Vol. 37 #2, December 1998

Ontario Mathematics Gazette

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Math Trails - An Award Winning Method for Connecting Mathematics with the World Around Us

Some Thoughts on the New Mathematics Curriculum for Ontario - part 2

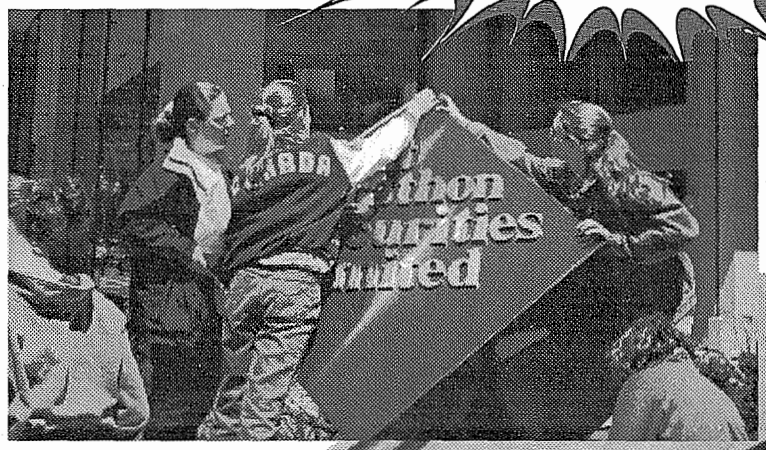
Simon Says Do This: An Indictment of Rote Learning

"Santa's Christmas Madness or Santa's Christmas Scam?"

OAME 1997/98 Financial Statement

Ron Lancaster's Math Trails are fun!

They are guided walks during which students answer mathematical questions related to architecture, public art, science, geography and pretty well anything encountered along the route. In this article Ron shares his thoughts about his Math Trails and hopes that other teachers will be inspired to create a Trail of their own.



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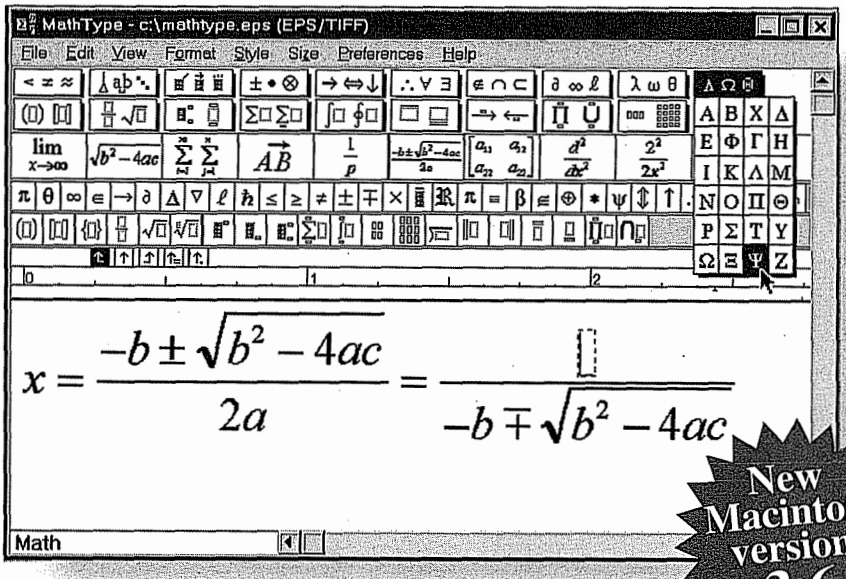
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Deadlines for advertisements are January 23 for March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.
Full page advertisements are to be on 8.5" by 11" paper with a minimum of 1/2" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

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Articles for the Gazette should be submitted to John Egsgard at the address on the left.

Deadlines for information to be included in the Gazette or the Abacus are: June 15 for the September issue, October 15 for the December issue, January 15 for the March issue, and April 15 for the June issue.

Notice to Authors

Your article should be typewritten or word-processed, double-spaced on one side of 21.6 cm x 27.9 cm paper with 3 cm margins. Diagrams and figures should be drawn by computer, if possible, or neatly drawn in black ink in camera-ready form. Please submit four copies to the editor and keep one as protection against loss. Submissions are blind-reviewed, so please send your full address and affiliation on a page separate from the manuscript. Computer files of text, and graphics where possible, will be requested later if your article is accepted.

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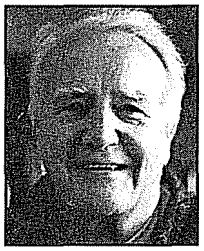
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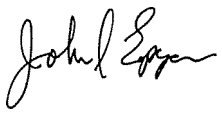
JOHN EGSGARD

You must have noticed that the September issue of the *Gazette* did not arrive until sometime in October. This was an indirect result of the fact that education is still being managed for the good of the politicians rather than for the good of the students. It was impossible to deliver something as educationally worthwhile to schools that had been closed politically. Would it not be wonderful if this never happened again!!

Despite the problems encountered by our schools OAME is continuing its work for the good of the students and their teachers. Read carefully president Ralph Connelly's message and Louis Lim's ***Directors' Dialogue***.

On our cover you will find some photos to accompany the lead article "**Math Trails - an Award Winning Method for Connecting Mathematics with the world Around Us**". To help us in our assessment of the revised mathematics curriculum we have two articles: "**Some Thoughts on the New Mathematics Curriculum for Ontario - Part 2**" which is a continuation of ideas presented in the September, 1998 issue, and "**Simon Says Do This: An Indictment of Rote Learning**". Classroom teachers will find "**Santa's Christmas Madness or Santa's Christmas Scam?**", "**About Problem Solving: Mathematically Amusing Numbers**", and "**The Devil's Advocate: Martial Arts**" helpful, interesting, and thought provoking.

I hope that the holiday season gives everyone the opportunity to recover from the politically caused problems of the fall.

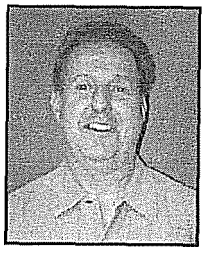


Error Correction

OAME *Gazette* Volume 37, #1 September 1998, *Letter to the Editor,* page 3, paragraph 2, line 1.

Should read: Consider y = f(x) changed into a(y+b)=f[c(x+d)].

▲ President's Message



RALPH D. CONNELLY

I've decided to use "my space" this issue to pay tribute to a very special group of people with the organization of OAME—the OAME Board of Directors. As President of OAME, I frequently find myself being called upon to respond to questions, comments, etc. that are outside my realm of expertise. (OK, I know there are some wags out there who believe that **everything** is outside my realm of expertise, but I'll ignore that for now!) Knowing that there are members of the Board that I can always call upon in such circumstances certainly makes my job much easier and more comfortable.

A recent letter I received contained a comment that caused me to reflect on the value of the OAME Board. Although the person was focusing on concerns regarding the new Secondary Curriculum—something by the way that I hope all OAME members involved in teaching at that level will take the time to do—the letter contained one sentence that "bounced around" in my head for a long time, "What are you people doing?"

It seemed that the "you people" being referred to was the OAME Board, so I thought I'd reflect on some of the things that the Board has done just within this past year. It should be noted that in virtually all of the identified accomplishments there were LOTS of others involved and this tribute certainly extends to everyone who made these accomplishments possible!

OAME had an absolutely fantastic Annual meeting in May at North Bay. The conference organizers are to be applauded for their efforts in making this a memorable, valuable, and enjoyable experience for everyone who attended.

A new document, *Teaching with Technology*, is now available through OAME, dealing with ideas for the use of a variety of technologies such as graphing calculators in teaching mathematics at the high school level.

A set of posters has been developed that give a clear and organized picture of the gr. 6-8 Ontario Mathematics Curriculum, organized by strands and sub-topics within strands. The posters will prove invaluable in planning the year's math program for teachers at those levels.

The document, *Linking Assessment and Instruction in Mathematics-Junior Years*, has been revised to reflect the expectations of the new Ontario Mathematics Curriculum, Gr. 4-6, and is now available through OAME.

The document, *Linking Assessment and Instruction in Mathematics-Primary Years*, is nearing completion. This is a joint project of OAME, CAPE (Consultants and Coordinators Association of Primary Education), and OMCA

Several members of the OAME Board gave of their time this summer to work on the Ministry of Education's Exemplars Project for Gr. 1-8 Mathematics.

When the Ministry of Education decided to go with a "Request for Proposals" format for developing the new Secondary Curricula, members of the OAME Board together with members of OMCA and the Fields Institute immediately took action to organize a proposal, and has thus been successful in ensuring that our organization has significant input into the new curriculum. Although it must be remembered that the new Curriculum is a product of the Ministry of Education and Training (MET), we are indeed fortunate that so many dedicated members of OAME have given of their time and talents to this project, to try to ensure that the curriculum reflects the best view possible of what high school mathematics should be.

To the "you people" of the OAME Board and beyond—"You people" are doing amazing things! And doing them in probably one of the most difficult "eras" of education, where teachers are being devalued and demoralized as never before! You've resisted the temptation to say "What's the use!" and instead provided much needed support for teachers at every level in this province. **WAY TO GO, FOLKS**!!!

Call for Gazette Editor

The Ontario Mathematics Gazette *is taking applications for a two year term as editor commencing with the September, 1999 issue.*

If you are interested or would like to recommend a colleague for this position, please contact Executive Directors Dave and Bonnie Alexander at the address on the OAME 98/99 Board on page 32.

▲ MATH TRAILS - AN AWARD WINNING METHOD FOR CONNECTING MATHEMATICS WITH THE WORLD AROUND US

RON LANCASTER
ST. MILDRED'S-LIGHTBORN SCHOOL



Ron Lancaster's Math Trails are fun! They are guided walks during which students answer mathematical questions related to architecture, public art, science, geography and pretty well anything encountered along the route. In 1998 Ron was honoured with an unprecedented third Hilroy Award for his Toronto Math Trails, created while he was on staff at St. Mildred's-Lightbourn School in Oakville. In this article Ron shares his thoughts about his Math Trails and hopes that other teachers will be inspired to create a Trail of their own.

With the onset of spring or on a fall day when the weather is summer-like, a question often asked by students is "Can we go outside today?". In my early years of teaching I tended to ignore requests to go outside. I suppose I was worried by several things. How would my Principal react? What would we do outside? Could I trust my students to stay on task? Could any lesson no matter how creative or engaging be able to compete with the warmth of the sun? These are all legitimate concerns for any teacher and they tend to be so daunting that the issue of going outside remains on the outside.

I began to think about creating a Math Trail seven years ago when I was working on a television series for TVOntario. Many of the forty shows from this series were filmed outside of the TVOntario studios in banks, restaurants, garden centers, the Ontario Science Center and even from a boat in the middle of the lake at Kelso Conservation Area. I was disappointed that my students were not with me when I did these shows as it would have been a wonderful opportunity for them to see and feel the presence of mathematics in the world around us.

About a year after the TVOntario series was completed, I began conducting a Mathematics Trail for teachers at the Phillips Exeter Academy Mathematics and Technology Conference in Exeter, New Hampshire. Vince Delisi, the Mathematics Department Head at the Country Day School, and I designed the Exeter Trail together and we created a series of mathematical questions that teachers worked on as they walked outside along a set route. We refined this Trail for several years and eventually it was published in the Mathematics Teacher (March 1997), a magazine for teachers published by the National Council of Teachers of Mathematics (NCTM). This Trail has become an annual feature at the Exeter Conference and Vince and I have taken hundreds of teachers on these walks. We have experimented with Math Trails on the campus of Phillips Exeter Academy, in a candy store, a toy store and elsewhere in the town of Exeter and Vince and I are convinced that a Math Trail could be done almost anywhere.

My first attempt at creating a Math Trail for my students began in the parking lot of St. Mildred's. My grade 7 students worked on algebraic and combinatorial questions related to licence plates, they calculated the aspect ratio for various car tires, they analyzed the numbers used to mark parking spots and they measured the glass surface area for a number of cars. About a week later a student who liked the licence plate questions brought me in a tie decorated with licence plates, along with a certificate that proclaimed that I had a lifetime licence to do math anywhere and anytime. With that type of encouragement I decided to venture out from the parking lot into the city of Toronto.

For the next four months I was a frequent visitor to the downtown area of Toronto. I walked everywhere and got in the habit of looking up, down and in every other direction imaginable. I eventually put together a handbook that contained pages and pages of math questions. John Kersley, a mathematics teacher at Hillfield Strathallan College joined me in Toronto for a dry run and he offered me a number of excellent suggestions. I also had several teachers, parents and students follow the Trail without me and I made a

number of important changes to the Trail based on their feedback.

On April 23, 1996 I met my three grade 7 classes at the Oakville GO Station and together we headed into Toronto. In order to help supervise the students I had over a dozen parents join me, along with five staff members. Also on the trip were all of my Calculus students and this turned out to be the best part of the day. The involvement of my senior students came at a suggestion from Dr. Rachelle Sender, Co-founder of the Women Inventors Project. Dr. Sender has done a great deal of work in the area of promoting science and math for young women and when I showed her the project, she suggested the idea of having my senior students act as mentors for my Grade 7 students. The mentoring worked exceptionally well throughout the day and for the rest of the year and the Grade 7 students felt a special connection to the older students in the school.

The setting for this Math Trail was the Financial District of Toronto and we were in the city for the entire day. In order to take full advantage of the presence of my Calculus students I decided to divide my grade 7 students into very small groups of three to four students. Each group started the Trail at a different place and this avoided having large groups of people in one place at the same time. Some of the problems were fairly simple and direct and I identified them as being questions that had to be answered on the day of the Trail. For the remaining questions, the students collected data or made observations and they knew that they would be given time after the outing to make use of this information. These questions tended to be ones that required a great deal of thought or time to answer.

Over the course of my nineteen years of teaching I do not recall any project that has been as well received by my students as this Math Trail. The Grade 7 students were thrilled with the interactive nature of the questions and they enjoyed the opportunity to learn and do mathematics in a setting outside of the classroom. In their evaluations they spoke about how important it was to them to see how what they were learning was connected to their lives and to other subject areas. They also placed a great deal of value on having senior students on hand to help them with the questions and guide them along the Trail. My Calculus students were also delighted with the outing and in addition to having an opportunity to review mathematical topics studied in earlier years, they enjoyed their mentoring role immensely. The parents and other staff members were



"How many circles can be formed from the various pieces in the CBC logo?" Photograph taken at the John Street entrance to the CBC Building in downtown Toronto.



Students were floored with this question that involved the floor of the Simcoe Building in downtown Toronto.



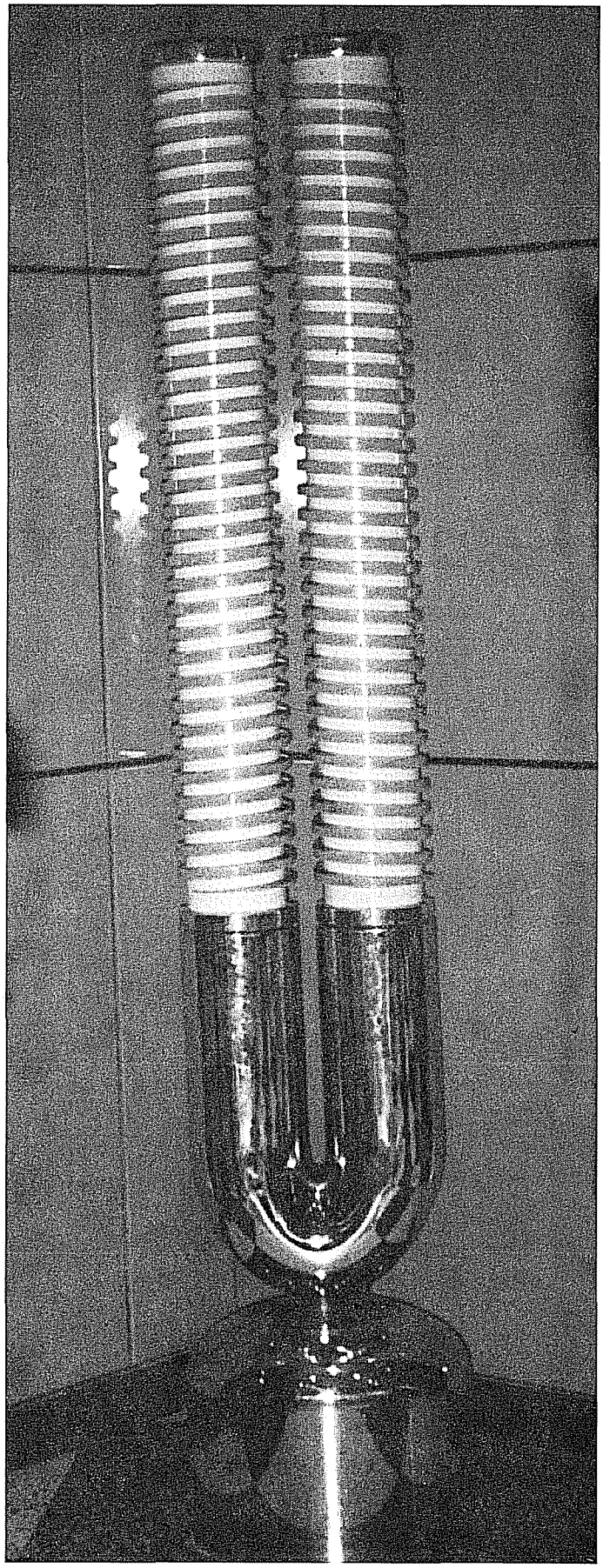
Grade 7 students, a Calculus student and a student from York Univesrity who co-presented a workshop on Math Trails with me at the 1998 OAME Conference in Toronto.

also very pleased with the day and the most common comment that I received was that they wished that they too could have had this type of experience when they were in school. This comment was also made by a number of my Calculus students and some of the students who had not done so well in mathematics through high school expressed the feeling that if they had been given this type of creative and engaging approach they might have done much better.

For about three weeks after our visit to Toronto my grade 7 students spent every period working on the questions from the Trail. Some of the questions involved content that they had already studied, while others involved new material that they had never seen. While I occasionally discussed questions with the class as a whole, I spent most of my time moving from group to group where I answered questions and added clarity to questions that could have been worded better. With some students I added in extra enrichment questions for them to work on. The classroom atmosphere was outstanding and it reached a climax the day they handed in their completed project. There was a sense of excitement at having worked through fifty pages of challenging problems and my sense of excitement lasted until I realized that I had to mark all of this stuff!

Over the summer I reflected on what I had learned from creating this Trail and I decided that in the upcoming year I would attempt to run the Trail again. Once the school year began I quickly realized that my new students had heard about the Trail and they were very keen to follow this Path to Math themselves. The year turned out to be dominated by Trails and the students took part in three of them. Between the Trails they worked through the content of the grade 7 course in a traditional chapter by chapter approach.

The setting for the first Trail of the year was the Eaton's Centre in downtown Toronto along with several office buildings nearby. Students studied the geometry of the Centre and they visited stores such as the Body



Right *A highly mathematical sculpture that spins round and round in the main hallway of the CBC Building in downtown Toronto.*

Cover *The dramatic five story tall window on the side of a newly renovated building at 100 University Street in downtown Toronto. "What measurements need to be made in order to conclude that this object is really a cube?" Piece of Public art at the corner of King Street West and York Street in downtown Toronto.*

OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration;

- demonstrations of excellence and creativity in mathematics education.
- specific contributions to mathematics education in the school, region, province or nation.
- demonstrates leadership in mathematics education in the school, region, province or nation.
- support of and contribution to the activities of OAME/AOEM.
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld form consideration.

OAME/AOEM LIFE MEMBERSHIP AWARD

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and accumulated 10 or more years of membership in OAME/AOEM.

KENNETH D. FRYER AWARD

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

DON ATTRIDGE AWARD

In recognition of the contributions made by Don Attridge, a founding member, former president and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

AWARD NOMINATION COVER PAGE

Send to: Peter Saarimaki

Intended Award (please check one)

❑ Life Membership ❑ Mona MacGregor ❑ Morley MacGregor ❑ Kenneth D. Fryer ❑ Don Attridge

Nominee Information:

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

Home Phone ( ) _____

Nominator Information:

Name of Nominator

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

Home Phone ( ) _____

Shop, the Nature Store and the Disney Store. In one of the office buildings they stood on a bathroom scale while moving up and down in a elevator and they created a graph of how their weight changed over time. In another building they studied various pieces of public art and used pillars with a mirrored surface to learn about anamorphic art and non-linear transformations.

The setting for the second Trail was St. Mildred's-Lightbourn School. This Trail was also used during an annual Open House. By following the Trail visitors to the school were able to become acquainted with the school as well as the grade 7 mathematics curriculum. The questions for this Trail involved student art on the walls, the structure of the building and the large photographs of graduating classes.

The setting for the third Trail was the Financial District and it was a modified version of the Trail from the previous year. One of the most popular questions on the Trail required the students to collect data for the distance between two students, one moving up an escalator and the other moving down another nearby escalator. Another popular question involved a physics experiment that was carried out on the side of a piece of public art near Roy Thomson Hall.

In addition to these three Trails, one of my grade 7 classes co-presented a Math Trail with me at the 1996 OAME Conference in Toronto. This session began with a brief overview of the concept of a Math Trail and then the forty teachers who attended the session took a walk in the city with my grade 7 students. The reaction from these teachers was highly positive and my students enjoyed meeting and speaking with teachers from all over Ontario.

In an effort to involve other people with these Trails I arranged to have a number of students from the Faculty of Education at York University participate in both Toronto Trails and I also invited a number of special guests to join us. The guests included math consultants and individuals involved in public art.

Last year I was curious to see if the questions that I had created for my grade 7 students could be used with students at other grade levels. I had students in grades 9 - 12 and in the Finite Mathematics course do the Trail and I decided not to modify any of the questions. What ended up happening was that these students answered the questions using strategies and techniques from their particular grade level and it was wonderful to see how students answered the same question in so many different ways.

One of things that I have particularly enjoyed about this project is the opportunity to help other teachers design Trails for their students. While I continue to give many workshops on graphics calculators, newspapers and magic, many of my workshops and consultant work involves the creation of a Math Trail. This year I have taken a year off from teaching and I will be traveling throughout North America as well as to Hong Kong, Singapore and Jakarta to work with teachers and to speak at conferences about Math Trails.

I am particularly excited about a Math Trail that I am creating for teachers at the 77th Annual NCTM Conference in San Francisco. The setting for the Trail will be the Museum of Modern Art and I have invited two other teachers, Kimberly Nick from the Bishop Strachan School in Toronto and Brigitte Bentele from the Trinity School in New York City to work with me on this project. I will also be accompanying students, parents and teachers from the Nueva School in California to the Museum in January 1999 and these individuals will be field testing the questions that we have created. They will also be developing new questions which is something that I encouraged my students at St. Mildred's to do as well.

I feel very fortunate to have been honoured with a third Hilroy Award for the creation of these Math Trails. I would be remiss if I did not offer my thanks to the various administrations under which I have worked throughout my teaching career. While at Oakville Trafalgar High School through the 1980s and at Applewood Heights High School in 1991-1992 I had a number of outstanding Principals who gave me the freedom to try new ways of teaching mathematics. Bob Loree, my Department Head at Oakville Trafalgar was an incredible mentor and I learned a great deal about teaching from him. At St. Mildred's I was also fortunate to work with several special administrators, including Ann Veale, the Vice-Principal and Lynda Duckworth who was Principal at the school and who is now retired. This type of support is absolutely essential for any teacher wishing to break from tradition.

I am on a part-time contract at the Bishop Strachan School in Toronto this year and one of my goals is take a number of students from the school on a Math Trail. While I am not sure of the dates at this time, teachers interested in joining us can reach me at ron2718@netaccess.on.ca

Happy Trails!

▲ Some Thoughts on the New Mathematics Curriculum for Ontario- part 2

GORD NICHOLLS
MANAGING DIRECTOR, THE CENTRE FOR EDUCATION IN MATHEMATICS & COMPUTING
UNIVERSITY OF WATERLOO

Gordon Nicholls has thirty -five years experience, including thirty-one years as a Head of Mathematics in secondary schools in North York and Cambridge, and a year as a lecturer at the Faculty of Education, University of Toronto. He has published numerous articles on mathematics education and co-authored two texts. For his work in mathematics education he has received the René Descartes Medal, and the Prime Minister's Awards for Teaching Excellence.

(This is the second letter written by the old teacher. This time he is writing to his daughter who is starting her sixth year teaching somewhere in southern Ontario.)

> *"I mean those who are rightly educated become good men. And we must never disparage education, which is the first and fairest thing that the best of men can ever have; it may take a wrong direction, but it can be reformed. And the work of reformation is the great business of every man while he lives."*
>
> From *The Laws* by Plato

Dear Ariadne

Last month I wrote a lengthy letter to Alexander about the need to learn from the past as we prepare to revise the mathematics curricula in Ontario. Now that you have several years experience I expect you would prefer to share ideas on what our goals should be and how we might attain a better mathematics curriculum so these goals could be achieved. Since there has been very little open discussion inside the teaching community about what should be in the new math programme I hope these thoughts will stir your imagination and in return you will respond with your dream of how we should try to organize the math programme for the start of the next century.

Let us begin with a little history. During WW II Canada was thrust into the position of being a major military and economic power, even though our population ranked us among the smaller countries of the world. The period from pre WW II to 1960 was a time of major transition in Canada. We went from a resource based economy that was part of the British Empire to a modern industrial economy which had economic, political and military might that was vastly greater than our numbers warranted. We grew in the joint anvils of depression and war to independence from Britain and into a major player on the international stage. Our leaders of the period envisioned a Canada that would play a leading role in world affairs and when necessary we would send troops to preserve peace. In the post war period our new industrial economy moved ahead at a rapid rate because it had not suffered the terrible war damage that occurred in both Europe and Japan. This period of prosperity lasted unabated until the industrial machinery of the war torn countries was rebuilt. By the early 1970's we were told that our prosperity was assured and the Prime Minister of the day advised us to prepare for a world of leisure. Unfortunately, concurrent with this political interpretation of our prosperity, the economic reality was an increasingly intense competition for manufactured products and within a few years an unemployment rate that had jumped to the 12 % level. Our political leaders of the 80's and 90's had to execute a quick about face as they looked for new strategies to deal with this intense world wide competition. As a result further transition has taken place in the last two decades and we have entered into both the GATT agreement and the NAFTA agreement. These agreements have lowered tariffs and placed Canadian business and workers on a level playing field with most of the major industrial countries of the world. We no longer put up tariffs to protect Canadian workers and products. The new strategy declares that Canadians are as well educated, as creative in the new technologies and as hard working as workers anywhere in the world. The corollary of this strategy is that our people must compete for sales and for jobs with all the efficient industrial countries.

By now you must be saying "So what does this have to do with a math curriculum Dad?" Let me see if I can show the connections. In the 1930's we had a high school math curriculum that concentrated on algebra[1]

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Judy Crompton, Past President OAME/AOEM

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March, 1999, the Nomination Committee will publish a slate of candidates, including qualifications, for consideration by the membership. Voting will be by mail and is open to all OAME/AOEM members. The election will be released at the Annual General Meeting of the OAME/AOEM in Richmond Hill in May, 1999.

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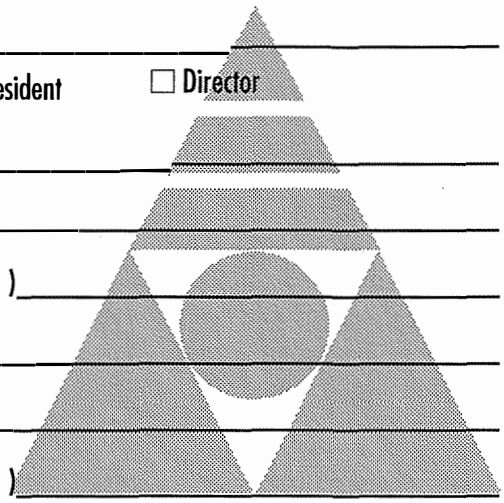
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and Euclidean geometry[2] in grades 9 to 12 followed by a grade 13 programme which included a major study of trigonometry[3] including extensive work on triangle and pure geometry problems along with identities for compound and half angles. Even though countries like France and Germany had introduced extensive analytic geometry courses and work with vectors into the high school as early as 1910, Ontario only had a minor introduction to the analytic geometry of the straight line in the grade 13 year. In the late 1930's we realized we were going to need thousands of young men who could navigate on the sea and in the air, who could use coordinate plots for artillery shoots and who were well versed in the use of electronic equipment. In 1938 Ontario finally introduced a revised high school programme which would bring substantial changes to the grade 13 year by 1941-42. The new analytic geometry for grade 13[4] was expanded to include the study of several loci and it also introduced the concept of tangent to a locus. The trigonometry course was modified to include a study of vectors[5] and their application to force problems. By 1940 the competition was military and we were fighting for survival. In order to compete we changed. We are now preparing for the next century and, whether we like it or not, once again there is an intense competition. Fortunately this competition is not leading to war; however, once again survival is at stake. In the new free trade zones that have been created we must compete at a competitive price or our workers will be displaced. No country can sustain long periods with large numbers of its people unemployed. Canadian business and industry have been working very hard for the last two decades tp put Canada in a competitive position. The new culture has returned us to our pioneer roots— it is work oriented and takes the positive outlook "we can do it". This leaves the schools of the country two choices. Either we prepare the students to operate in a competitive environment in a positive and healthy manner or we give the children a sheltered environment during their formative years and hope they will be able to adjust to the competitive environment of the adult world when they leave secondary school. As you know from personal experience, I think the preferred position is to attempt to prepare the children for the world they will

face. If we accept this position, then it will be necessary to emphasize values like doing ones best, completing work on time, striving for perfection, working rapidly and efficiently, creating a unique approach and finally using physical exercise to lessen tension and relax the body and mind when the pressure of school starts to cause turmoil.

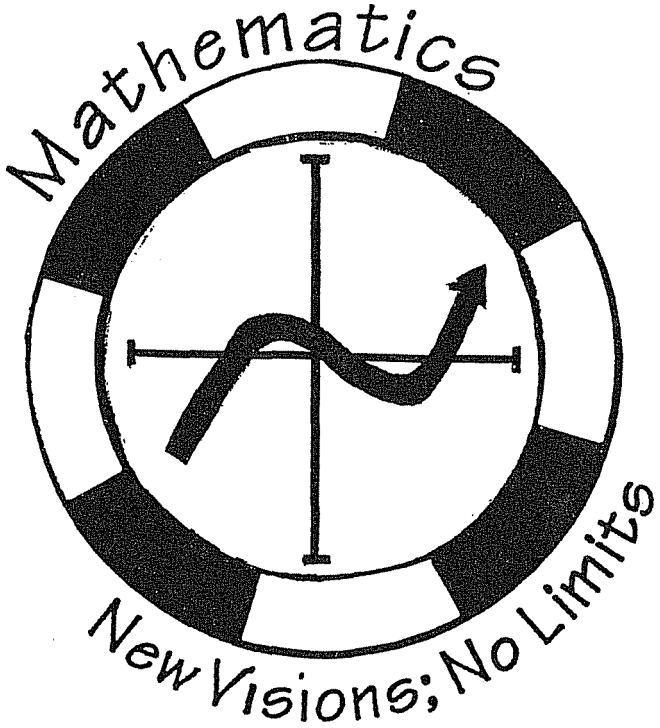
Can mathematics classrooms be of use to attain some of these objectives? My reply is yes. We can help students to grow in several of these directions and at the same time accomplish other objectives that are specific to mathematics. I think the following six general objectives should guide us as we prepare the new math curriculum for the 21st century.

1. **Illustrate the role of mathematics in our culture.** This would require including the history of some of the men and the really significant mathematical ideas of the last three millennia along with an analysis of the impact of math on western culture. I think each math course in the high school should have some history built into the course provided the mathematicians selected and the ideas they developed are an integral part of the course. Thus it would make sense to include some background on Thales in a synthetic geometry course; whereas it would be inappropriate to include him in a linear algebra section. Evaluation for this component could be in the form of an essay, book report or oral project.
2. **Mathematics as a way of thinking.** Students will learn that math is a way of thinking that can develop in two almost diametrically opposed directions. On the one hand intuition is used to search for patterns in either numeric or geometric form and hypotheses are posed about these patterns; however, the diametric approach develops rigorous and critical deductive arguments about specific and general cases. To develop the ability to argue deductively, we will need to build in stages from simple modus ponens arguments in grade 8 up to the multi step syllogisms that are used in more complicated proofs at the grade 12 level. On the other hand, if we want to encourage intuition, the math programme should allow for time to illustrate the difference between an inductive hypothesis and a deductive argument. Instead of dismissing the inductive approach as inferior, it will be important to illustrate that it is this approach that often acts as the catalyst to the creative idea. In order to show the power of the deductive argument , it should be introduced with great care and time should be spent to convince students that this type of reasoning was one of the great gifts that the Greeks gave to the world.
3. **Develop the power to think abstractly.** Students will learn to think abstractly over a period of several years. The process of abstraction begins as early as grade one when the child learns that the number '2' can represent two of many different types of objects. Early learning in arithmetic introduces the child to many examples of abstractions. For most teenage children the major examples of abstractions they will encounter in mathematics are the use of letters in algebra to represent unknown quantities and the general statements in geometry that represent conclusions about the many patterns the students have been able to observe in a variety of geometric situations. The long road from grade one to the abstractions taught in high school must be carefully presented in a sequence of steps that are suitably spaced. We know from experience that if the steps are not properly spaced or if steps are omitted then the learner very quickly gets confused and hopelessly lost. This is a major challenge in curriculum design and it must be done with great care.
4. **Different approaches to problem solving.** It is essential for students to learn different techniques for approaching math problems so they can appreciate that most problems in math can be solved in a variety of ways. As the student proceeds farther along in his/her studies more skills and techniques will be available. One of the best areas of mathematics to use for this objective is geometry. In synthetic geometry the theorems of Euclid often offer several approaches to the same problem. Once the student has available both Euclidean and Analytic techniques almost every geometry problem offers at least two solutions. It is also possible here to discuss which solution is more efficient or which is easier to construct. In many cases the Euclidean solution is more efficient, but also more difficult to create; whereas the analytic solution is attainable by following a systematic process which is considerably longer. This is one of the reasons why we should continue to include Euclidean geometry in the high school curriculum.

If we want our students to acquire different approaches to problem solving we must take care in the organization of the topics. In one widely used math text for grade 10 advanced level[6] the topic of analytic

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geometry was presented in such a way that many important connections to algebra were not suitably integrated. The following examples will illustrate how critical this integration is for the curriculum.

(i) After the length theorem is introduced it is possible to pose questions requiring one or two variables. The necessary algebra skills to handle these problems include the ability to solve one step radical equations and linear equations in one and two unknowns. Clearly these algebra skills should be in place so the students have access to these problems.

(ii) If the concept of locus and Descartes' method of first principles are taught as part of the introduction to analytic geometry, then the development of general formulas for the straight line and circle are possible along with related problems. One type of problem possible is to ask for the intersection of a circle with the x or y axis. This requires the student to solve a quadratic equation (by factoring provided the problems are restricted to rational solutions) and this means the student has an opportunity to use factoring skills and quadratic equation skills in the geometry part of the course. Once again the algebra skills must be available.

These two examples from the current grade 10 course illustrate the advantage of designing the analytic geometry part of the programme so that new algebra skills are introduced at the proper time and opportunity is provided for students to use these skills in other parts of the course. The new curriculum needs to do a better job in integrating the interplay between algebra and geometry and providing students with opportunities to use these techniques in suitably challenging problems.

In addition to emphasizing different approaches to problems, there are major topics in the curriculum that could be introduced using a problem approach. For example, in grade 9 students could be asked to solve arithmetic problems that are too difficult for most students when arithmetic methods are used. This provides the teacher with the natural place to introduce the concept of variable, the word 'equation' to represent a mathematical sentence and the need for some techniques to solve equations. This process is also one of the first examples the student will experience of building an abstract model (albeit a very simple one) to solve a problem. What follows would be about four weeks in a first algebra course. As algebra is developed to include quadratic equations and functions the student will experience many more examples where abstract models are created to solve problems.

If we put the stress on problem solving that I think is appropriate, then this stress must be reflected in the student evaluation. Since problem solving requires a delicate balance between skill mastery and approach strategy, care must be taken to construct questions and marking schemes that reflect this balance. Since you did not disagree with suggestions, in the letter I sent to Alexander[7], for what the ratio of skill to problem solving should be in student evaluation, then I will assume you agree on this point .

5. **Work Ethic.** We should stress with our students a work ethic that promotes completion on time, high quality in written work, creativity and giving one's best effort to the task at hand. It should be made clear to students and parents that the minimum amount of time spent outside of class for each academic subject should be 30 minutes for each class at the grade 9 level , increasing each year to about 60 minutes per class at the grade 12 level. the importance of homework preparation could be given further emphasis by showing a homework mark on the report card. This mark would also enable future employers to get a more complete view of the person they might hire.
6. **Practical skills for life in the 21st century.** The future is as variable as the many colours in a Thompson painting. We must be flexible when setting out a list of practical math skills and realize that many of the changes of the next decade are beyond the scope of our imagination. Some things that we can be reasonably sure about including in a math curriculum for every student are the following: a knowledge of simple and compound interest and the ability to understand annuity calculations in the form of mortgages, loans, and bonds; a knowledge of arithmetic and its use in a spread sheet to do personal and business budgeting; a knowledge of geometry and mensuration and the application to the practical calculations required for home maintenance; a knowledge of the concepts of average and variability and the application of these concepts to analyzing the distribution of data.

At this point in the letter I can just see you mumbling to yourself: "Dad, you are reaching for the stars! Although I agree with some of your objectives I need to

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see more specific ideas so I can form a concrete view of what you think we should do in the future." Here are a few specific suggestions Ariadne. Hopefully you will approve!

I. The balance between numerical and spatial topics.

If the math curriculum is to include a balanced exposure to the two major areas of mathematical thought, then we must specify some topics. Since we have over 2500 years of mathematical history the selection of topics is not an easy task. I would pick topics that give the students entry to four key developments in mathematics: geometry, analytical geometry, arithmetic and algebra. If there was more time available, then this list could be expanded; however, it is important to keep the list manageable and not to overload the students.

Since the following topics are intended for all students, it will be important to place them in the curriculum at times that are suitable to the maturity level of the student. Generally speaking, students in the 'academic' stream will study the topics sooner than students in the 'applied stream' and they will be expected to master these topics in a shorter period of time.

Topics to include for an understanding of spatial Relationships

- A Euclidean approach to angles, congruence, similarity, right triangles and circles.
- An Analytic approach to the basic geometric concepts of length, area, direction, ratio and the more complex concept of locus along with applications and problems.
- The use of trigonometry in problems involving the basic trig ratios, the Law of Sines and the Law of Cosines.
- Problems in the Euclidean and Trigonometry sections should include some examples of 3 dimensional problems. These problems would require separating the figure into different planes in order to set up the required geometric relationships.

Topics to include for an understanding of Numerical Relationship

- A thorough understanding of average, percent and

the use of these tools in conjunction with the concepts of variability, range and a measure of dispersion to analyze sets of data.

- The use of variables to describe problems in a symbolic language.
- A set of rules for operating with variables.
- show how algebra can be used to generalize the numerical patterns that appear in elementary sequence and series problems.
- sufficient fluency with the solution of linear and exponential equations so that annuity problems of the accumulated value and present value type can be solved.
- Graphical, chart and algebraic forms for the important growth functions used in economics and science: linear, exponential and logistic.
- Show how basic algebra can be used to custom design a spread sheet. Also include some of the graphing subprogrammes so information can be displayed visually.

II. Post Secondary Preparation.

The new curriculum must adequately prepare those students who will go on to post secondary education. The most recent statistics show that about 50% of secondary students go on to either college or university and that 40% actually hold a degree or diploma. These numbers are a strong argument for a solid academic programme in math for approximately half the secondary population. In addition to the topics listed in section 6 the following topics should be included in the programme for those students who plan to pursue post secondary math, science or technical studies at the college or university level:

- Extend the work on series and annuities to include the topic of constructing a life time earnings model that will require summation of an arithmetico-geometrico sequence. This topic could also include the Pascal Triangle and the Binomial Theorem, provided there will be time to introduce some elementary work with permutations and combinations.
- Extend the work on data analysis to include elementary probability and some simple statistics.
- Extend trigonometry to include work with functions, identities and trigonometric equations.
- The use of Calculus and Analytic Geometry in problems involving rate of change and in problems where summation is the fundamental concept.
- Rules for solving equations and how to apply these rules to solving problems. These problems should progress in level of difficulty over the years from grade 9 to 12 beginning with one variable linear problems and proceeding to linear/quadratic and eventually quadratic/quadratic and linear/cubic problems.
- Show how algebra can be used to describe a wide variety of functions- include polynomial functions up to the quartic level, exponential, logarithmic, logistic, reciprocal, radical, Cauchy, Gaussian, and the three basic trigonometric functions.
- Develop the necessary algebraic techniques to facilitate the ability to problem solve in algebra. The problems given should be of a significantly higher level of difficulty than those given in the applied level.
- high level of mechanical competence in algebra
- ability to construct specific examples and counter examples to illustrate a general situation.
- ability to observe specific examples and hypothesize a generalization about any pattern observed.
- ability to construct and explain a multi step deductive proof in problem solving
- ability to read and understand a proof presented in a text or by a peer. In particular students should be taught that part of the process of understanding is to be able to construct their own specific cases.
- Some facility with three dimensional problems.

III. External Evaluation

We must provide Ontario students with sufficient math expertise that they will feel positive about their place in the world and so they will be competitive in the new world economy . To accomplish these two rather contradictory objectives students need to know where they stand. Most students will feel good about their level of expertise if they can demonstrate their abilities on fair tests that are graded in a consistent way in all parts of the province. More attention needs to be given to evaluation and what the ratio should be for skills to problem solving to essay type questions. This was discussed in some depth in the first article[7]. The main point to reiterate here is the need for province wide

testing at certain key points in the school system. My preference would be to introduce external testing at the end of grade 8 and again at the end of grade 12. There will be those who will argue that this is too much pressure for our students. I would argue that the pressure currently being exerted on grade 3 pupils, where the children are only 8 years old, is indeed inappropriate. By grade 8 the students are on average 13 years of age and at this age they would be in a better position to handle the pressures and challenges of an externally set examination. In the new world economic order, competition and pressure are ever present and an introduction to these realities would not be inappropriate. The argument is even more compelling when the students reach age 18. An important part of the right of passage to adulthood, which occurs for most students in there grade 12 year, is the ability to learn to deal with these pressures. It is better to be taught techniques for handling pressure than to be suddenly forced into the situation in a working environment.

At this point Ariadne it is rather late and I have gone off in all directions with my ideas for a value structure for the schools and an ideal math curriculum. I suspect you will say I have been too thorough and it will be impossible to accomplish all the topics listed. If the Ministry officials pay attention to what was learned in the UTS experiment[8], where the content of grade 7 to 9 was taught in grade 7 and 8, then it is reasonable to hope that your generation of teachers will indeed do even more than what I have tried to specify as the minimum. In either case give it some thought and then send me your critical comments. I hope the weather is as fine as we have here. The early autumn colours have been superb and the days are still wonderfully warm. Enjoy your teaching and remember that old adage "do not smile till Christmas" is not to be taken too literally!

Love, Dad.

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(1) *High School Algebra*, by Crawford, Pub by MacMillan, 1919, 1937

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(4) *A New Analytic Geometry, by Durrant and Kingston*, Pub by MacMillan, 1940

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(7) *Some Thoughts on the New Mathematics Curriculum for Ontario*, by Gordon Nicholls, Pub. in the Ontario Mathematics Gazette, Vol 37 #1 , September, 1998- pg. 8

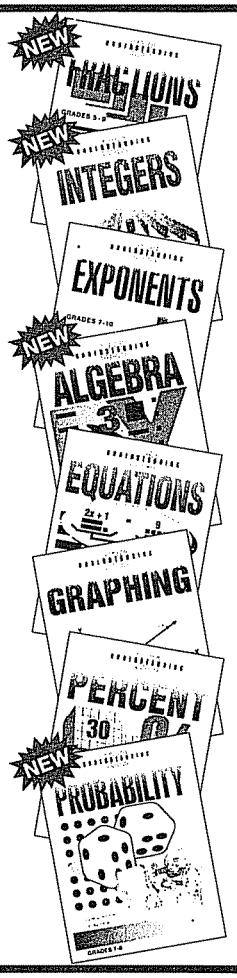
(8) See the unpublished University of Toronto Schools report on the *Compression Experiment* by A. Fleming

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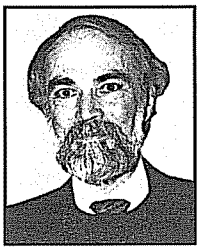
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▲ Simon Says Do This: An Indictment of Rote Learning



GARY FLEWELLING

Introduction

Rote learning occurs in both traditional and non-traditional classrooms. And, it occurs with worrisome frequency. I use the word 'worrisome' for the following reasons. Firstly, I claim that the phrase 'rote learning' is a pedagogical oxymoron. I also claim that rote learning ignores the needs of the learner, and that it is an ineffective way to learn. I further claim that rote learning impedes the learner from becoming a problem solver, that it is an anti-mathematical way of thinking, that it is a major source of math anxiety, and that it is a form of (unintended) psychological abuse. In this article I attempt to justify these claims and, in the process, re-emphasize the importance of teaching for understanding.

Definition

When I say, 'by rote', I mean, 'from memory, without thought of meaning, in a mechanical way.'

Some Beliefs About Rote Learning

Many think that rote learning has a significant role to play in the classroom ("Students often use rote learning as a kind of cognitive rest stop on the road to understanding." "Some things are best learned by rote." "It's the only kind of learning some of my students can handle." "It's great for training the memory.") Many, less enthusiastic, others see rote learning as an expedient or 'necessary evil' used by teachers to 'cover' an over-loaded course ("I'd like to teach for understanding a lot more than I do, But I just don't have the time in class to do it.") Many students graduate believing that most (all?) mathematics is learned by rote and that learning has a whole lot to do with memorizing, practicing, and regurgitating on request.

An Example of Rote Learning

A teacher (in the form of a real person, a text page, writing on a computer monitor, ...) presents students with the formula for calculating the length of a line segment (see fig 1). (Little or no attention is given to the development of the formula or to the concepts underlying the formula.)

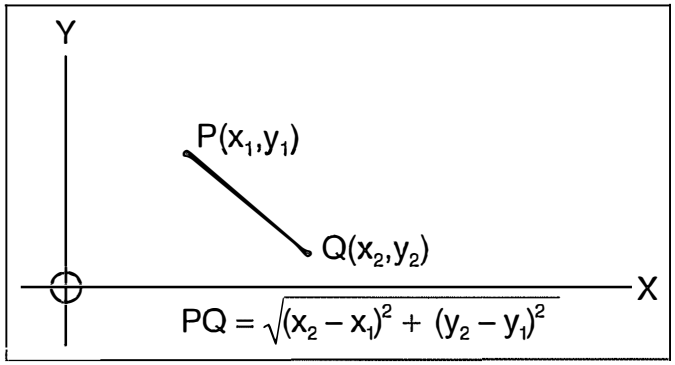


fig 1

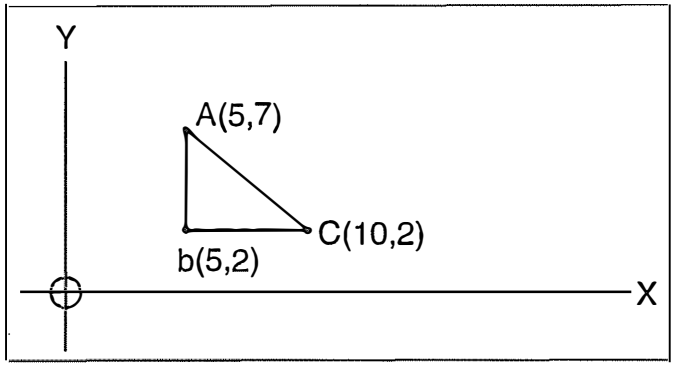


fig 2

Students are then walked through one or two sample problems to demonstrate how the formula is used.

Lastly, students are given a set of problems (like those in fig 2) to practice using the formula.

Students 'turn the crank' and let the formula grind out the length AC, and then the length AB, and then the length BC, treating each problem in similar fashion, sometimes calculating a sum instead of a difference, sometimes forgetting to square a difference, sometimes forgetting to calculate a square root, accepting without concern answers for AB or BC that are (impossibly) greater than AC. On the practice goes, all without understanding the concepts underlying the procedure, all without thought to meaning or reasonableness, all in a memorized mindlessly mechanical fashion.

An Example of Learning for Understanding

Students would have been better served in the above learning episode if they had first been given a well-designed series of fig 2-type problems (preferably in some context) and asked the following question. "How might the Pythagoreans have solved such problems (assuming that they had a rudimentary understanding of a Cartesian coordinate system)?"

After working on this task, and discussing it with others, students could be asked to summarize their conclusions in the form of a general rule or formula. The end result would be the identical or equivalent formula that the teacher presented to students in the first example. This time, however, it would be the student's formula, not the teacher's. In addition, there would be understanding about such things as, the formula's link to the Pythagorean theorem, when the formula need not be invoked, when an estimate would be sufficient, when signs would or would not be important, when square roots would and would not be needed, (perhaps) how the formula could be extended to handle 3D problems, etc.. Many students would not even feel the need to commit the formula to memory. They would know the length formula as a simple variant of the good old Pythagorean theorem (bringing the appropriate form forward, as required, in different problem situations.)

Comparing Rote Learning with Learning for Understanding

To me a rote-learning episode, like the first example, feels like being introduced to a stranger, shaking hands, and then being obliged to engage the stranger in awkward small-talk. A learning-for-understanding episode, like the one in the second example, on the other hand, feels more like embracing an old friend and comfortably sharing (verbal) intimacies and reminiscences.

Students will continually encounter the unfamiliar, the uncomfortable, and the unknown in their studies. But, a learning-for-understanding episode will give students the opportunity to turn the unfamiliar into the familiar, the uncomfortable in to the comfortable, and the unknown into the known. A rote-learning episode, a cognitive IOU from teacher to students, doesn't provide students with such an opportunity.

Rote Learning, an Oxymoron

Tom Kieren, recently retired educator and researcher at the University of Alberta, defines learning as, 'bringing forth a world of significance with others.'

Consider the first six words of his definition. Before anything can be brought forth that has significance for the learner (or teacher), there needs to be personal understanding, connections made with the familiar, ownership, relevance, knowledge of where it comes from, where it fits, what it does, why it is important, why it works, how it works, what it leads to, its limitations, its forms and descriptions in words, images, and notations that are a part of the learner's repertoire. Rote learning, on the other hand, with its lack of concern for meaning, and with its emphasis on acceptance and the procedural, isn't likely to bring forth from students anything more significant than questions like, 'Why am I doing this?' and 'What am I doing here?'

Now consider the last two words in Tom's definition of learning. 'With others' suggests meaningful interaction. This includes such things as communication (in all its forms) to share, give feedback, encourage, explain, direct, make suggestions, justify, clarify, summarize, extend, and evaluate. With rote learning the interaction is far simpler and more direct. The teacher gives and the student receives. The teacher presents and the learner (happily, meekly, or grudgingly) accepts. The teacher says "do this" and the student (hopefully) obeys. The teacher pours and the warm-blooded vessel (supposedly) fills.

Using Tom Kieren's language, rote learning, could be defined as 'bringing forth a world of insignificance at the insistence of others.'

Rote Learning Ignores the Needs of the Learner

Memory and the processes of memorization and recall are crucial ingredients in all learning and thinking. Rote learning uses them in a particularly poor way. With rote learning, little or no consideration is given to connecting the new with the familiar, to connecting procedures with related concepts, to memorizing things in a variety of forms using language and images that make sense to the learner and with which the learner is comfortable. With rote learning, little consideration is given to recalling and applying concepts and procedures in novel settings or in an integrated fashion with other concepts and procedures. With rote learning, concepts and procedures are generally placed in isolated slots in memory and recalled , restated, and applied in situations very similar to given simple example situations. Students

need strategies for storing and retrieving rules that go far beyond saying or doing something over and over and over again until it is memorized and then recalling it accurately and speedily (often with the aid of some awkward mnemonic.)

Rote learning is about accepting, not about understanding. Rote learning is about knowing how, not why. Rote learning is about inculcation, not education (Inculcate means, to cause someone to accept an idea, to impress by repeated statement or persistent urging, to implant, to ingrain, to instill. It comes from the Latin inculcare, to trample in < in + calx, calcis meaning heel).

Humans are tool making, sense making, tool using, sense using creatures. It is in our nature to want to understand, to construct meaning, to be in control of ourselves, to make sense of the world in which we live. Learners need to know what it is that makes a thing worthy enough or significant enough to warrant its own label or to warrant it being clearly expressed as a procedure, rule, or generalization worth remembering. Learners also need to develop skills for a life of independence and interdependence, not slavish dependence. Rote learning ignores these needs.

Rote Learning is an Ineffective Practice

I think that most teachers already know that rote learning is ineffective. Teachers are very familiar with students who memorize their math, without any concern for its meaning and who apply procedures in a mechanical unreflective way. These are the students who use the area of a rectangle formula to calculate a perimeter, who often get nine or ten out of ten on a short quiz and then forget it all over a weekend, who walk into science class, directly from math class, and fail to use



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the needed concept or skill just (rote) learned in math class. These are the students who also say, "Don't bother trying to get me to understand this stuff, just tell me what to do and I will do it." These are the students who have trouble getting help because they don't have the language skills or knowledge of what they don't know to enter into such a discussion.

The ineffectiveness of rote learning can be inferred from the following table, taken from The Third International Mathematics and Science Study, TIMSS. ('Lessons that are stated' are lessons that emphasize rote learning. 'Lessons that are developed', are lessons were the teacher is trying to teach for understanding.) Notice that nearly 4/5 of US math lessons involve rote learning, compared to about 1/5 of the lessons in German and Japanese classrooms. We all know which of these three countries scored the lowest on the TIMSS, the US.

Average Percentage of Topics in Eighth-Grade Mathematics Lessons that are Stated or Developed

	U.S.	GERMANY	JAPAN
LESSONS STATED	78	23	17
LESSONS DEVELOPED	22	77	82

Rote learning is the cause of much reteaching, term after term, year after year, summer school after summer school, hired tutor after hired tutor (I find it ironic that

teachers who would have students learn certain things by rote, because they don't have the extra time to teach these things for understanding, are later faced with having to find this extra time to remediate or reteach these same rote-learned things.)

Rote learning just doesn't deliver the goods.

Rote Learning is an Impediment to Problem Solving

Teachers know that many students have weak problem solving skills. This weakness can be seen in the tests and exams students write. (Problem solving is generally an insignificant component.) Look at the seat work and homework students engage in. (Most of it is drill, practice, and relatively low-order application.)

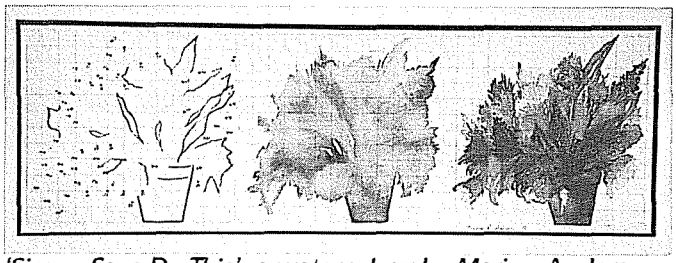
Rote learning has very little (anything?) to contribute to the development of problem solving skills. Problems worth their salt are seldom solved by 'turning the crank' on memorized procedures. Problem solving isn't about acting in an unreflective mechanical manner. Problem solving isn't about accepting. Problem solving (as the cliche goes) is about what you do when you don't know what to do. Problem solving is about understanding. It is about having the confidence to risk acting upon things. It is about planning, modeling, conjecturing, testing, proving, evaluating, and communicating. It is about using skills in an integrated, purposeful, and creative fashion. Rote learning, in contrast, is risk free. It is about being told what is true and about being told what to do.

Learning is often linked to two other things, practice and automaticity / mastery. It is the hope of many teachers that, **learning + practice = automaticity / mastery**.

For me practice means gaining experience (in using concepts, procedures and rules efficiently, accurately, effectively, purposefully, and creatively to understand situations, make decisions, solve problems, carry out investigations and experiments, communicate convincingly and clearly, ...) Unfortunately, because that which is (rote) learned means so little to students, the practice associated with rote learning must (by necessity) emphasize simple recall, the mechanical, simple tasks, and narrow superficial applications. This latter kind of experience doesn't help a student build or refine problem solving skills that have value in the real world.

Automaticity in or mastery of the 9x9 table, for example, isn't enough. Students need to know it, but they must also be able to connect these products to a variety of situations involving counting, number patterns, geometric arrays, repeated addition, measurement, comparisons, estimations, the fundamental principle of counting, sharing, etc. and they must be able to use these facts and connections to do significant things.

Rote-learning-plus-practice techniques train problem solvers as well as paint-by-numbers techniques train artists. People painting by numbers aren't thinking like artists. They aren't behaving like artists. They aren't feeling like artists. And, they aren't making art. Painting-by- numbers techniques keep would-be artists from becoming artists. Rote-learning-plus-practice similarly impedes the development of problem solvers.



'Simon Says Do This', a watercolour by Marion Anderson, from the author's collection.

I would prefer to see the following equation apply in the classroom.

learning for understanding + problems-based experience = better problem solvers

(where learning for understanding = problem solving)

Rote Learning is a Major Source of Math Anxiety

When material is presented to a student in such a way that no thought is given to meaning and it is practiced in a mechanical non-thinking manner, bad things tend to happen to that student. The student likely starts experiencing thoughts like the following.

So many isolated meaningless things to remember!

I wish my brain was good enough to make sense of it ail.

I got perfect on the last test, but I still don't really understand what I'm doing.

There's no choice or flexibility here.

I'm not in control.

I hope the questions on the test are like the examples, otherwise I'm lost.

I'm afraid to get help and show people how little I really understand.

I don't feel comfortable with this stuff.

I'd drop this subject if it weren't so important.

These aren't the thoughts of just the mathematically untalented or disenfranchised. I am amazed at the number of 'strong math types' who confess to having had similar thoughts (often adding that things didn't start making sense to them until grad school).

Students who 'fall behind', who can't cope with their math, are often prime targets for rote learning (in the form of remediation, extra drill, tutoring, summer school, etc.). I find it ironic that those for whom math has so little meaning are especially targeted to receive more of the same. Surely full access to reference sheets, calculators and computers would be mathematically more efficient and psychologically less damaging.

Rote learning is a significant cause of stress, anxiety, and loss of self-esteem. Rote learning is a health hazard.

Summary

The main task of teachers is to provide their students with opportunities to learn, with understanding. Insisting that students accept truths, solely on the authority of the teacher, text, or group, is not much of an opportunity to learn.

Students will not understand everything that is put before them. But, students have a right to expect that they will be given the opportunity to understand everything that is put before them. Things that can't be learned for understanding by a majority of students in a class should be shelved until they can be so learned or be presented to students as 'preliminary activities / explorations'.

Rote learning gives students the wrong idea of what learning is (the memorizing and practicing of meaningless rules and procedures), what they as students are (passive receivers of truth), what teachers are (transmitters of truth), and what mathematics is (a set of other people's truths and procedures to be memorized and used in a mechanical and unthinking way).

Conclusion

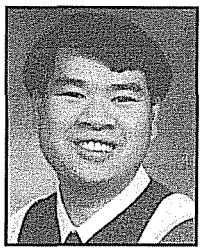
Place mathematical thinking at one end of a 'thinking continuum'. Walk to the other end of that continuum. There you will find rote learning.

Comparison of Learning for Understanding with Rote Learning

Learning for Understanding	Rote Learning
1. convincing	1. unconvincing
2. math as a way of thinking	2. math as a collection of procedures
3. working with things that make sense	3. working with the inexplicable
4. master	4. slave
5. addresses student needs	5. ignores needs of student
6. significant to learner	6. significance lost on learner
7. known to be true	7. accepted as true
8. student active	8. student passive
9. validated by student	9. validated by teacher
10. truth as constructed	10. truth as presented
11. student-owned	11. teacher-owned
12. wielded powerfully	12. wielded mechanically
13. student as rule maker	13. student as rule taker
14. described / explained in student language	14. described / explained in teacher language
15. teacher as educator	15. teacher as inculcator
16. remembered / reconstructible	16. often forgotten / not reconstructible
17. grows into being	17. pops into existence
18. considers student readiness	18. ignores student readiness
19. experiential	19. non-experiential
20. independence / interdependence	20. dependence
21. developed by end of lesson	21. presented at beginning of lesson
22. minimal reliance on memory aids	22. reliance on memory aids
23. painting without numbers	23. painting by numbers
24. learning via a problem solving process	24. impediment to problem solving
25. connected	25. isolated
26. thorough	26. superficial
27. develop procedures	27. follow procedures
28. a partnership	28. master-slave relationship
29. enlivens the mind	29. deadens the mind
30. enlivens the spirit	30. deadens the spirit
31. sense of personal efficacy / confidence	31. math anxious
32. bringing forth a world of significance with others	32. bringing forth a world of insignificance at the insistence of others

"Santa's Christmas Madness or Santa's Christmas Scam?"

LOUIS LIM
QUINTE S.S., BELLEVILLE, ON



Louis Lim is a third year Mathematics Teacher with the Hastings and Prince Edward District School Board. He is the ISOMA rep to OAME, writes the Director's Dialogue *for the* Gazette, *and enjoys presenting workshops at conferences. He is also a part-time Master of Education student at Queen's University interested in teaching Mathematics through technology. Louis is indebted to the many outstanding educators he has come in contact with, in particular, his mentor, Ron Lancaster.*

As a Mathematics teacher, I always cherish moments when class discussions alternate from teacher to students and students to students. One such instance happened in December 1997 when *"Santa's Christmas Madness"* scratch game cards were distributed across Canada. The media was quick to warn the public about this "scam", and not to phone the 1-900 number listed. I received one game card several days later, and scratched the play area only to realize that I had won $10 000! After reading the rules in the contest carefully, I decided to make up some questions to help convince my students that Mathematics is useful and relevant to them.

About half of my grade 9 Mathematics class were aware of the game cards. Several of my students were intrigued when I showed them that I had matched three $10 000, and therefore had won! I then took out another game card with identical outcome, and these students were now sceptical I had won two $10 000 with two game cards! One student pointed out that his cousin also had won $10 000 and was going to phone in. Others mentioned winning a trip for two while others had won a television.

We studied the game card in detail, and many students noted that there was a flat charge of $19.95 per call. I asked my students to work through the questions I designed, and to judge for themselves whether or not the contest was a "scam" as portrayed by the media.

#1. "You are a GUARANTEED WINNER when you uncover 3 identical Cash Prizes and your Prize Claim Number is one of the preselected Prize Winners."

(a) Explain the difference between "*and*" and "*or*".

(b) Since 3 "10 000" prizes were uncovered, is this ticket a guaranteed winner? Explain.

#2. (a) Answer the skill-testing question: $5 \times 8 - 20 + 5$.

(b) Suppose that you did not use Order of Operations. Will you still get the same answer as in (a)? Explain.

(c) Invent a problem, which when done without the Order of Operations, would give you a different answer.

(d) Wouldn't it be more appropriate to evaluate $10(0.5 \times 0.08 + 0.000202 \times 2)$ on your calculator, and then turn it upside down to read the solution? Try it!

#3. "Prizes available to be won & chances of winning respectively are $10 000: $\frac{1}{3\,400\,000}$; 52" screen TV: $\frac{1}{3\,400\,000}$; $1000: $\frac{30}{3\,400\,000}$; plane voucher which is redeemable for airfare for 2 to Hawaii."

(a) How many prizes are available in total?

(b) What is the actual probability that you'll win $10 000?

(c) Explain how "over one million dollars in cash prizes and gifts are available."

#4. (a) If each call costs $19.95, how much money would the company make if the entire 3 400 000 callers called in?

(b) How much profit would be made?

#5. If you believe that this is a scam and hired a lawyer, do you think you would have a good chance of suing the company for fraud? Explain in several sentences.

I plan to use this activity again in the last week of classes before the Christmas break. Student enthusiasm was high, as the game cards were relevant to their everyday life. I find it a great way to integrate order of operations, logical thinking, probability, number sense and writing all in one assignment. When a grade 9 student says, "Math saved me $19.95!", you know that students will cherish such experiences for many years to come.

▲ ABOUT PROBLEM SOLVING: MATHEMATICALLY AMUSING NUMBERS

JOHN GRANT MCLOUGHLIN
FACULTY OF EDUCATION
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What do 145, 370, and 2178 have in common? On the one hand, they are not related in any way; yet at the same time they are examples of numbers with curious (albeit distinct) mathematical properties. In fact, 145 equals the sum of the factorials of its digits. The numbers 153, 371, and 407 are the only other numbers greater than 1 that share a property with 370. I'll let you try to decipher the property that those numbers share. Meanwhile, 2178 is an example of a number that possesses a multiple which equals its reversal. Observe that 2178 x 4 = 8712. You may wish to find other such numbers.

Numbers possessing peculiar properties are often identified by such features. Number theorists and recreational mathematicians are inclined to define terms that may be used to characterize families of numbers sharing a property. For example, 220 and 284 are known as an amicable pair. Why? The proper divisors of 220, namely, 1,2,4,5,10,11,20,22,44,55, and 110 sum to 284. Similarly, the proper divisors of 284 (i.e., 1,2,4,71, and 142) sum to 220.

Automorphic numbers are numbers that appear at the end of their own square. The trivial examples are 0, 1, 5, and 6. It follows that any larger automorphic numbers must also end in one of the forementioned digits. In fact, some consideration of squares of numbers of the form (10b + 1) for b = 1,2,3,...,9 can be used to prove that all larger automorphic numbers must end in 5 or 6. More specifically, they must end in 25 or 76.

The remainder of this column draws upon ideas offered in Joseph S. Madachy's book entitled Madachy's Mathematical Recreations. The concept of multigrades is examined in considerable depth in Madachy's book. An introduction is provided here for those among you who like to play with numbers.

Multigrades refer to a special relationship between sets of numbers and their powers. Specifically the relationship deals with the respective sums of the separate digits raised to the same powers. Consider the case of {1,6,8} and {2,4,9}. It can readily be seen that the sum of the elements in both sets is the same. That is, the sum of the respective terms raised to the first power is 15. Now, consider the sums when each term is raised to the second power. Again we find that the sums are equal. We get 1 + 36 + 64 = 4 + 16 + 81. There are more advanced examples of multigrades cited in Madachy's book. For example, consider the sets {1,5,8,12,18,19} and {2,3,9,13,16,20}. Readers may wish to verify that the sums of the nth powers of the respective elements of these sets is equal for n = 1,2,3, and 4.

It may be surprising to learn that creating multigrades is relatively simple. It is critical to note that if all terms involved in a multigrade are increased by a constant amount, the multigrade feature remains in place. For instance, consider the sets provided in the initial example. Increasing all elements by 5 would produce {6,11,13}and {7,9,14}. Indeed the equality of the sums of the first and second powers continues to hold. More complex multigrades can be obtained by beginning with simple equalities such as 1 + 4 = 2 + 3. Adding 7 to both sides gives 8 + 11 = 9 + 10. By interchanging the sides we can produce {1,4,9,10}and {2,3,8,11}, thus, yielding a second-order multigrade. Checking the sums of squares, we find that both sets produce 198 as a total. This multigrade can then be extended by adding a constant to these elements, interchanging the terms, and proceeding in the same manner.

A couple of multigrades created by the late Royal V. Heath are remarkable because of their composition. A second-order multigrade uses each of the digits 1 to 8 exactly once in both sets, namely, {12,43,65,78} and {87,56,34,21}. Another consists entirely of numbers made up of 1's and 8's: {1118,1881,8181,8818} and {1181,1818,8118,8881}. Believe it or not -reversing the digits in each of the elements of this latter multigrade produces another multigrade!

This column on numerical curiosities represents the third and final piece in a series focusing attention on recreational mathematics. Questions or comments pertaining to any of the columns are welcomed.

REFERENCES

Madachy, Joseph S. (1979). *Madachy's Mathematical Recreations.* New York: Dover Publications.

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ALEXANDRU PINTILIE
MATHEMATICS TEACHER,
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When Alexandru Pintilie, B.Sc. Math, M.Sc. Stats, ex-A.S.A., does not teach math, he boasts about it, chaperons the soccer team, chauffeurs the chess team or lectures his teenage son.

I heard it one day on the radio. It is a story that seriously changed, if not the way I hold my lessons, at least the way I look at mathematics teaching.

The old owner and director of a Japanese martial arts school decided it is time he retires and leaves the school to one of his three sons. In order to decide which of them is worthy to assume the leadership of the school, the father decides to put them to the test. Each of them was to be invited in turn to the old man's office where a surprise was awaiting. Behind a door, a faithful servant was hidden. That servant was supposed to suddenly attack each of the sons with a wooden sword. The father wanted to know how each son would deal with that situation.

The eldest son, when suddenly hit by the attacker, did not fall down under the blow because he was very strong. In no time, he managed to disarm his opponent.

When his turn came, the middle son, being very swift, did not need more than a fraction of a second to react and avoid the blow and had no trouble afterwords with disarming his opponent.

Finally, the father invited the youngest son into his office. Before entering, the son said: "Father, behind that door there is somebody who will try to hit me with a wooden sword. I would like to know why."

Who do we consider to be a successful math student?

Is it the one who has solid technical skills and who knows and correctly applies methods and formulae? Is there any teacher who would not give such a student a mark in the nineties?

Is it the one who comes up fast with the right answer and who collects all the Cayley and all the AHSME honours?

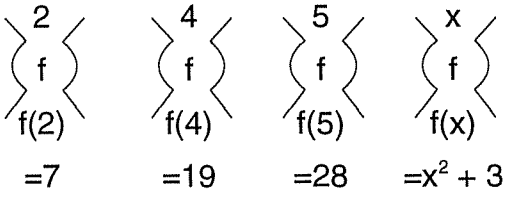
Is it the one who recognizes the nature of the problem and would not tackle it until he fully understands it? Such a student is not always given recognition because, although he can give three different solutions and two generalizations for a geometry problem, he is unable to factor $6x^2 + x - 15$ or to blurt out in two seconds the square of 47.

▲ TEACHING IDEAS FROM THE EDITOR

Distinguishing Between f and f(x) for Functions

To help my students distinguish between f and f(x) I use the idea of a function machine. Suppose a function takes each input, squares the input then adds 3. This would mean that an input of 2 gives an output of $2^2 + 3 = 7$. If the machine has the name f then the output for an input of 2 in f is called f(2). Similarly inputs of 4 and 5 would give outputs of f(4) = 19, and f(5) = 28, respectively. An input of x into function f would give an output of $f(x) = x^2 + 3$.

Diagrammatically



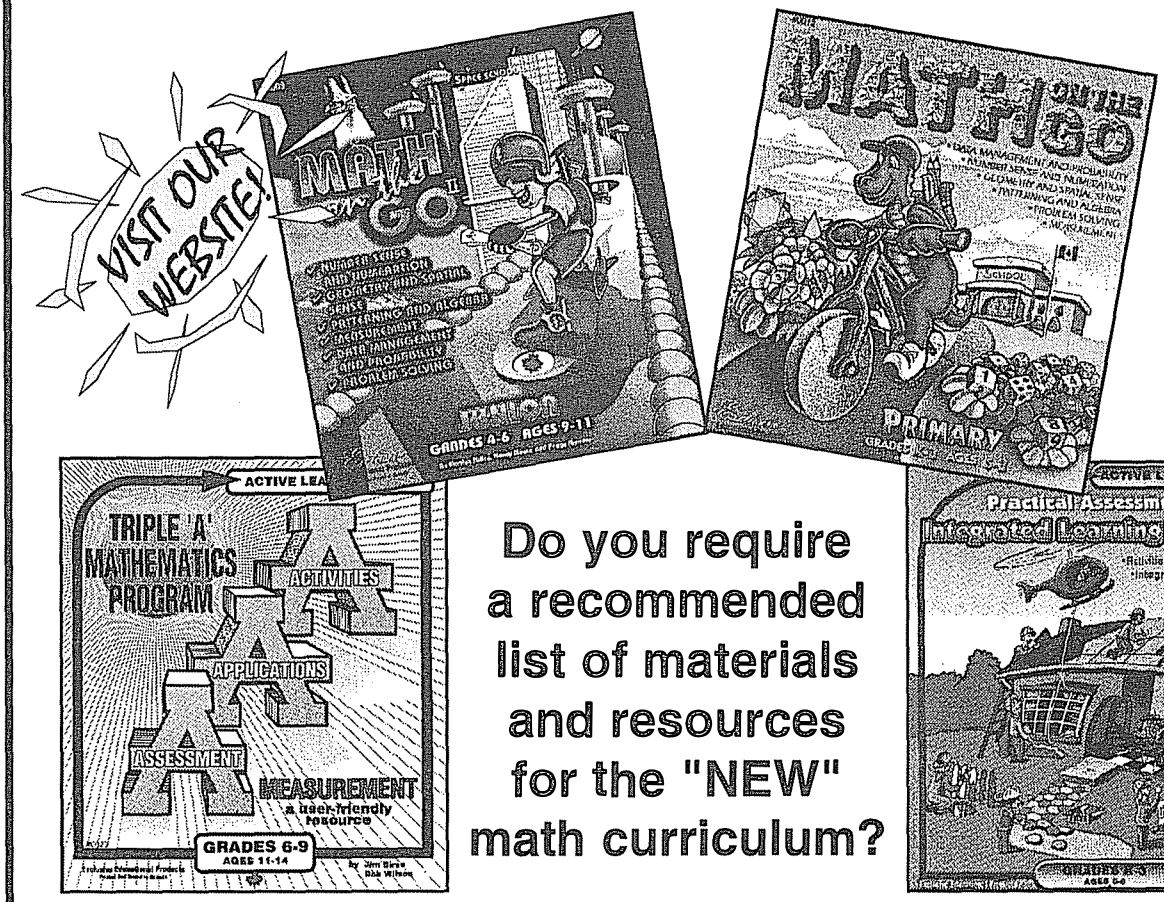
The above can be abbreviated by replacing the machine by an arrow so that we obtain the mapping notation

$f:2 \longrightarrow 7 \quad f:4 \longrightarrow 19 \quad f:5 \longrightarrow 28 \quad f:x \longrightarrow x^2 + 3$

When graphing functions ordered pairs are need so we replace the output f(x) by the variable y.

This leads to the graphing or ordered pair notation $f = \{(x,y) | y = x^2 + 3\}$.

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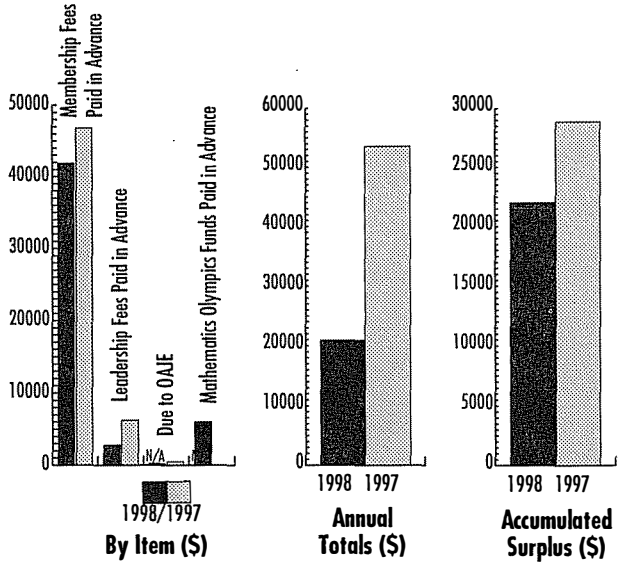
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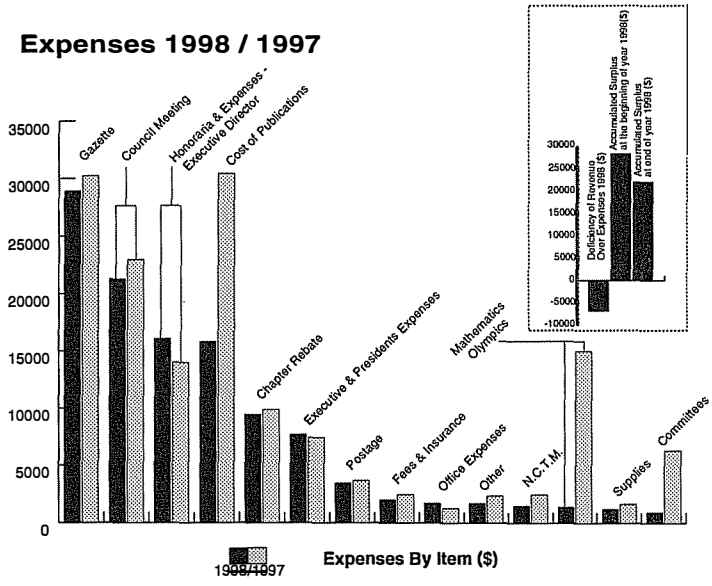
I have compiled the Balance Sheet of Ontario Association for Mathematics Education (Association Pour L'Enseignement Des Mathematiques) as at August 31, 1998 and the Statement of Operations and Accumulated Surplus for the year then ended from information provided by the Directors. I have not audited reviewed or otherwise attempted to verify the accuracy or completeness of such information. Readers are cautioned that these statements may not be appropriate for their purposes.

Markham, Ontario, September 17, 1998
ROBERT M. Davies Chartered Accountant.

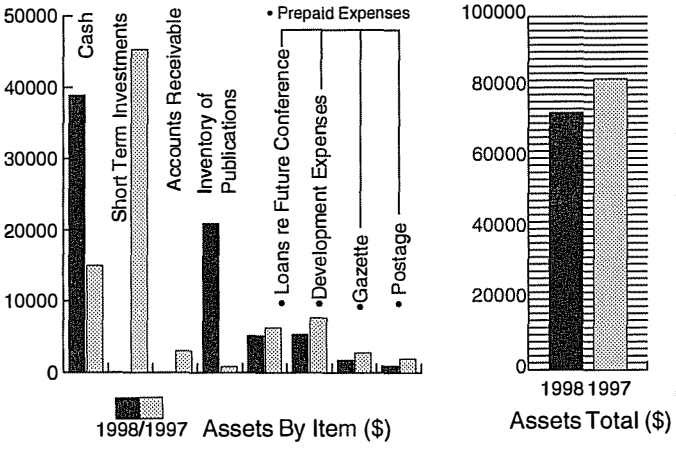
Liabilities and Surplus 1998 / 1997



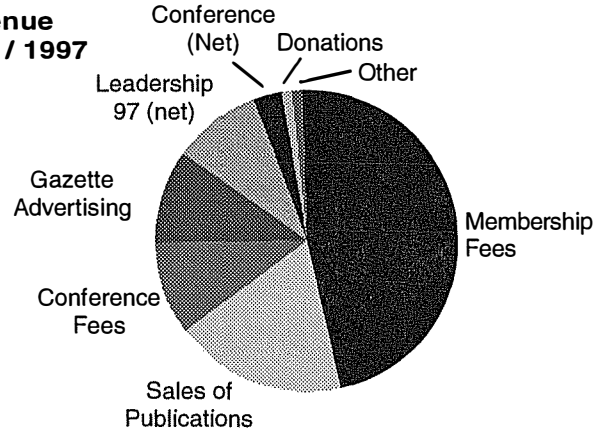
Expenses 1998 / 1997



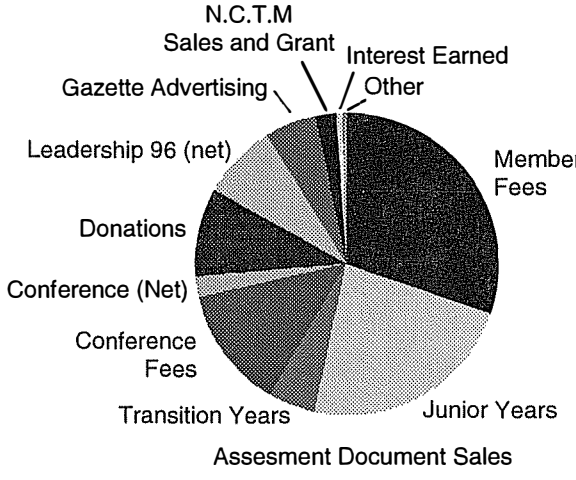
Current Assets 1998 / 1997



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1998 ($106,151)



1997 ($160,119)

▲ Directors' Dialogue

Louis Lim
Quinte S.S., ISOMA Representative to OAME

Introduction

As I sit in front of my computer writing this edition of the *Director's Dialogue*, I am very proud to say that I am a classroom teacher. Chatting with my fellow OAME board of directors at our meeting held on the evening of Friday, October 2, 1998 and all day Saturday, October 3, leaves me without any doubts that we, teachers, have the most important and rewarding career on our planet. I am proud that my colleagues and I continue to fight for the best interests of our students. It is not "business as usual", as some of my colleagues are teaching 7 out of 8 courses, and others have been legislated back to work only within the week since this board meeting took place. I find it remarkable that your board of directors were able to meet those two days and promote Mathematics education on your behalf.

President Connelly's Report

OAME president Ralph Connelly reflected on the many contributions OAME has had, and will continue to have, on Ontario educators. He reflected on the fine work the association has done to assist teachers with the changing curriculum and challenges: *Junior and Transition Years Assessment Documents*, *Scope and Sequence* posters for grades 6 to 8, *Teaching Mathematics Through Technology* document, OAME'98 in North Bay, the annual Leadership conference, and the soon to be released Primary Assessment Document.

Virus Alert!

If you purchased the *Teaching Mathematics Through Technology* document, take note: If you bought your copy **before** August 1998, disk #2 has a virus in the second probability activity. Be sure to use an anti-virus to detect and fix the problem. The executive directors, Bonnie and Dave Alexander, have stated that they will replace disk #2 if the purchaser sends it to them. Teachers purchasing the document after August 1998 will have the problem corrected with "The seal was broken to eliminate a virus on one of the disks".

Membership

Executive director, Dave Alexander, expressed concern with the continuing decline in OAME membership. The membership as of September 26, 1998 was 1095, while a year ago, it was 1388. Do not hesitate to contact me or any of the board members on how OAME can better meet your professional needs and demands. Suggestions are taken very seriously.

With the many recent retirements, retired teachers should note that annual fees are reduced to $30 per year. Speaking to several retired colleagues, they found it hard to go "cold turkey". I guess there will always be "teaching blood" in all of us many years after retirement! Also, encourage your new teachers to join OAME – I know that reading the *Gazette* and attending chapter and annual conferences have had huge impacts on my growth as a teacher. The networks I've established as a result of OAME are priceless!

Committees

AWARDS Do you know of an outstanding elementary or secondary teacher who deserves to be recognized by the OAME? If so, seriously consider nominating them for an OAME award. Awards include the Life Membership, and awards to honour Mona MacGregor, Morley MacGregor, Kenneth D. Fryer, and Don Attridge. Nominations should be submitted to Peter Saarimaki (see page 7), and are accepted up to January 31 for consideration for the calendar year; forms submitted after January 31 will be considered for the following year.

FUTURES The Futures committee confirmed the following: **OAME'99**: hosted by Y^4MA in Richmond Hill; **OAME 2000**: hosted by COMA in Ottawa; **OMO'99**: hosted by TE^2Y^2MS in Toronto; **OMO 2000**: hosted by QSLMA in Kingston.

A survey is being prepared to be given to elementary teachers. Their responses will give the OAME executive a better sense of meeting elementary teachers' needs, and hopefully encourage them to join our association. The futures committee has prepared a document to help coaches prepare their teams for the Ontario Mathematics Olympics (OMO). They plan to distribute them in mid-January, and will include sample problems, along with information on the team technology event.

MATH EQUITY A major initiative of the Math Equity committee has been addressing Anne Pedersen's concern of the representation of directors on the OAME board (34 secondary, 4 at the faculties of education, and only 5 from elementary). The executive committee has agreed to explore the possibility of fairer representation during the upcoming months. The committee was disappointed to report that they have been unsuccessful in getting Mike "Pinball" Clemens from the Toronto Argonauts to be the keynote at the OAME'99 Equity Breakfast. A keynote is being sought.

PROMOTIONS The Promotions committee has prepared a Faculty of Education kit. Included will be OAME publications, advantages of joining OAME, along with information on local chapters.

The committee has also produced an OMO kit. Included will be tips for teachers to prepare their teams, the technology requirements for one team event (along with strategies to become familiar), and a collection of activities for team preparation.

PUBLICATIONS You may have noticed that your September 1998 issue of the *Gazette* arrived in October. Council learned that the magazine was mailed late as a result of the lockouts of some boards. Dean Murray is pleased with the quantity of ads submitted in the *Gazette*. Be sure to quote the magazine when you purchase products. John Egsgard's term as editor comes to a close, and a new editor will be needed for the September 1999 issue. Are you up for it??

Have you checked out OAME's web site lately? (www.mathstat.uoguelph.ca/oame) Our web site coordinator, Doug Evans, states that the OAME membership form will be down loadable, an advertisement will appear for a new *Gazette* editor, and links to our

advertisers. Good news – OAME will be purchasing its own domain and site – you no longer have to remember the above site. Woo-hoo!!

EARLY/FORMATIVE YEARS The Early/Formative Years committee expressed great concerns about the process and the exemplars in the grades 1-8 document. They are in the process of expressing their concerns in writing to the Ministry of Education.

The committee is also looking towards writing an article for the *Gazette* related to the Statistics Canada web site, www.statcan.ca, and how that particular web site may be used to meet curriculum expectations. Peter Saarimaki will prepare a list of curriculum expectations at the grade 6 level which are met by the Statcan web site.

TRANSITION & SPECIALIZATION YEARS With the soon-to-be released grade 9 Math curriculum, it was decided by the executive that the transition and specialization years combine together to produce a "survival kit" for next year's grade 9 teachers. The committees spent Friday night brainstorming frequently asked questions (FAQs) of classroom teachers.

The document will include the FAQs with answers, a history of the development of the new curriculum (including Geoff Roulet's background paper, OAME's role), a gap analysis of the new curriculum (how it differs from the destreamed model), *Scope and Sequence* posters for grades 8-10 for both the applied and academic levels; and a complete exemplars unit (lessons, handouts, assessments, links with technology) to help teachers incorporate the vision of the curriculum. Dave Zimmer will serve as the over-all manager, leading the team of over 20 educators to make the document a reality.

RESOURCE SHARING

NCTM Frances Schatz reminded board members to spread the word that purchasing NCTM materials through her, the NCTM rep, generates 20% of the sales to OAME. She reported to council that she has received 153 individual orders, with a rebate cheque of $1072.72 to OAME. She used the rebate money to purchase more NCTM materials, and now has an extensive inventory. She will send out resources immediately if they are in stock. On behalf of the board of directors, I would like to whole-heartedly thank Frances for her outstanding efforts and endless hours not only processing the orders and maintaining inventory, but also for her numerous phone calls to NCTM to ensure we, Canadians, are treated fairly. My hat goes out to you, Frances!

JOHN KERSLEY Once again, John has given each director a bundle of valuable resources – pertaining from computer related articles (e.g. "Should home PC users worry about 2000?") to articles appearing in the *Abacus* from 20 years ago which are still relevant today. Contact any board member if you want to have a look at the resources.

Bookmark the following web site www.math.msu.edu/~winter/mjwhotlist1.html when you're on the Internet. It contains some interesting geometry and calculus links!

Here's a sample of the brain teasers we got: (i) A man gets out of his car but realizes he left the keys in the ignition. All the doors are locked and the windows are up but yet he was able to get his keys by himself. How?; (ii) A woman with no driver's license goes the wrong way on a one-way street and turns left at the corner with a no-left turn signal. A police officer sees her but does nothing. Why? Try them, and e-mail me for the solutions if you are stuck.

PROFESSIONAL DEVELOPMENT

LEADERSHIP'98 "Conquering the Challenge of Change" is slated to take place in Mississauga from February 25 to 27, 1999. The conference includes 7 hours of workshop time, numerous opportunities to network, keynote addresses from Mary Lou Kestell and Gerry Connelly, meals, two nights of accommodation, banquet, receptions, and publishers'/suppliers' display. The Leadership conference is designed to provide educators with the tools to be leaders in their classrooms, schools, and boards with the rapid changes in education. Be sure to contact either Bonnie or Dave Alexander to reserve a spot as a waiting list was created last year.

For the primary years, the focus will be on linking assessment and instruction. The workshop leaders are Sharon McPhail and Carolyn Robertson. They will focus on the new Primary years Assessment document to be published in early 1999.

There will be two secondary level workgroups focussing on the new secondary school Mathematics curriculum. Mary Howe and Emilia Veltri will lead the grades 9 & 10 workgroups while Myrna Ingalls and Kaye Appleby will lead the grades 11 & 12 workgroups. All four workshop leaders are members of the OAME board of directors, and all served on the writing team for the new Math curriculum.

Speak to your chapter rep about whether or not your chapter will be sponsoring a member to attend the conference free of charge.

OAME'99 Twenty-one chairs, with approximately ten committee members each, from Y⁴MA are working hard to make the OAME'99 conference, "*Mathematics: New Visions – No Limits*", from May 6 to 8, 1999 in Richmond Hill, a roaring success. They have gathered over 140 speakers, including keynotes Marian Small, Kay Toliver, Skid Crease, Don Fraser, and Phoebe Gillman.

Toliver ("Good Morning Miss Toliver" and "The Eddie Files" videos) has agreed to do a special two hour primary workshop for fifty elementary teachers in addition to her keynote address! For transition and specialization teachers, there will also be a half-day session at Canada's Wonderland at no extra charge. OAME members should expect to receive their program booklet by the end of January 1999. The cost for the entire conference is $135 for OAME members, so be sure to reserve your P.D. funds soon!

OAME 2000 Peter Lenczewski, on behalf of COMA chapter, reported to the OAME board that the theme of OAME 2000 will be "The A.R.T. of Mathematics – Assessment, Reform, and Technology". They are currently looking into a "conference within a conference" format similar to the "Math and Technology" conference in Exeter, New Hampshire.

OMO'99 The 1999 Ontario Mathematics Olympics is all set to go on Saturday, June 5, 1999 at York University for grades 7 and 8 students. TE²Y²MS, under co-chairs Stewart Craven and Brian McCudden, have organized activities ranging from individual tasks to team activities. Included will be the use of technology (either graphing calculator or CBL). Chapters will receive practice materials in January 1999 to assist with preparation.

Each chapter may send up to three teams. Speak to your chapter rep

for more information. The co-chairs were pleased to report to council that they have obtained a $5000 sponsorship from Imperial Oil. Way-ta-go TE²Y²MS (pronounced "teams")!!

AMTNYS'99 You may remember the joint partnership between AMTNYS (Association of Mathematics Teachers of New York State) and OAME in hosting a summer workshop back in summer 1995. AMTNYS will be hosting their 49th annual meeting at the Buffalo Convention Centre from November 11-13, 1999, and would like OAME members to attend or to present a workshop. For further information, contact conference coordinator, Bob Hazen

CHAPTER NEWS

Below are highlights of chapter reports submitted by chapter reps. to OAME. Don't hesitate to contact your chapter rep. to volunteer your services in a committee, event or to present a workshop at a conference. The networks and friendships you develop are long-lasting ... especially in our uncertain times when we need all the support we can get.

COMA is busy preparing to host OAME'2000. They have confirmed that the conference will take place at the University of Ottawa. The chairs and committees have been established, and the program is being organized. In addition, they are planning for their annual mini-conference on Monday, January 18, 1999 at Sir Guy Carleton High School. The conference will be geared towards the new Mathematics teacher. As if that weren't enough, they are also planning to hold their chapter Math Olympics on Saturday, April 24, 1999.

GOLDEN By the time you read this, their fall Primary/Junior workshops on November 11th, annual general meeting, and presentations to recruit new OAME members from the faculty of education at Brock University would have all taken place (the lag in time of writing this in October, and seeing it in print in December!). See chapter rep Elizabeth Pattison for more information if you're a Junior/ Intermediate teacher on how to attend the Leadership'98 conference free of charge (including accommodation)!

GVMA This chapter continues to produce quality publications. Two new publications include the revised grade 12 advanced exam package, and *Mathematics and Music Wave Shows* by Ray Nowak. Ray is not only a Math head in Peel, but he is also a professional musician. This book is a must for anyone who wants to see the relationships between Math and music! Check out the GVMA advertisement elsewhere in this issue of *Gazette* to order. Don't forget to frequently check their web site, www.golden.net/~gvma – they have an extensive collection of Math resource links! While you're at the site, have a look at their π buttons – they make excellent prizes when playing a Math game! 3.14159265358979323 ... (just a small piece of the π!!)

ISOMA continues to look for ways to keep its members informed about chapter happenings. In their newsletter, a database of e-mail addresses will be included. Their fall conference was held on November 4, (e-mail me for the details!) and they are busy preparing for their chapter Math Olympics, to be hosted again by Brad Stephens of Holy Trinity School.

MAC² postponed their three mini-conferences originally scheduled for October and November to assist elementary teachers implement the new grades 1 to 8 Math curriculum. They have stated that their conferences will take place once the contract situations get resolved. In the mean time, they are producing a comprehensive newsletter for their members this fall to communicate what is happening in Mathematics education, along with their plans for the year.

NWOAME Chapter rep. Wayne Fletcher reports that job actions have taken place in all but two of their school boards. As a result, they have not held any meetings and have postponed all activities. On the positive side, the chapter was able to financially support a member of their winning OMO team so that he could attend a summer Math camp in New York state last summer. Way to go, NWOAME!

SAME This busy chapter is currently hanging in mid-air, as they have been instructed not to have any meetings. They have set up an active bulletin board site, and will resume preparation for their spring conference in April once the political situation improves.

TE²Y²MS (pronounced "teams") hosted their fall conference, "Mathematics Education: Challenges and Opportunities" with many well known speakers. After hosting the awesome OAME'97 (aah – memories!), they are now working around the clock to host OMO'99 at York University on Saturday, June 5, 1999 for grades 7 & 8 students. See elsewhere in the *Director's Dialogue* for full details on the OMO!

WOMA is pleased that two of its members (Mary Howe and Kaye Appleby) will be acting as presenters for the Leadership'99 conference. Their fall conference, "*Sparks in the Darkness – Small Gems of Math Pedagogy*" is now history. They continue to publish a top quality newsletter, "*The Sine Post*" for its members, and will send copies to neighbouring schools to share with non-OAME members the benefits of joining. Plans are underway to provide enrichment sessions and Math Olympic competitions for elementary students in their chapter.

Y⁴MA has had to cancel their 11th annual fall conference due to the political uncertainty. They are planning their chapter Math Olympics, and look forward to sponsoring three teams to the OMO at York University in June 1999. They continue to be busy organizing the OAME'99 conference (see elsewhere for details!), and plan to outmatch their 1992 conference.

FINAL REMARKS

As a young teacher, I try not to get too upset or depressed with the political uncertainty of our days. I look towards my experienced colleagues for support to get through these challenging times. With the new four year Math curriculum, you can rest assured that OAME will be there to provide support and in-service. Attend a conference, network, volunteer your services in your local chapter, e-mail your colleagues (including me, please!), read past issues of the *Gazette*, and let's support each other during these rapidly changing times.

I leave you with a quote taken from the Quinte S.S. staffroom wall after last year's two week province wide protest: "A hundred years from now it will not matter what my bank account was, the sort of house I lived in, or the kind of car I drove ... but the world may be different because I was important in the life of a child."

Have a well-deserved Christmas holiday, and see you at OAME'99!!!

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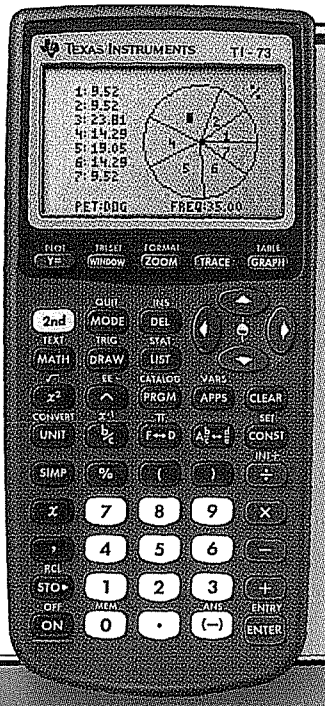
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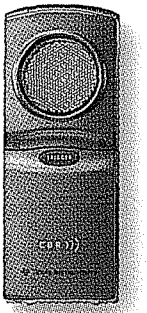
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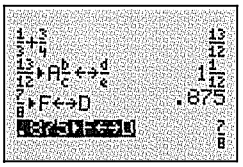
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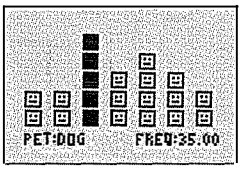
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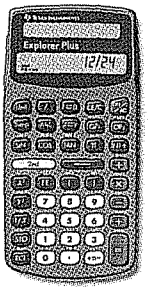
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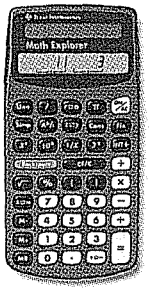
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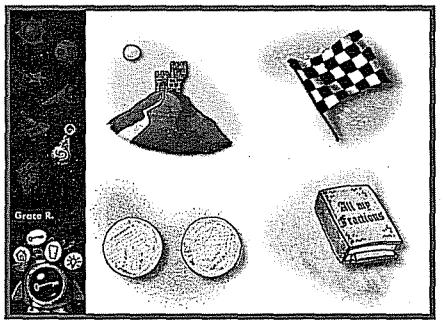
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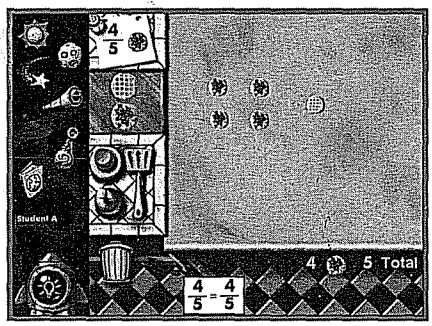
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Fraction Operations *Grades 4-6*

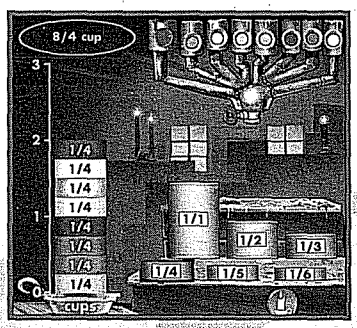
Students build on the concepts of fraction meaning and equivalence as they operate with fractions. Students see the purpose of constructing same size parts when they add and subtract using various models of fractions, and gain informal experience multiplying fractions when they create an area model to find a fraction of a fraction. They use measuring tools to explore division with fractions.



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