Vol. 37 #4, June 1999

Ontario Mathematics Gazette

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"by hand?"
(The old way)

Final Exam

1. State the Domain of $y = \frac{x-10}{\sqrt{x^2-16}}$.

2. Which relation is the inverse of y=(

3. Use a half-angle identity to find the e

a) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ b) $\frac{-1}{2}\sqrt{2-\sqrt{2}}$ c) $\frac{1}{2}$

4. If $f(x)=2x-5$ and $g(x)=$, find

a) $\frac{-(5x+11)}{x+3}$ b)

5. If $\sin\theta = -\frac{1}{2}$ and θ is

a) $\frac{-\sqrt{3}}{3}$ b) $\frac{\sqrt{3}}{3}$

6. Given $f(x) = \frac{x^2}{x+3}$ and

a) $\frac{x^2-2x}{x+3}$ b)

$\alpha = 1 \cdot \cos\beta = \frac{1}{2}$

"struggling with a word processor?"
(The hard way)

Final Exam

1. State the Domain of $y=\frac{x-10}{\sqrt{x^2-16}}$

2. Which relation is the inverse of $y=(x-2)^3$?

a) $y=\sqrt[3]{x-2}$ b) $y=\sqrt[3]{x}-2$ c) $y=\sqrt[3]{x+2}$ d) $\sqrt[3]{x}+2$

3. Use a half-angle identity to find the exact value of cos 67.5°:

a) $1/2\sqrt{2-\sqrt{2}}$ b) $-1/2\sqrt{2-\sqrt{2}}$ c) $-1/2\sqrt{2+\sqrt{2}}$ d) $\sqrt{3+2\sqrt{2}}$

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FINAL EXAM

1. State the Domain of $y = \frac{x-10}{\sqrt{x^2-16}}$.

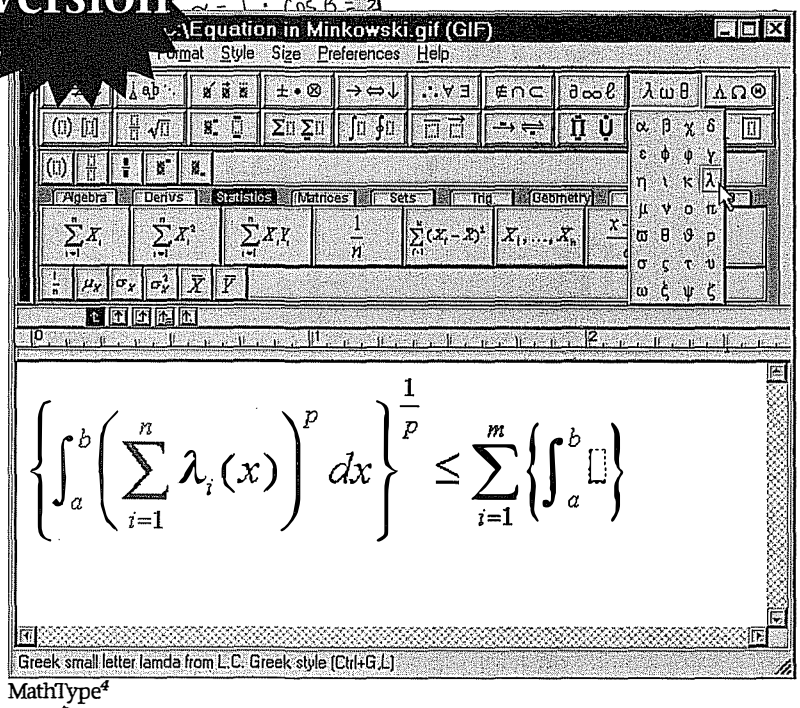
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▲ Editorial



JOHN EGSGARD

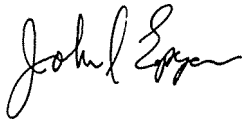
This is my final issue as editor of the Ontario Mathematics *Gazette*. I have enjoyed my term as editor. I have been happy to find a number of teachers at the elementary and secondary school levels who took the time away from their busy classrooms to share their ideas with their peers. Sharing of ideas is one of the keys to excellence in teaching.

I have many people to thank before I leave. My two associate editor Norm Viirre and Yvonne Bergsma; our *Gazette* designer Penny Clemens of *YDezign;* the publications committee chaired by Murray McLennan; the many people who wrote articles and reviewed articles; our faithful columnists John Grant McLoughlin, Alexandru Pintilie, Peter Saarimaki and Louis Lim, and our printer *Pole Printing*.

Although I have gathered the articles for this June issue I was not be able to proofread the issue and do the final touches as my wife Lyn and I were in France during months of April and May. Jack Weiner, the previous editor of the *Gazette*, graciously agreed to put the issue to bed and to write his own editorial.

This final issue contains a thought provoking article on the millennium, *Y2K: The Greatest Challenge Facing Our Schools*; 3 articles that may help in your teaching *Mathematics and Chinese: Building Beauty into Characters, Rubrics ... Their Purpose is to Drive Your Practice .. Not, To Drive You Crazy!,* and *Baseball Salaries: Does Money Buy Success?* There is also a research article *Opportunity to Learn Proof in Grade Twelve Mathematics Texts* as well as the *About Problem Solving* and the *Devil's Advocate* columns.

I am grateful to have had this opportunity to serve the excellent and dedicated mathematics teaching community of Ontario.



▲ Guest Editorial



JACK WEINER

Move over Ralph Connelly! Make room, Bob Smith!

Now I join the ranks of ex-editors returning to the job.

The *Gazette* has changed in my two year absence. There is a glossy cover, the photos actually do justice to authors and contributors, and the *ABACUS* not only is, but looks wonderful. Congratulations to editors John and Frances, to advertising manager Dean, and of course, to Penny, for the look!

But I do have one very serious concern. It has to do with a little exaggeration–a deviation from truth?–that appears on page 1. In the membership application, it says, "Join the over 2000 mathematics educators..." This number does not reflect reality, and hasn't for several years. According to Louis Lin in this issue's *Directors' Dialogue*, membership as of early February is down to 1164! (And that's not 1164 factorial!)

That is scary and worrisome, even embarrassing.

The only forum exclusively concerned with mathematics education in Ontario, Kindergarten through University and College, is OAME. This organization, as Ralph Connelly pointed out eloquently in his December, 1998 President's Message, produces at modest cost, tremendous resources for mathematics teachers and their classrooms. Yet most math teachers do not see it as worth their while to become (or remain) members of this organization.

We need more members. We need to represent the majority of mathematics educators in the province. Belonging has benefits as the converted already know. But our influence with, for example, the Ministry of Education, can only be diminished when we sheepishly acknowledge a membership below 1200. There is also the matter of funding OAME's activities. Diminishing membership is causing serious financial hardship.

So, how do we convince the tens of thousands of non-member math teachers to join?

I have several suggestions. But I will save them for September.

BECAUSE I HAVE A REQUEST.

Have you a strategy for increasing membership? Please write or email me (find details on how to reach me under "Directors-Term Expiring 2001" on page 36). In September's issue, I will include a list of all suggestions submitted and will turn this list over to the membership committee at the early October meeting of the Board.

For now, to each of the loyal 1164 and to new members who joined with conference registration, have a wonderful summer. Best wishes,

Jack

▲ Outgoing President's Message



RALPH D. CONNELLY

Well, some people just don't know when to give up! This *Gazette* issue will have Peter Saarimaki's first "President's Message", and I'm not SUPPOSED to have a "forum" in the *Gazette* anymore, but due to the good graces of this issue's guest editor, Jack Weiner, I've been given some space to mention a couple of items that OAME members should be aware of.

First, some great news from the National Council of Teachers of Mathematics (NCTM) Affiliated Group Services. OAME has been given a $1500 (and that's U.S.!) Professional Development Grant to help defray the costs of producing our professional development resource for teachers: *Linking Assessment and Instruction in Mathematics: Primary Years.* I'm sure you'll agree that the finished product is an exceptional resource for those involved in teaching mathematics at the Primary level, and we are extremely grateful to NCTM for providing us with this grant to help defray a portion of the Development costs. Thank you, NCTM, and rest asssured that this was money well spent.

And MORE great news from NCTM! Because of the fact that our Canadian dollar continues to be somewhat anemic compared to the U.S. dollar, the NCTM Board of Directors has agreed to "peg" the maximum exchange rate to be charged to Canadians for NCTM memberships, or for Canadian members ordering NCTM Educational Materials, at 25%! So, for example, an NCTM membership (including one journal), which costs $57.00 U.S., will now cost $71.25 Canadian—where as recently as one month ago, purchasing that same membership would have cost approx. $88.00 Canadian.

So I'd like to extend thanks to NCTM on behalf of OAME, both for the Professional Development Grant, and for demonstrating that there IS an interest in attracting and keeping Canadian members by making the "sting" of our dollar differential a little less painful.

OK—now I'll keep quiet. (If you believe THAT, I own this really neat little bridge down in San Francisco—the Golden Gate—that I'll be glad to sell you shares of, in Canadian dollars even!)

Ralph Connelly (the OLD president!)

▲ INCOMING PRESIDENT'S MESSAGE



PETER SAARIMAKI

Memo to OAME Members

Bon jour, shalom, and g'day, mates. Is your world turned up-side-down and nothing seems the same? Do the words look familiar but not make as much sense as they used to? Are you not sure whether you're coming or going? Well you're not alone, aaaannnnnd . . . I can see the Mounties on the horizon! Here's what the Mounties have been up to and how they can help.

Education in Ontario is certainly confusing, frustrating, exciting, challenging, . . . (add your own words here). And OAME is most assuredly in the thick of it. We continue to work to make sense out of it all and provide support and resources to assist teachers in delivering the curriculum (or -la as the case may be!). OAME is working on many fronts and I will deal with them in the order of their public "visibility". It should go without saying, that each area is important and no one area has pre-eminence over another.

Secondary Head-line-wise, Secondary programs are front and centre for the public. At the Secondary level, "Reform" is no longer the watchword, but rather "Implementation". Over the summer, many policies, supports and resources will come together to ready teachers for the beginnings of delivery for the new grade 9 curricula (it fits here because there are at least two!).

Our involvement and direction setting has been evident through the many long stages of this curriculum development, including the course profiles and beyond. The prime indication of our leadership role is evidenced by the following project.

OAME/OMCA Grade 9 Inservice Project The intent of the project is to guarantee that every teacher of Grade 9 mathematics in the province of Ontario has the information and tools necessary to deliver the new curriculum. It's BIG!

Phase I of the OAME/OMCA Project this spring ensured that at least two teachers in every secondary school in the province (that's 800 schools) received a common message and a package of resources designed specifically to the new Grade 9 courses. The training of the Point People and presenters was a joint venture involving OAME and OMCA with the financial backing of publishers, suppliers and the Ministry.

Phase II will introduce almost 600 teachers to the new technologies in the T3 Institutes this summer, through programs specifically designed for the Ontario Grade 9 program. Texas Instruments has been extremely supportive and is under-writing a major portion of these workshops for us. If you are not able to get in to one this summer, don't fret as OAME/OMCA and TI are in this for the long run.

Elementary This year, in partnership with CAPE, OAME produced the Primary version of Linking Assessment and Instruction in Mathematics. It was unveiled, without threats, to rave reviews at the Leadership Conference. It joined it's sister publication for the Junior division (revised) which has had to go to a third printing to keep up with demand. The Grade 6 - 8 posters, Technology binder and Transition Years book continue to help teachers and students.

Other Levels OAME continues to reach out to, contribute to, and provide support and leadership to a variety of other teacher-related organizations including the Ontario College of Teachers, OTF Curriculum Forum and the Ontario Curriculum Clearinghouse. In addition we continue to provide advice (sometimes unsolicited) to the Ministry and its many, varied projects.

We reach out to tertiary level math groups through our representatives on the Fields Mathematics Education Forum, through Directors on the Board, and with chapter events involving local Faculties of Education.

2000 UNESCO has declared 2000 as World Mathematical Year. With the focus on the special nature of 2000, this is a wonderful opportunity to give a higher profile to math and math events. More on this later, or check out http://www.math.jussieu.fr/~jarraud/wmy2000/ma2000.html.

September 1999 Even closer is September. The OAME/OMCA Project, Phase III, will continue to rollout this school year, with the focus expanding to preparations for the implementation of Grade 10 courses. As this happens, OAME will also continue to develop ways to assist teachers in our 4000 elementary schools who continue to grapple with the implementation of the 1 - 8 curriculum. We will be working with the College of Teachers to allow teachers to receive recognition for OAME sponsored professional development.

For all of these projects and others, OAME needs each and every member to be a participating proponent for mathematics. We need to communicate with our colleagues in school, with our networks across the Board, and with friends and relatives. Share articles from the Gazette and Abacus, attend PD sessions, and get involved in your local chapter. I plan to be out at as many local chapter events this year as I can and I expect to see you there too.

So have a restful, rejuvenating summer, and until we meet again, in person, on paper or through the Internet, bon jour, shalom and g'day mates!

Cheers, Pierre

▲ Mathematics and Chinese: Building Beauty into Characters



YING ZHENG

Ying Zheng is a Mathematics and Science teacher. She is currently teaching Mathematics at the Selwyn House School in Montreal, Quebec.

The application of mathematics is everywhere. Sometimes the influence of mathematics is so deeply ingrained in the development of certain things that one can not easily recognize it if not looking through the surface. An interesting example is the mathematical concept associated with the rules of writing Chinese characters. Illustrated here, in the simplest way, are a few examples that show how mathematics is used in giving Chinese characters the beauty and, sometimes, the sense.

Symmetry is very important in writing Chinese characters. The square appearance of all Chinese characters is an example of external symmetry. For instance, character *a* (pronounced as ying, meaning New and Novel; the same format will be used in explaining pronunciation and meaning hereafter) should not be written like *b* or *c*. In the latter two cases, although no components are missing, the wrong proportion among the components ruins the beauty of the character given by a square symmetry. Many Chinese characters also have internal symmetry as shown by characters *d* (mei, Beauty), *e* (wang, King) and *f* (chu, Out), where the left half and the right half are almost mirror images of each other.



a *b* *c* *d* *e* *f*

Sometimes, even the meaning and the sound of the character can be changed completely if the components are not accurately written. For example, character *g* (wei, No) will become *h* (mo, End) if the lengths of the horizontal lines are switched. Character *i* (tian, Sky) will become *j* (fu, Husband) if the curved line (an exponential curve?) does not stop exactly below the first horizontal line.



g *h* *I* *j*

That shows how the concept of **accuracy** is being practiced in writing Chinese characters. In fact, when I am trying to describe the writing of Chinese characters, I cannot stay away from using mathematical terms such as lines and curves. That is where interesting activities can be developed to get the students to use mathematical language in a culturally rich and stimulating environment.

Order is yet another important mathematical concept ingrained in the development of Chinese characters. When working with numbers if you switch the order of the numerals in the number 64, it becomes 46. Similarly, when you switch the order of the two components in character *k* (bu, Part), it becomes character *l* (pei, Accompany). When you switch the two components in character *m* (ming, Bright) and add a line beneath the right half, it becomes character *n* (dan, Courage).



k *l* *m* *n*

The meaning of Chinese characters also have an **additive** nature. The best example in this case can be given by the evolution of both the writing and the meaning of the following three characters *o* (mu, Wood), *p* (lin, Trees), and *q* (sen, Forest).



o *p* *q*

The meaning of some word combinations can also be deduced by adding the individual meaning together. For example, the meaning of the word combination rs (daren, Adult) comes from the addition of *r* (da, Big) and *s* (ren, Person).



r *s* *r* *s*

A good exercise in **logical** reasoning and **deduction** can be found in the following example:

Question: Given that character *x* means field, and character *y* means force, find the meaning of character *z* where *x* is written on top of *y*. (hint: *z* is an adjective.)



x *y* *z*

Answer: According to the additive nature of Chinese characters, *z* could mean field force. Since field labour was mostly performed by men in old times, *z* could mean man or, as an adjective, male.

The search and demonstration of the relationships among mathematics, language and culture can make very interesting lesson topics. Incorporating such topics into teaching will enrich the sometimes dry mathematics lessons. Students can also be encouraged to search for and present the connections between mathematics and their own heritage languages or other cultural symbols. Once the search gets started, it will be hard to predict where and when it will end. Imagine the outcome of such a project: mathematics becomes an interesting topic among peers, parents and siblings; everyone tries to speak in mathematical terms and to make sense out of them. Mathematics is no longer a scary subject but one that helps us to understand our past and be prepared for our future.

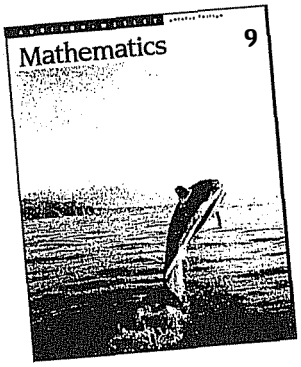
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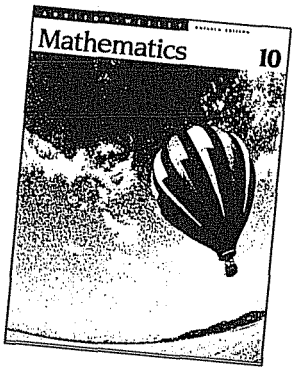
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Lynda has been an educator for 22 years. She has successfully fulfilled a number of roles in that time, including secondary school mathematics teacher, grade 5 classroom teacher, computer consultant, adjunct professor, mathematics coordinator, vice-principal, and now Professor of Mathematics Education. A winner of a number of professional awards, including OSSTF's Excellence in Education award for outstanding classroom teaching and the first Marshall McLuhan Award for visionary use of technology with students, Lynda is actively involved in research and assessment initiatives in these areas and continues to be an advocate of the braiding of technology and creativity into the fabric of mathematics education.



Peter has been teaching mathematics for 25 years with the Scarborough Board of Education - now the Toronto District School Board. Currently he is the Curriculum Coordinator responsible for Mathematics at the 140 Borough Drive Education Centre and a member of S.A.M.E.

Introduction

When I started teaching over twenty years ago, it was so simple. I took the book and went section by section to explain the concepts and skills needed to learn the material in that section. I prepared thoroughly and always reviewed the material before attempting to introduce it. I answered questions both on the materials and on my presentation. I had strategies and alternative approaches ready to provide clarification if necessary. I thought if I presented a concept clearly and the students listened, then they would "get it." "Telling it right" was enough. I provided students with homework and testing situations to practice the skills they learned. Clearly, then, if they were not successful it must be because they were not doing their part or were just not cut out of the right stuff. In general, it seemed that most of my students were doing fine. I was, I felt, an effective teacher. My successes far outweighed my failures.
(Gagnon, 1993, p.1)

Do you remember being in this class? Do you remember teaching this way? This passage evokes a powerful and familiar image. It describes what most conscientious mathematics teachers across the province do, and until recently, what students, parents, school administrators and policy makers expected us to do.

But things are about to change. In February, 1999, secondary mathematics educators were presented with a policy document which demands new practice for both instruction and assessment, and the vision contained within will not match these familiar images.

What does the new curriculum policy have to say about day-to-day mathematics instruction and assessment? In short, precious little.

Henry Pollak, a prominent contemporary applied mathematician, speaks fondly of "cross-country mathematics". By this he means that when one moves about a mathematical territory, it will be fraught with unexpected bumps, barriers, alternative routes and unmarked paths. The road to implementation of the new secondary mathematics curriculum will be much like Pollak's mathematical terrain, fraught with surprises, but we believe that we can negotiate it by working together, supporting each other, encouraging each other and taking advantage of every opportunity for sharing both our successes and our failures.

Our goal, through this article is to help friends and colleagues successfully negotiate some of the traumas of the initial dramatic shifts prescribed by the new policy statements. In this article, we do two things. We offer what we think is an exemplary rich assessment task and we present our interpretation of a mathematics rubric,

two of the driving factors in the new day-to-day mathematics instruction and assessment we are all being charged with implementing.

Background

Often, in education we see powerful ideas bastardized in the attempt to make them palatable and more accessible. This is what happened when complex theories such as whole language, child-centered and collaborative learning were popularized, sloganized and broadcast without care and precision. As a result these important contributions have been jeopardized, or in some cases negated in their impact on instruction and achievement.

It is our hope that two new "popular terms" - *performance based assessment* and *rubric,* do not become the newest additions to the mathematics educator's waste basket of "bad words".

Performance-based assessment using rubrics is far too important to mathematics education to be trivialized, and concomitantly, jeopardized. As mathematics educators, we cannot afford to have our peers turn away from these powerful and relatively new approaches and tools.

Recently, issues involving assessment have been driving instructional practice in most of the elementary mathematics classrooms across our province.

The workshops and preparatory sessions that were conducted to assist elementary teachers and parents of elementary-aged children understand the provincial Grade 3 Assessment of Reading, Writing and Mathematics have been enormously influential in changing attitudes about instructional practice, the nature of meaningful "problems" and expectations of students. The implementations of these assessments are influencing classrooms where we find more often that students are working collaboratively, extending one another's ideas, arguing for their own positions, criticizing one another's conclusions, making conjectures and applauding each other's triumphs. To achieve such behaviour, teachers have been constructing classroom settings where discussion about mathematics occurs and where students listen to one another seriously. Within mathematically promising classroom settings such as these, teachers and students encounter challenging and genuine dilemmas and are encouraged to develop positive mathematical behaviours such as risk-taking, creativity and tenacity.

As we enter the final few months before we are faced with the task of implementing the new secondary mathematics curriculum, it behooves us to look at the lessons learned by our elementary colleagues and thoughtfully adapt them and incorporate them into our own practice.

Secondary Reform - February 1999

The new secondary mathematics curriculum policy documents for Grades 9 and 10 will have been introduced in February, 1999. There will be some familiar elements - details about the overall and specific content expectations, information about the configuration and nature of specific grade-level courses, explicitly stated prerequisites and suggested routes through the courses towards graduation.

But there were also major differences. Perhaps most notable is the publication of a rubric that will delineate levels of performance and standards of achievement along a four-point scale as well as a breakdown of four categories of assessment: Problem solving, Understanding, Application and Communication. Secondary teachers will be required to use these scales and categories as a basis for the reporting of their students' performance just as their elementary colleagues are doing at present.

We believe that secondary mathematics teachers need to be thinking about this - now!

Why? Evaluation using a rubric makes assumptions about the type of instructional setting, the type of problem and the type of mathematics that will be the norm for our classrooms. Secondly, because all of us, for the first time in this province will be using the same bar to measure success. The explicit statements regarding the delineation of student performance will legislate levels of performance that will be tied to mark ranges on standardized report cards.

And so, now is the time for each of us to start thinking critically about the implementation of a rubric and the types of questions that it assumes. We must have some experiences in this arena if we are to help our colleagues and our students understand the changes in the nature of the set expectations and type of assessment that they will be facing.

Although it is neither easy nor palatable to admit, students, generally, have not had a clear model of our expectations - and neither have we! The key to the new

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curriculum and the styles of student assessment and evaluation experiences that it embodies is characterized by the fact that the expectations are very clear and success is predicated on students knowing how to perform. Simply stated, students need an uncluttered vision of what excellence looks like. They need to know when a question requires a simple, short answer as opposed to a full blown solution/discussion. They need to be able to discriminate between problem types and to know just what constitutes an appropriate response.

It has been said that "rich verbs" are at the heart of secondary mathematics education reform; however, we believe that there is not a shared understanding of what constitutes a "rich verb" or the "actions" intended by these verbs or their potential for richness.

However, we do have many examples and experiences to draw upon. Those students who have had the opportunity of preparing for and writing the University of Waterloo's Descartes contest have successfully experienced and performed in this expository environment. For years now, teachers who have coached and trained in this milieu have helped prepare the groundwork for the rest of us. They have been successful in communicating the elements of a good solution to their students. Clearly, these teachers value and model these elements and processes. What we need to do now is widen the net.

The notions of rubrics, rich verbs and expository mathematics environments are probably intuitively or implicitly understood by most mathematics teachers. What the new secondary mathematics curriculum is demanding is that we are not just thinking about what these mean, but that we articulate them explicitly to students and parents. After all, we can no longer ignore the fact that students will be coming to us from grade eight with experiences in participating in these sorts of assessment activities. Many of them will come from an environment of student-led conferencing where they have sat down with and communicated to their parents and teacher why their work was judged to be at a

particular level. These students will be familiar with rubrics and will be expecting to see them in their secondary schools.

We would like all teachers to have the comfort level and sense of familiarity of elementary teachers. We would like everyone to have tried to use a rubric, to have reframed a favourite question using rich verbs and to have modelled an expository mathematics environment.

Assessment

Assessment - we know it can be accomplished through many vehicles: observation check-lists, paper-and-pencil tests, open-ended problem sets, and portfolios, to name but a few. But it is the underlying principles that have been most instrumental in changing our elementary colleagues' attitudes towards and practices with respect to assessment. It is this explicit foundation that we, as intermediate and secondary educators must keep as our focus, for it embodies the notions of rich verbs and expository mathematics environments. We must demand that this foundation forms the backbone of any new textbooks and teaching resources. It is this explicit foundation that will reform our classrooms and our domain.

How would we describe this foundation? How does it frame and scaffold rich assessment tasks and inform mathematics instruction?

We believe that rich assessment tasks must:

- concretize a vision of mathematics that goes beyond correct answers;
- preserve mathematics as a subject that requires careful and thoughtful investigation;
- provide a vehicle through which all students can demonstrate their ability to apply core mathematical concepts/curriculum expectations;
- require the student to assume the role of an active mathematician;
- demand a high level of proficiency in students' ability to communicate their mathematical internalization of both process and content;
- exemplify the potential seamlessness of instructional and assessment activities; and,
- provide a lens through which teachers can reflect on their mathematics program, its effect on individual students and empower them to make necessary changes to instruction.

These underpinnings should sound familiar. Though the words may be different, they represent the spirit of the N.C.T.M. *Standards,* the first sections of the OS:IS guidelines, familiar descriptions of so-called "rich-learning tasks", and are central to current secondary curriculum reform.

What would an assessment activity that fulfills these criteria look like? In some cases, textbook questions, favourite problems and familiar exercises can be "tweaked" into becoming rich assessment tasks by adding specific verbs. In other cases they grow out of classroom experiences. Consider the accompanying tasks, designed for Grade 8 students as the culminating assessment activity for a Data Management & Probability Unit, and based on determining the extent to which they have demonstrated achievement of specific expectations from *The Ontario Curriculum, Grades 1-8.*

Hopefully, you will be able to make your own favourable judgments regarding their "fit" to the above criteria of a "rich assessment task".

Rubrics

We were introduced to the notion of the rubric at an assessment workshop six years ago given by a generalist who challenged us to design a 4-point rubric with descriptors for evaluating a week-end. At a second workshop, a year or so later, we were asked to do the same for a party. We couldn't help but feel that the power of the application of a rubric to mathematics instruction and evaluation was in danger of being lost in the effort to soften the theory with "fun" applications and the lack of subject-specific examples. Here was a trivialization just looking for a waste basket.

What a difference six years make! Rubrics seem to be everywhere these days. They are used to evaluate everything from reading to textbooks. Some are "canned" and some are custom-designed. Regardless of the source, many are of poor or questionable quality. We have seen teacher-designed mathematics rubrics that delineate measures for the quality of title pages, the "aesthetics" of graphs and the quality of word-processing. We have seen consortia-developed "one-size-fits-all" rubrics that are so generic that neither students nor teachers have any idea how to apply them to their work.

It is our hope that educators design and share assessment rubrics that have the integrity, staying-power and "goodness" of the mathematics itself. We hope that teachers will come to value and use rubrics in order to improve both their students' performance and their own teaching.

We do not believe that classroom teachers need to design problem-specific rubrics for each performance-based assessment that they use with students. Neither do we believe that the "one-size-fits-all" genre is satisfactory. And so, what we are proposing is a compromise.

It has been our experience, in workshops and the like, that when we ask teachers to "do" a problem, they scribble a few illegible notes on the corner of a page and then say they are finished. When we have asked them if that solution would be acceptable if submitted by a student, the answer is always the same, "no." In general teachers have internalized notions of their expectations for students, but they do not always explicate them. They may not know exactly what a good solution will look like, but they sure do know what they consider to be a bad solution!

The expectations of the revised curriculum with its rich-learning-task approach and performance-based demonstrations of learning, are new to all of us. As students, we were not expected to include so many different and explicit elements in our solutions. As learners, in order to become better teachers, we need opportunities to communicate our strategies, reflect on our reasoning, thoughtfully evaluate our solutions and demonstrate the integration of our "chunks" of knowledge. We need to practice these skills and value these approaches so that we can model them for our students. In short, intermediate and senior mathematics teachers need to "walk the (math) talk" if we are to be accountable to ourselves, our students and their parents.

Our rubric is a hybrid of the best elements from the Kentucky Portfolio Assessment Project, the Education Quality and Accountability Office (E.Q.A.O.) provincial assessments, the Ontario Curriculum: Mathematics Grades 1-8 document and our own applications of readings from the field.

We believe that our rubric is unique. It specifies important process and content elements, but it is not ready for use until the teacher completes the assessment task herself and then reflects on and explicitly expresses her expectations for the specific problem assigned. In the design of this rubric we have made the assumption that the process of *teaching* mathematics is analogous to the process of *doing* mathematics.

As students complete a number of these rich assessment tasks, and the accompanying rubrics, each with carefully communicated specific elements, they will assemble a portfolio that demonstrates how the broad goals of communication, application/creation of strategies, demonstrations of logical reasoning and competence with mathematical skills, processes and conventions can be manifested in different assessment instruments. We can help them to recognize different levels of performance, to accurately self-assess and clearly state the prerequisites for improved performance.

The Rubric in the Classroom

We feel it is advisable to use a 4-point rubric that includes elements of "look-fors" but we believe that the rubric also needs to delineate the specific attributes for a given problem.

For example, a rubric should lay-out the evidence that a student has a repertoire of, and can apply, problem solving processes and techniques. In general, the teacher would be examining the student's solution for evidence that he can:

- explain the task and/or restate the problem;
- identify the assumptions, clarify the tasks and select the relevant information;
- approach the problem with reasonable and appropriate responses;
- organize information in a variety of ways (e.g., creating a table, constructing a model, extracting patterns, extending available information);
- demonstrate the application of explicit strategies (e.g., estimation, making an exhaustive list, applying an algorithm, answering a simpler but similar problem, working backwards, modelling the problem);
- demonstrating the appropriate use of a selected strategy to develop a correct solution by generating a comprehensive product;
- make observations or generalizations based on the solution;
- connect or apply the solution to other strands within mathematics and to other domains.

Similarly, the teacher would take a second look at the solution, extracting evidence that the student knows and can use the reasoning process, e.g., that the student can:

- make valid observations based on available data;
- record data using appropriate formats, e.g., drawings, charts and/or graphs;

- organize data to illuminate, describe and extend patterns;
- formulate generalizations and conjectures based on the data;
- validate the generalizations using logical arguments or counter examples;
- make connections between known facts and the problem to be solved; and,
- validate his/her and other's thinking.

The teacher would take a third look at the solution, extracting evidence that the student knows and can use the mathematical communication process by:

- constructing an explanation that has sufficient details, provides a logical flow, is well structured, summarizes results, explains reasoning and extends the problem;
- using mathematical terms, notation and symbols to communicate ideas and thinking; and
- explicitly linking multiple representations, i.e., representing and connecting information via diagrams, tables, graphs, symbols.

Finally, the teacher would take a fourth look to find evidence that the student both understands and can make connections between the core concepts by:

- giving appropriate and correct answers to questions about the core concepts;
- successfully completing tasks (e.g., getting correct answers) that require the application of recently acquired skills and techniques in both familiar and novel situations;
- clearly and convincingly communicating a facility with the content specified by particular curriculum expectations within a strand;
- assembling individual curriculum expectations into conceptual, procedural or structural constructs which can be used to solve complex problems;
- applying individual content items to bridge conceptual, procedural or structural concepts between strands and between disciplines;
- using vocabulary and concepts from different disciplines to describe specific curriculum expectations.

By specifying all of these elements within the categories in the rubric, the student understands that these are global expectations and are valued in every performance-based demonstration of learning. The teacher's input assists the student in understanding what problem-solving, reasoning or communication "looks" like, however, in a specific task, i.e., in our example, it answer the question, "What does it mean to 'observe data'?" in the *Bears* problem. This accounts for the "empty boxes" in our template - the rubric will be customized for each open-ended assessment it is applied to, with the teacher specifying exactly what skills, processes, techniques and strategies she is looking for in the solution path. By collecting a portfolio of such problems, their associated rubrics and the evaluated solutions, students have a file that shows the demonstrations of their mathematical thinking over time. The collection should assist students to solidify concepts and processes and establish and organize them as a basis for future reference.

How We See It

We must begin this section by emphatically stating that few problems are "rubric-ready"! But we believe that most teachers have existing units, problem sets and investigations that are already, or could be "tweaked" into, rubric-ready assessments.

In fact, it is our view that a single problem, unless it is exceptionally designed, should not be assessed along multiple dimensions using multiple criteria. Our "family" of 5 problems, viewed holistically and collectively, however, will allow us to examine in a meaningful way, the paper-trail of students. The solution to the five problems taken as a "whole" will give us a "portrait" rather than a "snapshot".

Possible solutions to the problem set we have posed, juxtaposed with our own set expectations, would lead us to insert some of the following into the blank boxes in the template.

Problem Solving

- in Question (2), the numerical data supplied by the graph of male bear cubs is used to make informed deductions about the female cub population;
- in Questions (4) and (5), the student organizes information in a variety of ways;
- in Questions (2), (4) and (5), the student develops a correct solution by generating a complete, comprehensive product including evidence of a strategy;
- in Question (4) and (5), the student makes generalizations based on his/her solution.

Reasoning

- in Question (1), the student recognizes the implication of the symmetry in the graph of humans and infers from the symmetry (balance) that there are more male bear cubs;
- in Questions (2), (3) and(4), the student formulates generalizations and conjectures based on the data;
- in Question (5), the student deduces the probability associated with a single event from the given distribution.

Communication

- in Question (2), the student generates a properly labelled, well-scaled graph;
- in Question (3), the student provides a clear, concise discussion concerning the information conveyed (or not) by the graphs;
- in Question (3), the student supports his/her answer using proper sentence structure, clear language and appropriate mathematical vocabulary;
- in Question (4), the student explains clearly what the physical configuration of the graphs says about birth probabilities;
- in Question (4), the student generates a probability tree or systematic list to support and clarify; explanations.

Understanding/Connecting Content and Concepts

- in Question (3), the student identifies some assumptions made about the data, e.g., were the bears all born during the same season? do these numbers represent the number of bears born or the number of bears that survived to a certain age?
- in Question (4), the student relates the geometric symmetry in the graph to probability outcomes;
- in Question (5), the student extends the solution to question (4) to compare the reported distributions of bears to what we would expect if the probability of genders was equal;
- in Question (5), the student deduces and calculates values for theoretical probability correctly and accurately;
- the student demonstrates a clear understanding of what is being asked in Questions (4) and (5), by explaining and/or restating the problem.

Conclusions

You may be familiar with the comic strip "Calvin and Hobbes." In one installment, Calvin tells Hobbes that math is like a religion because you have to accept its ideals and results on faith. He explains, "No one can say how it happens. You either believe it or you don't."

With the new curriculum and its intended instruction and assessment, we have a wonderful opportunity to convince our students and the generations of teachers and students that will follow that Calvin's view of mathematics is wrong!

We believe that performance-based assessments using a rubric can do two things. First we believe that these tools have an important role to play in influencing teacher attitudes towards mathematics and its teaching. Second, and most importantly, we believe that they can help with the creation of an expository mathematics environment.

We look forward to hearing others in our community share the variety of ways in which they will encourage and support students as they are guided through the new intended curriculum.

References

Gagnon, A. (1993). Struggling. In Deborah Shifter (Editor) *What's Happening in Math Class? Reconstructing Professional Identities. Volume II.* New York: Teachers College Press.

Kentucky Department of Education. (1995). *Kentucky Mathematics Portfolio: Holistic Scoring Guide.*

Ministry of Education and Training, Ontario. (1997). *The Ontario Curriculum: Mathematics, Grade 1-8.*

Education Quality and Accountability Office. (1997). *Grade 3 Assessment of Reading, Writing and Mathematics.*

National Council of Teachers of Mathematics (NCTM). (1994). *The Assessment Standards for School Mathematics.* Reston, Va: NCTM.

National Council of Teachers of Mathematics (NCTM). (1989). *The Curriculum and Evaluation for School Mathematics.* Reston, Va: NCTM.

National Council of Teachers of Mathematics. (1991). *The Professional Standards for Teaching Mathematics.* Reston, Va: NCTM.

	LEVEL 1	LEVEL 2
Problem Solving	The student's work demonstrates a basic understanding of the problem, and includes given operations and strategies. The student's solution is largely unstructured, may contain minor errors and does not include observations or extensions.	The student product demonstrates an understanding of problems and the selection of appropriate strategies. The student accurately implements the strategy, provides an accurate solution but includes limited observations or extensions.
Reasoning	The student's work demonstrates understanding and control of simple mathematical ideas, relationships, procedures and concrete materials.	The student's product demonstrates understanding and control of most of the required mathematical ideas, relationships, procedures and tools.
Mathematical Communication	The student uses simple mathematical language, few conventions and a limited number of mathematical representations.	The student uses some appropriate mathematical language and conventions and a limited number of mathematical representations.
Understanding/ Connecting Core Content/Concepts	The student's work demonstrates some knowledge of content/concepts including skills, techniques, relationships, principles, procedures and tools.	The student's product demonstrates a knowledge of core content/concepts including skills, techniques, relationships, principles, procedures and tools, but showing limited connections.

Problem Solving

The student provides evidence that s/he can:

- explain the task and/or restate the problem;
- identify the assumptions, clarify the tasks and select the relevant information;
- approach the problem with reasonable and appropriate responses; organize information in a variety of ways (e.g., draw a chart, construct a model, look for patterns, gather additional information);
- demonstrate the application of explicit strategies (e.g., guessing & checking, making on exhaustive list, using an algorithm, answering o simpler but similar problem, working backwards, modelling the problem);
- demonstrating the appropriate use of the selected strategy to arrive at a correct solution by generating o comprehensive product;
- make observations or generalizations based on the solution;
- connect or apply the solution to other areas.

To evaluate problem solving in this problem, I am looking for the following evidence:

Reasoning

The student provides evidence that s/he can:

- observe data;
- record data using drawings, charts and graphs;
- use data to recognize patterns;
- make generalizations and conjectures based on the data;
- test the generalizations using logical arguments or counter examples;
- make connections between known facts and the problem to be solved;
- validate his/her own and others' thinking.

To evaluate reasoning in this problem, I om looking for the following evidence:

	LEVEL 3	LEVEL 4
Problem Solving	The student's work demonstrates a broad understanding of problems with alternate strategies. The student accurately and efficiently implements and analyzes strategies with correct solutions.	The student's work demonstrates a comprehensive understanding of problems with efficient and sophisticated strategies. The student accurately and efficiently implements and evaluates sophisticated strategies with correct solutions and includes analysis, justifications and extensions.
Reasoning	The student's work demonstrates perceptive understanding and control of the required mathematical ideas, relationships, procedures, tools, materials and models.	The student's work demonstrates perceptive, creative and complex understanding and control of mathematical ideas, relationships, procedures and materials tools and models.
Mathematical Communication	The student explains, makes conjectures and defends ideas. The explanation is organized, coherent and detailed. Ideas are effectively represented using a variety of modes and relationships are described.	The student uses sophisticated, precise and appropriate mathematical language and conventions throughout the solution process, from hypothesis formulation to conclusions and generalizations. The explanation includes explicit statements which link the wide variety of mathematical representations incorporated in the solution.
Understanding/ Connecting Core Content/Concepts	The student's product demonstrates a broad knowledge of core content/concepts and relationships, principles, procedures and tools and includes connections.	The student's work indicates a comprehensive knowledge of core content/concepts, relationships, principles, procedures and tools with connections throughout.

Understanding/ Connecting Core Content/Concepts

The student provides evidence that s/he can:

- give appropriate and correct answers to questions about core concepts;
- successfully complete tasks by applying taught skills and techniques in both familiar and novel situations;
- communicating competence and confidence with the content specified by particular curriculum expectations;
- assembling individual curriculum expectations into conceptual, procedural and structural constructs to solve complex problems;
- bridging strands and disciplines with content and concepts and using vocabulary and concepts from other disciplines to describe specific curriculum expectations.

To evaluate understanding of core content/concepts in this problem, I am looking for the following evidence:

Mathematical Communication

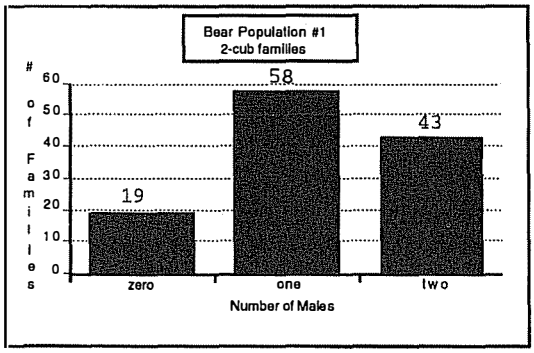
The student provides evidence that s/he can:

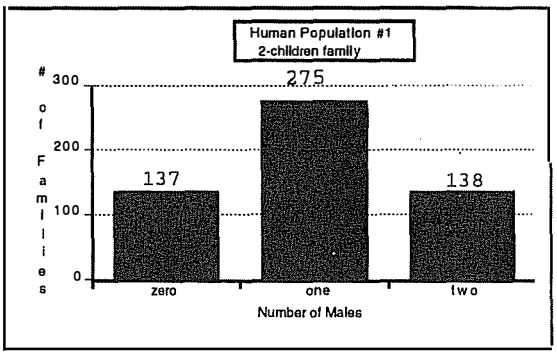
- construct an explanation that has sufficient details, provides a logical flow, is well structured, summarizes results, explains reasoning and extends the problem;
- uses mathematical terms, notation, conventions and symbols to communicate ideas and thinking;
- represents and connects information using diagrams, charts and graphs.

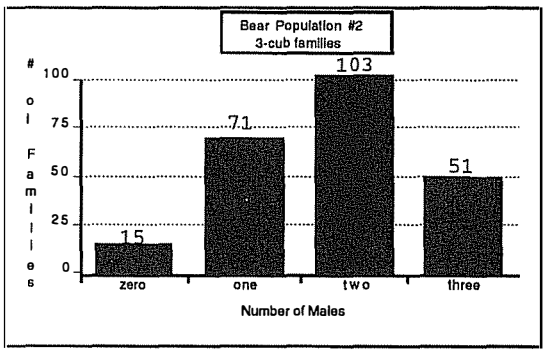
To evaluate mathematical communication in this problem, I am looking for the following evidence:

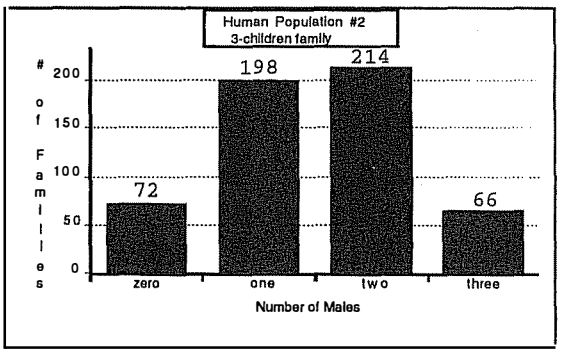
The Population Mystery

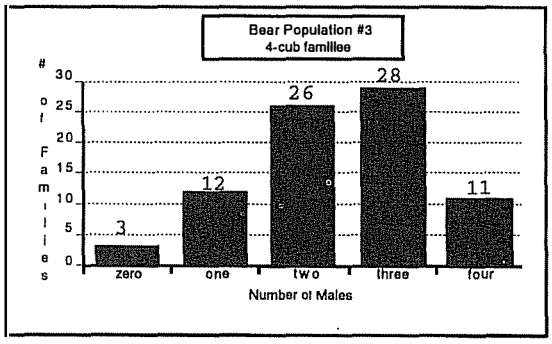
A team of researchers has been studying the population of a "mathematical" bear. They have been tracking bear families and keeping a record of the number of male cubs in each family. They think that they have collected some rather odd findings. To help understand their data, they have assembled some similar data for humans. They surveyed a series of human families with similar numbers of children and made notes of the number of boys. All of the results are shown below in graphical form. The researchers have concluded that the probability that a mother bear will give birth to a male cub is different from the probability that a human mother will give birth to a son.

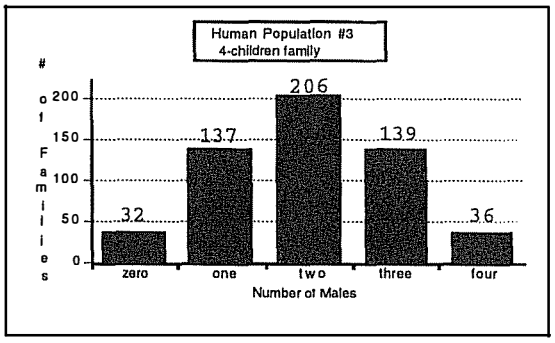












Using the information provided by these graphs and inferences that you can make from them, answer the following questions. Be sure to provide mathematical reasons in support of your statements.

1. Indicate any significant similarities, differences and patterns that occur in the graphs of the bear population and the human population.
2. The researchers didn't provide any graphs showing the number of female cubs in the bears families. Create your own graph showing the number of female cubs in the "three cub" families. Write a paragraph to justify the accuracy of your predictions.
3. Is the information provided by the graphs complete? Support your answer, including suggestions for improving the study.
4. The graphs showing the human populations appear more symmetrical than the bears' graphs. Is this symmetry significant? Why would the humans' graph have this symmetry?
5. A mother bear is about to give birth to a baby cub. What is the probability that this cub will be female? Give a detailed explanation.

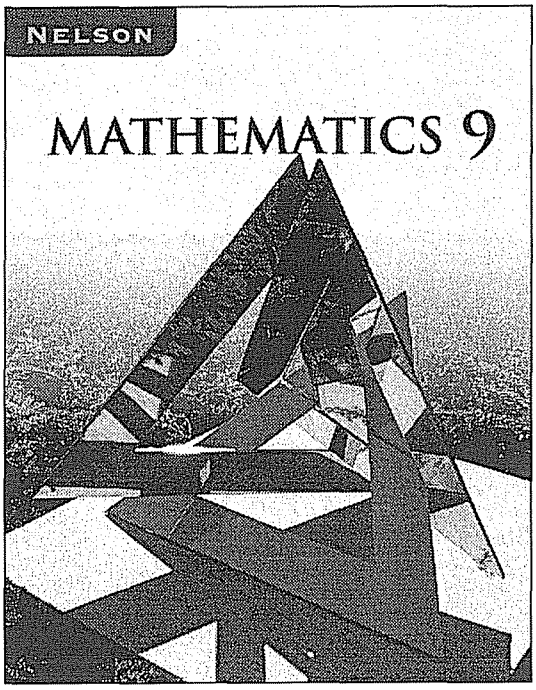
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▲ Baseball Salaries: Does Money Buy Success?



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A commonly observed event in major league baseball concerns some owners spending large amounts of money to secure or retain the players they believe are necessary in order to compete successfully on the field. Are these attempts to "buy success" worthwhile? We will suggest means by which this issue can be addressed in a mathematics classroom.

In this article two separate sets of major league data for 1997 are integrated — salaries and performance. Both sets of data are readily available from news sources.

These statistics can provide a rich source of data for student exploration. Such exploration should be encouraged because the process of forming and verifying conjectures involving real world data is an important task in any mathematics class.

Table I

NATIONAL LEAGUE			AMERICAN LEAGUE		
Team	Payroll (in Millions)	Games Won	Team	Payroll (in Millions)	Games Won
Atlanta Braves	50	101	Anaheim Angels	34	84
Chicago Cubs	40	68	Baltimore Orioles	52	98
Cincinnati Reds	30	76	Boston Red Sox	43	78
Colorado Rockies	42	83	Chicago White Sox	52	80
Florida Marlins	48	92	Cleveland Indians	47	86
Houston Astros	32	84	Detroit Tigers	20	79
Los Angeles Dodgers	44	88	Kansas City Royals	28	67
Montreal Expos	17.5	78	Milwaukee Brewers	22.5	78
New York Mets	34	88	Minnesota Twins	27.5	68
Philadelphia Phillies	30.8	68	New York Yankees	62.5	96
Pittsburgh Pirates	12.5	79	Oakland Athletics	24	65
St. Louis Cardinals	41	73	Seattle Mariners	37.4	90
San Diego Padres	35	76	Texas Rangers	51.8	77
San Francisco Giants	34	90	Toronto Blue Jays	51	76

Table I lists, for each major league team, the total team payroll at the beginning of the 1997 season, and the number of games that the team won during the 1997 season.

Is there a relationship between the team payroll size and team success?

We will use the data of Table I in four ways to address this question.

Method I

Rank the teams in each league according to team payroll and games won. These rankings (from high to low) are listed in Table II.

Table II

NATIONAL LEAGUE			AMERICAN LEAGUE		
Team	Payroll Rank	Games Won	Team	Payroll Rank	Games Won
Atlanta Braves	1	1	New York Yankees	1	2
Florida Marlins	2	2	Baltimore Orioles	2*	1
Los Angeles Dodgers	3	4*	Chicago White Sox	2*	6
Colorado Rockies	4	7	Texas Rangers	4	10
St. Louis Cardinals	5	12	Toronto Blue Jays	5	11
Chicago Cubs	6	13*	Cleveland Indians	6	4
San Diego Padres	7	10*	Boston Red Sox	7	8*
New York Mets	8*	4*	Seattle Mariners	8	3
San Francisco Giants	8*	3	Anaheim Angels	9	5
Houston Astros	10	6	Kansas City Royals	10	13
Philadelphia Phillies	11	13*	Minnesota Twins	11	12
Cincinnati Reds	12	10*	Oakland Athletics	12	14
Montreal Expos	13	9	Milwaukee Brewers	13	8*
Pittsburgh Pirates	14	8	Detroit Tigers	14	7

* = Tie

Observe that:

1. In the National League two of the top four payroll leaders made the playoffs (Atlanta and Florida); the other two playoff teams (San Francisco and Houston) ranked 6th and 8th in payroll.
2. In the American League two of the four top four payroll leaders made the playoffs (New York and Baltimore); the other two playoff teams (Cleveland and Seattle) ranked 8th and 10th. Money seems to help!

Table III

NATIONAL LEAGUE		AMERICAN LEAGUE	
Team	Salary Rank - Games Won Rank	Team	Salary Rank - Games Won Rank
Pittsburgh Pirates	14 - 8 = 6	Detroit Tigers	14 - 7 = 7
San Francisco Giants	8 - 3 = 5	Milwaukee Brewers	13 - 8 = 5
Houston Astros	10 - 6 = 4	Seattle Mariners	8 - 3 = 5
Montreal Expos	13 - 9 = 4	Anaheim Angels	9 - 5 = 4
New York Mets	8 - 4 = 4	Cleveland Indians	6 - 4 = 2
Cincinnati Reds	12 - 10 = 2	Baltimore Orioles	2 - 1 = 1
Atlanta Braves	1 - 1 = 0	Boston Red Sox	7 - 8 = -1
Florida Marlins	2 - 2 = 0	Minnesota Twins	11 - 12 = -1
Los Angeles Dodgers	3 - 4 = -1	New York Yankees	1 - 2 = -1
Philadelphia Phillies	11 - 13 = -2	Oakland Athletics	12 - 14 = -2
Colorado Rockies	4 - 7 = -3	Kansas City Royals	10 - 13 = -3
San Diego Padres	7 - 10 = -3	Chicago White Sox	2 - 6 = -4
Chicago Cubs	6 - 13 = -7	Texas Rangers	4 - 10 = -6
St. Louis Cardinals	5 - 12 = -7	Toronto Blue Jays	5 - 11 = -6

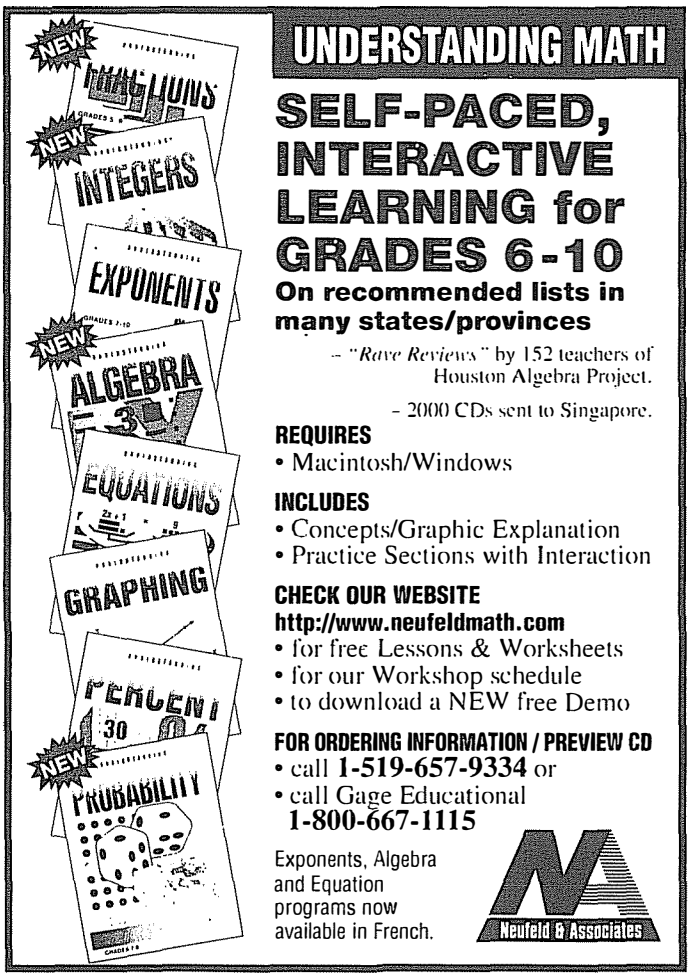
Method II

Discussions of team "success" sometimes involve more than the standings alone. There is also the consideration of whether the standings are consistent with the amount of money spent.

Compare the disparities between the rankings for payroll and games won. For each team, compute payroll rank minus games won rank. Call this the disparity index (DI). A positive difference indicates that the team "overachieved" while a negative difference indicates that the team "underachieved". If the difference is close to zero, the team achieved what might be expected, given the payroll. The disparity indicies are listed in Table III.

Observe that:

1. Five teams were significant overachievers (DI ≥ 5): Detroit, Milwaukee, Pittsburgh, San Francisco, and Seattle. Their payrolls were "bargains"; the owners should be praised for getting the most "bang for the buck".
2. Four teams were significant underachievers (DI ≤ -5): Chicago Cubs, St. Louis, Texas, and Toronto. The owners did not get what they paid for.
3. Seven owners got just about what they paid for (-1 ≤ DI ≤ 1): Atlanta, Baltimore, Boston, Florida, Los Angeles, Minnesota, and the New York Yankees. Note that inclusion on this list does not predict team success. Four of these teams (Atlanta, Baltimore, Florida, and New York) achieved success by spending large sums of money; Minnesota is an example of a team which spent little and accomplished little; is this good for the franchise – or the fans?



Method III

Compute the price of winning (PW), that is, the total team salary ÷ the number of games won. Table IV reports the calculations.

The three teams that paid the most per win were Texas, Toronto, and the New York Yankees, while the three teams that paid the least per win were Detroit, Montreal, and Pittsburgh. Compare this to the overachiever/underachiever lists (Table III).

Method IV

A method frequently used to determine the strength of the relationship between two sets of paired data involves the coefficient of correlation, if it has been previously introduced. The formula for this descriptive statistic for X - Y pairs is:

Table IV

NATIONAL LEAGUE		AMERICAN LEAGUE	
Team	PW (in Thousands)	Team	PW (in Thousands)
Pittsburgh Pirates	$158	Detroit Tigers	$253
Montreal Expos	$224	Milwaukee Brewers	$288
San Francisco Giants	$378	Oakland Athletics	$369
Houston Astros	$381	Minnesota Twins	$404
New York Mets	$386	Anaheim Angels	$405
Cincinnati Reds	$395	Seattle Mariners	$416
Philadelphia Phillies	$453	Kansas City Royals	$418
San Diego Padres	$461	Baltimore Orioles	$531
Atlanta Brewers	$495	Boston Red Sox	$551
Los Angeles Dodgers	$500	Cleveland Indians	$547
Colorado Rockies	$506	Chicago White Sox	$650
Florida Marlins	$522	New York Yankees	$651
St. Louis Cardinals	$562	Toronto Blue Jays	$671
Chicago Cubs	$588	Texas Rangers	$673

$$r = \frac{n(\Sigma XY) - (\Sigma X)(\Sigma Y)}{\sqrt{\left[n(\Sigma X^2) - (\Sigma X^2)\right]\left[n(\Sigma Y^2) - (\Sigma Y^2)\right]}}$$

Recall the following:

1. If r is close to +1 or -1, there is a strong linear relationship between the variables. If $r > 0$, this relationship is direct (small x's tend to be paired with small y's while large x's tend to be paired with large y's). If $r < 0$, the relationship is inverse (small x's tend to be paired with large y's and vice versa).
2. If r is close to 0, there is a weak, or non-existent, linear relationship between the variables.
   Using the data from Table I the results follow:
   - For the National League alone, r = 0.62.
   - For the American League alone, r = 0.43.
   - For both leagues together, r = 0.50.

   Are these correlation coefficients high enough to encourage team owners to continue to pay large sums of money in hopes of winning pennants?

Challenges:

1. For each league, calculate the average payroll cost for each win. Why can this **not** be done by averaging the numbers of Table IV? What data are needed to compute this? Collect the data and do the computations.
2. Perform similar analyses for other professional sports.

▲ Y2K: The Greatest Challenge Facing Our Schools

BY DAVID THORNBURG

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The world seems overwhelmed with hype regarding the "Year 2000 (Y2K) Problem" – a problem associated with potential errors in date calculations based on programmers' historical use of two digits to represent a year. Basically, the COBOL programmers of yore decided to save memory space by using the last two digits of a year, and placing a "19" in front. They probably thought that their code would become obsolete by the end of the century. But, as it turns out, lots of these old programs are still in use. So, as a result, some legacy software will think that the year 2000 is actually 1900.

According to some, this is going to mark the end of the world. The popular press has broadcast warnings of disasters ranging from airplanes falling out of the sky, to the caution that elevators might stop working at the stroke of midnight, December 31, 1999. Survivalists are busy stockpiling food, water and guns, and the great land rush in Montana and Idaho seems to have accelerated for those who "know" that life as we know it will soon end.

What drivel!

Hysteria regarding this "problem" has come home. I have received several letters from school systems asking if the Thornburg Center is Y2K compliant. I have assured them that we are. But now it is my turn to ask the question of America's schools. Are they Y2K compliant?

Before you answer, let me extend the definition a bit. As frustrating as it might be to get stuck in an elevator on January 1, 2000, I think there is a true Y2K disaster looming – a disaster that eclipses any caused by a bad date showing up in an interest calculation. A disaster that would make late Social Security checks look like a trivial event. I'm talking about making America's educational system Y2K compliant.

A truly Y2K compliant school will have taken measures to insure, by January 1, 2000, that each and every student will be graduating with the skills and habits of mind

needed to thrive in the 21st century. Any school that fails this Y2K compliance test has a lot of work to do in the coming months.

For example, the World Wide Web is the fastest growing communications - based educational technology in history. It took radio 38 years to reach 50 million users. Television took 13 years to reach this point, but the Web hit the 50 million mark in four years! According to a report from Jupiter Communications (Deconstructing the Digital Kid, 1998) the number of on- line teens will grow from 4.5 to 11 million between 1998 and 2002. But get this: The number of pre-teens on-line will jump from three to twenty million in the same time frame – almost a factor of seven!

So, here is the first Y2K question: Does each and every classroom in your school have meaningful access to the Internet? If not, you're out of compliance.

Of course, access requires appropriate hardware and software. Does every child have meaningful access to computers, network appliances and other informational technologies? Are these tools as commonplace as books and pencils? Does every teacher have in-class access to multimedia presentation tools, complete with high-brightness video projection equipment (preferably ceiling mounted)? If not, then you are out of compliance.

Now let's move to the harder issues: curriculum and pedagogy.

According to James Bailey *After Thought: The Computer Challenge to Human Intelligence, 1996* the greatest change in math education in recent centuries has been the translation of textbooks into English from Latin. The mathematics needs of the next century move far beyond the rote numerical computation skills and geometrical analysis little changed from the time of Euclid. Clouds are not ovals; streams are not lines, and a tabletop is not a plane. The geometry of fractional dimensions (the world of objects with fractal boundaries – clouds, etc.) has been well known for years now, as have its sister fields of chaos and complexity. The world of rapid change in which we live requires mastery of these "new maths." How are you doing in this area? Are you in compliance?

What about science education? The recent determination of neutrino mass has resulted in a new basis for explaining some of the "missing mass" of the universe. Do you insist on science textbooks that reflect this reality? Of course not. Textbooks can't be produced quickly enough to keep up with relevant scientific discoveries, but the Web can. Have you incorporated the Web into science instruction? Do students know how to gather information, determine its relevance, and determine its accuracy? If not, then you have a Y2K problem!

Staying with science for a minute, I had the honor of dining with a soybean farmer in Carlinville, Illinois a few months ago. I asked him what the greatest change in agriculture had been in the past generation. Without hesitation he said, "gene splicing." He went on to explain how genetically modified soybeans incorporated a herbicide-resistant gene from a bacterium, and how this allowed him to maintain high yields while easing maintenance. Years ago, when I worked on a ranch, "genes" were something you wore, not something you spliced. So here is the question: Can your students take part in a dinner conversation with a farmer on the merits of gene splicing – addressing not only the technical and scientific issues, but the ethical ones as well? That farmer is Y2K compliant. How about you?

The global marketplace is a reality. A case of economic sniffles in one country can produce a financial flu epidemic in another. Information passes effortlessly across boundaries, unaware of customs agents or physical borders. Information workers in one country can ply their craft in another through the use of telecommunications. The technology of global commerce is already in place. Human skills are needed: an understanding of the language and culture of people throughout the world. Foreign language skills are essential, as is the capacity to understand and collaborate with people from cultures different from our own. Looking at the emerging markets, it is clear that the Pacific Rim has a bright future, even if many countries on the western edge are in financial turmoil today. At what age do you start offering students courses in Spanish, Portuguese, Mandarin, Korean, or Japanese? When you explore cultural literacy, do you make comparisons between the legends of the countries of the Northern Hemisphere with those of the South? Are your students able to walk to a globe and pinpoint the location of countries mentioned in the previous evening's news? Where, exactly, are Singapore, Malaysia, and Beijing? Quick! Which has more land: The United States, Canada, or Brazil? What is that land used for? How does geography influence commerce? Why should an economic problem in Hong Kong affect the US stock market? These questions are a snap for graduates of Y2K compliant institutions.

A generation or two ago, large employers had tacit lifelong contracts with employees. That is long gone, replaced by a massive migrant workforce of contractors and entrepreneurs. Even employees inside Fortune 500 companies are expected to develop "intrapreneurial" skills. So, when do you teach students about bookkeeping,

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marketing, sales, time management, and the other skills needed to run a successful enterprise of any size? Y2K compliance requires it!

Let's leave curriculum and move to pedagogy. Y2K compliance means that every student will have the skills and habits of mind needed to be a lifelong learner. The notion of education stopping at the end of schooling is an anachronism. What are you doing to keep the love of learning alive in each and every one of your students? How are you working with your students' parents to help them help you with this task? What problem-solving skills are students being taught that have application beyond the world of school? Address these questions, and you'll be making progress toward Y2K compliance.

We have known how to help students learn more effectively since the time of Dewey, if not before. We all have access to the work of Howard Gardner, Jean Piaget, Lev Vygotsky, and other giants in pedagogical thought. And yet there are many schools that have scarcely moved beyond B. F. Skinner, especially in the realm of computer use by children. Does every faculty member in your school take advantage of the powerful pedagogical models that are free for the taking? Does the faculty know how to incorporate technology in ways that support the work of these great thinkers? If not, the clock is ticking. Y2K is just months away. And, let's not forget instructional models: Have you moved beyond the domain of what Ian Jukes calls "full-frontal teaching?" One of my colleagues, Norma Godoy, suggests that teachers should no longer be called "instructors;" they should be called "constructors" instead. This implies, of course, that educators focus on student projects and other strategies that develop and expand the thinking skills of all students. One of the benefits of this tectonic shift is that students immersed in a project-based curriculum will be able to transfer these skills from the classroom to the workplace. How is your Y2K compliance in this area?

Here is another issue to look at. Maslow is more important than Gardner. If our children are coming to school hungry and homeless, or are battered and abused by those with whom they live, the metaphysical analysis of split infinitives will be low on their personal scale of relevance. Children whose guardians think coke is something you sniff, not something you drink, have added challenges – especially if they were exposed to the consequences of drug abuse in-utero. How are you addressing Maslow's hierarchy of needs in your school? Y2K compliance officers will want to know.

Thinking beyond school (since our students will be spending the bulk of their lives outside the shelter of the

classroom), there are some significant issues to be addressed. First, there are thousands of high-paying jobs going begging because the skill level of the employment pool is too low. Unless something is done, this job shortfall will exceed one million by the year 2002. Imagine the Industrial Revolution without iron ore. We are in the midst of an Information Revolution and we don't have enough new recruits with the requisite skills. These are not just technical skills that are lacking. You can be the brightest person in the world, but if you can not express your ideas clearly to others, and lack the social skills needed to work in collaborative teams, you are unemployable. If you lack resiliency, creativity, tolerance for ambiguity, and the capacity to make good decisions quickly, you are employment- disabled. If you don't know how to mine the rapidly growing collection of information for relevant, accurate material germane to your work, you will get left behind. And, I'm not just talking about searching the Web, but about using paper- and colleague-based information resources as well. These are teachable skills. How are you doing with them? Are you Y2K compliant in this area?

Of course I can hear the huge "Yes, but" echoing across the country in the face of these proposals. "Yes, but we lack the financial resources to wire every classroom." "Yes, but we lack the resources to put technology in the hands of every child." "Yes, but we lack the staff development time needed to help our teachers learn a new curriculum and change their pedagogical practice." "Yes, but our curriculum is dictated by the state (or the entrance requirements of college, or the school board, or my cousin Fred) and there is nothing I can do about it."

Nothing we can do about it? Think about this. We have placed our children – our most precious assets in the world – on a train. This train is hurtling down the track at great speed, and we are headed for a train wreck of incredible proportions. Some might survive, but many will not. The train wreck isn't going to happen today – it won't happen for about a year, but it will happen. In the face of this oncoming train wreck, are you still saying there is nothing you can do to prevent it? Look in the face of any child and tell him or her, "I'm sorry, but there is nothing I can do." I can't tell a child this, can you?

So, where will I be on January 1, 2000? I might be in an elevator. Heck, I might even be on an airplane. I am not afraid of banks collapsing on me. My question is this: Should I feel safe visiting schools?

▲ Opportunity to Learn Proof in Ontario Grade Twelve Mathematics Texts

GILA HANNA
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GLENFOREST SECONDARY SCHOOL
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Introduction

This study investigates the occurrence of proof in representative textbooks used in grade twelve (age 17-18) advanced-level mathematics in Ontario. Proof is a prescribed topic at this level, as one of the Process Components described in the *Ontario Curriculum Guideline Mathematics Intermediate and Senior Divisions 1985, Part Three* (p. 18). In the advanced-level courses, concepts of proof are to begin in grade 10, with a more serious treatment of the topic in grade 12: "The grade 12 advanced-level course provides for the extension of this to more complex proofs." (*Guideline,* p. 18). This course is the first one for which the *Ontario Curriculum Guideline* (pp. 60-63) specifies the inclusion of a variety of formal proofs.

On mathematics textbooks there is very little recent research in general, as noted by Howson (1995), though textbook analysis could be of utmost importance in drawing attention to any discrepancies between curriculum aims and actual opportunities to learn. It is particularly striking that there has been almost no examination of the actual occurrence in mathematics textbooks of proofs, discussions of proof, and exercises requiring the construction of proofs, although clearly it is by such means that textbooks could play their part in conveying the theory and practice of proof in the classroom. So far, researchers have tended to focus instead on the nature and function of mathematical proof (de Villiers, 1990; Hanna, 1990), on student approaches

to proving (Coe and Ruthven, 1994; Hoyles, 1997), or on ways to teach proof more effectively (Dreyfus and Hadas, 1996; Hanna and Jahnke, 1993; Mariotti, Bartolini-Bussi, Boero, Ferri, and Garuti, 1997; Solow, 1982).

Method

The study focussed upon the two mathematics texts which, taken together, are used in over 80% of the grade 12 mathematics classrooms: *Foundations of Mathematics 12* (Dottori, D. et al., 1987) and *Mathematics 12* (Kelly, B. et al., 1988). These texts, which are referred to in the following as FM12 and M12 respectively, were examined for the frequency of occurrence of various kinds of proof and of items related to constructing a proof.

Since it was important to know where in the curriculum each kind of proof was used, the examination was organised by curriculum topic. These topics were chosen to conform as far as possible with the compulsory topics specified in the *Curriculum Guideline Mathematics Intermediate and Senior Divisions 1985 Part Three* (pp. 60-63) (Ontario Ministry of Education, 1985). Only the curriculum topics listed as compulsory in the *Curriculum Guidelines* formed part of this study: Algebra, Geometry, Functions and Relations, Exponents and Logarithms, and Trigonometry.

Definition of an Item

Both the textbooks studied are divided into chapters and sections within chapters. Most sections contain explanatory text and examples, followed by exercises. In this paper, the term "development section" refers to the explanatory text and examples in a textbook section, whereas the term "exercise section" refers to the accompanying set of exercises.

The term "development item" refers to an example, a paragraph, or a block of text dealing with a single idea or concept within a development section. "Number of development items" would then refer to the number of such items counted in the unit under discussion, whether it be the entire textbook or a portion of it, such as a section or topic.

The term "exercise item," on the other hand, usually refers to a single numbered question in an exercise section. This means that a question with multiple parts is counted as a single exercise item. This procedure worked well, allowing for more consistent counting and comparison within a given textbook (with the exception of the topic of geometry, as discussed later).

On the other hand, this procedure would have made it difficult to compare the two textbooks, of course, had this been a focus of the study. In FM12, for example, each trigonometric identity to be proven is numbered separately (p. 352), while in M12 several identities to be proven are grouped under a single question and therefore counted as one item (pp. 420-422).

Classification and Tabulation of Items

Items in both the development and the exercise sections were classified into three categories: (1) non-proof, (2) discussion (of proof), and (3) proof. The non-proof items are those that have nothing to do with proving in any way. These would include solving an equation, such as, factoring a polynomial, or calculating unknown angles in a geometric figure provided in the text. The statement of a theorem without proof or definition of terms would also be considered a non-proof item.

The discussion items are those which discuss the creation of a proof, or provide guidance in how to go about it. An example of a discussion item in a development section is: "To demonstrate that a pair of line segments, angles or triangles are congruent, we must show that they can be made to coincide by a suitable series of slides, flips and turns" (M12, p. 118).

Proof items in the development sections would include any item which provides a full or partial proof, such as one proving a trigonometric identity or a geometry theorem. Proof items in the exercise sections would include any in which the imperatives "prove" or "show" appear.

In tabulating the data, the number of proof, discussion, and non-proof items were recorded for each section of each chapter, for both textbooks. In addition, each proof was tabulated by type (category and sub-category), as discussed below.

Classification of Proofs

Proofs were considered to fall into two broad categories: direct proof and indirect proof. A proof is regarded as direct if the proposition proven is the conclusion of a sequence of deductions based upon accepted premisses, with each intermediate deductive step following necessarily from one or more of the previous steps. An indirect proof is one which establishes the truth of a proposition by showing that if the proposition to be proven were false, it would lead to a contradiction.



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On the basis of certain common features, the study also broke down direct proofs further into five sub-categories: basic, analysis, existence or construction, induction, and miscellaneous.

1. Basic direct proof:

A basic direct proof is one for which the textbook provides no plan or analysis, jumping immediately into the direct proof. An example is the proof of the Remainder Theorem in FM12 (p. 26).

2. Direct proof by analysis:

A proof was classified as proof by analysis if the textbook presented a "plan" for the proof, or if it worked backwards from the conclusion (theorem) to the hypothesis (i.e., if it posed the question: What do we need to know in order to prove the conclusion?).

3. Direct existence or construction proof:

In this type of proof the focus is upon the object which is to be constructed, or the existence of which is to be shown. If the statement to be proved from given premises is the assertion that an object X exists, or can be constructed, one must first effect its construction and then prove that the object constructed is in fact X.

4. Direct proof by induction:

This category covered proofs by mathematical induction, common in number and set theory, with applications in other branches of mathematics as well.

5. Miscellaneous direct proof:

This category was reserved for forms of direct proof that could not be said to belong to the first four categories, including visual proofs, informal (heuristic) arguments, and completed proofs presented for the purpose of eliciting a critique from the student.

Data-collection Quality Control

The data for the study were gathered by two research assistants experienced in both research and mathematics teaching, working independently of each other. A team of three researchers and two research assistants met weekly during the data collection phase to discuss the data and check the grids filled in by the two research assistants for consistency and accuracy. During these consultations the team also resolved issues relating to the method of counting exercise and development items as well as to the distinction between the five proof sub-categories.

Results and Discussion

Distribution of proof and discussion items across the curriculum

As mentioned, the number of proof items and discussion-of-proof items were group tabulated by curriculum topic for both the development and the exercise sections of the two textbooks studied. Table 1 shows the number of proof and discussion items by curriculum topic for the development and the exercise sections of each of the textbooks. The numbers in brackets are the number of proof and discussion items for each topic as a percentage of the total number of proof and discussion items for each textbook.

Table 1: Number and (%) of Proof and Discussion Items by Curriculum Topic, by Textbook and by Development or Exercise Item

	Foundations of Mathematics 12 (FM12)		Mathematics 12 (M12)	
Topic	Proof and Discussion Development Items	Proof and Discussion Exercise Items	Proof and Discussion Development Items	Proof and Discussion Exercise Items
Algebra	9 (9)	5 (2)	2 (2)	10 (4)
Geometry	58 (61)	166 (74)	88 (83)	181 (77)
Functions and Relations	3 (3)	6 (3)	1 (1)	9 (4)
Exponents and Logarithms	3 (3)	1 (0)	5 (5)	15 (6)
Trigonometry	23 (24)	46 (21)	10 (9)	19 (8)
Total Text	96 (100)	224 (100)	106 (100)	234 (100)

Not surprisingly, the overwhelming majority of such items, close to three-quarters in both texts, are geometry items. The next largest concentration of proof and discussion items is in trigonometry, with the remaining curriculum topics containing very few. Another interesting pattern is the distribution of proof and discussion items between the development and the exercise sections. In both textbooks the number and percentage of proof and discussion items in the exercise sections of a topic correlates very well with the number and percentage in the corresponding development section, with the number of exercise items being roughly double the number of the development items.

Proof and discussion items as a proportion of all items, by topic

For each textbook, each curriculum topic, and the entire text, Table 2 shows the total number of development and exercise items and their breakdown

into proof, discussion and non-proof items. The bracketed numbers are percentages of total items for a given topic. In geometry, for example, the 52 development proof items represented 38% of all 136 development items in that topic, for FM12. In the same row, the 6 discussion items accounted for 4%, and 78 non-proof items for 57%, of all geometry items.

Table 2: Number and (%) of Proof, Discussion, and Non-Proof Items, and Total Number of Items, by Curriculum Topic and by Textbook for Development and Exercise Items

	Development Items				Exercise Items			
Topic	Proof	Discussion	Non - Proof	Total Items	Proof	Discussion	Non - Proof	Total Items
Foundations of Mathematics 12 (FM12)								
Algebra	9 (10)	0 (0)	81 (90)	90	5 (2)	0 (0)	226 (98)	231
Geometry	52 (38)*	6 (4)	78 (57)	136	164 (56)	2 (1)	126 (43)	292
Functions and Relations	3 (3)	0 (0)	85 (97)	88	6 (3)	0 (0)	201 (97)	207
Exponents and Logarithms	3 (7)	0 (0)	39 (93)	42	1 (1)	0 (0)	96 (99)	97
Trigonometry	22 (20)	1 (1)	86 (79)	109	46 (18)	0 (0)	213 (82)	259
Total Text	89 (19)	7 (2)	369 (79)	465	222 (21)	2 (0)*	862 (79)	1086
Mathematics 12 (M12)								
Algebra	2 (2)	0 (0)	89 (98)	91	10 (4)	0 (0)	262 (96)	272
Geometry	48 (23)*	40 (19)	119 (57)	207	146 (40)	35 (10)	180 (50)	361
Functions and Relations	1 (1)	0 (0)	112 (99)	113	9 (3)	0 (0)	262 (97)	271
Exponents and Logarithms	5 (6)	0 (0)	78 (94)	83	15 (6)	0 (0)	232 (94)	247
Trigonometry	8 (6)	2 (2)	117 (92)	127	19 (6)	0 (0)	321 (94)	340
Total Text	64 (10)	42 (7)	515 (83)	621	199 (13)	35 (3)	1257(84)	1491

Note: Percentages in rows indicated by * do not add to 100, due to rounding

Table 2 shows that in FM12 about one-fifth of the items in both the development and the exercise sections are proof items. In M12 proof items represent only about ten percent of the development and 13% of the exercise items. On the other hand, M12 has considerably more discussion items, so that when these are grouped together with the proof items, the percentages of proof and discussion items in the two texts are closer, representing 21% vs. 17% in the development section and 20% vs. 16% in the exercise section, for FM12 and M12 respectively.

It is evident from the data that the curriculum topics with the highest percentage of proof items are geometry and trigonometry. Indeed, only for these topic were any discussion items found. Geometry obviously has the greatest proportion of proof and discussion items, close to half of the development and exercise items in both texts. In trigonometry, proof and discussion items were 20% and 18% of development and exercise items respectively in FM12 and 6% of both in M12.

Distribution of proof items in geometry

Geometry was the only topic in which a range of proof types was presented; in all other topics the textbooks made use of direct proof by analysis exclusively.

To analyse the use of proof in geometry it proved beneficial to modify the original classification scheme somewhat. Although both textbooks were initially examined for the two major categories of proof (direct and indirect) and for all five sub-categories of direct proof (basic, analysis, existence or construction, induction, and miscellaneous), the direct proofs actually found fell into only two sub-categories: basic and analysis. It should be noted in particular that no miscellaneous proofs were found. In other words, neither textbook made use of teaching devices such as visual proofs, heuristic arguments, or the presentation of correctly or partially completed proofs for critique by the student.

On the other hand, a new category of "unspecified direct" was added to the original five sub-categories to account for those exercise items that did not provide the students with the slightest clue as to how to go about constructing a particular proof. Thus four kinds of proof were found: basic direct, direct by analysis, unspecified direct, and indirect. For each of the two textbooks, Table 3 displays, for geometry, the development and exercise proof items by type of proof, in absolute numbers and as a percentage of the total number of proofs.

Table 3: Number and (%) of all Proof Items in Geometry, by Type, by Development or Exercise Item, and by Text

	Foundations of Mathematics 12 (FM12)		Mathematics 12 (M12)	
Kind of Proof	Development Proof Items	Exercise Proof Items	Development Proof Items	Exercise Proof Items
Basic Direct	12 (23)	1 (0)	15 (31)	2 (1)
Direct by Analysis	33 (64)	13 (8)	28 (59)	0 (0)
Unspecified Direct	–	138 (86)	–	133 (91)
Indirect	7 (13)	9 (6)	5 (10)	11 (8)
Total No. of Proofs	52 (100)	161 (100)	48 (100)	146 (100)

In the development sections of the geometry chapters of either text, well over a half of the proof items were direct proof by analysis. In the exercise sections, the overwhelming majority of the proof items did not assign to the student any specific kind of proof; all of these were classified as "unspecified direct proof," on the assumption that almost all students would create a basic direct proof, although some of the mathematically gifted students might conceivably construct another kind of proof. (In the case of trigonometric identities, exercise

proof items automatically classified as direct proof by analysis, on the assumption that most students would follow the method shown in the examples.)

Although both textbooks did emphasize proof by analysis in the geometry development sections, only in FM12 were there specific requests in these sections to do proofs by analysis. And even in FM12 only 13 items requested direct proof by analysis, as against 138 items in which no approach was specified. The lack of exercise items for proof by analysis is a concern, because proof by analysis, attributed to the Greek mathematician Pappus, is a method of working backwards from the conclusion to the hypothesis (Polya, G. 1973, p. 142), and working backwards in this fashion is an important problem-solving technique with wide applicability (Polya, G. 1973. pp. 225-230).

The case of indirect proof

Table 3 shows very little use of indirect proof in geometry in either of the textbooks. Both do have development sections which introduce the concept of indirect proof, but neither provides more than a few examples. In the exercise sections there are only nine such items in one text and eleven in the other; in the development sections there are seven and five. This is understandable, as grade 12 is normally the first time students are exposed to this method of proof. Perhaps more attention to this kind of proof is needed even at this level, however, since indirect proof is an important method frequently used by mathematicians, and the grade twelve advanced-level course is the only opportunity in the Ontario curriculum for university-bound students to learn it.

In both these textbooks the indirect proofs with greater mathematical significance occur primarily as theorems in the development sections, though in the design of curricula and textbooks it is considered important to have each topic include enough meaningful exercise items. In M12 there are some significant exercise items involving indirect proof, such as one on triangle inequalities. The exercise items in FM12 (p. 95, p. 105) are less significant, but more likely to be within the capabilities of students doing their first indirect proofs. Finding geometrically significant indirect proofs which most grade 12 advanced mathematics students would be capable of doing is certainly a challenge, yet it is a challenge that needs to be addressed.

Opportunity to observe or do at least one proof per section

To properly teach proof and proving, there should be frequent opportunity for students to do some proving (in the exercise parts), or to observe a proof (in the development parts). To assess the opportunity provided by the textbooks, a count was taken of the number of development and exercise sections that contained at least one proof item. In the majority of Ontario schools the length of one class period is 75 minutes. Most textbook sections take up one period of class time, so that having at least one proof item among the exercises for each section provides the teacher with an opportunity to assign a proof in that particular period.

Keeping in mind that the frequency of at least one proof in a section is a rather crude measure, there was a high degree of consistency between the two textbooks in the percentage of sections providing an opportunity to study a proof (at least one development proof item) and an opportunity to create one (at least one exercise item requiring the student to do a proof).

The greatest percentage of exercise sections with at least one proof occurred, of course, in geometry, 89% in FM12 and 85% in M12. This is primarily a reflection of the *Curriculum Guideline*, which uses the terms "proof" and "deduction" most often in relation to geometry. For the other four topics, algebra, functions and relations, exponents and logarithms, and trigonometry, the percentages of sections with at least one proof range from a low of 5 to a high of 35.

In both textbooks there is some correlation between the development component and the exercise component in terms of the proportion of sections with at least one proof, indicating that in general the exercise sections do provide opportunities for students to develop and for teachers to assess the understanding gained in the development sections. The percentage of exercise sections with at least one proof item is higher than the percentage of development sections (with the percentages being closer in FM12). In neither textbook, however, do the sections with at least one proof make up much more than one third of the sections, meaning that the teacher can assign students a proof from the textbook only in about one third of the lessons.

Conclusion

The study investigated the occurrence of proof in the textbooks which dominate the grade 12 mathematics classrooms in Ontario. It revealed that these textbooks are quite finely attuned, in the context of proof, to the specifications of the provincial *Curriculum Guideline*. In

the topic of geometry, where over 80% of the exercise sections contain at least one proof, the texts also do a reasonably good job of providing students with opportunities to learn proof. The study shows that such opportunities are much rarer in the other topics.

The examination undertaken by this study could not reveal, of course, how and to what extent the teachers use the textbooks, nor to what extent students actually read them.

Acknowledgment

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We wish to thank Ebby Madera, Sharon Silver, and Nathan Sidoli for their assistance in the development of this paper.

References

Coe, R. & Ruthven, K. (1994). Proof practices and constructs of advanced mathematics students. *British Educational Research Journal*, 20(1), 42-53.

de Villiers, M. (1990). 'The role and function of proof in mathematics', *Pythagoras* 24, 17-24.

Dottori, D., Knill, G., Stewart, J., & Gadoury, G. (1987). *The Foundations of Mathematics 12 (2nd edition)*, Toronto: McGraw-Hill Ryerson Limited

Dreyfus, T. & Hadas, N. (1996). Proof as an answer to the question why. *ZDM International Reviews on Mathematical Education*, 96(1), 1-5.

Hanna, G. (1990). Some pedagogical aspects of proof. *Interchange* 21(1), 6-13.

Hanna, G. & Jahnke, N. (Eds.) (1993). Aspects of proof. *Special issue: Educational Studies in Mathematics* 24(4) 329-438.

Howson, G. (1995). *Mathematics textbooks: A comparative study of Grade 8 texts.* TIMSS monograph no. 3. Vancouver: Pacific Educational Press.

Hoyles, C. (1997). The curricular shaping of students' approaches to proof. *For the learning of mathematics,* 17(1), 7-16.

Kelly, B., Alexander, B., & Atkinson, P. (1989). *Mathematics 12.* Don Mills, Ontario: Addison-Wesley Publishers Limited.

Mariotti, M.A., Bartolini Bussi, M., Boero, P., Ferri, F., & Garuti, R. (1997). Approaching geometry theorems in contexts: From history and epistemology to cognition. In E. Pehkonen (Ed.), *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education.*

Ontario Ministry of Education. (1985). *Curriculum Guideline Mathematics Intermediate and Senior Divisions 1985 Part Three.* Toronto.

Polya, G. (1973). *How to Solve It: A New Aspect of Mathematical Method,* New Jersey: Princeton University Press.

Solow, D. (1982). *How to read and do proofs.* Toronto: John Wiley & Sons.

▲ ABOUT PROBLEM SOLVING: THE THREE OPERATION INVESTIGATION

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Recently I have been working with a variety of investigative type problems. Here I will share an investigation that I created for use with a wide mathematical audience. In fact, the problem was initially prepared for a grade 5 class. Before proceeding into the investigation itself, two problems are offered as a means of understanding the mechanics associated with the idea. These problems were originally developed for use in the Newfoundland and Labrador Teachers' Association High School Math League. They can be thought of as "two operation problems".

Two Operation Problems

1) Consider a mathematical world in which only two operations exist. You can double a number or add 1 to a number. Each step consists of carrying out one of these operations. What is the minimum number of steps required to get from the number 1 to (exactly) 100?

2) Suppose that the two available operations are adding 1 or multiplying by 3. What is the minimum number of steps to get from 1 to 1000?

Both of these problems lend themselves to a trial and error approach; however, instinctively multiplying repeatedly at first leads to many unnecessary steps of adding 1. Hence, something is to be learned from being greedy. In fact, astute problem solvers will tend to recognize the merits of working backwards. The minimum number of steps in these situations are 8 and 9 respectively. You should verify these results before proceeding further.

It seems as though the operations could be arbitrarily selected and the targeted values could be readily

changed to develop a whole new family of problems. In fact, this is the case. Rather than shifting from addition and multiplication, I simply chose to allow another form of multiplication, namely, squaring a number. Rather than focusing on a specific target, I opted to consider a broad range of values. The following investigation is framed based upon these guiding principles. The presentation of the investigation is offered here in basically the same form that it was sent to the teachers of two grade 5 classes and some other interested individuals. The three questions could be extended or modified to invite many more possibilities for mathematical consideration. I shall refer to the investigation as the "Three Operation Investigation". Darren Hutchings (a B.Ed. student) and myself have begun to formalize our findings with respect to the questions (particularly number 3) posed here.

These findings will be shared with any interested readers. Your feedback or comments on your own variations of the investigation are certainly welcomed.

THREE OPERATION INVESTIGATION

For the purpose of this investigation, three operations will be permitted:

You can add 1 to a number.

A number can be multiplied by 2

a number can be multiplied by itself.

For example, if you have the number 12, it could become 13, 24, or 144 on the next step.

There are three opening questions to consider.

1. Starting at 1, what is the minimum number of steps required to obtain a particular result? Try out different numbers. For example, 100 will no longer require 8 steps. For starters, why not consider 100, 200, 300, 400, ... , 1000?

   (You may find the multiplication facts below to be helpful. I have written out the relevant times tables for 11 x 11, 12 x 12, ..., 22 x 22).

   Larger tables will not be helpful unless the investigation is extended to consider numbers greater than 1000.

2. Consider another perspective on the situation. Here you are given a specific number of steps and the challenge is to determine all numbers that can be obtained in that exact number of steps. For example, given 1 step you could obtain only 1 and 2.

   Suppose that you were given 2 steps. Work from the results for 1 step and consider where the next step could lead. After 1 step, the number is 1 or 2. So the 1 can again go to 1 or 2; however, the 2 can become 3 or 4. Interestingly enough we have generated only 1, 2, 3, and 4 as possibilities.

   Now for a third step, begin at these places: 1, 2, 3, and 4. See if you can generate all of the possibilities after the next step. The possibilities will soon become quite large. For example, 3 can become 4, 6, or 9.

3. List all of the numbers from 100 to 200 that can be obtained in at most 10 steps. For recording purposes, write the number of steps with each entry on the list.

   For example, the following sequence could be generated by adding 1, adding 1 again, doubling, doubling again, and multiplying the number by itself: 1,2,3,6,12,144. For recording purposes, 144 would appear on the list. Likely it would make most sense to write the list as shown below:

NUMBER OF STEPS

5 6 7 8 9 10

144

Note that one advantage is the "looking back" idea. For example, if 144 requires 5 steps, and 145 is not on your list, I know that 145 could appear under 6 steps, 146 under 7, 148 under 8, and 149 under 10.

Some numbers will be attainable in various ways and hence, different numbers of steps may figure into the sequences. For practical purposes, let us say that a number will appear on the list only once and that shall be under the minimum number of steps required. Consider 128 for example. It could be obtained as 1,2,4,8,64,128 or alternatively 1,2,4,16,32,64,128 or 1,2,3,4,5,6,7,8,64,128 or ... Since one of these cases required only 5 steps, we would enter 128 in that column.

Helpful Times Tables

11 x 11 = 121	12 x 12 = 144	13 x 13 = 169	14 x 14 = 196
15 x 15 = 225	16 x 16 = 256	17 x 17 = 289	18 x 18 = 324
19 x 19 = 361	20 x 20 = 400	21 x 21 = 441	22 x 22 = 484

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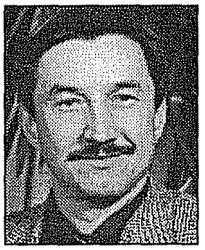
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▲ THE DEVIL'S ADVOCATE: THE QUEST FOR MEDIOCRITY



ALEXANDRU PINITILE

Not gods, nor men nor even booksellers have put up with mediocre poets. (Horace)

The word mediocrity comes from the Latin mediocritas which means average. The adjective "mediocre" is usually seen as a put down in spite of the fact that most of us are situated, in most respects, right under the bulge of the Gaussian distribution. Even before the statisticians took the trouble to quantify it, humans were aware of the trivial, run-of-the-mill character of their existence. On the other hand, our species takes pride in its dreams, legends and hopes, its appreciation of excellence, the quest for unlikely ideals. Whereas there is nothing wrong with being average, I believe that it is a shame to set mediocre goals and to worship mediocre idols.

Mathematics, just like poetry, has always been an area of human endeavour where OK is not OK and where "better" is the enemy of "good". Both involve passion, dedication and awe. If an accountant or a dentist would be excused if they do not look for beauty or the unexpected in their trades, I am appalled to see people who chose math as a profession take the approach "another polynomial, another dollar". Someone I know teaches and tutors mathematics while she proudly states that not only has she no appreciation for this subject but she has never met anyone who does.

I do not think that there are many English teachers that can write like Kipling but I expect most of them to be enraptured when they read "If" and to be excited to transmit the thrill to their students. I was speechless when I heard a future head of mathematics state that he would rather write a lesson plan for the ministry-guy than solve a geometry problem. Another math teacher confessed that he does not see any point in studying this subject and that the best way to be successful in it is to cram in some quick fix methods the night before the test.

Most of us, math teachers, do not possess extraordinary math aptitude but it is our duty to recognize and encourage talent. Honestly, I am thrilled when a student comes up with smarter methods of solving problems than the ones I put on the board. Actually I consider myself truly successful when some of the people I teach gain a better insight into a certain topic than I do. It is only normal that their nimble minds be more flexible than my brain which is too heavy with formulae and petrified by too many drills. Far too often we give tasteless and colourless tests that aim at rewarding the mediocre minds that do not know any better than how to reproduce problems we solved on the board, or memorize methods and formulae that make little sense to them. A grade 13 student I met once was telling me about being invited to Waterloo among the top 10 mathematicians in her grade, about her astronomical scores in the Euclid and about the modest marks she was getting in her math class. Tops in her math class were students who probably never heard of the Euclid or the Descartes papers, but who would not let their minds wander while solving gazillions of one or two-step exercises. When giving mindless tests, in order to avoid a huge class average, the teacher has to mark with a severity that often borders the absurd. I have heard horror stories about those who give zero marks out of five for not getting the right answer due to a simple calculation error, while giving five out of five for a right answer obtained by guessing.

I hope that I am wrong in my feeling that there are teachers that resent the outstanding student. They seem to see him as a fortunate outlier, not worth bothering with. If the majority are average or weak students, why waste time with the strong and motivated ones, right? Wrong! Gretzky sells not only hockey tickets and burgers but the game itself. How many parents wake up when it is still night to take their children to the arena for practice? Exceptional players provide motivation and a sense of excitement for millions. Exceptional math talents go often unnoticed by teachers and are lucky if their are not being called names by cool peers. In the Toronto Star's section "Young Street" one is more likely to find addresses for body piercing salons than articles on "nerdy" issues.

If gold rusts, what shall the iron do? - Canterbury Tales. (By the way, dollars to peanuts, I bet that, among high school graduates, there are more who have heard of Dennis Rodman than those who have heard of Geoffrey Chaucer.)

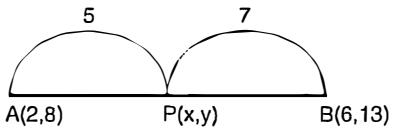
▲ Teaching Ideas from the Editor

The Remote Rule

Some years ago when I was in Grade 13 at St. Michael's college School in Toronto my teacher was Father Donald Faught. Father Faught taught us to think and to understand. He also emphasized that even the most brilliant among us could use little rules and gimmicks as memory aids. It was he who introduced me to the All Soup Turns Cold rule to help us remember that All trig functions were positive in the first quadrant, Sine was positive in the second, Tan in the third, and Cos in the fourth quadrant.

One of the rules that I have used often with my students is the Remote Rule for finding the coordinates of the point that divides a line segment in a given ratio.

Consider the segment A(2,8) and B (5, 13). We must find the coordinates of the point P that divides AB internally in the ratio 4:7



To find the x coordinate multiply the first ratio, 5, by the remote x-coordinate, 6, add to this the product of the second ratio, 7, by its remote x-coordinate, 2, then divide this result by the sum of the ratios, 5 + 7. "Remote" means the coordinate not connected physically to the segment associated with the ratio. A similar rule applies for the y-coordinate of P.

$$\text{Thus, } P=\left(\frac{5x6+7x2}{7+5},\frac{5x13+7x8}{7+5}\right)=\left(\frac{44}{12},\frac{121}{12}\right)=(3666...,10.08333...)$$

A similar rule applies for finding the Vector in 2 or 3 dimensions, where P divides a line segment AB in a given ratio.

▲ Directors' Dialogue

Louis Lim
ISOMA Chapter Rep. to OAME
Quinte S.S.

The following summarizes the OAME board meeting held on Saturday, February 13, 1999 at the Four Points Hotel in Mississauga.

Challenge

The first person who e-mails me with the correct solution wins a Math prize: "The year 1999 can be written as easily as 1, 2, 3, 4, 5". (Thanks to Stewart Craven for the neat problem.)

President's Report

President Ralph Connelly congratulated our web site coordinator, Doug Evans, on OAME's own web site **www.oame.on.ca**. It even has a counter!! Who will be visitor number 1000? Our president also reported that 6000 copies of the Primary Assessment Document have been printed. Write to the Executive Directors, Dave and Bonnie Alexander, to order your copy. We learned that a large portion of OAME's finance has gone to inventory and printing costs. It was reiterated that the Primary Assessment document, reprint of the Junior Assessment document, Scope and Sequence posters, and the technology document are not made for profit, but that money collected offset printing costs.

Executive Directors' Report

Dave Alexander reported that many chapters have had to cancel their conferences during this very difficult school year. As a result, there is clearly a decrease in membership in most chapters (with the exceptions of GOLDEN, PINE RIDGE, and COMA). Membership as of February 6th was 1164 (compared to 1397 at the same time last year). It is anticipated that the OAME'99 provincial conference, along with the many changes in both elementary and secondary levels, will generate some new members.

Committees

PROMOTIONS

John Kersley reported that the Promotions Committee is building activities to help coaches and students prepare for the Ontario Mathematics Olympiad (OMO). Included are hints on how to run the events. This committee is also working on listing the benefits of joining OAME, to be featured on our web site.

FUTURES

The Futures Committee is still looking for a sponsor to help finance OMO'2000. QSLMA is hosting the event, but it cannot run without a sponsor. Is your chapter interested in hosting OAME'2001 or a future OMO? See your chapter rep for information. Another concern of this committee has been the general trend of low teacher morale, and they brain stormed ways to combat it.

EARLY FORMATIVE YEARS

Check out the *Abacus* in this edition. You'll find great activities, taken from the Toronto District School Board's "Calendar Math 1999 – Getting Ready for Y2K", which can be done at home during the summer. In addition, you'll find an article written especially for junior level teachers on the StatCan web site (with reasons why to go there, what you'll find, and how to incorporate the data and charts in your curriculum.

TRANSITION AND SPECIALIZATION YEARS

As stated in the last *"Directors' Dialogue"* column, the transition and specialization years committees have joined forces. The original "Survival Package" focus has shifted due to the release of the Ministry course profiles. With OMCA (Ontario Mathematics Coordinators Association), OAME will produce an in-service package for grade 9 teachers.

Many of the original projects remain, but will not be sold as a complete package. For instance, the FAQs about the new curriculum will be placed on OAME's web site, the gap analysis will appear as an open-letter to teachers in the *Gazette*, and a complete two week unit (with lesson plans, BLM, assessment activities) will be available for a small fee to cover printing costs in the spring.

OAME and OMCA are also organizing in-service workshops throughout the province during the summer to assist teachers in implementing the new grade 9 curriculum. Check the OAME web site for further information.

PROFESSIONAL DEVELOPMENT

OAME'99

By the time you read this, OAME'99 is history. I have witnessed the incredible amount of dedication that the organizers and committees have worked to make such a conference a reality. We are indeed fortunate to have hard-working people work on our behalf to help us develop professionally.

OAME'2000

Peter Lenczewski reported, on behalf of COMA, that the theme of OAME'2000 is "ART of Mathematics – Assessment, Reporting, and Technology". Unlike previous provincial conferences, there will be half and whole day workshops geared to learning and practising certain skills (e.g., a particular computer software). Participants can stay at the University of Ottawa residence, and can enjoy the Tulip Festival running concurrently to the conference when not attending sessions.

LEADERSHIP 98/99

Executive Director Bonnie Alexander reported that there was an overwhelming response to the "Conquering the Challenge of Change" theme – more than 40 people on the waiting list! The trend during the past several years has been an increase each year on the waiting list.

LEADERSHIP 99/00

The next leadership conference is scheduled for February 24-26, Y2K. The organizers will need to decide whether to increase the number of sessions offered to avoid the growing trend of having to turn away teachers. Peter Saarimaki stated that a keynote who is able to look into the future of education, but not necessarily focus entirely on Mathematics, is being sought. UNESCO has declared the year 2000 "World Mathematics Year," and this conference will help prepare participants for the new millennium.

OMO'99

Again, when you read this, another OAME event has occurred. Stewart Craven distributed practice materials to each chapter rep to be given to the coaches. There are separate "school" teams and "all-star" teams to ensure fairness. The evening before the OMO, all teams met at York University to prepare for the technology portion of the event. Forty teacher candidates from York were trained to evaluate the solutions holistically using rubrics. The winning teams were supplied with Texas Instrument graphing calculators!!

CHAPTER REPORTS

COMA

COMA hosted a very successful mini-conference with 140 teachers attending 11 workshops on a miserable, rainy night (remember the awful weather back in January?). They are busy preparing to host OAME '2000 with the theme "The ART of Mathematics" (assessment, reporting, and technology). Peter Lenczewski reported

that there will be half day sessions on using technology to immerse participants in a skill.

GOLDEN

This chapter conducted workshops for both elementary and secondary levels. Elementary level teachers were provided with classroom tested ideas and activities for implementing the new K-8 curriculum. The secondary level focussed on implementing the upcoming grade 9 curriculum.

GVMA

Check out their frequently updated web site at **http://home.golden.net/~gvma**. They had excellent turnouts at both their fall and spring conferences. GVMA's new grade 12 advanced exam publication is now ready for sale. See their ad elsewhere in the *Gazette.*

ISOMA

ISOMA hosted an information technology workshop for its members in April. A major initiative has been compiling a list serve to include announcements of upcoming events, networks with other teachers to share teaching strategies and pedagogy, and communication for executive meetings.

MAC²

Bill Morrison reported that MAC² will be setting up a mentoring system via their soon-to-be running web site to assist new teachers. Their conference in April focussed on secondary school reform, with guest speaker Dr. Norma Henchey, a member of the Steering Committee of the cross Canada Exemplary Secondary Schools Project.

QSLMA

QSLMA hosted an elementary workshop for teachers in January focussing on grade 6 testing, and a spring conference in April. This chapter is busy preparing to host the OMO'2000. They are also in the process of instituting a book prize to be awarded to the top student graduating with three OAC's and pursuing a mathematics related career.

SAME

The SAME chapter hosted a very successful conference, "The New Curriculum — Survival Skills" in February with more than 100 participants. They will be sending two teams to OMO'99. Any SAME member interested in serving as counsellor should contact President Alan Chan.

SWOAME

Frank Di Pietro reported that SWOAME hosted "The Dinnerlude" which assisted its members to think about the upcoming grade 9 curriculum. They also celebrated "Math Awareness Week" during February 22 to 26, sponsored two teams to OMO'99, and hosted "Partners in Problem Solving".

TE²Y²MS

Most of this chapter's energy has focussed on putting the final touches to hosting the OMO'99 at York University. Committees working around the clock include hospitality, facilities, volunteers, registration, awards, curriculum writing, communications, and technical support.

WOMA

Mary Howe reported that WOMA hosted a series of four PD after-school sessions in March and April called "Grade 9 in '99'". The workshops covered changes in curriculum, teaching approaches, technology, and assessment to help implement the new grade 9 curriculum.

Y⁴MA

This chapter is to be congratulated for having hosted the annual provincial OAME conference for the second time this decade. They even had the energy to host their chapter math Olympics and send several teams to OMO!

As I put the final touches to this edition of the *"Directors' Dialogue"*, I enjoyed a day off from work as all schools in Belleville were closed due to an incredible snow storm (March 3) of almost 20 cm of snow. If teacher morale is low in your school, you might want to share the following quote which is displayed proudly in my living room:

The Essence of Teaching

"What nobler profession than to touch the next generation – to see children hold your understanding in their eyes, your hope in their lives, your world in their hands. In their success you find your own and so to them you give your all." (author unknown)

I wish you a relaxing and enjoyable summer. See you in September!

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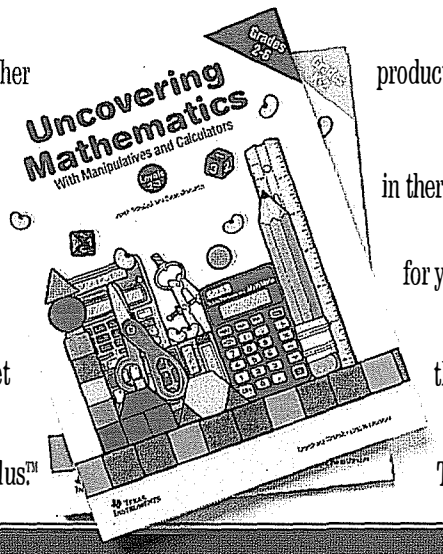
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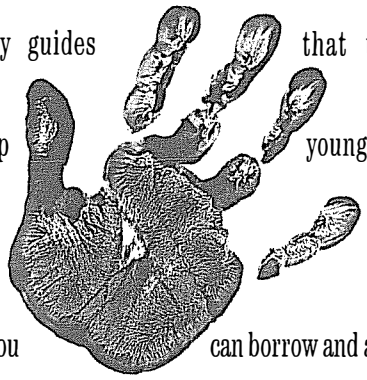
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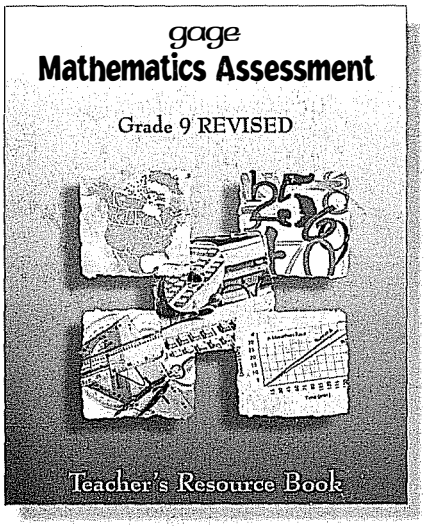
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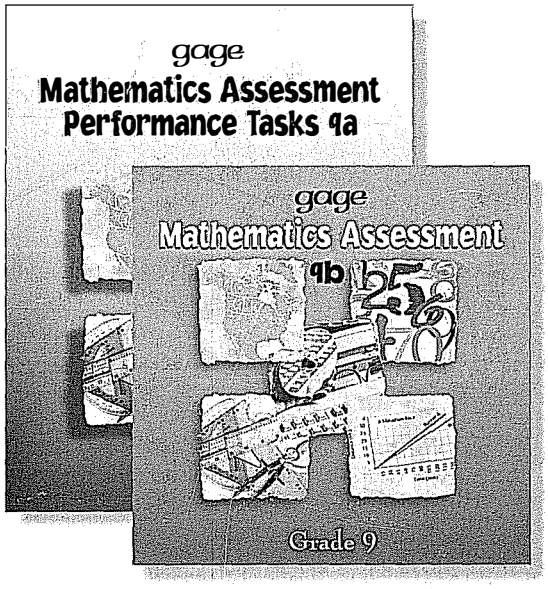
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