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Ontario Mathematics Gazette

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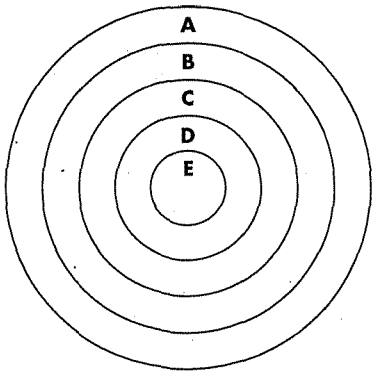
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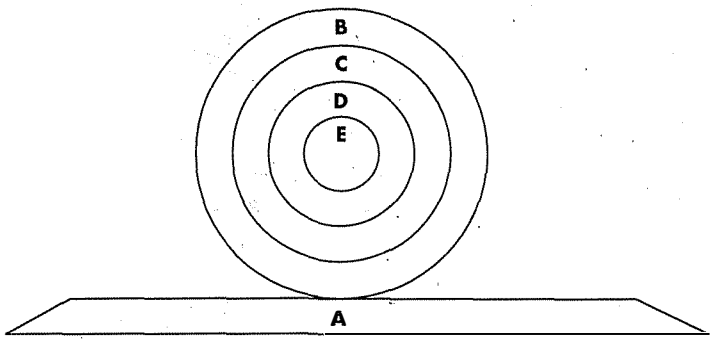
An Alternative Approach to Develop the Formula for the Area of a Circle

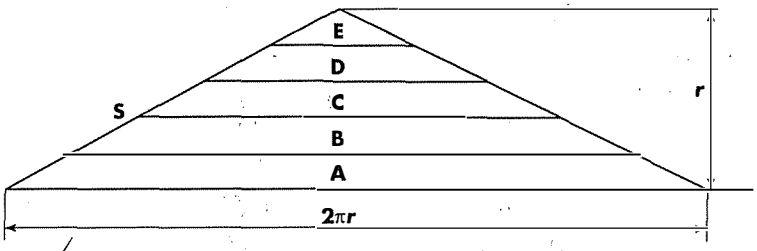
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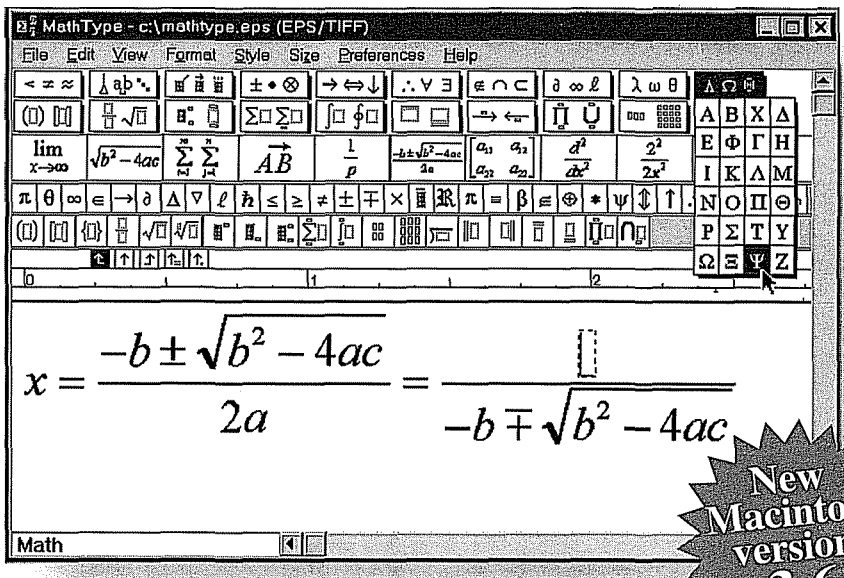
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▲ Editorial



JOHN EGSGARD

Education is still in a state of flux here in Ontario. As you will find in reading President Ralph Connelly's Ontario is not alone. California, with its population about equal to that of all of Canada, is going through an even greater upheaval. I hope that every mathematics teacher in Ontario will take the opportunity to comment on the proposed changes for the Ontario Secondary mathematics curriculum that can be found on the OAME web site. My big worry about these changes is that they may depend too much on the NCTM *Standards* which are taking a beating from many mathematics educators in the United States. But read the president of the NCTM's article on the NCTM *Standards*.

This issue of the *Gazette* is pointed primarily towards teaching ideas. *An Alternative Approach to Develop the Formula for the Area of a Circle* uses the diagrams displayed on the cover to re-examine the area of a circle. *Spatial Activities: Exploring Basic Properties of Shapes* supplies geometric ideas for use at the intermediate level. *The Palindrome Treasure Hunt* shows a way to interest your students by getting them notice the numbers on paper money. *Subtracting Fractions* is more theoretical than practical but will be intriguing to students as well. *Does Good Practice Make Perfect?* enlightens us on what some educators from the United States think about practicing mathematics to help learn and understand it. As usual we have a delightful article on problem solving *Seeking Mathematical Elegance Through Symmetry in Algebra* and a disturbing piece from our *Devil's Advocate* entitled *Who Cares?* The last articles are seeking comments from our readers. Please do send your comments in an email to at least one of the authors. Their email addresses are johngm@morgan.ucs.mun.ca for *Problem Solving* and alpint@bvg.on.ca for the *Devil's Advocate*.



▲ Letter to the Editor

Perhaps I am being picky, but it seems to me that in Step C, on page 24 of the September 1998 issue of the OAME *Gazette*, you have anticipated the solution in your check of the second derivative. Having found the values $x = 4$ and $x = -4$, the reason given for rejecting $x = -4$ should not be that lengths must be positive, but rather that the value of the second derivative at $x = -4$ is positive, resulting in a relative minimum value. It is, in my opinion, a dangerous habit to assume that because the problem asks for a maximum, and because the problems deals with physical quantities which must be positive, and because one of the possible values found is positive and one is negative that it follows that the solution to the problem, without doing a second-derivative test, must be the positive value found. It is quite possible to set up a function which would have a relative minimum at $x = -4$ but which might also have constraints (for example $0 \le x \le 10$) so that the maximum value of the function would occur at an endpoint.

Too often, because the check might be "tedious" or the solution is "obvious" we allow students to ignore this step in the solution of a problem. This is, in my mind, sloppy mathematics. We should, instead, be encouraging students to use appropriate methods of checking their work, whether that is simply checking the solution of an equation by substitution or by using the second-derivative (or other) test for maximum-minimum problems in calculus.

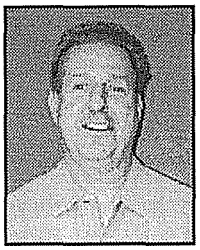
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▲ President's Message



RALPH D. CONNELLY

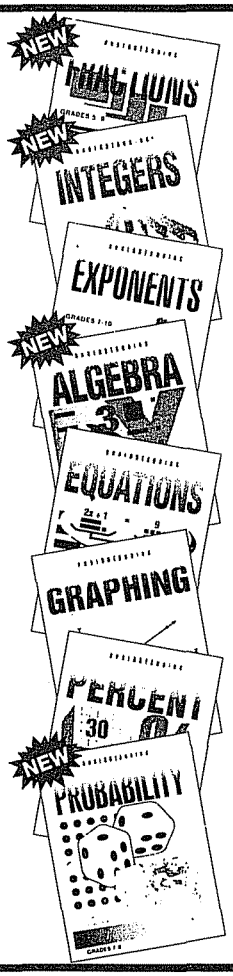
Regular readers of this column (both of you??) will recall the "time-warp" reference I made in my first President's Message. Hopefully by the time you read this, weather conditions will be very different than they are as I write this. I am at home, "grounded" by yet another vicious snowstorm which has combined temperatures of -15 degrees with high winds (wind chill reported as -40), lots of snow (another 12 cm so far, and the storm has just begun in earnest). I guess this should give me lots of time to write this message. Of course, the notes I had made for it are sitting in my office, and having once again left the task until the last minute (big surprise!), I'm going to have to wing it.

I'm not sure how many OAME members have been following, or are even aware of, the recent events related to school mathematics that are unfolding in California, but it's something that everyone involved with teaching mathematics at any level should be paying attention to. Because of the size and power of that State (their population is close to that of all of Canada) changes that happen there are almost certainly going to "ripple" through the rest of the country, and by proximity, to us as well. California has just adopted a new Mathematics Curriculum Framework that gives new meaning to "back to basics." As just a couple of very quick examples, calculators will no longer be allowed to be used for any mathematics until after Gr. 8. A very strong emphasis on "mastery" of "standard" paper-and pencil algorithms for operations on whole numbers, fractions, and decimals, as well as for square roots (yes, you read that correctly!) dominate the elementary grade curriculum. Topics such as Data Management and Probability have all but disappeared, and problem-solving "should be delayed until competence to mastery can be demonstrated with basic facts and skills". It would be a very big mistake to view this event as "a California problem." The very vociferous, influential, and politically savvy minority that lobbied successfully for the adoption of this Framework in California are at work everywhere (yes, even in Ontario!)

I believe in Ontario we have a very workable Gr. 1-8 Mathematics Curriculum; the "deliverables" that we've seen of the new Secondary Math Curriculum are overall very well done, and the work of EQAO and others have kept the central focus of problem-solving very effectively. However, it is now, more than ever, absolutely essential that teachers at all levels be given the necessary support in the way of professional development and resources to enable them to successfully deliver this curriculum at all levels. If we don't ensure that teachers have all the tools to be successful in implementing the curriculum as it currently stands, then you can be certain that the same forces that were at work in California will be on the scene to say:

"There, see, we told you it wouldn't work. Now here's what we suggest is needed." And we, too, could have a curriculum with it's own "Y2K" bug built in (i.e. that like California, we'll be "transported" back to 1900 by the end of the millenium!)

Enjoy the rest of the school year.



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▲ An Alternative Approach to Develop the Formula for the Area of a Circle

MAILVAGANAM KATHIRKAMANAYAGAN
WINSTON CHURCHILL COLLEGIATE INSTITUTE



Mailvaganam Kathirkamanayagan is a teacher of mathematics at Winston Churchill Collegiate Institute in Scarborough. He obtained his B.Sc at the Universiyt of Peradeniya, Srlanka and his Ph.D (Math) at the University of Texas at Arlington, Texas.

The formula for the area of a circle does not arise as naturally as the area of geometric shapes such as triangles, rectangles and squares. The easiest formula of area for teachers to explain to students is that of a rectangle. This is because, basically, a rectangle can be divided into a number of unit squares and its area computed by counting these squares. The next step is to show students the connection between the number of unit squares counted and the formula for the area of a rectangle.

From the concept of the area of a rectangle, the formula for the area of a triangle is easily developed. The area of a triangle is half the area of a rectangle of similar sides. However, the area of a circle is not intuitively connected to the formula for the areas of rectangles and triangles. As a result, it is very difficult and confusing for the students to visualize and fully appreciate the formula for finding the area of a circle.

The difficulty in connecting the area of a circle to that of a rectangle or triangle lies in the nature of their different boundaries of their shapes. The boundary of a circle consists of a continuous curved line where as a rectangle and triangle are both bounded by straight-line segments that can be separated into two basic components, namely, width and height. As a

consequence of a circle's curved boundary, the formula for calculating its area involves the constant and the squared term of the radius r.

To help students appreciate the formula for the area of a circle the method of ancient Greeks can be used connecting this formula to that of triangles. For this method, a circle is divided into n congruent isosceles triangles with vertices at the circle's centre and bases as tangents of the circle. This gives an approximation for area of $n\left[\left(\frac{1}{2}\right)r\left(\frac{2\pi r}{n}\right)\right]$ or πr^2.

It is then argued that the approximation becomes better as the number of triangles, n, grows. The purpose of this article is to show that it is possible to develop the concept of the area of a circle based on prior understanding of the formula for the areas of a rectangle and triangle. The ideas expressed in this article for explaining the formula for finding the area of a circle can be used to formulate a class activity that would help students to intuitively derive and verify this formula.

First, we shall visualize a circle as being made up of concentric rings as shown in Figure 1. Then we shall visualize a progressive unraveling of the strips defined by the rings. The first strip, defined as A, is unraveled in Fig. 3. The procedure for unraveling strip A is as follows. (Fig. 2) The strip is first cut along the line a-b ii) keeping the point c fixed, the strip is straightened out horizontally. Because the inner edge of the strip is shorter than the outer edge, and the unraveling process is symmetric, the unfolded strip has the shape of an isosceles trapezoid. The other strips are also straightened out one after the other as depicted in Figures 4 and 5. The length of the bottom of strip A is $2\pi r$ (the perimeter of the circle). After all strips are unraveled, the resulting figure is an isosceles triangle (Figure 5) with height equal to the radius, r, of the circle, and base of length $2\pi r$ (proof of this is given in appendix to this article). The area of this triangle is equal to the area of the original circle from which it was formed, and can subsequently be calculated as:

$$\text{Area of circle} = \frac{1}{2}\text{base x height}$$
$$= \frac{1}{2}(2\pi r)r$$
$$= \pi r^2$$

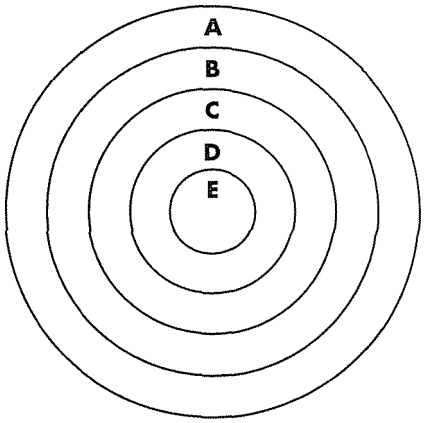


Figure 1

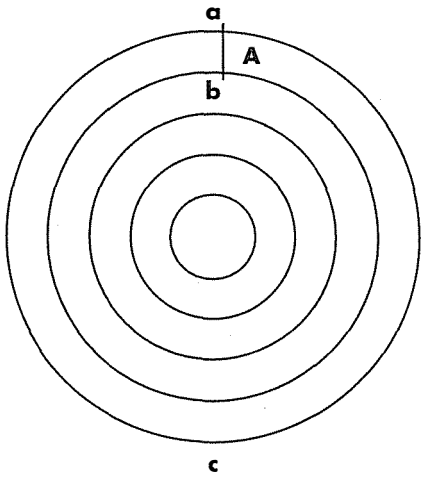


Figure 2

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The beauty of using this method to teach students the concept of the area of a circle stems from the fact that it relies upon concepts with which they are familiar. Secondly, a natural connection is established between the different elementary shapes. In addition, the formula for finding the area of a circle can be intuitively derived by the students.

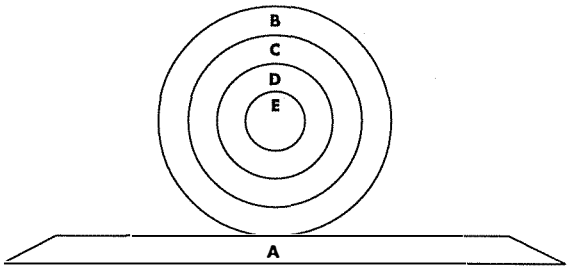


Figure 3

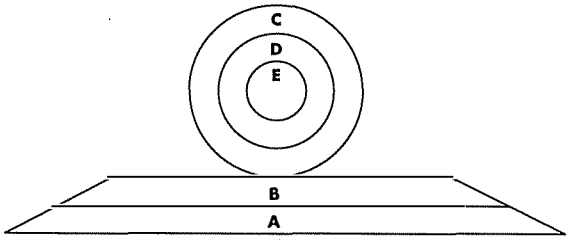


Figure 4

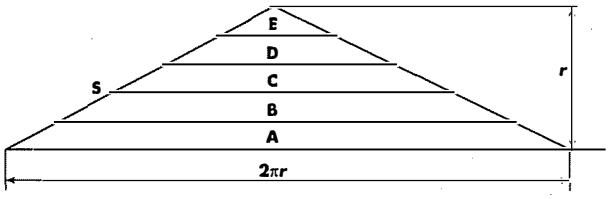


Figure 5

Appendix:

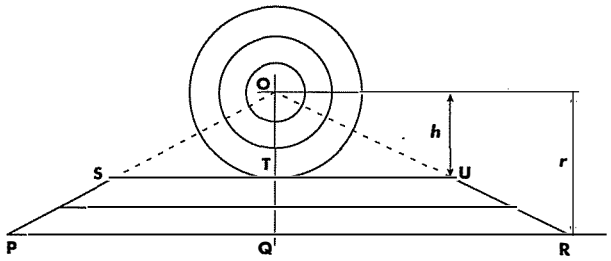


Figure 6

Since OT : OQ = h : r

= πh : πr

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Therefore, OSP and OUR are straight lines because of the equality of ratios.

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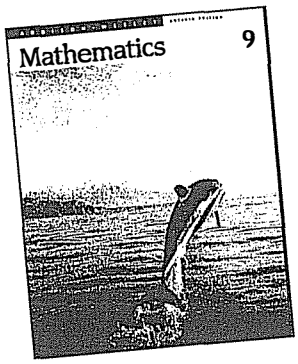
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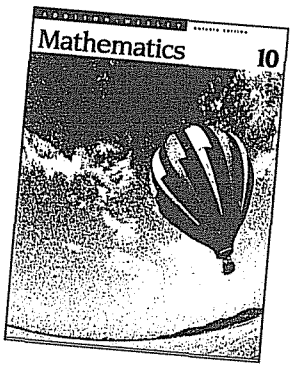
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▲ SPATIAL ACTIVITIES: EXPLORING BASIC PROPERTIES OF SHAPES

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Introduction

Very often students exhibit a variety of levels of geometric understanding. They manifest this disparity of understanding in many different ways. Some claim to already know what they are asked to prepare; by contrast, others may not even have a clear idea what a polygon is. Similarly, there are those who are confused about the conditions under which a parallelogram is a rectangle, whether or not a parallelogram is a trapezoid, and so on. In the midst of this classroom confusion, and in spite of the written and verbal pondering of many educators addressing the issue, we may well ask ourselves why it still persists. Are some students simply more naturally "mathematical", or are others simply lacking a key to mathematical understanding? Might there not be some practical, "decent", dedicated way of effectively remedying this problem?

Fortunately, we now have a sound school of thought that can explain such student responses: the van Hiele Levels of Thought Development in Geometry, pioneered by two Dutch middle school mathematics teachers, Dina van Hiele-Geldof and her husband, Pierre Marie van Hiele. Although originally presented in the late 1950s, their theory was not widely considered by North American educators until the mid 1970s, when Wirszup (1976) started writing about it. As such, over the past two decades there has been considerable scrutiny of van Hiele five levels of thinking, their existence, characterization, acquisition, and refinement at various levels (Usiskin, 1982; Mayberry, 1983; Burger & Shaughnessy, 1986; Gutierrez, Jaime, & Fortuny, 1991; Pegg, 1995).

The van Hieles identified five levels of reasoning which students go through in dealing with geometric concepts and figures. We know that the Piaget four Levels of Cognitive Development (Sensori-motor, Pre-operational, Concrete Operational, and Formal Operational) occur naturally and progressively during biological maturation. However, what the van Hieles identify as Levels of Thought Development in Geometry are not biologically achieved during a person's maturation; they can only be achieved by instruction and should be learned in their proper order: Levels 0, 1, 2, 3, and 4 (van Hiele, 1986). A brief description of these levels would be helpful here. Level 0: students identify and operate on shapes in their global appearances (wholistic); Level 1: students recognize shapes by their properties (part-whole); Level 2: students recognize relationships among properties and shapes (part-part; and, whole-whole); Level 3: students understand the deductive reasoning process; Level 4: students can work in different axiomatic systems. A more complete description of these five levels can be found in several National Council of Teachers of Mathematics (NCTM) publications such the *Mathematics Teacher* (see, for example, Shaughnessy & Burger, 1985, p 419-28) and the *NCTM 1987 Year Book* (see, M. Crowley, 1987, p. 1-16).

The NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) stated that the spatial sense is an intuitive feel for the individual surroundings and the objects in them. To develop spatial sense, children must have many experiences that focus on geometric relations; the direction, orientation, and perspectives of objects in space; the relative shapes and

sizes of figures and objects; and how change in shape relates to change in size. In particular, the *Standards* state that when children investigate the result of combining two shapes to form a new one, predict the effect of changing the number of the sides of a shape, draw a shape after it has been rotated a quarter or a half turn, or explore what happens when the dimensions of a shape are changed, they acquire deeper understanding of shapes and their properties. These type of activities promote spatial sense (NCTM, 1989, p. 49). These suggested activities would be related to van Hiele levels 0, 1, and 2.

The purpose of this activity is to implement the above NCTM position and to offer experiences in van Hiele levels 1 and 2. As a prelude for deductive reasoning - van Hiele level 3, there will be a set of exercises on justifying the identification of several resulting shapes through an intuitive reasoning.

The activity requires about two 40 minute class periods, depending on students' ability. Students may work individually, in pairs, or in small groups to promote peer collaboration.

The Activity

The activity consists of a set of mini activities utilizing spatial hands-on *shape-dissection* and *shape-composing* operations. The shape-dissection operation is based on the Theory of Dissection. For details on the Theory of Dissection, the reader can refer to Eves (1972). The shape composition is based on the basic transformations or motions: translation or slide, rotation or turn, reflection or flip, or any combination of them. The transformations are described in several contemporary text books such as Alexander, et al. (Mathquest 7, 1989, pp. 211-250; Mathquest 8, 1988, pp. 437-474); Ebos, et al. (Math In Context 9, 1993, pp. 477-507); and Knill et al. (Mathpower 9, 1994, pp. 533-563).

Set Up

Consider a right triangle drawn on a plain sheet of paper; choose the midpoints of the hypotenuse and one of its sides; join these two midpoints by a line segment; cut out the right triangle and cut along the line segment; make the resulting triangle #1 and the quadrilateral #2. The dissection is depicted in Figure 1.

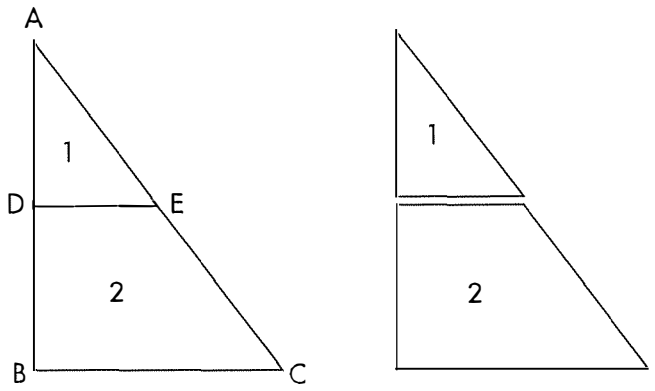


Figure 1

At this point, one can bring up some of the embedded basic properties of shapes for discussion. For example, the question "Is there a relationship between DE and BC? Explain" would imply an emphasis on the following (part-to-part relationships) and the underlying theorem within Δ ABC:

$$DE // BC;\ DE = \left(\frac{1}{2}\right) BC;\ \text{and}$$

and the segment between the mid-points of two sides of a triangle is parallel to the third side and half its length.

As a result of dissecting Δ ABC, an inquiry about the identification of the shapes of the resulting pieces,1 and 2, by their properties (part-to-whole relationships) would be made. This inquiry would create an opportunity to raise questions on the similarity aspects of the original Δ ABC and the resulting shape of piece 1, namely, Δ ADE (whole-to-whole relationships): "what parts of these two figures are equal? What parts are proportional?". It also would lead to recall of the characteristics of trapezoids in general and a right trapezoid in particular, namely DBCE.

In the following, we break down the activity into several sub-activities.

Activity 1(a)

Place the two pieces, 1 and 2, on any flat surface and compose the original triangular shape. Apply as you wish, a motion of slide, turn, or flip, or a combination of them on piece 1, 2, or both so that, without overlapping the pieces, you would compose a rectangular shape. Do not lift any of the pieces completely off the desk.

Discussion

A composed rectangular shape is depicted in Figure 2a.

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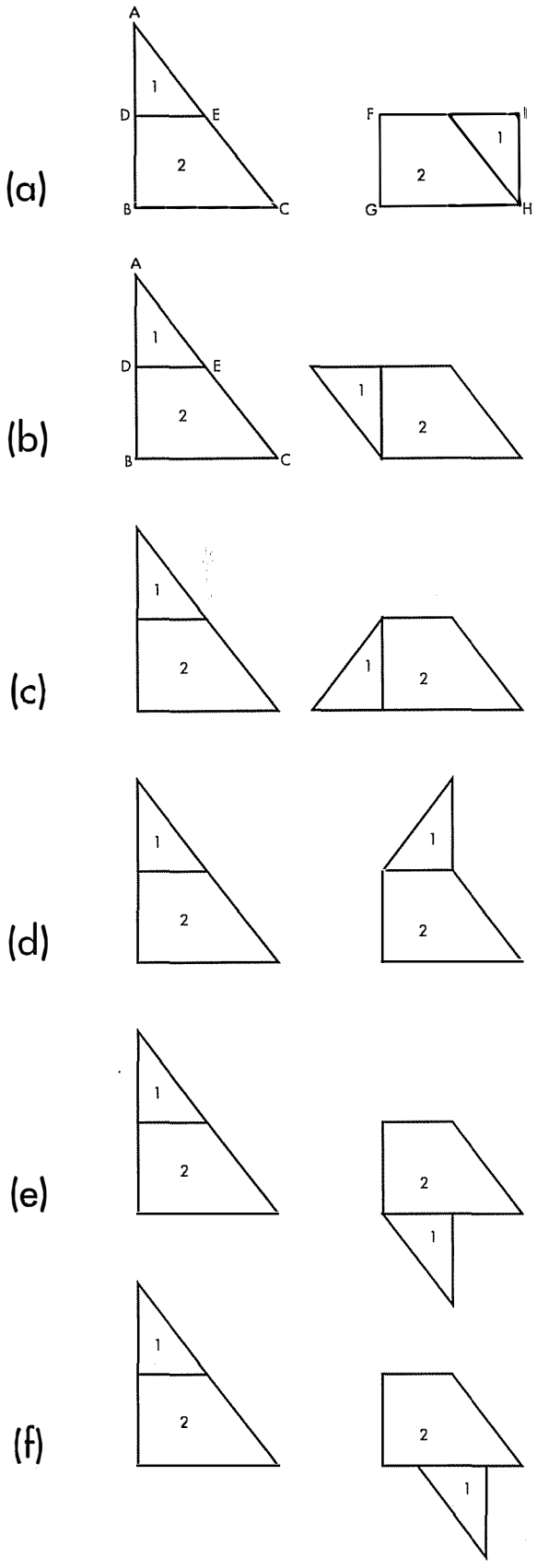


Figure 2

Figure 2a presents an arena for steering a discussion focussing on aspects and properties of the shapes involved: the original triangle, pieces 1 and 2, and the resulting shape. For example, the question of recognizing the resulting shape as a rectangle can be explored. A dialogue such as "try to justify that the resulting shape in Figure 2a is rectangular" would bring into the discussion the characteristics that are required for a quadrilateral to be a rectangle: a quadrilateral with (1) opposite sides are parallel to each other, and (2) four right angles. This requires highlighting the basic properties of the motions used and which recalling the characteristics of the three motions and their properties such as:

(i) the measures of the parts of a shape are invariant under each of the motions - the shape is preserved;

(ii) in general:

1 under the motion of a slide, the corresponding sides of a shape and its image are parallel;

2 under a 90° rotation, the corresponding sides of a shape and its image are perpendicular;

3 under a 180° rotation, the corresponding sides of a shape and its image are parallel;

4 under a 270° rotation, the corresponding sides of a shape and its image are perpendicular;

5 under a reflection, the line of reflection is the perpendicular bisector of the line joining any point of a shape and its corresponding image;

(iii)the relationships among the motions such as:

1 two consecutive reflections about two parallel lines is a slide; and

2 two consecutive reflections about two perpendicular lines is a 180° rotation; and so on.

Now, coming back to try to justify the resulting shape shown in Figure 2a as a rectangle, one may proceed as follows:

In Figure 2a, a 180° turn of piece 1 to the right (clockwise) about the point E

→ Δ ABC is transformed into the shape FGHI, and FJI is a straight line (as a result of a half turn); FI // GH (by the previous discussion above), GF // HI (by 180° turn)

→ ∠ B = ∠D = ∠F = ∠ I = 90° [∠ B = 90° is given]

→ the four angles ∠G, ∠ F, ∠ I, ∠H are right angles

→ FGHI is a rectangle.

Another crucial point for discussion would be an explanation of the question of whether or not the areas of Δ ABC and the rectangle FGHI are equal; and whether the perimeters of the two shapes are necessarily equal.

Activity 1(b)

Reassemble pieces 1 and 2 back into the original triangular shape. In a similar way of part (a), compose a parallelogram shape. Try to justify that the resulting shape of your composition of pieces 1 and 2 is a parallelogram shape.

A composition of a parallelogram shape is shown in Figure 2b.

A discussion on the conditions for a quadrilateral to be a parallelogram and intuitive steps to justify the identification of the resulting shape would be made, similar to those in part (a). Note that in each of the desired compositions of new shapes starting with case (a) above, there would be more than one way to arrive at the desired composition. For example, in Figure 2b, starting with the triangular shape, one may apply a half turn about the point D to the left on piece 1 and the desired composition follows. Or starting with the original shape, it would be sufficient to apply a half turn clockwise about point D on piece 2. A combination of more than one motion would be another possibility: starting with the original shape, make a rectangular shape as in part (a), see Figure 2a, and then slide piece 1 along the base a distance = GH; and so on.

The question of the relation between the area of the two shapes would be raised here. Correctly recognizing the relationship between the two areas is based on the understanding that nothing has been lost or added to the amount of the total size of the original shape by the shape-dissection and shape-motion operations.

Activity 1(c)

Starting with the original shape, use pieces 1 and 2 to compose a trapezoid shape. Justify that the result of your composition is a trapezoid shape.

Figure 2c shows a possible composition. A discussion analogous to that of part (a) would be raised focusing on the basic differences among the newly composed shapes of a rectangle, a parallelogram and a trapezoid. It is most important to highlight to the students whether or not a rectangle is a parallelogram and/or a trapezoid and vice versa. These lattice type interrelationships among the family of quadrilaterals are often unclear in the minds of students.

In a similar fashion, the rest of the activity may be introduced.

Activity 1(d)

Starting from the original shape, assemble a five-sided shape, a pentagon. Show your reasons why your shape is a pentagon. Figure 2d represents a possible composition.

Activity 1(e)

Reassemble the original triangle; compose a shape of a hexagon. Figure 2e shows a possible composition.

Activity 1(f)

Reassemble the original shape; compose a shape of a heptagon. Figure 2f depicts a composition.

Exercise 1

Review the results of the above compositions. Try to find out the maximum number of sides of a shape that is possible to assemble using pieces 1 and 2 with no overlapping. Find an algebraic rule for the maximum number of sides that a composed shape can possibly have.

Activity 2

In Activity 1, the original triangular shape in each of parts, (a), (b), (c), (d), (e), and (f) was the starting point. In this activity, it would be interesting to continue from the newly composed figure onward. In a sense, we would have a voyage in the space of polygons. But then we may be thinking of returning back home after all; thus the voyage would then has to be a returned journey:

A Return Voyage

Throughout the following steps, make a journal of your actions; make a full record of what you would have used.

(a) Using the pieces 1 and 2, reassemble the triangular shape; and then compose a rectangular shape;

(b) starting from the rectangular shape, compose a shape of a parallelogram;

(c) from the parallelogram, compose a shape of a trapezoid;

(d) from the trapezoid, compose a shape of a pentagon;

(e) from the pentagon you have, compose a shape of a hexagon;

(f) from the hexagon, compose a shape of a heptagon, and finally;

(g) assemble back the original shape.

Figure 3 represents a possible formation of the shapes listed above.

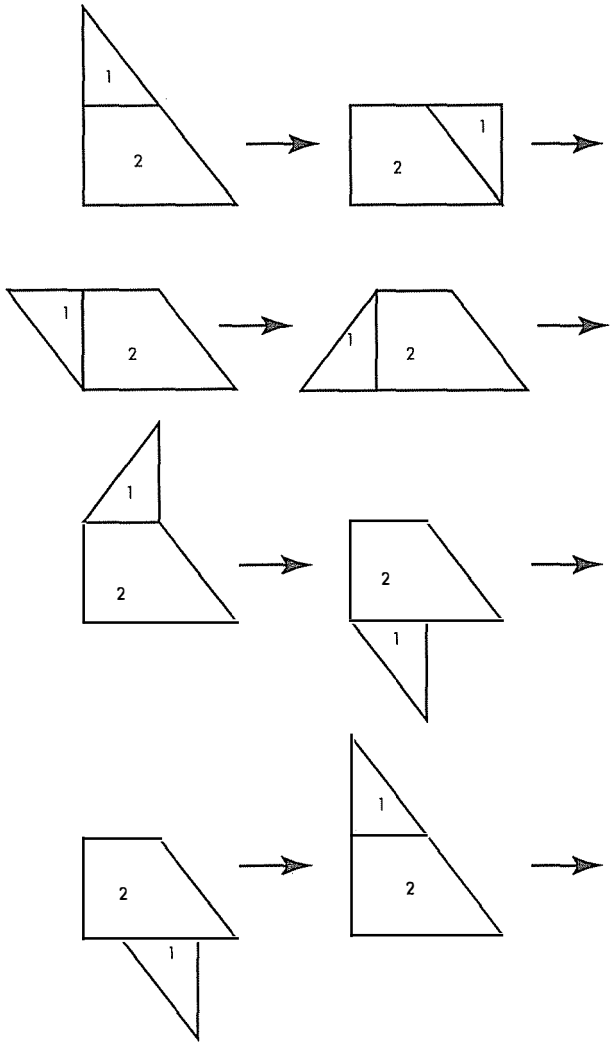


Figure 3

The following exercise would be a further application of the spatial hands-on experiences involved in the above activities. It would extend our experiences with surprises and imagination.

Exercise 2

Try to dissect the given shape in each of the following items into whatever pieces you wish so that you would be able to compose a right triangle shape.

(a) Start with *any* rectangular piece of a plain paper.

(b) Start with *any* parallelogram piece of a plain paper.

(c) Start with *any* trapezoid piece of a plain paper.

(d) Start with *any* regular pentagon, hexagon or heptagon piece of plain paper.

For *non-regular* pentagon, hexagon, etc. the composition into a right triangular shape would be a challenge probably suitable for upper high school grades and math recreational enthusiasts:

(e) Start with *any non-regular* (convex or not) pentagon, hexagon or heptagon piece of plain paper. Try to dissect the shape into whatever pieces you wish so that you would be able to compose a right triangle shape.

Epilogue

Most children, even before they come to school, can recognize and name some simple geometric figures. In early grades the curriculum does little to take children beyond this level because the interrelatedness of simple polygons is rarely the focus of activities. The problematic nature of modern school geometry was probably best described by Wirszup (1976) some two decades ago in a statement which refers extensively to van Hiele levels. He stated that the majority of our high school students are at their first level of development (p. 96); this is supported by Mayberry (1983) who found that preservice teachers are also at the first level of development. The second achievement results from the Third International Math and Science Study (TIMSS) released on June 10, 1997 and earlier in 1996 were not encouraging either. This unfortunate state of affairs is due in part to the scarcity of a recognized mathematical process available to youngsters for expressing interrelatedness. Given the state of affairs in school geometry, and based on the NCTM *Standards* position(1989, p. 49), we have proposed a class of mathematical operations based on dissecting simple shapes into parts, and composing the parts into new shape formations, while keeping certain fundamental attributes of the figures intact. This offers a process for developing understanding of interrelationships.

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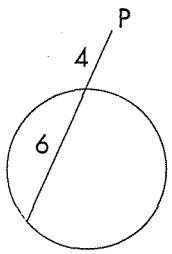
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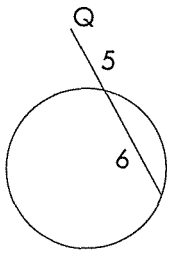
▲ TEACHING IDEAS FROM THE EDITOR

THE MAGIC NUMBER FOR A POINT OUTSIDE A CIRCLE

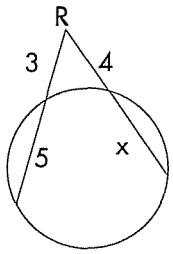
There is an interesting fact about a circle and a point outside the circle that I introduce in a special way. I draw a circle, mark in a point P outside the circle, and then draw a secant from that point to the circle. (Figure 1) I tell the students that the magic number for point P is 40. Then I draw another circle (Figure 2) and ask if anyone can tell me the magic number for the point Q. Whether or not a student guesses the magic number I draw 4 or 5 more diagrams. Usually most of the students in the class are able to figure out the magic number after the fifth diagram.



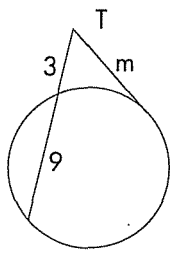
MN = 40
Figure 1



MN = *55*
Figure 2



MN = 24
Figure 3



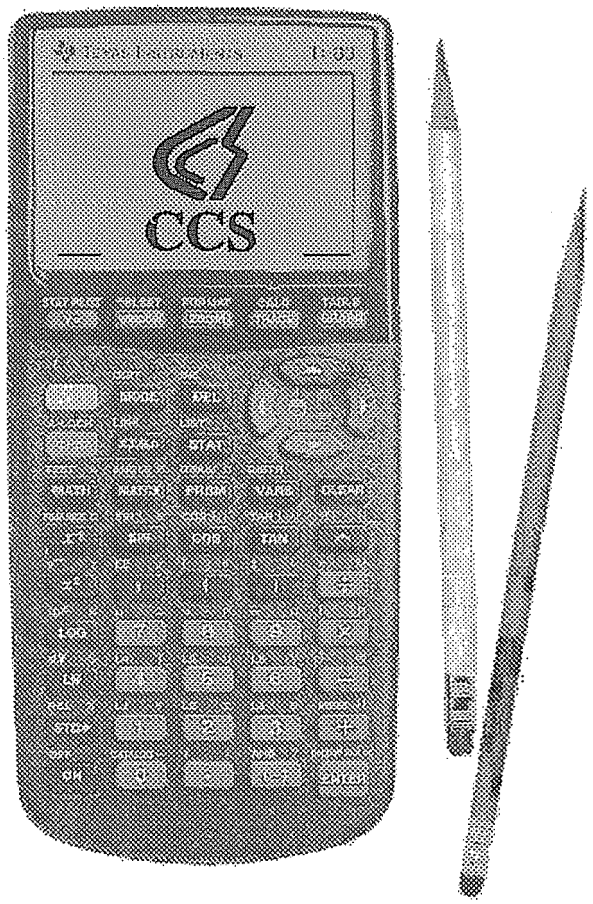
MN = 36
Figure 4

I then put problems like Figure 3 and Figure 4 on the board and ask the students to determine the variable x using the magic number. Even the weakest students are able to do the problem.

Finally we write down a rule for finding the number and then prove the rule is true.

While students are satisfied with the rule PA x PB = the magic number, it is important to emphasize that PA and PB are the distances from P to the points where the secant cuts the circle, that is, from P to the nearer point and P to the farther point. Note that in the tangent situation of Figure 4 the nearer and farther distances are the same so that m is the square root of the magic number.

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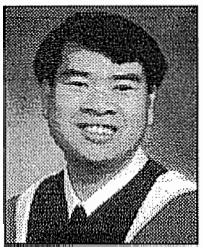
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▲ THE PALINDROME TREASURE HUNT

LOUIS LIM
MATHEMATICS TEACHER
& ISOMA CHAPTER REP.
QUINTE S.S., BELLEVILLE



Louis Lim is a third year Mathematics Teacher with the Hastings and Prince Edward District School Board. He is the ISOMA rep to OAME, writes the Director's Dialogue for the Gazette, and enjoys presenting workshops at conferences. He is also a part-time Master of Education student at Queen's University interested in teaching Mathematics through computer technology. Louis is indebted to the many outstanding educators he has come in contact with, in particular, his mentor, Ron Lancaster.

Numismatics and Mathematics are related – they both involve the study of beauty. I am too young to remember the craze during the late 1960s and early 70s when silver coins were being hoarded. But I have recently witnessed what I call the *"Palindrome Treasure Hunt"* at coin shows.

While coin collecting has always been popular, recently, there has been growing interest in collecting paper money. Many signature combinations, along with serial numbers, make intriguing challenges for the collector. For the past several years, I have seen many "radar" notes being displayed at dealer tables. A dealer told me that radar notes occur when the serial number reads the same back and forth. Aaah – palindromes! And "radar" is a palindrome!

In my grade 9 destreamed classes, I am faced with students of varying abilities and interests. I believe that my enthusiasm and love for the subject will overflow to some, but for others, I endlessly attempt to find the "click" which will hook them to Mathematics. My "Palindrome Treasure Hunt" is just one attempt.

I introduce the class with a transparency of three "radar" notes for the overhead. I ask the class to find all the similarities and differences of the three bank notes. We make a list, and often in each class, there will be a student who observes that something extraordinary has happened in the serial numbers.

At this moment, usually about half a dozen students instantly search their wallets for "radar" notes. On every occasion, so far, nobody has been able to find one. A student will often shout, "Why?", and the golden opportunity for the worksheet to search out the answer has arrived! I tell them not to give up, and should they find one, I would like them to bring it to class, and three bonus marks will be awarded. Also, good dinner table conversation!

In fact, I take the students down memory lane and remind them of the hoarding of the one dollar bill when they were being replaced by the "loonie" in 1987. I collected several hundred dollar bills, and only found one "radar" note in total! The joy of finding a "radar" note is indeed priceless!

Below is the worksheet the students work through:

#1. In the bank note, show in fig. 1, the serial number is very special. Paper money collectors treasure finding such an item since it can be worth $10 to $300 in addition to its catalogue value! Such a note is often referred to as a *"Palindromic Note."* What is a palindrome?

#2. (a) From the above serial number, choose *any* integer (two or three digits should do).

(b) **Reverse** the digits of your chosen integer.



Figure 1

(c) Add the integer chosen to its reversal.

(d) **Repeat** the process in (c) until another palindrome is obtained. (This can often be done)

#3. Palindromes also appear in words and expressions.

(a) Write a palindromic word.

(b) Write a palindromic expression or sentence.

#4. (a) Does our bank note contain any two-digit prime numbers in the serial number?

(b) If yes, **reverse** the digits of the two-digit prime number. Is this new number still prime?

(c) List all the two-digit prime numbers that are also palindromic when their digits are reversed.

#5. In our present issue of bank notes, each series is numbered from 0000000 to 9999999. A maximum of ten thousand notes may be palindromic. What is the probability of picking up a palindromic bank note? (Write as a fraction and then as a percent)

#6. *"Solid Numbered Palindromic Notes"* have serial numbers such as 1111111, 2222222. For every ten million consecutive notes produced, a maximum of _____ notes may have a solid number. What is the probability of finding a solid numbered palindromic bank note?

#7. *"Ascending/Descending (Ladder) Palindromic Notes"* have consecutive serial numbers such as 1234321, 3456543.

(a) Using all 7 digits, write all palindromic combinations that **first ascend** and then descend.

(b) Using all 7 digits, write all the **descending/ascending** palindromic combinations.

(c) What is the probability of finding an ascending/descending or descending/ascending palindromic bank note in a ten million consecutive sequences of notes?

#8. Keep your eyes out for palindromes when you save/spend your paper money! Imagine giving the cashier a $10 bill to buy some candy, and receiving a crisp $5 ascending/descending palindromic note in return! The 8th edition of *The Charlton Standard Catalogue of Canadian Government Paper Money* lists such a gem having a value range of $75 to $200!

(a) List one reason why the value range is so large.

(b) The above is a price range for an uncirculated (crisp) note. Here is a table for circulated notes:

CONDITION	VALUE OF UNCIRCULATED PREMIUM
Almost Uncirculated	80 %
Extremely Fine	60 %
Very Fine	40 %
Fine	20 %
Very Good	15 %

Calculate the price range of the $5 bill had it been found in very fine condition (this means it has been circulated, having some creases, but is fairly crisp and clean).

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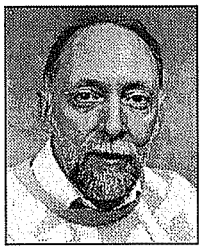
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▲ SUBTRACTING FRACTIONS

L. KEITH ROY
DEPARTMENT OF MATHEMATICS AND STATISTICS
LAKEHEAD UNIVERSITY
THUNDER BAY



Keith is Professor and former Chair of Mathematics and Statistics at Lakehead University. He has been a speaker at several OAME and NWOAME conferences and is the author of several previous articles in the Gazette.

Did you ever notice that

$$\frac{1}{5}-\frac{1}{6}=\frac{1}{30} \text{ and } \frac{1}{8}-\frac{1}{9}=\frac{1}{72}$$

The numerators are all 1's and the denominator in the answer is the product of the denominators of the fractions being subtracted. Also note that the denominators of the fractions being subtracted are consecutive positive integers.

In general, using a little bit of algebra, one can show that such results are not accidental.

$$\frac{1}{n}-\frac{1}{n+1}=\frac{n+1}{n(n+1)}-\frac{n}{n(n+1)}=\frac{1}{n(n+1)}$$

There are a number of similar shortcuts that could be discovered by students. Of course these shortcuts are not meant to supplant the tried and true methods of subtracting fractions but could be used to give students who are experienced with subtracting fractions an opportunity to investigate, experiment, and discover.

QUESTION 1

What would happen if the number in the numerators of the fractions being subtracted is not 1?

Notice that $\frac{4}{5}-\frac{4}{6}=\frac{4}{30}$ and $\frac{7}{8}-\frac{7}{9}=\frac{7}{72}$

Let a be a positive integer. Then

$$\frac{a}{n}-\frac{a}{n+1}=\frac{a(n+1)}{n(n+1)}-\frac{an}{n(n+1)}=\frac{a}{n(n+1)}$$

Alternatively,

$$a\left(\frac{1}{n}-\frac{1}{n+1}\right)=a\left(\frac{1}{n(n+1)}\right)$$

Again, as long as the denominators of the fractions being subtracted are consecutive positive integers and the numerators are equal, the denominator in the answer is the product of the denominators of the fractions being subtracted, and the numerator is unchanged.

QUESTION 2

What would happen if the denominators of the fractions being subtracted are positive, but not consecutive, integers?

It might be best to approach this question in two steps.

Notice that $\frac{1}{5}-\frac{1}{8}=\frac{3}{40}$ and $\frac{1}{4}-\frac{1}{9}=\frac{5}{36}$

Also $8-5=3$ and $9-4=5$

Let r be a positive integer and suppose that the numerators of the fractions being subtracted are both equal to 1.

$$\frac{1}{n}-\frac{1}{n+r}=\frac{n+r}{n(n+r)}-\frac{n}{n(n+r)}=\frac{r}{n(n+r)}$$

This time, if the denominators of the fractions being subtracted differ by a positive integer, the numerator of the result will be that positive integer, and the denominator again will be the product of the denominators of the fractions being subtracted.

Now notice that $\frac{4}{7}-\frac{4}{9}=\frac{8}{63}$ and $4(9-7)=8$

Also notice that $\frac{3}{5}-\frac{3}{11}=\frac{18}{55}$ and $3(11-5)=18$

Let r be a positive integer and suppose that the numerators of the fractions being subtracted are both equal to the positive integer a.

$$\frac{a}{n}-\frac{a}{n+r}=\frac{a(n+r)}{n(n+r)}-\frac{an}{n(n+r)}=\frac{ar}{n(n+r)}$$

The numerator of the result is the product of the difference of the denominators of the fractions being subtracted and the common numerator of the fractions being subtracted.

QUESTION 3

Notice that $\frac{5}{6}-\frac{4}{5}=\frac{1}{30}$ and $\frac{8}{9}-\frac{7}{8}=\frac{1}{72}$

In each of the above situations, the numerators differ by 1 and so do the denominators. Also, the numerator of the first fraction on the left equals the denominator of the

second fraction. Will the numerator of the result always be 1?

$$\frac{n}{n+1}-\frac{n-1}{n}=\frac{n(n)}{n(n+1)}-\frac{(n-1)(n+1)}{n(n+1)}=\frac{n^2}{n(n+1)}-\frac{n^2-1}{n(n+1)}=\frac{1}{n(n+1)}$$

So the numerator of the result will always be 1.

QUESTION 4

Suppose the numerator of the first fraction on the left equals the denominator of the second fraction. What would happen if the numerators differ by a positive integer and the denominators differ by the same positive integer?

$$\frac{n}{n+r}-\frac{n-r}{n}=\frac{n^2}{n(n+r)}-\frac{n^2-r^2}{n(n+r)}=\frac{r^2}{n(n+r)}$$

This time the numerator of the result is the square of the positive integer by which the numerators, and denominators, differ.

So $\frac{7}{9}-\frac{5}{7}=\frac{4}{63}$ since the numerators, and denominators, differ by 2.

QUESTIONS FOR THE READER

(a) Notice that $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$ and that $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$

Also notice that $\frac{5}{10}-\frac{5}{11}=\frac{5}{110}$

and that $\frac{5}{10}+\frac{5}{11}+\frac{5}{110}=1$

Find other examples of three fractions with the above characteristics.

What must be the relationship between the common numerator of the three fractions and the smallest denominator?

(b) Suppose the three fractions in (a) must have a total of 3 rather than 1.

Find examples of three fractions with such characteristics.

What must be the relationship between the common numerator of the three fractions and the smallest denominator?

(c) Notice that $\frac{2}{4}-\frac{2}{9}=\frac{10}{36}$ and that the total of the three fractions is one.

This time the denominators of the fractions on the left differ by more than one.

Find other examples of three fractions with the above characteristics.

What must be the relationship between the common numerator of the two fractions on the left and the smallest denominator?

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▲ ABOUT PROBLEM SOLVING: SEEKING MATHEMATICAL ELEGANCE THROUGH SYMMETRY IN ALGEBRA



JOHN GRANT MCLOUGHLIN
FACULTY OF EDUCATION
MEMORIAL UNIVERSITY OF NEWFOUNDLAND

This opening example could be found in many textbooks.

The sum of three consecutive numbers is 126. Determine the numbers.

Typically students let the numbers be x, x + 1, and x + 2. This leads to the equation

3x + 3 = 126. Hence, x = 41. It follows that the numbers are 41, 42, and 43.

The above example illustrates the tendency to let x be the smallest number in a list of numbers. Observe that if x had been the middle number (i.e., the representations x - 1, x, x + 1 would have been used) the equation 3x = 126 would have been produced, thus, giving a value of x in a slightly more direct manner. Now let me move the discussion to two examples of problems in which the idea of symmetry about x may prove to make significant improvements in terms of elegance.

Consider the following problem:

Show that the product of four consecutive natural numbers is always one less than a perfect square.

The usual approach may lead a solver to attempt to show that x(x + 1)(x + 2)(x + 3) is one less than a perfect square. Instead, consider the following technique. Now we have four consecutive numbers. Therefore, it is impossible to make the middle number x; however, the numbers can be represented as

$$\left(x-\frac{3}{2}\right),\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right) \text{ and } \left(x+\frac{3}{2}\right)$$

so that they produce a product that is symmetric about x. The product can be simplified by identifying the differences of squares that will always emerge when such symmetry is created through the selective representations of the numbers.

$$\text{Let } \left(x-\frac{3}{2}\right)\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)\left(x+\frac{3}{2}\right) = \left(x^2-\frac{9}{4}\right)\left(x^2-\frac{1}{4}\right)$$

$$= \left(x^4-\frac{10}{4}x^2+\frac{9}{16}\right)$$

$$= \frac{\left(16x^4-40x^2+9\right)}{16}$$

Recall that the objective was to show that the product is to be shown to be one less than a perfect square. Call the product P. Minimal manipulation produces the required result, as shown.

$$P = \frac{\left(16x^4-40x^2+9\right)}{16}$$

$$= \frac{\left(16x^4-40x^2+25-16\right)}{16}$$

$$= \frac{\left(16x^4-40x^2+25\right)-16}{16}$$

$$= \frac{\left(4x^4-5\right)^2}{4}-1$$

$$= k^2-1 \text{ where } k = \frac{\left(4x^2-5\right)}{4} = x^2-\frac{5}{4} = \left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)-1$$

Therefore, k is an integer and P is one less than a perfect square.

The symmetric feature allows for the multiplication to be conducted in an easier manner and with less likelihood of error. Further, the resulting expression is likely to be more friendly to manipulate for the purpose of factoring, or otherwise. One day I found myself posing a family of problems, one of which is presented here.

Determine all non-negative integers, x, for which x(x + 40) is a perfect square.

The decision to opt for a symmetric approach proved to be effective. I shall share the solution here. Perhaps some readers would like to share alternate approaches for solving this problem. My solution begins by letting k = x + 20, thus permitting symmetry to figure prominently. Since a perfect square product is required, the product is assumed to equal n^2.

$$n^2 = x(x + 40) = (k - 20)(k + 20)$$

$$= k^2 - 400$$

It follows that $k^2 - n^2 = 400$ or $(k - n)(k + n) = 400$. Since k, n are non-negative integers, the factors of 400 generate 8 pairs of simultaneous equations, namely:

$k - n = 1, \quad k + n = 400$ *

$k - n = 2, \quad k + n = 200$

$k - n = 4, \quad k + n = 100$

$k - n = 5, \quad k + n = 80$ *

$k - n = 8, \quad k + n = 50$

$k - n = 10, \quad k + n = 40$

$k - n = 16, \quad k + n = 25$ *

$k - n = 20, \quad k + n = 20$

Adding (k - n) and (k + n) gives an even quantity. Hence, the three systems marked * do not have integral solutions. The other systems gives the following values of k respectively: 101, 52, 29, 25, and 20. Since $k = x + 20$, the values of x which produce perfect square products, $x(x + 40)$, are 0, 5, 9, 32, and 81 in increasing order.

The beauty of mathematical solutions are often rooted in elegance or insight. The symmetric idea illustrated in this piece has represented one such personal insight that has provided me with a satisfactory sense of mathematical elegance in different situations. It has been shared as an example rather than a model. Your own comments and examples pertinent to self-discovery of mathematical elegance and insight are welcomed.

OAME Conference (Newsgroup)

OAME now has a conference (newsgroup) through ENO that can be used for discussions among members of the new curriculum or other issues that members want to talk about. You must be a member of ENO to participate, ENO membership is free to all educators in Ontario and can be done via their website **www.enoreo.on.ca**

The address for the OAME conference is:
enoreo.eduorg.oame

▲ Devil's Advocate Who Cares?

ALEXANDRU PINTILIE
MATHEMATICS TEACHER,
BAYVIEW GLEN SCHOOL



When Alexandru Pintilie, B.Sc. Math, M.Sc. Stats, ex-A.S.A., does not teach math, he boasts about it, chaperons the soccer team, chauffeurs the chess team or lectures his teenage son.

I admit that, in writing, this title creates confusion. Is it interrogative or is it rhetoric? Who cares about math teaching in my school, in Ontario, in Canada, on this ever shrinking planet?

Although it appears in March, this article was conceived during the last week of December when my mind also had the awkward task of deciding for a New Year resolution. One is not worried that much about having the strength for fulfilling one's promises . What makes a resolution hard to choose is whether it is worth the trouble to make it the highlight of one's immediate future.

Even if it is supposed to be only a job, the teaching profession becomes a passion that can either give you an incredible "high" or burn you to ashes. To feed its consuming frenzy, one needs fuel from both internal and external sources. One's enthusiasm has to reverberate in order to be kept alive.

Can the students provide the necessary feed-back? Do they understand the role of mathematics in their formation as individuals? Is their appreciation of the educator a reflection of the scholarly level of the lesson or do they use the same criteria that allow them to rate designer clothing, computer games and TV shows?

Maybe the parents care. They are mature and they certainly worry about the future of their children. An acquaintance of mine told me that her son was attending a private school in Montreal. A while ago, she, and a group of other parents, quite literally pounded the

principal's table until they convinced him to "release from duty" one of the math teachers. I asked her if she knew what the culprit was doing wrong. *"My son is getting low marks because he does not like his new teacher* - she answered - *and if I pay I expect to be listened to."* Of course, the discontented mother did not have any idea what and how the respective teacher was doing in the class, but her son's opinion was gospel. If the student is happy, the parent can relax upon coming home after a hard day's work..

The governing bodies' business is to care, isn't it? A colleague of mine - an outstanding math educator - was asked at a job interview if he can coach wrestling, but nobody bothered to see how strong his problem solving skills are. Actually, on very rare occasions candidates for math teaching jobs are asked to put pencil to paper. Higher up, the ministry inspectors are very thorough when rating schools. One should not call the outcomes objectives and God forbid that the teaching strategies are in anecdotal rather than point form (or the other way around). Even higher up, there is concern for improving our standards by cutting the preparation time and paying for TV ads that put teachers in a bad light - all for the low annual fee of $90 per beneficiary.

Our OAME representatives are also taking good care of the teaching flock. They never bother us with requests for contributions at OAME conferences. Let those who have better knowledge of both math teaching and the organizing committee to enlighten us!

While taking my Honours Specialist course, most of the participants -future leaders of the math teaching community - listened, nodded and smiled while instructors were questioning the utility of knowing the content in order to teach it, or extolled the merits of destreaming. The only criticism I heard during this course was directed towards Mrs. Jean Collins who dared show us some beautiful math problems!

(An absolutely true story. My wife travels by subway to work. Last year she used to travel in the same car with a lady who spent all her time between Finch and Bloor solving math problems. When my wife mentioned to me this interesting fact I said: *"It must be Mrs. Collins."* I was right.)

There must be someone who cares about math education. When I was encouraged to start this column I was so happy to be offered the chance to communicate with fellow math teachers! I tried to spark debate with my first article. I was having dreams (and nightmares) of an e-mail box full of letters ranging from the ones that would call me a clueless ignoramus to the ones that remind me what an obnoxious rascal I am, to the odd one that would find some truth in my words. The only message I received had kind words, but it came from someone who I also complemented for one of his contributions to the *Gazette.* Since then, the poor devil came up with two more articles that sparked the same passionate indifference.

Even a Devil has to end an article in a positive note. Last year, a young math teacher came for a job interview at Bayview Glen. In the interview she told us about her genuine love for the subject she teaches and that she expects no less from her students. While we dragged our feet in offering her the job, she accepted another position left vacant by a teacher who retired at another school. Guess whose shoes she is filling now?

Was it a coincidence or is there somebody up there that still cares?

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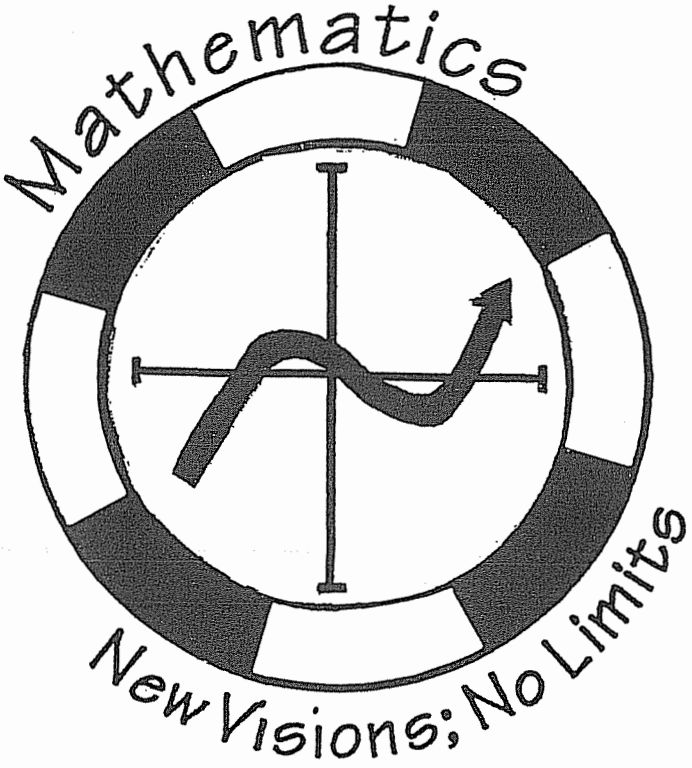
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OAME/AOEM AWARDS NOMINATION PROCESS

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration;

- demonstrations of excellence and creativity in mathematics education.
- specific contributions to mathematics education in the school, region, province or nation.
- demonstrates leadership in mathematics education in the school, region, province or nation.
- support of and contribution to the activities of OAME/AOEM.
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld form consideration.

OAME/AOEM LIFE MEMBERSHIP AWARD

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and accumulated 10 or more years of membership in OAME/AOEM.

KENNETH D. FRYER AWARD

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

DON ATTRIDGE AWARD

In recognition of the contributions made by Don Attridge, a founding member, former president and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

AWARD NOMINATION COVER PAGE

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▲ DOES GOOD PRACTICE MAKE PERFECT? SIX IN 10 EDUCATORS SAY NO; RESEARCHERS SAY YES

BY MATTHEW ROBINSON
1998 INVESTORS BUSINESS DAILY

Nothing attracts more ire from modern educators than asking children to memorize and practice, whether it be their spelling words or multiplication tables.

When polled last year, some six of 10 education professors objected to having kids memorize material.

These educators, who teach K-12 teachers, warn that practice, homework and direct, systematic instruction turn kids into automatons, stifling their creativity and ultimately dooming their ability to learn.

They even have a term for it: "Drill and kill."

The result? American kids spend less time working under instruction, and do less homework than their global peers.

But new findings in cognitive science and psychology support drills and practice.

"Nothing flies more in the face of the last 20 years of research than the assertion that practice is bad," asserted Professors John Anderson, Lynne Reder and Herbert Simon of Carnegie-Mellon University.

"All evidence . . . indicates that real competence only comes with extensive practice. By denying the critical role of practice, one is denying children the very thing they need to achieve competence," they wrote in a recent study. "The instructional problem is not to kill motivation by demanding drill, but to find tasks that provide practice - while at the same time sustaining interest."

That idea is causing a stir in the education world. Anderson, Reder and Simon are applying their findings in cognitive psychology to challenge the education status quo.

Kids, they argue, can learn better through "deliberate practice" -through hard work and constant feedback to master knowledge and tasks.

Of course, this is what most Americans think of when they are asked about education. They think of learning core knowledge that will be useful later in life.

"Nobody expects someone to be great without a great deal of practice and time in sports or music," Anderson said. "But it still seems that in the area of education, there is the notion that all we have to do is give a child a critical insight or inspiration and everything else will fall into place."

"Intellectual competence has to build up with the same kind of deliberate practice as musical talent or athletic ability," he added.

The education establishment's views on practice and memorization go back to the beginning of the century. They were popularized by progressive educators such as John Dewey, William Heard Kilpatrick and Carl Rogers.

Ideas in education drawn from these thinkers go by many names: rationalist or romantic theories of learning, constructivism, situated learning, project-style learning and discovery learning.

The theories have in common the belief that kids need only look inside themselves for knowledge. In this view, education is all about letting kids discover what they need to learn. In fact, some theorists claim that kids are actually hurt by direct, systematic instruction.

And such ideas get broad support in schools of education.

Fully 92% of education professors say that "teachers should see themselves as facilitators of learning, who enable their students to learn on their own." Only 7% think teachers should be "conveyors of knowledge who enlighten their students with what they know."

They also believe that direct instruction leads to "routinization." This, they think, drives out understanding.

"These valid objections to purely verbal, fragmented and passive education have . . . been used as a blunt instrument to attack all emphasis on factual knowledge and vocabulary," wrote E.D. Hirsch in "The Schools We Need."

California learned the hard way that romantic theories of education don't work as well as direct instruction.

The State Board of Education reversed its position in 1995 on whole language reading theory. Whole language says that kids learn to read the same way they learn to speak - by absorbing new words in the reading situation. Instead of teaching kids to sound out words and break down harder words, they would learn to guess words' meaning from context.

In other words, kids were largely left on their own to "discover" how to read and spell.

After almost a decade of practicing this method, the state scored at the bottom of national reading tests. The nation's biggest state now backs using direct instruction - including phonics instead of whole language to teach reading.

"Children vary in the amount of practice that is required for automaticity and fluency in reading to occur," the state's Comprehensive Reading Leadership Program found. "Some need to read a word only once to recognize it again with greater speed; others need more than 20 exposures. . . . Therefore, it is vital that students read a large amount of text at their independent reading level, and that the text provide specific practice in the skills being learned."

Translation: Learning requires reinforcement - practice and memorization - to master a subject.

"It's the missing link in American education," said Arthur Bornstein, a memory training expert in Los Angeles. "Schools tell kids what to learn, but not how to do it."

Bornstein coaches schools on how to help students remember what they're taught. Bornstein has watched schools slowly move away from stressing drills.

"It's a tragedy," he said, "but too many schools now assume kids will just pick up things as they go along."

Some experts argue that the education establishment's downplaying of deliberate practice helps explain American students' lower test scores.

In the Third International Mathematics and Science Study, American students were near the bottom. U.S. high school seniors scored 19th out of 21 nations in math. In science, they scored 16th.

American kids get nearly an hour less homework a day than the foreign average - 1.7 hours compared with 2.6 hours.

A study by James W. Stigler, professor of education at the University of California, Los Angeles, found that the average Japanese student gets instruction 90% of the time spent in the classroom. American kids get it only 46% of the time.

But cognitive research shows practice isn't just important for students. Teachers need it, too. Asian teachers spend much more time preparing to teach lessons. And although American teachers have smaller classes, they get less prep time.

But do drills stifle creativity, as many educators charge? No, says Temple University psychology professor Robert W. Weisberg.

Weisberg has studied the link between creativity and knowledge in artists such as Mozart, Picasso and Jackson Pollack. What he's found is surprising.

"It's a paradox," Weisberg said. "There is evidence that deep immersion is required in a discipline before you produce anything of great novelty. Before you look at significant achievement, expect to see 10 years of deep immersion to gain knowledge."

But, he noted, "There is this concept that genius has leaps of insight way beyond everybody else. If you look at the backgrounds of these people, there is much more of a progression. They don't make leaps - they build in small pieces."

Studies show that the brain actually changes with deliberate practice. A report in the journal Science shows that the cortical areas of the brain devoted to controlling the fingers actually expand for expert violinists.

Of course, repetitive drilling has its skeptics.

"If deliberate practice just meant rote memorization, then I wouldn't like it," said Boston College psychology professor Ellen Winner. "If (it) means working at something until you got it just 1right, like in a play, then I am all for it."

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P. Weygang
L. Wiggan
E. Wood
A. Worrall

Louis Lim

Louis is a Mathematics teacher with the Hastings and Prince Edward District School board, and has been the ISOMA chapter representative to OAME for two years. He writes the "Director's Dialogue" column in the *Gazette*, and is a frequent contributor to the *Gazette* and the *Mathematics Teacher*. Louis also enjoys presenting workshops at various chapter and OAME conferences, and is currently serving as a textbook reviewer to McGraw-Hill Ryerson. On the OAME Board of Directors, Louis will work hard to assist teachers to incorporate technology and cooperative learning in their classrooms, along with helping new teachers make the transition to our profession.

Anne Muller

Anne Muller is an elementary teacher with the Peel Board of Education. She has served as an OAME director for the last three years, during which time she was actively involved in the development of both the Junior and Primary *Linking Assessment and Instruction in Mathematics* documents. Anne is currently the Treasurer of CHAMP, and was the recipient of OAME's 1998 Morley MacGregor Award. She has conducted workshops on a variety of elementary mathematics topics for OAME and CHAMP conferences and is an elementary teacher representative on her board's Numeracy Work Team.

Bill Otto

Bill has been a member of the OAME Board of Directors for a number of years, serving as a member of the Specialization Years Committee and Chair of the Publications Committee. Bill has also been active in NWOAME Chapter, having been its Chair, its Chapter representative to the OAME Board, and coordinator of the Lakehead University/NWOAME Mathematics Students Awards Day. Bill is currently the Chair of Mathematics at St. Patrick High School in Thunder Bay.
Bill makes the statement: "OAME provides math teachers with a forum to develop, a strong voice to articulate, and the expertise to offer professional development and support to implement a vision for mathematics education in Ontario. I pledge to continue to work hard for you and OAME in this important role."

David Zimmer

David is a long-serving member of the OAME Board of Directors, having served as a Director, and more recently, as a Vice-President. He has acted as Executive Liaison to the Specialization Years Committee and as Chair of the Awards Committee. David has been active as a speaker and workshop leader, in his home chapter of CHAMP, provincially at OAME Annual and Leadership Conferences, and on personal invitation to schools and boards of education. Currently on leave from John Fraser Secondary School in Peel, Dave is busy writing mathematics materials for a Canadian textbook publisher.

Please mark your choices on the ballot

David Handley

David was a founding member of QSLMA chapter and continues to be involved in its activities. He also served on the OAME Board of Directors for a number of years. For the past twenty years, David has been active as a speaker and workshop leader. His mathematical interests include Calculus, History of Mathematics, mathematics contests, and adult education. David has worked as a referee for the OAME Gazette and the NCTM Mathematics Teacher. In the past, David has worked as a Special Education Coordinator, Head of Mathematics at Belleville C.I., and an organizer of elementary school mathematics contests in mid-eastern Ontario.

Myrna Ingalls

Myrna has been an active member of the OAME Board of Directors for a number of years and continues to serve as a member of the Specialization Years Committee. Myrna was the Chair of the Grade 11/12 writing team for the new secondary school mathematics curriculum and is currently a member of the Grade 9 Course Profile writing team. Myrna was recently named as a member of the steering committee of Fields Institute Mathematics Education Forum. A highly regarded workshop leader, Myrna is also Head of Mathematics at Unionville High School in York Region.

Jeff Irvine

Jeff has been an executive member of CHAMP chapter for the past ten years, serving as CHAMP president during 1996-1999. As Head of Mathematics at Brampton Centennial S.S., he has been very active in mathematics education in Peel, including being the Program Chair of the Peel Mathfest, co-author of the Peel Numeracy Scales, and a writer of the Peel Transition Years curriculum materials. Jeff has been a workshop leader for over thirty years at local, regional, and provincial conferences. He is the author of over twenty articles in the OAME *Gazette*, the CHAMP *News*, and both the Peel secondary and elementary newsletters.

John Kersley

John has taught mathematics and physics at Hillfield College for the past 25 years. He is a founding member of ISOMA and served on its executive for many years, including three years as the OAME chapter representative. For the past three years, John has been an OAME Director, serving on the Publications Committee and presently, as the Chair of the Promotions Committee. His interests are technology and mathematics and learning disabilities concerning mathematics.

Peter Lenczewski

Peter has been a member of the OAME Board of Directors for 13 years, four of those as Vice-President. He has been Chair of the Publications, Futures, and Promotions Committees. Peter was also Chair of the Facilities Committee at the last OAME conference in Ottawa. Peter has presented at several OAME and COMA conferences. He is currently COMA President and its chapter representative to the OAME Board of Directors. Retired from teaching in regular day school, Peter is the Mathematics Coordinator of the Ottawa-Carleton District School Board Continuing Education Department. His current focus is introducing technology into the Continuing Education Mathematics program.

PLEASE TURN OVER FOR PAGE 4

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Directors

John Dallon

John is Head of Mathematics at Orangeville D.S.S.. Over the past decade, as a teacher and department head, his direct leadership in secondary and elementary workshops has allowed him to share his love of Mathematics through alternative assessment, new technology and curriculum development. As OAME's Representative on the Ministry's Secondary Curriculum Feedback Panel, he was keenly aware of the challenges confronting today's mathematics teachers. As such, John believes that OAME must focus its support on helping classroom teachers in their daily activities.

Shirley Dalrymple

Shirley was a Vice-President of OAME during the year 1998-1999 and a long-serving chapter representative from Y4MA prior to that. Shirley has been very active as an organizer of professional development activities in Y4MA and, on the OAME Board of Directors, served as Chair of the Specialization Years committee for two years. Shirley is a highly regarded workshop leader. She was a member of the Grade 11/12 writing team for the new secondary school mathematics curriculum. Shirley is Head of Mathematics at John M. Denison Secondary School in York Region.

PLEASE SEE PAGE 3

CANDIDATE'S VITAE

President Elect

Mary Howe

Mary has long been active in the WOMA chapter of OAME. She has been a member of WOMA's executive since 1992 and has been the WOMA chapter representative to the OAME Board of Directors since 1994. Mary chaired WOMA's fall conferences in 1993 and 1995, and was Co-Chair of the 1996 OAME Annual Conference, held in London. Mary is Head of Mathematics at St. Thomas Aquinas C.S.S. in London. She is a gifted teacher and instructional leader whose mathematics department won the Kenneth Fryer Award for Outstanding Contributions to Mathematics Education in 1996. Mary regularly shares her ideas and expertise in workshops and through participation in OAME writing projects, including *Linking Assessment and Instruction in Mathematics: Transition Years*, and *Teaching Mathematics Through Technology.* Mary has participated in writing projects with EQAO, was Chair of the 9/10 writing team for the new secondary school mathematics curriculum, and is a member of the writing team for the Grade 9 Mathematics Course Profile.

Vice-Presidents

Shirley Dalrymple

Shirley was a Vice-President of OAME during the year 1998-1999 and a long-serving chapter representative from Y4MA prior to that. Shirley has been very active as an organizer of professional development activities in Y4MA and, on the OAME Board of Directors, served as Chair of the Specialization Years committee for two years. Shirley is a highly regarded workshop leader. She was a member of the Grade 11/12 writing team for the new secondary school mathematics curriculum. Shirley is Head of Mathematics at John M. Denison Secondary School in York Region.

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PLEASE TURN OVER FOR PAGE 2

What Are Standards?

GLENDA LAPPAN, NCTM PRESIDENT

Many different stances are taken about what standards for mathematics should entail. Some define "standards" as statements about the mathematics to be learned that also give levels of performance expected at each grade. Others think "standards" should specify very fine-grained behavioral objectives in mathematics for each grade. "Standards" hold many other shades of meaning that focus solely on the mathematics to be learned.

Teaching Matters. NCTM is an organization of teachers of mathematics at all levels. With that perspective, NCTM views standards as statements of criteria for excellence in school mathematics programs. Notice the word *program.* The NCTM *Principles and Standards for School Mathematics (PSSM)* (the draft of the updated *Standards*) examines the mathematics-content goals of a program, but it also shows problems, students' work on those problems, and teaching that helps reach the content goals.

Considering teaching to be a part of important standards is entirely appropriate for an organization of teachers. Our primary goal is to ensure a high-quality mathematics education for every student. Although content is a necessary and absolutely essential ingredient, it alone does not guarantee a high-quality program. We know that teaching shapes and influences what students learn. Teaching matters.

As a result, in NCTM's *Standards* documents, teaching approaches are crucial in the discussions and elaborations of the *Standards.* This is not to say that the Council espouses only one kind of teaching. In NCTM's *Professional Standards for Teaching Mathematics* (1991), a basic assumption is that "teaching is a complex practice and hence not reducible to recipes or prescriptions" (p. 22). Effective teaching can look very different in different classrooms. Good teachers use different strategies at different times for different purposes. *PSSM* gives teaching suggestions that come from real classrooms and real teachers.

Local Control. The Council believes that provinces, states and local districts have a responsibility to determine more-specific grade-level goals for mathematics that make sense for their own students. Therefore, NCTM's *Principles and Standards* document gives national vision and direction to help focus local efforts, not to preempt them. Even so, to help those who want more information as they develop their own curricula, *PSSM* includes more guidance on content matters than does the earlier NCTM *Curriculum and Evaluation Standards for School Mathematics.*

A Closer Look at an NCTM Standard. The *PSSM* draft, which was released in October for public review and comment, organizes NCTM's Standards into new grade-band levels – preK-2, 3-5, 6-8, and 9-12. Each grade band includes 10 Standards that are the same across *all* levels but have different applications at each level. Let's look at a Standard and see how it works.

Standard 1: Number and Operation (which is the same for all grade levels): Mathematics instructional programs should foster the development of number and operation sense so that all students:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand the meaning of operations and how they relate to one another;
- use computational tools and strategies fluently and estimate appropriately.

In the grades 6-8 chapter, bullet 1 above "understand numbers, ways of representing numbers, relationships among numbers, and number systems" is expanded on like this:

- Work flexibly with equivalent fractions, decimals, percents; compare and order these numbers efficiently and accurately, find their approximate locations on a number line; and choose appropriate and convenient forms of these numbers for solving problems;
- develop meaning for integers and be able to represent, compare, and order them;
- develop an understanding of large numbers, including the use of benchmarks to comprehend their magnitude; and recognize, understand, and appropriately use various representations for large numbers (e.g., exponential, scientific, and calculator notation);
- use number theory concepts (e.g., factors, multiples, prime factorization, relatively prime numbers) to solve problems and to understand ides about rational numbers;
- develop an understanding of the properties of the integer and rational number systems (e.g., order, density, and additive and multiplicative inverses);
- recognize and use commonly encountered irrational numbers (e.g. π and $\sqrt{2}$).

PSSM then goes on to discuss what kinds of experiences students should have had with these ideas in earlier grades, and what kinds of experiences are needed to help students achieve these specific goals in grades 6 through 8. This discussion of the growth of ideas over time is an important attempt to help teachers know how what they do at their grade level fits into the big mathematics education picture for their students. This picture focuses on both teaching and learning – the inseparable components of excellent mathematics programs. That's what the NCTM Standards are about.

But please see for yourself. Be sure to get a copy of the discussion draft of *Principles and Standards for Schools Mathematics* and let us know what you think. You can request a a copy of the draft through an electronic request form on NCTM's Standards 2000 webpage **www.nctm.org/standards2000/**, by e-mail at **standards2000-draft@nctm.org** (message must include your name, membership number, and telephone number), or by phone toll free at (888) 220-7952.

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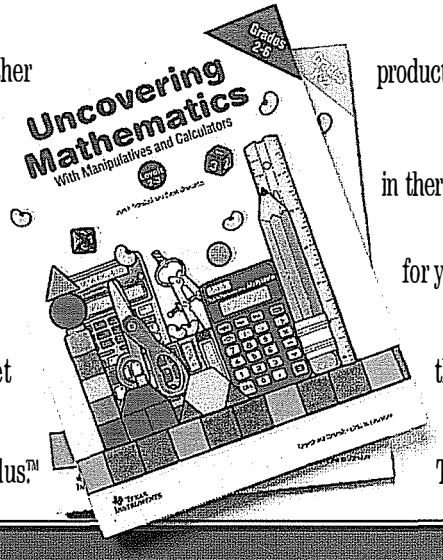
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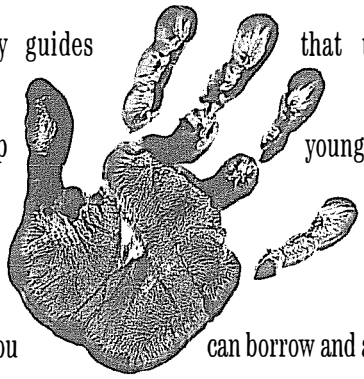
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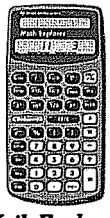


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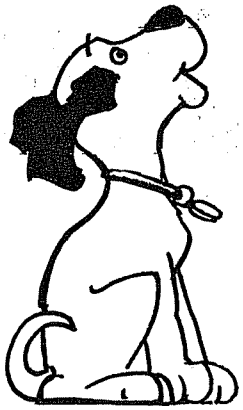
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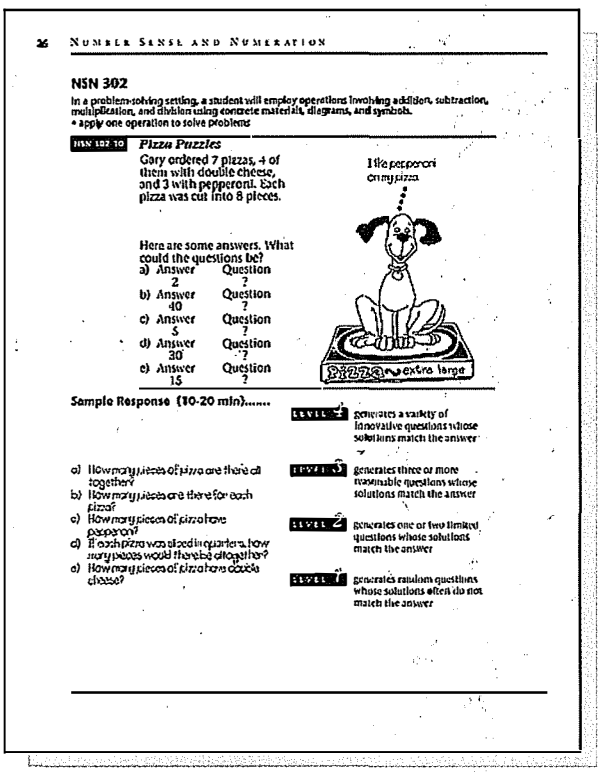
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Assess reasoning, knowledge and skills, problem solving, communication, attitude, and more...with classroom-tested activities kids enjoy.



- Directly correlated to learning expectations
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26 NUMBER SENSE AND NUMERATION

NSN 302

In a problem-solving setting, a student will employ operations involving addition, subtraction, multiplication, and division using concrete materials, diagrams, and symbols.
• apply one operation to solve problems

NSN 302-10 ***Pizza Puzzles***

Gary ordered 7 pizzas, 4 of them with double cheese, and 3 with pepperoni. Each pizza was cut into 8 pieces.

I'll be pepperoni on my pizza

Here are some answers. What could the questions be?

a) Answer 2 — Question ?
b) Answer 40 — Question ?
c) Answer 5 — Question ?
d) Answer 30 — Question ?
e) Answer 15 — Question ?

Sample Response (10-20 min).....

a) How many pieces of pizza are there all together?
b) How many pieces are there for each pizza?
c) How many pieces of pizza have pepperoni?
d) If each pizza was sliced in quarters, how many pieces would there be altogether?
e) How many pieces of pizza have double cheese?

LEVEL 4 generates a variety of innovative questions whose solutions match the answer

LEVEL 3 generates three or more reasonable questions whose solutions match the answer

LEVEL 2 generates one or two limited questions whose solutions match the answer

LEVEL 1 generates random questions whose solutions often do not match the answer

Sample activity page from
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