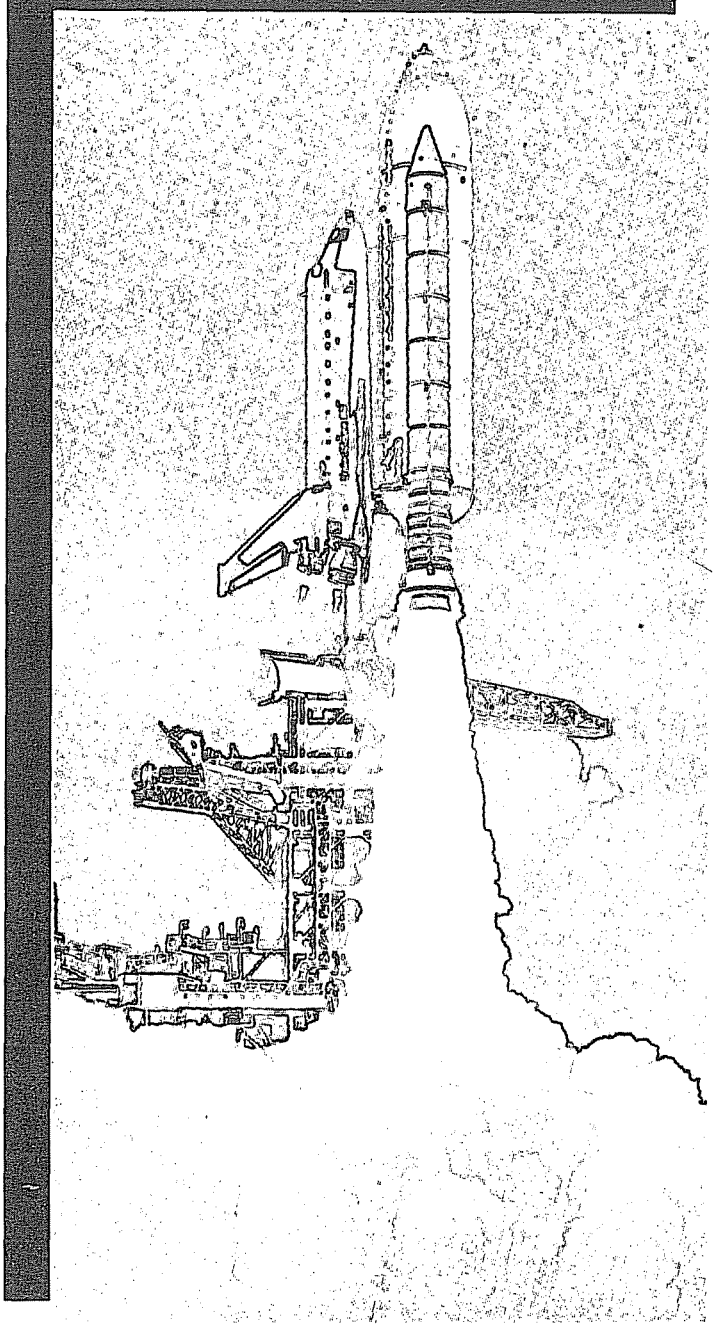
Vol. 34 #2, December 1995

Ontario Mathematics Gazette



Math and the Shuttle Launch - The Ontario Connection

(see page 31)

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

▲ Editorial



We live in interesting times.

For Ontario educators in particular, we live in dangerous times. Farewell to OAC's. Farewell to hard fought student-teacher ratios. Farewell to several hundred million dollars.

I refuse to be a harbinger only of doom and gloom. I think we can fare well! At least we can make the best with the resources at hand. I asked Todd Romiens to address this in a guest editorial. I asked Ron Scoins and Gary Flewelling to consider the future of an OAC-free curriculum in a two part Have Your Say.

I am not going to let this stuff get me down. The fall was my R. and D. semester here at U of G. As a result, my teaching batteries are recharged and I'm raring to go come January. Happy holidays!

Jack



Guest Editorial

TODD ROMIENS
MATHEMATICS CONSULTANT
WINDSOR BOARD OF EDUCATION

Send in the Clowns.

Well it seems to me we are in that position with the Ministry. Just when teachers are starting to get on board with many of the exciting educational changes we know are good for kids, the Ministry is cutting back on funding and putting boards in a position where support for new initiatives will be virtually impossible. There is little money for supplies, resumes and resource personnel. Also morale is being undermined with the elimination of OAC courses which could result in the loss of many teachers, the threat of more teaching time per teacher, more kids per class and a cut in salary. Toss in the fact that we have no curriculum defined by the Ministry for grade 7-9 and the curriculum for the specialization years of the future is a mystery.

Thank goodness for OAME! We are an organization that in conjunction with OMCA has been guiding the math community with the standards document, Focus on Renewal and the latest Assessment material. We are continuing to be the beacon that is looked to for directions in mathematics education. We must continue to be proactive and show the way. We must also continue to have dominant input to Ministry decisions on curriculum and pedagogical issues. Keep up the good work! ▲

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▲ President's Message

SUSAN STUART



Welcome to the December edition of the Gazette and Abacus. It is hard to believe that December has rolled around so quickly. It seems only last week that we were preparing for a new school year.

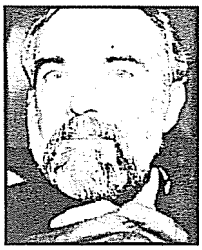
The fall term has given mathematics teachers in Ontario much to ponder. Last week at a meeting I overheard a Ministry of Education officer sarcastically describe the Minister of Education as a true politician who "manages his department through the media." Those interested and concerned about education - teachers, administrators, parents, Ministry officers - have learned through the newspapers and radio announcements that the OAC's will be removed from the secondary program and that provincial mathematics assessment will take place in grade 3 and grade 11 classrooms. The media discussion that followed these announcements emphasized the lack in specificity of Mr. Snobelen's plans. He stated that there would be no changes to the secondary school curriculum, yet declared that students would be ready for any and all college or university courses. Some parent groups supported the change, but said that they expected that the same content should be covered - just in earlier grades. Logic tells us that if you delete 25% of the Specialization Years advanced program, then major curriculum changes must occur. The ripple will be felt right back to elementary school classes. Jeri Lunney, an Ottawa teacher, recently pointed out to me that this year's grade 7 and grade 8's will graduate from secondary school the same year. They will be competing for college, university and job placements at the same time. How, asks Jeri, can we be assured that one group is not disadvantaged by the other? OAME is determined that any mathematics curriculum changes will be decided and developed by mathematics teachers in this province. The Specialization Years Committee—Todd Romiens (Chair), Ed Barbeau, Shirley Dalrymple, Doug Evans, Mary Howe, Peter Lenczewski, Ron Lewis, Rob McLeish, Geoffrey Roulet, Peter Saarimaki, Margaret Warren, and Rick White—has been involved in curriculum discussions for quite some time and will need to put their actions into high gear now that the announcement has been made. If you want your ideas to be considered, I encourage you to contact a Specialization Years Committee member (see the inside back cover of the *Gazette*) or log on to the OAME conference on the Village. Assessment has been a focus of the OAME Council. The Ministry of Education and Training has invited us, along with O.M.C.A., to attend a meeting to discuss the grade 3 provincial testing. We will ensure that OAME members are instrumental in its development and administration, and that the testing tools reflect both the tenor and the content of the Ontario Standard in Mathematics.

It was very disappointing to have cancelled the Leadership Conference. This conference has taken place for many years, and I think it is a sign of the financial times that many people were not able to attend. We apologize, as well, that the notices did not reach many of you until the last minute. The executive is exploring ways to make the future conference less expensive, in particular for cost of supply teachers. We hope that next year the Leadership will be alive and kicking!

On a brighter note, many exciting mathematics events are taking place with the help of OAME members around the province. The Junior Assessment material is well on its way thanks to the efforts of Nancy Moore (OAME) and Nancy Ianizzi (OAJE). The Transition Years Assessment Package is highly acclaimed and a great success with all those who have seen it. We are already going into a second printing. Congratulations to a hardworking group of teacher in Thunder Bay who put together a successful Math, Science and Technology conference called Rendezvous with the Future. It was enjoyed by many teachers. Thank you to Marilyn Hurrell who represented OAME on the planning committee.

On behalf of the OAME Council, soon to be known as the Board of Directors, I would like to wish you and your family a safe, happy and restful holiday. ▲

▲ Have Your Say 2001: An Educational Odyssey, Without OAC's



GARY FLEWELLING
INDEPENDENT MATHEMATICS CONSULTANT
ARTHUR, ONTARIO

I am not going to rail against a "done deal". The Ontario government intends doing away with Ontario Academic Credits (OAC's) in the year 2001. My intention here is to look at some of the ideas people are putting forward for coping with this OAC-free reality. Many of these ideas cause me considerable concern.

1) Amputation: The Simple Solution

Some think that the high schools should simply drop all OAC courses, save money, and leave all curriculum in Grades 10-12 unchanged. This would place no new demands on elementary or secondary students, teachers (provided they are not declared redundant) or budgets. It would be left to the universities and colleges to adjust to the new reality. My concern under this scenario is that senior mathematics would lose an opportunity for renewal and unrealistic demands would be placed on an increasingly impoverished tertiary education system.

2) More Than Trash Can Be Compacted: Let n be n-1

A lot of people talk about taking present 10-OAC course material and pushing it down into earlier grades. On CBC radio, for example, I heard a trustee suggest that we could compensate for loss of the OAC year by simply taking everything presently taught in grade n and teaching it in grade n-1. (I wonder why he didn't try for n-2, or more?) Such solutions ignore one small item, the learner.

3) There's Lots of Slack in the Transition Years

Many point to Grades 7 and 8, saying that much of what presently goes on in math in these grades is a wasteful repetition of junior work. They then use this observation to justify the idea of dumping a lot of early high school mathematics into these two grades. If this is true, I would suggest that the mathematics curriculum being implemented in those classrooms is not compatible with the mathematics curriculum described and intended in the Ministry's Common Curriculum, the Provincial Standards, or OAME/OMCA's Focus on Renewal. The mathematics curriculum described in those documents is comprehensive, demanding, relevant, and exciting. If anything, we should be doing fewer, not more, things in these grades and doing them better. (We should also remind ourselves that many things are in grade n because most students are not ready for them in grade n-c, $c \geq 1$.)

In a similar vein, many people are saying that destreaming has understretched and thus underprepared Grade 9 students entering Grade 10 mathematics at the advanced level. I can see this happening if most activity in grade 9 is targeted at the mid-range of students, if all students are expected to jump the same set of hurdles, or if all students are expected to do questions 1-21 on page 187 (and to do them in the same way). I can't see this happening if the learning activities are designed in such a way as to allow for a wide range of outcomes from a wide range of students. This is the kind of "rich" activity that you see in many other subject areas that are running successfully. This is the kind of activity that you see in a destreamed math classroom that is running successfully.

4) Repackage Four Years of Math Into Three Years

Many people think of all the things that are presently being taught in Grades 10-OAC mathematics and say that it is impossible to do it all in three years. They are probably right. But do we really want to teach it all in three years anyway? There are great gobs of math presently being taught that should not survive the next curriculum revision process. There are a great many things that should be replaced and many other things that should be approached differently. If numeracy is as important as the world says it is, then, like English, more than three courses with a math focus should be considered. For example, there might be a need for one course, at one or more levels of difficulty, in Grade 10 and one course in each semester of Grades 11 and 12. Some of these courses could be common. Other courses, for a variety of reasons, could be more specialized. My concern here is that an opportunity for curriculum renewal might lose out to a mere repackaging of, or tinkering with, our present decade-old curriculum.

5) Summary

The OAC's are going. I hope their demise will give us the opportunity to apply some new thinking to adjust to new realities and in the process renew and strengthen senior high school mathematics. ▲

▲ Have Your Say A Modest Proposal: Six-Sevenths Can Be Greater Than One!



RON SCOINS
UNIVERSITY OF WATERLOO

I support the Minister of Education's initiative to reduce the number of years of schooling required to earn a Secondary School Graduation Diploma. Let me list the reasons.

The reorganization of the school program that started with the introduction of the OAC's and culminated with the elimination of the Honours Secondary School Graduation Diploma was intended to result in the elimination of the fifth year of high school. It didn't. As a consequence, and as outlined below, we have the current situation in which far too many students are making unwise academic choices. The result is that they are not as well prepared for studies at college or university as they should be.

Based on the experience of teaching first year university students for over twenty years and senior high students for the 13 prior years, of conducting problem solving workshops with senior high school students throughout the past 21 years, of being an undergraduate academic advisor, and from close contact with teachers through the liaison program of the Faculty of Mathematics, I will share some of my observations.

The number of credits required for completion of the honour graduation year has gone from 9 to 8 to 7 to 6 over a twenty-five year period. Each time a reduction was made it was justified by arguing that the workload in each remaining course would be proportionally increased. It would allow students to gain a deeper understanding in areas of specialization. In my opinion, this has not occurred in the majority of schools. The result is that a large percentage of students are now only working at two-thirds the level of what is considered to be necessary for success at the tertiary level.

The change from a 27-credit SSGD followed by a 6-credit HSSGD to the present situation has resulted in a practice that many students take fewer courses spread over five years. It has also led to the widespread and misleading abuse called "upgrading". As a result, many students have a false sense of their capacity to learn a required body of material over a fixed period of time in a limited number of attempts.

Mathematics is not a subject that can easily be learned in a semestered environment. To gain a solid understanding of mathematical concepts requires a continuous exposure over a protracted period of time. Doubling the daily time of exposure reduces by one-half the number of evenings for this consolidation to occur. In a discipline that is cumulative in nature, this seems like an unwise strategy. It is also common for many students to have gaps of up to one year between math courses, to take prerequisite courses more than a year before attending university, and to be carrying only two courses in a semester. The most common problem experienced by first year university students, especially those from semestered schools, is coping with the workload. Over the past few years the normal first year course load in the Faculty of Mathematics has been reduced from 6 to the present 5 courses. But the workload is still a shock for many students.

There are many more observations that could be shared. Many of my friends and colleagues in the secondary schools are in agreement that changes need to be made. Change for the sake of change with no clear understanding of the consequences is certainly foolhardy but if changes are needed and from all indications are inevitable, we should seize upon the current situation as an opportunity to improve mathematics education in Ontario.

I believe a curriculum model that

- starts in grade 7,
- eliminates the duplication of the spiral approach,
- includes an accountability that ensures students have gained the prerequisite knowledge for entry to subsequent courses,
- has a syllabus that is manageable in the time allowed,
- emphasizes understanding as opposed to rote application of formulas or algorithms, and
- places problem solving as a central goal,

can result in students having a better foundation in mathematics than currently is the case. There already are schools in Ontario that have implemented a model that covers the content of grades 7 to OAC in six years.

As I stated at the outset, and for reasons including those outlined above, I believe mathematics education in Ontario can be improved. Members of the mathematics community must take a leadership role in school reform. It is incumbent on us to act with some urgency. Otherwise, we will miss the opportunity. ▲

values attributed to various occupations, traditional gender roles, educational attainment of males and females, female participation in mathematics, science and technology. At the same time, if other data sets concerning the labour force are made available, students can employ further analyses to develop supporting arguments for their positions.

The following applications and problems can all be approached in the manner just described. In each case the mathematical solution, when translated back into the terms of the original setting, invites debate and investigation of some social or values issue.

Group & Individual Differences

Some of the concerns expressed about research on racial differences appear to arise from confusion over the relationship of differences between group averages to individual differences. The problem in Figure 2 addresses this issue and can be used to introduce or develop the concept of standard deviation (MTB4G, MTT4G, MAT4A, MFNOAC).

Calculations of the means (Kalnu:180.02, Gurli:180.88) and medians (Kalnu:179.5, Gurli:182) show that in fact the group "averages" very slightly favour the Gurlis. The distribution of the data is such that more than half of the possible pairings will have a Kalnu representative with height equal to or greater than his Gurli opponent. The discovery, through simulation, of this apparent contradiction leads to a study of measures of deviation (standard deviations: Kalnu:10.41, Gurli:10.12) and the reliability of predicting individual differences from information about group averages. Students, when exploring this mathematical problem, regularly recognize its relation to spurious arguments about race and ethnicity.

The Environment

Mathematical models of biological and ecological processes can be employed to investigate environmental issues. (See Figure 3.) Again an initial focus on an application of mathematics can lead to discussions that raise serious issues and give students tools to better analyze conflicting views and information.

Population Comparisons

The villages of Kalnu and Gurli have been competing in various athletic events since 1729. This year the challenge is a wrestling match between randomly selected single representatives of each village. Mayor Hak of Gurli is confident of victory since a recent study by the eminent anthropologist, Dr. K. Smith, has shown that the men of Gurli are taller than those of Kalnu. He knows that when the wrestlers meet, the larger man from Gurli will win.

The data below give the heights, in cm, of all the men of Kalnu and Gurli.

Kalnu

175	180	171	194	194	164	171	178	173	187	187
182	184	187	189	171	172	179	179	198	170	189
195	165	167	178	179	175	189	188	183	185	185
188	189	174	176	177	177	193	202	198	159	166
180	183	186	175	188	160	157	170			

Gurli

195	176	184	184	188	186	182	174	193	169	165
170	190	172	170	175	178	185	187	162	190	185
183	166	170	163	198	173	173	174	182	184	188
201	187	187	188	164	193	191	177	177	201	178
176	177	178	174	200						

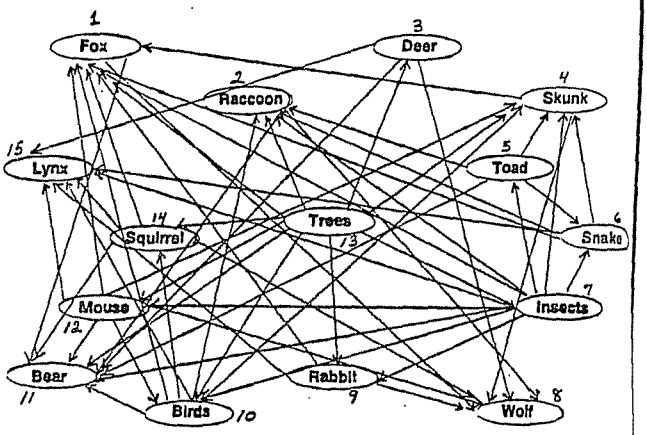
1. Is it correct to say that the men of Gurli are taller than those of Kalnu?
2. What measures might Dr. Smith have been using to make his claim concerning height?
3. Design a simulation for the selection of a wrestler to represent each village.
4. Use your simulation to check to see if Mayor Hak's confidence is justified. Simulate the selection of 10 different pairs of competitors. How often is the representative of Gurli taller?
5. Explain how the height data leads to Dr. Smith's statement and also generates your results in question number 4.

Figure 2.

Food Webs

Recent wet weather has caused a large increase in the mosquito population. People have been writing to the newspaper and phoning local politicians calling for a major spraying program. Environmentalists have been pointing out that any chemical spray that will eliminate the mosquitos will kill off all insects. Most people are not worried about this fact. Nobody really likes flies either. Should they be concerned? Investigate what effect the elimination of insects would have on the local animal population.

Food Web
Temperate Deciduous Forest



An animal or plant at the tail of an arrow is a food source for the animal at the head of the arrow.

1. How many links are there in the longest food chain?
2. In how many ways does the Raccoon rely on trees?
3. What happens to the food web if all the Insects are killed by chemical spraying?
4. What would happen if all the Skunks were trapped and removed from the forest?

Figure 3.

Taking The Next Step

EXPLORATIONS

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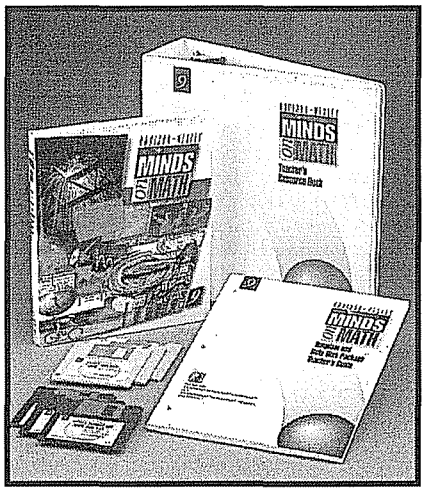
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The directed graph could be used to examine the results of eliminating insects, but a fuller investigation of the food web can be made by storing the information in a matrix (MFNOAC). If numbers are assigned to the points in the web (as above), then a matrix of 0's and 1's can represent the feeding habits of the animals. A 1 in position *i,j* (row,column) indicates that animal *i* eats animal *j*. Powers of the matrix may be determined using computer software such as *Matrices* from the NCTM (1988). The entries in different powers of the matrix record the number of multi-step food chains between pairs of animals. The original food web matrix with only 0's in row 7 and column 7 represents the web when insects have been eliminated. Powers of this new matrix show the altered chains and comparisons with the previous matrices indicate which animals would be effected. Such investigations emphasize the fragile nature of our environment and show that simple "solutions" to problems are not always best.

Mathematical Models

One of the problems associated with mathematical modelling is that too much faith is often placed in the resulting answers. The mathematical processes may be completely valid but the model may not fully represent the real world. The problem in Figure 4, developed for the OAC Finite Mathematics course (MFNOA), presents such a situation.

A Land Distribution Problem

In the country of Bahil there is a large essentially underdeveloped region called the high plains. This region of 500,000 km^2 now supports 150,000 members of the Uli tribe. The Ulis are a herding people who require approximately 2 km^2 per person to maintain their lifestyle. The Ulis now contribute to the Gross National Product of Bahil, about the equivalent of $5000 per year, for each of the 300,000 km^2 they occupy.

The land of the high plains could be much more productively employed raising cash crops such as wheat using modern farming methods. It is estimated that each km^2 could contribute $10,000 per year to the GNP. This development could be accomplished by allowing migration of skilled farmers from the rest of Bahil to the high plains at the rate of 5 migrants per km^2. Since the Ulis presently lack modern farming skills they could lose their means of support if enough land is converted to crop production. The government of Bahil is prepared to support the Ulis and this is projected to cost $8,000 per year for each Uli without land.

The government of Bahil has retained the social engineering firm, MFNOAC and Associates, to calculate the distribution of land that would contribute the greatest amount to Bahil's GNP. As the associate that has been assigned this task -

1. Find the land distribution that yields the greatest amount to the GNP.
2. Describe the new population mix in the region giving ethnic background and state of employment.

Figure 4.

The mathematical solution, obtained by a linear programming model, is that all the land presently occupied by the Ulis should be taken over for crop farming. Student objections to this answer produce the challenge to locate a mathematical error. This investigation leads to the observation that while the mathematical manipulations within the model may be correct it is very difficult (impossible) to construct mathematical models that incorporate moral positions. All solutions indicated by the application of mathematics should be tested against society's value systems before being acted upon.

Other Possibilities

Newspapers often provide opportunities for the exploration of mathematics along with social issues. This is especially true for Basic and General level classes. The front page of *The Toronto Star* for December 19, 1991 provided an excellent opportunity, with headlines announcing lay-offs at General Motors and the signing of Toronto Blue Jay's pitcher, Jack Morris, to a $10,800,000 contract. Division, ratio, and percent problems such as, "If Morris' contract was divided between the 2,650 workers at the closing Scarborough van plant, how much would each receive?" could provide arithmetic practice while raising serious questions about our collective priorities. The book, *Relearning Mathematics: A Different Third R - Radical Maths* (Frankenstein, 1989), contains many examples of this kind.

The body's reaction to alcohol and other drugs can be mathematically analyzed with a range of models (see Sandefur, 1992). Such topics support discussions of drug abuse and the significant problem of impaired driving. Similarly, models of the spread of infectious diseases can support efforts to inform our students about the dangers of AIDS and other sexually transmitted viruses.

Students may be assigned projects exploring correlations between statistics of a global nature, such as infant mortality rates and the availability of safe drinking water. Hudson (1990) gives examples of investigations of this type and Schwartz (1989), in *Mathematics and Global Survival*, presents problems involving ratio and proportion, percent, graphing, statistics and probability, all related to global social and environmental issues. Current data for such problems may be obtained from the annual publication, *World Military and Social Expenditures* (Sivard, 1993).

The thirteen general goals of education listed in the introductory pages of Ontario's curriculum guidelines make it clear that our task as teachers is more than instruction in subject content. Students are expected to develop: "a sense of personal responsibility in society", "esteem for the customs, cultures, and beliefs of a wide variety of societal groups", "respect for the environment and a commitment to the wise use of resources", and "values related to personal, ethical, or religious beliefs and to the common welfare of society" (Ontario Ministry of Education, 1985, p. 8). Mathematics education can play a part in the pursuit of these worthy goals.

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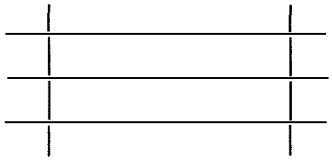
▲ PROPORTION TREES

JOHN BARTON
JOHN REED
CHARLES WYSZKOWSKI



John Barton, John Reed and Charles Wyszkowski have taught together at Port Perry High School for over twenty years. John Barton joined the staff in 1966 after graduating from R.M.C. and serving three years in the Navy. John Reed joined in 1974 as a graduate of McMaster. Charles Wyszkowski also joined in 1974 after working four years as an insurance adjuster upon graduation from U of T.

Proportion "trees" can be used at *any* level in any problem that *could* be solved through the setting up of a proportion ... problems involving ratios, percents, and rates as well as direct, inverse, joint and partial variation.



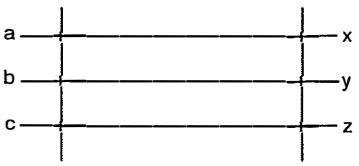
The "tree" itself has a minimum of two branches (no maximum except by type of situation).

The branches are labelled on the 'trunk' and must also show the units being used, where applicable.

For example, in working with ratios, no units need be shown as they are, by definition, the same in all parts of the question. However, in working with rates, the units must be shown.

Now, given a "tree", any two complete branches will provide a proportion.

So,



$$\frac{a}{b}\ \frac{x}{y},\ \frac{a}{c}\ \frac{x}{z},\ \frac{b}{c}\ \frac{y}{z}$$

And having the proportion should *usually* lead to only one

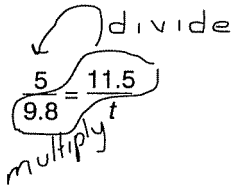
further line of calculations.

For example, given

$$\frac{5}{9.8} = \frac{11.5}{t} \; OR \; \frac{9.8}{5} = \frac{t}{11.5} \; OR \; \frac{t}{11.5} = \frac{9.8}{5}$$

then $t = 9.8 \times 11.5 \div 5$ (which is *one* calculator exercise).

i.e., you multiply the 2 diagonal "knowns" and divide by the third "known".



The position of the variable is *irrelevant.*

Ratio

Ratio - a comparison of quantities expressed in the *same* units.

For ratio problems the "tree" has 1 branch for "the whole thing" or "all of it together" and separate branches for each constituent part.

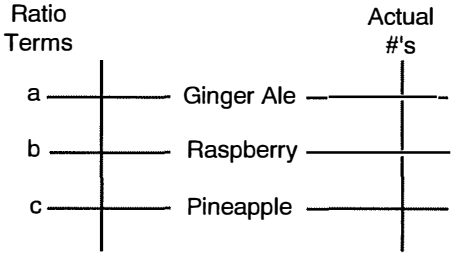
For example, a punch recipe calls for

7 L ginger ale
2 L raspberry juice
3 L pineapple juice

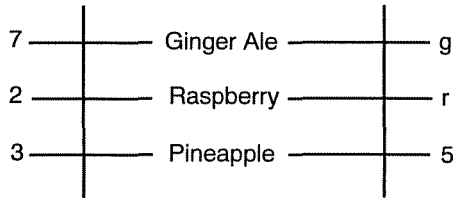
(a) How much ginger ale and raspberry juice is needed if 5 L of pineapple juice is used?

Note: (i)The quantities are all in the same units (Litres) and so we have a ratio.

(ii)The ratio terms are always placed all on one side of the "tree" (usually the left side).



(iii) The actual amounts are all put on the other side of the "tree".



(iv) All the results will be in the original units.

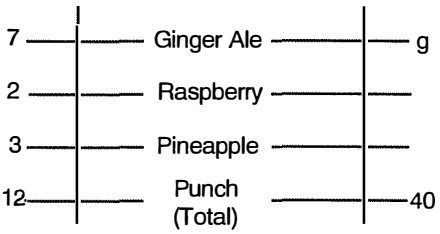
$$\frac{7}{3} = \frac{g}{5} \quad \text{and} \quad \frac{2}{3} = \frac{r}{5}$$

$\therefore g = 11.67$ and $\therefore r = 3.33$

Hence, approx. 11.67 L of ginger ale and 3.33 L of raspberry juice are required.

(b) How much ginger ale is required to make 40 L of punch?

Note: The introduction of "punch" requires an extra branch; it is "the whole thing" or "all of it" or "total", etc. And since it is made up of all of the other items, its ratio terms are the sum of all of the other ratio terms on the same side.



$$\frac{7}{12} = \frac{g}{40}$$

$\therefore g = 23.33$

Percents

Percents are merely a particular ratio, i.e., the second term is always 100. So 16.5% is equivalent to 16.5:100 .

The perennial trio of percent questions has usually been dealt with (starting in grade 7 or so) by using 3 different methods. For example ...

A Finding the percent

What percent of 2000 is 1500?

$$\frac{n}{100} = \frac{1500}{2000} = \frac{75}{100}$$

So, 1500 is 75% of 2000

B Finding the percent of a number

Find 25% of 420

25% of 420 = 0.25 x 420

= 105

C Finding the number

If 15% of a number is 30, find the number

15% of the number is 30

1% of the number is 30/15 or 2.

100% of the number is 100 x 2 = 200.

$\therefore$ the number is 200.

But these three types ...

(i) What is 4% of 1762?

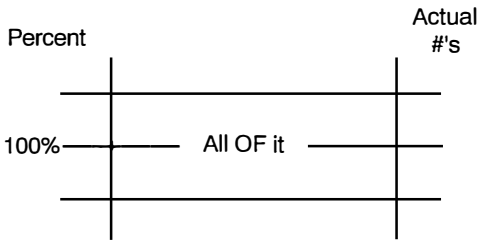
(ii) What percent of 13 is 560?

(iii) 82 is 61.8% of what number?

can easily be handled in *one* way using a "tree".

P.S. We fully appreciate that the first type can and should be handled quickly and directly ... but IN TIME! (The TIME is your judgement call.)

The "tree" for percent questions is as follows:



Three branches are used so as to accommodate percents less than 100% (put below) as well as percents greater than 100% (put above). (You could use just two branches but the visual impact of the relative positions is a teaching strategy that is effective for most students.)

Note: *(i) The percents are all put on one side of the "tree".*

(ii) The actual values are all put on the other side.

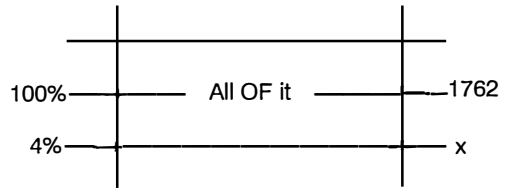
(iii) The middle branch is always *the "100%" and is labelled "All OF It" (or its equivalent), the emphasis being placed on the word "OF".*

The middle branch should be completed *before* any other entries are made on the "tree". This is the only surefire method of ensuring that the students do not mix up the numbers given in the question.

Whatever follows the "percent of", "% of" or just "of" in the question is put across from the 100%; this is the **All OF It,** the initial *whole.* The rest of the given information then falls logically into place, percents on one side and the actual numbers on the other side, keeping smaller values below and the larger values above.

(iv) ONLY 2 branches are chosen from the completed "tree" to set up the proportion. Students forgetting or ignoring this end up in impossible situations.

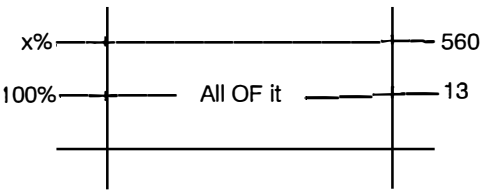
Example 1. What is 4% of 1762?



$\frac{100}{4} = \frac{1762}{x}$

$\therefore x = 70.48$

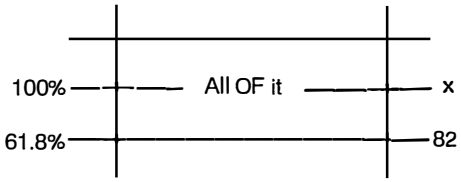
Example 2. What percent of 13 is 560?



$\frac{x}{100} = \frac{560}{13}$

$\therefore x = 4307.69$

Example 3. 82 is 61.8% of what number?



$\frac{100}{61.8} = \frac{x}{82}$

$\therefore x = 132.69$

CAUTION #1 - Some percent questions will NOT use the word "of"!

Jennifer earned $325 last year working at the Pizza Boat. She spent $165. What percent did she spend?

What to do? In all cases, throughout their lives, the students, when confronted with percent data, *must* know to ask, "Percent of what???".

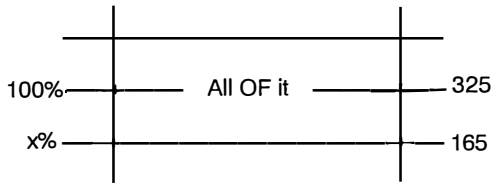
For example ...

Who receives more, Henry with a 32% raise or Barb with a 10% raise?

Of course there is no answer to this question because we have no idea about "32% of WHAT?" and "10% of WHAT?".

So in the problem with Jennifer, the question has to be asked, "What percent **OF WHAT** did she spend?"

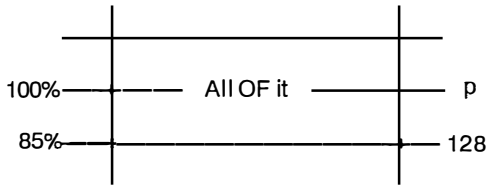
Answer: "What percent **OF HER EARNINGS** (i.e., $325) did she spend?"



CAUTION #2 - Some questions will have more than one "of" in them.

A pear is about 85% water. If the amount of water in a pear is 128 g, what is the mass of the pear?

There are 2 "of"s and neither of them is the "of" we want! We have to look at "A pear is about 85% water" and be able to see "85% **OF A PEAR** is water."



Retail Applications

Many simple retail questions crop up involving tax, discount and rates of discount. To be competent consumers the students need to have the realization that every price change involves 3 things: the original price, a change (down or up), and the new price.

With this simple awareness and the guidelines of ...

(a) identify the largest value ("original" or "new" price)

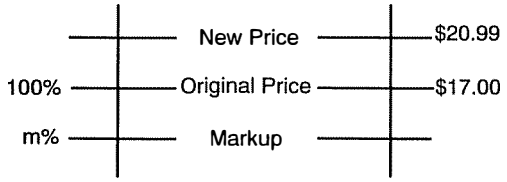
(b) the largest value goes on the top branch

(c) the 100% is always across from the "original" price

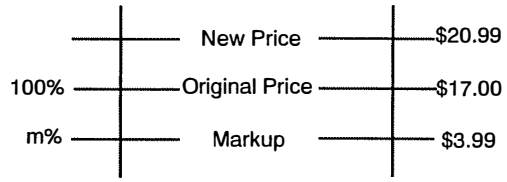
all such questions remain as easy to handle as the regular percent questions.

Observation: The bottom 2 branches always add up to the top branch.

Example 1. If Toronto Raptor T-shirts regularly selling for $17 become scarce and T's Tease decides to cash in on the demand by selling its shirts at $20.99, what is the rate of markup?



By the **observation** made earlier we can easily fill in another spot on the "tree", thus completing 2 branches. (*Either* spot can be used but one leads to simpler manipulation.)

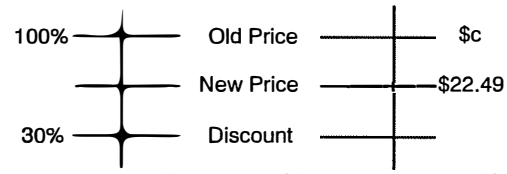


$\frac{100}{m} = \frac{17}{3.99}$

$\therefore$ m = 23.47

Therefore, the markup rate is 23.47% .

Example 2. U-Save department store advertised their school bag sale as **30% OFF!** If Marci chose one marked down to $22.49 what did the bag cost before the sale started?



Now complete one of branches yourself (again, pick the simpler one).

$\frac{100}{70} = \frac{c}{22.49}$

$\therefore$ c = 32.129

Therefore, the bag originally cost $32.13.

Business Applications

Markup, margin, markdown, rate of margin, etc. problems are all very easily handled with a "tree". They are all percent problems where the only question remains, "What is the 'All OF IT' ?"

Normally, the students are always given the business jargon, their definitions and the equations that relate them.

For example,

Markup = Selling Price - Cost Price

$$\text{Rate of Markup} = \frac{\text{Markup}}{\text{Cost Price}} \times 100\%$$

Selling Price - Margin = Cost

$$\text{Rate of Margin} = \frac{\text{Margin}}{\text{Selling Price}} \times 100\%$$

and so on ...

However the *bare necessities* are ...

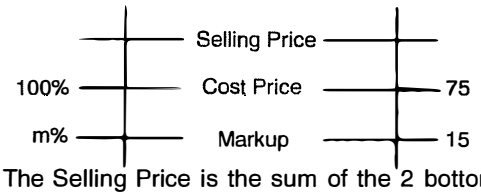
Markup	is *based on* the Cost
Markdown (Discount)	is *based on* the Selling Price
Margin	is *based on* the Selling Price

The first two are almost intuitive by the language; the third, margin, has to be learned.

Now, the "tree" designates the 100% or the "All OF IT" as whatever the calculations are *"based on"*.

If we use a 3 branch "tree" with the highest value always placed at the top then we get the added bonus of the 2 bottom branches always adding up to the top branch.

Example 1. If an article has a cost of $75.00 and a markup of $15.00, what are the rate of markup and the selling price?



I The Selling Price is the sum of the 2 bottom lines, i.e., $(75+15)

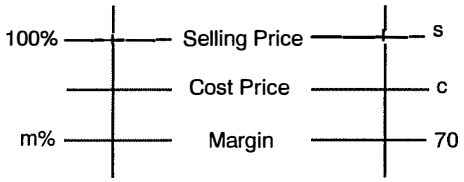
II

$\frac{100}{m} = \frac{75}{15}$

$\therefore$ m = 20

III The rate of markup is 20%.

Example 2. The margin on an item is $70. The rate of margin is 15%. What was the cost price and what was the selling price?



To start ...

a) you can immediately fill in the left side of the middle branch (top branch equals the sum of the lower branches),

OR

b) you can calculate the selling price by using a proportion.

Example 3. Lois can save $9.90 if she buys a calculator on sale at Robbery Row which is offering 30% discounts off the regular price. Find the sale price and the regular price of the calculator.

a) The regular price (selling price) can be calculated by using a proportion.



$\frac{100}{30} = \frac{s}{9.90}$

$\therefore s = 33$

b) The sale price (discount price) can then be found using the branch endings on the right side only.

$33 = d + 9.90$

$\therefore d = 23.10$

Therefore the sale price is $23.10 and the regular price is $33.00.

Rates

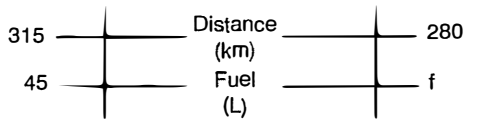
RATES -a comparison of two quantities expressed in *different* units. (Examples: 94 km/h or $2.75 a dozen.)

Rate questions are handled with a 2 branch "tree", each branch labelled with identification *and* units.

Note: (i) The given rate is entered on one side of the "tree".

(ii) The situation in question has its values entered on the other side.

Example 1. A car travels 315 km on 45 L of gasoline. How many litres will it take to travel 280 km?



$\frac{315}{45} = \frac{280}{s}$

$\therefore f = 40$

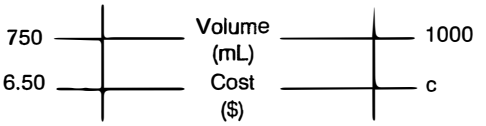
Therefore it will require 40 L of gasoline.

(iii) Both values on any one branch must *be in the same units.*

Example 2. The price of a 1 L bottle of syrup is $8.95. The price of a 750 mL bottle of syrup is $6.50. Which size is the better value?

To make a comparison we can either calculate the cost of 750 mL at the first price or we can calculate the cost of 1 L at the second price. *(Finding a unit cost at each price and then comparing those is a longer method but it is often better handled by the students.)*

Finding the cost of 1 L at the second price.



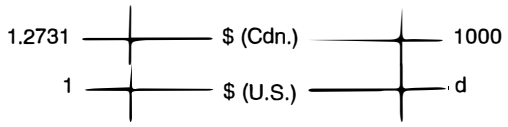
$\frac{750}{6.5} = \frac{1000}{c}$

$\therefore c = 8.67$

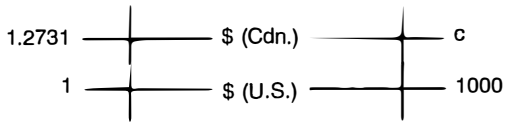
Therefore 1000 mL (i.e., 1 L) of syrup at the second price would cost $8.67 making the 750 mL bottle the better value.

More Applications

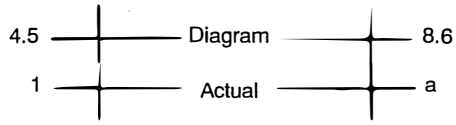
#1a. Looking forward to a trip to Disney World, Fiona went to the bank to buy U.S. dollars. If the exchange rate was $1 U.S. = $1.2731 Canadian, how much did Fiona get for $1000 Canadian? (Try it! Ans: $785.48 U.S.)



#1b. When Fiona's cousin from Indiana visited her, she brought $1000 in American money. How much Canadian money did she get in exchange?

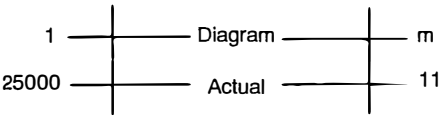


#2. If a science text illustration of an earwig used a scale of 4.5:1 how long was the actual earwig if the illustration was 8.6 cm long?



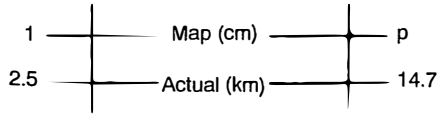
*All scale diagram ratios are understood to be diagram:actual.

#3. A map shows a scale of 1:25000. What is the distance on the map between two locations that are actually 11 km apart ?



$\therefore m = 1 \times 11 \div 25000$
$\therefore m = 0.00044$
$\therefore$ They are 0.00044 km or 44 cm apart

#4. A map shows a scale of 1 cm = 2.5 km. If two communities are actually 14.7 km apart, how far apart will they be on the map?



$\therefore p = 1 \times 14.7 \div 2.5$
$\therefore p = 5.88$
$\therefore$ They will be 5.88 cm apart on the map.

#5. Nickel metal will react with silver nitrate solution to give metallic silver and a solution of nickel (II) nitrate.

$Ni(s) + 2AgNO_3(aq) \Rightarrow 2Ag(s) + Ni(NO_3)_2(aq)$

If 15.32 g of nickel reacts with 88.68 g of silver nitrate in solution to produce 47.68 g of nickel nitrate,

(a) how much silver is produced?

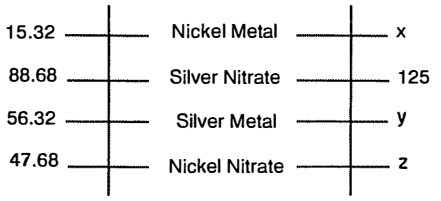
(b) how much of each material is involved in a reaction which uses 125 g of silver nitrate?

(a) $15.32 + 88.68 = x + 47.68$

$\therefore x = 56.32$

Therefore, 56.32 g of silver will be produced.

(b)



$\frac{15.32}{88.68} = \frac{x}{125}$

$\therefore x = 21.59$

$\therefore$ approx. 21.59 g of nickel metal will be involved.... and so on.

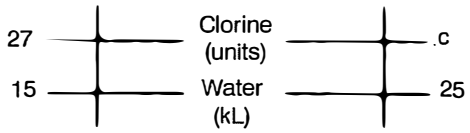
VARIATION PROBLEMS - these are linked to rate problems

DIRECT VARIATION

If $c \propto v$ then $c = vk$, $k \in R$ which means that we also have the *rate* c units/v units.

For example ...

The amount of chlorine needed for a pool is directly proportional to the size of the pool. If 27 units of chlorine are used for 15 kL of water, how much chlorine is required for 25 kL of water?



Final notes:

#1. The "proprtion tree" has been an extremely effective tool for weak students as well as for strong students.

#2. As mentioned at the outset, more applications of "Proportion Trees" exist, such as in Combining Rates, Inverse Variation, and Chemistry ("limiting reactant problems," etc.), not to mention perennial ratio and rate chestnuts such as bicycle gear problems and the following treasure ...

A cask is filled with 45 L of wine. 9 L are removed and the cask is refilled with water. Then 9 L of the mixture are removed and the cask is refilled with water again. What is the ratio of water to wine in the final mixture?

#3. You might, at some point, consider requiring only the calculated answer immediately after the "tree" diagram by having the students recognize the associated proportion right on the "tree".

#4. The "**Retail Applications**" tree has recently been modified by the authors to more obviously reflect the generic structural design of a "percent tree". Details available.

#5. Further information or examples may be obtained by contacting:

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▲ Helping Students to Construct and Link Problem-solving Models[1]

LUIS RADFORD
SCHOOL OF SCIENCE EDUCATION
LAURENTIAN UNIVERSITY
SUDBURY, ONTARIO, P3E 2C6



Luis Radford is an associate professor at Laurentian University, Sudbury, Ontario. He teaches in the preservice teacher training program. His main research includes the teaching and learning of algebra and the history of mathematics.

1. Introduction

In the last years, problem-solving has been considered an important element for the teaching of mathematics. For instance, the Provincial Standards in Mathematics have recently emphasized the role of problem-solving as one of the three key components of mathematics learning (Ontario Ministry of Education and Training, 1993, pp. 9-11. See also *Focus on Renewal of Mathematics Education,* OAME, 1993, p.4). The aim of this paper is to present a concrete approach for teaching strategies based in problem-solving methods.

Our starting point is the fact that, often, when students face a new problem they do not have an existing model of resolution. Generally, the construction of a new model which would allow them to solve the problem proves to be a very difficult task. The main idea of our approach is that, in this case, it is preferable to advance towards the process of problem-solving gradually. Instead of constructing a problem-solving procedure for the target problem, it would be better to first solve other simple problems related to it. The problem-solving procedures for the simplified problems will then allow the students to evolve towards a more complex problem-solving method which will, in turn, solve the target problem.

2. Helping Students to Construct and Link Problem-Solving Models

In order to give an idea of how our approach can be implemented in the classroom, we shall now present an example.

For our discussion, we will use the following problem[2]:

"Jane Kimble, a grocer, checked her supply of milk and counted 80 containers of milk. Some were 2L cartons and others were 3L bags. Altogether there was a total of 220L of milk. How many of the containers were 3L bags?

(Target problem)

This example belongs to a wide group of problems that we can write, using algebra with two unknowns, in the form of a linear system of equations: x+y =a; bx+cy = d. However, in the Curriculum Guidelines (Ontario Ministry of Education, 1985), the methods of algebra with two unknowns are introduced only in Grade 10 and, at the General Level, it is done either through graphical procedures (via intersection of straight lines) or through numerical ones (Ontario Ministry of Education, 1985, p. 47). How can we help students in Grades 7, 8 and 9 to construct an algebraic problem-solving model without using two unknowns?

In what follows, we shall suggest a possible didactic path which shows how we can help students in Grade 7, 8 or 9 to construct a sequence of problem-solving models moving from a problem-solving model based on concrete thinking to another one based on symbolic thinking. In order to do so, we shall begin by considering a simplified problem of the target problem.

The simplified problem follows:

"Jane Kimble, a grocer, checked her supply of milk and counted 30 containers of milk. Some were 2L cartons and others were 3L bags. Altogether there was a total of 83L of milk. How many of the containers were 3L bags?"

(Simplified Problem)

2.1 The Trial and Error Method

The simplified problem could be solved via trial and error. It is a simple method that has the advantage of requiring knowledge of only simple arithmetical concepts but it has the disadvantage that, to solve other similar problems, it can take a long time to find the answer. (The time will depend on the "size" of the numbers; in general terms, the bigger the numbers the less efficient the method). The didactic problem is then that of convincing students to move on to more complex procedures which can be done if we succeed in convincing them of the "power" of the new procedures. Here is an example of how this might be done.

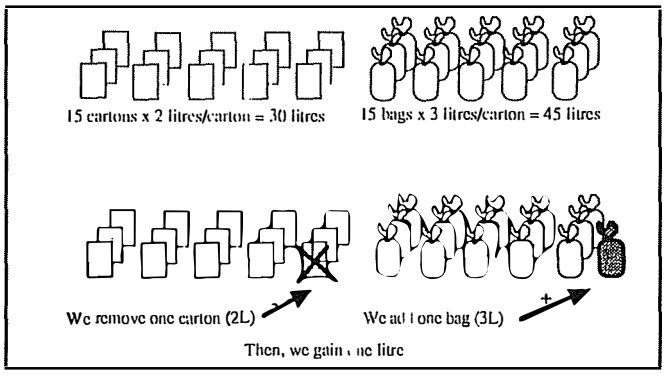
2.2 Towards a more complex problem-solving model: a manipulative-based method

In past times, when number symbols were not yet invented, the shepherd used to take a little bag containing as many

stones as he had sheep. When he returned home in the evening, he compared the stones with his sheep. If there were as many stones as sheep, he knew that he had not lost any of his sheep that day. Once symbols were invented to represent numbers, new procedures for counting and calculating emerged (Pallascio et al. 1993). This historical fact reminds us of the very important role played by manipulatives in the development of concepts. How can we help students move to a more complex problem-solving model using manipulatives?

Let's once again consider the simplified problem stated above. We may begin by supposing that the number of bags is the same as the number of cartons. Thus, we have 15 of each type of milk container. On a table placed in front of the students, we place 15 cartons or plastic boxes containing 2 objects each (two blocks, for instance, each one simulating a litre of milk). Beside this table, there is another table on which we place 15 bags, each containing 3 objects (three blocks, once again simulating litres of milk). At this point we can ask the students to calculate the total number of litres; they will find the answer to be 75 litres.

Instead of replacing the numbers of cartons and the number of bags, as done in the trial-and-error method, we will think in terms of how much we failed in our former assumption. We found 75 litres, but should have had 83 litres. So we missed the correct answer by 8 litres. This means that we must add some bags. Helped by manipulatives, students can see that if they add one bag, they must remove a carton in order to keep the total number of containers equal to 30.



We then face the following question: How many cartons must we replace by bags to obtain a total of 83 litres? In adding a bag and removing a carton, we gain 1 litre, but we still require 8 more litres. So we have to add 8 bags and remove 8 cartons from the tables. The students will discover that the solution is 23 bags and 7 cartons.

Changing some data in the simplified problem gives students practice in using the new arithmetic manipulative-based model.

2.3 An Arithmetic Problem-Solving Model

Until now we have used two problem-solving models to solve the simplified problem, namely the trial and error method (where the selected numbers are chosen in a non-systematic way) and an arithmetically organized manipulative-based method.

After using manipulatives and formulating other (similar) problems, the students can be introduced to an arithmetic-abstract problem-solving model, that is, a problem-solving model based only on arithmetic concepts without using manipulatives. In order to motivate this we need to confront students with a problem whose solution, using the current method, is not easy. We can change some data in our grocer's problem in such a way that the quantities are so large that it is tedious to solve the problem using manipulatives. Thus, consider the next problem which is, in fact, our target-problem (see the beginning of section 2):

> *"Jane Kimble, a grocer, checked her supply of milk and counted 80 containers of milk. Some were 2L cartons and others were 3L bags. Altogether there was a total of 220L of milk. How many of the containers were 3L bags?"*

Following the same thought process, the students should be encouraged to face the problem by manipulating ideas instead of manipulating concrete objects.

Suppose that the number of milk bags is the same as the number of milk cartons. We then have 40 of each. Calculate the number of litres. In the cartons, we have 40x2 = 80 litres; in the bags we have 40x3 = 120 litres. We then find that we have 80+120 = 200 litres of milk, but the problem requires a total of 220 litres. We are 20 litres short of the total. So our assumption that we have the same number of bags and cartons is wrong. We can then conclude that we need more bags than cartons. If we add one milk bag, we have to remove one milk carton (in order to keep the number of cartons and bags equal to 80), and we gain one litre of milk. But we need 20 litres, so we have to remove 20 cartons and replace them with 20 bags. So it remains, 40-20 = 20 cartons and 40+20 = 60 bags of milk.

This method of problem solving needs to be internalized by students. The teacher can then propose other problems changing some of the data in the last problem[3]. The teacher and/or students can also propose other problems of the form x+y = a; bx+cy = d. They can also discuss and construct problems of the type x±y = a; bx±cy = d.

The students will realize that the new problem-solving model has the advantage of allowing them to approach a relatively wide family of problems in a direct way; no trial and

---

[3] *An interesting problem is the following:*

> *"Jane Kimble checks her milk inventory. There are 80 containers of milk: some are 2L cartons and the others are 3L bags. There is a total of 195L of milk. What, then, must be the number of 3L bags?*

Let's suppose, as in the previous problem, that we have 40 milk bags and 40 milk cartons. We find that we have 200 litres instead of 195 litres. Thus, now we have 5 litres more, so we have to replace bags by cartons.

error is required, nor do they require manipulatives.

We have achieved a partial sequence of problem-solving models. We can note, at this point, that one of the biggest differences between the trial and error model and the previous arithmetic-abstract problem-solving model is that, in the latter, we use *hypothetical reasoning* by supposing that the number of bags equals the number of cartons. We then made calculations that allowed us to generate new data that we corrected later. In the trial and error method we simply repeat the same procedure with different quantities until we obtain the correct answer; hypothetical reasoning does not play any role.

Why is hypothetical reasoning important here? Because hypothetical reasoning is the logical base of algebraic thinking in solving word problems, as we will see below. Thus, the abstract arithmetical problem-solving model can help students to construct knowledge that will be useful to them when they learn algebra, and is a vital step in the construction of advanced problem-solving models.

In 2.4 below, we discuss an algebraic problem-solving model that uses only one unknown in solving the target-problem, which is, in fact a *natural generalization* of the arithmetic-abstract problem-solving model.

2.4 An Algebraic Problem-Solving Model

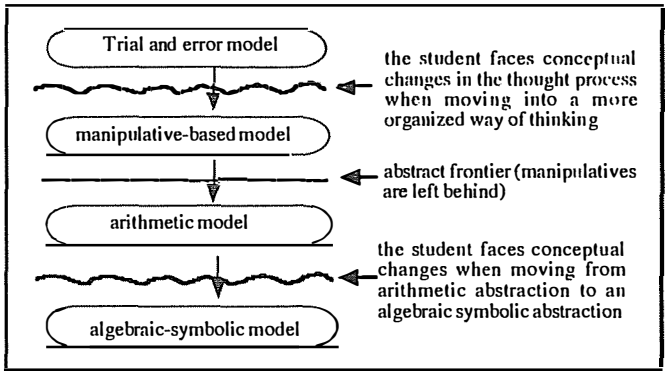
Instead of beginning with a numerical solution (dividing the total number of containers between the cartons and bags, and then deducing *the number* we have to add to the first and remove from the second) we can *suppose* (hypothetical reasoning) that we know this number already. Let x be this number. The *exact* number of cartons is not 40 (half of 80) but 40-x. In the same way, the *exact* number of bags is not 40, but 40+x. Given that a carton holds 2L and a bag holds 3L, we can then express the total number of litres as 2(40-x) + 3(40+x), where this quantity must equal 220 litres. We get the equation:

$2(40-x) + 3(40-x) = 220$

This is one type of equation usually taught in Grade 9.

This algebraic problem-solving model needs to be internalized by students. In order to achieve this, the teacher can propose other similar problems to be solved by the new model, as it was done before in the previous problem-solving model.

The following diagram illustrates the path that we followed in our example. One moves from the trial and error thinking to symbolic thinking. Each level develops problem-solving models that are used in the next level, but in a more complex way.



3. Concluding Remarks for Teaching

As we said before, the main idea of our approach consists in trying to construct a hierarchical progressive sequence of problem-solving models,

$M_1 \rightarrow M_2 \rightarrow \ldots \rightarrow M_n$

that start from a simple model M_1 and that arrives at a more complex model M_n. The choice of the models that precede the final model M_n depends on the M_n model itself and on the knowledge of the students. In our case, we aimed at a model capable of solving the grocer's problem (see beginning of section 2) using the algebraic knowledge presented in the Transition Years. Therefore, we should use algebra with only one unknown. In order to accomplish this, we used three pedagogical principles:

Principle 1: *to simplify the problem.*

Our first principle consists in simplifying the target problem. The simplified problem is then solved through an accessible problem-solving model which will allow, in the next step, to construct a new, more complex model.

Principle 2: *to convince the students.*

Our second principle is that of convincing the students of the necessity to go on to a more complex model of problem-solving. This can be done by presenting a few problems that are difficult or not appropriate to solve by using the present model as we did up above.

Principle 3: *to create a hierarchical link between problem-solving models.*

Our third pedagogical principle states that the next model to be constructed in the hierarchical sequence has to be based on the previous model. We can note, at this point, that the grocer's problem in section 2.3 can be solved by other methods (like almost any problem). One does not need to divide the number of containers in half. One can select any value x between 1 (even between 0) and the number of containers, that is, 80. Although other methods could solve the grocer's problem, the method of dividing the number of containers into two that we used is that which naturally leads to the algebraic problem-solving model at which we aimed. That is the reason why the

'dividing into two method' plays such a central role in our didactic sequence.

The previous principles can be applied to many situations. It remains, however, that the success of our approach in a mathematics class will depend upon the teacher's ability to choose an appropriate simplification of the problem and to obtain a suitable hierarchical sequence of linked models. It will be particularly necessary that the use of concrete models based on manipulatives be coherent with the abstract problem-solving models.

4. Some Historical Remarks

We have seen different methods in solving the same problem. The first one was the trial and error method. The third is a method developed by ancient Babylonian mathematicians, probably during the first Babylonian Dynasty (c. 1900 B.C.). It is called the *false position* method (in fact, in solving the problem, we start with a false solution). The last method is related to the historical emergence of algebra. We can find it in one of the most important books of Greek mathematics, the Diophantus' *Arithmetika* (c. 250). (See Radford, 1992). A detailed account of the historical links between the false position method and the algebraic one presented here can be found in Radford (1993).

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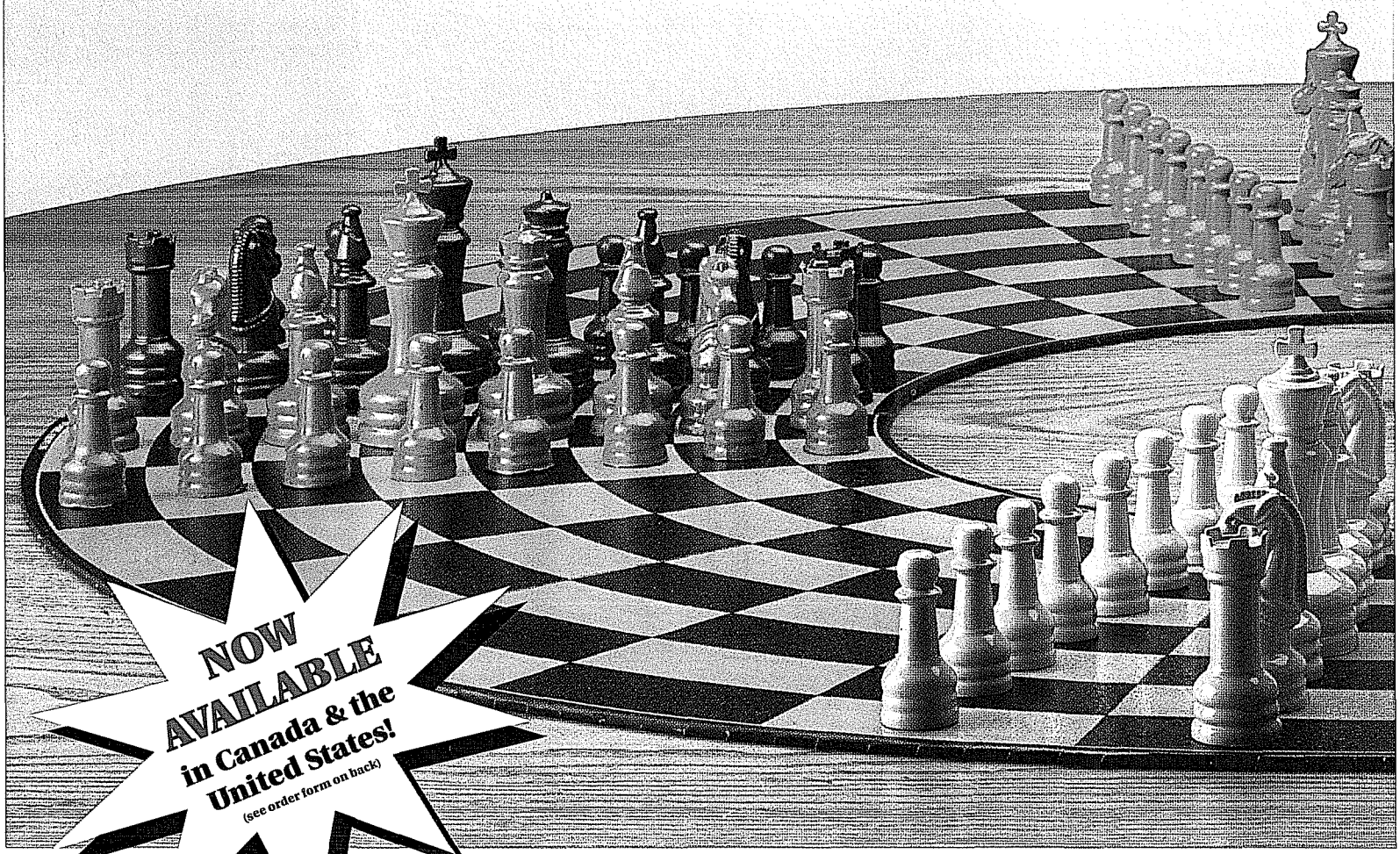
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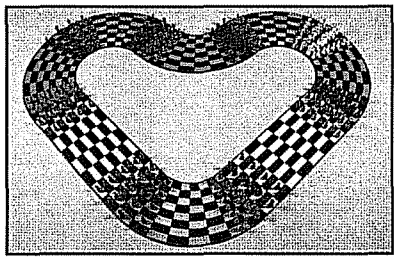
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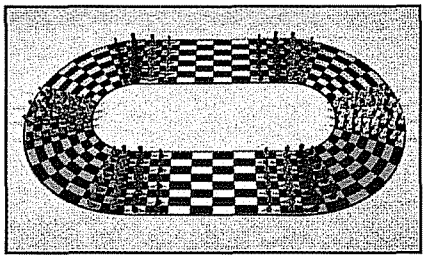


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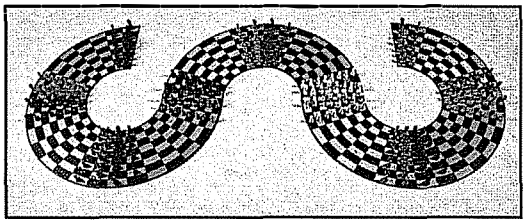
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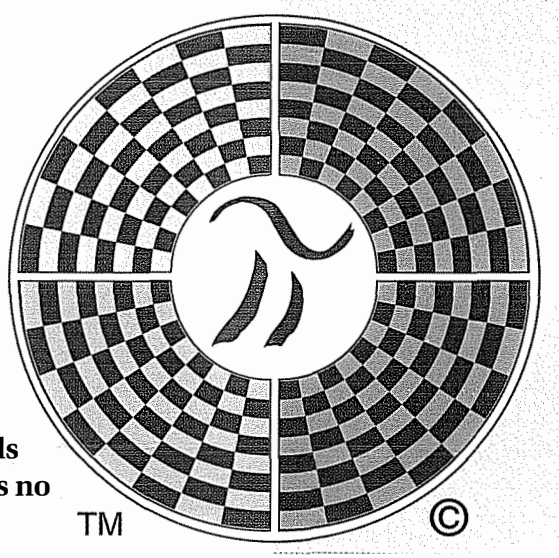


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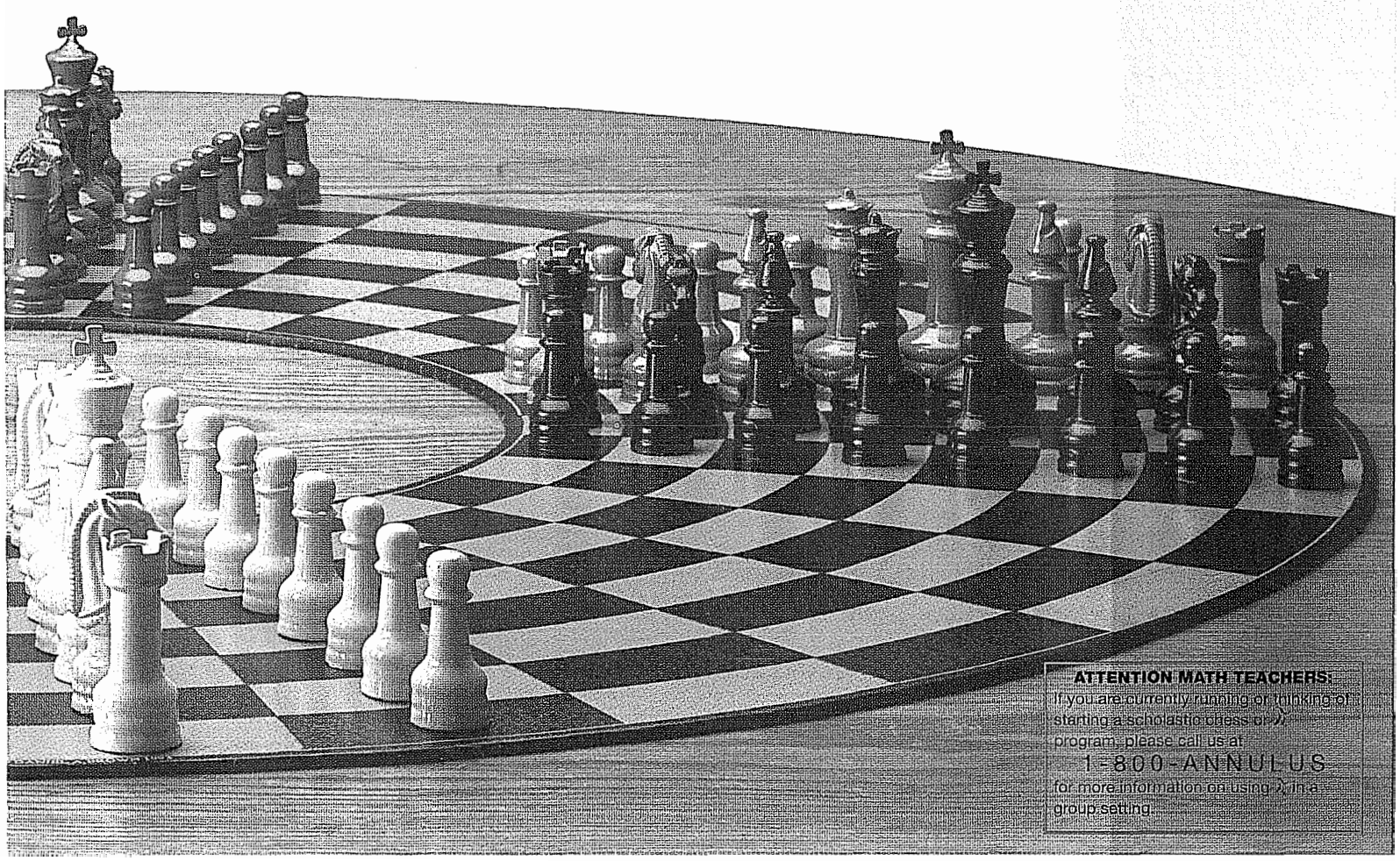
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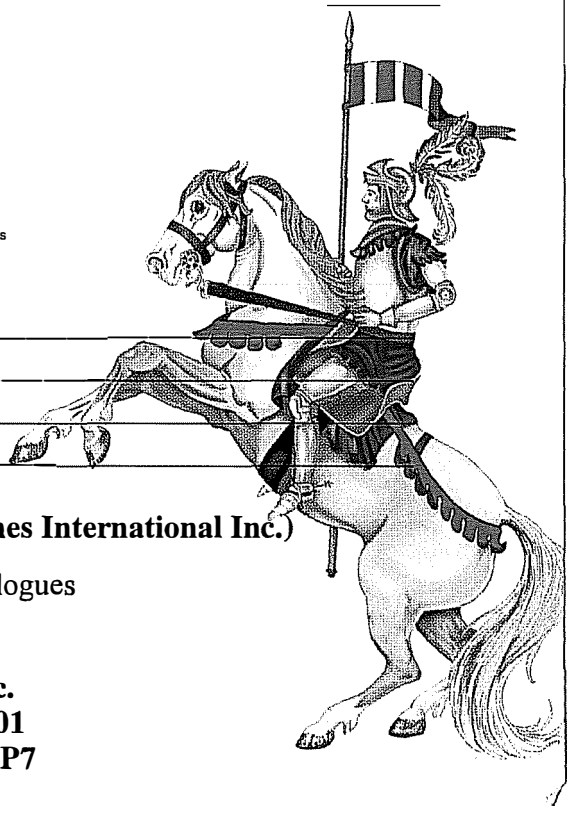
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An Alternative Way to Deal With Euler's Formula in your Algebra and Geometry Class

YSBRAND DE BRUYN



For the last five years Ysbrand de Bruyn has been teaching mathematics at Glenforest Secondary School in Mississauga. He has also taught for thirteen years at Toronto District Christian High school, Woodbridge, Ontario. Ysbrand has a MEd. and an MSc. in mathematics from the University of Toronto.

The development of Euler's formula,

$e^{i\theta} = \cos\theta + i\sin\theta$,

is dealt with in a limited fashion in the commonly used textbooks for the OAC Algebra and Geometry course. Two common explanations are provided.

The first approach requires the use of power series for the functions cosx, sinx and e^x, which are not justified, but simply presented as fact. The power series for e^{ix} is decomposed into its real and imaginary parts, which are then shown to be equal to the power series of cosx and sinx respectively. This method of explaining Euler's formula is provided in Stewart et al (1989, p. 401) and in Egsgard (1989, p. 456). This approach is alluded to in Anderson et al (1988, p. 318), but no details are provided.

The second approach provides some discussion about the behaviour of θ in $\cos\theta + i\sin\theta$: namely that it behaves like an exponent, but the reason for using the base e is not justified at all. This method is used in Del Grande et al (1989, p. 466).

Just as OAC level students are expected to provide well reasoned solutions to problems, it is equally important that significant formulae and theorems presented in class are justified, if not proven outright, using explanations that rely on previously taught concepts. In the first of the two approaches outlined above, the explanation for Euler's formula does not rely on previously taught concepts, and so is not completely satisfactory. For the second explanation of Euler's formula, the use of base e is not justified, a question which is not easily answered, but one that students do ask about.

The following explanation is a way of making Euler's formula seem reasonable to your Algebra and Geometry students. It should not, however, be considered a proof, but a way of making the base e in Euler's formula a reasonable one to use.

If your students have already taken the OAC Calculus Course, they should be familiar with the following limit:

[1] $$\lim_{n \to \infty} (1 + 1/n)^n = e.$$

Since most of your students have either forgotten this limit or would not have seen it, this limit can be motivated by means of a short calculator exercise, showing that $(1 + 1/n)^n$ tends to 2.718... when one uses larger and larger values of n. It can be pointed out that this is the number e which is also programmed in their calculators. Because many of the students taking the Algebra and Geometry course are unfamiliar with limit notation, the use of limit notation can be avoided if one rewrites [1] as $(1 + 1/n)^n \approx e$ (for large values of n), while making sure that students realize that this approximation becomes better with ever increasing values of n. Indeed all limits in the paper can be dealt with in a similar way. Raising both sides of [1] to the power x, we also have:

$$\lim_{n \to \infty} (1 + 1/n)^{nx} = e^x.$$

By a change of variable, N = nx (and assume for now that n>0), it is possible to define the exponential function as:

[2] $$e^x = \lim_{n \to \infty} (1 + x/N)^N.$$

This definition of the exponential function allows one to extend the domain of the function to imaginary exponents:

[3] $$e^{i\theta} = \lim_{n \to \infty} \left(1 + \frac{i\theta}{N}\right)^N,$$

provided this limit exists.

Using De Moivre's Theorem,

[4] $$\cos\theta + i\sin\theta = \cos\left(N\,\frac{\theta}{N}\right) + i\sin\left(N\,\frac{\theta}{N}\right),$$

$$= \left(\cos\frac{\theta}{N} + i\sin\frac{\theta}{N}\right)^N.$$

With the help of a calculator set in radian mode, it can easily be observed that the closer the value x is to 0, the better x approximates sin x. Indeed, the graphs of y = x and y = sin x can be seen to merge as x approaches 0. It can also be observed that cos x → 1 as x → 0. If we let x = θ/N, then for sufficiently large values of N, the above trigonometric approximations imply that:

$$\sin\frac{\theta}{N} \approx \frac{\theta}{N} \quad \text{and} \quad \cos\frac{\theta}{N} \approx 1.$$

Substituting the approximations in [5] into [4], it is reasonable that:

[6]
$$\cos\theta + i\sin\theta = \lim_{n \to \infty} \left(1 + \frac{i\theta}{N}\right)^N,$$

Comparing [6] with [3], it becomes reasonable to define the natural exponential function for imaginary exponents by:

$e^{i\theta} = \cos\theta + i\sin\theta.$

Even though there is no proof here that the right side of [4] is actually equal to the right side of [6], the purpose here is not to provide a rigorous proof, but simply to demonstrate a plausible argument, within reach of the typical student at the OAC level, the reason for using base e in Euler's formula. A formal proof of the limit is not necessary at the OAC Algebra and Geometry level. If one defines

$e^{i\theta} = \cos\theta + i\sin\theta,$

then only the reasonableness of using the base e and the use of the exponent $i\theta$ needs justification, since the rules for multiplication of complex numbers in polar form already implies that $e^{i\theta}$ follows the usual rules for exponents.

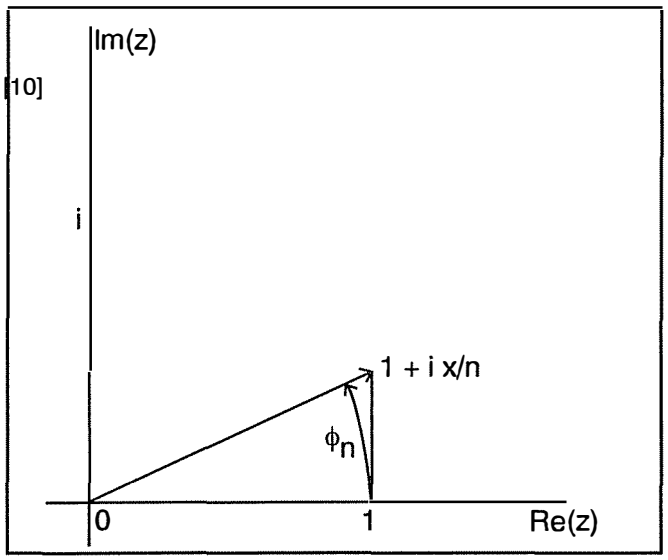
Appendix

For those interested, here is a proof that

$$\lim_{n \to \infty} \left(1 + \frac{ix}{n}\right)^n = \cos x + i\sin x .$$

Although the proof requires no more than OAC mathematics, it is not intended for OAC students in general, and is included only for completeness.

Let ϕn be the principal argument of the complex number $z = 1 + ix/n$ in polar form:

[10]


Then, in polar form, the complex number z can be written as:

$z = |z| [\cos \phi_n + i \sin \phi_n]$,
where the modulus $|z| = \left(1 + \frac{x^2}{n^2}\right)^{\frac{1}{2}}$.

Raising $z = 1 + ix/n$ to the power n, and applying De Moivre's Theorem we have:

[7]
$$\left(1 + \frac{ix}{n}\right)^n = \left(1 + \frac{x^2}{n^2}\right)^{\frac{n}{2}} \left(\cos\phi_n + i\sin\phi_n\right)^n$$

$$= \left(1 + \frac{x^2}{n^2}\right)^{\frac{n}{2}} \left(\cos n\phi_n + i\sin n\phi_n\right).$$

To see what happens as n approaches positive infinity, the two factors in [7] must be dealt with separately. It will be shown that the first factor tends to 1 as n tends to infinity, and that the second factor approaches $\cos x + i \sin x$.

For positive values of t, the expression $(1 + 1/t)^t$ is an increasing function of t (Dorrie, 1965, p. 45). Since its limit, as $t \to \infty$, equals e, we have $(1 + 1/t)^t < e$. In addition, since $1 + 1/t > 1$ for positive t, so also $(1 + 1/t)^t > 1$. If we now let

$t = \frac{n^2}{x^2}$, then the following inequalities hold:

$$1 \le \left(1 + \frac{x^2}{n^2}\right)^{\frac{n^2}{x^2}} \le e$$

Raising to the power $\frac{x^2}{2n}$, we get

$$1 \le \left(1 + \frac{x^2}{n^2}\right)^{\frac{n}{2}} \le e^{\frac{x^2}{2n}}$$

For any fixed value of x, $e^{\frac{x^2}{2n}}$ approaches $e^0 = 1$ as n increases indefinitely, and so we have:

[8]
$$\lim_{n \to \infty} \left(1 + \frac{x^2}{n^2}\right)^{\frac{n}{2}} = 1.$$

Now assume that $x > 0$ and that N is large enough so that for $n > N$, $\frac{x}{n} < \frac{\pi}{2}$.

Since

$$\tan\phi_n = \frac{x}{n}, \text{ and } \sin\phi_n = \frac{\frac{x}{n}}{\left(1 + \frac{x^2}{n^2}\right)^{\frac{1}{2}}},$$

the following inequalities are true for $n > N$:

$$\sin\phi_n \le \phi_n \le \tan\phi_n < \frac{\pi}{2},$$

$$\frac{\frac{x}{n}}{\left(1 + \frac{x^2}{n^2}\right)^{\frac{1}{2}}} \le \phi_n = \frac{x}{n}, \text{ and so}$$

$$\frac{x}{\left(1 + \frac{x^2}{n^2}\right)^{\frac{1}{2}}} \le n\phi_n < x.$$

In the case when $x < 0$, the above inequalities are reversed. We may now conclude that

$$\lim_{n \to \infty} n\phi_n = x.$$

Since the functions $\sin\theta$ and $\cos\theta$ are both continuous, the following limits also hold:

[9] $$\lim_{n \to \infty} \sin n\phi_n = \sin x, \text{ and}$$

$$\lim_{n \to \infty} \cos n\phi_n = \cos x.$$

Applying [8], [9], and [10] to [7] we obtain the desired limit:

$$\lim_{n \to \infty} \left(1 + \frac{ix}{n}\right)^n = \cos x + i \sin x.$$

Having shown that the above limit is $\cos x + i \sin x$ not only provides a sound basis for Euler's formula, but also enables the exponential function to be defined for real and imaginary values of t by the limit:

$$e^t = \lim_{n \to \infty} \left(1 + \frac{t}{n}\right)^n.$$

References

Anderson, Edwin. Crippin, Peter W.D. Davidson, Larry. Rachich, Frank J. Zorzitto, Frank A., *Algebra and Geometry,* Holt Rinehart and Winston of Canada Ltd., Toronto, 1988.

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Egsgard, John, Ginestier, Jean-Paul, *Algebra and Geometry: a Search for Meaning,* Gage Educational Publishing Co., 1989.

Stewart, James, Davidson, Thomas, M.K., Hamilton, O. Michael G., Laxton, James, Lenz, Patricia M., *Algebra and Geometry,* McGraw-Hill Ryerson, Toronto, 1989.

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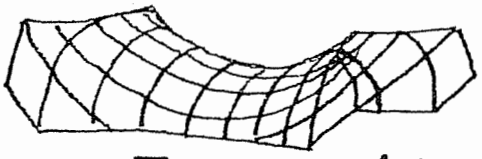
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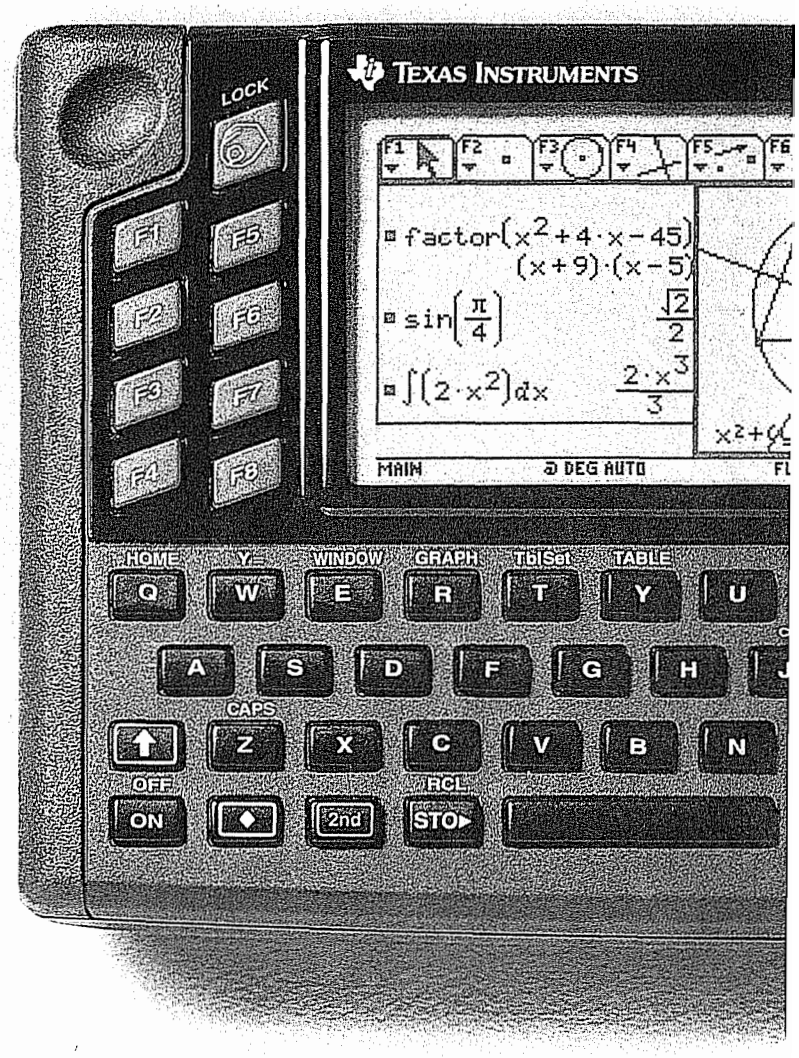
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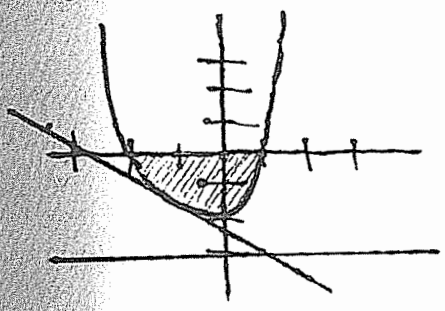
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▲ About Problem Solving: Oscillating Pairs Amidst Fun with Squares



JOHN GRANT MCLOUGHLIN
EDUCATION DEPARTMENT
OKANAGAN UNIVERSITY COLLEGE

1, 9, 16, 169, 196, 961 - it seems appropriate to call 1996 the "year of the perfect square." Perfect squares abound in this configuration. You may wish to pose some probability questions around this idea. By varying the possible number of digits and the flexibility in permuting them, an array of probability questions may surface.

Conditional probability problems could incorporate restrictions such as the digits must be different or given that the first digit is a 1, etc. The theme of perfect squares will form a basis for this column. Specifically we will consider two possible mathematical investigations. The initial Investigation is one of my own creations, whereas, the second appeared in an article by Jim Sherrill (1994) in Vector, a publication of the BC Association of Mathematics Teachers.

The first investigation considers the squares of the sums of digits of numbers. The idea is to pick a starting number and proceed to perform a repeated operation until a pattern appears. Let me illustrate this idea with an example. Suppose you start with the number 24. The sum of the digits is 6 and the square of 6 is 36. This becomes the second number. This procedure is repeated. The sum of the digits is 9 and the square of 9 is 81. Continuing we find that the sum of the digits is 9 and the square of 9 is 81. We have reached a point at which we will continue to get the result 81 indefinitely. Let me summarize by showing what the resulting sequence would look like:

24 36 81 81

Rather than repeatedly writing 81's, it will be sufficient to terminate the process when a number appears twice in succession. It will be said that the sequence terminates. One may wish to make an analogy to repeating decimals. Decimals with a period of 1 such as decimal representations of 1/3 or 2/9 would be deemed to terminate, for the purpose of our investigation.

Consider the patterns for the numbers 0 through 11 respectively. A summary of the results follows:

0	0					
1	1					
2	4	16	49	169	256	169
3	9	81	81			
4	16	49	169	256	169	
5	25	49	169	256	169	
6	36	81	81			
7	49	169	256	169		
8	64	100	1	1		
9	81	81				
10	1	1				
11	4	16	49	169	256	169

One can observe that numbers such as 0, 1, and 81 terminate sequences. Checking these numbers, we find that indeed they represent the squares of the sums of their digits. Are there other numbers that share this property? Consider much larger numbers as you look for patterns. While numbers such as 124876 or 2454563998 may appear quite large at first, it is important to keep in mind that it is the sums of their digits that are used to initiate the process. From a teaching point of view, we would like to get students to recognize that numbers such as 345, 27111, 9000003, and 57 will all produce similar sequences. These numbers may be thought of as a family of numbers because they share a common sum of digits, namely 12.

Consider the patterns created by starting numbers such as 2, 4, 5, and 11. They do not terminate according to the definition of termination as getting successive identical numbers in a sequence. Rather we find that they go back and forth between 169 and 256. These numbers share a curious property, namely, that the square root of one of them is the sum of the digits of the other, and vice-versa. Hence, this back and forth pattern will continue indefinitely. Realistically this mimics the pattern commonly found with repeating decimals that have a period of 2 or more. I will say that such patterns repeat. That is, a number previously found in the sequence appears a second time. Convince yourself that such an occurrence would ensure that the same sequence of numbers would continually reappear if the procedure was applied ad infinitum. The numbers 169 and 256 illustrate this idea. Such a pair of numbers will be called an oscillating pair. This pair will be considered to be the same as 256 and 169. The order will not make a difference in the oscillation. You can check that a number such as 949 would produce a sequence in which the 256 appears prior to 169. I challenge all readers to try to locate other oscillating pairs. Do other such pairs exist? Maybe you will discover repeating sets of numbers involving three, four, or more numbers. If so, then it is important to write them in the order they appear. However, if one identifies a triple (a,b,c), it would be considered equivalent to (b,c,a) but different than (a,c,b).

You are encouraged to pursue this investigation further by examining different possibilities. Some questions come to mind immediately. How many iterations may be required before a pattern terminates or repeats? Do all terminating patterns (aside from the trivial case of 0) end in 1 or 81?

The second investigation stems from Jim Sherrill making a curious observation while watching a game. He noticed that if he squared the number of minutes shown on the clock and then the number of seconds, the resulting squares were digital reversals of one another. Furthermore, the number of minutes remaining was a digital reversal of the number of seconds remaining. You can figure out how much time was left in the game. An example is provided to capture the essence of the concept. Consider the squares of 13 and 31. They are 169 and 961. The digital reversals are apparent. Sherrill neatly presented the result in the form of a square. He termed these arrangements "reversible squares".

$$\begin{array}{ccc} 13 & \rightarrow & 169 \\ \uparrow & & \downarrow \\ 31 & \leftarrow & 961 \end{array}$$

It should be noted that there was not 13:31 remaining in the game. The challenge remains open. Jim Sherrill provides a detailed summary of the findings of his investigation in the referenced article. If you would like a copy of the article, let me know and I'll try to accommodate your request.

Recent correspondence from readers has been very much appreciated. Eileen Shannon, a teacher at Westmount Secondary School in Hamilton, informed me that there are multiple solutions to the coins problem posed in the April 1995 column. This problem will be revisited in the next column. It is restated here so that you may (re)consider it in advance.

The total value of the nickels, dimes and quarters in a cash register is $19.95. Interestingly enough the total value of the quarters is an exact number of dollars with no change left over. Similarly the total value of the dimes is an exact number of dollars. There is a total of 167 coins made up of nickels, dimes, and quarters. Determine the respective number of nickels, dimes, and quarters in the cash register.

Any comments on the problem or any aspect of the investigations would be appreciated. I am particularly curious to receive any feedback on the investigation concerning oscillating pairs. Some results of your own investigations or comments on a class' experience with it would be interesting to read.

References

Sherrill, Jim, *Reversible Squares,* Vector, 35(2), 35-37, 1995.

▲

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▲ Council Comments



PETER SAARIMAKI
COORDINATOR OF MATHEMATICS
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Choose Your own Adventure

1. Council was an adventure, and so is this report. It's the old "Good News - Bad News" scenario. If you want to start with the good news, go to paragraph 17. If you want to start with some bad news, go to paragraph 12. If you want to start with something neutral, go to 24. If you do not want to decide, just read on. In any case, remember the famous, four-letter Beatles song (starting with H if you need a hint).

2. These notes were taken at the OAME Council meeting held Saturday, September 30, 1995. Comments about Council and its activities should be directed to the President. Comments about my comments can be sent to the editor. Now go to 20.

3. Susan Stuart called the meeting to order at 9:06 a.m. with 42 eager chapter reps, councillors and assorted others present, ready and willing to tackle the issues and challenges of the day. (Now, this is not the stuff you want to hear about, is it? There is still time to go back to paragraph 1 and actually make a decision.)

4. Good news reports came from the many chapters which are running Professional Development conferences throughout the fall. The chart following summarizes most - an impressive list, isn't it? We can't praise too highly the work the chapters do.

5.

CHAPTER	*EVENT and FOCUS*
CHAMP	Math contests for grades 9 - 11
CHAMP	Math contests for grades 7 and 8
CHAMP	MATHFEST - teacher conference
GOLDEN	Fall teacher conference
GVMA	Fall teacher conference
GVMA	Grade 9 OBE Math Program booklet
ISOMA	Fall teacher conference
MAC^2	Fall teacher conference
NOMA	Fall teacher conference
NWOAME	Pizza Party - Faculty of Education
NWOAME	Rendezvous with the Future (MST conf)
NWOAME	Dinnerlude - teacher conference
PINE RIDGE	Fall General Meeting
SWOAME	Dinnerlude - teacher conference
TEY^2MS	Math: The Shape of Things to Come - conference
WOMA	Fall teacher conference
Y^4MA	Fall teacher conference

6. There will be prizes for the first few people who contact me with the magic word from this report. You should not have to ask for help from anyone in this task. Go to 12.

7. The Executive Directors had some pretty gloomy news regarding budget and membership. If you wish to skip the bad news altogether, go into politics. If you wish to hear about the budget go to 18. Membership information is to be found at 16.

8. OAME 96 plans for London are just floating along. Will you be there? It promises to let you get high on math, ... and not just from all the hot air either. Since you are doing so well following this path, now is the time to go to 19.

9. John Kersley reported on the joint New York/Ontario conference at Scarborough College last summer. It apparently was a real learning experience! The bad news is that the attendance was very low. The good news was that OAME was not liable for any financial loss, and we did produce some recommendations for future joint ventures.

10. Joan Green, CEO of the EQOA (if you think you know what this stands for, go into administration), spoke to all councillors on Friday night. Her responsibilities include the provincial testing of literacy and numeracy. At the time of her talk, the Minister had not made any decisions, but Joan supported a broader view of assessment than traditional multiple-choice testing. She was presented with a copy of OAME's Transition Years Assessment pamphlet.

11. The Ontario Math Olympics for grade 7 and 8 students will be held in Scarborough, June 1, 1996. It was exciting to hear that Rockwell International is enthusiastic about sponsoring the event and wants to be involved. I think that qualifies as good news. Now go to 20.

12. We found it very hard to budge the budget onto the positive side. As you read elsewhere (if you followed the correct path without any help), membership fees are down, expenses are up and so are concerns for the organization. There was a long, emotionally draining debate on various possibilities, and the final decision was that, to balance the budget, chapter rebates on membership fees would be reduced from 30% to 24%. To be cheered up now, go to 6. If you can handle more bad news, go to 18.

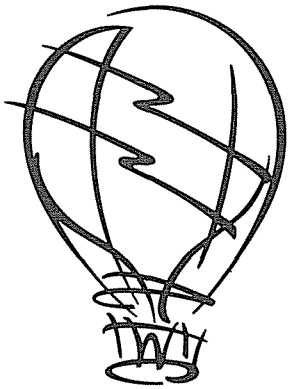
13. Following along hard on the successful production of the Transition Years Assessment document, a Junior project is under way. A large number of teachers and other educators are

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producing a range of activities and assessment tools for the Junior grades, that will be field tested in the winter months.

Volunteers will be needed to try out one or two activities, share the student results and their impressions. Contact Nancy Moore. Now go to 10.

14. Barry Onslow suggested chapters with a substantial bank balance may wish to invest some funds in OAME projects, such as the Junior Assessment Project, with interest free loans. This approach would produce a temporary rise in income (which is a corollary to outcomes based education). Now go to 13 to find out more about the project, or 12 for more on budget, or 28 for a good news story.

32. The Transition Years members are not resting on their laurels after the launch of the Assessment booklet. They are collecting personal reflections on problem solving and assessment to be compiled into a Gazette article. More committee reports may be found at 23 and 27, or read on.

35. The Math Equity breakfast at London in May will feature Nick Deleary, principal of an elementary school for the Chippewa of the Thames. He will be speaking on aboriginal education and aboriginal mathematics. Go to 26 for an aboriginal approach to geometry. Or read on.

16. The Executive Directors got to be the bearers of bad tidings. OAME membership is down to under 1400 (when it

was close to 2000 a few years ago).

This is disturbing in terms of teacher professional development. It is also a challenge as we looked at the financial implications - see 18. If you want a break now, go read the President's message. Remember to return to 17 when you are finished. An even more interesting pathway is via 28.

47. John Kersley again provided councillors with a resource package of interesting and useful handouts. What a help they are! Now go to 45.

17. Marilyn Hurrell reported, without stretching the point, for the Special Projects committee praising Judy Crompton for dealing so capably with Council's high expectations for materials in such a short time-line. Judy took it all in stride and the result is wonderful ... and the cost was held to a minimum by keeping overheads low. Judy received a standing ovation for taking Council's tall order and going up and over the top. Well done, Judy! And now a short pause while we regroup and move on to 23.

18. Did you know that, after producing the Gazette and reimbursing chapters, that what is left to Council of your $40 membership fee can be as little as $5.00, and in some cases members are actually subsidized by Council (five year memberships start looking like a really good deal).

37. You may send contest answers and your solutions to the imbedded problems (or to me) at <petesaar@village.ca>. If you are here, you should be finished reading this report. The correct path, the path of least resistance, the shortest distance between start and finish, ... well to put it briefly, you didn't get here from there.

19. Susan Stuart reported on some provincial initiatives across Canada. The Western provinces have developed a common framework for the math programs from 1 to 9 with a draft for 10 - 12. The Maritimes are similarly developing a common set of program expectations. It seems only Ontario and Quebec are going it alone. All have a common focus on assessment. If you want to continue in a pan-Canadian vein, go to 27. Otherwise go to 24.

20. Rob McLeish reported on the financial success of the first student Math Olympics, held last June in Ottawa. He had previously shared samples of chapter contest questions. Most chapter reports talked of their plans for team selection for next June. The kids really liked it and teachers found it very challenging and worth their time, so what are your grade 7/8 teachers doing to prepare? If you know the answer, go to the head of the class.

21. Dean Murray had some good news: *Gazette* advertising produced a profit for OAME. He reminded us all, and that means YOU too, to refer to the *Gazette* ad when ordering or requestion information from an advertiser. Now you really should hear some of the bad news to give you the motivation to enlist some new members. Go to 12, unless you've been there before, in which case you can go for coffee and then 16.

22. Dean Murray, on behalf of GVMA, had a mixed good/bad news report from OAME 95. It made money for OAME and GVMA (hurrah!) but not nearly as much as had been expected and hoped for (sob, sob). GVMA was praised for its hard work and dedication in running a conference that those attending found very worthwhile. For more on GVMA go to 5. For more on Conferences go to 5. For more on good news, go to 35. For morons, go to Queen's Park. [Note: the help function has been disabled]

23. The Early/Formative Years committee continues to produce useful, exciting activities for The Abacus. They are also looking to producing a Primary resource for teachers to connect the Provincial Standards to the classroom. Go back to 32 and try a different path this time.

24. Miscellaneous topics of discussion included revisions to the Constitution, NCTM Yearbook authors wanted, International Olympics team prospects for 1996, and Awards notices.

25. Help seemed to be the key word of the day. OAME needs help. Chapters are providing help to local teachers in great abundance. Go half way to 74.

26. An inclusive curriculum lets each student feel included. Many cultures, including the aboriginals, have cycles as a central theme. Why not start your geometry program with circles and curved lines instead of straight lines and boxes. You will be amazed where it leads and now many connections students can develop towards this approach. Now go to 8.

27. Specialization Years committee has a timely, relevant agenda: redesigning the secondary math program - not a simple task. Members are planning to compile a list of essential graduation outcomes through an e-mail conference.

The committee will accept any help that is offered, so if you have suggestions for what we should expect from graduating students, contact chair Todd Romiens. Do not go to 29.

28. Mary Donaghy, in reporting for Publicity and Membership, did not dwell on the dropping membership numbers nor the gloomy budget, but instead identified several plans the committee has to reach out to attract new members, to solicit charitable donations, to recognize excellence in students, and to make OAME-ers feel proud of their organization. This committee has re-abled the help function. Do not go to 29.

29. If you are reading this, you did not follow instructions nor the correct path. This is the end, J. Alfred Prufrock, and after looking at the balance between the good news and the bad, we can see that OAME is going on, not out, with a BANG. Now go to 37. ▲

CALL FOR AWARD NOMINATIONS

LIFE MEMBERSHIP

To qualify for Life Membership in OAME/AOEM a candidate shall have:

- 10 or more accumulated years of membership in OAME/AOEM
- demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular

Deadline for nominations: January 31, 1996.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award, to be presented annually to a person who does not necessarily meet the criteria for the Life Membership Award (i.e., not necessarily an educator or an OAME/AOEM member.) The award is aimed at recognizing people who have supported OAME/AOEM, such as publishers, educational suppliers, hotel personnel, etc.

At this time OAME/AOEM is requesting nominations for consideration for this year's award, to be presented at the Annual Conference.

Deadline for nominations: January 31, 1996.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM has established the Morley MacGregor Award, to be presented annually to an elementary school teacher of mathematics who has demonstrated excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME/AOEM is requesting nominations for consideration for this year's award, to be presented at the Annual Conference.

Deadline for nominations: January 31, 1996.

KENNETH D. FRYER AWARD

In Memory of the late Dr. K.D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, has established the Kenneth D. Fryer Award for Excellence, to be presented annually to a secondary school mathematics department demonstrating excellence in mathematics education as well as contributing to the overall development of students beyond classroom activities.

At this time OAME is requesting nominations for consideration by the committee for this year's award - to be presented at the Annual Conference.

Deadline for nominations: January 31, 1996.

Don Attridge Award

In recognition of the contributions made by Don Attridge, a founding member, former president and a previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the Spring of 1995. It will be presented this year for the first time. The award will be presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

Deadline for nominations: January 31, 1996.

AWARD NOMINATION FORM

Date of Nomination: _____

Award (Check One) ❑ **Life Membership** ❑ **Mona MacGregor Award** ❑ **Morley MacGregor Award**
❑ **Kenneth D. Fryer Award** ❑ **Don Attridge Award**

I/We nominate (Name of Nominee) _____

School, Organization or Business of Nominee _____

School, Organization, or Business Address of Nominee _____

Principal of Nominee (for the Kenneth D. Fryer Award) _____

Mathematics Department Head of Nominee (for the Kenneth D. Fryer Award) _____

Nominated by (Name, Address & Phone) _____

Please attach a separate sheet to the completed nomination form, outlining the reasons for your nomination and send to
David Zimmer, John Fraser Secondary School

NOMINATIONS FOR OAME/AOEM COUNCIL 1996-97

Each year, elections are held for President-Elect, 3 Directors and 7-10 Councillors for the Council of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 31, 1996, to:

Todd Romiens
Past-President, OAME

Include the nominee's qualifications with the Nomination Form. The Nomination Committee will make a final selection of those nominees who will stand for election. Their names and qualifications will be published in March, 1996. Voting will be by mail by members of the OAME/AOEM. The results of these elections will be made known at the Annual Meeting of the OAME/AOEM in London in May, 1996.

NOMINATION FORM

I nominate: _____

Address: _____

_____

For the position(s) of: _____ **President-Elect**

_____ **Director**

_____ **Councillor**

_____ **Nominator**

_____ **Address**

_____

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DEAN A. MURRAY
HEAD OF MATHEMATICS
MILTON DISTRICT HIGH SCHOOL



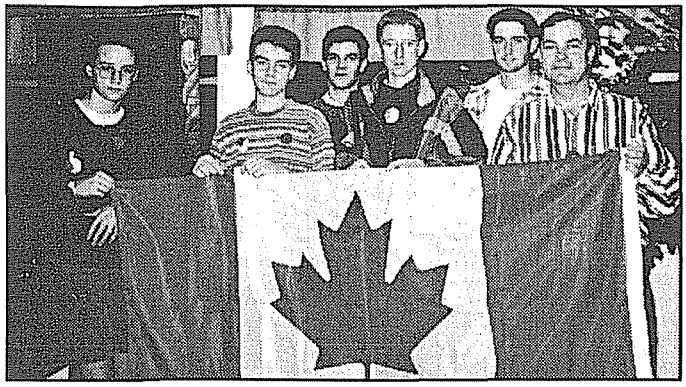
Dean Murray is head of Mathematics of Milton District High School. For years, he has chaired the Grand Valley Math Association's Program Committee and recently became President. Since this is not enough to keep him busy, he is also the Gazette Advertising Manager!

Approximately 73 staff and students from Milton District and White Oaks High Schools, as well as representatives from the 820 Milton *Blue Thunder* Squadron of the Royal Canadian Air Cadets together with parents/chaperones, left for Cocoa Beach, Florida on Thursday, November 9. The objective: to witness the fourth Canadian astronaut, Chris Hadfield, launched into space aboard the Space Shuttle Atlantis.

Major Hadfield is the first Canadian to perform the role of Mission Specialist on a shuttle flight. Chris graduated as an Ontario Scholar from Milton District High School in 1977. Earlier he attended White Oaks High School for grades 9 to 11.

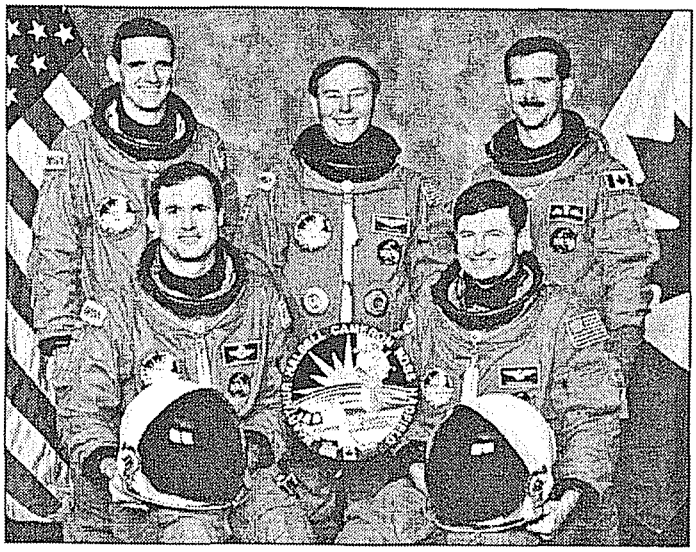
After leaving the Milton Go-station at 10:05 Thursday, November 9, we finally arrived at Cocoa Expo Sports Centre at noon the next day — 26 hours on a bus!! Needless to say, we were all very excited, and very much in need of a shower and sleep. However, adequate sleep for most of us would have to wait until we returned home. There were too many exciting things to do and see.

After lunch at a nearby restaurant, we spent the afternoon swimming, enjoying the sunshine and settling in. The less adventurous amongst us crept away to catch a few winks. Perhaps they were wiser, because the ride to the viewing site was scheduled for 4 a.m. the next morning.



Prior to Departure to viewing site at 5 a.m.
Left to right
Matthew Singleton, Ryan Kovacevich, Matt Kindree, Andrew Gates, Steve Liddon, Dean Murrary

Fast forwarding to 5 a.m. Saturday morning we arrived at the viewing site 2 miles (3.6 km) from the launch pad under a clear sky, and warm 70 degree Fahrenheit (21 degree Celsius) weather. With us was Marc Garneau, the first Canadian to travel into outer space. Several Miltonians took the opportunity to have their pictures taken with Major Garneau. Obviously, we were all very excited. Imagine our disappointment when the launch was "scrubbed". Unfortunately there were weather problems at three of the sites around the world used for emergency landing.



The Crew of *Atlantis*

Chris Hadfield top right corner

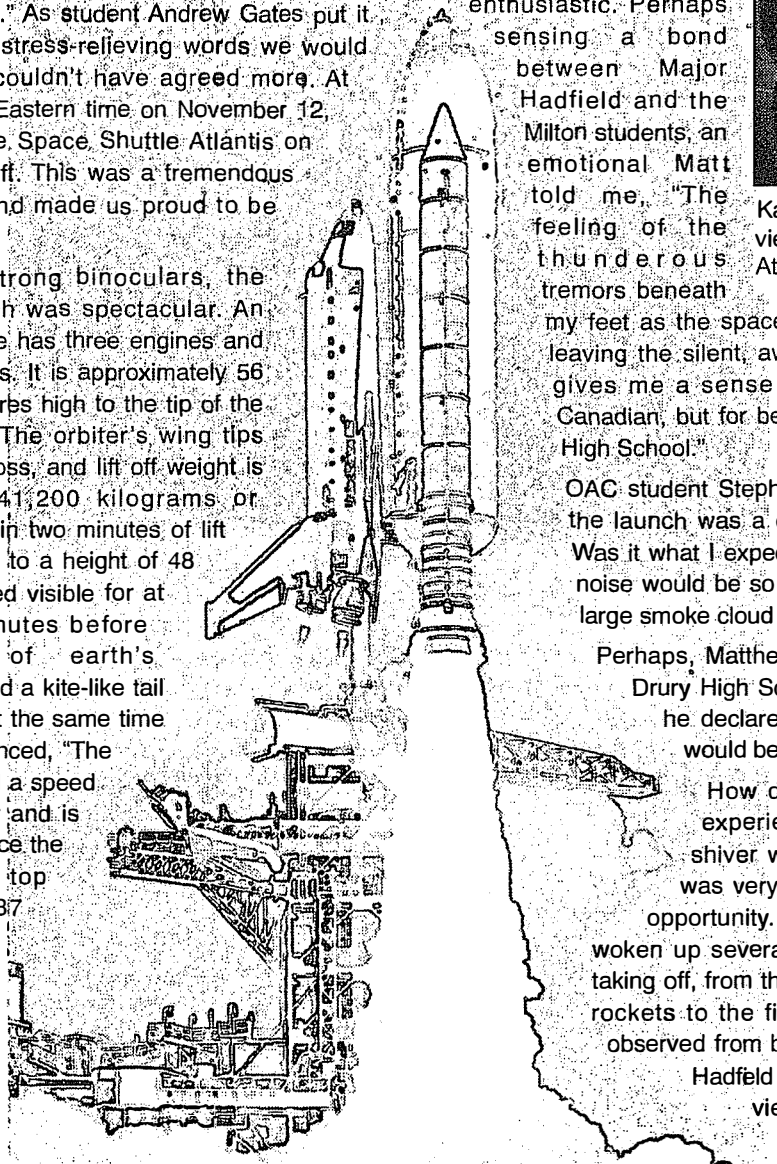
The launch was rescheduled for 7:30 a.m. Sunday. This time we had a 3:30 a.m. wake-up call. Sunday dawned much windier and much colder than the previous two days. Interestingly the probability of launch was higher than on Saturday since the other sites around the world now had clear visibility. It all depended on whether the dark clouds overhead dissipated over the next two hours. Why the concern over the weather at all sites? Apparently, the shuttle Commander must have a clear view of the geographical area below, for if by some mischance the shuttle runs into trouble and threatens a heavy-populated area, the Commander is charged with

the responsibility of detonating the shuttle, thus sacrificing himself and the rest of the crew.

For the next two and a half hours, we held our collective breath. Our organizer, Barry Chamberlain, who had all his fingers, both legs, and both arms crossed, together with the accompanying parents, Sue and Nick Klip, Richard Nagel, and Captain Rex Sutcliffe of the 820 Milton Cadet Squadron, were all in the same high state of anxiety.

At seven-twenty to our relief, the loud speaker boomed, "We have go for launch." As student Andrew Gates put it, "Those were the most stress-relieving words we would hear that morning." I couldn't have agreed more. At precisely, 7:30:43 a.m. Eastern time on November 12, 1995 we witnessed the Space Shuttle Atlantis on mission STS-74 blast off. This was a tremendous moment for all of us and made us proud to be Canadian.

Viewed through strong binoculars, the aftermath of the launch was spectacular. An assembled space shuttle has three engines and two solid rocket boosters. It is approximately 56 metres long and 76 metres high to the tip of the orbiter's vertical tail. The orbiter's wing tips measure 24 metres across, and lift off weight is an unbelievable 2,041,200 kilograms or 4,500,000 pounds. Within two minutes of lift off, Altaritis had climbed to a height of 48 kilometres, and remained visible for at least another few minutes before disappearing out of earth's atmosphere. It left behind a kite-like tail of clean white smoke. At the same time the loud speaker announced, "The shuttle has now reached a speed of 2,450 miles per hour and is increasing." In outer space the shuttle would reach a top speed of 27,887 kilometres per hour. Incredible!



All of us will take away fantastic memories from this experience: the exhilaration of watching the shuttle jet off into space like a great white bird; watching the Solid Rocket Boosters detach and fall into the ocean; even welcoming the particles of acid which burned souvenir dots into our clothing and irritated our eyes.

What did the students think? Ryan Kovacevich, a grade 10 Milton student who wants to be an aerospace engineer, wondered what Chris Hadfield was thinking as he was projected upwards into the skies. Ryan was visualizing perhaps a future for himself when he said, "Watching the launch has always been one of my greatest dreams."

Matt Kindree was also enthusiastic. Perhaps sensing a bond between Major Hadfield and the Milton students, an emotional Matt told me, "The feeling of the thunderous tremors beneath my feet as the space shuttle slowly left the ground leaving the silent, awe-struck crowd in amazement gives me a sense of pride, not only for being Canadian, but for being a member of Milton District High School."



Katie Hilts and Marc Garneau at viewing site before take-off of Atlantis.

OAC student Stephen Liddon exclaimed, "Viewing the launch was a once in a life time experience. Was it what I expected? Yes, but I never knew the noise would be so loud and that there would be a large smoke cloud afterwards."

Perhaps, Matthew Singleton, a student at E.C. Drury High School, voiced all our feelings as he declared, "It was not what I thought it would be; it was a lot better."

How do I personally feel about the experience? Without exaggeration, I shiver when I think about the lift-off. I was very fortunate to be given such an opportunity. Since returning home, I have woken up several times, visualizing the shuttle taking off, from the first thundering of the booster rockets to the fiery trail clearly visible as we observed from below. I kept wondering how the Hadfeld family must be feeling, as they viewed the awesome event from the family site one mile from the pad. Silently, I wished God's speed and a safe journey to all the astronauts.

Earlier I mentioned trip organizer Barry Chamberlain. Barry deserves much recognition and thanks from all of us for his work over the past one and a half years. As a result of his efforts, we were privileged to witness one of the most spectacular events of our times. ▲

▲ OAME/AOEM BALANCE SHEET AUGUST 31, 1995

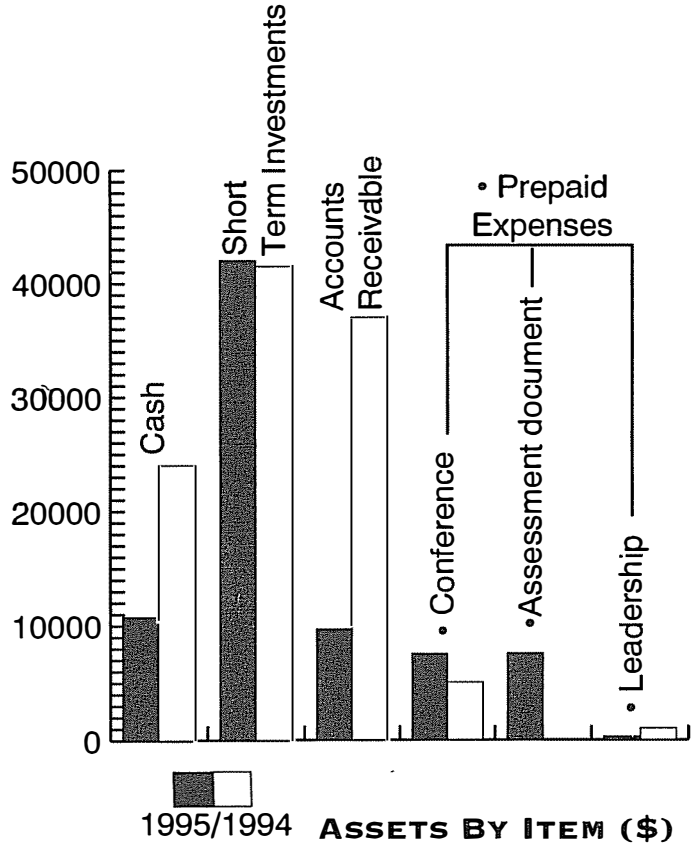
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NOTICE TO READER

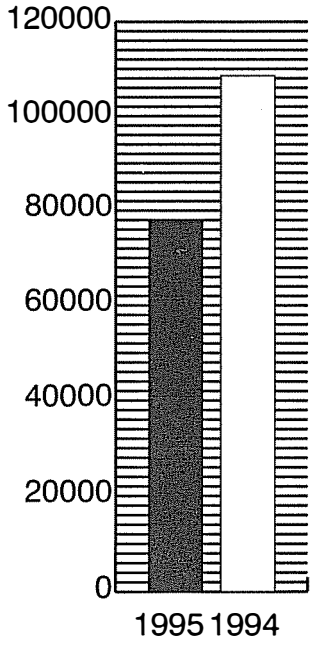
We have compiled the Balance Sheet of Ontario Association for Mathematics Education (Association Ontarienne Pour L'Enseignement Des Mathématiques) as at August 31, 1995 from information provided by the Directors. We have not audited, reviewed or otherwise attempted to verify the accuracy or completeness of such information. Readers are cautioned that these statements may not be appropriate for these purposes.

Markham, Ontario
September 19, 1995 ROBERT M. DAVIES
Chartered Accountant

CURRENT ASSETS 1995/1994

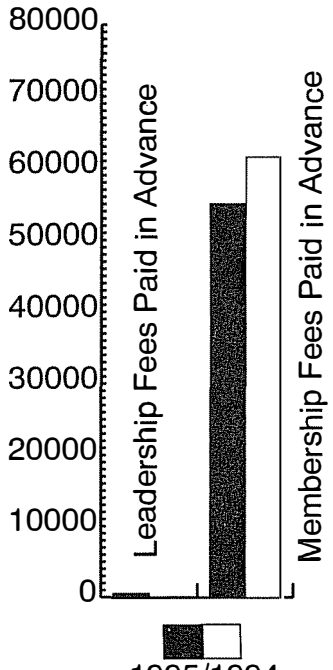


ASSETS BY ITEM ($)

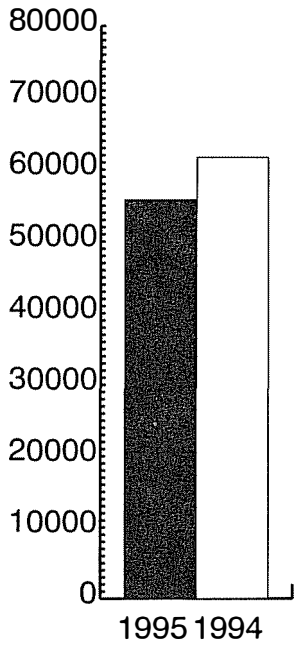


ASSETS TOTAL ($)

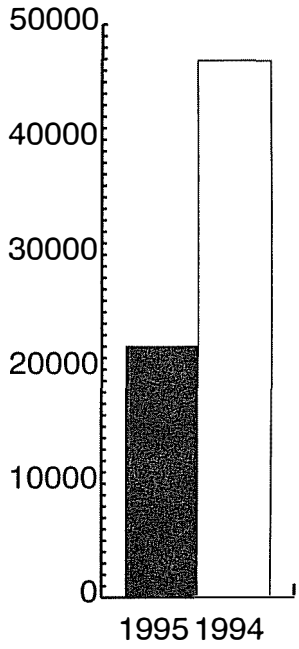
LIABILITIES AND SURPLUS



BY ITEM ($)

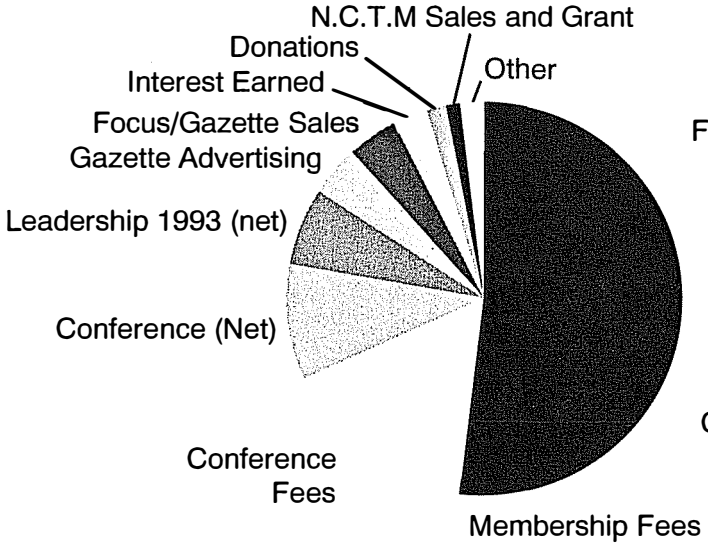


ANNUAL TOTALS ($)

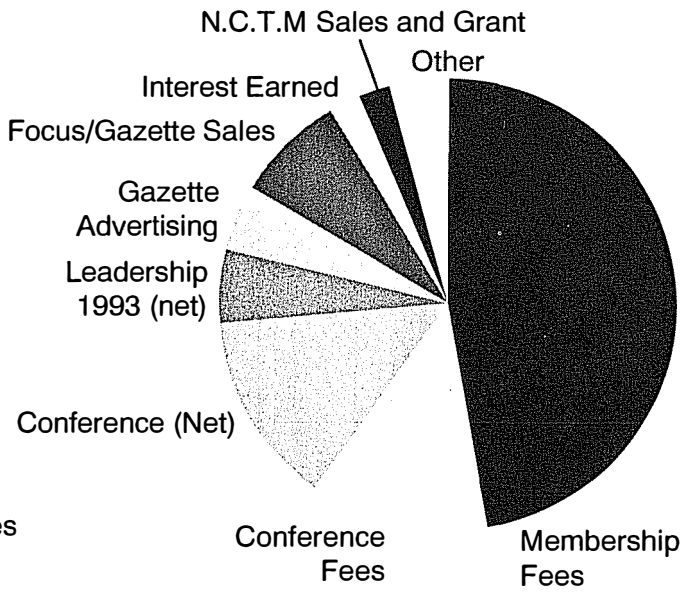


ACCUMULATED SURPLUS ($)
(NOTE 1)

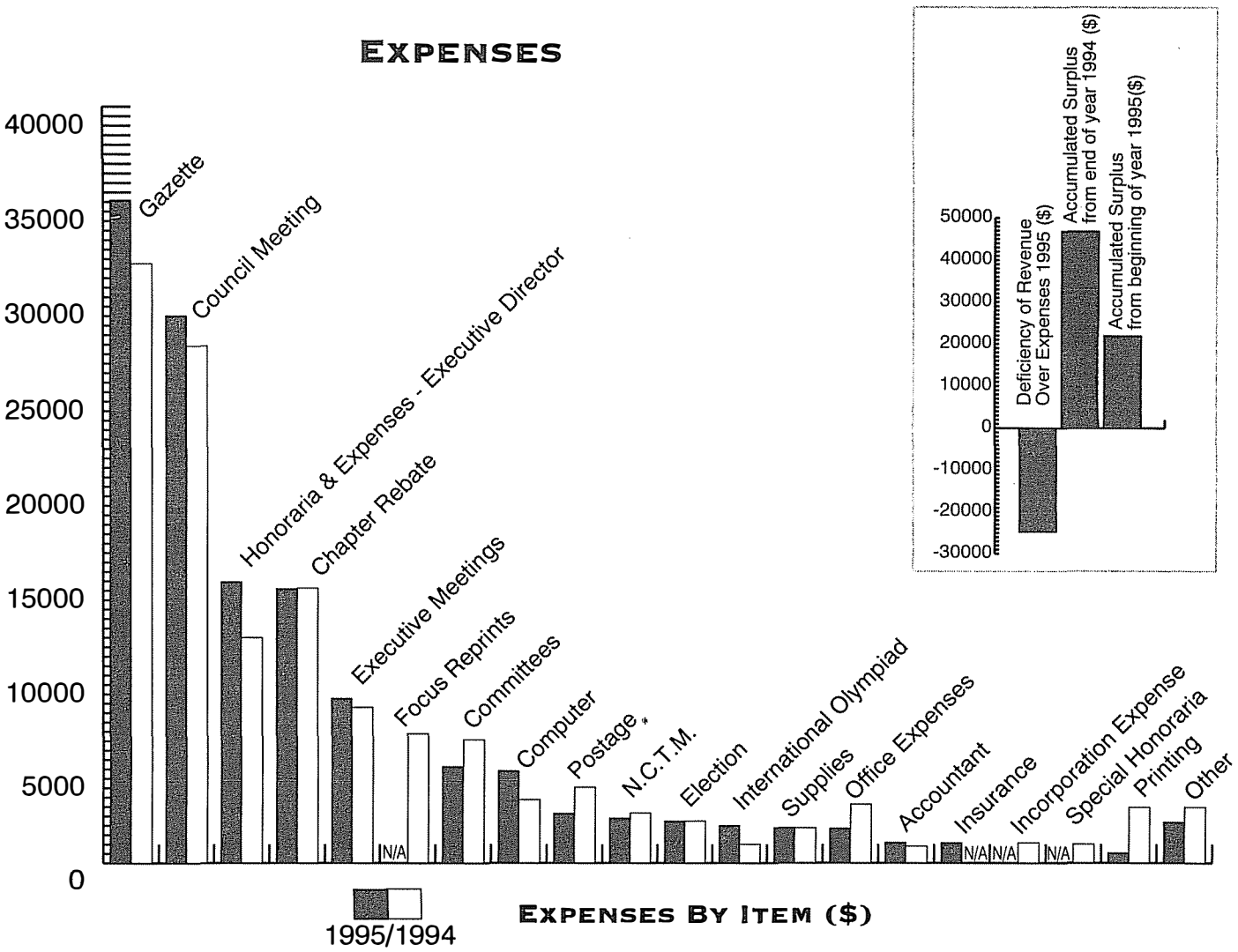
REVENUE



1995 ($107 541)



1994($140 238)



NOTES TO FINANCIAL STATEMENT

unaudited-See Notice To Reader
AUGUST 31, 1995

Operations and Significant Accounting Policies

1.The company was incorporated as a non-profit corporation without share capital in the Province of Ontario by Letters Patent dated January 22, 1993. As of September 1, 1993, all assets and liabilities of the prior unincorporated association were transferred to the company.

2.These financial statements have been prepared on the basis of the following policies:

(a) Revenue recognition-

(i) Membership fees are renewed in advance on their due date (four times per year) in association with Gazette mailings. Membership fee revenue is then pro-rated monthly and fees that apply after the year-end are shown on the Balance Sheet as Membership Fees Paid In Advance.

(ii) Revenue from the annual conference and the leadership conference are recorded at the time of the conference.

(b) Expenses-

Operating costs are recorded on the accrual basis.

(c) Fixed assets-

Fixed asset purchases are expensed in the year of acquisition.

(d) Income taxes-

The company is a non-profit organization and, accordingly, no income taxes have been provided in these financial statements.

As of January 1, 1995, the association was registered as a charitable organization under paragraph 149(1)(f) of the Income Tax Act.

O.A.M.E. 1995-1996 Council

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PAST PRESIDENT
Todd Romiens

PRESIDENT-ELECT
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DIRECTORS
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Peter Lenczewski

David Zimmer

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GOLDEN - Robert Saari

GVMA - Murray McLennan

ISOMA - John Kersley

MAC^2 - Bill Morrison

NOMA - Dan Charbonneau

NWOAME - Joe Hall

PINE RIDGE - Rick White

QSLMA - Douglas Evans

SAME - Anne Pedersen

SWOAME - Don Gray

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WOMA - Mary Howe

Y^{4}MA - Shirley Dalrymple

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COUNCILLORS - TERM EXPIRING 1997
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Shirley Fairfield

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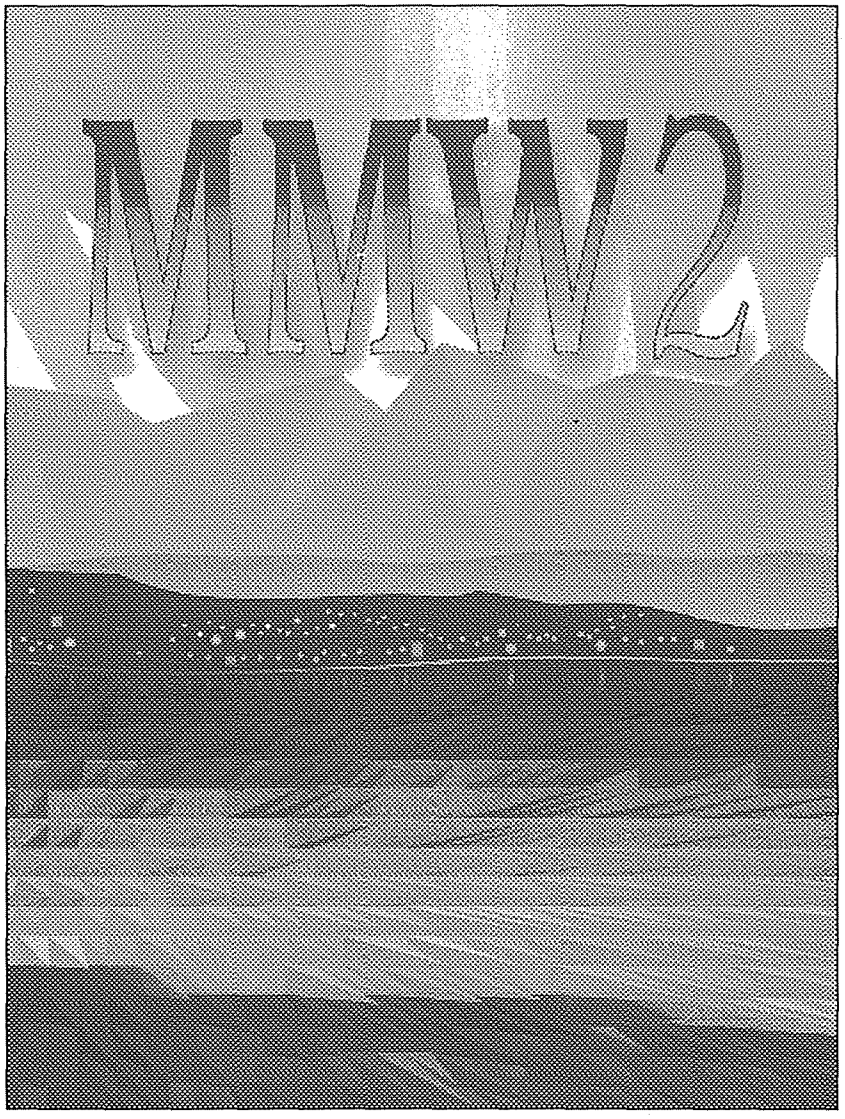
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NCTM REPRESENTATIVE
Mickey Sandblom

Editor's Note: Unfortunately, the changes for this page were misplaced and could not be recovered in time for publication. My apologies to Council.

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