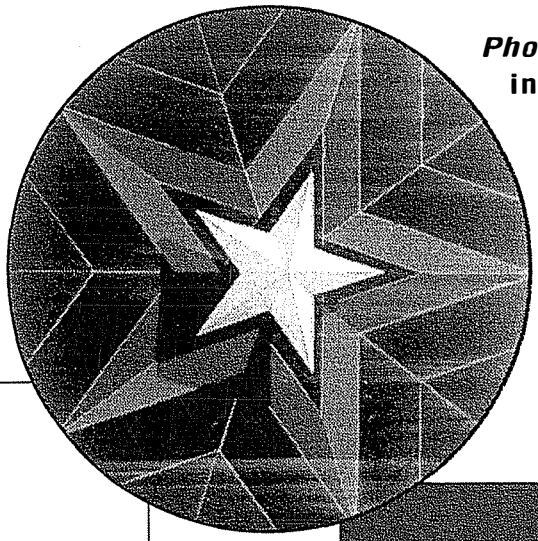
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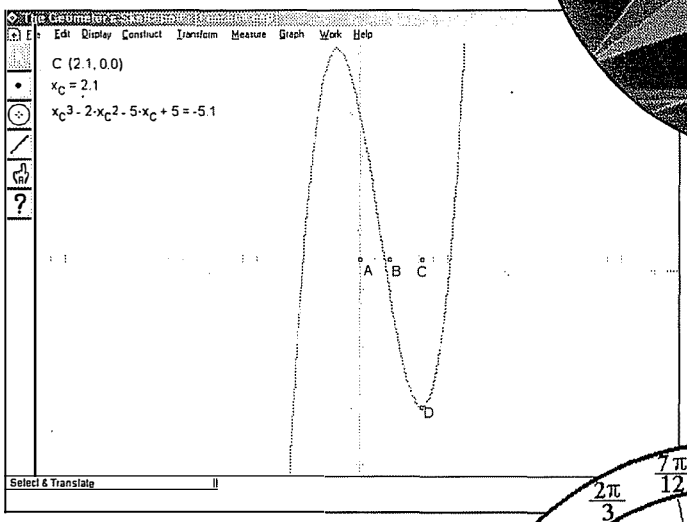
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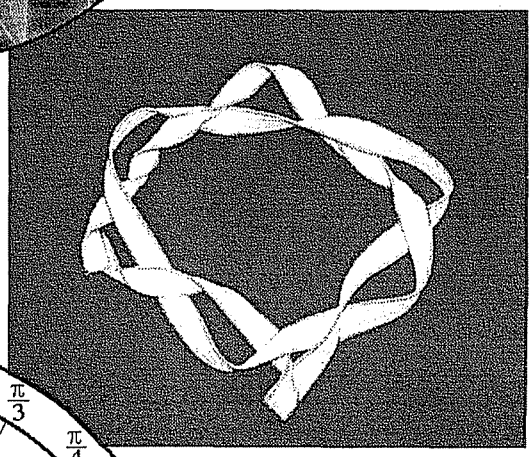


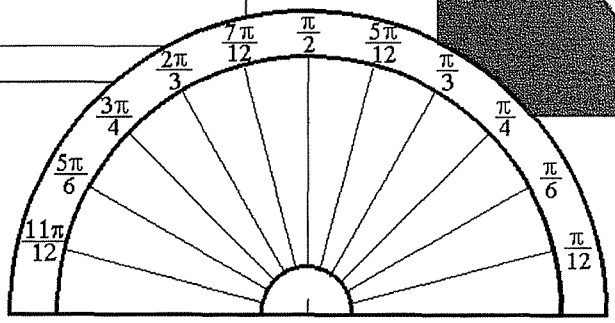
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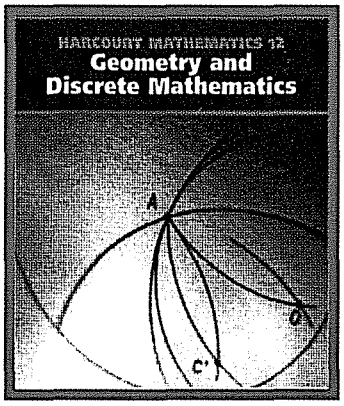
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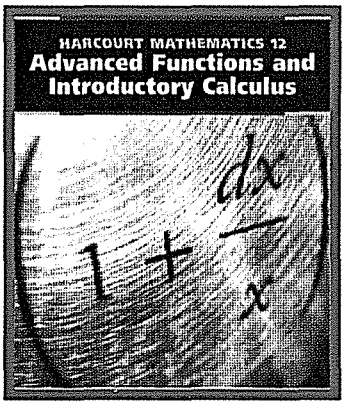
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The *Ontario Mathematics Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

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Refereed articles for the *Ontario Mathematics Gazette* should be submitted to **Louis Lim** at the address on the left. Deadlines are May 15 for the September issue, September 15 for the December issue, December 15 for the March issue, and March 15 for the June issue. Since articles are blind-reviewed, please send your full mailing and e-mail addresses on a page **separate** from the manuscript. Your identity must not appear anywhere in your manuscript. Submit **four** copies to the editor. Computer files of text and original photos and graphics, where possible, will be required later if your article is accepted for publication.

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▲ Editorial



LOUIS LIM, EDITOR
E-MAIL: Louis_Lim@edu.yorku.ca

"Erdös had an ability to inspire. He would take people who already had talent, that already had some success, and just take them to an entirely new level. His world of mathematics became the world we all entered" (Dr. Joel Spencer, as cited in *Wonders of Numbers,* by Clifford Pickover, p. 110).

The December 2001 issue of *Professionally Speaking* mentions that the current teacher shortage, throughout the entire province, will last until the end of the decade. The Ontario College of Teachers mentions that "by the year 2005, about 50 000 of Ontario's 124 000 full-time teachers in the publicly funded systems will have five years experience or less" (*Professionally Speaking*, September 2001, p. 23). Our profession is being injected with youth who are armed with the latest in educational research.

Although that is very exciting, studies have been conducted which show disturbing conclusions; for instance, "In the USA, 22% of all new teachers leave the teaching profession in the first three years because of lack of support and a 'sink or swim' approach" (DePaul, 2000, p. 1). Unfortunately, I did not find Canadian or Ontario data, but I believe that with so many new teachers entering our profession, if they are not supported and mentored properly, then we may not be able to retain them. Reality is that new teachers are often assigned the most difficult courses to teach; yet these probationary teachers are often evaluated in the same way as permanent teachers. In addition, the numerous government initiatives (e.g., teacher testing) and the negative portrayal of teachers in the media makes one wonder why anyone would want to become a teacher these days. Speaking with new teachers, responses I have gotten include: "I do it for the kids," "I want to make a difference," and "I've always wanted to be a teacher." Our new teachers are determined men and women, but with the constant "teacher bashing" in the media, the numerous changes to curriculum, instruction, and assessment, along with a system that can isolate teachers in their own classrooms, we "teachers in the trenches" need to support and mentor our new colleagues so that they

do not become discouraged.

Looking back on my six years of teaching, I have been very fortunate to work with some outstanding colleagues. During my first year of teaching at St. Mildred's Lightbourn School, I was assigned Ron Lancaster as my mentor. This independent school has a mentoring system in place where new teachers are assigned a "buddy" to work with. Ron was my department head in mathematics, and we were lucky to share the same preparation period. As a result, I spent hundreds of hours learning and working with one of the very best. We discussed innovations in mathematics instruction, teaching, and learning, along with immediate issues such as classroom management and parental involvement. Ron was an exceptional mentor since he was an excellent listener, allowed me to solve my own problems rather than tell me what to do, and truly believed that I could be an effective educator. Ron served as a trampoline to pull me back up whenever I felt discouraged.

During my second year of teaching, I was assigned Sue Catherwood as my mentor. She was my department head of mathematics at Quinte SS in Belleville. Sue exhibited similar qualities to Ron. She, too, was an excellent listener, always made herself available, and treated me as a true professional and colleague who had much promise. It was very emotional to say "good bye" to Sue after four years working in her department. As my mentor at Quinte SS, I always trusted her judgment and sought her advice and expertise.

Now that I have transferred to Unionville HS in Markham to assume the assistant headship, I feel that I have five solid years of working with outstanding people under my belt. It is time for me to assume a leadership role and mentor others (to return the favour). Unfortunately, we do not have anyone in our department who is new to the profession, nor am I hosting a faculty of education student this year. I do my part everyday by serving as "an ear" and offering support to the department.

Some schools have a mentoring program in place for new teachers, while others do not, so the new teacher may need to initiate the process. If there is no teacher in the school who is willing to serve as a mentor to the new teacher, then perhaps finding a colleague of the same age or an experienced teacher met at a conference could be helpful. I believe that this person should have similar interests, teaching philosophy, and is well respected by staff and students.

There is no doubt that I have been very fortunate to work with some exemplary mentors and colleagues. We arrive in our first teaching job as a "seed". We need people to help that "seed" develop, and with proper nourishment, to grow. A mentor has the capacity of turning a promising novice teacher with "raw ingredients" into an impacting and effective educator. It is our duty to help our younger colleagues succeed in these exciting, yet challenging times. New teachers need not experience the journey alone.

Be sure also to read John Egsgard's *What Makes a Good Mathematics Teacher? How Do We Find One?* as John reflects on his experiences teaching mathematics for 47 years. John also shares his vision of a mentoring program.

And lastly, the mathematics education community has lost one of its finest. Dr. John Del Grande, in his 80th year, passed away. I appreciate John Egsgard for allowing me to publish the eulogy of Dr. Del Grande, celebrating his life and accomplishments (see page 16).

References

DePaul, A. (2000). *Survival Guide for New Teachers: How New Teachers Can Work Effectively With Veteran Teachers, Parents, Principals, and Teacher Educators*. Jessup, MD: US Department of Education.

Harris, W. (2001, September). More Than the Teachers Are New. In *Professionaly Speaking* (pp.23-27). Toronto: Ontario College of Teachers.

Pickover, C. (2001). *Wonders of Numbers*. New York: Oxford University Press.

Teacher Shortage Consultation Warns Against Lowering Standards. (2001, December). In *Professionally Speaking* (pp. 38-39). Toronto: Ontario College of Teachers. ▲



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OAME/AOEM Awards Nomination Process

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Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration:

- demonstrations of excellence and creativity in mathematics education;
- specific contributions to mathematics education in the school, region, province or nation;
- demonstrates leadership in mathematics education in the school, region, province or nation;
- support of and contribution to the activities of OAME/AOEM, and
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

OAME/AOEM LIFE MEMBERSHIP AWARD

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and has accumulated 10 or more years of membership in OAME/AOEM.

KENNETH D. FRYER AWARD

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

MONA MACGREGOR AWARD

In recognition of the contributions made by Mona MacGregor, former OAME/AOEM Co-Secretary-Treasurer, OAME/AOEM established the Mona MacGregor Award. This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

MORLEY MACGREGOR AWARD

In Memory of the late Morley MacGregor, former OAME/AOEM Secretary-Treasurer, OAME/AOEM established the Morley MacGregor Award. The award is to be given annually to recognize an elementary teacher who has demonstrated excellence in mathematics education, as well as contributing to the overall development of students beyond classroom activities.

DON ATTRIDGE AWARD

In recognition of the contributions made by Don Attridge, a founding member, former president, and the previous Executive Director of OAME/AOEM, OAME/AOEM established the Don Attridge Award in the spring of 1995. The award is presented annually to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario.

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▲ LETTER TO THE EDITOR

Dear Editor:

I am most concerned that one item in the new grade ten provincial Mathematics curriculum exemplifies a disturbing philosophy. This is the introduction of Trigonometry in grade 10, in which the curriculum specifies that this topic be introduced by defining the trigonometric functions as ratios of the sides of a right-angled triangle.

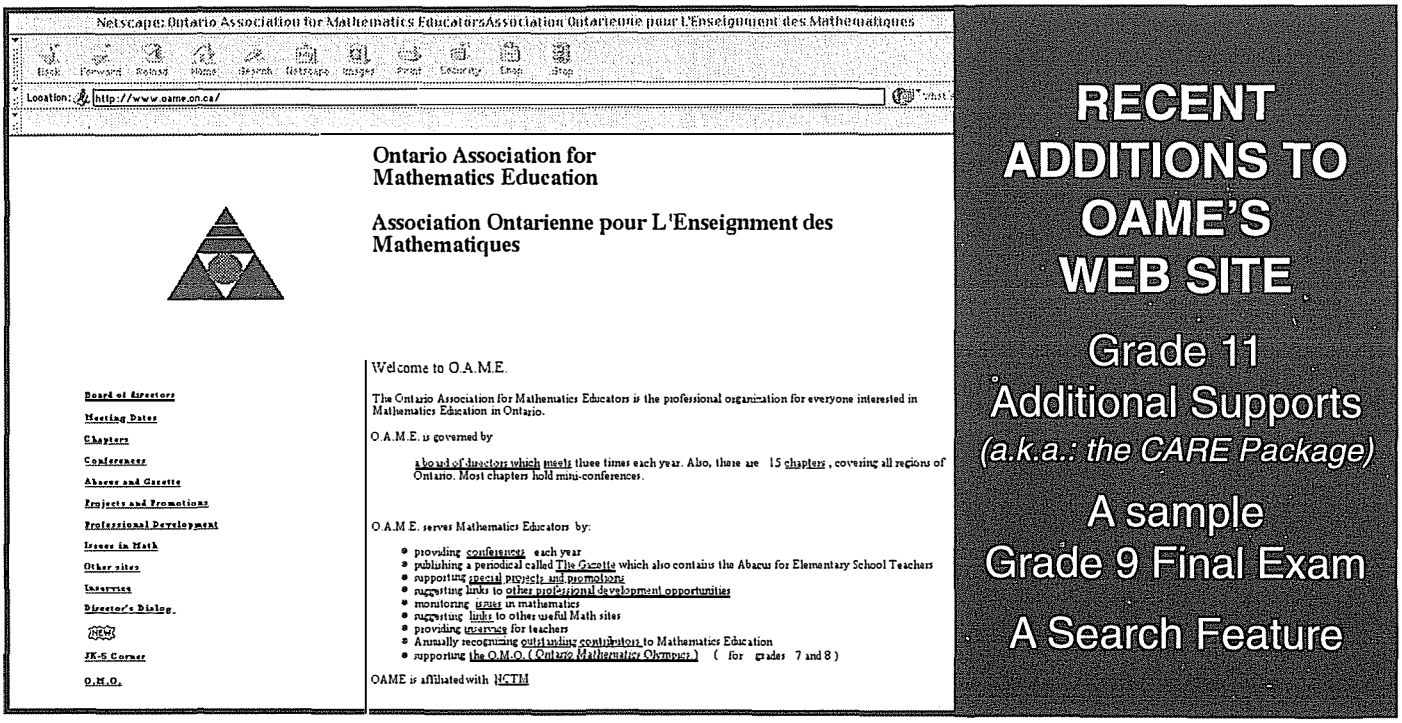
My first concern is that there has been a growing realization over the past years stating that this is not the best way to introduce these functions. Even in the grade thirteen texts of the 1960s, the definition alluding to the winding function was an attempt to actually tie the sine and cosine functions, named the circular functions, to the circle. The definition that the cosine and sine functions are the x and y coordinates of a point on the unit circle is not only more accurate and general, but is in fact, much simpler. It also directly connects the circular functions to the circle, and does not demand a redefinition of the functions at a later time to include negative angles or those greater than 90°.

I found my own experiences as a student learning Trigonometry by the right-angled triangle ratio definitions, and then later studying the concept of sine and cosine as functions, most confusing. It seemed to me originally that these were two separate trigonometries. Much later, by communicating with other teachers and reading articles and books, I understood how all aspects of trigonometry tied together very simply through the unit circle definition. I also believe that students find it much simpler learning from this definition, dealing with the right-angled triangle as a special restricted case.

My second concern, which is even more serious, is that the provincial curriculum has specified not only what should be taught, which is its purpose, but how it should be taught. This second effect is most concerning to me, regardless of how one likes to introduce Trigonometry. Teachers have there own ways of teaching concepts to students, and some are better than others. Not only that, but ways of introducing and developing topics in mathematics change and improve as imaginative, good teachers try new ideas to make concepts clearer and simpler. If the teaching method is specified, this development and evolution stops, and the improvement of teaching techniques is inhibited. That concerns me.

During the many years that I was involved in provincial curriculum development, we were always careful not to restrict good teachers by being so specific that new ideas cannot develop. In this case, the method of introduction, which is also of questionable merit, has been entrenched in the curriculum. That, I think, is a poor decision.

Tom Griffiths ▲



▲ PRESIDENT'S MESSAGE



SHIRLEY DALRYMPLE

I was very fortunate to have an opportunity to join educators across Ontario in the "Mathematics On-Line Working Meeting" hosted by the Field's Institute, University of Toronto. The goal of the meeting was for approximately 50 participants to identify the key issues and next steps for on-line learning in mathematics. The ideas shared were summarized in a white paper to help promote mathematics on-line learning as both rich and effective for students. The meeting provided participants with a lot of information, a vision of the future in mathematics on-line learning, and some ideas about what needs to be done. Here are some thoughts that I would like to share:

To live, learn, and work successfully in today's information rich society, our students that must be able to use technology effectively. Many of the students that we are teaching now, and most of the students we will be teaching in the near future, will take on-line courses to earn credits at college, university, or to train for their jobs. Many companies and educational institutions are already offering invaluable training in areas like leadership, management, and marketing in "easy-to-use" on-line programs designed for distance learning or for busy professionals. Also, most colleges in Ontario offer complete courses on-line.

However, students taking on-line learning courses in mathematics are often not successful. Those hoping to succeed on-line must make themselves familiar with the technology because anything that delays the learning or is difficult hinders the learning process. Methodology matters. Lots of content to be read on-line as well as rote and drill approaches have not been successful. Learning chat rooms and bulletin boards are not well used by students yet.

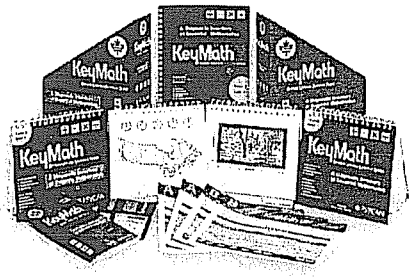
Some learning methods that technology uses well are games, simulations, and investigative supports. We have already started to enhance our teaching of mathematics with technologies such as graphing calculators, the CBR, spreadsheets, *Geometer's Sketchpad* and *Fathom!*. Brandon Hall, CEO of Brandon-Hall.com, Sunnyvale, California stated in *Training Magazine* (September 2001) that "The on-line experience provides tools that add value to all the traditional modes from classroom experiences to learning from books. It's about combining media and technologies for maximum impact." It has made me think that I would like to provide opportunities for my students to participate in on-line learning for small chunks of content and/or performance support tools. Simulations and games have a great deal of appeal to me because students are usually drawn to and motivated by them. I know that some of the materials that I would like to use already exist. My concerns are: How do I find these activities and incorporate them into my courses, and when will I have the time to do this? I believe if we can expose our students to on-line opportunities during elementary and high school that they will more likely succeed in the future on-line courses. One of the ideas shared at the on-line meeting was the building of a collection of sites organized by strands so teachers would be able to readily locate useful sites. This would certainly be helpful and save time. I will keep you posted.

My department is currently searching for sites that we can recommend to our grades 9 and 10 students who have weak prerequisite skills. We know that assigning more questions when they don't understand is not the solution, as they need both instructional intervention and practice. Games that improve skills would be well received. We are looking for web sites that can provide students with help to improve their skills in integers, power rules, polynomials, and so on. This would empower students to work on their own in school if they do not have access at home and enable parents to help their child at home if they do have access. We are also trying to convince our Board to help us by providing some money so a group of teachers could spend some time doing this. When we have assembled a collection together, we will share it with OAME, perhaps via the *Gazette*. I hope others will do the same. Together we are better.

I want to thank the members of Ontario Mathematics Coordinators' Association (OMCA) for their leadership in preparing resource materials for the new grade 12 courses. It is expected that materials will be ready for sharing in the spring and CDs will be sent to high schools in August or early September. These resources will certainly be welcomed as we prepare for six new courses in September. There will be a strong focus on both the Grade 12 Mathematics for College Technology Course (MCT4C) and the Grade 12 Mathematics for Everyday Life Course (MEL4E) because textbooks will not be available for these. This is especially important because the Mathematics for College Technology Course is an ideal alternate route to Advanced Functions and Introductory Calculus (MCB4U). At my school, we are recommending this route to any students who were successful but struggled in the 11U/C course, instead of repeating the 11U/C or trying MCB4U twice. This will also give them a valuable credit if they change their mind about their destination in their grade 12 year. OAME will also try to post materials on our web site as we get them.

I am looking forward to the Annual OAME Provincial Conference this year in Barrie. I am sure it will be very stimulating and timely. I hope to see many of you there. ▲

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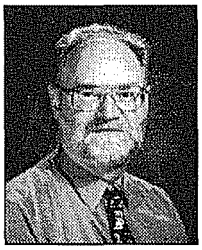
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▲ TECHNOLOGY CORNER – GRAPHING FUNCTIONS IN *THE GEOMETER'S SKETCHPAD*

FRED FERNEYHOUGH

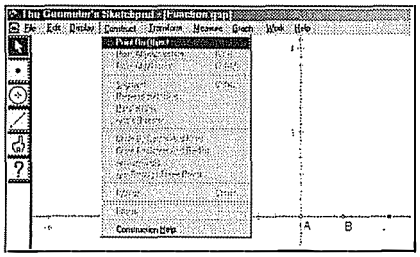


Fred Ferneyhough is Head of Mathematics at Central Peel SS in Brampton and is technology consultant for the McGraw-Hill Ryerson texts in Ontario. He is an avid user of, and advocate for, the use of technology in the mathematics classroom.

The literature that accompanies *The Geometer's Sketchpad* indicates that the software is not a function plotter. However, with a bit of manipulating, we can make it graph functions.

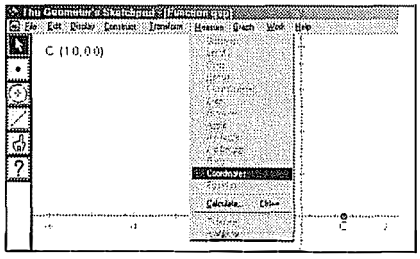
Start with a graph by selecting the **Graph** menu and selecting the option **Create Axis**. *The Geometer's Sketchpad* places two points on the grid. Point A is located at the origin while point B is placed at (1,0). By dragging point B, you can change the size of the window to a size that you like. Once this is done, it is a good idea to hide these points.

Click on the x-axis to highlight it and select the **Point on object** option from the **Construct** menu.

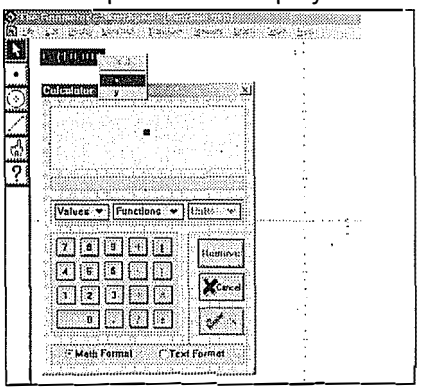


This will create a new point C on the x-axis. This will be your control point for the construction.

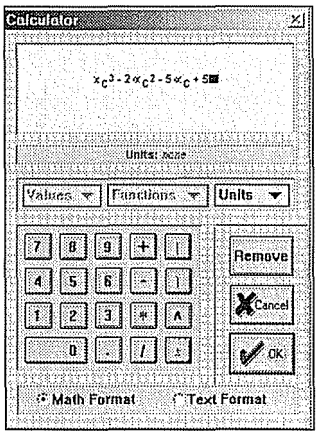
Highlight point C and select the option **Coordinates** from the **Measure** menu.



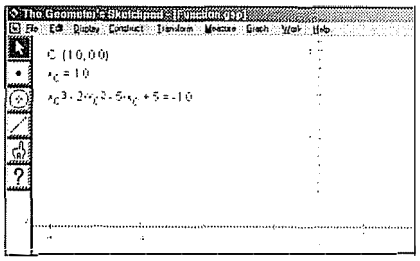
From the Measure menu pull up the **Calculate**... option. This displays what appears to be a scientific calculator on the screen, but it will do more than that. With the calculator on the screen, click on the coordinates of point C in the upper left corner of the screen. A small menu will pop up and display a choice of x or y. Move the mouse until the x option is highlighted and let go. On the calculator, you will see the character x_c representing the x-coordinate of point C. Click on the OK button and the x-coordinate will be displayed on the screen.



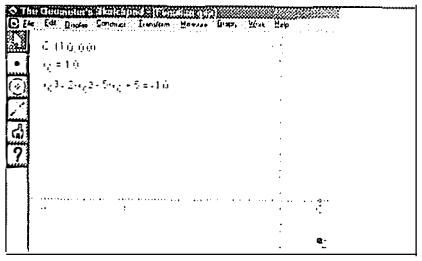
We will be treating this as the variable function to be plotted. Let's graph the function $f(x) = x^3 - 2x^2 - 5x + 5$. Each time that the variable is needed, click on the x-coordinate of point C that you have just created. The screen to the right shows what you should get when you are done. The multiplication symbols between the constants and the variable x_c are necessary. Click on the OK button again to get the next screen.



We are now going to treat the x_c as the x-coordinate and the last expression that we found as the y-coordinate of a new point. Highlight these objects in that order. From the **Graph** menu, select the option **Plot as (x,y)**.

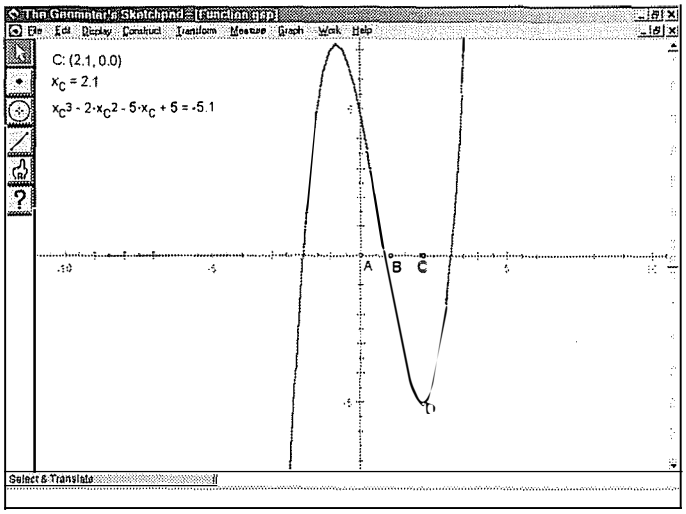


A point should appear on the screen at the coordinates that you have created. Finally, we are going to trace the path of point D as point C is moved along the x-axis. The best way to do this is to highlight point D first and then point C. From the **Construct** menu, select the

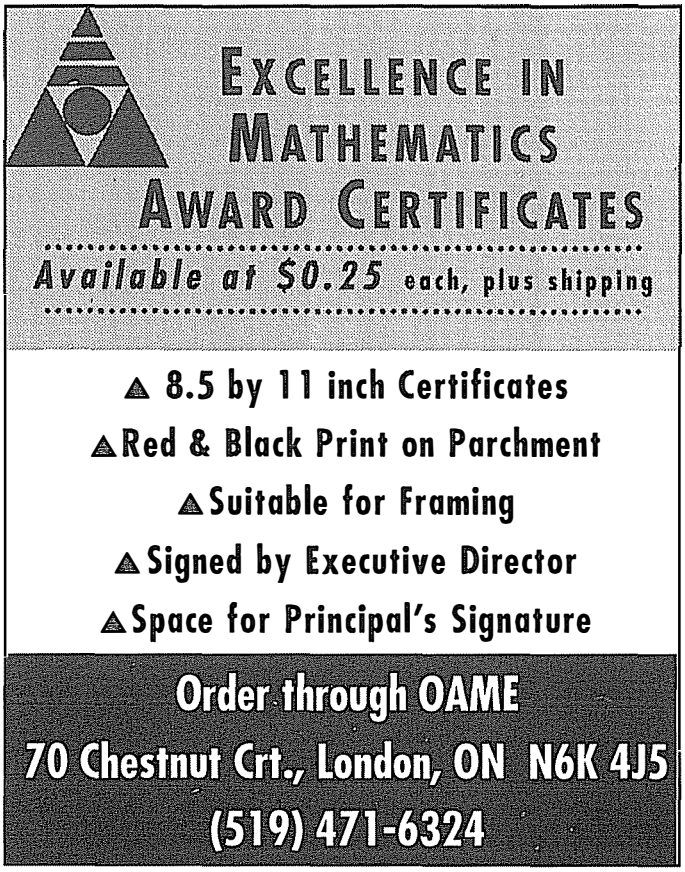


option **Locus**.

You should then get a curve drawn on the screen that is the graph of the function. Due to the placement of point B, you may not see the entire graph. Dragging point C will change the coordinates of point D on the screen and serves the same function as the TRACE button on the graphing calculator.



This is certainly not the best way to go about plotting functions, but if you have applications where you need to have a function plotted, this routine works nicely. ▲



▲ Guest Column: What is a Good Mathematics Teacher? How Do We Find One?

JOHN C. EGSGARD



John Egsgard had been retired for 5 years now after spending 47 years of enjoyable teaching. In this article, he hopes to transfer some of his love of teaching to the teachers who are just beginning their careers.

The following article is a summary of presentations and discussions I conducted in North America and Europe when I was president of the National Council of Teachers of Mathematics (NCTM). It seems to be appropriate for me to recall some of these ideas for our many young teachers of mathematics. More details can be found in the Proceedings of the Fourth International Congress on Mathematical Education, *pages 144 to 152.*

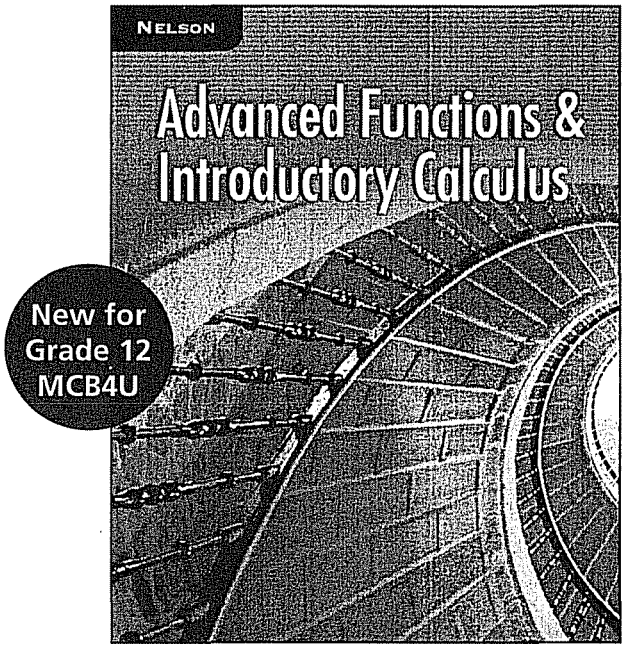
Good teachers of mathematics use their knowledge and love of mathematics, as well as their love and respect for their students, to lead these students to enjoy the study of mathematics.

I have said nothing about students being successful, for I believe that students who enjoy the study of mathematics will learn mathematics, and will be successful. My definition also makes it very simple to determine whether or not someone is a good teacher of mathematics. It is only necessary to discover the attitude towards mathematics of the students of that person. My definition of good mathematics teachers indicates that there are two key elements that will lead students to enjoy the study of mathematics. One is the love and knowledge teachers have of mathematics. The other is the love and respect teachers show their students. In other words, a personal relationship must exist between teacher and student, founded on the

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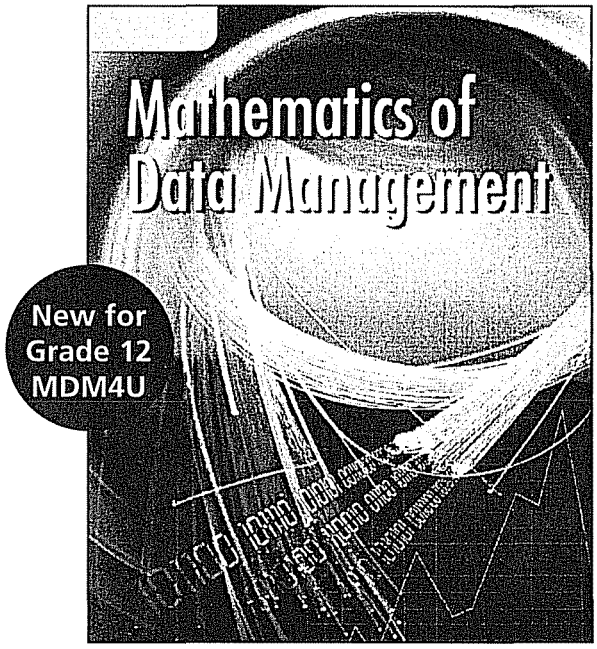
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mutual respect they show each other.

How do good teachers of mathematics show their love and respect for their students?

My experience is with high school students. In order to show these students that I loved and respected them, I tried to give each one as much individual attention as possible. To solve the dilemma of doing this in classes of varying size, I used the Socratic method of teaching that involves questions and answers – the questions being asked by both teacher and students and answered wherever possible by the students. This enabled me to ask most, if not all, of the students one question each day. This question provided daily personal contact with each student. Ordinarily, half of the class time was given over to students working alone on problems. Again this allowed me the opportunity of personal contact as I went from desk to desk answering questions, praising good work, encouraging students, and offering them hints. In addition, I was available before and after school to help students who had been away or who had difficulties that had not been cleared up during class time. In all of this, I found that praise and encouragement tended to help students try harder. Students want us to think that they are more than just students of mathematics so I found that I could increase my personal relationship with students if I showed a genuine interest in their other activities – academic, cultural, and athletic.

In the classroom, especially, the key to my effort to show respect was kindness – kindness to all students. Kindness is easy to give to someone who shows us respect. But kindness demands a true self discipline on the part of teachers as they try to encourage students who appear to be unaffected by their efforts. Meanness, lack of work, and rowdiness by the students must all be treated firmly but kindly. It is a slow and sometimes discouraging process trying to change students who dislike mathematics into being happy to be doing mathematics in one's class. Each year, I found students in my classes who only began to enjoy mathematics as the year was drawing to a close. The fact that these students did change gave me the confidence that I needed to continue working with all students no matter how hopeless it might have seemed.

Students Must Learn Mathematics Too

Yet all of the kindness and respect for students will not help unless the students find that they are learning mathematics, sometimes in spite of themselves. Indeed, successful learning and doing mathematics are absolute necessities for any student who is going to enjoy mathematics. For every time a student solves a mathematics problem successfully, especially if success seemed impossible to the student, that student has made a giant step on the road to the enjoyment of mathematics. For this to happen, the teacher has to teach so that students can learn. There are many mathematical ideas that are difficult in themselves, but good teachers of mathematics are able to peel away the unnecessary complications and present the core ideas so that these ideas can be understood. (This is why a solid mathematics background is so important.) Good mathematics teachers, through their questions and explanations, are able to help all of their students to understand and successfully do mathematics.

The previous discussion of good mathematics teachers leading students to enjoy the study of mathematics makes two assumptions: teachers have the necessary knowledge of mathematics and teachers enjoy mathematics themselves.

What is the Necessary Academic Knowledge for Good Teachers of Mathematics?

In general, as a minimum, teachers of mathematics must know all of the material that they will need to teach as well as the place of this material in the spectrum of the mathematics curriculum. This process of learning mathematics is never finished for good mathematics teachers as they must continually learn; for example, through reading the national and provincial mathematics education journals such as the Ontario Association for Mathematics Education (OAME)'s *Ontario Mathematics Gazette*, and the National Council of Teachers of Mathematics (NCTM)'s *The Mathematics Teacher*, *Teaching Children Mathematics*, and *Mathematics Teaching in the Middle School*; by attending conferences of the OAME and the NCTM, and by taking courses to keep their knowledge up-to-date.

How Do We Find Good Mathematics Teachers?

Much is demanded of good mathematics teachers. It is the obligation of the profession itself to see that mathematics teachers receive as much assistance as possible in their growth to become better mathematics teachers. If organizations, such as OAME and NCTM, are to be helpful to teachers at all levels (elementary,

secondary, and post secondary), teachers must lead the organizations from all levels. One of the gravest mistakes that organizations like these can make is to think that such representation is not needed. Mathematics education associations will be most helpful if the articles in their journals and the talks at their conferences are given by teacher experts who are communicating with people at their own level; namely, elementary, secondary, or post-secondary. I believe that teachers learn best from teachers who are experts at the same level. All of the good will in the world cannot make up for the fundamental error made in assuming that elementary teachers learn best about teaching elementary mathematics from secondary or post-secondary teachers, or that post-secondary teachers can help secondary teachers in their classroom practice. Teachers at the post-secondary level can and must assist elementary and secondary teachers to increase their knowledge of mathematics. It is only rarely that post-secondary school teachers are able to give worthwhile practical help, useful in the classroom, to elementary and secondary school teachers. Peers must aid peers. The good elementary school teacher must become an apostle assisting other elementary school teachers. The good secondary school teacher must be willing to share with other secondary school teachers. The good post-secondary school teacher must find ways to show other post-secondary teachers that good teaching is important and possible at that level.

The question now remains – Can one help a given individual become a good teacher of mathematics? First, let me make some statements that follow logically from my definition and my discussion on it:

1. Only a person with the required knowledge of mathematics for level X should become a teacher at level X.
2. Only people who are happy should become teachers of mathematics.
3. Only people who enjoy other people and want to help other people should become teachers of mathematics.
4. Only teachers who want to become mathematics teachers should become mathematics teachers.
5. Only teachers who love mathematics should become teachers of mathematics.

Note that this is a list of necessary conditions and not of sufficient conditions for determining whether or not an individual could become a good teacher of mathematics. In other words, no one lacking one of the five qualities should become a teacher of mathematics. But it does not mean that everyone who has these five qualities will become a good teacher of mathematics.

I believe that the best way to help someone with the necessary conditions to become a good mathematics teacher is to assign that person as an apprentice to an outstanding teacher. Many students who graduate from teacher training institutions in mathematics are not ready to become good teachers of mathematics. Indeed, many of these teachers give up after a short time in the classroom. Those who do go on to become good teachers of mathematics usually have had much assistance from their peers who are good teachers of mathematics.

The real learning about teaching and about whether or not one should become a teacher of mathematics comes in the classroom. I firmly believe that one learns to teach by doing, not by listening. An apprenticeship type of training for beginning teachers would require "experts" to whom they would be assigned in elementary or secondary schools. These "experts" would need to be successful teachers who are currently teaching in their own classroom each day. A maximum of three teacher-trainees would be apprenticed to each of these teachers for a minimum of one year. As in other apprenticeship programs, the trainees would be paid a nominal salary.

As I envision the program, the "expert" teacher and the trainee would be responsible for the same classes. Ample time would be set aside in the school day for them to prepare classes together, to decide what is to be taught, how it is to be taught, the questions that would be asked of the pupils, and so forth. Some days the teacher would teach all the classes, and on other days, the teaching would be shared with the trainee. Time would be spent daily to discuss both the teacher's and the trainee's classes so that the trainee could understand the why as well as the how.

Conclusion

I was a teacher of mathematics for forty-seven years. During that time, I had many students ask me why I had been content to teach the "simple" mathematics of the high school level when I could be challenged more by university level mathematics. They openly wondered why it was not boring to teach the same mathematics content year in and year out. My reply has been a simple one: "I teach students rather than mathematics." Even though the mathematics did not change from year

to year, my students did. Each year I had a new group of individuals to be with in my classroom. My years in the classroom were happy ones. I treated my students with respect and they reciprocated. We all had much fun in my classroom, and we all learned. But my respect for the students was not enough for them. They needed to be successful. Somehow in each of my classes, I was able to find something that every student could do well. I was fortunate in being able to do this in classes with four students and in classes with forty-five students. The fact that my students learned, and that they enjoyed learning, brought me great joy.

Teaching students is difficult. Many people try it and cannot handle the one-to-one and many-to-one relationships that are part of teaching. Yet, too much emphasis is given to the difficulties of teaching. We must encourage our young teachers to look for more joy in their teaching. We must assure them that it is there. One of the reasons I think that apprenticeship is an essential part of teacher preparation is that it gives the teacher trainees an opportunity to share in the joy of teaching – a joy that some may believe does not exist.

As you proceed in your own teaching career, think about my definition, adjust it, change it, but think about it. Always remember the following: Good teachers of mathematics use their knowledge and love of mathematics, as well as their love and respect for their students, to lead these students to enjoy the study of mathematics. ▲

"...a theorem as 'the square of the hypotenuse of a right angled triangle is equal to the sum of the squares of the sides' is as dazzlingly beautiful now as it was on the day when Pythagoras discovered it."

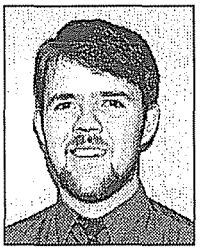
(Lewis Carroll)

"Mathematics...possesses...supreme beauty... such as only the greatest art can show."

(Bertrand Russell)

▲ Having Fun With Radians

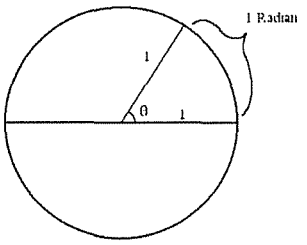
NOEL WALKER



Noel Walker is Department Head of Mathematics at Great Lakes Christian College, a private high school in the Niagara region. He is currently on sabbatical, splitting his time between studies at Brock University and working as an editor with Nelson Thomson Learning.

The first time I ever struggled to answer the classic question, "Why do we have to learn this?" came when I introduced radian measure to an OAC Calculus class a number of years ago. I discovered that high school students struggle to see the need for a new method of angle measure when degrees work well. While I can't recall my response at the time, I have since worked at finding a better one.

Why use Radians?



If aesthetics are worth anything, radian measure is a thing of beauty. On the unit circle, the angle subtended by an arc of length 1 is equal to 1 radian. Gorgeous! On an arbitrary circle, the angle is represented by the ratio of arc length to radius. Who wouldn't love a system that is so elegant! Unfortunately, that doesn't sell with the adolescent community so a better reason is needed.

Three separate tasks are made simpler with the use of radian angle measure. For starters, finding the area of a sector of a circle is awkward when using degrees to measure the angle $A = \frac{n}{360}\pi r^2$. One must, in essence, convert the angle n° to radians on the fly to complete the calculation. When using radians, the area is given by the simple formula $A = \frac{1}{2}r^2\theta$. Evaluating an Infinite Series that contains trigonometric functions is greatly simplified by using radians as the angle measure; however, this topic extends beyond the scope of secondary mathematics. Finally, radians also aid in the discussion

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of derivatives. The derivative of a trigonometric function cannot be justified without the Fundamental Trigonometric Limit; which cannot be evaluated properly in degrees.

The most obvious reason to teach radian measure is, of course, since the Ontario Ministry of Education Expectations requires it! Within the strand of Trigonometric Functions, in *The Ontario Curriculum Grades 11 and 12: Mathematics,* the second overall expectation states that students will "demonstrate an understanding of the meaning and application of radian measure" (p. 14). Five specific expectations require an understanding of radians.

One disturbing fact is that both calculating the area of a sector and finding derivatives of trigonometric functions are conspicuously absent from the Curriculum Expectations for grade 12 mathematics. It remains to be seen whether or not textbook publishers will choose to include either as optional topics in their books; never the less, to serve our students well, we need to introduce and reinforce this concept thoroughly to prepare them for post-secondary study.

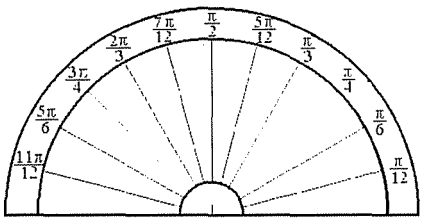
Although the idea of measuring angles with radians is likely as old as trigonometry itself, the word "radian" has only recently been added to the English language. It first appeared in print on June 5, 1873, in examination questions set by James Thomson at Queen's College, Belfast. He used the term as early as 1871, and it began to be adopted in the Mathematics community by 1874 (Cajori, 1919, p. 484).

If you get a great deal of resistance from your students when introducing radian measure, perhaps you can bait one of them into justifying why there are 360˚ in a circle! If you can get a few students in your class to be sufficiently hot under the collar about where radians originated, two great resources are http://mathforum.com/dr.math and http://mathforum.com/dr.math/problems/munroe12.3.97.html .

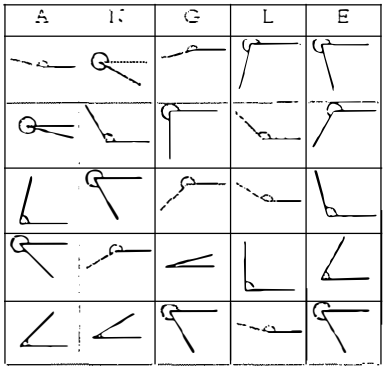
How Do You Make Radian Measure Meaningful?

Good pedagogy says that students need opportunities to construct meaning for authentic learning to take place. With that in mind, it should come as no surprise that students enter grade 11 understanding degree measure since they have already measured angles in degrees and can visually estimate angles in degrees. To make Radians real (no pun intended), students need opportunities to apply these skills within the context of radian measure. A couple of classroom tools that have helped me in the past include:

The Radian Protractor: Once the class is measuring angles in standard position, I usually introduce radian measure and then provide the students with their own Radian Protractor:



A digital copy of this image is available at http://www.noels_home.homestead.com/math.html . You can fit six on a page and then photocopy them onto a sheet of overhead acetate, which can be cut up individually. Once your class has had practice with the protractor, you can reinforce the concept with a little fun...



Radian Angle Bingo: Displayed here is one of a set of Bingo cards made with various angles. The complete set can be downloaded from http://www.noels_home.homestead.com/math.html. Use them like standard Bingo cards by calling out a letter and then an angle (some multiple of $\frac{\pi}{12}$). To increase the challenge, you can call out coterminal angles (equivalent angles that are greater than 2π).

In conclusion, there are a couple of Internet sites that have contributed to this article which you may find of interest: Jeff Miller, a teacher at Gulf High School in New Port Richey, Florida, has compiled an incredible web site full of stories on how different words came to be used in mathematics: http://members.aol.com/jeff570/mathword. html. One last Internet site that will blow your mind is Dave's Short Trig Course http://aleph0.clarku.edu/ ~djoyce/java/trig/. It features independent study materials, exercises, and a java applet that you will have fun playing with for hours.

References

Cajori, F. (1919). *A History of Mathematics.* New York: The Macmillan Company.

Ontario Ministry of Education and Training. (2000). *The Ontario Curriculum Grades 11 and 12: Mathematics.* Toronto: Queen's Printer for Ontario. ▲

OBITUARY
JOHN DEL GRANDE

The following is the eulogy for John Del Grande read by John Egsgard:

I have been asked to say a few words about John as a mathematics educator. Essentially, John was a teacher of mathematics – and a great one. He loved his students and he loved mathematics. He began his teaching in the mid 1940s while he was still an undergraduate at the University of Toronto. Several times a week, he would go to Ajax to teach calculus to returning veterans of World War II who wanted to become engineers. After graduating with a Master of Arts degree, he taught for a time at St. Andrew's College in Aurora, and then at Newmarket High School. After that, he began his lengthy career as coordinator of mathematics for the Borough of North York.

John's influence spread outside of his classroom. He was one of the small groups of mathematics educators in Ontario that helped introduce the "new math". He did this through his work with the Ontario Mathematics Commission. It was the Commission that persuaded him to write texts that would cover the new materials. John's textbook writing began in the 1950s and continued into the 1990s for a total of over 40 books. But the greatest work he did outside his own classroom was as coordinator of mathematics as he led the elementary and secondary schools of North York to become among the premier schools of mathematics in the province. Besides his work in Ontario, John gave lectures at math meetings throughout the world and wrote articles for many of the prestigious mathematics education journals in North America. Even in his last years, John assisted the University of Waterloo in preparing challenging mathematics problems for its contests.

John was a tireless worker. As a writer he led many writing teams to produce the excellent textbooks for which he was renowned. Even after his so-called retirement from North York, John continued to show his love of mathematics by obtaining a Doctorate in Mathematics Education from OISE at the University of Toronto. This love for mathematics was never ending. I visited him at the beginning of the year. In our conversation, he told me of a treatise that he had just completed in geometry.

John was a very close friend of mine who had much influence on me as a mathematics teacher. I shall miss him; so will the world of mathematics education. But we should not worry about him too much. For I can see him in heaven discussing mathematics throughout eternity with the other great mathematicians who will be with him. ▲

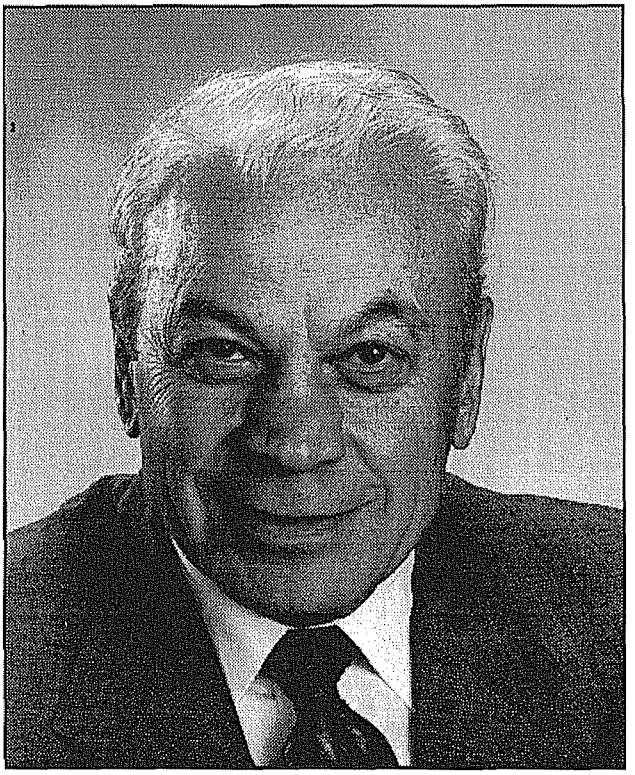


Photo courtesy of David Bates

▲ The John J. Del Grande Scholarship Established at OISE/UT

DAVID BATES

In July 2001, the Samuel Beatty Fund of the University of Toronto established the **John J. Del Grande Scholarship** to support the teaching of Mathematics in Ontario schools. The scholarship, in the amount of $1000, will be presented annually to a student entering the OISE/UT Preservice Program with Mathematics as a designated teaching subject and who has demonstrated academic excellence in Mathematics and a strong experience profile. This award honours a past chairman of the Fund, Dr. John J. Del Grande, in recognition of his dedication and outstanding contribution to Mathematics education.

John Del Grande, during more than forty years as an educator, was a teacher, research assistant, Mathematics department head, Mathematics co-ordinator, university lecturer, and author. He was a member of the National Council of Teachers of Mathematics (NCTM) since 1950 and of the National Council of Supervisors of Mathematics (NCSM) since its inception. In 1960-62, he was Vice-President of the Ontario Mathematics Commission whose mandate was to bring the "New Math" to Ontario. In 1961-62, he served as President of the Ontario Association of Teachers of Mathematics and Physics (now OATM). He founded the Mathematics Co-ordinators Association in 1968, and in 1978, instigated the Ontario Mathematics Co-ordinators and Consultants conference. From 1976 to 1982, he was the driving force behind a Computer Assisted Instruction program in Mathematics, a co-operative effort involving the North York Board of Education, the Ontario Institute for Studies in Education (OISE), and the Ontario Ministry of Education. Chairman of the Samuel Beatty Fund since 1975, John retired from this position in 1998 and continued to serve the Fund as a Trustee.

Throughout his career, John Del Grande was a prolific writer with more than forty titles under his name. In particular, he devoted much time and effort to writing curriculum for the elementary level and was noted for his work in introducing geometry (spatial experiences) to the early grades. The many local courses and conferences he organized over the years indicated his eagerness to help his fellow Mathematics educators as well as his willingness to examine new ideas and techniques. Because of his expertise and reputation, he was a frequent speaker at local gatherings, NCTM conferences, and international congresses (ICME).

In 1965, John Del Grande received the Award of the Association of Professional Engineers of Ontario; this award recognizes secondary school teachers in Mathematics and Science who, in the opinion of Ontario Secondary School Teachers' Federation, have contributed the most to the advancement and teaching of these subjects. Three years later, the University of Waterloo awarded him the René Descartes Medal for teaching excellence and contributions in Mathematics beyond the classroom. Finally, in 1982, he was the first recipient of the NCSM Gilbert Medal, considered by many to be the most prestigious award for a Mathematics educator in North America. He is the only Canadian ever to have received this honour.

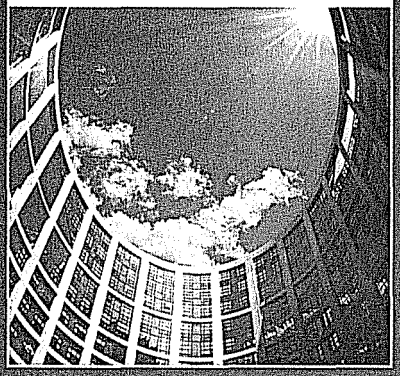
Sadly, John Del Grande passed away on Friday, November 23, 2001 after a lengthy illness. He was very pleased at the establishment of the John J. Del Grande Scholarship, particularly because of the support it provided for the teaching of Mathematics in Ontario, and it is unfortunate that he was unable to meet with **Giancarlo Brotto**, the first recipient of the award. Giancarlo completed his high school education at Brebeuf College, Toronto, and set his sights on a career teaching Mathematics. In May 2001, he graduated with distinction from the University of Toronto in an honours Specialist Program in Mathematics and its Applications with a teaching concentration. His experiences over the past six years as a tutor and substitute classroom teacher at Brebeuf, and a teaching assistant at U. of T., have strengthened his passion for teaching Mathematics in Ontario. He hopes to teach secondary school Mathematics upon completion of his degree from OISE/UT.

The Samuel Beatty Fund promotes and encourages the study of Mathematics in the University of Toronto and throughout the Province of Ontario. At the present time, the Fund provides, in addition to the John J. Del Grande Scholarship, approximately 25 in-course scholarships annually of $250 each. These are awarded on the basis of academic standing and financial need to students enrolled in specialist programmes in Mathematics, Computer Science, Statistics, and Physics at the University of Toronto. In addition, the Fund supports several outreach programmes for secondary school students. These programmes include SOAR, SIMMER, the Tournament of Towns, and several Computer Science and Statistics outreach activities. Travel assistance is also given to Ontario high school students who compete in the International Mathematics Olympiad (IMO) and the Physics Olympiad. ▲

Addison Wesley

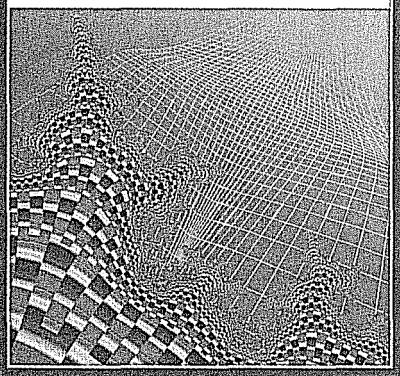
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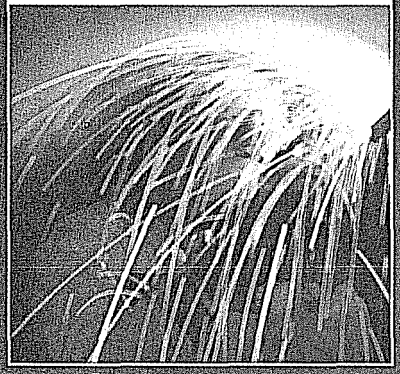
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▲ Under Construction: On Becoming a Constructivist in View of *The Standards*

CHERRY D. WARD

Cherry Ward teaches at Southeast Bulloch High School, Brooklet, GA. She enjoys helping students understand mathematics and discover its relevance in their lives.

Since the beginning of time, trends in the philosophy and methods of pedagogy have come and gone, with only minor variations from traditional instructional methods. These trends have included progressive movements, essentialist movements, teacher-centred instruction as opposed to student-centred instruction, drill and practice, project-based instruction, discovery learning, and many other movements (Cuban, 1993). In his studies, however, Cuban found that teachers have basically taught the same way, with the little variation, despite these instructional trends.

One can only speculate why significant change never took place. Many factors are involved in making changes in instructional methods within the classroom, but the central figure responsible for producing change is the teacher (Cuban, 1993; Sandholtz, Ringstaff, and Dwyer, 1997). Many educational movements never became widely accepted because teachers may have exactly followed the new guidelines for a brief time, whereas others changed only a few strategies; but soon, because of a lack of support, training, and continuity of approach, all fell back into the traditional methods of instruction.

As the old saying goes, "There's nothing new under the sun"; and so each new idea continues to present itself to educators as the answer to all their problems. Some of these methodologies are simply untested fly-by-night fads, whereas others are based on research and experience. The professional educator needs to examine and verify the claims of various trends before considering any change. Change that does not make an improvement is detrimental because it disrupts current instruction even more.

For mathematics teachers, the National Council of Teachers of Mathematics (NCTM) performs the service of determining the efficacy of various methods by conducting or examining extensive research (Hiebert, 1999). A product of its efforts is the *Curriculum and Evaluation Standards for School Mathematics* (1989) which advocates a need to change from a rote computational emphasis to an instructional approach that emphasizes critical thinking. An extension of this work is called *Principles and Standards for School Mathematics* (2000). NCTM affirms that students who actively participate in their educational processes have greater retention and become more independent learners than their peers. If teachers want to follow the ideas of the *Standards* documents, they must first learn about the research behind the NCTM's recommendations. When teachers understand the validity of the NCTM's approach, they are more likely to endure the difficulties encountered when implementing change.

Constructivism

A blend of educational theories is clearly behind the NCTM's *Standards*, all of which have been well studied and used by experienced educators. One philosophical alternative to traditional instruction that NCTM promotes is constructivism (Sandholtz, Ringstaff, and Dwyer, 1997; Brooks and Brooks, 1993). Constructivism as a philosophy is not new, but its application to modern education is still in the formative stages. Within constructivism, a variety of differing views are found. Instead of focusing on the differences, this article takes a generic look at how constructivist ideas can be used in the classroom. Simply stated, constructivism is "a belief that all knowledge is necessarily a product of our own cognitive acts" (Confrey, 1990, p. 107). By building on previously constructed knowledge, students can better grasp the concepts and can move from simply knowing the material to understanding it. Constructed knowledge promotes critical thinking, which allows students to integrate concepts within and between disciplines; to represent concepts in multiple forms; and to justify, defend, and reflect on the concepts.

The teacher plays an important role in helping students construct accurate knowledge. Students sometimes construct knowledge that is only valid in specific circumstances (Simon, 1995). The teacher then needs to offer additional situations that allow students to test their knowledge. When students realize that their construction does not work with the new information, they can make corrections and check again for validity.

Teachers must also be able to understand students' constructions that differ from their own. Davis and Maher (1990) tell of two students and their teacher who interpreted a problem differently. The problem involved two pizzas, each of which was cut into twelve pieces. The problem asked what fraction of the two pizzas was eaten if seven students ate one piece from each pizza. The two young boys used two pizzas as their unit and reported that $\frac{14}{24}$ of the pizza was eaten. The teacher, who was using one pizza as her unit, wanted them to answer with $1\frac{2}{12}$ of a pizza. The students' answer was not incorrect with respect to the presentation of the question, but the teacher was using a unit different from the one that the boys used, so their construction was incorrect to her. She explained her correct way to them but did not give them time to use their manipulatives to build the new knowledge by using her explanation. When given the same problem the next year, the boys used the knowledge that they had constructed and arrived at their original answer (Maher and Davis, 1990). This example demonstrates that the constructed knowledge was retained, whereas the information received through direct instruction was forgotten. Not only is checking for correct understanding important, but extra time should be given to construct the new information.

Communication between student and teacher is another essential aspect of constructivism. Had the teacher in the example been attuned to the boys' explanations, she would have realized that their answer was also correct. Instead, she was so focused on their different answer that she did not listen to their explanation. Communication must be present for the teacher to know how the student's knowledge has been constructed. Teachers need to realize that solutions are built from past constructions and therefore will probably differ from their own. They must be willing to accept this diversity as long as it is mathematically valid. To promote this communication in assessment, Noddings (1990) suggests that teachers add points for each step of correct thinking that students show rather than subtract points for incorrect work. Many students do not show their thinking because they lack confidence in it. If a correct answer only gets two points but the steps leading up to it are worth eight points, then the teacher demonstrates to the students that the thought process is more important than the answer.

Examples

What can the teacher do to promote the students' construction of knowledge within the classroom? An example that worked very well for my advanced algebra-trigonometry class of high school juniors and seniors was the "sine curves and spaghetti" activity (Peterson, Averbeck, and Baker, 1998). This activity was originally designed for middle school students, so I modified it to have my students actually construct the sine and cosine graphs from the unit circle. They marked their unit circle every 30 degrees. At each of these marks, they were to form a spaghetti triangle, if possible. I told them to measure and mark the length of the horizontal piece of spaghetti onto one graph and the vertical piece of spaghetti onto the other graph. To finish this activity in the ninety-minute class period, I encouraged the students to maintain a steady pace in performing all the necessary measurements and drawings. The students answered the wrap-up questions the following day.

The time spent dealing with such distractions as broken spaghetti, holes in paper from trying to work over carpet, and divided attention because of homecoming week were well worth it. This class no longer had difficulty remembering that cos θ was the x-coordinate for the unit circle and that sin θ was the y-coordinate. The understanding gained from this activity also transferred to trigonometry on right triangles and converting rectangular coordinates to polar coordinates.

Another simple lesson in construction involves the transformations of the trigonometric functions from their parent graph. This activity has more of the discovery style in its approach, but it is still constructing knowledge. Briefly stated, the activity involves having the students graph several functions along with the parent graph. They use a graphing calculator so that the activity moves along efficiently. From observing the graphs in relation to the parent graph, they determine the meaning of a, b, c, and d in the equations $f(x) = a \sin(bx + c) + d$, $g(x) = a \cos(bx + c) + d$, and $h(x) = a \tan(bx + c) + d$. Because the students must use critical thinking to construct the rules for transformations, they are more likely to remember them. If they do forget, however, they can simply reconstruct the situation at hand to determine what shift took place. They become less dependent on the teacher as the source of knowledge and learn to rely on their own strengths.

Resources

A primary concern in learning how to use a more constructivist approach in instruction is the time required to conceive and design the activities.

Because of technology, teachers have a tremendous amount of help at their fingertips. In addition to all the NCTM's publications, many sites on the World Wide Web include activities, lesson plans, and additional material that teachers can use. The teacher can go to the site, search for the needed concept or subject area,

and browse. Some of the more used sites are www.nctm.org, www.pbs.org, http://forum.swarthmore.edu, and www.ti.com. Even if an appropriate activity is not at one of these sites, the sites often include a list of links to other web sites. Another aid is to join a list-serve that addresses the area of concern. Most of the web sites listed have discussion groups that consist of teachers who are sharing ideas and asking for suggestions daily. A teacher who signs up becomes a part of a global mathematics community.

Since communication is also important in constructivism, the teacher must understand the student's thought processes. Questioning becomes imperative because it is a means to that end. The teacher should not ask questions in such a way that they lead the students to the desired response. The questions must encourage the students to further examine their work and reflect on its validity. Johnson's books (1982, 1986, and 1994) on making the mathematics classroom more motivational contain many valuable suggestions for improving questioning techniques. With the right questions, teachers can learn what the students think instead of what students think that teachers want to hear.

Conclusion

Constructivism offers promising new approaches to teaching. The research indicates that using constructivist methods – especially in the areas of instructional methodology, classroom communication, and questioning techniques – furnishes a more experiential learning environment in which the student can develop critical-thinking skills and improve transfer and retention of knowledge. Changing the present instructional methods will not be easy. To make a difference in education, teachers must become advocates for change. They must first evolve into confident, highly skilled mathematicians who are capable of relinquishing some control in instruction so that students are free to explore their knowledge. Teachers must next understand the philosophical foundation of the methods that they are promoting to ensure that all teachers, parents, and administrators support the change.

As agents of change, teachers must acquire leadership positions within the schools so that they can have a voice in budget decisions. They can urge to have money allocated for training, equipment, and in-service time so that they can become proficient in this form of instruction. And most important, teachers must build a community of teachers, within and between schools, to share strategies and offer encouragement. Finally, teachers must realize that they are the primary advocates who help students become more skilled in mathematical reasoning and ultimately construct the roads to the future.

References

Brooks, J. G., & Brooks, M. G. (1983). *In Search of Understanding: The Case for Constructivist Classrooms.* Alexandria, VA: Association for Supervision and Curriculum Development.

Confrey, J. (1990). What Constructivism Implies for Teaching. In Robert B. Davis, Carolyn A. Mayer, and Nel Noddings (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (pp. 107-22). Reston, VA: National Council of Teachers of Mathematics.

Cuban, L. (1993). *How Teachers Taught, 2nd ed.* New York: Teachers College Press.

Davis, R. B., & Maher, C.A (1990). The Nature of Mathematics: What Do We Do When We 'Do Mathematics'? In Robert B. Davis, Carolyn A. Mayer, and Nel Noddings (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (pp. 65-78). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal of Research in Mathematics Education 30*(1): 3-19.

Johnson, D. R. (1982). *Every Minute Counts: Making Your Math Class Work.* White Plains, NY: Dale Seymour Publications.

Johnson, D. R. (1986). *Making Minutes Count Even More: A Sequel to Every Minute Counts.* White Plains, NY: Dale Seymour Publications.

Johnson, D. R. (1994). *Motivation Counts: Teaching Techniques That Work.* Palo Alto, CA: Dale Seymour Publications.

Maher, C. A., and Davis, R.B. (1990). Teacher's Leaming: Building Representations of Children's Meanings. In Robert B. Davis, Carolyn A. Mayer, and Nel Noddings (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (pp. 79-92). Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics [NCTM] (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics [NCTM] (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

Noddings, N. (1990). Constructivism in Mathematics Education. In Robert B. Davis, Carolyn A. Mayer, and Nel Noddings (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (pp. 7-18). Reston, VA: National Council of Teachers of Mathematics.

Peterson, B. E., Averbeck, A., & Baker, L. (1998). Sine curves and spaghetti. *Mathematics Teacher 91*(7): 564-67.

Sandholtz, J. H., Ringstaff, C., & Dwyer, D. C. (1997). *Teaching with Technology: Creating Student-Centered Classrooms.* New York: Teachers College Press.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education 26*(2): 114-45. ▲

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▲ A New Approach to Teaching Mathematics in a Combined Grade Situation

LOIS WALKER
TRACEY VAN WILKEN

Lois Walker and Tracey van Wilken are teachers at Dickson Public School, in the Waterloo Region District School Board. The following article reflects their involvement in an Action Research Project with the Waterloo Region DSB.

We began the last school year each teaching a combined-grade class in a small school. Each class had over 26 students, several of whom were on an Individual Education Plan (IEP). We found it impossible to effectively plan program and evaluate student achievement for each grade while providing support to those needing extra help and working on IEPs. With large classes, classroom management issues, and insufficient amounts of material, it was challenging to provide hands-on activities and small group instruction. Our challenge was further increased because there were no other teachers teaching the same grade curriculum with whom to consult and/or plan. Earlier in the school year, we were able to combine our grades 1/2 and 2/3 classes to address the Social Studies and Science expectations. These units met expectations of all grades and provided an enjoyable and beneficial learning experience for the students. We were hoping to change our approach to teaching math in order to more effectively cover the curriculum and increase student involvement in lessons and activities.

Since we both had grade two students, we met and brainstormed ways that we could group them together for concept and skill lessons and have each of the other grades on their own for lessons. We developed a three-day rotation. While one grade had a lesson day, the other two grades were together, working on planned activities. There was a lesson room and an activity room. While one grade was having a directed lesson with one teacher, the other two grades were in the activity room with the other teacher, volunteers, and Educational Assistants as available. The students were divided into three colour groups. The green group addressed the Grade One curriculum expectations and consisted of mostly Grade One students and those on IEPs at the Grade One level; the yellow group addressed the Grade Two expectations, while the blue group addressed the Grade Three expectations.

Each Monday the students were introduced to their weekly job list for their colour group. During the week, in the activity room, they were responsible for completing a Problem of the Week, a computer activity, a paper-and-pencil task, and an activity using manipulatives. It was the students' responsibility to complete the activities on their job list and have them checked by a teacher or Educational Assistant. As each activity was completed, the students recorded comments and had their lists stamped, providing opportunities for journal writing and tracking. Once students had completed their job list, they moved on to the Extra Practice Activities listed on their Extra Practice record sheet. These hands-on activities addressed each of the five math strands in a fun format, and changed on a monthly basis. Again, the students were required to make a thoughtful comment upon completion, and have their work checked by an adult.

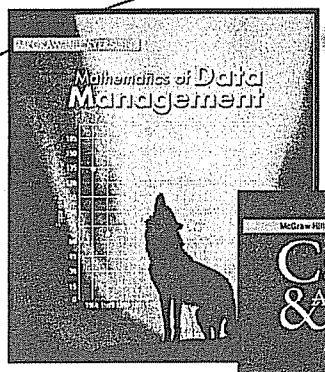
The lesson room allowed the teacher to focus on the curriculum expectations of the targeted grade in an intense fashion. The students were required to actively participate in the lesson. Student involvement and the use of manipulatives played a key role in this room. The pencil-and-paper tasks completed in the activity room reinforced the skills and concepts covered in an interactive way in the lesson room, and provided balance.

After completing four weeks of our action research project, the students were surveyed to see what they thought of the new style. Overall, the children preferred this new approach. They commented on having more turns during lessons and feeling proud when they helped other students in the activity room. One student commented, "It is fun to do something. There are not as many people because it is just Grade Three's. I get more turns." As well, he said, "It is fun to help someone who is smaller than you. I feel proud when I've helped somebody." Although the students felt it was difficult to catch up when they missed a lesson, most students

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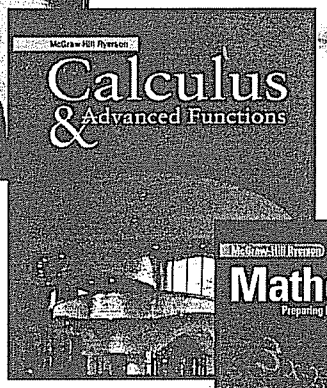
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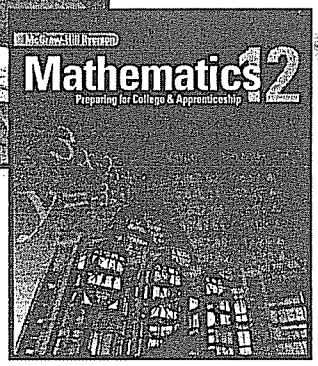
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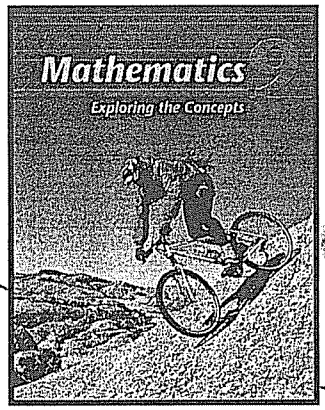
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commented that it was quieter and easier to concentrate during lessons. Many students appreciate that they had a teacher focusing on them and their learning for the entire lesson.

As teachers, we also sat down after four weeks of the action research project and interviewed each other. Generally, we found this new approach provided a positive learning atmosphere for the students and a less stressful teaching situation for us. We felt more prepared and organized and were able to provide more focused lessons. We now had two teachers teaching three math classes instead of each of us teaching two. Although the activity room required a great deal of organization, we had a greater focus, and there was only one lesson to prepare for each day. During the lessons, it was much easier to identify and remediate difficulties immediately. The students were unable to "hide in the crowd" due to decreased numbers and increased involvement.

Since the beginning of this new approach to teaching math, the students have demonstrated greater responsibility and ownership for their work. They have developed better independent work habits and seem to appreciate the opportunity to organize their work time and make choices. They co-operate, share materials, and effectively teach each other. As well, each student's computer time has increased. We are now able to provide lessons targeted to student's IEPs and give extra support to those who need it. Teaching lessons to a smaller group of students in one grade allows us to more easily monitor the progress of the students in that group.

This approach to teaching mathematics has been worthwhile and successful. We will continue to address the mathematics curriculum in this fashion. Next year, we plan to begin using this approach early in the year and will complete our long-range plans together for the overlapping grade. We are considering to modify this approach for other subject areas as well. ▲

"Many who have never had an opportunity of knowing much about mathematics confuse it with arithmetic, and consider it an arid science. In reality, however, it is a science which requires a great deal of imagination."

(Sonya Kovalevsky)

▲ PHOTO MATH - A NEW WAY OF PICTURING MATH

RON LANCASTER



Ron Lancaster is a middle and high school mathematics teacher, currently working throughout North America and Asia as an Independent Mathematics Consultant.

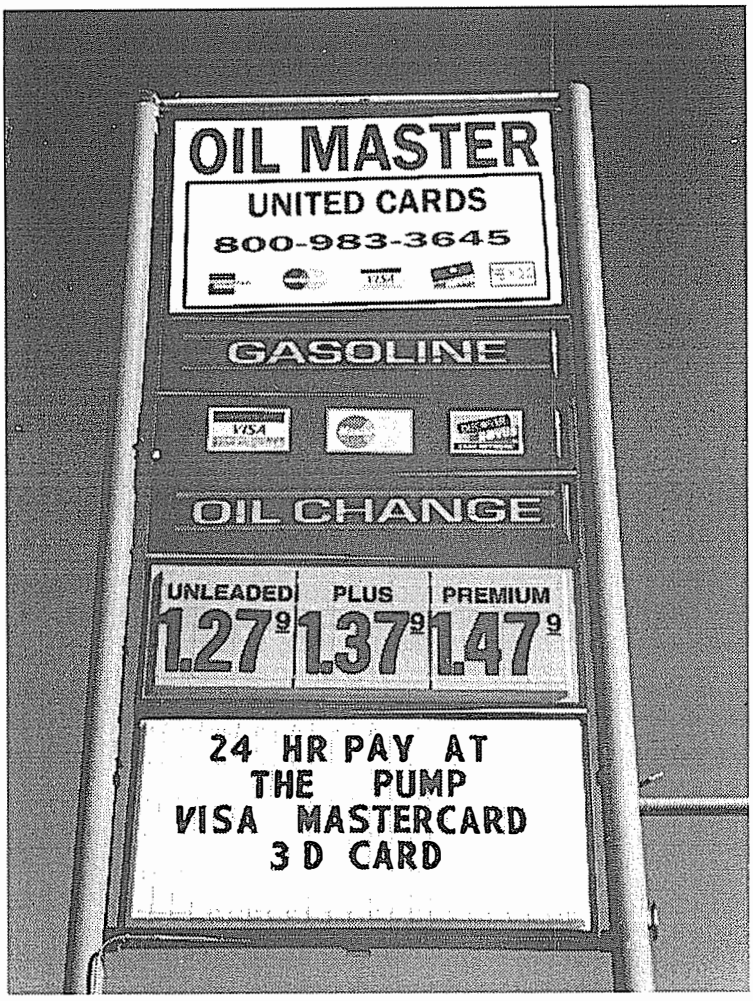
"A picture is worth a thousand words" is an expression that is often used to convey the importance of using images to make a point. The point of this column is to go beyond this notion and promote the idea that in fact "A picture is worth a thousand numbers." Photos with Math is what readers can expect to find in every *Photo Math* column and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario curriculum as well as the new NCTM document, *Principles and Standards for School Mathematics.* It is hoped that these questions will inspire students to be more creative and playful, and that in the end, students will develop an ability to see the world from a different perspective.

Teachers and students are encouraged to pose mathematical questions related to their own encounters and to submit these problems along with photographs to the author for possible publication in future columns. The author of this column does not presently have a cable modem and in order to avoid long periods of downloading items, submissions to this column should be sent via Canada Post to the address given above.

These photos were taken during a recent business trip to Kilgore, Texas. Kilgore is located in East Texas in an oil-rich area.

During my trip, I created a Math Trail at St. Luke's Church for teachers attending my workshops. After I prepared the Trail, I spent several hours driving around Kilgore searching for additional mathematical scenes. The following three photographs were taken during that drive.

Photograph #1



(a) Someone with a mathematical eye might look at these gasoline prices and view the 9 as an exponent instead of the intended $\frac{9}{10}$ of a cent. Using this exponential perspective, calculate the price of a gallon of gas for each of the three blends.

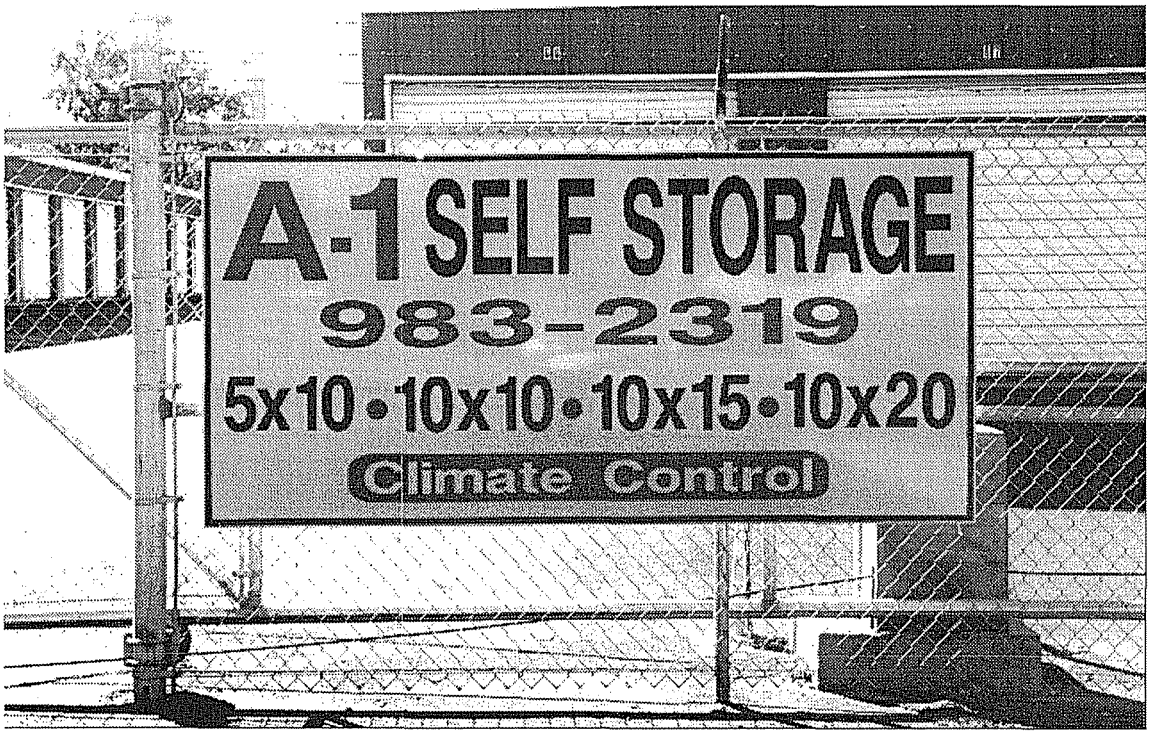
(b) The expressions given below continue the pattern of prices in the photograph. Find the value of each of these expressions and discuss the manner in which the answers change from one case to the next.

1.57^9, 1.67^9, 1.77^9, 1.87^9, 1.97^9, 2.07^9, 2.17^9, 2.27^9, 2.37^9, 2.47^9

(c) The author of this column once saw a sign that advertised gas for 1.08^9 dollars per gallon. Thinking in terms of exponents, he immediately knew that this was equal to 2 dollars per gallon. How did he know this?

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Photograph #2



(a) Let L and W represent the length and width of a storage space at A-1 Self Storage. Find L and W for each of the four spaces listed in the photo and record your results in the following chart.

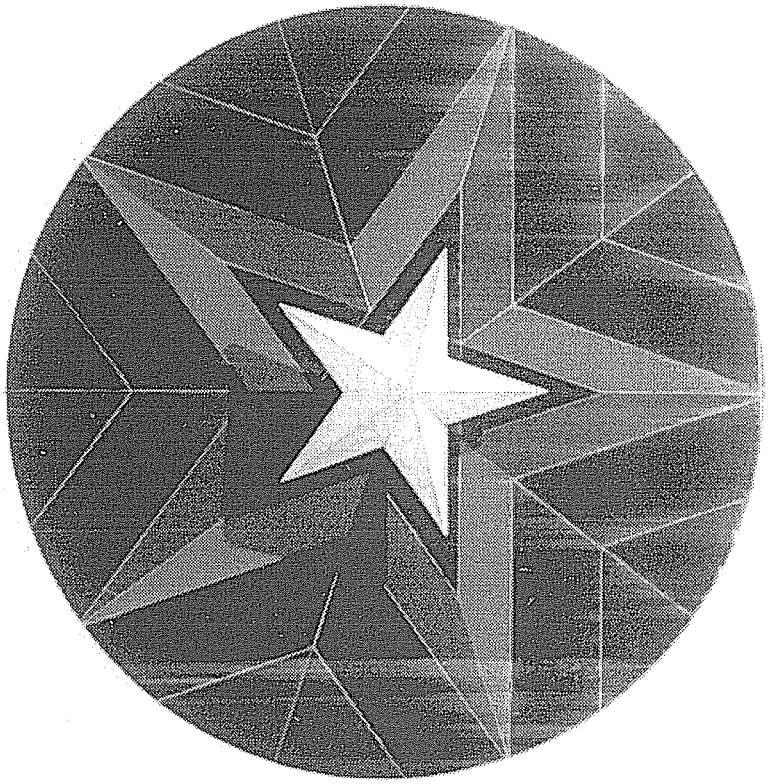
STORAGE SPACE	L	W
1		
2		
3		
4		

(b) Sketch the graph of L versus the storage space type along with the graph of W versus the storage space type. Is it possible to find two simple equations that can be used to produce these graphs?

(c) While it is possible to find equations that can be used to produce the graphs in part (b), it can be said with absolute certainty that these equations are not linear. There is, however, a way of slightly altering the data for the sizes of the spaces in the photograph so that the resulting graphs are perfectly linear. Make this change and then find the equations.

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Photograph #3



This huge and beautiful geometric design appears on a building in downtown Kilgore. Certain details of the design are sure to be lost during the scanning of the photo and in going from a colour to a black and white image. However, readers should be able to see that the design consists of a large circle, an inscribed pentagram, and a second smaller pentagram similar to the first.

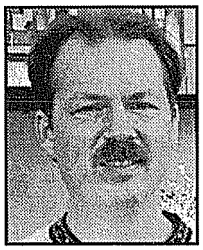
(a) Suppose the radius of the circle is 1 metre. Find the perimeter and the area of the inscribed pentagram.

(b) Suppose the area of the smaller pentagram is one-fifth of the area of the larger pentagram. Is it true that the perimeter of the smaller pentagram is one-fifth of the perimeter of the larger pentagram? Include some calculations to support your answer.

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▲ Tangrams Are Not Toys

CORNELIS DE GROOT



Dr. Cornelis (Kees) de Groot is an assistant professor of elementary education at SUNY New Paltz in New York State. His interests are related to learning transitions in mathematics and performance assessment. Dr. de Groot is director of the NYS K-8 Math Mentor Network and is the editor for Secondary Math Exemplars.

It is possible to assess much of peoples' thinking while observing their struggle to put a puzzle together. Ask anyone to put the seven Tangram pieces back together to form a square, and you will find her or him using trial and error strategies even that person admits to having done this before; adults and children alike. Although people have solved the Tangram puzzle, it is perplexing that each time they encounter the puzzle, it appears just as problematic as before. Each semester, I give this assignment to my pre-service elementary teachers and this ritual consistently repeats itself. Why does this phenomenon happen?

In this article, I describe paper-folding techniques that assist learners in discovering important underlying structures in the Tangram puzzle. The main ideas that I put forward are well suited for upper elementary and middle school teachers and maybe of interest to others as well. I will make the point that knowing these underlying structures enable learners to reconstruct the square from the seven discrete pieces, using mathematical reasoning and strategies. It is more effective to comprehend a structure than it is to remember the actual puzzle.

Manipulatives are occasionally characterized as toys, and sometimes starting as early as second or third grade, students are told that manipulatives, such as cubes or counting with fingers, are for little children. This negative characterization of manipulatives implicates that toys are not appropriate learning tools and that play or manipulation wastes time in the learning of mathematics. I think that this stance is problematic if we believe that the development of learning mathematics is conceptually facilitated by the development of insight into mathematical structures and mathematical relationships. It has been well documented that concrete experiences are needed for many children to accomplish the transition to more abstract mathematical knowledge (for example, Campbell & Johnson, 1995; Cobb, Wood, Yackel, & McNeal, 1993; and Thompson & Lambdin, 1994).

Relationships Among the Pieces

To begin the discovery of the underlying structures of the Tangram, I recommend that learners first develop a good sense of how the seven pieces (two small triangles, one medium triangle, two large triangles, a square, and a parallelogram) are related to each other. There are many activities in books or on the Internet that can assist with helping students form such relationships. You can find a selection of resources at the end of this article. It is particularly important that students realize that all the pieces can be constructed from the small triangle; that is, the medium triangle, the square, and the parallelogram are all composed of two small triangles and the large triangle is made from four small triangles. In figure 1, you can see that the seven pieces together are equivalent to 16 small triangles. This allows the learner to discern the part of the Tangram square that is represented by each piece. For example, the large triangles are each four sixteenths or one fourth of the Tangram square. The medium triangle, the small square, and the parallelogram are each two sixteenths or one eighth of the Tangram square, which leads to the making of a Tangram using these relationships and spatial and logical reasoning. This construction can be demonstrated using a square piece of paper.

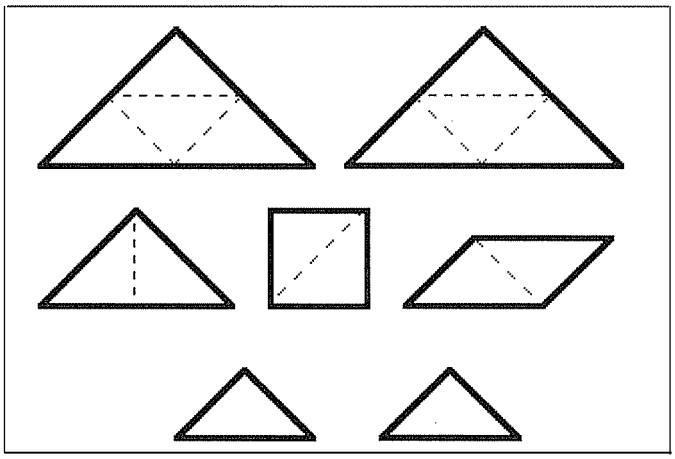


Figure 1. Seven Tangram Pieces are equivalent to 16 small triangles.

Unfolding the Tangram Structure

In this folding method, we will fold along or perpendicular to the diagonals of the square. Others have used a rectangular grid to reveal the structure of Tangrams (see, for example, the University of Exeter Centre for Innovation in Mathematics Teaching web site), but this does not lend itself well to working with the triangular shapes and the parallelogram. As a teacher, you can choose to pre-cut the Tangram squares, but I suggest that you use the diagonal folding methods, so often used in folding paper airplanes. This method might assist the students in viewing triangular pieces that fold on top of each other as congruent (see figure 2).

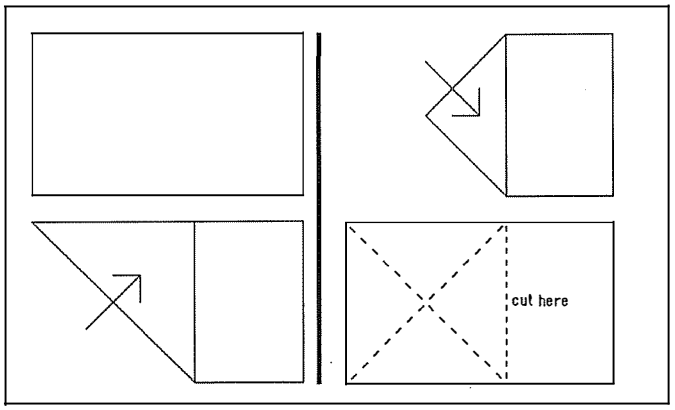


Figure 2. Making a square by diagonal folding.

Upon completion of the construction of the Tangram square, you will be able to identify four triangular regions, each representing one fourth of the entire square (see figure 3a). The two large triangles must be positioned in two of these regions. This is a good moment to investigate how many ways the two large triangles can be positioned in the Tangram square. It appears that there are only two possibilities: adjacent to or opposite of each other (see figure 3b). Determining the number of ways you can place given shapes is a very important step, since it will lead to an efficient strategy in re-construction of the puzzle later on.

Next, fold each vertex of the square to its centre and unfold it (see figure 3c). In this way, a smaller square has been formed, connecting all the midpoints of the original square. It is important to stop here for a moment and examine the size of this inner square. Students may see that the inner square is half the size of the Tangram square. They might also conclude that the two large triangles fit inside this square (see figure 3d). While this is true, it must be discussed that this strategy is not fruitful, since it will be impossible to place the square and the parallelogram in this construction. What can be discovered next? The medium triangle piece can be put in the space on each of the four corners and subsequently be flipped over to the inner square. This construction will show that the Tangram square is made up of eight medium triangles and that the inner square is made of four medium triangles. This in itself would

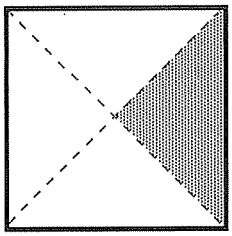


Figure 3a. One fourth of the figure is shaded.

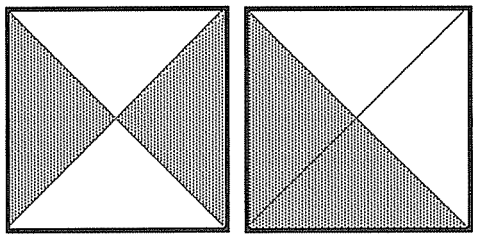


Figure 3b. Ways to place two large triangles.

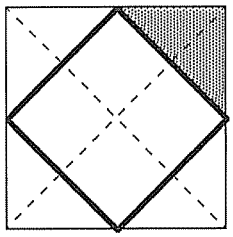


Figure 3c. One eighth of the figure is shaded.

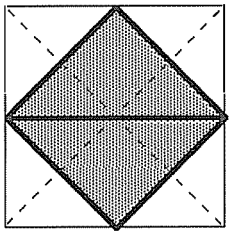


Figure 3d. Placing large triangles here prevents the placement of the square and parallelogram.

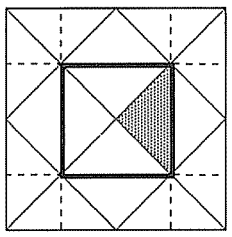


Figure 3e. One sixteenth of the figure is shaded.

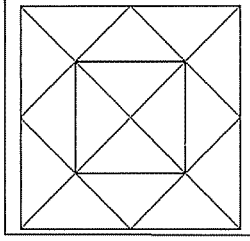


Figure 3f. The fundamental structure for the Tangram.

constitute a visual proof of the previously stated relationship.

Continue folding the vertices back to the centre and again fold the vertices to the centre of the smaller square. I found it was more convenient to fold when I first turned the paper over for this purpose. (You may recognize this construction from your childhood as a fortune telling game. Children often call this a "cootie catcher." It can be folded open and closed with one's thumb and index finger from both hands inserted.) Again, a smaller square is formed. This time, do not immediately unfold it. What is the relationship between this small square, the previous medium square, and the original large square? Unfold the paper completely. Several structures are visible (see figure 3e). First, the three squares that were previously discussed are recognizable. Second, do you see the 16 small triangles? To make this more apparent, have students trace the fold lines that are solid in the figure with a marker, and figure 3f will emerge from this.

We now have the fundamental structure underlying the Tangram. We have already determined where the two large triangles can go and where the medium triangles could fit. First, let us investigate where the square and the parallelogram might fit. The small triangle is the least of our concern, because it can fit in 16 places. You can see that there are only four spots for the square (see figure 4a). Note that due to the symmetry it does not matter which of these four spots is chosen. You can see that the parallelogram must go along the sides of the Tangram square (see figure 4b). There are two possibilities on each side, making a total of eight placements for the parallelogram.

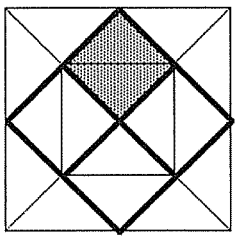


Figure 4a. Placing the square.

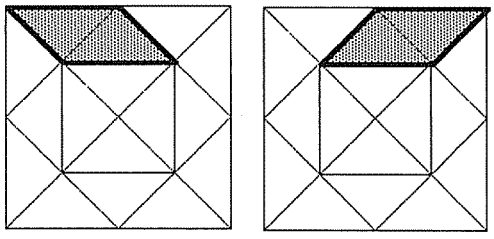


Figure 4b. Placing the parallel-ogram.

Putting It Together

We are now ready to reason backwards to a solution of this puzzle. We will start with the pieces that have the least number of possible placements and work our way up to those with the most: the two large triangles, then the square, then the parallelogram or medium triangle, and finally the two small triangles.

If we start with the two large triangles in opposing positions, we have two places left for the square (see figure 5a). Placing the square in one possibility forces the placement of the parallelogram in the space opposite the square (see figure 5b). There are only two possibilities for this. However, either choice presents a problem; it will be impossible to place the medium triangle anywhere (see figure 5c). We have now excluded the possibility of the two large triangles in opposition. Therefore, they must be adjacent!

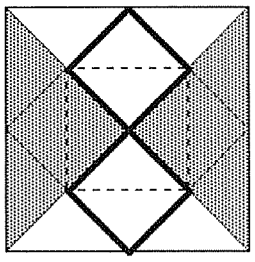


Figure 5a. The two big triangles are placed opposite. Two choices for the square.

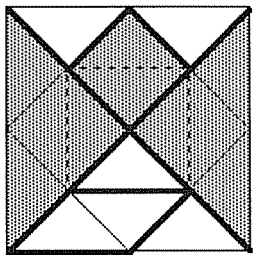


Figure 5b. The parallelogram must go in the space opposite the small square.

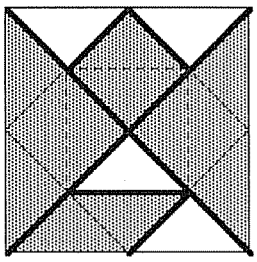


Figure 5c. Placing the medium triangle is impossible.

Start again by placing the two large triangles adjacent to each other (see figure 6a). There are two possibilities for placing the square. Because of the symmetry, it doesn't matter where it is placed (see figure 6b). In turn, this leaves only two possibilities for the parallelogram on the remaining side. One possible placement of the parallelogram will make it impossible to place the

medium triangle, but the other allows the placement of all the remaining pieces (see figures 6c and 6d). Now the puzzle is solved once and for all.

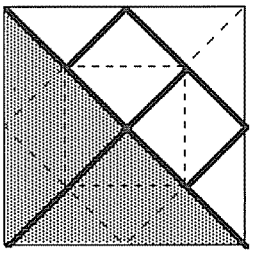


Figure 6a. Large triangles are adjacent and the small square can go in two places.

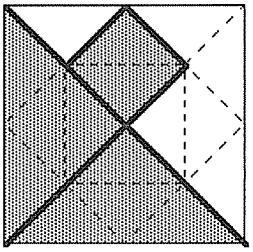


Figure 6b. By symmetry, the square can be arbitrarily placed in one of the two options.

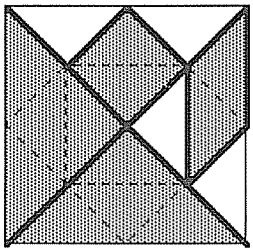


Figure 6c. Only two places for the parallelogram. This option prohibits the placement of the medium triangle.

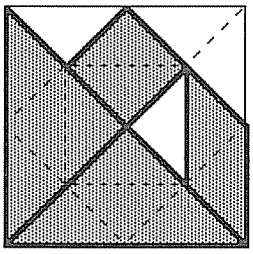


Figure 6d. The puzzle is solved.

The Payoff

We have reached a point where we can reconstruct the Tangram puzzle at any time using the underlying structure, the relationships among the pieces, and logical reasoning. This addresses a large number of important objectives all at once, making it very worthwhile to spend two or three lessons investigating Tangrams. The big payoff is that these strategies model particular mathematical ways of thinking and problem solving, allowing for a transition toward recognizing and describing the underlying structure, which in turn becomes embedded as the mental image for the Tangram. Using such a structure helps students to be successful at solving figures made with the Tangram pieces, as well as deciding whether or not it is possible to construct certain geometric shapes with all seven pieces or a subset of these. I have included a list of additional references for those who are interested in Tangrams and other mathematical puzzles or games. Enjoy!

References

Campbell, P. F., & Johnson, M. L. (1995). How Primary Students Think and Learn. In I. M. Carl (Ed.), *Seventy-Five Years of Progress: Prospects for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Cobb, P., Wood, T., Yackel, E., & McNeal, E. (1993). Mathematics as Procedural Instruction and Mathematics as Meaningful Activity: The Reality of Teaching for Understanding. In R. B. Davis & C.A. Maher (Eds.), *School, Mathematics, and the World of Reality.* Needham Heights, MA: Allyn & Bacon.

Thompson, P. W., & Lambdin, D. (1994). Research into practice: Concrete materials and teaching for mathematical understanding. *Arithmetic Teacher 41*(9): 556-558.

Additional Resources

Ajose, S. A. (1978). Doing arithmetic with tangrams. *Arithmetic Teacher 26*(4): 34-35.

Bohning, G., & Althouse, J. K. (1997). Using Tangrams to teach geometry to young children. *Early Childhood Education Journal 24*(4): 239-242.

Botermans, J., & Van Delft, P. (1995). *Creative Puzzles of the World.* Emeryville, CA: Key Curriculum Press. Available: http://www.keypress.com.

Dickoff, S. S. (1971). Paper folding and cutting a set of tangram pieces. *Arithmetic Teacher 18*(4): 250-252.

Dunkels, A. (1990). Making and exploring tangrams. *Arithmetic Teacher 37*(6): 38-42.

Oldfield, B. J. (1991). Games in the learning of mathematics part 2: Games to stimulate mathematical discussion. *Mathematics in School 20*(2):7-9.

Read, D. (1965). *Tangrams, 330 Puzzles*. New York, NY: Dover Publications.

Russell, D. S, & Bologna, E. M. (1982). Teaching geometry with tangrams. *Arithmetic Teacher 30*(2): 34-38.

University of Exeter Centre for Innovation in Mathematics Teaching web site: http://www.ex.ac.uk/cimt.

Math Forum at Swarthmore College web site, with an excellent list of links and resources: http://forum.swarthmore.edu/trscavo/tangrams/contents.html. ▲

"Mathematics reveals its secrets only to those who approach it with pure love, for its own beauty."
(Archimedes)

▲ Research on Patterns and Functions

JACK CARTER
CSU HAYWARD

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Brenner, Mayer, Moseley, Brar, Duran, Reed, and Webb (1997) studied how a prealgebra unit based on reform principles improved students' effectiveness in representing mathematical problems involving functional relationships. The study examined how students used symbols, words, and graphics to represent functions of two variables. The experimental unit emphasized multiple representations of concepts, meaningful contexts, problem-solving processes, and guided discovery, while comparison classes were taught from a text that focussed on solving equations and using equations to solve word problems. Brenner et al. concluded that students who received representational training performed better than comparison students in creating and coordinating multiple representations of functions.

Adams (1997) investigated how using the graphing calculator and participating in assignments to select appropriate window settings for the calculator's screen would affect students' concept of function. Four groups of students (calculator and assignment, calculator only, assignment only, and control) participated in the study. Analyses of the data indicated the students had difficulty with a wide variety of algebraic tasks related to domain, range, and scale settings for functions. Most students reported their image of a function was a graphical one supported by the vertical line test. Adams found that the assignment-only treatment was not appropriate for assisting students with identification, construction, and definition of function.

Giamati and Weiland (1997) explored the perceptual differences about patterns in mathematics expressed by Native American students, predominantly Navajo, who completed a series of patterning exercises and interviews. Since the Navajo language contains more verbs than nouns, the researchers expected the students' patterns to demonstrate aspects of transformational geometry that were culturally distinct. Isometries, similarities, and asymmetries were commonly employed in the students' patterns, but the students rarely depicted reflections, glide reflections, or static linear repetition. Instead the students often continued patterns in a circular way, in diagrams drawn in three dimensions, or in frames that were dynamic. The researchers concluded that their results had implications for patterning tests for giftedness and for understanding Navajo students' way of thinking.

O'Callaghan (1998) compared the effects of a technological (TE) and a traditional (TR) algebra curricular on students' understanding of the function concept. The TE approach emphasized realistic problem solving, conceptual knowledge, and technology. Results showed that TE students had better knowledge of the modelling, interpreting, and translating components as well as a better general understanding of functions. O'Callaghan concluded that the TE curriculum fostered student development of richer concepts of function and demonstrated promising outcomes relevant to understanding of functions.

Keller and Hirsch (1998) explored student preferences for graphical, tabular, or symbolic representations of functions in purely mathematical and applied settings. Students in the study were enrolled in either a traditional or a graphics calculator-based calculus course. Student responses on preference pretests were diametrically opposite with tables preferred in applied settings and equations preferred in purely mathematical settings. Responses on post-tests showed preferences had shifted so that students then favoured using graphs in applied settings. Students who used graphing calculators were more likely to have a graphical preference in both pure and applied settings. The researchers also found that students preferred graphical or tabular representations for functions on tasks with less formal language, and symbolic representations when tasks were expressed in more formal language.

Lloyd and Wilson (1998) documented how a teacher's conceptions of functions shaped the first implementation of a reform-based unit on functions. Their findings suggested that teachers' comprehensive and well-organized conceptions of functions acted as both a filter for the interpretation, and as a springboard for the implementation, of the curricular unit. The teacher's graphical proficiency and focus on the patterns of covariation by functions facilitated classroom

interactions that established links among representations and types of functions. They also noticed that studies of the process through which reform is interpreted and personalized can support teachers who are making long-term instructional changes.

References

Adams, T. L. (1997). Addressing students' difficulties with the concept of function: Applying graphing calculators and a model of conceptual change. *Focus on Learning Problems in Mathematics, 19,* 43-57.

Brenner, M. F., Mayer, R. E., Moseley, B., Brar, T., Duran, R., Reed, B. S., & Webb, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. American *Educational Research Journal, 34,* 663-689.

Giamati, C., & Weiland, M. (1997). An exploration of American Indian students' perceptions of patterning, symmetry and geometry. *Journal of American Indian Education, 36,* 27-48.

Keller, B. A., & Hirsch, C. R. (1998). Student preferences for representations of functions. *International Journal of Mathematical Education in Science and Technology, 29,* 1-17.

Lloyd, G. M., & Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education, 29,* 248-274.

O'Callaghan, B. R. (1998). Computer-intensive algebra and students' conceptual knowledge of functions. *Journal for Research in Mathematics Education, 29,* 21-40. ▲

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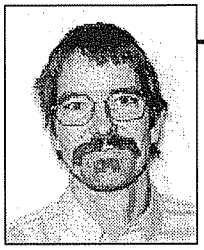
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▲ The Möbius Strip As a Rich Learning Task

TIM SIBBALD



Tim Sibbald teaches mathematics and physics at Clarke Road S.S. in London, Ontario. This is one example of the twisted thinking his students live with.

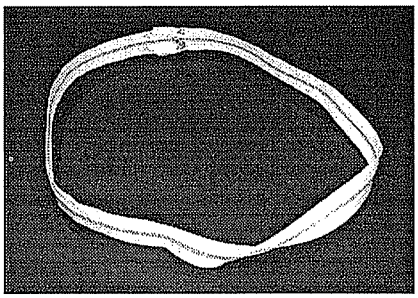


Figure 1: The Möbius Strip

The Möbius strip is perhaps the simplest object, from the field of topology, that can be used in the classroom. To construct one, take a long thin strip of paper and bring the ends together as if you are wrapping it around a cylinder. Then, twist one of the ends over (180˚) and tape the ends together. An example is shown in Figure 1. Do students recognize this to be the same as the recycling symbol?

In the classroom setting, you can have students mark each side. Then take a pencil and start to draw a line the length of one side. Before long, they will find that, without lifting their pencil, they are on the opposite side, and a short while later, back where they began. The line now appears on "both sides" of the strip. The Möbius strip has only one side! One application of this characteristic is to provide equal wear to both sides of a conveyor belt. Students can also follow one edge and determine that the surface has only one edge.

How can this small stunt be turned into a rich learning task? Firstly, you can introduce more than one twist. Then you will find that there is one side and one edge only when there is an odd number of twists. For even numbers of twists there are two sides and two edges. A further bit of intrigue, but that's only the beginning.

To continue, you can apply a pair of scissors and cut Möbius strips with different numbers of twists in half. Should you like the task that follows, you may wish to get a zipper that can be fully separated and sew velcro onto the ends to facilitate connecting after the twisting. Then undoing the zipper has the same affect as cutting. Alternately, a spool of cash register tape is a cheap alternative for long strips suitable for making strips with many twists.

After cutting strips with different numbers of twists, ask your students about the number of loops that have resulted. Then have them describe the loops. How many twists does each loop have? The loops are knotted in some cases - which? Can you describe the knots? How can we describe a knot? The observations are listed below:

PROPERTIES OF MOBIUS STRIPS CUT IN HALF			
Number of Twists	Resulting Number of Loops	Number of Twists for each Loop	Knot (yes/no)
1	1	4	Yes or No
2	2	2, 2	No
3	1	8	Yes
4	2	4, 4	No
5	1	12	Yes
6	2	6, 6	No
7	1	16	Yes

When students count the number of twists in each loop, they can separate the loops first by cutting one of the loops and, being careful to keep both sides of the cut oriented the same way, removing the other loops through the cut. In the case of one twist it has a knot if you follow the numerical pattern in the table, however, it is only a single twist which is not generally considered a knot.

Describing the knots is a challenge. Thinking of a good way of showing the common characteristics of related knots entails both problem solving and communication. Pictures of the knots arising from cutting the three twist, five twist, and seven twist strips are shown in Figures 2, 3, and 4 respectively.

To describe these knots, follow the line around from one spot. The three twist knot has six crossings, the five twist knot has ten, and the seven twist knot has 14 crossings.

This task is rich because of the different levels of difficulty that enter into it. Students will show a range of ability and organization preparing strips. Counting the loops gets more difficult with more twists as does determining the number of twists that each resulting loop has. Recognizing that there is a knot does not pose too much difficulty but describing the knots is a challenging task for problem solving and communication.

Should this whet your students' appetites, you may want to revisit the problem another day with the following twist. Instead of cutting the strips into two pieces, cut them into three! You may want students to mark the lines on the paper before making the strips but otherwise the method of

investigation is essentially the same.

What is the difference? In this case, students will get two or three loops. In some cases one loop has a knot (Which loop has the knot?). In the cases where there are three loops, it raises the question of how the loops are joined. Students have some difficulty appreciating what this question is asking. It is instructive to ask, "How many ways can three loops be joined?" and then have them return to their observations to see what happened in the investigation.

The results of the investigation are given below:

PROPERTIES OF MOBIUS STRIPS CUT INTO THIRDS		
Number of Twists	Resulting Number of Loops	Number of Twists for each Loop
1	2	1, 4
2	3	2, 2, 2
3	2	3, 8 (The 8 has a 3-Twist knot.)
4	3	4, 4, 4
5	2	5, 12 (The 12 has a 5-Twist knot.)
6	3	6, 6, 6
7	2	7, 16 (The 16 has a 7-Twist type knot.)

The knots which are observed are the same as the knots described for cutting the Möbius strip in half. Finally, the three loops are joined in pairs. The twists joining two loops are distinct from the twists joining the other pairs.

How does this problem fit into the curriculum? I tested it at the grade twelve general level to make the point that unusual patterns happen. They certainly discovered this and made several suggestions including the use of cash register tape and providing more assistance. (I left a substantial portion as homework.) In the new curriculum, the Möbius strip can be used to teach how to organize and interpret data in a table. The activity is tied to sequences in general and contains several examples of arithmetic sequences. It can be used as a problem solving task and also fits with modelling. Regardless of how you use it, you can expect to hear active communication as your students wrestle with how to describe knots made of twisted paper! ▲

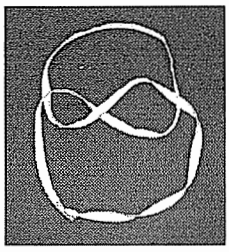


Figure 2: The Three Twist Knot

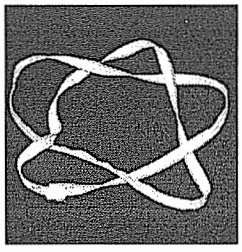


Figure 3: The Five Twist Knot

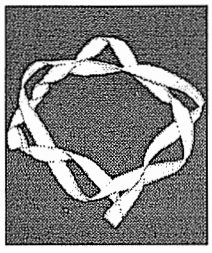


Figure 4: The Seven Twist Knot

▲ OAME/NCTM MARCH 2002 REPORT



FRANCES SCHATZ

"The most rewarding moments I had while teaching were when my students saw for themselves that they had the ability to learn mathematics and work problems out on their own. It happens with different students at different times. You can't predict when it will happen. You just know it will happen." NCTM's new executive director, Jim Rubillo.

Jim has been involved with NCTM for many years and has 36 years of teaching experience at the high school, community college, and university levels. "I always told my students, 'I don't really care about you today—I care more about what you're going to be like ten years from today. And maybe you won't like everything we do in this classroom today, but I hope you'll recognize its value over time.' That's part of the maturing process." Jim's vision for NCTM is that it is a council of educators who come together to discuss issues of importance and to make things better for teachers so that they can reach the young people in the classrooms. E-resources are the first solid step for continued growth of NCTM.

Currently, NCTM e-resources are available to NCTM members. Individual members are able to take advantage of current journals, student math notes, mathematics education dialogues, archives, search, and contact information. If you are interested in an NCTM membership, please contact me for current price and how to get the 25% cap on the exchange rate. When your membership is processed, you will receive a member login and password.

The National Council of Supervisors of Mathematics (NCSM) is an affiliated group of NCTM and Florence Glanfield at the University of Saskatchewan informed me that NCSM will lead two leadership symposias in conjunction with the NCTM Regional conferences in

Montreal and Regina. For more information about this organization or the symposias, you can write to Florence Glanfield, Department of Curriculum Studies, College of Education, University of Saskatchewan, Saskatoon, SK, S7N 0X1, or you can email her at florence.glanfield@usask.ca. I had the privilege of meeting Florence at various NCTM events; she puts a lot of time, energy, and thought into NCSM's workshops.

New Resources

In my last report, I talked about the *Navigations Series* for Geometry. PreK-2 is now available and the other three are expected to be ready for the Annual conference this year in Las Vegas. This first one (stock number 12140), demonstrates fundamental ideas of geometry using two- and three-dimensional shapes. There are blackline masters for student activities and includes a CD-Rom.

Mathematical History: Activities, Puzzles, Stories, and Games 2nd edition (stock number 12058) enriches grades 4-7 mathematics with easily duplicated sheets.

Web Bytes

Math Puzzles is a collection of interactive games that give practice in basic logic and arithmetic at www.mathispower.org/mathnav.html. Mike Kelley's Calculus-Help.com offers teacher-made activities in calculus topics, including a problem of the week, and links to useful other calculus sites at www.calculus-help.com. Homeroom Teacher offers links to free educational clip art, lesson plans, news, and more at www.homeroomteacher.com.

Important Dates to Mark on your Calendar

The 80th Annual NCTM Meeting, "Realizing the Vision of School Mathematics", will be held in Las Vegas, Nevada from 21-24 April 2002. Notice that the dates are from Sunday evening to Wednesday. There are more than 1000 sessions for professional development, over 200 exhibitors of math products and teaching resources, and most importantly, there are opportunities for networking and sharing of ideas. This year, you can look forward to two highlights: "A Cabaret of Melodies and Musings" by David McKillop from Halifax and "Mathematics and Fun" by Don Fraser from Toronto. Go to www.nctm.org/meetings/vegas/attraction.htm for more information and registration information.

The Regional Meeting, Montreal, Quebec 15-17 August 2002, is "The Joy of Learning Mathematics" to be held at the Delta,and Holiday Inn hotels.

The Regional Meeting, Regina, Saskatchewan, 17-19 October 2002, is "Making Mathematical Connections: 2002 and Beyond" to be held at the Delta Regina and Regina Inn hotels.

More information can be found at the NCTM web site, www.nctm.org.

REMEMBER: Each new individual membership with NCTM generates $5US for OAME. 20% of the price of publications or supplies by NCTM remains in Ontario for OAME work. ▲

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▲ Assessment Corner: Preparing Students for the Grade 9 EQAO Assessment

CHRIS SUURTAMM, RONA DOYLE, MARILYN HURRELL

All grade 9 mathematics students in Ontario encounter the grade 9 EQAO assessment of mathematics towards the end of their coursework. It is important for teachers to be familiar with the nature of the assessment as well as how to help students prepare for it. Rather than be a "shock" to the routines and learning in the classroom, the assessment should be an opportunity for students to be able to show what they know and can do.

Balanced assessment in a classroom is always the best indicator of a student's achievement in mathematics. Ongoing assessment by the classroom teacher is like a videotape that presents student growth and learning over time, provides rich data about student achievement, and can be the best measure of a student's achievement. On the other hand, the EQAO assessments are just a snapshot of student achievement at a certain point in time. Nonetheless, the EQAO assessment attempts to reflect a variety of assessment practices that teachers might use in a classroom. Hence, multiple choice, short answer, and extended response are used to try to emulate the variety of tasks that might occur in a classroom.

The EQAO assessment is designed to provide data that is useful to teachers, school, and board planning to improve student learning. However, schools, teachers, and boards should recognize that it is only one source of data. Unfortunately, the pressure of the public disclosure of the results without public understanding of the "big picture" causes a great deal of anxiety and seems to shift thinking into believing that the snapshot of student learning is more important than the richly detailed video of learning over time.

How is the Assessment Designed?

The questions on the EQAO assessment are directly linked to the grade 9 curriculum. This means that the questions are matched to curriculum expectations and the categories of the Achievement Chart: Knowledge/Understanding; Application; Thinking, Inquiry, and Problem Solving, and Communication. There are multiple choice, short answer, and extended response questions. The short answer questions each relate to a particular curriculum expectation, strand, and achievement chart category. The extended response tasks have multiple parts and address 1 or 2 strands, several curriculum expectations, and are scored across all four categories of the achievement chart. The short answer and extended response questions are developed by teachers, tested in a variety of grade 9 classrooms, refined, reviewed by EQAO, and further refined by other teachers. Thus, the EQAO assessment should be a fair match to the curriculum and a reflection of the instruction and assessment occurring in grade 9 classrooms across the province.

How Do Teachers Help Students Prepare for the EQAO Assessment?

The best possible preparation for EQAO is to work towards having a classroom that is true to the content and intent of the curriculum. Students should be engaged in the inquiry process, work with a variety of types of problems (both scaffolded and open-ended), communicate their processes and reasoning in a solution, and be comfortable using technology as a tool for problem solving. As well, it is important to integrate skills and review into the entire grade 9 course rather than spend inordinate amounts of time "up front" on review. Students will retain skills, and review material if presented "just in time" for them to use and extend the material in a new context. Students can get excited about math class by starting the course with something that is new. They can also get discouraged with math class if the first few weeks are spent covering material that they have already met as a stumbling block or have already mastered. These skills should not be dismissed but rather integrated within the course.

Getting Ready and Using Sample Materials

The EQAO web site (www.eqao.com) has many resources and support materials to help familiarize teachers and students with the assessment. These materials should be examined early in the course so that the teacher can determine how and when she/he might want to use the resources. For instance, the formula sheet that is distributed to students during the assessment is available so that teachers can distribute it at the beginning of their course. In this way, students will

become familiar with the type and placement of the formulas on the sheet. There is also a "Key Words" sheet that should be made available early in the course and modelled in assessments so that students are familiar with what is expected for instructional words such as "explain," or "compare".

The support materials also include sample questions. They are samples to show students what the questions might look like. These are not necessarily review material that a teacher might give out before an exam with the instructions: "If you can do these questions, then you will pass." Rather, teachers might want to model the style of questions in some of your own assessments, and as with any assessment, discuss the qualities of a good solution. Or students could work in groups brainstorming what should be included in a good response, recording on chart paper to facilitate whole class discussion. Sometime before the assessment, you may want a "dry run" with some of the materials. Discuss good test taking practices – for multiple choice, for short answer, and for extended response. Share solutions and have students look at samples of student work to determine what they think makes high quality work. Stress good communication of the problem solving process.

However, it is also essential to minimize anxiety. Practicing the process will help to reduce the anxiety. Being prepared well ahead of time also reduces last minute problems. This includes reading through the instructions ahead of time so that there is no confusion on that day. Talk about timing because timing generally does not play as strong a role in classroom assessment as it does in EQAO assessments. Perhaps discuss the idea that the teacher cannot really help them or give them clues with the questions, and explain why.

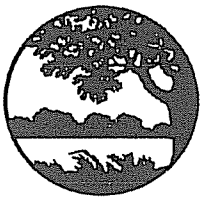
Concluding Comments

The EQAO grade 9 assessment is a reality that teachers and students contend with. Understanding the nature of the assessment, focusing on teaching the intent and content of the grade 9 curriculum, and giving students a "heads up" through a variety of uses of the sample materials will help students to show what they know and can do on the EQAO assessment. However, high-quality, well-balanced, diverse, ongoing classroom assessment is still the best indicator of student achievement. ▲

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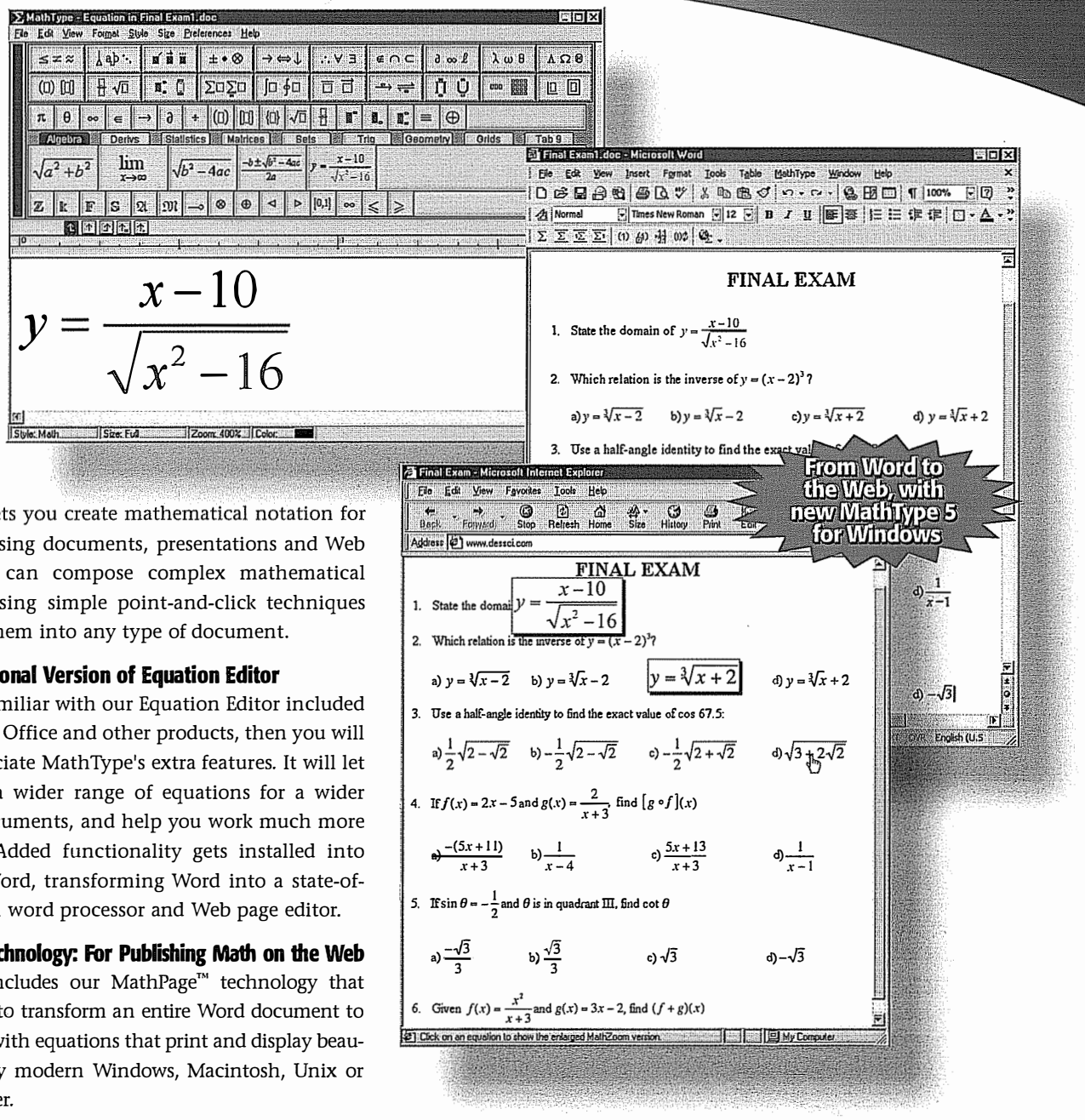
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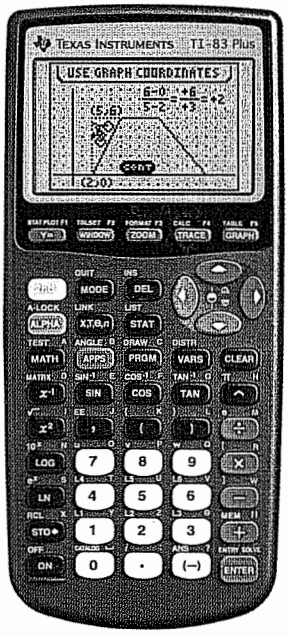
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