Vol. 39 #4, June 2001

Ontario Mathematics Gazette

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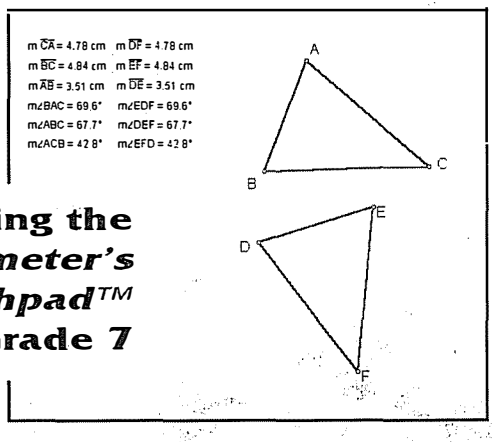
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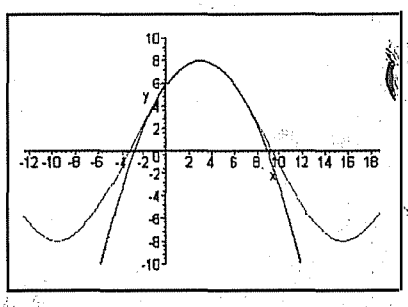
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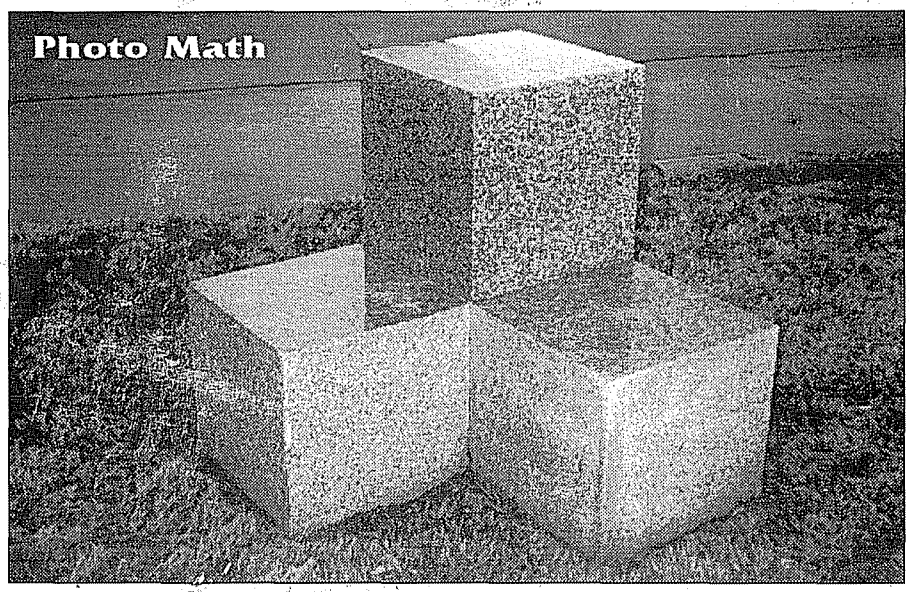
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A response to Peter Harrison's query





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Refereed articles for the *Gazette* should be submitted to **Louis Lim** at the address on the left. Deadlines are May 15 for the September issue, September 15 for the December issue, December 15 for the March issue, and March 15 for the June issue. Since articles are blind-reviewed, please send your full mailing address and e-mail address on a page **separate** from the manuscript. Submit **four** copies to the editor, and keep one copy to guard against loss. Computer files of text, and original photos and graphics, where possible, will be required later if your article is accepted for publication.

Articles must be original. Your article should be word-processed, doubled-spaced on one side with wide margins, not exceed 10 pages of text, and prepared according to the *Publication Manual of the American Psychological Association.* Diagrams and figures should be drawn by computer, if possible, or neatly drawn in black ink in camera-ready form. The editor reserves the right to edit articles for publication.

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▲ Editorial



LOUIS LIM, EDITOR

Books, Glorious Books!

"Do you know," the Devil confided, "not even the best mathematicians on other planets – all far ahead of yours – have solved it [Fermat's Last Theorem]? Why, there's a chap on Saturn – he looks something like a mushroom on stilts – who solves partial differential equations mentally; and even he's given up." [Arthur Porges, "The Devil and Simon Flagg" as cited in *Fermat's Enigma* by Simon Singh, p. 35]

As we approach the end of yet another challenging school year (new curriculum, extra half-course, new methods to assess students, negotiation for a new collective agreement, etc.), summer is a time to reflect on our school year. We survived, but is that enough? I think that the reason teachers are frustrated is that we are perfectionists – we want to do the best possible job educating our students, but due to the many changes in such a short period of time, we are struggling, frustrated, tired, and feel unappreciated. We want nothing but the best for our students.

As you get ready for a most deserving summer break, you might want to find some quality time to enjoy a book about mathematics. Imagine sitting at the beach reading about Hailstone Numbers (hope that's not the weather while you're at the beach!), or sitting on your patio reading *How To Lie With Statistics*, or sitting on a park bench with a cool breeze reading about how female mathematicians such as Sophia Germain and Sonya Kovalevsky fought hard to be recognized as mathematicians, due to their gender, while your own kids play frisbee with your canine family companion!

I have put together *"Lim's Top 10 Books About Mathematics"*. I love books – while I am pursuing my Master of Education degree at Queen's during the summer, you may find me during the evenings roaming the mathematics section at *Indigo* and *Chapters* bookstores. After teaching a full-day during the school year, you may find me at *Chapters* at the Quinte Mall in Belleville. While visiting my parents in Toronto during the holidays, you may find me at the *World's Biggest Bookstore* and *Indigo*. During the evening, you may find

me on the Internet surfing for books. I have several bookcases full of books about mathematics – I spend hundreds of dollars a year, and many have been added to my professional library by my super friend and mentor, Ron Lancaster. Don't fret – most of the books on my bookshelf are very readable, even for those without an undergraduate mathematics degree. I love reading books which are fascinating and comprehensible without using much brain power – a great way to relax after a hard day teaching students. I encourage you to visit your favourite bookstore and get a hold of the following *"Lim's Top 10 Books About Mathematics"*:

10. *A Mathematician Reads the Newspaper* by John Allen Paulos (Anchor Books)
9. *200% of Nothing* by A. K. Dewdney (John Wiley & Sons)
8. *Mathematics: The Science of Patterns* by Keith Devlin (Scientific American Library)
7. *What Are the Odds? Chance in Everyday Life* by Mike Orkin (W. H. Freeman and Company)
6. *Life by the Numbers* by Keith Devlin (John Wiley & Sons)
5. *e: The Story of a Number* by Eli Manor (Princeton University Press)
4. *The Joy of* π by David Blatner (Penguin Books)
3. *The Magic of Mathematics: Discovering the Spell of Mathematics* by Theoni Pappas (Wide World Publishing)
2. *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem* by Simon Singh (Penguin Books)
1. *Wonders of Numbers: Adventures in Mathematics, Mind, and Meaning* by Clifford Pickover (Oxford University Press)

Pickover's book is my favourite. It is 396 pages, contains 125 chapters (of about 1 to 2 pages each), with an extensive reference list for the reader to pursue topics in greater depth. I love the book since it is easy to read, one can start anywhere in the book, and I learned a wealth of mathematics. For instance, I learned about Hailstone numbers, Vampire numbers, Mozart numbers, Emordnilap numbers, apocalyptic numbers, along with special numbers such as 73 939 133. I enjoyed their lists of the 10 most influential mathematicians alive, the 10 most influential mathematicians who ever lived, 8 of the most influential female mathematicians, the 10 most important unsolved problems, 15 of the most famous transcendental numbers, and the 10 mathematical formulas that have changed the face of the world. Simply amazing!

To give you a taste of Pickover's book, I have summarized chapter 46, "Who Is the Number King?":

Hungarian mathematician, Paul Erdös (1903-1996), of no-fixed address, travelled around the world with a half-empty suitcase, arriving at local mathematicians' homes saying, "my brain is open." His purpose in life was "to do mathematics: to prove and conjecture." Mathematics was his life, and he supported students of mathematics by donating money from lectures or prizes he won, or rewarded those who were able to solve problems he posed. Arguably, Erdös is the second most prominent mathematician of all time (after Leonhard Euler) as he published in excess of 1500 papers.

From the chapter, I also learned that my Erdös number is infinity (but hope one day to lower it!). Erdös himself has a number of 0, and to have an Erdös number of 1, one needs to have published a paper with him. There are more than 4500 mathematicians with an Erdös number of 2, publishing with someone who has published with Paul Erdös. To give you a sense of Erdös' contributions to mathematics, Pickover notes that all recipients of the Fields' medal (mathematics equivalent to the Nobel Prize) have an Erdös number of 5 or less.

What an amazing human being to share with students – a person who devoted his life to posing and solving mathematical problems! We all need role models in our lives — why not an extraordinary legend like Dr. Paul Erdös?

Thank You ...

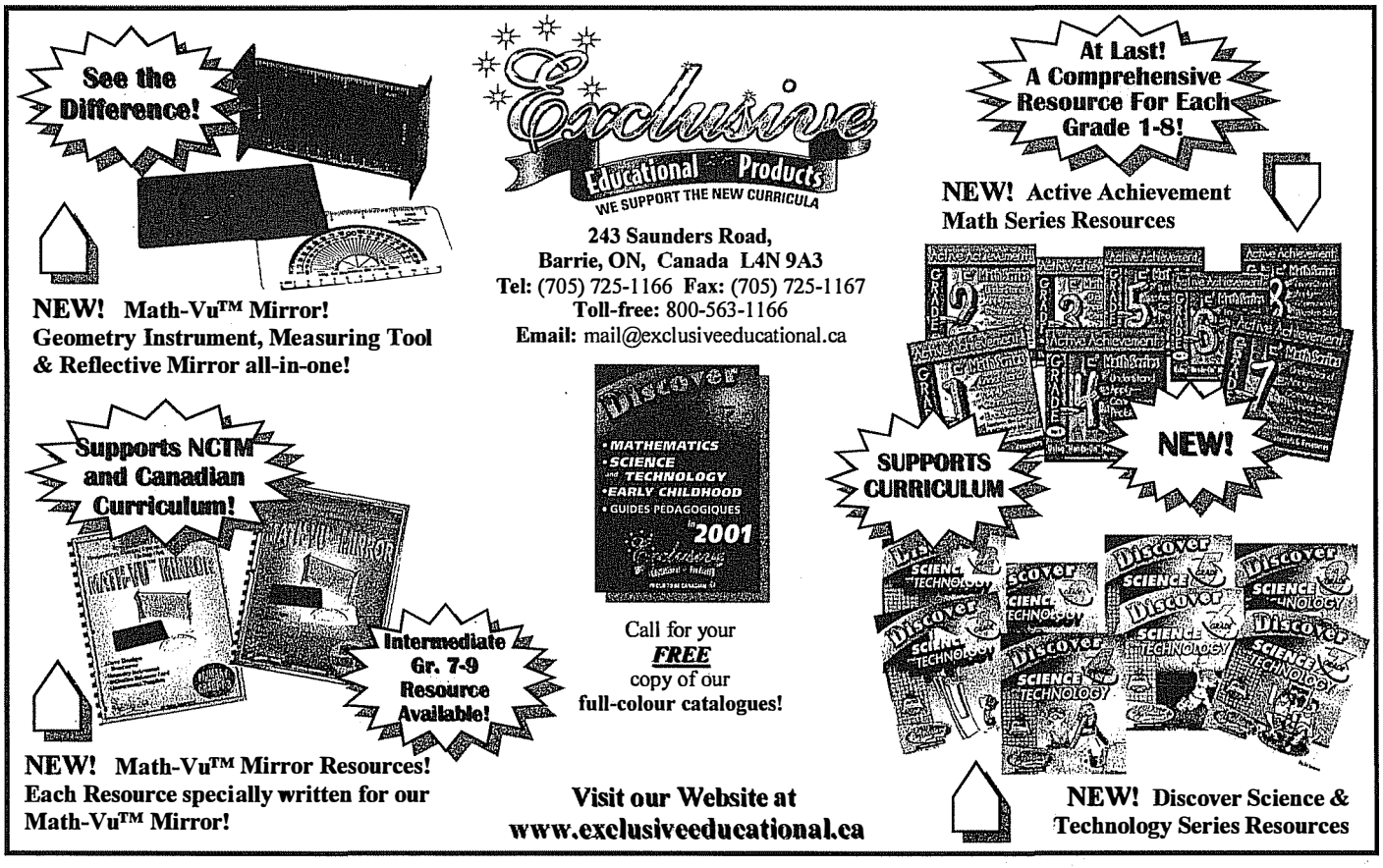
Assembling each issue of the *Gazette* is a collaborative effort. I am very fortunate to work with some truly amazing professionals. Firstly, I would like to acknowledge the contributions of my associate editors, **John Egsgard** and **Todd Romiens**, who edit each issue before it gets sent to the printers; I am amazed at their ability to find edits that I have not even considered. Also, a big thank you to my columnists, **Ron Lancaster**, **Marilyn Hurrell**, **Fred Ferneyhough**, and **Christine Suurtamm** who always produce strong submissions in the tight time-lines I provide them. The *Gazette* looks this good due to the expertise of our graphic designer, **Penny Clemens**. I enjoy her professionalism and patience. And let's not forget *Abacus* editor **Frances Schatz** who cheers me on with each issue, and **Dean Murray** who does an excellent job convincing advertisers

to place ads in our journal. And then there are the reviewers, a group of very dedicated educators, who take their role very seriously, in deciding which articles would be appropriate for our journal:

Anthony Azzopardi	George Gadanidis	Barry Onslow
Ed Barbeau	Pat Grew	Pat Rogers
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John Dallan	Peter Komlos	Eric Wood
Chris Dearling	Ron Lancaster	Kathy Kubota-Zarivnij
John Egsgard	Murray Major	
Gary Flewelling	Eric Muller	

And a BIG thank you to **Jack Weiner**, former editor of the *Gazette* and chair of the OAME Publications Committee. Sadly, Jack resigned as an elected OAME director several months before the end of his three year term. His workload and responsibilities have considerably increased at the University of Guelph. Thank you, Jack, for all your support and faith in me as editor of the *Gazette.* I know that you will still be only an e-mail away. We all appreciate your contributions to advance mathematics education in Ontario. You are truly one of a kind!

Enjoy the summer. See you in September with the unveiling of volume #40 of the *Gazette*! ▲



▲ Outgoing President's Message



MARY HOWE

Another year has flown by! Where did the time go? It seems that only yesterday we were celebrating the beginning of a new millenium! Despite the speed with which the year flew by, various wonderful things happened in the world of mathematics.

Many talented mathematics educators willingly and enthusiastically shared their time and expertise to help OAME provide professional development opportunities during the year. Although space dictates that I can't mention everyone, I would like to acknowledge several people and their contributions during the past school year:

In January, OAME's newest document, *Linking Assessment and Instruction in Mathematics: Middle Years*, was published. Many thanks to Barry Onslow, Ron Sauer, Sharon McPhail, and the many writing and field testing teams for their commitment and effort on this project. They have prepared a high calibre resource of activities to help teachers incorporate balanced assessment, evaluation, and teaching strategies in their classrooms. It is a "must have" for middle grade teachers. Thanks for a great job!

Many thanks to the OAME 2001 Conference Planning Committee for organizing an excellent collection of academic and social activities at the May conference in Scarborough. From the volunteers who worked on the organizational aspects of the conference, to the speakers who presented the sessions, everyone has made a difference in the lives of Ontario's teachers and students. In these days of rapid change, we welcome opportunities to meet with colleagues to discuss and learn innovative ideas in mathematics education. Thank you for providing such an excellent conference!

The OAME Leadership Conference in February was another outstanding opportunity for professional growth. Thanks to presenters Judy Crompton, Myrna Ingalls, Chris & Liisa Suurtamm, Barry Onslow, Lorraine Schroetter, Ruth Dawson, and Mike Davis for the great sharing and growth that happened during the three-day workgroups.

Several teams of grades seven and eight mathematics students will be participating in the Ontario Mathematics Olympics (OMO) finals in Thunder Bay in June, after having won their regional playdowns. A terrific job has already been done by Marilyn Hurrell, Wayne Fletcher, and their committee in planning an exciting weekend of social activities at the old fort together with math activities that will challenge the students' creativity and problem solving abilities.

Congratulations to all of our 2001 OAME Award winners who were presented with their awards at the Annual Conference. OAME is always pleased and proud to recognize excellence in mathematics teaching. The winners will be featured in the next edition of the *Gazette*.

I would also like to thank those people from the OAME Board, too numerous to be listed here, who served on committees at the Ministry, EQAO, Fields Forum, OTF, and OCC, articulating OAME's vision of mathematics. Your work is greatly appreciated!

After seven years of outstanding service as Executive Directors, Dave and Bonnie Alexander have decided to embark on a new chapter in their lives; full-time retirement. It is difficult to say "good bye" to two folks who have become such good friends and who have worked so tirelessly on behalf of Ontario's mathematics community. OAME will miss their unfailing enthusiasm, support, and organization. We wish them good health, fine wine, and lots of sunshine for travelling and boating.

Dave and Sue Hessey will serve as the new Executive Directors of OAME. Dave, a retired elementary school principal, and Sue, a Learning Coordinator with the Thames Valley School Board, reside in London, Ontario. They have many years of experience in mathematics education and are eager to begin working on behalf of mathematics teachers in Ontario. Please make them feel welcome when you see them!

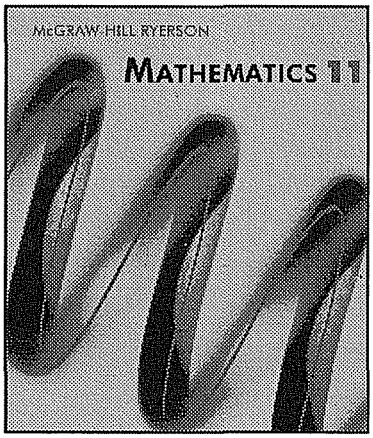
My sincerest thanks to the Executive, the OAME Board, and to all the members who provided support and help to me throughout my term as president. You made my job enjoyable. Thank you for your support! To those members who are not returning to the Board next year - Peter Saarimaki, David Zimmer, Carol Lemmon, Emilia Veltri, Wayne Fletcher, Jack Weiner, Ed Barbeau, Geoff Roulet - you will be missed. You are a VERY talented group of math educators! Thank you for your dedication to improving mathematics education over the years. Please keep in touch!

I would like to wish Shirley Dalrymple and her new OAME executive the very best in the next year. They are an enthusiastic and knowledgeable group of dedicated professionals. Look for great things to happen! ▲

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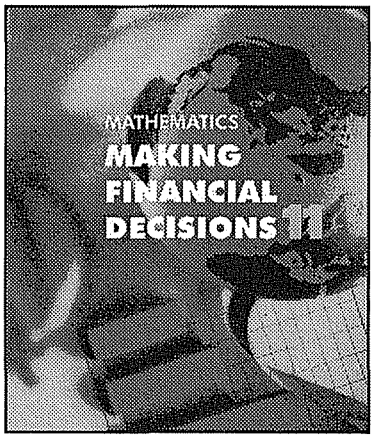
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▲ Incoming President's Message



SHIRLEY DALRYMPLE

"Today's mathematics curriculum must prepare students for their tomorrows. It must equip them with essential mathematics knowledge and skills; with skills of reasoning, problem solving, and communication; and most importantly, with the ability and the incentive to continue learning on their own." The Ontario Curriculum: Grades 11 and 12, *page 3.*

As illustrated in the curriculum guidelines, the new curriculum has a strong problem solving focus. In order for us to help our students meet the expectations, we have had to examine the way we teach and motivate students; it has challenged us to think about how students learn. The achievement chart has forced us to examine not only how we assess and evaluate students, but what is important to assess about students' learning.

Mathematics teachers around the province have been working through the implementation process, from grades 1 to 10. Elementary teachers have been implementing changes in all the grades at once. Secondary teachers have, for the last two years, been working on grade 9 implementation and have spent this past year working on grade 10. We are all becoming more comfortable talking about rich learning problems, rubrics, and assessment tasks. Both teachers and students have had some trials and some successes with a variety of performance activities, applications, communication of ideas, and many have had the opportunity to explore with technology. In particular, secondary students have been performing experiments with the CBR and doing investigations using *Geometer's Sketchpad*. We have lived through the EQAO testing: grade 3, grade 6, and grade 9. We have done a lot!

The French Scientist, Marie Curie, once said: "One never notices what has been done; one can only see what remains to be done." This quote reminds me of where we are now. Even though we have put in a lot of hours and understand that implementation is a process, we seem to always be talking and worrying about all the work there remains to be done. In elementary schools, teachers are faced with the challenge of new content, finding resources and support to assist their programs and their students, reporting, getting ready for EQAO testing, and the need to learn more about assessment. The job immediately facing secondary teachers is preparation for four new courses in grade 11, and looming in the background is the new grade 12 curriculum. We need resources and in-service, but mostly we need time because the task is enormous.

OAME promotes excellence in mathematics education throughout the province, and is committed to supporting its members across all panels. Please visit the OAME website, www.oame.on.ca, for up-to-date information on many of the projects in the works. Teachers from grades JK – 5 can find problems that are being developed and expanded for use throughout the grades. The grades 6 – 8 Committee is working on implementation with the *Middle Years – Linking Assessment and Instruction in Mathematics* document; watch for upcoming articles in the *Gazette*. The grades 9 – 10 Committee has created course outlines organized around the profiles with reference to the textbooks. They have completed Grades 9 and 10 Academic – Public Profile, Grade 10 Academic – Catholic Profile; other outlines such as the Grade 9 Academic – Catholic Profile, Grade 10 Applied – Public Profile, and Grade 10 Applied – Catholic Profile are under development and will be posted on the website when they are ready. This committee is also collecting summative assessment tasks for both grades 9 and 10 for teachers to access. The grades 11 – 12 Committee is working with OMCA to identify the "big ideas" in each of the new Grade 11 courses in order to create summative assessment tasks that will address them. The OAME Board is working to meet your needs for in-service and resources, especially around assessment.

The other big issue that we have to contend with is time. We don't have enough of it. If we did, working on new curriculum could even be fun! I wish I knew a way to help you get more time, but all I can do is share a suggestion that might help you. Network with other teachers in your department and in several other departments. Keep in touch by e-mail. Share assessment activities, projects, journals, ideas, and successes and frustrations. You will get useful activities and ideas, learn from different approaches, and feel like part of a team. Most of all, you will save time, which is at a premium. One last thought, take time for yourself and your family as your health and well-being are crucial; you are a vital component of quality mathematics education in Ontario. ▲

▲ LETTERS TO THE EDITOR

Dear Editor:

In addition to Canadian newspapers, I use the *New York Times* as one of my main sources for American and International news. The articles are lengthy and thought-provoking, and the paper seems to work from the premise that people who buy the paper are willing to take the time to delve into issues. I particularly like the Editorials, Letters to the Editor, and the Op-Ed page. Every Tuesday, they publish an entire section devoted to science, and many of the articles involve mathematics. Most teachers in Southern Ontario should be able to find a store where they can purchase a same-day copy of the paper for about $1.50 on weekdays and $6.50 on Sundays. The Sunday paper is huge and worth every penny.

The Letters to the Editor page is often decorated with small pieces of art work. These small art forms are used to draw readers to the Letters or to enlarge on the topic being discussed. On January 28, 2001, the *Times* ran a series of letters that were sent in by readers responding to an Op-Ed article entitled, "We Waste Our Children's Time" by Leon Botstein (January 25, 2001). The Letters were clustered under the title, "Try Thinking of Teachers as Managers," and appeared with art work by Isabel Klett that showed a teacher, a clock, and the quadratic formula written on a nearby blackboard. I was surprised that the formula was incorrect, although my sense is that the error was deliberate. It is not unusual for the art work to be satirical, playful, or controversial, so this error may have well been planned.

I sent the *New York Times* the following Letter to the Editor. Not surprisingly, it was rejected by the *Times*, but my hope is that mathematics teachers will not reject the challenge that I proposed in the letter. It is a wonderful exercise to explore the conditions under which the incorrect formula actually supplies correct answers.

I sent my Letter to Rick Soma, a teacher at the Horace Mann School in New York City, and asked him to work on my challenge with his students. His students are currently tackling it, but Rick did tell me that he immediately found a value of a which the incorrect formula produces correct results for any and all values of b and c. Before readers plunge into this problem, they may want to see if they can determine this value of a without any algebraic analysis. Rick suggested that readers then try to study the problem for specific values of one of the variables. For example, one could let b = 0 and then see what information can be obtained about a and c. I pointed out that if this approach is used then both sides of the equation could be thought of as a function of two variables. 3-D graphs could be obtained using MAPLE, Mathematica, or a TI-92, and solutions to my challenge could be obtained by searching for points where these graphs intersect.

Letter to the Editor – RE: "We Waste Our Children's Time" – Submitted to the New York Times

A good use of students' time in math classes across the nation this week would be to study the formula that accompanied the Letters to the Editor on January 28, 2001. The formula contained several errors, and a complex question to ask students is, "Under what conditions will the incorrect formula produce correct answers?" If students are going to spend more time in school then we must ensure that they will be engaged in rich questions of this nature as opposed to more time spent preparing for state tests.

$$-b \pm \sqrt{\frac{b^2 - 4ac}{2a}} \stackrel{?}{=} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Dear Editor:

In the March, 2001 issue of the *Gazette*, Professor Hosh Pesotan and I showed how the expression HJ can be beautifully simplified from the form in Peter Harrison's article, "Integrating Technologies–Hand-Held, Desk-Top and 'Skull-Based' to Solve a Problem in Geometry":

Peter determined that

$$HJ = 2\sqrt{2 - 2\cos\left(\frac{x}{2}\right)} + \frac{3}{2}\sqrt{8 + 7\cos\left(\frac{x}{2}\right) + \sqrt{15}\sin\left(\frac{x}{2}\right)}$$

and we simplified this to

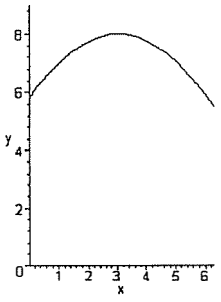
$$HJ = \frac{11\sin\left(\frac{x}{4}\right)}{2} + \frac{3\sqrt{15}\cos\left(\frac{x}{4}\right)}{2}$$

Now, we would like to respond to the author's question about why the graph appears to have a parabolic shape.

Here is a Maple definition and plot of HJ on the interval $[0, 2\pi]$.

> HJ: = x–>5.5*sin(x/4) + 1.5*sqrt(15)*cos(x/4);

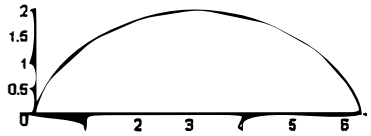
> plot(HJ(x), x = 0..2*π, y = 0..10);



This certainly looks, superficially, to have a parabolic shape. Many curves do, some dramatically.

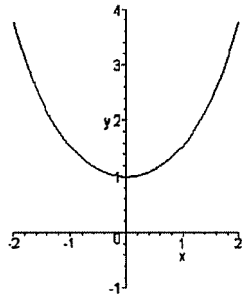
For example, the cycloid (parametrically defined by x = rt – rsin(t) and y = r – rcos(t)):

> plot([t - sin(t),1 – cos(t), t = 0..2*Pi]);



and the catenary, y = cosh(x):

> plot(cosh(x), x = –2..2, y = –1..4);



In fact, from Fourier Analysis, we know that any continuous function can be approximated to any degree of accuracy over a given finite interval by combinations of sines and cosines.

It is also a fact that given three non-collinear points on a continuous function with continuous third derivatives, the unique parabola determined by those points will very well approximate the original curve if the x values are sufficiently close. You obtain successively better fits with more points (obtaining a cubic with four points, a quartic with five, etc. These are the Lagrange Interpolation Polynomials. Higher degree requires higher continuous derivatives).

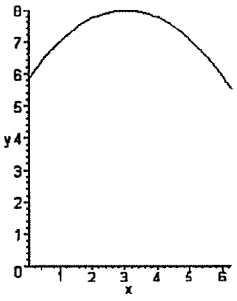
Therefore, on small enough intervals with sufficiently close points, it should not be too surprising that a sum of sines and cosines can have a parabolic appearance, or that conversely, a parabola should look somewhat like a sum of sines and cosines.

We admit the correlation that the author reports, 0.999, is surprising. However, if you try to extend the correlation to more points, further apart, the correlation will decrease. Let us illustrate this:

Below, we have defined the author's quadratic regression function as fpar(x) ("par" for parabola) and plotted fpar(x) and HJ on the same axes for x from 0 to 2π in Maple.

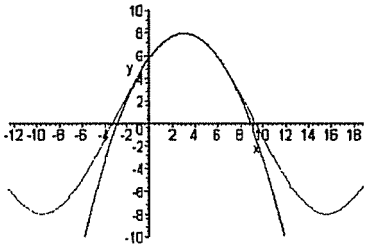
> fpar:= x-> -.2310696016*x^2 + 1.402381742*x + 5.836623924;

> plot([fpar(x), HJ(x)], x = 0..2*Pi);



We can hardly tell the graphs apart. However, on a larger interval:

> plot([fpar(x), HJ(x)], x = -4*Pi..6*Pi, y = -10..10);

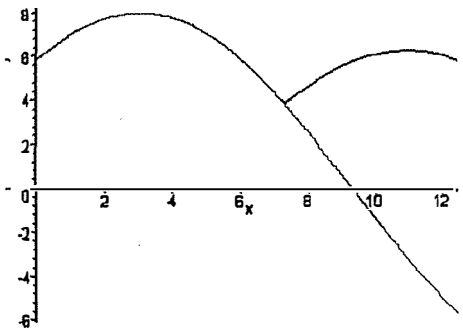


It becomes very clear that the two functions have very different graphs.

On another issue, to alleviate any worry that somewhere we have messed up in our algebraic simplification, below is a Maple plot of our formula for HJ and Peter Harrison's original expression on the interval x = 0 to 4π .

> ftrig:=x-> 2*(2 - 2*cos(x/2))^(1/2) + 1.5*(8 + 7*cos(x/2) + sqrt(15)*sin(x/2))^(1/2);

> plot([ftrig(x), HJ(x)], x = 0..4*Pi);



The functions are an exact match for x from 0 to a particular number somewhere between 7 and 8, and then they diverge abruptly. What is that number?

This is the interesting value of x to which Peter alludes at which the analysis would have to change. Why? Eventually, GJ shrinks to 0, and then the picture shifts and HJ is no longer HG + GJ.

We have some mathematical housekeeping to do first.

For reference, we have the following formulas valid for $\frac{x}{2} \in [0, \pi]$.

(1) $HJ = \dfrac{11\sin\left(\frac{x}{4}\right)}{2} + \dfrac{3\sqrt{15}\cos\left(\frac{x}{4}\right)}{2}$

(2) $HG = 4\sin\left(\frac{x}{4}\right)$

(3) $GJ = \frac{3}{2}\left(\sin\left(\frac{x}{4}\right) + \sqrt{15}\cos\left(\frac{x}{4}\right)\right)$

We must check that the formulas for HJ, GJ, and HG are still valid for $\frac{x}{2} > \pi$ while GJ is still positive.

In Figures 1, 2 and 3, we have $\frac{x}{2} > \pi$. In Figure 1, GJ > 0 and the formula HJ = HG + GJ still holds. In Figure 2, HJ = HG – JG, and in Figure 3, HJ = JG – HG. (Our thanks go to Peter Harrison for providing the figures, using the *Geometer's Sketchpad*™. JW and HP)

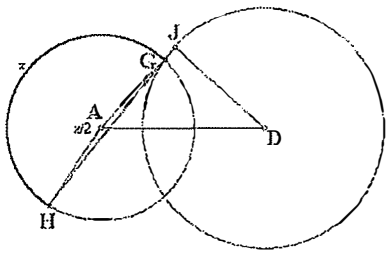


Figure 1

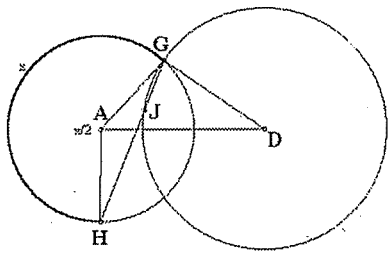


Figure 2

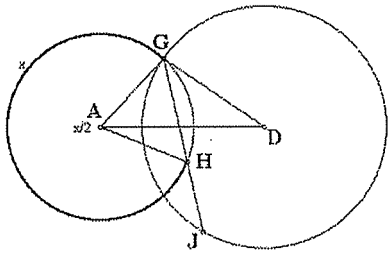
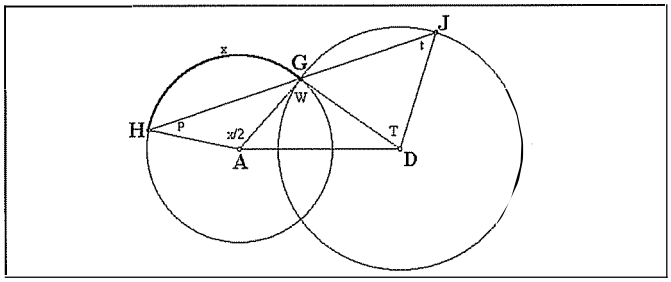


Figure 3

It is the situation in Figure 1 that concerns us.

Compare this picture with that under Note 3 in Peter's original article, reproduced below.

Note 3. (As appeared in March 2001 *Gazette*)



Now **please** refer back to the second column on page 16 in the March 2001 *Gazette*.

Equation ①, formerly $\frac{x}{2} + 2p = \pi$, becomes $\frac{x}{2} - 2p = \pi$. (This happens because the angle HAG is now $2\pi - \frac{x}{2}$, the reflex angle of $\frac{x}{2}$.)

Equation ③, formerly $t + p = \pi - W$, becomes $t - p = \pi - W$.

Following Peter's analysis step for step, you will find that we still have $T = 2W - \frac{x}{2}$. Therefore, Peter's formula for GJ, which in part involved the expansion of $\cos\left(2W - \frac{x}{2}\right)$, remains unchanged.

Also, since $\cos\left(2\pi - \frac{x}{2}\right) = \cos\left(\frac{x}{2}\right)$, his formula for HG and also HJ are still valid. Since his formulas still hold, so do our simplifications (1), (2), and (3).

We now consider the situation when HG = HJ; that is, HG is tangent to the larger circle at G, the points G and J coincide, and GJ = 0. We are seeking the x value at which this happens.

From (3), when $x = 2\pi$, $GJ = \frac{3}{2}$ and when $x = 4\pi$, $GJ = \frac{-3\sqrt{15}}{2}$. It may seem obvious from the nature of the problem that GJ is decreasing for x between 2π and 4π and will be 0 exactly once on this interval, but let us use a little calculus to make sure.

$$\frac{d(GJ)}{dx} = \frac{3}{2}\left(\frac{1}{4}\cos\left(\frac{x}{4}\right) - \frac{1}{4}\sqrt{15}\sin\left(\frac{x}{4}\right)\right)$$

$$= \frac{-3\sqrt{15}\cos\left(\frac{x}{4}\right)}{8}\left(\tan\left(\frac{x}{4}\right) - \frac{1}{\sqrt{15}}\right)$$

Hence, $\frac{d(GJ)}{dx} = 0$ when $x = \arctan\left(\frac{1}{\sqrt{15}}\right) \approx 0.26$.

Analyzing this derivative shows that GJ increases for

$$0 < x < \arctan\left(\frac{1}{\sqrt{15}}\right)$$

and decreases for

$$\arctan\left(\frac{1}{\sqrt{15}}\right) < x < 4\pi.$$

So the x value that we need for GJ = 0 must satisfy $2\pi < x < 4\pi$ and so $\frac{\pi}{2} < \frac{x}{4} < \pi$, that is, $\frac{x}{4}$ is a second quadrant angle.

Setting GJ = 0 yields $\sin\left(\frac{x}{4}\right) = -\sqrt{15}\cos\left(\frac{x}{4}\right)$ and so $\tan\left(\frac{x}{4}\right) = -\sqrt{15}$. Therefore (and remembering $\frac{x}{4}$ is a second quadrant angle), $\frac{x}{4} = \pi - \arctan(\sqrt{15})$ and so $x = 4(\pi - \arctan(\sqrt{15})) \approx 7.29391$, which is between 7 and 8 as predicted. Call this value x_0.

Now, here are two problems that you and some of your students may wish to pursue:

1) For $x \in [x_0, 4\pi]$, how do the formulas for HJ, HG, and GJ change?

   (Here are some starting points. For both intervals, you will find $\frac{x}{2} - 2p = \pi$.

   For awhile, HJ = HG – JG (Figure 2) until H and J coincide. After this, HJ = GJ – HG (Figure 3).)

2) Find triangles in Peter's original drawing under Note 3 which involve the angle measurement $\frac{x}{4}$. Using these, derive the "no square roots needed" formula for HJ directly.

Our thanks go to Peter Harrison for a neat article and the interesting questions he raised.

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▲ CALL FOR SUBMISSION OF ARTICLES

FRANCES SCHATZ

LOUIS LIM



Frances Schatz, Abacus *editor, is a freelance mathematics educator who has experience in teaching, contest coaching, editing, and writing. Frances is OAME's representative to NCTM, and conducts reviews for NCTM's publication,* Mathematics Teacher.



Louis Lim is editor of the Ontario Mathematics Gazette *and is an elected director of OAME. Louis is a mathematics teacher at Quinte SS with the Hastings and Prince Edward DSB, currently authoring his fourth textbook with McGraw-Hill Ryerson, and is in M.Ed. thesis work at Queen's University under the supervision of Dr. Lynda Colgan.*

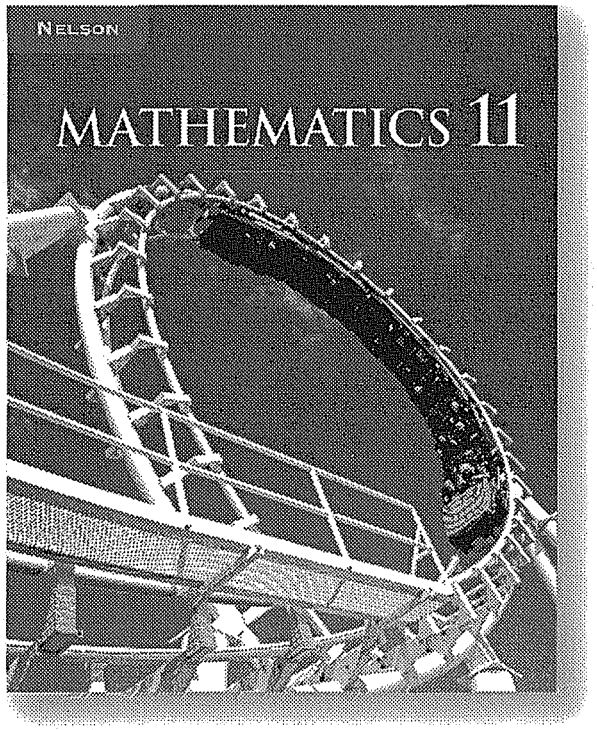
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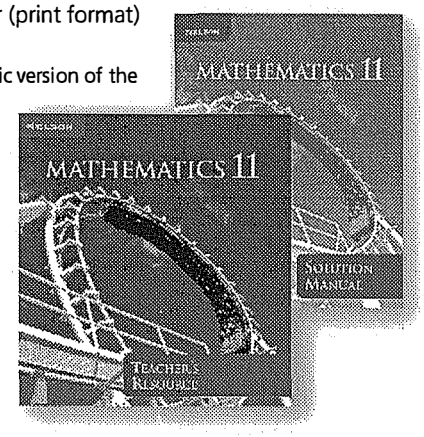
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activity, and like teaching, the more you work at it, the better you become. Ask a colleague to co-author, or involve some students in a project for a chosen topic. Let the curriculum guidelines serve as starting points. Choose your favourite lesson/topic or an activity that has given you positive results in the classroom. Articles that pertain to the implementation of the new mathematics curriculum (both elementary and secondary) are highly sought. Other areas of need include articles on assessment and evaluation, effective use of technology, and support for new teachers. Do not let page one of the *Gazette*, "Submission of Articles," discourage you from writing – your journal editors are very supportive and only an e-mail away.

How to Start

Keep a journal of daily observations with your classes. For instance, document your successes and struggles as you implement innovative instructional, curriculum, or assessment practices, and their effect on student learning and achievement. The journal allows for documentation that you might otherwise forget, serves as a springboard for ideas, and then publishable activities or articles may naturally emerge.

Brainstorm and Persist

Read past issues of the *Gazette, Abacus,* and other mathematical journals to locate articles that have a "hold your interest" effect. If you find an article or activity that is similar to yours, aim to make yours unique and original. Do not forget to cite this source in your references at the end of your article. It is important to give credit to sources of information. Have one or two colleagues read your draft. Ask them if some parts of your manuscript are not clear or unnecessary, and if they have any suggestions. Either make these people co- or secondary authors, or acknowledge their assistance in the introduction.

Ask the Editor

For the *Abacus*, you work directly with the editor, Frances Schatz. Submit your activity to the editor for comments. The editor can give you more information about suitable length, method of illustration if diagrams or photos are to be used, and other procedural information.

For the *Gazette*, submit four copies of your manuscript to Louis Lim. Your article will be blind-reviewed by several members of the editorial board; referees have approximately one month to review the submission. The referees will provide detailed feedback to improve/enhance your submission. Please do not be upset or discouraged if your manuscript comes back as "major (or minor) revision required." One needs to remember that the author(s), reviewers, and the editor all work collaboratively to make a manuscript with potential publishable; that in itself is a wonderful learning experience for all involved.

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Final Remarks

It is very rewarding to see your name and photograph in print. OAME has a membership of approximately

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It has been said that teachers positively influence the students they teach – writing for the *Abacus* or *Gazette* will allow you to have a positive effect on the teaching profession. Your editors look forward to working with you towards the goal of publication. Writing is a very rewarding experience that may lead to other opportunities. Please do not hesitate to contact the editors for further information or clarification. ▲

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▲ Assessment Corner: Understanding the Categories of the Achievement Chart

CHRIS SUURTAMM



Chris Suurtamm is currently teaching at the Faculty of Education at York University. She is also an experienced mathematics teacher. She is committed to helping teachers with the implementation of new curriculum and assessment practices. Her involvement includes Course Profile writing, EQAO reviewing, research in assessment, workshop leader, and author with Nelson Thomson Learning.

Mathematics teaching and learning is more challenging than ever! A quick look at key phrases in curriculum expectations, both in the elementary and secondary mathematics curriculum, show that students will: investigate, demonstrate, describe designs, determine properties, distinguish, present solutions, read and report, explain their thinking, and pose questions. Mathematics teaching and learning is more than just the dissemination and acquisition of knowledge. It requires a variety of instructional and assessment activities.

The Categories of the Achievement Chart, Knowledge and Understanding; Thinking, Inquiry, and Problem Solving; Application; and Communication, reflect the breadth of mathematical actions that should occur in a mathematics classroom. Understanding the meaning and differences between the categories are essential so that instructional and assessment techniques can cover this broad spectrum.

The Achievement Chart – it's like learning to use a "toolkit"

One way to think about the different meanings of the categories is to use the metaphor of learning to use a "toolkit".

Knowledge and Understanding relates to the practice required to learn to use specific tools. For instance, in learning to use a hammer, many nails need to be pounded. Through practice, a person is better able to hit the nail on the head, to hammer the nails in straight, and to learn to recover from mistakes. This is similar to students practicing skills so that they can apply them in context. However, skills can also be practiced within a context, much as the skill with a tool is increased through the creation and building of an object.

Application suggests that tools can be used on a specific task which may not be exactly the same as the task that was used for practicing or skill-building. Being able to apply a tool implies that a person would recognize which tool to select and how to use it appropriately in the context of the problem or job to be done. For instance, a student may select to use the Pythagorean Theorem to solve a problem. He or she also needs to use the theorem correctly, making the correct substitutions, and following through with the solution. Application thus relates to the selection of the appropriate tool and to its proper use.

Thinking, Inquiry, and Problem Solving is like fixing a major household problem or building a piece of furniture. It requires that a person knows which tools in the toolkit should be selected and applied. It also requires a plan. Unlike application, which relates to the use of one tool, this category requires the selection and sequencing of a variety of tools. It also incorporates reflection and evaluation as a person works with the tools to see if the plan needs adjustment or if there are tools that might work better.

Communication requires that a person be able to read blueprints or instructions that accompany a major project. Good communication includes the use of proper names for tools and procedures. Communication also incorporates the ability to tell, demonstrate, or write instructions and diagrams to tell others how to solve the problem.

Many teachers use the categories of the Achievement Chart as an organizing structure for their assessment data. Thus, teachers may choose to identify questions on a test or a quiz as pertaining to a particular category or categories. Also, teachers may use one or more categories of the Achievement Chart when building a checklist, rating scale, or rubric. In many cases, teachers may feel that a task incorporates many categories and it is often difficult to separate the various parts of the task into categories. This is a good sign that the task is probably multi-dimensional, engaging of students, and integrates their knowledge, skills, and problem solving ability.

Using Generic Rubrics

Generic Rubrics have been created for each of the categories of the Achievement Chart. A set of generic rubrics for each of the categories is available in the C.A.R.E. (Concerning Assessment and Reflective Evaluation) package on the OAME website (http://www.oame.on.ca/index.html). These Generic Rubrics provide teachers with components that they can use to build a rubric for a complex task. In building the rubric for the task, a teacher should decide which categories are addressed by the task and will be assessed. Once the categories are chosen, a teacher can review the generic rubrics to determine which rows pertain to the task. Those rows are then used to create the rubric for the task. In many cases, it will be necessary to make the criteria more task specific. This is especially true of the "Knowledge and Understanding" and "Application" rubrics that refer to a student's ability to perform a particular algorithm or procedure. To make the criteria more task specific, the particular procedure (e.g., calculate slope) should be stated in the rubric for the task.

In Summary

Understanding the categories of the Achievement Chart is a starting point. Classroom instruction and assessment that addresses the curriculum expectations and the categories of the Achievement Chart will include more than just showing students how to do something and then giving them a paper-and-pencil test to be sure that they can replicate what was shown. It also includes opportunities for students to explore mathematics, apply what they learn to new contexts, deepen their understanding through explaining and writing about what they are learning, and to make connections between mathematics and their world. ▲

▲ OAME/NCTM June 2001 Report



FRANCES SCHATZ

Thank you, OAME, for making it possible for me to attend the annual NCTM meeting in Orlando on April 4-7, 2001 as the Ontario Delegate. The whole conference was an awesome experience. There, a real teacher was defined.

A real teacher stands in line for a workshop while eating lunch in hand, only to be told after 20 minutes that the session can only take 25 people and the line has 40 people. A real teacher stands on the trolley so others can have a seat. A real teacher can walk from one end of the Orange County conference centre to the other end and make it to the next workshop on time. And finally, a real teacher joins the line-up at the printing shop to get a copy of an important handout which will only be available to pick up at a later time. This conference, which attracted 18 000 mathematics educators, offered over 1100 sessions on a first-come, first-served basis that replaced admission tickets of previous years. A number of Ontario teachers presented workshops that focussed on innovative teaching and learning strategies for the 21st century.

Leading this year's agenda was the unveiling of the first four books in the new *Navigations* series, which when complete, will contain more than 30 titles. (Don't fear: the 22 books of the *Addenda* series will continue to be valuable resource books.) The first books address the Algebra Standard, with one book for each grade band from prekindergarten through grade 12. The *Navigations* collection is an extension of NCTM's *Principles and Standards for School Mathematics,* which was released in April 2000.

Congratulations to Mary Bourassa, who received a MET (Mathematics Education Trust) FLAMeS award to attend the NCTM Annual Conference. This Future Leaders Annual Meeting Support award of $1000 is available to K-12 teachers. This money helps cover airfare, accommodation, registration, and substitute teacher costs. Mary is in her 7th year of teaching with the Ottawa-Carleton District School Board and is in her 5th year of teaching mathematics and computer science at Lisgar Collegiate Institute. She is an active member of the COMA chapter of OAME and the OCDSB Math Subject Council. Mary has given numerous workshops and likes to attend conferences to continue learning and keeping up-to-date with curriculum matters.

On a personal note, I had an informative luncheon meeting with Bob Robinson, who was a math consultant in the Hamilton area before his retirement. Now, he is very busy with various mathematical pursuits in Washington State. In 1985, Bob was Ontario's NCTM representative and he was responsible for encouraging me to join NCTM. During the conference, I proudly wore my 15 + years member ribbon.

At the 2001 NCTM Delegate Assembly, there was one major resolution passed by the delegates. O.R.01.03 "Be it resolved that the NCTM Board of Directors return to the policy of one complimentary speaker registration for presenters at NCTM Annual Conferences." This resolution was brought forward by the British Columbia Association of Mathematics Teachers and was strongly supported by the Canadian caucus. This year, presenters had to pay registration. This will be changed for next year's annual conference in Las Vegas.

NCTM encourages us to visit the website **www.figurethis.org** for family mathematics materials. *Figure This! Mathematics Challenges for Families* engages families in the math that grades 5-10 students should know. The challenges are exciting activities that can be done at home and will help show how math is used in the real world. A total of 80 mathematical challenges will be released and will be available on-line.

For more information about NCTM programs, visit www.nctm.org, or contact me if you need information about NCTM memberships, books or products. ▲

▲ Cards, Combinatorics, and Cooperative Learning

CHARLIE GALLANT



Charlie Gallant (PhD, Illinois, 1971) graduated from St. Francis Xavier University in 1961, and except for study leaves, he has remained there. His main interests are the mathematical preparation of teachers, and the uses of cooperative methods and of geometry in teaching. His greatest pleasures are derived from family and gardening.

Much of the learning that takes place in our classrooms is individual and/or competitive. It is much more difficult, of course, to have students work cooperatively. Now and then, we stumble upon an activity that produces superb cooperative learning opportunities. The following article, I believe, describes such an activity. The interaction works best if students are in groups of three with a calculator at the ready.

Teachers often use a standard deck of 52 playing cards and the game of 5-card draw poker to illustrate counting procedures and probabilities. Table 1 shows, for 5-card poker, the ranking of the various types of hands. For example, a flush beats out a straight because there are fewer flushes (5108) than straights (10 200) or equivalently,

the probability of a flush, $\frac{5\,108}{\binom{52}{5}} = .00199$,

is less than the probability of a straight, $\frac{10\,200}{\binom{52}{5}} = .00392$.

Table 1 – Five-card Poker

Type of Hand	Number	Reasoning
1. Straight Flush	40	$= \underline{40}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}$
2. Four of a kind	624	$= \underline{52}\ \underline{3}\ \underline{\frac{2}{4!}}\ \underline{1}\ \underline{48} = \binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1}$
3. Full House (Triple and one pair)	3744	$= \underline{52}\ \underline{3}\ \underline{\frac{2}{3!}}\ \underline{48}\ \underline{\frac{3}{2!}} = \binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$
4. Flush	5108	$= \underline{52}\ \underline{12}\ \underline{\frac{11\ 10\ 9}{5!}} = \binom{4}{1}\binom{13}{5} - 40$
5. Straight	10 200	$= \underline{40}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4} - 40$
6. Three of a kind	54 912	$= \underline{52}\ \underline{3}\ \underline{\frac{2}{3!}}\ \underline{48}\ \underline{\frac{44}{2!}} = \binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1}$
7. Two Pairs	123 552	$= \frac{\underline{52}\ \underline{3}\ \underline{48}\ \underline{\frac{3}{2!}}\ \underline{\frac{44}{2!}}}{2!} = \binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}$
8. One Pair	1 098 240	$= \underline{52}\ \underline{3}\ \underline{\frac{48}{2!}}\ \underline{\frac{44}{3!}}\ \underline{\frac{40}{3!}} = \binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}$
9. Bust (no pair, no flush, and no straight)	1 302 540	$= \underline{52}\ \underline{48}\ \underline{\frac{44}{5!}}\ \underline{40}\ \underline{36} - (40 + 5108 + 10\,200)$ $= \binom{13}{5}\binom{4}{1}^5 - (40 + 5108 + 10\,200)$
Total # of hands	2 598 960	$= \binom{52}{2}$

Table 1 shows, where possible, the counting done in two ways: by combinatorial formula and by, the more intuitive, permutation count. In a poker hand, order does not matter, so why use two methods of counting? Firstly, it enables us to check our answers.

Secondly, the combinatorial method, though more elegant, does not work in all cases, while the permutation method reveals more of what is actually taking place. Permutation means that order is important so the counting matches the way in which the cards are dealt; that is, in order, one at a time. For example, how many 5-card poker hands consist of four aces? The two solutions are:

a) $\binom{4}{4}\binom{12}{1}\binom{4}{1} = 48$; i.e., of the four aces *choose* four, of the twelve denominations remaining *choose* one, of the four cards in that denomination *choose* one.

b) $\underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{48} = 1152$ means that 1152 hands exist in which we have 4 choices for the first card, three choices for the second card, etc. An example of such a hand is $A^CA^DA^HA^SQ^C$. Clearly, the 1152 hands include $A^DA^SA^HA^CQ^C$, which is not good, since in poker, it is regarded as the same hand. Hence, we must "divide out" the order to yield $\underline{4}\ \underline{3}\ \underline{\frac{2}{4!}}\ \underline{1}\ \underline{48} = 48$.

This permutation count regards the order as important, counts on that basis, and then removes (divides out) the order.

Similarly, a full house can be formed in $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$ ways; i.e., of the thirteen denominations choose one; of the four cards in that denomination choose three, and of the twelve denominations remaining pick one, and of the four cards in that denomination pick two. Likewise, we argue that there are $\frac{52}{3!}\frac{3}{}\frac{2}{}\frac{48}{2!}\frac{3}{}$ ways to get a full house since the first card can be anything, there are three choices for the second card and two choices for the third card. However, this counting procedure would regard $A^HA^DA^S$ different from $A^DA^SA^H$. Hence, we must divide out the order; that is, divide by 3! There are only 48 choices for the fourth card since we must exclude all four cards involved in the triplet. Again the pair, for example, 8^C8^H as 8^H8^C must not be counted twice so we divide by 2!.

Now comes the fun. Working cooperatively, the various groups address two-card poker. What is the best hand? Defend your position, explain to your group, and then explain to the entire class. Continue ranking the hands. Proceed to three-card, and then to four-card poker. The result for five-card poker can be displayed via an overhead at all times, or given as a handout so that it serves as the paradigm to be imitated. Playing 5-card poker conditions one to certain rankings. Expect some surprises!

Table 2 – Two-card Poker

Type of Hand	Number	Reasoning
1. Straight Flush	$52 = \underline{52}\ 1$	
2. One Pair	$78 = \frac{52}{2!}\frac{3}{} = \binom{13}{1}\binom{4}{2}$	
3. Straight	$156 = (\underline{52}\ 4) - 52$	
4. Flush	$260 = \left(\frac{52}{2!}\frac{12}{}\right) - 52 = \binom{4}{1}\binom{13}{2} - 52$	
5. Bust	$780 = \left(\frac{52}{2!}\frac{48}{}\right) - (52 + 156 - 260)$	$= \binom{13}{2}\binom{4}{1}\binom{4}{1} - (52 + 156 - 260)$
Total # of hands	$\underline{1326} = \binom{52}{2}$	

It is surprising that one pair beats a straight, and also that a straight beats a flush!

Table 3 – Three-card Poker

Type of Hand	Number	Reasoning
1. Straight Flush	$48 = \underline{48}\ 1\ 1$	
2. Three of a kind	$52 = \frac{52}{3!}\frac{3}{}\frac{2}{} = \binom{13}{1}\binom{4}{3}$	
3. Straight	$720 = (\underline{48}\ 4\ 4) - 48$	
4. Flush	$1096 = \left(\frac{52}{3!}\frac{12}{}\frac{11}{}\right) - 48 = \binom{4}{1}\binom{13}{3} - 48$	
5. One Pair	$3744 = \frac{52}{2!}\frac{3}{}\frac{48}{} = \binom{13}{1}\binom{4}{2}\binom{12}{1}\binom{4}{1}$	
6. Bust	$16\,440 = \left(\frac{52}{3!}\frac{48}{}\frac{44}{}\right) - (48 + 1096 + 720)$	$= \binom{13}{3}\binom{4}{1}^3 - (48 + 1096 + 720)$
Total # of hands	$\underline{22\,100} = \binom{52}{3}$	

Again, a straight beats a flush and both are beaten by three of a kind!

Table 4 – Four-card Poker

Type of Hand	Number	Reasoning
1. Four of a kind	$13 = \frac{52}{4!}\frac{3}{}\frac{2}{}\frac{1}{} = \binom{13}{1}\binom{4}{4}$	
2. Straight Flush	$44 = \underline{44}\ 1\ 1\ 1$	
3. Three of a kind	$2496 = \frac{52}{2!}\frac{3}{}\frac{2}{}\frac{48}{} = \binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{1}$	
4. Straight	$2772 = (\underline{44}\ 4\ 4\ 4) - 44$	
5. Two Pairs	$2808 = \frac{\frac{52}{2!}\frac{3}{}\frac{48}{}\frac{3}{2!}}{2!} = \binom{13}{2}\binom{4}{2}\binom{4}{2}$	
6. Flush	$2816 = \left(\frac{52}{4!}\frac{12}{}\frac{11}{}\frac{10}{}\right) - 44 = \binom{4}{1}\binom{13}{4} - 44$	
7. One Pair	$82\,368 = \frac{52}{2!}\frac{3}{}\frac{48}{}\frac{44}{2!} = \binom{13}{1}\binom{4}{2}\binom{12}{2}\binom{4}{1}\binom{4}{1}$	
8. Bust	$177\,408 = \left(\frac{52}{4!}\frac{48}{}\frac{44}{}\frac{40}{}\right) - (44 + 2816 + 2772)$	$= \binom{13}{4}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1} - (44 + 2816 + 2772)$
Total # of hands	$\underline{270\,725} = \binom{52}{4}$	

The biggest surprises here are that a straight flush is no longer the best hand, and that two pairs beat a flush, but just barely! Again, three of a kind beats both a straight and a flush. Also, note that a three of a kind beats either a flush or a straight!

Following this presentation, a quiz (groups can work together) covers sampling of 2, 3, 4, and 5-card poker. For example, the probability of getting one pair in each type of poker is: two-card, .0588; three-card, .1694; four-card, .3042, and five-card, .4226. For an assignment, students are to consider 6-card poker! (The ranked hands are given in Table 5)

Table 5 – Six Card Poker

Type of Hand	Number	Reasoning
1. Straight Flush	36	$= \underline{36}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}$
2. Four and one pair	936	$= \frac{52}{4!}\frac{3}{}\frac{2}{}\frac{1}{}\frac{48}{2!}\frac{3}{} = \binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{2}$
3. Two Triples	1248	$= \frac{\frac{52}{3!}\frac{3}{}\frac{2}{}\frac{48}{3!}\frac{3}{}\frac{2}{}}{2!} = \binom{13}{2}\binom{4}{3}\binom{4}{3}$
4. Flush	6828	$= \frac{52}{6!}\frac{12}{}\frac{11}{}\frac{10}{}\frac{9}{}\frac{8}{} - 36 = \binom{4}{1}\binom{13}{6} - 36$
5. Four of a kind	13 728	$= \frac{52}{4!}\frac{3}{}\frac{2}{}\frac{1}{}\frac{48}{2!}\frac{44}{} = \binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1}$
6. Straight	36 828	$= (\underline{36}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4}) - 36$
7. Three pairs	61 776	$= \frac{\frac{52}{2!}\frac{3}{}\frac{48}{2!}\frac{3}{}\frac{44}{2!}\frac{3}{}}{3!} = \binom{13}{3}\binom{4}{2}\binom{4}{2}\binom{4}{2}$
8. Triple and one pair	164 736	$= \frac{52}{3!}\frac{3}{}\frac{2}{}\frac{48}{2!}\frac{3}{}\frac{44}{} = \binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}\binom{11}{1}\binom{4}{1}$
9. Three of a kind	732 160	$= \frac{52}{3!}\frac{3}{}\frac{2}{}\frac{48}{3!}\frac{44}{}\frac{40}{} = \binom{13}{1}\binom{4}{3}\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}$
10. Two pairs	2 471 040	$= \frac{\frac{52}{2!}\frac{3}{}\frac{48}{2!}\frac{3}{}\frac{44}{2!}\frac{40}{}}{2!} = \binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{2}\binom{4}{1}\binom{4}{1}$
11. Bust	6 985 044	$= \frac{52}{6!}\frac{48}{}\frac{44}{}\frac{40}{}\frac{36}{}\frac{32}{} - (36 + 6\,828 + 36\,828)$ $= \binom{13}{6}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1} - (36 + 6\,828 + 36\,828)$
12. One Pair	9 884 160	$= \frac{52}{2!}\frac{3}{}\frac{48}{}\frac{44}{4!}\frac{40}{}\frac{36}{} = \binom{13}{1}\binom{4}{2}\binom{12}{4}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}$
Total # of Hands	$\underline{20\,358\,520}$	$= \binom{52}{6}$

One pair is now the worst hand! A flush beats 4 of a kind. The idea of a full house is broadened to include "Four and a Pair," "Two Triples," and "Three Pairs," ranked in that order.

This experience appears to give a solid foundation to which students can refer, and from which they can move forward to understand other problems, and subsequent considerations that are often illustrated by such games.

Reference

Litwiller, B., & Duncan, D. (1977). Poker probabilities: A new setting. *The Mathematics Teacher, 70*(9), 776. ▲

▲ PHOTO MATH – A NEW WAY OF PICTURING MATH

RON LANCASTER



Ron Lancaster is a middle and high school mathematics teacher, currently on staff at the Bishop Strachan School as a mathematics consultant. Ron also works as an Independent Consultant throughout North America and Asia, and is an Instructor for Texas Instruments.

"A picture is worth a thousand words" is an expression that is often used to convey the importance of using images to make a point. The point of this column is to go beyond this notion and promote the idea that in fact, "A picture is worth a thousand numbers." Readers can expect to find photos with math in every Photo Math *column and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario curriculum as well as the new NCTM document,* Principles and Standards for School Mathematics. *It is hoped that these questions will inspire students to be more creative and playful, and that in the end, students will develop an ability to see the world from a different perspective.*

Teachers and students are encouraged to pose mathematical questions related to their own encounters and to submit these problems along with photographs to the Editor for possible publication in future columns. The Editor of this column does not presently have a cable modem, and in order to avoid long periods of downloading items, submissions to this column should be sent via Canada Post to the address given above.

In 1996 and 1997, I attended the *Teaching Contemporary Mathematics Conference* held annually at the North Carolina School of Science and Mathematics in Durham. During the 1996 conference, I took the following photographs in the area of the Durham Arts Council.

Despite the fact that travelling to the United States is expensive these days, due to the low Canadian dollar, I would highly recommend this two-day conference to teachers in Ontario. For more information, go to http://phywww1.ncssm.edu/green/math/index.htm.

Photograph #1



1. This sign appears at the entrance to a parking lot across the street from the Durham Arts Council.

   (a) Find the cost of parking a car in this lot for each of the following times. Compared to prices in your area, do you think that it costs a lot of money to park in this lot?

time	cost
30 minutes	
59 minutes	
1 hour	
61 minutes	
2 hours	
24 hours	

   (b) Suppose that at 6 p.m. a person exited from the lot and paid a total of six dollars for parking. When did this person enter the lot?

   (c) Graph the total cost of parking a car in this lot over the course of a 24-hour time period.

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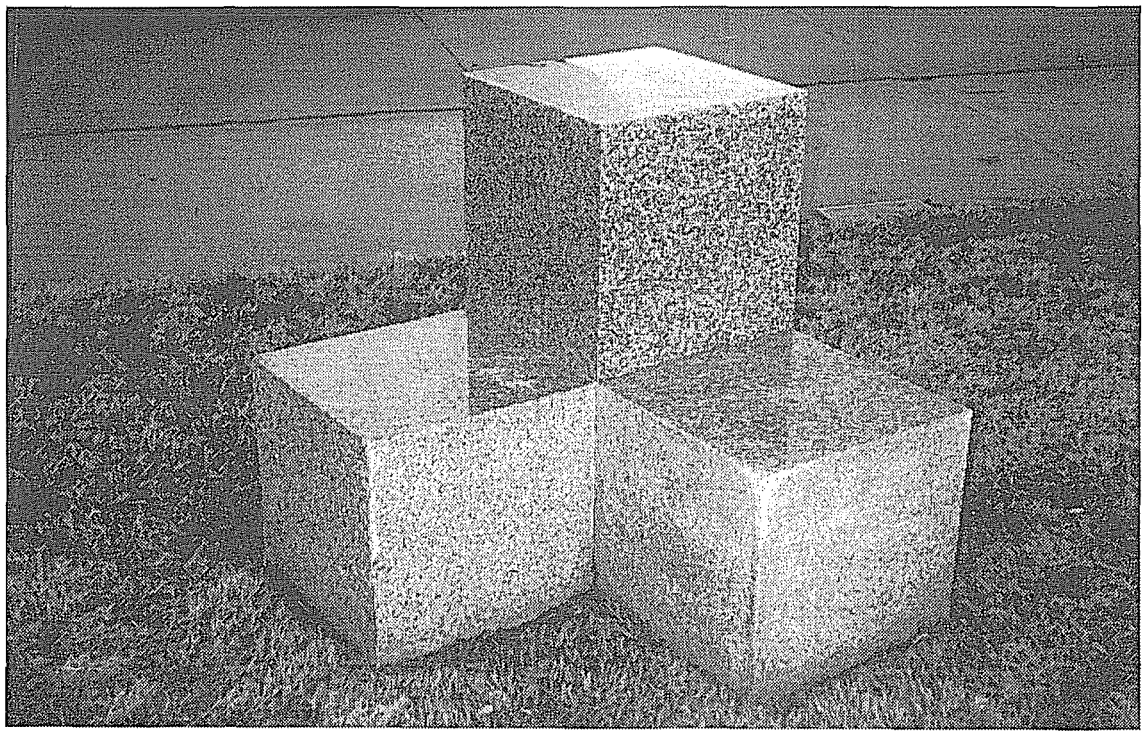
2. Each of the remaining photographs were taken in a park on the property of the Durham Arts Council. In each photo, you will see various concrete sculptures by local artist, Vernon Pratt. Mr. Pratt is a professor at Duke University and in 1988, he was hired by the Durham Arts Council to create and install these sculptures in the downtown area of Durham. Each of his concrete sculptures consists of 1 to 8 cubes and every one of them would fit into a large 2-by-2 cube. For example, the sculptures shown in photographs 2 and 3 were made from 4 cubes.

   (a) Find the total number of sculptures that can be made for each of the following cases:

number of cubes	number of sculptures
1	
2	
3	
4	
5	
6	
7	
8	

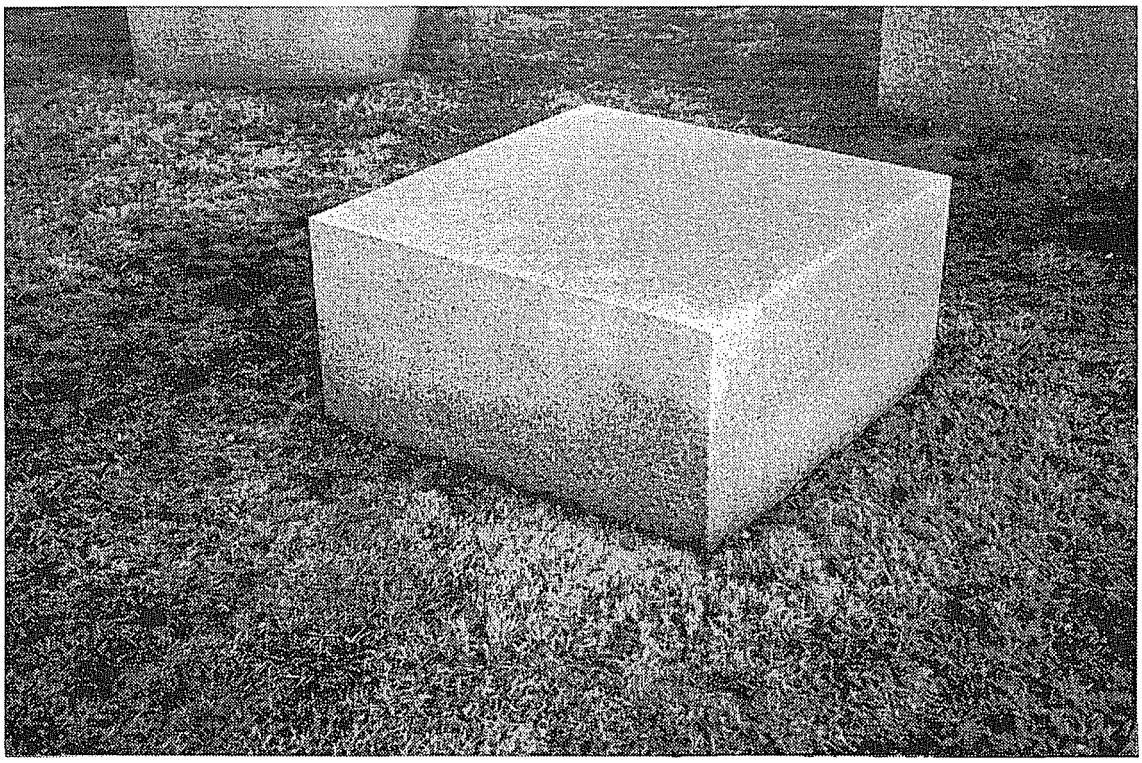
   (b) How many sculptures could be made if from 1 to 27 cubes were used along with the condition that they fit within a 3-by-3 cube?

Photograph #2

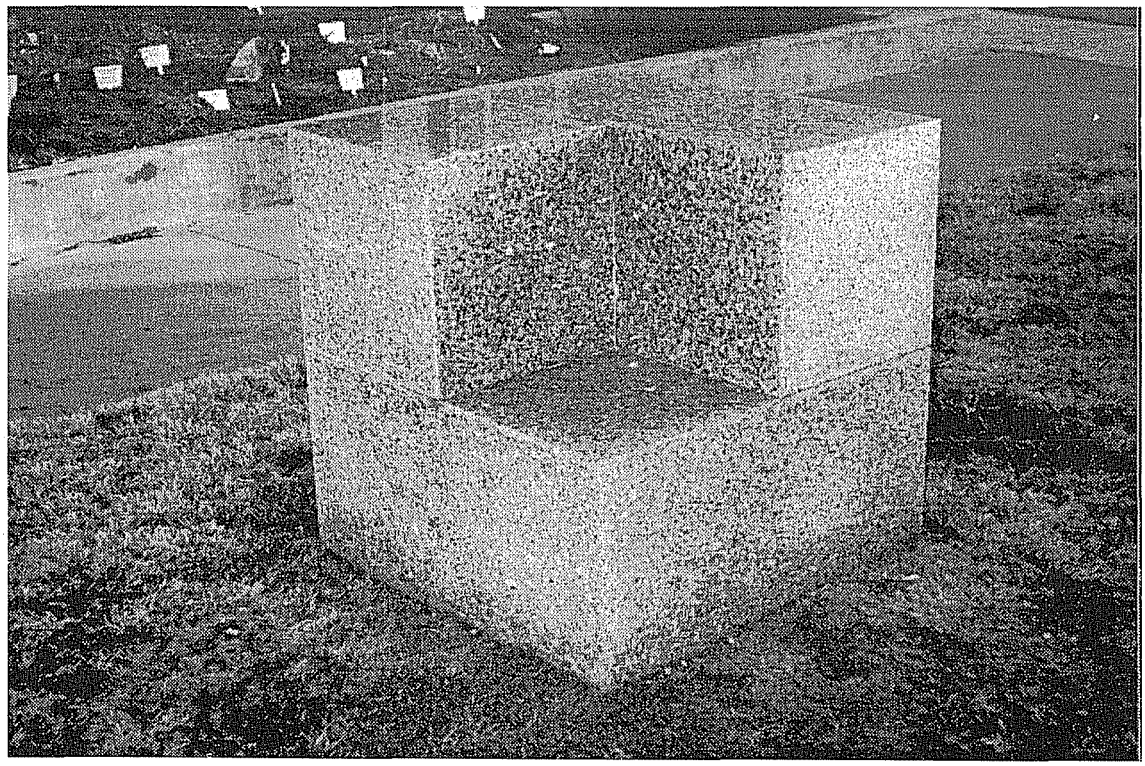


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Photograph #3

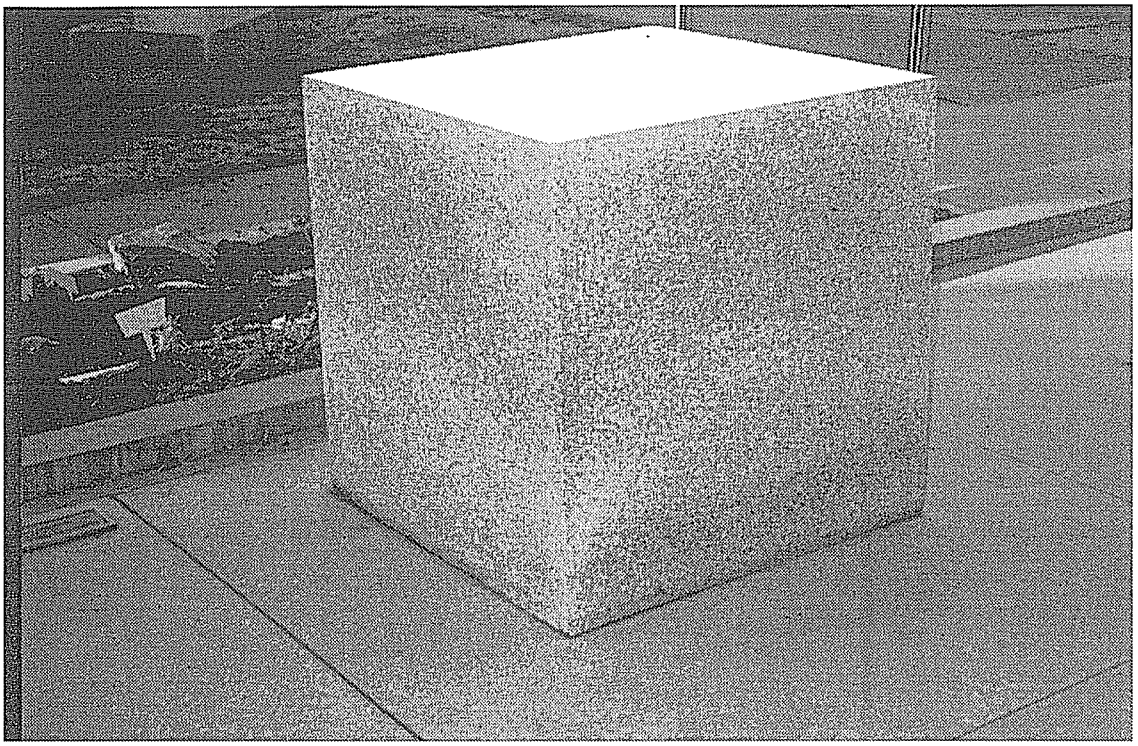


Photograph #4



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Photograph #5



Photograph #6



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Comments and partial answers

Photograph 1

Question 1

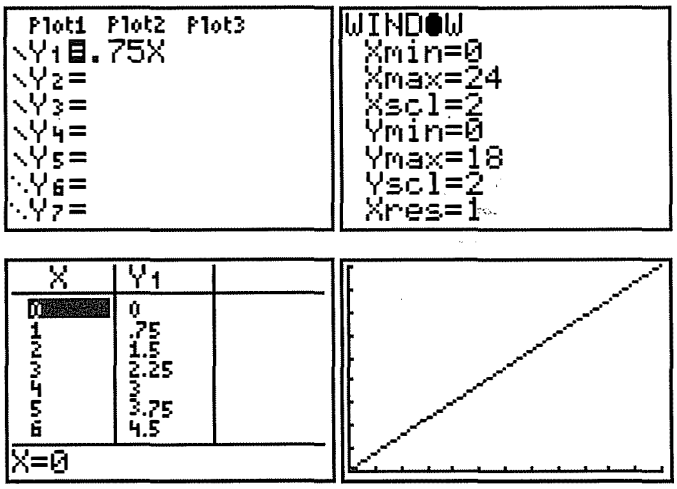
I frequently assigned this question to my grade 7 students at St. Mildred's-Lightbourn School in Oakville, and with few exceptions, most students picked up on the fact that the sign contains a major error. The decimal point should not be there and the owners of this lot clearly meant to say 75 cents per hour. I covered up the decimal point with tape during my trip in 1996 and it stayed put while I was in town. When I returned to the conference in 1997, I was disappointed to find the tape was gone, once again exposing the decimal point. I think they "missed" the point!

(a) The following answers were calculated using a rate of 75 cents per hour. Divide these costs by 100 to obtain the answers for the rate posted on the sign. Here is an interesting question for teachers: Should full marks be given to a student who uses the rate of .75 cents per hour?

time	cost
30 minutes	75 cents
59 minutes	75 cents
1 hour	75 cents
61 minutes	$1.50
2 hours	$1.50
24 hours	$18

(b) This person was charged for parking eight hours and readers will probably not be surprised to know that a number of my students gave an answer of 10 a.m. for this question. In fact, we cannot be certain when this person entered the lot – the best we can say is that the time of entry was between 10:00 and 10:59 a.m. (assuming the clock has no counter for measuring time by the second).

(c) It was not uncommon for me to find students creating the graph by using the formula y = .75 x, where y represents the cost in dollars for parking for x hours. The table of values for this function along with the graph are given. The next "step" of course is to ask if this graph is correct. If it is not correct, what "steps" should be taken to correct it?



Photographs 2, 3, 4, 5, and 6

Question 2

(a) This is a beautiful activity to try out with your students. I suggest that you have them work with small wooden blocks or plastic containers similar to the ones used to hold milk (containers of this type can be purchased at office supply stores). One of the first things your students will need to resolve is which cases should be included. You may want to try a different approach, but I have asked my students to adhere to the rules used by Pratt. The name he assigned to this installation is, "All of the Possibilities of Stacking Up to two^3 to Sit On," and the idea that he had in mind was to use only the cases that could be physically made. I am being deliberately vague here because I think that it would be a good exercise for teachers and their students to draft their own set of conditions and then find the number of possible cases.

(b) This is a difficult counting exercise and teachers may want to assign this problem to students in their Finite Mathematics course. A number of my students have worked on this problem but I do not know the correct answer. I have a collection of close to 1000 small wooden blocks, and a number of years ago, several students used a large number of them as part of a display for this problem. They felt that we would need more blocks if they were going to include an example of each case.

Readers are encouraged to send in comments and solutions to this problem for possible publication in a future column. Drawing pictures is one way of recording the various cases but it will be interesting to see if readers come up with alternative representations. ▲

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OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration;

- demonstrations of excellence and creativity in mathematics education.
- specific contributions to mathematics education in the school, region, province or nation.
- demonstrates leadership in mathematics education in the school, region, province or nation.
- support of and contribution to the activities of OAME/AOEM.
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

OAME/AOEM Life Membership Award

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and accumulated 10 or more years of membership in OAME/AOEM.

Kenneth D. Fryer Award

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Mona MacGregor Award

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Morley MacGregor Award

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

Don Attridge Award

In recognition of the contributions made by Don Attridge, a founding member, former president and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

AWARD NOMINATION COVER PAGE

Send to: Stewart Craven

Intended Award (please check one)

❑ Life Membership ❑ Mona MacGregor ❑ Morley MacGregor ❑ Kenneth D. Fryer ❑ Don Attridge

Nominee Information:

_____

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

_____

_____

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

Nominator Information:

_____

Name of Nominator

_____

_____

School, Organization or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

▲ A REVIEW OF: *REALIZING A VISION OF TOMORROW'S MATHEMATICS CLASSROOM: A HANDBOOK ON RICH LEARNING TASKS*

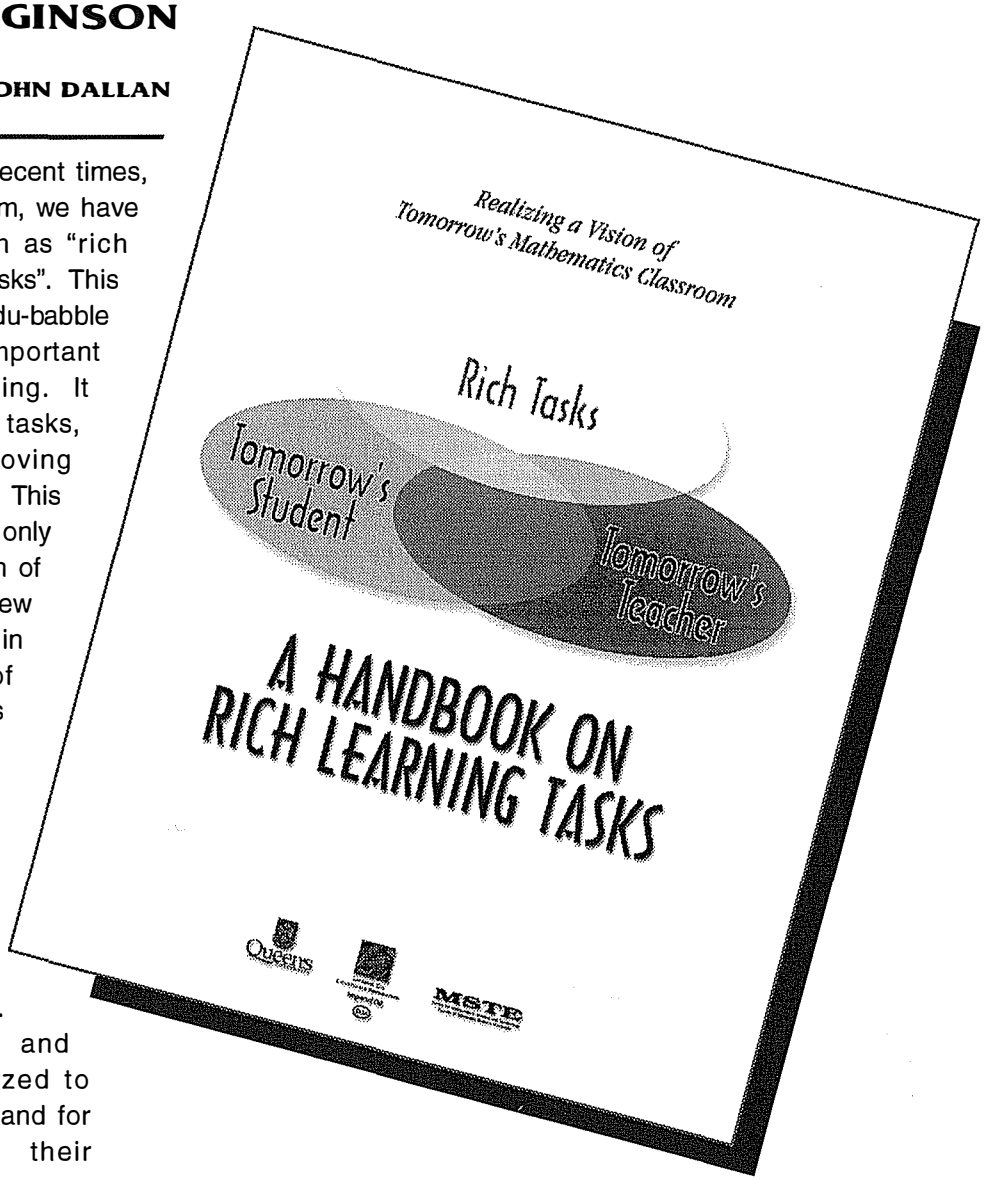
BY GARY FLEWELLING WITH WILLIAM HIGGINSON

REVIEWED BY JOHN DALLAN

Do not let the title scare you. In recent times, with the evolution of the new curriculum, we have been inundated with phrases such as "rich learning tasks" and "rich assessment tasks". This book synthesizes a great deal of the edu-babble and clearly identifies some of the important aspects of teaching for understanding. It contains numerous examples of rich tasks, related rubrics, and ideas for improving learning in the mathematics classroom. This short 80 page book was designed not only to assist educators in the clarification of some of the issues surrounding the new reforms, but also to improve learning in the classroom. Through the use of sample classroom activities, it breaks down some of the ambiguity around what constitutes a rich learning task and the good teaching methods needed to make an activity "rich" in learning.

The book is a blending of pedagogy with trouble shooting solutions for potential problems. Characteristics of rich learning and assessment tasks are both analyzed to highlight what makes them effective and for those factors which diminish their effectiveness. The book skillfully summarizes many of the ideas in easy-to-read comparative lists and checklists that help teachers to determine whether a task truly meets the expectations set out by the new curriculum.

Personally, I was pleased to see that the authors directed a significant emphasis on placing the "Simon Says" approach to teaching into a more appropriate perspective. There are times when many us may rely on rote learning in our attempt to "cover the curriculum," and a full chapter of the resource was dedicated to bringing closure on this rote approach to teaching. It was difficult to read the indictment of rote learning without realizing the number of times that I actually focussed on the immediate gains rather than the long term benefits for my students. That is a chapter that every teacher should read in order to reflect on one's own teaching practices.



There have been many instances (far too many to count) where I have created a task which I thought would be a wonderful and interesting learning and assessment activity for my students, but only to have it fall flat. Fortunately, the body of this handbook can come to the rescue by focussing on many aspects of the development and use of a rich, useful task. The authors examine issues such as structure and relevance as they relate to developing tasks and assessing student performance. They have provided teachers with a tool in the form of a checklist for judging the richness of any learning task. If I had read this resource sooner, I may not have had as many frustrating experiences.

What is a rich task without a rubric? I have a shelf piled high with different rubrics. Some were effective and some have simply migrated to the bottom of the stack. The authors of this book have obviously taken the time to thoughtfully develop quality rubrics that can be used as examples for teachers as they develop their own rubrics. Too often rubrics are developed for the sole purpose of organizing achievement into levels so that we can quickly collect information about our students. It was nice to see that the authors included a "Student's Rubric" that was written in language that is more appealing to students. Perhaps if we wrote more rubrics of this nature, students would take more time to use and reflect upon them.

The plethora of learning expectations that we are required to cover can often focus our attention on the less critical and general expectations, causing us to lose sight of the key concepts of the curriculum. As the mounds of rubrics, rich learning tasks, exemplars, and EQAO assessment materials begin to grow, it is often difficult to keep everything in perspective. Fortunately, this handbook provides a framework for realizing a meaningful and manageable vision of quickly changing mathematics classrooms. It is not only a resource for the new teacher who is just learning the "ropes"; it is also a resource for anyone looking to get a better grasp of the many reforms in mathematics education and to improve their own teaching skills. And even if you do not adopt the suggestions, at the very least, it can stimulate discussion and debate in your mathematics department. ▲

To purchase a copy of the resource, contact the Centre for Mathematics, Science and Technology Education, Faculty of Education, Queen's University, Kingston, ON K7L 3N6

▲ Developing Higher-Level Thinking Skills Through Math Journals

SARAH GERMANN



Sarah Germann is in her sixth year of teaching with the Waterloo Region District School Board. She is currently teaching half of the Grade 4 programme as well as the primary French programme at Trillium Public School in Kitchener. The article is a product of a lengthy professional development workshop entitled, Action Research. *This workshop, offered through the W.R.D.S.B., allowed teachers an opportunity to discuss and reflect on a personal question in their teaching.*

Quite honestly, I began to implement journals in my mathematics class to avoid panic and frustration during the grade 3 provincial EQAO assessment where students are required to explain and rationalize their thinking. In math journals, I asked students to explain their thinking about problems and solutions that we worked on during math classes. When I first used math journals two years ago, I was increasingly discouraged by students' apparent lack of effort to think beyond what had been discussed in class. Students would remark, "It's too hard," or "I don't get it." I was frustrated with the lack of time to help students develop and improve their written work. They were frustrated with their inability to communicate their ideas clearly in written form. Convinced that these activities were well within their capabilities, I was not willing to give up.

I have always provided my students with ample opportunities to explore, manipulate, and discuss concepts during mathematics classes. It is during discussion time that the students, as a class or in small groups, are able to explain their ideas. I have also modelled some written answers and provided them with

the opportunity to write. It is at this point that many get frustrated, saying, "It is too hard." I have stumbled upon this dilemma during both of my years teaching grade 3; however, my first group seemed to tackle their problems in a more positive way. These students were very strong academically. Very few of them struggled with language and they loved to be challenged. We enjoyed math journals and they came to be very proud of their work.

My grade 3 class this year has many students who are in earlier stages of language skill development, and they get extremely frustrated when asked to write in math class. At the beginning of the year, with the announcement of math journals, heads would drop and children would groan. There would be very little eye contact, and when they would actually start writing, many students would just sit and stare at the blank page.

Obviously, I needed to do something, so I tried an experiment with two of my students who are very strong in mathematics, but who struggle a great deal with writing in their math journals. On two separate occasions, I allowed these students to dictate entries for their math journals rather than write them. At first, both struggled to organize their thoughts, but they were able to rethink and revise their thoughts as they went along. They were more willing to try and elaborate on their thinking. It was wonderful to see their level of frustration decrease.

I also gave my students a questionnaire that would be submitted anonymously. I asked questions about reading and writing, and I also asked them to try and identify the most challenging aspect about math journals. The most interesting answer I got, though, did not have to do with mathematics. One particular child was frustrated about constantly erasing her work and trying to get her ideas on the page before they were gone. Another concern was the uncertainty of what students were supposed to write. They were afraid that their ideas would be classified as "stupid".

After reading the surveys, I initiated large group discussions around some of the information that they had provided. Together, we discovered that one of the major stumbling blocks was fear. Not only were they afraid of making mistakes with their ideas, but for many of them, writing was a challenge. I decided that they needed to gain confidence in expressing their own ideas.

We began writing more math journals together, and I allowed them to copy and include any of their own ideas. Most students just copied what we had done together, but to my delight, some added extra examples or different pictures. They were also given the opportunity to work in small groups to solve problems and explain their solutions. Choosing one person to scribe was a bit of a challenge, but their willingness to share the group's ideas was increasing. Shortly after this, more students were willing to attempt writing on their own.

As a relatively new teacher, I have underestimated the power of modelling how to write journals. I have also learned that I must mostly focus on mathematical ideas rather than on writing style. At the grade 3 level, sentence structure and spelling can be left for another time, and the students must be aware that the emphasis is with their ideas. It is too overwhelming for them to focus on both.

Given the proper tools and support, grade 3 students are capable of developing their higher-level thinking skills through math journals. They must learn to value their ideas and the ideas of others. Most importantly, their ideas must be treated with delicate hands in order to preserve and enhance their confidence and willingness to share. ▲

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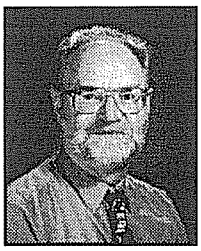
Mathtype™ was used to create formulas in this issue of the OAME/AOEM *Gazette.*

Courtesy of Design Science.

www.mathtype.com

▲ Technology Corner: Using *The Geometer's Sketchpad*™ to Explore the Triangle Congruence Postulates in Grade 7

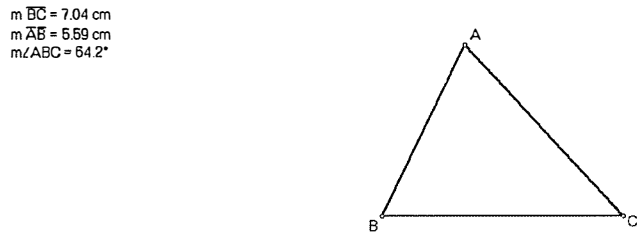
FRED FERNEYHOUGH
HEAD OF MATHEMATICS, CENTRAL PEEL SS



Fred Ferneyhough is co-chair of the Peel Math Heads' Association, and Technology Consultant for McGraw-Hill Ryerson's MathPower 9 *and* MathPower 10 *textbooks. He is an avid user of technology in the classroom.*

Recently, *The Geometer's Sketchpad*™ was ministry licensed for use throughout the province of Ontario. It has been widely implemented as a major part of the new grades 9 and 10 curriculum, and will be used in the new grade 11 curriculum. However, there is no reason why it could not be used in elementary grades as well. What follows is a set of activities that address the congruence expectation in grade 7 through *The Geometer's Sketchpad*™, "Identify through investigation the conditions that makes two shapes congruent."

Using *The Geometer's Sketchpad*™ To Explore the SAS Case



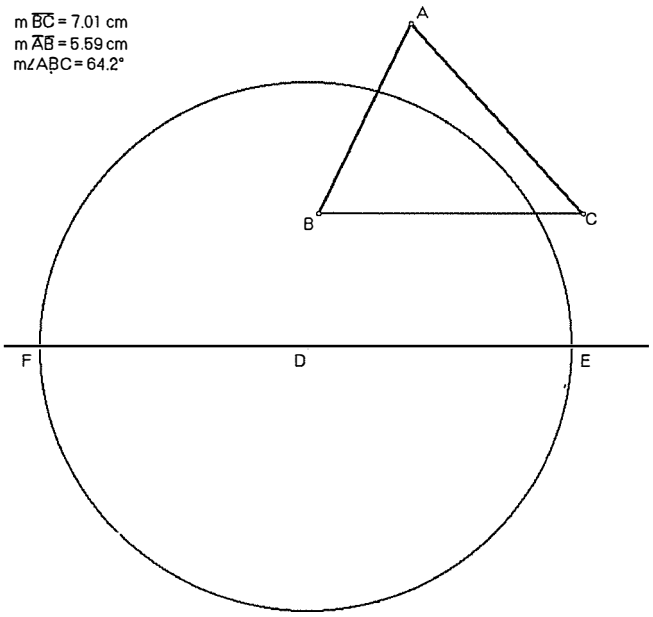
1. Construct an acute-angled triangle ABC in the top right corner of the screen. Measure side AB, side BC, and ∠ABC. Construct point D in the lower part of the screen.

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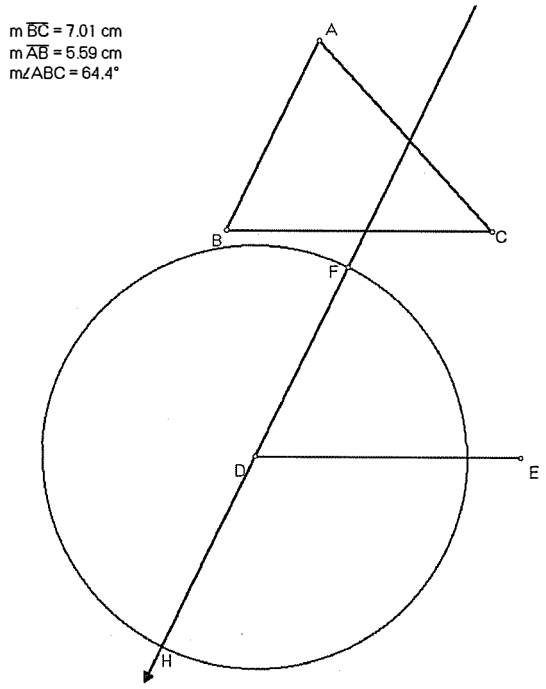
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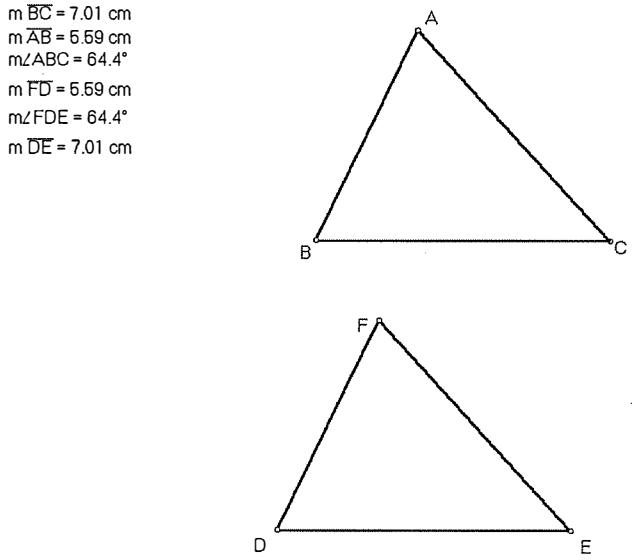
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2. Highlight point D and line segment BC. Select "Parallel line" from the Construct menu. Highlight point D and line segment BC again. Select "Circle By Center And Radius" from the construct menu. Highlight the circle and the line through point D. From the Construct menu, select the option "Point At Intersection". There will be two points of intersection. Pick one point to the right of D and be sure that it is labelled E. Highlight the other point, the circle and the line through point D. Select "Hide Objects" from the Display menu.

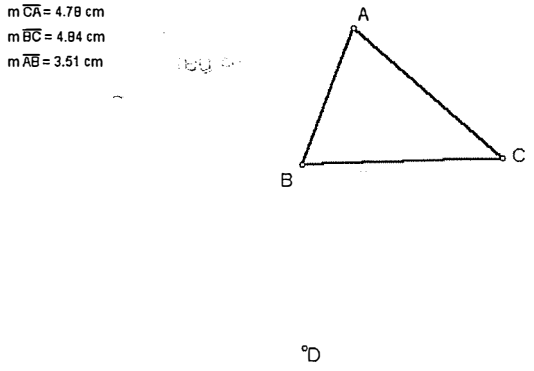


3. Highlight points D and E. Select "Segment" from the Construct menu. Highlight point D and line segment AB. Select "Parallel Line" from the Construct menu. Highlight point D and line segment AB again. Select "Circle By Center And Radius" from the Construct menu. Highlight the circle and the parallel line through point D. Select "Point At Intersection" from the Construct menu. Again, there will be two points of intersection. Select the point that will create an acute angle at D and label it F.

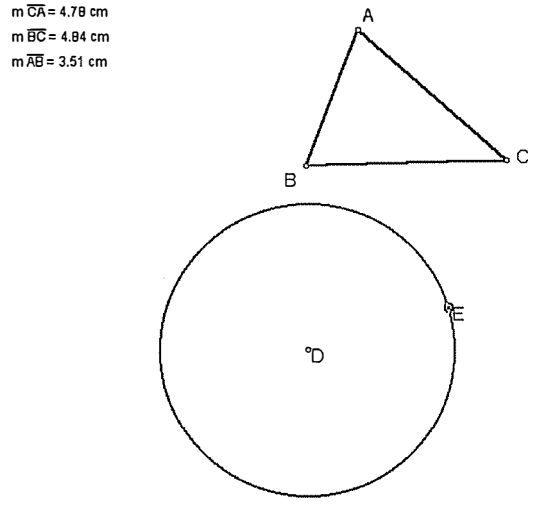


4. Highlight the other point of intersection, the parallel line through D and the circle. Select "Hide Objects" from the Display menu. Highlight points D and F. Select "Segment" from the Construct menu. Measure the lengths of DE and DF and the degree measure of ∠FDE. How do these values compare with the corresponding values in the original triangle? Highlight points F and E. Select "Segment" from the Construct menu.
5. Measure the third side in each triangle and the degree measures of the other two angles in each triangle. How do these values compare? What does this imply about the two triangles? Drag point C to different locations. Are the sides and angles still equal? Will they still be equal if the original triangle has an obtuse angle?

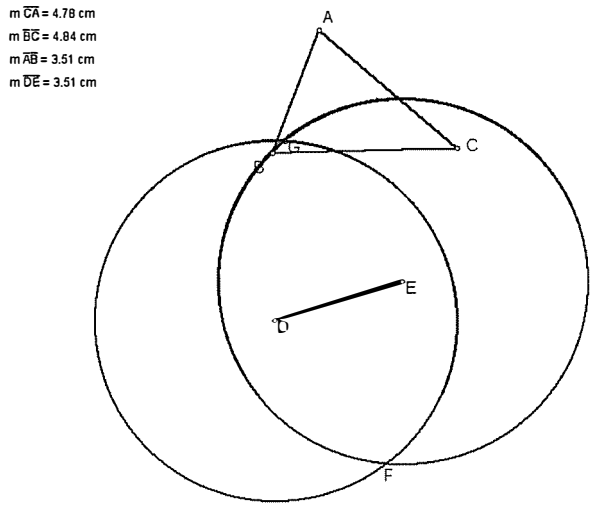
Using *The Geometer's Sketchpad*™ To Explore the SSS Case



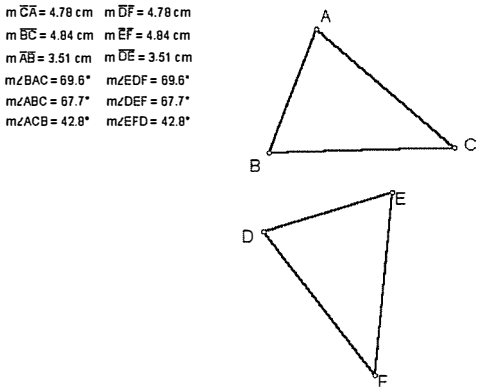
1. Construct any triangle ABC in the upper right part of the screen, and point D in the lower portion of the screen. Measure the lengths of all three sides of ΔABC.



2. Highlight point D and line segment AB. Select "Circle By Center And Radius" from the Construct menu. With the circle highlighted, select "Point On Object" from the Construct menu. Label this point E. Highlight the circle and select the "Hide Circle" option from the Display menu. Highlight points D and E. Select "Segment" from the Construct menu. Measure the length of line segment DE.

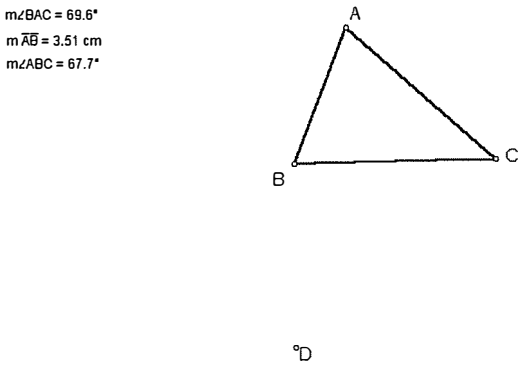


3. Highlight point D and line segment AC. Select "Circle By Center And Radius" from the Construct menu. Highlight point E and line segment BC. Select "Circle By Center And Radius" from the Construct menu. With both circles highlighted, select "Point At Intersection" from the Construct menu. There will be two points of intersection. Label one of these points F.

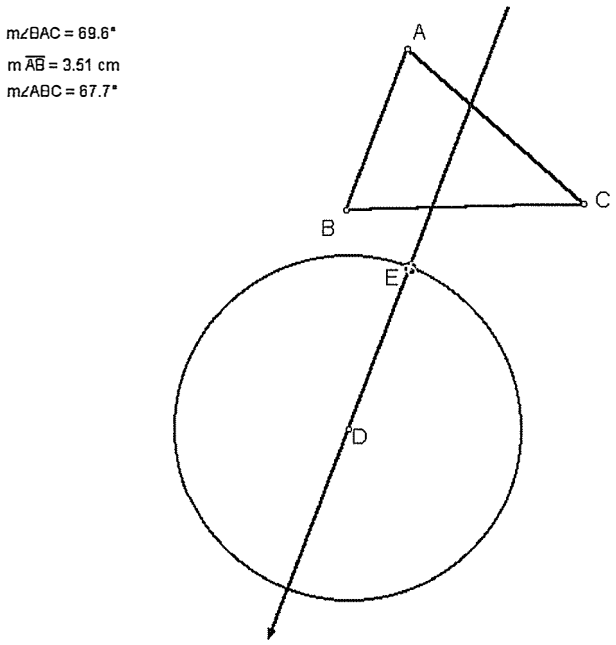


4. Highlight both circles and the other point of intersection. Select "Hide Objects" from the Display menu. Highlight points D, E, and F. Select "Segment" from the Construct menu to complete the triangle. Measure the lengths of DF and EF. How do these measurements compare with the lengths of the corresponding sides in ΔABC? Find the degree measure of the angles in each triangle. How do the measurements of corresponding angles compare?
5. Drag any of the vertices from ΔABC to a new location. What happens to the lengths of corresponding sides and degree measures of corresponding angles? What can you conclude about two triangles that have three pairs of corresponding sides that are equal?

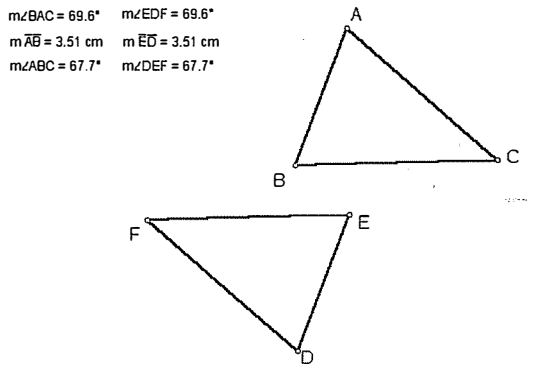
Using *The Geometer's Sketchpad*™ To Explore the ASA Case



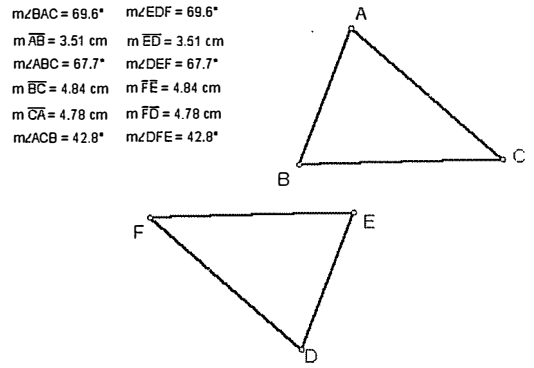
1. Construct an acute-angled triangle ABC in the upper right part of the screen, and point D in the lower portion of the screen. Measure the lengths of side AB and the degree measures of ∠BAC and ∠ABC.



2. Highlight point D and line segment AB. Select "Parallel Line" from the Construct menu. Highlight point D and line segment AB again. Select the option "Circle By Center And Radius" from the Construct menu. Highlight the circle and the line through point D. Select "Point At Intersection" from the Construct menu. There will be two points of intersection. Choose one and label it E. Highlight the circle, the line through D, and the other point of intersection. Select "Hide Objects" from the Display menu. Highlight points D and E. Select "Segment" from the Construct menu. Measure the length of DE. How does it compare to the length of AB?



3. Highlight point D and line segment AC. Select "Parallel Line" from the Construct menu. Highlight point E and line segment BC. Select "Parallel Line" from the Construct menu. Highlight both lines. Select "Point At Intersection" from the Construct menu. Label this point F. Highlight the two lines again. Select "Hide Lines" from the Display menu. Highlight points D, E, and F. Select "Segment" from the Construct menu to complete the triangle. Find the degree measures of ∠EDF and ∠DEF. How do these measurements compare with the measurements of the corresponding angles in ΔABC?



4. Find the measurements of other sides and angles in each triangle. How do the measurements of all corresponding sides and angles compare?
5. Drag any of the vertices from ΔABC to a new location. What happens to the measurements of corresponding sides and angles in the two triangles? What can you conclude about two triangles that have two pairs of corresponding angles and the contained corresponding side that are equal? ▲

▲ DIRECTORS' DIALOGUE



MARILYN HURRELL, NWOAME
LAKEHEAD DISTRICT SCHOOL BOARD
THUNDER BAY, ON

This edition of the Directors' Dialogue *summarizes the highlights of the OAME Board of Directors' meeting on Saturday, February 10, 2001 at the Four Points Hotel in Mississauga. Kirsten Boucher pointed out that this date is a palindrome!*

PRESIDENT'S MESSAGE: Mary Howe, President, and Shirley Dalrymple, President-Elect, welcomed those present. The weather was cold, with ice and snow, but most board members braved the worst to attend the meeting.

Mary reported on the following OAME initiatives:

- the Elementary School Exemplars Project has been restructured and some OAME members are on the writing teams;
- the Middle Years *Linking Assessment and Instruction in Mathematics* document will be ready for the Leadership conference at the end of February;
- the grades 11-12 committee has created a 4 page flyer outlining various paths through grades 11 and 12. This will be included in the March 2001 *Gazette* mail-out to OAME members;
- Myrna Ingalls has drawn up a flyer which explains the differences in the various grades 11 and 12 courses;
- work on the grade 11 profiles are underway;
- work on the elementary curriculum units have started.

A letter of resignation was received and accepted, with regret, from Jack Weiner. Jack has been an OAME board of director for many years and an editor of the *Gazette.* He will be missed at board meetings!

EXECUTIVE DIRECTORS' REPORT: Dave Alexander reported that membership, as of February 4,

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2001, is 1260. He feels that the retention rate is good, although some chapters (NWOAME, Y⁴MA, and ISOMA) are down in numbers.

The search for a new Executive Directors team was successful, and Sue and Dave Hessey will be introduced at the OAME conference in May. They have already begun to shadow Bonnie and Dave, and will serve as OAME's Executive Directors beginning September 2001.

LINKS TO OTHER ORGANIZATIONS:

NCTM: Frances Schatz pointed out that NCTM has an affiliates website: www.nctm.org/affiliates/

Pss..t!! Want to save money? NCTM has a 25% cap on foreign exchange for NCTM membership. Multiply the US$ price by 1.25 and mail NCTM a cheque in Canadian funds for that amount. Do not use your credit card to become/renew your NCTM membership as you will pay a whopping 51% exchange rate!

FIELDS' INSTITUTE: Shirley Dalrymple reported on the following:

- The Data Management task force is meeting with textbook publishers to discuss the new course. The task force will offer in-service and resources;
- The Secondary-Tertiary task force is working to inform colleges and universities about the issues that surround the double cohort and the new curriculum;
- The Teacher Education task force is concerned about teacher shortage and will encourage faculties of Education to increase enrolment in areas of need;
- The On-Line Learning task force is meeting to identify the role of the teacher in such learning and

▲ NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2002-2003

Each year, elections are held for a President-Elect, a Vice President (3 year term), and 6 to 9 Directors (3 year term) of OAME/AOEM. Nominations for these positions are to be submitted by 4:00 p.m., January 18, 2002, to:

Mary Howe, Past President

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2002, the Nomination Committee will publish a slate of candidates, including qualifications, for consideration by the membership. Voting will be by mail, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Barrie, May 2002.

NOMINATION FORM

I nominate: _____

For the position(s) of: ☐ President-Elect ☐ Vice-President ☐ Director

Nominee's Address: _____

_____

Telephone: H ( ) _____ W ( ) _____

Nominator's Name : _____

Nominator's Address: _____

Telephone: H ( ) _____ W ( ) _____

to ensure high quality development of the On-Line Learning.

MINISTRY OF EDUCATION: The Elementary School Exemplars Project is underway. Myrna Ingalls reported that transfer course descriptions and grade 11 Workplace Profiles are now available.

OMCA: Stewart Craven reported that OMCA is looking at the material needed for implementation of the new grade 11 courses. Subject to government funding, workshops are being planned around *The Geometer's Sketchpad*™ for grades 9 and 10, with a special session for grades 7 and 8.

ONTARIO TEACHERS' FEDERATION: The OTF meeting was also held February 10; watch for a report in the next *Gazette*.

PROFESSIONAL DEVELOPMENT:

- **OAME'2000:** Peter Lenczewski reported that the OAME conference, hosted by COMA, doubled COMA's membership and netted $28 000 for OAME. Mary Howe thanked Peter and all the committees for a highly successful conference;
- **Leadership'2001:** 121 teachers have enrolled for this conference;
- **OAME'2001:** the website is http://schools.tdsb.on.ca/same, and on-line registration is encouraged;
- **OMO'2001:** Wayne Fletcher and Marilyn Hurrell reported that all is going well for the Northern Adventure. So far, 18 teams have registered for the Ontario Mathematics Olympics in Thunder Bay;
- **OAME'2002:** The chairs for this annual provincial conference, hosted by MAC^2, are Jon Rae, Lyn Vause, and Linden Grey. The location is tentatively at York University.

COMMITTEE REPORTS:

AWARDS: Peter Saarimaki was awarded the Life Membership for his many years of contribution to OAME. Mary Bourassa has been awarded a FLAMeS Award by NCTM.

FUTURES/PROMOTIONS: The Pine Ridge chapter may host OAME'2003. Y^{4}MA and WOMA will host OMO in 2002 and 2003 respectively. Chapters are asked to consider hosting OMO'2004 and 2005.

The committee is reviewing the NCTM Outreach Kit. It is also updating the OAME conference planning handbook and developing a webpage that will feature OMO problems.

MATH EQUITY: Emilia Veltri reported that the committee will ask Howard Hampton to speak at the OAME'2002 equity breakfast. Stewart Craven is the new chair of the committee.

NOMINATION: Members who are leaving the OAME board this year were thanked for their years of service. New faces are needed and chapter reps are asked to promote this at their meetings.

PUBLICATIONS: Dean Murray thanked Jack Weiner for his 20 years of service to the OAME board. Doug Evans asked for website addresses of interesting articles for math teachers.

The Vice-Presidents' Task Force on Publications: The *Gazette* is meeting the needs of membership, but we need to expand the reviewer base and the solicitation of articles; this is a big job for the editor alone. Elementary teachers need to be encouraged to contribute to the *Abacus*.

Do you have an article that you would like to see published? Contact either Louis Lim, Editor of the *Gazette,* or Frances Schatz, Editor of the *Abacus*.

JK – 5: Laurie Moher reported that the committee will now post one problem on the website and show how it can be developed from JK to 5.

6 – 8: Carol Lemmon reported that the committee will write an article for the *Gazette* describing a connecting activity. The implementations of the Middle Years *Linking Assessment and Instruction in Mathematics* document was discussed.

9 – 10: Wayne Fletcher reported that the committee has completed course outlines around the course profiles for grades 9 and 10 Academic Public and grade 10 Academic Catholic. They are working on grade 9 Academic Catholic, and grade 10 Applied for both Public and Catholic. They will also create a "virtual binder" of summative tasks for all courses which teachers can access.

11 – 12: Myrna Ingalls reported that Mary Howe was given suggestions for her response to a Ministry of Education questionnaire re: implementation needs in

grade 11. She would like to gather a group to identify the big ideas in each course and then create summative assessment tasks for these big ideas.

CHAPTER REPORTS

These are few, perhaps due to the weather!

COMA: COMA held several mini-conferences throughout the year. They are sending two teams to OMO'2001 in Thunder Bay.

MAC²: The Annual Spring Dinner Conference will be held on April 12. Peter Saarimaki will be a speaker.

NOMA: NOMA held a very successful conference in October and have monthly meetings that focus on a relevant topic in math. NOMA plans to send a team to OMO'2001.

NWOAME: All energy is focussed on hosting OMO'2001.

PINE RIDGE: The Annual Meeting is March 26. Pine Ridge will send two teams to OMO'2001.

QSLMA: A reorganizing meeting is planned for May.

SAME: Energy is focussed on OAME'2001. SAME is also sending two teams to OMO'2001.

SWOAME: Plans are being made for a regional OMO.

TE²Y²MS: A successful mini-conference was held in February featuring Mary Lou Kestell as key note speaker. Local OMO is March 3, and three teams will compete in OMO'2001.

WOMA: The spring conference is April 11. This is a busy chapter with monthly enrichment sessions being held for kids from many grades. A busload of mathletes will participate in the American Regional Mathematics League Competition in Pennsylvania. Two teams are being sent to OMO'2001.

Y⁴MA: A successful fall conference was held. A regional OMO is planned and two teams will travel to OMO'2001 in Thunder Bay.

Thanks to Bonnie Alexander for her thorough Minutes. I used them to compensate for my late delivery of this column due to horrendous computer malfunctions.

Did I mention that Dave Handley, of QSLMA, won the last trivia contest on Thunder Bay? Here is an easy one: List, in order, the next 5 palindromic dates.

Marilyn ▲

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ISOMA - TBA
MAC² - Lisa Abernethy

NOMA - TBA
NWOAME - TBA
PINE RIDGE - Jacqueline Hill

QSLMA - Wendy Gallant

SAME - TBA
SWOAME - Fronk Di Pietro

TE²Y²MS - TBA
WOMA - John Ferguson

Y⁴MA - Rasey Mastrofrancesco

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