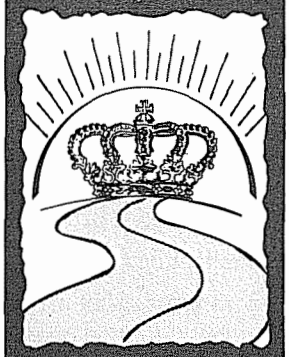
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ROY BLOMSTROM
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Roy Blomstrom is a teacher and writer whose educational works have appeared in Forum and Orbit magazines in addition to the Gazette. He claims his wife is the real math teacher in the family.

Imagine, if you can, being an Anglophone who is suddenly asked to teach oral French. I find myself in a similar position: I am an English teacher whose last math class ended in high school in June, 1964, and for the past two months I have been teaching a class of grade ten advanced mathematics. It has been, thus far, a wonderful experience — albeit a sometimes frightening one. And, I must add, I have not worked this hard at preparing lessons in two decades.

I came to be a math teacher because of a timetabling problem that arose last June. Would I mind teaching a math class, I was asked. Instead of instantly shouting, "Are you crazy?" and running from the room, I hesitated a fraction of a second too long. (I have since learned the importance of fractions.) Now I am a math teacher.

The first thing I learned about mathematics is the strangest of all: it is easy to learn, now. In high school I was, at best, a mediocre math student. I memorized my way through much of my math — and understood little of it. Now, as an adult, math makes more sense than it did when I was a student. Somehow, thirty years of buying groceries and building rec rooms and marking English papers, has rewired my brain so that it does math without overheating so much.

My wife, who is a bonafide math teacher, has helped me a great deal. "Were there some things called surds and indices?", I will ask. "Well, dear...", she'll begin, and I'll find out about another change in the math curriculum since I last sat in Mr. Henderson's class. It is hard to teach math without knowing the terminology and without an awareness of what my grade ten students were taught last year in grade nine.

The textbook (I have never, as an English teacher, worked from a textbook) is a godsend. The whole lesson does not have to be carried in my head. There can be "time out" for students to do classwork — the kind that does not have to be marked. I can assign homework that takes a long time to do and a short time to take up. The textbook actually has a section at the start of each unit that explains the concept and illustrates it by working out sample problems! It even has answers at the back. By night, when I prepare my lessons, the textbook is my teacher; by day it is my friend.

I have organized my classroom into pods (my wife makes me call them "subsets") consisting of four students apiece. I have nine pods. (Can you work out how many students I have?) When students do classwork from their texts they work in cooperation with the other members of the subset. If a student has difficulty, the first place to go is to the other members. If no one in the subset knows the answer, the student goes to another group to see if anyone there can help. I am the student's last hope. Before major tests, I give "subset tests" that allow the students to preview the items on the "real thing" (with the numbers changed, of course.)

The system has worked well. However, now that we are further along into the semester, I can see that there will soon be some problems. Some of the students are not catching on as fast as others. "Extra help" time is upon us.

In August, I worried about what to do when students fall behind, and decided that I would keep a "homework" book on the go to help them. I do the homework that I assign the students, and put the solutions into a binder that students can look at when they need to. The rule is that students may not copy out the solutions in the binder. They may look at the solution, mull it over, and then go back to their seats to work things out for themselves. I have found that the homework book becomes especially useful if I annotate the solutions. Now I routinely jot down little notes when I suspect that some of the steps in the solution are not going to be obvious. The homework book has also been of great benefit to students who miss a day or two of classes, and it has led me to the practice of providing annotated "answer sheets" when I hand back tests.

English and math classes are quite different in a number of ways. My English classes are lesson-intensive. Homework consists in the main of reading or of doing long-term projects. My math class is homework-intensive. The real learning begins when the lesson is over. Students who waste class time or do not work assiduously at homework every night have a very hard time of it. In my English classes, I can announce early in September that by a certain date a novel has to be read. Homework runs in flex-time. My math students, however, have to understand today's lesson today, and to do tonight's homework tonight. One of the crazier educational ideas, it seems to me, is the idea that math classes can be semestered effectively. They cannot. Math students need to have time to mull over their lessons and, especially, time to practice their work from September through June. Homework-intensive courses require long-range time for homework.

As an English teacher I wondered, at times, what marking math papers would be like. I assumed that the same question marked by different math teachers would get essentially the same mark, and that the marking of mathematics would be,

somehow, more ***objective*** than the marking of English papers. Wrong.

I didn't think about such things as the mark-value placed on a question, or about part marks, or about the weighting of marks based on the difficulty of the concept. I didn't consider giving marks for the use of correct mathematical form, and I didn't think about the difference between giving marks for what is done correctly and taking them away for what is incorrect. At a recent parents' night I found myself telling a parent that the student was actually doing much better than the marks indicated. It was the truth. The marks were wrong. The student understood the concepts well, followed form well, but was sloppy. Sloppiness is a lot easier to fix than stupidity.

Teaching math has forced me to face up to a question that rarely occurs in English classes. What mark should I give a student who didn't understand how to do something in September, but understands how to do it now? Should the September marks count? I've decided they shouldn't... but I haven't talked to my wife about it, yet.

I think English teachers and math teachers think of marks in quite different ways. To me, most math teachers mark *subtractively* — that is, they think of a problem as having a net worth expressed by the number of marks allotted to it on a test. A student receives full marks if the problem is correctly solved, less than that if the answer is incorrect or if there are errors of form. Some English teachers follow the same system, but many, however, mark *additively* — they give marks based on the student's ability to demonstrate skilful writing or to go past what constitutes good work, for example. Whereas one student's work receives "full" marks for being a thorough answer (free of spelling mistakes, even), another's may receive more than full marks because of the student's use of effective transitional elements and vivid use of language. I wish I could give my math students more opportunities to come up with clever solutions, so that I could mark their work more the way I mark the work of my English students.

Despite their differences, math and English do share some common characteristics. For instance, really good math students, I have noticed, have a "number sense" that is very much akin to the "language sense" my best English students have. In my grade ten class there are students who know that hiding inside the number 75, for example, is a perfect square, 25, multiplied by 3. These students fly through the work we do on radicals. They understand the math rather than memorize it. The best of my English students find words with the same ease that these math students find useful numbers. Asked for the word that means the feeling of ill-ease which arises from a suspicion that something bad or startling is about to happen, they instantly come up with a word like *tension* (did you, or did you think the word was *suspense*?). They haven't memorized the definition; they haven't even heard it before. They just know what it means.

I am not really sure, even after twenty-five years of teaching English, how the language sense is created. I suspect that (pardon the computer jargon) some brains are just "hard wired" to develop it if the right "program" is fed in. To this end, I will do such things as give spelling tests in which I do not tell the students which of the words on their spelling list I am asking them to spell: I will just say, "Write out the word that means a person who teaches at a university." Were I to say *professor,* my pronunciation would be too much of a clue to the correct spelling: I want the students to develop their own way of "hearing" the word inside their heads so that they will use that voice, instead of mine, as an aid to spelling. I also want the word to become part of the students' useable vocabulary, rather than to remain in their *recognition* vocabulary. For the same reason, in my English classes literary terms tend to be defined very loosely. In this case I am more interested in fostering the ability to recognize that a particular device has been used, than I am in having the students memorize a definition. I want them to be able to answer questions about the "how" and "why" of English — not so much the "what."

After two months of teaching math, I claim no expertise whatsoever at developing strategies to program my students' brains for math. I am trying a couple of things, though. For example, I sometimes give a math test that has no marking scheme. The marking scheme, I suspect, gives the same kind of information to the students that the clearly pronounced words on a spelling test would. Where time is important, however, such as on an examination or on an exam-preparation test, I have no qualms about including the marking scheme. I am also trying hard not to surrender to the temptation to give students magic formulas that bypass understanding. Every time a student asks me about cross-multiplying, for example, I launch into an explanation of what cross-multiplying is actually about — an explanation, I might add, that the students don't particularly want to hear.

One of the tools I use in my English classes to help me provide explanations, is my word-processor. I have a collection of *WordPerfect* "comment files" that I use when I mark essays, for example. Each file consists of a mini-lesson that I can print out — for example, a short how-to on using underlining, italic print, or quotation marks for titles. The files are so useful that I think I would create an equivalent set for math if I thought that I would be teaching the course more than once. This semester I just don't have the time. Sadly, I have found that a word processor isn't even much good for preparing math tests. While I've become quite proficient at using *WordPerfect's* equation editing capabilities, I haven't found an easy way to cope with all the diagrams I need to construct. My math department head swears by pencilled copies of all tests and assignments. He can, I must admit, erase old work and substitute new far faster than I can. And in a math class, anything that saves time is not to be overlooked — even if it is so "old tech."

Time works differently in math and English. Early in October my math class lost two full periods to Board of Education mandated C.A.T. testing of English skills. I've been trying to get them caught up to their peers in the other math classes ever since. Had I lost those two periods in English, I would not have worried at all about "catching up" — indeed, since English skills do not need to be acquired sequentially, I would most likely have just gone on to the next thing.

From my English teaching experience I know that students

need frequent and long-term reinforcement of things that they have been taught, and I have been trying to apply that experience to my math class. For example, most of my tests contain old material along with new. The students are slowly coming around to the idea that if they haven't yet learned the old stuff they still need to learn it. Many of them would just as soon leave it unlearned — out of sight, out of mind. Unfortunately for the students, it is quite difficult in an English class to pin a label on a concept, but it is not difficult at all in a math class. My students know *exactly* what it is that they don't know. And that lack of ignorance puts quite a damper on their bliss. I spend a lot of my one-on-one time fighting their discouragement.

I wish I could devote more of my time to two gifted math students that I have in my class. Right now they are just lock-stepping their way through the course with everyone else, doing whatever bonus questions come their way on the math tests. I'd like to be able to talk to them about math the way that I can talk to my English students about English. But I don't know enough. My mathematical knowledge lacks context.

Still, on balance, the year is going well. I hope I will be able to look back on four months of math as pleasantly as I look back on two. However, it is Sunday night, now, and I must end these reflections. Tomorrow we start co-ordinate geometry, and I have to talk to my wife.

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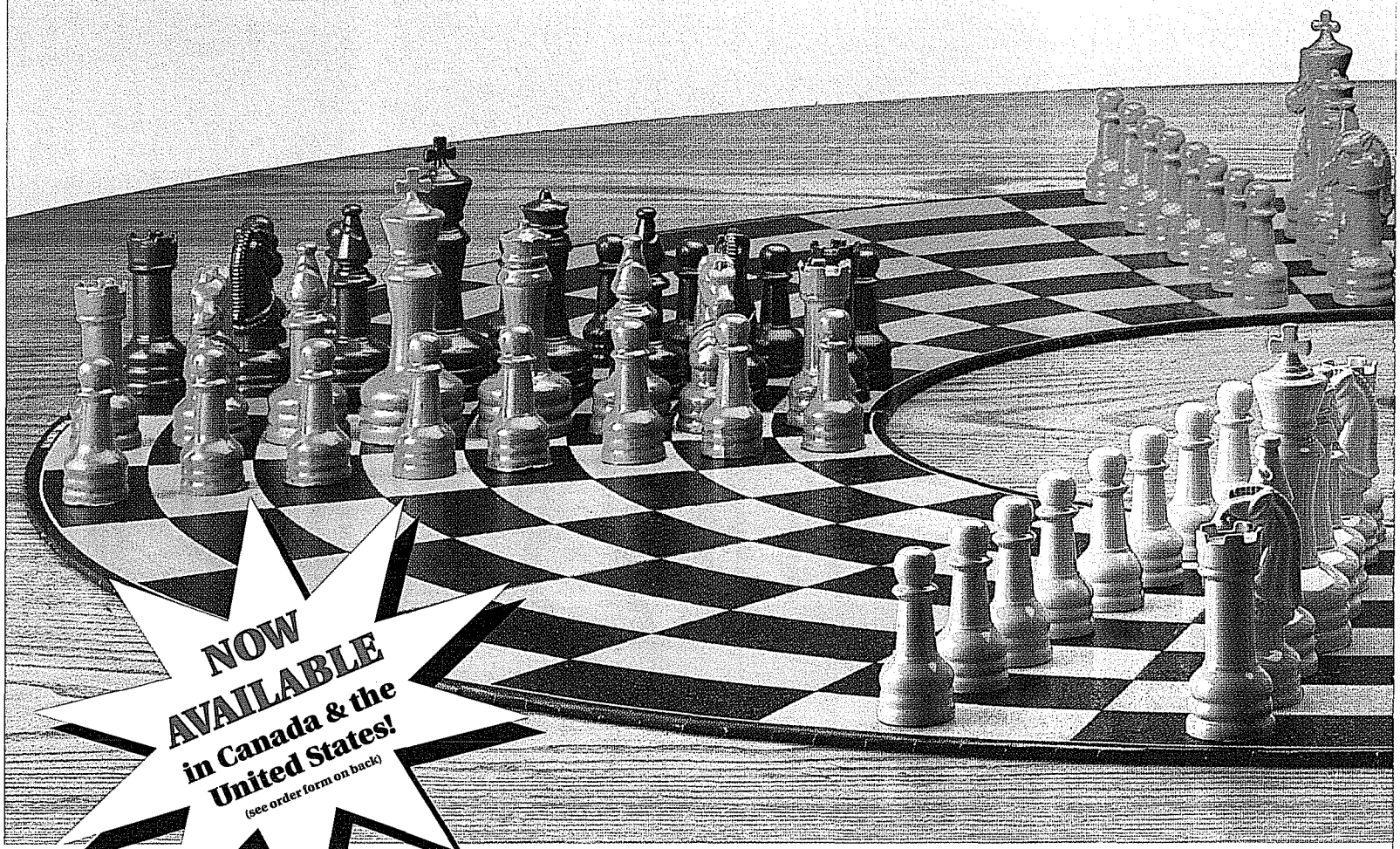
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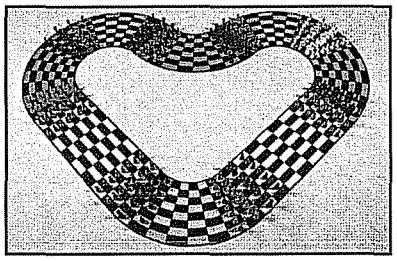
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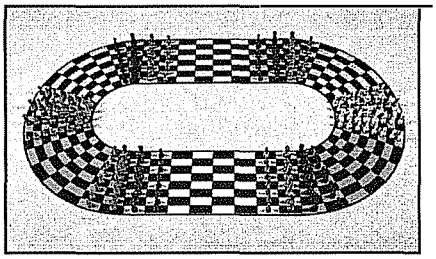


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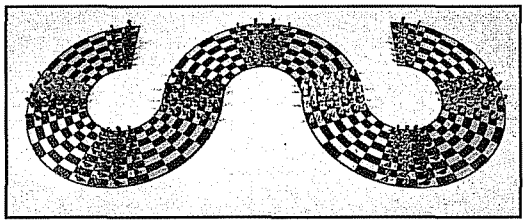
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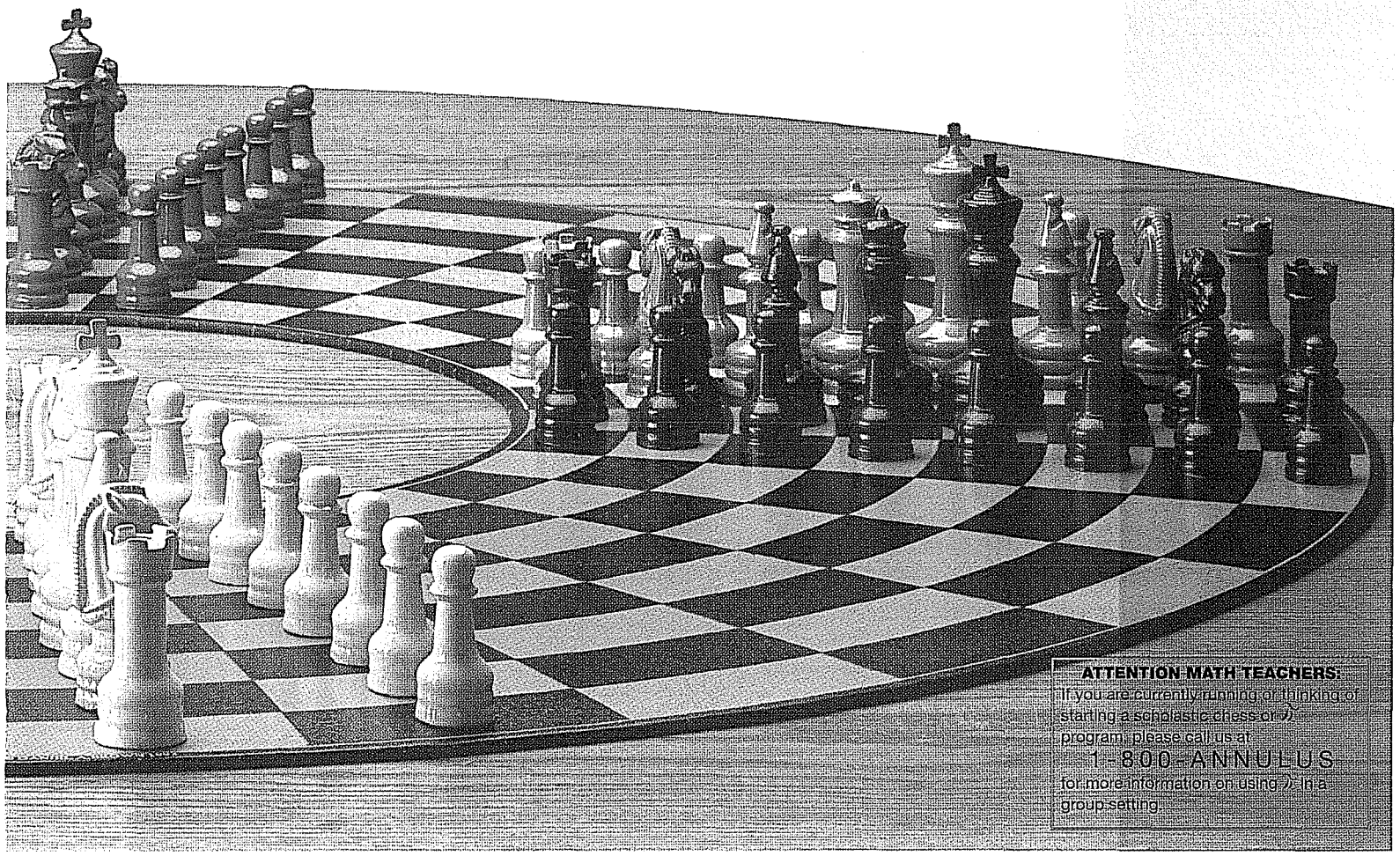
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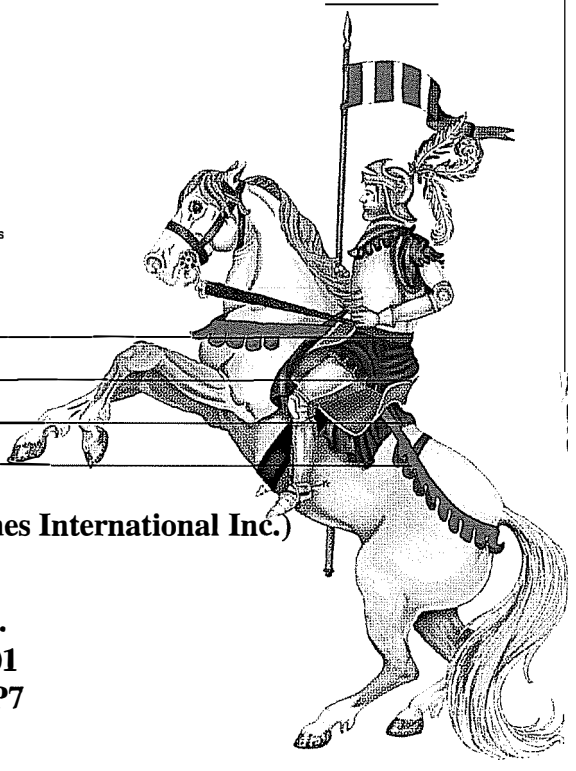
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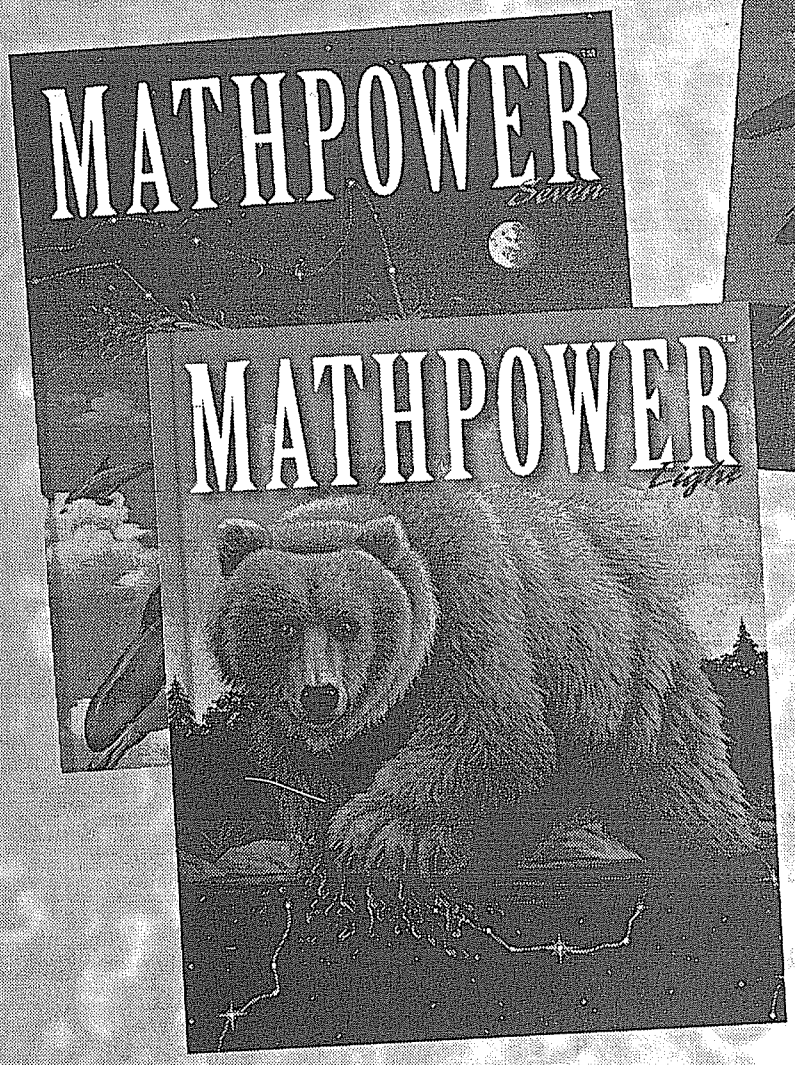
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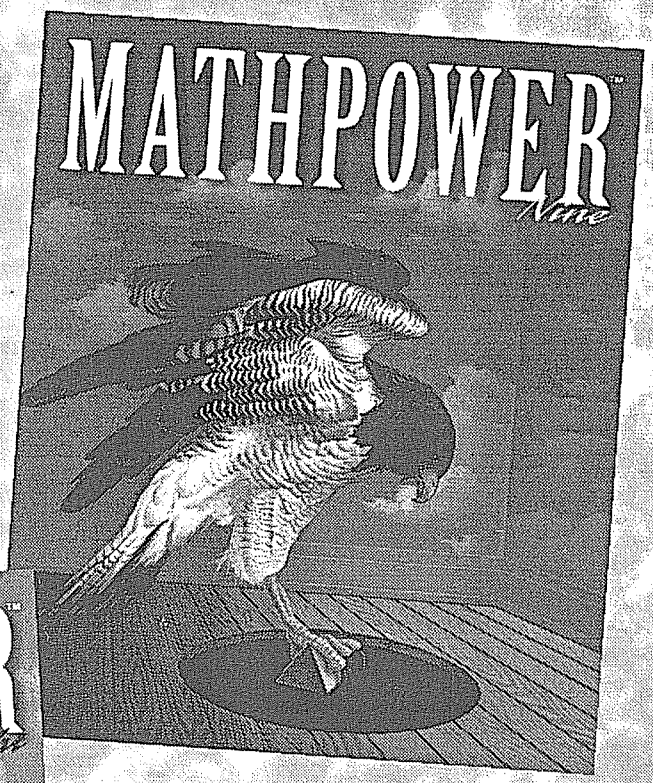
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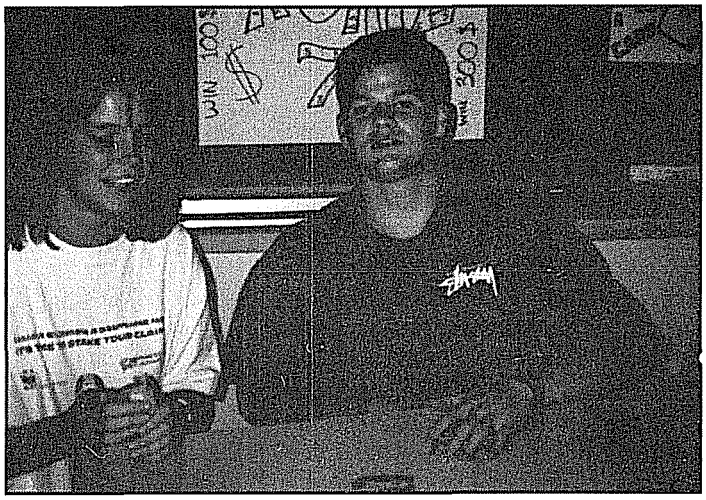
Students in my Finite Mathematics Class at Fort William Collegiate Institute in Thunder Bay, Ontario (NWOAME) held a Casino Day as part of their study of probability.

Working in pairs, students designed their games, worked out their expected winnings (or losses!) according to probability theory studied in class, and then invited fellow classmates and staff in to take a chance! Armed with monopoly dollars, prospective players entered the room which had been transformed into a Casino complete with music, costumes and special effects.

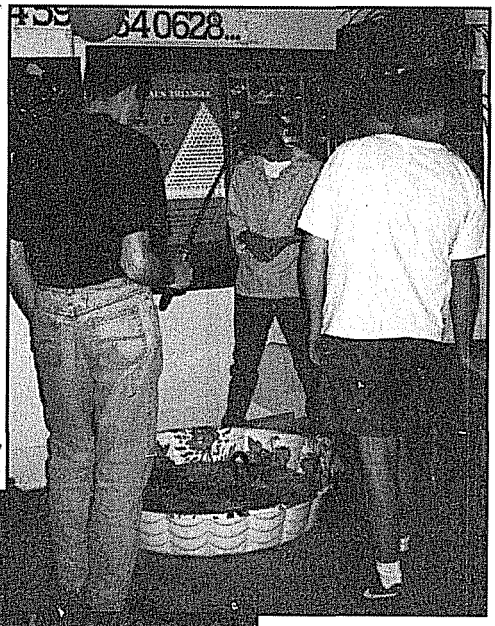
Games included "Toss Two", in which players had to toss two out of three tokens into jars with various sized openings to win, "Lucky, Lucky", a homemade dice board complete with lights and music, and various card and dice games.

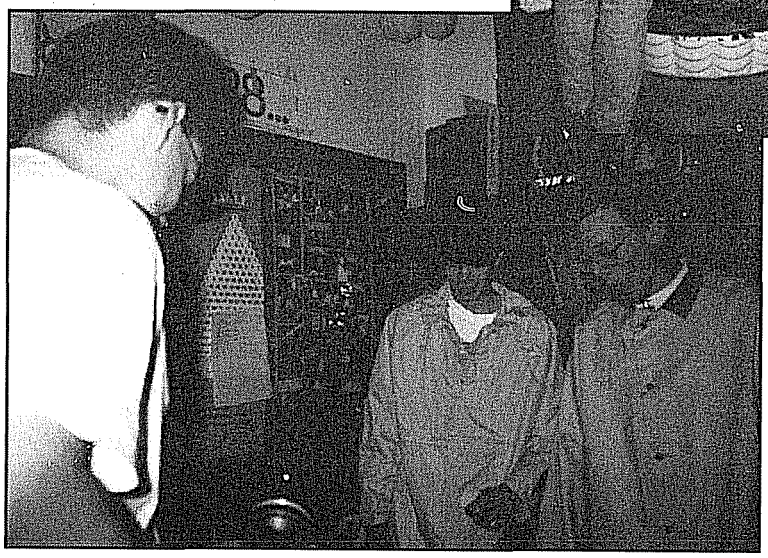
The most ingenious game was "In Golden Pond". Its creators created "fish" out of a deck of cards by attaching fins and eyes to each card, then laminating them. They attached a paper clip to each card and set up fishing poles with magnets for bait. Dressed in yellow slickers, they filled a small wading pool with water and lily pads, and players won or lost according to their "catch".

For those with "money" left over, lottery tickets were available, and a draw was held for a "Math Cheat Shirt" from GVMA.



Katie Burton and Ryan Landgraf invite players to draw "Royal 7's".

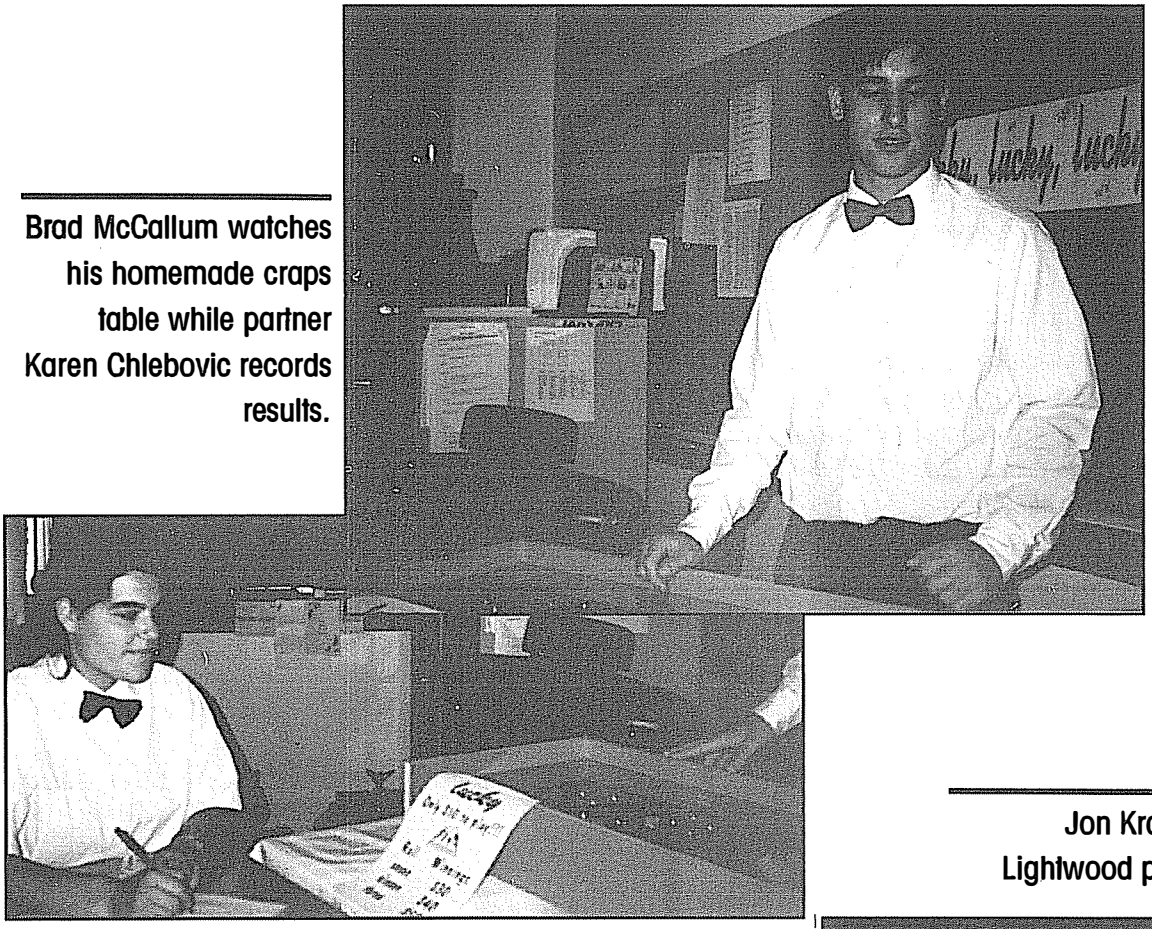




Brad Halabisky and Richard Wassanar watch as players try to pull in the BIG one. The boys reported that they lost their shirt on this game!!

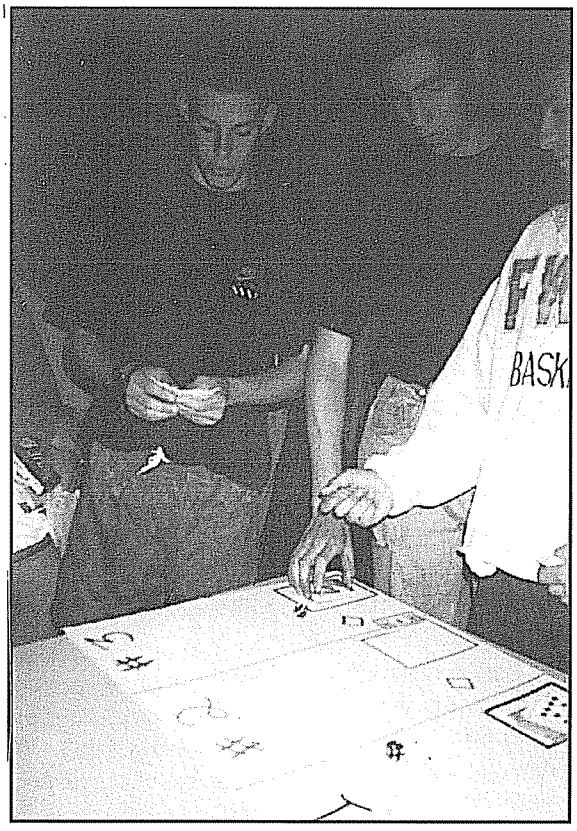
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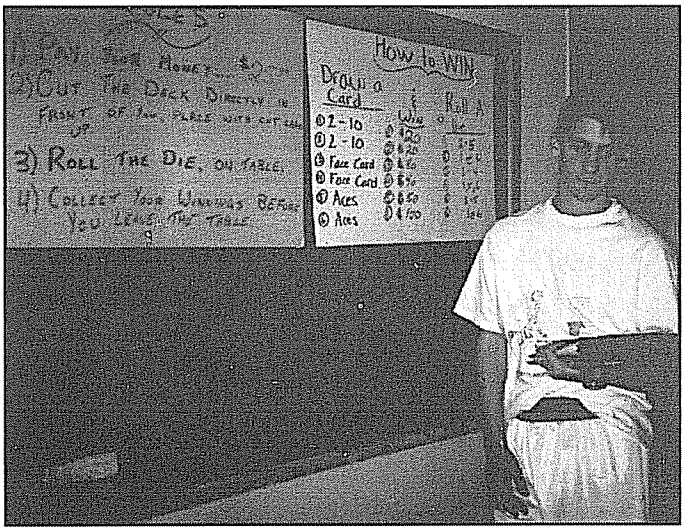


Brad McCallum watches his homemade craps table while partner Karen Chlebovic records results.

Jon Krawchuk and Ivan Lightwood play "DC Money".

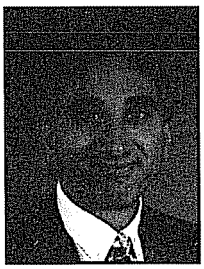


Scott Paddington keeps a close eye on fellow students trying their luck at his card game.



▲ MATH AND THE WORLD AROUND US

VINCE DELISI
THE COUNTRY DAY SCHOOL
KING, ONTARIO



Vince Delisi is the Department Head of Mathematics at The Country Day School. He is especially interested in using technology to enhance the teaching of mathematics. He is also the Math Resource Editor of the Gazette.

Here is a wonderful article that appeared in The Globe and Mail on January 18, 1995. It is reprinted with the permission of that paper. Following the article is a mathematics assignment based primarily on the graph in the article. I hope many of you and your students will have some fun with it.

Physics buys a ticket to the Super Bowl

BY STEPHEN STRAUSS
SCIENCE REPORTER
THE GLOBE
AND MAIL

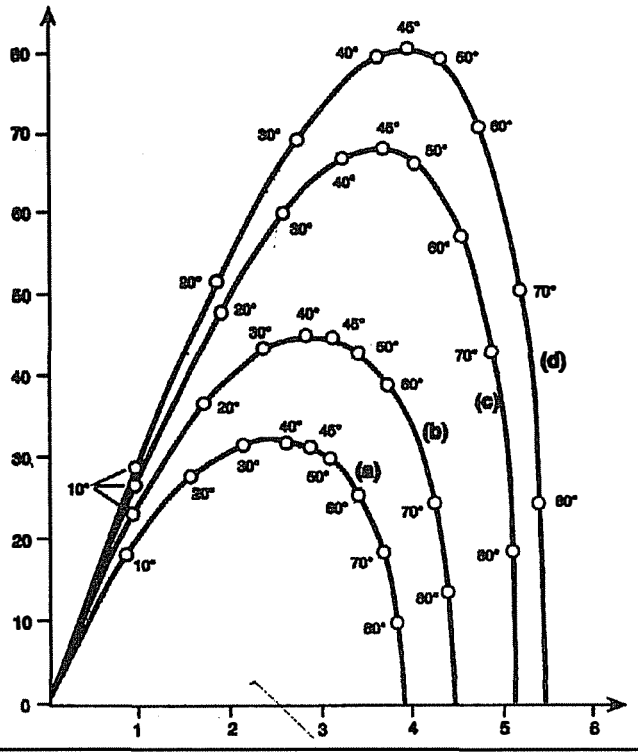
Tomorrow is Super Bowl Sunday, a time when football fans throughout North America take over the continent's television sets and when non-fans leave the room or desperately hope for a commercial break.

But perhaps there is another way - a scientific way - to view the bashing and mashing of players in the U.S. professional football championship game. This is to look at the physics that governs the trajectory of the thrown and kicked pig-skin, the speed with which the bullet pass hits a receiver's hands, the collision of linemen.

There is one proviso to this study: much of the physics of football is hard to quantify and harder to reflect upon. When asked what he considers the most interesting physics aspect in football, Peter Brancazio, a physics teacher at Brooklyn College in New York who wrote the 1984 classic book, Sport Science, can't come up with a quick reply. "There is not a lot you can say, but baseball, baseball I could go on about for hours," he laughs.

The physics of football that can be measured most definitively generally centres on the aerodynamics of the ball - the passes and punts. For example, in a 1985 article in the journal The Physics Teacher, Prof. Brancazio explored the "kicker's dilemma". This relates to what sports commentators call "hang time" - the number of seconds a kicked ball remains in the air.

The distance that a ball moves doesn't tell you the whole effect of a kick. It's hang time is more to the point. The longer the pigskin stays up, the longer the opposition must wait to catch it and the longer the kicking team has to race down the field to tackle the receiver.



The laws of projectile motion say it is impossible for a kicker to maximize simultaneously a ball's horizontal distance and duration of flight ("hang time"). When a football is kicked at a speed of 60 miles an hour, the distance, angle of kick and hang time are shown (a) if the ball is facing sideways as it moves, (b) tumbling end-over-end, (c) spiralling nose first and (d) theoretically, travelling in a vacuum.

A short hang time has the reverse effect. After a 70-yard, line-drive boot, your opponent may be able to catch the ball and return it 20 yards back up the field before encountering the first tackler.

The kicker's dilemma is that if he were looking for the absolute maximum in hang time he would kick the ball straight up - that is at a 90 degree angle. However, he also wants is to propel the ball forward - a task best accomplished by launching the weirdly shaped ball (an ellipsoid 11.1 inches long, 6.8 inches in maximum diameter) at a 45° angle. To accomplish both purposes a kicker must find a sawoff between

the two best kick angles.

The strategy you pursue in the tradeoff depends on whether you are punting the ball or kicking off (when the ball is launched from a tee on the ground). Generally speaking, the best kickoff is one that goes so deep into the end zone that the receiver doesn't even try to return it. On the other hand, a punt, when the ball is held by the kicker before he dispatches it, may take place farther down field, and its best result may be that the receiver is tackled just as he gets his hands on the pigskin (which, by the way, is made of cow hide and not pig hide).

Kickers generally don't study physics, but constant practice has made them nearly perfect. In his analysis of 44 kick-offs and punts, Prof. Brancazio found that the average kickoff was in the air 4 seconds and the average punt 4.2 seconds.

A trifle, you say. Yet one translation of this is that Prof. Brancazio's average kickoff travelled 63 yards, while his average punt went only 50 yards. Assuming that the average kickoff left the kicker's toe moving at 70 miles an hour, it turns out that it was launched at the golden 45° angle. Punts whose off-the-foot initial velocity was 60 mph to 70 mph were propelled skyward at a 55 to 60 degree angle.

While all of this takes into account the effects of gravity, the equation ignores the other major component in punter physics: air resistance. If you can kick a ball with a nice spiral, it slices or screws through the air with its nose pointing forward. Without a spiral, the oddly shaped football wobbles, becoming what is termed a "wounded duck" or "dying quail".

The difference between the travelling distance of a spiral kick and an inept wounded duck can be huge. Prof. Brancazio computed that at a 45° angle, an end-over-end kick launched at 55 mph would travel 37 yards. Its nose-first brother would travel 55 yards. The same spiralling kick without any air resistance would go 67 yards.

But kicking a ball so it will spiral is not easy. "An average athlete can learn to throw a decent spiral pass," suggests Prof. Brancazio. "However, kicking a spiral is another matter." To do so a punter swings his kicking leg out and into the ball while launching it with the top of his foot rather than the toe of his boot. Punters succeed, according to the 1985 calculations, about 60 percent of the time.

During kickoffs, when the ball is placed on a tee, even pros find the leg out-toe-in, top-of-foot-on-bottom-of ball too difficult. Therefore, kickoff players go for the end-over-end, non-spiral style because, while the ball doesn't travel the farthest possible distance, it is gyroscopically stable. Translation: It goes straight.

How does this relate to throwing the ball? The short answer is: ditto. Quarterbacks have some natural advantages over kickers. The ball slips off their hands into a natural spiral when they toss it in a wrist-rolling motion, which in baseball produces the curve ball. Prof. Brancazio counted average spins of 10 revolutions a second in well-thrown balls.

If you are tossing a short bullet pass to a receiver 10 or so yards down the field, hang time is not important. After watching a football game, University of Colorado physicist Albert Bartlett computed that a bullet toss launched at a 12° angle will move at a height of about 4 feet and be in the air only about 0.7 seconds. This figure may be a bit unrealistic because passers must hurl the ball over the upraised hands of linemen in the opposing team.

Nevertheless, by comparison, a "deep bomb" - a pass travelling 50 yards - launched at a 42° angle would reach a height of 33 feet and be in the air 2.9 seconds.

The short pass travels at 55 to 60 mph and presumably could hurt a receiver, but because the ball weighs so little - 0.91 pounds - it does not have the same impact as a car moving at similar speeds.

Even the pros sometimes launch a dying quail. Frank Ryan, who has a doctorate in math and who took the Cleveland Browns to the 1964 National Football League championship, once commented that not only is "the football one of the most oddly shaped ball-like objects that ever existed (but) the aerodynamics of it are almost beyond comprehension."

For many people the most interesting physics of the Super Bowl would surely be found in collisions between players, what commentators call "crunch time". Spectacular flips and bone-breaking tackles are favourites in TV game highlights, but from a pure physics perspective they are very difficult to model. Each one is influenced by angle, momentum, whether or not cleats are stuck in grass, and a host of other things.

Nonetheless, North Carolina State physics professor David Haase has done some first-principle calculations and translated collisions into homey comparisons: "Taking a hit from a 250-pound line-backer moving at 20 miles an hour would be like falling off the top of a 12 to 15 foot ladder, in protective pads."

If you were a receiver hit by two 220-pound defensive backs running at full tilt, "the impact on your body would roughly the equivalent of trying to catch a 16-pound bowling ball dropped from the 13th floor of a building."

Prof. Brancazio says the impact may not be ruinous. The best a student of the physics of crunch time can do is simply observe forces in action. If a lighter runner is hit straight on by a bigger defensive player, he will be driven back. But if he is a good runner, this backward hit may turn into a "bounce" that he recovers from and follows up by racing down the field.

The best runners have a low centre of gravity; that is, they run low to the ground. To be stopped, they must be hit low or tripped. If they are tackled high they might fight off the efforts.

The class of behemoth linemen is nearly impossible to quantify. It's the physics of pushing and shoving, or as Prof. Haase puts it, "It's the physics of sumo wrestling."

Which probably is a good place to stop and remind the true aficionados of physics in sport that spring training for baseball starts next month. And no amount of physics can tell us if there will be a baseball season this year.

Conics and the Super Bowl

1. Read the article, "Physics Buys a Ticket at the Super Bowl," reprinted from the Globe and Mail, Saturday, January 28, 1995. Look at the graph contained in the article and discuss the information conveyed with another person in your group.

   (a) Explain why "hang time" is important when a kicker kicks the ball.

   (b) In order for a kicker to be successful, explain what is more important, distance covered, hang time, or some combination of both.

   (c) Explain the meaning of the angles given on the graphs?

2. Recall (from grade 11) that the general equation for any parabola is $y=ax^2+bx+c$. Since all four curves go through (0,0), what must be true about the value of the parameter c in $y=ax^2+bx+c$?

3. Assuming that c=0, we can model the flight of the football by the equation $y=ax^2+bx$.

   (i) Find two ordered pairs on each of the curves in the article (the curves are labelled (a), (b), (c) and (d)).

   (ii) Use a system of equations to find the values of the parameters a and b. Use these results to state model equations for the flight of a football that is kicked at a speed of 60 mph (a) facing sideways as it moves, (b) tumbling end-over-end, (c) spiralling nose-first, and (d) theoretically travelling in a vacuum.

   Discuss how closely your equations model the actual data from the newspaper graph. Are the curves in the article parabolas? Discuss your reasoning clearly and make specific reference to the graphs in the article.

   (iii) Find the vertex (i.e., the maximum value) of each model parabola found in (ii).

   (iv) Once you have these model equations, you can find out any piece of information (i.e., you can interpolate) about the number of yards the ball travels and the related hang time. Give some examples of how these types of equations could be useful in helping football players and coaches analyse kicking strategies, and change game plans. Make specific reference to your equations.

4. In the seventh paragraph of the article, a football is called an ellipsoid (a 3-D object made from revolving an ellipse about its major axis) and its dimensions are given. Find an equation for an ellipse that has these dimensions and sketch a graph of this curve.

5. Comment on this assignment.

6. Follow-up Assignment for the next class.

In 3(ii) you found models for the experimental data by using 3 data points (i.e., (0,0) and the two that you chose). If you wanted a more accurate model, you could use more data points and find a method to obtain the data from the graph as accurately as possible. Use a TI-82 calculator to help tabulate the results. Do this in the following way:

(i) To set up the calculator,

press [ ] and [STATPLT Y=]

to get to the STAT PLOT feature. Press enter to format the STATISTICAL PLOTS in the following way: turn PLOT 1 on. The TYPE of graph should be the line graph. The x-axis will come from LIST 1 (Xlist: L1) and the y-axis from LIST 2 (Ylist: L2).

Press [QUIT MODE] to exit this screen.

Press [STAT] and [ENTER]

to get to the screen where you will enter data that you gather from the graphs in the article. Time will go in L1, Yards will go in L2 and the angle will go in L3. If there is any data in these lists now, you should clear it manually using the delete

[DEL] key

or use the CLEAR LIST [STAT] [ ] command.

Leave the calculator on while you go to the next set of instructions to gather the data.

(ii) Choose only one of the graphs from the newspaper (i.e., a,b,c, or d). Using a clear ruler with millimetre markings, measure (as accurately as possible) the number of millimetres in one unit on the time (seconds) axis (we will call this x) and the number of millimetres in ten units on the distance (yards) axis (call this y). Use a very sharp pencil and a ruler to draw lines to both axes from each point labled 10°, 20°,...80°. Consider the x-intercepts to be at (0,0) and the far right point on each graph. Enter 0 in each of the first lines of L1, L2 and L3. Measure each distance from the origin along the horizontal (seconds) axis on the newspaper in millimetres and enter into the first cell of L1 on your calculator. To make this accurate, you must divide this measurement by the unit length that you found above (we called this x). The resulting decimal value should be calculated on the correct line of L1. Continue for each data point. Repeat for each distance along the vertical axis, but place these measurements in L2. Be sure to take your measurement, multiply by 10 (since each vertical unit is 10 yards) and then divide by the value we called y above.

Your calculator display should now look something like this:

L1	L2	L3
0	0	0
1	27.982	10
1.8696	49.541	20
2.8261	66.514	30
3.7478	76.606	40
4.1739	77.523	45
4.4522	76.606	50
4.8783	67.89	60
5.3478	48.532	70
5.5652	22.936	80
5.6522	0	90
	_____	_____
L1 (12) =		

(iii) Now use your calculator to generate the equation of a quadratic curve (a parabola) which fits these data points as closely as possible. The process that the calculator goes through to do this is called Quadratic Regression and the curve is called a regression curve.

Press STAT ▶ ENTER.

The values of the parameters a, b, and c will be displayed.

(iv) I now want you to have the calculator graph the raw data, the model equation that you calculated from the two data points in 3 (ii) and the Quadratic Regression curve which the TI-82 calculated. First format the graph window by

pressing WINDOW.

Set the Xmin=0, Xmax=6, Xscl=1, Ymin=0, Ymax=90 and Yscl=10 to correspond to the way the newspaper graphs are set up. To enter your equation,

press Y=.

Let Y1= the equation that you found in 3(ii). There is an easy way to enter the equation of the regression equation in Y2. With the cursor flashing beside Y2,

press VARS ▶ ▶

Now press GRAPH

to superimpose all 3 graphs. The first graph is the actual data, the second is the model equation from 3(ii) and the last one is the regression equation which was calculated from all 11 data points.

(v) Discuss the differences and similarities between the two models. When would it be better to take the time and use more points? How could you use these models if you were the special teams (kicking) coach of a football team?

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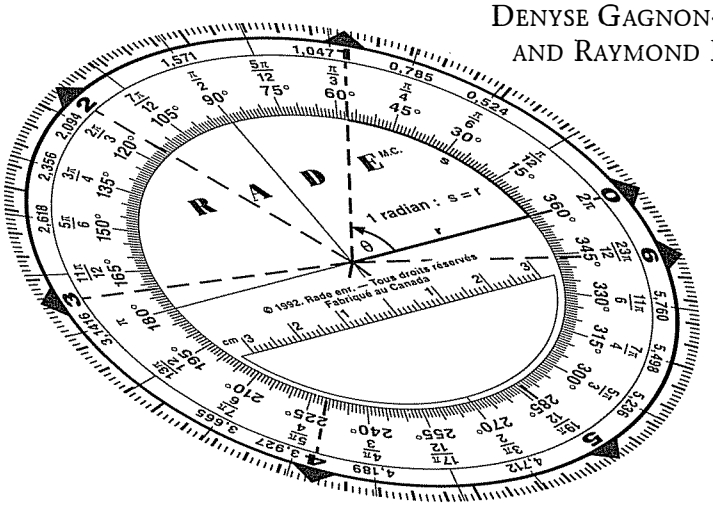
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▲ About Problem Solving: Making Math with 1995



BY JOHN GRANT MCLOUGHLIN
EDUCATION PROGRAM
OKANAGAN UNIVERSITY COLLEGE
3333 COLLEGE WAY
KELOWNA, BC V1V 1V7

The number 1995 is a curious one. Observe that 19 is a prime divisor of 95. Furthermore, it is the largest prime divisor of 95. The next such year to share this property will be 2323. What other properties of 1995 may be evident to students who consider this number from such a slant? The sum of its digits is 24. The product of its digits is 405. Would these latter two properties uniquely define the number 1995? Let's attempt to answer that question.

We know that many numbers would have digits that sum to 24. It is more productive to consider numbers that have digits that combine to form a product of 405. The number 405 may be expressed as a product of prime factors, namely, 3x3x3x3x5. Note that the sum of the four 3's and 5 equals 17. Most significant is the fact that 17 is less than 24. That is, a number such as 33335 would satisfy the requirements for the product and be 7 short of the required sum. This shortfall can easily be addressed by adding seven 1's to the string of digits, thus, not affecting the product. A number such as 111111133335 would have the same product and sum of digits as 1995. The actual string of digits may be ordered in numerous ways. You may wish to figure out exactly how many possibilities exist.

Suppose that we had represented a pair of 3's as a 9 for the purpose of the investigation. Then we would have identified a number such as 3395 that satisfied the product requirement. It looks visually similar to 1995. Observe that 3x3 equals 1x9 and hence the products are bound to be equal. The sum of the digits in 3395 is 20. Therefore, we may insert four 1's into the string to meet the requirements. Again a multitude of possible arrangements exist.

It appears that there is only one other family of digits that would satisfy the requirements. This family is made up of one 1, one 5, and two 9's. It is the combination that forms numbers such as 1995. Returning to the source of the problem, 1995, I would like to identify some of my own observations. Basic numerical operations and the properties of numbers may be combined to form rich sources of problems. I had no idea how many numbers would share these sum and product properties when the investigation began. Initially it was the fact that 19 is a factor of 95 that struck me. I proceeded to look for other properties that this number may share with others. An elementary school class could attempt to make a list of numbers that share a common sum of digits. One may even wish to restrict the number of digits to coincide with the level of numerical understanding in terms of place value. Consider a problem such as listing three digit numbers with digits that sum to 10. The students begin to recognize that not all mathematical questions have a single answer. Patterns abound with such activities. Although it is unlikely that students will successfully list all solutions, such a problem invites them to continue seeking patterns, organize information and ultimately discuss the role of mathematical process. Secondary students may be encouraged to compute the number of solutions without taking such an exhaustive approach. Students at all levels should be encouraged to identify numerical properties and participate in the problem posing process. A wealth of ideas may be found in the book entitled "The Penguin Dictionary of Curious and Interesting Numbers" by David Wells.

Here is another idea that may be developed from 1995. The numbers 1, 9, 9, 5 may be arranged to form mathematical expressions equivalent to 1, 2, 3 and so on. This activity provides an excellent vehicle from which one can informally assess the mathematical skills and knowledge in a junior/intermediate classroom. Each expression must include the four digits: 1, 9, 9, and 5. They may be combined to form different numbers. Any mathematical symbols may be used. Expressions such as the following may appear:

$87 = 9 \times 9 + 1 + 5 \quad 23 = 19 + 9 - 5 \quad 72 = (5 - 1) \times (9 + 9)$

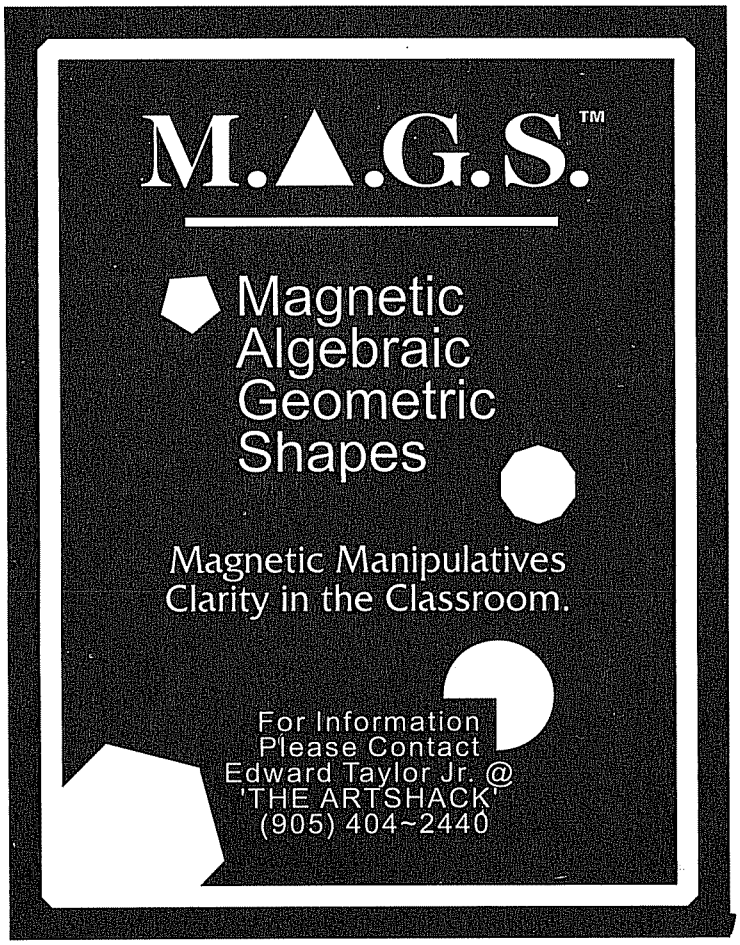
Proper use of brackets indicates that students may be well versed with using them. On the other hand, it is likely that false equations such as $59 = 9 + 1 \times 5 + 9$ may inform teachers of a need to review order of operations.

Keep in mind that no restrictions are imposed in terms of the use of symbols. Usually when someone uses a square root symbol, it opens up a wide array of possibilities. Note that one may not use squaring or cubing by writing a 2 or 3 as the exponent. That is because there are no 2's or 3's. However, one of the 9's may be combined with a square root symbol to do cubing. You may find it helpful to use this in creating an expression for 34. If your mathematical repertoire includes repeating decimals, factorials, and such things, this will only serve to assist you in this activity. From experience with my elementary methods course, I can assure you that expressions can be made for all integers from 1 to 100. How much further can one go without encountering a break?

The 1995 theme prevails in the January, 1995 Mathematics Teacher calendar. The collection consists of an excellent assortment of problems prepared by Philip Brown of Exeter Academy in New Hampshire. I would encourage you to take a

look at it if you are interested in finding a variety of problems well suited to the secondary school level. Before closing, I would like to leave you with another problem. It may seem routine at first glance. My intent was to compose a problem that did not simply lend itself to a system of n equations and n unknowns. Rather it is intended to challenge prospective solvers to draw in other aspects of mathematical thinking. Any comments on this problem or any other aspect of the column are appreciated.

> The total value of the nickels, dimes, and quarters in a cash register is $19.95. Interestingly enough the total value of the quarters is an exact number of dollars with no change left over. Similarly the total value of the dimes is an exact number of dollars. There is a total of 167 coins made up of nickels, dimes and quarters. Determine the respective number of nickels, dimes, and quarters in the cash register.



▲ Council Comments



PETER SAARIMAKI
COORDINATOR OF MATHEMATICS
TORONTO BOARD OF EDUCATION

Introduction

It seems my estimate of readership was marginally over-stated last issue. This column apparently attracts only 48% of *Gazette* readers, not "over half" (although only 3% more *would* make it true). Maybe I can be assuaged by the fact that 92.5% of our readers haven't "voted" yet. You can still offer your comments and suggestions. Would it help if I imbedded a FREE OFFER within these reports?

These notes were taken at the OAME Council meeting held Saturday January 21, 1995. Comments about Council and its activities should be directed to the President. Comments about my comments can be sent to the editor.

General

Did you know ... that each chapter of OAME receives a rebate (equal to 30% of your membership fee) from OAME for each person who has identified himself or herself as a member of the chapter? This helps the local chapters put on events in your area, so make sure you identify your affiliation when renewing your membership this year.

OAME 95 Conference

Members should receive their program brochure by late February. This year's conference, running from May 11 to 13, has over 50 workshops for the primary and junior divisions and over 100 for transition and specialization years. To top it off, Saturday morning features a panel discussion on the Royal Commission on Learning and the future of education in Ontario. The panel will consist of Commission co-chair Gerald Caplan, Guelph NDP MPP Derek Fletcher, Conservative and Liberal Education Critics Dianne Cunningham and Charles Beer. Originally Minister of Education and Training David Cooke was to appear but unfortunately, he had a scheduling conflict.

There will be several featured speakers and special events with publishers' displays and social opportunities to network across the province. As a special attraction, the one day fee will allow you to attend the Saturday panel as well.

The **Awards** Committee announced that more members will be able to show their appreciation to the winners this year. The presentations will precede the keynote speakers at the conference.

Equity

Ron Lancaster will be the keynote speaker at this year's Equity breakfast at the Annual conference. All are welcome to join in this exciting start to the day.

Special Presentation

In recognition of the many dedicated years of service to the OAME, most lately as co-executive directors, Don and Carol Attridge were presented with a breathtaking Inuit sculpture at lunch time. The thoughts of many were expressed most eloquently by Mickey Sandblom, a past-president and now NCTM rep.

Publications

Vince Delisi has been selected as the new resources editor, so if you have heard of any new resources and you'd like to know what others think of them, let Vince know. You can also volunteer to be a reviewer.

Assessment column editor Jennifer Weiner is looking for final exams or other assessment items to use as samples. See the masthead for contact information.

ERIC has asked to receive copies of the OAME Gazette for inclusion in their electronic database. This will expand OAME's contacts world-wide.

You may also notice the increase in adverts in this issue. This brings more revenue to OAME and helps members become aware of new products. (See also the report from Publicity and Membership.) Thanks are extended to new *Gazette* advertising manager Dean Murray for his quick work.

Summer Conference

OAME and AMTNYS are co-sponsoring a conference July 28 - August 1 to be held at Scarborough College. AMTNYS is the Association of Mathematics Teachers of New York State. This will be an excellent time to meet our American neighbours mathematically. The costs will be kept low (possibly $75 for registration and $31 for residence). Talk to your chapter rep or call John Kersley for more information. (See the notice on page 18.)

Special Projects

As the start of the OAME/OMCA Assessment Project, 11 activities have been developed and sent out for field-testing to about 60 teachers across Ontario. By spring, rewrites will begin and publication of the assessment activities, with sample student work, is planned for June.

COMMITTEES

Early/Formative Years

An **exemplary** unit, with six activities, has been developed on volume and capacity for the next *Abacus*. These have been related to the Provincial Standards and include literature connections, home activities and assessment items as well.

Transition Years

What comments did/do you have on the draft brochure for Transition Years parents (see page 20 in the Dec. 94 *Gazette*)? The committee is also planning an annotated bibliography of activities suitable for students in these critical years.

Specialization Years

Now that you've seen the Royal Commission Report on Learning, what do you think should or will happen to the Specialization Years? How do we prepare our students for post-secondary study in four years? Wither goes Calculus, or Algebra and Geometry?

Publicity and Membership

The final version of a joint OAME/NCTM membership flyer is now available for chapters to distribute at PD sessions to encourage members to recognize the benefits of both organizations.

New OAME logo pins and rings are being developed for official presentations and possibly for sale as well. Would you be interested in buying, and wearing, an OAME pin or ring? Let your chapter rep know. How about discounts, coupons and reduced rates with your membership card? These are all being investigated.

CHAPTERS

One regret I have is that most of the wonderful activities planned by chapters will be over by the time you read these notes in the *Gazette*. Each chapter puts on some marvellous Professional Development. Keep in touch with your local chapter to be aware of their plans, **before** they happen.

CHAMP

Math contests are *BIG* in this chapter. In November, over 5000 students from grades 8 to 11 participated in CHAMP's eighth annual contests. The students represented 165 schools across the four boards and will be recognized at an awards ceremony on April 13 as part of Springfest workshops with a social time.

COMA

Besides fall and winter mini-conferences, a newsletter, and special forums focussing on different grades, the members here are busy planning the **first** annual Ontario Mathematics Olympics. Chapters will be sending their team(s) to Ottawa for the competition on Saturday June 3, 1995. (How is your chapter selecting the team? Which of your students are you encouraging and helping in their preparation?)

GOLDEN SECTION

The fall conference was held at the new Embassy Suites and provided teachers and student teachers a chance to meet socially and participate in workshops looking at new TVO programs, hands-on activities and links between literature and mathematics.

GVMA

This busy chapter has been working on the very quickly approaching 1995 OAME Annual conference. All OAME members should have received the Program booklet by late

February. Early information shows a very comprehensive selection for elementary and secondary teachers. Our thanks go to the hard-working members of GVMA for their dedication to the planning of this conference. Come out to beautiful Guelph this May for the time of your (mathematical) life. And to top it off, Saturday Morning features a panel on the future of education in Ontario with Gerald Caplan and representatives from all three major political parties.

ISOMA

A widespread group, they still managed to attract most of their members to the fall conference. They hold preparation evenings for students to help them train for the Waterloo contests and select their representatives for the Ontario Math Olympics.

NOMA

Math, "Just Do It" is the theme for the spring conference Friday, April 28. This follows a successful fall conference with workshops in both English and French. NOMA is considering hosting the 1997 OAME conference. It's been a while since we had the chance to visit some of our more northern areas, so let's hope.

NWOAME

After "Fishing the D-Stream", this chapter, with Lakehead University, will hold a Math Awards Day on June 2. Next Oct. 19-21 why not "Rendezvous with the Future", an MST conference in Thunder Bay? Congratulations are extended to Marilyn Hurrell and Bill Otto for their on-going initiatives.

SAME

The spring conference will be separate but concurrent during the week of April 27, one for K to 6 and the other for 7 to OAC. Several members are working on the OAME/AMTNYS summer conference.

SWOAME

Plans are well on the way for a Springtime Dinnerlude to be held during a weekend in April. Four teachers from the two Windsor Boards, who attended the Leadership conference in the fall, have been involved in SWOAME activities and are field-testing some of the Assessment project materials. Local teachers have also been responding to requests in the Gazette with their comments on the Transition Years brochure.

TEY^{2}MS

Plans are underway for a student-parent conference this spring. The day will include individual competitions as well as collaborative team events for students, with opportunities for parents to experience and discuss issues in mathematics education.

WOMA

A busy chapter: 130 elementary and secondary teachers attended the fall conference. The spring conference will be hosted by Lambton County. Hot air balloon rides will be featured at the OAME 96 Conference "The Skys the Limit." (This certainly appeals to us basket cases and other lighter-than-air heads.)

The chapter has allocated finances to support their student teams going to the Ontario Math Olympics in Ottawa.

Y^{4}MA

Mark your calendars for Oct. 17, 1995, and look forward to their fall conference, but don't miss the AGM on May 31 and the local Math Olympics (along with the International one in July).

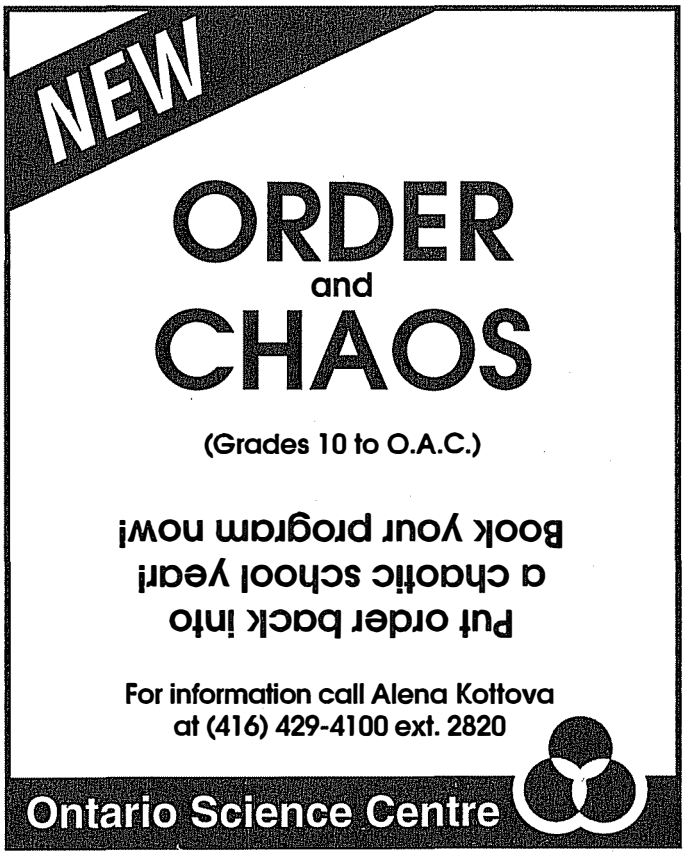
General

A welcome was extended to Dave and Bonnie Alexander as they settled right in to their duties as executive directors.

The release of the Royal Commission on Learning came after Council, but obviously affects us all. The short version was available the very day of its release to teachers on the Electronic Village. The EV conference *commission* has the text of the report as well as all the recommendations.

The release of the "new" Common Curriculum and Provincial Standards is also noteworthy. Share your views by joining the new **OAME conference** on the Electronic Village. Let your point of view be heard/seen and hear/see what others think. The next few years will be critical to mathematics in Ontario, especially for the secondary years.

You may send any comments to me via e-mail at petesaar@village.ca.



ASSESSMENT COLUMN

JENNIFER WEINER

The assessment tool for this issue is a grade 12 business math final examination, used in an Ontario high school this past January. It should be noted that the students were allowed, and in fact required, to use both their textbook and an income tax guide during this exam. Any business math text should include the necessary charts and tables, in case you are interested in using this exam as a practice exercise for your students. (There was also a cartoon pertaining to the bonus question included in the exam, but we are unable to include it for copyright reasons.)

PLEASE NOTE: THIS EXAM WAS NOT PREPARED SPECIFICALLY FOR THIS COLUMN. IT IS NOT AN OAME CREATION, AND WE ARE NOT ADVOCATING FOR OR AGAINST ITS USE AS A MEANS OF ASSESSING STUDENTS. IT IS SIMPLY AN EXAMPLE OF WHAT'S "OUT THERE".

Although it should really go without saying, all of you are invited to comment on any item in this column (or anywhere else in the *Gazette,* for that matter) in the form of a LETTER TO THE EDITOR. Letters could either be addressed directly to me, or sent to Jack. I am interested in your comments and suggestions, and as usual, in your submissions. Submissions can be sent to: Jennifer Weiner, c/o Dept. of Mathematics & Statistics, University of Guelph, Guelph, ON, N1G 2W1.

MTB4G1 Business Mathematics

Final Examination
January 1995
Time: up to 2 hrs.

Instructions:

1. This is an open text book exam, so that you can have access to all the tables and charts needed. Unless you know the work well, you will need ALL the available time.
2. Write on one side of the foolscap only. Put final answer on exam sheet, where possible.
3. When finished, staple the exam sheet on top of the foolscap. DO NOT FOLD!
4. Graph paper is available on request. Put this in the appropriate place in your answers.
5. Show all work on foolscap (for possible part marks).

Marks

1. Fill in the chart:

	Decimal	Fraction	Percent
6	0.2	—	_____
	_____	—	$\frac{1}{8}\%$
	_____	$\frac{3}{400}$	_____

2. a) 150 increased by 20% equals _____

6 b) 120 is what % more than 80? _____

c) 33 1/3% of 12 is what % of 16? _____

3. When I was 25 I bought 180 shares of Standard Paper at $25\frac{1}{2}$. Five years later I sold them at $30\frac{1}{4}$.

7 a) Including commission what was my total investment? _____

b) What were the net proceeds? _____

c) What was my capital gain (or loss)? _____

4 4. Find the approximate yield on a $7\frac{3}{4}\%$, $8 000 bond due in 10a when the current market price is $78\frac{1}{2}$.

3 5. The downtown hotel was insured for $750 000 with the risk being shared among the VISTA Insurance Co. ($250 000), FELIX Insurance ($400 000) and UPTOWN Insurance ($100 000). A small fire causes a loss of $150 000. How much will each company pay?

FELIX _____ VISTA _____ UPTOWN _____

6. Derek Morris read his hydro meter on Valentine's Day (6765) and again a month later (6848).

4 a) How many kwh of energy did he use? _____

b) What did it cost if the first 500 kwh cost 7.63¢ /kwh and the remainder cost 4.38¢ /kwh? _____

4 7. Martha Washington used $97m^3$ of water last month. Her charge is 39.82¢ /m^3 for the first $14m^3$ used and 25.54 ¢/m^3 for the rest. Find the total water service charge if the sewer charge is 115% of the water use charge. _____

8. a) Prepare an amortization schedule for the FIRST FOUR payments on a mortgage of $39 000 payable in quarterly instalments at $400 plus interest at 15 1/4%. The total cost of the property is $95 000. [on foolscap]

6 b) How long will it take to pay off the mortgage? _____

c) What is the owner's equity after 5 years? _____

9. Prepare a "STATEMENT OF ADJUSTMENTS" to find the balance due to the seller on the closing date, given the following information: [ALL ON FOOLSCAP]

12 a) Closing date – 31 May 1995

b) Purchase price – $87 500

c) Deposit – $1 900

d) Purchaser assumes existing mortgage of $24 600 (balance at end of April). Interest rate is $12\frac{3}{4}\%$.

e) No taxes have been paid for 1995, the mill rate is 77.5 mills and the house is assessed at $15 300.

f) A fire insurance policy taken out on November 30, 1994, will be transferred to the new owner. The annual premium is $178.75.

g) The oil tank (200L capacity) is quarter full. Oil costs 48.3 ¢/L.

h) The last water bill of $16.10 for two months was paid by the previous owner on May 31.

10. Evaluate:

10 a) $\dfrac{2^9 \times 4^7}{8^3} =$ c) $27^{\frac{1}{3}} =$ e) $\dfrac{2^{-2} + 3^{-2}}{2^{-1} + 3^{-1}} =$

b) $\dfrac{5 \times 5^0}{5 - 5^0} =$ d) $81^{-\frac{3}{4}}$ or $81^{-0.75} =$ (show your work)

11. Solve:

7 a) $4x + 6 = 18$ $x =$ _____

b) $16x + 3 = 14x + 7$ $x =$ _____

c) $6a - (a-3) - 11 = a + 5 - (a-7)$ $a =$ _____

12. Solve:

14 a) $y = 4x - 10$ (1)
$3x - 2y = 8$ (2)

(x,y) =

b) $x^2 - 4x - 21 = 0$ $x =$ _____

c) $x^2 + x - 1 = 0$ $x =$ _____
(to 2 decimal places)

13. a) △PQR is right angled at P, PQ = 10, QR = 26; find PR _____

6 b) △ABC is right angled at B, a=c=10. How long is the hypotenuse? _____

14. a) Find the mean, median, and mode for the following selected sample of monthly sales: $15000, $21000, $48000, $27000, $19000, $14000, $34000, $31000, $15000, $17000, $23000, $18000.

7

mean _____ median _____ mode _____

b) Which average best represents this data? Explain. _____

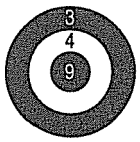
_____

15. Using your General Tax Guide, state on which line of the income tax return you would list the following accounts.

15

1.	Scholarships, fellowships, bursaries and artists' project grants	_____
2.	Tuition fees and education amount transferred from a child	_____
3.	Age amount	_____
4.	Other income	_____
5.	CPP or QPP contributions by an employee	_____
6.	Moving expenses	_____
7.	Spousal amount	_____
8.	Additional personal amounts	_____
9.	Pension income amount	_____
10.	Tuition fees	_____
11.	Educational amount	_____
12.	Allowable portion of medical expenses	_____
13.	Charitable donations	_____
14.	Provincial tax	_____
15.	Total payable	_____

16. (BONUS) Three darts are thrown at the target shown below. Assume that each dart lands within one of the rings or within the bull's–eye. How many different points are possible?



WHEN YOU HAVE FINISHED
OR
10 MINUTES BEFORE THE EXAM ENDS ...

Take 5 minutes to check over your answers for any "obvious" mistakes. Put your sheets of foolscap in order, with the exam papers on top.

Hand in: 1. EXAM

2. TEXTBOOK (keep your tax guide)

Total 111 Marks

New! New! New!

NOW AVAILABLE!

New! New! New!

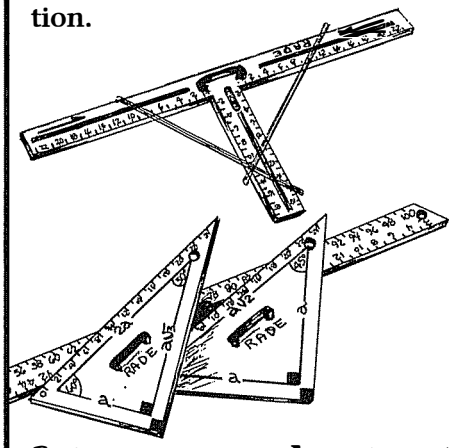
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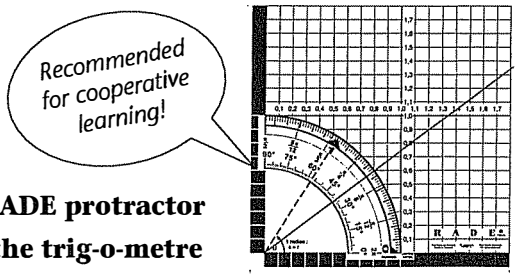
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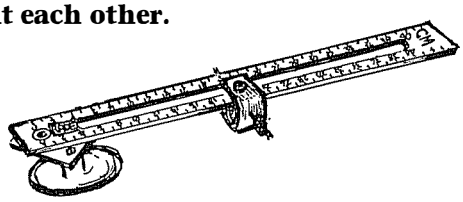
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TRIG-O-METRE	300 0767	$27.90
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▲ A Review of TVO Tapes: Linear Systems & Quadratic Functions

JOHN KERSLEY
HILLFIELD-STRATHALLAN COLLEGE
HAMILTON

Introduction

TVO has developed a series of video tapes on various mathematics topics for secondary schools geared to the general level student. Each series consists of set of 10 minute programs on a specific topic.

Linear Systems

I reviewed these tapes with two grade 10 classes: one class had studied the solution of a linear system of equations and the other class had just started the topic. The videos use two female robots/aliens to introduce the mathematics. The presentations were clear and easily comprehended by the majority of the students. Most students liked the characters and with the excellent computer graphics, the videos kept the interest of the majority of the students. A few students, however, felt that the tapes were a bit simplistic and were a bit below their grade level but most did feel the pace of the programs was good. A few mentioned that the tapes did reinforce what was learned in class. Students could see what they were doing in class.

Quadratic Functions

This series was reviewed with a grade 11 class. Again, the students appreciated the use of the robots and the computer graphics. Similar to the linear system tapes, the presentations and explanations were clear and easily understood. The pace was easy to follow. Students liked the idea of solving questions visually. A few students did mention that, in places the tapes were a bit repetitive. However, this was done to emphasize the important points.

Overall

Along with other available resources, manipulatives, computer software, etc., these tapes can be an effective way in illustrating and explaining the mathematical concepts. I feel they can be used to introduce the material as long as the playing of the tapes is spread over an appropriate period of time a class is on the given topic. The tapes can also be used to review certain aspects and reinforce what was discussed in class. Used appropriately, these tapes can offer a refreshing change to the standard approach and, thus keep students' interest.

Teacher's guides accompany each of the series. These guides contain program descriptions, objectives, objectives and pre-and post-viewing activities. These guides can be used to explain further or to challenge the students.

For further information, contact: *TVO's Customer Service Center, toll free at 1-800-INFO-TVO, or fax (416) 484-4425.*

▲ Letters to the Editor (cont'd)

Dear Editor:

There is now a conference for OAME members and other interested teachers on the Electronic Village. To join this conference all one need do is:

- select JOIN from the main menu;
- type OAME as the conference name;
- hit ENTER or RETURN.

(Editor's note. The rest of this letter is actually a personal response to me concerning a previous electronic communication. In jest, I chastised Dave for spelling and grammatical errors.)

I tried to clean up my keyboarding in response to your overly critical appraisal of my last email message. i hope you are satisfied with this effort. It's been spell-checked, style-checked and dry cleaned. Only took 3 hours to type it in.

By the way, I DID notice the case error in the previous paragraph and deliberately chose NOT to correct it just to see if you were paying attention.

Dave Zimmer
John Fraser Secondary School, Mississauga

Editors Reply:

Thanks David. Your valuable info. arrived just in time to make this issue of the Gazzette. good Job.

Jack

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Mickey Sandblom

▲ OAME 95 Conference Presents...

Panel Discussion: The Royal Commission on Learning and the Future of Education in Ontario ▲ Saturday May 13, 9:30am–12:00



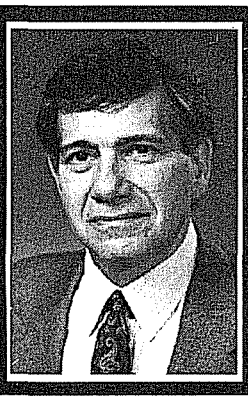
Charles Beer was first elected to the Ontario Legislature in the riding of York Mackenzie in 1987, and was re-elected on September 6, 1990. He is currently Official Opposition's Critic for Education and the new area of Children's Services. He is also Chair of the Standing Committee on Social Development.

During his first year in office, Mr. Beer chaired the Select Committee on Constitutional Reform, which studied the Meech Lake Accord. From October 1988 to August 1989, he served as Parliamentary Assistant to the Minister of Education and served in that portfolio until his appointment to Cabinet. In August 1989, Charles Beer was appointed Minister of Community and Social Services Minister Responsible for Francophone Affairs.

Mr. Beer was raised in Newmarket, Ontario, and educated at the University of Toronto and York University. He pursued post-graduate studies in Canadian history at l'Universite Laval, and is also a graduate of the National Defence College in Kingston.

Gerald Caplan, a former co-chairman of Ottawa's Task Force on Broadcasting, has been at the centre of every major policy debate in Canada for the past twenty-five years. Mr. Caplan has written a syndicated weekly newspaper column on public affairs. He co-chaired, with Monique Begin, the Ontario government's Royal Commission on Learning, the focus of this OAME Conference panel. He has served as the NDP's national director, managed the party's back-from-oblivion election campaign in 1984 (winning 32 seats) and has held the ear of every major New Democratic leader, from Audrey MacLaughlin to Premier Bob Rae. A powerful speaker, Mr. Caplan provides audiences with a ring-side seat in the corridors of Canadian politics.

Mr. Caplan has a doctorate in history and has held professorships both in Canada and abroad. With the Royal Commission on Learning, he has been charged with the task of improving education in Ontario. He is doing so with enthusiasm and dedication.





Dianne Cunningham is the Member of Provincial Parliament for the riding of London North.

A former teacher, educator and self employed professional with her own management and consulting business, she became involved in politics in 1973 when she was elected as trustee for the London Board of Education. She served on the Board for fifteen years, including two terms as Chairperson. In April, 1993, she was awarded the "Woman of Distinction" Award by the London YM-YWCA for her involvement in the London community.

In 1988, Mrs. Cunningham successfully contested the provincial by-election in the riding of London North. She was re-elected to the Ontario Legislature in the general election of September, 1990.

She has served as opposition critic for a number of Ministries and was also Chief Opposition Whip. She is now critic for the Superministry of Education and Training and, in this capacity, was one of the chief architects of the Progressive Conservative Party's discussion paper on Education, entitled, "Blueprint for Learning in Ontario".

Mrs. Cunningham ran for the leadership of the P.C. Party in May, 1990. In March of 1993, she was honoured by being named Deputy Leader of the Progressive Conservative Party of Ontario.

Derek Fletcher is MPP for Guelph and Parliamentary Assistant to the Minister of Consumer and Commercial Relations. He was elected to the Ontario legislature in 1990

Mr. Fletcher is a member of the Standing Committee on General Government and the Standing Committee on Regulations and Private Bills. He also serves on three New Democratic caucus committees; the Rural Advisory Committee, Environment Committee and the Parliamentary Assistants for Small Business Committee. He has also served on the Standing Committee on Administration of Justice and on the interministerial Parliamentary Assistants' Anti-Drug Strategy Task Group.

Mr. Fletcher is a former labour council president and public school trustee, and was one of the founding members of Guelph's race relations committee. He served as president of the Guelph and District Labour Council from 1984 to 1988, and as a trustee on the Wellington County Board of Education from 1985 to 1990. His special interests in these posts include worker health and safety, environmental issues, quality of education, and racism and discrimination in the workplace and school.



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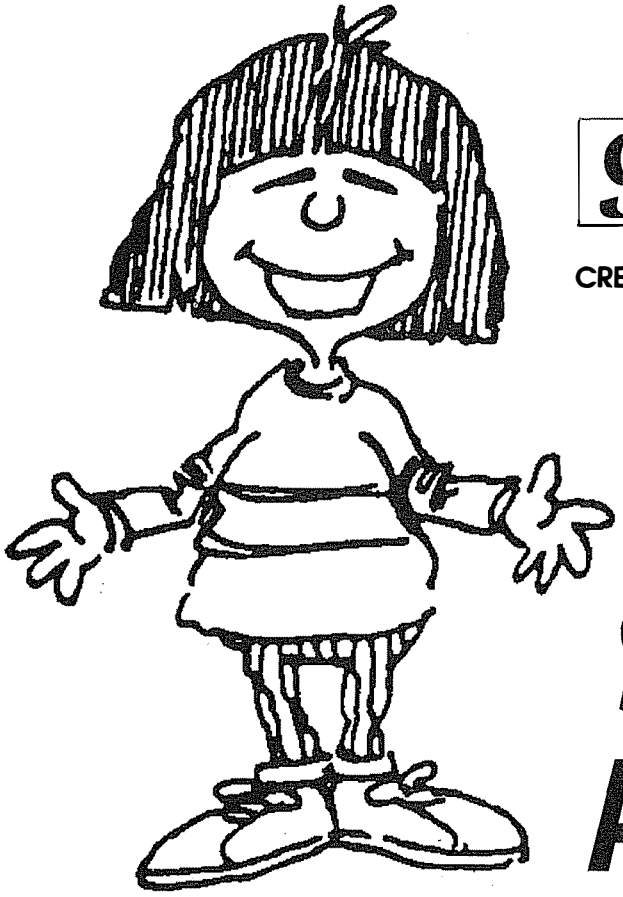
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