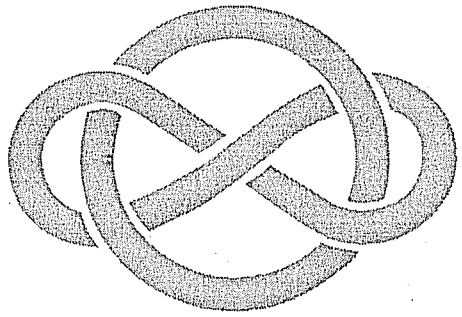
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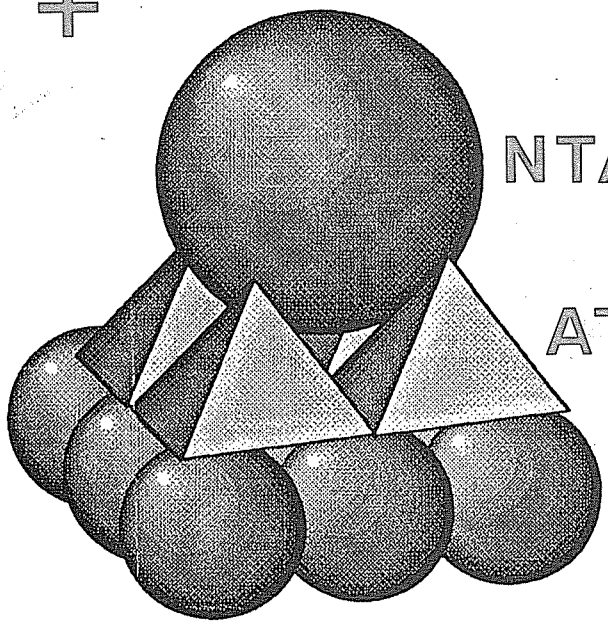
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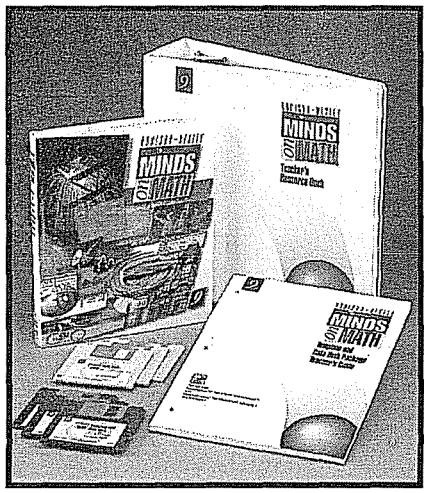


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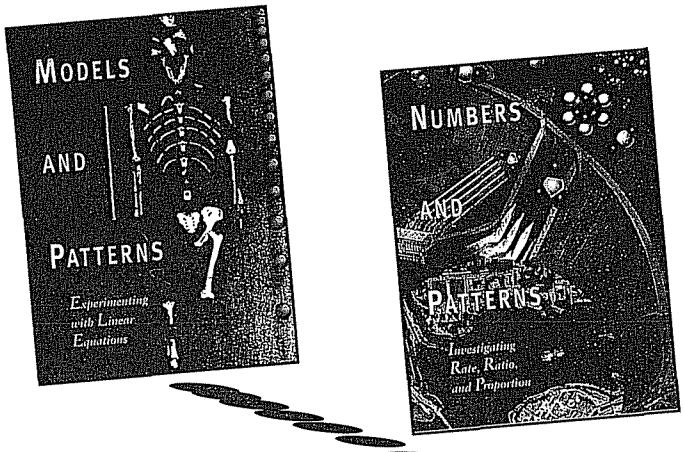
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Editorial



I want to share with you a very special moment, the kind of moment that for me captures one of the important, positive, and incredibly satisfying roles a teacher can play in the life of a student.

The start of the fall semester is one of my great joys in life. Campus during the summer is relaxed and informal, enjoyable. But things are pretty deadly after five o'clock and on weekends. There's nobody around except us workaholics. And you can't get a decent cup of coffee after hours.

But the day after Labour Day, this place comes alive. Students from frosh to grads are lugging books, chowing down, hugging old friends and new. There are orientation barbecues and concerts galore! It's so exciting. Of course the elevator comes only once an hour and stops on every floor (and these elevators, I'm convinced, were built by the lowest bidder—they're very slow!!) but that's a small price to pay.

So I'm heading to my office on this, the first official day of fall registration when I nearly bump into one of our new old grad students. New, since he's just starting his masters of mathematics program. Old, since he has just graduated from our undergraduate program.

"Hi Rob. How was your summer? And congratulations on starting your masters. You'll do great."

"Hi Professor Weiner. Summer was really good. And..., well, uh, Jack, can I talk to you a minute?" This sounded ominous. Rob looked very nervous.

"Sure. I was just headed to my office." I fumbled with my keys. "You would think after 15 years, I would know which key is which! Have a chair." Rob sat down . "So what can I do for you?"

He averted his eyes a little, almost shyly. "I don't need anything thanks. I just, well, there's something I have wanted to tell you for the last few years but everytime I tried to bring it up, I got a little emotional and that's a bit embarrassing and.... well, let me just say it straight out.

"When I was a first year math student, I was pretty frustrated. I had totally made up my mind to change programs entirely. I didn't like the first calculus course in the fall and was struggling with your course in the winter.

"One night around seven, I was with a friend in the math lab. We were both having trouble with Integration by Parts. You stuck your head in the door and asked how things were going. You then spent forty minutes or so working with us. After that, Integration by Parts seemed easy and for the rest of the semester, I found everything in the course much easier to understand. I started to enjoy math again. And, well the upshot is, here I am, starting a masters in math. I wouldn't be doing this if it wasn't for your help that night. I always wanted to thank you for that."

We all have stories like this to share. They help reaffirm why we love our job.

Jack

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▲ PRESIDENT'S MESSAGE



We cannot leave the future of education to the experts. They are not infallible. Collectively, we all have to take responsibility for the future. The future doesn't just happen. We must learn all we can from the past and use it to shape the years to come. S.J. Bezuszka, *from Heart and Mind: A Classroom Odyssey.*

A colleague of mine at Nipissing University lent me a copy of Stanley Bezuszka's, *From the Heart and Mind.* Father Bezuszka, as many of you know, offers the reader insights into the teaching and learning of mathematics, humorous stories of teachers and students and comments on the state of education. Many of his quotations reminded me of special teachers whom I have had the privilege of meeting through OAME. The above quotation made me think of our organization. As I prepared to write this column, I reflected on the role of our association and this quote seemed to capture our belief that this is the mathematics educator - the teacher, the consultant, the coordinator - who should be directing the changes that take place in the mathematics classrooms of the province. The tasks that the members of OAME have been involved in during the past years and the plans that they have for this year prove their commitment to the future of education.

Reading over Todd Romien's President's Messages from last year makes me realize how incredibly active this organization has been. Thanks to Todd's leadership and the hard work of our committees, many exciting projects were started. In June the 1995-96 Council met to formulate plans for the coming year - a vision of strengthening partnership in mathematics education in order to bring the best education possible to our students seemed to be reflected in many of these projects.

Scarborough College was the site of the Joint International Math Conference, July 28 - 31. Organized by OAME and the American Mathematics Teachers of New York State, the conference provided teachers the opportunity to share ideas and listen to outstanding speakers and presenters. Although the participation was lower than hoped, the teachers who took part were given an outstanding experience. A sincere thank you is extended to John Kersley, Chris Swan, Shirley Dalrymple, Ron Lancaster, and Anne Pedersen for all their hard work. The teachers who attended certainly benefitted from their efforts.

Joan Green, head of the Education Quality and Assessment Office, will join us at our first council meeting in September. Councillors will have the opportunity to question her about provincial testing and to share their ideas with her. We will certainly make sure that she is given a copy of the OAME/OMCA assessment booklet which was prepared for Transition Years Teachers by a joint committee. If you want to hear more about the discussion with Joan, contact your chapter representative or a councillor in your area after the September 29/30 meeting. And talking about assessment...be sure you get a copy of the booklet for yourself and encourage your colleagues to do the same. Junior teachers should watch for news about the assessment materials being prepared for their level by OAME and OAJE joint workgroups.

Thanks to the tremendous efforts of Jeri Lunney, Rob McLeish and a team of dedicated teachers in Ottawa, many teachers and students are looking forward to the second annual Ontario Mathematics Olympics. SAME (Scarborough) has volunteered to host the 1996 Olympics. Watch future *Gazettes* for more details and contact your local chapter representative to find out if there will be a 'playdown' in your area — better still, volunteer to organize that playdown for your chapter!

Another area of the province that will be 'math-busy' during the next few months is London - the site of OAME '96. If you live in that region, call a committee member to see if you can lend a hand. The last few months are always the most hectic when planning a conference, and I am sure they could use your help.

On a sad note, Jack Weiner is completing his final year as Editor of the *Gazette* and has decided it's time to move on to other projects. As we encourage members to consider applying for the position, we will be reflecting on the impact that Jack has had on the growth of this publication - no one can argue about the immense size of the shoes that need filling!* Jennifer Weiner's commitment to the improvement of the *Abacus* will be sorely missed. Few know of the difficulties that she has had in filling those pages. OAME members can benefit from the expertise of these two dedicated professionals for only a few more issues.

Have a wonderful school year and interesting math teaching!

**Editor's Note: Size 11 on a 5 ft. 6 in. frame. Go figure!*

Letters to the Editor

Dear Editor,

Readers of the *Gazette* might be interested in recent changes and other news concerning Mathematical Sciences at Lakehead University.

Effective July 1, 1995 the Department of Mathematical Sciences became the School of Mathematical Sciences containing two Departments: the Department of Computer Science and the Department of Mathematics and Statistics. This change reflects the maturity of our offerings in computer science and the professional nature of our courses and programs. Dr. John Griffith, a former editor of the *Gazette,* has been appointed Director of the School, Dr. Maurice Benson becomes Chair of the Computer Science Department, and I have been appointed Chair of the Department of Mathematics and Statistics.

Our co-op program in computer science has been modified to include two eight-month work terms as requested by several of the employers that provide co-op placements. Senior students who are not enrolled in the co-op option now have the opportunity to pursue an eight-month work term in order to gain practical experience before graduation. Thirty-one students were given co-op placements in spring/summer 1995.

Dr. Bill Eames retired in June of 1994. Students and faculty have enjoyed his sense of humour and benefited from his wisdom since 1966 when he came to Lakehead University.

Sincerely,
L. Keith Roy
Chair, Department of Mathematics and Statistics
Lakehead University

Dear Editor,

Thanks for another great OAME *Gazette.*

I agree wholeheartedly with Mary Lou Kestell's comments in her letter in the June *Gazette.* We have enough homegrown negativism and don't need more from across the water. By all accounts the British education system could do with innovative, enthusiastic educators like Mary Lou - Tony Gardiner eat your heart out! Eric Muller's report about Morocco included some interesting universal math problems but surely there was a misprint in the birds problem - if 100 birds were purchased for 20 rials then they must all be sparrows (at 5 per rial). Maybe we could instruct students to attempt these problems without putting pen to paper, thereby simulating conditions in the Atlas mountains. Wouldn't it be wonderful if problem solving became a social activity here. I guess women weren't involved - were they at home addressing other pressing questions like, "What's for dinner?"

Sincerely,
Ann Bobker
Halton Board of Education

Call for Editors:

The Ontario Mathematics *Gazette* and the *ABACUS* are taking applications for two year terms as editors commencing with the September, 1996 issues.

If you are interested or would like to recommend a colleague for either position, please contact Executive Directors Dave and Bonnie Alexander at the address on the masthead (inside front cover).

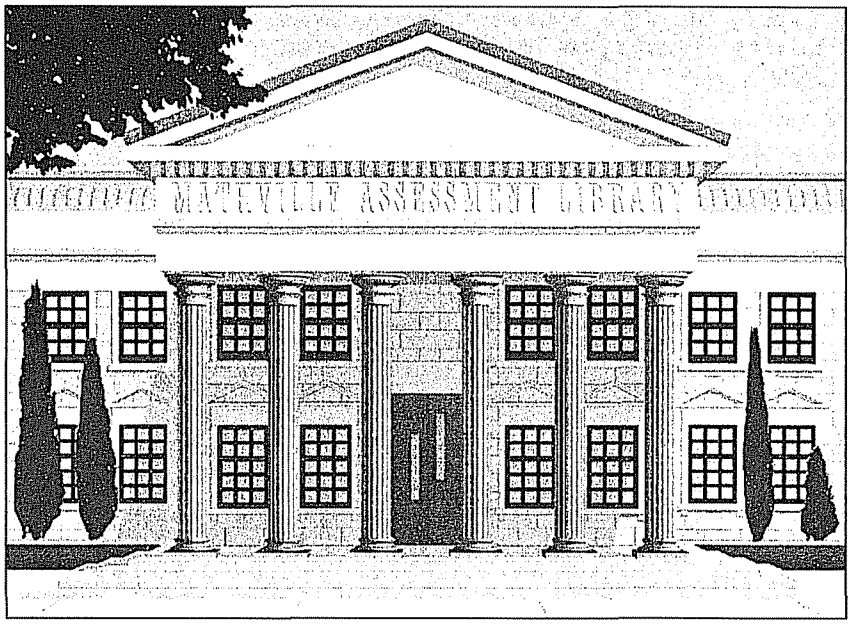
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▲ Exploring Geometric Properties Though Defining and Solving a Triangular Shape Whenever Measures for the Three Sides (SSS) are Known: A Logo Module for the Computing Classroom

MEDHAT H. RAHIM
FACULTY OF EDUCATION
LAKEHEAD UNIVERSITY



Educated at the Universities of Baghdad, London- England, and Alberta, Medhat has taught mathematics, statistics, and Mathematical Methods at the Universities of Alberta and West Indies-Trinidad. Currently, he teaches Mathematical Methods at Lakehead University. His interests include use of manipulatives and technology in teaching and learning mathematics.

Abstract

Based on the National Council of Teachers of Mathematics Standards (NCTM, 1989) and the National Assessment of Educational Progress reports (NAEP, 1988), a computer module through LOGO dealing with the problem of defining and solving different types of triangles will be presented. In particular, the module covers the *Side-Side-Side* (SSS) case. The main purpose of the software is to introduce a computer environment that allows students (a) to gain experience and knowledge of the necessary and sufficient conditions in constructing triangles and determine the measures of their unknown segments and angles, and (b) to create a rich environment for investigating geometric relationships (NCTM Standards 1989, p. 114).

Introduction

The secondary school results for the fourth mathematics assessment of the National Assessment of Educational Progress (NAEP) that appeared in two articles (Brown et al. 1988a + 1988b) indicate that, generally, secondary school students seem to have some knowledge of basic algebraic and geometric concepts and skills yet they often are not able to apply this knowledge in problem solving situations, nor do they seem to understand many of the structures underlying these mathematical concepts and skills (Brown et al. 1988b, pp. 346-347). Similarly, in other areas of mathematics, such as measurement, students are lacking conceptual knowledge that would enable them successfully to do the application, problem solving and reasoning items on the assessment (Brown et al. 1988a, p. 248). For example, on four choice multiple choice items on "Properties of Figures" concerning the *relationships among the three sides of a triangle* "Choose the set of three numbers that cannot be lengths of sides of a triangle", only 9 percent of the seventh grade students answered correctly. Eleventh grade students who had no geometry performed as well as seventh grade students on this item with 8 percent answering correctly, while only 33 percent of the eleventh grade students with geometry selected the correct response. (Brown et al. 1988a, pp. 340-341)

In North America, therefore, mathematics professional organizations, researchers, and curriculum developers have been working hard through research and national assessments to identify weaknesses in reasoning about triangles and to suggest remedies. The most recent documents on such tasks, in addition to the NAEP 1988, have been the National Council of Teachers of Mathematics (NCTM) documents (1) *Curriculum and Evaluation Standards for School Mathematics* (1989) and (2) *Professional Standards for Teaching Mathematics* (1991). The NCTM Standard 12: Geometry indicated that

> *Computer software that allows students to construct geometric figures and determine the measures of arc, angles, and segments creates a rich environment for investigation of geometric properties and relationships. (p. 114)*

With these NCTM recommendations and the NAEP results in mind and the fact that triangles are the seeds for all other polygons (every polygon is dissectible into a finite number of triangles), the work described here has been developed to deal with various aspects of triangular shapes, polygonal shapes, and the kind of misconceptions such as those indicated above concerning triangles in particular and polygons in general. Further, this work deals with other fundamental aspects that are vital for high school students, namely the trigonometric aspects of triangular and polygonal shapes.

Purpose

Generally, this paper presents an interactive and simulative software through Logo in a computer assisted instruction (CAI) fashion dealing with basic and visual representations of (a)

preliminary basic concepts and properties of polygonal shapes as wholes, (b) part to whole interrelationship among a polygon and its sides and angles, (c) part to part interrelationships among sides, among angles, and among sides and angles of a polygon, and (d) whole to whole interrelationships among simple (convex and non convex) polygonal regions.

Although there are several software packages that can be used for simulation purposes in geometry, such as *Cabri,* Geometer's *Sketchpad,* and Geometry *Inventor,* these are only application tools for visual exploration. Logo, in addition, is particularly more appealing because it offers, *through programming,* a rich environment for investigation of geometric properties and relationships.

The work is aimed at elementary and middle school levels, except for the trigonometric aspects of it, for which upper high school levels are targeted. For elementary and middle school levels, teachers in teacher directed investigations may use the software as a computer application to explore and experience part to part and part to whole interrelationships of polygons. This work is part of a larger work that would examine, through Logo, a set of propositions on *Shape Transforms* among polygons embedded in recent works of Rahim (1986), and Rahim and Sawada (1986, 1989, & 1990). The set of propositions on Shape Transforms is based on a merge of two aspects in mathematics, *Dissection Theory* (see Eves, 1972) and *Transformation operations* or *Isometries* in addition to the *recursion* concept (possible repetition of these two operations.) It, therefore, represents a new approach in geometry called *Dissection Motion Operation* approach or DMO for short). For example, consider the following situation.

> *A right angle triangle can be transformed through a sequence of dissection and motion operations into a* ***rectangle*** *and the resultant rectangle can further be transformed into a* ***parallelogram.*** *This parallelogram can be transformed further into an* isosceles acute triangle. *(Rahim and Sawada, 1986, p. 238).*

Note that DMO is an *area-invariant* recursive operation, that is, the area of the original shape and the resultant shape are equal.

For this session, the case of triangular shapes is considered and various aspects of triangles will be presented. We start first with the necessary and sufficient conditions to define a triangle, and then we will entrench these properties into Logo procedures that will handle these conditions through an interactive mode. For elementary and middle school levels, the teacher may guide the student through the software to explore, understand, and learn information which will help them to deal with properties and interrelationships of various types of triangles. For high school levels, in addition, the content of the procedures which constitutes the software, may be used for classroom discussion and lab practice in the computing classroom.

The three well-known cases in trigonometry for defining triangles are: The *three sides* are given - the SSS case, the *two angles and the side in common* are given - the **ASA** case (the case when other than the common leg is given is identical to this case), and the *two sides and the enclosed angle* are given - the **SAS** case.

The SSS case: The three sides of a triangle are known.

The aim here is that a set of *three numbers* is going to be provided by the student, as a user, through an interactive mode with the microcomputer. And, as such, this paper is primarily directed toward the development of a set of Logo procedures which can be used to check the interrelationship among the three numbers provided by the user in terms of (a) whether or not they make a triangle, (b) solving the triangle in case the set does make a triangle, (c) simulating and sketching the triangle, and (d) providing the information calculated.

Trigonometric Tools

To build a module to implement the task above, we are required to create: procedures to defining triangles, procedures for solving triangles, and procedures to sketching triangles of interest.

Recall that in our case, SSS, to define a triangle is to understand the necessary and sufficient information about the three sides. Their appropriate measures have to be provided so that the three angles can be calculated and the desired triangle can be sketched. That is, in addition to the three sides, the three angles should be known and the triangle is said to be solved.

In order to manage all these steps using Logo, first of all the *starting position and the heading* for the triangular cursor (turtle) has to be chosen on the plane (screen). One of the ways to do this is the following procedure, **START.POSITION.HEADING (OR START.P.H)**, given below. It allows the user to choose a position and a heading as he/she wishes and makes them known too as a "new home" and a "new direction" to the triangular cursor. This known "new home" is for future cases in which a return to the initial position and heading would be needed. It can be deleted otherwise by removing the two "make" statements at the tail of the procedure. Note that the following procedures are in *Apple Logo II version*, they can easily be converted into other Logo versions.

```
TO START.POSITION.HEADING;
CS ST PU
PR[I AM GOING TO ASK YOU TO PROVIDE ME WITH
    THE X-Y POSITION]
PR[AND THE HEADING OF THE TRIANGULAR
    CURSOR OR THE TURTLE, OK?]
PR[]PR[]PR[]
WAITFORUSER
PR[TYPE IN AN X-COORDINATE, WITHIN
    (-140<X<140)]
SETX FIRST RL
PR[TYPE IN A Y-COORDINATE, WITHIN
    (-120<Y<120)]
```

```
SETY FIRST RL
PR[TYPE IN A HEADING FOR THE TURTLE,
    (FROM 0 TO 360)]
SETHEADING FIRST RL
MAKE"START1.POSITION POS
MAKE"START1.HEADING HEADING
PL]
END

TO WAITFORUSER
PR[PRESS RETURN TO CONTINUE_]
MAKE"WAITING RL
TEST:WAITING=[]
IFFALSE(WAITFORUSER)
END
```

The restriction on the ranges of X and Y can be changed or removed depending on the level of students' awareness of the possible 'loss' of the triangular cursor from the screen. The restriction on the HEADING may include (OR FROM 0 to -360). Teachers can utilize this procedure so that students can explore and visualize plotting and directions on the X-Y plane.

After picking a position and a heading on the screen to start from, the task which follows is then to create and activate a procedure for defining a triangle through computer interactions with the student. The defining procedure DEF.SSS.TRI would request the student to provide measures for the three sides of a desired triangle. Below is the DEF.SSS.TRI procedure:

```
TO DEF.SSS.TRI
PR[THING OF THREE NUMBERS THAT CAN BE]
PR[MEASURES OF THREE SIDES OF A TRIANGLE]
WAITFORUSER
PR[TYPE IN A LENGTH FOR THE FIRST SIDE]
MAKE "SIDE1 FIRST RL
PR[TYPE IN A LENGTH FOR THE SECOND SIDE]
MAKE "SIDE2 FIRST RL
PR[TYPE IN A LENGTH FOR THE THIRD SIDE]
MAKE "SIDE3 FIRST RL
TEST(OR (:SIDE1 + :SIDE2 + :SIDE3)/2<:SIDE1
    (:SIDE1 + :SIDE2 + :SIDE3)/2 = :SIDE1)
IFTRUE[PR[SORRY.YOUR SIDES DO NOT
    ASSEMBLE A TRIANGLE: SIDE2 + SIDE3 MUST
    BE GREATER THAN SIDE1. TRY
    AGAIN]DEF.SSS.TRI]
TEST(OR(:SIDE1 + :SIDE2 + :SIDE3)/2<:SIDE2
    (:SIDE1 + :SIDE2 +:SIDE3/2 = :SIDE2)
IFTRUE[PR[NO,SORRY. SIDE1 + SIDE3 MUST BE
    GREATER THAN SIDE2
TRY AGAIN]DEF.SSS.TRI]
TEST(OR(:SIDE1 + :SIDE2 + :SIDE3)/2<:SIDE3
    (:SIDE1 + :SIDE2 + :SIDE3)/2 = :SIDE3)
IFTRUE[PR[NO. SIDE1 + SIDE2 MUST BE GREATER
    THAN SIDE 3.
TRY AGAIN]DEF.SSS.TRI]
END
```

The procedure will provide immediate suggested responses for the student whenever the three numbers provided cannot make a triangle. It will give an immediate feedback on incorrect responses. Otherwise, it will accept the three imputs as a set of three numbers that makes a triangle.

An illustration for why the procedure above has been set the way shown is needed. A general triangular shape depicted in Figure 1 below may be helpful.

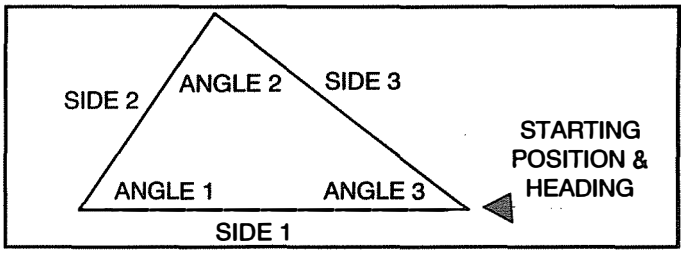


Figure 1

Teachers can use the content of this procedure, through the LOGO environment, to discuss the embedded propositions and rules concerning the conditions for three numbers to make a triangle. In particular, teachers would highlight the following proposition:

> *that the sum of any two sides of a triangle should be greater than the third side, or that half of the circumference of a triangle should be greater than the length of any of its sides,*

That is, if the relation

(:SIDE1 + :SIDE2 + :SIDE3)/2<:SIDE2 holds then

(:SIDE1 + :SIDE2 + :SIDE3)/2 < :SIDE2 —>

:SIDE1 + :SIDE2 + :SIDE3 <2(:SIDE2) —>

:SIDE1 + :SIDE3 <:SIDE2 (impossible to make a triangle).

Similarly for the other two inequalities in the procedure DEF.SSS.TRI above, the same result follows when '=' replaces '<' in the three inequalities.

Having been successful in picking a working set of three lengths, the task now is to introduce a procedure to solve the proposed triangle, that is to calculate the three unknown angles. Whenever appropriate, depending on the grade level of the class, computing teachers may use the procedure below, SOLVE.SSS, as a means for discussing trigonometric ratios and rules that are necessary for the task, e.g. the application and use of the Cosine Law [for three sides a, b, and c of a triangle, $a^2 = b^2 + c^2$ - 2bc(Cos A) from which the ARCCOS function is derived.

At this point let's consider the first version of SOLVE.SSS, labeled as SOLVE.SSS1:

```
TO SOLVE.SSS1
MAKE "ANGLE1 ARCCOS (:SIDE1*:SIDE1 +
    :SIDE2*:SIDE2-
:SIDE3*:SIDE3)/(2:SIDE1*:SIDE2)
MAKE "ANGLE2 ARCCOS(:SIDE2*:SIDE2 +
    :SIDE3*:SIDE3-
```

```
:SIDE1*:SIDE1)/2*:SIDE2*:SIDE3)
MAKE "ANGLE3 180-:ANGLE1-:ANGLE2
END
```

This procedure, SOLVE.SSS1, works in cases, but not in others, and needs some adjustment. After some usage, the user would probably realize the problem. The required adjustment has something to do with the way the Cosine ratio and its principal values (see Figure 2) have been built in the core of the Logo language which is out of the scope of this session. However, an elaboration here may be helpful. The reader, through some usage of the procedure SOLVE.SSS1, would discover that, in *some cases,* the turtle will not make a closed shape. Rather, it will draw a "zigzag" like figure rather than a triangle. This may happen whenever a calculation of the measure of an angle is involved and the output of the measure is negative. The negative measure causes the turtle to head to turn to the opposite (left) direction and the resultant sketch will not be a closed figure. It is not difficult to observe, as the figure is being drawn on the screen, that the turtle will turn to the left rather than to the right an angle equals ANGLE 1. The same may happen when the calculation involves ANGLE 2.

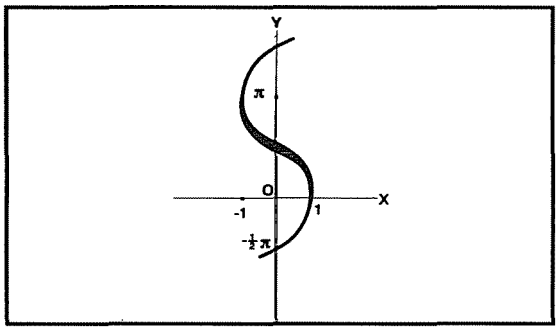


Figure 2

To make this article a more complete piece of work, the above procedure, SOLVE.SSS1, may be modified into an adjusted one, SOLVE.SSS, which will be the one used in the later super procedures. This does not mean this is the only adjustment for the procedure to make it work in every case.

A modified version for the procedure SOLVE.SSS1 is the following:

```
TO SOLVE.SSS
MAKE "ANGLE1 ARCCOS (:SIDE1*:SIDE1 +
    :SIDE2*:SIDE2
-:SIDE3*:SIDE3)/(2*:SIDE1*:SIDE2)
TEST(AND :ANGLE1<0:ANGLE1>-180)
IFTRUE[MAKE "ANGLE1 :ANGLE1 + 180]
TEST(AND :ANGLE1<-180:ANGLE1>-225)
IFTRUE[MAKE "ANGLE1:ANGLE1 + 360]
TEST (OR :ANGLE1 = -180 :ANGLE1 = -225)
IFTRUE [MAKE "ANGLE1:ANGLE1 + 360]
MAKE "ANGLE2 ARCCOS(:SIDE2*:SIDE2 +
    :SIDE3*:SIDE3
-:SIDE1*:SIDE1)/(2*:SIDE2*SIDE3)
TEST(AND :ANGLE2<0 :ANGLE2>-180)
IFTRUE[MAKE "ANGLE2 :ANGLE2 + 180]
TEST(AND :ANGLE2<-180 :ANGLE2>-225)
IFTRUE[MAKE "ANGLE2 :ANGLE2 + 360]
TEST(OR :ANGLE2 = -190 :ANGLE2 = -225)
IFTRUE[MAKE "ANGLE2 :ANGLE2 + 360]
MAKE "ANGLE3 180 - :ANGLE1 - :ANGLE2
END
```

The remaining part of the task is to create a procedure to draw the triangle based on the new information, namely, the calculated values of the angles of the triangle. At this stage students can easily recognize the completion of the task; it is merely a set of commands that will lead the turtle to move forward and to turn according to the given lengths of the sides and the calculated values of the angles. The procedure SKETCH.TRI below will do the simulation:

```
TOSKETCH.TRI
FD :SIDE1
RT180-:ANGLE1
FD :SIDE2
RT180-:ANGLE2
FD :SIDE3
RT180-:ANGLE3
END
```

The procedure SKETCH.TRI can draw any solved triangle.

There remain only two more procedures to fully complete the work:

(a) The procedure INFO.SSS below will provide all the information about the triangle, the given sides and the calculated angles; and

(b) the (ARCCOS:X) procedure acting on the values of the variable:X from 1 to -1 handling the values of the corresponding angle and specifically when ARCCOS acts on the values 0 or -1:

```
TO INFO.SSS
PR[WOULD YOU LIKE TO SEE THE INFORMATION
    ABOUT THIS TRIANGLE ON THE SCREEN?
    TYPE Y OR N]
IF READLIST=[N][PR[OK, BYE NOW.]STOP]
TEXTSCREEN
PR[]
PR[THE INFORMATION YOU GAVE ARE:]
PR(SE[FIRST SIDE = ]:SIDE1[,][SECOND SIDE =
    ]:SIDE2[AND][THIRD SIDE = ]:SIDE3}PR[]
PR[THE CALCULATED INFORMATION ARE:]
PR(SE[1ST ANGLE=]:ANGLE1[,]2ND
    ANGLE=]:ANGLE2[,][3RD ANGLE=]:ANGLE3)
END

TO ARCCOS:X
TEST(:X=0)
IFTRUE[OUTPUT 90]
TEST(:X=-1)
IFTRUE[OUTPUT -180]
OUTPUT ARCTAN (SQRT 1 - :X*:X)/:X
END
```

Figure 3 below illustrates the ARCCOS procedure

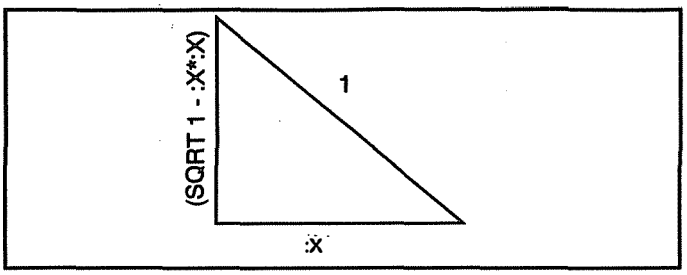


Figure 3

Finally, all subprocedures required for the superprocedures SSS are ready to use and the SSS procedure below will execute all the above requirements:

```
TO SSS
CS
START.POSITION.HEADING
DEF.SSS.TRI
SOLVE.SSS
SKETCH.TRI
INFO.SSS
END
```

Remarks and Suggestions

After the development of a working set of such procedures, the superprocedure SSS will activate the *whole* task as many times as needed. The activities described above would lead students, *through* teacher directed *investigations,* to consider, explore, and conceptualize visually and in an interactive manner relationships among the three sides of a triangle and among its corresponding angles (part to part interrelationships); relationships among the measures of the three sides and the type of a triangle they determine; and relationships among the sides of the triangle which determine whether or not a triangle does exist (part to whole interrelationships) such as:

(1) the sum of two sides of a triangle is greater than the third. Equivalently, *half* the circumference of a triangle is greater than any of its sides,

(2) for measures of the three sides, if the sum of two measures ≤ the third measure, the three measures will not make a triangle,

(3) three equal sides make an equilateral triangle and the corresponding angles will be equal,

(4) two equal sides make an isosceles triangle and the two corresponding angles will be equal,

(5) three non equal sides make a scalene triangle and the corresponding angles will be unequal,

(6) a Pythagorean triple such as (a, b, c) such that $a^2 + b^2 = c^2$ will necessarily make a right triangle,

(7) largest angle is opposite to the longer side and smallest angle will oppose the shortest side,

(8) conditions which determine the formation of right, obtuse, and acture triangles based on the Cosine Law and Pythagoras.

Teachers may intentionally suggest sets of three numbers in a chart like format, some of which would not make a triangle. As well, they may suggest other sets which would cover all types of triangles such as equilateral, isosceles, scalene, and include Pythagorean triples so that discussions in the computing classroom about the conditions of defining three numbers that make a triangle and the relationships among them that effect the formation of a triangle would be initiated.

It would be helpful for the teachers to suggest the following mini project for students to implement and explore the consequences: the teacher may suggest a Pythagorean triple such as 60, 80, & 100. Then he/she may ask the students to do the following sequence of mini tasks:

(a) Use the above set of numbers and observe the type of triangle that would be presented on the screen.

(b) Use the same set but with slightly different position and heading; try to keep your triangles at about the centre of the screen.

(c) Use the same set of numbers with the third number slightly different, say, use 60, 80 & 120 and observe the changes in the formation of the resulting triangle.

(d) Use the same set of numbers but this time make the third number slightly smaller than 100, i.e. use, say, 60, 80, & 90. Observe the changes in the formation of the resulting triangle.

(e) Write a conclusion about your visual observations of the simulations of the different resultant forms of triangles.

Instruction for organizing the execution of the software:

1. You are required to have two disks:

(a) the Apple Logo II programme disk, and

(b) a formatted disk called VOLUME (a volume here is a formatted disk on which you keep procedures or files of information) and each volume must be given a name. Let's give this volume the name SSS.

II. Now, you should save each of the logo procedures listed above except the procedure "SOLVE.SSS1" because it is replaced by the procedure "SOLVE.SSS". Briefly, you have to save on the SSS disk the procedures:

START.POSITION.HEADING, WAITFORUSER, DEF.SSS.TRI, SOLVE.SSS, SKETCH.TRI, INFO.SSS, ARCCOS:X, and SSS.

III. To organize the running of the programme, a special set of auxiliary procedures which organizes the execution of the programme is needed.

The following is a set of these auxiliary procedures involved in the execution:

START.SSS, FRAME.SSS, INTRO.SSS, BEGIN.SSS, MAIN.SSS, AND PREPARE.SSS. The detail of each is listed below.

It is important to save each of the following procedures separately. That is, whenever you finish typing and saving a procedure on your formatted disk, SSS, erase the procedure from the work space before typing and saving the other one and so on.

```
TO START.SSS
CT TS REPEAT 3[PR[]]
PR[LOADING THE SYSTEM, PLEASE
    WAIT...]PR[]PR[]
PR[LOADING...]
LOAD "WAITFORUSER
LOAD "FRAME.SSS LOAD "INTRO.SSS LOAD
    "BEGIN.SSS
LOAD "PREPARE.SSS LOAD "MAIN.SSS PR[]
WAITFORUSER INTRO.SSS
END

TO FRAME.SSS
TEXTSCREEN CLEARTEXT PR[]
PR[TITLE: A MODULE FOCUSES ON:]
PR[INVESTIGATION OF GEOMETRIC PROPERTIES]
PR[THROUGH DEFINING, SOLVING AND]
PR[SKETCHING TRIANGLES WHENEVER]
PR[MEASURES FOR THE THREE SIDE ARE
    PROVIDED]
PR[]PR[]PR[BY...AUTHOR...ADDRESS ETC...]PR[]
 END

TO INTRO.SSS
TS CT FRAME.SSS PR[] WAITFORUSER
TS CT PR[PLEASE TYPE IN YOUR NAME]PR[]
PR[YOU MAY QUIT THE PROGRAMME BY TYPING
    Q]
MAKE"NAME FIRST READLIST
TEST:NAME=[Q]
IFTRUE[CT PR[BYE BYE NOW]STOP]
PR SE[THANK YOU]:NAME
PR[IT'S MY PLEASURE TALKING WITH YOU]PR[]
PR[THIS IS A MODULE ON:]
PR[(1) DEFINING TRIANGLES WHENEVER]
PR[MEASURES FOR THE THREE SIDES ARE
    PROVIDED]
PR[]
PR[(2) SOLVING TRIANGLES]
PR[]
PR[(3) GRAPHING TRIANGLES]
WAITFORUSER
PR[]
BEGIN.SSS
END
TO BEGIN.SSS
PREPARE.SSS
MAIN.SSS
END
TO PREPARE.SSS
CT TS
REPEAT 4[PR[]]
PR[...A MODULE ON...]
PR[INVESTIGATION OF GEOMETRIC PROPERTIES
    OF TRIANGLES:THE-SIDE SIDE SIDE CASE]
PR[....]
PR[]PR[]
PR[LOADING SSS...]
LOAD "SSS
END

TO MAIN.SSS
CT TS
PR[MENU: SIDE-SIDE-SIDE (SSS)]
PR[]PR[]
PR[0) EXIT]
PR[1) DEFINE A TRIANGLE]
PR[]PR[ENTER 1 TO START, 0 TO EXIT]
MAKE "ANS READCHAR
IF:ANS= "0[CLEARTEXT STOP]
IF:ANS= 1[SSS]
PR[]
WAITFORUSER
MAIN.SSS
END
```

VI. To run this module do the following:

1. Insert the Apple Logo II programme disk in the disk drive and switch on the computer.

2. Press RETURN when you see the large title apple logo on the screen; you should see now a greeting message "Welcome to Logo" with a question mark and a blinking cursor beside it.

3. Take out the Apple Logo II disk from the disk drive and insert the DATA DISK (which is named as volume SSS) and type SETPREFIX"/SSS and press the RETURN key (note that there is a *one space* between X and "). You should see a question mark and the blinking cursor below the line you typed.

4. Type LOAD "START.SSS and press RETURN. You should see START.SSS DEFINED.

5. Type START.SSS and press RETURN. You should see LOADING THE SYSTEM, PLEASE WAIT...

LOADING...

you should wait till you see the message PRESS RETURN TO CONTINUE...

6. From now on follow the instructions.

Remark: If you press 0 (zero) to exit the programme and immediately changed your mind, you still can return back to the programme by typing MAIN.SSS followed by RETURN. Do the same whenever you make a typing error and the computer responds by a question mark (e.g. if you type letter "0" as zero).

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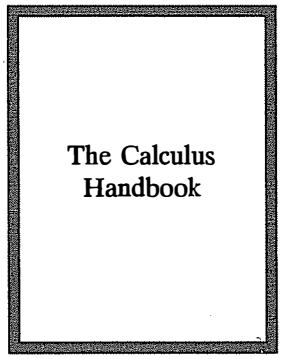
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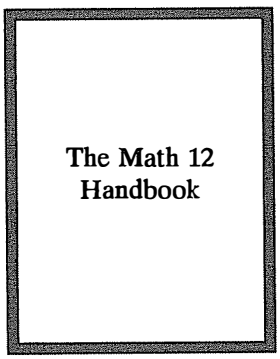
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▲ Master Minds At Work: The '95 International Math Olympiad

MIKE ZICHOWSKI
NORTH YORK BOARD OF EDUCATION



Mike Zichowski is a veteran math teacher and workshop host of 19 years with the North York Board of Education. He is currently pursuing a career as a freelance writer/videographer.

OIM 1995 IMO

The universal language of mathematics was showcased at York University this summer from the 13th to the 25th of July, when over 4000 students from 73 countries and 350 officials and volunteers converged on the campus for the 36th International Mathematical Olympiad (IMO). This is the pinnacle of mathematics and problem-solving competitions for high-school students around the world. For the first time in the event's 36 year history, Canada hosted the competition.

The Tait McKenzie gymnasia were the site of two grueling exams, each 4 1/2 hours in length, that the participants wrote over the course of two days. Each day's paper consisted of 3 mathematical problems set by an international jury at the University of Waterloo. According to one of the many invigilators, York University math professor Hans Joshi, "University math majors would have a very difficult time with these questions."

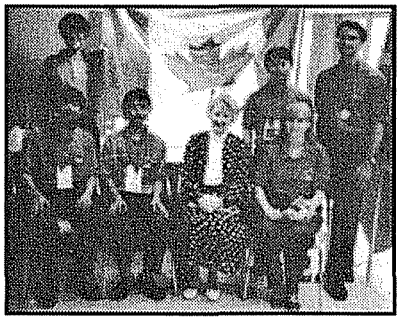
Dr. Bruce Shawyer, Chairman for the 1995 IMO, said "These competitions are one way of making mathematics education more relevant, challenging and fun for students. They also provide Canadians with the opportunity to measure their abilities against the best students from around the world." Dr. Graham Wright, Executive Director of the Canadian Mathematical Society, commented, "Looking around the gym and seeing all the different cultures and languages represented here, we begin to see the true power of mathematics. Mathematics provides us with the common bond, a bond that allows us to work together, no matter what our culture, our background, our languages."

The growth of this competition and the increased importance so many countries attach to it (as evidenced by the dozen foreign television crews present) sends a clear message that math and the development of top level mathematics and applied scientists is now a global priority. York University, by hosting an event of this size, provided an opportunity for business, government, educators and individuals to draw attention to the increasing importance of mathematics.

Participating teams consisted of six students and two leaders. This year across Canada more than 200,000 students in grades seven to twelve completed in local provincial and national math contests to earn a spot on the Canadian team. Past Canadian teams have done consistently well in the IMP competitions, placing in the top 25. Since 1981 Canadian students have been awarded 8 gold, 14 silver and 31 bronze medals and regularly finished ahead of countries with larger populations. This year's results included 2 silver and 2 bronze medals for Canada. China led the medal parade, with 4 gold and 2 silver, followed by the Romanian and Russian teams. At the 1994 IMO in Hong Kong where 69 countries participated, Canadian students earned 1 gold and 3 bronze medals. Three members of the Chinese team wrote perfect papers. Gold, silver and bronze medals are awarded to approximately the top half of the 400 competitors.

Astonishingly only 16 females participated at this year's competition. Nevertheless, one female competitor from Iran did write a perfect paper.

Next year, New Delhi, India will be the venue of the 37th IMO, but as one veteran competitor from Chile remarked, "York's facilities and friendly atmosphere will be difficult to beat. I might come here to study later."



The 1995 Canadian IMO team together with York University President, Susan Mann.
Back Row Left to Right:
Byung Kyu Chan, Cyrus Hsia, Frederick Latour
Front Row Left to Right:
Donny Cheung, Lawrence Tang, Susan Mann, Alyssa Ker



Toronto math teachers take a breather from their invigilator duties at the closing ceremonies.

NOMINATIONS FOR OAME/AOEM COUNCIL 1996-97

Each year, elections are held for President-Elect, 3 Directors and 7-10 Councillors for the Council of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 31, 1996, to:

Todd Romiens
Past-President, OAME

Include the nominee's qualifications with the Nomination Form. The Nomination Committee will make a final selection of those nominees who will stand for election. Their names and qualifications will be published in March, 1996. Voting will be by mail by members of the OAME/AOEM. The results of these elections will be made known at the Annual Meeting of the OAME/AOEM in London in May, 1996.

NOMINATION FORM

I nominate: _____

Address: _____

_____

For the position(s) of: _____ **President-Elect**

_____ **Director**

_____ **Councillor**

_____ **Nominator**

_____ **Address**

_____

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▲ About Problem Solving: Mathematics Contests



JOHN GRANT MCLOUGHLIN
EDUCATION DEPARTMENT
OKANAGAN UNIVERSITY COLLEGE

Mathematics contests take various forms. The Scarborough high school math leagues, junior high math days in western Newfoundland, and the Euclid contest are a few that come to mind. Perhaps you are familiar with a local endeavour that you would like to share with readers.

The W.J. Blundon Contest is an annual contest written by senior high school students in Newfoundland and Labrador. The contest is organized by the Mathematics and Statistics Department at Memorial University of Newfoundland. Jack Suvak, a math professor in the department, is responsible for putting the contest together. My familiarity with the contest stems from my teaching experience there. Since 1990, I have also provided input into the actual paper with various problem proposals. The provincial nature of this contest makes it less likely that readers have seen it before. Hence, a copy of this year's contest is included in the column. Note that all questions require full written solutions.

This column is intended to serve as a forum for exchanging ideas. Please write and share examples of interesting contest initiatives in your area. Samples of contests are also welcomed. Of course, you are encouraged to share the W.J. Blundon Contest problems with students. I will send complete solutions to anyone who requests them. Simply send your request together with a self-addressed, stamped envelope, to my Kelowna address above.

The Twelfth W.J. Blundon Contest

February 22, 1995

1. a) From a group of boys and girls, 15 girls leave. There are then left two boys for each girl. After this, 45 boys leave. There are then 5 girls for each boy. How many boys and how many girls where in the original group?

   b) A certain number of students can be accommodated in a hostel. If two students share each room then two students will be left without a room. If 3 students share each room then two rooms will be left over. How many rooms are there?

2. How many pairs of positive integers (x,y) satisfy the equation

$$\frac{x}{19}+\frac{y}{95}=1$$

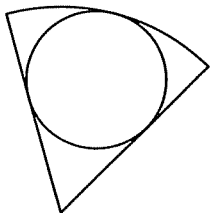
3. A book is to have 250 pages. How many times will the digit 2 be used in numbering the book?

4. Without using a calculator:

   a) Show that $\sqrt{7+\sqrt{48}}+\sqrt{7-\sqrt{48}}$ is a rational number.

   b) Determine the largest prime factor of 9919.

5. A circle is inscribed in a circular sector which is one sixth of a circle of radius 1, and is tangent to the three sides of the sector as shown. Calculate the radius of the inscribed circle.



6. Determine the units digit of the sum

$$26^{26}+33^{33}+45^{45}.$$

7. Find all solutions (x,y) to the sytem of equations

$$x+y+\frac{x}{y}=19$$

$$\frac{x\,(x+y)}{y}=60$$

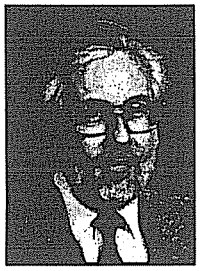
8. Find the number of different divisors of 10800.

9. Show that n^4-n^2 is divisible by 12 for any positive integer $n>1$.

10. Two clocks now indicate the correct time. One gains a second every hour and the other gains 3 seconds every 2 hours. In how many days will both clocks again indicate the correct time?

How Many Ways?

ARTHUR B. WENK



Arthur Wenk teaches OAC mathematics courses at St. Andrew's College in Aurora, Ontario. In addition to classroom games, he has invented campus-wide treasure hunts, "The Finite Quest" and "The Finite Challenge," in preparation for final examinations.

NOTE: Donna Bird Ericksen, Martha L. Frank, and Ryan Kelley, in inventing the game of WITPO (Erickson, Frank and Kelley,The Mathematics Teacher, April, 1991), have provided not only a good deal of fun for their students, but also a source of ideas for other teachers. Mike Zichowski has worked out a variation of the game for team play (Zichowski, Ontario Mathematics Gazette, December, 1993). The present article applies the principles of the game to combinatorics.

Combinatorics, comprising permutations, combinations and a certain amount of set theory, does not generally require sophisticated algebraic skills. Rather, the obstacle many students encounter in this subject consists of interpreting problems and identifying appropriate paradigms for solving them. Calculating P(n,r) or C(n,r) or n! presents no particular difficulty, particularly with the availability of calculators that perform these functions directly. Recognizing which formula to use, on the other hand, may prove a troublesome task for many students. As with problem-solving generally, one improves by doing lots of different kinds of problems and learning to recognize the relatively small number of models underlying the apparently large number of problem descriptions.

"How Many Ways?" is a game designed to present students with a variety of problems in combinatorics. Students who might balk at working fifty exercises on a homework assignment happily go through that many questions in the context of a competitive game. The form of the game follows that of WITPO.

The game requires a die, a playing board, playing tokens (my students use the ends of mechanical pencils, coins, bent paper clips, etc.), and a set of 3" x 5" question cards. (The playing board and suggested questions and answers are reproduced with this article.) To prepare the game, mount the playing board on a piece of bristol board or heavy cardboard (it helps to enlarge the board, if you have a copying machine that does this), and tape or glue the questions and answers to opposite sides of the index cards. I find it helpful to use different coloured cards for each copy of the game, to avoid problems of sorting them out should the cards ever be dropped. Teachers may wish to add their own questions to represent particular topics.

Students should be divided into two-person teams. (They may compete individually if you prefer, seated around the playing boards.) The roll of a die determines which team plays first. A player from the team to the left selects a card from the shuffled deck of questions and reads it aloud. The players on the answering team confer, then give their response. If they answer correctly, they roll the die, advance their token the indicated number of squares on the board, and carry out any instructions printed on the destination space. A team making a correct response continues to play until they give an incorrect answer. The turn then passes to the next team on the left. The first team to reach, or overshoot, the Finish square wins. Used question cards are placed on the bottom of the stack.

Students should be encouraged to explain their correct responses, should any player not understand how they were obtained. In the case of an incorrect response, an opponent should explain how to obtain the correct answer. Teachers should walk around while the game is in progress and be prepared to explain questions that stump all the players at a particular table.

My classes play the game after spending a class period reviewing two chapters on combinatorics and before taking a test on this material. Working in two-person teams permits students who are uncertain about the details of a particular problem to receive immediate help from one of their peers, either a partner or an opponent, if both members of the team need help. I find that this informal tutoring helps the "teacher" as much as the learner.

References

Erickson, Bird, Donna, Frank, Martha L., and Kelly, Ryan, WITPO (**W**hat **I**s **t**he **P**robability **O**f)., *Mathematics Teacher,* April, 1991, 258-264.

Zichowski, Mike, The Revised WITPO Game, *Ontario Mathematics Gazette,* December, 1993, 16-21.

START	☛	go back 1 space		Go ahead 3 spaces
				Return to START, and role die again
End of turn	Go back 3 spaces		☛	Role Die again
Exchange tokens with right-hand opponent				
☛	Go ahead 1 space		Role die again	
				Role die again: if even, go ahead; if odd, go back
End of turn		Go back 3 spaces		☛
Exchange tokens with left-hand opponent				
☛	Go ahead 1 space		Go back 3 spaces	**FINISH**

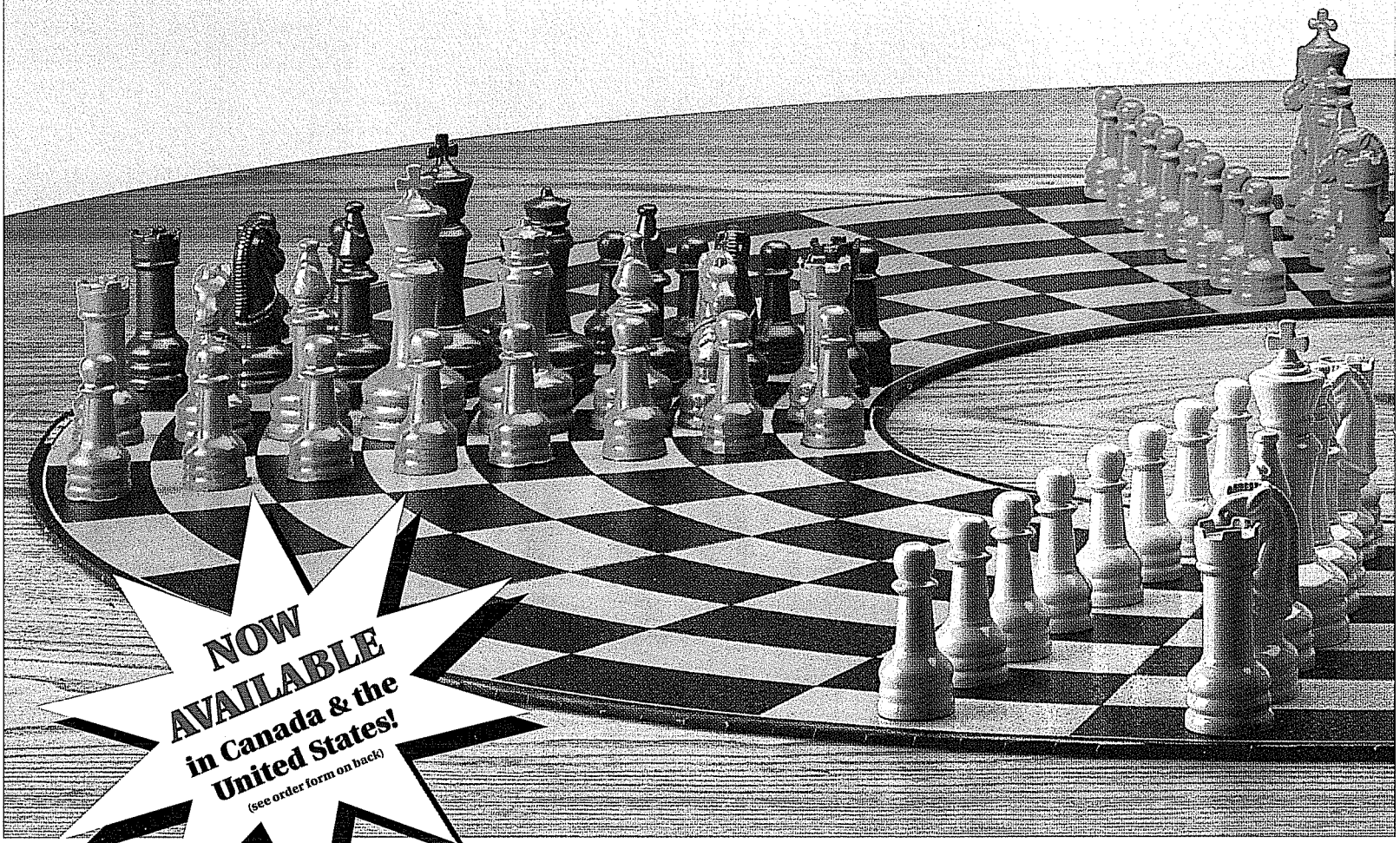
How many ways are there for Jack to dress in shirt, tie and slacks if he has 5 shirts, 10 ties, and 6 pairs of slacks?	5 x 10 x 6 = 300
How many ways are there to select a card that is either a black face card or a red face?	6 + 2 = 8
How many ways are there to arrange the letters of the word COMPUTER? Use factorial notation.	8!
How many ways are there for a math club with ten members to select a president and a vice-president?	10 x 9 = 90
How many ways are there to arrange the letters of the word EQUATOR if the letter A must be in the middle position? Use factorial notation.	6!

How many ways are there to berth six boats side-by-side if the two catamarans must be together?	5! x 2 = 240
How many ways are there to arrange the letters of the word MISSISSIPPI? Use factorial notation.	$\frac{11!}{4!4!2!}$
How many ways are there for a football team of eleven players to form a circular huddle (use factorial notation)?	10!
How many ways are there to form three-digit numbers using only the numbers 1 to 8 if the number 5 must be included?	$8^3 - 7^3 = 169$
How many ways are there to string 10 pearls on a strand of nylon if a knot is to be placed in the strand when all the pearls have been strung?	$\frac{9!}{2} = 181\,400$


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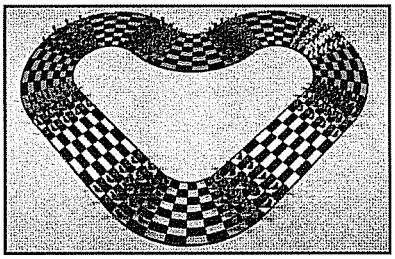
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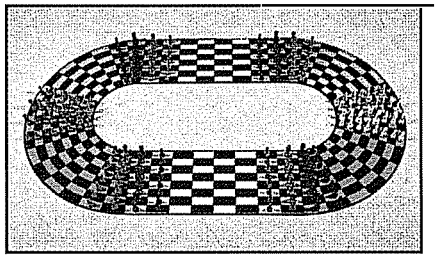


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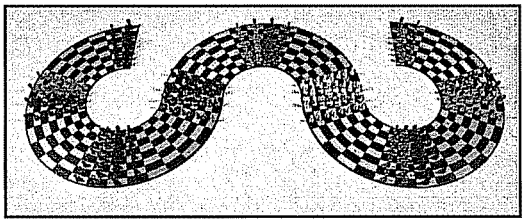
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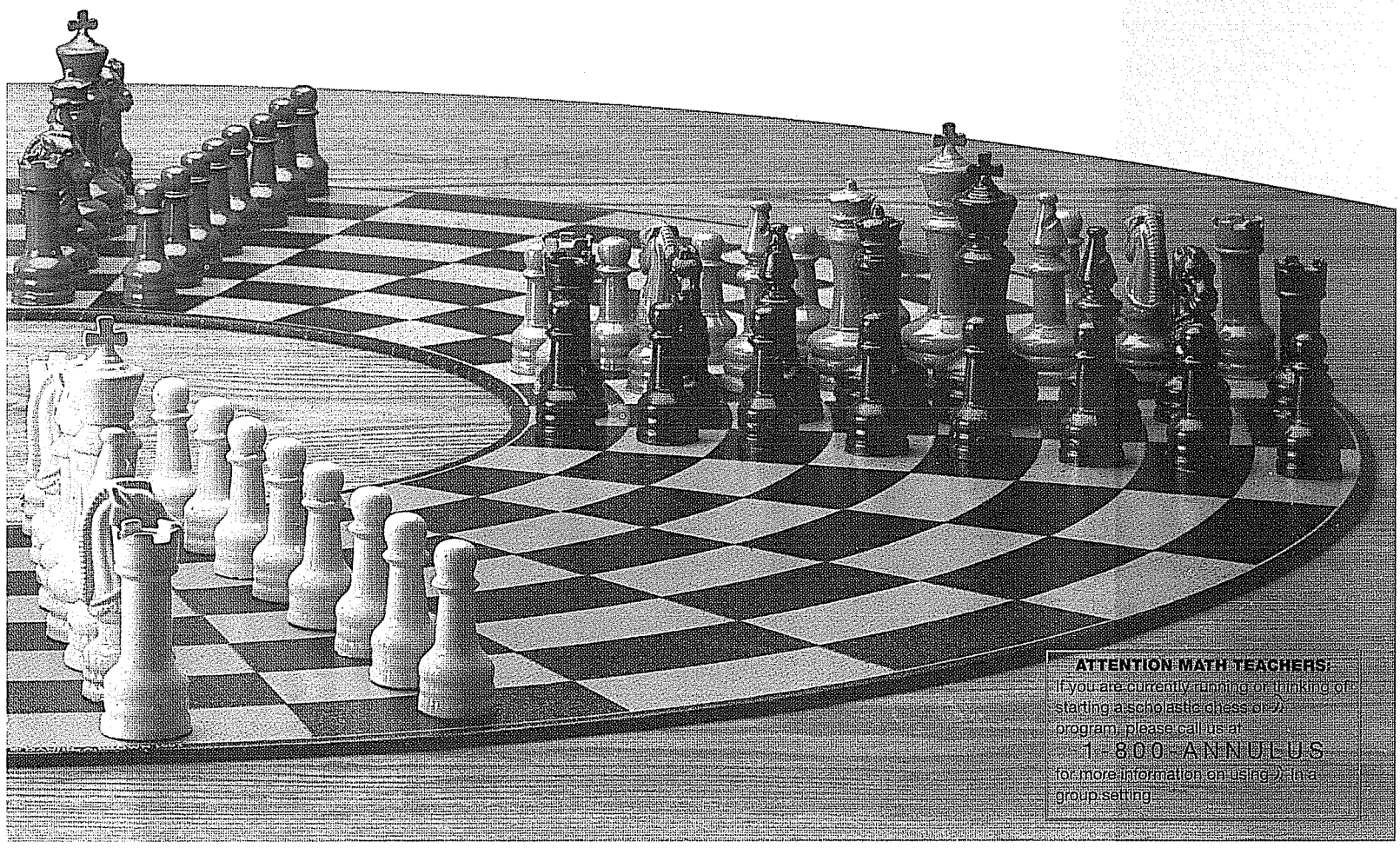
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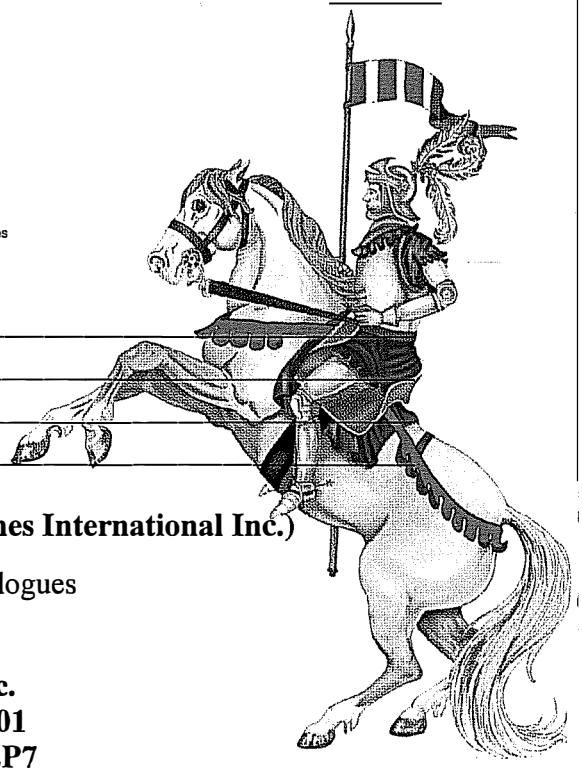
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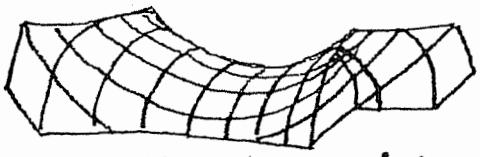
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How many different cars can be ordered given two choices of engine size, four choices of exterior colour and three choices of upholstery?	$2 \times 4 \times 3 = 24$
How many ways are there to answer a multiple choice test consisting of 20 questions and 4 choices for each question? Use exponential notation.	4^{20}
How many ways are there for a member of parliament to visit each of the 15 towns in her riding once? Use factorial notation.	$15!$
How many ways are there for a school to form a five-person mathematics team from a class of seventy students? Use bracket notation.	$\binom{70}{5}$
How many MayFest Committees of five students can be chosen from a school of 450 students if one particular student must be on the committee? Use bracket notation.	$\binom{449}{4}$
How Many poker hands of 5 cards can be dealt from a deck of 52 cards so that each hand contains 2 spades and 3 hearts? Use bracket notation.	$\binom{13}{2}\binom{13}{3}$ $=22\ 308$
How many ways are there to arrange the letters of the word COMMITTEE? Use factorial notation.	$\frac{9!}{2!2!2!}$
How many subsets of size 5 from the universal set {1,2,...,8} contain both 1 and 2? (6 3) = 20	$\binom{6}{3} = 20$
How many ways are there to form seven-digit telephone numbers if the first two digits cannot be one or zero?	$8^2 \times 10^5$ $= 6\ 400\ 000$
How many ways are there for a club of six members to choose three-person committees?	$\binom{6}{3} = 20$

How many six-digit Canadian postal codes can be formed with alternating letters and numbers (e.g., L4G 3H7) if the codes must begin with a letter?	$26^3 \times 10^3$ $= 17\ 576\ 000$
How many ways are there to form a five-member Concert Committee from a class of thirty students? Use bracket notation.	$\binom{30}{5}$ $= 142\ 506$
How many ways are there to choose three books from a shelf containing seven different books? Use bracket notation.	$\binom{7}{3} = 35$
How many ways are there to deal a poker hand of five cards from a deck of 52 cards? Use bracket notation.	$\binom{52}{5}$ $= 2\ 598\ 960$
How many ways are there to form a study group of two boys and two girls from a class of 8 boys and 7 girls?	$\binom{8}{2}\binom{7}{2}$ $= 588$
How many ways are there to deal a poker hand of five cards with two acres and three queens?	$\binom{4}{2}\binom{4}{3}$ $= 24$
How many ways are there to take marbles out of a bowl containing a red marble, a green marble, a clear marble, a striped marble and a dotted marble?	$2^5 - 1 = 31$
How many subsets are there for a set with five elements?	$2^5 = 32$
How many ways are there to choose a delegation of at least one member from a group of six representatives?	$2^6 - 1 = 63$
How many divisors does 18 have, other than 18, if the prime factorization of 18 is 2 x 3 x 3?	$(2)(3)-1 = 5$

LOOK, symbolic manipulation!
solve $ax+bx-c=0$
result $x=\frac{c}{b+a}$

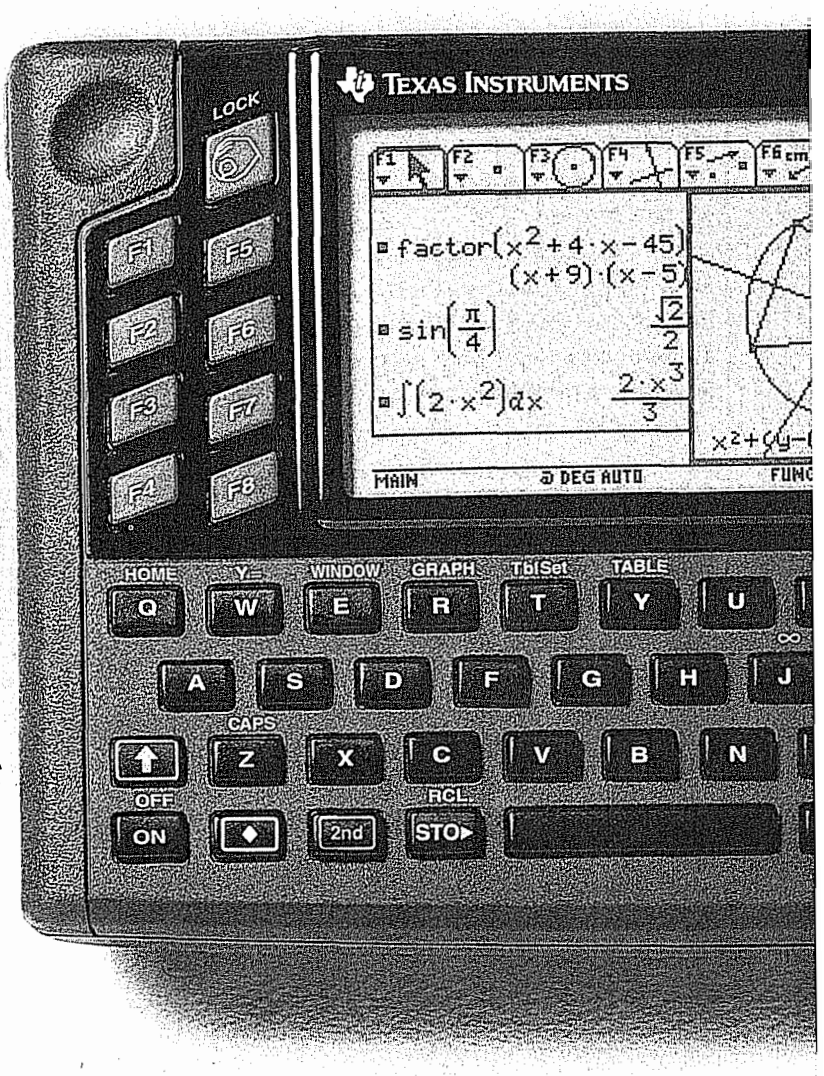
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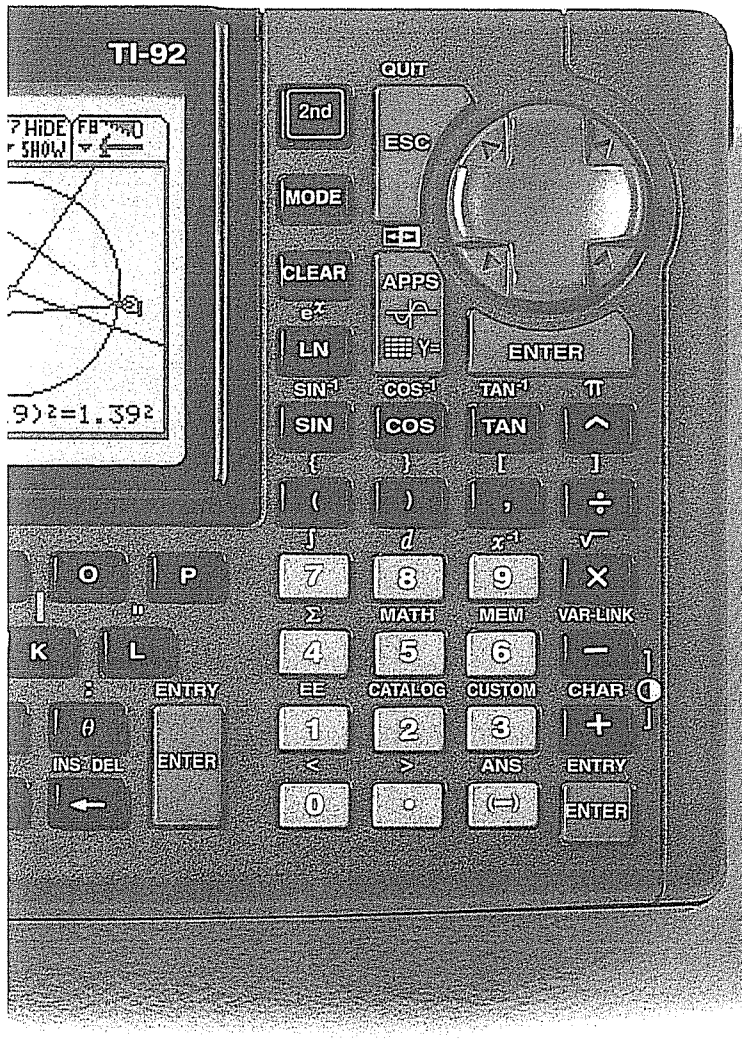
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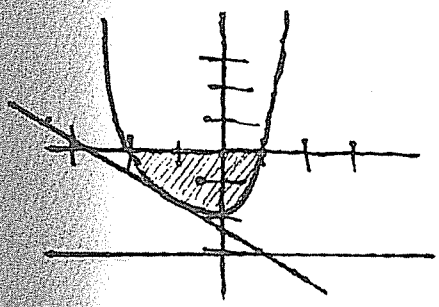
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TEXAS INSTRUMENTS

How many ways are there for a spade flush to be drawn in poker? A spade flush is a hand of five spades.	$\binom{13}{5} = 1287$
How many ways are there for a flush to be drawn in poker? A flush is five cards from the same suit.	$4 \times \binom{13}{5} = 5148$
How many ways are there for a quartet (soprano, alto, tenor, bass) to be selected from a chorus of six sopranos, five altos, three tenors and three basses?	$\binom{6}{1}\binom{5}{1}\binom{3}{1}\binom{3}{1} = 270$
How many ways are there to form subsets from the set {red, orange, yellow, green, blue, indigo, violet}	$2^7 = 128$
How many factors does 9000 have, other than 9000, if the prime factorization of 9000 is $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$?	$(4)(3)(4) - 1 = 47$
How many ways are there to choose at least one nut from a bowl containing seven filberts, six cashews, five pecans, four pistachios, three almonds, two macadamia nuts and a peanut? Use factorial notation.	$8! - 1 = 40\,319$
How many ways are there for twelve teenagers to ride in three cars, four people to a car? Use bracket notation.	$\binom{12}{4}\binom{8}{4}\binom{4}{4} = 34\,650$
How many ways are there for twelve teenagers to ride in three cars, four people to a car, if Beth and Sean ride in the same car? Use bracket notation.	$\binom{10}{2}\binom{8}{4}\binom{4}{4} = 3150$
How many ways are there for King Arthur and his twelve knights to sit at the Round Table? Use factorial notation.	$12!$
How many ways are there for King Arthur and his twelve knights to sit at the Round Table if King Arthur himself sits in the seat at the north?	$12!$

How many ways are there to select marbles from a bag containing two red marbles, three green marbles and four striped marbles?	$3 \times 4 \times 5 - 1 = 59$
How many ways are there to choose two-person teams from a class of ten students?	$\frac{10!}{2^5} = 113\,400$
How many ways are there to award gold, silver and bronze medals to 14 contestants in the 100-metre dash?	$14 \times 13 \times 12 = 2184$
How many handshakes take place at a meeting of ten business people if all of them shake hands with each other?	$\binom{10}{2} = 45$
How many three-digit positive integers are there?	$10^3 - 10^2 = 900$
How many ways are there to arrange four inmates and four guards in a line, if inmates and guards must alternate? Use factorial notation.	$4! \times 4! \times 2$
How many ways are there to arrange the letters of the word PLECTRUM if the two vowels must be together? Use factorial notation.	$7! \times 2$
How many ways are there to build a 3-scoop ice cream sundae if there are 31 flavours of ice cream to choose from and each scoop can be topped with marshmallow, whipped cream, butterscotch sauce of hot fudge?	$31^3 \times 4^3 = 1\,906\,624$
How many ways are there to arrange the letters of the word QUEUES if the consonants must be at the ends? Use factorial notation.	$\left(\frac{4!}{2!2!}\right) \times 2 = 12$
How many ways are there for a full house of three aces and two kings to be drawn in poker?	$\binom{4}{3}\binom{4}{2} = 24$

A Handy Trick for Producing Matrices with Simple Inverses

W. G. S. HINES
MATHEMATICS AND STATISTICS
UNIVERSITY OF GUELPH



Gord Hines is a Professor of Mathematics and Statistics at the University of Guelph. His research interests include exploring the interplay between genetics and game theory in models of biological populations.

In preparing exercises in matrix algebra for students, it is sometimes desirable to produce integers-only matrices whose inverses are also integer-only. A few simple standard results from matrix theory readily lead to a procedure for generating such matrices, which have the added advantage of looking arbitrarily complicated or simple, as you wish. A simple modification produces matrices whose inverses involve only integers and some fractions with pre-selected denominators. The unmodified method also can be used to produce 'nice' sets of simultaneous linear equations for elementary work, and to produce matrices with an important property in statistical work - 'positive definiteness'.

1. The basic method

i) If symmetry is not required, the basic method is simple and yet completely general:

1) Take a (square) matrix of the desired size, with ones down the diagonal , i.e., the identity matrix.

2) Add any (integer) multiple of any row (or column) of the matrix to any other row, (or column) repeating as often as desired.

For example, start with a 3 x 3 identity matrix I:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and add +3 times the first row to the second and then -2 times the first row to the third to obtain an invertible matrix, M_1

$$M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Continue, say by adding -1 times the second row to the first and +1 times the second row to the third to obtain a new, slightly more complicated invertible matrix, M_2:

$$M_2 = \begin{pmatrix} -2 & -1 & 0 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{or} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}.$$

Finally, add 2 times the third row to the first and 1 times the third row to the second to produce another matrix M_3:

$$M_3 = \begin{pmatrix} 0 & 1 & 2 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

which is

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -2 & -1 & 0 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

or, substituting in from above for the second matrix, M_2,

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

as you might want to verify. Notice that each of the matrices in the product has all entries integers and has a determinant of +1.

The inverse of M_3 is

$$M_3^{-1} = \begin{pmatrix} 1 & 1 & -3 \\ -3 & -2 & 8 \\ 2 & 1 & -4 \end{pmatrix}$$

with the desired property that all of its entries are whole numbers, as are those of M_1 and M_2.

(You might want to check that the matrix claimed to be an inverse of M_3 does have the property that

$$M_3 M_3^{-1} = I = M_3^{-1} M_3$$

by carrying out one or both of those matrix multiplications.)

ii) If symmetry is required, only a slight modification is required.

1) Take a (square) matrix of the desired size, with ones

down the diagonal.

2) Add any (integer) multiple of any row of the matrix to any other row, and then add the same multiple of the column corresponding to the first of those two rows (the one multiplied and added to the other row) to the corresponding second column.

For example, again start with

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and add +3 times the first row to the second and then -2 times the first row to the third:

$$M_{1\ \text{intermediate}} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}.$$

Then add +3 times the first column to the second and then -2 times the first column to the third:

$$M_{1\ \text{sym}} = \begin{pmatrix} 1 & 3 & -2 \\ 3 & 10 & -6 \\ -2 & -6 & 5 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

As a next step, you might choose to add -1 times the second row to the first and +1 times the second row to the third, producing

$$M_{2\ \text{intermediate}} = \begin{pmatrix} -2 & -7 & 4 \\ 3 & 10 & -6 \\ 1 & 4 & -1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & -2 \\ 3 & 10 & -6 \\ -2 & -6 & 5 \end{pmatrix}.$$

Now add -1 times the second column to the first and +1 times the second column to the third

$$M_{2\ \text{sym}} = \begin{pmatrix} 5 & -7 & -3 \\ -7 & 10 & 4 \\ -3 & 4 & 3 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} -2 & -7 & 4 \\ 3 & 10 & -6 \\ 1 & 4 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Now add 2 times the third row to the first and 1 times the third row to the second

$$M_{3\ \text{intermediate}} = \begin{pmatrix} -1 & 1 & 3 \\ -10 & 14 & 7 \\ -3 & 4 & 3 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5 & -7 & -3 \\ -7 & 10 & 4 \\ -3 & 4 & 3 \end{pmatrix}$$

Finally add 2 times the third column to the first and 1 times the third column to the second, to give a symmetric matrix, $M_{3\ \text{sym}}$:

$$M_{3\ \text{sym}} = \begin{pmatrix} 5 & 4 & 3 \\ 4 & 21 & 7 \\ 3 & 7 & 3 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} -1 & 1 & 3 \\ -10 & 14 & 7 \\ -3 & 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}.$$

(There is a shortcut method of calculating symmetric matrices like $M_{3\ \text{sym}}$. Written out in full, M_3 can be expressed as

$$M_{3\ \text{sym}} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$$

Notice that the last matrix is the transpose of the first one, that the second last is the transpose of the second one, and that the fourth is the transpose of the third, and recall that the transpose of any matrix product, say AB or ABC, is the product of the transposes in reverse order:

$(A\,B)^T = B^T A^T$

and

$(A\,B\,C)^T = C^T B^T A^T$,

so that

$$M_{3\ \text{sym}} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\left[\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\right]^T$$

$$= M_3 M_3^T$$

$$= \begin{pmatrix} 0 & 1 & 2 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 4 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 4 & 3 \\ 4 & 21 & 7 \\ 3 & 7 & 3 \end{pmatrix}$$

In general, any matrix produced by the method of the previous subsection, such as M_3, when multiplied by its own transpose, will produce a symmetric matrix which has an inverse with integer-valued components. In fact, there is also a relationship between the inverses involved:

The inverse of $M_{3\ \text{sym}}$ is

$$M_3^{-1}{}_{\text{sym}} = \begin{pmatrix} 14 & 9 & -35 \\ 9 & 6 & -23 \\ -35 & -23 & 89 \end{pmatrix}$$

Since $M_{3\ \text{sym}} = M_3 M_3^T$,

$$M_{3\ \text{sym}}^{-1} = (M_3^{-1})^T (M_3^{-1})$$

$$= \begin{pmatrix} 1 & -3 & 2 \\ 1 & -2 & 1 \\ -3 & 8 & -4 \end{pmatrix}\begin{pmatrix} 1 & 1 & -3 \\ -3 & -2 & 8 \\ 2 & 1 & -4 \end{pmatrix}$$

iii) Technical note: Why does it work? We've used a few properties of matrices and their determinants and inverses. The relevant properties are:

The determinant of the initial (identity) matrix is +1.

Adding any (integer or other) multiple of any row (or column) to any other row (or column) does not change the determinant of a matrix.

The determinant of a matrix of integers is always an integer.

By Cramer's rule, the entries in the inverse of a matrix of integers are the ratios of various determinants of integer-only matrices to the determinant of the original matrix, and so are all integers if that determinant is +1 or -1.

The requirement that the determinant of the matrix used be +1 or -1 is not only sufficient to ensure that the inverse is also integer-only, it is also necessary. The determinant of the product of two matrices is the product of their individual determinants, so that the determinant of the product of two integer-only matrices is an integer, and so is greater than 1 in absolute value unless the two matrices each have determinants of +1 or -1. The determinant of the product of a matrix and its inverse is + 1, since that product is the identity matrix - which has determinant +1. Thus if both a matrix and its inverse are integer-only, the determinant of each must be ±1.

2. Some variations and applications

i) Because of the integers-only form of the matrices constructed above, we can be sure that any set of equations such as

$0X + 1Y + 2Z = 1234$

$4X + 2Y + 1Z = 56789$

$1X + 1Y + 1Z = -1995$

will have integer solutions for X, Y and Z, and will do so even when the right hand sides of these equations are changed to other integers.

ii) For variety, fractions can easily be introduced into the inverse produced by multiplying any given row (or column) of the chosen matrix by some integer: this has the effect of causing the corresponding column (or row) of the inverse to be divided by the same integer. For example, doubling the second row in M_3 to produce

$$M_{3\ \text{modified}} = \begin{pmatrix} 0 & 1 & 2 \\ 8 & 4 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

produces a matrix with inverse

$$M_{3\ \text{modified}}^{-1} = \begin{pmatrix} 1 & 1/2 & -3 \\ -3 & -1 & 8 \\ 2 & 1/2 & -4 \end{pmatrix}$$

Similarly, the matrices produced by the division of individual rows or columns of the original matrix by small integers will have the corresponding columns or row, respectively, of their inverses multiplied by the same integers.

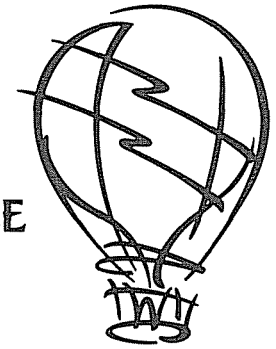
iii) Much of applied statistical analysis rests on the use of symmetric 'positive definite' matrices, such as some matrix A, say, with the property that for all possible vectors x, the product x^TAx must be positive if x differs from zero. This property is essential in practical problems as its absence can lead to absurdities during the analysis, and frequently to panic on the part of the student carrying out the analysis under examination conditions. The basic method for obtaining symmetric matrices outlined above in fact automatically produces positive definite matrices (with fairly manageable numbers) without the need to construct a large data set and to perform extensive calculations, or to calculate eigenvalues for some arbitrarily constructed and hopefully positive definite matrix. To confirm the positive definiteness, it's enough to note that the symmetric matrix can always be written as the product of a matrix and its transpose, say $A = BB^T$, so that $x^T A x = (x^T B)(B^T x) = y^T y$, where $y = B^T x$. Since $y^T y$ is the sum of the squares of the entries in y, this term is positive unless all entries in y are zero. Since A has an inverse, $x = A^{-1} B y$, and so x is then also zero in all of its components if y is, and we have satisfied the requirements that A be positive definite.

OAME / AOEM

Annual Conference

Congrès Annuel

TOUT est POSSIBLE



the SKY'S the LIMIT

London Convention Centre
May 9 - 11, 1996
Hosted by W.O.M.A.

ASSESSMENT COLUMN

JENNIFER WEINER

The assessment tool for this issue is an OAC Algebra and Geometry final examination, used in an Ontario high school last year.)

PLEASE NOTE: THIS EXAM WAS NOT PREPARED SPECIFICALLY FOR THIS COLUMN. IT IS NOT AN OAME CREATION, AND WE ARE NOT ADVOCATING FOR OR AGAINST ITS USE AS A MEANS OF ASSESSING STUDENTS. IT IS SIMPLY AN EXAMPLE OF WHAT'S "OUT THERE".

You are invited to comment on any item in this column (or anywhere else in the *Gazette,* for that matter) in the form of a LETTER TO THE EDITOR. Letters could either be addressed directly to me, or sent to Jack. I am interested in your comments and suggestions, and as usual, in your submissions. Submissions can be sent to: Jennifer Weiner, c/o Dept. of Mathematics & Statistics, University of Guelph, Guelph, ON, N1G 2W1.

INSTRUCTIONS:

1. Calculators are necessary.
2. Full marks for full solutions.
3. The exam is in two parts: Part A short answer, Part B full solutions are required.
4. The exam is out of 100 marks.
5. Answer all questions in Part A in the spaces provided, and in Part B, on the paper provided.
6. ESL students may use a dictionary for translation.

PART A: SHORT ANSWER
[30 marks]

Answer in the spaces provided. Each question is worth 2 marks.

1. Given the points A(4,3,5) and B(6,2,7) state a vector
   (a) parallel but not equal to $\overrightarrow{AB}$ 1.(a)_____
   (b) perpendicular to $\overrightarrow{AB}$. 1.(b)_____

2. Given $\vec{a} = (4,5,7)$ and $\vec{b} = (5,-1,2)$ find $\vec{n}$, where $\vec{n}$ is an unit vector in the direction of $\vec{a} \times \vec{b}$. 2._____

3. If $\vec{a}$ and $\vec{b}$ are unit vectors and the angle between them is 60°, determine $|2\vec{a} \cdot \vec{b}|$. 3._____

4. What is the fifth term, t_5, in the expansion of
   $$\left(x^2 + \frac{3}{x}\right)^8.$$ 4._____

5. Given the matrix $\begin{pmatrix} 4 & 5 \\ 6 & 8 \end{pmatrix}$
   (a) What is the image of $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$? (a)_____
   (b) State A^{-1}. (b)_____
   (c) Find the point that A maps to $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$. (c)_____

6. Describe the geometric effect of each of the following transformations:
   (a) $\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ 6.(a)_____ 6.(b)_____

7. The following reduced augmented matrix represents a system of equations of three planes:
   $$\begin{pmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & 5 & 11 \\ 0 & 0 & k^2-9 & k-3 \end{pmatrix}$$
   For which value(s) of k will this system have
   (a) no solution? 7.(a)_____
   (b) an infinite number of solutions? 7.(b)_____
   (c) an unique solution? 7.(c)_____

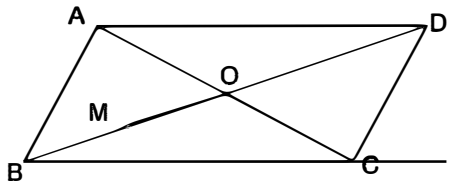
8. Given $z_1 = 4 + 3i$ and $z_2 = 7 - 6i$
   Determine: a) $\frac{z_1}{z_2}$ b) $z_1\bar{z}_2$ 8.(a)_____ 8.(b)_____

END OF PART A

PART B: FULL SOLUTIONS ARE REQUIRED
[70 marks]

1. ABCD is a parallelogram with the diagonals AC and DB intersecting at O.

   If M is the midpoint of $\overrightarrow{BO}$ with $\overrightarrow{MB} = \vec{a}$ and $\overrightarrow{BA} = \vec{b}$ then in terms of $\vec{a}$ and $\vec{b}$, find



   (i) $\overrightarrow{BD}$ (ii) $\overrightarrow{AO}$ (iii) $\overrightarrow{CM}$ [5 marks]

2. If A(4,3,2), B(5,-2,3), and C(7,-5,5) are the vertices of ΔABC, find:

   (a) the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$;

   (b) the measure of angle PQR, in degrees, correct to 1 decimal place;

   (c) the area of ΔABC. [7 marks]

3. Grace is cruising in her new sleek jet at an air speed of 490 km/h and heading in the direction, N 60° E. If the wind is from the north at 92 km/h, determine her ground speed and the actual direction of travel. [5 marks]

4. If A divides $\overrightarrow{XY}$ in the ratio 4:7 and B divides $\overrightarrow{AY}$ in the ratio -6:5

   (a) in what ratio does X divide $\overrightarrow{BY}$?

   (b) If O is any point not on the line through A and B, express $\overrightarrow{OY}$ as a linear combination of $\overrightarrow{OX}$ and $\overrightarrow{OB}$. [5 marks]

6. (i) Explain why $3x + 2y - 4z + 1 = 0$ and $6x + 4y - 8z - 6 = 0$ are equations of 2 parallel planes.

   (ii) What is the distance between them? [5 marks]

7. It is time for a break so write down the time and then place a pen/pencil on the desk pointing to your right and another pen/pencil beside it but pointing away from you. Now extend your left arm in the direction of the cross product of the first pen with the second.

   When someone comes say, "Give me a break, please." [1 mark]

   (Ed. note: The student was given an individual bite size sample of a "Kit Kat" chocolate bar.)

8. (a) Determine the Cartesian equation of the plane containing the points, A(1,1,1), B(3,-2,1), and C(-1,8,3).

   (b) Determine the intersection of the line, $r = (1,-1,3) + t(2,1,5), t \in \mathbf{R}$, and the plane in (a). [7 marks]

9. Find the intersection, if any, of the planes given by: $2x - 3y - 6z = 3$

   $x + y - z = 8$

   $x - y + 2z = 9$. [5 marks]

10. Prove by mathematical induction:

$$\sum_{k=1}^{n} (3k^2 + k) = n(n+1)^2 \quad \forall n \in \mathbf{N}.$$

    [6 marks]

11. (a) Express $\left(1 + i\sqrt{3}\right)^{10}$ in the form a + bi.

    (b) Solve $z^5 - 32 = 0$ [9 marks]

12. For the conic given by $5x^2 + 2\sqrt{3}xy + 3y^2 = 36$

    (a) Name the conic.

    (b) Determine an angle of rotation that will eliminate the xy term from the given equation.

    (c) Applying the rotation found in (b), determine the image equation.

    (d) Sketch the given curve. [11 marks]

13. Given that $\vec{a}$, $\vec{b}$, and $\vec{c}$ are linearly independent vectors find x, y and z if $(x-4)\vec{a} = (x+2y)\vec{b} + (3x-y+z)\vec{c}$ [4 marks]

TOTAL MARKS 100

CALL FOR AWARD NOMINATIONS

LIFE MEMBERSHIP

To qualify for Life Membership in OAME/AOEM a candidate shall have:

- 10 or more accumulated years of membership in OAME/AOEM
- demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular

Deadline for nominations: January 31, 1996.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award, to be presented annually to a person who does not necessarily meet the criteria for the Life Membership Award (i.e., not necessarily an educator or an OAME/AOEM member.) The award is aimed at recognizing people who have supported OAME/AOEM, such as publishers, educational suppliers, hotel personnel, etc.

At this time OAME/AOEM is requesting nominations for consideration for this year's award, to be presented at the Annual Conference.

Deadline for nominations: January 31, 1996.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM has established the Morley MacGregor Award, to be presented annually to an elementary school teacher of mathematics who has demonstrated excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME/AOEM is requesting nominations for consideration for this year's award, to be presented at the Annual Conference.

Deadline for nominations: January 31, 1996.

KENNETH D. FRYER AWARD

In Memory of the late Dr. K.D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, has established the Kenneth D. Fryer Award for Excellence, to be presented annually to a secondary school mathematics department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME is requesting nominations for consideration by the committee for this year's award - to be presented at the Annual Conference.

Deadline for nominations: January 31, 1996.

Don Attridge Award

In recognition of the contributions made by Don Attridge, a founding member, former president and a previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the Spring of 1995. It will be presented this year for the first time. The award will be presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

Deadline for nominations: January 31, 1996.

AWARD NOMINATION FORM

Date of Nomination: _____

Award (Check One) ❑ **Life Membership** ❑ **Mona MacGregor Award** ❑ **Morley MacGregor Award**
❑ **Kenneth D. Fryer Award** ❑ **Don Attridge Award**

I/We nominate (Name of Nominee) _____

School, Organization or Business of Nominee _____

School, Organization, or Business Address of Nominee _____

Principal of Nominee (for the Kenneth D. Fryer Award) _____

Mathematics Department Head of Nominee (for the Kenneth D. Fryer Award) _____

Nominated by (Name, Address & Phone) _____

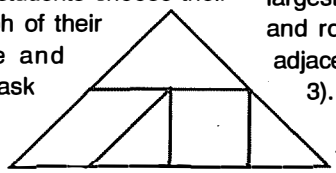
Please attach a separate sheet to the completed nomination form, outlining the reasons for your nomination and send to
David Zimmer, John Fraser Secondary School

▲ Tangrams - A Universal Teaching Tool*

BONNIE V. WHITLEY
MATHEMATICS CHAIR
WILLIAM BYRD HIGH SCHOOL
RANOAKE COUNTY, VIRGINIA

Bonnie has been teaching mathematics for twenty-five years. She loves the challenge of bringing fresh activites to enliven her classes.

Tangrams are a valuable universal teaching and learning tool because they can be used across the K-12 mathematics curriculum. In kindergarten or first grade read the story of the Chinese emperor whose broken mirror formed the original seven tangram pieces. Next let the K-1 students choose their favorite shape and make a live bar graph of their choices. Tape the triangle, square and parallelogram shapes to the board and ask students to stand under their choice.



In other elementary mathematics classrooms students could investigate which shapes can be formed from just two pieces of the tangram. Can you form a square, triangle, rectangle, parallelogram, and trapezoid from just two small right triangles? Try to form all five polygons from just three tangram pieces.

Tangrams can go home to become a family mathematics project. See the article printed from "IDEAS", Marea W. Channel, Arithmetic Teacher, Vol. 41, No. 3, November, 1993.

In elementary or middle school classes some teachers use the tangram pieces to compare areas. For example, how does the area of one small right triangle compare to the area of one large right triangle? See Andrey Dunkel's article, "Spatial Sense", in The Arithmetic Teacher, February, 1990, p. 38-42.

Tangrams can be used to extend students' creativity skills by asking them to create tangrams of their own. See an explanation of a tangranimal world in "From the File of", by Shelley Day, The Arithmetic Teacher, April 1988, p. 51.

At the high school level, I use tangrams to strengthen students' visual perception. First by working with the actual shapes, students must visualize a whole geometric shape made from the seven individual pieces-a "parts to whole" process. Then drawing the tangrams forces students to fine tune the accuracy and details inside the whole figure. Students begin with the whole figure and then draw in the individual parts-a "whole to parts" process. Looking at the students' drawings of tangrams tells me if students need help with the drawing process.

To begin the tangram project, I give the students the worksheet (figure 1) and have them cut out the pieces of the tangram. I explain that all seven pieces must be used to form the five polygons-square, rectangle, triangle, trapezoid, and parallelogram. I encourage the students to try to form the square. Usually someone in the class forms the square without any help. If so, I let that person show the class how they made the square by putting the tangram pieces on the overhead projector. Then I have students draw the outline of the square and insert the interior smaller parts.

At this point, I explain the criteria for grading. I stress that the first five tangrams required must all be drawn on the same sheet of typing paper (as stated in #2 of figure 1.) I show students the diagrams in figure 2 with the outlines drawn but without the interior pieces. I do not want students to trace the shapes.

If no one is able to form the square after five or ten minutes, or before the class is over, I give the students the hint I want them to use to solve the tangram. All five shapes can be formed by moving the two large triangles and leaving the five smaller pieces intact as in figures 3-6.

To form all five shapes place the five smaller figures in this triangular shape and keep them there. Now move only the two largest triangles to form the rectangle from the square. Slide and rotate the two large triangles so that the hypoteni are adjacent to two consecutive sides of the former square (figure 3).

Next, to form the parallelogram from the rectangle, slide the large left triangle to the right of the rectangle and line it up with the other large triangle so that two of their legs are touching (figure 4).

To form the right triangle from the parallelogram move the right most triangle and match the legs at the top of the former parallelogram (figure 5).

To make the trapezoid from the parallelogram pick up the right large triangle and flip it over (figure 6).

Over the years several students have found alternate ways to place the five smaller polygons that will work with all the shapes except the square (figure 7). I am always pleased with variations that students find on their own such as the ones in figure 7 by Nadia Duertler. Maybe as students become more acquainted with tangrams in other grades, finding various ways of forming the five polygons from a base figure and writing descriptions of how to form them will become the focus in high school.

In the last requirement of the project, I want to remind students of the creative process. Many students believe they are not creative. By giving students a procedure to follow (figure 8), they are less anxious and more willing to try to be creative. Students will move the tangram pieces into some arrangement, add details and a title to make the arrangement more recognizable. I have included some examples of students' work.

Cont'd on page 34

*Reprinted from the Virginia Mathematics Teacher, Volume 22, Number 1, Fall, 1995.

▲ THE ONTARIO MATHEMATICS OLYMPICS '95



JERI LUNNEY
CARELTON-OTTAWA
MATHEMATICS ASSOCIATION

Imagine if you can, 120 girls and boys from across Ontario travelling to Ottawa to do mathematics together on a Saturday. These grade seven and eight students were at Lisgar Collegiate in Ottawa, participating in the first Ontario Mathematics Olympics, June 3, 1995. It was a wonderful day with twenty-nine teams from all across Ontario competing in eight Olympic events. The teams travelled form Colbalt, Kingston, London, Mississauga, North Bay, North York, Ottawa, Pickering, Picton, Sarnia, Scarborough, Sudbury, Thornhill, Toronto, Unionville, Whitby and Windsor. Most teams were winners in regional events held earlier this year. Not even rain could dampen the spirits of everyone involved.

Until now, the students of Ontario have had access only to paper and pencil mathematics contests, in which students competed as individuals. The Ontario Mathematics Olympics provides a different experience for students, offering them the chance to work in teams towards a common goal. It is critical that students have the opportunity to:

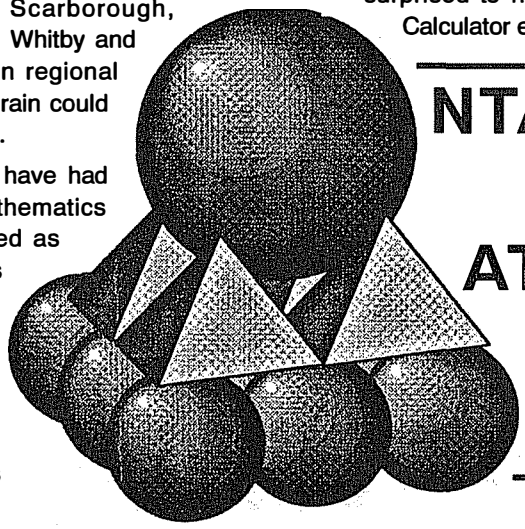
- become effective problem solvers
- work with others on problems
- be able to tackle new problems with enthusiasm and understanding
- believe in the usefulness and value of mathematics
- enjoy doing mathematics

The teams were each composed of four students: two girls and two boys, of whom two were grade seven and two were grade eight students. Students engaged in a series of hands-on problem solving activities intended to test the creative and cooperative aspects of their mathematics abilities. The Olympic events were challenging, provocative and fun for all, requiring teamwork and decision-making.

Accompanied by parents, teachers and siblings, the competitors arrived at 8 o'clock for breakfast. Over 200 people were fed by the staff at Lisgar cafeteria. The Olympic events opened at breakfast with a placemat activity and other warm-up activities. The prizes for the placemat activity were Carl and Otta, the teddy bear Olympic mascots. After the breakfast activities, the students, clad in their Olympic t-shirts, were taken by student helpers to their first event.

The teams of four had been broken into pairs for the morning events and rotated through the five fifteen-minute activities: Colour Logic, Puzzling Perspectives, Match It and Tangram Teasers. After a pizza lunch, the four-person teams were reunited to participate in three thirty-minute activities: The Puzzle -3D Model, Tower of Power and 4-Function Calculator. Parents and teacher supervisors were able to enter the classrooms and see their students in action. The student helpers kept the events running smoothly and on time.

Three celebrity teams were invited to participate in the last event in the afternoon and then stay for the awards ceremony. The twelve local celebrities, all of whom were sponsors of the Ontario Mathematics Olympics, included people from Addison-Wesley, Bell Northern Research, the Carleton Board of Education and the University of Ottawa. Those of you who know Ann Jones, Assistant Director of Education for the Ottawa Board of Education and a former OAME President, will not be surprised to hear that her team won the 4-Function Calculator event!



NTARIO

ATHEMATICS

LYMPICS

At 2:05 pm, students, parents and teacher supervisors met again in the cafeteria for drinks, snacks and music while the scores were finalized. All proceeded to the auditorium for the awards presentations. By 4 pm, the teams were saying good-bye, some back to their hotels, others on their way home. Many students, teachers and parents took the opportunity to see the sights in their capital city.

The Ontario Mathematics Olympics 1995 Steering Committee thanks all those who helped in any way to make the day a success. Without the commitment of chapter representatives, the corporate sponsors, teacher supervisors, school administrators, board coordinators and consultants, other school board personnel, parents and especially the students, this day would not have taken place. Let's ensure that the Ontario Mathematics Olympics is an annual event.

Steering Committee

Ron Gaudreau Colleen Irvin Jeri Lunney Rob McLeish
Mitch Phomin Steve Snell Tom Solc

Announcing the Winners!

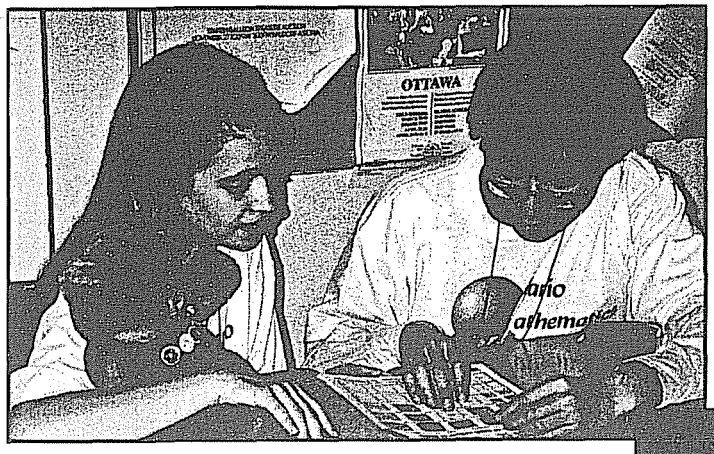
First Place Team:
Scarborough All-Stars #1, Scarborough (SAME)
Cally Cheung, David Pritchard, Nicolas Lee, Peggy Wong

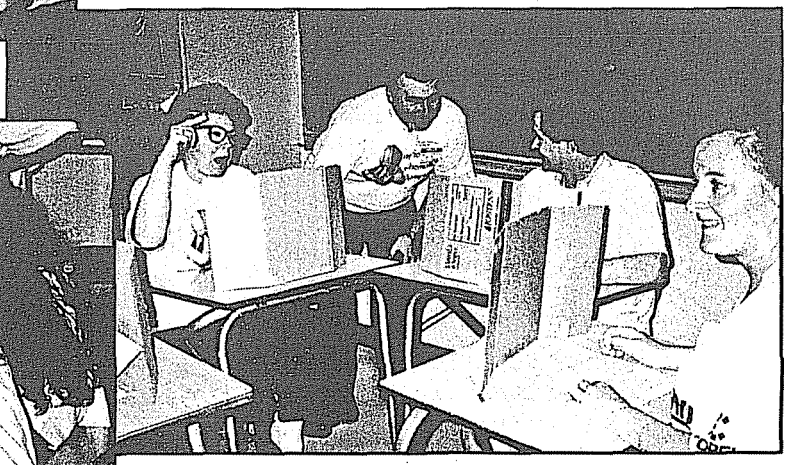
Second Place Team:
Upper Canada College #2, Toronto (ISOMA)
Heather Dunlop, Jackie Baker, Ashiq Aziz, Amyas Au

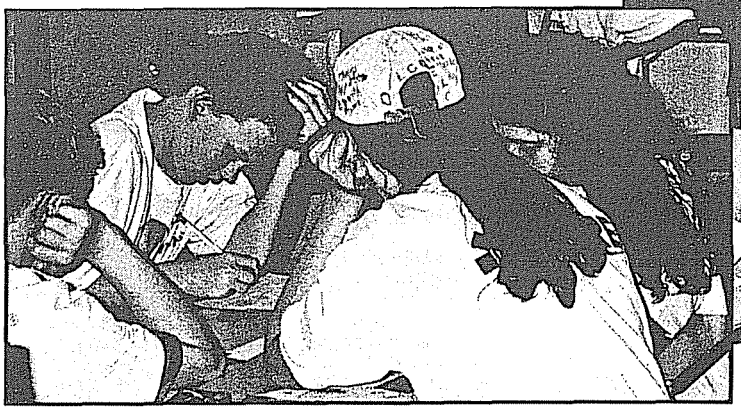
Third Place Team:
St. Patrick's Intermediate School, Ottawa (COMA)
Colin Hayman, Aldus Fong, Aisha Seid, Karen Splinter

Individual Event Winners

Event	Chapter/School	Names
1) Colour Logic	NOMA Dr. Carruthers P.S. #1	Jeff Bell, McLean Edwards
2) Puzzling Perspectives	COMA St. Patrick's Intermediate	Aisha Seid, Karen Splinter
3) Match It	QSLMA Western Area Immersion Unit	Jennifer Valberg, Laura Beckwith
4) Coordinate Chaos	SAME Scarborough All Stars #1	Cally Cheung, David Pritchard
5) Tangram Teasers	COMA St. Patrick's Intermediate	Colin Hayman, Aldus Fong
6) The Puzzle: 3-D Model	NOMA Dr. Carruthers P.S. #2	David Alldred, Tiiu Vail, Heulwen Rankin, David Jackowski
7) Tower of Power	SAME Scarborough All Stars #1	Chally Cheung, Nicolas Lee, David Pritchard, Peggy Wong
8) 4-Function Calculator	TEY^{2}MS TEY^{2}MS All Stars #1	Laura Wagner, Curtis Ching, Jonathan Wu, Jennifer Kwok











▲ COUNCIL COMMENTS



PETER SAARIMAKI
COORDINATOR OF MATHEMATICS
TORONTO BOARD OF EDUCATION

These notes were taken at the OAME Council meeting held Saturday June 17, 1995. Comments about Council and its activities should be directed to the President. Comments about my comments can be sent to the editor.

General

Susan Stuart extended a warm welcome to new, former, and returning members to Council at this, the beginning of our Council year. She expressed a special greeting to Bill Morrison from MAC^2, our newest chapter.

OAME/95 conference

Preliminary reports from GVMA repeated their appreciation for the opportunity to host the conference this year at Guelph. Even with some glitches in services, registration went smoothly and teachers took advantage of a wide range of excellent sessions. The conference reached its goal of over 700 paid registrants with more than 150 speakers and workshop presenters also attending sessions. A lot of hard work by a dedicated group. Congratulations to all concerned in GVMA

OAME/96 conference

Kaye Appelby reported that London plans are in full swing for next spring, including hot air balloon rides, chapter banners, and special, low hotel rates. Have you marked the dates in your calendar yet? It promises to be yet another wonderful opportunity for professional development, networking, and just good times. May 9, 10 and 11, 1996. See you there!

OAME/97 conference

Council approved the possibility of varying the dates of the annual conference to allow a request from TEY^{2}MS to host the 1997 conference in Toronto. With the shortened time-line, hotel space is limited in April and May so that Council was asked to allow the planners some flexibility of dates.

Conference Planner

Lorna Morrow and Jan Scully have prepared a comprehensive guideline for annual conference planning. It was gratefully received by several groups planning or considering hosting future annual conferences.

OMO - The First Annual Ontario Student Math Olympics

The Carleton-Ottawa people are to be congratulated on a successfully run olympics for grade 7 and 8 students. The real evidence of success has been the enthusiastic students returning to their home chapters. Special thanks were extended to Rob McLeish and Jeri Lunney (a past president of OAME) for their hard work and the work of their committees.

HOT FLASH!! - there are reports of a major company wanting to fund the 1996 OMO, which SAME has offered to host. Be sure your chapter has plans to select and prepare teams to compete. Maybe YOU will become INVOLVED!!

Mission Statement and By-Laws

Susan Stuart spoke of continuing OAME's continuing pro-active role in mathematics education, especially in light of provincial initiatives to look at the Specialization Years and compulsory testing. She is following up on contacts with Universities, CAATs, business and industry.

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As well, certain changes were proposed for some of the OAME by-laws and award descriptions, but you'll have to ask your chapter rep for more information, since I don't think most people care to hear about it here.

Resources

John Kersley has volunteered to gather interesting materials for Council members and distributed a varied collection right off the bat. These will be shared with chapters by the reps and can then be passed on to others as the demand arises.

COMMITTEES

Early/Formative Years

Ron Sauer reported this always eager, industrious group still plans to contribute articles and activities to the *Abacus*. Also this year, they will be providing support to the Junior assessment project.

Transition Years

The *Gazette* will be the beneficiary of committee plans to produce a series of articles to demonstrate methods to integrate mathematics and provide connections to other subjects.

Specialization Years

Todd Romiens will chair this vital committee as the province starts to review and revise the program and curriculum expectations for grades 10 to graduation, which now seems to be after grade 12. The committee includes councillors from the secondary and post-secondary panels.

Special Projects

As a continuation of the soon-to-be-released *Linking Assessment and Instruction in Mathematics for the Transition Years,* plans are already underway to create a junior level resource package. Watch for more details in the fall.

The aforementioned Transition Years resource is scheduled for release this fall. GET ONE for yourself and you'll breathe a sigh of relief when you discover how easy it is to provide interesting activities for your students ... AND ... also be able to assess and evaluate students using the NCTM and the Provincial Standards.

CHAPTERS

One regret I have is that most of the wonderful conferences planned by chapters for the fall will be over by the time you read this issue. Each chapter puts on some marvellous professional development. Keep in touch with your local chapter to be aware of coming events.

CHAMP

Mathfest95 is planned for Thursday, November 2nd and will include two workshop opportunities, dinner, publishers' displays and networking nodes. On Nov. 15 and 16 students in grades 7 and 8 will write CHAMP's math contests in preparation for the OMO.

COMA

While hosting a regional and the provincial OMO, this group still produced a newsletter and ran some forums aimed at specific audiences.

GVMA

Dave Davidson is retiring as chair of GVMA. Dave was very proud to receive an OAME Life Membership award. Fall conference plans were tentatively set for November 2 or 9.

ISOMA

The fall conference is planned for Wednesday, October 25 at Royal St. Georges College, Toronto, with a stellar line-up of presenters.

NOMA

The fall conference planned for Sept. 22 and 23 in Sudbury. Speakers include Dave Zimmer, Ron Lancaster, Don Fraser, Dr. M. Wadlinger and Trevor Brown.

NWOAME

The Math, Science and Technology conference is scheduled for Oct. 19 - 21. Coming in December and February are Dinnerludes.

SAME

A full slate of officers was elected at the AGM in May, when those present were addressed by Rollit Goldring, Program Department Superintendent, Scarborough Board, and Lynda Colgan, math coordinator (and one of the most famous students of a little known commentator).

SWOAME

A dinnerlude, with some MEALMATH thrown in, is planned here too this fall. The chapter is planning to sponsor four teachers to attend the Leadership Conference.

TEY^{2}MS

The first newsletter came out in June. It described the boards included in this chapter, congratulated the Student Olympic teams, published some of the local cooperative problems for teachers to try, and introduced the executive. (Apart from the usual president, vice-president, etc., one position is described as the Reality Checker - sounds like the meetings can be out of this world!!)

Plans are well under way for the third annual fall conference on Thursday, October 26th, *Math: The Shape of Things to Come,* with keynoter Chester Makischuk.

Y^{4}MA

The fall conference is planned for Tuesday, October 26th. Plans are already underway to hold three competitions this year to select the student teams for the 1996 OMO.

Did you know?

- that ordering NCTM materials through our rep, Mickey Sandblom, doesn't cost you any more than directly, BUT makes money for OAME?
- that the Ontario Curriculum Clearinghouse catalogue is in all schools and lets you find out about all sorts of wonderful resources from around the province?
- that the oldest known mathematical puzzle is about cats? (Does this mean they've always been a problem?)
- that you can buy copies of the OAME Certificate of Merit for your students at a very reasonable price and make them feel honoured at the same time?

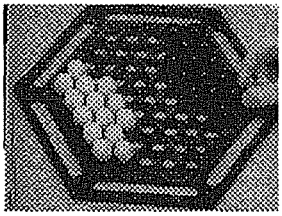
You may send your comments to me at petesaar@village.ca.

ABALONE GAMES CANADA LTD.

abalone

MATH GAMES FOR THE CLASSROOM!

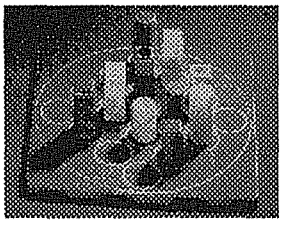
ABALONE



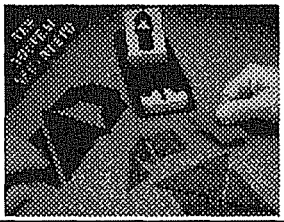
Abalone is a purely strategic game that has become a classic in Canadian Schools. Two players attempt to be the first to eject six of the opponent's marbles off the hexagonal board in a mental duel that requires concentration, observation, planning and wits. Winner of the MENSA Select Seal and Best Bet by the Canadian Toy Testing Council (CTTC). For ages 8 to adult.

QUARTO!

1995 Best Bet by the CTTC, MENSA Select. The objective is to form a straight line with 4 pieces with at least one attribute in common, (a Quarto!) Each piece features 4 attributes: height (tall or short) color (dark or light) shape (round or square) and density (solid or hollow). But beware! It is the opponent who choses the piece you will play. A formidable strategic challenge. For ages 8 to adult.



TANGOES



The classic Chinese tangram converted into a creative excercise for two players or teams. Players attempt to replicate the shape on a card using all seven pieces, before the opposing player or team. Great for pattern recognition, proportions, geometry and problem solving.

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Tangrams - A Universal Teaching Tool

Cont'd from page 29

Many commercial books and a commercial tangram puzzle are now available. However, the inexpensive seven paper tangram pieces are an effective teaching tool to increase spatial sense, perspective in drawing, and creativity.

Tangram Project

1. (Two points each) Use all seven tangram pieces to form each of the following: square, rectangle, parallelogram, trapezoid, right triangle.
2. (One point each) Draw all five shapes, listed above, on one sheet of typing paper.
3. (Ten points) On a piece of typing or construction paper create a tangram picture of your own. Be sure to include a title, add details, color the tangram, and put your name on the back.

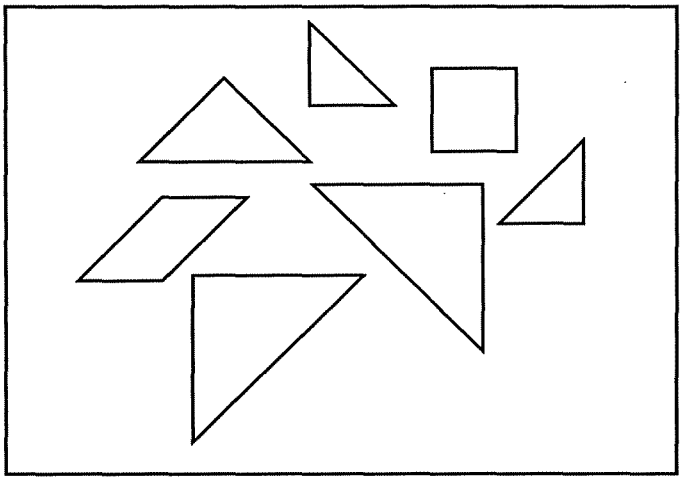


Figure 1

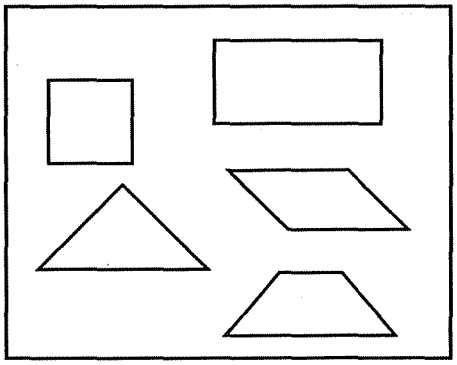


Figure 2

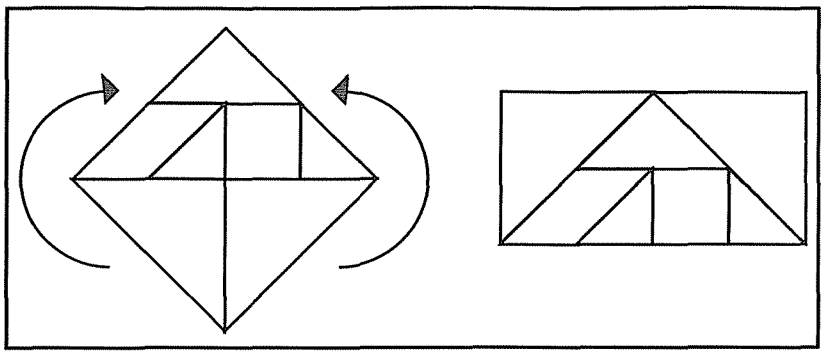


Figure 3

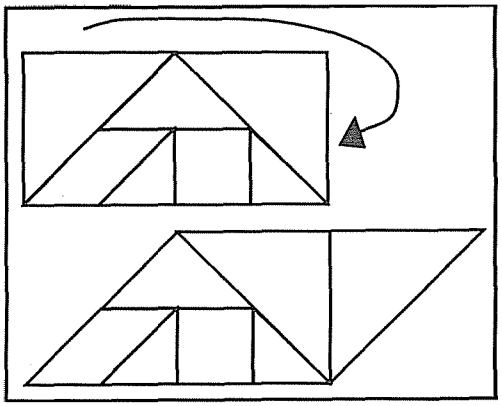


Figure 4

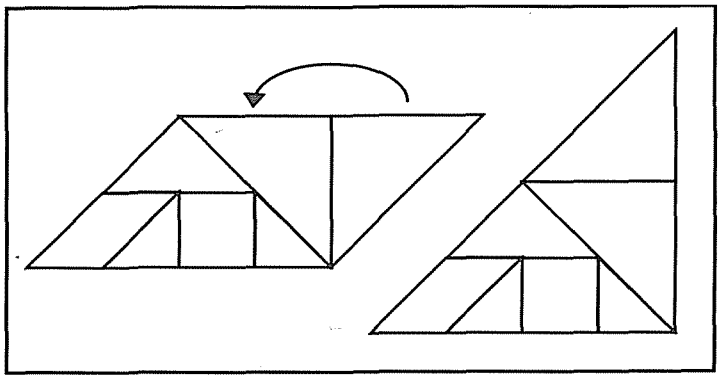


Figure 5

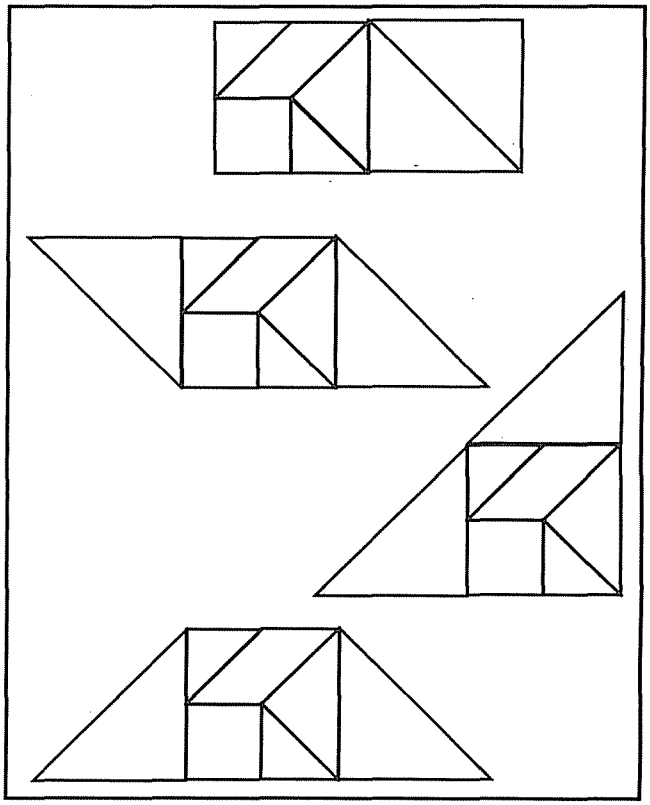


Figure 7

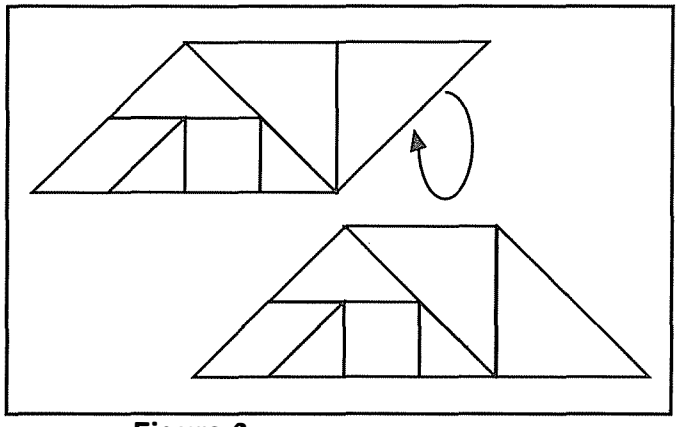


Figure 6

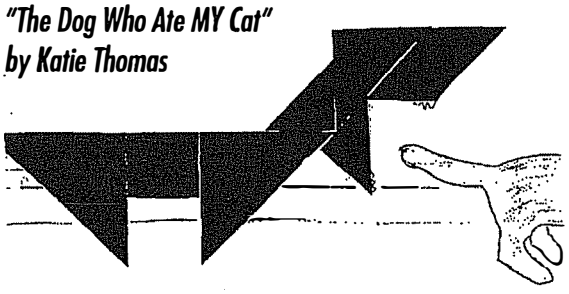
Developmental Skills Institute

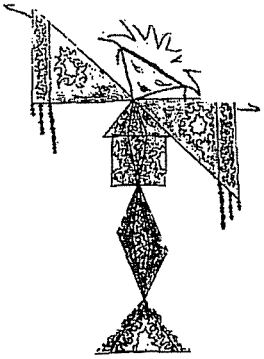
	GOAL	PLAN →				
		Model · · · · · · · · · · *Purpose* · · · · · · · · · · *Synthesis* · · · · · · · · · · · · · *Symbol*				
T A S K	**Create a symbol.**	**Name the thing.**	**State what you want to communicate.**	**Find information on the thing.**	**Transform the thing with the information.**	**Title the new thing.**
		Step 1	**Step 2**	**Step 3**	**Step 4**	**Step 5**

Figure 8

Tangram Creations

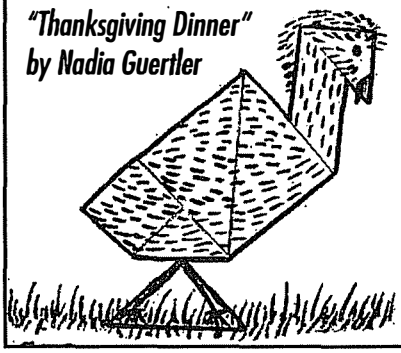
"The Dog Who Ate MY Cat"
by Katie Thomas

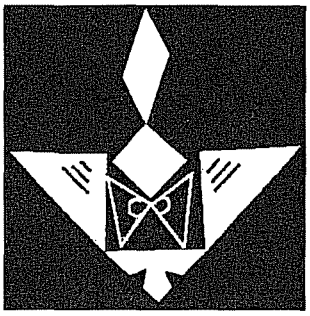




"The Lady"
by Shelly Bruner

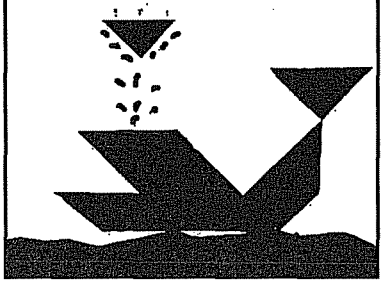
"Thanksgiving Dinner"
by Nadia Guertler



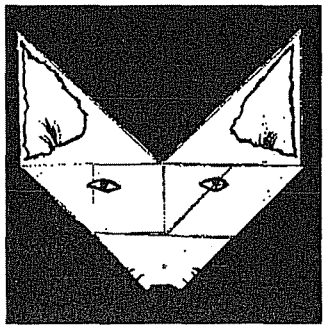


"The Stingray"
By Jason Dickerson

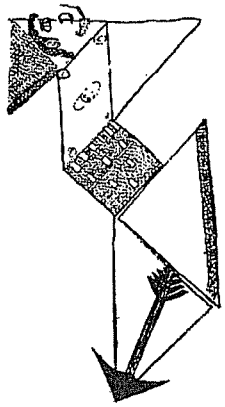
"The Whale"
by Julie Vaden



"Artic Fox"
by Scott Stephenson



"A Mad Man's Dream"
by Lyndon Arbogast



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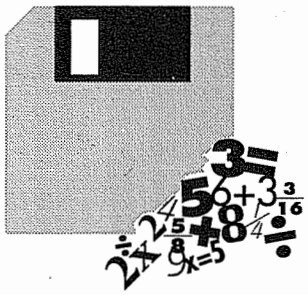


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