Vol. 38 #2, December 1999

Ontario Mathematics Gazette

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Men as Trees Walking

Implementing Journal Writing in the New Grade 9 Mathematics Curriculum: Why & How?

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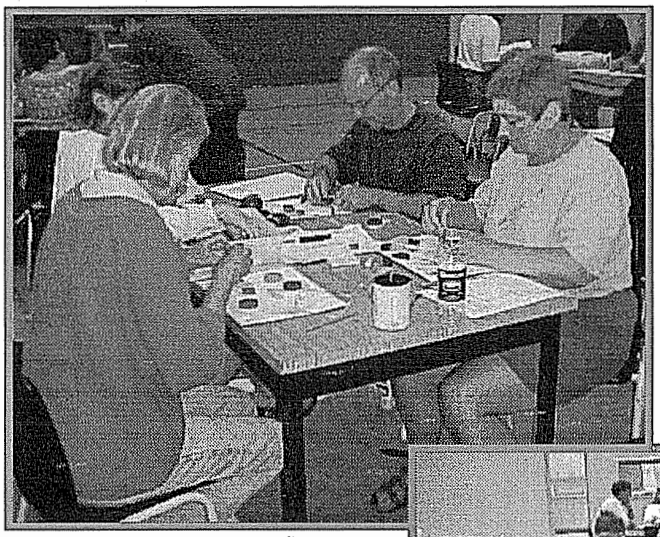
THE DEVIL'S ADVOCATE: My Pet Peeves

Here are what participants had to say about the Exemplars Project and the use of rubrics in evaluation:

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"Nice to see that I am on track with marking. All examples from the course will be used."

"The knowledge of rubrics does not stop at math. I can extend it to other subject areas."





At the end of the course, participants made comments such as:

"This was an excellent course - the presenters were knowledgeable and willing to share their materials and expertise. The opportunity to network was invaluable."

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"by hand?"
(The old way)

Final Exam

1. State the Domain of $y = \frac{x-10}{\sqrt{x^2-16}}$.

2. Which relation is the inverse of y=(

3. Use a half-angle identity to find the e

a) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ b) $-\frac{1}{2}\sqrt{2-\sqrt{2}}$ c)

4. If $f(x) = 2x-5$ and $g(x) =$ find

a) $\frac{-(5x+11)}{x+3}$ b)

5. If $\sin\theta = -\frac{1}{2}$ and θ is

a) $\frac{-\sqrt{3}}{3}$ b) $\frac{\sqrt{3}}{3}$

6. Given $f(x) = \frac{x^2}{x+3}$ and

a) $\frac{4x^2-2x}{x+3}$ b)

"struggling with a word processor?"
(The hard way)

Final Exam

1. State the Domain of $\frac{y=x-10}{\sqrt{x^2-16}}$.

2. Which relation is the inverse of $y=(x-2)^3$?

a) $y=\sqrt[3]{x-2}$ b) $y=\sqrt[3]{x}-2$ c) $y=\sqrt[3]{x+2}$ d) $\sqrt[3]{x}+2$

3. Use a half-angle identity to find the exact value of cos 67.5°:

a) $1/2\sqrt{2-\sqrt{2}}$ b) $-1/2\sqrt{2-\sqrt{2}}$ c) $-1/2\sqrt{2+\sqrt{2}}$ d) $\sqrt{3+2\sqrt{2}}$

"with MathType!"
(The right way)

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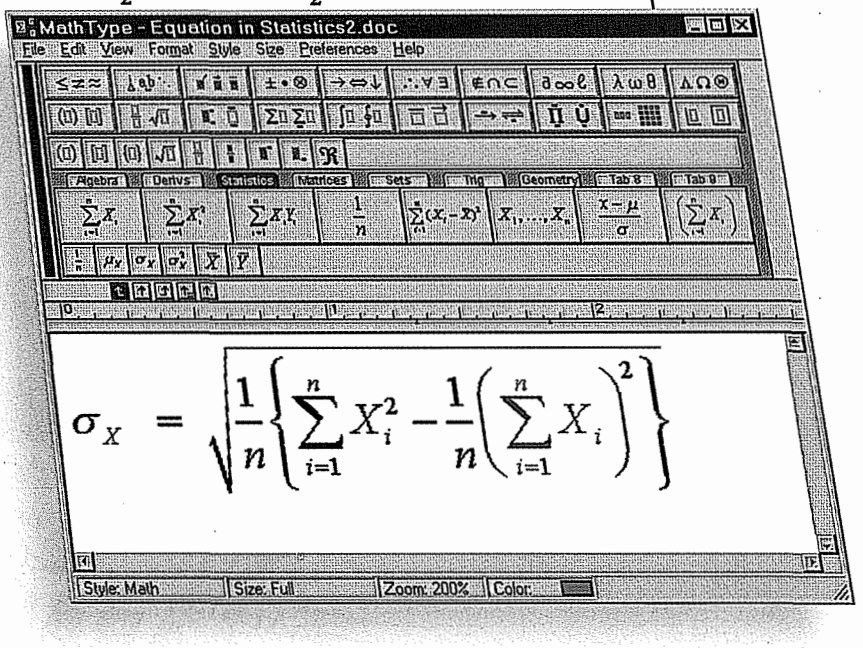
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Your article should be word-processed, double-spaced on one side of 21.6 cm x 27.9 cm paper with 3 cm margins. Diagrams and figures should be drawn by computer, if possible, or neatly drawn in black ink in camera-ready form. Please submit **four** copies to the editor and keep one as protection against loss. Submissions are blind-reviewed, so please send your full address and affiliation on a page **separate** from the manuscript. Computer files of text, and graphics where possible, will be requested later if your article is accepted.

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▲ Editorial

CO-EDITOR, TODD ROMIENS



It is with great pleasure that I assume the task of co-editing the *Gazette* with Louis Lim. After rising through the ranks of chapter rep, councillor, director, and president, it is hard to know how to continue to make a meaningful contribution to our organization. I'm sure that the role of co-editing will provide an opportunity for me to do that.

Everyone should be extremely pleased with the dedication that a select few have to the publication of our journal. From those who submit articles for publication, to Frances Schatz who does an outstanding job with the *Abacus*, Dean Murray who works tirelessly to solicit ads to help us financially, and to Penny Clemens who does a super job in getting the entire issue to press. They all help to make the job of editing considerably easier.

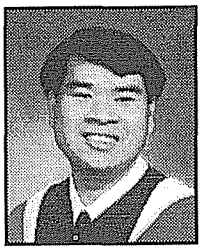
This issue again contains thought provoking articles from our colleagues in the math education community. Ed Barbeau spins an interesting web around our current curriculum; our co-editor, Louis Lim, reflects on the use of journals in his classroom; Paul Lessard and his team gives us some insight to the writing of exemplars, and Harold Reiter and Douglas Faries show us samples of the new American Mathematics Competition that Canadian and American students will have an opportunity to write in the year 2000. These articles, along with the *Abacus* material and regular columns, should make for interesting and helpful reading.

I wish you all the best of the holiday season, and encourage you to submit articles for publication and to get your colleagues to join OAME. We are especially interested in how you are coping with the new curriculum, and ideas you have which are working for you in its implementation.



▲ EDITORIAL

CO-EDITOR, LOUIS LIM



It is an honour to serve as Co-editor of the *Ontario Mathematics Gazette.* During the past several years as an OAME board of director, I have consistently expressed concerns that elementary and secondary teachers are not submitting articles for possible publication, yet these same teachers are more than willing to present at conferences. Speaking with the previous editor, John Egsgard, I believe that fear of having one's article rejected is a major factor in not submitting an article to the *Gazette.*

According to another previous editor, Jack Weiner, three criteria that your submission should have are: (i) the mathematics is accurate; (ii) the article is of interest to our members, and (iii) the article is well-written. I suggest writing your article on a topic you are particularly proud of sharing with the mathematics education community. That may include a research article, or innovative classroom activities that demonstrate students doing and understanding mathematics (with photos!).

Please send four copies of your manuscript to either Todd Romiens or I. Your article will be blind-reviewed, and will come back as "publish", "publish with modifications", or "reject". To provide that "push", I am willing to discuss possible topics, or provide a voice for you to bounce ideas. As you will soon discover, there is great professional satisfaction in knowing that your published work has influenced other teachers (and their students). Todd and I look forward to receiving your articles.



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▲ PRESIDENT'S MESSAGE



PETER SAARIMAKI

REPORT to OAME Members

The report cards are coming!! The report cards are coming!! But we don't expect anything humorous, right?

Whether you're elementary or secondary, pre-school or tertiary, reporting can be a trying time: trying to be on time, trying to summarize a lot of assessment data, trying to find the right "words", trying to convey to students and parents what you've discovered about student performance, etc. etc.

"Parents have a right to know at stated intervals how their child is "getting along." The report to parents should give the necessary information regarding the child's progress, interests and attitudes. Such reports demand, as does the whole program, that the teacher make a careful study of each child. To say that such reports will take too long to prepare is to suggest that the teacher is too busy with Education to think about the pupils." (my emphasis) from Programme of Studies for Grades I to VI of the Public and Separate Schools, 1941.

As ever, looming large, is the challenge of honestly and completely reporting on the many facets of students' performances, over a long period of time, encompassing a vast range of expectations for a curriculum with numerous strands and categories. This multitude of components, often described as the warp and woof of the program, has led many a teacher to unravel and feel they are barking up the wrong tree (sorry, just couldn't resist!).

My Report

No, you're not going to see my report card! Besides, Mom always hid them! Instead, I would like to report to you on some current musings and readings that have spoken to me of late of the "teacher condition" in the here and now.

Conditions for Learning

Rereading Baker's *How Big is the Moon?* reminded me of Brian Cambourne's conditions for learning, which I'd like to share with you here.

Brian Cambourne said, originally in Language,

Learning and Literacy, that children learn about maths best, when:

1. They are immersed in real purposes
2. They see problem solving demonstrated or modelled
3. Significant adults expect them to learn
4. They are given the responsibility of choosing what and how they'll explore
5. We acknowledge approximation as a relevant condition for maths learning
6. They receive feedback about their explorations
7. They are part of a secure environment which encourages sharing and cooperation.

Of course this idea of creating an environment that supports math learning is not new. In the Ontario "Grey Book", actually the 1941 Programme of Studies, we read that:

> *"Learning takes place most efficiently when the interest of the learner is aroused. Interest, which must not be construed to mean 'diversion' or 'amusement', is the foundation of learning. Learning takes place when pursued in connection with a worth-while purpose, and when related to a real situation."*

Learning from the Tree of Mathematics

Many recent math presentations refer to Dan Kennedy's prize-winning NCTM article *Climbing Around on the Tree of Mathematics*, usually picking up on his "there's too much for anyone to know it all" message. But he has other messages, just as relevant to the Ontario scene.

Dan uses his arboreal metaphor to discuss the litter of dead leaves from centuries of curriculum material on the forest floor. As we implement the "new" curriculum, we need to be sure we truly discard the old and no longer necessary items. Otherwise we run the risk of ensuring the curriculum only produces qualified graduates for the University stream to the detriment of other career paths.

If our curriculum already contains far too much material to cover in the year, we must keep the dead leaves from just accumulating. We need to look to the role technology can play in furthering students' comprehension of the expectations.

Numeracy Task Force

Britain recently published a scheme to rescue primary school maths, because they had found, amongst other conclusions, that some children were spending four-fifths of a lesson working by themselves. As well, results on the national curriculum maths tests showed too few students were reaching (their) acceptable level 4.

Started in September, students now have a daily numeracy hour (45 - 60 minutes depending on pupils' ages), a strong emphasis on mental arithmetic and times tables, and a training program for teachers. The teacher training, for every primary maths teacher, is on effective methods and includes demonstrations, and started with a three day training course last summer for principals, consultants and one teacher from every school.

What would help you deal with the question: "If 60¢ US is $1 CDN, why isn't $1 US the same as $1.40 CDN?" What kind of inservice would you benefit from right now? What opportunities is your Board providing? What about your OAME chapter? If they are not offering anything, why not volunteer to help organize some sessions?

Report Cards

BTW, do you have an opinion on the current provincial report cards? (Silly question!!) Soon the Ministry will be looking for input when they plan to revise the current report cards, both elementary and secondary. OAME needs your opinions and views so we can represent the best view of teachers.

Should report cards list the strands? The categories? Both? Something else?

Should reporting cover a certain cut of the program each term? Or a bit of everything each term?

Send me your thoughts, opinions, and especially suggestions on how reporting can be made manageable while remaining true to the vision of mathematics.

2000

A reminder as we enter the year 200, whether we struggle to fight the "millennium is next year" fight, or accede to public enthusiasm, UNESCO has declared 2000 as World Mathematical Year. With the focus on the special nature of 2000, this is a wonderful opportunity to give a higher profile to math and math events. Check out http://www.math.jussieu.fr/~jarraud/wmy2000/ma2000.html.

Is your school, community, or OAME chapter planning any public celebrations of mathematics? What will you and your classes do that's special, and mathematical this year - the first of the 2000's?

Happy New Year and have a happy math millennium!!

Cheers, Pierre

▲ A Shared Experience With The Ontario Mathematics Exemplars Project

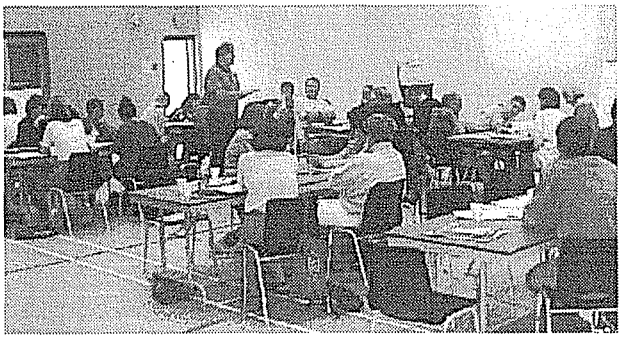
PAUL LESSARD
GEORGE FAWCETT
JULIAN D'ANGELA

Paul Lessard is Numeracy Consultant for the Hamilton-Wentworth Catholic District School Board. He has been involved with the Mathematics Exemplars Project since October 1998.

George Fawcett is a former Department Head of Mathematics for the Hamilton-Wentworth District School Board . He has been involved with the Project since February 1999.

Julian D'Angela, formerly a Department Head of Mathematics, is a Vice-Principal at Bishop Ryan Secondary School, Hamilton-Wentworth Catholic District School Board, and has been involved with the project since June 1999.

On July 26, thirty-one participants from seventeen different school boards, at all three levels, Primary, Junior and Intermediate, met with presenters Paul Lessard and George Fawcett at Pine Glen Elementary School in Huntsville to get an in-depth look at the Ontario Mathematics Exemplars Project (see Appendix A). Initially, only a few of the participants had heard of the project, although at least half of the participants had some degree of familiarity with rubrics as an assessment tool. At the end of the four-day session, SC60 (see Appendix B), participants had moved from the novice position to a state of readiness, eager to use existing learning tasks, use rubrics, choose other learning tasks, and to create their own rubrics.

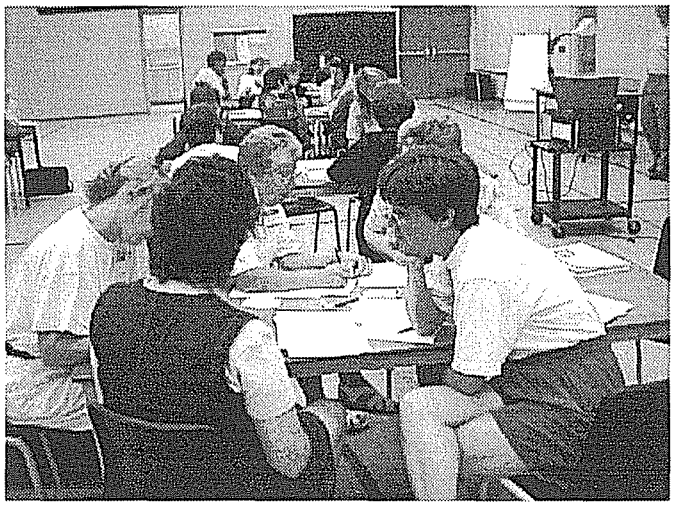


Here are what participants had to say about the Exemplars Project and the use of rubrics in assessment:

"I feel more confident in creating rubrics (myself or with colleagues) and in their implementation."

"Nice to see that I am on track with marking. All examples from the course will be used."

"The knowledge of rubrics does not stop at math. I can extend it to other subject areas."



To understand how this transition occurred in a time frame of five hours per day, over a four-day period, a brief outline of the events are necessary. On day one, participants were given an overview of the Exemplars Project; then they quickly turned to the task of marking! Each participant marked a common student paper, using a prepared rubric with four levels. They worked through a process that we call pair share/table share to reach consensus on the level of the student work. Through extensive discussions, all participants were surprised and reassured to learn two things:

i) there was close agreement among all groups/tables, regardless of division;

ii) their ideas and opinions were shared by their colleagues, regardless of division.

That marked the beginning of the confidence-building process that would culminate in the activities on day four.

On the second and third days, the main focus was on grade-specific tasks. Each teacher used a prepared rubric to assess four student papers, again using the pair share/table share process. The comfort levels continued to increase as teachers made decisions about the level represented by each paper and shared their reasons for each assessment. All participants became more confident that - "Yes, I am on track here!" Over these two days, discussions focussed on what made the student tasks "levelable"; that is, what in each task provided for level 4 performance, and how might each task be extended to make it more powerful. Also as part of their growth towards confidence to develop their own personal rubrics, teachers chose a problem solving idea from a collection of suggestions, adapted it for their grade level and developed an appropriate rubric from their perspective. They then used the table share process to "fine-tune" their rubric through discussions. Finally, each group presented their task and rubric for further discussion and dialogue.



On day four, the group focussed on a grade specific task. Each teacher developed an appropriate rubric for the grade specific task by working through the discussion process, and used this rubric to assess two common student papers. Participants were pleasantly surprised again to see consensus on the assigned levels for these student papers, even though different rubrics were used. Participants now believe that they possess the confidence, in discussions with grade partners in their own schools, to develop learning tasks, create appropriate rubrics, assess student papers, and to be comfortable with the reasons for their evaluation decisions.

At the end of the course, participants made comments such as:

"This was an excellent course - the presenters were knowledgeable and willing to share their materials and expertise. The opportunity to network was invaluable."

"We had opportunities for discussion, sharing and growing together. We saw we were on the "right course" in scoring . . . it was good to be positively reinforced."

"Paul and George are very knowledgeable in this subject area. They presented the information in an extremely confident manner. Their flexibility, good senses of humour and energy were much appreciated!"



As presenters, we were immensely pleased with the group's re-examination of the curriculum expectations using new evaluation tools, and we were equally impressed with the group's enthusiasm - for discerning the characteristics of "levelable" problems, for their use of rubrics in assessment, and for the moderation process in their decision making to build consistency and confidence. This experience, and others like it, bodes well for the future of the Exemplars Documents in Ontario Elementary Schools.

The teachers in Huntsville are eagerly awaiting the release of the Exemplars Documents - with their forty rich investigative tasks. The tasks have a similar format. Each task begins with a full-class component where the teacher introduces the scenario. The teacher will often review the background knowledge involved and pose a question that the students will discuss as a large group.

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Next the class breaks up into small groups to explore an extension to the original problem using concrete materials, active learning and group interaction. Having gone through an extensive investigation of the mathematics surrounding the problem, then each student is prepared to begin the "On My Own" section. Here the student is presented with a situation similar to the previous investigation(s), where the problem is given a real-world context; each student must internalize the problem and devise a strategy. During the investigation, the student is often confronted with a need to justify the strategy chosen and the relevance of the solution given the real-world context. There is usually one additional twist to the task; another piece of information is often introduced, which may require rethinking by the student. Again the student is challenged to explain the thinking, justify the strategy and the relevance of the answer. The Exemplars Documents will contain each investigative task, teacher's instructions, selected answers and samples of student work in each of the four levels. Look for the release of these documents early in 2000!

Appendix

A: The Ontario Mathematics Exemplars Project, is a project developed through the leadership of the Ontario Ministry of Education, coordinating hundreds of teachers from a cross section of Ontario District School Boards. Rich investigative tasks and level-specific rubrics were created; field tests using these tasks and rubrics were completed across the province. Groups of teachers marked the papers and chose representative papers for each of the four levels. This foundational work by hundreds of educators makes the Exemplars Project a reality; ultimately the results will be published, with one book per grade, containing the tasks, student samples by level and teacher comments explaining the indicated levels. It should be noted that the teachers of Ontario will be receiving additional Exemplar Resource Documents: Reading, Writing and Science. This session SC60 was most timely in helping teachers prepare to get the most value from the soon-to-be-released Mathematics Exemplar Document.

For further information about the Exemplar Projects at the Elementary Level, please contact the Curriculum and Assessment Branch of the Ministry of Education and Training, Mowat Block, 16th Floor, 900 Bay Street, Toronto ON M7A 1L2.

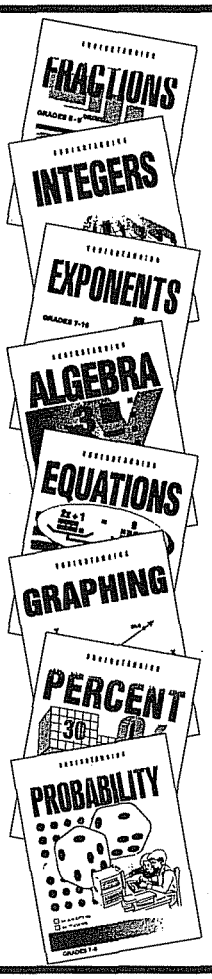
B: SUMMER INSTITUTES Session SC60 Math Exemplars-Each Strand-Unique Assessment Strategies was one of the Summer Institutes, an excellent program funded by the Ontario Ministry of Education, and implemented by OECTA/OTF. There were approximately 53 different topics offered throughout Ontario during the summer of 1999, with about 2500 participants from elementary schools. Many teachers considered these summer sessions as a major opportunity for professional development. Certainly, participants in SC60 have clearly stated their plans to attend future sessions, as comments such as the following indicate:

"Keep it up. Reduced PD days makes this a necessity."

"Summer is when we have time to attend and absorb!"

"More courses for summer and opportunities for (teachers from) northern boards to attend."

"Please don't limit us to only two (courses in a summer)!" ▲



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UPDATED SEPTEMBER 1999

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Men as Trees Walking

ED BARBEAU
UNIVERSITY OF TORONTO



Ed Barbeau has been teaching mathematics at the University of Toronto for 32 years. He is keenly interested in keeping the lines of communication open between the high school and university mathematics communities, and between mathematicians and the general public. He has often been a guest on CBC's Quirks and Quarks and welcomes invitations to visit high schools.

This article is based on a plenary talk given on May 29, 1999 at the Summer Meeting of the Canadian Mathematical Society in St. John's, NF.

"Men as trees walking" – that was the phrase that came into my mind as I examined the draft *Principles and Standards of School Mathematics* **(1)** recently issued by the National Council of Teachers of Mathematics. The phrase is from the Bible (Mark 8:22-26), where a curious miracle is described. A blind man touched by Christ could at first see only "men as trees walking" and required a second touch in order to see clearly.

One has the same feeling about the *Principles and Standards*; while it is generally praiseworthy, it is hard to bring the document into clear focus. It contains a mixture of recommendations about topics, processes, teaching styles and general philosophy, but as a lecturer of first-year university students, I did not get a clear sense of what I would be able to count on from the students in front of me. It seems that there is such a plethora of ideas put forward that perhaps a second healing touch is needed to tease out clearly the main threads.

The central issue became clear to me one evening as I watched Rob Buckman on TV Ontario present a documentary on alternative medicine. Many people were alienated from traditional medicine which they found too reductionist and narrowly focussed. Whatever the specifics, alternative medical regimes were attractive because they were holistic, proceeded from a broader worldview and significantly involved the patient in the diagnosis and choice of treatment. Cases that might seem identical to a traditional doctor might receive quite different treatments for patients in different environments. Surely something like this is behind the pressure for reform in education. Teachers and students are reacting against a curriculum which seems to be reduced to a list of topics and processes, against an imposed canon robbing students and teachers of their autonomy. Whatever the details might be, we want students to be intimately involved in an educational process that cares about who they are and what characteristics they bring to the mathematics class.

But the charge that traditional education (as traditional medicine) has consistently lacked the human touch is far too stereotypical; it has resulted in a number of ghosts that have haunted recent educational reform. Let us raise a few of them.

Ghost 1: failure, streaming, elitism.

There is no doubt that school was a brutal experience for many students in the past, but the fact remains that success in mathematics depends on a certain level of ability and application. To deny students the opportunity to fail is also to deny them the opportunity to enjoy success. This is not a call to ignore students who are floundering, but rather to create conditions that do not neglect the imperatives of learning mathematics and the possibility for achievement, but do provide support in which these might be realized and students can move on in confidence. Many students now are uncertain about what they can do or should know; even good students are denied the chance to demonstrate what they are capable of. While there is much that can be done to make mathematics more generally accessible, it needs to be recognized that the subject is often difficult and beyond the ability or interest of some segment of the population.

Ghost 2: the syllabus, list of prescribed topics, facts and procedures.

The charge here is that having a *list* is too confining and leads to an emaciated mathematics that is reduced to a disembodied set of facts and routines. It is not clear why this should *necessarily* be the case. In fact, the criticism seems to be misplaced; it should be directed at matters of design. In the hands of a teacher familiar with mathematics, the syllabus can serve as a set of markers and goals that frame what will happen in the class. An

uncertain or ignorant teacher will hold to the list without any regard to connections and deeper understanding.

Ghost 3: drill, rote learning and memorization.

Memorization was an important part of traditional education, and many cultures put a great deal of emphasis on what children should *remember.* Children seem to have good memories, and it seems foolish not to take advantage of this. But they need to be taught to evaluate what is worth memorizing, how mastery can be reached and how they can exploit the coherence of mathematics to leverage their knowledge of a few facts into fluency over a larger domain. The most able children can do this naturally; others need to have the issue explicitly addressed, so there is an underlying equity issue involved here for students without natural talent.

Ghost 4: arithmetic.

This is a word that seems to have fallen upon hard times in the curriculum stakes. But it is through arithmetic that most ordinary citizens engage the connection between mathematics and the world, and lack of numeracy can present a severe handicap. Arithmetic has come to symbolize mindless memorization and manipulation. We need to detach it from this calumny and exalt it to the level of mathematical richness that it deserves.

Ghost 5: paper-and-pencil algorithms.

This also is tarred with the brush of drudgery, and the advent of modern technology has provided detractors of traditional calculation with a pretext for summarily discarding it. The issue is really how traditional arithmetic should be handled in a modern curriculum. Perhaps we need to see them more as an additional means of accustoming students to working with figures or as examples of algorithms. Long division seems to be particularly suitable for indicating how one can move from the idea of tallying a continued subtraction to a mechanical algorithm which is fast and accurate; one can point out that it was a human invention and replaced a method that was decidedly inferior (the "scratch method"). What has changed is that the importance of paper-and-pencil methods as a practical technique is reduced and we now have alternatives for reaching children encountering difficulties.

Ghost 6: word problems.

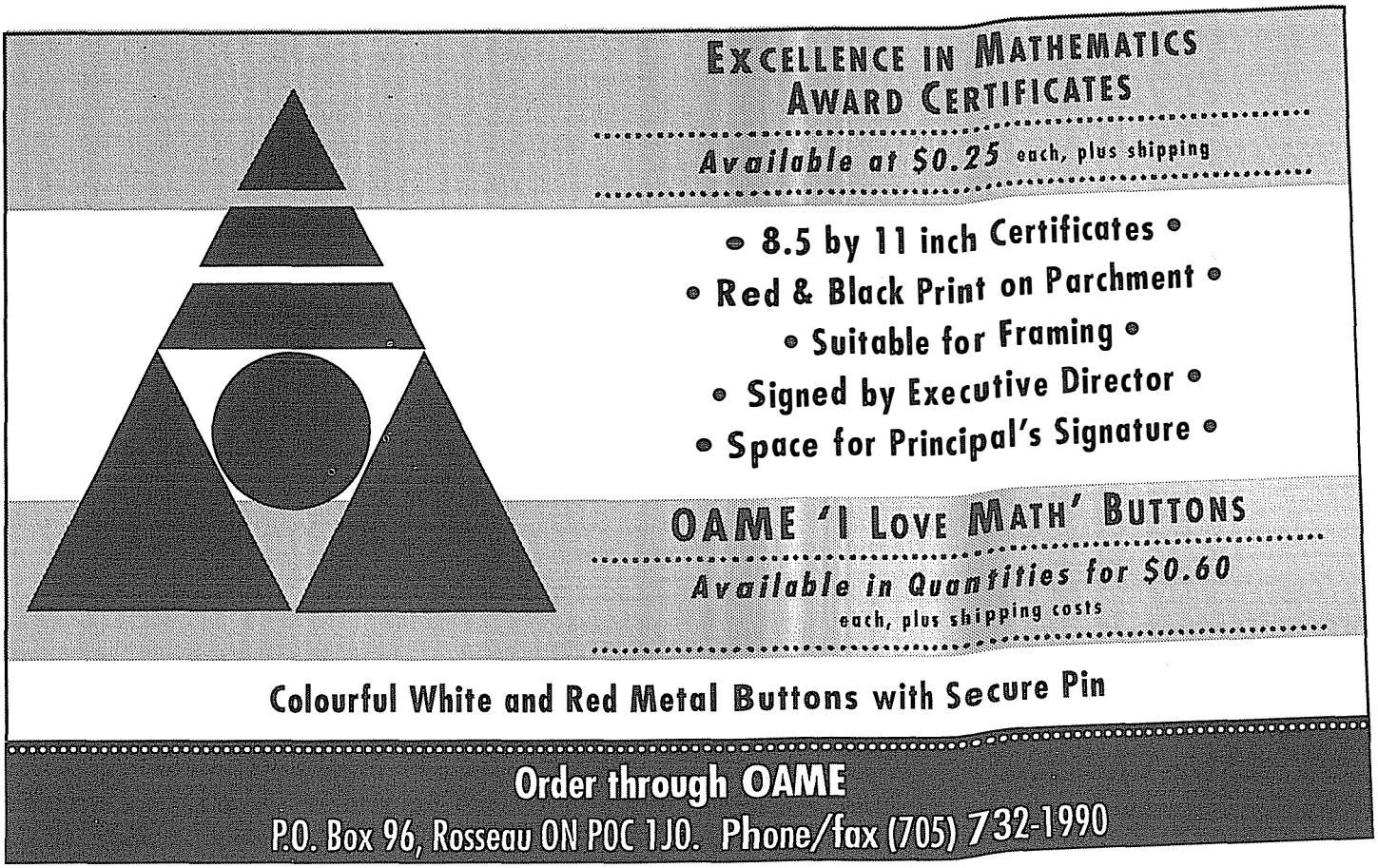
Traditional word problems are criticized as being artificial, but one can argue that this is precisely the point of most word problems. They provide a toy situation in which certain points about interpretation, and formulation can be made. The question again is not whether we should retain word problems, but how appropriate they are to the situation and how we plan to move beyond them.

Ghost 7: authoritarianism.

This ghost has two manifestations, depending on whether you are referring to the teacher, who, we are told, should be the "guide on the side" rather than the "sage on the stage", or to the subject itself in which pupils are oppressed by the tyranny of "one right answer". Without disputing the advantages of the more open and friendly classrooms that modern students enjoy, it remains the case that a teacher's effectiveness depends on what she knows and that sometimes students need to submerge their egos and pay attention to what she has to say. In the same way, it seems mischievous to deny the power of mathematical certainty, especially given the diversity of the ways in which one might think about concepts and approach problems. (One feels that the work of Gödel and Lakatos, however lauded by serious mathematicians, have had a particularly pernicious effect on some mathematics educators seduced by the vamps of relativism.) There are many ways to encourage the individuality of pupils without permitting them to believe that black is white.

All of these Ghosts are the traces of essential components of mathematical education in the past which had better be part of the future as well. Children are going to either succeed or fail at any worthwhile task, and the question is how humanely the failure is handled and whether pupils are held back or advanced for frivolous reasons. Mathematics is an hierarchical subject and we need to spell out what students need to master at each stage; the issue of the syllabus is one of design and focus. We cannot deny the need for practice; the question is whether the student has the strategies and perspective to learn and memorize efficiently and effectively. Arithmetic and the standard algorithms are as important as ever; what we need is to be sure that they are put into the proper context and conceptual framework.

These Ghosts are accompanied by a number of Sirens that drive a lot of educational reform. Like the ghosts, the sirens also speak to important aspects of mathematics education. Here are some of them.



Siren 1: problem solving and investigation.

This siren calls us away from the ghost of drill, of dry and unilluminating exercises. There is nothing wrong in wanting our children to solve problems and explore mathematical situations, but we can run into serious distortions if we do not take the trouble to ground them mathematically and psychologically.

Any problem we present to children should be carefully analyzed for its mathematical content and appropriateness to the maturity of the pupils. Strategic decisions need to be made as to what mathematics has to be presented beforehand as background and what can be brought out in the analysis of the problem. Let me give an example that I have used with students and prospective teachers.

Let ABCD be a unit square and let E and F be the respective midpoints of sides BC and CD. The three line segments AE, AF and BF partition the square into five regions. It is required to determine the area of each region.

There is actually quite a lot in this. At what point should this example be introduced, before or after the pupils see the area formula (half-base-times-height) for triangles? This will govern how they might approach the problem. If the students are to try a structural rather than a formulaic approach, they might need at least implicit understanding of isometries and might need to understand that areas of nonoverlapping sets add, that areas are invariant under rigid transformations and that they vary as the square of the factor of a dilatation. Should some of this be discussed ahead of time, or can it emerge as the problem is covered? If the students exploit the similarity of the two subtriangles of ABE, they may need to know that AE is perpendicular to BF; what tools can they be expected to deploy? Will students who assign letters to the five regions have the necessary algebraic understanding to proceed, or is this a nice vehicle to introduce them to this approach? Finally, will the pupils be able to handle the fractions? Are the fractions appropriately expressed? When we take all of this into consideration, this example could take quite a bit of time to do properly - has that been anticipated by the teacher?

The problem-solving approach to the curriculum has a great deal to be said for it, but it does require imagination by the teacher to envisage what might happen and, importantly, what can possibly go wrong. It

is risky, and we should be sure that teachers are equipped to accept the risks.

Siren 2: relevance; real-life.

Mathematics was seen to be alienating because of the artificiality of what we were asking students to do. We should make reference to the daily lives of the pupils and concern ourselves with what they will need to succeed in their later careers. But what passes for relevance often raises the question "for whom?". It seems that children often have the phony relevance of a tedious arithmetic problem dressed up with clowns and cookies or are brought into the world of adult concerns in the name of relevance. *Play* is part of the world of a child, and the unadulterated world of mathematics gives lots of opportunity for this. I have never been aware that some nice number, geometric, topological or combinatorial novelties have been beyond the pale for the children I have had the pleasure of presenting mathematics to.

Siren 3: patterns.

No mathematician can gainsay the importance of a sense of structure in doing mathematics, and the ability to recognize pattern is an important part of this. What has been forgotten in a lot of modern educational reform is that the power of patterning is seen in its use in analyzing and generalizing mathematical situations. Much of what passes for patterning is a kind of teacher "guess-my-rule" game and rather ad hoc examples. There is no excuse for this. It is hard to progress through the curriculum without finding natural opportunities to exploit patterns, and teachers need to be alert to this and make them explicit to their pupils at the right time. One aspect that seems to be neglected is giving the students the mathematical voice to describe and analyze patterns.

Siren 4: data analysis.

We do not have to look very far to realize how much of our daily lives is governed by a flood of data of one sort or another. So there is certainly a duty to help pupils become number-wise and to interpret what they read astutely. But the danger is that we do not just cover a lot of canned techniques and introduce jargon, which may in some cases stand in the way of good discussion and analysis.

Siren 5: technology.

First, let me agree that through modern technology we are undergoing changes in our lives that are at least as profound as those induced by the invention of the printing press and the Industrial Revolution. But any revolution, no matter how pervasive, is not completely disjoint from the past. Many of the issues raised by technology are old ones, and the charge of misuse and mindlessness can be, and has been leveled against Arabic numeration, algebraic symbolism, logarithms, slide rules, mechanical adding machines and numerical algorithms of all types whether executed on paper or on a bench with pebbles. Certainly, the modern computer has greatly expanded both the range of the mathematics we *can* do as well as the mathematics we *want to* do; our curriculum should reflect this. But we do not need idolatry. Technology is *there*, part of our environment, as are books and pens, and a general purpose of education is to produce students who can understand and work within their environment. There are core mathematical issues that are independent of technology, but can be greatly informed by our use of technology - it is in this spirit that we should embrace the use of calculators and computers in our classrooms.

All of the sirens speak to important aspects of the modern mathematics curriculum, but they all involve subtle issues and bring the risk of being trivialized. The big question is whether we will have teachers in front of our children who will handle the issues in a sensitive, moderate and intelligent way.

In designing a curriculum, we need to keep both the Ghosts and Sirens in mind, for each of them not only speak to important goals but also carry a warning of distortion and counterproductivity. There are a number of factors of which we need to be cognizant.

1. For a body of mathematics, I believe that there are three stages that a learner must pass through, *initiation, formalization* and *consolidation*. In the stage of initiation, the learner encounters the ideas in a somewhat haphazard way, feeling her way about, exploring; there has to be some motivation, some reason that the person is interested in the material at hand. At the formalization stage, the ideas are drawn together and organized; at this point, the pupil should learn proper concepts, processes and conventions. There may indeed be a lot of artificiality; the payoff should be a growth in the knowledge and mathematical power of the learner. In the consolidation stage, the learner reflects upon what has gone before, contextualizing it, detecting relationships and connecting it to other material. It may be only here that the learner truly appreciates the reasons for what she has been taught before.

OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration;

- demonstrations of excellence and creativity in mathematics education.
- specific contributions to mathematics education in the school, region, province or nation.
- demonstrates leadership in mathematics education in the school, region, province or nation.
- support of and contribution to the activities of OAME/AOEM.
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld form consideration.

OAME/AOEM LIFE MEMBERSHIP AWARD

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and accumulated 10 or more years of membership in OAME/AOEM.

KENNETH D. FRYER AWARD

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

DON ATTRIDGE AWARD

In recognition of the contributions made by Don Attridge, a founding member, former president and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

AWARD NOMINATION COVER PAGE

Send to: Shirley Dalrymple

Intended Award (please check one)

❑ Life Membership ❑ Mona MacGregor ❑ Morley MacGregor ❑ Kenneth D. Fryer ❑ Don Attridge

Nominee Information:

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Name of Nominee (Principal and Department Head for K.D. Fryer Award)

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School, Organization or Business Address

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Nominator Information:

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Name of Nominator

_____

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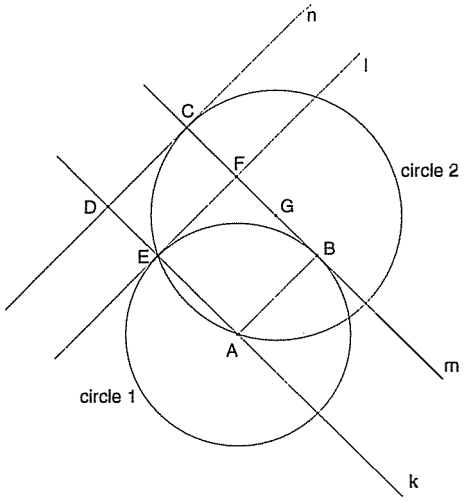
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Traditional education has emphasized the second of these stages while modern practice seems to focus on the first and third without the buttress of the second stage. A good curriculum allows for all three stages to occur. The first signifies to the pupil that the subject matter *could* belong to them, the second provides the tools to *allow* it to belong to them and the third ensures that it *does* belong to them.

2. Have a strong focus and a slender core for each course in the curriculum. Make sure that context is established, the purpose of the material becomes clear, there is enough depth to support its assimilation and allow students develop the necessary skills and understanding to proceed.
3. Have one or two attainable goals for each year; plan to achieve them and *move on*. This means that the sort of sterile spiralling that now occurs in schooling should cease, but does not preclude returning to previous material to inform and consolidate it.
4. Have enforceable entry requirements for secondary and tertiary courses. No teacher should be asked to deal with a student who does not have reasonable prerequisites for the material to be studied. There is an important pedagogical purpose in requiring students to review and organize in their minds the work of several months or a year; it is this process that makes purposeful curriculum progress possible.
5. Change pace. The diversity of the mathematical enterprise should be reflected in our curriculum, whether with respect to subject matter, level of discourse or application. There are many ways in which people can think about or do mathematics. Therefore, we need space to help students find their mathematical voices and texture their ways of thinking about and doing mathematics.
6. Orchestrate the material. Increase the level of complexity judiciously, make sure that foundational material is covered and the student is psychologically prepared for what is to follow. We need to analyze in much more detail what students are asked to do; this is where members of the University community can be particularly helpful. Too often problems are thrown at pupils with little appreciation of what needs to be in place to embark upon them. A sound curriculum needs both exercises and problems of many different intensities.



Instruction for Constructing a Golden Rectangle

1. Draw two points, A and B, and construct a segment AB.
2. Construct a circle (circle 1) with center A, passing through B.
3. Construct a line k, passing through A and perpendicular to the segment AB.
4. Let E be the point of intersection of circle 1 and line k. Construct a line l, passing through E and parallel to the segment AB.
5. Construct a line m, passing through B and perpendicular to the segment AB.
6. Let F be the point of intersection of lines l and M. Construct the midpoint (point G) of the segment BF.
7. Construct a circle (circle 2) with centre G, passing through E.
8. Let C be the point of intersection of line m and circle 2. Construct a line n, passing through C and parallel to the line l.
9. Let D be the point of intersection of lines k and n.
10. Rectangle ABCD is the golden rectangle.

Figure 1 - Golden Rectangle Construction

This is one area in which the *Principles and Standards* seems to be particularly weak. There are a number of examples thrown in, that upon closer analysis seem to involve a great deal of which the proponent might not be aware. On page 92 is a set of instructions for constructing a golden rectangle (see figure 1 above). This seems as though it might be pretty heavy sledding for a typical student, and any teacher who embarks on this without careful thought is wading into a treacherous swamp. Why should pupils be interested in a golden section, or in ways of constructing such a thing? Will most pupils muster the necessary level of concentration to negotiate the ten-stage unmotivated set of instructions and understand why it works? This example is meant to illustrate the connectedness of mathematics, but it seems to me to require such a level of maturity and mastery of some basic algebra that it could easily spin out of control in the hands of any but the most adept teacher.

7. Meet different needs. This is a reason we can give for teaching anything at all that applies to mathematics. Some mathematics is taught so that students will be able to accept the privileges and responsibilities of

citizenship, some to situate them in the rich culture that they are entitled to inherit, some to provide recreation and additional options towards a full and rewarding life, and some for professional preparation. These needs can sometimes be met by the same piece of mathematics, but all should inform the curriculum that we set.

8. Foster sound practice and mental attitudes. How well students perform in mathematics seems to depend on the worldview that they bring. In designing a curriculum, we should encourage an attitude of mind that involves the following:
   (a) Awareness of structure; appreciation and exploitation of symmetry;
   (b) Flow of ideas, analysis and reasoning;
   (c) Corroborative quality of mathematics; inner consistency;
   (d) Shifting of perspective;
   (e) Checking and monitoring one's work;
   (f) Organizing and polishing one's work;
   (g) Mathematical register; expressing ideas appropriately; the place of heuristic and formalism;
   (h) Sense of context;
   (i) Attainment of power through understanding, reasoning and technical facility;
   (j) Visualization; appropriate imaging;
   (k) Appreciation of symbolic representation: numeration, algebra, diagrams;
   (l) Making distinctions: classification, equivalence, isomorphism, congruence;
   (m) Grasping the interplay between concrete and abstract; progression to higher order structures that in turn become "concrete".
9. Finally, we come to the subject matter itself. The centrality of arithmetic and algebra must be affirmed. No student can succeed at the secondary level without a good grasp of arithmetic, nor succeed at the tertiary level without a good algebraic grounding. But these are not all. Geometry and combinatorics are also indispensible. While students should see how mathematics can be applied in different areas, it is not clear to me that this should necessarily occur in the mathematics class. Science, shop, civics, geography and music are areas in which important mathematical concepts can be conveyed in an appropriate and powerful setting. And we should not forget how many important mathematical concepts and processes underly some recreations and puzzles. In fact, if the elementary and secondary curricula are well-designed, one could devote the middle school years largely to recreational and cultural issues as a way of consolidating what should have been learned at the elementary level and preparing the ground for a more sophisticated high school program in which symbolism, algorithm, analysis and reasoning have important roles.

I would like to close with two courses that might be given at the levels of grades 9 and 10. The first is designed to give a systematic introduction to algebra and the second to geometry. It is assumed that students have gone through an initiation stage in both areas, that they have some familiarity with formulae and the use of letters to represent numerical quantities, and that they have had the opportunity to play around with geometric objects either through tactile or computer models. While technology is not explicitly mentioned in either syllabus, it is understood that it can play a large and appropriate role. These syllabi would be accompanied by resources for the teacher that lay out alternative approaches as well as useful exercises, problems and investigations, and that make explicit the pedagogical and mathematical goals that are brought to light through the material.

Course 1: Linear and quadratic functions

1. The linear equation ax = b. Problems that lead to an equation of this form.
2. The equation of the straight line; slope; various forms.
3. The solution of a system of two linear equations in two unknowns using numerical, algebraic and graphical techniques. Consistency of two linear equations.
4. Factoring difference of squares.
5. The quadratic: factoring over reals and integers, completion of the square, quadratic formula, discriminant and its significance; complex numbers (real quadratic having complex conjugate roots); relation of sum and product of roots to coefficients; first and second order differences; the factor and remainder theorem (for quadratics).
6. Graphing of quadratic functions; comparing the graphs of the functions f(x), f(x - c), f(ax), f(x) + c; similarity of all graphs of quadratic functions (role of completion of square)

This is the entire prescription for a full-year course. In some classes, it may be necessary for the teacher to cover only this material and nothing else, spending the

time and introducing whatever additional materials are necessary to ensure that students become fluent in the essential components. Normally, one would hope that the teacher would extend the material in some way, either through the introduction of applications, extended investigations or additional topics. It may be that the more able students in the class are given modules to work on independently possibly with the support of additional pamphlets or computer software. I do not see much point in introducing algebra tiles to the class at large - they amount to an additional code interpolated between the student and the standard code - but they may be useful for individual students who have trouble getting an initial grasp of algebra.

Other topics that might be introduced are: motion of projectiles, history of solving equations, linear and quadratic diophantine equations, Pell's equation, quadratic residues, first and second order recursions (characteristic equation), analysis of the dynamical system $x \rightarrow \lambda x(1 - x)$, polynomials of degree exceeding 2; complex numbers (conjugates, modulus, geometric interpretation of sum, product and inverse, roots of unity up to the fourth); calculus of finite differences applied to polynomials interpolation and extrapolation; linear inequalities.

Course 2: Geometry of triangles and circles

1. Triangles: congruence theorems; ambiguous case; Pythagorean theorem; similarity.
2. Trigonometry: six standard ratios and basic identities ($\cos^2 x + \sin^2 x = 1$; $\sec^{2x} = 1 + \tan^2 x$); sine, cosine and tangents of angle sums and differences; double angle formulae; conversion formulae between sums and products; Law of Sines; Law of Cosines
3. Circles: subtended angles; cyclic quadrilaterals; tangent theorems
4. Coordinate geometry: circles; area of triangles; family of lines passing through pair of intersecting lines; circles passing through intersection of a pair of circles; radical axis; simple loci problems

Again, the teacher would have the discretion of reinforcing these core topics for students encountering difficulty, amplifying the material through applications and investigations, or moving to additional topics. These could include the following: complex numbers (used in deriving geometric and trigonometric results); vector geometry; statics; loci (conjectures, verification and proof); solid geometry; advanced Euclidean geometry; transformations and their uses (isometries and similarities; composition of isometries; isometry determined by three points; isometries generated by reflections)

The modern school, even at the secondary level, has to serve students all across the spectrum of intelligence, ability, motivation and interest, often in the same class. The only way that the system can cope with this situation is that students are trained early to become autonomous is choosing their goals and more self-propelled in accessing the resources needed to learn what they need to know and in seeking validation for their knowledge. Any system of education predicated on the teacher being the sole source of knowledge and evaluation for the student is bound to fail; the more able and motivated will fall far short of their potential and others will not be able to really engage the material. Students must first reflect within themselves what they value and how far they are willing and able to go in achieving their goals. A successful school regime depends on certain beliefs and characteristics of students as well as the knowledge and experience of teachers; unless we recognize the need to start with this, then our principles and standards will become an exercise of merely trying to stave off the most trenchant critics from all sides and leave us seeing "men as trees walking".

Reference

1. *Principles and Standards for School Mathematics: Discussion Draft* (October, 1998) National Council of Teachers of Mathematics ▲

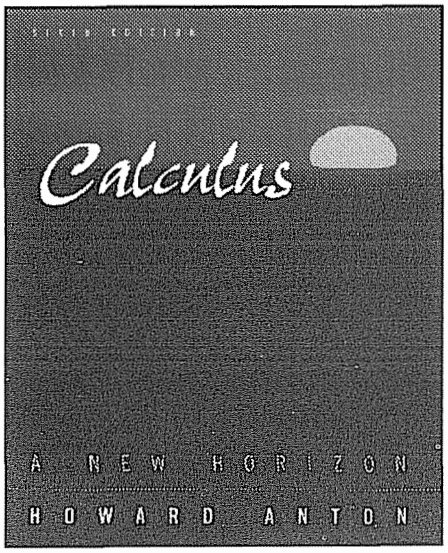
We know

$$\frac{1}{3}+\frac{1}{5}=\frac{3+5}{3\times 5}=\frac{8}{15}$$

show that if 8 and 15 are 2 sides of a right angled triangle, the hypotenuse is a whole number, and also equal to the next odd number after 15.

Can you prove this true for any 2 fractions with consecutive odd numbers in the denominators and 1 in the numerators?

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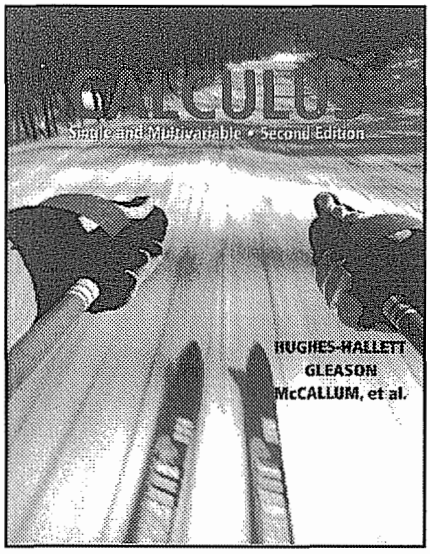
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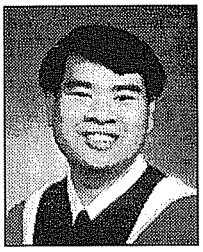
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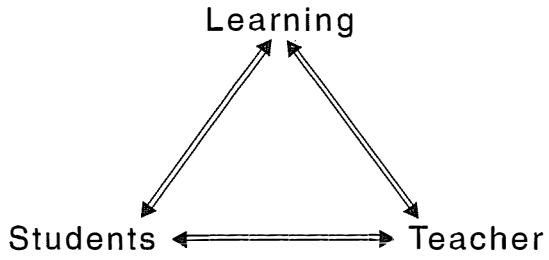
▲ Implementing Journal Writing in the New Grade 9 Mathematics Curriculum: Why & How?

LOUIS LIM
QUINTE SECONDARY SCHOOL, BELLEVILLE, ON



Louis Lim is a fourth year Mathematics teacher with the Hastings and Prince Edward District School Board. He is currently an elected OAME director, serves as Co-editor for the Gazette*, writes the Director's Dialogue for the journal, and enjoys presenting workshops at conferences. Louis is 50% completed his Master of Education degree at Queen's University, and during the past summer, took "Measurement and Evaluation". Under the direction of Dr. Lynda Colgan, he researched the "why's" and "how's" of implementing multiple assessment strategies in the new grade 9 mathematics curriculum for his course project.*

Jenn Kerr Stenmark, in *Assessment Alternatives in Mathematics: An Overview of Assessment Techniques That Promote Learning* (1989), states: "Communication in mathematics has become important as we move into an era of a 'thinking' curriculum ... writing in mathematics classes, once rare, will now be vital" (p. 11). Ed Bernard (1994) argues that our mathematics classes should prepare students to be successful in their futures: "It is important that they can communicate what they are doing to fellow workers, clients, supervisors ... the business world and post-secondary schools want graduates who can communicate clearly and creatively ... who can describe as well as apply what they know" (p. 25). The National Council of Teachers of Mathematics (NCTM) lists "communication" as one of the ten curriculum standards in their soon-to-be released *Standards 2000* document. Thus, implementing journals in the mathematics classroom can create what I call the "triangle of learning":



Journals put student learning at the forefront of assessment. Borasi and Rose (1989) argue that journals (i) allow students to express, reflect, and consolidate their knowledge, feelings and beliefs of mathematics, make connections, become aware of strengths and misconceptions; (ii) journals also provide immediate feedback for the teacher to assess students' learning and misconceptions (rather than discover them on a summative evaluation such as a unit test), and (iii) strengthens the relationship of students and teacher as journals provide open dialogue (p. 352). Waywood (1992) states that "there is evidence that a number of years of journal use changes students' attitudes to mathematics and learning mathematics" (p. 37).

Although there is no denying that implementing journals in the mathematics classroom will create more work for the already overworked teacher, the research of Borasi, Rose, and Waywood state that there are many gains in the process. Not only do students enhance their communication skills, but teachers can become more effective in the classroom by addressing students' difficulties as expressed in their journals. Perhaps most satisfying for me as a classroom teacher is that students will feel the teacher truly cares about them through the supportive feedback in the journal. Could that be equated with fewer discipline problems?

As a teacher, I am sometimes guilty of equating assessment and grading. If assessment is to improve and enhance student learning, then I do not believe that journals should be graded for a mark. Rather, journals could be used to **assess** student learning. Thus, I discourage correcting spelling and grammar as the focus is for students to reflect, express and consolidate their learning. Enhancing rapport with students is difficult when every spelling and grammatical error is corrected

in red ink.

Besides frequent comments and feedback, students could be provided with the "ICE" model, created by Queen's University professor Robert Wilson: "a portable growth model that attempts to mirror the usual pattern of development when a learner acquires a new skill. At first, the learner focuses on one or two central **I**deas, then makes some **C**onnections with those ideas and his or her own experience, and may, given time **E**xtend those learnings to the level of maxims" (p. 135).

I believe that we can use Wilson's "ICE" model to generate the types of communication that we expect in students' journal writing. Using the research papers of Borasi, Rose, and Waywood, I have created general descriptors for students to write about:

IDEAS	CONNECTIONS	EXTENSIONS
• Express, reflect (feelings) • Share your thoughts • Engage in a dialogue • State math concepts and procedures accurately • Clarify misunderstanding • Explain concept • Summarize • Define	• Connect learning of mathematics (unfamiliar to familiar) • Apply to increase understanding (e.g., problem solving) • Relate • Integrate concepts • Become aware (e.g., strengths, difficulties)	• Create new problem situation • Explore • Extend learning outside of class • Learn about oneself to find positive change • New awareness to own life • Solve extension or supplementary problems

To demonstrate the power of "ICE," I have created a bank of questions for journal entries using the Relationships unit in the Addison Wesley textbook, *Mathematics 9:*

Ideas

(2.1) Pick five different rides at Canada's Wonderland or another amusement park, and sketch a height vs. time graph. Also, explain what is occurring at each phase of the graph.

(2.2) A friend is having difficulty deciding which variable goes on the x-axis, and which goes on the y-axis. Clarify that confusion.

(2.2) When should you join the points on a graph with a smooth curve? When not? Explain your reasoning.

(2.3) Explain what is meant by a "mathematical model."

(2.4) What do you feel are the advantages of using a graphing calculator (or Calculator-Based Laboratory (CBR)) in the mathematics classroom?

(2.6) Your parents want to know what you learned in mathematics class today. Describe what is meant by "proportional situation" so that they understand what you learned.

Connections

(2.2 & 2.6) State one advantage and one disadvantage to each of the following representations: tables of values, graphs, equations.

(2.2) When two quantities are graphed, must they always produce a straight line? If not, provide a real-life example which is non-linear.

(2.3) Which model, in relating animal and human life, best depicts the relationship of age? Explain your choice. Also, why would one want to use the other models?

(2.5) Design an experiment to determine if eating breakfast is a factor towards success in school.

Extensions

(2.2) Reflect on the concepts of the unit. Describe five real-life situations of how quantities are related.

(2.2) Looking in the newspaper, or watching the evening news, apply your understanding of relationships using the concepts of the unit.

(2.5) Design an experiment of interest to a medical doctor which demonstrates relationships.

To successfully implement journals in the mathematics classroom, I believe that it needs to be introduced at the beginning of the semester. Students should be aware that journals will play a vital role in learning mathematics, and be given a copy of the types of descriptors generated using "ICE." Margaret McIntosh (1991) reassures us in her research that "most mathematics teachers find that if they persevere in having students write in mathematics class beyond the first week or two of students' moaning and groaning, they see positive results for both learning and teaching in mathematics" (p. 432).

I believe that students should write in their journals during class time at least twice a week (10 minutes each). To encourage writing at the "ICE" levels, I suggest providing the questions for students to write about, rather than leave it totally open-ended. I believe that journals provide students with excellent opportunities not only to reflect, but to consolidate their learning. To maintain confidentiality, I believe the journals should be stored in a secure place. It is important that teachers provide regular feedback in the journals every couple of weeks. As stated, journals

provide extra work for teachers (e.g., having to read them), but the many gains from this assessment strategy make me inevitably ask, "As professionals, can we do without journals?"

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▲ The American Mathematics Competitions →10

HAROLD REITER
UNIVERSITY OF NORTH CAROLINA CHARLOTTE
DOUGLAS FAIRES
YOUNGSTOWN STATE UNIVERSITY

On Tuesday, February 15, 2000, students across the United States and Canada in grades 10 and below will take part in the first **AMC→10,** the new American Mathematics Competitions contest for middle and high school students. The new contest supplements the two currently offered competitions, the American High School Mathematics Exam (now to be called **AMC→12**), and the American Junior High School Mathematics Exam (now to be called **AMC→8**). Students in grade 8 and below can participate in the **AMC→8**, while all middle and high school students can take the **AMC→12**.

Why another contest? We hope this new exam will encourage many more 9th and 10th graders to participate in the American Mathematicians Competitions. The table below lists the number of students at each grade participating in AMC exams in the years indicated:

Grade	1995-96	1996-97	1997-98
6	12 857	12 844	13 279
7	76 049	75 925	76 209
8	113 116	113 041	111 912
9	33 814	33 125	30 288
10	64 829	63 103	57 061
11	89 354	87 388	78 050
12	77 769	76 794	70 839

A motivating factor for establishing the new exam was the observation that there was a huge decline in student participation from the 8th grade to 9th grade. It should be pointed out that the totals given for the 8th grade students are for their participation in the AJHSME American Junior High School Mathematics Examination, which is not open to 9th graders. Ninth graders have been limited to the AHSME American High School Mathematics Examination.

The Committee on American Math Competitions feels that the decline in the number of ninth grade participants may be due to the inaccessibility of the AHSME to this group. Most students do not study exponential functions, logarithms, or trigonometry until after the 11th grade. Consequently, younger students have very little chance on a handful of the current AHSME problems. On the other hand, we also feel that students in grades 9 and 10 can profit from participating in problem solving adventures. Many of these students have enjoyed successful participation in the AJHSME. The new exam will consist of problems accessible to 9th and 10th graders. We hope that the students in 9th and 10th grades will have a good experience taking this exam and will be encouraged to continue with the program.

The American Mathematics Competition exams are sponsored by a host of organizations including the National Council of Teachers of Mathematics (NCTM) and the Mathematical Association of America (MAA). The **AMC→12** will also be a 25-question, 75-minute exam. Previously, the exam consisted of 30 questions with a time limit of 90 minutes. The **AMC→12** and the **AMC→10** will take place on the same day.

Students will not be allowed to take more than one of them. It is expected that there will be from 6 to 12 common problems. Students will qualify for the American Invitational Math Exam (AIME) in the usual way, that is, by scoring at least 100 on the **AMC→12.** In addition, the top 1% of **AMC→10** participants will qualify for the AIME.

The **AMC→12** is currently celebrating its 50th anniversary. To commemorate this milestone, the three most recent chairs of the AHSME committee have selected their favorite problems from each of the 50 years. This commemorative edition of AHSME can be found at **http://www.unl.edu/amc/anniver.pdf** on the world wide web. Registration information is also available at **http://www.unl.edu/amc/**.

The new **AMC→10** will be a 25-question, multiple-choice contest, with 75 minutes allowed. Correct answers will be worth 6 points and blanks will be worth 2 points, so the top possible score will be 150, just as it is for the **AMC→12.** Following is a sample **AMC→10** exam with answers following.

DID YOU KNOW:

Florence Nightingale was the first

person to use circle graphs?

She used them to represent data she collected on

helping to prevent disease during the Crimean war.

AMC→10 Sample Contest

1. The number halfway between $\frac{1}{6}$ and $\frac{1}{4}$ is

   (A) $\frac{1}{24}$ (B) $\frac{1}{5}$ (C) $\frac{2}{9}$ (D) $\frac{5}{24}$ (E) $\frac{3}{14}$

2. The marked price of a coat was 40% less than the suggested retail price. Alice purchased the coat for half the marked price at a Fiftieth Anniversary sale. What percent less than the suggested retail price did Alice pay?

   (A) 20% (B) 30% (C) 60% (D) 70% (E) 80%

3. The mean of three numbers is ten more than the least of the numbers and fifteen less than the greatest of the three. If the median of the three numbers is 5, then the sum of the three is

   A) 5 (B) 20 (C) 25 (D) 30 (E) 36

4. What is the largest number of obtuse angles that a quadrilateral can have?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

5. Consider the sequence

   1, –2, 3, –4, 5, –6, . . . ,

   whose n^{th} term is $(-1)^{n+1} \cdot n$. What is the average of the first 200 terms of the sequence?

   (A) –1 (B) –0.5 (C) 0 (D) –0.5 (E) 1

6. What is the sum of the digits of the decimal form of the product $2^{1999} \cdot 5^{2000}$?

   (A) 5 (B) 7 (C) 10 (D) 15 (E) 50

7. Find the sum of all prime numbers between 1 and 100 that are simultaneously one greater than a multiple of 5 and one less than a multiple of 6.

   (A) 52 (B) 82 (C) 123 (D) 143 (E) 214

8. Two rectangles, A, with vertices at (–2, 0), (0, 0), (0, 4), and (–2, 4), and B, with vertices at (1, 0), (5, 0), (1, 12), and (5, 12), are simultaneously bisected by a line in the plane. What is the slope of this line?

   (A) –4 (B) –1 (C) 0 (D) 1 (E) 2

9. A two-inch cube (2 x 2 x 2) of silver weighs 3 pounds and is worth $200. How much is a three-inch cube of silver worth?

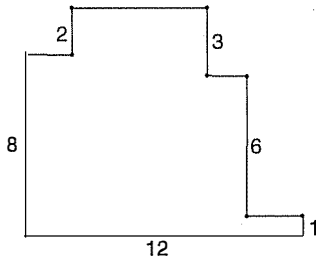
   (A) $300 (B) $375 (C) $450 (D) $560 (E) $675

10. The outside surface of a 4 x 6 x 8 block of unit cubes is painted. How many unit cubes have exactly one face painted?

    (A) 88 (B) 140 (C) 144 (D) 192 (E) 208

11. The adjacent sides of the decagon shown meet at right angles. What is its perimeter?

    (A) 22 (B) 32 (C) 34 (D) 44 (E) 50



12. Certain positive integers have these properties:

I. the sum of the squares of their digits is 50;

II. each digit is larger than the one to its left.

The product of the digits of the largest integer with both properties is

(A) 7 (B) 25 (C) 36 (D) 48 (E) 60

13. At the end of 1994 Walter was half as old as his grandmother. The sum of the years in which they were born is 3844. How old will Walter be at the end of 1999?

(A) 48 (B) 49 (C) 53 (D) 55 (E) 101

14. All even numbers from 2 to 98 inclusive, except those ending in 0, are multiplied together. What is the rightmost digit (the units digit) of the product?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

15. How many three element subsets of the set

{ 88, 95, 99, 132, 166, 173 }

have the property that the sum of the three elements is even?

(A) 6 (B) 8 (C) 10 (D) 12 (E) 24

16. A point is chosen at random from within a circular region. What is the probability that the point is closer to the centre of the region than it is to the boundary of the region?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

17. The student lockers at Olympic High are numbered consecutively beginning with locker number 1. The plastic digits used to number the lockers cost two cents apiece. Thus, it costs two cents to label locker number 9 and four cents to label locker 10. If it costs a total of $145.86 to label all the lockers, how many lockers are there at the school?

(A) 1200 (B) 2100 (C) 2431 (D) 2573 (E) 7293

18. In the equation A + B + C + D + E = FG, where FG is the two-digit number whose value is 10F+G, and letters A, B, C, D, E, F, and G each represent **different** digits. If FG is as large as possible, what is the value of G?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

19. What is the maximum number of points of intersection of the graphs of two different cubic polynomial functions with leading coefficients 1?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 6

20. The graphs of $y = -|x - a| + b$ and $y = |x - c| + d$ intersect at points (2, 5) and (8, 3). Find a + c.

(A) 5 (B) 7 (C) 8 (D) 10 (E) 13

21. A sealed envelope contains a card with a single digit on it. Three of the following statements are true, and the other is false.

I. The digit is 1.

II. The digit is 2.

III. The digit is not 3.

IV. The digit is not 4.

Which one of the following must be correct?

(A) I is false (B) II is true (C) II is false (D) III is false (E) IV is true

22. A circle is circumscribed about a triangle with sides 3, 4, and 5, thus dividing the interior of the circle into four regions. Let A, B, and C be the areas of the non-triangular regions, with C being the largest. Then

(A) $A + B = C$ (B) $A^2 + B^2 = C^2$ (C) $A + B + 6 = C$ (D) $4A + 3B = 5C$ (E) $\frac{1}{A^2} + \frac{1}{B^2} = \frac{1}{C^2}$

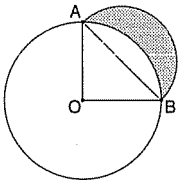
23. The digits 2, 4, 5, 6, 8, and 9 can be distributed among the lettered squares in the array so the the sum of the entries on each of the rows and columns is the same number K. What is K?

(A) 15 (B) 16 (C) 17 (D) 19 (E) 21

7	a	b	1
c			d
3	e	f	10

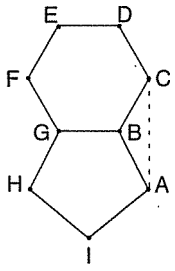
24. In a circle with center O, $\overline{OA}$ and $\overline{OB}$ are radii and $\angle AOB$ is a right angle. A semicircle is constructed using segment AB as its diameter as shown. The shaded portion of the semicircle outside circle O is called a *lune*. What is the ratio of the area of the lune to the area of the triangle?

A) $\frac{\sqrt{2}}{\pi}$ (B) 1 (C) $\frac{\pi}{\sqrt{3}}$ (D) $\frac{\pi}{\sqrt{2}}$ (E) $\frac{2\pi}{3} - 1$



25. A regular hexagon and a regular pentagon have a common edge as shown. Find the measure of the angle BAC.

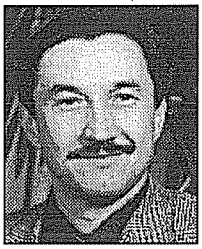
(A) 24° (B) 30° (C) 36° (D) 45° (E) 48°



Answers

1. D 2. D 3. D 4. D 5. B 6. A 7. C 8. D 9. E 10. A 11. D 12. C 13. C 14. D 15. C 16. A 17. B
18. B 19. B 20. D 21. E 22. C 23. D 24. B 25. A

▲ THE DEVIL'S ADVOCATE: MY PET PEEVES



ALEXANDRU PINTILIE
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For a foreign-born Canadian there is no greater pleasure than to discover those little colloquial treasures of the language that the English teacher in your natal country never mentioned to you. For instance, I did not know that a hot dog was actually a "tube steak", or that *The English Patient* is a "chick flick". I was getting on my high horse without knowing it and for a long time I failed to notice the chip on my shoulder. Becoming a Canadian has more to do with getting the "bon-mots" than knowing the rules of hockey.

It took me a while to understand the idea of "pet peeve". I believe that it is something that "peeves me off" and at the same time offers me a good opportunity to bore others with my critical opinion on the matter, while getting high on delusions of superiority.

I am sure that other teachers have noticed and had a good laugh encountering the following oddities, worthy of Lewis Caroll's pen.

THE COSINE RULE. In a number of textbooks it is given as a set of 3 formulae: $a^2 = b^2 + c^2 - 2bc \cos A$; $b^2 = a^2 + c^2 - 2ac \cos B$; $c^2 = a^2 + b^2 - 2ab \cos C$. Well, what if we apply the formula in the triangle MNP? I propose that we make the students memorize all P(26,3) possible permutations of this formula and pray that no one will ever use a or b for sides of a triangle.

FOIL. If a grade 9 student cannot grasp the idea of multiplying each term in one bracket by each term in the other, he is either severely mentally handicapped or he does not pay attention, neither case warranting reinventing algebra. Anyway, multiplying a 3 term polynomial by a 9 term one would become impossible because the acronym would need more letters than can be found in the English alphabet.

THE SQUARE OF A BINOMIAL.

Textbooks (and teachers!) ask students to learn two variations of these formule: $(x+y)^2 = \ldots$ and $(ax+by)^2 = \ldots$. How about $(ax+y)^2$ or $(a + bxy)^2$? It is easier to memorize than to understand, isn't it?

BEDMAS. Why don't we tell the students that operations are performed in the order of complexity, from the most complex (exponentiation) to the simplest (addition and subtraction)? There is no need to explain brackets; they were invented to be self-explanatory. Items inside a bracket belong together not only in mathematics, but also in Linguistics, Computer Sciences, Chemistry, etc.

MEANINGLESS RESTRICTIONS. When dividing a polynomial (A, the dividend) by another polynomial (B the divisor) , one obtains two other polynomials: Q, the quotient and R the remainder such that $A = BQ + R$ and $\deg(R) < \deg(B)$. No restrictions for the variable(s) are needed here! Not only does the equality hold for all values of x (be they real, complex or members of fancier spaces) but we are also talking polynomials, not polynomial functions.

ALGEBRA TILES. Algebra is, by definition, the part of mathematics that develops abstract thinking. Hands-on algebra is a non-sense, like non-contact wrestling or waterless swimming.

I believe that the above examples have originated from our tendency to underestimate our students' ability. No matter how low our standards, there will always be students who will outsmart us, by going even lower. If Jimmy can master complicated strategy games, he can understand the order of operations without the crutches provided by silly acronyms! ▲

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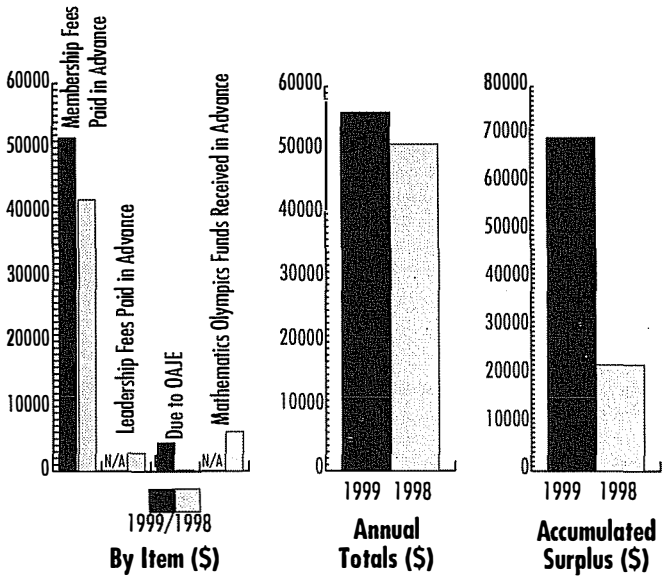
FINANCIAL STATEMENT

Notice to Readers

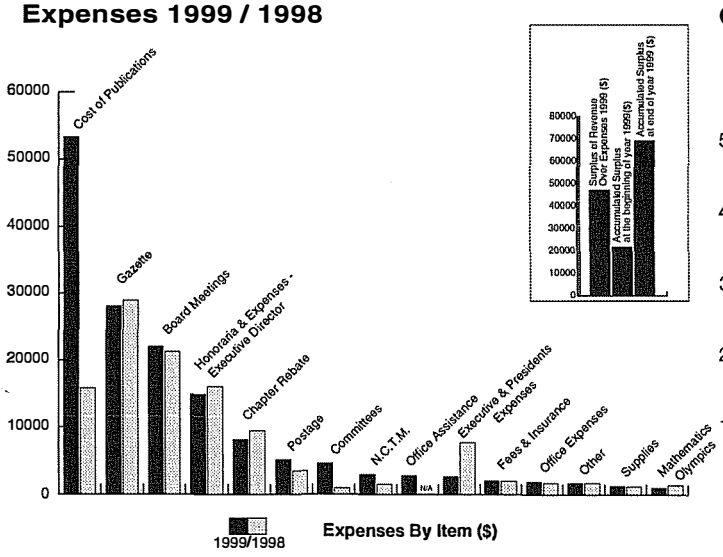
I have compiled the Balance Sheet of Ontario Association for Mathematics Education (Association Pour L'Enseignement Des Mathematiques) as at August 31, 1999 and the Statement of Operations and Accumulated Surplus for the year then ended from information provided by the Directors. I have not audited reviewed or otherwise attempted to verify the accuracy or completeness of such information. Readers are cautioned that these statements may not be appropriate for their purposes.

Markham, Ontario, September 24, 1999
ROBERT M. DAVIES Chartered Accountant.

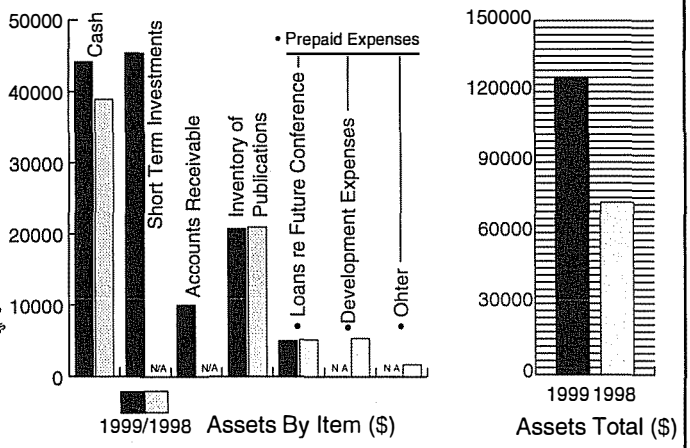
Liabilities & Surplus 1999 / 1998



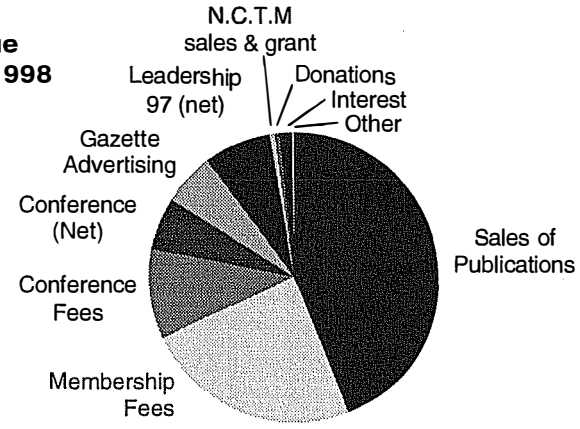
Expenses 1999 / 1998



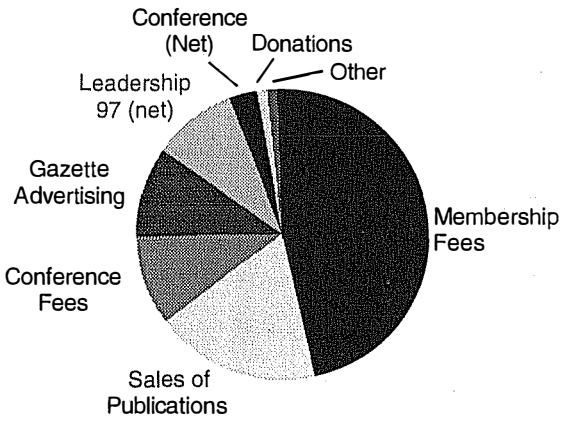
Current Assets 1999 / 1998



Revenue 1999 / 1998

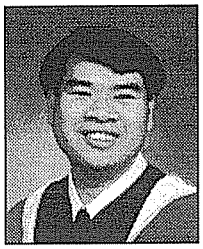


1999 ($199,627)



1998 ($106,151)

▲ DIRECTORS' DIALOGUE



LOUIS LIM
QUINTE SECONDARY SCHOOL
BELLEVILLE, ON

This edition of the "Director's Dialogue" summarizes the Saturday, October 2 OAME board meeting at the Four Points Hotel in Mississauga.

PRESIDENT SAARIMAKI'S REPORT

Peter Saarimaki reported that OAME hosted the NCTM Leadership Conference, where NCTM affiliates (such as OAME), sent one or two executives, totalling 25 executives across Canada. Peter noted that OAME, with 1400 members, is the largest mathematics education association in Canada. He ended his report by sharing the huge success of the unique OAME/OMCA Summer Institutes which featured many OAME and OMCA members (see the section "Professional Development" for further information).

SAD NEWS ...

The board learned that Executive Directors Dave and Bonnie Alexander have announced their resignation beginning September 2001. Gord Cooke attempted to make a motion to deny their request, but the board learned that the constitution does not allow their resignation to be rejected. It is hoped that the new Executive Directors will work along side Dave and Bonnie to ease into their roles.

GOOD NEWS ...

Dave Alexander reported that OAME is in good financial shape. As of September 26, 1999, there are 1383 OAME members (similar to September 1997). With the massive number of retirements, it is very important that faculties of education encourage teacher candidates to become members of our association.

More than 6000 documents were sold through the OAME office through the summer, generating $88 000 in revenue! If you don't have the Assessment documents, Primary video, Math Tech document, and Transition Years posters, be sure to order them through the Executive Directors. They are all written by and for OAME members.

NCTM REPORT

OAME-NCTM Rep, Frances Schatz, shared with the board of directors several interesting resources. In particular, she noted that the NCTM 1998 Yearbook, "Teaching and Learning of Algorithms in School Mathematics", was edited by Ontario's very own Lorna Morrow. A new NCTM resource, "Developing Mathematically Promising Students", focussing on programs for the gifted, creative, high/low level achievement students, was also shared. Be sure to order materials through Frances (see her address on the last page of the *Gazette*) as OAME gets a small rebate from your total order. For new or renewing memberships (NCTM membership with one journal is $88.00 Canadian) remember to make your Canadian **cheque** payable to OAME-NCTM to receive the new 25% exchange cap (paying by VISA does not give you the cap!). On behalf of OAME, I would like to congratulate and thank Frances and her husband, Frank, for their tremendous efforts!

NEWS FROM THE FIELDS INSTITUTE

Shirley Dalrymple reported that OAME is well-represented at the "Field's Forum" which meets every 6-8 weeks (OAME members include Myrna Ingalls, Shirley Dalrymple, David Zimmer, Eric Muller, Walter Whitely, and Lynda Colgan). Current initiatives include looking at the revision of the grades 11 and 12 mathematics policy, along with addressing the current math teacher shortage with a recent grant received.

PROFESSIONAL DEVELOPMENT

OAME/OMCA INSERVICE WORKSHOPS

Peter Saarimaki, on behalf of Todd Romiens, shared with the board of directors a *Power Point* presentation which summarized the very successful OAME/OMCA Inservice Workshops. Phase I occurred last June when two teachers from each school were introduced to the new curriculum. Phase II occurred during the summer as week-long institutes to familiarize teachers with the new grade 9 curriculum, assessment, technology, and rich learning tasks. More than 500 teachers from 430 secondary schools (reaching 30 000 grade 9 students!) benefited from this unique professional development opportunity. To continue to support teachers, check out the discussions web site at **http://media.mediacentre.com/grade9math**.

LEADERSHIP 2000

Once again, we can expect a complete sellout of the Leadership Conference. The theme for the February 2000 conference is "World Mathematical Year 2000:

Preparing for the Future". Elementary teachers can focus on developing mathematical literacy, or teaching and learning mathematics through inquiry, while secondary teachers can focus on developing strategies for successful implementation of the new grades 9 and 10 courses, or an in-depth look at the new grades 11 and 12 courses. With keynote speakers, Queen's University Dr. William Higginson, and CityTV's David Onley, you can expect tremendous professional development!

OAME'2000

Mark your calendars for May 10 to 13, 2000! The COMA chapter is busy organizing OAME'2000 at the University of Ottawa. The "conference within a conference" format has generated a healthy number of workshops for each of the Assessment, Reform, and Technology strands. While in Ottawa, be sure to catch the Tulip Festival. For up-to-date information check out the OAME web site at **www.oame.on.ca**, and click on the link to OAME'2000.

OMO'2000

QSLMA, under chair, Cindy Harvey, are busy working to make OMO'2000 a success. A unique element of this provincial Olympics will be Queen's University Professor Joan McDuff's focus leadership class organizing major parts of the OMO.

COMMITTEE REPORTS

AWARDS

Check the Awards Nomination Form on page 13 to nominate a mathematics educator for one of OAME's prestigious awards. Be sure to submit your nomination form to Shirley Dalrymple before January 31, 2000 to be considered for the calendar year.

FUTURES

The Futures committee confirmed the following:

OAME'2000: hosted by COMA in Ottawa;

OAME'2001: hosted by SAME in Scarborough;

OMO'2000: hosted by QSLMA in Kingston.

Several chapters have stated interest in hosting OAME'2002 and 2003, along with OMO'2001. Hopefully, my next *Director's Dialogue* will confirm some (or all of them). As an aside, since QSLMA hosted the annual provincial OAME conference in 1974, 1984, and 1994, is it safe to assume that they will be hosting OAME'2004?

PROMOTIONS

Kirsten Boucher reported that the Promotions committee has updated the OAME membership brochure, put together a Communications Kit on how to deal with the media, have created an OMO package of activities to be made available to chapter reps, along with acting on the suggestions of increasing membership (see Jack Weiner's September 1999 Editorial).

PUBLICATIONS

Dean Murray's hard work as Advertising Manager has generated $11 914.96 revenue for OAME during the past year. The Publications committee learned that the colour of the *Gazette* title page depends on the ad located on the back cover. We urge you to take a close look at the products and support our advertisers. Have you recently checked out OAME's web site at **www.oame.on.ca**? It has a new look! The committee made a major apology to Mary Lou Kestell, who was not acknowledged for her very thoughtful "Letter to the Editor" in the September 1999 issue of the *Gazette*.

JK-5

Laurie Moher reported that the JK-5 committee will prepare an article on "Communication in Mathematics" for a future issue of the *Gazette*, along with submitting several activities to the *Abacus* on play-based learning. The committee would also like to collaborate with OMCA on possible revisions to the Provincial Report Card which will be sent to the Ministry.

6-8

Barry Onslow has agreed to serve as Project Manager for the new grades 6-8 assessment document. Teachers are being sought to field test the items. Concerns were raised that the majority of articles in the *Gazette* have focussed on the secondary level, and an initiative of the 6- 8 committee is to submit articles to the *Gazette* and *Abacus* which would be of interest to elementary teachers.

SECONDARY

Discussion took place of what teachers need now and in the future. A group of teachers will field-test the "Cagey Problem" in the course profiles, and an article will appear in the March 2000 *Gazette* which consolidates their experiences of that activity. Focus on the new grades 11 and 12 curriculums have been delayed until the February 2000 board meeting with the current rewrite of the senior level courses.

CHAPTER NEWS

At a time with so much change, I strongly urge you take full advantage of your chapter affiliation. Chapters are always looking for members to get involved — either through serving on the executive, organizing or speaking at mini-conferences, sending feedback to chapter reps, etc. I can assure that your valuable contributions will benefit mathematics

education — is there any greater self-satisfaction?

COMA

COMA has been very busy preparing for OAME'2000. With the tremendous amount of time and energy being devoted to the annual provincial conference, their chapter conference has been replaced by three forums focussing on the implementation of the new curriculum. By the time you read this, Myrna Ingalls would have spoken to COMA members on the use of the course profiles. At the end of January, several COMA members will share their experiences of the new grade 9 course, and in mid-April, a forum will take place to discuss the upcoming new grade 10 courses.

CHAMP

Their annual fall conference, Mathfest'99 consisted of two workshop sessions, publishers' displays, dinner, and door prizes. Hopefully, you'll still be able to participate in its annual mathematics contests for grades 7-11 students during the second week of December. This workshop is designed to encourage students in solving recreational problems and to assist them in preparing for the "Canadian Mathematics Challenge" in the new year.

GOLDEN

The Golden Section increased by 24 members as a result of hosting an OAME/OMCA Summer Institute during the past summer. To continue to support grade 9 teachers, evenings have been set aside to provide materials and technological support. Several workshops (again history when you read this) were examining "Algebraic Models and Rates of Change", and a tour of the *Geometer's Sketchpad* by Peter Harrison. For their elementary teachers, workshops were conducted on assessment. E-mail chapter rep Elizabeth Pattison for future chapter happenings.

GVMA

Their fall conference took place on November 4 in Kitchener. Ron Scoins was awarded a "Distinguished Teacher of the Year" award by the University of Waterloo, and Jim Schurter, a former GVMA secretary, was awarded a Descartes Medal.

ISOMA

ISOMA held its fall conference at Bishop Strachan School on October 26. Sessions focussed on the new guidelines and technology, a mathematics trail by Ron Lancaster, and a Music and Math presentation by Cynthia Church. Beginning January 2000, be sure to check out ISOMA's web page at **www.appleby.on.ca**, along with their electronic version newsletter published in the fall and spring. Considerable efforts are being made to improve and increase communication with its members who span throughout the province.

NOMA

NOMA held a very successful conference, "MATH4Y2K", on September 17 and 18. Over 200 participants were in attendance, including more than 100 teacher candidates from Laurentian and Nipissing Universities. Stewart Craven delivered the keynote, and presenters included Brenda Holly, Anne Pedersen, Ron Lancaster, Shirley Picknell, Myrna Ingalls, Chris Kirkpatrick, Carol Lemmon, Judy Dussiaume, Sheila Giroux, Jim Hill, Bob Smith, and Karen Burgess. NOMA also thanked Dan Charbonneau (organizer) and Ron Lancaster (instructor) for hosting an OMCA/OAME Summer Institute in Sudbury during late August.

NWOAME

November 5 and 6 saw NWOAME's fall conference take place. Sessions included the *Geometer's Sketchpad*, primary assessment, and graphing calculators. Wayne Fletcher reported that the chapter's regional Olympics have gained popularity, and the chapter will continue its tradition of sending a team to the OMO (this time in Kingston, June 2000). Although not finalized, the NWOAME executive has passed a motion to apply to host the provincial OMO in 2001. Recall the heart-warming story in the September 1999 *Director's Dialogue* where I summarized how the NWOAME community united together to raise funds to send a team to OMO'99 (in Toronto).

PINE RIDGE

Jacqueline Hill reported that the Durham schools are still in turmoil (recall that their secondary teachers taught 7 out of 8 last year). Now that they have a contract, their federation has permitted them to run a mini-conference. Plans are currently underway for the mini-conference to be held in the spring of 2000, and the focus will be on the new grades 9 and 10 curriculum.

SAME

SAME hosted their fall conference on October 19, "Embracing Change", which featured key note speaker, Mary Lou Kestell. The chapter is pleased to report that planning for OAME'2001, "2001: A Math Odyssey", is progressing well. Committees are being filled, and the Delta Hotel Toronto East is being booked. May this chapter have the "same" success as Y^{4}MA's OAME'99 conference! (apologies for not recognizing the board member who created that pun as I forgot who said it)

SWOAME

SWOAME spends most of its energy helping members implement the new grade 9 curriculum. Phase I took place on June 8, 1999, when two teachers from

every high school in the Windsor-Essex region were inserviced. Phase II, an OAME/OMCA Summer Institute, took place from August 30 to September 3, with Ron Lancaster as the instructor. Plans for the upcoming year (phase III) include: three workshops, a "Dinnerlude" in February 2000, a local Math Olympics to select two teams for the OMO in June 2000, sponsor a "Math Awareness Week" in February, and to develop activities to participate in the year 2000 (the math year).

WOMA

Like other chapters, WOMA's fall conference has occurred. It was held on October 28 in London with featured speaker, George Gadanidis. Starting October 24, Margaret Kemp and Tom Griffiths again held mathematics enrichment sessions for grades 7 and 8 students every third Saturday of the month. Also, Tom has been busy every Monday night conducting contest preparation workshops for grades 9 to 11 students, and Walker Schofield joined Tom on Tuesday nights preparing grades 11 to OAC students for the Canadian Open (not golf!), the Euclid and Descartes contests. Be sure to attend WOMA's spring conference — the chapter is celebrating its tin anniversary (10 years)!

"Education is not the filling of a pail, but the lighting of a fire."

William Butler Yeats. ▲

OAME-NCTM Report

FRANCES SCHATZ, 56 OXFORD STREET, KITCHENER, N2H 4R7 PH: (519) 578-5948, FAX (519) 578-5144 OR EMAIL:FRANCES.SCHATZ@SYMPATICO.CA

Feature Books

Teaching and Learning of Algorithms in School Mathematics: **Lorna Morrow** This NCTM 1998 Yearbook is edited by Ontarioís mathematics teacher, consultant and writer, Lorna Morrow. This yearbook attempts (and does it very well) to answer questions about the place of algorithms in todayís mathematics curriculum and to stimulate new questions as we continually adapt school mathematics for the twenty-first century. As I read this book I find myself having more confidence about topics that are appearing on Ontarioís new grade 9-12 mathematics courses.

Future Basics; Developing Numerical Power by Randall Charles and Joanne Lobato. This is an excellent resource for those interested in rethinking the K-9 mathematics curriculum and in promoting numerical power at a time when mastery of paper-and-pencil algorithms is shifting to technological tools.

Developing Mathematically Promising Students (NCTM publication #700) is new for 1999. This book attempts to address curriculum questions and answers regarding programs for the gifted, creative or high-ability/low achievement students.

For information or pricing on these books or other NCTM materials, please contact me.

OAME earns 20% on purchases made through OAME-NCTM.

Individual NCTM memberships can be obtained or renewed through me, the OAME-NCTM representative. The price (One journal included) is $88.00 Canadian made payable to OAME-NCTM. The only way to obtain the membership with the new CAD cap of 25% is via cash or Canadian cheque to OAME-NCTM. Remember, each new NCTM membership through the representative earns OAME $5.00.

Shaping the Principles and Standards

NCTM has used the feedback from thousands of readers of the draft copy to shape the final version of this document. The variety of perspectives expressed made the incorporation of the many suggestions a complex task for the writers. NCTM hopes that the Principles and Standards for School Mathematics will serve as a valuable support of the mission to improve the mathematics education of all students. This document will be officially released on Wednesday, April 12, 2000 at the annual meeting in Chicago.

NCTM members who are current as of April 1, 2000 will receive a copy of the final document as a benefit of membership. Be sure you join or renew to have your membership current on that date.

NCTM requires authors for the ***2002 Yearbook: Making Sense of Fractions, Ratios and Proportions***. For guidelines you can go to http://www.nctm.org/publications/yearbook.html or contact me. ▲

▲ NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 1999-2000

Each year, elections are held for a President-Elect, a Vice President (3 year term) and 6 to 9 Directors (3 year term) of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 21, 2000, to:

Ralph Connelly, Past President OAME/AOEM

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2000, the Nomination Committee will publish a slate of candidates, including qualifications, for consideration by the membership. Voting will be by mail and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Ottawa in May, 2000.

NOMINATION FORM

I nominate: _____

For the position(s) of: ☐ President-Elect ☐ Vice-President ☐ Director

Nominee's Address: _____

_____

Telephone: H ( ) _____ W ( ) _____

Nominator's Name : _____

Nominator's Address: _____

Telephone: H ( ) _____ W ( ) _____



OAME 1999-2000 BOARD

PRESIDENT
Peter Saarimaki

PAST PRESIDENT
Ralph Connelly

PRESIDENT-ELECT
Mary Howe

EXECUTIVE DIRECTORS
David & Bonnie Alexander

VICE PRESIDENTS
Shirley Dalrymple

Bill Otta

David Zimmer

CHAPTERS & REPRESENTATIVES
CHAMP - John Radger

COMA - Ron Bender

GOLDEN - Elizabeth Pattison

GVMA - Murray McLennan

ISOMA - Ian Robertson

MAC² - Lyn Vause

NOMA - Carol Lemmon

NWOAME - Wayne Fletcher

PINE RIDGE - Jacqueline Hill

QSLMA - Gordon Cooke

SAME - Kirsten Boucher

SWOAME - Frank Di Pietro

TE²Y²MS - Kathy Kubota-Zarivnij

WOMA - Linda Alton

Y⁴MA - Rosie Manefrancesca

DIRECTORS -
DIRECTORS - TERM EXPIRING 2000
Marilyn Hurrell

Laurie Maher

Anne Pedersen

Susan Stuart

Margaret Warren

DIRECTORS - TERM EXPIRING 2001
Edward J. Barbeau

Dan Charbonneau

Stewart Craven

Geoffrey Roulet

Jan Scully

Emilio Veltri

Jack Weiner

TERM EXPIRING 2002
John Dallan

Myrna Ingalls

John Kersley

Peter Lenczewski

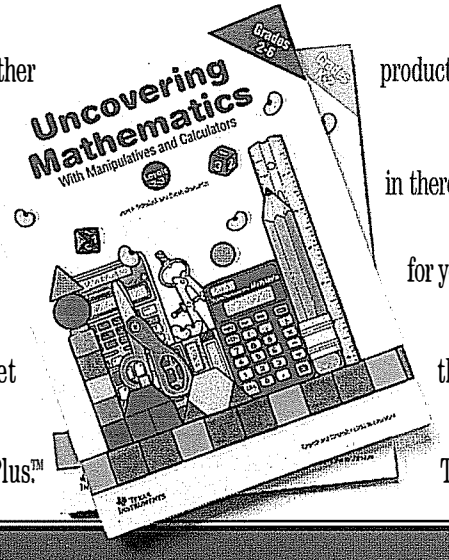
Louis Lim

Anne Muller

NCTM REPRESENTATIVE
Frances Schatz

WEB SITE COORDINATOR
Doug Evans

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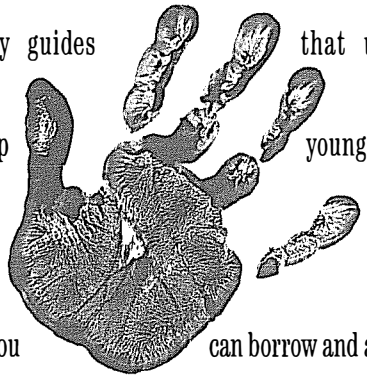
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(Math Explorer™)

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