Vol. 38 #3, March 2000

Ontario Mathematics Gazette

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Math Phobia Can Be Beaten!

Canadian Statistics in the Classroom: Resources for Activities in the Junior Classroom

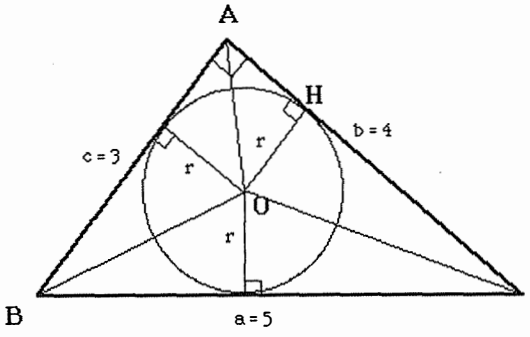
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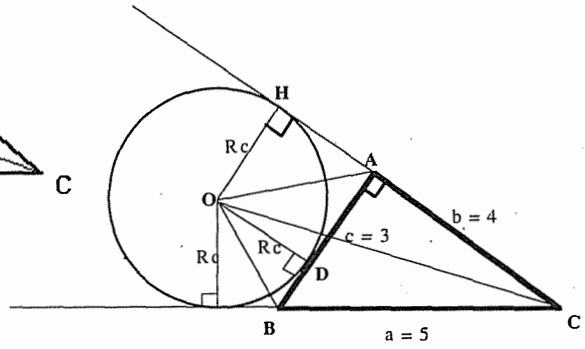
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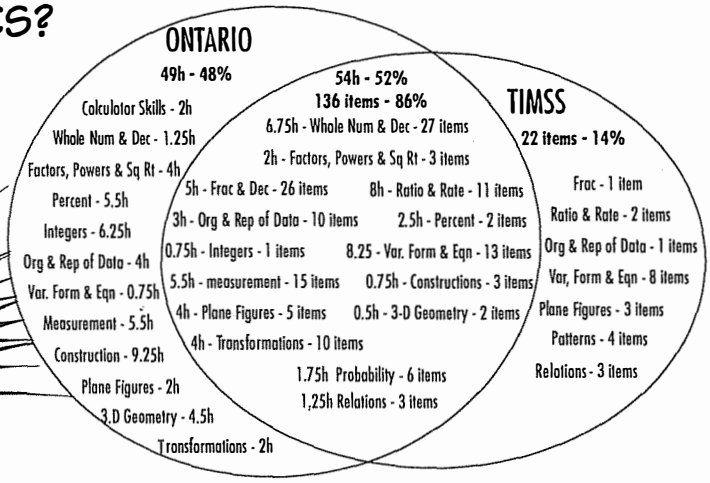


Figure 1: Ontario Grade 8 Mathematics Curriculum (1985) & TIMSS

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The *Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 601349.

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Deadlines for information to be included in the *Gazette* or the *Abacus* are: June 15 for the September issue, October 15 for the December issue, January 15 for the March issue, and April 15 for the June issue.

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▲ Editorial

CO-EDITOR, TODD ROMIENS



The first session of grade 9 math has been completed. I'm sure teachers did a great job, even though there were many kinks to be ironed out with both the curriculum and its implementation. In talking with many teachers of the grade 9 courses, I was very impressed with their ability to employ technology, rich problem solving techniques and various forms of assessment in teaching the new courses. This was due in part to the opportunities for professional development provided since last May by "OMCA", "OAME", "OTF", and summer institutes sponsored by Texas Instruments. I believe this is the first time so much attention has been paid to the implementation of the curriculum as opposed to the curriculum itself.

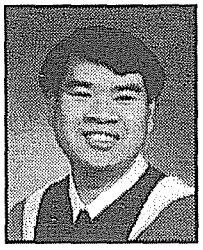
The *Gazette* would like to publish examinations that were given in grade 9, at the end of the course. If any of our readers has an exam, or a rich learning task that they would like to share, it would be greatly appreciated. Please send them to me or Louis as soon as possible.

The Provincial Review for grade 9 is in the beginning stages. Items are being field tested and assessed. I believe that the first official test will be administered in January 2001. It is important that these test items maintain the integrity of the philosophy of our new grade 9 courses. Even if the results of the first few tests are not as good as we would like (as happened with grade 3), don't make the tests easier to accommodate the results. We are making positive changes in the way kids do and think of what mathematics is, we must continue to encourage teachers and students to move in these positive directions by making external testing reflect this philosophy. ▲



▲ EDITORIAL

CO-EDITOR, LOUIS LIM



Through taking Dr. Bill Higginson's "Issues in Mathematics, Science, and Technology Education" M.Ed. course at Queen's University this past fall, I was able to focus a significant portion of my studies on the grade 8 TIMSS results. I wanted to know what the countries that scored very high did, and whether, we, in Ontario, could adapt some of their policies in our own educational system.

Time and time again, I read about the huge investment countries such as Japan place on teacher development. A Japanese teacher summarizes it best: "How could any teacher be expected to do a good job when there is no time outside of class to prepare and correct lessons, work with individual children, consult with other teachers and attend to all the other matters that arise in a typical day at school?"

With many collective agreements up for renewal in the spring, I am worried that teaching 7 out of 8 could be a reality. Clearly, the top performing countries do not spend their entire day in the classroom. In fact, Japanese teachers spend approximately 60% of their day in the classroom; the rest is spent collaborating with other teachers on lesson planning, marking, and preparation.

At a time with so much change (e.g., numerous teacher retirements, implementing a new curriculum), teachers need support -- support to deliver the best possible curriculum for students, and support that what we do as educators matters, and matters a lot. It is even more difficult than ever for department heads (where they exist) to support their new teaching colleagues when they, too, must teach 6 out of 8.

If there's a message from the TIMSS results, it is the need to focus our energies on teacher development. OAME does a great job in hosting annual and provincial conferences, the recent summer institutes, leadership conferences, etc. but teachers must be encouraged to attend them and be able to share their new learning with colleagues. Without time, we cannot collaborate with colleagues and professionally develop -- improvement of student learning happens when our energies focus on quality (not quantity) of instruction. ▲



▲ PRESIDENT'S MESSAGE



PETER SAARIMAKI

Is That Your Final Answer?
A Tangential Response!

I've borrowed this highly recognized (and equally disparaged) phrase from a familiar figure (not geometric) in Ontario's mathematical circles (and spheres of influence) - Don Fraser. This was the title of one of Don's recent talks (the first half anyway - the title not the talk!).

Don has a style of delivery that effectively uses cartoons, jokes and subtle humour to get people involved, thinking and creating. That takes a special talent and personality that most of us can't pull off, but we still need to get our students involved through some means. And what are they, you justifiably ask? How do you *interest* students in money topics? How do you *shape* them up and get them *round* to your way of thinking in geometry? How do you get them to *move* ahead in transformations and *reflect* on their learning?

First, I'd like to share some thoughts from Marilyn Burns, originally published in the April 1993 issue of *Instructor,* that I have been using in recent workshops, namely her list of:

"The 12 Most Important Things You Can Do to Be a Better Math Teacher"

1. Set the following expectation for your students: Do only what makes sense to you.
2. Have your students explain their reasoning in all instances.
3. Encourage children to talk with one another during math class.
4. Make writing an integral part of math learning.
5. Embed math activities in contexts.
6. Use manipulative materials whenever possible.
7. Bring the quality and richness often apparent in students' writing and art into their math work.
8. Make calculators available to all children at all times.
9. Let children push the curriculum rather than having the curriculum push the children.

10. Keep an eye out for instructional activities that are accessible to students with different levels of interest and experience.
11. Remember that confusion and partial understanding are natural to the learning process.
12. Take delight in students' thinking.

Each of these suggestions leads to a wealth of ideas and further questions (remember the quiz show?). Choosing a random number as an example, let's consider #3 (chosen because of the proximity of the two symbols!!).

Math talk has so many rich possibilities. In the book by the same name (published by the Mathematical Association (U.K.)), Hilary Shuard identifies some ways to initiate maths talk, including using phrases such as: *Go on . . . , What made you think of that?, Tell John how it works, What do you think, Saleem?*, and even *I hadn't thought of that.* Provide students these prompts so their group work encourages thinking and communicating. Remember that talk does not have to lead to recording: it can be a valuable activity in its own right.

But to get back to a theme here (and just what is the theme you ask?). Well, like the quiz show, this is really a series of questions that are not related to each other, other than by juxtaposition.

I want to use a phrase that Fraser has been heard to use, when he was not trying to read an Eraser, "and just what **was** the **question**?" Socrates, and many others, thought that good teaching involved coming up with good questions - ones that got the "right" answers and thus moved the lesson inexorably towards its pre-determined conclusion. We know now though, that good questions need to come from the learner. Students need to be engaged in the problems and contexts to encourage them to pose questions, just as the quiz questions make the contestant think. We want them to be problem posers, not class problems.

Quiz shows seem to be sweeping the ratings these days, with shows such as the *Millionaire, Greed, Twenty One* (did you hear of the young students showing off their counting skills who went "19, 20, blackjack"?), and *Win Ben Steins' Money*. Now there's got to be an interesting question in all that: What's a rating? How often can you get one? How do you sweep them? Just who is Ben Stein?

Quiz shows also appear to be very profitable with very little up-front expenses in the way of preparation. Profitable for some contestants, the advertisers and the TV stations (although apparently getting insured *against* winners is not profitable). What does it cost to run such a show compared to sit-coms or sporting events? How much do advertisers pay? How do TV profits compare to the movies?

Using contexts that students are turned onto ensures their interest and involvement. Consider the mathematics of quiz shows: what other math is there? Why isn't there a popular math quiz show? Well the answer to that is easy: because they haven't designed one yet.

Have your students create their own math quiz show. Working in groups, have them design a set of questions at a variety of levels (maybe less than on the millionaire show). They must choose, answer and support their questions at each level, leading them to consider whether it is harder to multiply two-digit numbers or divide by one-digit numbers, whether it is easier to rearrange formulas or use the Pythagorean theorem, whether it is nobler to suffer the slings and vectors of outrageous applications, . . .

BTW, is there a "right" answer to any of these questions? Or are we more interested in the questions students conceive and how students defend their choice? (and it should be obvious that that's a rhetorical question, eh?)

And the moral is . . .? Students are capable of creating amazing questions and generating well thought out solutions, especially when they are involved in the situation, have ownership in the context, and have good role models.

To end, I want to share with you my favourite definition of problem solving (from a conference I attended in England many years ago when I learned about buns and bunnies, leapfrogs, and the bellringer problem . . . but that's a whole other story) which states that:

problem solving is when you get stuck . . .
and stay engaged.

So stay engaged with your students, go off on interesting tangents and pretty soon you'll be asking them, "Is that your final question?" ▲

Cheers, Pierre

▲ Math Phobia Can Be Beaten!

SHARON BLOMFIELD
TRILLIUM PUBLIC SCHOOL
WATERLOO REGION DISTRICTSCHOOL BOARD

Sharon Blomfield is a junior-level teacher at Trillium Public School in Kitchener. She pursues professional growth through her involvement in the Action Research Project in the Waterloo Region District School Board.

An agitated student approached me one day. Her face was red and tears were forming. "I don't get this question!" she sputtered. The mathematical problem was not difficult and I knew that it was well within her capabilities. Now, I knew exactly what to do to help her.....

Over my years as a junior teacher, I have encountered many who are very reluctant mathematic's students. During a lesson, their heads are down. They avoid eye contact, hoping that I will not call on them. When asked to answer a question, they use a variety of techniques to avoid answering. During individual working time, they show very little ability to tackle problems successfully or follow mathematical procedures. If they do ask for help, instead of a specific question, they will often state their request for help as, "I don't get it." They are not engaged in the learning process and, so, it is not surprising that they feel, and are, relatively unsuccessful in achieving the expected learning.

It has been my sense that most student difficulties with mathematics result from the self-fulfilling prophecy, "I'm not good at math, so I won't try." And of course, these students don't do well. It has been quite unproductive to explain mathematical procedures to them as long as their self-concept has been clouded by their negative feelings. When I was able to convince these students to set aside the "I can't," I achieved greater success with them.

Last year, I knew that I would be participating in an Action Research project that my board sponsors in conjunction with OISE-UT. In Action Research, teachers are encouraged to articulate and study an issue of

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particular concern to them in their own practice. This topic was timely. It had been bothering me for years. I needed to find a more effective way to help these students.

From the beginning of the year, I continued a strong emphasis on helping students understand, visualize and relate mathematics to their own experiences. I encouraged those students who appeared to be reluctant to set aside negative feelings and to involve themselves more fully in our activities. I continued, however, to have a number of students whose engagement in the learning process and achievement was less than I expected.

Asking the Right Questions

As I began the formal phase of Action Research in January, I soon decided that I needed to inform and engage my students in this study. I spoke to the class about the issue of finding math "scary". I said that we would be beginning a voluntary "Math Club" for students who would like to "feel better about math." I said that, while we might occasionally meet at recess, most of the club's activities would be conducted during regular class. The "club" would certainly not involve great quantities of assigned homework, although students might choose to do extra work at home. It would involve my interviewing students to talk about their feelings about math. It might also involve meeting as a group to discuss feelings or to work on problems.

Some students were eager to participate. I interviewed them individually several times over the next few months. Many of them asked for specific help, during class, or at recess. A very few requested work to take home. Others whom I knew to be very nervous about math were more reluctant. I approached them quietly, saying that, while I was not pressuring them to "belong to the Math Club", I would be interested in speaking to them privately about their feelings. All agreed to have several chats with me.

In the end, "Math Club" was little more than a label, an avenue to express and share feelings. Membership in the club was very open and fluid. Yet, I came to see that it was perhaps the most effective tool I used to help students increase their mathematical achievement.

What They Told Me

My initial interviews, not surprisingly, revealed a great deal of fear. Most students said they were afraid that classmates would laugh at them if they made a mistake. Some students thought that they were the only person in their class who felt so afraid. When I asked if they had actually experienced ridicule for making a mistake, most said that they had not. Only one student recalled such an experience, in grade 1!

I next asked students how their fears affected their learning. One student said, "I was so scared, I didn't hear. I just sat and looked at the page"; another, "When there's a question I should know, I put up my hand and then I'm afraid that I'm not right, so I say that I forget"; a third student usually did nothing during assigned seat work, and would fill in the correct answers as they were being taken up.

I decided that we needed to bring the issues of fear of mathematics and fear of ridicule into the open. In a whole class discussion, I asked if anyone had ever had a fear of trying something new, especially in mathematics. Of course, the vast majority of the class admitted to this. Some of the "Math Club members" shared their feelings in this discussion, but many preferred to listen.

I then asked if any students would be willing to discuss this issue with a small group of like-minded classmates. Four girls volunteered. When asked how they felt about math, each of them said, "Scared." One student added, "...and embarrassed." "You think your friend will say that you should know that." I asked if any physical feelings were associated with this fear. "Yeah!" said one student, surprised that I would mention this. "It's like there's somebody walking through my stomach saying, 'Don't do math!'" Others replied, "My tummy starts to hurt," or, "I get hot in the face."

One student in this group suggested, and together we created, a display chart entitled, "Strategies to Overcome a Difficult Challenge."

Changes Start to Happen!

As time progressed, I made close observations of student behaviour when I introduced a new topic or challenge. I also conducted periodic interviews as well as on-going observations. Bit-by-bit, some of the most frightened students became more active learners. I saw students immediately look at our "Strategies" chart, or at other classroom displays. They often took out concrete materials to help with calculations. One student, who previously in the year had never handed in a test because she had only barely begun each one, one day completed an entire test to a good standard and handed it in with pride. Another student told me, "Before, I used to guess. Now I think about it." The student who had

previously spoken about someone walking through her stomach, one day said to me unprompted, "That little marching guy is going through my stomach again, but this time, he's telling me to do math." Those students who became more engaged in assigned tasks improved in their daily work and test results.

Once I asked my students what teaching techniques had helped them the most. The use of concrete materials was mentioned frequently. "You help me to understand what I'm doing," was often stated, a reference to my emphasis on concept development. The use of a variety of display charts also helped. One student said, "Math Club helped me a lot." (It had been only two days since I had first mentioned Math Club.)

Another said, "The homework you gave me". On two occasions, I had given her three simple calculations to complete after she asked me for homework. These last two students were obviously affected by something other than the amount of academic help received in that short time!

The degree of change varied from student to student, with some still "hiding" from my eye during lessons and seat work. As time went on, it became apparent that some students were moving ahead much more substantially and quickly than others. When I examined my data, it became clear that the most consistent difference between those students who showed greatly improved self-confidence and achievement and those who did not show much change, was in their willingness to share their feelings. Those students who showed improvement were those who had openly admitted their fears in front of classmates. Those students who continued to hold onto their fears were those who had only spoken to me privately about this issue.

In time, however, nearly all reluctant students did show improving confidence. I feel sure that the quieter ones were positively influenced by the example of their more openly expressive peers and the obviously greater success that they were having. We frequently talked about our feelings toward mathematical tasks in whole-class, small group and individual discussions. Students could articulate their feelings and could suggest effective strategies to deal with challenging problems.

Late in the course of this study, I knew that we were achieving results when the student who had through much of the year never even begun tests and other independent assignments, one day correctly completed several multiplication and division calculations and then wrote a detailed, logical and complete explanation of these procedures. She handed it in to me with shining eyes. The student who had spoken about "the little guy in my stomach", also completed this assignment successfully. "What about that little guy who runs through your stomach?" I asked. "He's still there", she said. "But now it feels like he's just sitting around, doing math himself."

Together We Learned

My students were quite interested in the fact that I was writing about them. Several times, I shared with the class early drafts of this article. One day, discussion turned to my image of the fear of mathematics as a wall or mountain between the student and the task. I asked the students what their image was. One student said that "it was like a maze". Another said that it was "like a big emptiness". A third student's image was that of "a see-through wall. When you don't try, it gets darker and darker and eventually you can't see through it at all." A fourth said it was "like you're in jail and you can't get out unless you try." A classmate added, "When you start thinking and thinking, it's like you find the key."

Math phobia is real. It creates a barrier that lies between the student and the learning of mathematics. Early attention must be paid to reluctant and fearful mathematics students. If their fear is not addressed, this barrier grows. My students showed me that a vital key to successful intervention lies in the emotional realm. A teacher must create a safe atmosphere and encourage students to share openly these feelings; all must work together to create this safe place. The process of acknowledging and reducing negative feelings must take place before a focus on academic outcomes can be fully productive. In my classroom in the future, there will definitely be more frequent discussion about the feeling side of mathematics so that reluctant students' molehills do not become mountains!

.....I knew that it would be quite useless to try to explain the problem to her right then. So I took the paper from her hands and asked, "What are you feeling?" "What?!" she asked. "What feelings do you have?" I repeated. Eventually she sputtered out, "Frustrated!" I told her to take a few minutes in a quiet space and to come back when her frustration was gone and she was able to listen calmly to my instructions. In a few minutes she was back. I handed back her paper and together we read the instructions. "Oh, is that what you do?" she said, and off she went. In a few minutes, she had successfully completed the task. ▲

▲ PRODUCT REVIEW: *MATHMANIA*

REVIEWED BY GORD COOKE

Gord Cooke is a secondary mathematics teacher who loves teaching with technology. When he is not giving Sketchpad workshops, he can be found sailing the St. Lawrence River and Lake Ontario.

Mathmania is a quarterly publication whose goal is to provide creative classroom-ready activities that can help teachers and their students enjoy doing math. Published quarterly, February, April, September and November, each issue contains eight to ten activities designed for grades 6-9. It is also available at a reduced rate in electronic format from its website www.mathadventures.com.

The activities in Volume 4, Number 1 (February 1999) include "Math Workout", "Why do Elephants Have Big Ears", "Interview with a Mathematician", and "Yesterday's Math News". The final article "Mathematically Speaking" is a crossword puzzle using many of the terms encountered in the earlier pieces and which appears to be a regular feature.

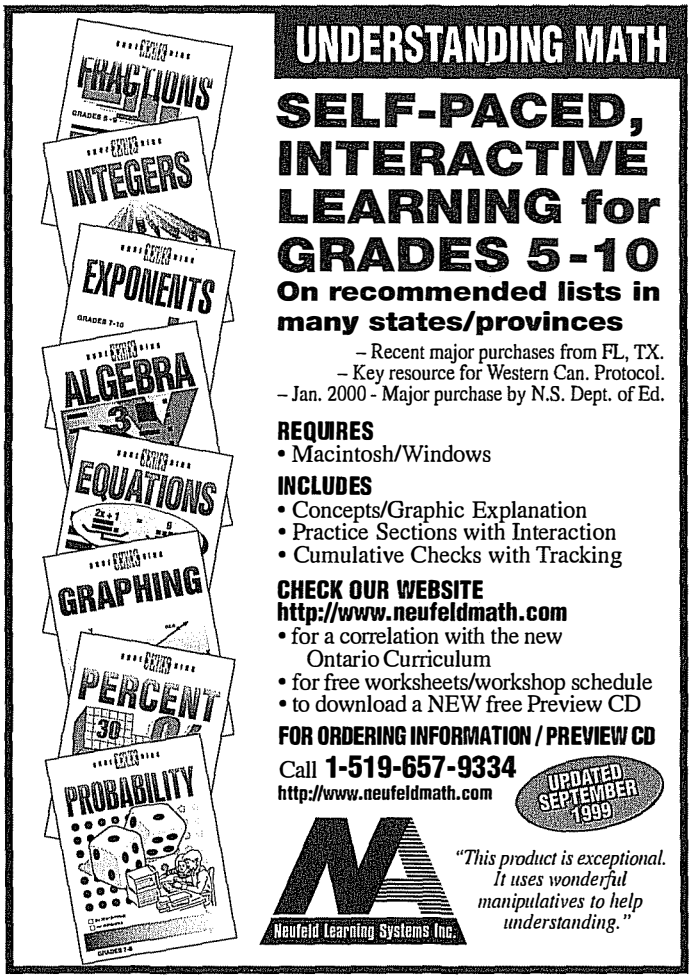
Most activities have a personal point of view. "Michelle wakes up Saturday morning..."; "Have you noticed that..."; "Doug Metcalfe has worked in the construction industry since 1970..." Students can see themselves in the activities as active participants or as a third party in a conversation or interview. Interviews are not only conducted with real people doing real math but with mathematical objects like circles and fractions who discuss their characteristics and history.

"Why do Elephants have Big Ears" is an investigation into the relationship between surface area and volume ideally suited for the new grade 9 curriculum. Because most elephants have a somewhat irregular shape, students consider a simpler model, a cube. Students are helped to understand the role that surface area plays by creating tables and analyzing ratios. In addition to providing clues which lead to the answer of the original problem, there are additional questions about sponges and dehydration. An extension of this activity might be to graph the values developed in the table leading to discussions of linear vs non linear relations.

The familiar problem of maximum area for given perimeter is particularly well developed in the article "A New Home for Scruffy". A full page story leads into the problem in which students are asked to note all words with a mathematical meaning. These reappear in the crossword at the end of the issue.

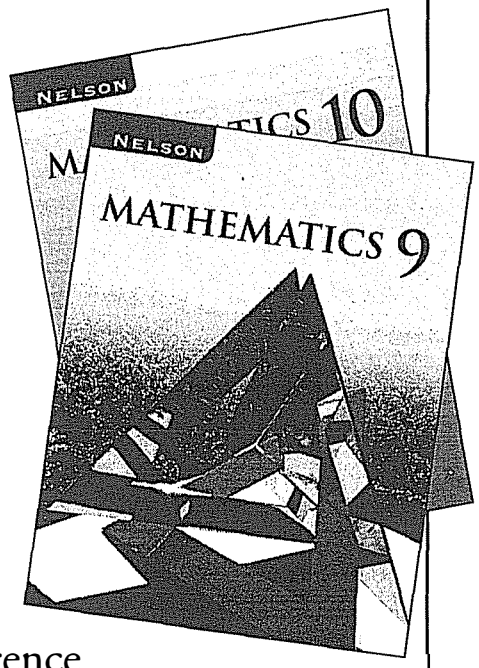
Mathmania falls into the "gotta have it" category at least for each school or math department. The inside cover includes a short pedagogical article which is especially appropriate to a system like ours which is in the process of reform. It helps put the activities in the broader context of rich learning tasks and constructivist theory. My only complaint is that some of the clip art is a little tired, but the overall quality makes this a negligible objection.

A two year subscription, eight issues, is $28, but for an even better deal, try the electronic version for $12 sent via email as PDF documents. For more information contact the editors of *Mathmania*, George and Janette Gananidis



Nelson Activities OAME 2000

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▲ Canadian Statistics in the Classroom: Resources for Activities in the Junior Classroom

SUSAN STUART
NIPISSING UNIVERSITY



Susan was chair of the 1998-99 OAME Early/Formative Committee. The committee felt that it is important to highlight a Canadian web resource for elementary teachers. A matching activity can be found at the end of the article.

Classroom teachers are constantly on the watch for resources that will enable them to make mathematics interesting and relevant for their learners. As teachers browse through journals frequently found in staff rooms of elementary schools, such as *Teaching Children Mathematics* (NCTM), they discover worthwhile activities but often find that the materials are based on American situations and statistics, measures are given in Imperial units or data is out-dated. It is time consuming to adapt these activities to make them suitable for our children, and Canadian information is frequently not available for substitution.

One source of information that is worthwhile for grades 4 - 8 teachers to have a look at, is the Statistics Canada web site. These resources have been developed using data collected from the Canadian census and from scientific and geographical knowledge. Up until now, this rich source of data has been shared with secondary school teachers, and the focus of the Statistics Canada educational department has been to assist subject-specific teachers and librarians to develop lessons and units for grades 9 to OAC. Both the Statistics Canada website and E-STAT, a CD-Rom available from StatsCan, are crucial databases for any up-to-date study of Canada in a secondary economics, family studies, geography, or history classroom, but are they resources that elementary school teachers should also have a look at?

This article will give you a brief glimpse at what the website contains and what role it might play in your classroom.

The Statistics Canada website, **www.statcan.ca**, is a well-organized site that provides information about Statistics Canada and its services. Logging into the homepage, you will find these choices:

Message from the Chief Statistician of Canada.

Daily news: Highlights of newly released data, schedules for major releases and announcements of new products and services.

Census: Free tabular data from the 1996 Census; including the Statistical Profile of Canadian.

Communities and information on the 2001 Census.

Education resources: Programs and products to integrate Canadian statistical information into teaching and learning.

In depth: Selected articles from our analytic periodicals.

Service centres: Information on our regional offices, libraries and other distributors of Statistics Canada data.

Employment opportunities.

Links to other sites.

Canada quiz: Explore our site and learn about Canada and Canadians.

Français

Canadian statistic: Free tabular data on aspects of Canada's economy, land, people and government.

Products and services Catalogue; CANSIM and Trade statistics ($); downloadable publications ($); research papers; list of recently released products; and other services.

Concepts, definitions and methods.

About Statistics Canada.

Are you in a Statistics Canada survey?

Year 2000 Seminars and conferences.

The site certainly seems to contain a wealth of information and I was anxious to explore its suitability for an elementary classroom. I clicked on Canadian Statistics, and after a few more clicks, came to the following headings:

The Economy: Latest indicators, Economic conditions, International trade, Primary industries, Manufacturing and construction, Communication, transportation and trade, Finance and services.

The State: Government, Justice and crime.

The Land: Geography, Environment.

The People: Population, Health, Education, Families, households and housing, Labour, employment and unemployment, Culture and leisure.

Statistical Profile of Canadian Communities contains additional free information, from the 1996 Census of Population, for approximately 6 000 communities in Canada.

I believe that grades 5 and 6 students would benefit from exploring some of these topics and with each click of the mouse, they would find current Canadian statistics. Within "Canadian Statistics: The Land", there is data about province and territory sizes, Great Lake sizes, river lengths, and tributaries. The user has access to weather conditions in Canadian capitals and major cities around the world, including precipitation (snowfall, total precipitation and number of wet days) and high and low temperatures. What elementary teacher has not spent hours trying to find this type of current information? Having the following up-to-date information would make it possible for classroom teachers to develop activities that require learners to find average temperatures and temperature ranges, to record statistics on line and bar graphs, and to compare local weather patterns with those of major cities in Ontario.

AVERAGE ANNUAL TEMPERATURE			COLDEST MONTH			WARMEST MONTH		
	Average high	Average low	Month	Average high	Average low	Month	Average high	Average low
St. John's	8.6	0.8	February	-1.4	-8.7	July	20.2	10.5
Charlottetown	9.5	0.8	January	-3.4	-12.2	July	23.1	13.6
Halifax	10.7	1.4	February	-1.5	-10.6	July	23.4	13.2
Fredericton	11.0	-0.6	January	-4.0	-15.4	July	25.6	12.9
Québec City	9.0	-1.0	January	-7.7	-17.3	July	24.9	13.2
Montréal	10.9	1.2	January	-5.8	-14.9	July	26.2	15.4
Ottawa	10.7	0.8	January	-6.3	-15.5	July	26.4	15.1
Toronto	12.6	5.2	January	-1.3	-7.9	July	26.5	17.6

http://www.statcan.ca:80/english/Pgdb/Land/Geography/phys08b.htm (in part)

Moving away from the *Canada Statistics* section of the website, I clicked my way through bits and pieces of the Census section. I found profiles of selected cities, towns and villages of Canada. Do your students need to find out about the town or city that they live in, or about a far away place such as Inuvik? Just enter the information and select the type of information you require. You can request community population sizes for 1991 and 1996 (the census years) with the percentage population change, discover land area, investigate the education level or average income of a specific population, types of work, family sizes, and types of dwellings. All data are given by gender, age, and total, and specific populations can be compared to the total Canadian population data. I was curious about my hometown of Sudbury and found that the population decreased by 824 between 1991 and 1996, slightly less than 1% of the total population. I believe that junior level students would be interested in finding out information about the area of the province in which they live. In the process, they would have opportunities to develop number sense, practice computational skills, and deal with data in many forms.

The Statcan web site is timely. For example, last year it provided information and activities about "The Great Ice Storm of 1998". Recently, I enjoyed activity ideas found in the *Teachers' Kits Lesson Plans* that have been based on the 1996 Census results. This site provides elementary and secondary teachers with classroom materials dealing with the results of the survey (population counts, age and sex, families, immigration and citizenship, language, income, and labour force). The activities are produced in PDF format and you will need Adobe Acrobat reader to access the files.

One drawback of the *Population Counts* page, **www.statcan.ca/english/kits/counts.htm**, is that it records all large numbers using the comma rather then a space to separate the three-digit periods. Teachers should adjust this when presenting the information to the students. The first of the elementary activities presents the learners with a map of Canada showing national, provincial, and territorial population figures and requires them to make comparisons. This section of the web site is well-worth some time exploring and using. Some other activities that can be found are:

The Perfect Principal

This activity is meant to be used as a quick assessment tool at the beginning of the year or before entering a mathematics unit on Data Management and Probability. During this activity students are required to examine survey data on the desired attributes of a perfect principal, describe the information that can be extracted from the given data and calculate the mean, median and mode for the set of data provided. **www.statcan.ca/english/kits/princ1.htm**

OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration;

- demonstrations of excellence and creativity in mathematics education.
- specific contributions to mathematics education in the school, region, province or nation.
- demonstrates leadership in mathematics education in the school, region, province or nation.
- support of and contribution to the activities of OAME/AOEM.
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld form consideration.

OAME/AOEM LIFE MEMBERSHIP AWARD

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and accumulated 10 or more years of membership in OAME/AOEM.

KENNETH D. FRYER AWARD

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

DON ATTRIDGE AWARD

In recognition of the contributions made by Don Attridge, a founding member, former president and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

AWARD NOMINATION COVER PAGE

Send to: Shirley Dalrymple

Intended Award (please check one)

❑ Life Membership ❑ Mona MacGregor ❑ Morley MacGregor ❑ Kenneth D. Fryer ❑ Don Attridge

Nominee Information:

_____

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

_____

_____

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

Nominator Information:

_____

Name of Nominator

_____

_____

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

Analysing the Job Market for Babysitting in Your Neighbourhood

Grades 6 to 9 students research the demographic profiles of a neighbourhood in a large urban centre or community, by retrieving data that will help them analyze the job market for babysitters. They will develop a better understanding of how population dynamics can affect them, either directly or indirectly. **www.statcan.ca/english/kits/baby1.htm**

'Snack Attack'

This activity is meant to be used as a quick assessment tool for the beginning of the year or before entering a mathematics unit on Data Management. With minimal changes this activity can be used to assess students' skills in different grades (3-7). Students count the different types of snacks in a bag of *Bits and Bites* and use the primary data to answer a number of questions related to data analysis. Data are used to create graphs or tables. Later, grades can be given a choice as to what type of graph to draw, which measure of central tendency to use etc. **www.statcan.ca/english/kits/snack1.htm**

What is the Average Height of Your Class?

This simple Internet exercise can be used by grade 1 and up to reinforce lessons on measurement (specifically height), data analysis, and measures of central tendency, graphing and self-concept for intermediate students. **www.statcan.ca/english/kits/height.htm**

The Statistics Canada web site has great potential to be a rich Canadian resource for elementary teachers. This potential can only be fulfilled if Canadian classroom teachers make use of the information that it presents and add to the site with material of their own. Although Statistics Canada has the staff to develop some lessons and activities, the most valuable source for lessons and activities comes right from your classrooms. I would challenge all elementary teachers to use the site to develop rich problems. Then, share the results by sending the problems and sample students' work to (1) Statcan for inclusion on their site (2) to O.A.M.E. for publication in the *Abacus* or (3) to me for a follow-up article (see Board of Directors list for address). By helping one another through the sharing of ideas, elementary teachers can take a great resource from the web and transform it into powerful mathematical experiences for their learners.

Descriptions of these activities were provided by Statistics Canada, Nov, 1999. ▲

▲ Call for Applications for OAME Executive Directors

The Ontario Association for Mathematics Education is looking for new Executive Directors to take over the full responsibility of day-to-day operations September 2001. The appointment is to be made February 2001 to allow for a staged transfer of duties from the current Executive Directors.

Responsibilities include:

- Maintenance of the membership database
- Liaison with the printer and mailing house for *Gazette* mailings
- Managing the finances and the financial records
- Supervision of the sale of resource documents
- Arrangements for Board meetings, Executive meetings and Annual General Meeting and recording of minutes of each
- Arrangements for the Annual Leadership Conference
- Acting as an advisor to the Executive and Board regarding Constitution, By-laws, and terms of reference

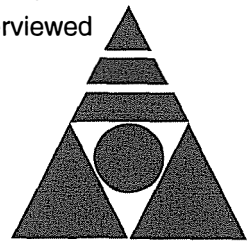
The position requires:

- some experience with OAME at the provincial or chapter level
- computer literacy, including e-mail
- available space for the OAME office records, supplies, and equipment
- availability to communicate with members and educators during working hours
- bookkeeping skills
- organizational skills

Ideally the position would be filled by a couple rather than an individual since many of tasks require the cooperation of two people.

There is a modest monthly fee paid to the Executive Directors as well as travel, telephone, and other expenses incurred in conducting OAME business. Candidates should apply in writing, stating qualifications, prior to November 1, 2000 to: Peter Saarimaki, Chair of Search Committee, 67 Erinlea Crescent, Scarborough ON M1H 2S8.

Successful candidates will be interviewed by a panel in early 2001.



A STATCAN ACTIVITY FOR GRADE 5

Population by Mother Tongue, Showing Age Groups, for Canada, Provinces and Territories, 1996 Census - 20% Sample Data

MOTHER TONGUE	TOTAL - AGE GROUPS	UNDER 15	15-24	25-44	45-64	65 AND OVER
CANADA						
TOTAL - MOTHER TONGUE	**28 528 125**	**5 899 200**	**3 849 025**	**9 324 345**	**6 175 780**	**3 279 770**
English	16 890 615	3 936 505	2 400 635	5 469 760	3 304 820	1 778 895
French	6 636 660	1 246 945	875 930	2 198 120	1 572 295	743 350
Non-official languages	4 598 285	612 275	515 890	1 533 410	1 225 500	711 195
English and French	107 940	29 025	14 750	28 925	20 335	14 905
English and non-official language	249 545	62 665	35 325	80 140	44 190	27 225
French and non-official language	35 840	9 220	4 770	11 420	7 090	3 340
English, French and non-official language	9 225	2 555	1 705	2 550	1 555	845
ONTARIO						
TOTAL - MOTHER TONGUE	**10 642 790**	**2 213 575**	**1 399 530**	**3 483 420**	**2 299 610**	**1 246 660**
English	7 694 630	1 803 360	1 068 935	2 471 850	1 520 780	829 705
French	479 285	68 255	53 475	164 520	130 720	62 315
Non-official languages	2 296 570	296 575	251 460	791 330	618 795	338 415
English and French	33 940	9 255	4 610	9 140	6 405	4 530
English and non-official language	130 730	34 800	20 100	44 030	21 090	10 720
French and non-official language	5 335	715	550	1 915	1 455	715
English French and non-official language	2 300	630	400	625	375	270

Permission granted for educational purposes and adapted from Statistics Canada Internet Site

EXPECTATIONS:

- explain the choice of intervals used to construct a bar graph or the choice of symbols on a pictograph;
- display data on graphs (e.g., line graphs, bar graphs, pictographs, and circle graphs) by hand and by using computer applications;
- relate division to fractions (e.g., 16 divided by 3 is 5 1/3);

ACTIVITY:

English and French are the two official languages of Canada.

1. In small groups, discuss and decide what information this chart provides. List at least three things. Compare your list with other groups.
2. The chart uses many large numbers. Sometimes it is difficult to compare information when you have numbers as large as these. We can use our calculators to make the comparisons simpler.
   For example: The chart reports that 5 899 200 people under the age of 15 live in Canada and 3 936 505 of them speak English as their 'mother tongue'. That is 3 936 505 / 5 899 200 = 0.67 (or about 2/3).
   Use your calculator to find the fraction of 'under 15 year-olds' who speak French and other languages. What fraction of the 'under 15 year-olds' speaks three languages? Are the fractions the same for Ontario?
3. Discuss what information from the chart your group would like to display. Think about
   - What type of graph or chart would best show your information?
   - How you will create your graph or chart - by hand or computer.
   - What intervals you will use on your graph. Why?
4. Gather data from students in your class and/or school to find out how the languages they speak compares to all of Canada or to Ontario. Display your data and describe your findings.
5. Log on to the Statcan website to get similar information about the other provinces and territories. Have each group in the class choose a different area of Canada and compare the data to Ontario or to the classroom statistics.

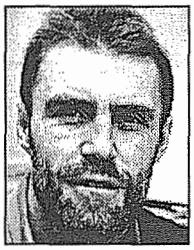
TEACHER NOTES:

With the students, discuss the languages that might be included in 'non-official' category and the types of information that can be compared using a graph. (e.g., a) number of 'under 15' that are included in each language group, b) population in each age group in Ontario or in Canada.)

Rather than work with the very large numbers, students should use calculators to reduce the comparisons to simple fractions or decimals. If the students express all the comparisons as decimals, they could then represent the data by colouring corresponding sections of a 10 x 10 grid.

▲ TIMSS: What Can We Learn About Ontario Mathematics?

GEOFFREY ROULET
FACULTY OF EDUCATION, QUEEN'S UNIVERSITY



Geoff Roulet teaches pre-service and M.Ed. courses in Mathematics Education. His research interests include the development of learning environments and activities in which students can discover their own mathematics.

In November 1996 the first results, those for 13 year-old students, from the Third International Mathematics and Science Study [TIMSS] were released (Education Quality and Accountability Office [EQAO], 1996; Beaton, et al., 1996). As usual, newspaper stories focussed on how our pupils measured up to those in the 45 other jurisdictions participating in the study. The international ranking of our students is of interest, but TIMSS provides us with much more data. This information can be combined with the performance measures to tell us about school systems, students' lives, teachers, curricula, and instructional methods around the world. This article will explore some of these issues at the Grades 7 and 8 level.

An Overview of TIMSS

The TIMSS program collected data at three school levels or populations (Martin, 1996).

Population 1. Students enrolled in the two adjacent grades that contained the largest proportion of 9-year-old students at the time of testing, third and fourth-grade students in most countries.

Population 2. Students enrolled in the two adjacent grades that contained the largest proportion of 13-year-old students at the time of testing, seventh and eighth-grade students in most countries.

Population 3. Students in their final year of secondary education. As an additional option, countries could test a special subgroup of these students - students taking advanced courses in mathematics - in Ontario, students who were taking or had taken both OAC Calculus and OAC Algebra and Geometry.

Forty-six jurisdictions in 40 different countries participated in the complete Population 2 study and different subsets of this group opted to take part in the research at the other two levels. Ontario joined all phases of the TIMSS study, both as part of the Canadian sample and as a jurisdiction on its own. Thus our students' performance and factors in our school system and society at large, may be compared with those in other Canadian provinces and other countries.

The student surveys and paper-and-pencil tests were completed in the spring of 1995, but the TIMSS data

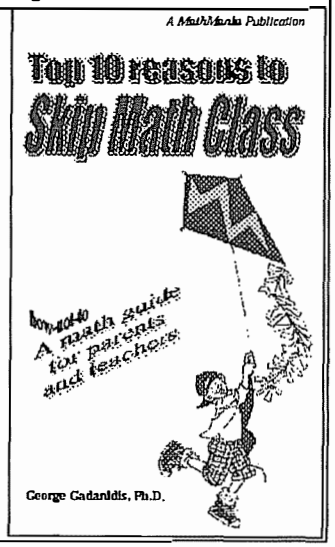
Top 10 Reasons to Skip Math Class ***A how-not-to math guide for parents and teachers***
38 pages; $5 (shipping and taxes included)

What would you do if you suddenly discovered that the math taught in school actually taught children not to think mathematically?
Highly recommended for people who can laugh and cry at the same time.
Guaranteed to double its value when it gets banned.

CAUTION !!
- ❖ This book was written by a math consultant; and
- ❖ It is not approved by the Ontario Ministry of Education.

Don't conform! Buy a copy or two by sending a fax, email, or letter to:

MathMania, 2145 Whitfield Road, Port Perry, ON, L9L 1B2
Fax: 905.985.9498 email: mathmania@mathadventures.com
Visit our web site for more neat stuff: http://www.mathadventures.com



collection began in 1988 with an analysis of mathematics and science curricula in the participating countries. The multi-stage approach to the study provides information at three levels of the curriculum development and delivery process (Martin, 1996):

- the intended curriculum, the mathematics instructional and learning goals as defined at the system level,
- the implemented curriculum, the mathematics curriculum as interpreted by teachers and made available to students,
- the attained curriculum, the mathematics content that students have learned and their attitudes towards the subject.

For the development of assessment items, all participating countries were permitted to submit questions employed in their national testing and system review programs. Participating jurisdictions identified the match between submitted items and their curricula and through an iterative process an international consensus on appropriate item pools was developed. The TIMSS question sets can be taken as representing an international core curriculum, topics that most jurisdictions address to some extent, but not perfectly matching the program of any one nation (Garden & Orpwood, 1996).

All test items were constructed in English and then translated into 30 other languages and adjusted to fit jurisdictional cultures. Potential items were pilot tested in the participating nations.

All jurisdictions were expected to follow a sampling protocol that was developed and overseen by staff of Statistics Canada. Random samples of students for each test were to come from in excess of 75% of the respective populations. Although some countries had difficulties meeting the high sampling standards, at the population 2 level, 61% of the nations were deemed to have followed the protocol. In particular, the participation rates of Singapore and Japan, the two top achieving nations, were 95% and 87% respectively, while Canada's was 84% (Martin & Mullis, 1996). There have been suggestions (Ontario Secondary School Teachers Federation [OSSTF], 1997) that the TIMSS results are invalid and biased against Ontario, but the careful attention to test construction and sampling indicate that this is not the case. It is true that our pupils' achievement does not indicate their knowledge of the Ontario curriculum. What TIMSS does give us is a comparison of Ontario students' abilities to answer a set of mathematics questions deemed appropriate by educators working in over 40 different countries. There may exist valid explanations for Ontario's performance, but each of these should be taken as a question rather than an excuse. For instance we might explore the following issues:

- How does the Ontario mathematics curriculum compare with TIMSS and the curricula of other jurisdictions?

  Are we satisfied with the differences or should we adjust our curriculum to be closer to that of the top performing nations?
- How does Ontario mathematics instruction compare to that in other jurisdictions?

  Should we adopt the teaching approaches used in the top scoring countries?
- What factors in Ontario's educational system might affect TIMSS performance?

  Should we adopt school structures similar to those of top performing jurisdictions?
- How do Ontario teachers' qualifications compare to those in other jurisdictions?

  Should we require qualifications similar to those in countries with higher scores?
- What student demographic characteristics might affect performance?

  How could the Ontario school system be changed to accommodate for potentially negative factors in student backgrounds?

TIMSS Results

The newspapers and politicians love to talk about Ontario's ranking relative to other jurisdictions, but care must be taken in interpreting average success scores. Naturally occurring sampling errors mean that any one country's achievement scores could be expected to fall in a range that overlaps the results of neighbouring entries in the ranking lists. Thus the best we can do for each TIMSS population is identify three groups relative to Ontario: those jurisdictions that scored significantly above, those with effectively equal results, and those with achievement results significantly below. The numbers of jurisdictions in each of the above three groups for all four TIMSS populations and sub-populations at the senior level are shown in Table 1 (Robitaille, Taylor & Orpwood, 1996, 1997; Robitaille,

Taylor, Orpwood & Donn, 1998). These numbers and the Above:Below ratios in the bottom row of Table 1 show that the lowest ranking for Ontario occurred with the 13 year-old (Grades 7 and 8) population.

RESULTS RELATIVE TO ONTARIO	9-YEAR-OLDS GRADES 3 & 4	13-YEAR-OLDS GRADES 7 & 8	GRADUATES LITERACY GRADE 12 & OAC	GRADUATES MATHEMATICS SPECIALISTS
Above	13	20	3	1
Equal	7	12	10	10
Below	9	12	10	7
Ratio	13:9	20:12	3:10	1:7
Above:Below	= 1.44	= 1.67	= 0.30	= 0.14

Table 1: TIMSS Results: Ontario Mathematics Ranking.

A closer look at the full range of Grade 8 TIMSS data can suggest reasons why Ontario's relative performance was weakest at this point in our school system.

Ontario Grade 8 Mathematics Curriculum and TIMSS

The population 2 TIMSS assessment items were grouped into six content areas: fractions and number sense; geometry; algebra; data representation; analysis and probability; measurement; and proportionality. Table 2 gives the Ontario, Canadian, and International Grade 8 averages on the full TIMSS test and each of the five content areas individually.

CONTENT AREA	ONTARIO	CANADA	INTERNATIONAL
Full TIMSS Test	54	59	55
Fractions & Number Sense	60	64	58
Geometry	52	58	56
Algebra	48	54	53
Data Rep. Analysis & Probability	64	69	63
Measurement	46	52	51
Proportionality	44	48	45

Table 2: Grade 8 TIMSS Performance by Content Area.

The numbers show that relative to the International and Canadian performance, Ontario did least well in algebra and measurement. An examination of the Ontario curriculum, the types of questions typically presented to Ontario students, and the TIMSS test items suggests some reasons for our relatively weak performance.

A content analysis and comparison with the Ontario Grades 7 and 8 curricula has been carried out (Nagy, Russell, Bordignon, Roulet & Higginson, 1998). At the time of the TIMSS test, the spring of 1995, *The Common Curriculum* (Ontario Ministry of Education and Training, 1995) had just recently been put in place. Implementation of this new curriculum was incomplete and many classes were still employing textbooks written to comply with the previous mathematics curriculum guide (Ontario Ministry of Education, 1985). Thus the analysis of TIMSS items was carried out against the 1985 curriculum and each item was identified as falling within or beyond the Ontario courses.

Those questions that applied to the Ontario program were identified with the appropriate unit of the Ontario course and an estimate of the required teaching time was made. Similarly the time required to address all Ontario course topics not tested by TIMSS was also estimated. Figure 1 compares the content of the population 2 TIMSS test to the Ontario curriculum.

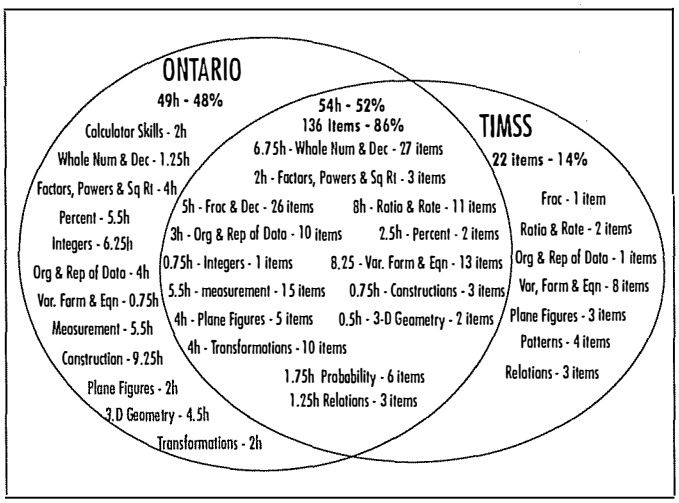


Figure 1: Ontario Grade 8 Mathematics Curriculm (1985) & TIMSS

The vast majority (86%) of TIMSS items were deemed to fit the Ontario curriculum, but on the other hand almost half of the Ontario Grade 8 course was not assessed by TIMSS. Some major Ontario Grade 8 topics, such as Percent, had few TIMSS questions since in most countries this concept is addressed at an earlier grade. Some topics, such as Calculator Skills, are included in the curricula of only a few nations. If an Ontario Grade 8 teacher had followed the curriculum as set out in the 1985 guideline, one-half of their mathematics class time would have been dedicated to instruction that would not have had a direct benefit for TIMSS performance.

The poorest match between the Ontario curriculum and TIMSS test occurred for the topic of Variables, Formulas and Equations (Algebra). Here 8 of 21 questions were deemed to be beyond the Grade 8 course as set out in the 1985 *Guideline*. This mismatch

may have been increased by teachers' interpretations of the Ontario curriculum documents. Although 13 TIMSS items fell within the Ontario unit of Variables, Formulas and Equations, for which the guideline suggests 9 hours of instruction, only 20% of Ontario teachers reported giving this amount of time to the topic. In fact, 42% indicated that they did not include the unit in their courses. In many other jurisdictions this confusion as to what topics are on or not on the curriculum does not exist. For instance, in Singapore 100% of the Grade 8 teachers identified Algebra as a course topic and provided it with more than 10 hours of course time. With the large number of topics in the Ontario curriculum creating time pressures, and, prior to the release of the 1997 curriculum (Ontario Ministry of Education and Training), the relative freedom of choice provided our teachers, the implemented curriculum often varied considerably from the intended. With many Ontario Grade 8 teachers lacking strong backgrounds in mathematics it is not surprising that a significant number may have opted to drop the more abstract topic of algebra.

Even when Ontario teachers provided more than 10 hours of study in Algebra, student performance was not great. The average achievement on Algebra items by Ontario pupils receiving such instruction was 41% while Singapore students had an average of 79%. Pupils in Singapore did significantly better than Ontario students on both mechanical (TIMSS Item L16) and problem solving (TIMSS Item T1) algebra questions (see Figure 2) (International Association for the Evaluation of Educational Achievement [IEA], 1996).

TIMSS POPULATION 2 ITEM L16

Find x if $10x - 15 = 5x + 20$

Answer: _____

TIMSS POPULATION 2 ITEM T1

There are 54 kilograms of apples in two boxes. The second box of apples weighs 12 kilograms more than the first. How many kilograms of apples are in each box? Show your work.

Figure 2: TIMSS Algebra: Variable, Formulas & Equations Items.

A close examination of the curricula of the Asian countries shows why their students perform well on questions such as Item T1 above. Stevenson and Stigler (1992, p. 187) provide an example of a Japanese fourth-grade question (Figure 3).

There is a total of thirty-eight children in Akira's class. There are 6 more boys than there are girls. How many boys and how many girls are in the class?

Figure 3: A Japanese Grade 4 Problem.

The children did not employ algebra to solve this problem, but represented the class by a strip of paper, divided it into 38 equal parts, took off six for the extra boys, and by folding the remaining length in half found the number of girls. With such concrete experiences in the primary and junior grades, the transition to algebra is simplified.

In the other area of relative weakness for Ontario, Measurement, all TIMSS questions were considered as fitting the Ontario curriculum. Although Ontario students appeared to understand the basic measurement concepts they were not able to apply them in problem settings such as in Item K5 (Figure 4). A significant portion of the Measurement items were of this type:

The length of a rectangle is 6 cm, and its perimeter is 16 cm. What is the area of the rectangle in square centimetres?

Answer: _____

Figure 4: TIMSS Measurement Item

That Ontario students are capable of more than just following set routines is shown by their achievement on the Patterns questions. For instance, Item J18 (Figure 5) was one question in which Ontario Grade 8 pupils out performed their Singapore counterparts. Our students' abilities to solve such questions is probably due to the patterning activities they experience in the primary grades.

TIMSS POPULATION 2 ITEM J18

The table represents a relation between x and y.

What is the missing number in the table?

A. 2
B. 3
C. 4
D. 5
E. 6

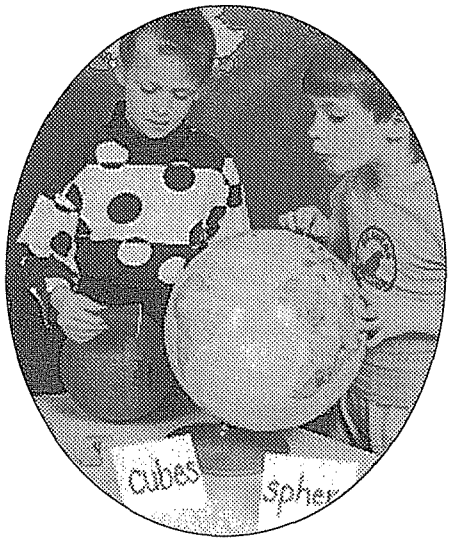
x	y
1	1
2	?
4	7
7	13

Figure 5: TIMSS Algebra: Pattern Item.

In general it seems that Grades 7 and 8 students in other jurisdictions participated in more problem solving activities than Ontario pupils and their study of algebra was more extensive. The new *Ontario Curriculum Grades 1-8: Mathematics* (Ontario Ministry of Education

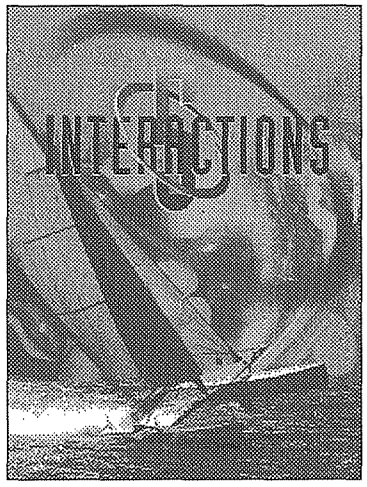
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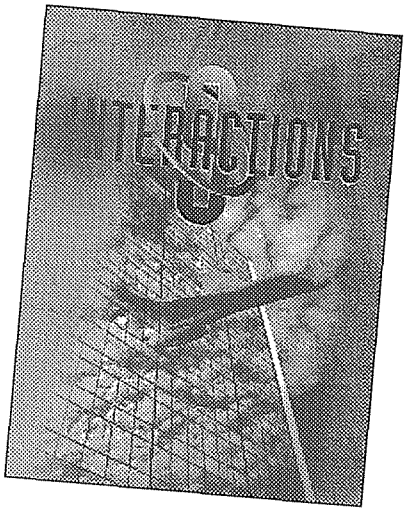
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and Training, 1997) increases the algebra content of the senior elementary courses, but it is not at all clear that the guideline calls for more significant problem solving. In addition, the curriculum is still very extensive and is likely to generate time pressures that force teachers to continue to be selective in their topic coverage.

Comparisons of Teaching

As part of TIMSS, the United States Department of Education, National Center for Educational Statistics and the National Science Foundation funded a special study (Stigler & Hiebert, 1997) to closely observe and analyze the teaching in 231 Grade 8 classrooms in Germany, Japan and the US. The data from this research allows us to compare what typically happens during mathematics lessons in these countries to what occurs here in Ontario. The picture of Japanese mathematics instruction that emerges from this study contrasts sharply with the typical image of Asian classrooms as places where pupils silently participate in endless drill and practice.

Japanese mathematics lessons were designed to promote student understanding while US and German teachers focussed on the development of skills. In the US, mathematics was presented by the teacher, while in Germany, topics were developed through teacher led lessons. The typical lesson pattern in both countries was teacher demonstration and explanation of a procedure followed by individual student practice. These observations echo those made during Ontario's provincial review of Grade 8 mathematics which reported that "the most commonly used methods for teaching Grade 8 mathematics are presentation of information to the class by chalkboard or overhead projector and assignment of individual work" (Ontario Ministry of Education, 1991, p. 2).

In contrast to the US, German, and Ontario situations, Japanese teachers do not dominate their classrooms and the lesson sequence is distinctly different. Typically, students are presented with a problem situation for which they attempt to generate a solution. Methods are shared and the class collaboratively develops



approaches and understandings of the mathematics involved. "Whereas students in the US and German classrooms must follow the teacher as she leads them through the solution of example problems, the Japanese students have a different job: to invent their own solutions, then reflect on those solutions in an attempt to increase understanding" (Stigler & Hiebert, 1997, p. 1.2).

The level of mathematical thinking called for in Japanese classrooms is high. In over half the lessons observed, students presented alternate solutions and explained their thinking. "Japanese students spent nearly half of their time inventing new solutions and engaging in conceptual thinking about mathematics" (Stigler & Hiebert, 1997, p. 3.1). In almost two-thirds of the lessons the reasons behind mathematical processes were deductively developed. That such teaching is not a new development in Japan can be seen by a study of the recent translation (Becker & Shimada, 1997) of a 1977 Japanese text illustrating and explaining the open-ended approach for that country's teachers. In most cases, although the Japanese instructional methods incorporate many of the mathematics education reform ideas promoted by the National Council of Teachers of Mathematics [NCTM] (1989) and Ontario Association for Mathematics Education [OAME] (1993), the mathematics topics are quite traditional. Thus Japanese students have for a number of years been studying the mathematics of the TIMSS assessment, but through approaches that promote understanding and problem solving abilities.

School Systems, Teacher Assignments and Professional Sharing

In general, Ontario teachers are as well or better qualified that those in other countries (Epp & Ross Epp, 1998). In Singapore and Japan, as in Ontario, there are a variety of routes to obtaining teacher qualifications, but the end result is that in all three jurisdictions the majority of teachers have four or five years of post secondary education. All three jurisdictions also provide specialized programs for those preparing to teach at different school levels. It is in the definition of these levels where major differences are found.

In separating their school systems into divisions, the vast majority of countries participating in TIMSS designate Grades 7 and 8 as part of their secondary panel (Robitaille, 1997). This second level of schooling may be further divided into lower and upper secondary or junior and senior high school, but still in these cases, 12 and 13 year-old students are placed in a secondary school environment. In particular, Singapore has three divisions, primary, orientation, and secondary, comprised of grades 1 to 4, 5 and 6, and 7 to 11 respectively, and Japan follows a 6-3-3, primary, lower-secondary, upper-secondary pattern. When placed at the secondary school level, students of ages 12 and 13 attend classes led by teachers with education and training directed toward specific subjects. In Ontario, although we include Grades 7 and 8 in the Intermediate Division, these grades are most often treated as part of the elementary school system, and instruction is provided by generalist teachers. Thus, in most countries, including Singapore and Japan, the top achievers in the TIMSS assessment, teachers responsible for Grades 7 and 8 mathematics can focus on this one subject.

The demands on Ontario teachers at the senior elementary grades is increased by their number of contact hours. Although teachers in Asian countries spend more hours per day at school, they are in their classrooms for fewer hours. TIMSS data show that Canadian Grade 8 teachers have an average of 27 hours per week of student contact while Japanese teachers spend an average of 20 hours in the classroom. Asian teachers spend their out of class time, approximately one-third of their school day, in groups, cooperatively planning lessons. In contrast, Ontario teachers plan curricula and lessons and assess student work in relative isolation with little chance for professional sharing. Ontario teachers further reduce their opportunities for professional growth by spending fewer hours on professional reading than the majority of teachers participating in TIMSS. In fact, 16% reported that they never consulted the professional literature.

Some of the relative weakness in Ontario's Grades 7 and 8 TIMSS achievement scores may be attributed to the more favourable Asian teaching conditions: the opportunity to focus on mathematics and to participate in collaborative course and lesson development. I am not being critical of teachers presently teaching Grades 7 and 8 in Ontario, but I believe that if we wish to increase the intellectual level of study in: history and geography, science and technology, health and physical education, the arts, language, and mathematics; in the senior elementary years we need to move to a system that provides classes led by subject specialists. Ontario Junior-Intermediate qualified teachers graduate from faculties of education with particular subject

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concentrations and Intermediate-Senior teachers are also qualified to teach Grades 7 and 8. Using the Ontario certification system presently in place it should be possible to join the rest of the world and provide mathematics classes taught by professionals who have qualifications, confidence, and interest in the discipline.

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Initial ideas for this article were developed during participation in an EQAO funded project, *Ontario in TIMSS: Secondary Analysis and Recommendations,* directed by Graham Orpwood (York University) and, for mathematics, conducted by Philip Nagy, Howard Russell, Catherine Bordignon (University of Toronto/OISE), Geoffrey Roulet, and William Higginson (Queen's University). ▲

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Glenn has classroom teaching experience with the Peel Board of Education and has extensive experience as a tutor. He knows the importance of home study in the goal of success with grade 11 mathematics and his videos and workbook cover this curriculum thoroughly and very accurately. He designed his *VideoMath* program to reinforce math skills needed to complete MAT3A and to continue with advanced studies. This program is not a recognized Ontario credit course and I'm sure any user of the program would be quick to add 'it is easier to understand than the text I use'. When I supply teach I use the video that fits the current study topic the best. All of my private tutorial students in MAT3A are using the 365 page workbook. Everyone of them are studying the well-written examples, doing the practice exercises and finding the hints section and answer keys very helpful. In fact, the use of the workbook has meant better marks in class and fewer tutorials which gives the young person independence in math studies.

The voice on the video with light music in the background is comfortable listening. The diagrams on the video are neat and the lesson/solutions are clearly spoken. All the worked examples on the video are also printed in the workbook. The student learns through hearing, seeing and feeling as he/she listens, reads and writes. All the material is user-friendly even for the most anxious math student. And yes, the program does lots of problem solving to show applications.

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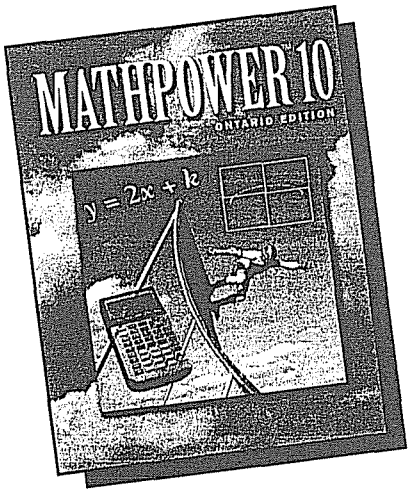
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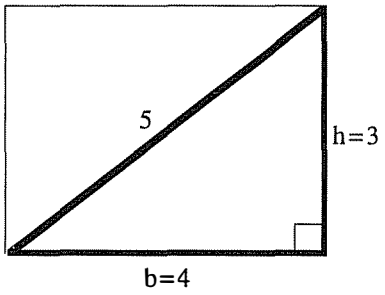


André Lemaire was born in the village of Rulles, Belgium. He came to Canada in 1967 and received his undergraduate degrees in Ottawa. He teaches mathematics and physics at Garneau High School. His hobbies include skydiving and physics.

The triangle is the king of polygons since any polygon can be constructed using triangles. The triangle is irreducible since it is the only geometrical figure to keep its shape, which is why the triangle is so important in architecture. Its role is to make any construction rigid (called triangulation). If one wants to build something with efficiency, that is light and solid, one must use triangles. Along with circles, triangles are the most studied geometrical figure.

In the following text and diagrams, I demonstrate 7 methods to calculate the area of the triangle. In order to illustrate these different methods, and to ensure the reader can compare them easily, I will use the same triangle. The methods illustrated in this paper are universal, which means that they work for any kind of triangle. To simplify the comparisons, I use a right-angled triangle with sides 3, 4 and 5, the famous Pythagorean triple.

METHOD #1: "THE CLASSIC"

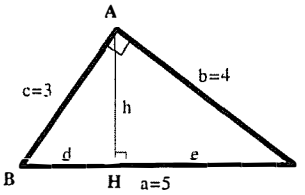


Let A= area, b = length, h = height.

Rectangle $A = b \times h$. Triangle $A = \frac{b \times h}{2}$.

Therefore, $A = \frac{4x3}{2} = 6 \text{ units}^2$.

METHOD #2: HERON'S FORMULA



a, b, c, are the 3 sides opposite to the 3 angles A, B, C.
h is the height
H is the foot of the height
d = BH
e = HC

Note: In this case, we will find the value of h with respect to the sides, then we will use the formula from method 1.

CALCULATION OF h

$b^2 = h^2 + e^2$

$e = a - d$

$h^2 = c^2 - d^2$ $\qquad$ $e^2 = a^2 - 2ad + d^2$

then $b^2 = c^2 - d^2 + a^2 - 2ad + d^2$

or $b^2 = a^2 + c^2 - 2ad$ (1)

or $h^2 = c^2 - d^2$ (seen above) and

from (1) $\quad d = \frac{b^2 - a^2 - c^2}{-2a}$

or $d = \frac{a^2 + c^2 - b^2}{2a}$

therefore $d^2 = \frac{(a^2 + c^2 - b^2)^2}{4a^2}$

hence $h^2 = c^2 - \frac{(a^2 + c^2 - b^2)^2}{4a^2}$

so $h^2 = \frac{4a^2c^2 - (a^2 + c^2 - b^2)^2}{4a^2}$

this above numerator is a difference of 2 squares

we obtain $h^2 = \frac{(2ac + (a^2 + c^2 - b^2))(2ac - (a^2 + c^2 - b^2))}{4a^2}$

remove the inside brackets of the 2 numerator factors

we get $h^2 = \frac{(2ac + a^2 + c^2 - b^2)(2ac - a^2 - c^2 + b^2)}{4a^2}$

then $h^2 = \frac{((2ac + a^2 + c^2) - b^2)(b^2 - (-2ac + a^2 + c^2))}{4a^2}$

and $h^2 = \frac{((a+c)^2 - b^2)(b^2 - (a-c)^2)}{4a^2}$

the 2 numerator factors right above are both a difference

of 2 squares which yields when removing the inside brackets :

$$h^2 = \frac{(a+c+b)(a+c-b)(b+a-c)(b-a+c)}{4a^2} \quad (2)$$

let $p = \frac{1}{2}$ perimeter of ΔABC which gives $a + b + c = 2p$, and consider the 4 factors of the numerator of (2):

we have $a + c + b = 2p$; $a + c - b = 2p - 2b$;

$b + a - c = 2p - 2c$; and $b + c - a = 2p - 2a$ (3)

and with (3) in (2), we get:

$$h^2 = \frac{2p(2p-2b)(2p-2c)(2p-2a)}{4a^2}$$

$$\text{or } h^2 = \frac{16p(p-b)(p-c)(p-a)}{4a^2}$$

divide by 4 and put in order: $h^2 = \frac{4p(p-a)(p-b)(p-c)}{a^2}$ (4)

the area of ΔABC is: $A = \frac{a \times h}{2}$

from (4), we get: $A = \frac{a}{2} \times \frac{\sqrt{4p(p-a)(p-b)(p-c)}}{a}$

simplify and we obtain: $A = \sqrt{p(p-a)(p-b)(p-c)}$ (5)

knowing that $p = \frac{1}{2}$ perimeter, then for ΔABC,

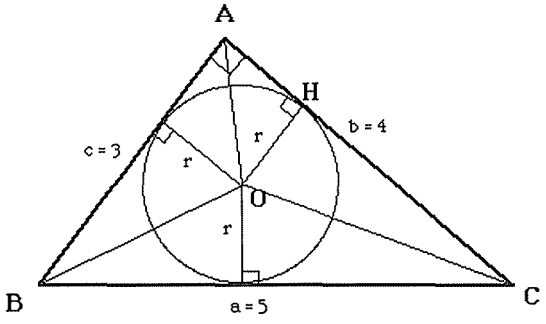
$$p = \frac{5+4+3}{2} = 6$$

and from (5) we get the area of ΔABC:

$$A = \sqrt{6(6-5)(6-4)(6-3)} = \sqrt{36} = 6 \text{ units}^2.$$

METHOD #3: THE INSCRIBED CIRCLE

(using the inscribed circle radius and the 3 sides)



r is the radius of the inscribed circle of triangle ABC

Since ΔABC is the sum of the 3 smaller triangles, ABO, BCO and CAO, area ΔABC = area ΔABO + area ΔBCO + area ΔCAO.

On the other hand, the 3 radii in the diagram have been chosen going from 0 to the points of tangency of the inscribed circle and the 3 sides of the ΔABC; therefore, these radii are perpendicular to the sides where they arrive. Thus, these 3 radii are respectively the heights of the triangles ABO, BCO and CAO.

area of $\Delta ABO = \frac{c \times r}{2}$

area of $\Delta BCO = \frac{a \times r}{2}$

area of $\Delta CAO = \frac{b \times r}{2}$

The area of these 3 triangles above have the sum of their area equal to the area of the ΔABC.

Thus, the area of $\Delta ABC = \frac{a \times r}{2} + \frac{b \times r}{2} + \frac{c \times r}{2}$

or area of $\Delta ABC = \frac{r}{2}(a+b+c)$ (1)

Example

Calculation of r using the ΔCAO:

AH = r (2) because $\measuredangle BAC = 90°$ and is bisected to give $\measuredangle OAH = 45°$.

Since $\measuredangle OAH = \measuredangle AOH \Rightarrow OH = AH = r$, also in ΔABC $\tan C = \frac{3}{4}$ then: $\measuredangle C = \tan^{-1}\frac{3}{4}$ (3)

CO is the bisector of $\measuredangle ACB$, then from the ΔCHO we can say: $\tan\frac{C}{2} = \frac{r}{CH}$ therefore $CH = \frac{r}{\tan\frac{C}{2}}$ (4)

since AH + CH = AC = b = 4 then from (2) and (4) we can say: $4 = AH + CH$ and $4 = r + \frac{r}{\tan\frac{C}{2}}$

and including (3) we get $4 = r + \frac{r}{\tan\frac{\tan^{-1}\frac{3}{4}}{2}}$ (5)

let $\tan\frac{\tan^{-1}\frac{3}{4}}{2} = Z$ then (5) becomes $4 = r + \frac{r}{Z}$

putting to the same denominator we get: $\frac{4 \times Z}{Z} = \frac{r \times Z}{Z} + \frac{r}{Z}$

or $4 \times Z = r \times Z + r$ or $4 \times Z = r(Z + 1)$ so $r = \frac{4 \times Z}{Z+1}$

finally we get:

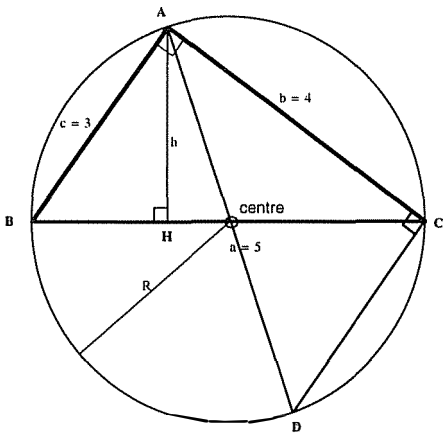
$$r = \frac{4\tan\frac{\tan^{-1}\frac{3}{4}}{2}}{\tan\frac{\tan^{-1}\frac{3}{4}}{2} + 1}$$

incredible but true, right here the numerator is exactly equal to the denominator, (you can make the calculation yourself) hence r = 1. Therefore, from the formula found at the previous page (formula (1)) and knowing that the

inscribed circle radius is equal to 1 the area of the Δ ABC $= \frac{1}{2}(5+4+3) = 6$ units^2.

METHOD #4: THE CIRCUMSCRIBED CIRCLE

(using the circumscribed circle radius and the 3 sides)



R = radius of the circumscribed circle

AH = h = the height of ΔABC.

The ΔABC is a triangle with a right angle $\measuredangle$BAC. The side opposite to the angle BAC, BC, is the diameter of the circumscribed circle. The reader should recall that inside a circle, a right angle on the circumference intersects a chord which is the diameter of the circle. Therefore, BC = a = 2R = 5 (1).This is exactly what happens to the angle DCA. This angle is a right angle because it intersects the chord AD which is the diameter by construction. Hence AD = 2R.

On the other hand, the angles ABC and ADC are equal since both are angles on the same circumference intersecting the same chord AC.

From this fact, the ΔABH and the ΔADC are similar because they both have 2 angles respectively equal making their corresponding sides proportional.

Thus, we can say: $\frac{AB}{AD} = \frac{AH}{AC}$

or AB x AC = AH x AD (AD = 2R)

therefore: c x b = h x 2R

multiplying both sides of the equation by a we get:

$$a \times b \times c = a \times h \times 2R$$

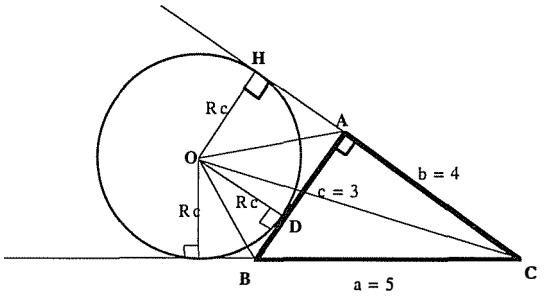
but since area of $\Delta ABC = \frac{a \times h}{2} \Rightarrow$ 2 area of Δ ABC = 2A = a x h, then a x b x c = 2A x 2R

finally we get the area of Δ ABC: $A = \frac{a \times b \times c}{4R}$ (2)

from (1) we have shown that BC = a = diameter = 2R = 5 therefore R = 2.5 (3). If we put (3) into (2), the area of the Δ ABC is $A = \frac{5x4x3}{4x2.5} = \frac{60}{10} = 6$ units^2.

METHOD #5: THE EXINSCRIBED CIRCLE

(using the 3 sides and one of the 3 exinscribed circles of the triangle)



An exinscribed circle of a triangle is a circle exterior to the triangle, tangent to one side of that triangle and tangent to the extension of the 2 other sides.

In extending the 2 sides of each triangle angle, one can build 3 circles exinscribed to the triangle. In this case, we choose arbitrarily the exinscribed circle opposite to the vertex C and tangent to the side AB. The following demonstration utilizes the exinscribed circle radius also holds for the 2 others exinscribed circles respectively opposite to the vertices A and B.

area ΔABC = area ΔCBO + area ΔCAO - area ΔABO (1)

In this case, the radii Rc, of the chosen exinscribed circle are also the heights of triangles CBO, CAO and ABO, because the radius of a circle arriving to the point of tangency is perpendicular to the tangent to the circle at that very point. These tangents are the side AB and the extensions of the sides CA and CB of the ΔABC.

then we have: area $\Delta CBO = \frac{CB \times Rc}{2} = \frac{a \times Rc}{2}$

area $\Delta CAO = \frac{CA \times Rc}{2} = \frac{b \times Rc}{2}$

area $\Delta ABO = \frac{AB \times Rc}{2} = \frac{c \times Rc}{2}$

then from (1) area $\Delta ABC = \frac{a \times Rc}{2} + \frac{b \times Rc}{2} - \frac{c \times Rc}{2}$

and the area of Δ ABC $= \frac{Rc}{2}(a+b-c)$ (2)

let a + b + c = 2p therefore $p = \frac{1}{2}$ perimeter of ΔABC

then a + b – c = 2p - 2c or a + b – c = 2(p – c) (3)

or $p - c = \frac{a+b-c}{2}$

therefore $p - c = \frac{a+b+c-2c}{2}$ or $p - c = \frac{a+b+c}{2} - c$ (3b)

if we put (3a) in (2) we get area $\Delta ABC = \frac{Rc}{2} \times 2(p-c)$

or area of ΔABC: A = Rc (p – c) (4)

Since there are 3 exinscribed circles, we name their radii Ra, Rb and Rc, each one being opposite respectively to the angles A, B and C and tangent respectively to the sides a, b and c.

putting (3b) in (4) we get:

$$A = Rc \times \left(\frac{a+b+c}{2} - c\right) \text{ since } p = \frac{a+b+c}{2}$$

$$\text{or } A = Rc \times \frac{a+b+c-2c}{2}$$

$$\text{finally we get } A = Rc \times \frac{a+b-c}{2}$$

$$\text{for the other circles } A = Ra \times \frac{b+c-a}{2}$$

$$\text{and } A = Rb \times \frac{a+c-b}{2}$$

$$\text{then we get } (Rc+b)\tan\frac{C}{2} = Rc$$

$$\text{then } Rc\tan\frac{C}{2} + b\tan\frac{C}{2} = Rc$$

$$\text{isolating } Rc: b\tan\frac{C}{2} = Rc - Rc\tan\frac{C}{2}$$

$$\text{or } b\tan\frac{C}{2} = Rc\left(1-\tan\frac{C}{2}\right)$$

$$\text{finally we obtain } Rc = \frac{b\tan\frac{C}{2}}{1-\tan\frac{C}{2}}$$

if we replace C by its value from (7) and b with 4

$$Rc = \frac{4\tan\frac{\tan^{-1}\frac{3}{4}}{2}}{1-\frac{\tan^{-1}\frac{3}{4}}{2}} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$

from (5) and (9), the area of the ΔABC: $A = 2 \times \frac{5+4-3}{2} = 6 \text{ units}^2$ with Rc

with the same procedure we can obtain: $A = 3 \times \frac{5+3-4}{2} = 6 \text{ units}^2$ with Rb and

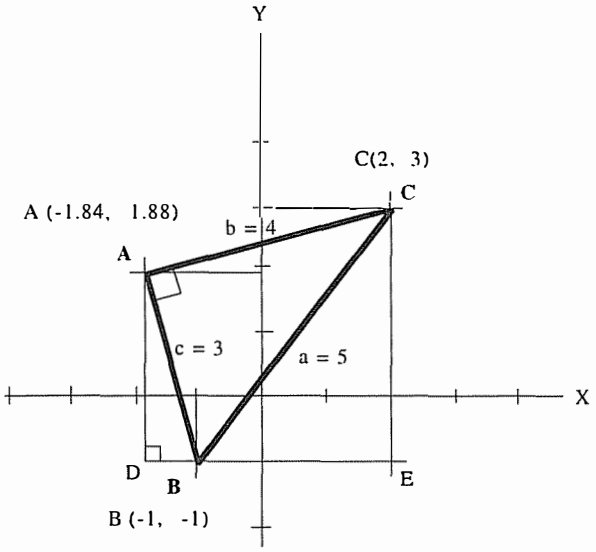
$A = 6 \times \frac{4+3-5}{2} = 6 \text{ units}^2$ with Ra

thus for this ΔABC: Ra = 6 Rb = 3 Rc = 2

METHOD #6: THE COORDINATES OF THE VERTICES' DETERMINANT

(using the determinant of the vertices' coordinate)

Locate the triangle in 3 out of the 4 quadrants of a Cartesian Plane. The coordinates of 2 of the vertices will be easy to verify. For the 3rd one, I will calculate the coordinates using trigonometry.



One can verify that BC = a = 5, because BE = 3 and EC = 4 from the chosen coordinates of B and C.

Now, calculate the coordinates of the vertex A:

Since the triangles CBE and CBA are equal, (because their corresponding sides are equal), the $\measuredangle CBE = \measuredangle CBA$ so $\measuredangle ABD = 180° - 2 \times \measuredangle ABC$.

For ΔABC $\tan B = \frac{4}{3}$ then $B = \tan^{-1}\frac{4}{3}$

and the $\measuredangle ABD = 180° - 2\tan^{-1}\frac{4}{3}$

for the $\measuredangle ABD$ let $Z = 180° - 2\tan^{-1}\frac{4}{3}$ (1)

from triangle (with right angle) ABD, we have $\cos Z = \frac{BD}{3}$

then BD = 3 cosZ (2)

and $\sin Z = \frac{AD}{3}$ then AD = 3 sin Z (3)

if we substitute Z by its value (1) with (2) we have

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$$BD = 3\cos\left(180° - 2\tan^{-1}\frac{4}{3}\right) = 0.84$$

then the abscissa of A is equal to: -1+(-0.84) = -1.84 (see the diagram)

with (3) we get $BD = 3\cos\left(180° - 2\tan^{-1}\frac{4}{3}\right) = 0.84$

then the ordinate of A is equal to 2.88 + (-1) = 1.88 (see diagram).

Now use the determinant of the coordinates of the vertices to calculate the area of the ΔABC.

Note: The determinant is an arrangement of values in rows and columns. In a determinant, there are always as many rows as columns. Therefore, we obtain a square table which has surprising properties. One can use it to calculate the solutions of a linear system of equations for instance. In this particular case, we will use it to calculate the area of the Δ ABC.

Using the co-ordinates of the vertices of the triangle, the area of any triangle is:

$$A = \frac{1}{2}\begin{vmatrix} Xa & Ya & 1 \\ Xb & Yb & 1 \\ Xc & Yc & 1 \end{vmatrix}$$

$$= \frac{1}{2}((Xa \times Yb \times 1) + (Xb \times Yc \times 1) + (Xc \times Ya \times 1) - (Xc \times Yb \times 1) - (Xa \times Yc \times 1) - (Xb \times Ya \times 1))$$

In this case, using the co-ordinates of the vertices of Δ ABC, we obtain:

$$A = \frac{1}{2}\begin{vmatrix} -1.84 & 1.88 & 1 \\ -1 & -1 & 1 \\ 2 & 3 & 1 \end{vmatrix}$$

which we find respectively in the 1st column, the abscissae of the vertices A, B and C, and respectively in the 2nd column, the coordinates of the same vertices A, B and C. In the 3rd column, we only have the number 1.

To solve the determinant, I will use the Sarrus rule, where we rewrite at the bottom of the determinant the 2 first rows. The determinant is equal to the sum of the products of the triplets of the descending (from left to right) diagonals minus the sum of the products of the triplets of the ascending (from left to right) diagonals. (Check the following diagram).

$$A = \frac{1}{2}\begin{vmatrix} -1.84 & 1.88 & 1 \\ -1 & -1 & 1 \\ 2 & 3 & 1 \\ -1.84 & 1.88 & 1 \\ -1 & -1 & 1 \end{vmatrix}$$

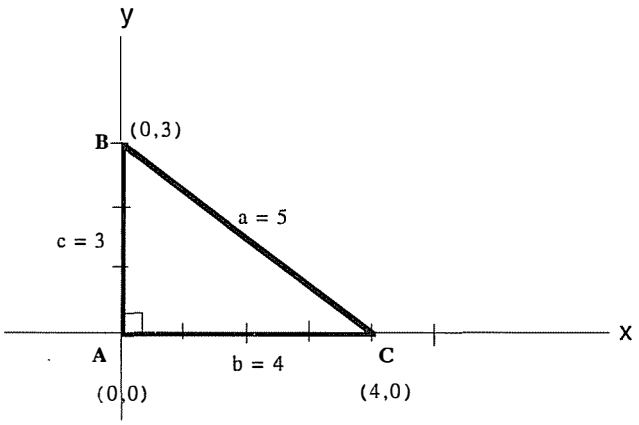
$$A = \frac{1}{2}((-1.84 \times (-1) \times 1) + (-1 \times 3 \times 1) + (2 \times 1.88 \times 1) - (2 \times (-1) \times 1) - (-1.84 \times 3 \times 1) - (-1 \times 1.88 \times 1))$$

$$A = \frac{1}{2}(1.84 - 3 + 3.76 + 2 + 5.52 + 1.88)$$

$A = 6$ units^2 (which is the required area)

METHOD # 7 THE INTEGRAL ONE

To find the area of the right angle triangle with sides 3, 4 and 5 on the interval $0 \le x \le 4$, one locates the triangle with one side of the right angle on the x axis and the other side of the right angle on the y axis.



The side BC is a line segment whose function is $y = \frac{-3}{4}x + 3$ where $\frac{-3}{4}$ is the slope and +3 is the y-intercept. The area of the ΔABC is the result of integrating the function $y = \frac{-3}{4}x + 3$ in the region from x = 0 and x = 4 . This gives us the area under the curve that is the area of ΔABC limited to the axis between the $0 \le x \le 4$ region.

$$\text{the area of } \Delta ABC = \int_0^4 \left(\frac{-3}{4}x + 3\right)dx$$

$$= \frac{-3}{4 \times 2}x^2 + 3x \Big|_0^4$$

$$= \frac{-3 \times 16}{4 \times 2} + 3 \times 4 - \left(\frac{-3 \times 0}{4 \times 2}\right) - (3 \times 0)$$

$$= -3 \times 2 + 3 \times 4 - 0 - 0$$

$$= -6 + 12$$

$$= 6 \text{ units}^2$$

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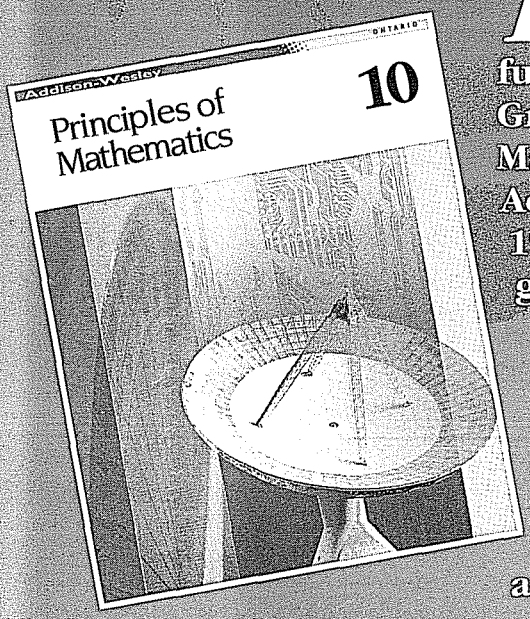
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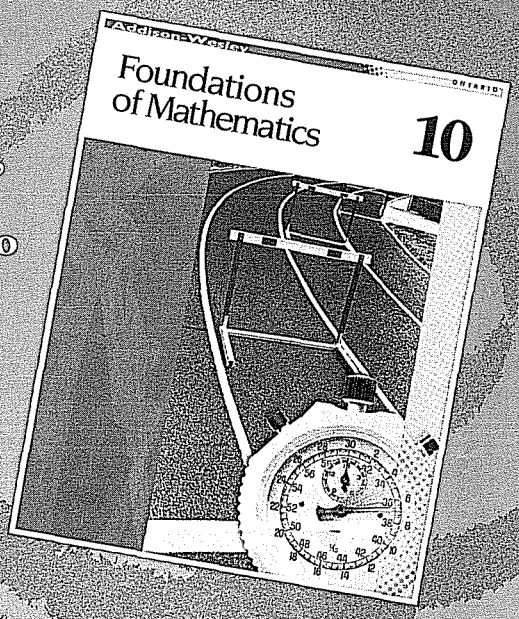
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3. Use a half-angle identity to find the

a) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ b) $-\frac{1}{2}\sqrt{2-\sqrt{2}}$ c)

4. If $f(x) = 2x-5$ and $g(x) =$, find

a) $\frac{-(5x+11)}{x+3}$ b)

5. If $\sin\theta = -\frac{1}{2}$ and θ

a) $\frac{-\sqrt{3}}{3}$ b)

6. Given $f(x) = \frac{x^2}{x+3}$ and

a) $\frac{4x^2-2x}{x+3}$ b)

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Final Exam

1. State the Domain of $y=\frac{x-10}{\sqrt{x^2-16}}$

2. Which relation is the inverse of $y=(x-2)^3$?

a) $y=\sqrt[3]{x-2}$ b) $y=\sqrt[3]{x}-2$ c) $y=\sqrt[3]{x+2}$ d) $\sqrt[3]{x}+2$

3. Use a half-angle identity to find the exact value of cos 67.5°:

a) $1/2\sqrt{2-\sqrt{2}}$ b) $-1/2\sqrt{2-\sqrt{2}}$ c) $-1/2\sqrt{2+\sqrt{2}}$ d) $\sqrt{3+2\sqrt{2}}$

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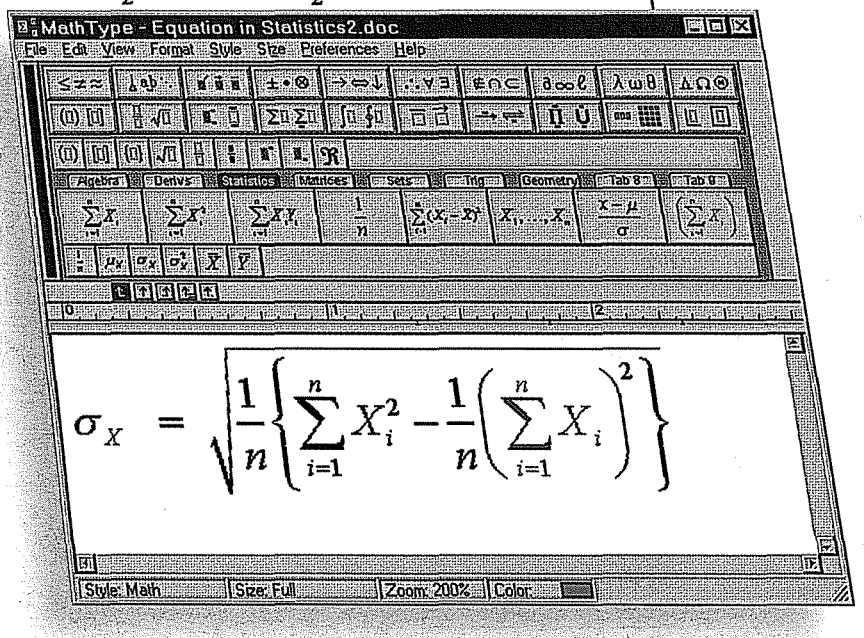
2. Which relation is the inverse of $y=(x-2)^3$?

a) $y=\sqrt[3]{x-2}$ b) $y=\sqrt[3]{x}-2$ c) $y=\sqrt[3]{x+2}$ d) $y=\sqrt[3]{x}+2$

3. Use a half-angle identity to find the exact value of cos 67.5°:

a) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ b) $\frac{-1}{2}\sqrt{2-\sqrt{2}}$ c) $\frac{-1}{2}\sqrt{2+\sqrt{2}}$ d) $\sqrt{3+2\sqrt{2}}$

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