Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceed 10 pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including websites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

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The *Ontario Mathematics Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

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Advertisements for the *Ontario Mathematics Gazette* should be sent to **Dean A. Murray** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

Advertising Rates

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▲ Editor's Message

MARILYN HURRELL, EDITOR



As I am writing this, the wind is howling, it is -30˚C and I have three feet of snow in the yard and six foot high snow banks. This winter season truly has tested us all with both regional and international weather disasters taking place. I for one, look forward to the warmer weather that March will bring.

This issue of the *Gazette* sees two changes.

Thank you to Joel Yan for the columns he provided over the past year, introducing resources for teachers which are available through Statistics Canada. Please continue to visit www.statcan.ca for updates.

Mary Lou Kestell, a former President of OAME, joins John Egsgard and Todd Romiens as an Associate Editor of the *Gazette*. Mary Lou is currently working as Student Achievement Officer with The Literacy and Numeracy Secretariat.

Mention Florence Nightingale and you will most likely, in a *Blink!* (see Lynda Colgan's column), envision her as a nurse in the Crimean War. But Florence had another talent as described in the article: *Florence Nightingale, the Passionate Statistician*.

Michael Lazure, a mathematics teacher in Kingston, ON has summarized the use of *Finite Differences from Grades 9 to 12*. Louis Lim encourages teachers to invite Mathematicians into their classrooms and Jack Weiner describes a *Friday Special* problem which came about as he tried to help a student claim his prize.

Chris Suurtamm's column, *Assessment Corner*, appears in this issue and discusses the importance of ongoing formative assessment. Read and use Gary Flewelling's article on *The Games We Play in the Classroom*.

Be sure to visit the updated OAME website www.oame.on.ca which has gone live in the last few months and offers OAME members many special features.

Enjoy this issue of your *Gazette*.

Marilyn

Non-Graded

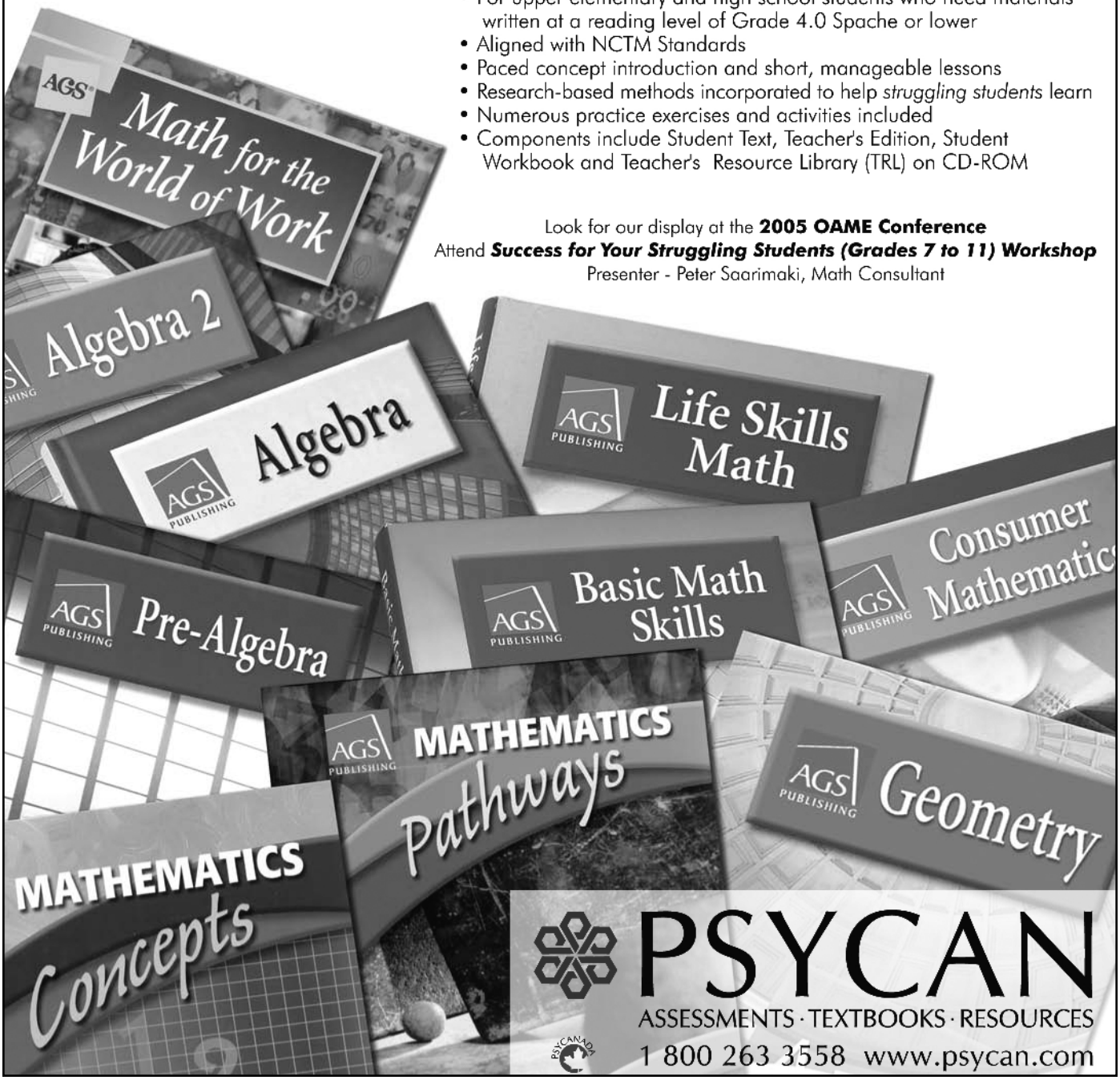
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▲ From the Editor's desk: March 2005 OMG

I do some math tutoring at a locally owned franchise, and saw this notice pinned up in the staff room. I am presenting it to you, the readers of the OMG *since I think it contains information which may help in those teacher-parent interviews. Marilyn*

Outfit Your Child for Success by Appealing to Their Unique Learning Style

Understanding a child's learning style can help a parent and teacher personalize instruction or homework to ensure peak performance in school and build a child's self confidence.

There are three widely recognized learning styles, also known as learning channels: visual, auditory and kinesthetic - tactile. The following tips can assist parents and teachers in determining what type of learning style their child exhibits. When reading the following tips, keep in mind that a child may exhibit more than one of the following learning styles.

Visual Learners

If you notice that the child easily remembers how things look and can describe something he/she sees in great detail, but has a difficult time following verbal directions, he/she is likely to be a visual learner. To help the child learn more successfully, you may want to:

- Encourage the child to take written notes to help him/her remember concepts
- Keep highlighter markers and coloured paper on hand to help the child liven up notes and study materials
- Urge him/her to pay close attention to graphs, charts, maps and pictures
- Use illustrations to help the child recall information easier
- Create flash cards with the child to remember facts

Auditory Learners

If the child prefers listening to stories, music and lessons taught in class, or moves his/her lips when reading, he/she may be an auditory learner. To help the child learn more successfully, you may want to:

- Purchase a tape recorder and encourage the child to record notes and chapter summaries whenever possible
- Read key concepts and review questions aloud with him/her
- Urge the child to recite aloud important dates, key terms and facts
- Help the child make up songs or rhymes in order to increase his/her ability to recall facts
- Have the child verbally repeat information to you

Kinesthetic - Tactile Learners

If you notice that the child is always on the move and has a hard time sitting still or paying attention, he/she is likely a kinesthetic - tactile learner. To help the child learn more successfully, you may want to:

- Incorporate hands-on learning activities whenever possible
- Encourage the child to study in different positions (e.g., sit in a chair, on the floor, at a table, etc.);
- Take short breaks, even just stretching, after ten minutes
- Urge the child to take notes on important facts and rewrite his/her notes several times
- Ask the child to write key words or phrases with his/her finger, on the table or in the air
- Allow the child to walk around while studying for a test

As you work with the child throughout the school year, it's important to remember that he/she will be better prepared for classes if parents and teachers understand the child's learning styles and use this knowledge to help him/her excel in school and on homework. ▲

Reprinted with permission from Ian Hamilton, owner and Director of Education at the Sylvan Learning Centre in Thunder Bay ON.

Srinivasa Ramanujan (1887 - 1920)

One day this Indian mathematician rode with the British mathematician Godfrey Harold Hardy (1877 - 1947), in a taxi which was marked '1729'. "A very boring number", said Hardy. "But quite the opposite!" answered Ramanujan immediately. "It is a very interesting number in that it is the smallest number which can be written as the sum of two cubic numbers in two different ways." What are the two different ways?

▲ President's Message

GORD COOKE



Gord Cooke is Vice Principal and Special Education Resource Teacher at Thousand Islands Secondary School in Brockville Ontario. He also served a stint as English Language Coordinator for the Grade 9 Assessment of Mathematics at EQAO.

I found the following at www.ldonline.com in an article by Nanci Bell:

"Why can't everyone think with numbers? Why do some children learn math readily, handle money and time concepts with ease, retain information from year to year, and think with numbers effortlessly? What cognitive processes do some have that others do not?"

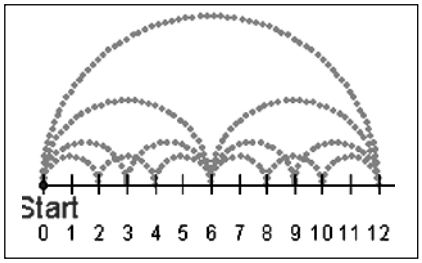
Mathematics is cognitive process-thinking-that requires the dual coding of imagery and language. Imagery is fundamental to the process of thinking with numbers. Albert Einstein, whose theories of relativity helped explain our universe, used imagery as the base for his mental processing and problem solving. Perhaps he summarized the importance of imagery best when he said, "If I can't picture it, I can't understand it."

For the people who "get" math, the language of numbers turns into imagery. They use internal language and imagery that lets them calculate and verify mathematics; they "see" its logic.

Imaging is the basis for thinking with numbers and conceptualizing their functions and their logic. The Greek philosopher Plato said, "And do you not know also that although they [mathematicians] make use of the visible forms and reason about them, they are thinking not of these, but of the ideals which they resemble…they are really seeking to behold the things themselves, which can be seen only with the eye of the mind?"

This certainly describes some of the efforts of a group of Ontario teachers who came together in another of Tom Steinke's summits. This time we were working on authoring *GSP* sketches which could be used to help remediation efforts in areas other than Geometry. Led by Nick Jackiw and Nathalie Sinclair the group was taken along a learning journey that included technical aspects of sketches and pedagogical considerations on how to use the sketches in a remedial setting.

One of my favourites was a simple activity called Jump Along. Students set two parameters: n jumps of size m. Animation produces n semicircular jumps on a number line. Here is an investigation into the ways to get to 12.



Questions like, "What do prime numbers look like?" and "What kind of numbers have jump paths that don't cross each other?" help students make the kind of image/language connections referred to in the Bell article.

To see the results of the summit, go to www.vlearning.ca and follow the links to the PRISM sketches. Happy imagining.

I would like to thank everyone who has supported me during this past year. Special thanks go to the OAME Executive, Stewart, Kathy, Anna, Dan and Judy. Finally Sue and Dave Hessey deserve my utmost thanks and appreciation for everything they do for this organization.

Gord ▲

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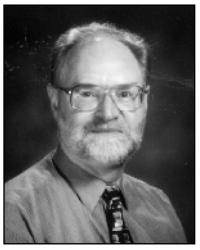
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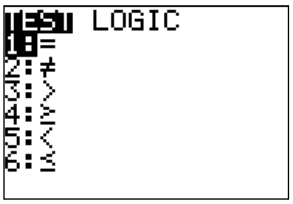
▲ Technology Corner: Using the TI84 for Algebra and Trigonometry in the MCR3U and MCF3M courses

Fred Ferneyhough

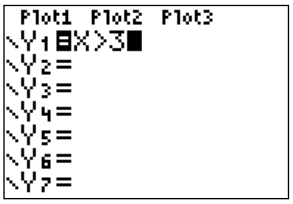


Fred Ferneyhough is a mathematics department head at Harold M. Brathwaite SS in Peel and an author and tech consultant for several books. He is an avid user of and advocate for the use of technology in the mathematics classroom.

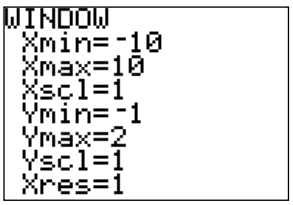
In the grade 11 U/C course, there are several algebraic topics that provide opportuninties to use the TI84 Plus to advantage. In this column, we will examine three such topics – inequalities, rational expressions and trigonometric identities.



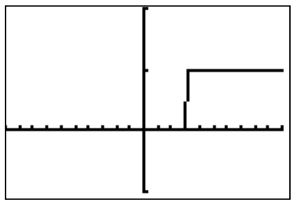
Above the MATH key on your calculator is the word "TEST". If you press 2nd and MATH, you can access the menu shown to the left. This menu contains the six logical operators that compare two quantities. These can be used in the functions window.



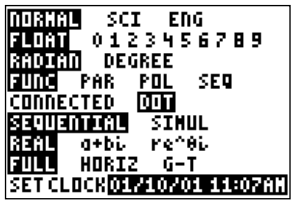
The simplest type of inequality is shown in the first function variable. The logical operator ">" has been inserted from the TEST menu shown above. Any of the six operators can be used. When the device compares the left side to the right side, it assigns a result of 1 if the statement is TRUE and 0 if the statement is FALSE.



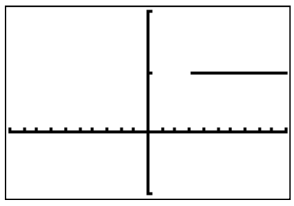
Set the window as shown to the left. Since the result can only be 0 or 1, the range has been compressed so that a more accurate reading can be made.



When the "function" is graphed, we get the result shown. The jump in the graph represents the calculator's default display. Using the TRACE feature and moving across the screen will show that the y-value is 0 just before the jump and the y-value is 1 just after the jump. The problem is that that y-values of 0 cannot be seen and the jump is annoying.



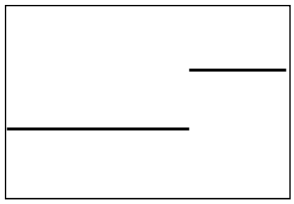
To get a better view of this outcome, there are two changes that we can make. First, press MODE and change the setting in the fifth row from CONNECTED to DOT. The jump is due to the fact that the calculator displays graphs by connecting successive points with a line segment.



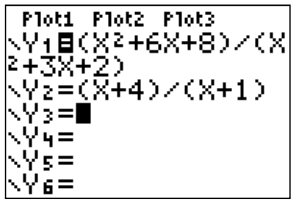
Now, the graph has the jump removed. This display is similar to the type of graph that we do when we graph inequalities above a number line. However, another improvement can be made by turning off the axes.



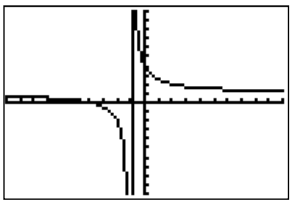
Press 2nd and ZOOM , move down to the fourth row and press ENTER on the phrase "AxesOff".



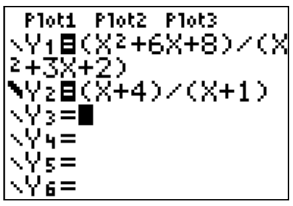
Finally, the graph shows both sets of y-values correctly without interference from the axes. You can enter any type of inequality into the function window. Try graphing the expression "$y_1 = -x < 4$". This will allow students to see that this is equivalent to "$y_1 = x > -4$".



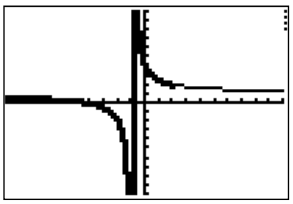
Next, we move on to rational expressions. Let's not even talk about students' abilities to factor expressions so that they can simplify an expression. Aside from checking the answers in the back of the text, how do they know if they have the correct answer? Try this. Put the original expression in Y_1. In Y_2, enter the result that you got after the attempt at a solution.



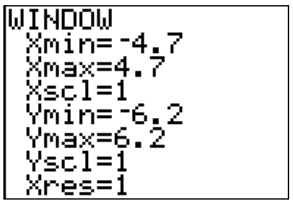
In the function window, the second function has been inactivated by pressing ENTER on the equal sign. The graph to the left shows only the first expression. Notice that it looks like a vertical asymptote appears in the function at x = –1. However, this is just the calculator attempting to connect sucessive points again. There are better ways to plot a vertical asymptote.



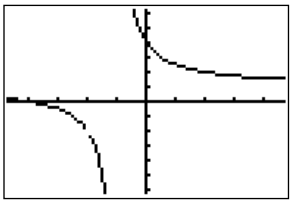
Now, go back to the functions window and activate the second function. In addtion, move to the left of the function and press ENTER once to change the style. This will cause the second function to be displayed in a heavier pattern.



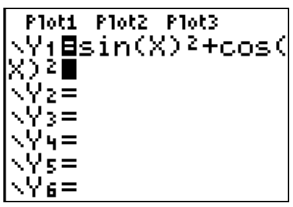
As the function was being plotted, the ENTER key was pressed. This stops the graphing until the ENTER key is pressed again. It can be seen here that the second function overlaps the first, so the expressions must be equivalent. When we teach this topic, we attempt to teach students about restrictions on the variables.

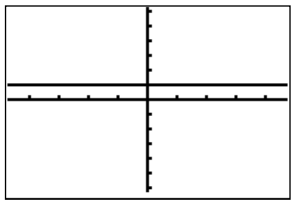


Let's go back to the first graph and change the window. As mathematicians, we know that the original function has two types of restrictions – a vertical asymptote at x = -1 and a hole at x = -2, y = -2. The vertical asymptote is due to the factor remaining in the expression, while the hole is due to the common factor that was removed. This window will allow us to see something different.



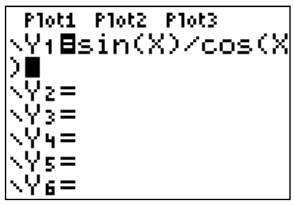
Use the TRACE feature to move across the graph. The values in the window allow the x-coordinates to increase by 0.1 When the value of x is –2, there is no y-value shown. In the diagram, we can see the gap in the graph at (–2,–2). As the x values approach –1, the y-values change rapidly on either side of the value x = –1 and there is no y-value at x = –1. Anyone want to start talking about limits??



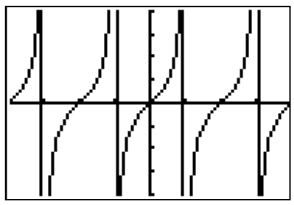


Trig functions can also be demonstrated on the calculators. Before proving the results, or developing them, we can display them graphically. Enter the expression as shown in the diagram. We know that this is the basic Pythagorean identity for trig expressions, $\sin^2 x + \cos^2 x = 1$.

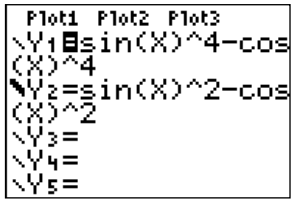
In the graph we see a horizontal line suggesting that the result is 1. Now, we can take some time to develop the identity using any traditional method.



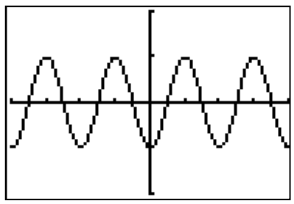
In the graph I tried the same thing with the identity for tan(x). I entered sin(x)/cos(x) and graphed it.



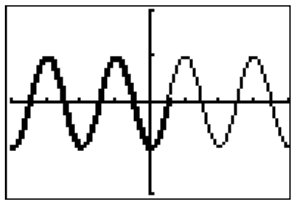
Imagine my surprise when one of my students said "hey sir, that looks just like the tan graph!" I almost had tears in my eyes.



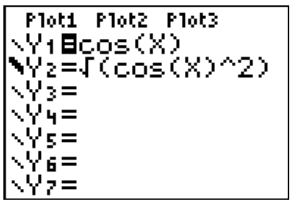
For trig identities, I have been trying the same thing as was shown above for rational expressions. Enter the left side of a proposed identity into Y_1 and the right side of the expression into Y_2. Deactivate the second function.



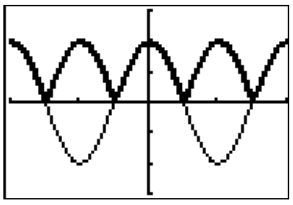
Once again, only the first function is graphed. I was hoping to get questions about why this looked like a sine graph, but I guess that was too much to hope for.



Now, go back to the functions window and activate the second function. Notice the the style was also set to a heavier setting as before. The graph shows that the expressions overlap, suggesting that the two expressions are identical.



The final example shows how to handle problems where students are asked to determine if an expression is an identity or not. An example is shown with the left and right sides entered into different functions. Algebraically, students may conclude that the left and right sides are equivalent.



If both functions are graphed together with different styles, we see that the graphs are different, suggesting that the expressions do not form an identity.

There are varying opinions on how much algebra students should be required to do in our current curriculum. Most teachers still think that there is a lot of value in stressing algebraic manipulations. This approach allows us to view algebraic problems graphically and, if we use the tables of values, numerically as well. ▲

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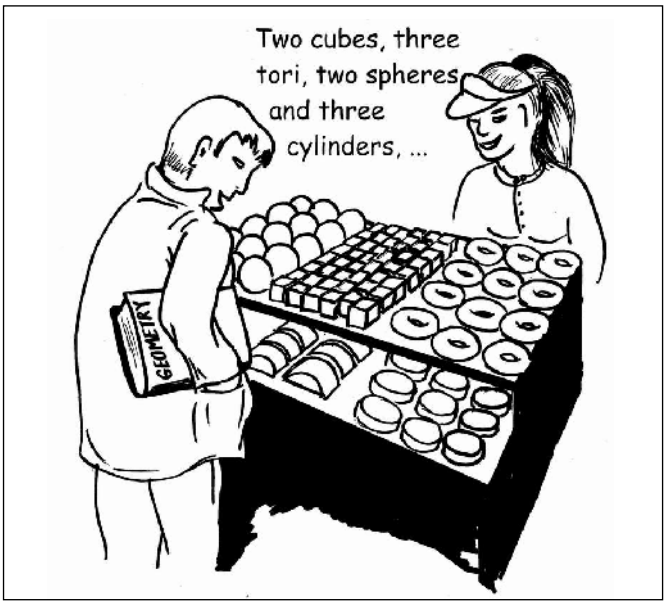
E-mail: gvmath@golden.net

▲ Hey It's Elementary: Turns Out You CAN Judge a Book by it's Cover!!!



LYNDA COLGAN

"It's no wonder that truth is stranger than fiction. Fiction has to make sense." *Mark Twain*



httpwww.pims.math.capiimagescakes.jpg

A grocery store in my home town recently advertised salmon fillets on a 'two for one' special. I went to the store, stood in line, and then asked the clerk for four fillets. I then explained that I wanted them wrapped in two packages of two so that I could freeze one package for use later. The clerk patiently wrapped the two packages of fillets and proceeded to charge me the full price for four pieces of fish. When I inquired about the 'buy one get one free' special, the clerk looked at me exasperatedly and said (all that was missing was a Homer Simpson, 'duh?'), "Yes. But you bought four." We stood staring in disbelief at the other's dim-wittedness, until I finally said (in my best teacher voice), "Now let's think about this together. In one hand I have two fillets, which is a perfect, 'buy one, get one free' set, and in the other hand I have another 'buy one, get one free set' so clearly, I should have been just charged for two fillets. Simple, right?" The response? "Just go to the cashier.

She'll deal with it at the check-out. All of this math is making my head hurt, ma'am." And with that he turned away. There was a woman standing beside me who had witnessed the episode from start to finish. She turned to me and said, "It makes you wonder what they're teaching them in school, doesn't it?"

Now, I'll admit, I was astounded that this young man who must have completed a few years of high school had made such a gaff, given so trivial a problem. I was also disappointed by his anti-mathematics sentiments. In spite of my feelings, it never occurred to me to hold this young man up as the public face of public education. However, the woman beside me had made a very quick judgment about schools in general based on very little information, in fact, based on one unsuccessful and very public authentic mathematics assessment.

In cognitive psychology terms, the woman had used her adaptive unconscious – the part of the brain that lets us think without thinking. Malcolm Gladwell, author of the book, *Blink*, suggests that the adaptive unconscious should be compared to a giant computer that quickly and quietly processes a lot of the data that we need in order to keep functioning as human beings. He uses this example: When you walk out into the street and suddenly realize that a truck is barreling toward you, do you have time to 'problem solve' by thinking through all your options? Clearly not. The only way that human beings have survived as a species, Gladwell claims, is attributable to the fact that we have developed another kind of decision-making apparatus that allows us to make high-level, sophisticated decisions quickly based on little information. He likens it to us having our own automatic pilot system.

Gladwell suggests that whenever we meet someone for the first time, interview an individual for a job, or react to a new idea – in fact whenever we are faced with making a decision quickly and under stress – we rely on the adaptive unconscious, our personal autopilot, to make instant, unconscious and often uncannily accurate decisions.

To illustrate the power of the adaptive unconscious, Gladwell describes the experiments of Nalini Ambady. Ambady once gave students three ten-second videotapes of a teacher – with the sound turned off – and found that they demonstrated no hesitation about rating the teacher's effectiveness. When Ambady cut the videotapes back to five seconds 5 seconds, the results were the same. In fact, the results were almost identical when the students were shown just two seconds of videotape. Surprising? Yes, but the second part of the experiment was even more dramatic. When the students' snap decisions were compared to the annual evaluations of those same professors made by their students after a full semester of classes, the results were essentially identical. A person watching a silent two-second videotape, of a teacher he or she has never met, will come to conclusions about how effective that teacher is that are very similar to those of a student who has sat through a teacher's class for an entire semester.

Gladwell gives a second illustration. He suggests that when we first pick up a book, within two seconds, we process the design of the cover, associations we have with the name(s) on the cover and the first few sentences and generate an immediate impression. Our judgement – is made by a flurry of thoughts, images and preconceptions. If it looks right, we go with it.

Gladwell is very careful to avoid the use of the word intuition to describe these snap decisions. He contends that they are based on thinking that moves a little faster and operates a little more mysteriously than the kind of deliberate, conscious decision-making that we usually associate with 'thinking'.

Blink got me thinking about the new textbooks for elementary mathematics that have been published recently in Ontario. Classical decision theory assumes that all decisions are rational, and the assumption has always been that textbook selection, whether by committee or individuals or any combination of both, was based on the formal establishment of criteria and a thorough evaluation of all of the available options. However, if Gladwell is correct, it may be that decisions about textbooks are more instinctive than previously thought.

The importance of textbooks cannot be underestimated. In fact, the only direct educational expenditure that is greater is the cost of teacher salaries. In Ontario, the provincial government provided $500 million in 2001 to purchase textbooks in support of the new curriculum then being introduced into high schools. An Ontario educational ministry official stated at that time that elementary schools spend on average $75 per student per year on textbooks.

More significantly, textbooks have assumed a significant role in elementary mathematics. In fact, many researchers have shown that the textbook is the de-facto curriculum. As Canadian mathematics researcher David Robitaille, leader of The Third International Mathematics and Science Study (TIMSS) stated, "Teachers of

mathematics in all countries rely very heavily on textbooks in their day-to-day teaching, and this is perhaps more characteristic of the teaching of mathematics than of any other subject in the curriculum. Teachers decide what to teach, how to teach it, and what sorts of exercises to assign to their students largely on the basis of what is contained in the textbook authorized for their course."

If Gladwell is correct, then we do not need more textbook selection committees, criteria, rubrics, checklists or time to ensure that decision makers make the most appropriate textbook selections for ourselves as teachers, and for the most important individuals in the entire equation – our students. 'Thinking Without Thinking,' as posited by Gladwell means that we call upon our deeply-rooted expertise and reflections on accumulated experiences whenever we need to make a fantastically complex decision in a matter of seconds.

In fact, Gladwell uses an interesting incident to illustrate the potentially negative effect of too much information (which may explain why so many teachers who had never had any problems assessing and evaluating students before the era of the rubric, were suddenly stymied when confronted with lists and lists of delineated criteria):

> *A random group of consumers were invited to rank 44 different jams according to specific texture and taste criteria. The consumers' rapid ratings were very similar to those found by a panel of food experts.*
>
> *Then another non expert group was asked to rank the jams. But this time, they had to write out detailed explanations of their reasons. Result: The rankings were turned on their head. A clear winner in the first polls ended up last, and other top finishers were lost in the shuffle.*
>
> *This experiment (like others in the book) illustrates situations in which "introspection destroyed people's ability to solve insight problems. By making people think about jam, (the testers) turned them into jam idiots". Forcing the consumers to carefully describe their preferences had, in fact, clouded their ability to know their own minds.*

As Gladwell posits, the difference between good decision-making and bad has nothing to do with how much information we can process quickly, but on the few particular details on which we focus during that metaphorical blink of an eye during which we make judgments. Since it can never be forgotten that all rapid judgments are a function of expertise and experience, they can turn out to be 'bad' because of erroneous data or negative experiences. Gladwell emphasizes that our ability to 'think in a blink' can be manipulated and clouded by unconscious prejudice and he is careful to explain the ways in which flawed snap judgments can yield devastating results, particularly when outside influences, emotions, prejudices and other factors come into play.

To make a decision is to invoke what Gladwell calls the "the giant supercomputer in (your) unconscious". Each of us needs to think about this the next time we pick up a new textbook and make a snap decision. Our judgment is not arbitrary. The flurry of data that gets processed in our unconscious mind and enables us to leap to conclusions may be non-rational, but it is not irrational. University students appear to have a power of knowing what effectiveness looks like when it comes to evaluating teachers, because after years of schooling, they have built a repertoire of reliable indicators. Teachers recognize reliable indicators of student success, often accurately evaluating student work before formal assessment steps begin. And, members of the public – like the woman beside me in the grocery store – also have (personally) reliable indicators on which their judgments are made.

We need to remember that, "The power of knowing, in that first two seconds, is not a gift magically given to a fortunate few." Gladwell argues that there is in all of our brains, a mighty backstage process, which works its will subconsciously, giving us the capacity to sift huge amounts of information, blend data, isolate telling details and come to astonishingly rapid conclusions. Gladwell is emphatic that although we may not have much control over whatever bubbles to the surface from our unconscious, we do, control the environment in which rapid cognition takes place and by extension, we can control rapid cognition. The control, he says, rests in our mental images.

The best image I have ever read about mathematics education comes from the NCTM Standards. Were the experiences and expertise it embodies to form the foundation for all of our judgments about the mathematics classroom, we could be assured that everything from the textbooks in them to the evaluations of the products generated by them would cause both my sales clerk and my queue companion to blink their way

to dramatically different judgments. It's worth revisiting … just in case you're about to pick up a new textbook for consideration.

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodations for those who need them and challenges for those who stand to benefit from them. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. ▲

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▲ FRIDAY SPECIAL



JACK WEINER

Fridays at the University of Guelph, five minutes before the end of class, I say, "Pens down, books closed, it's "Friday Special" time!" Then I show on screen an article or cartoon or letter or…something that has to do with math, the "real world" and has absolutely nothing to do with class, assignments, or tests. It is math for fun. Often, it is some flagrant example of how the media abuse math. I hope you use some of these. Make them "Monday Morning" specials or "We need a break!" specials. If one falls flat, blame me. If it works, you take the credit.

A few years ago, I received a letter from a mom, advocating on behalf of her teenage son. It seems he entered a contest to win a Honda Nighthawk. He won second prize: $200 in CD's. In order to claim his prize, he had to answer the…SKILL TESTING QUESTION!

The question was the usual bit of arithmetic: $22 + 14 \div 3 \times 4 - 3$.

He submitted the correct answer $37\frac{2}{3}$.

However, Honda informed him, "We are sorry. We can't give you the prize because your answer to the SKILL TESTING QUESTION was incorrect. You see, 22 + 14 is 36, which divided by 3 is 12. Multiply this by 4 to get 48. Finally, subtract 3 to arrive at the correct answer of 45".

Honda didn't obey "order of operations", otherwise known as "BEDMAS"! Mom asked me to intercede on her son's behalf. I wrote a letter to Honda. I explained that "...the order of operations is the logical consequences of the axiomatic properties of the real numbers."

I then offered a numerical example. In **your** class set up the example up this way. Ask for a volunteer. "You have $100 saved and are working part-time at MacDonald's. How much can you save each week for ten weeks?" Say the amount is about $40.

Now everyone will agree that the student will, after ten weeks, have $100 + $40 x 10 = $500 saved.

However, according to Honda's arithmetic, the student would have $140 x 10 = $1400. Hmm, I thought. Maybe we should all use Honda's method. Maybe that is

Professor Weiner: Here is a copy of the Honda Ballot with the "skill testing" question.

FEEL FREE!

to win a Honda NightHawk



1st prize Honda Nighthawk 250cc and Canada Safety Council Course (Approx. value $3,650.00).
2nd prize 10 Compact Discs (Approx. value $200.00). See reverse for rules.

Name: _____
Address: _____
Phone: _____

Winners must correctly answer the following skill testing question:

22 + 14 ÷ 3 x 4 – 3 = _____

Honda has stated that 45 and 37½ are both correct answers.
Would you please answer the question & explain how (why?) you arrive at your answer?
Would you also state your opinion on whether both 45 and 37½ are correct answers.
Thank you very much for your time. Your assistance with this is greatly appreciated.

how Honda prices their cars!

I mailed the letter and sent a copy to the mother. My friend and really smart colleague, Gary Flewelling, suggested that while helping out is fine, my time is valuable and I should charge a consultation fee. I explained this in a note to mom which I attached to her copy of the Honda letter. "My consultation fee is $1000 – $500 x 2." So in reality, she owes me …But according to Honda, I should receive…

Several weeks later, I contacted her. "Did your son receive his prize?"

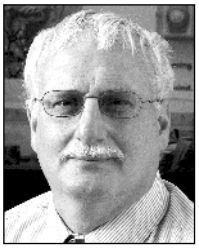
"Yes," she said, "and thanks for your help."

"What about my consultation fee?"

With a smile in her voice, she replied, "The cheque is in the mail!"

Best wishes, Jack ▲

▲ OAME/NCTM March 2005 Report



JOHN KERSLEY

NCTM Notes

Membership Information and Benefits www.nctm.org

NCTM has a great website. Similar to the OAME website it has an area for all to access and also an area for members only. This member-only site contains much relevant information on most useful resources for your students and teacher development. Of course, to access everything on the NCTM website, one needs to have at least an individual membership. Membership dues contribute to the development of resources and materials, the many various conferences and the delivery of NCTM services. Contact me for a current Canadian individual membership form. The Canadian price is $90 which includes choice of one journal plus $22.50 for postage to Canada. It is paid by a cheque written in Canadian dollars. Discounts are available for 2 and 3 year memberships. If you apply online with a credit/debit card, you will pay the full exchange rate. It will cost more.

NCTM Resources www.nctm.org/catalog

A new resource from NCTM is a publication series, entitled, "Empowering the Beginning Teacher of Mathematics". There are three books in the series:

- *Empowering the Beginning Teacher of Mathematics: Elementary School* #12720NB5
- *Empowering the Beginning Teacher of Mathematics: Middle School* #12721NB5
- *Empowering the Beginning Teacher of Mathematics: High School* #12722NB5

New teachers encounter challenges both inside and outside the classroom. These books can be used as resources to help the beginning teachers and/or to augment programs already in place in your school or board. The books highlight six categories: professional growth, curriculum and instruction, classroom level assessment, classroom management and organization, equity, and school and community. The books contain articles, lists of tips, personal testimonies, notes and

quotable thoughts.

The *Navigation* series of books continue to be best sellers. These books put *NCTM's Principles and Standards* into action. They outline the development of concepts and skills from the earliest years through high school graduation. The books are written at the following levels: Pre-kindergarten – Grade 2, Grades 3 – 5, Grades 6 – 8 and Grades 9 – 12. Topics for each level include algebra, geometry, data analysis and probability.

These resources are excellent additions to your school's mathematics library.

NCTM Conferences www.nctm.org/meetings

If you would like to attend one of the best comprehensive and interesting conferences for mathematics education, then consider the annual NCTM conference which will be held in Anaheim, California, April 6 – 9, 2005. It is entitled, "Embracing Mathematical Diversity". NCTM members should have already received a preview brochure. The conference offers over 1000 presentations. Something for everybody and Anaheim is also the home of Disneyland.

Though smaller, but just as interesting and worthwhile are the regional conferences to be held in the fall of 2005:

- October 6 – 8, 2005 "Discover the Connections: Instructions and Assessment" in Hartford,
- October 20 – 22, 2005 "The Role of Assessment: More than Tests!" in Birmingham and
- November 10 -1 12, 2005 "Mathematics at its Peak" in Denver

Members get a very sizable discount on advance registration rates.

Interesting Websites for you to visit as suggested by NCTM

- www.bc.edu/~caseyb/oview.html linking literacy and mathematics, PreK – 2
- www.mathblues.com a new online magazine for high school: Articles, problem-solving tips, contests and humour
- www.balancedassesment.concord.org offers hundreds of innovative mathematics assessment tasks for teachers, grades K – 12
- www.testpreview.com free resources and practice exams for students preparing for standardized tests in high school, college and graduate school ▲

▲ Get Interactive

KEVIN MAGUIRE



Kevin Maguire is an Instructional Leader in Mathematics/Numeracy for the Toronto District School Board. Kevin is also an OAME Director. His particular interests include teaching with technology and elearning.

In this space, websites of interest to teachers of mathematics will be shared. The recent announcement that the Ministry of Education has negotiated take-home rights for students for *The Geometer's Sketchpad* has sparked this review of some of the biggest, and best, *GSP* sites, especially for students.

Sketchmad

http://www.mth.msu.edu/~nathsinc/sketchmad/ *Note the new address for this site, which was mentioned in the previous column.

Key Curriculum Press: The Geometer's Sketchpad

http://www.keypress.com/sketchpad/ Seems like the logical choice for the first site – where it all began. Key Curriculum maintains a vast library of links, and welcomes contributions from teachers and students from all over the world. Classroom Activities, under *General Resources*, in the left navigation bar, is a good place to start.

***GSP* – Hastings and Prince Edward District School Board**

http://www.hpedsb.on.ca/sg/quinte/gsp.htm This very comprehensive website has been compiled over the years by Bill Lundy, System Educational Technology Support" (SETS) person for Mathematics in the HPEDSB. Not only are there a huge number of links (by grade) to *GSP* sites, there are also similar compilations for other software – select "Up" in the blue navbar on the left to see the general software page.

NRICH – Interactive Geometry Challenges

http://www.nrich.maths.org.uk/mathsf/journalf/rb_interact_geom.html To get to the dynamic geometry section on this page, scroll down past the "Mathematical Challenges"(or try some of them!). The Dynamic Geometry Problems show a wide variety of ways to use GSP to solve problems.

Center for Technology and Teacher Education: Geometry and GSP links

http://teacherlink.org/content/math/relatedlinks/sketchpad.html This site has been a rich resource of geometry links for a number of years. The links included on this page demonstrate the versatility of *GSP* – from Buffon's Needle to Geometry in Art.

MathsNet: Geometry

http://www.mathsnet.net/geometry/index.html Always a leader in dynamic and interactive geometry for students, this site contains a large number of explorations that students can try, as well as a number of online "courses": "Interactive Transformations", "Interactive Shape" and "Interactive Construction". ▲

▲ Upcoming Events

For further information on OAME Chapter Conferences contact the Chapter Representative and for NCTM conferences, contact John Kersely, OAME/NCTM Rep.
See the last page in this *Gazette* for contact information.

Chapters now have websites that are linked to the OAME website: see these for full information on upcoming events.

OAME Chapter Conferences and Events

April 6, 2005	Peterborough	PRMA
April 14 2005	Fanshawe College	WOMA
April 21 2005	Barrie	MAC^2 Spring Conf.
April 27 2005	Brock University	Golden
June 4 2005	Laurentian U/Science North Sudbury	NOMA: OMO 2005
Fall 2005	Thunder Bay	NWOAME Math Fest

OAME Annual Conference

May 12 - 14 2005	NEXUS: Building Mathematical Connections York University	hosted by TEAMS

NCTM Conferences

April 6 - 9 2005	Anaheim, California	83rd Annual Conference

▲ Assessment Corner: Who Says Formative Assessment Doesn't Count?!

CHRIS SUURTAMM



Chris Suurtamm is an Assistant Professor of Mathematics Education at the University of Ottawa. She is an experienced mathematics teacher who is committed to helping teachers with the implementation of the new curriculum and assessment practices.

I frequently talk to teachers about assessment and recently, I have been asked "Is it true that formative assessment doesn't count?" or "How can students take formative assessment seriously if it doesn't count?". I have been very surprised by these questions because there is nothing in the definition of formative assessment that states that it doesn't "count". In fact, it "counts" considerably and is a form of assessment that will contribute the most to improved student learning. So, let's examine formative assessment and what it means to "count".

What is formative assessment? Formative assessment refers to the gathering and use of information about students' ongoing learning by both teachers and students to modify teaching and learning activities. Formative assessment includes those assessment activities that provide teachers with evidence of the progress of student learning during the course of learning. It is distinct from summative assessment, which takes place after classroom activities to see what learning has taken place. It includes the gathering of evidence about students' skills and understanding of concepts. Formative assessment may include observations, conferencing, questioning, quizzes, progress reports on projects, journal entries, or the submission of written work. Marks can be given for these assessments, or not. In Ontario, students are evaluated on their achievement of curriculum expectations. Certainly, formative assessments will help to guide students and teachers as to the level of achievement of the expectations. As such, the recording of formative assessments provides the teacher with

information about student progress towards meeting expectations. There may be further assessments, particularly summative, that examine these curriculum expectations but the "in progress" status of achievement need not be ignored.

What is most important about formative assessment is that students be provided with feedback about what they are doing well and what areas they need to improve. Research studies have shown that if students are only given marks, their learning is not necessarily improved. Feedback improves student learning when it provides each student with specific guidelines on strengths and weaknesses (Black & Wiliam, 1998). Such feedback not only should give each student guidance on how to improve, but it should be followed by giving students opportunities to work on that improvement. Formative assessment has long been recognized as part of good teaching practice. Further to this, there are compelling research results to indicate that the practice of formative assessment may be the most significant single factor in raising the academic achievement of all students—and especially that of lower-achieving students (Black & Wiliam, 1998). Formative assessment should focus on what students know and can do and provide them with suggestions for improving their learning rather than provide negative feedback about what they are unable to do. Thus, pupils can begin to see themselves as successful or potentially successful. Often, students who do not think that they have the potential to be successful will not invest any effort in learning because it might lead to disappointment. In many cases, students who do not see themselves as potentially successful believe that they cannot learn, cease to take school seriously, and become disruptive and unproductive (Fuchs, Fuchs, Karns, Hamlett, Katzaroff & Dutka, 1997). Formative assessment that focuses on what students know and can do and prompts students towards improvement helps to create a classroom culture of success (Shepherd, 2000).

When formative assessment becomes an integral part of classroom practice, student achievement is enhanced (Black & Wiliam, 1998; Fuchs & Fuchs, 1986). Formative assessment not only helps to guide students to make improvements, it also informs teachers in determining directions to take to guide student learning or to alter classroom instruction. Effective formative assessment becomes a continuous classroom focus, quite indistinguishable from teaching and curriculum. Teachers are continuously engaged in ongoing assessments of the student learning and in turn of their teaching in their classroom. Teaching and learning are interactive. Most teachers agree with the extensive research that the transmission style of learning does not develop student understanding of mathematics and that students need to be actively engaged in the learning process. As such, they also need to be actively engaged in the assessment process.

There are challenges to cultivating this integrated view of teaching, learning, and continuous assessment. Research indicates that formative assessment often is not recognized as significant by teachers, principals, parents, or the general public, and is seldom articulated or featured as a priority. More information and professional development are necessary to help teachers, students and others develop experience in how to make use of formative assessment. For instance, involving students in the assessment process helps students to understand some of the goals of instruction and the elements in successful learning. Engaging students in their own and peer assessment also helps teachers share the responsibility of determining student progress and growth. In addition to helping students learn how to learn, there are pedagogical payoffs when students begin to improve their ability to peer- and self-assess. Student participation becomes a key component of successful assessment strategies at every step. If students are expected to effectively participate in the process, then they need to be comfortable with the criteria for good work, to assess their own efforts with a view to the criteria, and to share responsibility in taking action based on feedback. Students often see classroom lessons as a series of activities that are unconnected to their own learning goals and they often do not have a clear overall picture of the purpose of the instruction. Self-assessment encourages students to connect to the overall goals of the actions in the classroom.

Summary

Assessment is formative when it is used to adapt teaching and learning (Black & Wiliam, 1998). An effective formative assessment program requires more than just the addition of a few observations and an extra quiz in the middle of a unit. It requires recognition of the goals of the unit, appropriate check-points where learning can be assessed and appropriate assessment tools. Formative assessment helps teachers and students decide the next steps in the teaching and learning process. It requires that teachers and students examine student work with a view to making suggestions

for next steps and providing feedback to improve student learning. Although student work may be marked and recorded, the marks are merely indications of student progress at that point in time. The feedback in the form of comments and next steps and the opportunities for improvement will make the difference in student learning.

Many studies suggest that improved formative assessment helps all students but particularly low achievers. Thus it reduces the range of achievement while raising achievement overall (Black & Wiliam, 1998). In Ontario, there have been many guidelines regarding summative assessments but more work needs to be done to guide teachers in formative assessment. What forms it should take, how it should be used, how teachers can use it to provide feedback to students, and its role in a student's formal evaluation are all issues that should be addressed. Formative assessment can make significant change in student achievement. External testing such as EQAO does not necessarily improve student learning. It is what teachers and students do in classrooms that improves student learning. Providing students with meaningful feedback throughout the learning process is essential and this is what really "counts" as assessment.

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Adam Riese (1492 - 1559)

This German mathematician was Germany's best known Rechenmeister of that century. He is credited with replacing the old system of computation using counters by the more modern written computation. He posed the following problem: Three bachelors wanted to buy a house for 204 Gulden. The first bachelor contributed three times as much as the second bachelor, who contributed four times as much as the third bachelor. How many Gulden did each bachelor pay?

▲ Logical Reasoning Through Classification

LYNN M. GORDON CALVERT

This is an excerpt from an article which first appeared in delta-K*, Volume 39 Number 2 June 2002.* delta-K *is the Journal of the Mathematics Council of the Alberta Teachers' Association.*

Logical reasoning skills developed through sorting, classifying and ordering activities are essential precursors to recognizing, extending and generalizing patterns and provide a foundation for understanding formal and abstract algebra in later years.

.....It is valuable for [students] to recognize that classification schemes are all based on similar forms of logical structures, whether in mathematics, science, biology or everyday life.

Examples of Classification Systems include Taxonomy of Living Organisms, Blood Types, Fingerprint Classification.

Postal Codes in Canada are another example of a classification system. For example, examine the PC such as T5J 4J4.

The first character represents a particular province or territory (or major area within the province). T is for all locations in Alberta. The second character is 1 – 9 for urban locations 0 for rural. Five represents a major area within the city of Edmonton. The third character represents a postal station or a city post office in urban places or a set of post offices in a rural geographic area. J represents an area in downtown Edmonton.

In urban centres, the fourth character refers to one side of the street on a block face, a large business, an office building or another destination. In rural areas, it refers to a particular community.

Fifth and sixth characters continue to pinpoint the location further to a particular mail drop. The last three characters of the given postal code narrow in on the location of Canada Post Corporation in Edmonton.

Pointing to the variety of classification schemes used and created daily begins to make explicit the underlying logic, principle methods and attributes by which data may be classified. Such knowledge provides a foundation for enhancing generalization skill and logical reasoning in mathematics. ▲



http://www.oame2005.ca

The Toronto Educators Association for Mathematics (TEAMS) is proud to present the 32nd Annual OAME 2005 Conference
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David Onley	CityPulse and CablePulse24 science specialist
James Stigler	UCLA Psychology professor

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Elaine Simmt (University of Alberta) The Art of Teaching Math	**Margaret Sinclair, Ron Owston, Janet Murphy, Ray Bowers, Paul McGuire, Pat Margerm** (York University) Panel Discussion: - Online Professional Learning		**Tom Steinke** (Ottawa-Carleton C.D.S.B.) Technology
Marian Small (U. of New Brunswick) Patterning & Algebra	**Chris Suurtamm** (University of Ottawa) Assessment	**Akihiko Takahashi** (DePaul University, Chicago) Lesson Study	**Walter Whiteley** (York University) Connections

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▲ Photo Math - A New Way of Picturing Math

RON LANCASTER
INDEPENDENT MATHEMATICS CONSULTANT



Ron Lancaster is a Lecturer in Mathematics Education at the Ontario Institute for Studies in Education at the University of Toronto. He is also an independent mathematics consultant and works with teachers, students and educational organizations in North America, Asia and Israel.

"A picture is worth a thousand words" is an expression that is often used to convey the importance of using images to make a point. The point of this column is to go beyond this notion and promote the idea that in fact "A picture is worth a thousand numbers". Photos with Math is what readers can expect to find in every Photo Math column and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario curriculum as well as the new NCTM document, *Principles and Standards for School Mathematics*. It is hoped that these questions will inspire students to be more creative and playful and that in the end, students will develop an ability to see the world from a different perspective.

Teachers and students are encouraged to pose mathematical questions related to their own encounters and to submit these problems along with photographs to the author for possible publication in future columns. Photos can be sent via e-mail or by regular mail to the author of this column.

I asked Paul Lessard, Julian D'Angela and George Fawcett what motivated them to create their Great Math Adventure Seminar Series for teachers of grades 1 to 9. They responded from several points of view but all of them centered on the beauty and power of math that kids encounter each and every day when caring and insightful adults ask kids to think about, discuss and reflect on their observations rather than just recording the adults' point of view. It is an opportunity for young math authors to build curriculum. This sets the environment where kids can say "Aha! I can see the math!"

They talked about how many math stories were crystallized in concrete and lasting ways through everyday math happenings like: Canadian Stamps, Dominoes, Dollar Store, Chinese Checkers, Girl Guide Cookies and Decks of Cards. These everyday encounters help kids to realize that math tastes good, looks good and feels good. Students were able to see, hear and do math and respond as Emeril would say "Bam! We'll be right back to kick it up a notch!"

The math photos that they share with us through the *Gazette* continue to highlight the lasting beauty and robust power that fills each of us as we help kids explore, describe, discuss, clarify and celebrate the abundance of math they encounter each and every day. They also felt that so much has been written and rewritten about expectations and achievement charts that it is important to remember that maybe the most enduring indicators will emerge as teachers challenge their students to respond to the beauty of math nestled in each and every math adventure. Photo math collections open the door to a variety of math adventures.

They closed the discussion by saying that they were looking forward to continuing to help teachers help reluctant readers and problem solvers discover, enjoy and succeed in mathematics. Photos remind us that this visualization process is a critical element in helping reluctant learners and are, most likely, equally captivating to both genders. Their website, www.marjaneducational.ca, is available for your convenience so conversations around picturing math differently, adventures in math and the beauty in math may continue.

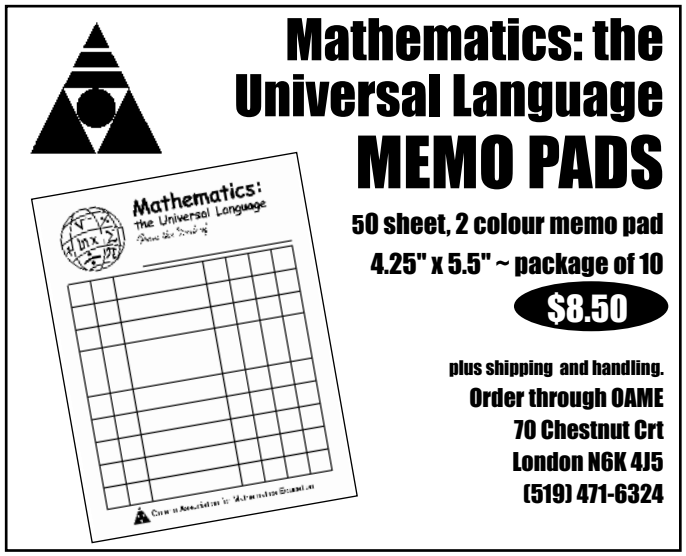


Photo 1



a) Use this photo to discuss line symmetry with your partner.

b) What does each of the letters in the word MAXIMUM have in common?

c) Which letters on the sign have horizontal line symmetry?

d) Do any letters have both horizontal and vertical line symmetry?

e) Use a chart to classify the letters in the alphabet using the headings: Vertical Symmetry, Horizontal Symmetry, Both Line Symmetries, No Line Symmetry.

f) What type of symmetry does the whole word HIDE have? Explain your thinking to your partner.

g) With your partner find five whole words that have horizontal symmetry.

h) Extend this activity to include vertical word symmetry and rotational symmetry.

i) Teachers: remember to ask your students "What does the '40' mean?" (teachable moments . . .)

Photo 2

This soup display was originally stacked in the shape of a rectangular prism. One case of soup and some cans of soup were removed.

a) Estimate the number of cans of soup in this photo.

b) With your partner, compare your estimates and discuss how you arrived at your estimates.



c) With your partner determine the actual number of cans in this display. Discuss the accuracy of your estimates and whether your methods of estimation could have been improved.

d) Determine the number of cans of soup in the original display. If a can of soup costs $1.19, estimate the cost of the soup in the original display.

e) If each can has a volume of 284 ml, estimate the volume, in litres, of the soup in the original display.

f) With your partner compare your estimates for parts d) and e) and discuss the methods used.

g) Suppose your cafeteria bought the soup in the original display to serve for "Soup Day". Assume that 40% of the students in your school have soup on Soup Day and that one can of soup serves two students. How many Soup Days could be served with the soup that was purchased? Explain your thinking.

Photo 3

a) In the outlined section of the maintenance access cover, work with your partner to investigate the number of different sized trapezoids.



b) What is the total number of trapezoids in the outlined section?

c) What is the total number of trapezoids inside the larger circle?

d) Make a list of characteristics that trapezoids have.

e) What other polygons can you identify and describe in this photo?

f) How many different sized regular hexagons are in the photo?

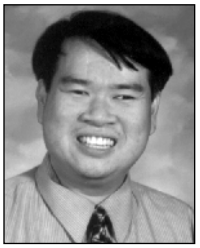
g) Use these regular hexagons to discuss the concept of similar figures with your partner.

h) With your partner, identify four pairs of similar trapezoids. ▲

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▲ "What does a Mathematician Look Like? What Does a Mathematician Sound Like?" Inviting Mathematicians to Speak With High School Students

Louis Lim
Head of Mathematics
Unionville High School
York Region District School Board



Louis Lim is department head of mathematics at Unionville High School and is a PhD student at York University. He is currently an editorial panel member of the NCTM's Mathematics Teacher *journal. Louis has contributed to the mathematics education community through his involvement as workshop presenter; textbook writer; writer with Ministry of Education projects (revised mathematics curriculum, student exemplars, and prior learning assessment and recognition); Additional Qualifications (AQ) instructor; and was editor of the OAME* Gazette *from 1999 to 2003. Louis is committed to making mathematics relevant and enjoyable for students through the linking of research into practice.*

With the elimination of OAC, some post-secondary institutions have modified the content of their first year mathematics courses (see for instance Weiner, 2002). Research has been conducted with how students in the double cohort fared in first-year university mathematics (e.g., Kajander & Lovric, 2004). Tutorials and papers have been written to help students make the transition from high school to university mathematics (e.g., Lovric's *Mathematics Review Manual* and Barbeau's, *Guidance for Teachers and Students Coming to University*). Hence, attempts have been made to "reach out" to incoming students.

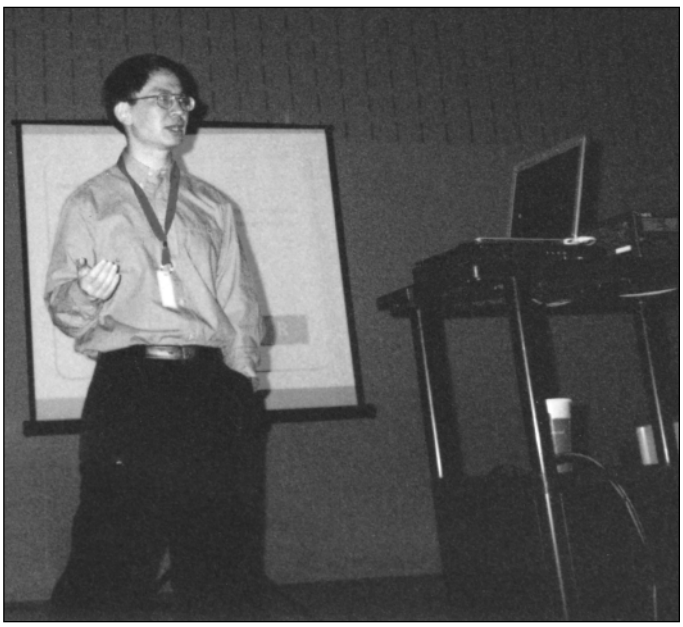
As a high school mathematics teacher, I enjoy following grade 12 students' application processes to universities and community colleges. I often wonder what I can do to help alleviate some of the stress they experience. Students are very excited, yet anxious, about what lies ahead for them. Students have mentioned that they are nervous about learning in large classes and wonder if their professors will care about their learning.

For spring 2004, I invited in two mathematics professors from York University, Professor Walter Whiteley and Professor Jianhong Wu. My goal was to convince students that professors are indeed approachable and genuinely care, so students would be more willing to ask questions after lectures or during office hours.

I advertised the talks at my school as, "What Does a Mathematician Look Like? What Does a Mathematician Sound Like?" In fact, both these mathematicians were recipients of the York University system-wide teaching award of excellence, a most significant honour considering the number of faculty at York!

I had the good fortune of taking three half-courses with Dr. Wu and consider him one of the most amazing professors I had at York. He made difficult concepts comprehendible and interesting to learn. He is currently the Canada Research Chair in Applied Mathematics and was featured in a publication, *York U*. The article described how he is leading a team to build a mathematical model related to the spreading of SARS. Although I have not taken a course with Dr. Whiteley, I heard him when he was a keynote speaker at the fall 2002 Y^4MA mathematics conference and as a guest speaker in a mathematics education course I took back in the mid-1990s.

I am most appreciative that both Drs. Whiteley and Wu accepted my invitations, considering their busy schedules. In fact, Professor Wu response was, "I do have one principle, and namely I shall never turn my

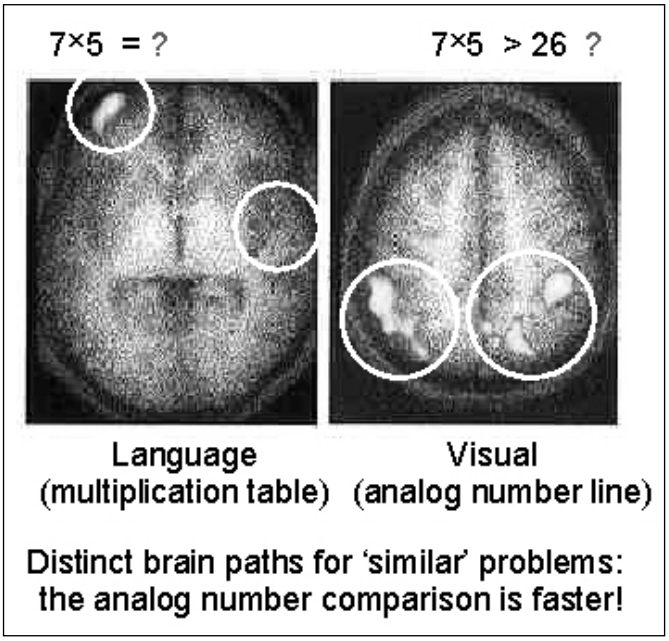


Dr. Wu, speaking with students at Unionville High School.

back to students." Having never organized such an event, my principal suggested I speak with the department head of physical and health education to book the small gym and with the student in charge of setting up audiovisual equipment for assemblies. The teachers in my mathematics department were also very supportive, witnessed by having all grades 11 and 12 classes running during period 4 attend both talks. One of the computer studies teachers even took his class to one of the talks! Flyers were posted in the school, and daily announcements were made leading up to the talks. Students not taking mathematics period 4 or during this semester, were invited to sign up so I could "field-trip" them. In total, approximately 200 students attended each talk.

Dr. Whiteley, being a Visual Geometer, focused on visual reasoning, discussing how people learn and do mathematics. He talked about transformations and symmetry as he used the *Geometer's Sketchpad* on the overhead. The students were particularly interested when he talked about how three-day old babies do arithmetic! Dr. Wu talked about his experiences on a national research team project called, "Transmission Dynamics and Spatial Spread of Infectious Diseases: Modeling, Prediction, and Control." He shared information on how statistical data analysis and simulation can be used to mathematically model the spreading of infectious diseases such as SARS and the West Nile virus. Both talks were highly relevant to students, especially because we have experienced, first hand, the SARS outbreak. Students have learned geometry since kindergarten but never seen how mathematicians would use it to model solutions in the real world. I was pleased that students stayed behind to ask the speakers questions.

The initial year of the guest speaker series, "What Does a Mathematician Look Like? What Does a Mathematician Sound Like?" was a success. For the 2004-05 school year, I have invited Professor Ron Lancaster (OISE/UT) to speak about the beauty and magic of mathematics to grades 9 and 10 students and Professor Jack Weiner (University of Guelph) to speak with grades 11 and 12 students about first year university calculus. I am currently searching for other speakers. In particular, I would like to invite a female mathematician and a community college mathematics professor. I encourage other high school mathematics departments to host similar events for their students. A phone call to a university or community college mathematics department will lead to potential speakers. We need to enhance the communication between high schools and post-secondary institutions.



A slide from Dr. Walter Whiteley's workshop. The brain image is likely from Brian Butterworth's Science*, May 7, 1999 v284 i5416 p928. For additional information, Dr. Whiteley suggests the related link, http://www.unicog.org/docs/NumbersPage.htm.*

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Brahmagupta (c. 628)

He was the most prominent Hindu mathematician in the 7th Century. In his book Cutta ca, he writes: If one reduces the number of days by 1, divides the difference by 6 and adds 3, the result is always one-fifth of the original number of days. What is the original number of days?

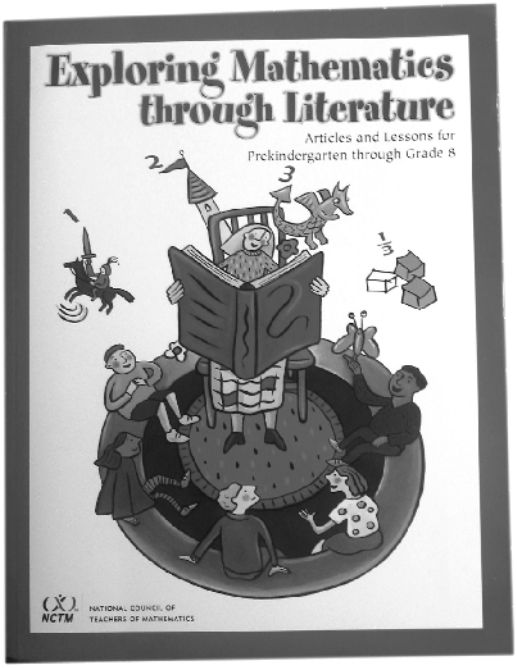
▲ BOOK REVIEW: *EXPLORING MATHEMATICS THROUGH LITERATURE: ARTICLES AND LESSONS FOR PREKINDERGARTEN THROUGH GRADE 8*

A REVIEW BY: JOANNE HARRIS
RETIRED GRADE 8 MATHEMATICS TEACHER



Joanne Harris is a recently retired grade 8 Mathematics teacher. She believes that making connections to the real world, to other subjects and to previous knowledge is one of the keys to helping students learn.

The book *Exploring Mathematics through Literature* is a compilation of articles that have previously been published by the National Council of Teachers of Mathematics (NCTM). The stated purpose of the book is to provide classroom examples that demonstrate how to use children's literature to teach mathematics effectively in prekindergarten through eighth grade. This purpose has been met very effectively for PreK to grade 6 but in a limited manner for grade 7 and grade 8 due to the small number of examples that were used.



The editors' of the book selected articles that they felt had a strong potential for teaching mathematical content through literature. The author of each original article was contacted and asked to create an addendum. The format selected for the addendum was the same for each article included in the final copy of the book. It identified the grade range that the article was written for, the mathematical topic(s) included, the title(s) of the children's books used in the article and materials needed to do the activity or unit in a classroom. Following this there was a discussion of the mathematics involved in the unit or lessons, teacher notes and suggested questions to use, and references and recording sheets in the form of BLM's that could be used when teaching the unit. No specific mention is made of potential assessment strategies.

The material in the book is organized by content strand, so if you were looking for something that could link measurement and literature you would look under measurement in the index. The same chart that identifies the strand addressed by the article then also identifies the grade that the article was written for.

The variety of children's books used by the various teacher/authors and the way they describe the use in their particular classrooms is well worth reading. The teacher's who have written articles that were included in this book have concluded that a high quality, well designed children's book can become another effective tool to be used by the mathematics teacher.

Exploring Mathematics through Literature demonstrates how children's books can be used to initiate a discussion about a mathematical concept, or how children's books can be a source for problem solving or the basis for problem solving ideas, or how children's books can act as an example to follow when creating your own book with a mathematical theme.

Questions a teacher should be asking when trying to identify a book's potential for use as part of a mathematics and literature unit include: Is the mathematics worthwhile? Is it accurate? Is the book appealing? Is it well illustrated?

The majority of the material in this book is written for the primary or junior classroom. Of the 30 articles in the book only four identify grade 7 and/or 8 as the target audience. With the increased emphasis on Literacy in Ontario classrooms today, this book would make an excellent addition to a school's professional library. However, because of the limited number of grade 7 and 8 applications I would hesitate to recommend it's purchase by a middle school.

Exploring Mathematics through Literature is edited by Diane Theissen. It is available through the National Council of Teachers of Mathematics at a cost of $34.95 (US) for non-members and $27.96 (US) for members. ▲

▲ MathSwap ~ An Online Library of Resources for Teachers of Mathematics

JUDITH KOELLER



After completing studies in cryptography, Judith Koeller worked as a consultant for a cryptography company, and as a developer for a math software company. She currently works as a lecturer at the University of Waterloo, teaching courses and integrating math resources and technology.

The Centre for Education in Mathematics and Computing is best known for the Canadian Mathematics Competitions. Approximately 200 000 students across Canada participate annually in contests such as the Euclid Contest and the Canadian Computing Competition. In fact, the CEMC offers a broad range of services, events and resources for teachers of mathematics and computer science.

At a recent teacher workshop hosted by the CEMC, it was striking to see how many excellent resources are created by teachers across Ontario and Canada. At this one event, lesson plans, math problems, TI-80 programs and ideas for using math problems were all presented. Many teachers are happy to share their materials with each other, and we benefit from sharing instead of always re-inventing the wheel.

That is why one of the CEMC's newest ventures is the MathSwap website, www.mathswap.ca. MathSwap is a free website for high school mathematics teachers to share materials with others.

The MathSwap library includes all kinds of resources:

- Worksheets
- Discovery activities
- Lesson plans
- Math problems
- Programs and files (for *GSP*, TI-80, *Fathom*, etc.,)
- Math project ideas
- Enrichment & math project ideas
- Anything that may be of use to another teacher.

The submissions that we receive will be reviewed and classified according to their fit with the Ontario curriculum. In Fall 2005, the site will include a search engine to allow teachers to quickly find materials appropriate for their topic and course.

We are currently inviting teachers to share a resource or two that can be added to the library. To send in a resource, you can fill in the form at www.mathswap.ca, or email jakoelle@uwaterloo.ca. You can also sign up to receive an email once the search engine is available.

By contributing a resource or two, you will help to build a library of free resources for math educators. ▲

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▲ Comparing the Games We Play in the Classroom

GARY FLEWELLING



Gary is an independent mathematics consultant. He is interested in all aspects of mathematics education including teacher education, curriculum development, resource development, assessment, problem-solving, the use of technology, and creative thinking. When not involved in things mathematical, he is usually found with his wife, in outback Australia, looking for gemstones.

In this paper the author compares the two games played most frequently in our classrooms, concluding that (although both games may prepare the student for success in school) only one of the games prepares the student for success in life.

The Games We Play

There are two fundamentally different games that we play in our classrooms. I call the game that most of us have experienced, the *knowledge* game. I call the other game, the *sense-making* game.

It is important that we be able to recognize each game and understand their differing characteristics and purposes because, in my opinion, only one of these games, the sense-making game, prepares students for life.

You can identify which game is being played in a classroom by examining the learning tasks the students and their teacher focus on and the way in which they engage in these tasks.

Prompts for Mathematical Activity

Elaine Simmt (2002) states, 'just as important as solving problems is specifying them in the first place. Hence, placing an emphasis on "problem solving" in school mathematics, that is looking for the solution to a pre-specified problem, misses this key aspect of human cognition observed in my research. The teacher needs to provide students not problems to solve but with prompts for mathematical activity. In this paper I use the prompt shown in Fig 1a.

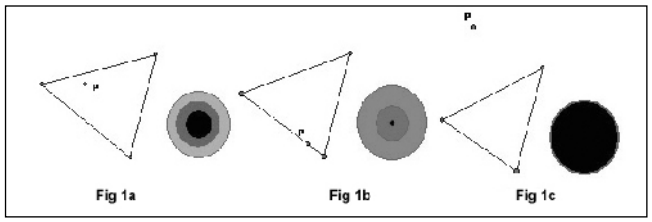


Fig 1a. The sketch is drawn using The Geometer's Sketchpad™*. As point P is dragged, 3 differently colored concentric discs change in size, as shown in Fig 1b and 1c. Readers can explore the sketch themselves by going to the web site, http://hydra.educ.queensu.ca/sketchmad/sketches/index.html, referred to in Flewelling and Sinclair 2004.*

Like any situation, this sketch can generate very different tasks, prompt very different kinds of mathematical activity. Below I describe two possible tasks and how they might unfold in the classroom. The task, entitled *Triangle-Land*, is engaged by students playing the sense-making game. The second task, entitled *The Triangle Connection*, is engaged by students playing the knowledge game. Though both tasks focus on the connection between the position of point P and the behavior of three discs, they make very different demands on the student, foster very different dispositions, and ask the student to pursue very different goals.

Description of Tasks Unfolding

Triangle-Land

1. Setting the Stage / Launching the Investigation

   "You are a scientist who has landed at point P in Triangle Land. You are charged with the task of investigating how the sun near this world behaves. Start your investigation by dragging point P and studying the behavior you witness."

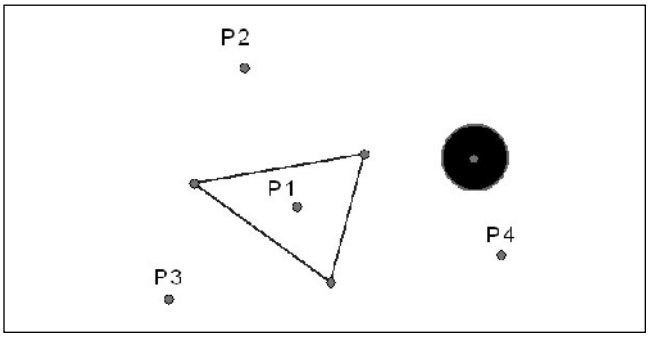
   - Teacher gives students (working independently or in small groups) the opportunity to explore the situation, get familiar with its elements, their behavior, possible connections / relationships.
   - Teacher advises students to record interesting observations / discoveries and be prepared to share them.
   - Teacher encourages students to ask questions such as, When does that (interesting thing) happen? Why does it happen? What is this phenomena connected to?

2. Sharing Observations / Identifying Focus Problems

   - Teacher facilitates student sharing of observations.

- Discussion leads to focusing on the following problem: "When does the sun display one color? Two colors? Three colors?"

3. Conjecturing: Connecting / Predicting/ Testing / Proving
   - Students continue investigation focused around above questions.
   - Students connect particular sun displays with particular locations of dragged point. Geometric significance of these points identified.
   - Area of individual discs connected to the areas of triangular regions.
   - Predicts sun's behavior for various positions of point P.



"I know the exact location of 3 points outside the triangle (and 1 point inside) where the sun is the same single color. And I can prove it!"

   - Provides area proofs for assertions.

4. Communicating / Demonstrating / Evaluating
   - Students share findings, providing evidence for claims.
   - Demonstrate proofs of assertions
   - Evaluate convincingness / elegance of demonstrations
5. Assessing Student Performance
   - Teacher provides new (related) sketch for students to investigate individually.
   - Students write report related to investigation. Teacher provides rubric listing elements of a successful report.
   - Teacher assesses student performance on task (and report.)
6. Reflecting On / Extending the Investigation
   - Students are encouraged to create their own sketches for others to investigate.
   - Are there other points of interest outside Triangle-Land?

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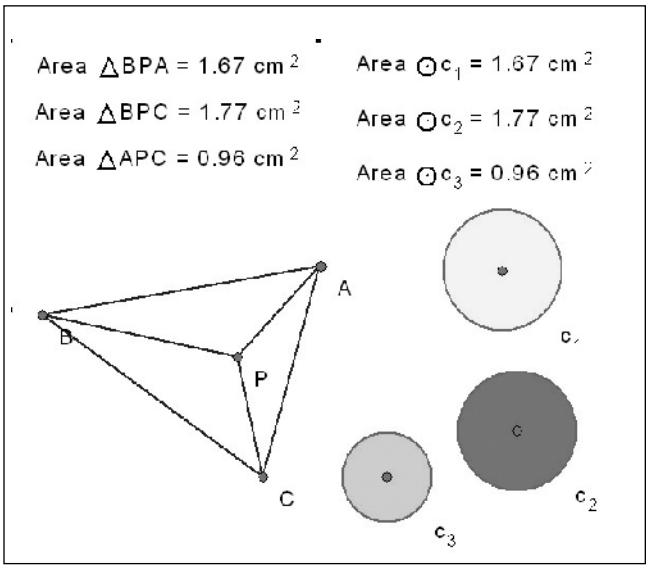
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Triangle Activity Sheet

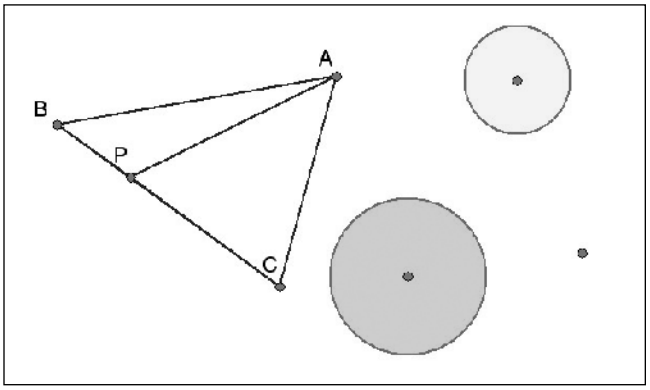
1. Launching the Task
   - Teacher introduces students to sketch, demonstrating disk size color change.
   - Teacher directs students to work through the following set of 13 sub-tasks, indicating that doing so will allow them to understand the behavior witnessed in the sketch.

TASK

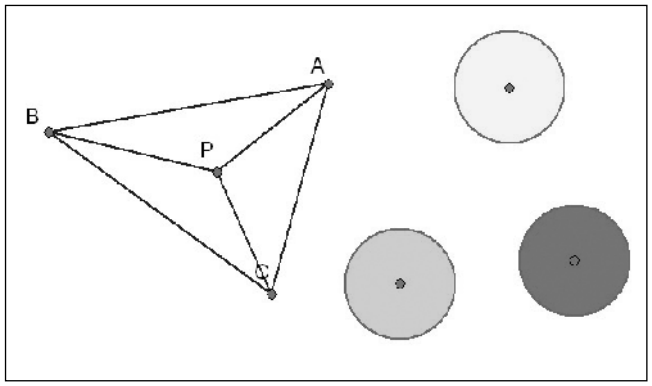
1. Label the vertices of the triangle A, B, and C.
2. Join point P to points A, B, and C.
3. The disk is actually three overlapping disks. Separate them by dragging their centers.
4. Measure the areas of ΔPAB, ΔPAC, and ΔPBC.
5. Measure the areas of the circles.



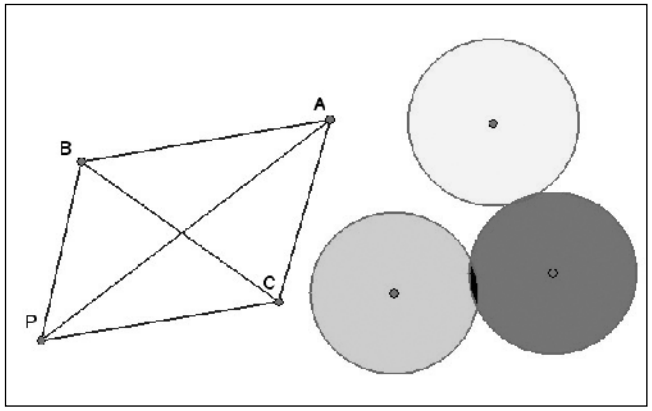
6. Compare the results in parts 4 and 5 as you drag point P.
7. Drag point P onto one side of the triangle. Explain why one circle shrinks to a point.



8. Drag point P onto a vertex of the triangle. Explain why two circles shrink to a point.
9. Drag point P to a position inside the triangle where the 3 disks have the same area.



10. Demonstrate that the point found in part 9 is located at the intersection of the medians of ΔABC.
11. Drag point P outside the triangle to a point where the three disks are again of equal area. Prove that quadrilateral PABC is a parallelogram.



12. Prove that ΔPAC = ΔPAB = ΔPBC. (This proves that the three disks are equal in area.)
13. Sketch where you would expect to find two other points outside the triangle where the disks are again equal in area. Explain your reasoning.

2. Wrapping Up the Activity
   - Teacher 'takes up work'.
   - Teacher directs students to summarize their findings in their notes.
   - Teacher assesses student performance.

A (brief) Comparison of the Two Tasks

Triangle-Land

1. Goals of Task
   - (Student's) Make sense of situation.
   - (Teacher's) Develop (and assess) skills / dispositions in anticipation of future sense-making episodes, incidentally reinforcing geometric concepts and procedures.
2. Cognitive Demands
   - Use knowledge in a purposeful, integrated, creative, authentic manner.
   - Meta-cognize, monitor progress, reflect on actions / products, consider implications.
   - Use imagination / intuition.
   - Think critically.
3. Dispositions Fostered
   - Anticipate challenge / ambiguity / uncertainty.
   - Sensitized to notice things, search out patterns / relationships, make connections, be discerning, ask questions, search out / articulate problems, make predictions, want to know why.
   - Inclination to justify, argue, critique, clarify, convince.
   - Take more control over learning / how task unfolds, initiate action, persevere.
4. Beliefs / Attitudes Fostered
   - Subject is an important way of thinking / a product of human / personal activity.
   - Subject is exciting and pleasurable.
   - 'I am primary agent for my learning'.

The Triangle Connection

1. Goals of Task
   - (Students) Follow teacher instructions / answer teacher questions.
   - (Teacher's) Reinforce / assess geometric concepts and procedures.
   - (Both) Success on current task.
2. Cognitive Demands
   - Use knowledge in a narrow, isolated, mechanical, artificial manner.
   - Check accuracy / correctness / completeness of work.
   - Think convergently.
3. Dispositions Fostered
   - Anticipate situations / problems / tasks involving clarity, simplicity, certainty, correctness, closure.
   - Sensitized to respond to questions, to answer quickly / clearly / accurately, to find / apply needed concepts / procedures (to familiar situations).
   - Inclination to obey / conform / acquiesce / follow.
4. Beliefs / Attitudes Fostered
   - Subject is an important collection of truths / rules / tools developed by others.
   - Subject anxious.
   - 'Teacher primary agent for my learning'.

The Triangle-Land task is a much richer task. I define a 'rich task' as one that gives students the opportunity to use (and learn to use) their knowledge in an integrated, creative, authentic, and purposeful fashion to inquire, investigate, experiment, and problem solve and, in so doing, acquire knowledge with understanding, and in the process, develop the beliefs and dispositions of a life-long sense maker. (A comprehensive treatment of rich learning tasks can be found in *Teaching With Rich Learning Tasks: A Handbook* by Flewelling with Higginson, 2003.)

A Comparison of the Two Games

Herbert Spencer (19th century philosopher and coiner of the phrase 'survival of the fittest') once said, "The great aim of education is not knowledge but action". The knowledge game is essentially about the acquisition of knowledge (stuffing with rules) with time divided between acquiring, reinforcing, and demonstrating acquisition. The sense-making game is about action, sense-making action (playing with rules).

The cognitive demands of each game are different. Experience with the knowledge game is not a prerequisite for playing the sense-making game. In fact, playing the knowledge game fosters the development of beliefs, practices and dispositions that act as impediments to playing the sense-making game. The knowledge game handicaps the student for playing the game waiting to be played beyond the classroom.

Whitehead (1929) talks about 'the aimless accumulation of knowledge, inert and unutilized'. When he talks about the 'aimless accumulation of knowledge', he is talking, I believe, about the knowledge game. When he talks about 'inert and unutilized knowledge', I believe he is talking about the product of the knowledge game and its lack of usefulness in / transfer to sense making.

The Curriculum Connection

Most mathematics curricula I am familiar with are structured in ways that promote the knowledge game and discourage the playing of the sense-making game. A curriculum working group of the Canadian Mathematics Educators Study Group, at their May 2003 annual meeting, at Acadia University, Wolfville, Nova Scotia seems to concur. They state in their draft 'manifesto' (about Canadian mathematics curricula), "we believe that the structure of these curricula is an obstacle to student learning of mathematics. Over-specified and fragmented lists of expectations misrepresent what mathematics is and militate against deep and authentic engagement with the subject".

We find that:

- students coming out of high school mathematics must be able to engage effectively with complex problems; they require the ability to 'think mathematically'—that is, to investigate the mathematics in a situation, to refine, to expand, and to generalize;
- students' mathematics concepts must be woven into a connected set of relationships;
- students must be able to independently encounter and make sense out of new mathematics.

Conclusion

Both games, played well, can prepare the student for success in school. But we must ask ourselves the following two questions: "What do we want students to be successful at?" and "What do we want students to be prepared for?" If our answer to the second question is, 'Life!' then our answer to the first question must be, 'Sense making!'

If we really want to graduate life-long sense makers then we will have to ensure that students are with teachers who engage them routinely in rich learning tasks, in classrooms where the sense-making game is played.

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[1] *This site will soon be transferred to the OAME website: http://www.oame.on.ca*

▲ OAME Website Members Only Section Now Available

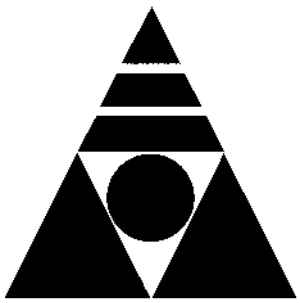


GREG CLARKE
OAME WEBSITE COORDINATOR

Have you checked out the OAME website lately?

There have been a number of developments in the past few months, and as of mid-January, 2005, OAME is pleased to announce that the long-awaited Members Only section is now open for business! This part of the OAME website contains electronic back issues of the *Gazette* and the *Abacus*, including a searchable Tables of Contents. Other features of this secure area include account information, address update feature, minutes of previous OAME board of directors meetings, and a section that will allow you to email various OAME board members. You will be able to renew your membership on-line by credit card in our new secure area. Look for even more features in the future as this section continues to be developed.

To enter the Members Only area, you need to activate your online account using your OAME membership number (this 5 digit number can be found on the mailing label of your *Gazette*) and an email address. Go to the OAME website (http://www.oame.on.ca) and click on the link for Members Only. If your membership number or email address do not match what's on file with OAME, contact the Executive Directors and they will be happy to help you out (EDs@oame.on.ca)



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Online Shopping has also come to the OAME website with the addition of a secure Shopping Cart area. Credit cards are accepted online for a number of items including new memberships, OAME memo pads, Linking Assessment and Instruction in Mathematics documents, and shortly a new line of math-related items.

As the OAME website continues to evolve, please send your comments and suggestions to the website coordinator, Greg Clarke, who can always be reached by email at web@oame.on.ca ▲

▲ Florence Nightingale, the Passionate Statistician

Jill T. Thomley
Sarah J. Greenwald
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Jill E. Thomley is an assistant professor of statistics at Appalachian State University in Boone, NC, USA. Dr. Thomley is an applied statistician whos is involved with a wide variety of research projects in computational science, geology, biology and political science. Over the past few years she has been studying the history of statistics for use in both a senior capstone course in statistics as well as material for bringing insight, added depth and multidisciplinary perspectives to topics in introductory and mid-level courses. She has been involved in a number of presentations and workshops on AP statistics and dealing with data for middle grades and high school teachers.

Sarah J. Greenwald is an associate professor of mathematics at Appalachian State University in Boone, NC, USA. Her research area is the Riemannian geometry of orbifolds and she is also interested in the geometry of the universe, mathematics and popular culture, and women and minorities in mathematics. She enjoys creating classroom activity sheets in order to engage students with mathematics related to The Simpsons *and has taught a class on women and minorities in mathematics with portions dedicated to both mathematical content and equity issues.*

This article first appeared in The Centroid*, the Official Journal of the North Carolina Council of Teachers of Mathematics. Vol 30 No 2 Fall 2004.*

Most American schoolchildren are familiar with the legend of the "Lady with the Lamp". Florence Nightingale's contributions to statistics are less well-known than her nursing career, but they are still highly noteworthy (Spiegelhalter, 1999), since much of her celebrated success in nursing reform would not have been possible without them. In addition, not until the middle of the twentieth century would there be another woman statistician who achieved a similar level of recognition and influence as Florence (Salsburg, 2001). Incorporating her mathematical achievements into the classroom can be especially rewarding because, unlike many other women mathematicians, students are already familiar with Florence. Examining her mathematics in the context of her other achievements also provides a rich arena to highlight connections to other subject areas.

During the Crimean War, Florence Nightingale revolutionized the profession of nursing. She and her nurses dramatically raised standards of patient care, including sanitation and nutrition, which drastically reduced deaths from infection and disease in military hospitals. Her efforts inspired noted author Henry Wadsworth Longfellow to create a poem that cemented her image as the "Lady with the Lamp" (Dossey, 2000).

Florence realized that the problems she found in Crimea were not limited to wartime hospitals. Armed with statistical data she had collected, Florence turned her attention to large-scale reforms of public health. It was Florence's devotion to the systematic collection and vivid presentation of data to further social change that led her biographer Sir Edward Cook to dub her the "Passionate Statistician" (Cook, 1913).

Florence's Early Life

Florence Nightingale was born in 1820 in Florence, Italy, to wealthy British parents. The Nightingales were among the "upper ten thousand"; the economically and socially advantaged segment of society that ruled England. Both she and her sister Parthenope were thoroughly schooled in reading and writing by their governess. At seven years of age Florence was already a prolific and keenly observant letter-writer; by the age of ten she had written her own autobiography, in French. She was also noted in her family for compiling very detailed lists of nearly everything around her, many of which appeared in her journals and letters.

Beginning at age twelve, Florence was educated by her father William Nightingale. As a devout Unitarian, her father rejected the contemporary philosophy that women were too fragile for the rigors of higher education. Florence studied history, politics, ethics, mathematics, philosophy, and composition. She also had a particular aptitude for languages, becoming fluent in Latin, Greek, French, German, and Italian. At the same time, young Florence learned all the manners and household skills expected of privileged Victorian girls. She was formally presented to the Queen Victoria in May 1839 (Dossey, 2000).

Florence's studies under her father were equivalent to a Cambridge college education (Dossey, 2000). At the time, only men were allowed to attend. When she was twenty, she had reached the limits of William Nightingale's mathematical ability (Diamond and Stone, 1981). Her insistence on further instruction from an outside source was met with resistance from her mother. Here Florence got support from her Aunt Mai, who located willing, educated tutors for her niece. One was

James Sylvester, who is noted in mathematics for developing the theory of invariants with Arthur Cayley (Stinnett, 1990). Florence learned algebra, geometry and arithmetic, which she in turn taught to several children before starting training as a nurse (Lipsey, 1993).

Along with her mathematical tutoring, Florence conducted a self-guided course in social and health statistics. The creation of the first public register of births, deaths and marriages (when Florence was seventeen) by the General Registry Office was an unprecedented event that made social statistics a popular topic for conversation (Florence Nightingale Museum Trust, 2003). Prior to this, such records (if they existed) were usually in the hands of local church parishes. Florence also studied hospital blue books and any other data she could obtain. One of her most ambitious projects, done in 1853, was to gather her own data by sending questionnaires to hospitals regarding health administration, which she then laboriously analyzed (Diamond and Stone, 1981).

Florence the Nurse

In nineteenth century England, nursing was not considered a suitable profession for respectable women. Nonetheless, feeling called by God, Florence pursued it. When she was 29, she rejected a marriage proposal from her longtime suitor Richard Monckton Milnes. Unfortunately, despite progressive attitudes about education, her parents still felt marriage was necessary for a woman of their class, and they turned deaf ears to her request to study nursing. Two years later, a meeting with Elizabeth Blackwell, the first woman to earn a medical degree in the United States, caused Florence to reassert her position. She was finally permitted to go to Germany for training (Dossey, 2000).

Florence became the head of a charitable hospital for sick gentlewomen. Women of Florence's class were expected to perform volunteer service, and the clients of the home were women from good families. During this period her social connections let her meet and interact with doctors who were on the forefront of medical innovation. Among them was John Snow, who used the new Registry Office data and map plots of cholera deaths to establish that cholera is a water-borne illness (Dossey, 2000). Though it is not documented, one can only imagine that this effort impressed Florence, who was extremely interested in this kind of data use.

Florence was also deeply involved in the treatment of cholera patients, taking a leave of absence from her position to do so. Then, in 1854, Florence was appointed as the "Superintendent of the Female Nursing Establishment in the English General Hospitals in Turkey" by the Secretary of War. This was an unprecedented honor for a woman. With 40 volunteer nurses, she went to the military hospital at Scutari and from there into the public eye and history books.

Florence the Statistician

Florence's fascination with statistics and her desire to nurse were both present from her childhood. In Crimea she found the opportunity to meld her two passions into a single pursuit. One of her first acts was to institute uniform statistical recordkeeping to replace haphazard and contradictory military journals (Kopf, 1916). These data would form the foundation of her later work when she returned home. The Royal Commission, charged with formalizing Florence's reforms, would come to rely heavily on her statistical analyses, and it had as one of its goals the establishment of a "statistical department for the army" (Dossey, 2000). She also pressed for more uniform data collection on the civilian front, both in hospitals and in the national census. Both of these proposals were "too advanced for their time" (Dossey, 2000; Kopf, 1916) since hospitals were not used to the rigorous collection of data, and her proposals were either implemented briefly in some hospitals or not at all.

In an era where even statisticians believed that statistics should be "the driest of all readings," (Heyde and Seneta, 2001), Florence drew upon her extensive mathematical training and became an innovator in standardized data collection, tabulation and graphical displays (Stinnett, 1990). Florence thought that both visual and auditory communication was necessary to understand the real meaning of data: "the diagram which is to affect thro' the Eyes what we may fail to convey to the brains of the public through their word-proof ears" (Diamond and Stone, 1981). Her closest associate and collaborator for twenty years was medical statistician William Farr, who shared her practical view of applied statistics, though he remained rather critical of what was at the time an unconventional reliance on graphs (Diamond and Stone, 1981). She was also influenced by the Belgian statistician Adolphe Quetelet, considered to be the founder of social statistics (Dossey, 2000).

Florence's most well-known invention was the polar area chart (erroneously referred to as "coxcomb" charts in some texts), an example of which is shown in Figure 1. Very few significant precedents existed for this kind of display (Kopf, 1916).

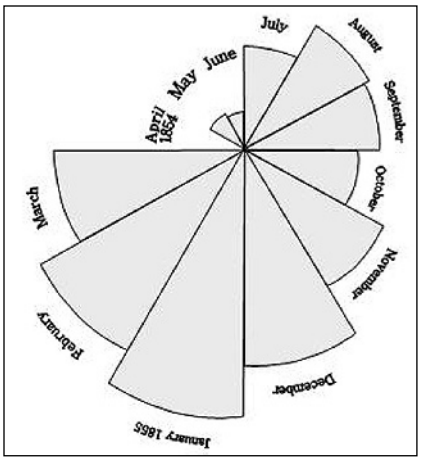


Figure 1 A partial reproduction of one of Florence's polar area charts showing military fatalities due to disease in the Crimean War.

Polar Area Charts

Polar area charts are variations on pie charts, which were first used by William Playfair in 1801 (Small, 1998). In a pie chart, the circle or "pie" has a constant radius. The total area is proportionally divided among categories to show their relative frequencies. For example, if one category has twice as many items in it as the others, then its piece in the pie chart is twice as large as any other piece.

In a polar area chart, the circle is divided into angles or "wedges" of the same size for each category. The radii of the wedges vary. Each radius is equal to the square root of the frequency for that category. The square root is used for the radius because the area of a circle is πr^2; using the square root of the frequency for radius means that the wedges in the polar area chart are still proportional to one another, like in the pie chart.

Table 1 US Olympic Medals 1896 - 1908

Olympics	Gold	Silver	Bronze
1896	11	6	2
1900	20	14	19
1904	80	86	72
1908	23	12	12

Table 1 shows the numbers of gold, silver and bronze medals the United States won in the first four modern Summer Olympic Games.

Figure 2 shows a standard pie chart for the percentages of each the three medal types the United States won in the 1896 Olympics. We would need a separate pie chart for each year if we wanted to compare all four years.

Figure 3 shows a polar area chart for the same data for all four years. Each quadrant or quarter of the circle represents one Olympics. The radius of each colored slice or wedge is the square root of the gold, silver or bronze medal count, and the three slices for each year are stacked on top of each other. As in the pie chart, gold medals are shown in white, silver medals in gray and bronze medals in black.

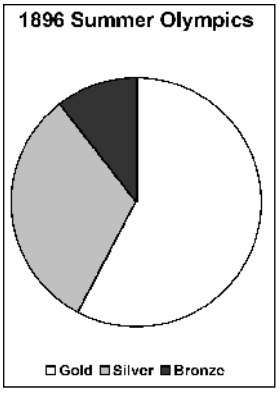


Figure 2 Pie Chart for US Medals in the 1896 Summer Olympics.

The advantage of the polar area chart for this data is that it displays not only the relative percentages within each year, but it also gives an indication of the total number of medals won, something the pie charts do not do. In 1904, for example, the very large wedges show that far more medals were won in that year than in any of the other three years. The relative size of the wedges also shows that the U.S. won nearly equal percentages of gold, silver and bronze that year. In 1908, the totals for silver and bronze were the same, and this is represented by the striped wedge.

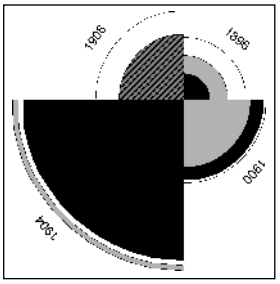


Figure 3 Polar Area Chart for US Medals in the 1896, 1900, 1904 and 1908 Summer Olympics.

Though they were innovative and dramatic, polar area charts were subsequently replaced by pie or bar charts in common usage because of the laborious hand calculations that were required to find square roots in the days before calculators. Figure 3 requires 12 square root calculations. Many of Florence's diagrams representing yearly data about military deaths due to disease, wounds and other causes had 36 square roots, three for each month. Then, the charts themselves had to be drawn by hand with rulers and curved templates, making pie and bar charts more attractive options in terms of time and effort.

Technology was a common problem for statisticians of the late 19th and early 20th centuries: ideas about how to effectively summarize and visualize large amounts of data often preceded the ability to actually implement those ideas. Modern computers allow people to make a wide range of graphs with a few clicks of a mouse, so statisticians are now revisiting many of the ideas first proposed a century ago.

Florence's Legacy

Florence Nightingale was the first woman elected as a Fellow of the Royal Statistical Society, and she was also

made an honorary member of the American Statistical Association. She received the St. George's Cross from Queen Victoria for her *Notes on Matters Affecting Health, Efficiency, and Hospital Administration of the British Army*, one of the first published documents that made use of her statistical data and charts (Stinnett, 1990). Karl Pearson, the developer of the Pearson product moment correlation coefficient and one of the grandfathers of modern statistics, said that Florence was a "prophetess" in the area of applied statistics (Stinnett, 1990).

In the later years of her career, Florence tried unsuccessfully to establish an Oxford professorship in applied statistics, which would have been the first position of its kind. Unfortunately, like many of her ideas, this was not fully embraced in her lifetime. Her methods are now standard practice in many fields, especially in health services auditing, quality control, epidemiology, (Spiegelhalter, 1999) and the national census of countries such as the United States (Kopf, 1916).

NCTM Standards

The activities presented address many points in the NCTM *Principles and Standards for School Mathematics*, especially with regard to the middle school curriculum. Students in sixth through eighth grades are expected to acquire a thorough grounding in algebra and geometry. Pie, bar and polar area charts all rely on geometric proportionality to display data, and comparisons of pie and polar area charts in particular can generate interesting discussions about area. For example, why do we use the square root of counts in the polar area chart rather than the counts themselves?

The NCTM middle grades standards also emphasize the ability to manipulate real-life quantitative data, including looking at how data is collected and interpreted. In grades three through five, students learn the basics of data collection and how to make simple representations such as bar charts. In the middle grades, there is increased emphasis on comparison, integration and explanation. The students must know how to manipulate not only whole numbers but fractions, percentages, exponential relationships and ratios to represent and explain quantitative relationships, all of which are necessary for the construction of pie, bar and polar area charts. As in the worksheet presented below, students need to be able to create the graphs and also explain what those displays mean, answer questions based on the data, and form conjectures about causes and relationships.

Activity Sheet: Florence Nightingale's Polar Area Diagrams

The following activity sheet gives students a chance to practice making pie and bar charts and construct Florence's polar area chart firsthand. The data provided looks at male and female athlete participation in the last eight Summer Olympics. The numbers are of a similar magnitude to those found in Florence's Scutari data. There are interesting trends in the data that require the students to draw conclusions using the related history. Participation by men dips markedly in 1980, the year that the United States boycotted the Summer Games in Moscow. Participation by women, on the other hand, shows steady increase, probably thanks to the enactment of Title IX in 1972 in the U.S. and the women's movements in many countries during the same era.

Historical information about the Summer Olympics, including for example the number of events, pictures of the posters and medals, and competition highlights can be found at the website of the Olympic Movement (International Olympic Committee, 2004). The questions given at the end of the worksheet can be done individually or used for class discussion. These questions are designed to encourage the critical thinking and interpretation skills needed to explain the data, which is as important as the presentation.

Square Root Activity

Have your students search the internet for information about methods of finding square roots by hand, which were taught in American schools until about World War II. Ask them to use these methods to try to find the square root of a small number or a perfect squares such as four, then for a larger number. Discuss with students how the invention of hand calculators and personal computers has changed the teaching, learning and practice of mathematics.

Florence Teaches Math Activity

When teaching mathematics, Florence posed the following types of questions to her students: How tall is the reindeer? Are you as tall? How tall are you? How far is the highest point of Europe from the equator? How far do you come to school? If you walked the same number of miles each day that you come to school, how long would it take you to walk to the equator? (Lipsey, 1993). Have your students solve some of these problems, either individually or in small groups. Then ask them to create other problems of their own that are similar.

Women Statisticians Activity

Have your students research the lives of other prominent female statisticians such as Gertrude Cox (who helped create the statistics department at North Carolina State University) and the very prolific Florence Nightingale David (whose parents were close friends of Florence Nightingale). Discuss their contributions to the field of statistics.

Census Activity

Suppose your students were asked to conduct a census of the students or classes at your school, similar to the U.S. decennial census. Have students research how the U.S. Census Bureau performs the census and create a standard data collection sheet for your school census that anyone can use. Discuss as a class what their census should look like and what kind of questions should be asked so that they would be able to analyze the data to make summaries and comparisons? Information and a number of free lesson plans that include large, full-color maps are available from the Census Bureau (United States Census Bureau, 2000).

Invent Your Own Graph Activity

Ask students to try to invent a new type of graph or picture-based method of displaying data. Compare these to the graphs they already know and discuss the similarities and differences.

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Activity Sheet: Florence Nightingale's Polar Area Diagrams

Florence Nightingale is known by legend as the "Lady with the Lamp", an innovator in the field of nursing. Born at a time when most colleges accepted only men, she insisted on learning mathematics from the time she was twelve years old, and then put those skills to good use analyzing data she collected herself in military hospitals. She wanted to clearly show that disease, not wounds, caused most of the British Army's deaths during the Crimean War (1854-1856). Most other statisticians at the time thought statistics should be very plain (and maybe even boring), but Florence decided to use interesting graphs to make her points stand out and to show people what was going on so that she could convince them to make changes in the way things were run.

Florence Nightingale was the first woman elected as a Fellow of the Royal Statistical Society, and she was also made an honorary member of the American Statistical Association. Her methods are widely used today, especially in health care and the census.



Portrait of Florence Nightingale, circa 1858, from the Library of Congress Prints and Photographs Division [#LC-USZ62-5877].

One of Florence's inventions was called a polar area chart, which is very similar to a pie chart. In a polar area chart, the circle is divided into angles or "wedges" of the same size for each category. The radius of each wedge is then equal to the square root of the count (or frequency) for that category. The square root is used for the

radius because the area of a circle is $\pi r2$, and using the square root of the frequency for radius means that the wedges in the polar area chart are still proportional to one another, like in the pie chart. See Figure 1, p. 32.

Each wedge represents the number of deaths due to disease for the British Army in Crimea. There are no fatalities at the start of the war in April 1854, but as the months pass the numbers increase and the radii of the wedges get bigger until January 1855. She explained that this was due to the fact that, as fighting went on, the hospitals became more crowded and dirty, so there was more disease. The winter was also very cold – notice that January is the largest piece of the graph.

The data given below show the number of men and women athletes from all countries who have competed in each Summer Olympics since 1972. How many athletes participate? How has their participation changed over time? Are there more participants or fewer? Are the numbers of men and women similar? What are the causes of these trends or changes? These questions are similar to the kind Florence was trying to answer with her data.

1) On a separate sheet of paper, make a pie chart for each year to show the percentage of women and men athletes.

Year	Host City	Host Country	Women		Men	
1972	Munich	West Germany	1058	15%	6065	85%
1976	Montreal	Canada	1247	21%	4781	79%
1980	Moscow	USSR	1124	22%	4093	78%
1984	Los Angeles	USA	1567	23%	5230	77%
1988	Seoul	South Korea	2186	26%	6279	74%
1992	Barcelona	Spain	2708	29%	6659	71%
1996	Atlanta	USA	3523	34%	6797	66%
2000	Sydney	Australia	4069	38%	6582	62%

2) On a separate sheet of graph paper or using a computer program, make a comparative bar chart showing the number of women and men athletes for each year.

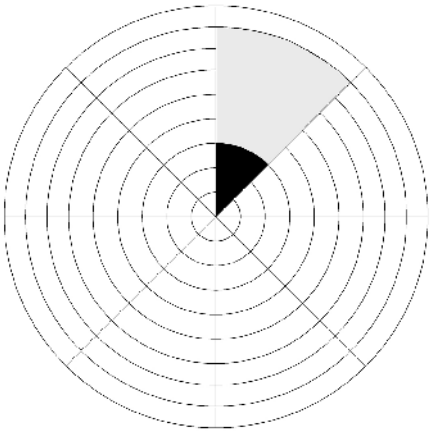
3) Create a polar area chart for the data using the table and the grid on the following page.

- Find the square roots for all the counts in the table.

		1972	1976	1980	1984	1988	1992	1996	2000
Women	Count	1058	1247	1124	1567	2186	2708	3523	4069
	Square Root	33							
Men	Count	6065	4781	4093	5230	6279	6659	6797	6582
	Square Root	78							

- For each year, draw the radius for the women's count and shade in the wedge.
- For each year, find the radius for the men's count (which is larger). Shade in the extra area covered by the men's wedge with a different color. When you are done it should look like the women's wedge is on top of the men's wedge for each year. The year 1972 has already been done for you. Label each section with the appropriate year.

There are eight equal sections on the grid, one for each year. The circles on the grid represent the tens digit (e.g., 10, 20, 30). For convenience, round all your square roots to the nearest tens place when drawing the graph.



4) Answer the questions below using the tables and your graphs.

a) Describe participation by men athletes over time. Are there more men participating now than in the past, fewer men participating now, or are the numbers staying the about same? Explain your answer.

b) What year had the lowest participation by men? Can you find a historical reason why this is true?

c) Describe participation by women athletes over time. Are there more women participating now than in the past, fewer women participating now, or are the numbers staying the about same? Explain your answer.

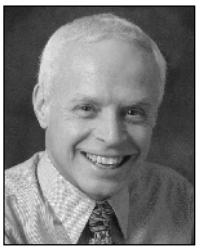
d) In 1972, the United States Congress passed a law called Title IX. One of things it did was to create more opportunities for women athletes in high school and college. Can you use this to help explain the reason for your answer to part (c)?

e) What can you say about the total number of participating athletes for each year?

To quote Florence Nightingale, she believed that statistics is "...the most important science in the whole world; for upon it depends the practical application of every other science and of every art; the one science essential to all political and social administration, all education, all organization based on experience, for it only gives results of our experience." ▲

▲ Finite Differences

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Secondary students at most grade levels are well aware that given the equation of a relationship they can generate an appropriate table of values (set of data points) and graph the relationship. Students may not be aware that hidden within a set of data points is all the information necessary to determine the mathematical relationship between x and y for certain types of relationships. Finite differences reveal the nature of the relationship (linear, quadratic or higher degree) depending on the number of differences required to obtain a constant difference. Finite differences can also reveal to our students that much more information about a relationship is contained within a table of values other than the obvious points to be plotted on a graph. Given an appropriate set of data points, finite differences can be used to generate the actual terms of the relationship. I use finite differences in both the grade 9 Applied Mathematics and the grade 12 Advanced Functions and Introductory Calculus courses that I teach. I suggest that finite differences could be used at all grade levels. This article will present sample problems appropriate for each grade level of the secondary curriculum.

Grade 9 Applications

- Finite differences will tell us if a set of data points represent a **LINEAR RELATION** or not.
- If the finite differences on the left, Δ x, are constant then the **run** is constant.
- If the finite differences on the right, Δ y, are constant then the **rise** is constant.
- If **both** the **rise** and **run** are constant

$$\left(\text{slope } m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}\right)$$

the **slope is constant** and the points represent a **Linear Relation**.

Example 1.

Given the set of data points $\{(0,4),(1,6),(2,8),(3,10),(4,12)\}$

Determine the finite differences and state whether the points represent a linear relation or a non-linear relation. Determine the slope.

Δ x RUN	x	y	Δ y RISE
	0	4	
1 – 0 = 1			6 – 4 = 2
	1	6	
2 – 1 = 1			8 – 6 = 2
	2	8	
3 – 2 = 1			10 – 8 = 2
	3	10	
4 – 3 = 1			12 – 10 = 2
	4	12	
the run is a constant 1			the rise is a constant 2

The points DO represent a LINEAR RELATION.

- The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$.
- The values of x do not have to be consecutive.
- The y-intercept b is found at the data point where x = 0.
- The equation of the line can be determined if both the slope and y-intercept are known.

Example 2.

Given the set of data points $\{(-3,6),(0,4),(3,2),(6,0),(9,-2)\}$

Determine the finite differences and state whether the points represent a linear relation or a non-linear relation.

Determine the slope, the y-intercept and the equation of the line.

Describe the slope and the slant of the line.

Δ x RUN	x	y	Δ y RISE
	– 3	6	
0 – (–3) = 3			4 – 6 = –2
	0	4	
3 – 0 = 3			2 – 4 = –2
	3	2	
6 – 3 = 3			0 – 2 = –2
	6	0	
9 – 6 = 3			–2 – 0 = –2
	9	–2	
the run is a constant 3			the rise is a constant –2

The points DO represent a LINEAR RELATION.

- The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{3}$.
- The y-intercept is 4.
- Since the equation of the line is y = mx + b, then the equation of the line is $y = \frac{-2}{3}x + 4$ for this set of data.
- The slope is negative and the line slants downward to the right.

Grade 10 Applications

If the y-intercept cannot be found in the table of values then we may use any data point and the equation of the line to find the y-intercept.

Example 1.

Determine the finite differences and state whether the points represent a linear relation or a non-linear relation. Determine the slope, the y-intercept and the equation of the line. You will use the equation of the line to determine the y-intercept.

Δx RUN	x	y	Δy RISE
	– 2	– 1	
2 – (–2) = 4			2 – (–1) = 3
	2	2	
6 – 2 = 4			5 – 2 = 3
	6	5	
10 – 6 = 4			8 – 5 = 3
	10	8	
14 – 10 = 4			11 – 8 = 3
	14	11	
the run is a constant 4			the rise is a constant 3

The points DO represent a LINEAR RELATION.

- The slope is $m = \frac{\text{rise}}{\text{run}} = \frac{3}{4}$.
- Since the equation of the line is y = mx + b and one point is (6, 5) then the equation of the line is:

$$5 = \frac{3}{4}(6) + b, \text{ (b is unknown.)}$$

$$5 = \frac{9}{2} + b$$

$$b = \frac{1}{2}$$

The equation of the line is $y = \frac{3}{4}x + \frac{1}{2}$.

Successive finite differences can be used to determine each term in a relationship.

- Let A be the constant finite difference Δx_1.
- Let n be the number of finite differences necessary to obtain a constant value Δy_n.
- Let B be the constant finite difference Δy_n.
- The term is $\frac{B}{n!}\left(\frac{x}{A}\right)^n$, recall 5! = 5 x 4 x 3 x 2 x 1.

Example 2.

Given the set of data points $\{(-3,-3),(-1,9),(1,21),(3,33),(5,45)\}$ determine the terms of the relationship using finite differences.

Δx_1 $x_2 - x_1$	x	y	Δy_1 $y_2 - y_1$
	–3	–3	
2			12
	–1	9	
2			12
	1	21	
2			12
	3	33	
2			12
	5	45	
A = 2			B = 12 n = 1

The slope is m = 6. The general term is $\frac{B}{n!}\left(\frac{x}{A}\right)^n$.

The first term is $\frac{12}{1!}\left(\frac{x}{2}\right)^1 = 6x$.

Since this term is degree 1 then only a constant term k remains and the relationship is y = 6x + k.

We may use any point from the set of data to determine k: using the point (–1, 9) 9 = 6 (–1) + k, and k = 15.

The relationship is y = 6x + 15.

Grade 11 Applications: The Quadratic Function

Example 1.

Given the set of data points $\{(-4,29),(-1,8),(2,5),(5,20),(8,53)\}$ determine the terms of the relationship using successive finite differences.

Δx_1	x	y	Δy_1	Δy_2
	–4	29		
3			–21	
	–1	8		18
3			–3	
	2	5		18
3			15	
	5	20		18
3			33	
	8	53		
A = 2				B = 18 n = 2

The general term is $\frac{B}{n!}\left(\frac{x}{A}\right)^n$.

The first term is $\frac{18}{2!}\left(\frac{x}{3}\right)^2 = x^2$.

Now determine new y values by subtracting the first term x^2 from the original y values f(x). (next page)

We then complete a finite difference table in order to determine the second term.

Δx_1	x	$f(x) - x^2 = y$	Δy_1
3	–4	$29 - (-4)^2 = 13$	–6
3	–1	$8 - (-1)^2 = 7$	–6
3	2	$5 - (2)^2 = 1$	–6
3	5	$20 - (5)^2 = -5$	–6
	8	$53 - (8)^2 = -11$	

A = 3 B = – 6
n = 1

The general term is $\frac{B}{n!}\left(\frac{x}{A}\right)^n$.

The second term is $\frac{-6}{1!}\left(\frac{x}{3}\right)^1 = -2x$.

Since this term is degree 1 then only a constant term k sremains and the relationship is $y = x^2 - 2x + k$. We may use any point from the set of data to determine k: using the point (–1, 8) $8 = (-1)^2 - 1(-1) + k$, and k = 5.

The relationship is $y = x^2 - 2x + 5$.

Grade 12 Applications: Polynomial Functions of any Degree

- Successive finite differences can be used to determine each term in any relationship if y is a function of x where each term is a positive integer power of x.

Example 1.
Given the set of data points
$\{(2,-11),(4,-37),(6,-47),(8,-17),(10,77),(12,259)\}$
determine the terms of the relationship using successive finite difference tables.

Δx_1	x	y	Δy_1	Δy_2	Δy_3
2	2	-11	-26	16	24
2	4	-37	-10	40	24
2	6	-47	30	64	24
2	8	-17	94	88	
2	10	77	182		
	12	259			

A = 2 B = 24
n = 3

The general term is $\frac{B}{n!}\left(\frac{x}{A}\right)^n$.

The first term is $\frac{24}{3!}\left(\frac{x}{2}\right)^3 = \frac{1}{2}x^3$.

Now determine new y values by subtracting the first term $\frac{1}{2}x^3$ from the original y values f(x).

We then complete a finite difference table in order to determine the second term.

Δx_1	x	$f(x) - \frac{1}{2}x^3 = y$	Δy_1	Δy_2
2	2	$-11 - \frac{1}{2}(2)^3 = -15$	–54	–32
2	4	$-37 - \frac{1}{2}(4)^3 = -69$	–86	–32
2	6	$-47 - \frac{1}{2}(6)^3 = -155$	–118	–32
2	8	$-17 - \frac{1}{2}(8)^3 = -273$	–150	–32
2	10	$77 - \frac{1}{2}(10)^3 = -423$	–182	
	12	$259 - \frac{1}{2}(12)^3 = -605$		

A = 2 B = –32
n = 2

The general term is $\frac{B}{n!}\left(\frac{x}{A}\right)^n$.

The second term is $\frac{-32}{2!}\left(\frac{x}{2}\right)^2 = -4x^2$.

Determine new y values by subtracting the second term $-4x^2$ from the previous y values f(x). We then complete a finite difference table in order to determine the third term.

Δx_1	x	f(x) + 4 x 2 = y	Δy_1
2	2	–15 + 4 (2) 2 = 1	–6
2	4	–69 + 4 (4) 2 = –5	–6
2	6	–155 + 4 (6) 2 = –11	–6
2	8	–273 + 4 (8) 2 = –17	–6
2	10	–423 + 4 (10) 2 = –23	–6
	12	–605 + 4 (12) 2 = –29	

A = 2 B = –6
n = 1

The general term is $\frac{B}{n!}\left(\frac{x}{A}\right)^n$.

The third term is $\frac{-6}{1!}\left(\frac{x}{2}\right)^1 = -3x$.

Since this term is degree 1 then only a constant term k remains and the relationship is $y = \frac{1}{2}x^3 - 4x^2 - 3x + k$.

We may use any point from the set of data to determine k: using the point (2, -11) $-11 = \frac{1}{2}(2)^3 - 4(2)^2 - 3(2) + k$ and k = 7.

The relationship is $y = \frac{1}{2}x^3 - 4x^2 - 3x + 7$.

Grade 12 Applications: Advanced Finite Differences

Successive finite differences can be used to determine each term in any relationship if in each term of the relationship x and y are related in the following way:

- $y^m \propto x^n$ where m and n are positive integers.
- Let m be the number of finite differences necessary to obtain a constant value Δx_m
- Let A be the constant finite difference Δx_m
- Let n be the number of finite differences necessary to obtain a constant value Δy_n
- Let B be the constant finite difference Δy_n
- The relationship is $\left(m!\frac{x}{A}\right)^n = \left(n!\frac{y}{B}\right)^m$
- In the examples on the previous pages m = 1 and the relationship reduces to

$\left(1!\frac{x}{A}\right)^n = \left(n!\frac{y}{B}\right)^1 \therefore y = \frac{B}{n!}\left(\frac{x}{A}\right)^n$ as seen before.

Example 1.

Given the set of data points

$\{(1,-1),(4,8),(9,45),(16,128),(25,275),(36,504)\}$

determine the terms of the relationship using successive finite differences

Δx_2	Δx_1	x	y	Δy_1	Δy_2	Δy_3
	3	1	–1	9		
2					28	
	5	4	8	37		18
2					46	
	7	9	45	83		18
2					64	
	9	16	128	147		18
2					82	
	11	25	275	229		
		36	504			

A = 2, m = 2 B = 18, n = 3

Since $\left(m!\frac{x}{A}\right)^n = \left(n!\frac{y}{B}\right)^m$

$\left(2!\frac{x}{2}\right)^3 = \left(3!\frac{y}{18}\right)^2$

$(x)^3 = \left(\frac{y}{3}\right)^2$

$(x)^{\frac{3}{2}} = \left(\frac{y}{3}\right)$

The first term is $y = 3x^{\frac{3}{2}}$

Now determine new y values by subtracting the first term $3x^{\frac{3}{2}}$ from the original y values f(x). We then complete a finite difference table in order to determine the second term.

Δx_2	Δx_1	x	$f(x) - 3x^{\frac{3}{2}} = y$	Δy_1	Δy_2
	3	1	$-1-3(1)^{\frac{3}{2}} = -4$	–12	
2					–8
	5	4	$8-3(4)^{\frac{3}{2}} = -16$	–20	
2					–8
	7	9	$45-3(9)^{\frac{3}{2}} = -36$	–28	
2					–8
	9	16	$128-3(16)^{\frac{3}{2}} = -64$	–36	
2					–8
	11	25	$275-3(25)^{\frac{3}{2}} = -100$	–44	
2					–8
		36	$504-3(36)^{\frac{3}{2}} = -144$		

A = 2, m = 2 B = –8, n = 2

Since $\left(m!\frac{x}{A}\right)^n = \left(n!\frac{y}{B}\right)^m$

$\left(2!\frac{x}{2}\right)^2 = \left(2!\frac{y}{-8}\right)^2$

$(x)^2 = \left(\frac{y}{-4}\right)^2$

The second term is y = –4x.

Since this term is degree 1 then only a constant term k remains and the relationship is $y = 3x^{\frac{3}{2}} - 4x + k$. We may use any point from the set of data to determine k: using the point (4,8) $8 = 3(4)^{\frac{3}{2}} - 4(4) + k$ and k = 0.

The relationship is $y = 3x^{\frac{3}{2}} - 4x$.

Finite differences enable the students to determine the nature and degree of the relationship between x and y and therefore to decide whether the relationship is linear, quadratic or of higher degree. Finite differences also give the students an opportunity to experience some elementary data analysis because they can actually determine the terms of the relationship. Some of the mystery of regression analysis found on the modern scientific calculators is revealed as simple arithmetic steps within the reach of most high school students. ▲

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