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Editor

▲ **Louis Lim**

Associate Editors

▲ **John Egsgard**

▲ **Todd Romiens**

***Abacus* Editor**

▲ **Frances Schatz**

Design, Tech Art, and Production

▲ **Penny Clemens**
Graphic Designer, Cambridge, ON

Printing & Binding

▲ **Pole Printing,** Box 69, 89 King Street
Forest, ON N0N 1J0 (519) 786-5112

Advertising Manager

▲ **Dean Murray**

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▲ Editorial: A Prime Year – What's all the Fuss?

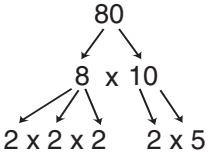


LOUIS LIM, EDITOR

It's Christmas day and I'm giving my golden husky, Grover, his "walkie break" in the fresh, white snow that has been falling all day. I cannot believe that you'll be reading my editorial in the spring since Christmas day is windy and snowy and spring seems so distant away. While Grover is enjoying the wintry weather, which his thick golden fur can withstand, I am thinking about what is in store for me as the palindromic year, 2002, ends and the prime year, 2003, begins.

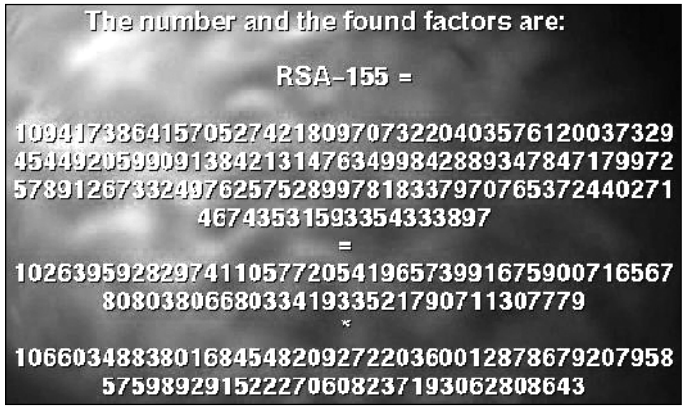
The theme of the 2003 OAME annual conference is "Primed for Math". Tara Tasker did a wonderful job creating the cover for the program booklet, consisting of 149 numbers and our job is to find the one number that is not prime. In November, I received my program booklet at school on our PD day. During the morning workshop, I could not resist trying to find the composite number. My teaching colleague, Julie Shaw, also attempted to find it. At the time of writing this editorial, I have still not found the composite number. It is very tempting to input each number into the "Prime Number Calculator" located at http://homepages.ihug.com.au/~shane007/maths/prime.html!

The Oxford Dictionary of Current English defines *prime number* as a "natural number other than 1 that can be divided exactly only by itself and unity." Looking through my copy of *The Ontario Curriculum: Grades 1–8 Mathematics*, students learn about prime numbers in the number sense and numeration strand in grade 8: "represent composite numbers as products of prime factors (e.g., 18 = 2 x 3 x 3)". I recall my own experiences as a student constructing factor tress such as 80 = 2 x 2 x 2 x 2 x 5 = 2^4 x 5:



So what's all the fuss for the organizers of the OAME provincial conference to choose a "prime" theme? I went on the Internet and performed a Google search on prime numbers. There were over 79 400 hits returned! One particular web site that caught my attention was the Great Internet Mersenne Prime Search (GIMPS), www.mersenne.org/prime.htm. Currently, there is a $100 000 award, from the Electronic Frontier Foundation, for the first person to discover a ten million-digit prime number. Anyone with a computer can go to that web site and download the free software to participate in GIMPS. Michael Cameron, a 20 year old Canadian, found the largest prime number to date on November 14, 2001, $2^{13\,466\,917} - 1$, a prime number consisting of 4 053 946 digits. Cameron was one of 130 000 computer users who collaborated together on the GIMPS project, and it was his 800 MHz AMD T-Bird PC running part-time for 45 days that discovered the 39th known Mersenne prime number; a Mersenne prime is a prime number of the form $2^p - 1$, where p is prime (the first few Mersenne prime numbers are 3, 7, 31). I must ask the question…will an *Ontario Mathematics Gazette* reader find the ten million-digit prime number? Time will tell!

Clifford Pickover, in *Wonder of Numbers*, describes how a group of scientists used 300 personal computers and a supercomputer to break the RSA-155 code, which is used to secure credit card transactions and the sending of e-mails in Europe. The 155-digit number is factored into two prime numbers consisting of 78-digits each:

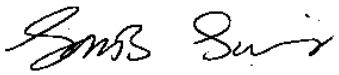


http://www.certconf.org/presentations/1999/finalssmdraft/img44.htm

According to the web site www.rsasecurity.com/rsalabs/challenges/factoring/rsa155_faq.html, we should not be alarmed that this rather impossible feat has been cracked; rather, we are assured that a person who is an expert in number theory and has the finance to purchase a very expensive supercomputer can crack an RSA key of 512 bits, but the costs would far outweigh the benefits. Such an activity reminds us of the need for encryptions to be changed on a regular basis so the dangerous "black box" on the *Sneakers* video does not become a reality.

By the time you read this, you'll be in the "prime" year of 2003. I'm convinced that prime numbers are very important in our lives. As students of mathematics, we have learned the pure aspects of mathematics through constructing factor trees, using the Sieve of Eratosthenes, applying the Fundamental Theorem of Arithmetic (every natural number can be written as a product of primes), and learning that Euclid proved that there is an infinite number of primes. But for encryption, it is the theories of prime numbers that make it possible.

Enjoy 2003, a "prime" year! ▲



Upcoming Events

For further information, please contact the chapter representative on the last page of the *Gazette* for OAME conferences and Mathematics Olympics, and Frances Schatz for NCTM conferences.

OAME Conferences

(all chapter conferences are held after school)

April 3, 2003	Oakville	ISOMA Chapter Conference
April 10, 2003	Kitchener	GVMA Chapter Conference
April 10, 2003	London	WOMA Chapter Conference
April 14, 2003	Ottawa	COMA Chapter Forum Series
May 8 – 10, 2003	Oshawa	30th Annual OAME Conference
November 13, 2003	Kitchener	GVMA Chapter Conference
April 28 – May 1, 2004	University of Waterloo	31st Annual OAME Conference

Mathematics Olympics

April 12, 2003	Toronto	ISOMA Chapter
April 12, 2003	Toronto	TEAMS Chapter
April 26, 2003	Ottawa	COMA Chapter
May 14, 2003	Sudbury	NOMA Chapter
June 13 – 14, 2003	London	Ontario Mathematics Olympics

NCTM Conferences

April 9 – 12, 2003	San Antonio, Texas	81st Annual Meeting
November 20 – 22, 2003	Edmonton, Alberta	Canadian Regional Conference
April 21 – 24, 2004	Philadelphia, Pennsylvania	82nd Annual Meeting

▲ Letter to the Editor

Dear Editor:

In the September 2002 edition of the *Ontario Mathematics Gazette*, I noticed in Marilyn Hurrell's *Directors' Dialogue* that the *Don Attridge Award*, "to recognize an exceptional and creative secondary school mathematics teacher who enhances the teaching of mathematics in Ontario" has been renamed *Award for Exceptional and Creative Teaching in Secondary Schools*. This move is something I support and have strongly lobbied for during the last several years.

It has been a most significant honour and a great pleasure to have an Ontario Association for Mathematics Education award presented annually in my name. However, I came to believe that the name for this type of award should not be linked to any one individual person, in this case, myself. There have been so many exceptional secondary school teachers over the history of OAME who could have merited having their name attached to this award.

In conclusion, I commend the Ontario Association for Mathematics Education for this change and salute all future winners as well as those who have won this award in the past.

Sincerely,
Don Attridge ▲

▲ From the Editor's Desk...Celebrating March 14th

Making a Pi Necklace
by Diana Funke

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As a 7th grade math teacher, I like to make mathematics as visual as possible. For Pi Day, my students make Pi necklaces.

I use Pi as a way of introducing my students to the idea of an irrational number. After studying decimals that terminate or repeat, I ask them to bring in a can and we compare the circumference to the diameter by dividing C by d. This is how they find out what Pi is all about.

Cont'd on page 23

▲ President's Message

MARGARET WARREN



Getting them ready!!! This is a phrase I have heard numerous times from many different sources throughout my career as an educator. Getting them ready for the Grade 3 EQAO assessment . . . getting them ready for the Literacy Test . . . getting them ready for the real-world . . . getting them ready for university . . . getting them ready for grade 9 . . . getting them ready for the next year . . . getting them ready for the exam. Sometimes, I wonder if they are even ready to get ready! I know that the person who is getting them ready is not working from any real knowledge but rather assumptions about what and how they think they are getting them ready. It also worries me that while we are so busy getting them ready we may not even be getting them ready for something that is best for them.

I believe children learn developmentally and that their learning builds on their previous knowledge. During the time I was a program coordinator, I worked with a team that developed a learning continuum based on the new curriculum and I emphasized the importance of each teacher focussing on what the expectations for their grade as well as what the expectations for the grades above and below were. I remember being at one of my daughter's soccer games last year and having a discussion with a parent. Her daughter (who was in grade 7) was at home studying for a math exam. The parent agreed with me that it seemed a bit young to have a math exam but said that the teacher explained she was getting them ready for grade 9. Interesting that at the same time, we were engaged in discussions at my high school in which we decided to cancel exams for most grade 9 courses and bring the assessment methods more in line with what the new assessment policy advocates.

Recently, my own daughter finished her first introduction to integers and algebra in grade 7. She was manipulating very lengthy and complex integer expressions involving order of operations and exponents. The expectations went well beyond those outlined for grade 7. The teacher believed that she was preparing the students for grades 8 and 9 and found it somewhat surprising that I had a different perspective. My daughter

was not only very confused but was also developing a dislike for mathematics.

Teachers of OAC courses have always worried about how well their students will be prepared for university. In fact, during the recent curriculum revisions teachers were afraid to let go of any content for fear that the students would not be ready for first year Calculus. Integration survived long after the OSIS curriculum revision simply because teachers believed they were doing their students a special service and because the universities had not changed their expectations. Although the new math curriculum encourages the use of technology throughout all of the courses, I personally know teachers who resist using it especially in the senior grades because they are afraid that universities won't allow it. So once again, we are so busy getting students ready for university that we abandon methods and curriculum that students will enjoy and experience success with.

Two teachers teaching the same course – one giving harder tests and assignments than the other and generally believing to be doing a better job of getting them ready for university. If one set of marks is higher than the other, which students get the privilege of going to university – the students in the class that is supposedly well prepared or the students from the class that is supposedly not so well prepared. And then – what good does all that getting ready do for the students???

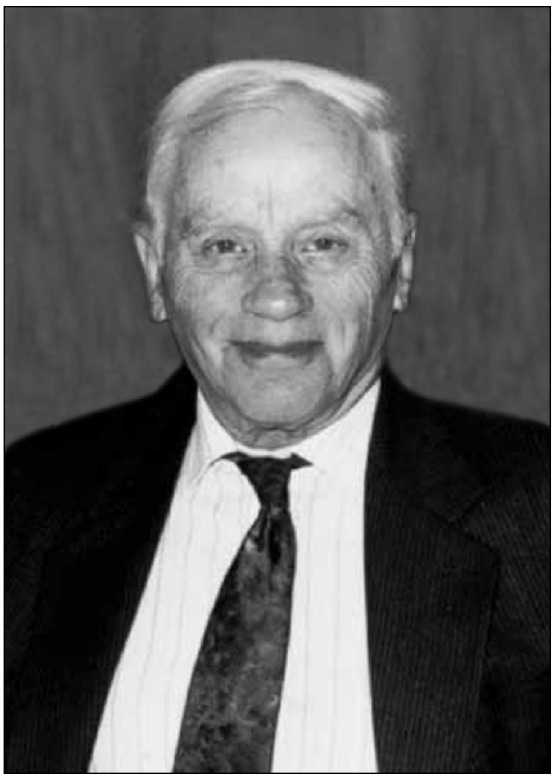
Getting students ready . . . what does that really mean? Should we care about doing it? How would we best do it? No question some basic knowledge and skills would be essential. However, I had all of the knowledge and skills that had been taught to me in high school and I was anything but ready for what I met at the University of Waterloo in first year mathematics. I am sure that my teachers believed they had done a fine job of getting me ready and I think they had, but not in the ways that they thought. I loved learning and I knew how to learn. I had a positive attitude. I was a problem solver and I was determined. I was not afraid to seek out help. These were the characteristics and skills that ensured success for me in university – not the explicit set of mathematics knowledge and skills that I had acquired.

If we inspire students to enjoy problem solving, to have confidence about understanding math, to see the relevance of what they are learning, and if the experiences we give them are suitable to the expectations for the grade level they are in then we will have succeeded in getting them ready . . . for whatever may follow! ▲

wintergreen

▲ Tribute to Paul Sherk (1914 – 2002)

PAUL SHERK has passed away. His work and spirit, however, will not be forgotten. Paul was a man whose dedication and interest in the well-being of young people lasted long after he finished his formal teaching career. Paul was always seen at the award ceremonies in Scarborough for math contest winners, taking pictures, and talking with students. Paul was always at the OAME conferences, with his ubiquitous camera, capturing the essence and beauty of the moment and ensuring that it was never lost. Paul appreciated what was important and wanted to make things last.



To young teachers in Scarborough, Paul was an icon. They knew that Paul was different and that he was looking for understanding in mathematics that went beyond the norm. Who will forget the construction of his elliptical pool table or the many models he helped construct for his students? The Scarborough Math Awards were unusual because Paul had made the trophies out of wood. His photography, woodwork, and teaching were all part of the same package. He was trying to create a foundation that would last.

Paul's teaching career began in 1942 when he was accepted for on-the-job teacher training in St. Catharines. Paul then moved Stouffville where he taught until 1949. Next, he moved to Smith Falls where he stayed until 1955. From there, he went to Athens District High School where he was principal until 1961. At Athens, he worked hard to save the school by increasing student enrollment to its present number of 350. His final teaching assignment was at Cedarbrae Collegiate in Scarborough where he remained as mathematics department head until 1977 when he formally retired. Paul was awarded the Descartes Gold Medal, presented by the University of Waterloo, in 1976 for his outstanding contributions to high school mathematics throughout the province.

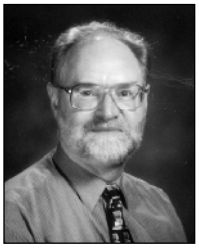
Paul met his wife Norma, of over 61 years, when they were students at Queen's University in 1937. They had five children, all of whom have made significant contributions to their chosen fields. Paul had many grandchildren (and great grandchildren), and until his stroke in January 2002, he attended all their secondary and post-secondary graduations. Paul's Christian beliefs were always of prime importance in his life. He held various positions in church life including Sunday school teacher and deacon.

Upon Paul's passing, it behooves us to think what is important in teaching and life. Paul was exceptional in all aspects of his life because he took the time to do things properly. He took his responsibilities seriously and worked for the betterment of others with no object of self-gain. He was thoughtful and caring. He knew what he believed and put these beliefs into action. The beliefs were simple and straightforward. Care for others and believe in those around you. And in the end, Paul cared and there is nothing more we can ask of anyone.

Written by: Peter Crippin, Faculty of Mathematics, University of Waterloo

▲ Technology Corner: Using Technology to Develop Multiple Solutions in Calculus

FRED FERNEYHOUGH



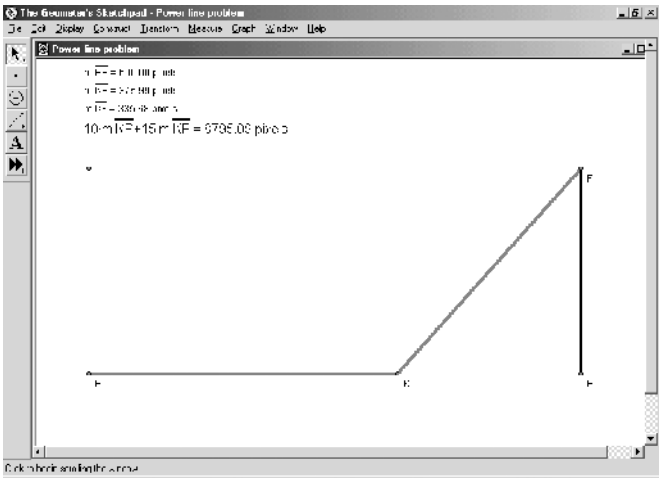
Fred Ferneyhough is Head of Mathematics at Central Peel SS in Brampton and is technology consultant for the McGraw-Hill Ryerson texts in Ontario. He is an avid user of, and advocate for, the use of technology in the mathematics classroom.

When students in lower grades ask what Calculus is all about, we are often at a loss to provide an answer that they can understand. When the grade 9 courses were first implemented, a team of teachers at Central Peel Secondary School used several traditional problems from calculus as applications of the Pythagorean Theorem and non-linear relationships. Multiple approaches not only increase students' chances of understanding concepts but also address the curriculum guidelines of solving problems using different models. Consider the following Calculus problem that can be solved in several ways:

A power plant is located on the south bank at point P. A factory is located on the north bank 600 m downstream at point F. The bank is 250 m wide. It costs $10/m to lay cable along the riverbank and $15/m to lay cable underwater. Find the location of point K on the south bank so that the factory's cost of laying cable is minimized.

Solution 1 – Using Traditional Calculus Methods

We find a cost function and its first derivative. By setting the first derivative equal to 0 and solving for the variable, we come up with a solution that places point K at approximately 376.4 m downstream. That results in a cost of $8795.08 to lay the cable. In my experience, students have not done well with this type of solution due to weak algebraic skills; that has been true of students from both the old and new curricula.



The dotted lines represent the banks of a river (of course, we make the claim that the banks are perfectly straight), while the solid lines represent the path of the cable.

Solution 2 – Using The *Geometer's Sketchpad*

The diagram above was created using the *Geometer's Sketchpad* and could be replicated by grade 9 students. In the solution, point K is dragged along line segment PB to determine the minimal cost, using the calculator built in the *Geometer's Sketchpad*. The answer is shown in pixels since the two measurements used in the calculation are lengths measured in pixels. Students using this approach for the first time seem to like the method since the solution is visual and the students control the placement of point K. Note that the optimal solution is given in the diagram as 375.99 m downstream.

Using this type of diagram, students are required to build an expression for the cost from the lengths in the diagram and the unit rates for laying cables. After completing the unit on algebra and linear equations, we have found that grade 9 students understand the following algebraic development of the expression for costs:

Cost = 10PK + 15KF

Let x represent the length of PK. ∴ KB can be represented by (600 – x). Using KB and BF as the legs of a right-angled triangle, we get:

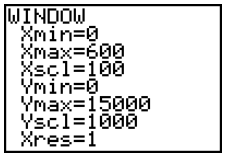
$$KF = \sqrt{(600-x)^2 + 250^2}$$

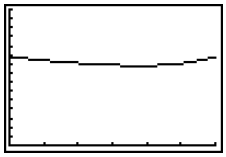
$$\therefore \text{Cost} = 10x + 15\sqrt{(600-x)^2 + 250^2}$$

Solution 3 – A Graphical Solution Using the TI-83 Plus

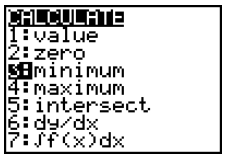
Many mathematics teachers have been using the TI-83 Plus for several years and are accustomed to the features on the calculator that will locate a local maximum and/or local minimum. To begin a graphical solution to this problem, store the expression $10x + 15\sqrt{(600-x)^2+250^2}$ in Y1.

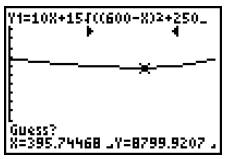
After some trial and error, students may find a window that is useful for this graph and gives the graph on the far right.

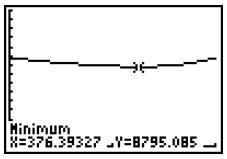




The CALC menu is used to find the minimum value for a graph:







From the final screen it can be seen that the minimum cost is $8795.09 when the point selected is 376.4 m downstream.

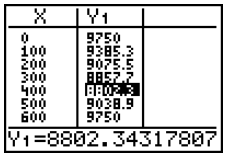
Solution 4 – A Numeric Solution Using the TI-83 Plus

One feature of the TI-83 Plus that could be used to explore relationships is the table of values.

In the next sequence of screens, the optimal value is found by closing in on the best result. In each pair of screens, I have included the Table SetUp values as well as the resulting table.

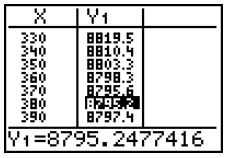
Start with an increment in the values for x of 100. The optimal solution in this table occurs at x = 400 in the table, but the best answer could be anywhere between 300 and 500. So, we will use x = 300 as the next value for TblStart and change the increment (ΔTbl) to 10.



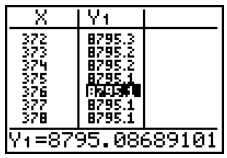


Now, the best solution appears for x = 380 but the optimal solution could be between 370 and 390. So, adjust the TblStart value to 370 and the increment to 1.







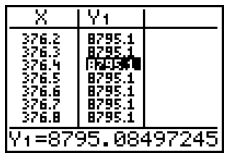


The best estimate is now x = 376 but the optimal solution could be between 375 and 377. So, adjust the TblStart value to 376 and the increment to 0.1.

Note in this iteration and the next that several y values appear to be equal in the table. However, when the cursor is placed on these values, more accuracy is seen at the bottom of the screen.

Finally, we get the solution that was obtained in the algebraic approach.



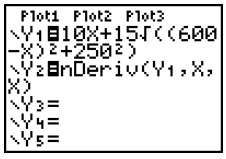


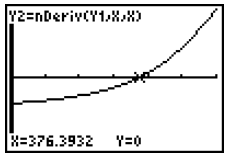
In each iteration above, the level of accuracy increases from hundreds to tens to units to tenths. If more accuracy is required, more iterations could be performed.

Solution 5 – Using the nDeriv Feature in the TI-83 Plus

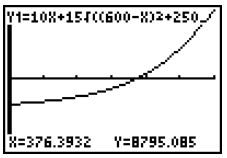
Many teachers have discovered that the TI-83 Plus will graph derivative functions.

The nDeriv function is used here to find the numerical derivative for all values of x and plots the function.





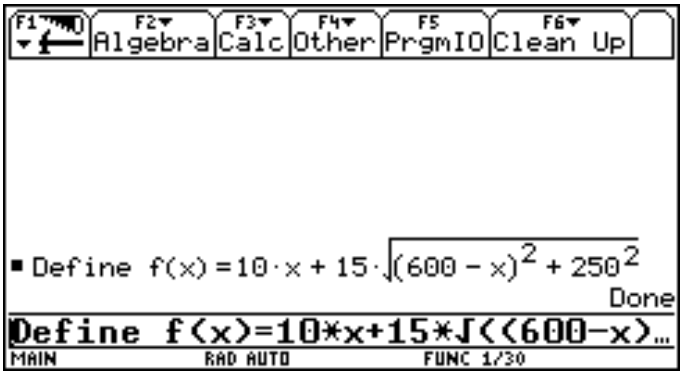
By changing the window, we can look at this function and use the CALC menu to find the zero of the function. The screen will show the value of the cost function at the zero for the derivative function.



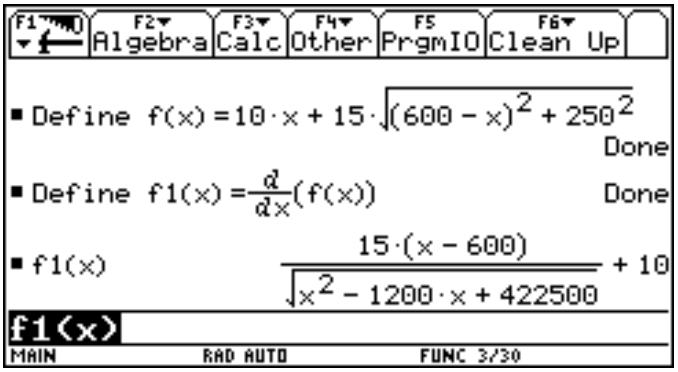
Solution 6 – Using the Computer Algebra System on the TI-92 Plus

Due to the messy algebra in solution 1, you may choose to use a computer algebra system to get students past the difficulties with algebra. However, this should not be used in all problems. Students still require good algebraic skills and it would be irresponsible for us to ignore this need.

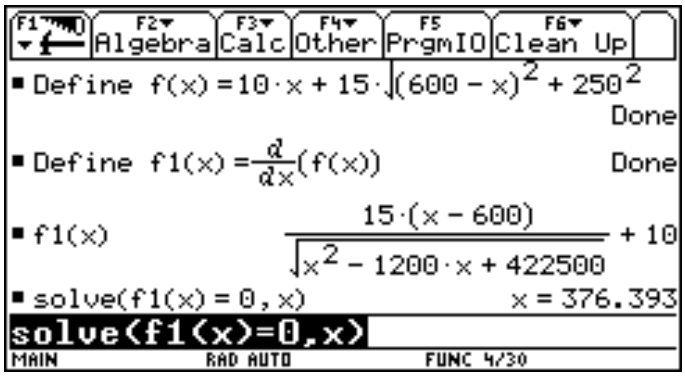
First, we define a function to represent the cost of laying the cable across the river.



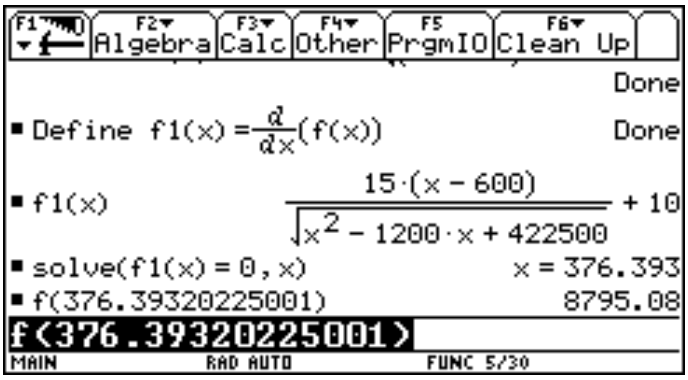
Next, define the first derivative function and display it on the screen.



Use the algebra menu to set the first derivative function equal to 0 and find a solution for x.



Finally, substitute the solution into the original function to find the cost.



Clearly, technology allows us to explore problems in Calculus and provides more students access to a solution that they can understand. ▲

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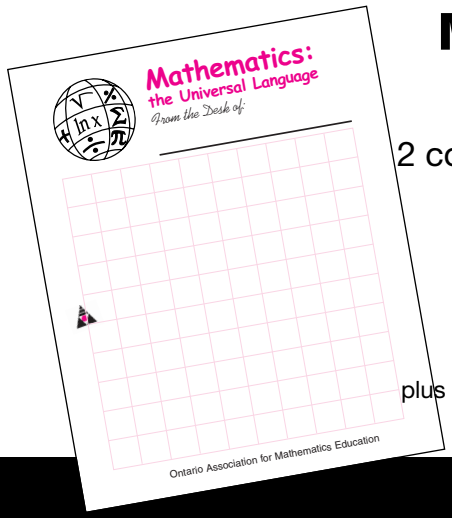
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▲ Hey, It's Elementary...: Move Over Oprah... There's Something Radical Happening on the Book Club Scene in Ontario!

LYNDA COLGAN
E-MAIL: colganl@educ.queensu.ca



Lynda Colgan has been at the Queen's Faculty of Education for four years, teaching in the B.Ed., M.Ed., and Ph.D. programs. Her primary focus is the classroom and her research is aimed at improving the mathematics learning experiences of the child by supporting teachers and parents.

CANADA READS

Is it possible to find a single book that captures the imagination of an entire country?

Absolutely. *Oprah's Book Club* was the first to remind us that reading *en masse* was an important and satisfying thing to do. *Oprah's Book Club* has been enormously successful in raising the literacy and learning quotient among ordinary citizens. From Kentucky to Kennebunkport, people of all ages and backgrounds meet, voluntarily and enthusiastically at kitchen tables or around fireplaces, delving into thought-provoking novels like Rohinton Mistry's *A Fine Balance* and Joyce Carol Ootes' *We Were the Mulvaney's*.

Not to be outdone by our neighbours to the south, last summer, CBC radio launched *CANADA READS* (our soon to be annual Newfoundland to British Columbia read-a-thon). In April 2002, CBC Radio host, Mary Walsh, led five panelists on a competitive quest to choose one book for the nation to read together. Former Prime Minister Kim Campbell, actor Megan Follows, musician Steven Page of the *Barenaked Ladies*, and writers Leon Rooke and Nalo Hopkinson, each had an opportunity to nominate and defend their choice of one work of Canadian fiction as **the** title **all** Canadians should read. The shortlist of books and nominators were:

- *The Handmaid's Tale* by Margaret Atwood - chosen by Kim Campbell
- *Whylah Falls* by George Elliott Clarke - chosen by Nalo Hopkinson
- *The Stone Angel* by Margaret Laurence - chosen by Leon Rooke
- *A Fine Balance* by Rohinton Mistry - chosen by Megan Follows
- *In The Skin of a Lion* by Michael Ondaatje - chosen by Steven Page

During a week's worth of spirited debate and impassioned conversation, the panel participated in a daily "battle of the books", voting one book off the list until only a single title remained. The winning title was unveiled on April 23rd (Canada Book Day[1]) by Roch Carrier, our National Librarian, as the book that all *CANADA READS*.

CANADA READS, the first event of its kind for a country, was innovative, engaging, and struck a chord with Canadians. Shortly after *In the Skin of a Lion* was honoured as being that title that *CANADA READS*, there were competitions. One of the first challenges to be issued was by "Chesley Area Friends of the Library". This group challenged other "Friends" groups and libraries across Canada to monitor how many of their patrons read Ondaatje's winning novel. The Chesley community "Friends" group would post each community's final tally as a percentage of their population on a web site, keeping a running tally, finally announcing a "winner" on Labour Day. A significant number of communities participated in the "Chesley Challenge" and each reader is to be congratulated for his or her effort and interest in the book and this unique Canadian initiative[2].

[1] Canada Book Day coincides with UNESCO-declared World Book Day. The celebration takes place on April 23 each year, a symbolic date for world literature when prominent authors including Shakespeare, Cervantes, and Nabokov were either born or died.

[2] You can check out the results of the "Chesley Challenge" at http://www.friendsoflibraries.ca/canadareads.htm.

More significant, however, was the number of individuals who chose not to compete, but to share their common (or not so common) reading experiences. One of the most important offshoots to the *CANADA READS* project has been the formation of book clubs, most face-to-face, but some on-line, where readers do not race to finish the book but gather instead to reflect and study the author's work, celebrating and debating ideas, opinions, and insights.

The reader who competed to finish the winning book in record time so that the next person on the list could also check off that he had "read" *In the Skin of a Lion* could probably tell you that the book is really a story about the lives of immigrants who built Toronto in the early part of the 20th Century. In contrast, members of local and virtual book clubs discovered through study and discussion, the ways in which at its core, *In the Skin of a Lion* is a novel about love and hate, politics and anarchy, rich and poor.

The *CANADA READS* book clubs underscore the fact that studying, reflecting, and learning deeply are not solitary endeavours. Other members of our community play a formative and necessary role in our own learning and the construction of meaning is mediated by social relationships. Book clubs concretize[3] the notion of connectedness and emphasize that sharing – responsibility, membership in a community, beliefs, values, expectations and knowledge – is the most important element in learning, developing, and maturing, professionally and personally.

The Ontario Mathematics Community Reads

The phenomenal success of the *CANADA READS* book clubs has its own meteoric parallel in the Ontario mathematics community[4]. Just as *In the Skin of a Lion* captured the imaginations of Canadians from coast to coast, teachers from Peterborough to Oakville and Windsor to Thunder Bay, are meeting enthusiastically and purposefully to study and work through John van de Walle's book, *Elementary and Middle School Mathematics: Teaching Developmentally*. Many clubs are using as their catalyst for discussion and organization a book study facilitator's guide written by the mathematics team from the Halton District School Board: Jane Bennett, Mike Davis, Ruth Dawson, Craig Featherstone, and Don Jones.

The principal goal of any workshop, summer institute, or conference for educators is to transform teaching and learning. The aim of these energetic math book study groups is no different – a better mathematics learning community for children achieved by helping teachers understand how specific practices and uses of resources/materials encourage children's mathematical learning and development.

What is different is that the book clubs are making a huge difference where it matters – in the life of a child. In traditional conferences, the success of professional development usually rests with an outside, expensive expert being parachuted in to speak, bringing the "truth" to teachers. In the book club model, however, "truth" is created by the members themselves as they interact and discuss their own professional learning. The book study model acknowledges the power of relationships, and requires each member to be responsible in some way for the learning of every other participant. By raising difficult issues and telling personal stories in a supportive environment, group members find connections with some of the ideas in the book and with each other.

---

[3] Seymour Papert, a student of Jean Piaget, has argued persuasively that the computer is a powerful instrument for enabling students to bridge the gap between "concrete" thinking and "formal" thinking. He argues that the computer can "concretize" formal mathematics, bringing it closer to a youngster's experience and produce more efficient mathematics instruction. "Concretize" is used in this context to capture the notion that book clubs epitomize the tangible model we strive for in describing a community of learners.

[4] To my knowledge, the first mathematics book club in Ontario began in 1997 under the leadership of Mary Lou Kestell. Mary Lou selected What's Your Problem: Problem Posing and Problem Solving K-2 *by Penny Skinner and supplemented the readings with articles from* Teaching Children Mathematics *so that both Primary and Junior-division teachers could meaningfully participate. Funded by an Industry Canada grant, and supported by Penny Skinner herself (who participated on-line from Australia), the book club met regularly in face-to-face and on-line formats over a two-year period. The writers and moderators for the on-line version were Mary Lou (the Professional Development voice), Val Brocbank-Stevens (as the voice of the Junior teacher), and Monica Pupa (nee Caldwell), representing the primary teacher's perspective. Mary Lou's pioneering work in this innovative professional development model was shared at meetings of The Ontario Mathematics Coordinators Association. Thanks, Mary Lou, for laying the groundwork.*

Pearson

The book study model has strong conceptual roots[5] in Vygotskian and social constructivist theory. First, the book club member must learn how to talk about the assigned chapter during a community sharing time. As part of her response journal, the member accommodates her initial understandings by expressing her new knowledge in her own words. When she next experiments with the strategies in novel ways as part of her own classroom work, she makes another sophisticated developmental transition, assimilating her new knowledge into her teaching repertoire and evaluating it. When she examines with her colleagues the results of her own classroom experience, there is a sense of shared ownership for the observable and documentable changes to practice. Within the Book Club setting, teaching and learning are distributed and interactive, becoming the responsibility of each and all of the participants.

Professional development dollars continue to diminish from school board budgets. Educators and administrators are finally facing the fact that most workshops and isolated sessions do little to motivate or sustain significant change to teacher practice and professional beliefs. Hundreds of teachers throughout Ontario stand before eager students to help them meet the high standards that the province has adopted. It makes sense to consider the book club option as a powerful model to help teachers themselves deepen their own knowledge and skills.

For further reading:

Ball, D. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to know, *Phi Delta Kappan 77*(7), 500–508.

Goldberg, S. M., & Pesko, E. (2002). *The teacher book club*. Available: http://www.ascd.org/readingroom/edlead/0005/goldberg.html.

National Council of Teachers of Mathematics. (2002). *Study shows high-quality professional development helps teachers most.* Available: http://www.nctm.org/news/articles/2002-03nb_sixaspects.htm.

Sparks, D., & Loucks-Horsley, S. (1989). Five models of staff development for teachers. *Journal of Staff Developments, 10*(4), 40 – 55.

Wertsch, J. V. (1985). *Cultural, communication, and cognition: Vygotskian perspectives.* Cambridge University Press. ▲

[5] The major theme of Vygotsky's theoretical framework is that social interaction plays a fundamental role in the development of cognition. Vygotsky believed that every function in a learner's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the learner (intrapsychological). Vygotsky contended that this applies equally to voluntary attention, to logical memory, and to the formation of concepts.

▲ Math Magazines: Developing Students' Communication Skills

JOANNE HARRIS



Joanne Harris teaches grades 7 and 8 mathematics at Centennial Middle School in Georgetown. She is an avid quilter and loves the action of a great hockey game. She is always on the lookout for ways to make her mathematics program more relevant for students.

Extra! Extra! Read all about it! Latest issues of Math magazine hit the stands! Using a magazine format and working individually, grades 7 and 8 students at our school are encouraged to look at Mathematics as integrated rather than as separate strands. Students are challenged to see and describe how Mathematics is used in their daily lives, in entertainment and the arts, and to report on it.

For the specific assignment, each strand is represented in some aspect in the final product. The project, with its accompanying rubric, was assigned at the start of the second term and the project was due at the beginning of the third term. That allowed adequate time for students to produce a quality project and for teachers to properly evaluate it. At midterm, a "check point" was held for students to bring their work "in progress"; this was a time management strategy to prevent work from being left to the last minute. Students unable to produce any work at the checkpoint had a note sent home to parents/guardians. These notes were returned with parent/guardian and student signatures.

To focus students' thoughts, the types of articles for a complete magazine were specifically assigned.

The first requirement was a research article. Students could select a mathematical person, a math-related instrument (grade 7 only), a math-related career (grade 8 only), or a mathematical topic of personal interest. The range of choices allowed for many opportunities to explore. Students had access to the Internet for research purposes as part of their Information Technology program. A few students, though, had difficulty applying their writing and research skills learned and used in Language Arts to the Mathematics classroom.

The second requirement was a crossword puzzle. The web site www.puzzlemaker.com has a puzzle maker that many students found helpful. This component reviewed basic terms across the strands. Students created the clues so others completing the puzzle would think of specific mathematical terms (e.g., the answer to "the measure of the distance around the outside of a circle" is circumference). Using the Puzzlemaker application encouraged students to focus their energies on the definitions or clues rather than the actual creation of the puzzle.

The Art section was the next requirement. Students created a line design, a tessellation, or a quilt design. Grade 7 students were encouraged to create a tessellation design. Many students took advantage of computer technologies such as the *Geometer's Sketchpad* to construct their designs. In some cases, students used paint programs to complete the colouring of the final product. Information was made available about how to construct a basic line design and pictures of finished samples were shown in the classroom. A few quilting magazines were also made available to encourage students who wanted to try that form of expression.

The fourth requirement was the "Dear Math Maniac" column, an advice column. To make marking easier, students were encouraged to select from a list of questions posted in the classroom. To answer the question, students explained what something is, or described how to do something. A sample question was, "Dear Math Maniac: I don't understand the difference between 3x2 and 3^2. I thought they both gave an answer of 6 and my teacher said I was wrong. Signed, Puzzled in Parry Sound." Students completed a minimum of three "Dear Math Maniac" questions.

The next section was "Entertainment". Students created an original Math-related comic strip, game, or Mathematical jokes. Many students turned to the Internet for ideas and it was amazing how much mathematical humour was accessible.

The final portion of the project was "Problem Central". In this section, students created three different types of word problems with solutions. Students were reminded to keep their answers separate from the questions. As the magazines were to be read by grades 7 and 8 students, the problems selected had to be appropriate for that age group.

When the project was assigned, samples of *Games* magazine, word search, crossword puzzle, and other puzzle-based magazines were made available in the classroom for students to view.

Students demonstrated creativity in naming their magazines, using titles such as *CosMATHpolitan* and *MATHa Stewart Living*, and often selected a theme that ran throughout their articles. Other teachers in the school became involved. Students used Information Technology classes to research or create their crosswords, Language Arts teachers helped students edit, and Art teachers encouraged students to draw original cartoons.

Although assessment at the end was a challenge, there was value in waiting for the final product, rather than accepting the sections one at a time. However, this alternative could be considered as a strategy with students who possess weak time management skills.

Would we do it again? Yes, but with some changes. Some of our students needed more direction, especially with the research article. We are just beginning to see the benefits of working with other staff members and would probably try to get them on board right from the start. Overall, this project made students look at Mathematics and communicate about it in different ways.

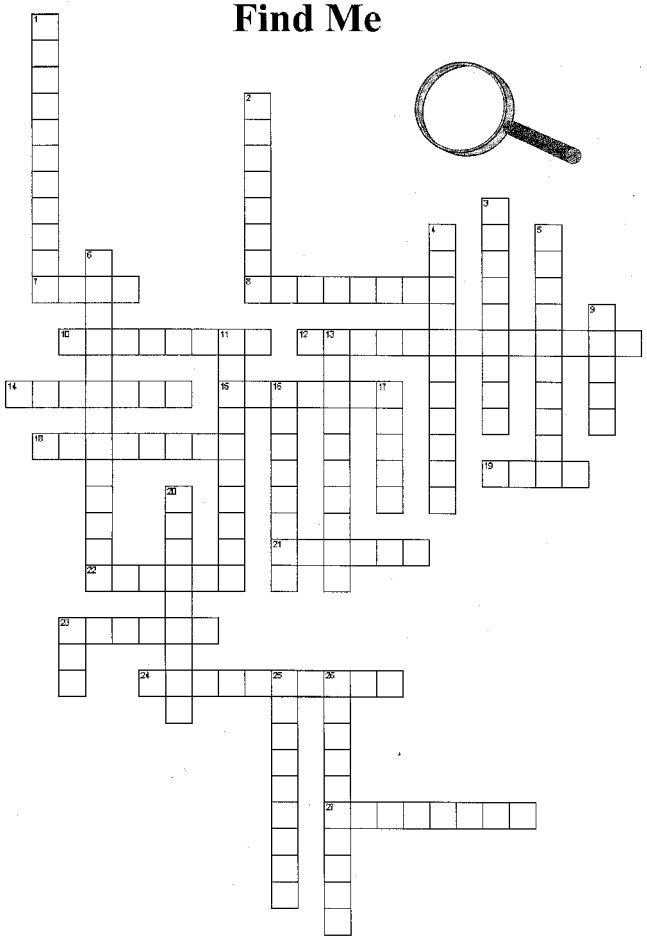
Thank you, **Jan Crowfoot**, for presenting the original concept in a workshop at the Fall 2001 CHAMP Math4U conference. Using her structure, we developed specific details and questions to meet our specific needs.

Thanks also to **Victoria Little** and **Chandra Massey** for providing permission for me to share their work with the mathematics education community (see page 16). ▲

Rubric used to assess each magazine

Criteria	Level 1	Level 2	Level 3	Level 4
Problem Solving • Problem Central	• creates a limited variety of appropriate problems • infrequently selects the appropriate problem solving strategy • solution have limited accuracies	• creates some appropriate problems • sometimes select appropriate strategies to solve problems • solutions are generally correct	• consistently creates an appropriate variety of problems • chooses appropriate strategies and is consistently able to provide accurate solutions to the problems	• always creates an appropriate variety of problems • always selects the most appropriate problem solving strategy to provide solutions that are always accurate
Understanding of Concepts • Crossword Puzzle	• provides clues that are partially correct and demonstrates a limited understanding of the mathematical vocabulary	• sometimes creates clues that are correct and demonstrates an understanding of the basic mathematical vocabulary	• consistently creates appropriate clues that demonstrate understanding of the basic mathematical vocabulary	• thoughtfully creates correct and creative clues that demonstrate thorough understanding of the basic mathematical vocabulary
Application of Mathematical Procedures • Art Feature • Entertainment	• applies both Art and Mathematics concepts to create a design that reflects a limited understanding of design, patterning, creativity, and original presentation • applies limited Mathematical terms correctly in the entertainment feature	• sometimes applies Art and Mathematics concepts to create a design that reflects good design, patterning, creativity, and original presentation • applies some Mathematical terms correctly in the entertainment feature	• consistently applies both Art and Mathematics concepts to create a design that reflects good design, patterning, creativity, and original presentation • consistently applies Mathematical terms correctly in the entertainment feature	• insightfully applies both Art and Mathematics concepts to create a design that reflects outstanding design, patterning, creativity, and original presentation • always applies Mathematical terms correctly in the entertainment feature
Communication of Required Knowledge Related to Concepts, Procedures, and Problem Solving • Dear Math Maniac • Research	• provides answers to Dear Math Maniac using limited appropriate Mathematical terminology and symbols • completes a research project with limited use of standard conventions	• provide some answers to the Dear Math Maniac questions using appropriate Mathematical terminology and symbols • completes a research project using some of the standard conventions	• consistently provides clear and precise answers to the Dear Math Maniac questions that use the appropriate Mathematical terminology and symbols • completes a research project using standard conventions	• always provides clear and precise answers to the Dear Math Maniac questions using appropriate Mathematical terminology and symbols • completes an original research project clearly using standard conventions

Find Me



Across
7. To average something.
8. A problem with an answer.
10. Two lines that never touch are_____.
12. The distance around a circle.
14. A triangle that has no sides the same.
15. A number multiplied by 100 is your .
18. 9 x 7 = 63 63 is a_____of 9 x 7.
19. The amount of space a shape covers.
21. An angle greater than 90 degrees.
22. A number in the middle of a group of numbers.
23. A____is when you ask a series of multiple choice questions.
24. When 2 shapes are exactly the same but reversed.
27. How much an object can hold.

Down
1. Reducing a number as much as you can by dividing it.
2. Likely to increase or decrease in size, number, etc.
3. A triangle that has 2 sides the same.
4. The bottom number of a fraction.
5. When you subtract, you find the_____.
6. A four sided figure where all sides are parallel.
9. An angle less than 90 degrees.
11. A problem with no answer.
13. An expression showing that two numbers are not equal.
16. A shape that is turned.
17. A mark used to identify classification.
20. The distance around an object.
23. The number obtained by adding two or more numbers together.
25. Things are____when the shape and dimensions are the same.
26. An official examination.

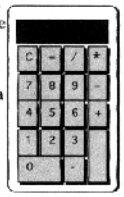
28 of 28 words were placed into the puzzle.

Visit Puzzlemaker at DiscoverySchool.com

DiscoverySchool.com

Dear Math Mania!

Dear Mania, I don't know how to get to a decimal number to a percent number! Example: 0.80 = ?% From: Percent Trouble



Dear Percent Trouble, okay all you have to do to change a decimal to a percent is move the decimal over to the right two times. Put the percent sign after the final number. There you have it, you changed a decimal to a percent! That's all!

Dear Math Mania, I have a problem, I don't know how to reflect a shape properly, can you help me? From: Reflect Problem!

Dear Reflect Problem, to reflect a shape or object first you have to draw your shape, see example A. Next draw a mirror line beside the shape, this will be your line you reflect over, see example B. Next count or measure how far away each point is from the mirror, and draw the dots of each point of your shape on the other side of the shape. Next connect the lines, see example C. There you go! You now know how to properly reflect a shape!



A)  B)  C) 

Dear Math Mania, I don't know how to get the area of a square, do I multiply it by three, add two or do something else? Help me! From: Area Of A Square = ?



Dear Area Of A Square = ? First what you need to do is measure the sides on your square. Each of your sides are 2cm long. Next write under your square A = . The A stands for area = . Next write A = S2, S2 stands for the side of your square multiplied by the other side of your square. Then write how long your sides are. Example: A = 2cm multiplied 2cm. Now write what your question equals. Example: 2cm multiplied by 2cm = 4cm. Then add the square sign 2 and your done! Now you can find the area of a square.

4

JOKES

How do you make a foolish person busy all day??????
Put him in a round room and tell him to sit in a corner.

Math and alcohol don't mix, so... **PLEASE DON'T DRINK AND DERIVE.**

What has 3 feet and 1 leg??????
A yard stick.

What did one math book say to the other??????
Don't bother me, I've got my own problems.

Mathematics is made up of 50% formulas, 50% proofs and 50% imagination.

To mathematicians, solutions mean finding the answers, but to a chemist, solutions are all mixed up.

What is one of little mermaids' articles of clothes??????
Algebra (Algae-bra)

Old mathematicians never die; they just loose some of their functions.

How many lines of symmetry does a circle have??????
180 lines (360 divided by 2)

Why is 6 afraid of 7??????
Because 7 8 9. (seven ate nine)

▲ Polyhedra Summative Task for MAP4C

TIM SIBBALD



Tim Sibbald teaches mathematics and physics at Clarke Road Secondary School in London, Ontario.

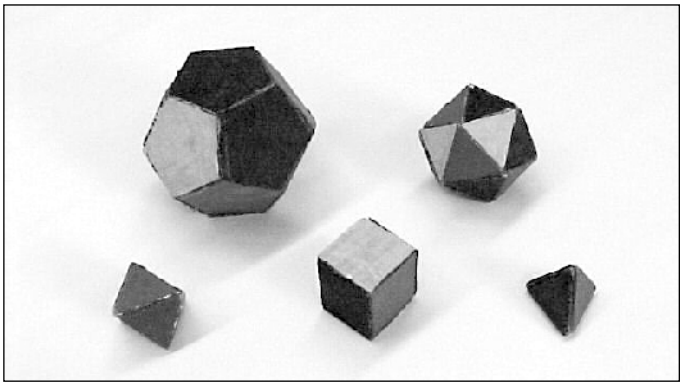
The use of polyhedra in mathematics is not new; however, a review of its application in the new grade twelve mathematics curriculum is due. How does it match the expectations? How can it be used as a summative task? The focus of this article is on the College and Apprenticeship Mathematics (MAP4C) course, but the content may be adapted to other courses as well.

Polyhedra come in many varieties so it is necessary to simplify the focus to two particular groups of polyhedra. The first, which will be most useful for the formative stage of teaching, is the Platonic Solids. A second group that will be more challenging, but important for the summative task, is the Archimedean Solids. Both of these groups are said to be "convex", meaning that if you take any two points on the surface then all points on the line joining the two points are inside or on the solid.

The Platonic Solids

The Platonic solids have every face congruent to every other face. As a result, every edge length is identical. There are five such objects: the tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron. The "tetra"-hedron is composed of four identical triangles, the cube (sometimes called a hexahedron) is six identical squares, the "octa"-hedron has eight identical triangles, the "dodeca"- hedron has twelve pentagons, and the "icosa"-hedron is made up of twenty identical triangles.

It may be useful to have students find the prefixes "tetra", "dodeca", and "icosa" in the dictionary. They could also come up with other words that use the same prefixes. The prefixes can later be used to develop reasons for the names of the Archimedean solids.



The Platonic Solids (Clockwise from bottom: Cube, tetrahedron, octahedron, isosidhedron, dodecahedron.)

The first step in studying the solids is to build the models. There are a number of schemes for building the models. Kits are available for the relatively simple models but they incur cost. Nets that can be printed on paper (see the resources listed at the end) are available; however, they give the impression that you need a net to make the object. A doweling and PlayDough, or toothpick and gumdrop, approach would likely be successful but it would be wise to make the polygons first. The approach used by the author was to make plexiglass templates of the polygons that were then used to cut the shapes out of cardboard. A hot glue gun was used to assemble the solids. This has the advantage in that the construction arises naturally (no one has to invent the net before you can do this) and the glue dries relatively fast. Care should be taken to address safety with respect to the hot glue; we borrowed the hot glue guns from the visual arts department and found that they had a low and high setting to address this very concern! Note that we have already addressed one expectation: "Design and construct physical models of things, satisfying given constraints and using concrete materials."

While constructing each object, students could build the net by tracing the location where they intend to join the next polygon; that would address the expectation: "Create nets, plans, and patterns from physical models related to a variety of applications." It can also be used to promote the idea that there are many nets that give the same object. In some cases, students may be able to address the question of how many different nets exist.

The surface area can be computed in two ways. Firstly, one might start by having students measure the surface area. To do this, they will break each polygon into the isosceles triangles that compose it. They can then measure the height of a triangle, which will be the same for all triangles in one kind of polygon - for the

Platonic solids it is the same for all faces. The base length is simply the edge length used throughout the structure. They can measure the base (or edge) length and calculate the measured surface area; this addresses the expectation, "Demonstrate accuracy and precision in working with metric measures." The same procedure could be completed using the net.

In addition to measuring the surface area, students can calculate the exact surface area and estimate the volume, this will address the expectation: "Solve problems related to .... surface area and volume of prisms, pyramids, ... including problems involving combinations of these objects." The exact surface area requires that students calculate the area of a polygon given just the edge length; that can be done by breaking the polygon into isosceles triangles where the size of the centre angle is calculated. From this, the tangent of half the centre angle relates the height to half the known base length. Alternately, the cosine law could be used.

The volume calculation is complicated since the distance from the middle of a face to the centre of the solid is not known. To address this problem, have students measure/estimate the distance. Arriving at an exact value is feasible for a dedicated student but too awkward and time consuming for the class. With the distance to the centre estimated as "h" and the area of the base of a prism being "A", the volume of a prism is calculated using the formula $V = \frac{Ah}{3}$. The volume of a prism is then multiplied by the number of such prisms that occur in the solid (i.e., the number of faces).

The calculated surface areas and volumes for an edge length "L" are given below to facilitate checking students' results. Note that the explicit value of $\tan(30^\circ) = \sqrt{3}$ has been used.

With the class divided into groups working on a particular solid, then the results can be brought together and a discussion begun:

- Why is the surface area of an octahedron exactly twice the surface area of a tetrahedron?
- Why is the surface area of an icosahedron three times the surface area of a tetrahedron? Why isn't this true for their volumes?
- Why does the surface area always depend on L^2, whereas the volume depends on L^3?
- The tetrahedron and octahedron are different shapes but have the same volumes. Explain how that can be.
- For a given edge length, which shape will have the largest surface area?
- How will the results be different if "L" is measured in inches instead of centimetres? (Try it!) (Note that the expectations mention working with area in imperial units and also conversion between imperial and metric.)

TEACHER ANSWER TABLE

Name of the Solid	Surface Area	Volume
Tetrahedron	$\sqrt{3}\,L^2$	$\frac{\sqrt{2}\,L^3}{3}$
Cube (Hexahedron)	$6\,L^2$	L^3
Octahedron	$2\sqrt{3}\,L^2$	$\frac{\sqrt{2}\,L^3}{3}$
Dodecahedron	$\frac{15\,L^2}{\tan(36^\circ)}$	Exact formula is awkward. Approximately $2.6327\,L^3$
Icosahedron	$3\sqrt{3}\,L^2$	Approximately $2.1817\,L^3$

The Archimedean Solids

The Archimedean Solids may have faces that are different polygons, however, at every vertex there must be the same combination of polygons. For example, if one vertex joins a square, a triangle, and a pentagon, then all vertices on the solid must have these three polygons meeting. There are exactly thirteen of these solids that allows for diversity of outcomes in the classroom and considerable latitude for variety.

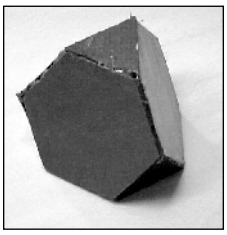
As mentioned earlier, the names can be justified using a basic knowledge of the prefixes. An icosidodecahedron, for example, has twelve ("dodeca")

THE ARCHIMEDEAN SOLIDS

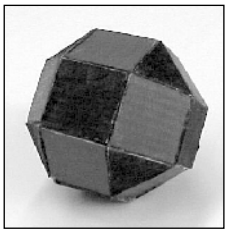
Name of Solid	Description of each vertex.
Truncated Tetrahedron	Two hexagons and a triangle.
Cuboctahedron	Two squares and two triangles.
Truncated Cube	Two octagons and a triangle.
Truncated Octahedron	Two hexagons and a square.
Rhombicuboctahedron	Three squares and a triangle.
Great Rhombicuboctahedron	An octagon, a hexagon and a square.
Snub Cube	A square and four triangles.
Snub Dodecahedron	A pentagon and four triangles.
Icosidodecahedron	Two pentagons and two triangles.
Truncated Dodecahedron	Two decagons and two triangles.
Truncated Icosahedron	Two hexagons and a pentagon.
Rhombicosidodecahedron	A pentagon, two squares and a triangle.
Great Rhombicosidodecahedron	A decagon, a hexagon and a square.

pentagons and twenty ("icosi") triangles. An astute observer will note when the prefix "icosi" occurs immediately before hedron, it is changed to "icosa" as in icosahedron. A colleague has suggested that this is due to the similarity in pronunciation between "i" and "h" in Greek.

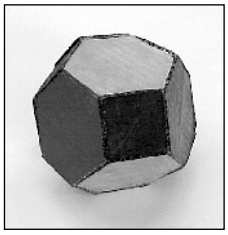
The surface area calculations can be completed for the Archimedean solids, but unlike the Platonic solids, it is necessary for students to consider each type of polygon separately.



Truncated Tetrahedron



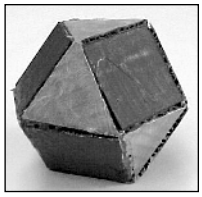
Rhombicuboctahedron



Truncated Octahedron

The construction of nets can also be completed. Students could be asked to explain how the pattern that is evident in each three dimensional model might be made clear on a net. The catch being that a net has cut edges and these connections would have to be conveyed using the net. Students should be able to develop their own approaches to showing the missing connections on their nets.

As a summative task, the students can be asked to determine the surface areas. They could also estimate the volume although it will seem trickier to many students because it is more difficult to estimate the height of the pyramids that compose the solid.



cuboctahedron

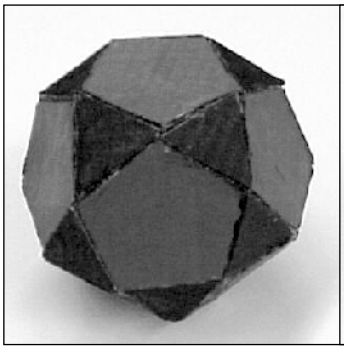
- Do more sides mean more area?
- The truncated dodecahedron and great rhombicosidodecahedron have the highest ratio of area to sides. Suggest a reason.
- Suppose professor Lou Knee discovered a new solid with 15 squares and 15 hexagons. What surface area do you think the shape might have? Explain why.
- If you doubled the edge length, how would the surface area change?
- The Archimedean solids are ideal for discovering the Descartes-Euler formula:

$$V + F = E + 2$$

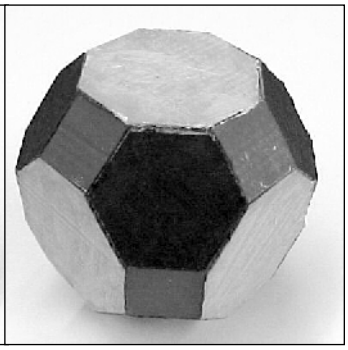
where V is the number of vertices, F is the number of faces and E is the number of edges. For a truncated cube, for example, V = 24, F = 14, and E = 36, so V + F = 24 + 14 = 36 + 2 = E + 2. The act of counting vertices, faces and edges is not difficult but requires a certain organizational aptitude.

SURFACE AREAS OF ARCHIMEDEAN SOLIDS

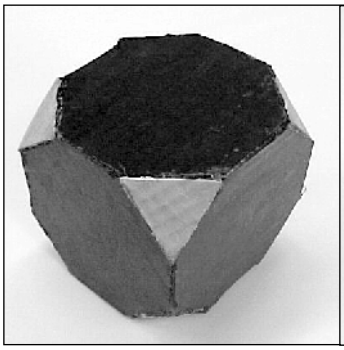
Name of Solid	Polygon Composition						Total Surface Area ($x L^2$)
	3	4	5	6	8	10	
Truncated Tetrahedron	4			4			12.124
Cuboctahedron	8	6					9.464
Truncated Cube	8				6		32.434
Truncated Octahedron		6		8			26.785
Rhombicuboctahedron	8	18					21.464
Great Rhombicuboctahedron		12		8	6		61.755
Snub Cube	32	6					19.856
Snub Dodecahedron	80		12				55.286
Icosidodecahedron	20		12				29.306
Truncated Dodecahedron	20					12	100.990
Truncated Icosahedron			12	20			72.608
Rhombicosidodecahedron	20	30	12				59.306
Great Rhombicosidodecahedron		30		20		12	174.292



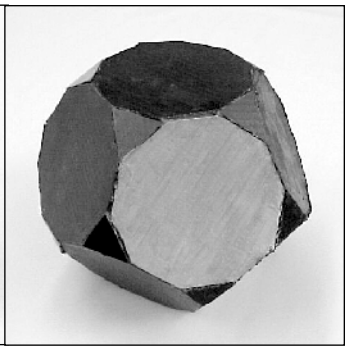
Icosidodecahedron



Great Rhombicuboctahedron



Truncated Cube



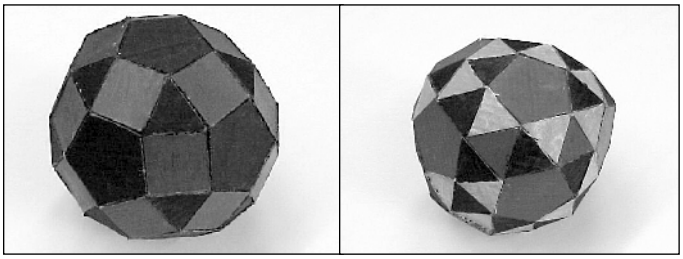
Truncated Dodecahedron

If a class composes all the data they have about the polyhedra, then this formula can be verified. While it is not explicitly in the curriculum expectations, it could be used as a bridging point to the expectations "evaluate any variable in a given formula" and "rearrange a formula to isolate any variable." Perhaps some students will appreciate the mathematical aesthetics simply to avoid the algebra!

Beyond the Curriculum

Since students are likely to complete work with polyhedra at different paces, it is a good idea to have a few additional ideas that will provide enrichment for some students.

Find the minimum number of colours necessary to paint the shapes so that the same colour never appears on both sides of an edge.



Rhombicosidodecahedron *Snub Dodecahedron*

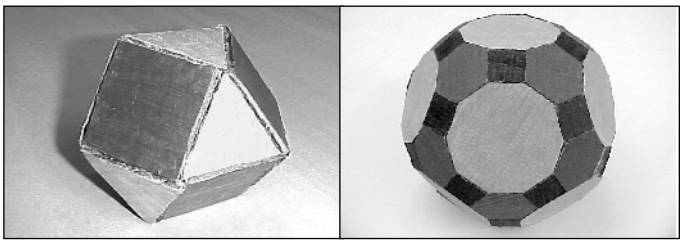
This colouring problem is related to the traditional four colour problem, but the surface is not flat. In fact, the truncated icosahedron requires five colours. Perhaps the best strategy is to have paint available since the result of colouring the models is quite artistic and suitable for display. A tetrahedron requires four, a cube requires three, an octahedron requires two, a dodecahedron and an icosahedron both require four. Among the Archimedean solids, the rhombicosidodecahedron is surprising since it only requires two colours in spite of having 62 faces!

Consider the shape that is created if you replace every face with only the point in the centre of the face and then you join these. This concept is called the "dual" of the shape.

The concept of the dual is mentioned in some of the web sites listed in the resources. Since each face is replaced by one point, the number of vertices in the dual must equal the original number of faces. This type of reasoning will limit the number of possibilities for the interested student. A tetrahedron, for example, has four faces, therefore, its dual has four vertices and must be a tetrahedron. An octahedron and cube are the duals of each other. Similarly, a dodecahedron and icosahedron are duals of each other.

Handedness is a factor for two shapes. An exceptional student could be challenged to make both versions.

The snub cube and snub dodecahedron each come in two different versions. Essentially, from a vertex of a square and pentagon respectively, there are two different directions that one could traverse the neighbouring triangles to find the next square or pentagon. By constructing a square or pentagon with its neighbouring layer of triangles, it will become apparent that there is a choice of where to situate the next square or pentagon.



Snub Cube *Great Rhombicosidodecahedron*

If the models are tossed around the classroom, students will soon realize the remarkable strength inherent in the shapes they have built. This can be used as a bridge into an independent study project of Buckminister Fuller and his notion of "tensegrity". This arises in two key areas of the modern world. Firstly, in chemistry where chemical bonds create strength by their geometric arrangements, and secondly, in architecture where many students are familiar with the idea of struts in a roof. When this idea is taken into three dimensions, the domes at Expo-67 and Toronto's Ontario Place arise. You might want to provoke student thought by pointing out the dome in Toronto, while it appears to be made of hexagons, has three pentagons! Otherwise it couldn't be closed up the way it does.

The architectural idea can also be examined from the point of view of designing a house based on one of the shapes. This has been done and information about "dome-homes" is available. Students could do a critical analysis of what is available. Some companies are in the southern United States; will they have considered snow? Are the energy savings quoted consistent with the difference in area to volume ratio for a dome versus a cube? (and is the usable space reflected in the comparison?) How could one lay out the interior of a dome home so that rectangular appliances fit reasonably? What would students suggest you do for plumbing on the second story?

One web site, www.geodesics-unlimited.com, is particularly noteworthy because it has detailed information on the relationships between the Platonic and Archimedean solids. They explain duality, snubbing, truncation, etc. They also explain the difference between domes derived from Platonic and Archimedean solids (geodesic domes), and other methods.

Conclusion

The use of polyhedra models in the secondary curriculum is appropriate and an exciting opportunity for students in the college preparation course. The questions presented here suit a wide range of abilities and can be adapted to other courses in the secondary curriculum (time permitting). The relative ease of breaking concepts into the component parts by using Platonic solids or the polygonal faces should facilitate progress by all students.



Truncated Icosahedron

Reference

Wenninger, M. J. (1966). *Polyhedron models for the classroom.* Washington, D.C.: National Council of Teachers of Mathematics.

Additional Resources

http://www.intent.com/sg/platonics.html and
http://www.intent.com/sg/archimedeans.html
This site has layouts for fold up models of the Platonic and Archimedean solids respectively.

http://www.scienceu.com/geometry/facts/solids/
Examples of all of the Platonic and Archimedean solids. This includes sound files that can be played to assist in learning the correct pronunciation of the names.

http://www.cs.washington.edu/homes/dougz/polyhedra/
Superb pictures of all of the Platonic solids and Archimedean solids along with printable versions of the nets for all of the models.

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/phi3DGeom.html
Defines the dual property of polyhedra. Provides the coordinates of the Platonic solids.

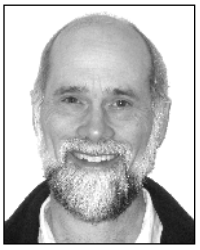
http://www.friesian.com/polyhedr.htm
This site has a table detailing the number of vertices, edges, and faces for the Platonic and Archimedean solids. It has sketches of each and also lists the dual of each. A small section at the end contains information about solids in four dimensions.

http://www.peda.com/poly/
This company sells die cast models of some of the solids and also software for creating virtual models and nets. ▲

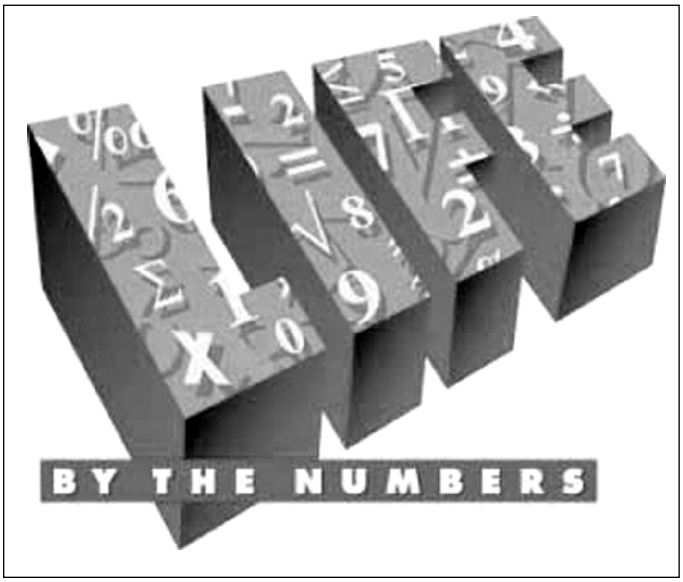
exclusive

▲ Video Review: *Life by the Numbers*

ROBERT SHERK
HEAD OF MATHEMATICS, SYDENHAM HIGH SCHOOL, SYDENHAM, ONTARIO



Robert Sherk is the Head of the Mathematics Department at Sydenham High School and is president of the Quinte St. Lawrence Mathematics Association (QSLMA), a chapter of the OAME. He has recently written for a number of publishers and is particularly interested in applications of technology in the new mathematics curriculum.



The *Life by the Numbers* video consists of a series of segments from a prime time mathematics series designed to present connections between mathematics and the real-world to a general audience. These segments were selected by teachers for use in their math classrooms based on their usefulness in presenting applications of specific topics in the intermediate/senior curriculum. Although these programs lack the visual appeal and dynamic presentation that may be needed to capture and maintain the attention and interest of students, the programs are worth using in classes as an alternative to traditional classroom activities. They could be used to highlight applications of concepts and/or for enrichment. Topics include those covered primarily in the grade 9 and grade 12 Data Management courses or are extension or enrichment topics. There is a definite focus on computer-related applications in most of the segments.

- **Module 1 *Mathematics and Monsters*:** Deals with the mathematics of scale and the relationships between surface area and volume or mass of objects (11 minutes);
- **Module 2 *New Geometry*:** Models naturally occurring shapes in plants (branches and blossoms) using fractals (9 minutes);
- **Module 3 *Artificial Life*:** Uses a computer-generated simulation to model evolutionary changes in an artificial ecosystem (9 minutes);
- **Module 4 *Linear Perspective*:** Uses the basic rules of geometry to create three-dimensional perspective drawings that model in two-dimensions what we see in everyday life (7 minutes);
- **Module 5 *Pictures From Numbers*:** Uses computer-generated graphics based on mathematical models to create images for an Imax film about outer space (10 minutes);
- **Module 6 *Crunching Numbers*:** Collects and analyzes data taken from videotapes of past football games, enabling coaches to develop game plans for future games (5 minutes);
- **Module 7a *To Spin or Jump*:** Uses computers to analyze videotapes of ice skaters to determine the best techniques for making the triple axel jump in figure skating (5 minutes);
- **Module 7b *A Fine Line*:** Careful measurements of a variety of biological factors enable bio-mechanical scientists to assist Olympic athletes to train and develop the most energy efficient techniques in their sporting events (8 minutes);
- **Module 8 *House Rules*:** Applies probability theory to gambling, including the use of computer simulations to predict outcomes (9 minutes);
- **Module 9 *Reading Minds*:** Uses probability theory and statistical sampling techniques to analyze opinion poll results and predict future trends in public opinion (6 minutes);
- **Module 10 *Ground Truth*:** Creates accurate topographical maps using satellite-imaging techniques for use in flight simulators and military strategy planning (7 minutes);

- **Module 11** ***Circles, Angles, and Echoes*****:** Uses geometric and trigonometric techniques, coupled with data obtained from aerial photographs and ground level measurements, to create maps of remote areas including those below the surface of the ocean (10 minutes);
- **Module 12** ***Machines of the Future*****:** A look into the MIT Media Lab where they are developing software to assist in surfing the Web and hardware to make our lives easier. This module also includes an overview of the development of Boolean algebra and its application to computers (11 minutes);
- **Module 13a** ***Networking Earth*****:** Uses mathematical models to develop efficient telecommunication systems and networks that minimize costs and connection wait times (6 minutes);
- **Module 13b** ***Data Mining*****:** Uses Graph Theory to analyze data and draw conclusions based on that data (6 minutes).

This two-hour video is available for purchase from the Texas Instruments on-line store, http://education.ti.com/us/global/promo/ml-video.html, or by phoning 1-800-TI-CARES (1-800-842-2737). The video costs $4.95 US and $4.00 shipping. The 32-page teacher guide is available for free, by downloading it from http://education.ti.com/us/global/forms/ml-guide.html. ▲

Cont'd from page 4



I have them make a Pi necklace to reinforce the idea that some numbers never repeat or end. We usually use from 100 to 300 beads, depending on the size of the bead. They assign a color to each digit (including 0) and then string beads of those colors into a necklace, using the digits of Pi as their guide. Some students make their own beads with polymer clay and others string store-bought beads of all sizes. The first bead, representing the number three, is bigger than the rest. (In the illustration, the necklace has a big silver triangular "bead" as the whole number 3 part of Pi.)

At 1:59 P.M. we all stop what we are doing and wish everyone a Happy Pi Day!

Diana Funke, Mathematics teacher
Davisville Middle School
North Kingstown, RI
ride1033@ride.ri.net ▲

▲ Photo Math – A New Way of Picturing Math

RON LANCASTER



Ron Lancaster is a middle and high school mathematics teacher, currently working throughout North America and Asia as an Independent Mathematics Consultant.

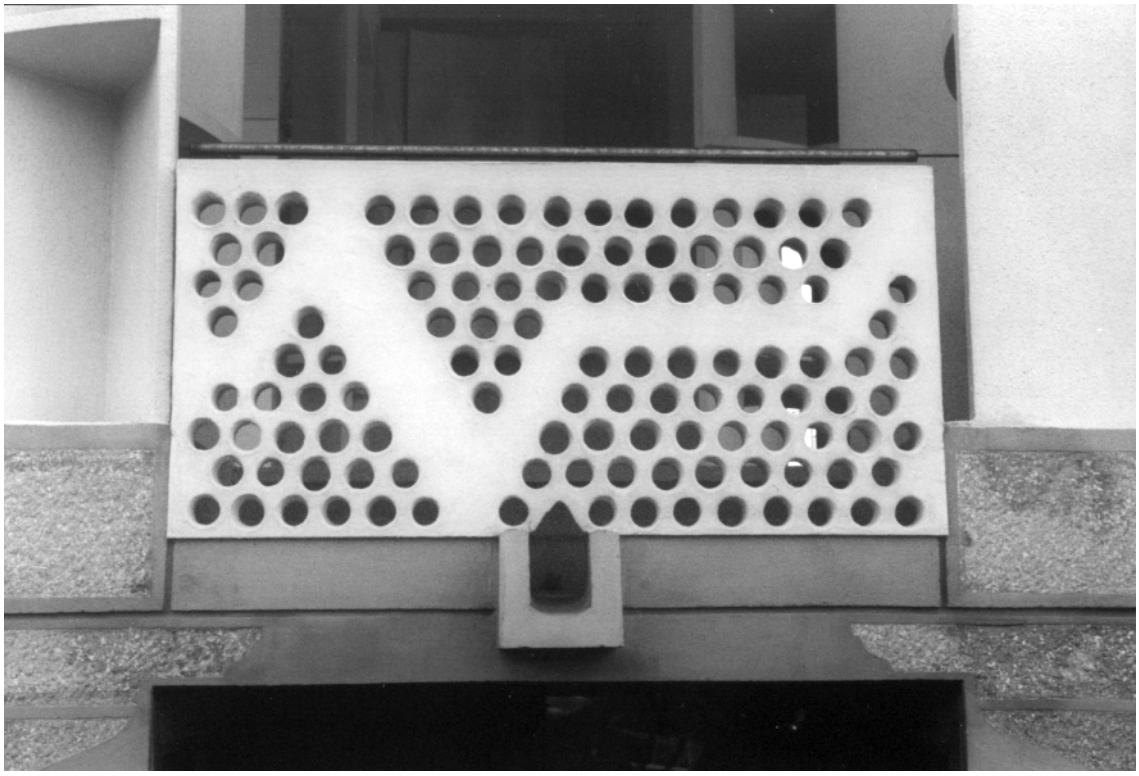
A picture is worth a thousand words is an expression that is often used to convey the importance of using images to make a point. The point of this column is to go beyond this notion and to promote the idea that in fact "A picture is worth a thousand numbers". Readers can expect to find Photos with Math in every *Photo Math* column, and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario curriculum as well as the NCTM document, *Principles and Standards for School Mathematics*. It is hoped that these questions will inspire students to be more creative and playful, and that in the end, students will develop an ability to see the world from a different perspective.

Teachers and students are encouraged to pose mathematical questions related to their own encounters and submit these problems, along with photographs, to the columnist for possible publication in a future column.

Introductory Comments

The following photographs were taken in Tel Aviv during a trip I took in January 2003. I was in Israel to work with a software company. I had been in Israel in August 2002, and on both occasions, I had some interesting conversations with friends back home about the risks associated with travel to the Middle East at this time. One night, I received a message from a friend who expressed a great deal of concern for my safety. I had just gotten back to my hotel after a midnight walk to a local food store. The streets were filled with people and I felt as safe as I do when I go for walks in my own neighbourhood or in Toronto. I watched CNN and spent time reflecting on the major disconnect between the images shown on the screen and the manner in which people live their lives. I do not mean to imply that there is no danger, but it seemed to me that most viewers would probably grossly overestimate the probability of getting into trouble in Israel. Teachers who use the questions accompanying these photos might want to discuss with their students the manner in which the media distorts the probability that viewers will encounter bad things in their lives. For more information about this idea, I would recommend Michael Moore's latest film *Bowling for Columbine* and the book *The Culture of Fear: Why Americans Are Afraid of the Wrong Things* by Barry Glassner.

Photograph #1



Study the design of the circles shown in the photo. This design was found on the concrete side of an apartment balcony. Each balcony had a different design of circles and the overall effect was quite beautiful.

(a) Without actually counting the number of circles, estimate the total number. Then count each one and compare this result with your estimate.

(b) By grouping the circles in different ways, find several ways of counting the total number of circles. Note that your method for counting should be more sophisticated then simply counting them one-by-one.

(c) Without performing any calculations, guess if the total area of the circles is less, more, or equal to 50% of the total area of the face of the balcony. What measurements would you need to do in order to calculate the precise percentage of space taken up by the circles?

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Photograph #2



Study the design on the sign shown in the photograph.

(a) How does the structure of the phone number shown in the sign differ from a typical phone number in North America? Carry out some research to see if all phone numbers in Israel have a structure similar to the one shown here. In particular, see if they all consist of nine digits.

(b) Without performing any calculations, guess if the area of the octagon is less, more, or equal to half of the area of the large circle. Measure the length of the side of the octagon and the diameter of the circle and use these measurements to find the area of the octagon and the circle. Calculate the ratio of the area of the octagon to the area of the circle and see if your estimate was correct.

(c) Using the *Geometer's Sketchpad*, create a dynamic sketch of an octagon inside a circle. Be sure that the centres of both shapes coincide. Measure the area of these two shapes and then calculate the ratio of the areas. Change the length of the side of the octagon, leaving the size of the circle as is. Find the length of the side of the octagon for which the ratio of the area of the octagon to the area of the circle is 1:2.

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Photograph #3



In the above photo, study the beautiful combination of a trapezoidal window and a rectangular door that leads into a store. Note that the width of the window (at the top) is the same as the width of the door.

(a) Without performing any calculations, decide if the area of the trapezoidal window is more, less, or equal to the area of the door. Is it possible for you to be absolutely certain that you have picked the correct choice?

(b) Find the length of the window (at the bottom) for which the area of the window is two thirds of the area of the door.

(c) Using the *Geometer's Sketchpad*, create a dynamic sketch of a trapezoid beside a rectangle similar to what you see in the photograph. Change the length of the bottom edge of the window and study the relationship between the areas of the trapezoid and the rectangle. Find the specific case for which the ratio of these areas is 2:3. How does this case compare to the one you found in part (b)?

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▲ OAME/NCTM March 2003 Report

FRANCES SCHATZ



NCTM.org Has a New Look

NCTM redesigned its home page and audience pages late in the fall for teachers, leaders, and families. These pages, available at http://www.nctm.org, now highlight lessons, activities, articles, grants, and weekly problems. E-resources such as *Illuminations*, *ON-line* math journal, access to journal articles, and many other interesting web sites need a login and password. One receives this privilege with an NCTM membership.

Canadian NCTM Membership

The Canadian price for a one-year NCTM individual membership, which includes a journal, web site access, and discounts on NCTM publications, is $107.50. A cheque in Canadian dollars sent with the membership application is the only way to get the membership at the 25% exchange rate. If any other means of payment is used, one has to pay the current exchange rate (around 60%). There is a discount available for 2-year or 3-year membership as shown on the web site – multiply the U.S. dollar amount by 1.25.

81st Annual Meeting, San Antonio, Texas, April 9-12, 2003

You can register for *Building Mathematical Communities* at www.nctm.org/meetings/registration or call toll free 1-888-241-8406. The Annual Meeting offers 1200 sessions and 200 exhibits and the opportunity to share ideas with mathematics educators from the United States, Canada, and other countries. I have attended the last two conferences and experienced days packed with lively sessions and dialogues with colleagues.

Speak at the 82nd Annual Meeting in Philadelphia

The most memorable experience of my career in teaching was being a speaker at the Las Vegas annual meeting in 2002. I recommend making a proposal to speak on your favourite lesson topic in Philadelphia, Pennsylvania, April 21-24, 2004. Information and instructions are available at http://www.nctm.org/meetings/philadelphia-faq.htm. Deadline to submit your proposal is May 1, 2003. (OAME annual conference and NCTM Regionals also look for speakers; both are great starting points.)

Mathematics Education Trust Awards

Applications are due by December 5, 2003 (can be sent anytime now) for grants, awards, or scholarships. MET provides funds to support classroom teachers, sponsors activities for prospective teachers and affiliates, and recognizes the lifetime achievement of leaders in mathematics education. Visit http://www.nctm.org/about/met/.

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Sandra Unrau, President of Mathematics Council Alberta Teachers' Association, invites mathematics educators to attend *Taking Mathematics to the Nth Degree*. Participants will experience exponential growth in teaching practices. Plan to attend this conference in Edmonton, Alberta, November 20-22, 2003. More will be provided in the June 2003 *Gazette*.

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▲ Assessment Corner: Asking the Right Questions

CHRIS SUURTAMM



Chris Suurtamm is an Assistant Professor of Mathematics Education at the University of Ottawa. She is an experienced mathematics teacher who is committed to helping teachers with the implementation of the new curriculum and assessment practices.

I spend many days sitting at the back of classrooms observing student teachers hone their skills as developing teachers. I notice that they often work very hard at developing engaging activities or interesting lessons but are still working at asking questions that encourage students to think. I believe that since many of the student teachers have just completed a university degree and are accustomed to delivering presentations in a university class, they are accustomed to classroom demonstrations of their knowledge. Teaching mathematics is more than sharing your own knowledge. It is also encouraging and developing mathematical thinking in others. This involves creating a discourse in the classroom through good questioning, listening, and use of student responses.

"Asking the right questions" has many purposes. It helps to develop mathematical thinking and it also serves as formative assessment. Asking the right questions helps students to develop critical mathematical thinking skills, encourages communication skills as students explain their thinking, and helps students make connections. Good questions encourage discussion rather than seek single correct answers.

> *Mathematics is often thought of as the discipline of "the right answer". Many of us, however, are uncomfortable with this designation because it can interfere with our efforts to help students express their mathematical thinking, learn from mistakes, experiment effectively, and pursue their mathematical interests. How does the effort to "get the right answer" fit with these goals? (Basden, Boone, Fetter, Koenig, Lanius, Mabbott, McKinstry, Renninger, Salehi, Stein, Underwood, & Weimar, 2003).*

Teachers often ask questions that require a single expected response. Students feel as though they are trying to guess what the teacher already knows. This contributes to the impression that there is one correct answer and only the teacher knows it. Students hesitate to answer for fear of being wrong and the occasional misuse of student responses increases this fear. Conceptual learning in mathematics involves more than getting the right answer.

Good questions should be a central focus of teaching because such questions probe student understanding and emphasize that there are many different approaches to a problem. Through questions that begin with words such as describe, explain, compare, and justify, students have the opportunity to express their creativity and originality as well as their comprehension. This provides good feedback to the teacher as well as stimulates responses from other students.

Questioning helps teachers to informally assess student understanding as an aid to informing instruction, and to assess student performance on problem solving tasks. Teachers gain insight into new approaches to problems and concepts, and recognize the level of students' comprehension. The teacher's role as facilitator includes developing and asking probing questions, listening intently to student responses, and on occasion, paraphrasing and summarizing student thinking. Such discourse techniques also might require that teachers mediate competing perspectives while valuing each individual's mathematical thinking and language and recognizing the student's perspective.

Questions During Problem Solving

We often provide students with rich activities and tend to "stay out of the way" while students work. However, teachers often recognize that students do not "pull out the math" and do not necessarily have the kind of mathematical conversation we expect. During problem solving tasks or investigations, a teacher can circulate and ask probing questions to students as they work to help students focus and move forward. As well, questioning and observing students in a problem-solving context can give a teacher an in-depth understanding of a student's ability to think, communicate ideas, and make connections.

Questions that allow a teacher to recognize and follow a student's thinking can take many forms. For example,

"Explain the problem in your own words" and "Describe what information is given in the problem" may help a group or individual get started on the problem. During the process of problem solving, questions such as, "Explain the steps you used to solve the problem" or "What do you think will happen?" will help a group or individual think about how they should continue to proceed.

> *The value of classroom discussion of ideas cannot be overemphasized. As students describe and evaluate solutions to tasks, share approaches, and make conjectures as members of a community of learners, learning will occur in ways that are impossible otherwise. Students begin to take ownership of ideas and develop a sense of power in making sense of mathematics (Van de Walle, 2001, p. 437).*

Questions during observation of problem solving are an integral part of the instructional and assessment process. Worthwhile observations are often the result of good questions.

Questions to Guide Instruction

Open-ended questions during a whole class discussion can provide significant feedback about student understanding and help students engage and think through the lesson. Good questions also provide feedback as to how well the instruction is being received. Through discourse, a teacher can grasp the mathematical needs of the class. A teacher can determine what students know and what misconceptions they might have.

For instance, questions during a lesson could ask students to make comparisons through "How do the two graphs compare?" If this does not stimulate a response, the teacher could stimulate and support student thinking by scaffolding with questions such as "Are they linear?" or "Do they increase?" The questions should be responsive to students rather than prescriptive, focussing attention on particular key concepts. Other examples of key questions might include:

- Would you explain why you did this?
- How do we know that we have a correct answer?
- Where have we seen this before?

Making use of student questions is also important. For instance, rather than answering a student's question, a teacher could refer the question to the class. If a student states, "I don't understand why we have two answers and only one is okay" when solving a contextual problem, the teacher could turn it back to the class and ask if anyone can explain why.

A summary of a lesson helps to support the inquiry process if the teacher acknowledges the points that students made throughout the lesson. The summary should reflect the flow and language of the lesson and connect to the experience in the classroom and the conversation of the students. Connecting the ideas to the people who brought up significant points during class will help the students relate to the information in the lesson and increase retention. Further discussion in the subsequent lessons will be increased if students feel that their responses are valued and form an integral part of the lesson. Use of student responses also reinforces the notion that the teacher does not need to be the only one with the right answers.

Summary

Teaching is a very demanding task. It requires the ability to design engaging lessons, carry out such lessons, stimulate student thinking, and respond to student perspectives. And during all of this action, teachers are constantly reflecting and assessing student understanding and the task at hand. It is a balancing act.

> *We want them [students] to discover that they can solve substantive mathematical problems, and assess whether a solution makes sense, but not to be overwhelmed by the challenge. We hope to establish the understanding that every idea proposed is valued, and we expect each person in the classroom to assume the responsibility for evaluating whether or not an idea makes mathematical sense. On the other hand, we do not want our students to feel that there is no educational authority in the class. We want students to engage in sense-making, as opposed to simply following prescribed rules, but we do not want to deny them the pleasure and confidence that come from mastering techniques and mimicking experienced professionals (Basden et. al., 2003).*

Including key questions as part of the design of a lesson will help to ensure that mathematical thinking is probed and key concepts are developed.

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▲ Out of the Mouths of Babes

(Psalms 8:2)

LYNDA COLGAN



Lynda Colgan has been at the Queen's Faculty of Education for four years, teaching in the B.Ed., M.Ed., and Ph.D. programs. Her primary focus is the classroom and her research is aimed at improving the mathematics learning experiences of the child by supporting teachers and parents.

Tell us what good work looks like and tell us what comes next.

A grade 3 student's advice on how to make schools better.

Imagine this plea stated with the wide-eyed innocence and naiveté that only an eight-year old can genuinely possess. And then ask yourself, "Who is going to be the one to tell her that it is enormously difficult to do what she asks?"

I remember seeing this poignant quote on the cover of a pamphlet from the Education Quality and Accountability Office (EQAO) a number of years ago. Joan Green, who was the Director of the testing agency at the time, often used it at the start of workshops and sessions in which she so eloquently spoke about the potential value of large-scale assessments in implementing and sustaining curriculum reform efforts. She used the child's seemingly simple request to emphasize that a principal focus of the EQAO was transparency; i.e., making the *implicit* qualities of good mathematics tasks and solutions *explicit*.

Of course, the EQAO was not the only body working towards this end. The Ministry of Education for Ontario (following the lead of the National Council of Teachers of Mathematics [NCTM], and acting upon substantive and high-quality research from around the world) attempted to equilibrate the realm of assessment, evaluation, and grading with the inclusion of an "Achievement Chart" in its new elementary and secondary curriculum policy documents.

The "Achievement Chart", better know as "*the* rubric" was intended to describe four distinct levels of performance "calibrated" to "standards" of demonstrated success and delineated along a learning continuum. The rationale for the four-point scale and the accompanying descriptive rubrics was to support a shift to a more qualitative system for teaching, doing, assessing, and evaluating mathematics. By "laying bare" the distinguishing qualities of mathematical content and process for solutions along a continuum using accessible yet precise language, illuminated by descriptions of tangible characteristics at each milepost, it was anticipated that learners would be helped to grow both in their understanding and capacity to achieve success. At the same time, it was believed teachers, parents, and students would come to have a more common and shared understanding of what success looks like in a math classroom and on a piece of graded work, be it project, report, activity, test, or some other form of summative assessment.

The challenge of dissecting mathematics solutions in order to create rubrics that clarify expectations, pinpoint strengths and weaknesses in student understanding, and provide a shared language for discussing student achievement has proven to be enormously difficult. Provincial coordinators and consultants have been over-extended as they attempt to explain to thousands of front-line teachers the subtle differences among the categories. School board personnel are busily sharing resources and developing scalable strategies for using the provincial achievement chart as a framework for more tailor-made, grade-specific, and kid-friendly rubrics.

In spite of the fact that individual educators from the board-level to the classroom have been working tirelessly to make exemplary resources available to teachers across Ontario, there are still many educators who do not have consistent access to quality resources that have been prepared with attention to assessment theory, mathematics or children's mathematical development. These teachers often come to rely on "shareware" rubrics – print and web-based assessment charts.

No doubt, it would be possible to find poor examples of any assessment instrument, including simple tools such as checklists and quizzes. However, most educators, seasoned as well as new, have personal and professional experience with these resources and can make critical judgements about their quality, instantly eliminating poor examples from a range of samples. On

Mathematics Problem Solving Scoring Guide

	Emerging	Developing	Proficient	Exemplary
Conceptual Understanding **Key Question**: *Does the student's interpretation of the problem using mathematical representations and procedures accurately reflect the important mathematics in the problem?*	1. Your mathematical representations of the problem were incorrect. 2. You used the wrong information in trying to solve the problem. 3. The mathematical procedures you used would not lead to a correct solution. 4. You used mathematical terminology incorrectly.	1. Your choice of forms to represent the problem was inefficient or inaccurate. 2. You used some but not all of the relevant information from the problem. 3. The mathematical procedures you used would lead to a partially correct solution. 4. You used mathematical terminology imprecisely.	1. Your choices of mathematical representations of the problem were appropriate. 2. You used all relevant information from the problem in your solution. 3. The mathematical procedures you chose would lead to a correct solution. 4. You used mathematical terminology correctly.	1. Your choice of mathematical representations helped clarify the problem's meaning. 2. You uncovered hidden or implied information not readily apparent. 3. You chose mathematical procedures that would lead to an elegant solution. 4. You used mathematical terminology precisely.
Strategies and Reasoning **Key Question**: *Is there evidence that the student proceeded from a plan, applied appropriate strategies, and followed a logical and verifiable process toward a solution?*	1. Your strategies were not appropriate for the problem. 2. You didn't seem to know where to begin. 3. Your reasoning did not support your work. 4. There was no apparent relationship between your representations and the task. 5. There was no apparent logic to your solution. 6. Your approach to the problem would not lead to a correct solution.	1. You used an oversimplified approach to the problem. 2. You offered little or no explanation of your strategies. 3. Some of your representations accurately depicted aspects of the problem. 4. You sometimes made leaps in your logic that were hard to follow. 5. Your process led to a partially complete solution.	1. You chose appropriate, efficient strategies for solving the problem. 2. You justified each step of your work. 3. Your representation(s) fit the task. 4. The logic of your solution was apparent. 5. Your process would lead to a complete, correct solution of the problem.	1. You chose innovative and insightful strategies for solving the problem. 2. You proved that your solution was correct and that your approach was valid. 3. You provided examples and/or counterexamples to support your solution. 4. You used a sophisticated approach to solve the problem.

	Emerging	Developing	Proficient	Exemplary
Computation & Execution **Key Question**: *Given the approach taken by the student, is the solution performed in an accurate and complete manner?*	1. Errors in computation were serious enough to flaw your solution. 2. Your mathematical representations were inaccurate. 3. You labeled incorrectly. 4. Your solution was incorrect. 5. You gave no evidence of how you arrived at your answer.	1. You made minor computational errors. 2. Your representations were essentially correct but not accurately or completely labeled. 3. Your inefficient choice of procedures impeded your success. 4. The evidence for your solution was inconsistent or unclear.	1. Your computations were essentially accurate. 2. All visual representations were complete and accurate. 3. Your solution was essentially correct. 4. Your work clearly supported your solution.	1. All aspects of your solution were completely accurate. 2. You used multiple representations for verifying your solution. 3. You showed multiple ways to compute your answer.
Communication **Key Question:** *Was I able to easily understand the student's thinking or did I have to make inferences and guesses about what they were trying to do?*	1. I couldn't follow your thinking. 2. Your explanation seemed to ramble. 3. You gave no explanation for your work. 4. You did not seem to have a sense of what your audience needed to know. 5. Your mathematical representations did not help clarify your thinking.	1. Your solution was hard to follow in places. 2. I had to make inferences about what you meant in places. 3. You weren't able to sustain your good beginning. 4. Your explanation was redundant in places. 5. Your mathematical representations were somewhat helpful in clarifying your thinking.	1. I understood what you did and why you did it. 2. Your solution was well organized and easy to follow. 3. Your solution flowed logically from one step to the next. 4. You used an effective format for communicating. 5. Your mathematical representations helped clarify your solution.	1. Your explanation was clear and concise. 2. You communicated concepts with precision. 3. Your mathematical representations expanded on your solution. 4. You gave an in-depth explanation of your reasoning.
Insights **Key Question:** *Does the student grasp the deeper structure of the problem and see how the process used to solve this problem connects it to other problems or "real-world" applications?*	1. You were unable to recognize patterns and relationships. 2. You found a solution and then stopped. 3. You found no connections to other disciplines or mathematical concepts.	1. You recognized some patterns and relationships. 2. You found multiple solutions but not all were correct. 3. Your solution hinted at a connection to an application or another area of mathematics.	1. You recognized important patterns and relationships in the problem. 2. You found multiple solutions using different interpretations of the problem. 3. You connected your solution process to other problems, areas of mathematics or applications.	1. You created a general rule or formula for solving related problems. 2. You related the underlying structure of the problem to other similar problems. 3. You noted possible sources of error or ambiguity in the problem. 4. Your connection to a real-life application was accurate and realistic.

Figure 1. This is but one example of the many rubrics that fail to describe discrete categories. Like many public-domain rubrics, it is also unnecessarily long.

the other hand, rubrics are still relatively new to the world of education (and in particular, mathematics education). Since many teachers, parents, and students have limited first-hand experience with them, they may lack both theoretical and practical knowledge about the distinguishing characteristics that define a quality instrument. This reality, complicated by the fact that the Internet alone can provide immediate access to an enormous repository of rubrics with no quality-control mechanism, opens the door to the very real possibility that bad rubrics could end up in the classroom.

My survey of a number of shareware rubrics that have been made available to teachers by colleagues or through standard routes such as workshops or teacher web sites, suggests that a large number are in the form of tables that have "characteristics" along one axis and "levels" along another. The "characteristics" are fairly consistent and most boil down to fields captured by terms such as *conceptual understanding, strategies and reasoning, computation and execution,* and *communication*. The "levels" can be numbers such as one, two, three, or four, or words such as "emerging", "developing", "proficient", or "exemplary", and they can be in ascending or descending order, depending on the author.

Although superficial **differences** such as organization and design of the charts are far from consistent, the real problems arise from deeper, more substantive **similarities**. The first concern is that the separation of the categories is not "clean". Often, there is considerable overlap in the wording. This "fuzziness" leads to confusion for teachers, students, and parents. For example, in the sample rubric shown (Figure 1), we see that similarly worded descriptors appear in two discrete categories: *conceptual understanding* and *strategies and reasoning: you chose mathematical procedures that would lead to an elegant solution* and *you chose innovative and insightful strategies for solving the problem*, respectively. Similarly, under the category *conceptual understanding*, we read "you used mathematical terminology precisely" while under *communication* we are told "you communicated concepts with precision".

The "fuzziness" does not end with the categories. In fact, in many cases, the descriptors are also blurry. In an effort to develop qualitative descriptors, most authors of rubrics have tended to fall back on quantitatively-focussed phrases that contribute little to making implicit qualitative characteristics of a solution, explicit. For example, in Figure 2, we see that the "more is better"

Mathematical Writing to Inform Rubric

Criteria	Score Point 4	Score Point 3	Score Point 2	Score Point 1
Information about Problem/ Mathematical Process	• well developed • thoroughly informs reader about problem/process • mathematical process clearly presented w/lots of mathematical support	• fairly well developed • informs reader about problem/process • mathematical process clearly presented w/ mathematical support	• little development • minimal information about problem/process • mathematical process not clearly presented w/ little mathematical support	• little written • confusing or inaccurate
Mathematical Details	• plenty of specific numerical details that thoroughly explain topic	• some specific mathematical details that adequately explain topic	• numerical details given, but either wrong details or do not help explain topic	• numerical details not given
Organization/ Audience	• clearly organized all the way through • uses many signal words (first, next, etc.) • all paragraphs have topic sentences	• organized most of the way through • uses some signal words (first, next, etc.) • most paragraphs have topic sentences	• little organization • few signal words used (first, next, etc.) • few paragraphs have topic sentences	• no organization evident • no signal words used • no topic sentences
Use of Mathematical Terminology	• lots of mathematical terminology correctly used	• mathematical terminology correctly used	• some mathematical terminology, but not correctly used	• no mathematical terminology
Connections to Math Knowledge	• reasoning thoroughly connected to mathematical knowledge (definitions, laws, properties, etc.)	• reasoning connected to mathematical knowledge (definitions, laws, properties, etc.)	• did not correctly connect reasoning to mathematical knowledge	• no attempt to connect reasoning to mathematical knowledge

Figure 2. Many descriptors fail to describe qualitative differences among samples of work along the learning continuum.

notion is used to communicate growth in learning. The distinction between a level four evaluation and a level three evaluation requires a judgment that differentiates a "*well* developed" solution from a "*fairly well* developed" solution or a "mathematical process with *lots* of mathematical support" from one with just "mathematical support".

How can we expect such uninformative descriptions to achieve consistency with respect to summarizing degrees of student success in mathematics classrooms?

The use of rubrics was also intended to eliminate long-standing and questionable evaluation and grading practices such as the inclusion of non-academic factors in the determination of final marks. However, it is not uncommon to find rubrics like Figures 3 and 4 that include descriptors of everything from student behaviours to creative expression as criteria for measuring mathematics achievement.

Math Grading Rubric

Project Piece	10	8	6	4	2	Individual Comments
Choosing A Topic	Student used class time wisely, was able to manouver through the various resource links to choose their topic and gather their data. The student was able to print out the data they had chosen for their project. All directions were followed.	The student was observed not using class time wisely or was having extreme difficulties following the links to collect their data. Please see far right for details.	The student was observed not using class time wisely AND was having difficulties following the links to collect their data. Please see right for details.	The student was unable to print out their data that they had chosen to use for their project. They had extreme difficulties following links to locate data. Please see right for details.	The student spent all of their class time playing around rather than working. They could not follow the links or print out their data on their own. Please see right for details.	
Frequency Chart	Frequency chart includes all categories, the tallies, associated with each category, and the 'Frequency' or number to go with each category. It is a neat, organized, and complete chart written in pencil on it's own sheet of paper. The data 'printout' sheet was included. All directions were followed.	The sheet is missing one or two of the preceeding pieces of information. Please see the far right for details.	The sheet is missing three pieces of the preceeding pieces of information. Please see the far right for details.	The sheet is missing four items. The data included on the frequency chart does not correspond to the data 'printout' sheet. Please see the far right for details.	The sheet is totally illegible. It shows data that is very different from that on the data 'printout' sheet or the sheet was not included. Please see the far right for details.	
Mean, Mode, Median, and Range	Mean, mode, median, and range are complete and accurate. The data is neat, organized, and shows information from the data collected on the rough draft. The data is on it's own sheet and in pencil. All directions were followed.	The sheet is missing one or two of the preceeding list. Please see the far right for details.	The sheet is missing three pieces of the preceeding pieces of information. Please see the far right for details.	The sheet is missing four items. The data included is inaccurate and is sloppy. Please see the far right for details.	The sheet is totally illegible. It shows data that is very different from collected data. Please see the far right for details.	
Stem-and- Leaf Plot	It is a neat, organized, and complete chart written in pencil. It is on it's own piece of paper. The data shown matches the data collected. All directions were followed.	The sheet is missing one or two of the preceeding list. Please see the far right for details.	The sheet is missing three pieces of the preceeding pieces of information. Please see the far right for details.	The sheet is missing four items. The data included is inaccurate and is sloppy. Please see the far right for details.	The sheet is totally illegible. It shows data that is very different from collected data. Please see the far right for details.	
Box-and- Whisker Plot	The chart is neat, complete, organized, and in pencil. It is on it's own sheet of paper. The data used is the same as the data collected. All directions were followed.	The sheet is missing one or two of the preceeding list. Please see the far right for details.	The sheet is missing three pieces of the preceeding pieces of information. Proper places are not labeled. Please see the far right for details.	The sheet is missing four items. The data included is inaccurate and is sloppy. Please see the far right for details.	The sheet is totally illegible. It shows data that is very different from collected data. Please see the far right for details.	

Figure 3. Several descriptors refer to behaviours rather than demonstrations of learning.

Tessellation Rubric			
Score 3	Score 2	Score 1	Score 0
• More than four colors included • Original shape and three transformations used • Entire page filled • Additional lines/colors added to shapes in recognizable pattern • No blank holes or gaps	• Four colors included • Original shape and two transformations used • Less than 100% but more than 80% page filled • Some additional lines/colors added to shapes, but not in recognizable pattern • Few blank holes or gaps	• Three or less than three colors included • Original shape and one transformation used • Less than 80% but more than 70% page filled • No additional lines/colors added • Several blank holes or gaps	• Other

Figure 4. There is essentially no mathematics in this rubric.

Though the use of a rubric was never intended to eliminate mathematics from the assessment and evaluation process, it is possible to find some rubrics that have done just that. Figure 5 is illustrative of the "beautifully-wrapped but empty gift box syndrome" – in this case a child can earn a math mark for doing everything *but* math.

Somewhere between the blurring of categories and descriptors and contradictions between intent and product, the huge influx of rubrics has muddied the already miry waters of assessment and accountability.

The challenge is to create and share a set of accessible, efficient, easy-to-use, transparent rubrics that can enrich program planning, classroom instruction, assessment practices, and simply grading issues.

I believe that the **ICE**[1] model, developed by Robert Wilson and further illuminated by Sue Fostaty Young, has the potential to be an accessible tool and practical resource for helping mathematics educators achieve their goal of transparency and common understanding, and so I offer a summary here to other concerned members of the mathematics community.

As you will see, the **ICE** model helps to clarify the steps in the development of learning, but its foundations are not new. You will find that **ICE** has a familiar "ring" because it is based on cognitive and assessment theory and the familiar work of experts from Bloom to Piaget.

[1] *ICE is an acronym for **I**(deas), **C**(onnections), and **E**(xtensions), the categories in the model.*

Making A Math Boardgame: Positive and Negative Integers

Teacher name: Mrs. Hauck

Students Names _____ _____
Period_____ Date _____

CATEGORY	4	3	2	1
Attractiveness	Students designed game box that contains all game pieces. Game should look like a game that could be purchased at a store	All game pieces are contained into game box created by students Some effort was put into designing game box.	Students put little or no effort into designing a game box	No game box provided
Rules	Rules were written clearly enough that all could easily participate.	Rules were written, but one part of the game needed slightly more explanation.	Rules were written, but people had some difficulty figuring out the game.	The rules were not written.
Accuracy of Content **Answer key for each game card must be included**	All information cards made for the game are correct.	All but one of the information cards made for the game are correct.	All but two of the information cards made for the game are correct.	Several information cards made for the game are not accurate
Creativity	The group put a lot of thought into making the game interesting and fun to play as shown by creative questions, game pieces and/or game board.	The group put some thought into making the game interesting and fun to play by using textures, fancy writing, and/or interesting characters	The group tried to make the game interesting and fun, but some of the things made it harder to understand/enjoy the game	Little thought was put into making the game interesting or fun
Cooperative work	The group worked well together with all members contributing significant amounts of quality work.	The group generally worked well together with all members contributing some quality work.	The group worked fairly well together with all members contributing some work.	The group often did not work well together and the game appeared to be the work of only 1-2 students in the group.
Group knowledge of content	All students in group could easily and correctly state several facts about the topic used for the game without looking at the game.	All students in the group could easily and correctly state 1-2 facts about the topic used for the game without looking at the game	Most students in the group could easily and correctly state 1-2 facts about the topic used for the game without looking at the game.	Several students in the group could NOT correctly state facts about the topic used for the game without looking at the game.

Figure 5. Although the rubric purports to assess understanding of positive and negative integers, it gives more credence to superficial qualities than conceptual development.

The ICE model simplifies and crystallizes the stages in which learners grow from a state of beginning to one of competence and expertise. More importantly, it allows us to explicitly value the simultaneously discrete and connected ways in which mathematical skills and processes grow and operate.

The first step in the learning process is that of ***I****deas*. ***I****deas* are defined as the fundamentals of new learning and include building blocks such as the steps in a process, the necessary terminology/vocabulary, definitions, details, and facts. ***I****deas* are only information.

In the world of elementary math, this translates into recognizing numbers, knowing that a triangle has three sides, or associating heavy clouds with the probability of rain. Practice exercises and assessment questions ask the student to: *show* a "greater than sign", *identify* the parallelogram, *state* the slope of the line shown on the graph, *graph* the ordered pair (2, 6), *measure* this angle using a protractor, *match* the unit "*cm*" with the appropriate expansion, or *fill in the blanks* for "in the expression $5x^2$, 5 is called the _____, x is called the _____, and 2 is called the _____". Though ***I****deas* are "bits and pieces" of knowledge that are best described in terms of simple recognition or recall, the notion extends to the solution of simple, algorithmic problems using routine rules.

How do ***I****deas* apply to the provincial curriculum expectations and the standard rubric for elementary mathematics?

My analysis[2] of the Ontario Curriculum Grades 1 – 8 shows that approximately 35% of the curriculum expectations are stated using verbs associated with ***I****deas*. Examples are: *read and print number words to twenty; identify the basic parts of a graph – labels, scales, title, data;* and *identify the two-dimensional shapes of the faces of three-dimensional figures*. Though "closed" and often discounted as "just" examples of lower-order thinking because they require little more than factual responses to demonstrate comprehension or retention, ***I****deas* provide the crucial raw materials that students need in order to proceed to the next level along the learning continuum, conceptual understanding.

I*deas* are used to complete basic content tasks, an important facet of the *Understanding of Concepts* category of the provincial rubric. Ideas are also used in the *Communication* category, and are demonstrated by students' use of accurate terminology, appropriate mathematical symbols, and representations. The ***I****deas* of *Problem Solving* are demonstrated by the student's attempt to replicate "steps" such as underlining important

[2] The author would like to emphasize that the Ontario Curriculum does not prescribe either an instructional or assessment blueprint. The interpretations are those of the author based on the language of ICE and other models of thinking such as Bloom's taxonomy. The author extrapolated the percentages of I(deas), C(onnections) and E(xtensions) by considering the total number of expectations from grades 1-8. The 35/50/15 distribution at any one grade level could be different than what the author has calculated for the document as a whole.

words and organizing information. The ***I**deas* of *Application* are demonstrated when students' apply their skills to familiar situations and use routine drawings, graphs, charts, and formulae to record data or to make observations.

***I**deas*-level demonstrations are set as a basic minimum requirement for success, fitting best with Level two of the provincial achievement rubric. Major student errors and the continuous need for teacher support set Level one achievement apart.

When we are able to establish and articulate relationships among the isolated bits of knowledge in our "***I**deas*" bank, making connections between what we already know and what we are now learning, we experience "***C**onnections*". There are two kinds of ***C**onnections*: those made between and among the discrete bits and pieces of knowledge and those made by the learner between the material and their own real-world experiences.

It is commonly acknowledged that when we construct something in the physical world, we require tools, materials, and good old-fashioned elbow grease. We can extrapolate this metaphor to describe the way in which we construct knowledge. We apply active thinking (effort) to the tools we already have – our existing ideas and knowledge – to understand the new things we see, hear, touch, or experience. In the following diagram (Figure 6, adapted from John Van De Walle) we imagine that black dots represent existing ideas. The lines joining them represent connections or relationships among the ideas. The white dot is an emerging idea. Existing ideas must be connected to the new idea in order to give it meaning and this requires that the learner actively endeavours to wrestle with these new ideas, working to fit them into existing networks. As learning occurs, these networks of dots are rearranged, augmented, or otherwise modified so that ideas fit better with what is known.

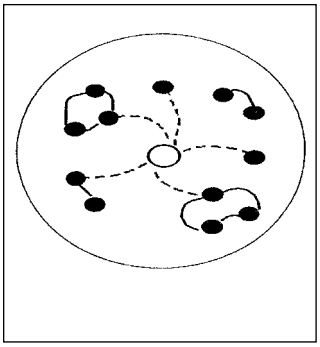


Figure 6. We use the ideas we already have (black dots) to construct a new idea (white dot), developing in the process a network of connections between them. The more ideas used and the more connections made, the better we understand. John Van De Walle, p.27

In mathematics, ***C**onnections* are explicitly made when we translate between and within various representational modes (i.e., pictures, manipulative models, written symbols, real-world situations, and oral language) and contexts. We have reinforced the notion of ***C**onnections* in Ontario by emphasizing the teaching and learning of "big ideas" in mathematics; i.e., ideas that are integrated into webs of information and knowledge that is tied together so that learners are not experiencing isolated skills, concepts, rules, or symbols. ***C**onnections* apply to all grade levels and across all

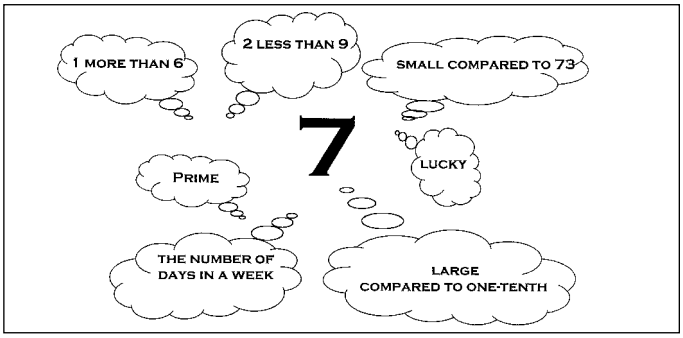


Figure 7. Consider the concept of seven as constructed by a young child. It is first likely connected to the counting procedure and understood to be less than 10 and more than 2. However, the web of connected ideas can grow large and involved. Van de Walle, p.29

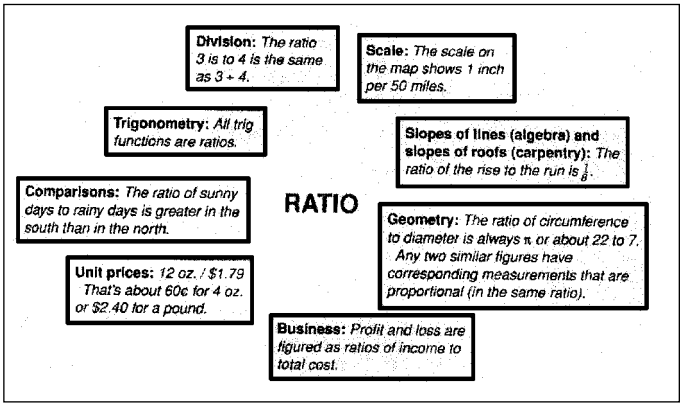


Figure 8. A potential web of associations that could contribute to the understanding of "ratio". Van de Walle, p.29

strands. Two webs of connections are shown below (Figures 7 and 8).

If ***C**onnections* are webs, then in the mathematics class, we can expect that children will be asked to "Estimate the answer to $\frac{12}{13}+\frac{7}{8}$"; "Compare squares and rectangles"; or answer questions like "In how many ways can two triangles be different?"

I found that approximately 50% of the mathematics curriculum expectations for Grades 1 – 8 are stated using verbs that invite learners to attempt to make ***C**onnections*. Examples are: *identify relationships between objects by*

stating shared attributes; use events from meaningful experiences to describe probability; compare experimental results with predicted results; discuss trends in graphs to draw conclusions about the data.

***C**onnections* also apply across the categories of the Ontario Achievement Chart. The ***C**onnections* of *Communication* are demonstrated by the students' purposeful use of one mode of representations to inform or clarify another. The ***C**onnections* of *Problem Solving* are demonstrated by the selection and application of specific and labelled strategies. The ***C**onnections* of *Application* are demonstrated by the students' ability to use known facts, concepts, and procedures to solve a novel problem or make a generalization. The ***C**onnections* of *Understanding of Concepts* are demonstrated when the student combines procedures or concepts within and across strands or disciplines.

***C**onnections*-level demonstrations are set as the standard for success; i.e., level three of the provincial achievement rubric.

The highest "level" in the **ICE** model is called the "***E**xtensions*" category[3]. At this point on the learning continuum, students are able to use their learning in novel ways apart from the initial learning situation to *predict how …*; *state the implications of …*; *propose solutions for …*; or *validate his and others' thinking*. Clearly, this best describes achievement in the level four category of the provincial rubric.

Approximately 15% of the mathematics curriculum expectations for Grades 1 – 8 are stated using verbs associated with Extensions. For example, *evaluate arguments that are based on data analysis; ask "what if" questions, discuss and defend the choice of a pattern rule;* or *create problems giving rise to first-degree equations with one variable*.

As with ***I**deas* and ***C**onnections*, ***E**xtensions* also apply across the domains identified by the provincial achievement chart. The ***E**xtensions* of *Communication* are demonstrated by the students' use of multiple representations to verify, assess, and evaluate evidence and solutions. The ***E**xtensions* of *Problem Solving* are demonstrated by the students' ability to pose problems and to invoke informed but original experimental designs or experiential methods. The ***E**xtensions* of *Application* are demonstrated by the students' ability to use old definitions and laws to predict new generalizations and conclusions by relating knowledge from several areas. The ***E**xtensions* of *Understanding of Concepts* are demonstrated when the student is able to reorganize prior knowledge and skills creatively or generatively to describe or illustrate new generalizations or concepts across strands or disciplines.

STRANDS	IDEAS	CONNECTIONS	EXTENSIONS	TOTALS
Number Sense & Numeration *Expectations selected from Ontario Curriculum Document and categorized by verb as I, C, or E.*	Typically scores from quizzes, homework. Procedural skills remain important but should be weighted in proportion to other goals that reflect what the curriculum values.	Students see how mathematical ideas build on one another in a useful network of connected ideas.	The verbs associated with "extensions" suggest that children need opportunities to think deeply,	The application of **ICE** ensures that a grade is more than an average of scores.
Spatial Sense & Geometry *Expectations selected from Ontario Curriculum Document and categorized by verb as I, C, or E.*	Ideas are at the beginning of the learning continuum.	Students apply ideas in a context or situation.	Extension verbs challenge students to ask questions and reason about important ideas.	**ICE** helps to align assessments with the true goals and objectives of the program.
Patterning & Algebra *Expectations selected from Ontario Curriculum Document and categorized by verb as I, C, or E.*	Ideas (only information) are at the lowest level on a rubric.	Students recognize that mathematics is not a laundry list of isolated rules and formulae.	Extension solutions demonstrate a student's facility in using a variety of mathematical representations to add to their own understanding and to better communicate ideas to others.	The final grade is dependent upon the quality of the instruments that are used to generate the grade. **ICE** can serve to keep a balance in those instruments.
TOTAL	35	50	15	100

Figure 9. Generating a grade through the use of the ICE model reflects the developmental nature of learning and can still fit nicely into administrative expectations about marking.

The Ontario curriculum document requires that teachers provide opportunities to learn that are representative of these different levels of learning. In fact, I believe it suggests an instructional blueprint – 35% of instructional, practice and homework time should be devoted to Ideas, 50% reserved for ***C**onnections*-type activities and questions, and 15% dedicated to ***E**xtensions* problems and investigations – if we use the verbs of the expectation statements as markers of where learners should be along the **ICE** learning continuum.

The provincial achievement chart requires that there be opportunities for students to respond to a variety of question types during tests, assignments, examinations, and performance assessments, and be measured using a range of qualitative and quantitative tools. **ICE** provides a possible framework for a balanced approach that could

[3] The use of the word "extension" is problematic and it is one that has caused discomfort in the math education community because it implies that these behaviours are not intended for all students and is often associated with "bonus" questions. It might be better to think of this category as the one of "Elaboration" although it is not within my purview to impose such a change to another's model.

ensure that all learners are respected and given a chance to experience diversified learning opportunities

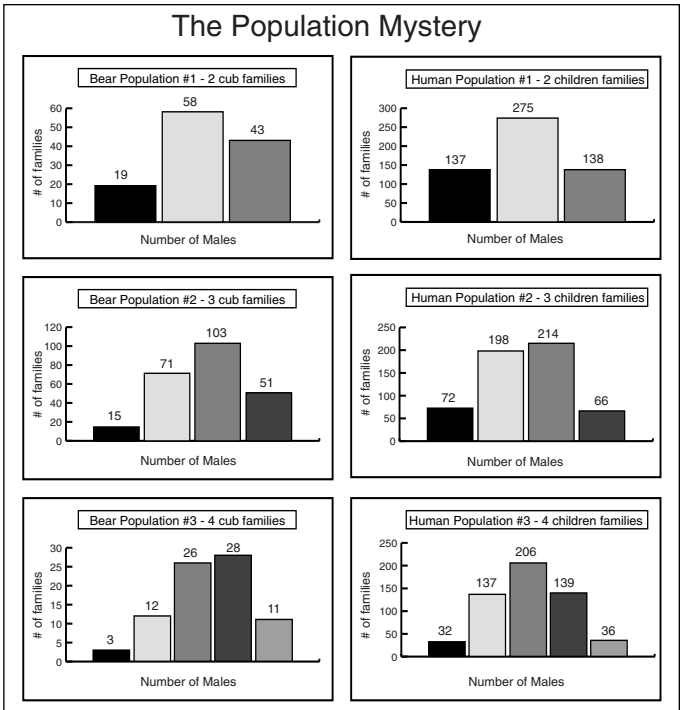
ICE could also help teachers to create balanced assessment instruments by assigning and explicitly labelling question-types and weightings according to the instructional blueprint, or by designing rubrics that articulate the steps in the development of learning, the demonstration of problem solving, and the communication of both.

***I**deas*-level evaluation would be accomplished through the use of some single-point scores (for "right" or "wrong" questions) and some multiple-point scores (for solving multi-step but routine problems, or performing standard algorithmic computations). ***C**onnections*- and ***E**xtensions*-type evaluation could be made using either multiple-point scores or rubrics determined by the achievement chart depending on the task and its context.

Figure 10 shows an intermediate division assessment task (created by Peter Harrison, Riverdale C.I.). Given statistical information about human populations and bear populations in the form of a set of graphs, students are asked to analyze the data critically as presented, looking for clues that will help them to evaluate the validity of a researcher's conclusions. They are asked to evaluate the relationship between probability and statistics by applying probabilistic arguments to interpolate and extrapolate values/decisions from the visual data. Finally, they address the probability issues that surround us daily by illustrating the importance of justifying real-world conclusions using mathematically-defensible arguments. This particular "problematic situation" is designed to assess students' ability to integrate skills from *graphing* (displaying information) with those from *statistics* (discussing data numerically) and *probability* (the mathematics of outcomes).

Some students may use a narrative approach to address the family of problems. These students may describe how the graphs are actually paired off so that bear families and human families with the same number of cubs or children are next to each other on the page. These same students may write that the human graphs appear to be more "balanced"; whereas the bear graphs appear to be "lop-sided". These students are working at the level of ***I**deas*.

Other students will recognize the implication of the symmetry in the graph of humans and infer from the lack of symmetry (balance) that there are more male bear cubs. They may extend their discussion to say something really quite significant about graphs and probabilities by stating that a symmetrical graph tells us that there is a 50-50 chance of a baby being a boy and discuss why it is likely that the non-symmetrical shape of the bears' graphs has caused the researchers to question the birth probabilities for the bears.



THE POPULATION MYSTERY

Using the information provided by these graphs and inferences that you can make from them, answer the following questions. Be sure to provide mathematical reasons in support of your statements.

1. Indicate any significant similarities, differences, and patterns that occur in the graphs of the bear population and the human population.
2. The researchers didn't provide any graphs showing the number of female cubs in the bears' families. Create your own graph showing the number of female cubs in the "three cub" families. Write a paragraph to justify the accuracy of your predictions.
3. Is the information provided by the graphs complete? Support your answer, including suggestions for improving the study.
4. The graphs showing the human populations appear more symmetrical than the bears' graphs. Is this symmetry significant? Why would the humans' graph have this symmetry?
5. A mother bear is about to give birth to a baby cub. What is the probability that this cub will be female? Give a detailed explanation.

Figure 10. A team of researchers has been studying the population of a "mathematical" bear. They have been tracking bear families and keeping a record of the number of male cubs in each family. They think that they have collected some rather odd findings. To help understand their data, they have assembled some similar data for humans. They surveyed a series of human families with similar numbers of children and made a note of the number of boys. All of the results are shown below in graphical form. The researchers have concluded that the probability that a mother bear will give birth to a male cub is different from the probability that a human mother will give birth to a son.

These same students may examine the graphs more closely, noticing that the human graphs have more features than just balance/symmetry. Observing the "hump" in the middle, the short bars off to the ends and the taller bars towards the middle, these students could make comparisons to coin tossing experiments in which it can be shown that when two coins are tossed, it is common to get one head and one tail more often than either two heads or two tails and extrapolate to gender – boy, girl, head, tail. These students deduce the probability associated with a single event from the given distribution and explain clearly what the physical configuration of the graphs says about birth probabilities, generating a probability tree or systematic list to support and clarify written explanations. These students are working at the level of ***C****onnections*.

Other students will examine the quantitative information contained in the middle graphs; i.e., that there were 15 bear families with no male cubs and 72 human families with no male children. They will use the numerical data supplied by the graph of male bear cubs to make informed deductions about the female cub population. The bottom graphs tell us that in families with 4 cubs or children, there were 3 families with no male cubs and 32 families with no boys. They will focus on how the graph tells us that there are 550 families in all and that 25% of them should have zero boys; i.e., 137.5. The students who use this approach will notice that in the 3 cub families, there are 51 families with three males, 15 families with three females, and 15 families with zero male cubs. They will also be able to conclude that the 3 cub family graph tells us that in the 71 families with one male cub, there must be two female cubs. They will find the total number of male cubs (15 families with none, 71 families with one, 103 families with two and 51 families with three) or 0 + 71 + 206 + 153 = 430 male cubs. Using the fact that there are 240 families with three cubs each, they can calculate the probability that a cub will be male is $\frac{430}{720}$ or roughly 60%. This group of students is working at the level of Extensions.

Finally, **ICE** can also assist in the difficult process of linking grading to learning. Each term, every elementary teacher in Ontario is required to report on a minimum of three strands of the mathematics curriculum by reducing all of the assessment data she has acquired about an individual student into a single letter or number grade for each strand plus anecdotal comments.

As part of one's long-range planning, it is possible to generate a term-by-term template that is organized by unit or strand (an example is provided in Figure 9) and "weighted" according to the distribution of "Ideas", "Connections", and "Extensions" represented in the curriculum. This can be an invaluable visual checklist to determine that a range in types of learning is explicitly planned, and of course, once the expectations are categorized, it is then possible to devise representative test questions/performance assessments. This front-end approach to planning assessment and instruction informs decisions about teaching strategies, task selection, and instrument design.

Individual pages are then made for each student on which quiz and test scores and rubric summaries from throughout the term are recorded. By tabulating evidence from across the spectrum – fact-based questions to problem solving and inventing – the resultant overall letter or number grade communicates *how well* rather than *how much* a student learned.

We have many books, resources, and articles that are statements of philosophy and purpose for assessment. We have how-to prescriptive guides that tell us about norm-referencing and criterion-referencing. Often, these resources fail to speak directly and practically to the classroom teacher. I hope that this brief overview of **ICE** will be welcomed as a practical model for understanding the learning continuum, some of its milestones, and the nature of the terrain in between. I also hope that it might allow us to generate rubrics that clearly label and describe those milestones to facilitate the journey along the learning curve.

With sincere thanks to **Sue Young** and **Bob Wilson**, the originators of ICE, for their review of this article and constructive help in clarifying critical points.

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