Vol. 39 #2, December 2000

Ontario Mathematics Gazette

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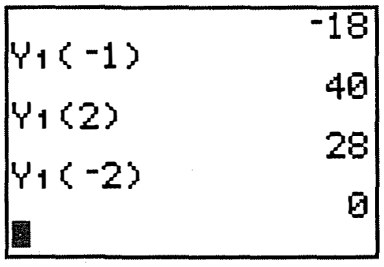
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SketchMad resources

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extended investigations especially designed for the Ontario curriculum

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A collection of pre-made sketches and Java sketches contributed by Ontario educators.

SketchMad in Ontario
Notices of conferences, workshops and professional development opportunities

SketchMad links
Annotated links to resources for Sketchpad on the Internet.

SketchMad
A Support Centre for using The Geometer's Sketchpad in Ontario

Tips on getting started with Sketchpad
Questions and answers to help you get started with Sketchpad in your classroom.

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The *Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 601349.

Editor
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Deadlines for advertisements are January 23 for March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.
Full page advertisements are to be on 8.5" by 11" paper with a minimum of 1/2" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

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Refereed articles for the *Gazette* should be submitted to **Louis Lim** at the address on the left. Deadlines are May 15 for the September issue, September 15 for the December issue, December 15 for the March issue, and March 15 for the June issue. Since articles are blind-reviewed, please send your full mailing address and e-mail address on a page **separate** from the manuscript. Submit **four** copies to the editor, and keep one copy to guard against loss. Computer files of text, and original photos and graphics, where possible, will be required later if your article is accepted.

Articles must be original. Your article should be word-processed, doubled-spaced on one side with wide margins, not exceed 10 pages of text, and prepared according to the *Publication Manual of the American Psychological Association.* Diagrams and figures should be drawn by computer, if possible, or neatly drawn in black ink in camera-ready form. The editor reserves the right to edit articles for publication.

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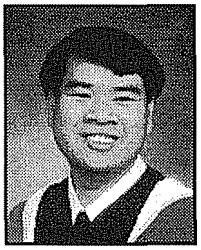
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▲ Editorial



LOUIS LIM, EDITOR

It is an honour to assume the role of full-editor of the *Ontario Mathematics Gazette*. I have thoroughly enjoyed the past year co-editing the *Gazette* with Todd Romiens.

When Todd stated his desire to step down as co-editor, I kept my fingers crossed that he would agree to stay on as an Associate Editor. Knowing his time commitment as an education professor at the University of Windsor, I am most appreciative of his flexibility and willingness to continue working with me. Thanks, Todd!

Our previous *Gazette* editor, John Egsgard, supported my application for the position of editor when I applied, and I have never forgotten what he said to me: "If you become editor, I would be willing to serve as your Associate Editor." John was true to his words, and has agreed to serve as the other Associate Editor. Thank you, John!

During the upcoming two years, I hope to continue the tradition of past editors. I hope you are enjoying Ron Lancaster's "PhotoMath" column, Marilyn Hurrell's "Directors' Dialogue", and Frances Schatz's "OAME/NCTM Report". This issue is the unveiling of the "Technology Corner" which Fred Ferneyhough has volunteered to write. The layout of the *Gazette* has improved vastly over the years, and that is the result of the hard work of our Graphic Designer, Penny Clemens.

The *Gazette* is only as good as you make it. I welcome your feedback, letters to the editor, innovative classroom-ready activities, and articles. With so many rapid changes taking place, we need to support each other as we strive to provide our students with the best learning experiences possible. Our membership is approximately to 1400, so the time you take to write will make a difference to advance mathematics education in Ontario. It is also very professionally satisfying to see your name and photo attached to an article!

Be sure to take advantage of all that OAME has to offer (see Mary Howe's "President's Message" in the September 2000 *Gazette*). The countdown is on for OAME'2001 in Scarborough! ▲



▲ PRESIDENT'S MESSAGE



MARY HOWE

Due to the immensity of change that has taken place in the curriculum from kindergarten to grade 12, teachers have developed a thirst for support materials to help them with the implementation of the new curriculum and strategies for assessment. To fill this need, a wide range of materials have been prepared over the last year for the use of all teachers of mathematics.

Ministry of Education Initiatives:

Several projects have been completed recently through Ministry initiatives and funding:

Ten Ministry-funded integrated mathematics units for the junior grades were completed recently, and can be downloaded from the web site planner.media-x.com. These units, developed using the Curriculum Planner, feature mathematics integrated with other curriculum areas for both single and combined junior grades. Ministry-funded inservice has already been planned in most Boards.

Mathematics Exemplars have been produced for every strand in each elementary grade, as well as one exemplar for grade 9 mathematics. District teams of teachers will have delivered the exemplar inservice during October and November throughout Ontario.

Another Ministry project, Course Profiles for Applied, Academic, and Essentials mathematics courses, have been completed for grades 9 and 10. Both the Public and Catholic Profiles can be downloaded from the web site www.currriculum.org or copied from the CDs that each secondary school received in October. The grade 11 Workplace Course Profile should be completed in time for use in semester two.

In a new initiative announced by the Ministry in September, mathematics units, both Public and Catholic, will be written for each grade in elementary school, as well as for each combined grade. Teams of teachers from across the province will write the units which will be released for implementation by the beginning of the next school year.

OAME Projects:

To assist Ontario mathematics teachers in bringing the mathematics curriculum and assessment changes to life in their classrooms, OAME will release its document *Linking Assessment and Instruction in Mathematics: Middle Years* in February 2001. This document will complement those already written for the primary and junior grades, featuring rich instructional and assessment tasks that allow teachers to weave the expectations from several strands into their classroom instruction in contexts that are realistic and interesting to children. The documents can be purchased from OAME. Check the web site www.oame.on.ca for more details.

The CARE package, Concerning Assessment and Reflective Evaluation, was prepared by OAME and OMCA members for use at the OTF Summer Institutes. The CARE Package features strategies to help teachers implement the profiles with their text resource, ideas for balancing assessment and instruction, discussion about the language and categories of the Achievement Chart, and suggestions for student evaluation. The package can be downloaded from the OAME web site.

The annual OAME Leadership Conference, February 22 - 24, 2001 features junior, intermediate, and secondary workshops in which the speakers will illustrate the links between teaching strategies, assessment and evaluation, and the new curriculum. Teachers will remain in their divisional workgroups for the entire three day conference, building their capacity as leaders to offer professional development when they return to their own schools and Boards. The planning committee has been extremely fortunate to obtain Joan Ferrini-Mundy, editor of the recently published NCTM document *Principles and Standards of Mathematics* as the keynote speaker.

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Mathtype™ was used to create formulas in this issue of the OAME/AOEM *Gazette*.

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What happens next?

A great deal of time, energy, and money have gone into preparing materials for the mathematics classroom. Now, the focus must be shifted to professional development based on using and adapting these resources. The Ministry has funded release time for teachers to receive inservice, but far more time is necessary. Single session professional development is a start, but real change will take place only after teachers can return to a second or third round of inservice after they have tried activities in their own classrooms. The growth that takes place from reflection and discussion of successes and failures will foster improvement as well as provide support for teachers as they implement change.

Also of great importance, is the need for teacher planning time to prepare exemplary instruction. Effective change requires time, not just over the long term, but on a daily basis as well. Increasing teaching time while decreasing planning time will not support effective implementation of the new curriculum.

What can you do?

Use the best parts of the materials that are available. Adapt and modify the resources to make them your own. Share your efforts with others. Take advantage of all professional development opportunities. Attend your local Association's mini-conference and the OAME Leadership and spring conferences. Ask questions. Read everything you can. Participate in professional chat groups on-line. Keep the lessons that always worked well. Make changes to allow your students a wide range of opportunities for success. And most of all, share your classroom successes that bring students both understanding and enjoyment of mathematics. ▲

Don't Miss:



Scarborough – May 9-12 , 2001

OAME 2001 will be held at two locations:

Delta Hotel Toronto East
Kennedy road at Hwy 401
Scarborough

Howard Johnson
Markham Road at Hwy 401
Scarborough

We look forward to seeing you there!

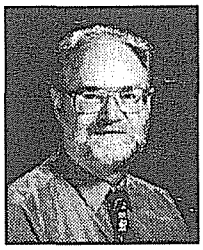
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▲ Technology Corner: using the TI-83 Plus for the factor theorem

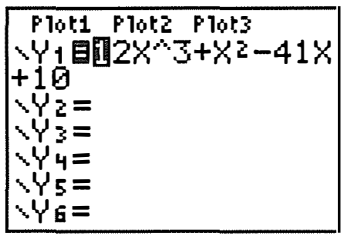
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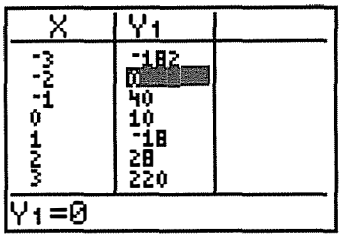
Fred Ferneyhough is co-chair of the Peel Math Heads Association, and Technology Consultant for McGraw-Hill Ryerson's MathPower 9 *and* MathPower 10 *textbooks. He is an avid user of technology in the classroom.*

Welcome to the Technology Corner. In this column, I hope to share some of the successes that I have had in using technology in the mathematics classroom, using TI-graphic calculators, The Geometer's Sketchpad, spreadsheets, and other software. Questions, comments, and suggestions should be sent to me at dybydx@home.com.

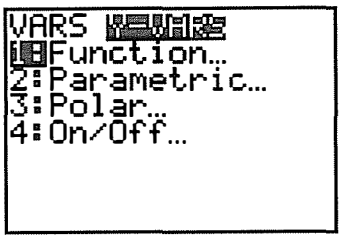
The Factor Theorem in MAT 4A is sometimes viewed as one of the most important concepts from that course. A class that I taught a few years ago worked out the following method of using the TI-83/83 Plus to help solve problems involving the tedious calculations which need to be done to begin a solution.



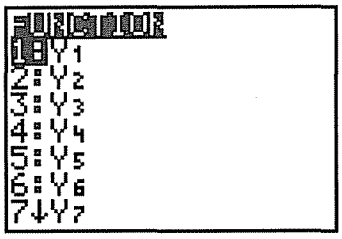
First, let's take the function $f(x) = 12x^3 + x^2 - 41x + 10$. We all have cooked up examples like this one. It has one integer and two rational roots. Define the function as y_1 in the "y=" editor. The first step in the solution is to find a value of x so that the function has a value of 0. There are two different ways to do this on the TI-graphic calculators:



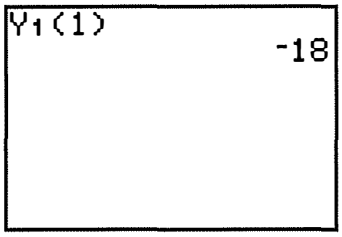
The first method is to look at the table of values that the graphic calculator generates by pressing the 2nd key and the GRAPH key. From the graphic, we see that $y_1 = 0$ when $x = -2$, so we conclude that $x = -2$ is a root of f(x), and $(x + 2)$ is a factor of f(x). Students can then continue to do the division and find the remaining factors and roots.



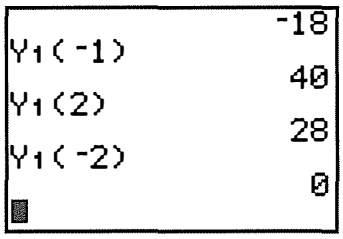
A second method involves using a feature of the machine that many people tend to overlook. The TI-83 Plus does have the ability to handle functions and evaluate them for values of x. To accomplish this, start in the home screen, press the VARS key and move the cursor to the right to Y-VARS.



Hit ENTER and you should get the screen to the left which gives you a choice of the "y=" functions. Hit ENTER on y_1.



The expression y_1 will be pasted onto the home screen. You can then add brackets with a value enclosed. The calculator will evaluate the function using the value.



With trial and error, students should be able to find an integer value that returns a function value as 0. As before, y_1 has a value of 0 when $x = -2$.

I like these methods because they take the tedium out of finding function values. Lessons can still be presented in the same manner, and this method lends itself to extensions involving functions that have either all rational roots or higher degree. ▲

▲ A Modified Probability Exercise



DAVID R. DUNCAN
AND BONNIE H. LITWILLER
PROFESSORS OF MATHEMATICS
UNIVERSITY OF NORTHERN IOWA
CEDAR FALLS, IOWA

Teachers of probability are always seeking ways in which familiar problems can be modified to make them more challenging. We will demonstrate this by solving a sequence of probability problems in two situations. Situation I involves routine use of the concepts of "and", "or", and "independence"; situation II will be more complex.

Situation I:

Suppose that two baseball players (Al and Bill) come to bat consecutively. Let E be the event that Al hits a home run and let F be the event that Bill hits a home run. Suppose further that P(E) = 0.1 and P(F) = 0.05, and that E and F are independent events. Find the probabilities that:

A. Al and Bill both hit home runs in a given sequence of consecutive plate appearances (first Al, then Bill).

B. At least one of the two players hits a home run in a given sequence of consecutive plate appearances.

The solutions to these problems are routine for many students:

Problem A:

Since E and F are independent events,

$$\begin{aligned} P(E \text{ and } F) &= P(E) \bullet P(F) \\ &= (0.1) \bullet (0.05) \\ &= 0.005 \end{aligned}$$

Problem B:

$$\begin{aligned} P(E \text{ or } F) &= P(E) + P(F) - P(E \text{ and } F) \\ &= (0.1) + (0.05) - 0.005 \\ &= .145 \end{aligned}$$

Situation II:

Let E and F be the independent events as defined in the original situation. However, let us assign the probabilities in a different way:

- The probability that ***only*** Al hits a home run in the consecutive plate appearances of the two players is 0.1.
- The probability that ***only*** Bill hits a home run in the consecutive plate appearances of the two players is 0.05.

Now, consider the same two problems (A and B) as stated in Situation I. To solve these problems, P(E) and P(F) must be calculated. Denote these probabilities as x and y respectively. According to the statement of the modified problem, P(E and not F) = P(E) • P(not F) = 0.1 and P(F and not E) = P(F) • P(not E) = 0.05.

These two equations can be rewritten as x(1 - y) = 0.1 and y(1 - x) = 0.05. Solving for y in the second equation yields $y = \frac{0.05}{1-x}$.

Substituting this value of y into the first equation yields:

$$x\left[1-\frac{0.05}{(1-x)}\right] = 0.1$$

$$\text{Then } x\left[\frac{1-x-0.05}{1-x}\right] = 0.1$$

$$0.95x - x^2 = 0.1 - 0.1x$$

$$x^2 - 1.05x + 0.1 = 0$$

Using the quadratic formula:

$$x = \frac{1.05 \pm \sqrt{(-1.05)^2 - 4(1)(0.1)}}{2}$$

$$x = \frac{1.05 \pm 0.8382}{2}$$

$$x = 0.9441 \quad \text{or} \quad x = 0.1059$$

Then y = 0.8945 or y = 0.0559, using $y = \frac{0.05}{1-x}$.

There are two possible x-y solutions to this problem:

Solution 1: x = 0.9441 and y = 0.8945

Solution 2: x = 0.1059 and y = 0.0559

Now let us solve the modified problems:

Modified Problem A:

Solution pair 1: P(E and F) = (0.9441)(0.8945)
= 0.8445

Solution pair 2: P(E and F) = (0.1059)(0.0559)
= 0.0059

Modified Problem B:

Solution pair 1: P(E or F) = P(E) + P(F) - P(E and F)
= 0.9441 + 0.8945 - 0.8445
= 0.9941

Solution pair 2: P(E or F) = 0.1059 + 0.0559 - 0.0059
= 0.1559

A slight modification in the original situation greatly increased its complexity.

How could two greatly differing solutions be possible in Situation II? The explanation lies in the fact that the two stated probabilities are for exactly ***one*** home run (0.10 for E and not F; and 0.05 for F and not E). There are two ways that these low probabilities could occur:

- P(E) = 0.1059 and P(F) = 0.0559.

  For these relatively low individual probabilities, the most frequent outcome is that ***neither*** player hits a home run. There is little probability "left over" for the "exactly one" events:

  P(E and not F) = (0.1059)(0.9441) = 0.1000

  P(F and not E) = (0.0559)(0.8941) = 0.0500

- P(E) = 0.9441 and P(F) = 0.8945.

  For these relatively high individual probabilities, the most frequent outcome is that ***both*** players hit home runs. There is little probability "left over" for the "exactly one" events:

  P(E and not F) = (0.9441)(0.1055) = 0.0996

  P(F and not E) = (0.8945)(0.0559) = 0.0500

In the baseball model, the (0.9441, 0.8945) solution pair is quite unrealistic; however, the algebra doesn't know about baseball. Both solution pairs are possible for the problem as stated.

The solutions of Situation II differ from those of Situation I in two noteworthy ways. First, the quadratic formula is needed in Situation II; secondly, a pair of solutions occurs for each of A and B, rather than the unique solutions of Situation I. Both of these changes deserve special attention as possibilities in probability situations.

Challenges:

1. Use the graphing calculator to find values of x and y.
2. Find other routine problems that can be modified to be more challenging. ▲

▲ NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2001-2002

Each year, elections are held for a President-Elect, a Vice President (3 year term), and 6 to 9 Directors (3 year term) of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 19, 2001, to:

Peter Saarimaki, Past President

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2001, the Nomination Committee will publish a slate of candidates, including qualifications, for consideration by the membership. Voting will be by mail, and is open to all OAME/ AOEM members. The election results will be released at the Annual General Meeting of the OAME/ AOEM in Scarborough, May 2001.

NOMINATION FORM

I nominate: _____

For the position(s) of: ☐ President-Elect ☐ Vice-President ☐ Director

Nominee's Address: _____

_____

Telephone: H ( ) _____ W ( ) _____

Nominator's Name : _____

Nominator's Address: _____

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▲ What Is Mathematics?

SANDERSON M. SMITH
CATE SCHOOL, CARPINTERIA

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The title presents a fascinating question. I am a mathematics teacher and should be able to readily respond to the question. After all, I do teach the subject. My somewhat overly simplified answer is that mathematics is the language of our universe. Be you evolutionist, creationist, or holder of some other belief, empirical evidence overwhelmingly suggests that our universe is mathematically designed. Still, my response to the question is clearly unsatisfactory. Among other things, it does not address questions such as, "Why does mathematics work?" and "Where does it come from?"

Despite my love and appreciation for the magnificent academic discipline of mathematics, I do not have a good answer to the question, "What is mathematics?" Since the time of Plato, mathematicians and philosophers have addressed this question. Answers, many of them contradictory, have been produced for over two thousand years, but none have survived the test of time and critical analysis.

Do we, as teachers of mathematics, simply present the academic discipline of mathematics as an established, unchangeable, and irrefutable block of knowledge that everyone should learn? Do we accept this interpretation of mathematics? Do our students view mathematics this way? Regretfully, I think the answer to these questions is all too frequently, "Yes."

Contrary to this static viewpoint, I believe it is possible that students can gain a tremendous appreciation for mathematics if they understand that the question, "What is mathematics?" had been analyzed and debated since the time of the Pythagoreans, a mystical cult that surfaced around 550 BC. I also believe that humans love and appreciate a good mystery. If this is so, why not present the truth in a mathematics classroom? After all, the truth is very simple: The very nature of mathematics is a mystery!

The purpose of this writing is to present a brief summary of types of mathematical thought that have surfaced since the ancient Greeks realized that humans have the mental capacity to reason. Prior to doing this, it is important to note three discoveries that shocked the mathematical world. These, and a few other discoveries, shattered the historical beliefs of many intellectuals who thought that their version of mathematics had a firm foundation. While a perfect analogy is not possible, imagine yourself having purchased and living in a luxurious tenth floor condominium in a very desirable location, and then find out the foundation of the building is unstable. Here are the shockers:

Shocker #1: *The discovery (invention?) of non-Euclidean geometries in the 19th century.*

Many schools of mathematical thought had accepted the laws of Euclidean geometry (studied in modern schools) as indubitable truths about the universe. Meaningful geometries that did not conform to the laws of Euclid's famous historical work, *The Elements*, were discovered independently by Hungarian Janos Bolyai (1802-1860) and Russian Nikolai Lobachevsky (1792-1856). For example, in some non-Euclidean geometries, Euclid's famous parallel postulate ("Through a point external to a line, there exists, in a plane, exactly one line parallel to the given line") is false.

Shocker #2: *The discovery (invention?) of quaternions by Irish mathematician William Rowan Hamilton (1805-1865).*

A quaternion is a meaningful type of number that does not "behave" like the numbers we commonly use. Among other things, in quaternion arithmetic, x times y does not equal y times x. Mathematicians learned that the basic laws of common algebra, accepted as truth for twenty centuries, are not universal truths.

Shocker #3: *Gödel's Incompleteness Theorem.*

In the 20th century, German Kurt Gödel (1907-1978) proved that consistency can never be established by methods of mathematical proof. Every logical system must contain statements that can't be proved. In other words, a formal mathematical system could never prove its own consistency. Something must be accepted on pure faith. (In modern day geometry, the terms *point*, *line*, and *plane* are never formally defined. And, a postulate such as "In a plane, two non-parallel lines intersect at a unique point" is accepted as true without proof. However, there are simple non-Euclidean geometry models where this postulate is not true.)

These discoveries, and some others that have not been mentioned, are important in understanding why many theories about the nature of mathematics have failed to pass the test of time. What follows is a very brief summary of some historical schools of thought which made attempts to answer the question, "What is mathematics?":

Platonism

Greatly influenced by the earlier Pythagoreans, Plato (c.427-c.347) asserted that mathematics represents a separate universe of abstract objects existing outside of what we know as time and space. Mathematical objects (such as numbers) aren't created by humans. They always existed. Mathematical truth possesses absolute certainty. Platonism asserts that a mathematician is an empirical scientist who can only discover what is already there. He or she can't invent new numbers.

Problems with this view: Platonists never really explain how flesh-and-blood mathematicians come to interact with the external universe of mathematics. The discovery of non-Euclidean geometries and quaternions contradicted the "absolute truth" view of the Platonists.

Formalism

German mathematician David Hilbert (1862-1943) headed this group. Formalists assert that mathematics must be developed through axiomatic systems. Formalists and Platonists agree on the principles of mathematical proof, but Hilbert's followers don't recognize an external world of mathematics. Formalists argue that there are no mathematical objects until we create them. Humans create the real number system by establishing axioms to describe it. All mathematics needs is inference rules to progress from one step to the next. The Formalists tried to prove that within the framework of established axioms, theorems, and definitions, a mathematical system is consistent. In the mid-twentieth century, formalism became the predominant philosophical attitude in mathematics textbooks.

Problems with this view: Gödel's Incompleteness Theorem contradicts the consistency philosophy of formalism. It has been pointed out that accepted results from theorems were used before axioms were created to establish the theorems. The modern emphasis on the concrete and the applicable is not consistent with the formalist philosophy that you don't really do mathematics until you state a hypothesis and begin a proof.

Logicism

English mathematicians and philosophers Bertrand Russell (1872-1970) and Alfred North Whitehead (1861-1947) co-founded this school of thought. This school claims that mathematics is a vast tautology. All of mathematics is derivable from principles of logic. Many of the logistic ideas are similar to those of the Formalists, but the Logicists believe that mathematics can be deduced from logic alone. Among other things, the Logicists attempted a logical construction of the real number system, whereas the Formalists constructed it axiomatically. Logicism also uses mathematical sets in its logical development.

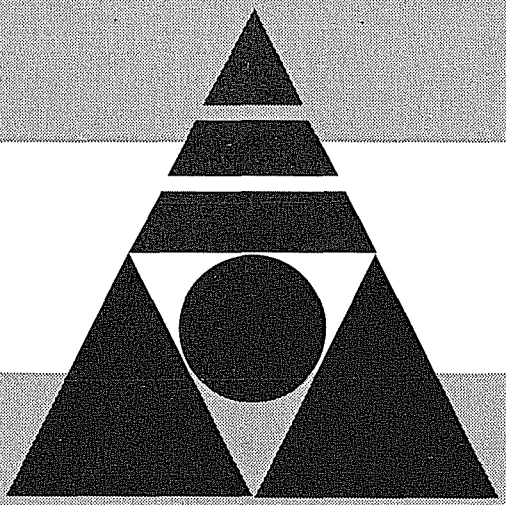
Problems with this view: Logicism, despite many attempts, could not successfully resolve paradoxes that arose in set theory. Gödel's Incompleteness Theorem was a deathblow to the "math is a tautology" philosophy expounded in *Principia Mathematica*, a monumental work constructed by Russell and Whitehead.

Intuitionism (sometimes called Constructionism)

Building on the philosophies of Immanuel Kant (1724-1804), Dutch mathematician Luitzen Brouwer (1881-1966) emerged as the leader of this school of thought, which differs considerably from those previously discussed. Intuitionists claim that mathematics originates and thrives within the mind. Human minds intuitively possess the forms of space and time. The natural numbers are given intuitively, and they represent the fundamental datum of mathematics from which springs all meaningful mathematics. Mathematical laws are not discovered by studying nature; rather they are found in the recesses of the human mind.

Problems with this view: The intuitionist view doesn't give any insight as to why mathematics works. We don't know how intuitive knowledge is held in the brain. Mental representations of concepts such as love and hate differ considerably from human to human. Is it realistic to assume humans share the same intuitive view of mathematics? Why do we teach mathematics if it is all intuitive? I suspect that most folks would not associate mathematics with the word fiction, which represents that which is feigned or imagined. Yet consider the following:

- $\sqrt{2}$ is a symbol that appears frequently in algebra and geometry texts. The symbol represents an irrational number (non-repeating decimal) that no human has ever written out in complete decimal form. In fact, humans have never viewed the complete



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decimal representation of any irrational number, including π and e. Does that mean irrational numbers are fictional?

- Has anyone ever seen a segment of length one-centimetre actually divided into infinitely many parts? The wonderful branch of mathematics we know as calculus accepts representations of this without any human observing what is actually represented. Again, are we talking fiction?

Here's an interesting question about the amazing number π which makes numerous appearances in mathematics textbooks: Is there a sequence of 100 consecutive zeros in the decimal representation of π? We can note that modern computers have calculated π to billions of decimal places, and no such sequence has been found. But what about the next billion places, and the next ten billion after that? The point is, we simply don't know, although probability certainly suggests that we would eventually find such a sequence.

In conclusion, despite the fact that I have taught mathematics for many years, I really cannot explain what it is, where it comes from, why it works, or how we can make such amazing use of things that could be classified as fictional. Mathematics is, to a great extent, a mystery. As a thinker, I can only say that my personal philosophy of mathematics takes bits and pieces from each of the historical schools of mathematical thought. As previously mentioned, I see mathematics as a language. As we become more and more proficient with this language, we will better understand the universe that we inhabit.

I don't believe humans will ever discover all of the mathematics that exists. The mystery of our existence in a mathematically designed universe is what makes living interesting and exciting. As teachers, shouldn't we let our students know that the question, "What is mathematics" has yet to be answered?

> *"Mathematics is not a perfect gem, and continued polishing will probably not remove all flaws. Still, it is the most precious jewel of the human mind and must be treasured and husbanded."*
>
> *Morris Kline, New York University*

Related Readings

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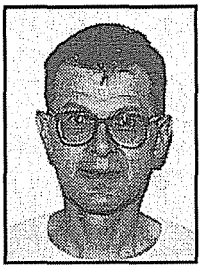
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A Graph Sketching Problem

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Ed Barbeau has been teaching mathematics at the University of Toronto for 32 years. He is keenly interested in keeping the lines of communication open between the high school and university mathematics communities, and between mathematicians and the general public. He has often been a guest on CBC's Quirks and Quarks, *and welcomes invitations to visit high schools.*

During the last academic year, I taught one section of a large-enrolment (over 1200 students) calculus course, and got a renewed sense of the preparation of many students for university mathematics. These were students who were not planning a specialization in mathematics, but whose high school grades were sufficient to gain entry to the university. Many of them were planning to apply to the faculty of medicine, and were taking the course to fulfil a prerequisite for other science courses. Unfortunately, in most cases, students did not handle the material well. While one can make the usual complaints about lack of technical facility, the issue is somewhat deeper and has to do with students' attitudes and intellectual curiosity. On a strictly technical level, it is probably the case that in their high schools the syllabus was well covered by their teachers, and that the difficulties of the students arose from a faulty perception of mathematics engrained over a whole school career.

I have always felt that a good diagnostic tool for how well students fare with calculus is graph-sketching, and so put a lot of emphasis on this skill both in my teaching and examining. I encourage students to sketch graphs of functions they encounter in any calculus problem, particularly in optimization. Being able to draw a graph not only requires sufficient algebraic facility to analyze the function or equation involved, but the ability to make judgments and form inferences on the basis of a limited (but self-corroborating) body of information. So I insisted that the final examination have a graph-sketching problem; this was agreed on by my fellow instructors, provided that I mark it.

I would like to use an examination question to indicate possible reasons for students to be failing. I hope to give a more textured picture of what students in university are called upon to do, in the hopes that it might inform what might be on the syllabus in high school and how it might be handled. Perhaps most importantly, it might have a bearing on the advice that we give to our students if they are to succeed in higher level mathematics, as many things become impossible without a proper attitude and willingness to engage mathematics assiduously and productively.

The problem was to sketch the graph of the function for $y = \sec x - \tan x$ for $0 \le x \le 2\pi$.

I will first present a number of ways in which the problem can be done, and then comment on what the students actually did. For convenience, let $f(x) \equiv \sec x - \tan x$.

Solution 1.

Observe that

$$f(x) = \sec x - \tan x = \frac{1}{\sec x + \tan x}$$

$$f'(x) = \sec x \tan x - \sec^2 x = -\sec x\, f(x)$$

$$= \frac{-\sec x}{\sec x + \tan x} = \frac{-1}{1+\sin x}$$

$$f''(x) = -\sec x \tan x\, f(x) - \sec x\, f'(x)$$

$$= -\sec x \tan x\, f(x) + \sec^2 x\, f(x)$$

$$= -\sec x[\tan x\, f(x) - \sec x\, f(x)] = \sec x[f(x)]^2.$$

From these expressions, we can read off the following information:

$$\lim_{x \to \frac{\pi}{2}} f(x) = 0;$$

$$\lim_{x \to \frac{\pi}{2}} f'(x) = \frac{-1}{2};$$

there is a vertical asymptote at $x = \frac{3\pi}{2}$; $f'(x)$ is always negative where defined; $f''(x)$ is positive for

$0 \le x < \frac{\pi}{2}$ and $\frac{3\pi}{2} < x \le 2\pi$, and negative for $\frac{\pi}{2} < x < \frac{3\pi}{2}$. Thus, the function f(x) is always decreasing, has a removable discontinuity when $x = \frac{\pi}{2}$, and a discontinuity when $x = \frac{3\pi}{2}$; it is convex on the two intervals defined by $0 \le x < \frac{3\pi}{2}$ and $\frac{3\pi}{2} < x \le 2\pi$, and concave on the interval defined by $\frac{\pi}{2} < x < \frac{3\pi}{2}$.

Solution 2.

Writing the function f(x) in terms of sines and cosines, and then simplifying yields $f(x) = \frac{1-\sin x}{\cos x}$, so that, by the quotient rule,

$$f'(x) = \frac{\sin x - 1}{\cos^2 x}$$

$$f''(x) = \frac{\cos^3 x + 2\cos x \sin^2 x - 2\cos x \sin x}{\cos^4 x}$$

$$= \frac{\cos^2 x + 2\sin^2 x - 2\sin x}{\cos^3 x} = \frac{(\sin x - 1)^2}{\cos^3 x}.$$

Again, it is easy to read off the signs of the derivatives. However, for behaviour at the point $x = \frac{\pi}{2}$, we can use l'Hôpital's Rule to find that

$$\lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0;$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{\cos^2 x} = \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{-2\cos x \sin x} = \lim_{x \to \frac{\pi}{2}} \frac{1}{-2\sin x} = -\frac{1}{2}.$$

Once again, we can use this information to sketch the graph.

Solution 3.

We can begin as in Solution 2, but make use of $\cos^2 x = 1 - \sin^2 x$ and a cancellation to obtain for the first derivative

$$f'(x) = \frac{-1}{1+\sin x}$$

so that, more easily, we find that

$$f''(x) = \frac{\cos x}{(1+\sin x)^2}.$$

We can put the information together to sketch the graph as before.

Solution 4.

We can take a more structural approach to the situation by making the substitution $2u = \frac{\pi}{2} - x$, from which we find that

$$f(x) = \frac{1-\sin x}{\cos x} = \frac{1-\cos 2u}{\sin 2u} = \frac{2\sin^2 u}{2\sin u \cos u} = \tan u.$$

This, of course, does not apply to $x = \frac{\pi}{2}$ itself, where f(x) is not defined, but it does make clear that the discontinuity there is removable and tells us how we can remove it. The graph that we are seeking is the graph of $y = \tan x$ after it has been reflected in the line $x = \frac{\pi}{4}$ and dilated by a factor of 2 in the horizontal direction.

There were 1268 students writing the examination. I sampled the performance of 250 of them, and the number of students receiving various grades out of 10 are given in the following:

10	2	4	17
9	1	3	63
8	2	2	81
7	2	1	60
6	1	0	13
5	6	NA	2

Many students simply calculated the first and second derivatives of f(x) and then essentially threw in the towel. A lot of them were not comfortable with the tangent and secant functions, and so converted the functions involved to sines and cosines.

Since it was important to find the sign of the derivatives, it would have been helpful if students were able to recognize such expressions as $\sec^2 x - 2\sec x \tan x + \tan^2 x$ or $1 - 2\sin x + \sin^2 x$ as perfect squares; remarkably few made this simplification, or any other "obvious" simplification which would have cut down on the work. In general, the students had poor command of straightforward algebraic manipulation.

But manipulation for what? What most of the candidates really lacked was a sense that the purpose is to reveal information. Any particular expression for a function will make some of its properties manifest, but other properties will remain latent. It may be that students were drilled in the usual routines of factoring and expanding, but few were offered the occasion to do these things purposefully.

There were other indications of lack of facility. Most students proceeded along the lines of the first two solutions above (without noticing the role of f(x) in the expressions for f'(x) and f''(x), and using, in effect, implicit differentiation). No one analyzed the behaviour of f(x) near $x = \frac{1}{2}\pi$ by noticing that $f(x) = (\sec x + \tan x)^{-1}$, and most students were at a loss to deal with this issue.

Those who used sines and cosines could have managed to do this using l'Hôpital's Rule, but almost no one did this, despite the fact that this rule was covered in the course. (The students are maladept at relating materials from a different part of the course.)

One striking characteristic of many of the solutions was that, even when students did find correct information about the nature of the function and its derivatives, they produced graphs that were inconsistent with at least some of this information. In particular, expecting a vertical asymptote to the graph at $x=\frac{1}{2}\pi$, they drew the graph as increasing on the interval $\left[0,\frac{1}{2}\pi\right)$ even though they had recognized the derivative of f(x) as being negative there; there was no taste for recognizing or exploring this apparent anomaly.

What was not seen in any of the solutions? No one thought to examine how the first derivative behaved near $x=\frac{1}{2}\pi$ although several did surmise that there might be a level inflection point of f(x) there. This level of thoroughness is not to be expected in the general run of students (and was not required in the marking), but one might hope that a few of the top students would give a solid and complete analysis. A solution along the lines of Solution 4 was beyond the pale. There seems little in the school regime to encourage that sort of structural approach to problems.

While the examination was not the vehicle to assess what preliminary thinking might have occurred before the students began to tackle the question in earnest, it is highly unlikely, based on my experience, that very many "cased" the situation in advance to get a rough idea of what to expect. This might involve sketching the graphs of y = sec x and y = tan x then thinking what the graph of the difference function might be. They might have noted that, since sec x + tan x is increasing without bound on the interval $\left[0,\frac{1}{2}\pi\right)$, the function f(x) would decrease on the same interval to zero, or that, on the interval $\left[\pi,\frac{3}{2}\pi\right]$, sec x and tan x have opposite signs and are both unbounded, so that their difference would tend to $-\infty$. Such a preliminary look has the advantage that students go into the problem with some expectations and some idea of where the significant behaviour occurs, and have something against which to measure the results of their analysis.

How does one diagnose what is going on? A traditionalist would say that the students are poor in basic skills and have not had sufficient practice. A reformer would counter that the students are not in touch with the mathematics and need a curriculum that induces them to engage mathematics more intimately while they are learning it. In Ontario, this debate has resolved itself into a curriculum that tries to present material more holistically. But a holistic curriculum is not of much value if it remains shallow, and it can only remain shallow unless it is backed up by some solid competence on the part of the student. One needs to go some distance into a topic before one is able to analyze, contrast, and make connections.

Those who are attempting to bring in a problems-based course (one based on, to use the jargon, "rich learning experiences") are definitely on the right track, provided that a great deal of attention is paid to students being thorough in their analysis, that technical weaknesses are identified in the process and handled through carefully crafted and orchestrated exercises (not just the routine drill that often occurs), and that at the end, what has been learned is consciously reviewed and consolidated. When problems are suitably selected, a discussion of the merits of alternative approaches can occur, with the payoff that students can pick up the idea that mathematical judgments are important, both in making the technical work manageable and in the clarity of the solution. Skill-development and problem-solving should go hand-in-hand. A syllabus supported by reams of disembodied exercises or superficial and flaky problems will confound the most dedicated teacher. The new Ontario curriculum gives us the occasion to bring the technical and the holistic into harmony.

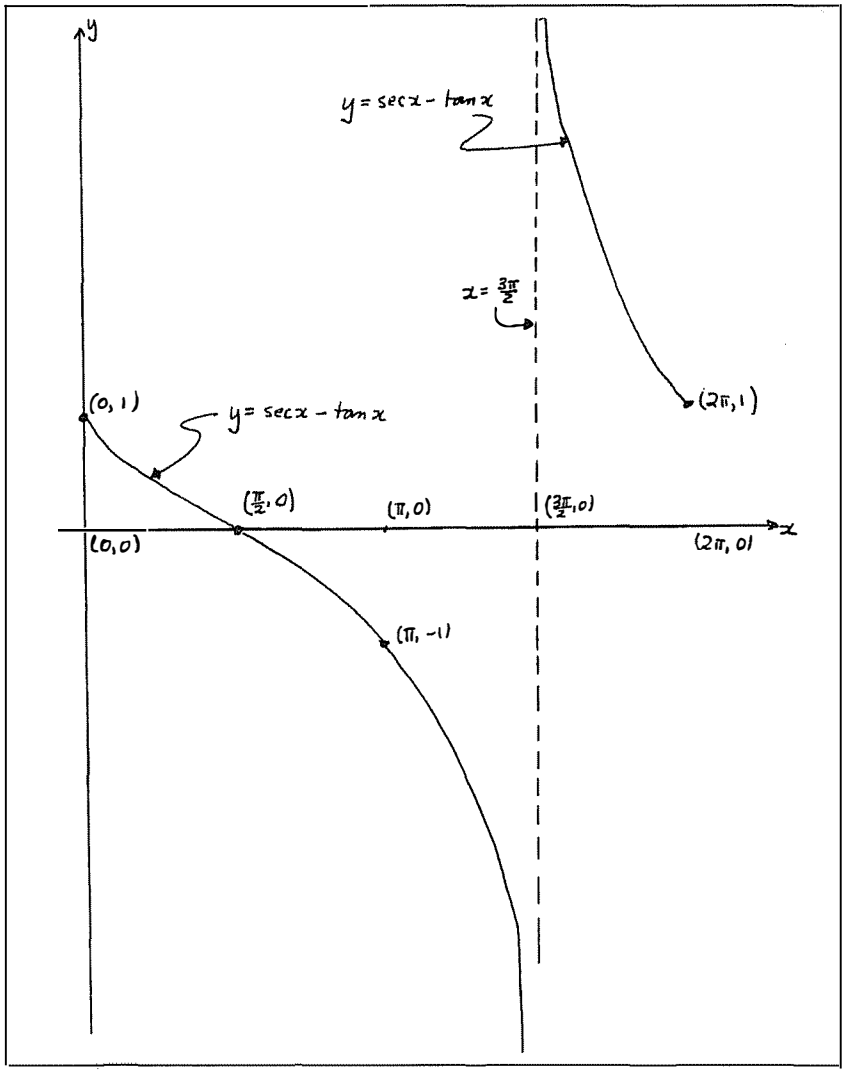
To be more specific, I would encourage much attention to the sketching and analyzing of graphs wherever the curriculum permits. Graphs are means of amplifying information given by other means; producing them can help tune up the cognitive skills of students. In an applied problem, students should try to predict from the information what the graph of a relevant function might be like, before checking with a calculator or paper and pencil. If the graph turns out not to be what was expected, then students should analyze the situation to see what is out of kilter. More attention can be paid to means of transforming a graph into graphs of related functions or equations. Some may argue that this is far too sophisticated an approach for a high school class. Unfortunately, as I tell my students, that is not how God

made the world. Either they will learn to do mathematics with their feet on the ground and their minds in gear, or they will ultimately fail. How much better that they are faced with the issue while they are young and have time to rectify the situation!

Now that we have technology to get a precise graph by simply plugging in the function, why should we bother with sketching graphs by hand? I want to emphasize that a graph sketch is precisely that - a sketch. One wants to get a holistic view of some analytic expression, and for this purpose, one marshalls sufficient information to identify its chief characteristics and render them in a diagram. One does not necessarily have to have complete accuracy, but precision that is good enough. Indeed, a good piece of advice for students first looking at a problem of this type is to defer using calculus. For example, in the present example, f(x) is the difference of two functions whose graphs should already be known, and so the student should be able to surmise how their difference might behave. However, some questions might not be settled; for example, the difference between sec x and tan x is positive when $0 < x < \frac{\pi}{2}$, but does this difference get steadily larger or smaller? We might expect a maximum or minimum in a certain interval, but where exactly? Might there be points of inflection? Once these questions are raised, then one can turn to the calculus to provide more precision.

Producing graphs with software and sketching them by hand are two quite different enterprises. The software graph will produce greater precision, but will be indifferent to significant features or scale; having access to such graphs will make possible many such tasks as solutions of equations that are beyond the reach of standard techniques. But a student who can produce sketches efficiently gains a feel for the properties of various types of functions, such as trigonometric, exponential, logarithmic, or polynomial, and a sense of significant features and qualitative behaviour. Technology goes hand-in-hand with paper and pencil, each informing and monitoring the results of the other.

There are several social forces ranged against us in achieving this goal, whatever are our best intentions. The first is that students' attitudes are probably set before they come to high school, and many students emerge from an elementary regime where teachers are not comfortable with mathematics, do not have much of their own experience with the subject to draw upon, and therefore present material at the most basic utilitarian level as a lot of procedures to be remembered. The second is an anti-intellectualism that afflicts a significant part of the student body, where students are often put down by their peers for working hard or making achievements. The third is a narrow perspective on learning that looks for immediate and practical payoffs rather than understanding and coherence. In facing these challenges, the mathematics community needs to be united behind the propositions that problem-solving and analysis are important ingredients *along with* a high level of manipulative and judgmental skills for a mathematics curriculum that will engage our science- and engineering-bound students, and that properly learned, mathematics can be satisfying to a large number of students. ▲



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RON LANCASTER



Ron Lancaster is a middle and high school mathematics teacher, currently on staff at the Bishop Strachan School as a mathematics consultant. Ron also works as an Independent Consultant throughout North America and Asia, and is an Instructor for Texas Instruments.

"A picture is worth a thousand words" is an expression that is often used to convey the importance of using images to make a point. The point of this column is to go beyond this notion, and promote the idea that in fact "A picture is worth a thousand numbers." Readers can expect to find photos with math in every Photo Math *column, and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario curriculum as well as the new NCTM document,* Principles and Standards for School Mathematics. *It is hoped that these questions will inspire students to be more creative and playful, and that in the end, students will develop an ability to see the world from a different perspective.*

Teachers and students are encouraged to pose mathematical questions related to their own encounters, and to submit these problems along with photographs to the author for possible publication in future columns. The author of this column does not presently have a cable modem, and in order to avoid long periods of downloading items, submissions to this column should be sent via Canada Post to the address given above.

In 1998 I made a proposal that I called "A World-Wide Search for the Number 77" to the organizers of the 77th NCTM Annual Meeting. It was essentially a scavenger hunt in which teachers and their students would search within their neighbourhoods or anywhere else for examples where the number 77 puts in an appearance.

I suggested that photographs of the places where the number 77 was found could be taken and submitted to the NCTM, and I could be the person who received and handled the items. Instead of a photograph, submissions could have consisted of a sales receipt for say 77 cents, or a song by the musical group the 77s (I am not making this up, there really is a group called the 77s and one of their big hits was the song 88) or anything else related to the number 77 (no road signs please!). Submissions could also have involved mathematical properties of the number 77. For example, every number greater than 77 can be written as the sum of integers, the sum of whose reciprocals is equal to 1. If you are intrigued with this result, the representation for the number 78 is given below. It would be a great exercise for you and your students to prove that 77 cannot be so represented, or to find the representation for say the number 79.

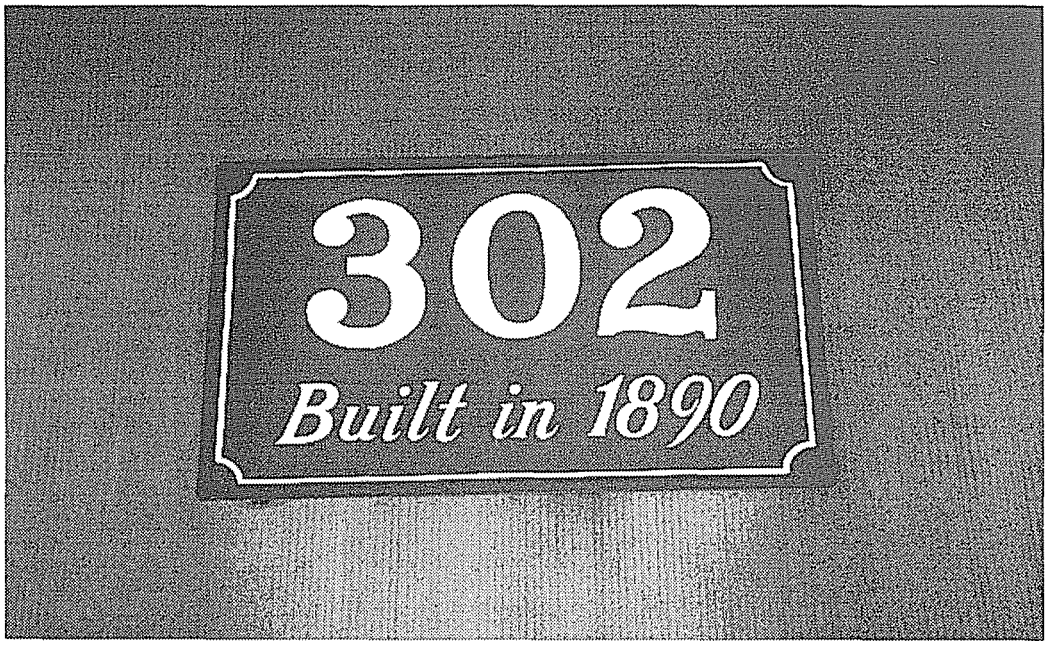
$$78 = 2+6+8+10+12+40$$

$$\frac{1}{2}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}+\frac{1}{40}=1$$

The NCTM Program Committee liked the fact that this activity would allow teachers and their students all over the world to participate in an activity that would promote the notion that mathematics is everywhere and that would encourage everyone to be more observant about their surroundings. I had suggested that items could be put on display at the NCTM annual conference in San Francisco and published in the three NCTM journals. In the end, though, they felt the proposal would be very difficult to manage and coordinate by a small group of people. I wonder if this idea would work on a smaller scale at an OAME Annual Conference? Hmmm...

I had also recommended that participants in this scavenger hunt be encouraged to create mathematical questions related to their submissions. In order to give the Program Committee a sense for what I had in mind, I created a series of examples all based on a walk that I had in New York City. I began my search for the number 77 on East 77th and West 77th in Manhattan. I then took a 120 block walk from the Upper East side of Manhattan all the way down to the tip, and the photographs from this column will give you a sense for what I found on my trip.

Photograph 1



1. This sign appears on an outside wall in front of a beautiful old building at 302 West 77th Street in New York City. Use only the digits 1, 8, 9, and 0 to form an expression that is equal to 302. You may use the operations of addition, subtraction, multiplication, and division, as well as juxtaposition (putting two numbers side by side as in 90 or 18), exponents, square roots, factorials, the greatest integer function, and decimal points. All four digits must be used. To help you get started, here are two expressions for which the answer is not 302, but 77:

$$81-\sqrt{9}-0!$$

$$80^{1}-\sqrt{9}$$

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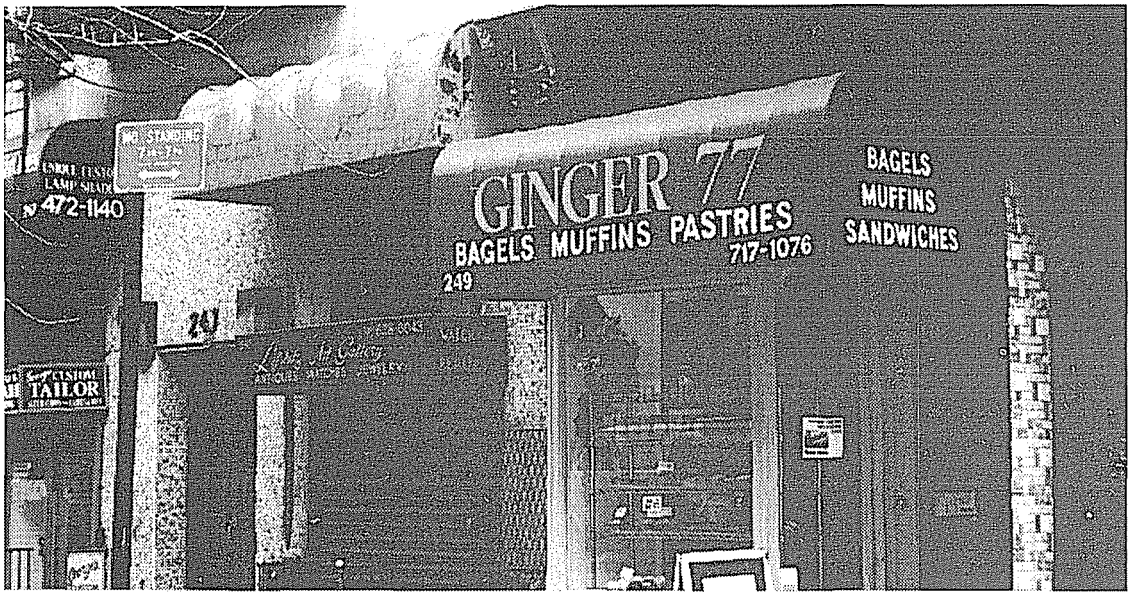
Photograph 2



2. This parking sign is for a garage located on West 77th Street in New York City, just down the street from the building at 302. The tax on parking in Manhattan is between 10% and 20%. Assuming that the after-tax cost of parking at this garage is a multiple of 25 cents, determine the tax rate.

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Photograph 3



3. (a) Find a word to put between Ginger and 77 so that it makes sense when combined with Ginger or with 77.

   (b) The phone number for Ginger 77 begins with 717. How many other phone numbers could be assigned that begin with 717?

   (c) How many phone numbers beginning with 717 end in a four-digit number that
   (i) is divisible by 77?
   (ii) contains exactly two 7s?

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Photograph 4



4. This No Standing sign, 7AM – 7PM Mon thru Fri, is located in front of Ginger 77.

   (a) What is the difference between a sign that says "No Parking" and one that says "No Standing"?

   (b) Select the percent from the choices given below that best describes the percent of time that "Standing" is not allowed over the course of a week. Make your selection by estimating the percent, and then use a calculator to verify your choice.

   (i) 0% (ii) 25% (iii) 50% (iv) 75% (v) 100%

   (c) Re-answer the question posed in part (b) for each of the time intervals given below. Rather than answering this question one case at time, you might want to work with a general case and develop a formula for the percentage of time that "Standing" is not allowed over the course of a week. When you are finished, describe how this percent changed from one case to the next.

   (i) 8 AM - 6 PM (ii) 9 AM - 5 PM (iii) 10 AM - 4 PM

   (iv) 11 AM - 3 PM (v) 12 PM - 2 PM (vi) 1 PM - 1 PM

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Photograph 5



5. (a) The Canal Jean Co. is located at 504 Broadway in New York City and it is a very popular store, especially amongst teenagers. If the text were removed from the outdoor banner shown in the photograph, the checkerboard pattern would consist of 77 small squares. By taking 4 contiguous squares, it is possible to make a 2 by 2 square. How many 2 by 2 squares are there on the banner? How many 3 by 3 squares are there? How many squares are there altogether?

   (b) For an m by n checkerboard pattern, is it possible to find values of m and n for which the total number of squares (of all sizes) is equal to 77?

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Comments and partial answers

Photograph 1

Question 1 One possible solution is $\frac{\sqrt{9}}{.01}+\left[\sqrt{8}\right]$.

Readers are encouraged to search for other solutions and to submit them to the author of this column for possible publication in a future issue of the *Gazette.* Recall that the greatest integer function, denoted by [x], is defined as the greatest integer that is less than or equal to the value of x. Here are a few examples to help you understand its definition:

$[3.4567] = 3,\ [23.987] = 23,\ [8] = 8,\ [-6.4573] = -7$

Photograph 2

Question 2 I have used this problem (and a more detailed version of it) for many years with students in grades 7 and 8, and it has been interesting to see how they approached the problem. Students familiar with the basics of algebra have tried to set up an equation but have had difficulties with completing it. "Mr. L, what do I make it equal to?" has been the standard question asked of me. In fact, when I have used this question with my high school students this question has usually been the immediate and only response. In the end, most students have resorted to a method of guessing and many have created a program (using a graphics calculator or a computer) to eliminate the guess work in the guessing. The answer turns out to be 18.25%, and anyone who thinks taxes are high in Canada may want to park that idea.

Photograph 3

Question 3

(a) The word is ROOT.

(b) There are one thousand other phone numbers that begin with 717.

(c) Starting with the number 717-0000 and ending with 717-9933, there are 130 phone numbers that end in a four-digit number divisible by 77.

Starting with the number 717-0077 and ending with 717-9977, there are 324 phone numbers that contain exactly two 7s.

Photograph 4

Question 4

(a) Readers are encouraged to seek out an answer to this in the community in which they live, and to compare their findings with other locales to see if there is a uniform definition of these terms throughout Ontario.

(b) Of the 168 hours in a week, 'No Standing' is in effect for 60 hours. This is approximately equal to 36% of the time. Note that two of the choices are close to this answer, and not surprisingly, my students have had trouble choosing between an estimate of 25% and 50%. The most popular choice has been 50%.

(c) The answers for the six cases are given below, and you should note that from one case to the next, the percent decreases by a constant amount. While the drop appears to be 6%, it is in fact equal to $-5\frac{20}{21}\%$.

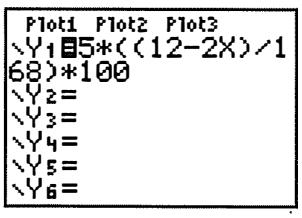
time interval	number of hours during which No Standing is in effect	percent of time during which No Standing is in effect *(approximate answers)*
8 AM - 6 PM	10	30%
9 AM - 5 PM	8	24%
10 AM - 4 PM	6	18%
11 AM - 3 PM	4	12%
12 PM - 2 PM	2	6%
1 PM - 1 PM	0	0%

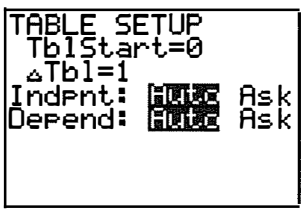
One way of generalizing the times is to use the 24 hour clock, letting the time at which the rule goes into effect be 7 + x, and the time at which it ends be 19 – x. The rule will then be in effect for 12 – 2x hours per weekday, and the percent of time during which the rule rules will be

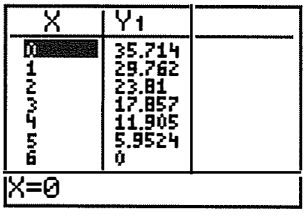
$$\frac{5(12-2x)}{168}\cdot 100\cdot$$

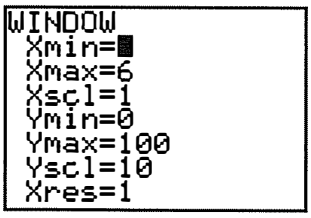
Note that this is under the assumption that $0 \le x \le 6$.

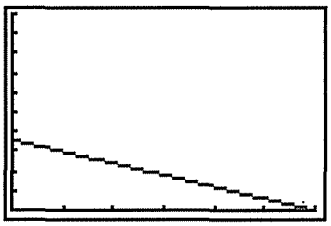
Teachers wanting to give their students some practice at working with a graphics calculator may want to use this problem for that purpose. Screenshots for a TI83 Plus are given below:











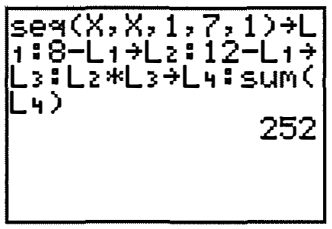
Photograph 5

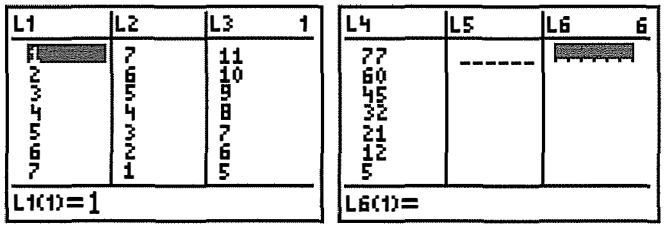
Question 5

(a) The number of 2 by 2 squares is equal to 60 (6 x 10), and the number of 3 by 3 squares is 45 (5 x 9). The total number of squares of all types is 252. See the chart given below for a listing of the various types of squares:

type of square	total number
1 by 1	77
2 by 2	60
3 by 3	45
4 by 4	32
5 by 5	21
6 by 6	12
7 by 7	5

Teachers wanting to give their students some practice at working with a graphics calculator may want to use this problem for that purpose. Screenshots for a TI83 Plus are given below. For readers who require an explanation of the formulas for L_2 and L_3, here is a hint: For the 7 by 11 checkerboard pattern, the number of a by a squares is (8 – a) x (12 – a).





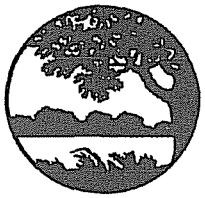
(b) This problem is left as an exercise for interested readers. ▲

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OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration;

- demonstrations of excellence and creativity in mathematics education.
- specific contributions to mathematics education in the school, region, province or nation.
- demonstrates leadership in mathematics education in the school, region, province or nation.
- support of and contribution to the activities of OAME/AOEM.
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

OAME/AOEM LIFE MEMBERSHIP AWARD

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and accumulated 10 or more years of membership in OAME/AOEM.

KENNETH D. FRYER AWARD

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

DON ATTRIDGE AWARD

In recognition of the contributions made by Don Attridge, a founding member, former president and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

AWARD NOMINATION COVER PAGE

Send to: Shirley Dalrymple

Intended Award (please check one)

❑ Life Membership ❑ Mona MacGregor ❑ Morley MacGregor ❑ Kenneth D. Fryer ❑ Don Attridge

Nominee Information:

_____

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

_____

_____

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

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Home Phone ( ) _____

Nominator Information:

_____

Name of Nominator

_____

_____

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

▲ The SketchMad Collaborative: A project to support Ontario teachers using "The Geometer's Sketchpad"







WILLIAM HIGGINSON
NATHALIE SINCLAIR
AND NICK JACKIW

William Higginson and Nathalie Sinclair are at the Faculty of Education at Queen's University (higginsw@educ.queensu.ca and nathsinc@educ.queensu.ca) where Bill is an Associate Professor and Nathalie is a third year doctoral candidate in mathematics education. Nick Jackiw has, on several occasions, been a visiting scholar at the Centre for MSTE at Queen's. He is the creator of Sketchpad and has been intimately connected with its evolution for over a decade. He is currently the Chief Technology Officer of KCP Technologies in California.

The mathematics education community has now had about two decades to explore the relationship between computers and the teaching and learning of mathematics. From that experience, there appear to be four general statements that can be made with little fear of contradiction. First, the relationship between the discipline of mathematics and the various technological devices of the information age is exceptionally close. There may be some truth in the frequently articulated reassurances given to the mathphobic that, "you don't have to be good at math to use computers." This be as it may, these black boxes are, at their core, mathematical entities and one stands little chance of understanding their structures and modes of operation–beyond a very superficial level–without a firm comprehension of a number of fundamental mathematical principles. Second, the potential of these devices–computers, CD Roms, videos, and well-designed materials developed for them–as tools for the realization of a wide range of curricular aims, is enormous. We know this because of the glimpses we have had when a rare confluence of factors–mainly, knowledgeable teachers with strong technological and institutional support–have come together under the watchful eyes of a competent research team. Third, the widespread realization of this potential has been elusive. For every one of the positive glimpses referred to above, there have been dozens, if not hundreds, of examples of underutilization. Even in cases where significant efforts have been expended to obtain software, it often has been used sporadically, if at all. Fourth, the major factor in determining whether or not computers will be effectively used in math classrooms is the amount and quality of support available for the teacher. The computer systems and related software in this area are complex and sophisticated. Afternoon workshops and thick manuals dropped on a desk are not, in themselves, close to being sufficient for teachers to use these materials effectively with their students.

In light of this situation, it seemed natural for a group of us in the newly-formed Centre for Mathematics, Science and Technology Education at the Queen's Faculty of Education to make the support of mathematics teachers trying to use computers in their classrooms a high priority. In particular, we decided to use some of the funds made available to the Centre (2000 – 2004) by the Imperial Oil Philanthropic Foundation to set up a support site for the use of "The Geometer's Sketchpad™". Our reasons for choosing Sketchpad were twofold: first we thought of it as a particularly rich curriculum resource, and second, it was, unlike much software, universally available to the publicly supported schools of Ontario through a provincial license.

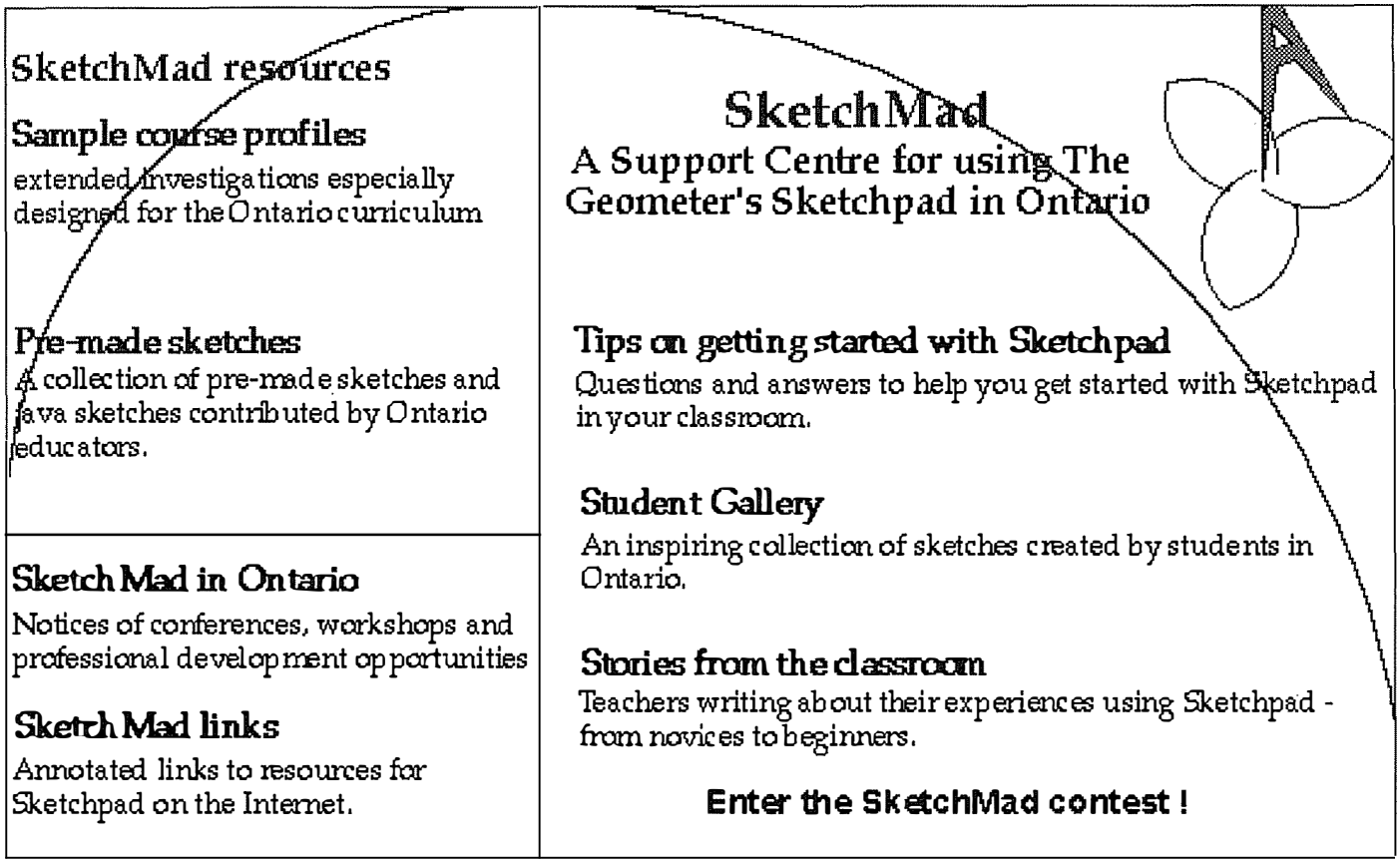
We have named our initiative "The SketchMad Collaborative". Our title is consciously whimsical–and thereby, perhaps, easy to remember. The easiest way to find our site is to enter "SketchMad" into an internet search engine like "Google". There aren't many other "SketchMad" sites so you can find yourself at http://hydra.educ.queensu.ca/sketchmad/ quite quickly. The emphasis on collaboration underlies the fact that this is a team effort–contributions are solicited from all Sketchpad users. We are delighted with the quality of our early-stage materials such as Peter Harrison's fine piece on the solution of quadratic and cubic equations. The SketchMad initiative is one of two extensions of work carried out at the Faculty, the "Tomorrow's Mathematics

Classroom" Vision Project, [http://tilp.educ.queensu.ca/faculty/tmc/]. The second 'extension' project, led by Gary Flewelling, is a "Handbook on Rich Learning Tasks". Central to the Vision project was a tripartite conception of the discipline of mathematics: as at tool, as a language, and as an art. The 'mad' part of our title reflects this latter conception of our field. Some Sketchpad users are passionate about it in a very positive way–they think of it as a creative tool. Many Ontario teachers are more likely to be at the 'mad at' stage: here is yet another externally compelled burden. A measure of our success over the next two years will be the extent to which some of them begin to move toward the 'passionate about' version of being SketchMad.

A Tour of the SketchMad Site

As global awareness and interest in Dynamic Geometry software has grown in the past several years, more and more teachers are finding applications for this technology outside of–as well as within–Euclidean geometry. Research conducted in the United States has suggested that The Geometer's Sketchpad has potentially useful applications from the early junior grades through to the level of current mathematics research. The Ontario Licensed Sketchpad CD-ROM comes with two curriculum modules (in PDF format), both of which focus primarily on geometry. (One, *Exploring Geometry*, is geared toward secondary level students; the other, *Geometry Activities*, more for intermediate grades.) While many of the supplemental curriculum materials available in print about Sketchpad address a similar content and age focus, a quick internet survey finds teachers using Sketchpad to introduce concepts in trigonometry, algebra, calculus, and probability to their students. Our hope with the SketchMad site is to harness some of this richness in ways that are particularly appropriate to Ontario teachers, by focusing both on the areas where the new curriculum explicitly calls for Dynamic Geometry software, and on areas where we believe it can motivate or contribute to classroom discussion on topics further afield.

Although there are now thousands of sites on the Internet that have resources for teachers and students using Sketchpad, the SketchMad site is devoted to Ontario teachers and students. You will not only find technical, curricular, and strategic resources on the site to help you use Sketchpad in your classroom, but you will also have the opportunity to read about and see what other Ontario teachers and students are doing with Sketchpad. From the front page (as shown in the image below), you will be able to follow links to these areas:



1. **Tips on getting started with Sketchpad**. Here you will find some answers and suggestions to questions that teachers have frequently asked about how to get started with Sketchpad in their classrooms–with references and links to resources that are Ontario specific. If you can't find the answer to your specific question on this page, you will find a link at the bottom of the page that will allow you to submit it to us–we hope to provide answers as quickly as possible!
2. **Sample course profiles.** This area contains a collection of Sketchpad activities that are directly correlated to the Ontario high school curriculum. They have been created and contributed by Ontario mathematics educators. Many of them have been tried out in the classroom too. We encourage you to either comment on these activities, or submit activities that you have tried in your classroom and that you would like to share with other teachers. You will find that some of the activities have sample rubrics and student work included.
3. **Pre-made sketches.** Sometimes it's much easier for you and your students to be able to work from pre-made sketches–effective sketches can sometimes be difficult or time-consuming to create. Here you will find sketches that Ontario teachers have used in their classrooms for investigating areas such as triangle centres and trigonometric functions. You can simply download them and put them on your computer lab server so that all your students can interact with them.
4. **Student Gallery.** This is a collection of sketches that have been created by Ontario high school students. Both you and your own students might find these sketches inspiring. We encourage your students to submit their own sketches–we'd love to show off the kinds of things that Ontario students are creating with Sketchpad!
5. **Stories from the classroom.** Sometimes the hardest part of using a new technology is to imagine how it might all work in your classroom. Here you will find a collection of stories that Ontario teachers have written that describe in detail how they have used Sketchpad with their students, from the beginning to the end, and how their students have responded.
6. **SketchMad in Ontario and SketchMad links.** These two sections provide links to select Sketchpad resources on the Internet as well as to workshop opportunities and Sketchpad contacts in Ontario.

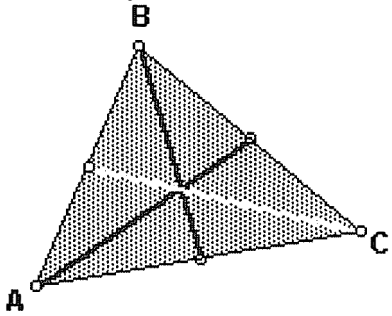
We have also recently launched the SketchMad contest. We have invited students and classrooms from across Ontario to submit sketches that investigate the theme of "offspring triangles," as shown in the following image. This contest is open until December, but we will be running a new one in the spring. Winning entries will be displayed on the SketchMad web site. ▲

The Activity
Fall 2000

Polygons and their Offspring

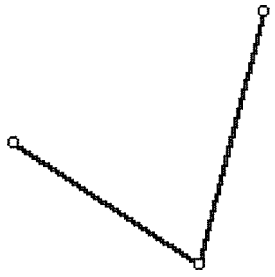
Consider the "offspring" of a polygon (an offspring can refer to any geometric object that is derived from the polygon. For example, the medians of a triangle, the diagonals of a quadrilateral, perpendicular bisectors, angle bisectors, etc.). What relationships, properties or patterns can you find?

Here is an example investigation. Construct the three medians of a triangle. Use these three segments to create the "offspring" triangle so that the three medians become the sides of a new triangle. Find the area of both the original triangle and the new one. How do they relate?



original triangle

Area ABC = 7.995 cm^2



offspring triangle

Area GJI = 5.996 cm^2

Ontario Association for Mathematics Education PUBLICATIONS

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▲ OAME/NCTM December 2000 Report



FRANCES SCHATZ

Thank you, OAME, for sponsoring me to the NCTM Canadian Leadership conference at Mount St. Vincent University in Halifax, Nova Scotia. The two day event gave the twenty delegates (Mary Howe and myself from Ontario) an opportunity to network, to find out the what's and why's about changes that are occurring at NCTM regarding affiliates, and to preview (with group discussion) a section of the new *Principles and Standards* that was released in April this year. Mike Fulton (Ontario and west provinces) and Sharon Henderson (eastern provinces), who serve on the NCTM Affiliate Services Committee, led the workshops. Rita Janes, NCTM board member from St. John's, Newfoundland, was our keynote speaker. All of the delegates appreciated the hospitality of the Mathematics Teachers Association of Nova Scotia.

OAME is a partner affiliate with NCTM. The affiliation fees are being put to very good use to establish a web site and resource centre for affiliates. This project was started in the spring 2000 to house an exciting new area just for leaders and teachers of NCTM's affiliates. This **Resource Centre** replaces the *Affiliate Handbook*. NCTM will be creating an archive of contributions from Affiliates. This will include stories of how affiliates can reach pre-service students, develop new leadership, and offer alternative conference formats. I encourage you to take a tour of the web site: www.nctm.org/affiliates/resource/.

The most interesting site I have found is *Illuminations.* Go to www.nctm.org/affiliates/, then click on teachers' corner and illuminations. *Illuminations* has something for teachers and students at all grade levels. You will find math investigations, internet-based lesson plans developed by teachers, and a way to search for information in *Principles and Standards.*

Are you interested in preparing an article for the *2003 NCTM Yearbook* and *Classroom Companion* booklets? (Remember, Louis Lim, editor of Ontario's *Gazette,* is always on the search for articles for Ontario teachers too).

You can go to the web site www.nctm.org/publications /yearbook.htm for the guidelines for preparing articles or the call for manuscripts. The 2003 yearbook will be *Learning and Teaching Measurement.* If you are unable to access the web site, please send a request to me for specific information you would like to receive.

New Resources

#722 Mathematics Assessment: Cases and Discussion Questions for Grades 6-12, Classroom Assessment for School Mathematics K-12 Series. This is part of a series of six books to complement the *Assessment Standards for School Mathematics.* This book features stories written by math teachers describing their experiences with classroom assessment and includes facilitator notes.

#740 Multicultural Mathematics Materials, 2nd edition. This new title features games and activities from around the world to enhance individual students' ethnic identity and heritage with an appreciation for cultural diversity. Activities are reproducible.

Membership

Membership prices changed June 1, 2000. If you are interested in an NCTM individual, senior, or student membership, please contact me by phone, fax, or email for up-to-date information. If the membership is paid by cheque or cash in Canadian funds, there is a 25% cap on exchange rate. Processing of memberships takes 6-8 weeks. The membership brings you a choice of *Teaching Children Mathematics*, *Mathematics Teaching in the Middle School*, or *Mathematics Teacher.* In addition, members can download *Dialogue Notes* and *Student Math Notes.*

NCTM also reminds us about **Recognition Awards** available to teachers. The Canadian Prime Minister's Awards recognize the efforts of outstanding teachers in all disciplines. Guidelines and nomination forms for 2000-2001 are available at the web site www.schoolnet.ca/pma or write to **The Prime Minister's Awards for Teaching Excellence**, Industry Canada, Room 713, Heritage Place, 155 Queen Street, Ottawa, ON K1A 0H5. ▲

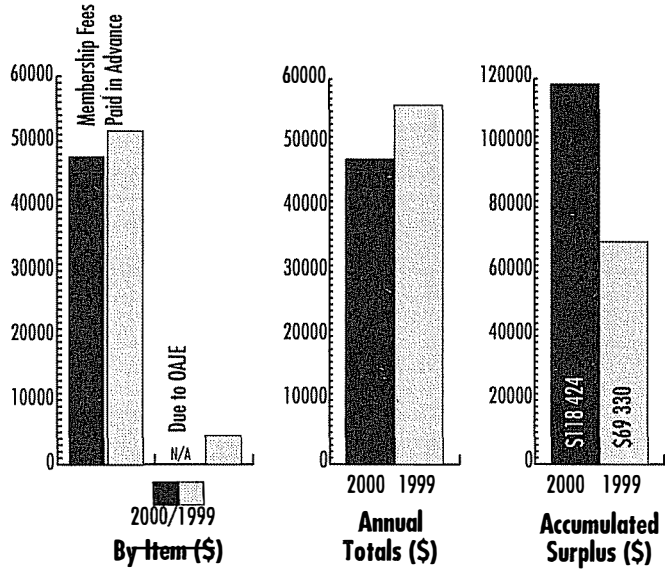
FINANCIAL STATEMENT

Notice to Readers

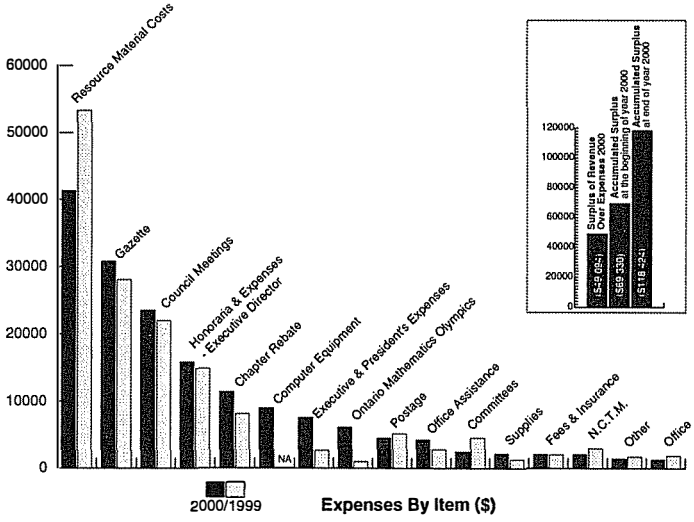
I have compiled the Balance Sheet of Ontario Association for Mathematics Education (Association Pour L'Enseignement Des Mathematiques) as at August 31, 2000 and the Statement of Operations and Accumulated Surplus for the year then ended from information provided by the Directors. I have not audited reviewed or otherwise attempted to verify the accuracy or completeness of such information. Readers are cautioned that these statements may not be appropriate for their purposes.

Markham, Ontario, September 25, 2000
ROBERT M. DAVIES Chartered Accountant.

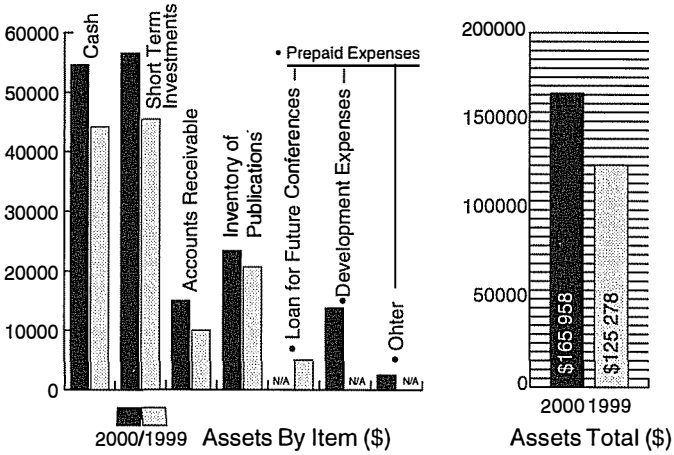
Liabilities & Surplus 2000 / 1999



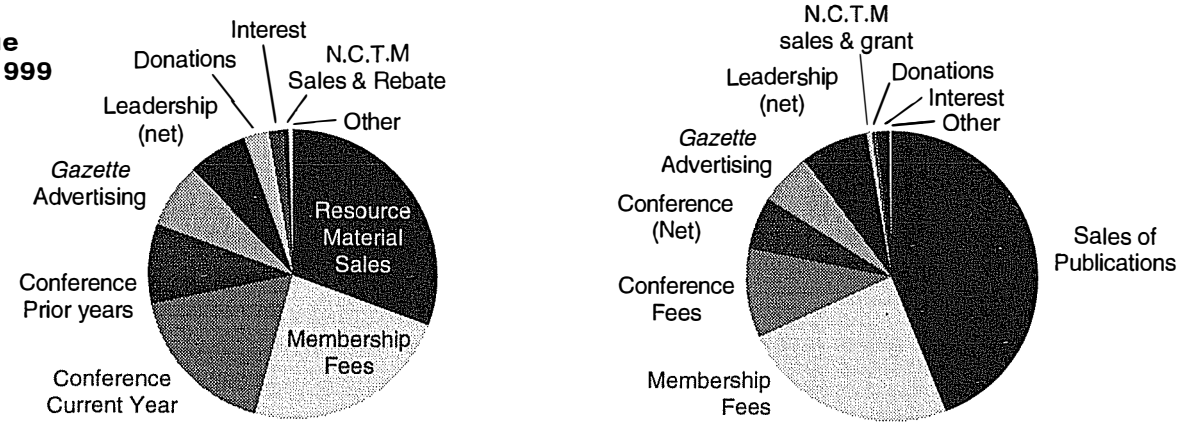
Expenses 2000 / 1999



Current Assets 2000 / 1999



Revenue 2000 / 1999



2000 ($214 947) **1999 ($199 627)**

▲ Directors' Dialogue



MARILYN HURRELL, NWOAME
LAKEHEAD DISTRICT
SCHOOL BOARD
THUNDER BAY, ON

This edition of the "Directors' Dialogue" summarizes the Saturday, September 30, 2000 OAME Board of Directors' meeting at the Four Points Hotel in Mississauga.

PRESIDENT'S REPORT

Mary Howe reported that the integrated Elementary Units for grades 4 - 6 have been posted on the Ministry of Education web site, planner.media-x.com. Summer Institutes, sponsored by the Ontario Teachers' Federation, were well attended. Thanks to Myrna Ingalls and her committee for presenting their C.A.R.E. (Concerning Assessment Reflective Evaluation) package to mathematics teachers.

EQAO has posted training materials for the Secondary assessment on its web site, www.eqao.com, and is looking for markers in February and for the summer of 2001. The Ontario Mathematics Coordinators' Association (OMCA) is conveying to EQAO concerns that the investigations component of the grade 9 assessment was delayed to June 2000.

EXECUTIVE DIRECTORS' REPORT

Dave Alexander reported that the current OAME membership as of September 24, 2000 is 1243 . Chapter reps have a list of members in their chapter who have not renewed, and these people will be contacted.

The search is on for new Executive Directors for OAME as Bonnie and Dave Alexander are going to retire! We wish them all the best. Peter Saarimaki is chair of the Search Committee, and recommends that once again the Executive Directors' position be handled by two people who can work in close collaboration. Applications close December 1, and the appointment will be made in early 2001.

PROFESSIONAL DEVELOPMENT

NCTM:

Frances Schatz thanked OAME for sponsoring her to the Canadian NCTM Leadership Conference this past July in Halifax. She gained insight and renewed energy to deal with some of the issues encountered as OAME/NCTM representative. Frances encourages new, current, and former members of NCTM to continue their memberships via the NCTM web site: http://www.nctm.org/membership/ as she will no longer be handling them.

Frances was very pleased to hand OAME a cheque for $1257 from the sale of NCTM materials during the past year to OAME members. Be sure to order your NCTM materials through Frances as OAME receives 20% of the sales. Visit www.nctm.org to check out the latest NCTM titles.

FIELDS INSTITUTE:

Shirley Dalrymple described the Fields institute and its mission. The institute is located at the University of Toronto, and is a centre for world class research in mathematics. The institute is named in honour of Canadian Mathematician, John Fields (1863-1932). The Fields' medal, awarded for international discoveries in mathematics, is recognized as the highest honour in the mathematics community (equivalent to the Nobel Prize in other disciplines).

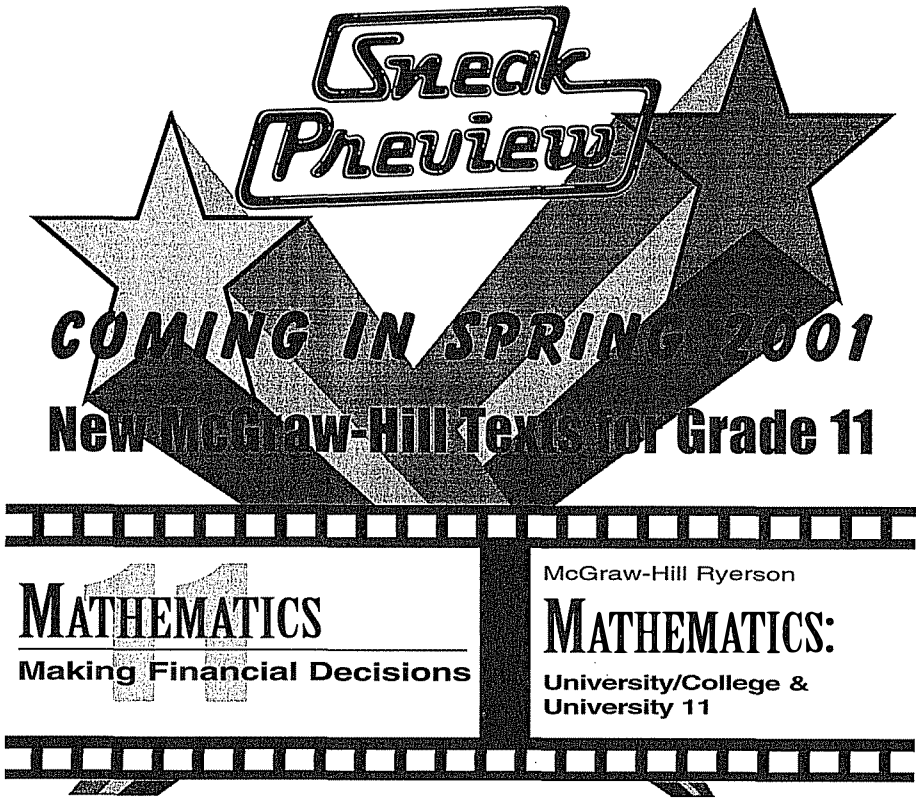
The Fields Mathematics Education Forum is a group that meets regularly to discuss issues that influence mathematics education in the province. Members of this forum include representatives from faculties of education, university and college mathematics departments, OMCA, and OAME.

Currently, there are three task forces at work:

1. Mathematics Teacher Education: This group will address the current teacher shortage, and provide strategies to attract more math students into teaching.
2. Transition from Secondary to Post Secondary Education: This group is made up of representatives from high schools, universities, and colleges, and will deal with the impact the new curriculum has on Secondary School Reform and entrance to post secondary institutions.
3. The new Grade 12 Data Management course: This group will pull together resources for this new course, which is very different from anything currently being taught.

Leadership 2001:

Mary Howe reported that a flyer for the leadership conference was sent with the September *Gazette.* She noted that several OAME Directors, past and present, will be conducting the workshops. The keynote speakers are Joan Ferrini-Mundy, past NCTM Director and chair of the writing teams for the recently published NCTM *Principles and Standards for School Mathematics*, and the always popular Don Fraser, Professor Emeritus at OISE/UT. Workshops include grades 4-6: Communication in Mathematics in the Junior Division; grades 6-8: Linking Assessment and Instruction in Mathematics in the Middle Grades; grades 9-10: Learning Math Through Inquiry, and grade 11: Key Issues in the Grade 11 Math Curriculum.



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OAME 2001:

Kirsten Boucher reported that plans are well underway for this conference. Dates are May 9-12, at the Delta Hotel Toronto East and Howard Johnson at Markham and 401. Check out their web site at http://schools.tdsb.on.ca/same for the latest information. Watch for your copy of the program booklet in the mail in early spring.

OAME 2002:

Lisa Abernethy reported that plans are beginning for OAME 2002, hosted by MAC^2 at York University. The first meeting is October 16, and the theme will involve palindromes. An ad will be placed in the OAME 2001 program.

OMO 2001:

Marilyn Hurrell spoke with chapter representatives on Friday night, and presented them with a package that includes air travel, accommodation for three nights, meals, and an awards banquet at Old Fort William. The package has mathletes and coaches arriving in Thunder Bay on Thursday, June 7, 2001, and departing on June 11 at noon. A "Northern Experience" is planned for the Friday. The cost of the package is $415 per person. Additional sponsorship is being sought. The theme of OMO 2001 is "A Northern Odyssey, a Superior Experience for Your Mathletes". Chapter reps were enthusiastic about the plans. It was stressed that regional math olympics should be held by mid-April, as names must be given to airlines a month in advance of June 7 to benefit from the special airfares.

PRESENTATION OF THE MONA MACGREGOR AWARD

President Mary Howe and President-Elect Shirley Dalrymple recognized the work done by the Mona MacGregor Award recipient, Penny Clemens, OAME's Graphic Designer. Congratulations, Penny!

COMMITTEE REPORTS

AWARDS:

Shirley Dalrymple asked that nominations for awards in 2001 be sent to her. These include Life Membership, Mona MacGregor, Morley MacGregor, Don Attridge, and the Kenneth D. Fryer awards.

FUTURES/PROMOTION:

John Kersley reported that his committee will address concerns raised over chapter regions in the membership brochure, and update the brochure over the next two years. Chapters have been approached to host the annual OAME conference and the OMO from 2003 to 2005.

MATH EQUITY:

Wayne Fletcher reported on behalf of Emilia Veltri that the Equity Breakfast at OAME 2001 will be scheduled as a workshop so that sufficient time will be available for the speaker. Cost of the breakfast will be $15. Dr. Julia O'Sullivan, the Equity Breakfast keynote speaker at OAME 2000, has been asked to return. The committee is currently seeking articles on math equity to be included in handouts at the breakfast/workshop.

PUBLICATIONS:

A letter of resignation from Todd Romiens as Co-Editor of the *Gazette* was read and accepted, with regret. Louis Lim will serve as the full editor of the *Gazette,* beginning with this issue. Louis appointed Todd and former *Gazette* editor, John Egsgard, as Associate Editors. Jack Weiner reported that starting in December, the table of contents, President's Message, and one article from the current issue of the *Gazette* will be posted on the OAME web site, www.oame.on.ca. He also mentioned that Frances continues to need *Abacus* articles.

Doug Evans, web master for OAME, presented a report on the capabilities of the web site. These include:

1. Chapters can have links to the web site and advertise their activities.
2. All committees can have Doug set up addresses for them so that e-mail can be sent directly to the committee chair. Committees and chapters can also send him information about special projects. He can also advertise resources that are for sale.
3. He can set up membership and mini-conference registration forms that can be downloaded and returned electronically.
4. Announcements can be made using the web site (e.g., suggestions for speakers, articles, resources).

To reach Doug, e-mail him at oame@oame.on.ca or devans@rideau.net.

CURRICULUM/POLICY COMMITTEES

JK-5:

This committee plans to make effective use of the OAME web site. They will submit five problems each month to the OAME web site beginning in January 2001. They will meet in December to write six months worth of problems.

Gr 6-8:

Lisa Abernethy reported that this committee will submit an article to the March 2001 *Gazette* outlining an activity from the course profiles and including reference rubrics. They also plan to produce a training package for the OAME web site to be used in implementing the soon-to-be-released *Middle School Linking Assessment and Instruction in Mathematics* OAME document.

Gr 9-10:

Jacqueline Hill reported that this committee will produce comparisons of the three existing grade 9 academic textbooks, using the course profiles. They also plan to do the same for the grade 10 textbooks. They will be advertised in the *Gazette* and offered for sale.

The committee hopes to hold a day long workshop for chapter reps on the C.A.R.E. package before the February meeting.

The committee expressed concerns with the delay in the investigation portion of the grade 9 provincial assessment, and has prepared a letter for the OAME President to send to EQAO conveying this concern.

Gr 11-12:

Myrna Ingalls reported that the 11-12 committee has chosen two projects:

1. Create a poster to highlight the wide variety of jobs accessible to students who graduate from the four high school streams. The guidance association will

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kirsten.boucher@etel.tdsb.on.ca
paul.zolis@etel.tdsb.on.ca

or visit our website at: http://schools.tdsb.on.ca/same

be approached to see if they can help with this initiative.

2. The second project is bigger. Since there are two pathways (11U and 11U/C) to calculus, the committee will approach OMCA to develop criteria for which grade 11 course to take. The committee will develop a clear articulation of the vision of the similarities and differences that should exist between 11U and 11U/C mathematics, and then develop some assessment activities to illustrate them. Some people will be approached to develop a first draft of this vision, and present it at OAME 2001 as a workshop titled, "Two Pathways to the New Calculus".

CHAPTER NEWS

COMA: Rep Ron Gaudreau. Elections have taken place. Co-presidents for 2000-2001 are Peter Lenczewski and Allan Harvey. A mini-conference is planned for Monday, January 22, 2001 at Sir Guy Carleton S.S. Regional math olympics will take place in April.

GVMA: Rep Cathy Eagles. The fall conference is planned for 4:30 p.m. Thursday, November 2 at Eastwood Collegiate and the Schwaben Club. The featured speaker will be Myrna Ingalls who will outline the grades 11 and 12 math menu. Elizabeth Matlock, Darren Luoma, Gord Plumridge, Peter Crippen, and Colin Campbell will conduct workshops that focus on practical tasks and assessment in the new grade 10 classroom.

MAC²: Rep Lisa Abernethy. The fall mini-conference will be held on November 2. Workshops will include K-3 and 4-6: (exemplars and activities), and 7-8: OMO preparation. Three newsletters are planned for this year. E-mails have gone out to schools in the region asking for volunteers to serve on OAME 2001 committees. A regional math olympics competition is planned with top teams advancing to the OMO in Thunder Bay. During the winter, workshops on technology in both elementary and secondary classrooms are planned to assist teachers in both curriculum implementation and EQAO administration.

NOMA: Rep Carol Lemmon. The fall conference is planned for October 13 and 14. Details can be found on the web site www.cs.laurentian.ca/serge/noma. NOMA hopes to have monthly sessions where teachers can get together and do activities found in the new curriculum.

NWOAME: Rep Wayne Fletcher. NWOAME will focus on running OMO 2001, along with increasing membership. A spring conference is under consideration. Math awards will be scaled down in 2001 to make way for OMO 2001. Planning for OMO 2001 is well underway with all committees in place.

PINE RIDGE: Rep Jacqueline Hill. The Annual General Meeting will be held in November and a spring conference is planned. The chapter olympics will be promoted, and hopefully teams will be sent to OMO 2001.

QSLMA: Rep Wendy Gallant. QSLMA is regrouping after their successful OMO 2000. A meeting will be held in October.

SAME: Rep Kirsten Boucher. SAME is working on OAME 2001.

TE²Y²MS: Rep Kathy Kubota-Zarivnij. The regional math olympics will be held on March 3, 2001, and a mini-conference is planned for February 1 from 4:30 to 8:30 p.m. at East York Collegiate.

WOMA: Rep John Ferguson. A fall conference, focusing on grades 7 to 9, is planned for November 2 at the University of Western Ontario's Faculty of Education. Tom Griffiths and Marg Kemp continue to hold monthly mathematics enrichment sessions for grades 7/8 students at UWO. OAME sends best wishes for a fast recovery to former WOMA Rep Linda Alton.



Y⁴MA: Rep Rosie Mastrofrancesco. The chapter will support OMO 2001 by sending three teams to the competition in Thunder Bay. Fundraising activities will continue throughout the year. A fall conference is planned for October 19, 2000 at St. Elizabeth C.H.S. The theme is "Connecting Math to the Learner's World: The Art of Thinking".

CLOSING REMARKS

OK, here goes this edition's contest (still no answer to the September quiz):

E-mail me any three facts that you can find on Thunder Bay, the site of OMO 2001! ▲

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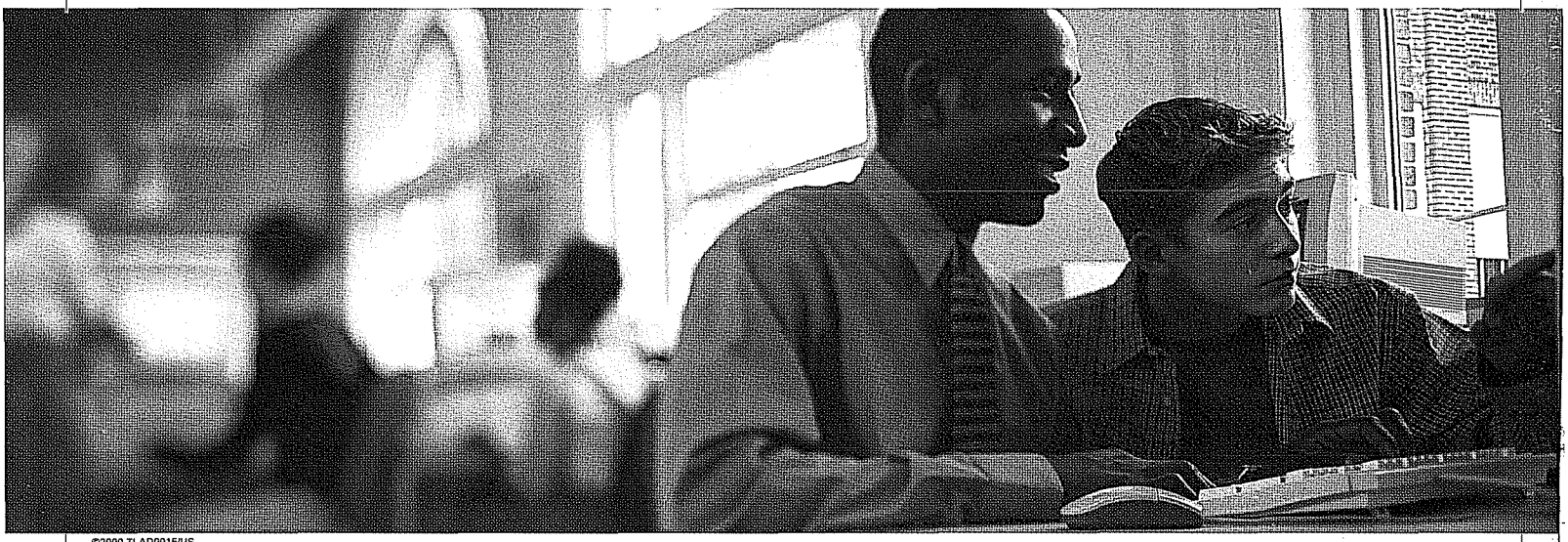
By Dr. Austin J. Connolly (1992)

TAP is linked directly to the KeyMath-Revised/UCN and is valuable for students at-risk, or those needing remediation. TAP isolates and remediates problems identified by KeyMath-Revised/UCN.

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