Vol. 39 #3, March 2001

Ontario Mathematics Gazette

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Parents: A Rich Resource

OMO 2001: A Northern Odyssey - A Superior Experience For Your Mathletes

Pythagorean Misteak

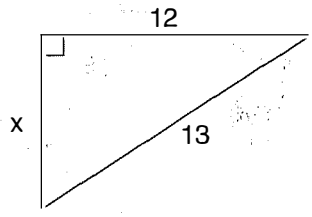
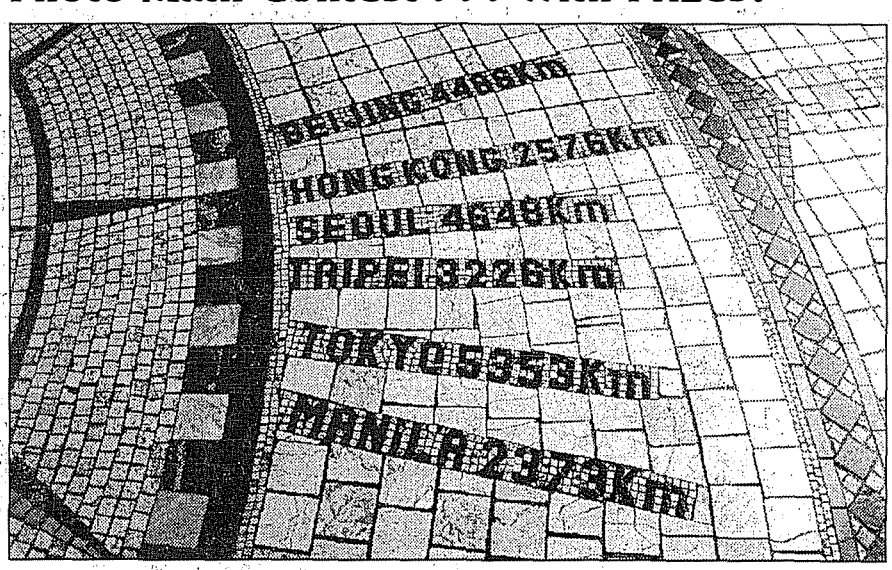
The Problem	The Solution
	$x = \sqrt{12+13}$ $x = \sqrt{25}$ $x = 5$

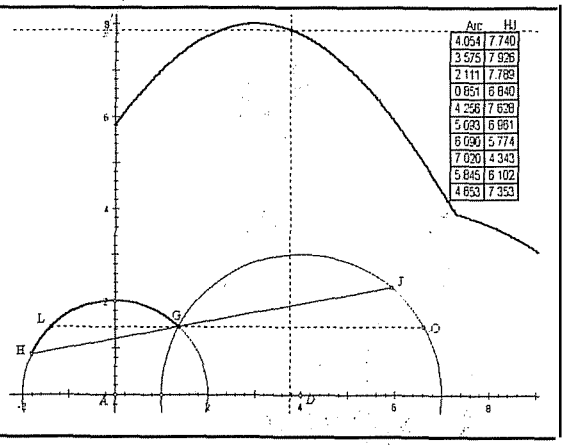
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▲ Editorial



LOUIS LIM, EDITOR

"Education – the ability to learn, to change as things change, to adapt and cope and learn again – that is the gift that truly keeps on giving."
Ken Dryden

OAME plays a significant role in my professional growth as an educator. The annual conference, this year from May 10 to 12 in Scarborough, will be an opportunity for me to reflect on my practices, pick up many innovative ideas, and discuss pertinent issues as I continue to implement the new secondary mathematics curriculum. I have attended the annual OAME conference since 1997, and have especially enjoyed the keynotes such as Ken Dryden, Ivars Peterson, Ron Lancaster, David Foot, Kay Toliver, Marian Small, Don Fraser, Nick Jackiw, and Dan Kennedy. I found each of these speakers so inspirational that I even had some of them autograph my OAME programme booklet after their sessions.

My mentor and math buddy, Ron Lancaster, convinced me that I should share my innovative practices and ideas with others. Ron was my department head at St. Mildred's-Lightbourn School from 1996-97, and we spent hundreds of hours discussing mathematics and mathematics education. I enjoy meeting new people, and quickly discovered that conducting workshops are "give and take" – "give" in the sense of my own contributions to advance mathematics education through workshops, and "take" since participants offer many superb ideas in return. It has been wonderful for people in subsequent years to mention that they attended a session of mine and used my ideas with success. I'm scheduled to conduct five workshops at OAME'2001 – it would be great to personally meet some loyal readers of the *Gazette*.

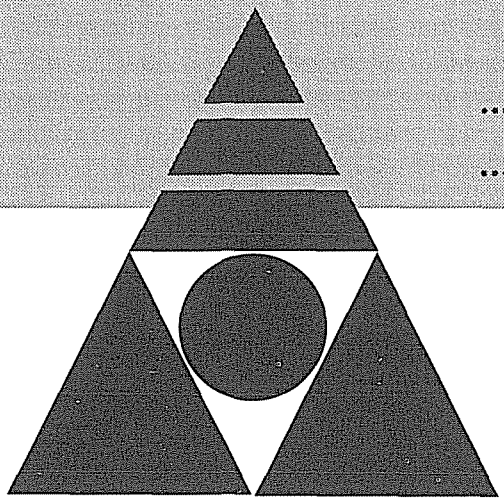
Although I sometimes feel guilty leaving my students to attend conferences, I believe that the ultimate "winners" are our students. Many of the innovative activities that I use in my lessons were picked up from OAME conferences. I feel rejuvenated and re-energized once I return from the conference. I enjoy visiting the crowded publishers' displays to see the newest resources – and quickly observe that many teachers spend their **own** money to provide their students with the

best learning experiences humanly possible. I have discovered that students have unique ways of saying "thank you". Here are several which come to mind: they visit you at lunch time to see how your new classes are going; they interrupt your present class by bringing you the last piece of the birthday cake from their class party; they threaten not to take next year's math course if they don't have you as their teacher, and let's not forget the presents at Christmas (some are quite silly – but I keep all of them!).

I'm counting down the days until OAME'2001. I would like to take this opportunity to recognize the hard work of the conference co-chairs, Kirsten Boucher and Paul Zolis, and all their committee members who have worked selflessly during the past several years to ensure that the conference is beneficial and memorable for each of us. Thank you! They have gathered an impressive calibre of speakers, many who are at the front lines of developing new curriculum, textbook authors, experts in assessment, technology, and innovative practices.

Enjoy all that OAME'2001 has to offer – the keynote speakers, speakers, the publishers' displays (and unveiling of the new grade 11 textbooks!), social events, and networking. Every OAME annual conference reaffirms that each of us has entered the most important profession in the world. As Ken Dryden said, *"A goalie, good or bad or neither, has a big impact on every game because of the nature of the position he plays. A teacher is the same. A forward or defence man must be a star to have the same impact. So must be a lawyer or an accountant. In your chosen profession, in what you do, you will make a difference . . .* ***you*** *will be that favourite teacher in somebody's head, the image that they carry with them all of their lives. That is a pretty remarkable legacy to have."*

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▲ President's Message



MARY HOWE

We are living in times of unprecedented educational change. We must ensure that our teenage stakeholders and their parents are aware of the changes and their implications.

At the secondary school level, the introduction of destination-based grades 11 and 12 courses provide students with a broad variety of choices in mathematics. In order to make appropriate choices, students must be knowledgeable about the possible pathways and the destinations to which they lead. Math teachers and guidance counsellors have a very important role to play in ensuring that students and their parents are aware of the available choices, and where each destination-based course leads.

The insert that accompanies this issue of the *Gazette* provides valuable information that should be shared with grade 10 students, their parents, and guidance counsellors. Grade 10 Parents' Night and Teacher Advisor Program (TAP) sessions provide excellent opportunities to introduce the new grades 11 and 12 courses, and to suggest possible pathways to university, college, and the workplace.

In the past, students were often under the misconception that the doors to university would be closed if they did not take advanced level mathematics. As a result, students often struggled, and frequently failed because they studied (or often memorized) mathematics that was meaningless and too difficult for them.

In the new curriculum, students are encouraged to take mathematics courses that will allow them success along the destination-based pathway they have chosen, rather than stretch higher than they can achieve in order to keep the fictitious doors open. Thus, the new curriculum provides more flexibility for students if they wish to change their chosen pathway. Crossover courses are available after grade 9 for students wishing to change from an Applied to an Academic course, and vice-versa. Commencing after grade 10, transfer courses allow students to change from one type of course to another, and provides partial credit for the additional learning. As a result, the doors are no longer closed to a student wishing to change to a different destination pathway. We, as teachers, must convey that message to our students.

Upon examining the four page *Gazette* insert, you will find a sampling of university and college programs and their corresponding grade 12 mathematics prerequisites. Post secondary programs that require no mathematics prerequisites are also listed. Share these lists of careers and corresponding mathematics requirements with your students and their parents. Students are often unfamiliar with the variety of career opportunities that are available, particularly at community college.

Refer to the graphic titled, "The Ontario Curriculum: Possible Pathways in Mathematics", and note that the majority of students are expected to take the grade 11 University/College (11 U/C) mathematics as a prerequisite for most grade 12 university and college courses. The fast-paced grade 11 University (11U) mathematics course is designed for those students who have a high level of achievement in mathematics and who are pursuing a university program focused on mathematics. The 11 U/C course provides students with additional time that will allow them to consolidate skills while they explore and develop understandings of new concepts in a variety of destination-based applications. Students taking the grade 11 U/C course will develop a solid foundation that should enable them to do better in calculus than if they had taken the 11 U course. Thus, it is crucial that secondary teachers encourage the bulk of their students to take the grade 11 U/C pathway instead of 11 U.

Please copy the *Gazette* insert, use it as a basis for discussion at a department meeting, share it with guidance counsellors, and distribute it to your students and their parents as they begin to make next year's course selections. A worthwhile activity for the guidance and mathematics departments in your school would be to examine the requirements of your local universities and colleges and alter the list so that it is specific to your region and students' interests.

The curriculum guideline states that "today's mathematics curriculum must prepare students for their tomorrows." We, as teachers, must ensure that our students are knowledgeable and able to make informed decisions about their futures and the mathematics courses that will lead them to success. OAME will continue to support teachers, parents, and students in this direction. ▲

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▲ Letter to the Editor

Dear Editor:

I was thrilled to read Fred Ferneyhough's new *Technology Corner* column in the December 2000 issue of the *Gazette.* Fred is well-known for his knowledge about technology, and he is to be congratulated for his willingness to share his innovative ideas.

In Ferneyhough's inaugural column, he studied two different methods for using a TI-83+ graphics calculator, along with knowledge about the Factor Theorem, to solve a cubic equation. I was inspired by Ferneyhough's column to compile and share with readers of the *Gazette* a collection of other methods that I have used with students to solve such equations. Nine different methods are given here along with questions for students. I would suggest that readers follow my comments with a TI-83+ on hand.

For convenience, let's use the same equation that Ferneyhough used in his column:

$12x^3 + x^2 - 41x + 10 = 0$ (1)

Method 1

This is an extension of the second method that Ferneyhough presented in his column. Using the Factor Theorem, students can search for roots of this equation by evaluating the quantities

f(1), f(2), f(5), f(10), f(-1), f(-2), f(-5), and f(-10) (2)

where $f(x) = 12x^3 + x^2 - 41x + 10$. Typically, students stop when they find an expression that equals to 0, and then they follow a different route to find the other roots.

Ferneyhough demonstrated how the TI-83+ could be used to evaluate these quantities, and it turns out that it is possible to evaluate all of them in one step. Begin by entering f(x) as shown in Figure 1, and enter the expression as shown in Figure 2. Press the ENTER key, and in one step, you will obtain answers to all of the expressions in (2). You will need to scroll to the right to view some of the answers.

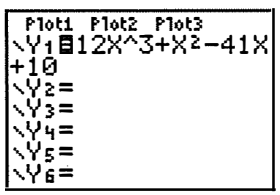


Figure 1

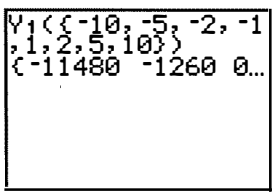


Figure 2

One way to avoid using brace brackets is to enter the numbers into List 1 as shown in Figure 3 (note that the number 10 should be entered even though it does not appear in the figure), and then evaluate the expression as shown in Figure 4. The answers can be obtained by scrolling to the right or by viewing List 2 in the spreadsheet.

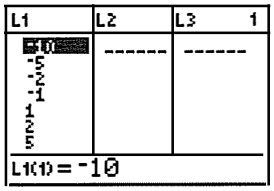


Figure 3

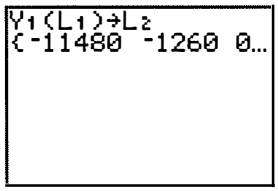


Figure 4

Method 2

An interesting method for evaluating the expressions in (2) is depicted in Figure 5. Note that this method assumes that f(x) has been entered in the y= editor. Students could be asked to explain why these steps create the desired outcome, and to look for other mathematical topics where this strategy could be applied.

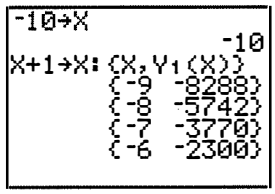


Figure 5

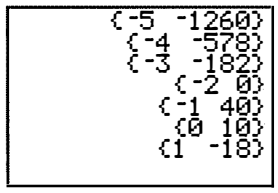


Figure 6

Method 3

The TI-83+ has the capability of deciding if a statement is true or false, and as shown in Figure 7, this feature can be used to make decisions related to the Factor Theorem. The presence of a "1" in List 2 points to a root of the equation. To avoid having to scroll to the right to see the rest of the entries in List 2, students can view the results (see Figure 8) by opening the spreadsheet of the TI-83+ (press the STAT key, followed by the ENTER key). Results that are not shown can be viewed by scrolling down.

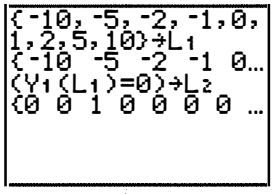


Figure 7

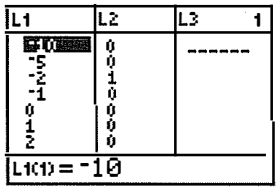


Figure 8

Method 4

The idea of using a conditional statement can be turned into a graph as shown in Figure 9. Note that the

animated graph style has been used (it creates a rather neat effect of jumping up at x = –2). Use the WINDOW settings as shown in Figure 10. It would, of course, be more logical and fitting to make Xmin equal to -10 and Xmax equal to 10. Try this and you will see that the graph misses the action at x = –2. Students could be asked to explain this behaviour and to suggest ways of getting around this. If you are not familiar with the reason, the problem stems from the fact that the screen of the TI-83+ is 95 pixels wide. One could argue that this is not the best way of evaluating the expressions from (2), but the graph creates a nice visual effect of thinking about this problem.

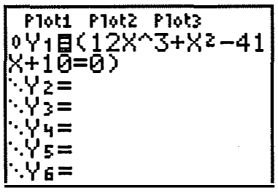


Figure 9

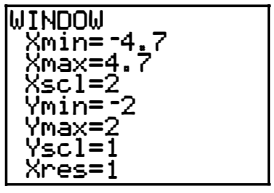


Figure 10

Method 5

Parametric equations can be used to create another way of visualizing this problem. Begin by selecting the settings as shown in Figures 11 through to 14, and then enter the functions as shown in Figure 15. Then, press the GRAPH key and you will obtain the result as shown in Figure 16. Students could be asked to explain this seemingly peculiar result.

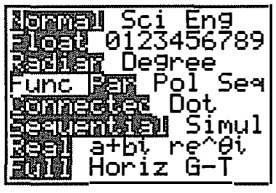


Figure 11

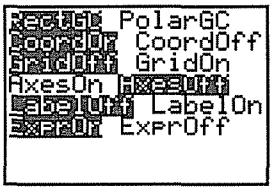


Figure 12

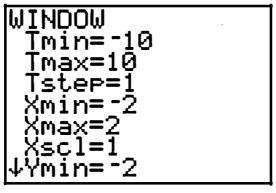


Figure 13

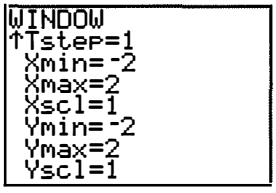


Figure 14

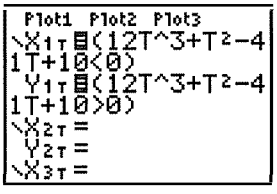


Figure 15

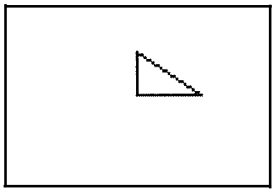


Figure 16

The appearance of a triangle can be explained by studying a table of values for the parametric equations. Use the set-up as shown in Figure 17 and then look at the TABLE (see Figure 18). You will need to scroll down to see all of the results.

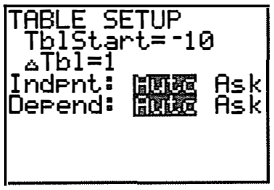


Figure 17

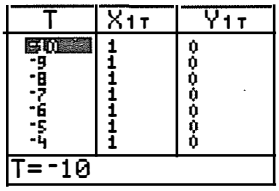


Figure 18

Method 6

Polar graphs can also be used to create yet another unusual display of viewing the results from evaluating the expressions in (2). Begin by selecting the settings as shown in Figures 19 and 20, and then enter the function as shown in Figure 21. Then, press the GRAPH key and you will obtain the result as shown in Figure 22. Students could be asked to explain this seemingly curious result (why the ellipse?).

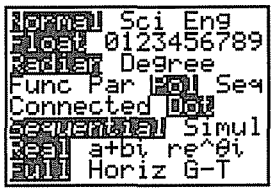


Figure 19

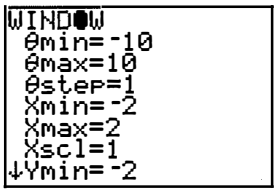


Figure 20

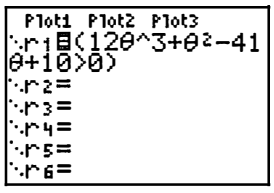


Figure 21

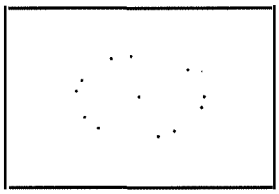


Figure 22

Students could also be asked to predict the shape of the graph that will be displayed when the > symbol is changed to the < symbol. They could also be asked to explore what happens if the value of θstep is changed from 1 to say 0.1.

Method 7

Figure 23 shows an unusual application of random numbers to the Factor Theorem. See Figure 24 for a continuation of these results. Here are a few questions for students to consider: What are the chances that this method will produce a random number, X, between 1 and 10 for which $Y_1(X)$ is equal to 1 on the first step? What are the chances that such a value of X will be obtained within two steps? (three steps? four steps?

etc.) How could this process be modified so that the random numbers are all factors of 10? That is precisely what students have done in the past. They have picked a factor of 10, tried it, and if it did not work, they tried another, and another, etc.

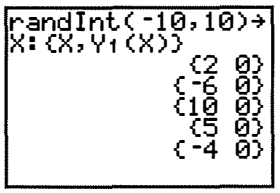


Figure 23

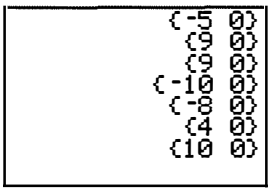


Figure 24

Method 8

Writing a program for the TI-83+ is yet another method for evaluating the expressions in (2). A list of the commands for one such program, called INTROOTS, is given in Figures 25 and 26. This program simply checks every possible integer value of X between -10 and 10. Note that this program assumes that f(x) has been entered in the y= editor. Students could be asked to explain the purpose of every step of the program and then encouraged to create a more efficient version or one that includes text that guides the user along from the beginning to the end.

Program INTROOTS

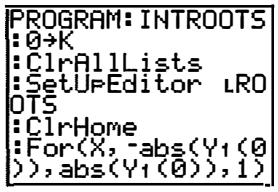


Figure 25

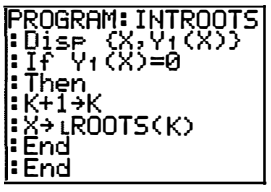


Figure 26

When the program has finished, the results can be viewed by looking at the spreadsheet of the TI-83+ (see Figure 27).

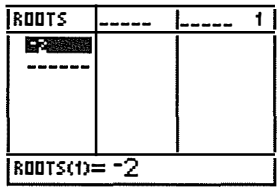


Figure 27

Here is a question that will probably challenge your very best students: Is it possible to find an equation with integer roots for which this program fails to work? The answer is a resounding "yes". Try running the program for the equation $x^3 - 5x^2 + 6x = 0$. This equation has three integer roots, but the program will only detect one of them. Do you see why? Students could be challenged to create a program that avoids this pitfall.

One could argue that this program is not the best way of solving equations, but it is possible that through this exposure, some students will be inspired to create their own programs for other purposes.

Method 9

Figures 28 and 29 show how the notion of finding the maximum value of a function can be used to select the expression in (2) that is equal to 0. Note that the range of the function defined in the y= editor is the set {0, 1). The maximum value of this function is 1 and it occurs when x = – 2. I leave you with the following challenge: Why has the TI-83+ failed to produce the correct answer to our problem? What is the significance of the number 9.999993584?

I have a TI-83 that I no longer use since switching over to a TI-83+. I will award this unit to the teacher or the student who sends me a note that clearly and correctly explains why the TI-83+ does not return the correct answer to our problem. In the case of a tie, I will use the randInt feature of my calculator to choose the winner.

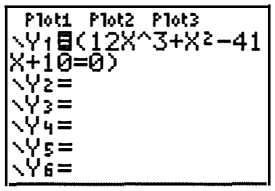


Figure 28

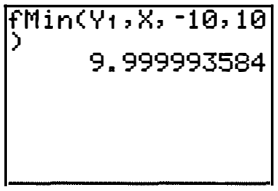


Figure 29

Summary

When presenting these methods to teachers, their initial reaction is often one of worrying that they would never have discovered these methods on their own. This may or may not be true, but what is certain is that these teachers have been able to apply this new knowledge to other problems that involve the use of a TI-83+. In sharing these methods with their students, they have equipped them with the ability to do the same thing; that is, allowing them to be able to choose from a large collection of possible strategies, some of which increase the level of insight as opposed to simply finding the answer to a problem.

Submitted by:
Ron Lancaster

▲

▲ OAME/NCTM March 2001 Report



FRANCES SCHATZ

OAME continues to be a partner affiliate with NCTM with great rewards. Just recently, the OAME executive received word that NCTM has approved a grant of $1500 to assist in the publication of the *Linking Assessment and Instruction in Mathematics: Middle Years* document. This book will be another invaluable resource written by Ontario educators for Ontario's mathematics teachers.

OAME pays affiliation fees to NCTM. NCTM's website, a resource centre for affiliates, can be accessed at www.nctm.org/affiliates/. In addition, an individual membership to NCTM provides you with a journal related to the level of mathematics you are teaching, along with 20% off on NCTM catalogue books and supplies. PLEASE order your catalogue materials through me, your OAME-NCTM representative, as OAME gets a 20% rebate as well. The rebate helps cover some of the affiliation fee.

OAME' 2001 will be in Scarborough, May 10-12. Since this conference is being held at two locations, one hotel for elementary, and another hotel for secondary, you will find the NCTM display for elementary and secondary at the Delta Hotel publishers' display. There will be a shuttle bus between the two conference locations, so please find time in your conference schedule to browse the full display. Directions and information will be near the OAME display in the Howard Johnson display area. Once again, you will find a selection of books that will be of interest to Ontario mathematics teachers.

New Resources

#740 Multicultural Mathematics Materials: 2nd Edition

Canadian non-NCTM member price is about $36.00. Marina Krause's popular first edition, published in 1983, led to this revised edition. Here is a book that will introduce children to cultural diversity by featuring games and activities from around the world. This book contains convenient, reproducible activity pages for classroom distribution in grades 1-10.

#744 Teaching Mathematics to the New Standards: Relearning the Dance

This book is a fascinating account of the author's attempts to change the way she teaches elementary school mathematics. Ruth Heaton is involved in a never ending process of investigation while teaching. Canadian non-NCTM member price is about $30.00.

NCTM Memberships

For up-to-date information on individual, student, senior, or institution memberships, you can check memberships on the website www.nctm.org. I recommend that you complete the application form given in US dollars, then calculate the Canadian funds by US total x 1.25. This caps the exchange rate at 25%. Write a cheque or money order in Canadian funds for the calculated amount. At the top of the form, please indicate that your affiliate is OAME and mail the form with payment directly to NCTM. If you pay the membership by credit card, you will be paying the full exchange rate which is currently a 1.51 factor instead of 1.25. On the website, you will have access to *Student Math Notes* and *Dialogue*. Canadian memberships take 6-8 weeks to process, and the membership starts with the journal one month after you join.

Speaker Request for NCTM Annual 2002 Meeting

Carol Edwards, program chair, is seeking speakers for this annual meeting to be held in Las Vegas, Nevada, Sunday, April 21 to Wednesday, April 24. The major venues will be the Venetian, the Sands Expo Centre, and Harrah's. "Realizing the Vision of School Mathematics" is the theme for this conference and K-12 teachers are requested to conduct workshops. Information about the types of sessions, criteria for selection, answers to questions, and proposal forms are available online at www.nctm.org/meetings or through fax-on-demand (800)220-8483, document #455. Send your proposal assoon as possible. If you need assistance with your proposal,contact Lorenda Wieder .▲

A Pythagorean Misteak

ANDREW JEACLE
HEAD OF MATH AND CO-OP
GLOUCESTER HIGH SCHOOL



Andrew Jeacle is the Head of the Mathematics and Co-op Education departments at Gloucester High School in the east end of Ottawa. He wishes to thank Prof. Jack Weiner and Prof. John J. Hubert at the University of Guelph for showing him how much fun math can be - both to do, and to teach.

An incorrect method which generates a wrong answer is widely recognized as a "mistake". A "misteak", on the other hand, is a seemingly incorrect method which leads to the correct answer. Ever since Professor Jack Weiner introduced them to me back in first year Calculus at the University of Guelph, I have always enjoyed hearing about math misteaks. Two of my favourite misteaks are:

- the upside down quadratic formula

$$\left(x = \frac{2c}{-b \pm \sqrt{b^2 - 4ac}}\right), \text{ and}$$

- the family of radicals where

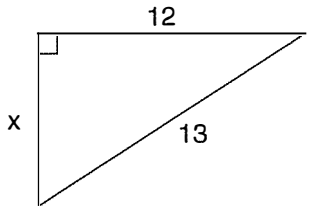
$$2\sqrt{\frac{2}{3}} = \sqrt{2\frac{2}{3}} \text{ or } 4\sqrt{\frac{4}{15}} = \sqrt{4\frac{4}{15}}.$$

Misteaks are most enjoyable when they are stumbled upon by some well meaning, but generally misinformed, student. These are the students who love to apply "rules" in order to solve a problem. Unfortunately, they occasionally memorize the rule incorrectly, and generate some alternate method which, of course, is bound to fail. On the oh-so-rare occasions when these student ramblings actually work, you have a misteak.

As I was marking final exams last June, I noticed that a student had made a misteak that I had never encountered. It is so beautiful in its simplicity that I am certain that the work I am about to present is common knowledge to some people. However, for those who have never encountered this little curiosity, allow me to share it with you.

The Problem

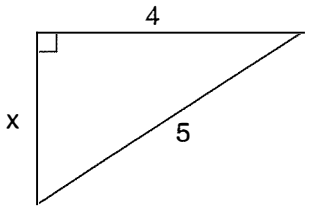
Solve for x:



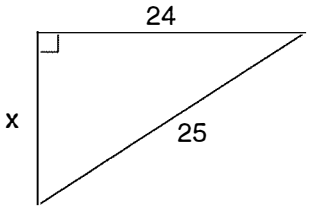
The Solution

$$x = \sqrt{12+13}$$
$$x = \sqrt{25}$$
$$x = 5$$

As I read the first line, I was prepared to dismiss the solution as yet another incorrect hybrid of Pythagoras' most famous work, but as I glanced ahead, I noticed that the answer just happened to be correct. In order to prove to myself that it was indeed a fluke, I tried this unconventional method of solution with the first two Pythagorean triples that came to mind:

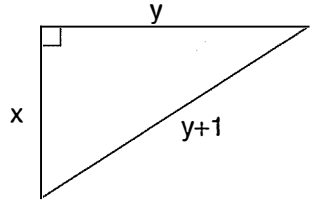


$$x = \sqrt{4+5}$$
$$x = \sqrt{9}$$
$$x = 3$$



$$x = \sqrt{24+25}$$
$$x = \sqrt{49}$$
$$x = 7$$

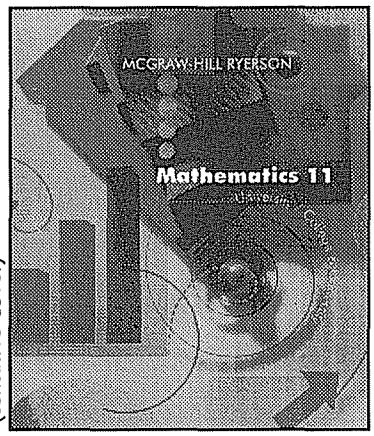
Hmmm! Time to look a little deeper. I began by considering the general case where the hypotenuse measures one unit more than one of the other sides of the right triangle:



Coming Soon!

from McGraw-Hill Ryerson

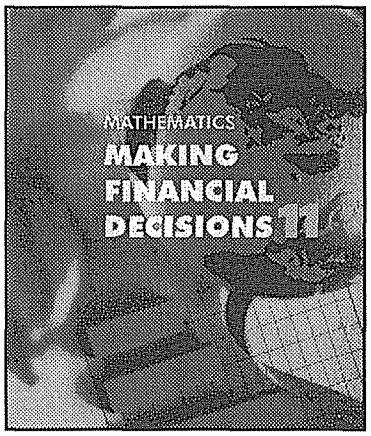
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The Pythagorean Theorem says that

$$x^2 + y^2 = (y+1)^2.$$

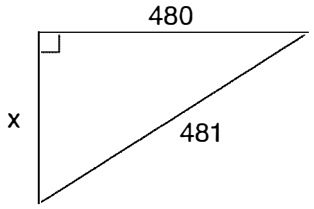
Solving for x:

$$x^2 + y^2 = y^2 + 2y + 1$$
$$x^2 = 2y + 1$$
$$x = \sqrt{2y+1}, \text{ since } x > 0 \text{ and } y > 0$$

Written another way,

$$x = \sqrt{y + (y+1)}$$

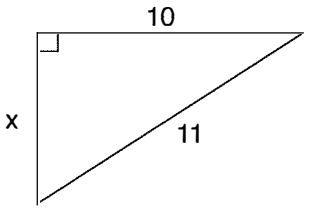
Eureka! So the following solutions are indeed acceptable (not to mention correct!):



$x = \sqrt{480 + 481}$

$x = \sqrt{961}$

$x = 31$



$x = \sqrt{10 + 11}$

$x = \sqrt{21}$

Next, I decided to isolate y in this relation, in order to generate Pythagorean triples more easily:

$$x^2 = 2y + 1$$
$$2y = x^2 - 1$$
$$y = \frac{x^2 - 1}{2}$$

When I input a rational value for x in this relation, I am certain to get a rational value for y. (When working with $x = \sqrt{2y+1}$, I have to be careful what I input for y if I wish to make x rational). So to generate Pythagorean triples of the form (x, y, y+1), I can choose any value for x, say

$$x = 9$$

then solve for y:

$$y = \frac{9^2 - 1}{2}$$
$$y = 40$$

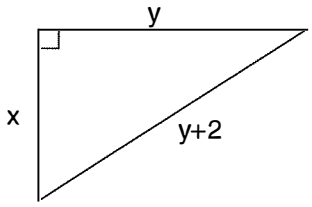
Since y is 40, then (y+1) is 41 and my triple is (9, 40, 41). If I let x = 73, my triple is (73, 2664, 2665). Choosing an even value for x gives non-integer values for y and y+1, but of course the relationship still holds

$$\left(4, \frac{15}{2}, \frac{17}{2}\right).$$

A nice exercise for students would be to prove that

$$x^2 + \left(\frac{x^2-1}{2}\right)^2 = \left(\frac{x^2-1}{2} + 1\right)^2.$$

Other families of right triangles could also be considered. For example:



In this case, x can be shown to equal $2\sqrt{y+1}$. Triples can be generated by choosing values of y that are one less than a perfect square. Working with a perfect square of 64 (selected at random), y = 63, so (y+2) = 65 and $x = 2\sqrt{63+1}$. The triple is (16, 63, 65). Working with 196, I can generate the triple (28, 195, 197). Working with 121 generates (22, 120, 122). Note that an odd perfect square (such as 121) will generate a triple which is a multiple of the type discussed earlier. For example (22, 120, 122) is simply twice of (11, 60, 61).

Easier still, if I start with the lower number (call it x), the triple is $(2x, x^2 - 1, x^2 + 1)$. For example, if I pick x = 38, my triple is (76, 1443, 1445).

As we continue the tour through my mind, my next thought was to summarize the two types of triples I had studied:

Triples of type (x, y, y+1) are generated using

$$\left(x, \frac{x^2-1}{2}, \frac{x^2+1}{2}\right),$$

where it is best to use an odd integer for x.

Triples of type (x, y, y+2) are generated using $(2x, x^2 - 1, x^2 + 1)$, where it is best to use an even integer for x.

Sure enough, a little more work shows that triples of the type (x, y, y+k) are generated using

$$\left(kx, k\left(\frac{x^2-1}{2}\right), k\left(\frac{x^2+1}{2}\right)\right).$$

From here, I went off on a few tangents, discovering a few more properties and unravelling a few more questions. I even did an admittedly less than exhaustive search of Pythagorean related websites, most of which focussed on the $(2mn, m^2 - n^2, m^2 + n^2)$ triple, a much more powerful generator. But while some of the new discoveries were interesting, I found myself getting further and further away from the glorious simplicity of the original misteak. What sticks with me are the simple Pythagorean generators that I developed, and most emphatically, the easy way to solve for x in a right triangle with sides x, y, and y+1. ▲



By a happy accident the paper used for this first covering consisted of sheets of Ostrogradsky's lithographed lectures on the differential and the integral calculus, bought by my father in his youth. These sheets, spotted over with strange incomprehensible formulae, soon attracted my attention. I remember how, in my childhood, I passed whole hours before that mysterious wall, trying to decipher even a single phrase, and to discover the order in which the sheets ought to follow each other. By dint of prolonged and daily scrutiny, the external aspect of many among these formulae was fairly engraved on my memory, and even the text left a deep trace on my brain, although at the moment of reading it was incomprehensible to me.

When, many years later, as a girl of fifteen, I took my first lesson in differential calculus from the famous teacher in mathematics in Petersburg, Alexander Nikolaevitch Strannoliubsky, he was astonished at the quickness with which I grasped and assimilated the conceptions of the terms and derivatives, "just as if I had known them before." I remember that this was precisely the way in which he expressed himself, and in truth the fact was that at the moment when he began to explain to me these conceptions, I immediately and vividly remembered that all this had stood on the pages of Ostrogradsky, so memorable to me, and the conception of space seemed to have been familiar to me for a long time."

— Sonya Kovalevsky

▲ Integrating technologies — hand-held, desk-top and "skull-based" to solve a problem in Geometry

PETER HARRISON



Peter Harrison has been teaching mathematics and computer science for 27 years. He has been a department head, a teacher specially assigned, a mathematics coordinator, and a textbook author. Presently, he is teaching real-living students, and is enjoying his new motor scooter, fly fishing, and single malts.

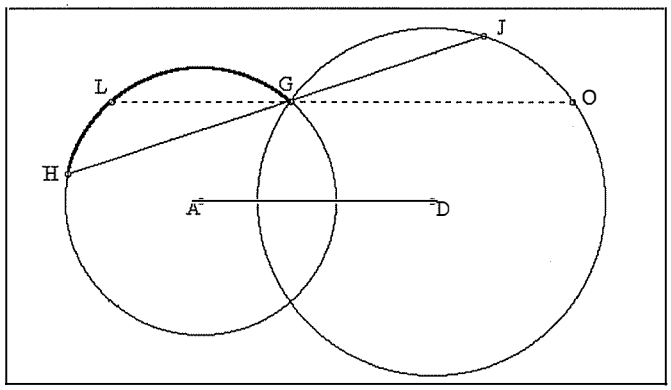
In this article, the author explores an old problem decked-out in new garb and discovers several fascinating features that may easily have been overlooked by a more traditional pencil-and-paper treatment.

I seem to spend a lot of my spare time (!) these days playing around with the *Geometer's Sketchpad* and old geometry problems. Here is one problem that has robbed me of a considerable amount of sleep:

> *Two circles intersect at G. Prove that of all straight lines through G, and terminated by the circumferences, the greatest is that which is parallel to the line joining the centres.*
>
> *(A Modern Geometry for High Schools,* W.J. Lougheed and J.G. Workman, Macmillan, 1940, page 93 - You just have to love the language. Talk about communication!)

It is quite easy to mount this problem using Sketchpad. As you can see, I have constructed two circles, a variable segment HJ, passing through G (one of the two points of intersection between the circles), and the fixed segment LO which is parallel to the segment

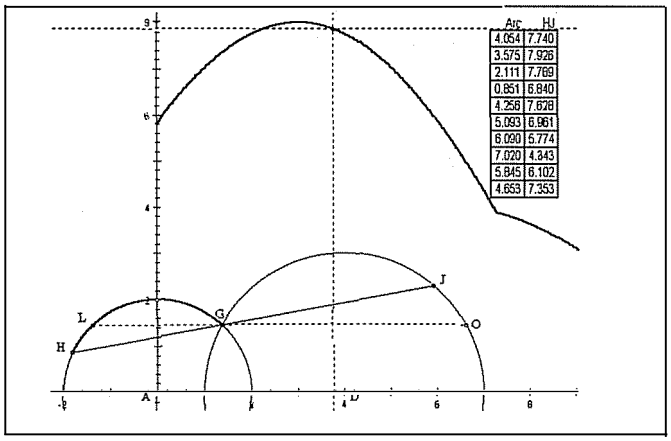
joining the centres A and D. By measuring LO and HJ, Sketchpad gives a pretty convincing report on their relative sizes. HJ always seems to be shorter than LO.



Now, this is all well and fine for honing my Sketchpad skills, but as you might imagine, this is not what has kept me awake. By and large this result is not too exciting and probably does not justify the effort needed to construct a proof. Mind you, this problem is from a student text, and I am frank enough to admit that it took me a while to actually prove it (see note 1).

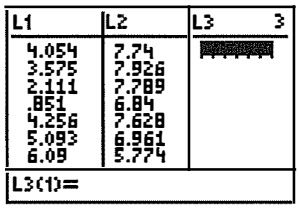
The problem that has been bothering me is the following: (and dear colleagues I would welcome any insights that you might be good enough to offer!)

I decided to approach the problem technologically. I created two circles: one with centre at (0,0) with radius 2, and another with centre at (4,0) with radius 3. I then graphed the length HJ against the arc length GH (measured counterclockwise). As point H moves around its circle, the corresponding length of HJ sketches an interesting path (see note 2). Its two parts are easily explained, with the "corner" neatly corresponding to a particular event in the story of point H.



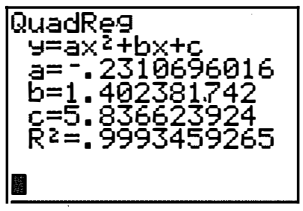
Now, you have to agree that that first hump looks seductively like part of a parabola.

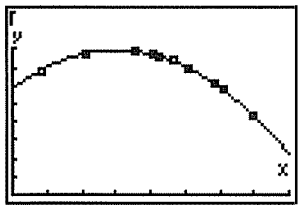
With that in mind, I copied the table into my TI-83 and performed a quadratic regression. Here is what I found:



L1	L2	L3 3
4.054	7.74	
3.575	7.926	
2.111	7.789	
.851	6.84	
4.256	7.628	
5.093	6.961	
6.09	5.774	

L3(1)=





Now, the correlation factor of 0.9993459265 would surely tempt anyone to declare that this is indeed parabolic. And herein lies the cause of my sleepless nights: How can the graph of this function be parabolic?

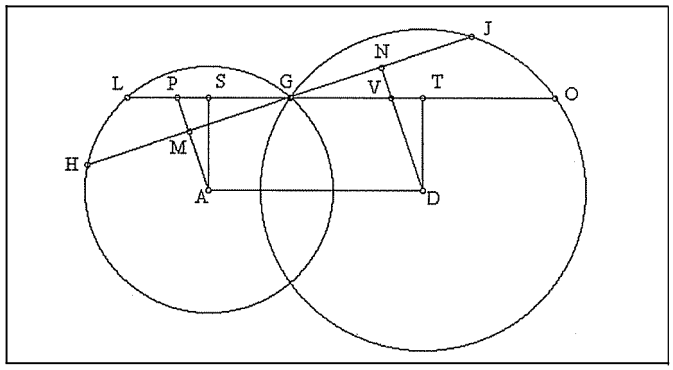
I have managed to grind out the following function:

$$|HJ| = 2\sqrt{2 - 2\cos\left(\frac{x}{2}\right)} + \frac{3}{2}\sqrt{8 + 7\cos\left(\frac{x}{2}\right) + \sqrt{15}\sin\left(\frac{x}{2}\right)},$$

where x is the arc length GH (see note 3).

But here my trigonometry fails me utterly (and please don't hold Paul Sherk, my all-time favourite math teacher, responsible for this!). I am at a complete loss to explain how this function can possibly be quadratic. Can anybody help?

Note 1.



Create M and N as midpoints of HG and GJ, and S and T as midpoints of LG and GO respectively.

Extend AM to P on LO, and let V be the point of intersection of LO and DN.

Notice that PMG and GNV are right angles with PG and GV as hypotenuses. That makes PA and VD parallel, and angles APS and DVT equal. Thus, triangles PSA and VTD are congruent (ASA).

$GN \leq GV$, $MG \leq PS + SG$, and $PS = VT$

$\therefore MG + GN \leq PS + SG + GV$

But M, G, and N are collinear, as are P, S, G, and V. This means that $MG + GN = MN$, and after we rewrite $PS + SG + GV$ as $SG + GV + VT$, we see that $SG + GV + VT = ST$.

$\therefore MN \leq ST$

$\therefore 2MN \leq 2ST$

$\therefore HJ \leq LO$

Note 2.

To create the graph using the *Geometer's Sketchpad,* I did the following:

1. Open a new sketch.
2. From the **Graph** menu, select **Create Axes**.
3. Using the **Graph** menu, select **Plot Points** to plot the points (1, 0), (4, 0), (2, 0).

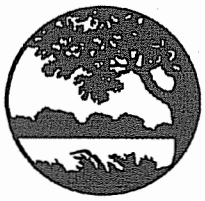
   Unless you move the point (1, 0), one unit on the x-axis should equal one centimetre. You should check that by creating a line segment between points A(0, 0) and B(1, 0) and measuring the length of that segment. Select the segment and pull down the **Measure** menu. Click on **Length**. (If your answer is displayed in inches, go to **Preferences** (in the **Display** menu) and change the settings. You will need this scale to create meaningful data.
4. Create each circle by selecting its centre and a point on the circumference; i.e., (0, 0) and (2, 0), and (4, 0) and (1, 0). Pull down the **Construct** menu and click on **Circle by Center and Point**. You might want to drag the origin a little to the left.
5. Create the point G by selecting both circles and clicking on **Point at Intersection** under the **Construct** menu. Label the upper point G. Make sure that you "de-select" the two points of intersection before you go onto the next step.
6. Select the circle with centre A(0, 0), pull down the **Construct** menu, and click on **Point on Object**. Label this point as H.
7. Create the arc GH. Select the point G, shift-click on the circle, and shift-click on the point H. Pull down the **Construct** menu and select **Arc on Circle**. Note that you select the arc in an anti-clockwise fashion. If you pull down the **Display** menu, you can highlight the arc by clicking on **Thick** to the right of **Line Style**.
8. Select the arc. Pull down the **Measure** menu. Click on **Arc Length**. The length of the arc GH will

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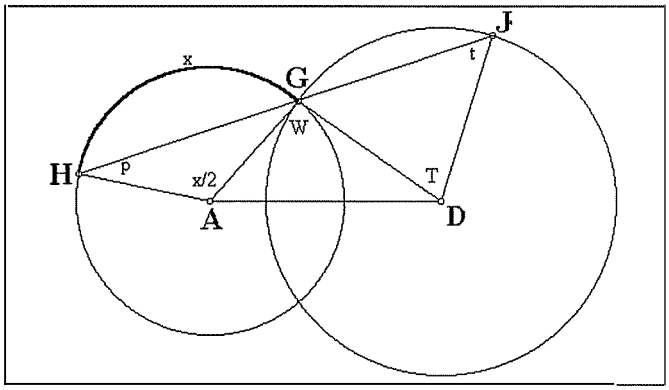
appear in your sketch. If you move point H, this measurement will change.

9. Construct a ray from point H through point G. Select the **Ray** tool by dragging on the **Line** tool in the **Drawing** menu on the left side of the screen. Click on point H and drag the cursor to point G. Release the button. Create the point J by selecting both the ray HG and the circle centered at (4, 0), and then click on **Point at Intersection** in the **Construct** menu. You should then create a segment from H to J and hide the ray. This will look a lot better.
10. Click on the segment HJ and measure its length by pulling down the **Measure** menu and selecting **Length**. The length of HJ will appear in your sketch. If you move point H, both of the measurements – arc length GH and segment length HJ will change. Play with this for a while. There are a couple of rather nice discontinuities to explore.
11. Select the measurement of arc length GH (not the arc!) and pull down the **Graph** menu. Select **Plot Measurement**. Indicate your choice - **On Horizontal (x) Axis**.
12. Select the measurement of segment length HJ (not the segment!) and pull down the **Graph** menu. Select **Plot Measurement**. Indicate your choice - **On Vertical (y) Axis**. The horizontal broken line may not be visible. Move the point H to bring the line into the screen. You will need to move the origin into the lower left corner of the screen.
13. If you move the point H around the circle, you should see the horizontal and vertical lines moving. Their point of intersection will trace out the graph of the length of HJ as a function of the arc length GH. If this is actually working for you, you have done a great job. Congratulations!
14. Now, select both of these new lines and construct the point of intersection. Select the point of intersection and the point H (in that order!), and under the **Construct** menu, click on **Locus**. This will create your curve.
15. To create some data points, select both the arc length GH measurement and the segment HJ measurement (in that order), and under the **Measure** menu, click on **Tabulate**.

    You can change the appearance of the table by selecting the table and clicking on **Flip Direction** in the **Measure** menu.

    Move the point H. You will notice that even though the measurements change, the entries in the table do not. Select the table, and under the **Measure** menu, select **Add Entry**. Repeat these actions several times until you create a reasonably sized data set.

Note 3.



Let the length of the arc GH be x. Therefore, the angle HAG will be $\frac{x}{2}$ (recall HA = 2).

Let the angle DJG (and hence DGJ) be t, angle GDJ be T, and angle AGD be W.

Let the angle AHG (and hence AGH) be p.

We have the following:

$2p + \frac{x}{2} = 180$ ①. $2t + T = 180$ ②. $p + t = 180 - W$ ③.

Rearranging ③ gives us $p + W = 180 - t$ or $2p + 2W = 360 - 2t$ ④.

Rearranging ② gives us $2t = 180 - T$ ⑤. Combining ⑤ with ④, we now get $2p + 2W = 360 + T - 180$ or $2p + 2W - 180 = T$ ⑥. Combining ① and ⑥ yields

$$\left(180 - \frac{x}{2}\right) + 2W - 180 = T \text{ or } 2W - \frac{x}{2} = T.$$

Remember, I constructed the triangle AGD to have side lengths 2, 3, and 4. You can easily show that

$\cos W = \frac{-1}{4}$ [using the cosine law];

$\sin W = \frac{\sqrt{15}}{4}$ [using the identity $\sin^2\theta + \cos^2\theta = 1$];

$\cos 2W = \frac{-7}{8}$; $\sin 2W = \frac{-\sqrt{15}}{8}$ [using the double angle formulae].

Using the cosine law,

$$HG = \sqrt{4+4-2x2x2\cos\left(\frac{x}{2}\right)} = \sqrt{8-8\cos\left(\frac{x}{2}\right)}$$

$$= 2\sqrt{2-2\cos\left(\frac{x}{2}\right)}$$

and $GJ = \sqrt{9+9-2x3x3\cos T}$

$$= \sqrt{18-18\cos\left(2W-\frac{x}{2}\right)}$$

since $T = 2W - \frac{x}{2}$. This last expression simplifies to

$$3\sqrt{2 - 2\cos\left(2W - \frac{x}{2}\right)}.$$

Expanding $\cos\left(2W - \frac{x}{2}\right)$, we get:

$$\cos\left(2W - \frac{x}{2}\right) = \cos(2W)\cos\left(\frac{x}{2}\right) + \sin(2W)\sin\left(\frac{x}{2}\right)$$

$$\cos 2W = \frac{-7}{8}, \quad \sin 2W = \frac{-\sqrt{15}}{8}$$

$$\therefore \cos\left(2W - \frac{x}{2}\right) = \frac{-7}{8}\cos\left(\frac{x}{2}\right) - \frac{\sqrt{15}}{8}\sin\left(\frac{x}{2}\right)$$

Therefore,

$$GJ = 3\sqrt{2 - 2\cos\left(2W - \frac{x}{2}\right)}$$

$$= 3\sqrt{2 - 2\left(\frac{-7}{8}\cos\left(\frac{x}{2}\right) - \frac{\sqrt{15}}{8}\sin\left(\frac{x}{2}\right)\right)}.$$

This will simplify to $\frac{3}{2}\sqrt{8 + 7\cos\left(\frac{x}{2}\right) + \sqrt{15}\sin\left(\frac{x}{2}\right)}$.

Finally,

$$HJ = 2\sqrt{2 - 2\cos\left(\frac{x}{2}\right)} + \frac{3}{2}\sqrt{8 + 7\cos\left(\frac{x}{2}\right) + \sqrt{15}\sin\left(\frac{x}{2}\right)}.$$

Comments from Professors Hosh Pesotan and Jack Weiner, Department of Mathematics and Statistics, University of Guelph

The expression for HJ can be wonderfully simplified. The author of the article has determined that

$$HJ = 2\sqrt{2 - 2\cos\left(\frac{x}{2}\right)} + \frac{3}{2}\sqrt{8 + 7\cos\left(\frac{x}{2}\right) + \sqrt{15}\sin\left(\frac{x}{2}\right)} \quad (1)$$

where the first summand is HG and the second is GJ.

Let us use the identities

$$\sin\left(\frac{x}{2}\right) = 2\sin\left(\frac{x}{4}\right)\cos\left(\frac{x}{4}\right)$$

$$\cos\left(\frac{x}{2}\right) = 2\cos^2\left(\frac{x}{4}\right) - 1$$

as well as old faithful $\sin^2\theta + \cos^2\theta = 1$.

First, working inside the square root portion of HG,

$$2 - 2\cos\left(\frac{x}{2}\right) = 2 - \left(4\cos^2\left(\frac{x}{4}\right) - 2\right) = 4 - 4\cos^2\left(\frac{x}{4}\right) = 4\sin^2\left(\frac{x}{4}\right)$$

and so the HG portion in (1) becomes

$$HG = 2\left(4\sin^2\left(\frac{x}{4}\right)\right)^{\frac{1}{2}} = 4\sin\left(\frac{x}{4}\right) \quad (2)$$

Note that since $\left(\frac{x}{2}\right)$ is a sector angle, $0 \le \frac{x}{2} \le 2\pi$, so $0 \le \frac{x}{4} \le \pi$. Therefore, when we take the square root of $\sin^2\left(\frac{x}{4}\right)$, we necessarily obtain a positive answer; no absolute value is necessary.

Second, working inside the square root portion of GJ, again back in (1),

$$8 + 7\cos\left(\frac{x}{2}\right) + \sqrt{15}\sin\left(\frac{x}{2}\right)$$

$$= 8 + 7\left(2\cos^2\left(\frac{x}{4}\right) - 1\right) + \sqrt{15}\left(2\cos\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right)\right)$$

$$= 8 + 14\cos^2\left(\frac{x}{4}\right) - 7 + 2\sqrt{15}\cos\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right)$$

(this next step is so cute!)

$$= 1 - \cos^2\left(\frac{x}{4}\right) + 15\cos^2\left(\frac{x}{4}\right) + 2\sqrt{15}\cos\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right)$$

$$= \sin^2\left(\frac{x}{4}\right) + 2\sqrt{15}\cos\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right) + 15\cos^2\left(\frac{x}{4}\right)$$

$$= \left(\sin\left(\frac{x}{4}\right) + \sqrt{15}\cos\left(\frac{x}{4}\right)\right)^2$$

Now an important note: When $\frac{x}{2} = \pi$, HG actually passes through A and is a diameter of the smaller circle. Why? If $\frac{x}{2} = \pi$, then from (2), $HG = 4\sin\left(\frac{\pi}{2}\right) = 4$. But in ΔHAG, $HG \le HA + AG = 4$. (Remember, HA and AG are radii.) We must have HG = HA + AG, and so HAG is a straight line.

For our concerns here, $\frac{x}{2} \le \pi$. (Otherwise, we can no longer consider HAG a triangle with $\angle A = \frac{x}{2}$). Therefore, $0 \le \frac{x}{4} \le \frac{\pi}{2}$. This guarantees $\sin\left(\frac{x}{4}\right) + \sqrt{15}\cos\left(\frac{x}{4}\right) \ge 0$ and so the square root portion of GJ way back in (1) is

$$GJ = \frac{3}{2}\sqrt{\left(\sin\left(\frac{x}{4}\right) + \sqrt{15}\cos\left(\frac{x}{4}\right)\right)^2}, \text{ that is,}$$

$$GJ = \frac{3}{2}\sqrt{\sin\left(\frac{x}{4}\right) + \sqrt{15}\cos\left(\frac{x}{4}\right)}. \quad (3)$$

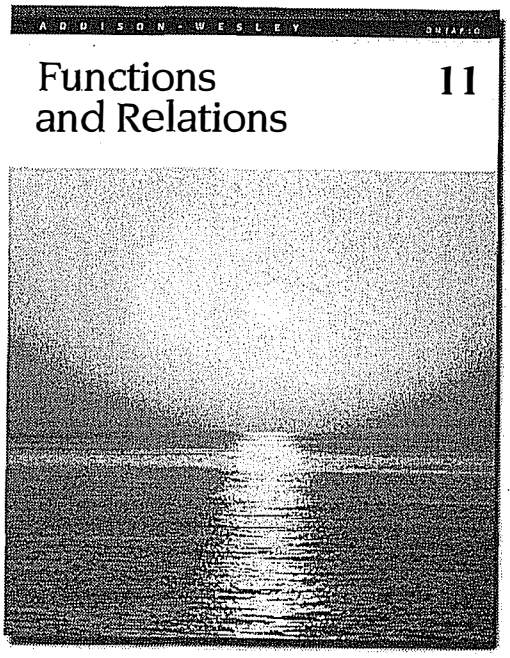
Putting it all together in (1), and collecting like terms, we have

$$HJ = \frac{11\sin\left(\frac{x}{4}\right)}{2} + \frac{3\sqrt{15}\cos\left(\frac{x}{4}\right)}{2}.$$

▲

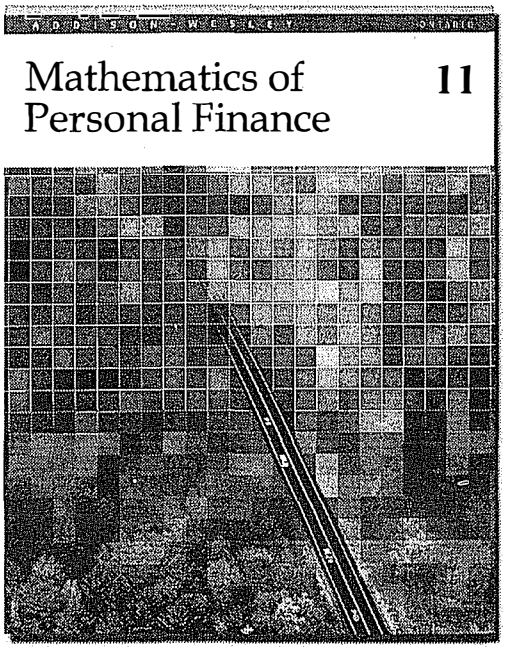
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▲ Parents: A Rich Resource

DR. GEORGE GADANIDIS

DR. ANN KAJANDER



George Gadanidis teaches at the Faculty of Education, University of Western Ontario. He has organized several workshops for parents. He is the author of MathMania *Magazine.*



Ann Kajander teaches for Lakehead Public Schools, at Lakehead University, and in the Kindermath *project. She conducts workshops for in-service elementary teachers and parents.*

Why Involve Parents?

One of the most rewarding experiences we have had as teachers has been our involvement in organizing and leading mathematics workshops for parents. Our overall goal in these workshops has been to involve parents in meaningful and enjoyable mathematical experiences, and to offer parents ideas for providing similar experiences for their children.

Parents have come to our workshops eager to learn how to help their children develop mathematically. Parents have been willing to try new approaches, and they have been enthusiastic about sharing their own ideas and experiences of what has worked well with their own children.

Students benefit when parents display a positive attitude towards mathematics and regularly do mathematics activities with their children: "They are more likely to feel confident in their abilities; to enjoy and learn more from the mathematics that they experience at school; and to develop a sense of richness, usefulness, and pervasiveness of mathematics" (Kliman, 1999, p.140).

What Do Parents Want?

"I was never any good at math myself..."

This is a common opening line given by many parents in conversation about their child's mathematical progress. It is not surprising that parents who lack confidence with mathematics express concerns about their own abilities to help their children. Yet, even parents who enjoy mathematics may express concerns about their lack of understanding of the mathematics their children are learning in school.

Some of the new topics in the curriculum may be unfamiliar to parents. As well, the use of the open-ended approach in problem-solving situations may leave them uncertain how best to help their child.

Parents may also feel uncomfortable about approaching the classroom teacher for help, as they may feel awkward or embarrassed about their own lack of understanding. As conversations between parents and the teacher tend to be connected to the progress of a particular child, parents may feel uncomfortable engaging in curriculum discussions with the teacher. Some parents have been very relieved to find that evening sessions were available for their benefit.

Following is a list of some broad areas that we have found many parents interested in discussing:

New Content

Parents' own previous experiences in math may be problematic (Merttens, 1995, p.3). Many parents have simply never learned topics found in the new curriculum; such as probability, data management, or spatial reasoning. The result is that they feel unable to help their children in these areas.

New Pedagogy

The current approach to learning is often confusing to those first experiencing it. Many of us were brought up with the notion that there has to be a single right answer in mathematics. Open-ended tasks done for homework can leave parents uncertain how to help their child find the "right" approach.

New Assessment and Reporting

There has been an increased emphasis with assessment in Ontario. We now have province-wide mathematics testing for grades 3, 6, and 9 students, with a shift from traditional testing of facts and skills, to testing for understanding and problem solving. Although the focus on higher level cognitive skills is a welcomed

change, there remains some confusion about assessment practices. For example, the Ontario Achievement Chart, which is used to assess student performance, reflects a complex interplay of expectations, assessment categories, and achievement levels.

With a shift towards levels of performance rather that simple averages, parents may be left wondering how a level 3 correlates to 75%. It can also be frustrating for the teacher who is still grappling with new assessment strategies to have to defend them to parents.

New Terminology

Recent changes in terminology, although meant to clarify concepts, may be confusing to parents. For example, parents report feeling flustered by the change to "regrouping" from "borrowing", and to "simplified" from "reduced" fractions. When their children tell them that these are the "wrong" terms, parents may feel inadequate and/or confused.

Recent research shows that a supportive home environment is important to student success (Ochoa, 2000). It is therefore worth-our-while to help parents become more knowledgeable about the changes.

Suggested Responses to Parents

Whether through workshops or individual consultation, parents need to be reassured that their confusion is justified. Here are some suggestions we have found useful in responding to parents:

- Minimize the importance of terminology and encourage flexibility. Encourage parents to listen to their child's use of, or explanation of, a new term and continue along with its use without feeling inadequate.
- Although attending a workshop might be a good way to brush up on new content, simply listening to their child allows parents to get a sense of new areas of learning.
- Encourage parents to have fun with open-ended tasks brought home. A brief explanatory note from the teacher might be helpful. Parents need to understand that the problem-solving process is important, and that various approaches are often possible to a valid solution.
- Parents naturally want their children to move beyond the facts and skills of mathematics. They want their children to understand what they are doing and to be able to apply these skills and understanding in problem-solving situations. Share with parents strategies that you use to assess higher level cognitive skills. Also, share with them samples of students' work. An excellent resource for assessment ideas and samples of students' work at levels 1 to 4 is the *Linking Assessment and Instruction in Mathematics* series published by OAME.

One parent recently asked, "How did my child get level 3s when she always gets every multiplication question right on the tests?" In order to validate the usefulness of the levels, we need to ensure that some higher level thinking activities are provided where students have the opportunity to demonstrate level 4 work.

How Can We Involve Parents?

While workshops for parents do require a certain amount of effort, the benefits can make them worthwhile. Here are two ways to provide parents with experiences and ideas for helping their children develop mathematically:

- **Math workshops for parents** are an excellent venue for involving parents with rich mathematical experiences that help them develop an appreciation of mathematics. We try to let the experiences in the workshops speak for themselves. When we ask parents to compare and contrast these experiences with the ones they had in school, they end up sharing many of the points we want to highlight. For example, parents often make positive comments about the problem solving activities in which they participate, about the importance of trying different approaches, and about the benefits of communicating and justifying their thinking. It is much more effective if parents, rather than the presenters, make these points. The workshops also help model how parents can interact with their children during math activities.
- **Family Math evenings for students and parents** are an excellent way to get parents and students doing mathematics together. There are many ways to organize such evenings. Here are some features of successful family math evenings that we have experienced:
  1. Start with a brief introduction to the evening. Perhaps the school principal or a parent from the school community council could welcome everyone and outline the structure and rationale for the evening.
  2. Parents visit their children's classroom where they

do math activities prepared by the teacher and students in the preceding weeks. As the students are familiar with the activities, they take on a teaching role with their parents.

3. There are other math centres around the school that children and parents can participate. These can include a computer centre, a tessellation centre, a math puzzle centre, etc. Also, there is a challenge with a prize, such as guessing the number of jellybeans in a jar.
4. There is a handout for parents to take home at the end of the evening. The handout includes general information about the math program, tips to help children develop mathematically, and a collection of activities that parents and children can do together at home (or in the car, or in the grocery store, etc.).

In addition to such evenings, parents can be supported in other ways. For example, some parents appreciate a brief note from the teacher which accompanies an open-ended activity to be done at home. While teachers have expressed frustration over the low success rate of "newsletter" style handouts sent "home in the backpack", the shorter explanatory note accompanying a particular assignment seems to be more popular. Children are more likely to be careful with instructions attached to a particular assignment, and parents may be motivated to read the explanations for guidance about the goals and purposes of the task and how much intervention is appropriate. A series of such correspondence might be very helpful for parents over the course of a year.

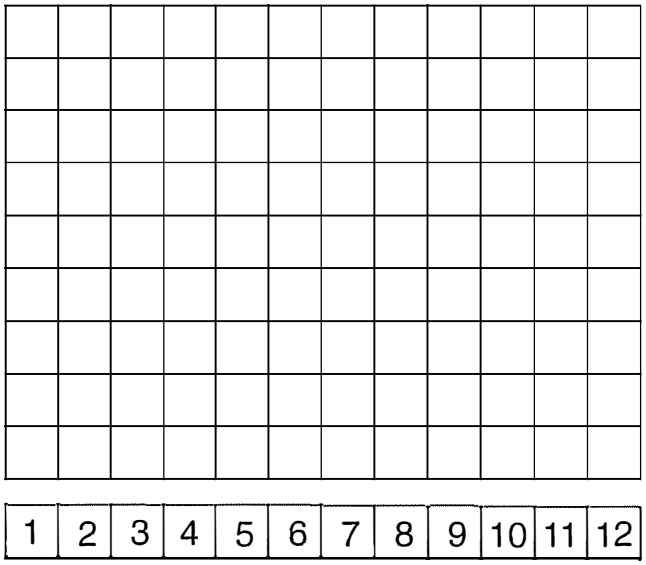
Parents appreciate suggestions of games and activities that they can do at home. Many parents express lack of time as a major issue, so activities that do not take time away from other things, such as estimating totals while standing in a store check-out line can be very popular. A math discussion that takes place as part of an activity that is already going on, like shopping, cooking, or driving in a car, can be a great way to do math in a meaningful context.

Math Activities for Parents

We use a wide variety of activities in our workshops with parents. We try to create a balance between activities that challenge and motivate parents, with activities that are appropriate for their children. Following are four examples:

Place Your Bets

Give parents a grid like the one below and 12 pennies. Tell them that you will roll two dice and find the sum. If a player has a penny in the column represented by the sum, they can remove it (at most, one penny is moved per roll). You will roll the dice twelve times. Tell them that the winner of the game will have the fewest pennies left on their grid. Give them a couple of minutes to decide where on the grid they want to place their pennies. They can place more than one penny in each column. It is best to have parents work in pairs or in small groups so that they can exchange ideas.



1	2	3	4	5	6	7	8	9	10	11	12

As a second part to the activity, ask parents to roll a pair of dice 36 times and colour in the squares depending on the sums they get. By creating a bar graph, parents develop a visual image of the outcome distribution.

The activity is a great way to introduce probability concepts, and to help parents realize the importance of hands-on, informal experiences before mathematical concepts are introduced more formally to their children.

A Modelling Career

Pose the following problem to parents:

Suppose you want to develop a way to measure a person's left-handed and right-handedness. How could you do it? Is it possible to come up with a numeric value?

Share with parents one idea; for example, you could throw 10 tennis balls and ask a person to catch 5 with their left hand and 5 with their right hand. The number of balls one catches with each hand could represent a measure of left-handedness and right-handedness. Is

that a good model? What other methods could we use? Organize parents into small groups and ask them to solve the problem. This problem involves parents in the process of modelling, which is at the heart of the new mathematics curriculum.

Food and Car Games

Car games can be a creative way to get at "what did you do in math today?" Examples might be, "At this speed, how long will it take to drive to x which is y km away?" or numeracy questions like, "If we bought 24 chocolate bars and 60 lollipops, how many would each of 12 friends (then 10 friends) get in a lootbag?" If the parent chooses something "too hard" or "too easy", the child will let the parent know. And it is more fun than flashcards.

Food games are great during snack time. Try having parents cut up an apple during a workshop, and discuss what fraction "a quarter of a half" looks like.

Take a Walk

Send parents on a walk around the library in search of mathematics. Pick one of the math strands as a theme and ask questions like the following:

Number Sense and Numeration: Can you find the fraction one half? What is the smallest decimal you can find? What the biggest number you can see?

Geometry and Spatial Sense: What shapes do you see? Can you find a square? A trapezoid? A cylinder?

Measurement: How many basketballs would fit in this room? What is the largest area you see? What fraction of the volume of the room do we make up?

Patterning and Algebra: What patterns do you see? Can you see a pattern no one else can see?

Data Management and Probability: What never happens in this room? What always happens? What has a 25% chance of occurring?

Once parents start seeing mathematics around them, they never stop — and neither do the children.

Looking Ahead

Parents are a rich resource that we should be taking greater advantage of in our efforts to improve mathematics education. As we work with parents, and as we make mathematics more meaningful and more exciting, children will benefit.

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OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration;

- demonstrations of excellence and creativity in mathematics education.
- specific contributions to mathematics education in the school, region, province or nation.
- demonstrates leadership in mathematics education in the school, region, province or nation.
- support of and contribution to the activities of OAME/AOEM.
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

OAME/AOEM Life Membership Award

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and accumulated 10 or more years of membership in OAME/AOEM.

Kenneth D. Fryer Award

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Mona MacGregor Award

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Morley MacGregor Award

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

Don Attridge Award

In recognition of the contributions made by Don Attridge, a founding member, former president and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

AWARD NOMINATION COVER PAGE

Send to: Shirley Dalrymple

Intended Award (please check one)

❑ Life Membership ❑ Mona MacGregor ❑ Morley MacGregor ❑ Kenneth D. Fryer ❑ Don Attridge

Nominee Information:

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

Home Phone ( ) _____

Nominator Information:

Name of Nominator

School, Organization or Business Address

Telephone Number ( ) _____

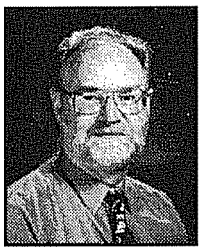
Fax Number ( ) _____

Home Address _____

Home Phone ( ) _____

▲ Technology Corner: Graphing Trigonometric Functions on the TI-83+

FRED FERNEYHOUGH
HEAD OF MATHEMATICS, CENTRAL PEEL SS

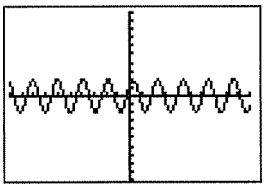


Fred Ferneyhough is co-chair of the Peel Math Heads' Association, and Technology Consultant for McGraw-Hill Ryerson's MathPower 9 *and* MathPower 10 *textbooks. He is an avid user of technology in the classroom.*

Welcome to the Technology Corner. *In this column, I hope to share some of the successes that I have had in using computer technology in the mathematics classroom, such as TI-graphic calculators, The* Geometer's Sketchpad, *and spreadsheets. Questions, comments, and suggestions should be sent to me at dybydx@home.com.*

With the advent of the use of graphic calculators in the new curriculum, we now see expectations that specifically address manipulation of the graphing window. A good application of that arises in MAT 4A, where we spend a fair amount of time asking students to graph functions of sine and cosine functions. My experience has always been with mixed results, until recently. Part of the problem was that I asked students to graph the base function, for example, f(x) = sin(x), over the domain $x \in [-2\pi, 2\pi]$, and then asking them to graph a transformation of that function over the **same** domain.

Doing that on a graphic calculator would be an inappropriate use of technology. Students would simply enter the equation and the domain to graph the function, showing no understanding of the nature of the function or the effect of transformations. The graph of a transformed trigonometric function in the standard viewing window with domain $x \in [-10,10]$ and $y \in [-10,10]$ is shown:



Several years ago, I had a superb 4A class who suggested graphing a number of cycles of the transformed function rather than specifying a domain for the graph. To set that up, I emphasized the concepts of amplitude, period, and phase shift, relating them to the basic transformations that we did in MAT 3A, and repeated at the beginning of MAT 4A.

We generalized that, if we are going to show two cycles of the function, the first cycle will begin at x = (phase shift − period) and will end at x = phase shift. Likewise, the second cycle will begin at x = phase shift and end at x = (phase shift + period).

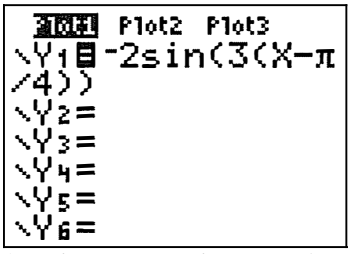
As an example, try the function $y = -2\sin\left(3\left(x - \frac{\pi}{4}\right)\right)$.

Before we started graphing, we determined that the amplitude is |-2| = 2, the period is $\frac{2\pi}{3}$, and the phase shift is $\frac{\pi}{4}$ units to the right. Then, the endpoints of the two cycles were determined and written on the board:

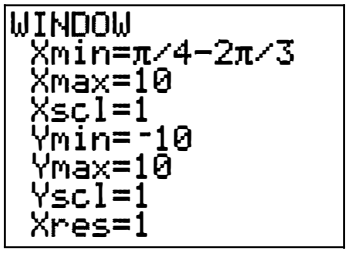
First cycle begins at $x = \frac{\pi}{4} - \frac{2\pi}{3}$, ends at $x = \frac{\pi}{4}$.

Second cycle begins at $x = \frac{\pi}{4}$, ends at $x = \frac{\pi}{4} + \frac{2\pi}{3}$.

Now, we take all of this information to the calculator. First, press the y= button at the top left corner of the calculator and enter the equation:



Next, fix the domain and range by pressing the WINDOW button. You can enter an expression into the WINDOW screen:

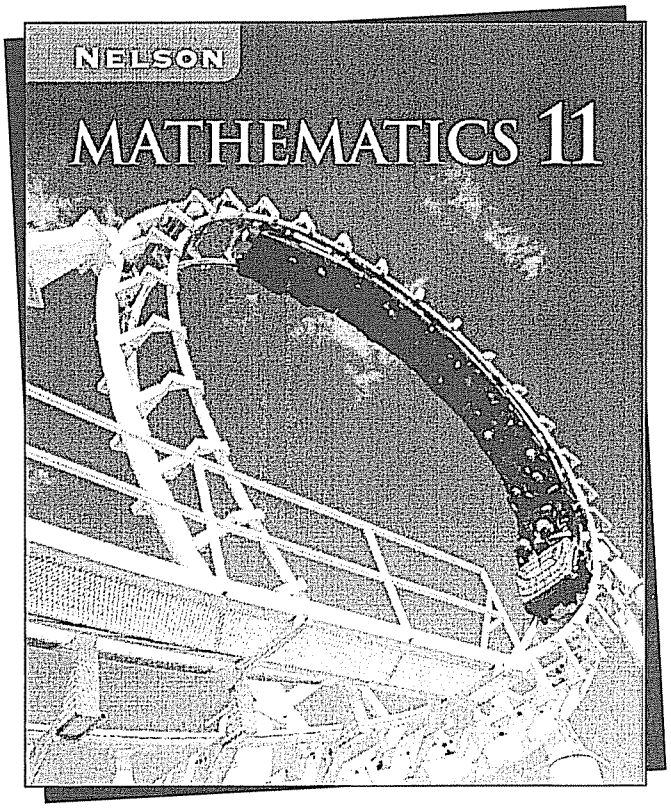


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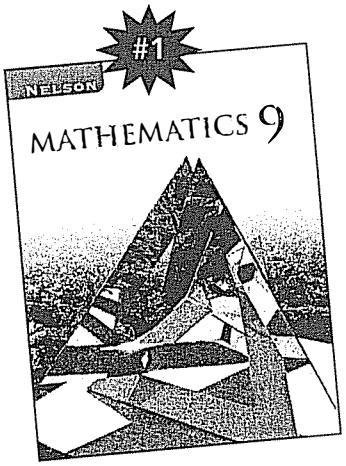
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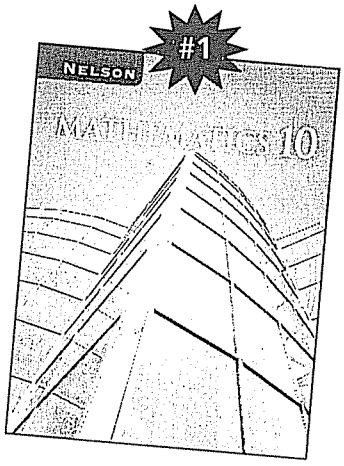
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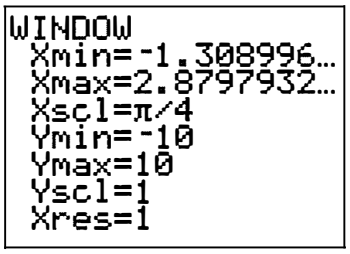


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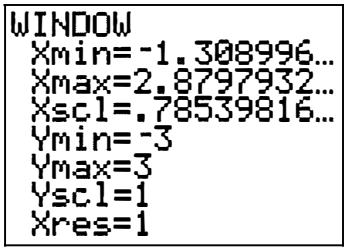
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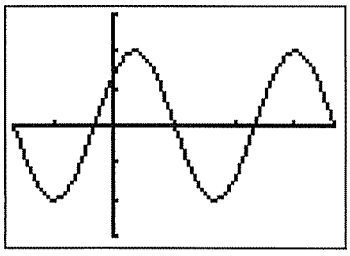
After you hit ENTER, this line will change to a decimal. In the same way, enter the expression for Xmax as the right endpoint of the second cycle. Once again, the value will be converted to decimal form as you hit the ENTER key. For the x-scl, we chose $\frac{\pi}{4}$, although some students made a good case for using $\frac{\pi}{12}$.



Finally, correct the range. My students suggested using a range on the calculator that would leave a gap of one unit between the top and bottom of the screen and the graph itself. This means setting the Ymin and Ymax values as shown:



At this point, we can call up the graph. If everything has been done correctly, we should see exactly two cycles of the function:



Having the students fix the domain and range on the calculator to show a fixed number of cycles of the wave forces students to think about the transformations involved in such graphs. ▲

▲ PHOTO MATH – A NEW WAY OF PICTURING MATH

RON LANCASTER



Ron Lancaster is a middle and high school mathematics teacher, currently on staff at the Bishop Strachan School as a mathematics consultant. Ron also works as an Independent Consultant throughout North America and Asia, and is an Instructor for Texas Instruments.

"A picture is worth a thousand words" is an expression that is often used to convey the importance of using images to make a point. The purpose of this column is to go beyond this notion and promote the idea that in fact, "A picture is worth a thousand numbers." Readers can expect to find photos with math in every Photo Math *column, and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario mathematics curriculum, as well as the new NCTM document,* Principles and Standards for School Mathematics. *It is hoped that these questions will inspire students to be more creative and playful, and that in the end, students will develop an ability to see the world from a different perspective.*

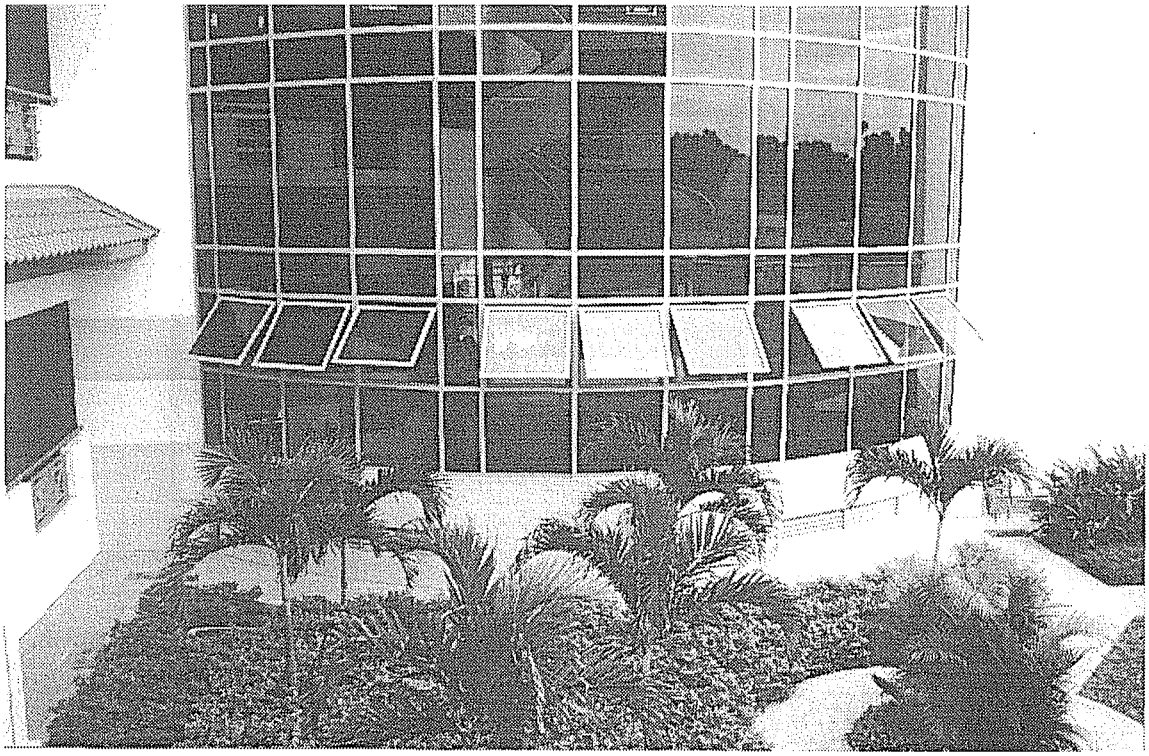
Teachers and students are encouraged to pose mathematical questions related to their own encounters, and to submit these problems along with photographs to the Editor for possible publication in future columns. The Editor of this column does not presently have a cable modem, and in order to avoid long periods of downloading items, submissions to this column should be sent via Canada Post to the address given above.

Contest... Prizes... It's your turn to create the questions!

One of the stated goals of the *Photo Math* column has been to promote the idea of "seeing mathematics everywhere" through the use of photos accompanied by mathematical questions. This issue is quite different from previous ones in that *no questions* have been included with the photographs. It is *your turn* to create the questions! Teachers and their students are encouraged to compose and answer questions related to the photos, and then to forward them to the Editor for possible publication in a future column. Prizes will be awarded for best submissions!

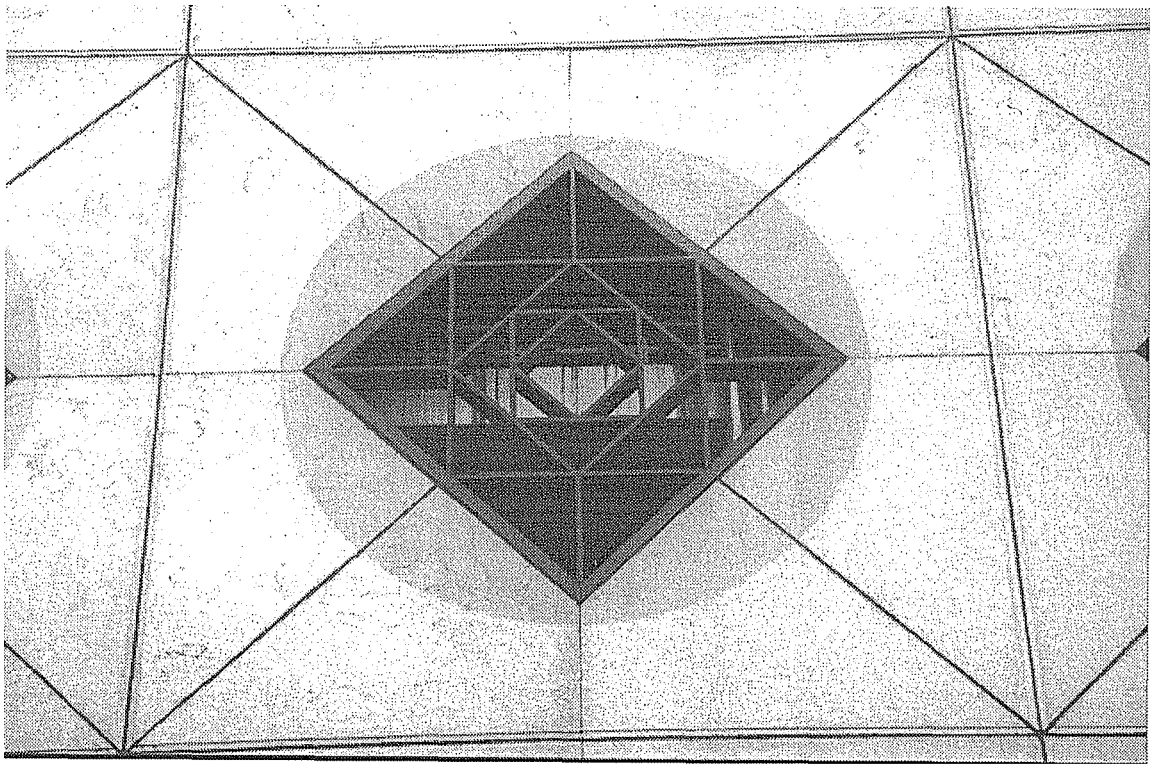
The five photographs that follow were taken by the Editor during a recent trip to Singapore.

Photograph #1
Main building of the Grassroots Club, Singapore



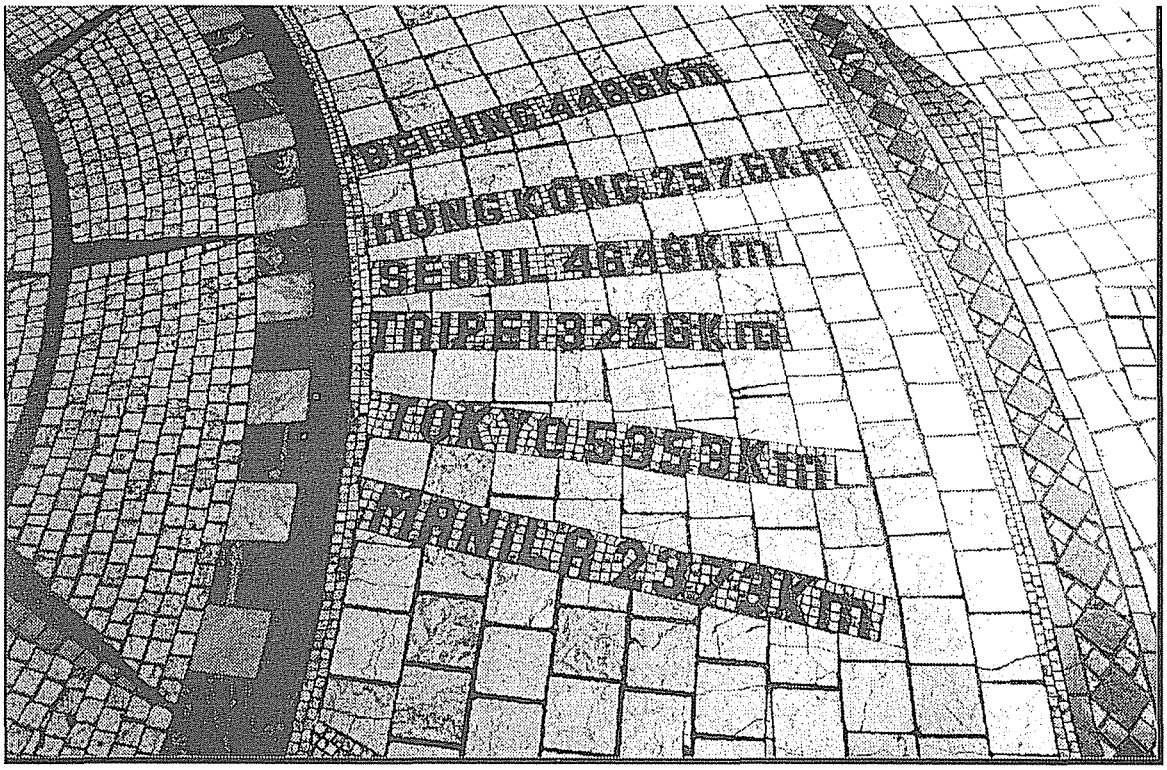
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Photograph #2
Design on commercial building in Singapore



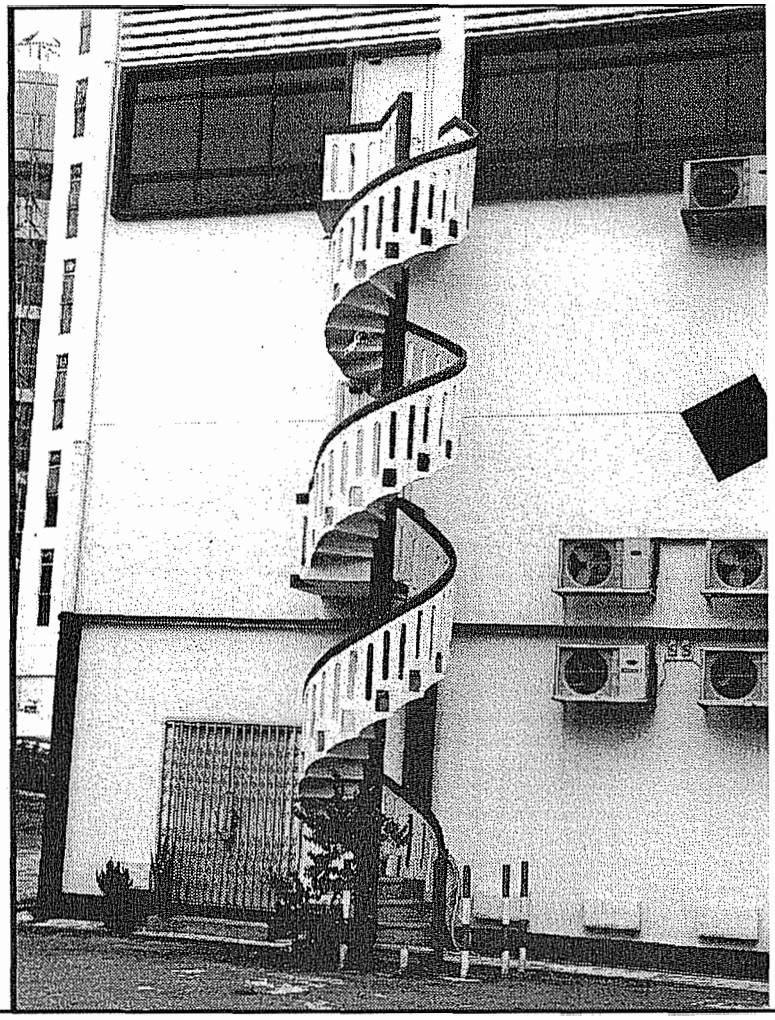
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Photograph #3
Sidewalk mosaic of distances between Singapore and other cities in Asia



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Photograph #4
Back entrance to Singapore Red Cross Building



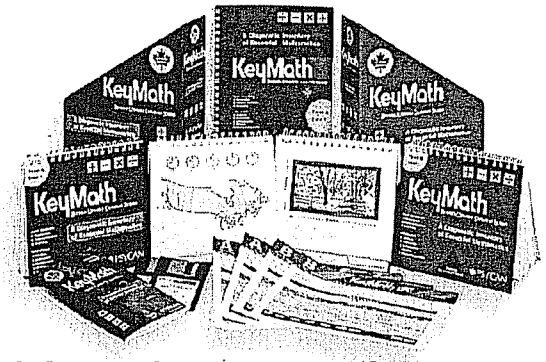
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Photograph #5
The Ascott Hotel, Singapore



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▲ OMO 2001: A Northern Odyssey

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MARILYN HURRELL
OMO CO-CHAIR

The Ontario Mathematics Olympics (OMO) provincial finals will be held from June 8 to 11, 2001 in Thunder Bay.

"Where?" you may ask. "Does Thunder Bay have polar bears and igloos?" Well, the snow and ice have generally melted by the end of May, and our temperatures can actually rival the warm spots of the world. In mid-June, you can expect temperatures from 20 to 25°C.

"But how can we get there?" Well, you can drive up to Sault Ste. Marie, and then have a glorious drive along the north shore of Lake Superior. It is about a 16 hour drive from Toronto, but the scenery is worth it. Then, of course, you will have to drive back ... or take a bus ... or the OMO 2001 Organizing Committee has put together packages, involving air travel from Toronto on Air Canada at a guaranteed low rate of $250, but you must be booked by February 28. West Jet has good fares from Ottawa and Hamilton, but again, you must book early as seats are limited.

We are hoping for at least one team from every OAME chapter, for a total of about 130 grades 7 and 8 students. With every team, we plan on having two coaches and some parents, but we would be pleased with whoever can make the trip.

Our travel package/plans will see teams arrive in Thunder Bay on the afternoon of Thursday, June 8. They will be picked up at the airport and transported to our host site, Lakehead University. After registering and settling into residence (a single room for each person!), there will be a pizza party where teams meet each other and familiarize themselves with the campus. Lakehead University has a unique setting, built around the shores of Lake Tamblyn and surrounded by native forests. Lights out at 11 p.m.!

Friday morning, after breakfast in the residence cafeteria, the group will board buses, bound for a "Northern Adventure". No more said just yet. But bring warm clothes, sturdy shoes or hiking boots, a hat, and some bug spray. Oh, and don't forget your bathing suit! Boxed lunches will be provided. After a BBQ supper around Lake Tamblyn, practice sessions will be held to familiarize students with the graphic calculator and computer software that will be used on Saturday. Snack time, then lights out at 10 p.m.

On Saturday, June 10, the OMO will take place! Sessions will be held from 9 a.m. to 12 noon. After a lunch break, sessions will resume and end at 3:30 p.m. Students, coaches, and parents will be offered a walking tour of the Lakehead University campus and the adjoining Confederation College. About 4:30 p.m., buses will depart for Old Fort William, a wonderful replica of a North West Trading Company post. There, visitors will tour the buildings, the canoe-making shed, watch as natives prepare their traditional foods, the farm, and watch traditional dancing. The awards banquet is set to begin at 6:30 p.m. The Hospitality Committee is hard at work on entertainment for this event. It will likely be a "late" night with awards, sponsor recognition, and dancing!

After a final brunch on Sunday, June 11, visitors will be transported back to the airport for departures between 10:30 a.m. and 1 p.m., depending on the airline used. We hope that they take lasting memories of the good times they had in Thunder Bay and at OMO 2001.

OMO 2001 is co-chaired by Marilyn Hurrell and Wayne Fletcher, both OAME directors. The enthusiastic committee of about 20 organizers, include Bill Otto and Emilia Veltri, OAME directors, and Joe Hall, former OAME director. Most of the committee members are also members of OAME. There is a good mix of elementary and secondary teachers.

Local and provincial sponsorships continue to be sought. So far, Esso, a 3 year sponsor of this event, has once again donated $5000. Bowater, a local paper company, has donated $2000. Lakehead University is a true partner, offering rock bottom prices on food and accommodations along with and free meeting rooms. Texas Instruments, the Thunder Bay telephone company, and the City of Thunder Bay are also sponsors. Many of the OAME chapters which have said they will participate are also fundraising for their teams.

We are having a good time planning this event; we know that all who attend will also have a good time. Be there or B^2! ▲

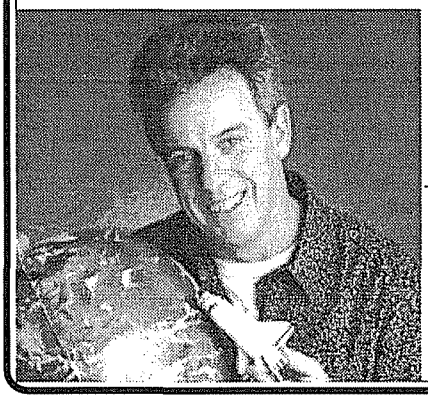


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Thursday, May 10th, 2001 3:30 p.m.

Bjarni Tryggvason

The Canadian Astronaut Program was established in 1983, when the United States invited Canada to fly an astronaut on their Space Shuttle. Bjarni was one of the six Canadian astronauts selected from more than 4000 applicants.

On August 7, 1997 Bjarni Tryggvason, as a Canadian Payload Specialist, participated in his first flight, Mission STS-85 aboard Space Shuttle Discovery.

Come hear this dynamic speaker on 'Space Day',
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