Vol. 38 #1, September 1999

Ontario Mathematics Gazette

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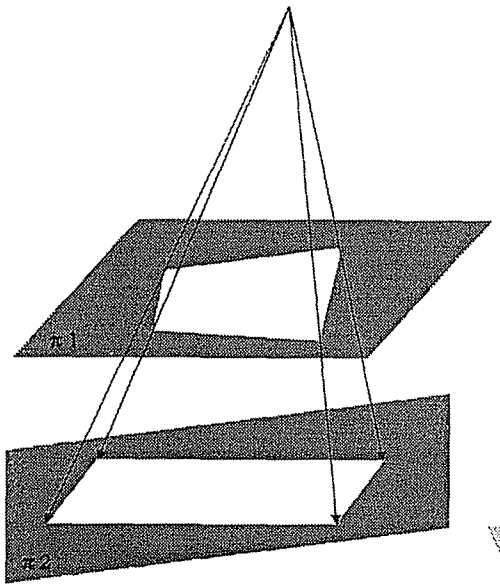
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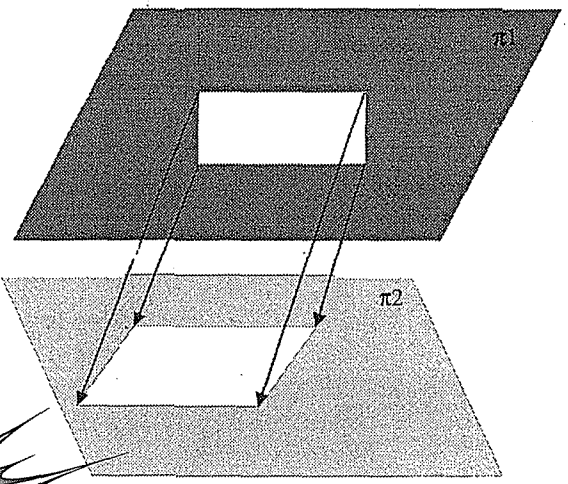
Sunlight, Shadows and Linear Transformations

Announcing the Three Best Kept Secrets in Math Resources: Music, Video, and the Equation Editor

Extended Pythagorean Theorem Meets Geometer's Sketchpad

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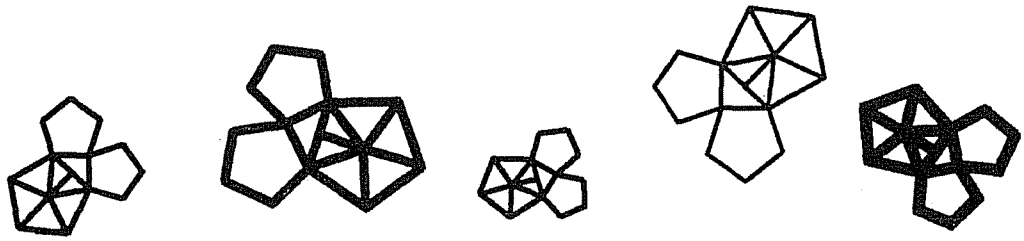




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"by hand?"
(The old way)

Final Exam

1. State the Domain of $y = \frac{x-10}{\sqrt{x^2-16}}$.

2. Which relation is the inverse of y=(

3. Use a half-angle identity to find the e

a) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ b) $-\frac{1}{2}\sqrt{2-\sqrt{2}}$ c)

4. If $f(x)=2x-5$ and $g(x)=$ fun

a) $\frac{-(5x+11)}{x+3}$ b)

5. If $\sin\theta = -\frac{1}{2}$ and θ is

a) $\frac{-\sqrt{3}}{3}$ b) $\frac{\sqrt{3}}{3}$

6. Given $f(x) = \frac{x^2}{x+3}$ and

a) $\frac{4x^2-2x}{x+3}$ b)

"struggling with a word processor?"
(The hard way)

Final Exam

1. State the Domain of $y=\frac{x-10}{\sqrt{x^2-16}}$.

2. Which relation is the inverse of $y=(x-2)^3$?

a) $y=\sqrt[3]{x-2}$ b) $y=\sqrt[3]{x}-2$ c) $y=\sqrt[3]{x+2}$ d) $\sqrt[3]{x}+2$

3. Use a half-angle identity to find the exact value of cos 67.5°:

a) $1/2\sqrt{2-\sqrt{2}}$ b) $-1/2\sqrt{2-\sqrt{2}}$ c) $-1/2\sqrt{2+\sqrt{2}}$ d) $\sqrt{3+2\sqrt{2}}$

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FINAL EXAM

1. State the Domain of $y=\frac{x-10}{\sqrt{x^2-16}}$.

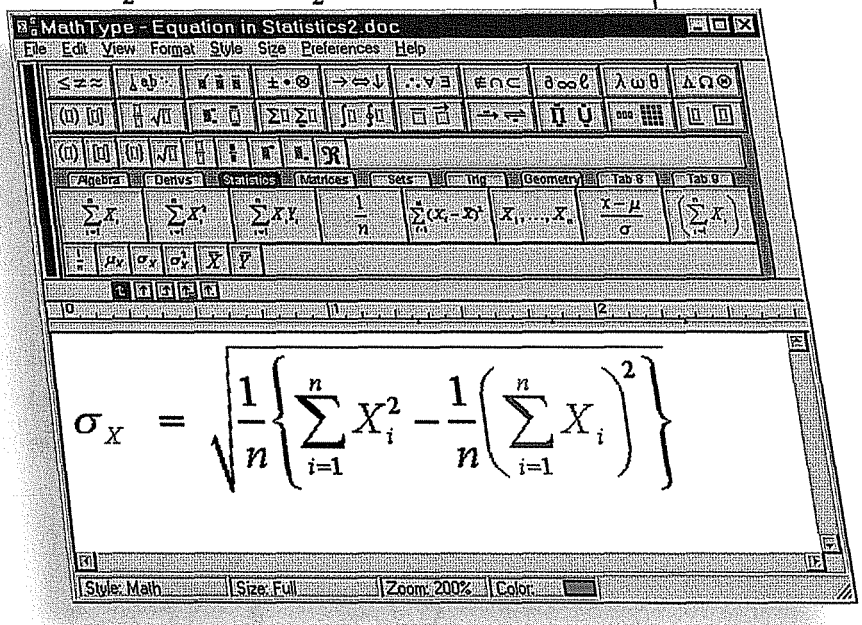
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▲ Guest Editorial

JACK WEINER



In my June editorial, I lamented the sad state of OAME membership and asked for your advice. OAME is doing an important job and doing it well. How can we convince our mathematics teaching colleagues to join us, first in support, and then in helping us accomplish our tasks even better?

Kevin Spry, head of mathematics at Centre Dufferin District High School in Shelbourne, offered some thoughtful comments.

"In talking with other math teachers about joining OAME, I have found that some are not members because they do not see the full benefits of membership. Many math teachers see the *Gazette* and discounts for attending the annual conference and chapter conferences as not worth the $40. Often this is because the *Gazette* is shared around the department. Also, the school boards often pay the full registration cost. So by spending $40 to be a member they save the board $35. I do not know how true it is for all boards but in the case of someone in my department, the Professional Development fund would pay the full registration cost for a non-member, but not the cost of registration and membership—it was $5 more! Apparently conferences could be covered but not memberships!

"Hence, rightly or wrongly, it was remarked, 'Why pay money to save the board money?' Here is a suggestion: why not charge the full conference fee for both members and non-members? Hence the board pays the full registration cost. But as an incentive for people to be members of OAME, offer a 'rebate' to members who pay their $40 membership fee and attend the conference...

For example, I believe the conference costs about $135 for members and $170 for non-members...Instead charge $170 for all (the boards paying for many people) and have OAME provide a $35 gift certificate for members who have paid $40 and attend the conference. It could be used to offset conference fees if the attendee paid his/her own way or to purchase items from vendors...

"This might be an inducement for people to be members and to attend the conference."

Kevin goes on to suggest some chapters reorganize along board lines (possibly, two or more boards together) to encourage more participation by all educators at a local

level. This way, he argues, people can have the opportunity for grassroots support of OAME at a truly local level and feel more "ownership" of the organization. As it stands, Kevin's department maintains ties to GVMA but teachers have to travel 1.5 hours each way to attend the semi-annual conferences.

The amazing Frances Schatz (the Abacus, NCTM rep, Descartes medalist, and more, more, more!!) commented that we once again need to bring more elementary teachers into OAME. She urges school memberships should be available and widely publicized.

Could OAME set up an alliance with STAO (the science teachers' association) and similar organizations in other subject areas? As a group, a package membership in all the organizations could be offered at a special price. Within each school, one teacher could act as the official OAME member, another as the STAO member, and so on. Each of these teachers would share the resources from his/her organization with colleagues and, ideally, all classrooms would then reap the benefits of all the subject associations.

Good ideas. Here's my favourite. Let's set up a partnership with a major chain, such as Business Depot or Radio Shack. To keep things hypothetical, let's say for the moment we have negotiated an alliance with Radio Depot.

"Radio Depot and OAME
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The payoff for us: Radio Depot agrees that the $40 membership can be used as a voucher for any purchase (sales items included–let's not be cheap) of $200 or more. The payoff for Radio Depot: the chain, very publicly, is seen to be supporting a great cause. Moreover, every teacher, annually, has a reason to make a "Radio Depot kind of product" purchase. So membership will pay for itself, and Radio Depot gets a sale that might otherwise have gone to Business Shack.

In fact, this would work even better if we formed the Radio Depot alliance together with our sibling provincial mathematics associations.

Lot's of great stuff in this issue. Guest editing has been fun but I gladly hand the reigns over to Todd Romiens and the incredibly enthusiastic, energetic, talented and young Louis Lim. (Three of these adjectives apply as well to Todd, but you already knew that.) Good luck to you both. I know the *Gazette* will prosper under your leadership.

Hey, this is the year. Glue down your keyboards and hang on to those mice. See you almost halfway through Y2K in Ottawa.

Best wishes,



▲ Letter to the Editor

Comments on the March 1999 *Gazette* Vol 37 #3

I agree with Ralph Connelly's notes in the March 1999 *Gazette.* Referring to the "back to basics" movement in California he says, "It would be a very big mistake to view this event as "a California problem."

When I worked as a curriculum coordinator between 1993 and 1998, my superintendent passed on tonnes of paper that had come to him from parents who were intently following, via the internet, the arguments in California. I cynically assured him that we could surely not be directed "back to the basics" because in Ontario Mathematics education had never left the basics.

I heard the criticism from California, but I went farther. I looked in students' notebooks. I saw that those who did well on standardized tests, in our board, were doing mathematics with manipulatives, discussing, and dialoguing in order to reach a deeper understanding of their work — which they then could explain. And they were also doing some practice questions. Those who were doing badly on the standardized tests were, on a daily basis, instructed to open their texts to a given page and do questions 1 to 28, with no dialogue and no interaction with teachers or among themselves. They were also told to leave out anything marked "Problem Solving" and leave out any questions involving the use of a calculator. As a kind of evaluation, the solutions were marked with a "✔" or an "✗" and given a numerical value. Students then moved on to the next set of questions — feeling like winners or losers because of the marks attached to their work, but not feeling in control of changing it.

This last scenario is the one I imagine when I hear the "back to basics" people talk. This is not rich education; nor is it necessary education. Further, it is harmful. That kind of mathematics education is what has made it socially acceptable, in our culture, to hate mathematics.

If Matthew Robinson (*Gazette*, page 25, Vol. 37 #3, March 1999) had asked me if I objected to having students memorize material, I too may have said yes depending, of course, on the context. I believe that there is definite value in memorizing the times tables and number stories of, say, 18 — but I do not believe that doing so is mathematics. And I do not believe that it can be the whole or even the main focus of a mathematics

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program. Still, in a rich mathematics program, I do not object to practice that leads to routinization. But drill in place of understanding does no one any good.

Similarly, education that allows students simply to "discover" is not enough either. The new elementary mathematics programs are successful when teachers allow students time to carry out an inquiry, explore new concepts, and then work with those concepts to model and formulate ideas, and to manipulate and transform solutions. The class of students has discussions along with the teacher about inferences that can be drawn, and about the similarities among solutions. Conclusions and communication used to summarize and even generalize solutions allow students to demonstrate real understanding of mathematical ideas. If there is appropriate numeric or algebraic practice that goes along with presenting a precise solution, then there is also time made for the related practice. Importantly, practice is not done in isolation.

My reading of the Stigler research that followed TIMSS is completely the opposite of Robinson's. He says, "the average Japanese student gets instruction 90% of the time spent in the classroom. American kids get it only 46% of the time." I saw the transcripts and watched the video. I saw Japanese students struggle, in small groups, with a problem, discuss the issues in order to come to understand the problem more deeply, present ideas to classmates, work in small groups to refine their solutions, and discuss as a whole group all the parts of the problem and its solution. If that represents "getting instruction for 90% of classtime", I am for it!

On the other hand, the 46% of instructional time for American students showed the teacher standing at the front of the room, solving a problem or two on the blackboard, and showing approaches for students to mimic. I then saw students doing 20-30 similar questions for homework during the other 54% of the time. If this is what is meant by "discovery learning", I am against it! Robinson and I need to get together. We may be on the same side.

On the OAME web site Geoff Roulet, Gord Cooke, and Louis Lim have published an article about the new Grade 9 and 10 Mathematics curriculum called *"Something Old, Something New"*. In pointing out the new ways of teaching dictated in the guidelines the article says, "students will be actively involved in their lessons. They will pose problems, formulate hypotheses, design and carry out experiments, describe situations that could be modeled by given graphs and equations, and develop skills through problem solving in a variety of contexts." This does not seem like 90% teacher directed instruction. I do suggest that anyone looking for images of teachers doing what is right for students, read the Stigler reports (on the NCTM website at www.nctm.org), read the transcripts of the lessons, and read *"The Open Ended Approach"*, another new NCTM publication about teaching in Japan.

I refer as well to the Glenda Lappan article (*Gazette*, page 27, Vol. 37 #3, March 1999). Lappan says, "Although content is a necessary and absolutely essential ingredient, it alone does not guarantee a high-quality program. We know that teaching shapes and influences what students learn. Teaching matters." I agree whole-heartedly and believe that those are the two foundations upon which the movement to improve mathematics education in Ontario has built its platform. The new curriculum describes a mathematics program that is driven by the problems, the investigations, and the inquiries in which teachers invite students to engage. The content that is learned following the dissonance that has been created by the search for understanding is taught because there is a reason to learn it — to close a gap. Mathematics teachers become the directors of learning in much the same way music teachers direct a band or choir. They choose a repertoire and conduct the group through a learning process, focusing on different groups for different practices, until the students can play together and each has learned what was necessary to contribute.

The *"Something Old, Something New"* article points out, in the second paragraph, that for experienced math teachers, there is little change in mathematical content. I fear that readers will conclude that there is no reason to read further. I think the authors really are saying that, although there will be little new mathematics content for teachers to learn, there is a great deal to be learned about style of instruction and centre of control in the classroom. The message cannot be that the new curriculum will allow teachers to preserve the good old mathematics.

A friend's daughter attends a high school in southern Ontario. Judged by success in the Waterloo Mathematics Contests the mathematics department in this school would be considered one of the best in the board and in the province. Teams from this school always have a place on the top ten lists. This is not to be sneered at because, for the 100 or so students who participate in these contests, there are valuable lessons. However there are 2000 students in the school. My friend's adolescent daughter, in 1999, is doing drill and practice on mathematics content that was not even in the 1985 curriculum. Her grade 10 mathematics notes are filled

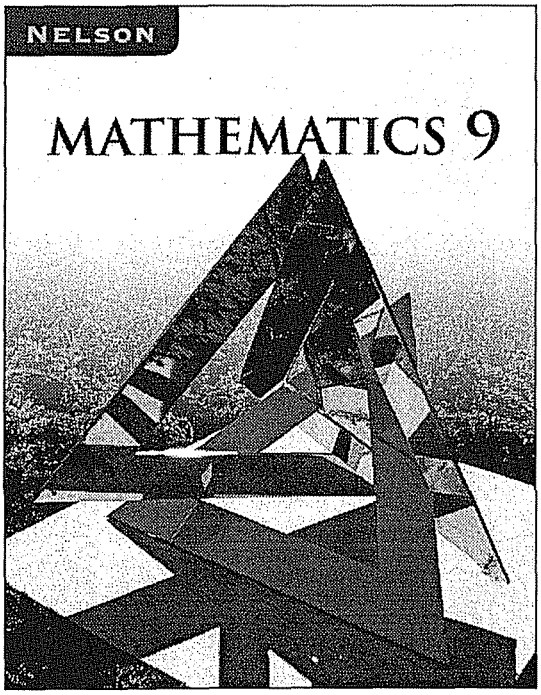
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with paper and pencil responses to questions from textbook pages. She has rationalized the denominator in 30 to 60 questions, but has no idea of the value or comparison of an answer of $\sqrt{2}$ or 1.4142135... . She has no appreciation of the factors she has been finding for the 75-150 factoring questions she has done on a similar number of specially created polynomials. This has nothing to do with her world, expanding her knowledge, or exciting her heart to learn. She feels lost in a desert — no oasis in sight. There is not one example of a rich problem in her notes.

My friend's daughter is intrigued by the big ideas in mathematics but argues that her biggest challenge is to guess what the teacher wants, because many arbitrary rules need to be met each day — none having to do with any big ideas. From what the teenager measures, she has 90% instructional time and the rest is just busy work. There is no time to discuss and come to understand, and she doesn't believe that these things are important, or even possible, in mathematics. My friend's daughter plans to be an investment banker and her mother called me to help her try to convince her daughter to continue to study mathematics. All I could say was "you'll need it in the future." Her comeback: "I'll learn it then." I have not yet succeeded in convincing this articulate young woman to stay in mathematics.

Mathematics education deserves better instruction, better program and more appreciation. It needs to be taught like an art. Yes, there needs to be some practice and perfection-making time. But there also needs to be some romance, some enticement to learn. The new secondary curriculum in the province of Ontario deserves to be implemented in its entirety. It was written by our colleagues in a heroic effort. The curriculum requires a change in belief. It requires mathematics teachers to believe that there is more to mathematics education than "mastering the basics" and it requires them to allow and encourage exploration and investigation into the way people work when they are thinking mathematically. It requires that students get to play the game as well as do the drills — or play the music as well as practise on the instrument. I, for one, desperately hope we can meet the challenge.

Capacity building will be the key. We all have to adapt an open and committed attitude to excellence, to teaching more mathematical thinking to more people and to lighting the lamp of mathematical knowledge in all. It may start with a little less attention paid to proper form and notation but it will not end there. It will end in a glorious role for mathematics in the future of all Ontario students. Please join the movement "beyond the basics"!

▲ President's Message



PETER SAARIMAKI

The theme this issue is "correspondence"1 as a form of communication. Through the following anecdotes I would like to encourage everyone to consider themselves a part of OAME and at the same time point out items of interest and/or concern to the organization and its members.

1. The first example concerns a beautifully hand-written letter (and with electronic tools these days we see these all too infrequently) from an elementary teacher who is concerned that all of OAME's time, talents and energy are being funneled into secondary issues. The message seemed to be "don't forget elementary teachers." Consider also the parallel concern to "make sure there is sufficient elementary representation in OAME."

The Board of Directors for many years has anguished over the small numbers of elementary teachers who are members of OAME and the small number who become Directors. Neither fact has dissuaded the Board from pursuing elementary issues. OAME has produced constructive, rational, and practiced responses to Ministry discussion papers incorporating the experiences and expertise of classroom teachers.

Likewise in the past few years, OAME has produced both the Junior and Primary Linking Assessment and Instruction resource documents by working in conjunction with OAJE and CAPE. The point being that, while we look for ways to entice/encourage/enthrall elementary teachers into OAME, the Directors do not forget the OAME mandate to deal with ALL years of the curriculum and will go to many lengths to produce relevant support for teachers.

The teacher's concerns were shared with OAME's two elementary committees and informed their plans for the year. My response to that elementary teacher included OAME's actions and an encouragement to become personally involved in OAME at the local level and provincially.

2. A second concern came via an email regarding our secondary activities. This time it was not that we

weren't doing enough but rather than we were letting others do our work for us. The issue centered on the Grade 9 Summer Institutes that were run across the province. The writer (are emailers "writers", "typers" or "keyboarders"?) was upset that OAME would be seen to be promoting a commercial venture. The writer thought the Summer Institutes were really T3 (Teachers Teaching with Technology) Institutes sponsored by TI (Texas Instruments).

In actual fact, although TI is a major sponsor of OAME activities including the Summer Institutes, the workshops were specifically designed to focus on the new Ontario Grade 9 curriculum. They were advertised and run with an OAME/OMCA focus and utilized monies from other sponsors, as well as the Ministry of Education.

"Optics" is an important part of organizations these days, and especially to OAME. We ARE concerned with our image. We DO want to ensure that educators are aware of what OAME can do for them. We WANT to be recognized for the work we ARE doing and not have people assume that someone else must be doing all the work. BTW, the Summer Institutes were a resounding success and have enabled us to achieve our goal of providing assistance to schools all across2 the province, and not just in areas that are fortunate enough to still have consultant support.

3. My third example came by FAX. It was from the OTF and addressed to "federation associations." That rang the alarm bells of several executive types who saw it. Coincidentally, at the same time, I was attending the Canadian NCTM Leadership conference with delegates from provincial math associations from BC to the Maritimes. I realized that Ontario's position is unique in that we are NOT a subunit of the teacher's union, nor the government.

In BC, NB and other provinces, teachers have a portion of their federation/union dues assigned to a subject association of their choice. In that way, every teacher belongs to at least one group, although the math groups are still struggling to encourage elementary teachers to see the light and forgo the language association every few years.

In Ontario we're autonomous from the Federation, but that means our only real link is through the OTF Curriculum Forum which discusses issues and makes recommendations, which most often seem to go nowhere. We are at the whim of OTF's budget makers. Individual teachers need to be encouraging their federation reps to support the work of the Curriculum Forum. Together, you can help support OAME's projects.

4. My last example was a phone call from a Ministry representative. Somewhat typically, it was a call at 9:30 a.m. announcing a meeting for 1:00 p.m. the same day! After decades of close work on many provincial documents, it still seems the time-lines of Ministry jobs and projects are unreasonably short.

Other recent examples include all the time-lines for curriculum responses, where teachers often had only days to read, discuss and submit comments to our official responders. This sounds like a "good news, bad news" joke (but I'll leave it to you to remember your own best): the short times are "bad" but the continued involvement is "good." Our saving grace is still the fact that OAME continues to be pro-active and is therefore ready for the last-minute calls for opinions and suggestions.

BTW, the Ministry meeting was to see about forming a partnership of groups all concerned with curriculum development and implementation. Other participants included reps from Curriculum Consortia, Federations, other subjects, and various project groups at the Ministry. A concern I had was that the Ministry just wants to "use" us and then be able to say "this or that document was developed WITH OAME." Time will tell as we will continue to dialogue and move mathematics forward on the provincial agenda.

These examples have not included the people who personally accost me in the streets (thankfully nonexistent so far), but they do serve to point out the range of opportunities that exist to make your concerns, interests and suggestions known to the organization. Please continue to communicate or correspond (on a one-to-one basis). Take the time NOW to "write" me about things that concern you.

Cheers, Pierre.

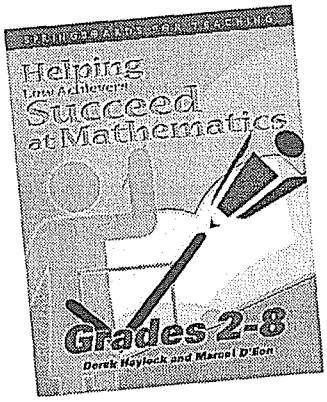
1 i.e., the "I have something to say to you" type, not "the function such that for every element of one set there is a unique element of another set".

2 A total of 14 Institutes provided enough spaces to accommodate teachers from almost a half of the publically funded schools in Ontario.

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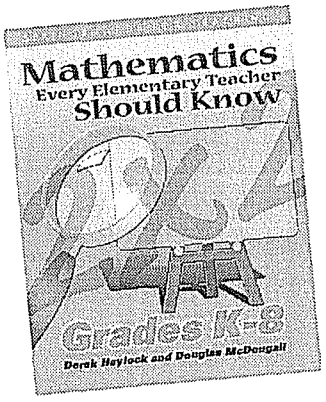
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▲ Reflections on Doing Mathematics

TOM JENKYNS AND ERIC MULLER
DEPARTMENT OF MATHEMATICS BROCK UNIVERSITY



Eric Muller is a Professor of Mathematics at Brock University with a long history of deep involvement in Mathematics Education at all levels. His Excellence in Teaching and Contributions to Mathematics Education have been recognized by several awards and honours. He has served the OAME in many capacities and is director of the concurrent B.Sc./BEd program at Brock. He holds a B.Sc. and M.Sc. from Natal University and a Ph.D. (in Theoretical Physics) from Sheffield University.



Tom Jenkyns has taught Mathematics and Computer Science at Brock for several years. His Ph.D. is from the University of Waterloo and his research is primarily in Combinatorics and Optimization. Tom regards Eric as a master teacher and a teaching master.

Introduction

Every decade or so Ontario goes through a review of the mathematics school program. We may disagree with the political motivations of such reviews, nevertheless they do offer an opportunity to improve the teaching and learning of mathematics. At the core of such improvements is an understanding of the subject, mathematics, and how people do mathematics. One of us (EM) is involved with the Brock concurrent B.Sc./B.Ed. program that aims to prepare future J/I teachers with a solid background in and an enthusiasm for mathematics and the sciences. In a first exercise of a third year course that looks at issues in the teaching and learning of mathematics, students are asked to reflect on what is mathematics and what it is to do mathematics. Many students find this a frustrating exercise. They have been involved with mathematics for most of their lives and yet have rarely reflected on the nature of the discipline and how they have succeeded in it. Mathematics is a human activity, and as mathematics teachers we are responsible to apprentice young minds in our art. We believe that it is time for us to lead our students more explicitly in the ways we do mathematics. This article therefore looks at how we (Eric and Tom) do mathematics as we build on an example taken from two *Gazette* articles (ref. 1 and 2). We sincerely hope that as you read this article you will reflect on how you do mathematics and that you will communicate this to your students. To awaken the imagination, we build on parallels between the activities of doing mathematics and of climbing mountains – both are creative and challenging. Not everyone may reach the top, but most people can get some enjoyment from the activity, even if it means staying to paths on the lower slopes of the mountain.

Raising Questions

Looking back at our childhood and teenage years we recall that our teachers and parents were particularly successful in encouraging us to question. In mathematics raising questions is fundamental not only in perking our initial interest but also at every stage of our attempts to solve a problem. The mathematical question that attracted our attention was posed by John Grant McLoughlin (ref. 1). *How many distinct triangles can be constructed with sides of integral length and specified perimeter? For example, given a perimeter of length 7 there are only two possible triangles: those with sides of (1,3,3) and (2,2,3).* This question also perked the interest of Helmut Paddags (ref. 2) who turned it into a valuable classroom experience. What were the factors that attracted us to this particular question? We had a sense that we could make some headway with the problem, it was a challenge but it did not appear overwhelming. We made connections to problems that appeared to have similar structure, we had a feeling that there was something significant underlying this problem, and certainly the problem just looked like fun. How can we get our students involved in mathematics? We have found, that for the majority of our students, we have to be quite explicit about all those factors which help us to get involved in and make progress in mathematics and with a specific problem.

Preparation

Even for the simplest of climbs, an experienced mountain climber does some preparation; boots, good socks, hat, water, chocolate, clothes to keep one dry and some to keep one warm, and so on. Through research and experience the climber will select equipment that

takes into account safety, the nature of the climb and the constraint of weight. A considerable amount of mental equipment is also brought to the climb. This is built up through previous experiences of similar climbs and from knowledge passed on by other climbers. During the climb the person must be vigilant and able to adapt and respond to unexpected situations, whether it be from the geography of the mountain, or from a turn of the weather. Tackling a problem in mathematics also requires preparation. The physical equipment of pencil, paper, calculator and computer is obvious. It is much more difficult to determine what mental equipment will be needed to tackle a particular problem. This comes mainly from experience. When we first considered this problem, we looked to our experiences with various types of triangles and to our extensive experience at pattern recognition. We also turned to our ability to introduce notation and to our knowledge of discrete functions including sequences and series. With this equipment we felt we were ready to tackle the problem.

Unfortunately, having the tools without experience as to when and how to use them is unlikely to get us very far, either in mountain climbing or in doing mathematics. As teachers we are faced with a real balancing act, between the student's development of specific mathematical skills (knowledge of the tools of mountain climbing) and their exposure to problems where they are likely to be used (experience on different climbs). What makes this preparation more complex is the fact that different routes up the mountain may expose very different challenges. Similarly in mathematics individuals may approach a problem in different ways and may be equally successful at solving it. We were to find out later that this was the case for this problem.

It is time to turn our attention to the problem at hand. The initial goal for our climb, the peak we were aiming for, was to prove that there were no other cases than the ones developed by Paddags and his class.

Exploration and Alternative Approaches

We repeated much of what Paddags and his class had accomplished; it was necessary for us to experience this exploration. We were fascinated by the results suggested by their work; our exploration raised for us new questions that suggested alternative approaches, though many led to impasses. The fact that someone else had already started up the mountain did not dampen the zeal for us to do it ourselves, in fact, it made it more interesting. There may be an alternative path, we may recognize patterns not spotted by those who have done it before. We needed to develop an ownership of the problem and we needed to get different perspectives on the problem. We used numerical, geometrical and algebraic skills. Doing mathematics does require an ability to move among representations and to learn from the insights that each new representation may bring. New opportunities arose when we generalized the problem and looked at four kinds of triangular triples with integer sides a, b, and c and a given perimeter n, $M=\{(a,b,c) : 0 \le a \le b \le c \le a+b\}$, the set of all triangular triples. This set contains all triangles including degenerate ones (those with zero area), for example (0,0,0) and (2,2,4). By removing degenerate triangles from M we obtain $N=\{(a,b,c) : 1 \le a \le b \le c < a+b\}$, the set of all non-degenerate triangular triples, considered in the original problem.

From the set M we select the set of all scalene triangular triples $O=\{(a,b,c) : 1 \le a < b < c \le a+b\}$, whose sides are all of different lengths including, for example (2,3,4), but also including some that are degenerate, for example (1,2,3). By removing the degenerate triangles from O we obtain $P=\{(a,b,c) : 1 \ a < b < c < a+b\}$, the set of all triangular triples which are both scalene and non-degenerate.

We let $T_\alpha(n)$ be the number of each of the four types of triples (α is M, N, O, or P) with perimeter length n. We found the striking repetitions of pattern

n	1	2	3	4	5	6	7	8	9	10	11	12	13
T_M	0	1	1	2	1	3	2	4	3	5	4	7	5
T_N	0	0	1	0	1	1	2	1	3	2	4	3	5
T_O	0	0	0	0	0	1	0	1	1	2	1	3	2
T_P	0	0	0	0	0	0	0	0	1	0	1	1	2

Why bother looking at related questions? If we take the time to study nearby mountains we may find one which is joined by a col to the one we are planning to climb, and this could provide easier access to the top. Indeed we did find, after careful reflection, that one of the related problems provided a simpler proof to the problem we were looking at. The exploration also provided a valuable check for the sequences that we had generated. We realized that we could easily compute the number of triangles that are both degenerate and non-scalene. They are few and they have a repeating pattern, because, for n odd there are none, and for n even there are either two or one. This comes about because, if 4 is a factor of n there are two, for example if n=12 we have (0,6,6) and (3,3,6), and when 4 is not a factor, for example if n=10 we have only (0,5,5). This

sequence is obtained by computing $T_Q = T_M - T_N - T_O + T_P$. We are reassured to find what we expected

T_Q 0 1 0 2 0 1 0 2 0 1 0 2 0

This new and interesting correlation between sequences did not provide us with a new route to achieving our goal. However we did gain new insights from exploring the number in each of the subsets of the various types of triangles. We found that a small thing like choosing different orders to list the triangles suggested different possible patterns.

Patterns, patterns, patterns, mathematics is sometimes called the science of patterns[1]. That observation is useful only if we understand what a science is. In a science the situation is sufficiently well specified and detailed that an experiment can be repeated and the results of all such repetitions can be predicted to agree to within a specified level of error. A certain level of confidence is developed and the results are accepted as scientifically true. In other words the probability that someone will obtain a different set of results under the same conditions is sufficiently small that it is set to zero. Mathematics on the other hand seeks to demonstrate that the probability is zero, not just small but actually zero. There is no possibility of getting a different result. This is the, often elusive, mathematical proof. Paddags and his class obtained a set of six conjectures, and they have reached the final step in a science: they have convinced us that all the cases they could think of were covered and satisfied the conjectures. But are there any outstanding cases? We still have some climbing to do before we get to the top of the mountain.

Reaching a Peak - the Mathematical Proof

For the great majority of people, a few hours on the slopes observing and enjoying the change of scenery is all that they are prepared to undertake or are able to do and enjoy. Serious mountain climbers have the more ambitious goal of reaching a peak. In mathematics education, students should be exposed to situations where they will find satisfaction exploring mathematical ideas, developing conjectures and achieving mastery of mathematical techniques. Students with the capacity, interest and tenacity to move from the conjectures to proofs should be challenged to reach for the peak and be rewarded by the exhilaration of success.

We urge you to read the beautifully illustrated book by Keith Devlin called Mathematics - The Science of Patterns (ref. 6)

At the time when we were working on this final assault of the mountain of integer triples we were not aware that two other groups of climbers (ref. 3, and 4) had already reached the top. They had started from different base camps and had chosen separate routes. We would like to share with you how they got there.

The two other groups of climbers noticed a rather natural connection between the geometrical problem and an equivalent one of partitioning integers into parts. Here is a brief summary of their route:

A standard piece of equipment mathematicians turn to is notations and symbols. We introduce the triples (a,b,c) that define non-degenerate triangles with integer perimeter n. Without loss of generality we order the sides so that $a \le b \le c$. The authors (ref. 3 and 4) change to a different mathematical representation; they move from geometry to number theory, and use partitions of the number n (see ref. 5) as their climbing tool. As examples both (3,3,4) and (1,2,7) are partitions of the number 10. The first partition can be used to represent the sides of a non-degenerate triangle while the second partition cannot be used that way as it fails the triangle inequality ($a+b > c$). Partitions provided for the other climbers a first clue for a possible path to the top. They observed that the number, $T_N(n)$, of triangles with integer perimeter n is equal to the number of partitions of n into three parts, $p_3(n)$, minus the number of cases that do not satisfy the triangle inequality. Since $p_3(n)$ was well known to them, what they had to do was to determine the number of cases that do not satisfy the triangle inequality. But is this simpler than determining $T_N(n)$ directly? These climbers were really skilled in the use of their partitioning equipment, and used it again to search for those partitions that fail the triangle inequality. They switched back to the geometrical representation and noted that the triangle inequality fails when the sum, $j = a + b$, of the two smallest sides is less or equal to ½. How many of these cases are there? For every j, the number of such cases is just the number of partitions of j into two, $p_2(j)$, which again was well known to them. They now could see a path to the peak. They had an expression for $T_N(n)$ purely in terms of their partitions equipment, namely

$$T_n(n) = p_3(n) - \sum_{j=1}^{\frac{n}{2}} p_2(j)$$

The concept was developed. They had the essentials for a proof of a formula that was true for all n. To get their formula into a more useful form they did a considerable amount of algebraic manipulation and simplification.

We started our climb from a different base camp. We had the path that Paddags and his class had traveled, and they had gone a substantial way up the mountain. Would their path lead us to the top? We tackled their first conjecture, if n is odd, then $T_N(n) = T_N(n+3)$. Using the same notation as before, since (a,b,c) is a triangular triple with perimeter of length n, then (a+1,b +1,c+1) is a triangular triple with perimeter n+3. For every triple of perimeter n there will be one of perimeter n+3 by extending each side by one. For example (3,5,7) is a triple with perimeter 15 and (4,6,8) is a triple with perimeter 18. However there may be some other triangles with perimeter n+3 that do not have such a match. Therefore $T_N(n+3) \geq T_N(n)$.

Our aim is to show that this inequality is in fact an equality. We could show that there are no additional triangular triples besides the ones obtained by a direct match. Another approach is to show that $T_N(n+3) \leq T_N(n)$. We decided on the latter and noted that since n is odd n+3 must be even. We therefore switched our consideration to an ordered triple (A,B,C) with perimeter of even length, A+B+C. We know that $0 < (A+B)-C = (A+B+C)-2C$.

But both (A+B+C) and 2C are even, therefore so is (A+B)–C and since it is greater than zero and 2 is the smallest even integer, we have $(A+B)-C \geq 2$, or $C \leq (A+B)-2$, and therefore $C-1 < C \leq (A-1)+(B-1)$.

We have constructed a triangular triple (A–1,B–1,C–1) with perimeter n–3, and using the same argument as before we must have that $T_N(n-3) \geq T_N(n)$.

This proof got us a little further up the mountain and we felt quite good. Unfortunately the remaining conjectures did not suggest similar individual proofs. We had to back-track. We were not equipped to tackle the cliff that lay ahead.

We returned to exploring all non-degenerate triangles of a given length and looked at the systematic listings of the lengths of the sides. For example for n=16 we have the triples

(4,6,6)	(2,7,7)
(5,5,6)	(3,6,7)
	(4,5,7)

One of us (TJ) has great equipment in discrete mathematics - concepts and techniques used for ordering, sorting, generating subsets, etc. Would any of this equipment be useful for this problem? What struck us were the limits imposed on the lengths of the sides of triangles. Could we break the problem down to simpler cases? For instance in the above example the longest side can only have length 6 or 7. This suggested fixing the value of c and exploring for the corresponding number of triples. We introduced the notation $T_N(n,c)$ as the number of triples of perimeter length n and longest side c. From our example $T_N(16,6) = 2$, and $T_N(16,7) = 3$ and therefore $T_N(16) = T_N(16,6)+T_N(16,7) = 5$. This approach would only work if we could determine what values of c were possible and if we could get an expression for $T_N(n,c)$. We explored some more, looking for patterns among the possible c values and keeping track of the numbers $T_N(n,c)$. Things started to fall into place.

The largest possible value that c can take is obtained by noting that $c \leq (a+b)-1$, and therefore that $2c \leq (a+b)+c-1 = n-1$, from which get that $c \leq (n-1)/2$.

But c is an integer, therefore the largest possible value of c is $[[(n-1)/2]]$, where $[[x]]$ is called the floor function and is the largest integer less or equal to the number x.

Similarly the smallest possible integer value of c is obtained from $n \leq c+c+c = 3c$, and $c \geq n/3$.

Because c is an integer the smallest possible value of c is $\lceil n/3 \rceil$, where $\lceil x \rceil$ is called the ceiling function and is the smallest integer greater or equal to x.

Now that we had limits for c could we get an expression for $T_N(n,c)$? The clue here is to note that $T_N(n,c)$ is just the number of a (or of b) values in the range of its possible values. For simplicity we have assumed that $a \leq b \leq c$. What is the range of possible values of a? Its smallest value occurs when b is largest, when b = c, therefore the smallest value of a is n-2c. The largest value of a is obtained by noting that $2a \leq a+b = n-c$, and therefore that $a \leq (n-c)/2$ with largest value $[[(n-c)/2]]$. We now just count the number of values of a and get $T_N(n,c) = [[n-c)/2]] - (n-2c) + 1$.

What remained for us to do was to sum these values and simplify the result. With quite a bit of algebra we developed the formula, for n even, $T_N(n) = \lfloor n^2/48 \rceil$, where $\lfloor x \rceil$ is the nearest integer to the number x.

Thus with our proof that $T_N(n) = T_N(n+3)$ for all odd integer n, we attained our goal and we had reached the top of the mountain. A check of the expressions

developed by Paddags and his class showed that they had generated all possible cases - a job well done.

Neighbouring Peaks

Once we got to the top of the mountain we took a rest and marveled at the world around us. We were excited to find that there were neighbouring peaks and that we were surrounded by interesting questions. Suppose we classify triangles according to their longest side c and let $U_\alpha(k)$ denote the number of triangular triples of the four types we described earlier. The first few values of these functions are

k	1	2	3	4	5	6	7	8	9	10
U_M	2	4	6	9	12	16	20	25	30	36
U_N	1	2	4	6	9	12	16	20	25	30
U_O	0	0	1	2	4	6	9	12	16	20
U_P	0	0	0	1	2	4	6	9	12	16

The number of degenerate and non-scalene triangles ($U_Q = U_M - U_N - U_O + U_P$) generates a sequence

U_Q	1	2	1	2	1	2	1	2	1	2

which provides some confidence in the accuracy of the four sequences. The formulation and subsequent proof were less of a challenge for us than the one for fixed perimeters. We were surprised when we found it easiest to develop a formula and proof for the case when triangles were classified according to their middle side b. Why didn't we think of tackling this problem first! Denoting $V_\alpha(j)$ as the number of triangular triples of the four types we described earlier, we found the first few values of these functions to be

b	1	2	3	4	5	6	7	8	9	10
V_M	3	6	10	15	21	28	36	45	55	66
V_N	1	3	6	10	15	21	28	36	45	55
V_O	0	1	3	6	10	15	21	28	36	45
V_P	0	0	1	3	6	10	15	21	28	36

We suggest you look for a formula for V_N. Note that when we fix the middle length the possible cases for the shortest and longest sides are very much constrained. Good luck.

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interesting mathematical problem? We enjoyed spending a considerable amount of time looking for ideas, searching for clues, drawing, exploring patterns, matching structures, and so on. Our hope was that these activities would provide insights and thereby point to paths up the mountain. We also spent much time in formalism, in rewriting and simplifying expressions, and in searching for appropriate notation. However we found that the latter were the means to achieve specific objectives that had been formulated from the exploration. A challenge for us as teachers of mathematics is to provide activities, time and motivation for our students to explore and develop mathematical ideas. This is where the mind is at its best and technology is there to serve. We then have to balance this time for exploration with time for our students to develop an understanding and facility in using the tools of mathematics. We believe that our teaching will be most successful when students are provided with experiences where the tools and techniques are used to reach some objective, for example, to generate a different insight, or for ease of communication, or as an alternative representation. Our experience suggests that doing mathematics requires both the ability to explore and generate ideas, and the ability to develop the mathematics to bring these ideas to fruition. Our challenge as mathematics educators is to provide an environment that is rich in the exploration of mathematical ideas and yet is balanced by a disciplined approach to the development of a sound understanding of mathematical notation, concepts, and techniques.

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▲ Sunlight, Shadows and Linear Transformations

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The first contact that many students have with the concept of a linear transformation is an introductory course in linear algebra. Even though examples of simple linear transformation of the plane such as reflections in the x and y axes, stretches, rotations and the like are usually provided, the formal definition of a linear transformation remains a mystery for many students. To some students, the sudden appearance of the formal definition is akin to pulling rabbits out of a hat. Indeed for other students, the definition of a linear transformation, namely, that for vectors **a** and **b**, and scalar k, a linear transformation T must satisfy the two conditions:

(i) $T(\mathbf{a}+\mathbf{b}) = T(\mathbf{a}) + T(\mathbf{b})$, and (ii) $T(k\mathbf{a}) = kT(\mathbf{a})$,

is just another one of those meaningless rules in the game of mathematics. Fortunately, it is possible to provide students with a concrete geometric context to give meaning to the two preceding conditions in the formal definition of a linear transformation. By representing a linear transformation as an affine projection from one plane to another, it is possible to visualize a linear transformation as a "shadow" transformation from an "object" plane to a "shadow" plane. In doing so, affine projections, and hence also linear transformations, can be characterized as those transformations which map parallelograms to parallelograms. The usual definition of a linear transformation can then be derived geometrically, by the way in which it maps parallelograms to other

parallelograms.

From an intuitive point of view, a linear transformation is almost the same as an affine transformation. One distinction is that affine transformations map points, while linear transformations map vectors. A second distinction is that the null vector (origin) is a fixed vector for a linear transformation, while under a general affine transformation, the origin point can move to any other point in the plane. This difference entails that an affine

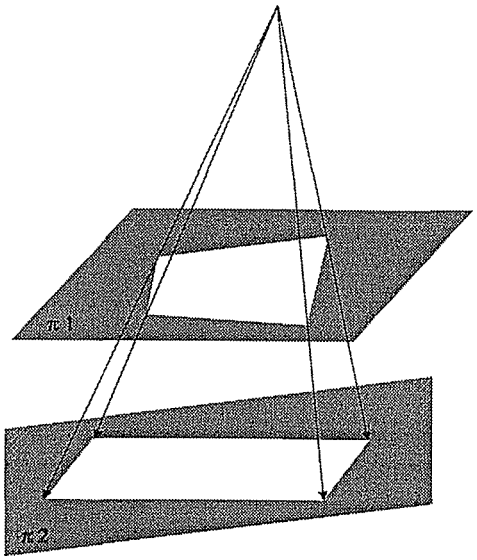


Figure 1

transformation can be thought of as the composition of a linear transformation followed by a translation. Because affine and linear transformations are so closely related, while a plane affine transformation is "uniquely determined by its effect on any one triangle" (Coxeter 1989, 203), a plane linear transformation is completely defined by its effect on any two non-zero, non-parallel vectors. Two nonparallel vectors, of course, form a "triangle" when they are placed tail to tail and their arrow heads joined together.

The set of invertible linear transformations is thus a subset of a set of transformations called affine transformations, which in turn is a subset of the set of projective transformations. Projective transformations first arose in dealing with problems in perspective in renaissance art (Kline 1967, 215-219). A projective transformation is a projection created by a point source of light shining through one plane onto another. The mapping of a geometric shape such as a quadrilateral from one plane to another under a projective projection can be visualized as the "shadow" image of the quadrilateral on the second plane by means of the point

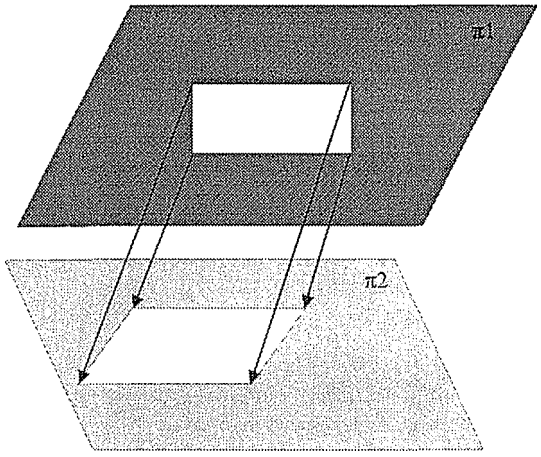


Figure 2

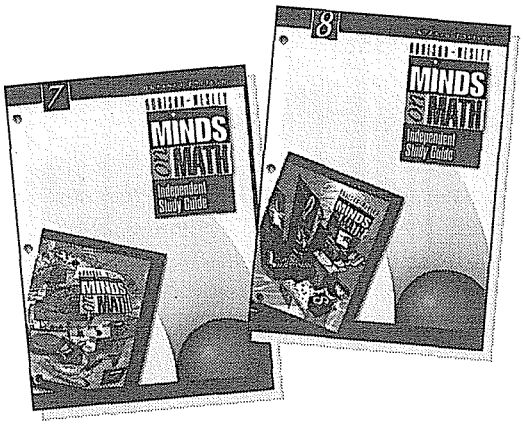
source of light (Figure 1).

An affine transformation can be considered to be the composition of any number of affine projections. An affine projection (also referred to as an oblique projection) of a plane figure from one plane to another is accomplished by means of parallel rays from the object plane to the image plane (Dorrie 1965, 304). An affine projection can be visualized as a "shadow transformation", whereby the image of an object is formed by "shadowing" the object from the object plane to the image plane by means of parallel rays of light. Figure 2 shows an affine projection of a rectangle from one plane to another plane. A typical example of an affine projection is the image of a window on the floor or a wall formed by sunlight streaming through the window on a sunny day. Any other affine transformation is a composition of a number of affine projections. By making some observations in simple experiments with shadows of plane figures in sunlight, it is possible for students to gain an intuitive understanding of the nature of affine transformations, and consequently linear transformations. In addition, the usual definition of linear transformations can be derived in a meaningful way.

SUNLIGHT SHADOW ACTIVITIES

The following activities are ones for which the resulting observations and conclusions will lead students to an understanding of the normal definition of a linear transformation as well as an intuitive notion of what linear transformations are about. These activities can be done by students on their own as an assignment and need not necessarily be done during class time.

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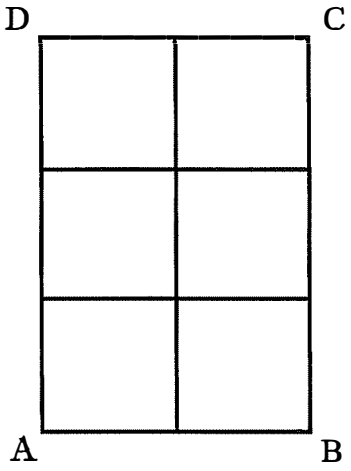


Figure 3

Activity I.

1. Given a south facing window such as the one pictured in Figure 3, sketch the image that this window would make on the floor at various times during a sunny day: at local high noon, early in the morning, in the afternoon, and when the sun is almost due west. Name the shapes of the possible images that you think will occur during the day?

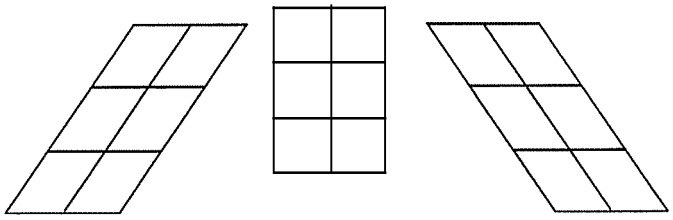


Figure 4

Figure 4 shows, simultaneously, the images that would appear on the floor in the morning, at noon, and in the afternoon. The rectangular image is the one that occurs at high noon. Since the bottom and top edges of the window are parallel to the intersection line between the image plane (floor) and the object plane (window wall), the images of the top and bottom edges are all equal to each other and also to the top and bottom edges of the original window.

2. Any one of the corners of the window shown in figure 3 is a right angle. Based on your observations in question 1, what are the possible values, in degrees of the image of a right angle under sunlight projections?

   Any angle from 0° to 180° is possible as an image of a right angle under sunlight projections.

Activity II.

1. Cut a square hole in a cardboard sheet. On a sunny day, hold this sheet in front of a second cardboard sheet so that the image of the hole is seen on the second sheet. By holding the two sheets at different angles to each other and to the sunlight, obtain the following different images: another parallelogram with a different shape, a rectangle, a square, and a rhombus.

   *This activity will develop in the student a sense that any parallelogram can be the image of a square under sunlight transformations. This will be useful in developing the matrix of a plane linear transformation since the two columns of the matrix of the transformation are the images of the orthogonal basis vectors, **i**=(1,0) and **j**=(0,1), which are two adjacent edges of the square whose vertices are: (0,0), (1,0), (1,1) and (0,1).*

2. Repeat the activity in number 1 with a parallelogram hole cut out of a cardboard sheet.

   This activity, although duplicating some of the observations in Activity I, reinforces the fact that the kinds of transformations under investigation are ones which map any given parallelogram to any other parallelogram.

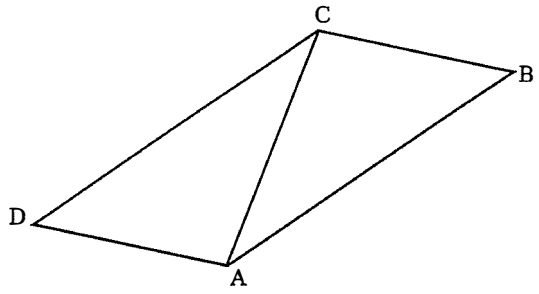


Figure 5

Activity III.

Suppose a window in the shape of an arbitrary parallelogram with one diagonal slat as shown in Figure 5 is in a south facing wall. Sketch some of the possible image shapes that would occur on the floor at various times throughout the day.

This activity will later lead to the sum rule in the formal definition of linear transformations.

Through these experiments, your students will be convinced that an affine or shadow transformation maps any given parallelogram onto another parallelogram. Since a linear transformation is an affine transformation

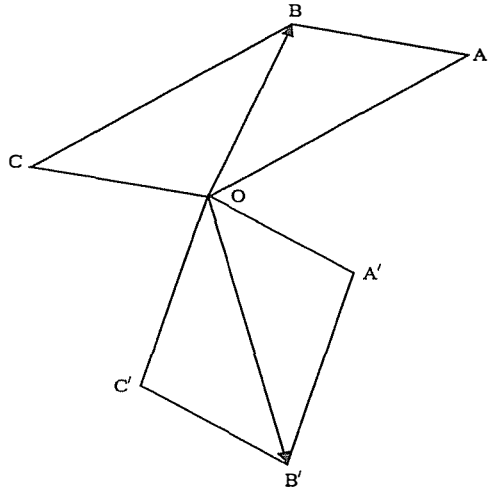


Figure 6

which leaves the origin fixed, it is useful to represent intuitively at least, plane linear transformations as those transformations which map parallelograms to parallelograms. A consequence of this view of linear transformations is that the two usual rules in the definition of a linear transformation now arises in a natural way. Indeed, suppose S is the transformation which maps any parallelogram OABC onto the image parallelogram OA'B'C' as shown in figure 6, then the following must hold. Since the diagonal OB' of the parallelogram OA'B'C' is the image of the diagonal OB of parallelogram OABC, which in turn is the vector sum of the edge vectors $\vec{OA}$ and $\vec{OC}$ we have:

$$\vec{OB'} = S(\vec{OB}) = S(\vec{OA} + \vec{OC}).$$

Also OB' is the vector sum of the two edges OA' and OC' of parallelogram OA'B'C', which in turn are the images of OA and OC respectively:

$$\vec{OB'} = \vec{OA'} + \vec{OC'} = S(\vec{OA}) + S(\vec{OC}),$$

and so,

$$S(\vec{OA} + \vec{OC}) = S(\vec{OA}) + S(\vec{OC}),$$

which is the sum rule in the definition of a linear transformation.

The scalar multiple rule in the definition of a linear transformation can be explained from the students observations to question 1. Figure 7 shows the window of Figure 1 along with a possible image superimposed. It can be observed that the original and image "windows" each comprises six congruent parallelogram "panes" respectively. Since OC' and OD' are each the images of OC and OD respectively (see figure 7), we have:

$$\vec{OC'} = S(\vec{OC}) = S(3\vec{OD}), \text{ and } \vec{OC'} = 3\vec{OD'} = 3S(\vec{OD}).$$

Hence we obtain an instance of the second rule in the definition of a linear transformation:

$$S(3\vec{OD}) = 3S(\vec{OD}).$$

A few final remarks concerning this method of deriving the rules for the definition of a linear transformation. First, it is only an intuitive device to assist

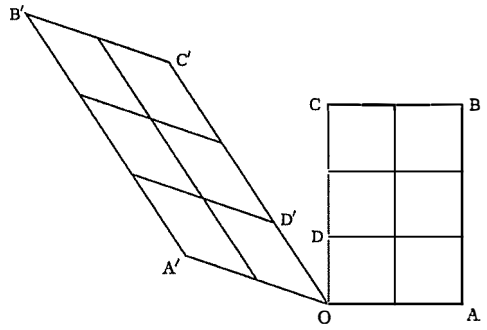


Figure 7

students in accepting the definition of a general linear transformation as a reasonable one that has some physical meaning, but it obviously cannot be seen as a rigorous derivation of the standard definition of linear transformations. Second, it should be noted that only invertible linear transformations are also affine transformations. In the case of the experiment in Activity III above, if the object plane (the card with the parallelogram hole), is gradually turned until it becomes parallel to the direction of the rays of sunlight, the image parallelogram would become narrower until it becomes a line segment. In this circumstance, one has a non-invertible linear transformation, but not, in the strict sense an affine transformation. Third, if you wish, the term affine transformation or affine projection does not really need to be used, the term projection or shadow transformation would be more meaningful to most students.

References

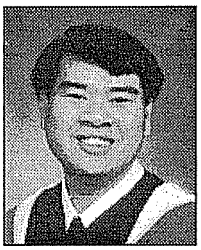
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▲ Announcing the Three Best Kept Secrets in Math Resources: Music, Video, and the *Equation Editor*

LOUIS LIM
QUINTE S.S., BELLEVILLE, ON



Louis Lim is a fourth year Mathematics teacher with the Hastings and Prince Edward District School Board. He is currently an OAME Director, writes the Director's Dialogue for the Gazette, and enjoys presenting workshops at conferences. He is also a part-time Master of Education student at Queen's University interested in teaching Mathematics through technology. Louis is indebted to the many outstanding educators he has come in contact with, in particular, his mentor, Ron Lancaster.

Math, Music and Mayhem Audiocassette

Perhaps the most impacting (and best kept secret) in Math resources is Dave Mitchell's Math, *Music and Mayhem* audiocassette. The cassette contains Math songs, stories, and activities suitable for grades 7 to 12. I learned of this invaluable resource from my teaching colleague, Dave Handley, who played me the "SOHCAHTOA" song.

I have used the "Circle Song", "BEDMAS", and "Quadratic Formula" songs with amazing results. My grades 9 and 11 students enjoyed singing along when I provided them with the lyrics. They appreciated that the songs made math fun, and were more willing to tackle the abstract concepts. When it came time for their unit test and midterm examination, *every* student in my grade 9 classes remembered the formulae for circumference and area of a circle, and *every* student in my grade 11 advanced class knew the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

After deriving a formula, I give each student the lyrics to the Math song. For the first time, we listen without singing. I then play the song a second time, and the students are now asked to sing along. With the rhythm now unconsciously memorized, the class sings it a final time, but without the music. I should warn you, though, to tell your students not to sing during a test or examination!

For instance, in the "Circle Song",

Here is a circle, it knows how to get around.
It has a radius from centre to rim.
And its diameter's a line that goes from side to side
While passing through the centre.
Now, isn't that simple?
Pi r-squared sounds like area to me,
When I need a circumference, I'll just use Pi d.

students who recite the final two lines to the tune of the music are then able to recall the formulae for circumference and area of a circle. Having taught the grade 9 destreamed course twice, I have always been frustrated that students are unable to remember their formulae on tests and exams. That problem has now been rectified!

I still overhear students, two months since I played the songs, able to sing them from memory. As one student so enthusiastically said minutes before the quadratic equations test, "The song really does work! I couldn't get the (quadratic formula) song out of my head when I was in bed!" News even spread to the guidance office, and the head of guidance shared with me the positive influence the Math songs have had on my students' attitudes toward mathematics.

The activities and stories provided on the audiocassette are very applicable to the transition years. For instance, "Card Ordering" is a great problem-solving activity that requires students to arrange ten cards from ace to 10 in the particular sequence so that the following happens:

- move the top card to the bottom of the deck – the new top card is an ace – set the ace aside;
- move the top two cards to the bottom of the deck, and turn over the new top card – it is card 2 – set it aside;

- move the top three cards one at a time to the bottom of the deck – turn over the new top card – which is card 3 – set it aside;
- continue until all the cards, up to 10, have been set aside.

Using *Math, Music, and Mayhem* is a creative and original way to make mathematics come alive in the classroom. I strongly encourage you to purchase, for $12 (including shipping and taxes), the audiocassette:

Mr. Dave Mitchell, Cameron Heights C.I.
301 Charles St. East, Kitchener, ON N2G 2P8

Math in the Movies

The next time you're on the Internet, check out the web site http://world.std.com/~reinhold/mathmovies.html. I found *The Math in the Movies Page* an incredible resource for using videos in the math classroom. The site lists movies which contain math scenes, along with short reviews and ratings for the math content.

Using videos in the math classroom can mean simply showing a scene for several minutes which helps get across a concept in a more interesting way than a chalkboard lesson. For instance, when introducing the chord properties in the circle unit of the grade 11 advanced course, I allow Mel Gibson, in *The Man Without A Face*, to explain how to find the centre of a circle using only compass and ruler. I reproduce the instructions below for each student. After Mr. Gibson (his character is Justin McLeod and he tutors Chuck Norstadt) has finished his explanation, I replay it with several pauses in the video. Students are now able to find the centre of a circle by constructing the perpendicular bisector of a chord:

#1. Draw a circle.

#2. Draw chord AB.

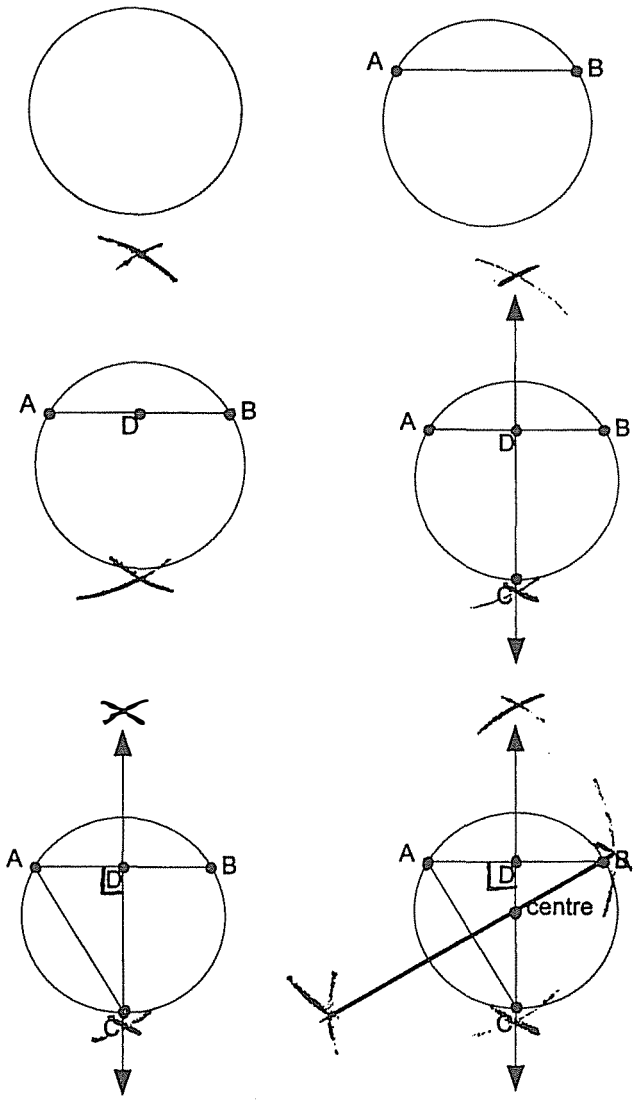
#3. Bisect chord AB at D.

#4. Draw a straight line from D to form straight line DC.

#5. Mark in right angle $\angle ADC$, and draw in chord AC.

#6. Bisect AC to locate the centre of the circle.

Before the semester ended, my grade 11 advanced class watched the entire video. I challenged them to describe, in detail, all the mathematics they could find in the movie. Besides finding the centre of a circle, other rich math concepts included an explanation of Euclid's proposition 47 (Pythagorean Theorem), along with Norstadt attempting to dig a hole three cubic feet. One



Steps 1 - 6

student shouted, "Is that possible?" and we later reviewed some measurement concepts. Others were able to find a golden rectangle, reflections, and even a fireplace in the shape of a parabolic curve.

The web site lists over 25 movies with math scenes, and is updated on a regular basis. I have used *Sneakers* (cryptography and factoring) in my grade 9 class, and *Stand and Deliver* in my OAC calculus class. I am currently looking into the scene where Meg Ryan, in I.Q., cannot dance with Tim Robbins: "she can only walk half the distance between them and then half again and half again and she will never reach them." Perhaps the next time students watch a movie or borrow a video they will keep their "mathematical eyes" open. Who knows what they will find and share with you!

New CONTEST FORMAT NAMES

NEW CONTEST, NEW FORMAT, NEW NAMES

The American Mathematics Competitions is pleased to announce a new contest as well as new names and a modified format for two of the current exams. The AJHSME is now the American Mathematics Contest ⇨8 (AMC ⇨8) and the AHSME is now the American Mathematics Contest ⇨12 (AMC ⇨12). The new contest is the American Mathematics Contest ⇨10 (AMC ⇨10), for students in grades 10 and below. This new contest will give more young students a chance to successfully participate in a significant mathematical problem solving experience.

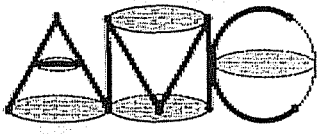
Why should my school sign up? Because the AMC ⇨10 and AMC ⇨12 provide an excellent opportunity to challenge your students' mathematical abilities. It is but a means for furthering mathematical interest and development.

The AMC ⇨10 and AMC ⇨12 will each be 75 minutes long and will consist of 25 questions each. Each correct answer is worth 6 points and a blank is worth 2 points. The AMC ⇨10 and AMC ⇨12 will have several questions in common and will be given at the same time, on the Tuesday before the third Monday in February (the current AHSME date). The students should choose between AMC ⇨10 and AMC ⇨12. Students in 10th grade and under may take either the AMC ⇨10 or AMC ⇨12, but 11th and 12th grade students may not take the AMC ⇨10. The school team score will be determined from the AMC ⇨12. To qualify for the American Invitational Mathematics Examination a student must score at least 100 points on the AMC ⇨12 or be in the top 1% of the AMC ⇨10 participants.

The registration fee for one or both contests is $30.00. One bundle of ten AMC ⇨12 is $12.00 and one bundle of ten AMC ⇨10 is $10.00. The first bundle of the AMC ⇨10 will be free for the year 2000 only.

<u>2000 AMC exam dates:</u>
AMC⇨8 - TUESDAY, November 14, 2000
AMC⇨10 - TUESDAY, February 15, 2000
AMC⇨12 - TUESDAY, February 15, 2000
AIME - TUESDAY, March 28, 2000
USAMO - TUESDAY, May 2, 2000

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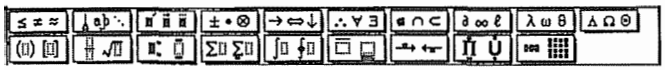
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Corel Word Perfect 8's Equation Editor

Great news! *Corel Word Perfect 8* has recently been Ministry licensed, so we now have free access to the *Equation Editor* for educational purposes. To create equations and Math symbols in earlier versions of *Word Perfect* was more of a hassle than an asset. It is now easy and fun to produce attractive and professional looking handouts and tests which include Mathematical equations and symbols.

Instead of having to type in functions such as OVER, SUP, SUB, etc. as we did in *Word Perfect* 5.1 and 6, version 8 has ready to use templates to create fractions, radicals, sums, integrals, matrices, and brackets through point and click.



For instance, to include $\lim_{x \to 5} \frac{x^2}{\sqrt{x+4}}$ in your document using Corel Word Perfect 8:

- go up to the menu bar and select INSERT followed by EQUATION;
- click on [ ] and select ;
- using your keyboard type in the letters "l", "i", "m". Notice that the letters are no longer italized as the *Equation Editor* recognizes "lim" as a Mathematical operation;
- press tab key;
- type x. Click on [ ] and select →, and then type in the number 5;
- press the tab key;
- click on [ ] and select ;
- type in x. Click on [ ] and select . Type in the number 2;
- press the tab key twice.
- select [ ] and select . Type in x+4.
- in the menu bar, select FILE, and then UPDATE.

I have noticed that students no longer ask for clarification of an equation or symbol during a test. Your job is to point and click, while the *Equation Editor* worries about italicizing, proper centring, and spacing. The days of manually writing in the radical sign, division sign, or fraction in your word processed documents are over!

▲ Extended Pythagorean Theorem Meets Geometer's Sketchpad

BRUCE CLOAD AND MARLON COATES



Bruce Cload is a mathematics instructor for the B.Sc./B.Ed. program at Brock University. this elite concurrent education program trains enthusiastic mathematics teachers for grades 4 through 10.



Marlon Coates is a student currently enrolled in the B.Sc./B.Ed. program. Marlon is slated to graduate in June 1999 and looks forward to undertaking a career as an elementary teacher.

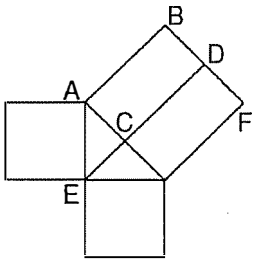
Introduction

As many school children can recite, "The sum of the squares on the sides of a right angled triangle is equal to the square on the hypotenuse." This is the Pythagorean Theorem. Here, we will investigate the lesser-known chant, "The sum of the equilateral triangles on the sides of a right angled triangle is equal to the equilateral triangle on the hypotenuse" and other such recitations for pentagons, octagons, and, in fact, any regular polygon. In particular, we will discuss how students, through the use of the computer program *Geometer's Sketchpad,* can discover, manipulate, and bring to life such geometrical theorems.

Extended Pythagorean Theorem

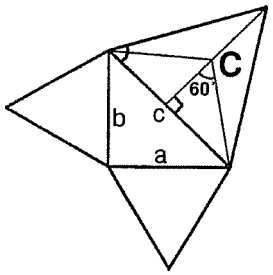
Euclid's proof of the Pythagorean Theorem (for a historical discussion see [1]) entails constructing a square on each of the base, height, and hypotenuse of a right angled triangle and then dividing the square on the hypotenuse into two rectangles. One rectangle is equal

in area to the square on the base and the other to the square on the height.



Construct perpendicular ED from right angle at E to side BF. Prove rectangle ABCD is equal in area the square with side AE.

If instead of squares we construct equilateral triangles, what happens? Does the sum of the areas of the triangles on the legs equal the area of the triangle on the hypotenuse? First let's draw the picture where a and b are the lengths of the sides of the right angled triangle and c the length of the hypotenuse. Let C be the incentre of the equilateral triangle on the hypotenuse.



By trigonometry, the areas of the triangles are

$$\left(\frac{3a^2}{4}\right)\cot(60^\circ),\ \left(\frac{3b^2}{4}\right)\cot(60^\circ),\ \text{and}\left(\frac{3c^2}{4}\right)\cot(60^\circ)$$

for the base, height and hypotenuse respectively.

Since $a^2 + b^2 = c^2$ by the Pythagorean Theorem, then

$$\left(\frac{3a^2}{4}\right)\cot(60^\circ)+\left(\frac{3b^2}{4}\right)\cot(60^\circ)=\left(\frac{3c^2}{4}\right)\cot(60^\circ).$$

The answer to the above question, then, is yes.

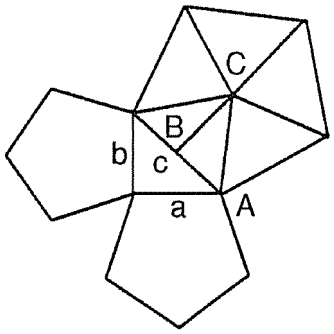
But why stop at equilateral triangles? This procedure also works for regular n-sided polygons (n-gons). Take a right angled triangle with base a, height b, and hypotenuse c, and construct n-gons on each side. The area of the n-gon on the hypotenuse is

$$\left(\frac{nc^2}{4}\right)\cot\left(\frac{360^\circ}{2n}\right).$$

The sum of the areas of the n-gons on the base and height is

$$\left(a^2+b^2\right)\left(\frac{n}{4}\right)\cot\left(\frac{360^\circ}{2n}\right).$$

By the Pythagorean Theorem, these two expressions are equal. For example, consider the diagram for n=5, that is, pentagons. Let the lengths of the sides of the right angled triangle be a, b, and c, and let C be the centre of the pentagon on the hypotenuse.



Angle BCA is 36°. Triangle ABC therefore has area equal to

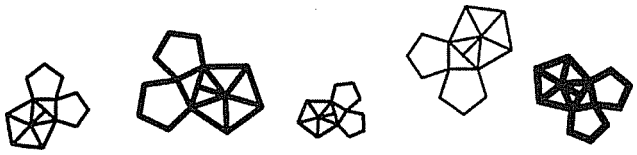
$$\left(\frac{c^2}{8}\right)\cot(36^\circ).$$

So the pentagon on the hypotenuse has area equal to

$$\left(\frac{5c^2}{4}\right)\cot(36^\circ).$$

Extended Pythagoras Together with the Sketchpad

Geometer's Sketchpad [2], a computer program from Key Curriculum Press, enables students to perform and manipulate geometrical constructions dynamically. The commands are menu- and mouse-driven and can be mastered in about an hour. Using *Geometer's Sketchpad*, students can construct a right angled triangle with regular polygons on each side. They can measure the areas of each polygon and verify the extended Pythagorean Theorem for their construction. More importantly, they can rotate, shrink, expand, and flip their construction in real time while the area of each polygon in the changing construction is continuously displayed. Thus, they will observe the sum of the areas of the polygons on the sides equal to the area of the polygon on the hypotenuse under all of their manipulations. For example, consider the pentagon case:



After initial explorations, one can ask the students to consider various questions:

1 What happens when n is large? Is there a problem when drawing the polygons? See what happens

when n is greater than or equal to 10. Construct the cases for decagons (n=10) and dodecagons (n=12).

2 In the Pythagorean Theorem, there is a natural way of dividing the square on the hypotenuse into two rectangles. Is there a natural way in the n-gon case of dividing the polygon on the hypotenuse into two (non-regular) n-sided polygons, one equal to the polygon on the base and one equal to the polygon on the height?

3 Can we extended the chant further? Perhaps – "The sum of the figures on the sides of a right angled triangle is equal to the figure on the hypotenuse, provided each figure is geometrically similar to the other figures and the area of each figure is proportional to the square of the side on which it is based." For example, this chant is true for a semi-circle on each of the three sides with diameter equal to the side. Are there other such regular figures?

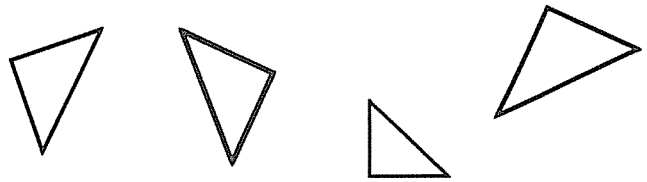
4 What happens if the triangle is not a right angled triangle? Investigate this on the computer by shifting the triangle. Can you think up a chant? (An advantage of *Geometer's Sketchpad* is that one can fix a portion of a construction [e.g. a point or a line] while shifting the rest of the construction.)

Effectiveness

Once mastered, the software is very effective for exploring a problem quickly: imagine answering question (1) in section 3 by hand-drawing the decagons and dodecagons. There are, however, drawbacks to projects using such software. Since computer output is easily copied (more so than written assignments), in assigning grades one must evaluate the process rather than the end product. To demonstrate the software, unless one's school is equipped with overhead projection facilities, one need either videotape a demonstration using a high speed camera, use fixed slides on acetate, or corral volunteers to work with small groups of students on a computer. For pacing a project, one must take into account the different levels of computer literacy among students and be sure to have extensions on lessons for those who are advanced. (We will note here that since the purpose of such a project is the exploration of geometrical ideas rather than the exploration of *Geometer's Sketchpad*, one should facilitate the student's use of the program through the scripting tool.)

Conclusions

Computer-based geometric manipulation breaks through preconceived ideas on familiar topics. For example, in the first two diagrams in this paper, we drew the right angled triangles in their familiar representations, with the right angle in the lower right- or left-hand corner of the triangle. A reason for this is, perhaps, blackboard or typesetting convenience. Using *Geometer Sketchpad*, however, will embed in student's minds many different images of a right angled triangle. Also, through this sort of manipulation, students gain a more intuitive understanding of the validity of a geometrical result than if they were presented with a formal algebraic argument. Structural development of a solution is discovered; to students this development is equivalent to proof. In addition, students will discover different meanings and representations for a mathematical expression. For the Pythagorean Theorem students can compare geometric meaning with algebraic meaning the square on the side vs the square of the side. Thus, students are encouraged to think in different conceptual frameworks and to examine ideas in different notations.



In this paper, we have discussed exploring the Pythagorean Theorem with *Geometer Sketchpad*. As computers take their place beside the blackboard as a teaching tool, computer based geometrical manipulation will take its place beside algebraic manipulation as a means to expose mathematics, and, moreover, to enliven mathematics. For the teacher as for the student, there are new challenges to be overcome and new mistakes to be made but that is part of the joy of teaching and the thrill of learning.

References

V. J. Katz, *A History of Mathematics: An Introduction*, Harper Collins, 1993.

Key Curriculum Press, Innovators in Mathematics Education, *The Geometer's Sketchpad.* There are many web sites with information and projects on Geometer s Sketchpad. The main Key Curriculum Press one is http://www.keypress.com/sketchpad/index.html.

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▲ ABOUT PROBLEM SOLVING: PLAYING WITH WORDS

JOHN GRANT MCLOUGHLIN
FACULTY OF EDUCATION
MEMORIAL UNIVERSITY OF NEWFOUNDLAND

The context for the discussion that follows is the English language. The experiences of those readers who consider placing other languages at the forefront of solving these literal problems are welcomed. Please share your insights.

I'll begin with a basic counting activity. Write any word. Then write (as a word) the number of letters in the word that was written. Then again write (as a word) the number of letters in the numerical word just written. Continue until you are satisfied that you have encountered an "honest number" that tells you how many letters appear in its spelling. For example, if you begin by writing "FIREFIGHTER" or "MISSISSIPPI", as two students did recently in a local grade five class, you would then write ELEVEN. Subsequently, the words would read: SIX, THREE, FIVE, FOUR, FOUR. Is there a word that requires more steps to complete the task? It seems to me that eleven letter words may be the most far from honest!

Let's shift the attention to alphabetical order. If that 'i' in eight(y) would move over a few places, the letters would be in alphabetical order. Can you find the only number which is spelled with its letters in alphabetical order?

How about assigning values to letters as they appear: A = 1, B = 2, C = 3, D = 4, . . . , Z = 26. An addition activity is to create words with values of exactly 100, for example. The value of MATH would equal 13 + 1 + 20 + 4 = 38. Of course, MATHEMATICS would have greater value. In fact, it's relatively close to 100 in value. There are many 100 point words. Places with values of 100 are not so uncommon either. For starters, you may wish to find two American cities with NBA teams that meet this criterion.

The challenge may readily be modified for use with a calculator. Try to find a word with a value as close to 1,000,000 as possible when the operation is changed to multiplication.

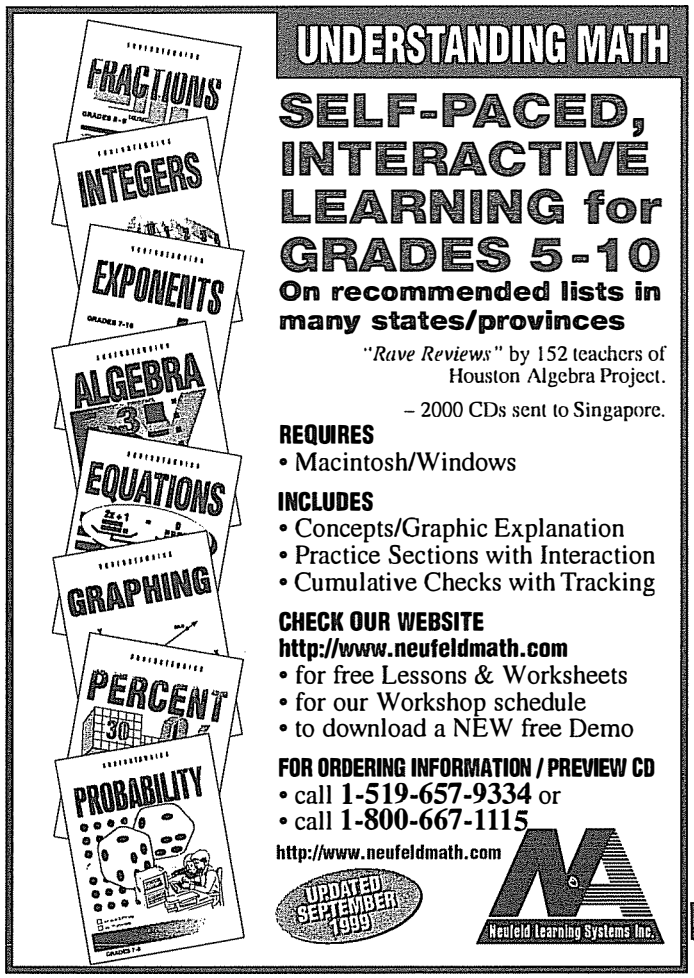
I'd like to offer one more challenge. The following problem appeared in the October 1960 (issue 31) publication of *Mathematical Pie*. The experiences of Bryan Dye's students with this challenge enticed me to share it here:

> *The eight letters in FORTY-SIX are different but this is neither the largest nor the smallest number that can be expressed in words in which all the letters are different. What are the largest and smallest numbers that you can find expressed in words in which all the letters are different?*

The challenges posed in this feature offer opportunities for participation at various levels. Results and feedback on any of the challenges would be welcomed from readers or their students. I am particularly interested in responses to the final two challenges neither of which have been definitively resolved.

Reference

Dye, Bryan (1996). Numbers in words. *Mathematics Teaching* 156, 5.



▲ THE DEVILS ADVOCATE: Do You Like Brahms?



ALEXANDRU PINTILIE

How many of you remember this film? It's an old, old movie from the long bygone times when deductive geometry was not regarded to be elitist, and the mathematics teacher could not be bothered with entertaining and befriending his fun-loving students. It was right after the mythical times of the ten-mile-walking-to-school-through-knee-high-snow era, and just before the epochal discovery that grammar and spelling are a nuisance.

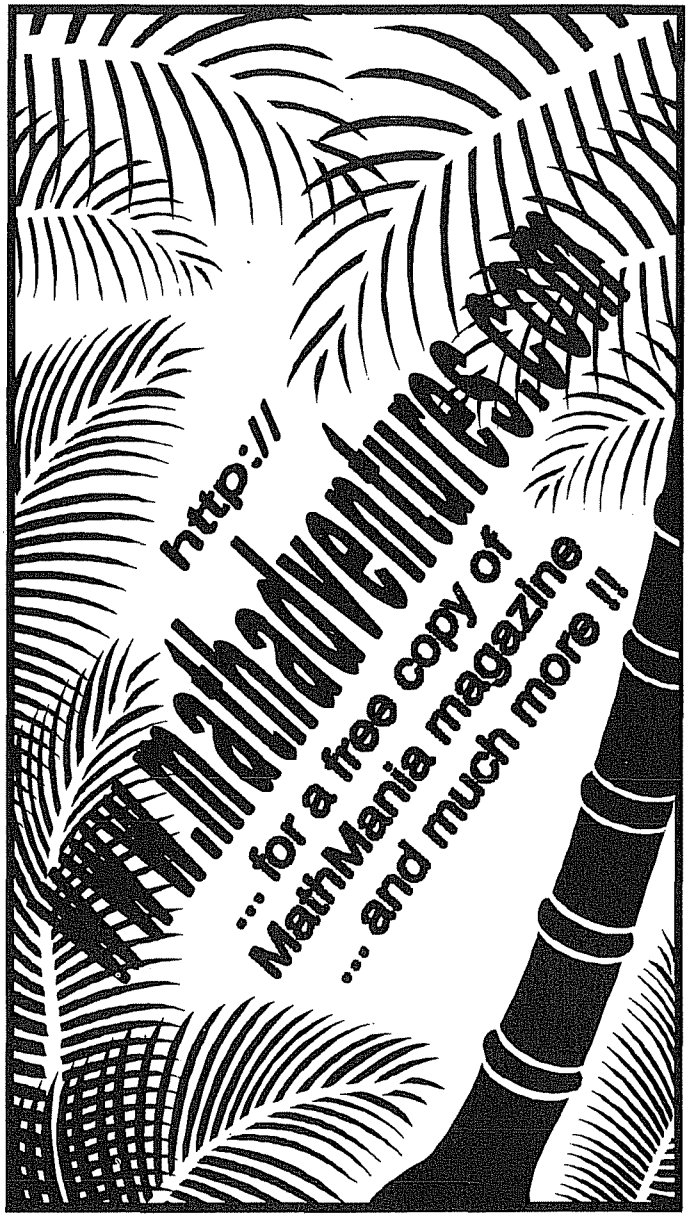
From what I can remember, the film is about the importance of matching musical tastes in a romantic relationship. (It never occurred to me at that time, but now I realize that the whole story is a rich learning task in which the student can draw a graph having the correlation of musical tastes on the x-axis and the ecstasy derived from the relationship on the y-axis. Just imagine the manipulatives one can use for such a lesson! The rubric would be so open-ended that it would make figure skating judging look like rocket science.)

Musical tastes make all the difference. Imagine the absurdity of a newspaper ad announcing a "Musical Concert". Is it Chopin, Kenny Rogers or Marilyn Manson? Do I dust off my black suit, do I put on my cowboy boots or should I get some serious piercing? If you think it is a difficult choice for the members of the audience, how about the performer who faces a mixed audience? The piano virtuoso might be tempted to dress in gangsta' rap attire, try some line dancing steps while singing an Alannis Morrissette tune enriched with Christian Gospel lyrics.

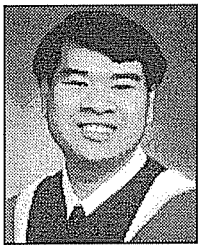
I believe that he should stick to his scheduled Brahms recital. This is what he is supposed to do and this is what he does best. The connoisseurs in the audience will enjoy the recital. Others, with an open mind, are there for a new experience and will maybe acquire a taste for a different kind of music. Entering a concert hall, will be a good reason for a third group to have the patience to listen, and, even if they don't enjoy the experience, to show respect for the artist's effort. A fourth group will be there to be seen in select company.

The fact that the majority prefers Guns and Roses to Brahms is no reason to have the Royal Conservatory of Music shift focus. Neil Postman speaks of education as a "subversive activity", because it counterbalances prevailing fads and dogmas. It was its role to teach Darwinism when the Christian Scriptures constituted the only truth; it is its duty to preserve spirituality in a world ruled by the bottom line. The fact that a lot of our young people live by the wisdom of Dennis Rodman and Kurt Cobain and would rather spend their summer flipping burgers than reading would actually offer further reason why school should expose them to classical music, Euclidian geometry and great literature.

It is our duty to play Brahms. Whether they like it or not.



▲ Directors' Dialogue



Louis Lim
Quinte S.S., Belleville, ON

This edition of the "Director's Dialogue" summarizes the Saturday, June 13 OAME Board of Directors meeting at the Four Points Hotel in Mississauga.

Our President Speaks From the Heart

OAME president Peter Saarimaki described his mission statement for the upcoming year as coming from the HEART. H stands for health of the organization. Peter expressed concerns about OAME's declining enrolment, and the fact that fewer people are getting involved in OAME, and those who are involved are doing more and more. E signifies emotion, and we need to share our enjoyment and satisfaction of Mathematics and teaching with others. A is for arteriosclerosis, and Peter described the analogy that blood circulates in and out, and thus communication should occur through open channels. R stands for ripple, and our president reminded us that we have a huge effect on others when "we drop a small pebble." And finally, T represents testing. What would you like to be tested on since we accomplish so much together?

New Gazette Editor Named!

Todd Romiens was named the new editor of the *Gazette* and will begin his term with the December 1999 issue. Yours truly (Louis Lim) will support Todd, and was named the Associate Editor. Here's an analogy taken from hockey: When Todd was president of OAME in 1994/95, I was a teacher candidate at York University, and always read his "President's Message" in the *Gazette.* I am very excited to be able to work with one of my heroes.

Retiring Editor Honoured

John Egsgard was honoured by the OAME board for his dedication and hard work in serving as *Gazette* editor from 1996 to 1999. I was most proud to see John recognized as he always "took extra good care of me" as I experimented with my writing potential by submitting articles to the *Gazette.*

Read All About It ... NCTM

Frances Schatz, our NCTM representative, shared some fantastic news with the board of directors. When purchasing your NCTM membership or resources, be sure to pay by Canadian cheque. NCTM has capped the exchange rate at 25%. Therefore, total up your purchases, and add 25% Canadian. Frances also shared the good news that $1500 worth of NCTM resources was sold at the OAME'99 conference in Richmond Hill, with 20% going to OAME!

Past-president Ralph Connelly shared the fantastic news that NCTM has awarded OAME a $1500 Professional Development Grant for the production of *Linking Assessment and Instruction in Mathematics: Primary Years*. Contact executive directors Dave and Bonnie Alexander at to purchase your copy.

News From the Fields Institute Math Forum

Shirley Dalrymple reported that a major concern of the Fields' Institute Math Forum was the shortage of mathematics teachers, and that excellent math teacher candidates are choosing careers in industry. Eric Muller and Walter Whitely are co-chairing a committee aimed at bridging the gap between the faculties of education and departments of math. One initiative is to develop new math courses (e.g., recreational, problem solving, history of math) which would be of interest to prospective teachers.

Professional Development

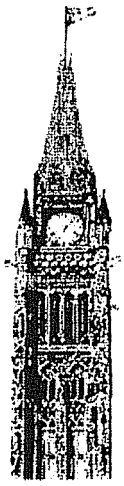
OAME'99

Wow — what a conference! Thanks to co-chairs Jan Scully and Ann Rubin and their many committees for putting together such an amazing conference. With 1200 participants expected, 1400 showed up for the three days, with 517 new OAME members!

OMO'99

OMO'99 chair, Stewart Craven, shared many wonderful experiences that resulted from OMO'99 at York University. Firstly, he stated that 114 students from 12 chapters participated, but 114 is not divisible by 4. Huh??? The solution is located at the end of this column.

Stewart also shared a heart-warming story from the NWOAME chapter. When the community discovered that there were no funds to send a team to Toronto for



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the OMO, both of the boards in Thunder Bay contributed $600, and Bowater, a local paper mill, contributed $1000. There's more ... one individual donated some air miles points, and the Red Lake community raised funds to send their Red Lake team member to Toronto.

The top school team hailed from the Pine Ridge chapter, while the top all-star team was from ISOMA. OMO is alive and kicking for next year, thanks to the financial support of Texas Instruments and Spectrum.

OMCA/OAME IN-SERVICE WORKSHOPS

By the time you read this, phases I and II would have happened. Phase I brought together 58 educators for an in-service on May 4 and 5. These point people and presenters then returned to their respective boards to train teachers using the resources provided. Phase II consisted of 15 summer institutes across Ontario for almost 600 teachers. Sponsored by Texas Instruments, these five day workshops provided teachers with the necessary tools to deliver the new curriculum using technology. Check out the OAME/OMCA web site at and join in the discussion.

CEREAL HUNT

What is the name of the new cereal with a mathematical name? The first person to e-mail me with the correct answer wins a prize (and it's not cereal!) Thanks to John Kersley for bringing samples of the cereal for the board members to munch on! Please note that this contest is not open to OAME board of directors present when the cereal was distributed!

COMMITTEES

AWARDS

Congratulations to the following recipients of OAME awards:

- OAME Life Membership Award: Judy Crompton
- Kenneth D. Fryer Award: Dr. Dennison S.S. Math Department
- Mona MacGregor Award: Mickey Sandblom
- Morley MacGregor Award: Barbara Nicholson
- Don Attridge Award: Jan Scully

(For more details, see page 35.)

FUTURES

The following have been confirmed:

- OAME 2000: hosted by COMA in Ottawa
- OAME 2001: hosted by SAME in Scarborough
- OMO 2000: hosted by QSLMA in Kingston

The futures committee stated that they have accumulated lots of information to support future hosts of the OMO. Two chapters have tentatively offered to host OMO 2001 and OMO 2002 respectively, and a motion will take place at our next board meeting in October.

PROMOTIONS

Kirsten Boucher reported that the Promotions committee has five initiatives for the upcoming school year: (i) put together a communications kit on how to deal with the media; (ii) a package of OMO activities for chapters to use; (iii) search for ways (e.g., perks) to increase membership; (iv) generate a logo contest in the *Gazette* and sell t-shirts, with OAME getting a percentage of the profits, and (v) update the OAME membership brochure.

PUBLICATIONS

Congrats to Dean Murray, the *Gazette* advertising manager, for generating $10 250 in advertising revenue to OAME. Be sure to purchase resources from our advertisers, and also mention that you saw their ad in the *Gazette.*

As of June 20, the OAME web site, has had 1450 visits. OAME's web site coordinator, Doug Evans, is looking into including chapter web sites on the OAME web site, as well as having links to our advertisers.

JK-5

Several of the major initiatives of the JK-5 committee will be looking at the exemplars in detail, and writing an article for the *Gazette* on its history, how to use it, as well some cautionary measures to consider. They will also look into a detailed study of the early years (e.g., JK/SK), and submit activities to the *Abacus* for possible publication. With the newly released Primary Assessment document, they will find ways to promote it, and to provide in-service in using the document.

6-8

The grades 6-8 committee listed reporting, teaching math through the strands, and assessment activities being difficult to implement as three major challenges facing grades 6-8 teachers, along with what OAME could do to support them. Their projects during the upcoming year include an update of the linking assessment document, provide elementary members with articles in the *Gazette* and web site, John Dallan will write an open

letter as a grade 9 teacher to grades 7 and 8 teachers, reformat the grade 6-8 "Scope and Sequence" posters, and consider the possibility of offering summer institutes which link technology with the assessment activities.

SECONDARY

When the board meeting adjourned at 3:15 p.m., the Secondary Committee went back to work. They drafted an urgent message which should have reached you by the time you read this column; namely, (i) use the course profiles as your main guide to the grade 9 program; (ii) learn to assess and evaluate in new ways, and (iii) smoothing the way for implementation of the new grades 10 to 12 courses. Rest assured, OAME will provide support materials such as "Making the Most of the Grade 9 Profiles," develop a resource package, create support materials to help teachers, parents and students decide on grade 10 course selections, as well as leadership in assessment and evaluation.

CHAPTER NEWS

CHAMP

Reserve the evening of Thursday, October 28 for Mathfest'99! The conference will focus on implementing the new grade 9 curriculum, and will include two workshop sessions, publishers' displays, dinner, and door prizes. The CHAMP Mathematics contests for grades 9 to 11 will take place in November. E-mail chapter rep John Rodger for more info.

COMA

The very successful Carleton-Ottawa Math Olympics (COMO) were held on Saturday, May 1 with a total of 52 teams (up from 39 last year). The event was even highlighted on the CHRO news! Peter Lenczewski reported that all committees for the OAME 2000 annual conference have been formed, and they are making great progress in their Sunday morning meetings.

GOLDEN

On Wednesday, April 14, more than 160 teachers (a record!) from Fort Erie to Hamilton descended upon Brock University to hear Judy Crompton, Myrna Ingalls, and John Dallan discuss the key messages of the new grade 9 curriculum. Plans for the upcoming year were (when I write this column "will be") made in August, so e-mail chapter rep Elizabeth Pattison for the latest details.

GVMA

Their fall conference is scheduled for November 4. Check out their web page at for the most recent update. Be sure to purchase GVMA merchandise which includes π-buttons, and their most recent publication, Grade 12 Sample Exam Plus Supplementary Problems.

ISOMA

Appleby College organized their "Introducing Technology into the Curriculum" conference on Thursday, April 9. More than 50 teachers from 12 schools attended and learned about the school's use of graphing calculators, IBM laptops, advanced presentations projection units, and interactive whiteboards. Congrats are in order to Brad Stephens and the Holy Trinity School Math Department for once again organizing an extremely successful ISOMA Math Olympics for grades 7 and 8 students. Plans are underway for a fall conference in the first week of November, along with an updated web page.

MAC^2

Two local Math Olympic competitions were held in April, and they were successful and popular with both students and parents. MAC^2 produced a newsletter for its members, and for the upcoming year, they intend to increase communication with members through more newsletters and their web site. You can email chapter rep Lyn Vause for an update on their upcoming fall conference.

NOMA

NOMA sponsored two of its members to attend the OAME Leadership'99 Conference in February. In return, these members shared their professional growth at their chapter's "Spring Thing'99" mini conference which was attended by 168 teachers. By the time you read this, their "MATH4Y2K" fall conference may have happened (depending on when you read this column). It is scheduled for September 17 and 18 in Sudbury, and will include Stewart Craven as the keynote speaker.

NWOAME

More than 90 teachers attended NWOAME's spring conference with sessions on the new grade 9 curriculum, technologies in the classroom, Impact Math, and two primary sessions on assessment and problem solving. Wayne Fletcher reported that 14 new members joined NWOAME at that conference, and membership is at an all-time high! At the conference dinner, two members, Marilyn Hurrell and Joe Hall were honoured, and the chapter wished them both well in their retirement.

QSLMA

QSLMA enjoyed a very successful spring conference

OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration;

- demonstrations of excellence and creativity in mathematics education.
- specific contributions to mathematics education in the school, region, province or nation.
- demonstrates leadership in mathematics education in the school, region, province or nation.
- support of and contribution to the activities of OAME/AOEM.
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld form consideration.

OAME/AOEM LIFE MEMBERSHIP AWARD

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and accumulated 10 or more years of membership in OAME/AOEM.

KENNETH D. FRYER AWARD

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

DON ATTRIDGE AWARD

In recognition of the contributions made by Don Attridge, a founding member, former president and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

AWARD NOMINATION COVER PAGE

Send to: Shirley Dalrymple

Intended Award (please check one)

❑ Life Membership ❑ Mona MacGregor ❑ Morley MacGregor ❑ Kenneth D. Fryer ❑ Don Attridge

Nominee Information:

_____

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

_____

_____

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address_____

_____

Home Phone ( ) _____

Nominator Information:

_____

Name of Nominator

_____

_____

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address_____

_____

Home Phone ( ) _____

focussing on the theme "Assessment in Mathematics" at the Royal Military College in Kingston. This chapter is also busy organizing OMO 2000, and needs all the help they can get from its members. Contact Cindy Harvey at Also, check out QSLMA's frequently updated web site at **http://cc.recorder.ca/~gordcook/**

SAME

After hosting a successful spring conference entitled "The New Curriculum — Survival Skills," plans are underway for a fall conference on Tuesday, October 19. The theme will be "Embracing Change," and a Chinese Auction throughout the evening will be held to raise funds for SAME. "The Anne Pedersen Mathematics Bursary" was named in honour of retiring teacher Anne Pedersen's dedication and commitment to Mathematics education. The bursary will fund students who attend Saturday morning enrichment math classes at the grades 4-8 levels. This busy chapter has just begun organizing for OAME 2001 with a most appropriate name, "2001: A Math Odyssey." Stay tuned for their soon-to-be released web site!

SWOAME

SWOAME hosted a successful "Dinnerlude" with 65 teachers in attendance. Todd Romiens and Katie Branovacki provided after-dinner talks on secondary school reform. They followed up by sending two teachers from each school to their Phase I grade 9 in service initiative on June 8 by providing teachers with an overview of the new grade 9 curriculum, and technology activities. After sponsoring two teams to OMO'99, this chapter is committed to organizing and planning the "2000 SWOAME Mathematics Olympics." Check out their web site at **www.uwindsor.ca:7000/math/rcaron.htm**

WOMA

Linda Alton reported that the "Grade 9 for '99" workshop series were well-received as more than 100 teachers from 40 schools attended the four workshops. Topics covered include rubrics and assessment, technology, and differences between the applied and academic levels. WOMA took an active role to provide funds for the Junior Math Challenge for grades 4, 5 and 6 students of the Thames Valley board. In addition, this chapter also supported grades 7 and 8 enrichment workshops one Saturday each month, averaging about 35 students. Be sure to read their most recent edition of SinePost, along with checking out their web site at **www.math.uwo.ca/woma/**

Y^{4}MA

This chapter is now recovering from an extremely successful OAME'99 conference. As I listened to Jan Scully's OAME'99 report at the board meeting, she mentioned that Y^{4}MA will apply to host it again in the year 2002 — but that was only a joke! Hmmm ... I've been serious up until here in the chapter news. I better stay on task ... here goes ... Their fall conference, "Building the Bridge Between Teaching and Learning," is scheduled for Thursday, October 7 and will centre on new assessment strategies and how to implement them. This fall conference will be a bit different from the previous years, as participants will attend one in-depth workshop on assessment (primary, junior, intermediate, or secondary).

CLOSING REMARKS

Here's to a great school year! Get involved in your chapter (e-mail your chapter rep for more info), read previous issues of the *Gazette,* and attend conferences. See you in Ottawa for OAME 2000 — The ART of Mathematics!

Solution: There were 28 teams of four, and one team of two.

OAME-NCTM Report

SUBMITTED BY FRANCES SCHATZ

The Canadian Leadership conference for provincial representatives was held in Toronto at the Delta Chelsea Hotel July 8-10, 1999. Two full days were devoted to leadership activities and Standards 2000. OAME hospitality was wonderful.

There were many opportunities for affiliated group representatives to share successes and challenges in providing leadersip to mathematics education. Sharon Henderson, Canadian NCTM representative for Quebec and east, and Ivan Johnson, Ontario and west representative, were our leaders. This conference certainly made me more aware and appreciative of OAME's organization, professional development activities and member services. My book order /membership forms were given as examples of an initiative to make ordering NCTM materials easier.

NCTM displays can be requested for chapter workshops, and mathematics teacher meetings. Frank and I have a variety of new materials on hand and you will find us at OAME meetings and conferences. OAME gets 20% of sales on NCTM published materials. OAME also receives $5 on new memberships ordered through me, your NCTM representative. For more information contact me (see OAME board on page 36).

▲ OAME 1999 AWARDS

This year's recipient of the Mona MacGregor Award is Mickey Sandblom.

Mickey joined OAME in 1978. From 1980 to 1984, he was the COMA Chapter Representative on OAME Council and served on the Computer and Membership Committees. He continued from 1985 to 1993 as an elected member of Council and then from 1994 through 1997 as Council's NCTM representative. For several years, Mickey was on the OAME Executive, serving as president in 1991. Among many other contributions, he co-chaired OAME 85 in Sudbury and served as OAME representative on the OAIP Intermediate Advisory Committee of the Ministry of Education from 1988 to 1992.

This year's recipient of the Morley MacGregor Award is Barbara Nicholson.

Barbara has taught grades K through 8 since beginning her teaching career in 1978. For several years, she worked in the Toronto District School Board as a program leader and later, a vice-principal. She has incorporated reading and writing strategies in her teaching of mathematics. This "mathematics across the curriculum" approach has proved effective in reducing math phobia for many students. Barbara initiated the Elementary/Middle Math Action Committee and has participated in the writing of curriculum guidelines. Her workshops and seminars, in which she shares her philosophy and vision, have been popular and inspiring. She is currently lecturing at York University on the teaching of mathematics and is a councillor with Y^{4}MA.

This year's recipient of the Don Attridge Award is Jan Scully.

Jan is currently a K through OAC mathematics consultant with the York Region District School Board. In a previous life, she was head of mathematics at Markville Secondary school, where her department successfully ran creative, forward thinking and innovative programs designed to meet the needs of the new curriculum and the students. In fact, under her leadership, that school was a previous winner of the K. D. Fryer Award. Jan has co-authored many publications including a text book series. She has been for years and continues to be a dynamic workshop presenter at conferences sponsored by OAME, Y^{4}MA, chapters across the province, and NCTM. She has instructed in mathematics teaching at several university faculties of education. She has co-chaired two(!) OAME conferences, first in 1992 and again this year. One quote that has come true in her classrooms: "I want students to enjoy mathematics."

This year's recipient of the OAME Life Award is Judy Crompton.

Judy has been on OAME Council for many, many years, including a term as President in 1997-98. She is also a past president of OMCA. She was the development leader for the Focus on Renewal, the Ontario document that has been the foundation of the ensuing work on both elementary and secondary curriculum in Ontario. She has been a leader in the movement to link assessment and instruction in mathematics resources. Judy had been a driving force behind the Fields Institute consortium which sought the Ministry of Education contract to write the new secondary curriculum. When that group's proposal was accepted, Judy became Project Manager and the resulting documents are a testament to her abilities, insight, and hard work. She accomplished this with calm and poise and logic, as well as diplomacy that won over many adversaries.

This year's recipient of the Kenneth D. Fryer Award is Dr. Dennison Secondary School.

The mathematics department of Dr. Dennison Secondary School of Newmarket, whose department head is Shirley Dalrymple, works as a team, running a daily math help room. The teachers have created a Math Wall of Fame, awarding certificates to students achieving over 85% each semester. It has hosted the York Region Board's Ontario Mathematics Olympics for the past three years. Moreover, members of the department have run many workshops for Y^{4}MA, OAME and the York District School Board.

The *Gazette* is also pleased to inform you that this year's Descartes medallists are OAME members of long-standing: Bob McRoberts and Jim Schurter.

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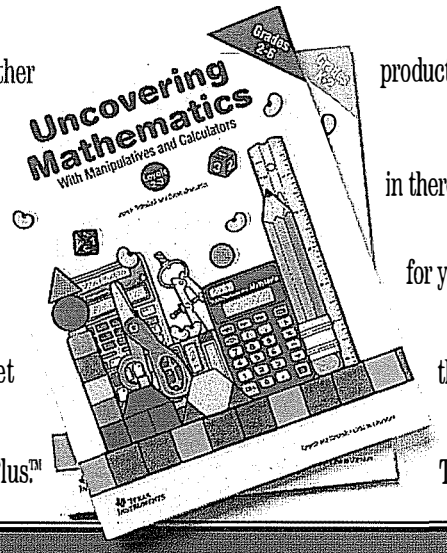
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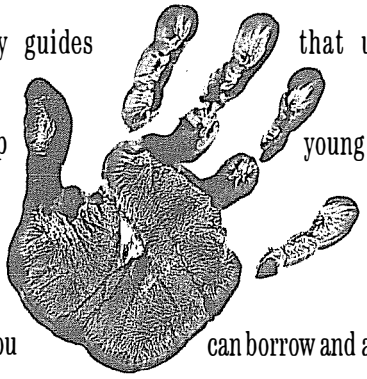
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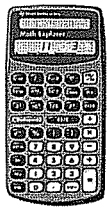
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Gage Mathematics Assessment 6

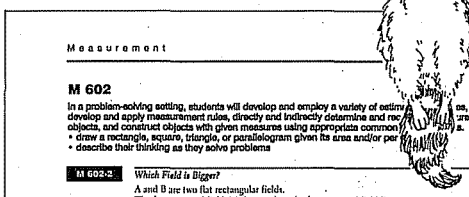


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Measurement

M 602

In a problem-solving setting, students will develop and employ a variety of estim... develop and apply measurement rules, directly and indirectly determine and re... objects, and construct objects with given measures using appropriate common...
- draw a rectangle, square, triangle, or parallelogram given its area and/or per...
- describe their thinking as they solve problems

M 602-2 *Which Field is Bigger?*

A and B are two flat rectangular fields.
The fence around field A is longer than the fence around field B.
Lots of people think that this means that field A is always bigger in area than field B.

1. Make a model or sketch to help you show that field A's area could be smaller than field B's.
2. Make a model or sketch to help you show that the two fields could have equal areas.

• Students will have more success if they work with a specific example. For example, Field A with perimeter 12 and Field B with perimeter 10.
• Provide students with 1-cm square grid paper. (blackline master).
Also consider having interlocking cubes available for students to use.

Sample Response (30-40 min).....

LEVEL 3
1. P = 26, A = 12
P = 20, A = 16
2. A, B, A

LEVEL 4 • gives clear and convincing demonstrations of the truth of statements in both questions

LEVEL 3 • may need some encouragement to demonstrate answers through specific examples

LEVEL 2 • may needs assistance to pick specific examples through which they can present answers

LEVEL 1 • needs assistance to understand problem and develop a solution strategy
• may confuse perimeter and area

Sample activity page from
Gage Mathematics Assessment 3

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