Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceeding 10 numbered pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including websites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

PERMISSION TO REPRINT: Permission to reprint any part of this publication for instructional use or for inclusion in an affiliate publication must be obtained through the Editor Marilyn Hurrell. Full credit must be given to the author and to the *Ontario Mathematics Gazette.*

▲ Table of Contents

Up Front

▲ Editor's Message / Marilyn Hurrell 2
▲ President's Message / Kathy Kubota-Zarivnij 3

Regular Features

▲ The *Abacus* . . . . . .8 page insert
▲ Technology Corner: Grade 9 Graphing Projects / Fred Ferneyhough 6
▲ Hey, It's Elementary...Mathematical Learning Disabilities? They're Real, They're Little Understood, but There Is Help / Lynda Colgan 8
▲ Friday Specials! / Jack Weiner 14
▲ OAME/NCTM Report / John Kersley 16
▲ Get Interactive / Kevin Maguire 17
▲ Assessment Corner: Making Sense of Assessment / Chris Suurtamm 18
▲ Photo Math – A New Way of Picturing Math / Ron Lancaster 20
▲ Director's Dialogue / Jacqueline Hill 38

Special Features

▲ Book Review: *Young Mathematicians at Work: Constructing Multiplication and Division* / Reviewed by Lyn Vause 11
▲ OAME Awards, 2005 / Judy Dussiaume 35
▲ *Growing Up Mathematically*: A New OAME Resource . . .37

Articles

▲ Up, Down, Sideways: Difficulties of a Daughter and Dad in Learning Mathematics / Daniel H. Jarvis 22
▲ Supports for Effective Implementation of the Revised Grade 9 / 10 Curriculum: The Role of Manipulatives in Making Algebra Meaningful / Ann Kajander 26
▲ An Equation Game / Michelle Triggs 29
▲ Learning Algebra with Computer Algebra Systems in Our Revised Ontario Math Curriculum / Tom Steinke, Kevin Spry, Fred Ferneyhough 31

Notices

▲ OAME Playing Cards 13
▲ OAME Publications for Mathematics Educators Linking Assessment & Instruction in Mathematics: Primary Years 20
▲ OAME T-shirts 29
▲ Upcoming Events 35
▲ OAME Memo Pads 38
▲ 2005-2006 OAME Board 40

Advertisements

▲ Pearson Education . . . . . .Inside Front Cover
▲ Texas Instrument 5
▲ OAME 2006 15
▲ Wintergreen 32
▲ GVMA 34
▲ Psycan . . . . . .Inside Back Cover
▲ Thomson Nelson Learning . . . . . Back Cover

The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

▲ About the *Ontario Mathematics Gazette*

The *Ontario Mathematics Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

Editor

▲ **Marilyn Hurrell**

Associate Editors

▲ **John Egsgard**

▲ **Todd Romiens**

▲ **Mary Lou Kestell**

***Abacus* Editor**

▲ **Trevor Brown**

Design, Tech Art, and Production

▲ **Penny Clemens**
Graphic Designer, Cambridge, ON

Printing & Binding

▲ **Pole Printing,** Box 69, 89 King Street E.
Forest, ON N0N 1J0 (519) 786-5112

Advertising Manager

▲ **Dean Murray**

Submission of Advertisements

Advertisements for the *Ontario Mathematics Gazette* should be sent to **Dean A. Murray** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising Rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARILYN HURRELL, EDITOR



Welcome back to another year of doing what you all do best, and that is to educate children.

When I receive a manuscript which an author has entrusted to me for possible publication in an issue of the *Gazette*, I immediately send it out to a minimum of three reviewers for a blind review (no identification of author or reviewer). There is a checklist which the reviewer uses to rate the article as ready to publish, publish with revisions or not publish. There is also a comment sheet and reviewers are invited to make additional comments on the manuscript. These reviews are then photocopied by me and sent back to the author. The process can take anywhere from a month to several months. Most reviewers receive 1 - 3 manuscripts per year to review and all are very reliable and conscientious in their duties. Reviewers are classified by grade level and particular areas of interest.

I am very pleased to thank the reviewers who have helped over the past year, and will continue to help make your *Gazette* an interesting, diversified and useful journal:

Kirsten Boucher, Alan Chan, Dr. Ed Barbeau, Dr. Lynda Colgon, Barbara Canton, Prof. Dave Cosh, Stewart Craven, Dr. Ed Antoz, Dr. Ralph Connelly, Peter Crippen, Ruth Dawson, Chris Dearling, John Egsgard, Pat Grew, Tom Griffiths, Dave Handley, Elaine Harvey, Joanne Harris, Jeff Irvine, Anna Jupp, Dr. Ann Kajander, John Kerseley, Peter Komlos, Murray Major, Pat Margerm, Fran McLaren, Jamie Pyper, Louis Lim, Todd Romiens, Dr. Geoff Roulet, Walker Schofield, Jay Speijer, Kevin Spry, Dr. Christine Suurtamm, Lynn Strangway, David Shepherd, Shirley Scott, Dr. Margaret Sinclair, Robert Sherk, Mary and Lawrie Sakamoto, Tara Tasker, Prof. Jack Weiner, Prof Kathy Kubota-Zarivnij, Anne and Rod Yeager, Mary Bourassa, Fred Ferneyhough, Jacqueline Hill, Laurie Moher, Lynda Ferneyhough and Kevin Maguire.

If you are interested in becoming a *Gazette* reviewer, please contact the editor.

June's brain teasers read: "by adding one line, make each of these true statements" 5 + 5 + 5 = 500 and 10 10 10 = 950. The first one was flawed and should have read 5 + 5 + 5 = 550. The solutions are: 545 + 5 = 550 and 10 T0 10 = 950 (o'clock)

Thanks to all who tried the impossible. My face is red.

Marilyn

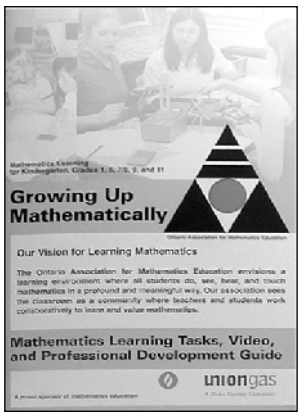
▲ PRESIDENT'S MESSAGE

KATHY KUBOTA-ZARIVNIJ



Kathy Kubota-Zarivnij is currently program coordinator of mathematics for TCDSB, senior elementary mathematics textbook author, mathematics AQ course instructor, and doctoral candidate. She has been an OAME director for 5 years and served on the OAME Executive for 3 years. She has a variety of mathematics education experiences, such as: elementary school administrator; in-service and pre-service mathematics teacher, and mathematics curriculum and program consultant, writer, and/or workshop presenter for different organizations, such as CCC, EQAO, MoE, NCTM, OAME, OCC, and TLP.

Introducing ... *Growing Up Mathematically*, OAME's Vision for Learning Mathematics!!



OAME envisions learning mathematics as a continuum that occurs across all grades levels. If students are to experience learning as connected, teachers can only make this happen if they have an awareness of the mathematics learning that happens before and after their grade level. Elementary teachers often make assumptions about what they should be "preparing" students for in high school and secondary teachers just as often make assumptions about what kinds of experiences students have had in elementary schools.

Therefore, *Growing Up Mathematically* brings to life the continuum of learning mathematics. Through glimpses into real classrooms in the video teachers can identify and discuss how the use of technology and manipulatives and the development of mathematical communication, reasoning, and problem solving become more sophisticated as students grow up through the years. The classroom-ready mathematics tasks incorporate the development of mathematical skills, knowledge, and processes.

OAME's vision project, *Growing Up Mathematically* was conceived as an expression of OAME members' shared vision of mathematics education, diversity of mathematics knowledge and experience, and creative possibility. It is based upon the simple idea of a student growing up through the years and what her/his experience in mathematics should be over those years. OAME board and executive members, representing all grade divisions: primary, junior, intermediate, senior, contributed to the creation and field testing of grade-specific, classroom tasks, recording of video images of mathematics learning from K to 11, and the creation of a professional development guide. The classroom experiences and sample student work are intended to be used by teachers and their students as living examples of what mathematics looks like, sounds like, and feels like according to the vision articulated by OAME. These resources can be used as stand-alone pieces or more powerfully as a set of connected components that depend and build upon each other.

The Vision Statement

The OAME vision for learning mathematics was written, reviewed, and revised by members of the Board of Directors and is based upon principles and beliefs that are aligned with those presented in the Ministry of Education curriculum documents. OAME believes that the learning of mathematics is based upon these beliefs and principles.

- Learning is structured around the key concepts of mathematics.
- Learners build on prior knowledge and understanding, and work at a developmentally appropriate level to construct new personal meaning.
- Learners are better able to transfer and to use knowledge meaningfully when new ideas are presented within rich and meaningful learning contexts, and when learners make connections both within and outside of school.
- The learning process is enhanced when new ideas are presented in a variety of ways and when all learners have access to "hands-on" and "minds-on" activities and opportunities. This includes the use of manipulatives, computers, calculators, and hand-held data-collection devices.
- Learning is enhanced when students experience a variety of instructional approaches, ranging from direct instruction to inquiry-based learning.

- Learners require opportunities to interact with each other during the learning process, to teach others, and to explain and justify the processes of mathematics. Learning is enhanced when the learners are encouraged to communicate their understanding.
- Effective mathematics education requires strategies that engage and motivate learners, that help students develop positive attitudes about themselves and about mathematics, and that encourage learners to make positive contributions to the learning environment.
- Learning is enhanced when students and teachers understand learning expectations and assessment standards and participate actively in assessing and evaluating their personal growth.
- Learning is enhanced when teaching, coaching, assisting, supporting, and feedback is planned and designed to improve specific learning and mental processing skills and to accommodate different learning styles and needs.
- Learning is enhanced when teachers plan and reflect together.
- Learning is enhanced when all participants in the process are active partners within a community of learners (e.g., parents, educators, students, and the community)."

These beliefs and principles are embodied in the *Growing Up Mathematically* video, mathematics tasks, and professional development guide.

The Video

The video focuses on the process components of mathematics and demonstrates the growing sophistication with which learners use materials, technology, vocabulary, and problem solving strategies. All segments in the video were filmed in five Ontario classrooms spanning the grades from Kindergarten to grade 11.

In these classrooms, students engage in mathematical learning experiences through the use of concrete materials and manipulatives, visuals, technology, and/or math picture books. These students build on their prior knowledge of key mathematical concepts and connect these concepts to their world. They understand the purpose of their learning through clearly defined expectations, goals, and assessment criteria. In the video, these students engage in inquiry, pose questions, and actively discuss their understandings with one another. The video is organized by the following topics: Setting the Stage, Organization Classrooms, Using Manipulatives, Using Technology, Teacher Questioning, Student Communication, Whole Class Debrief, Rich Assessment.

Mathematics Learning Tasks

Classroom ready learning and/or assessment tasks were developed and field tested. Specific teaching/learning and assessment strategies are included. As well, student work samples are provided for each set of tasks, so that teachers and students can see the possibilities of student mathematical thinking and doing. The mathematics tasks included are: Kindergarten - *Eleven is Eleven;* Grade 1 - *Ten Black Dots;* Grade 5 - *Who Won?;* Grade 7/8 - *Station-to-Station, Exploring Angle Sums in Polygons;* Grade 9 (Academic) - *Popcorn, Bungee Jump:* Grade 9 (Applied) - *Popcorn*; Grade 11U/C - *The Hula Hoop Experiment: Going Around in Circles, Pendulum;* Grade 11C - *Decisions, Decisions, Decisions.*

Professional Development

It is hoped that the vision statement, the video, the classroom learning tasks and the assessment ideas will promote many rich conversations between and amongst teachers. The professional development component of this resource is intended to inspire and focus those conversations. There are five professional development modules: Engaging Students in Learning Mathematics, Developing Mathematical Understanding, Developing Mathematical Communication, Consolidating and Sharing and Conversations About Student Work. Each module should take about 50 minutes and can be used in relation to the grade specific mathematics tasks.

Check Out *Growing Up Mathematically*!

So, think about the possibilities of using OAME's newly released mathematics education resource. The vision statement, video, mathematics learning tasks, and professional development materials could be used to promote conversations about teaching and learning mathematics between and amongst groups of mathematics educators and/or parents. As well, these materials can be used to celebrate the mathematics thinking and doing that Ontario students and teachers engage in!!! Enjoy!

KKZ ▲

INSTANT









The TI-Navigator system wirelessly links the teacher's PC with up to 40 graphing calculators from the TI-83 Plus and TI-84 Plus families.

Instant Assessment

The TI-Navigator™ classroom learning system places instant assessment tools in teachers' hands, while engaging math and science students in a collaborative learning environment. It is designed to support instructional practices shown by research to improve student engagement and learning.*

The TI-Navigator system's wireless, two-way communication provides a whole new level of instant interactive learning that allows teachers to:

- *Instantly* view every calculator screen in class with a simple keystroke
- *Instantly* assess students' understanding through quick polling
- *Instantly* distribute, gather, and grade student assignments

Give your classrooms the *instant* advantage.
Visit TI's booth or education.ti.com/Nav

tools | training | curricular materials education.ti.com

E-MAIL: ti-cares@ti.com CALL: 1.800.TI.CARES

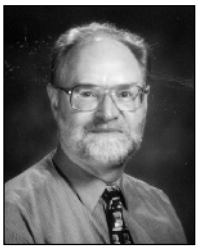
©2005 Texas Instruments AD5935

* Based on related research on classroom communications systems and specific pedagogy. For more information visit education.ti.com/research.

Texas Instruments

▲ Technology Corner: Grade 9 Graphing Projects

FRED FERNEYHOUGH



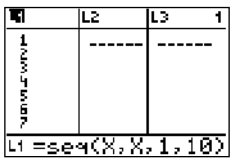
Fred Ferneyhough is former head of mathematics at Harold M. Brathwaite SS in Brampton. He remains a teacher there as well as an avid user of and advocate for the use of technology in the mathematics classroom. He is also an author and tech consultant for several books and a National Instructor for Texas Instruments.

As I do workshops, I'm always surprised that features that we use commonly in our school are not well known to others. In this column, we look at some mathematical relationships and operations that can be studied through the use of lists on the TI programmable calculators.

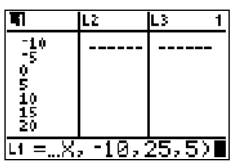
One problem that arises for many users is encountering lists stored by previous operations or users. Rather than deleting each number or clearing each list separately, you can clear them all at one time. To get to this menu press [2nd] followed by the [+] key. Scroll down to option 4 and press [ENTER]. This will paste the command into the home screen. Press [ENTER] again to execute the command. All of the lists are now empty.



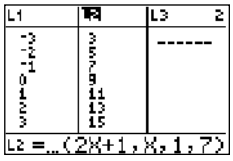
There is a shortcut for putting a sequence in a list. I never show students how to do this – I just do it and wait for them to ask me how to do it. In the list editor, move the cursor up until the list name L1 is highlighted as shown. To get the sequence command, press [2nd] and [STAT]. The phrase "seq(" will appear on the entry line. Complete the operation by filling in parameters (X,X,1,10). I read this as "the set of numbers x, such that x begins at 1 and goes up to 10". Press [ENTER] and the list will fill with the first ten whole numbers. Try this with any sequence.



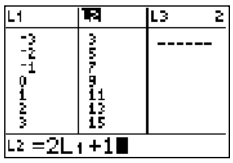
A variation of that command is shown in the next screen. In this screen, an extra parameter has been added at the end (X,-10,25,5). This increases the step value so that each number is increased by 5. You could read this as "the set of numbers x, such that x begins at –10 and goes up to 25 by 5". Without this extra parameter, the device assumes a default step value of 1.



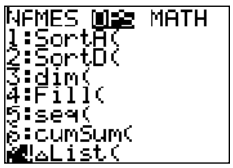
There are two ways to get a relationship between the first and second lists. In this screen, we have entered the numbers from –3 to +3 in list 1 and we want to set up the scatter plot with a linear relationship y = 2x + 1. The sequence command is used again, but the first parameter is the expression "2x + 1". The drawback is that you have to state how many terms to use for the fourth parameter, in this case, seven.



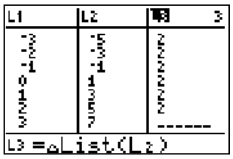
To set up a relationship with the first variable (independent) in L1 and the second variable (dependent) in L2, rather than use the sequence command, the operation shown allows you to treat the first list as you would a variable. The formula is applied for every value in List 1 and the result stored in list 2. The advantage here is that you do not have to state the number of terms in L1. The calculator will calculate and show values for all independent values.



When you are teaching grade nine and introduce the concept of first differences, the ΔList command is invaluable. If the y values are stored in List 2, take your cursor to List 3 and scroll up until the list name is highlighted. Press [2nd] and [STAT] to get at the LIST menus, and scroll over to OPS again. We want option 7.



You complete the command by entering the name of the list which contains the y-values. Press [ENTER] to complete the command and we would conclude that the relationship is linear since all of the first differences are equal. This saves a lot of work and aggravation. In grades 10 – 12, you can use lists 4 through 6 to get second, third and fourth differences depending upon the degree of the relationship being studied.

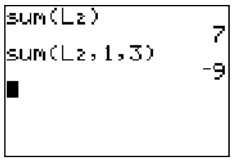


Return to the home screen by pressing [2nd] followed by [MODE], the shortcut for [QUIT]. In the LIST menu is a submenu for mathematical operations accessed by moving over to the title MATH. Each of the operations has some potential for teaching various topics in either statistics or sequences and series. Option 5 allows us to look at

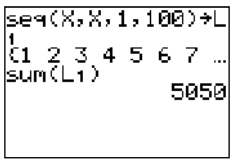


series in a different manner. Press [ENTER] to paste the command into the home screen.

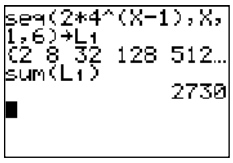
The commands in the next section refer to List 2 in the screen on page 6. The first command finds the sum of the terms in the list, which also represents the sum of an arithmetic sequence with a = -5, d = 2 and n = 7. The second sum finds the sum of terms 1 through 3. Any values can be added to this command to find sub-totals of sequences.



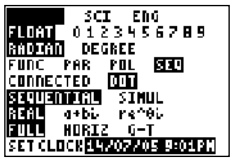
Just in case you don't have enough options, this screen provides a way to work on arithmetic sequences and series without ever going into the lists. The first command puts the first 100 numbers into list 1 and the second finds their total. The right arrow is accessed by using the [STO▸] key.



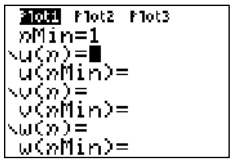
Using these commands with geometric sequences and series is just as easy. The first command stores a geometric sequence with a = 2, r = 4 and n = 6 in List 1. As before, the second command finds the sum of the sequence terms that form a series.



Have you ever wondered why the key [X,T,Θ,n] has four different variables on it? Each of them corresponds to a different mode of operation. The final choice is quite useful to us in this work as it switches the calculator into Sequential mode. To access this, press the [MODE] key and scroll down to the fourth line. Highlight SEQ in the fourth line and press [ENTER] to choose it. Change the next line to DOT at the same time.

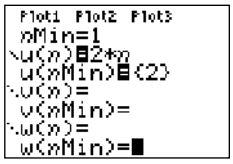


Initially, you don't notice anything different. However, when you press the [Y=] key, you get a completely different presentation. Also, in this mode, pressing the [X,T,Θ,n] key will now show the variable n on the screen rather than x. There are three different sequential variables shown as u, v and w. You can find these characters on the keyboard above the digits 7, 8 and 9 respectively. Pressing the [2nd] key followed by any of these digits allows you to refer to these functions from the home screen. Each sequence has information on two lines of the screen.

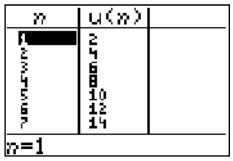


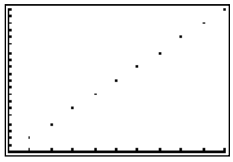
Using this mode of the calculator requires a slightly different way of thinking – always a good thing! The value of nMin is set by the calculator to 1 by the line nMin = 1. This value will be applied to each of the three sequences u, v, and w defined below that line. This means that the first term will be called term 1.

Look at the screen with \u(n)=2*n as the first line and u(nMin)=(2) as the second. The first line defines the relationship of the terms in the sequence while the second line gives the value of the first term. See the table of values and the graph of the sequence in the next two screen captures. The terms are numbered 1, 2, 3, and so on. The values of the terms are 2, 4, 6, and so on.



The table of values and the graph show the changes accordingly. The window looks completely different as well and requires some experimentation. The reason for changing the mode to DOT can now be seen. A connected set of points for a sequence does not make sense.



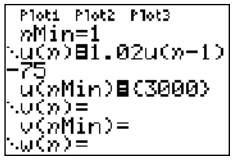


The manner in which this relationship has been defined represents an arithmetic sequence rather than a function. The window looks different too. When you changed the mode to DOT the graph changes to represent a sequence by a set of ordered pairs or points rather than a connected set of points or a line.

Consider this new screen with definitions for v and w. For the sequence v you see \v(n)=4*2^(n-1) and v(nMin)=(4). For the sequence w you see a recursive definition for the sequence with \w(n)=3w(n-1) and w(nMin)=(1). These are two different ways of defining sequences which will require further experimentation to gain understanding.



What follows is a problem that can be solved using the recursive sequence definition funtionality of the calculator. Scientists in the Arctic have approached you as a consultant for a problem they have detected. They have determined that there are 3000 Beluga Whales in the area and that their number will grow at the rate 2% per year. The local hunters have asked that they be allowed to take 75 animals per year for their purposes. The scientists are concerned that this approach would

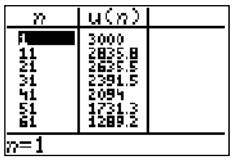


deplete the population over time. Examine a mathematical model of this problem and make recommendations.

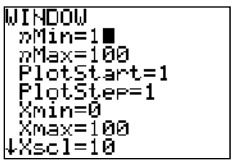
We set the calculator with the statement nMin = 1 to indicate that the sequence begins with the first term. For the second line, define each term of the sequence as a 2% increase over the previous population (1.02u(n-1) an increase of 2% over the previous term in the sequence u(n-1)) less the number the hunters want to take, 75 per year.

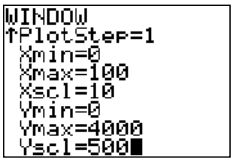
In the third line, set u(nMin) = 3000 as this is the initial population of the whales. The calculator adds in the brace brackets. For the second line, define each term of the sequence as a 2% increase in the previous population less the number which the hunters want to take.

The table of values was set to begin at n =1 with an increment of 10. This is shown in the WINDOW screen dump. Obviously, this population is decreasing. The presence of decimal values could be avoided if you have the opportunity to teach your classes about the "ipart" (integer part of a number) functionality.

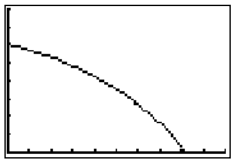


The calculator will display a graph and a table of values for the sequence. In order to see the graph, set the window values for sequence graphing as indicated in the next two diagrams. The values for nMin and nMax indicate the number of terms to be displayed. Similar values are used for Xmin and Xmax. The values for Ymin and Ymax are set to display the population values.





Press the GRAPH key to get the final screen. Determine how to advise the scientists and the hunters. Find and justify the number of whales that can be taken by the hunters that satisfies both parties. What types of conditions should be monitored in order to ensure that this model is effective?



The list and sequence operations allow our students to look at sequence and series in a completely different way and explore problems that are very unusual. If you have a TI-83 manual handy, check out Chapter 6 and explore the different options for graphing such as Time and Web graphing. ▲

▲ Hey It's Elementary: Mathematical Learning Disabilities? They're Real, They're Little Understood, but There Is Help.



LYNDA COLGAN

He who has not first laid his foundations may be able with great ability to lay them afterwards, but they will be laid with trouble to the architect and danger to the building.

Niccolo Machiavelli (1469 - 1527), *The Prince*

Innovations and curriculum foci in Ontario have validated what many elementary educators have known for years – the younger the child, the more daunting the task of teaching that student.

The launch of the Ministry of Education's Early Math Strategy (EMS) and its ancillary strand-by-strand resources serve to emphasize that the most important and long-lasting teaching of a child's life occurs during the formative early years. The EMS underscores the importance of teaching developmentally-appropriate content in stage (not age) -appropriate ways during the early primary division in order to enable the cognitive construction of many all-important, fundamental building-block conceptions. The focus on early math has been a powerful validation of the contributions made by primary teachers to every child's ultimate educational destination. The implicit message conveyed by each of these resources is that the seeds sown in the early years in the form of knowledge and affective attributes have implications for every grade thereafter and well beyond graduation. When children are not guided carefully along the mathematics learning pathway, their cognitive constructions become somewhat Tower of Pisa-like: structures cobbled together with bricks and mortar comprised of misconceptions. And, like the infamous tower, those poor mental constructions have the potential to withstand the test of time and the elements, defying logic with their enduring strength in spite what appears to be a perilous balance.

By finally giving primary teachers the resources,

knowledge and heart-felt respect they have so long deserved, we have better equipped them to provide exceptional mathematics programs for the children entrusted to their care. This work acknowledges that the primary grades demark a critical and sensitive period in children's mathematical development and that we have to lay solid groundwork for learning through these years.

That is why we, as a community committed to improving the mathematical life of children, must be careful not to switch gears abruptly by finding some new pet project on which to bestow our limited resources. We must not become complacent by thinking that "Early Math has been done in Ontario". While many of us have learned an enormous amount about early math because of recent initiatives, we must remember that a little knowledge is a dangerous thing.

What developmental psychologists and cognitive researchers know about early math is at a very early stage (compared to almost a half-century of work on language development, for example). It will take time and energy (and a lot of money) for us to learn about these recent advances and translate them into classroom-ready interventions and programs. But before any of that can happen, we need to spread the message that what researchers don't know about early math learning is staggering. We must refuse to accept that early mathematics development is less important than early language development and so we must become advocates, lobbying for early math's expanded attention – financially, pedagogically and professionally – in classrooms, boardrooms, and research settings.

Did you know that severe difficulties in learning about numbers and arithmetic are probably as widespread as disorders of literacy development? There is no single mathematics disability. In fact, mathematics disabilities are as varied and complex as those associated with reading. That means that between 6 and 7 per cent of school-age children may have a very real, but often undiagnosed learning disability that can best be defined as a deficit in the representation or processing of specifically numerical information. It also means that the comment "not trying hard enough" or "not motivated" may not be appropriate descriptions of what is going on in math class with "Joshua", the struggling student known to each of us. Joshua may have been impeded by a little known and less understood – but no less real – learning disability in mathematics.

One class of mathematics disability is dyscalculia. Brian Butterworth, a cognitive scientist, researcher and author of *The Mathematical Mind* believes that dyscalculia is akin to colour-blindness. It is innate and has nothing to do with the way we were taught, our anxieties or our work habits. Dyscalculia is based on the idea that humans are born with specialized neural circuits for recognizing and mentally manipulating numerosities (cardinal values). In fact, research shows that infants, even in the first week of life, are sensitive to the numerosity of a visual display. Since our mental capacity functions as a kind of starter kit for understanding numbers and arithmetic, when our in-born "number module" fails to develop normally in the brain due to injury or genetics, we become dyscalculic. Furthermore (although mathematics educators have argued for years that there is no "math gene") dyscalculia appears to be heritable. Studies of genetically abnormal populations have suggested a possible locus on the X chromosome, though this does not mean that all cases of dyscalculia are inherited.

Over the past decade, researchers have been trying to learn as much as possible about the genetic, neural, and cognitive deficits that underlie mathematics disabilities (MD) in order to be able to identify and then remediate this specific form of learning disability (LD). This has been done by taking the theories and techniques used to study normal mathematical development and using them to study children with low achievement scores in mathematics. These and other studies indicate that mathematics learning disabilities are not related to IQ, motivation or other factors that might influence learning.

David Geary, a leading developmental psychologist who specializes in mathematics learning, explains in his book *Children's Mathematical Development*, that there has been very little research on learning disabilities in mathematical domains outside of the areas of number, counting, and arithmetic. Thus, little can be said, at this time, about whether learning disabilities exist in the areas of geometry and algebra, for instance and, if they do exist, what their form or developmental course is.

What is known is that many MD children have difficulties getting basic facts into long-term memory and difficulties remembering, or accessing, the facts that are eventually stored in long-term memory. It appears that these difficulties are very similar to word finding difficulties that are common in some children with a specific type of reading disability.

Second, it appears that some children with MD can get facts into and out of long-term memory without too much difficulty but have trouble inhibiting other facts when they try to remember the answers to specific problems, such as 2 + 3. These children will not only remember 5, they might also have 4 (the number following 2 3 in the counting sequence) and 6 (the answer to 2 x 3) pop into

their heads at the same time. With too many facts being remembered, these children take longer to remember the correct answer because they may have to consider all of the answers that they remembered and then pick one of these, resulting in their making more errors.

The other consistent finding is that many children with MD use immature problem-solving procedures to solve simple arithmetic problems, i.e., procedures that are more commonly used by younger children without MD. For example, the least mature strategy for solving simple addition problems is called counting-all. To solve 5 + 3, most younger children will uplift five fingers on one hand, counting "one, two, three, four, five, "and then uplift three fingers on the other hand, counting "one, two, three." They will then recount all of the uplifted fingers starting from one. A quicker and more mature procedure is to simply state the largest number, five in this example, and then count-on a number of times equal to the value of the smaller number, as in "five, six, seven, eight".

As a group, children with MD use less mature counting strategies more frequently and for a longer period of time than do other children. Children with MD also tend to make more errors when using counting procedures to solve arithmetic problems. This delay in the adoption of mature counting procedures to solve arithmetic problems appears to be related to difficulties in keeping track of information during the counting process and a rigid conceptualization of counting.

The good news is that children with MD seem to be able to catch up to their peers in their ability to use counting procedures to solve arithmetic problems by the middle of the elementary-school years with the right interventions. In other words, for many children, MD appears to be a developmental delay and not a more fundamental deficit. Nonetheless, there appears to be a subset of children with MD who show difficulties in the use of counting procedures throughout the elementary school years which persist into adulthood.

This is why the five years of commissioned field-based research by the Department of Education for Western Australia to produce a state-wide professional development resource for its own teachers and students (then discovered by the broader mathematics education community and now commercially available as *First Steps in Mathematics*) is so timely, so worthy of consideration and such a boon to the early mathematics movement in Ontario. The original team, Sue Willis (a respected researcher at Murdoch University), Wendy Devlin and Lesley Price (both experienced primary classroom teachers and research assistants) and too many others to name individually, set out to generate a resource to scaffold the Western Australian Curriculum Framework. Their quest began with an intensive review of the literature and a decision to build (with the blessings of the original authors) on the work of pioneers in the field of mathematics learning development (and concomitantly, mathematics disabilities), including giants such as Herbert Ginsberg and Arthur Baroody. As a result of the informed longitudinal work by the research team, a suite of practical tools has been carefully crafted over time, with expert voices (including children's) contributing constructive criticism toward the achievement of the goals of the original task. As a result, the Department of Education was able to take great pride in a collaborative product whose grand total greatly exceeded the sum of its parts. When applied knowledgeably and concurrently, the resources are shown empirically to assist the classroom teacher in developing instructional techniques that remediate, or at least compensate for, a learning difficulty.

The purpose of *First Steps in Mathematics*[1] is to evaluate a child's ability and style of doing math. Using interesting and empirically-reliable tasks developed and validated by leading researchers like Malcolm Swan and Herbert Ginsberg, the professional development for *First Steps in Mathematics* equips the primary teacher with the skills and knowledge to conduct a one-to-one mathematics interview with a child in order to systematically observe not only what mathematics a child does, but how the child does the mathematics, and quite possibly, how she corrects it. The tools and training provided help the teacher to understand why it is as important to keep in mind that you are searching for what the child does that does work at the same time as you are probing to find out why, what the child does, doesn't work.

The program reinforces the fact that a good mathematics interview should include the use of manipulatives, (e.g., coins, base ten blocks, geoboards, cuisenaire rods, tangrams) and illuminates why a calculator is an important tool and can be used to uncover the difference between comprehension and computation difficulties.

The program emphasizes that a teacher who is equipped to learn from a one-on-one interview with a young child needs to remember to look at a broad spectrum of mathematical ideas. In addition to

[1] Outside Western Australia and its Department of Education and Training, First Steps in Mathematics is operated by Steps Professional Development and Consulting in the United States, the United Kingdom, and Australasia and by Pearson Professional Learning in Canada.

computation, for example, the teacher should explore the child's ability to make predictions based on understanding patterns, to sort collections of blocks or objects in a logical way, to organize space with flexibility, and to measure.

To aid in making a diagnosis which will result in useful recommendations, the program advises the teacher to look carefully at strengths and weaknesses. Note whether the child talks to herself, whether she draws a picture to help her understand a situation, or whether he asks you to repeat. See if the child has a mathematics "proofreading" capacity by asking him to estimate before he computes. The list of strategies goes on.

In some cases, MD can be attributed to the fact that learners have lost (or never had) the connection between mathematics and meaning. *First Steps in Mathematics* helps the thinking teacher to learn why professional judgments about children's mathematical learning or mathematical learning disabilities do not often occur with clarity and simplicity. Immersion in the multiple, complementary components of the program helps teachers to see why and how there can be combinations of difficulties which may include language processing problems, visual spatial confusion, memory and sequence difficulties, and/or unusually high anxiety. With the awareness that math understanding is actively constructed by each learner, *First Steps in Mathematics* helps the informed teacher to intervene in this process to advocate for or provide experience with manipulatives, time for exploration, discussion where the "right" answer is irrelevant, careful and accurate language, access to helpful technologies, and understanding and support.

For me, it is wonderful to see a project that started in the hallowed halls of academe resonate so strongly with the 'subjects' in the original study. Let's hope the pitter-patter of little feet can become a success stampede for many more young mathematical travelers and their guides.

References

Baroody, A. (1987). *Children's Mathematical Thinking.* New York: Teachers College Press.

Butterworth, B. (1999). *The Mathematical Brain.* London: Macmillan.

Butterworth, Brian. (2005). *Developmental Dyscalculia. http://www.mathematicalbrain.com/pdf/HMC26.PDF* Chapter 26 of *The Handbook of Mathematical Cognition*, edited by Jamie ID Campbell, Psychology Press: Pages 455 -467: (500k).

Geary, D. C. (1994). *http://www.amazon.com/exec/ obidos/tg/detail/-/1557982589/qid=1038069100/sr=1- 2/ref=sr_1_2/103-1305118-0103838?v=glance &s=books Children's mathematical development: Research and practical applications.* Washington, DC: American Psychological Association.

Ginsburg, H. (1977). *Children's Arithmetic: The Learning Process.* New York: Van Nostrand. ▲

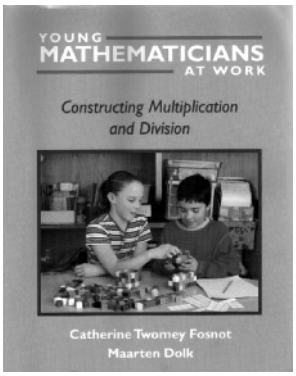
▲ Book Review: *Young Mathematicians at Work: Constructing Multiplication and Division*

REVIEWED BY LYN VAUSE



Lyn Vause is a Consultant with the Simcoe Muskoka Catholic Board, presently on secondment to the Ministry of Education. She is also presently working on her doctorate at the Ontario Institute for Studies in Education

Mathematics success serves as a gatekeeper at both the personal and public levels of our social experience. Personal professional career opportunities, social economic prosperity and democratic decision making are all factors of a civil society that are hampered or enhanced by the numeracy level of the general population. And although we know that mathematics learning holds such power over the fate of private individuals and economies in general, we still retain an old-fashioned, elitist view of mathematics that supports only a small fraction of the student population becoming truly numerate (e.g., at a level that allows for complex decision making around the environment, the economy, war, poverty etc.). By maintaining a view of mathematics achievement as biologically pre-ordained for the chosen few we close the gate on the many who could do well in mathematics if beliefs and understandings about mathematics were changed to respect the view that all children can learn mathematics with appropriate support and time and that good teaching instruction is what makes the difference. Many high achieving countries recognize that mathematics can become accessible for many more children once good instruction is in place. This book by Catherine Twomey Fosnot and Maartenn Dolk is about just such good teaching. In particular, how do we help students develop a deep understanding of mathematics from their very first introduction to this subject and throughout their elementary experience. On the first page of their book,

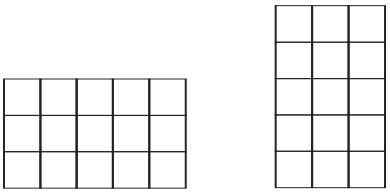


Young Mathematicians at Work: Constructing Multiplication and Division, they reveal the most important element of good teaching, namely learning:

"Interestingly, in some languages, learning and teaching are the same word. In Dutch, for example, the distinction between learning and teaching is made only by the preposition. The verb is the same. Leren aan means teaching; leren van means learning. When learning and teaching are so closely related, they will be integrated in learning/teaching frameworks; teaching will be seen as closely related to learning, not only in language and thought but also in action. If learning doesn't happen, there has been no teaching. The actions of learning and teaching are inseparable." (page 1)

The above quote articulates Fosnot's and Dolk's vision of mathematical teaching and learning. Based on their research with the Freudenthal Institute in the Netherlands, the authors collaborated on a series of books that provide guidance on how children mathematize their world and how teachers can encourage this mathematization by providing problem-solving contexts appropriate to children's developmental levels. In this book, *Constructing Multiplication and Division*, the authors paint a picture of the 'landscape of learning' for multiplication and division. Using vignettes from the classroom, problem-solving based instructional practices are explicitly and comprehensively detailed. The reader is taken on a journey through the landmarks of good instruction, namely, teaching with the big ideas in mind, the use of strategies and models and the importance of teacher pedagogical content knowledge. This last point, the pedagogical content knowledge, has always been the area of most contentious discussion in the education community and widely acknowledged to be the most bleakly amenable to change. This book, however, will give teachers a solid foundation for teaching multiplication and division while developing their own understanding of the concepts that are embedded in these two operations. At the same time, through transcripts of classroom discussions and samples of children's work, the authors give the readers a lens into the classrooms of highly proficient teachers. The teachers engage children in learning while constantly scaffolding the learning into that 'zone of proximal development' that Vygotsky alerted us to many years ago but which is often elusive for teachers who do not understand the mathematics and/or the children's position on the developmental landscape.

The authors' framework for learning mathematics is the use of strategies, big ideas and models. Strategies form the schema for new learning. When a child is first introduced to multiplicative problems, they will initially use prior strategies such as counting by ones or skip counting, then move to more efficient strategies such as grouping by tens, etc. The teacher's role is to provide the opportunity for the child to move along a trajectory of ever more efficient strategies with an eye at all times on the child's understanding of the math taking place through such strategies. Underlying this progression of strategies are the essential 'big ideas' inherent in the mathematics. Schifter and Fosnot (page 10) refer to big ideas as "the central, organizing ideas of mathematics principles that define mathematical order". They explain that *"as mathematical ideas developed through the centuries and across cultures, the advances were often characterized by paradigmatic shifts in reasoning. These ideas are "big" because they are critical to mathematics and because they are big in the development of children's reasoning."* (page 10) Unitizing is one such big idea. It is a conceptual leap to realize that the digits in a number increase by a rate of ten as they move to the left and decrease by a rate of ten as they move to the right. Unitizing is a significant shift in understanding – children who have just learned to count to ten by one object at a time now have to think of a set of ten objects as one thing which requires them to almost *"negate their original idea of number. It is a huge shift in thinking for children … and was a huge shift in mathematics that took centuries to develop."* (page 11) Finally, children need mental models to help them understand the mathematics. Such models also provide children with tools for thinking about mathematics. Arrays for learning multiplication are a helpful model that makes obvious some of the more difficult concepts in multiplication. For example, a 3 by 5 array can be turned 90 degrees to become a 5 by 3 array. In an instant, the commutative property of multiplication can be seen, understood and integrated into a mental model.

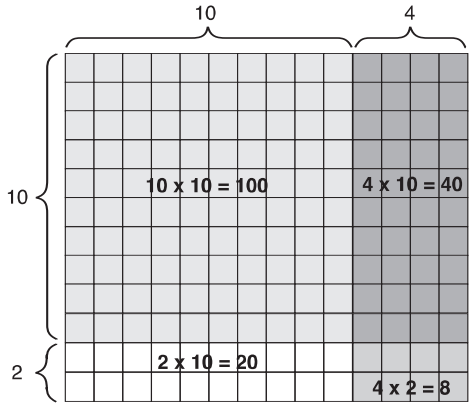


3 x 5 array 5 x 3 array

The distributive property can be ascertained and children can develop a mental model of what is actually

occurring when they use the multiplication algorithm 12 x 14, as shown :

12 x 14 = 14 is composed of 10 and 4
12 is composed of 10 and 2



Of further interest in this book, is the creative use of "strings" in daily mini-lessons to help students develop mental leaps between mathematical ideas. The mini-lesson is done orally with the teacher scaffolding the students thinking by moving from known math ideas to the unknown. For instance, one string that is demonstrated in a lesson in the book is the following:

5 x 6	2 x 7
30 x 6	40 x 7
35 x 6	42 x 7

Each problem is put on the board one at a time and the children are given time to develop their own strategy for each solution. For 5 x 6, many children will know the answer immediately. 30 x 6 may also be fairly easy. Then the students can extrapolate their knowledge of these two facts to the harder calculation of 35 x 6. After the mini-lesson, the students draw up posters describing and showing the strategy they have developed.

Halving and multiplying by powers of 10 is another strategy that children are introduced to through the strings approach. For example, here are two sets of strings: ¼ x 80, 0.25 x 80, 25 x 80 or 1 x 60, ½ x 60, 0.5 x 60, 0.50 x 60, 50 x 60.

The instructional technique that will be the most powerful for teachers is the use of the Math Workshop. The workshop consists of an open-ended investigation and a Math Congress. The investigations depend on the use of problem-solving contexts that are relevant to children and that build on prior knowledge. The authors state that problems that are likely to be mathematized by learners have at least three components: the potential to be modeled (candy boxes, crayons etc.); the situation allows children to realize what they are doing, for instance imagining packing crayons into boxes; the situation prompts the learner to ask questions, notice patterns, wonder, ask why and what if questions.

The Math Congress process provides the opportunity for students to share their thinking and for the teacher to make decisions about which ideas to focus on, to develop the commmunity's norms with regard to mathematics and to stretch and support individual and group thinking. Fosnot and Dolk bring up an important point about the sharing of children's ideas and one that is worth pointing out because I have fallen into this trap myself. They say,

"No matter how clearly one child explains to another, the ideas cannot be directly transmitted with language – the learner must construct them. Often when teachers stop lecturing and begin to facilitate more learner talk, they tend to believe that if a child just explains it well, others will understand. If this were true, the teacher might as well explain it herself! Mathematics cannot be learned through transmission – we each need to construct that understanding for ourselves, and that requires inferring and setting up relationships." (page 49)

Young Mathematicians at Work: Constructing Multiplication and Division, is an excellent book for the teacher who wants to hone her/his own skills in mathematics while at the same time becoming more deeply aware of how children learn the complex concepts in multiplication and division. This book is only one in a series. There are also books on Addition and Subtraction and on Fractions, Decimals and Ratios. Together the three books form an excellent basis for teachers who know that teaching is not enough, it is the learning that is most important. As the authors state in the conclusion, *"Knowing the landscape, having a sense of the landmarks – the big ideas, the strategies and the models – helps us plan the journey. We need to structure the environment to bring children closer to the landmarks, to the horizon – to enable them to act on their world mathematically."* (page 159). ▲

CUSTOM DESIGNED PLAYING CARDS

- large 7" x 4.5" deck of 52 cards
- plus 4 extra cards with card tricks and math games on them for K-12 use in classrooms.
- 58 cards will be enclosed in a "tuck" box.

$12 per deck plus shipping and handling.

Order through OAME 70 Chestnut Crt., London, ON N6K 4J5
Ph / Fax (519) 471-6324 Or Order ON-LINE www.oame.on.ca



▲ Friday Special

JACK WEINER



Fridays at the University of Guelph, five minutes before the end of class, I say, "Pens down, books closed, it's "Friday Special" time!" Then I show on screen an article or cartoon or letter or...something that has to do with math (well, usually), the "real world"(well, most often), and has absolutely nothing to do with class, assignments, or tests (always!) It is math for fun. Often, it is some flagrant example of how the media abuse math. I hope you use some of these. Make them "Monday Morning" specials or "We need a break!" specials. If one falls flat, blame me. If it works, you take the credit.

After the tragedy of Walkerton, we were all so much more conscious of water quality and water safety that assurances from proper authorities became common. Witness the ad placed in the *Toronto Star* on December 3, 2001 (six months post Walkerton):

Toronto tests its water 300 000 times per year? That seems really high. I mean that seems REALLY HIGH!!

Let's bring a numeracy eye to this number. The ad claims the water is tested every 4 hours. So, there are 6 tests per day or 365 X 6 = 2190 tests per year. Are we missing something here? How does this translate to 300 000 tests per year?

I know! There must be several testing stations. In fact, there must be $\frac{300\,000}{2190} \cong 137$ stations. Hmm.

I realize that Toronto is a BIG city. But even so, does it have 137 testing stations?

Luckily, the people responsible for the ad gave us a phone number.

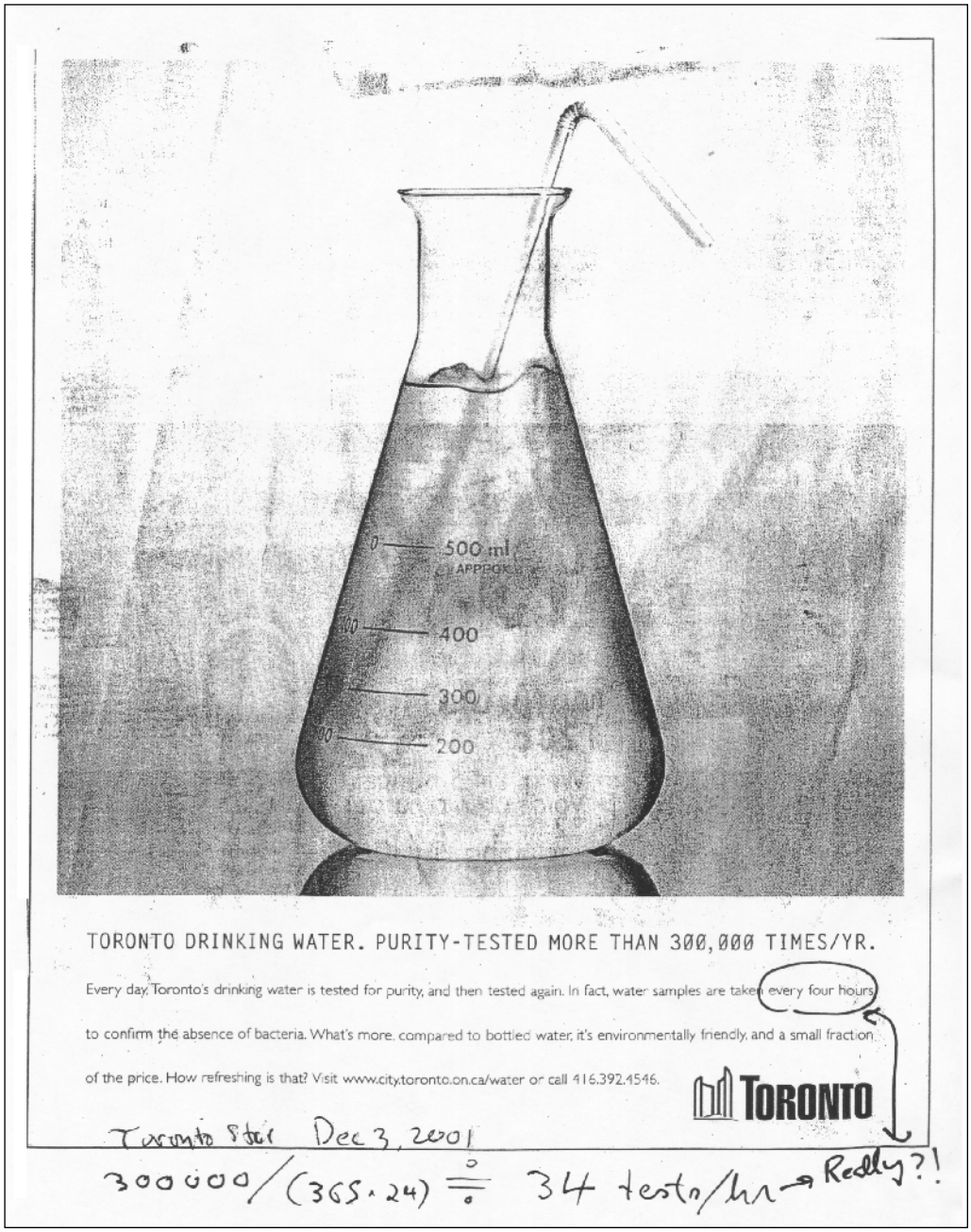
I called. I got voice mail. I left a message.

"Do you really have 137 testing stations?"

As I write this, it is June 21, 2005. I am still waiting for a reply. It could drive you to drink!*

Best wishes, Jack

*But don't drink and drive! ▲



OAME Annual Conference www.oame2006.ca OAME Annual Conference www.oame2006.ca

1 2 3 4 5 6 7 8 9 10 ...

WOMA presents

3 Days of Workshops
May 11-13

On-Site Accommodation
Modern, Inexpensive!

Keynotes
Cathy Fosnot
Jack Weiner

Banquet with
Harness Racing, Slots
Imax

Session
Proposals
Being Accepted
www.oame2006.ca
until
Oct 31!



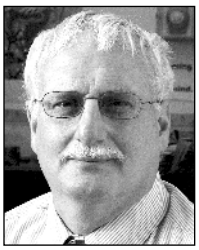
Plan to Attend OAME 2006

Every 1 Counts

Fanshawe College, London www.oame2006.ca

OAME Annual Conference www.oame2006.ca OAME Annual Conference www.oame2006.ca

▲ OAME/NCTM September 2005 Report



JOHN KERSLEY

Like every company or business, NCTM reviews its policies and cost structures. And, the inevitable happens. This summer NCTM has announced a modest increase in its membership rates. Full details can be found at **www.nctm.org/members**. NCTM has a variety of memberships including either individual or group memberships, one to suit your needs. NCTM has expanded its services in its 'members only' section. Members have access to the ON-MATH journal, presenting ideas for teaching and learning mathematics at all grade levels, REFLECTIONS, providing the opportunity to view, analyze, and reflect on instructional practices in the teaching of mathematics through video examples and FIGURE THIS, featuring mathematics challenges, hints, and solutions for middle-grades children and their families. A member is also allowed to download a limited number of articles from NCTM's monthly journals.

In addition, NCTM has expanded its rebate program for its affiliates. OAME is an official affiliate of NCTM. Currently, a $5 rebate is available to affiliates for each *new* NCTM membership submitted. Now, there will be two additional categories of rebates to affiliates. There is a $3 rebate for each NCTM membership *renewal* attained through an affiliate's conference or membership renewal cycle. Rebates for new members and renewals of memberships apply to NCTM Full Individual memberships and E-Memberships. The rebates do not apply to student or retired (soon to be renamed *emeritus*) memberships. NCTM also approved a rebate to affiliates for K–8 school memberships. Affiliates will have the opportunity to receive a $5 rebate for each new K–8 school membership they submit. The K–8 school membership includes a one-year subscription to either *Teaching Children Mathematics* or *Mathematics Teaching in the Middle School*. Contact me when you are renewing your NCTM membership. You will be supporting OAME.

TeachMath.org is a new NCTM website that furnishes information on how to become a mathematics teacher. Mathematics teachers are in demand throughout North America. For any student thinking of becoming a math teacher, this site contains valuable information.

OAME is always looking for people to submit articles on teaching ideas, experiences, thoughts, philosophy, etc for the *Gazette*. It is a very worthwhile experience and I encourage anyone to submit a manuscript. NCTM is also looking for authors for two upcoming publications. The editors of *Teaching Children Mathematics* invite teachers to share ideas and experiences related to measurement instruction in grades preK – 6 for its October 2006 focus issue: *Teaching and Learning Measurement*. The editors are particularly interested in manuscripts that include photographs and samples of children's work. Details can be found at **www.my.nctm.org/erources/tcm/calls_tcm.asp**.The editors of NCTM's 70th Yearbook (2008) *Algebra and Algebraic Thinking in School Mathematics* are looking for articles that address the nature of algebra and algebraic thinking in the curriculum, research on the learning and teaching of algebra, classroom practices and approaches to the teaching of algebra at different levels. The editors encourage collaboration among classroom teachers, college or university educators, researchers, supervisors, etc. Details can be found at **www.nctm.org/publications,yearbook.htm**.

NCTM publishes a tremendous number of resources for mathematics teachers. A list of them can be found at **www.nctm.org/catalog**. For example, popular resources for grades 4 to 8 include *Mathematics Assessment: A Practical Handbook for Grades 6 –8*, a collection of samples, thoughts and tips in assessment. *Menu Collection: Problems Adopted from Mathematics Teaching in the Middle School*, is a set of problems for warm-ups, challenges and critical thinking in mathematics, and *Teaching with Student Math Notes, Volume 3*, is a set of twenty issues showing the power and beauty of mathematics. Please contact me for pricing.

Websites of Interest

- Mathematics Programs **www.mcdougallittell.com**
  Several types of problems from grades 5 to calculus
- Round the Rug Math Series
  www2.bc.edu./~caseyb/oview.html
  Linking literacy and mathematics, pre-K to 2

- The Math Idea of the Month Club
  www.mathideas.mhschool.com
  Ideas and lessons via monthly mails
- Math and Engineering
  www.TeachEngineering.com
  Lesson plans K – 12 to engage students in the study of science and mathematics

Regional NCTM Conferences and Expositions

- Hartford, Connecticut — October 6-8, 2005
- Birmingham, Alabama — October 20-22, 2005
- Denver, Colorado — November 10-12, 2005

NCTM Annual Meeting

- St. Louis "Asking Questions-Generating Solutions" April 26-29, 2006 ▲

▲ Get Interactive

KEVIN MAGUIRE



Kevin Maguire is an Instructional Leader in Mathematics/Numeracy for the Toronto District School Board. Kevin is also an OAME Vice-President. His particular interests include teaching with technology and elearning.

In this space, websites of interest to teachers of mathematics will be shared. One of the fundamental aspects of math these days is the use of rich problems in the classroom. Below are a selection of websites that provide a variety of rich, investigative experiences for students.

Mathematics Education Reform: Lessons & Investigations

- **http://math.edreform.net/portal/math/teacher resourcesandsupports/lessonsinvestigations**

This site highlights interesting inquiry-based websites from K-12. Middle school teachers might be interested in "Petals Around the Rose". For those teachers familiar with this problem, they might follow with interest a teacher's description of exactly how she uses this problem with her class.

Daily Tantalizers Math Sample Activities

- **http://www.etacuisenaire.com/dailytantalizers/activities.jsp**

A number of rich activities for Grades 1-8 are provided. Hints and solutions are also available. Try the following:

- Grade 2: # 134 (A different way to subtract)
- Grade 5: # 11 (A Fermi Problem)
- Grade 8: # 125 (Algebraic modelling)

The Math Forum @ Drexel : Problems of the Week

- **http://mathforum.org/pow/**

A large variety of problems are posted by grade groupings. Students are invited to submit their solutions. All problems are rich and lend themselves to a variety of solutions.

PBS Teacher Source

- **http://www.pbs.org/teachersource/math.htm**

This site offers a wealth of integrated lesson plans and investigations. Lessons and investigations are organized by grade groupings.

NCTM I-Math Investigations

- **http://www.blaine.k12.wa.us/EStandards/Illuminations/imath/index.html**

i-Math Investigations are ready-to-use, online, interactive, multimedia math lessons. Complete i-Maths include student investigations, teacher notes, answers, and related professional development activities. ▲

▲ Editor's Note: Call for Columnist

This will be Kevin Maguire's last column. If you are interested in authoring this column, please contact the editor by September 30th, 2005.

▲ Assessment Corner: Making Sense of Assessment

Chris Suurtamm



Chris Suurtamm is an Assistant Professor of Mathematics Education at the University of Ottawa. She is an experienced mathematics teacher who is committed to helping teachers with the implementation of the new curriculum and assessment practices.

Many teachers perceive that there have been rapid changes in what is required in their classroom assessment and evaluation practices. These changes may include such things as a greater use of rubrics, designing performance tasks, organizing marks by content strand or process category, and using a combination of levels and marks. In some cases, teachers' assessment practices have been naturally evolving as their repertoire of instructional practices expands. Thus, many see current changes in assessment as an evolution rather than a revolution. However, for others, many of the assessment ideas are new, appear as externally mandated, and their suggested use feels more like a revolution than an evolution. In any case, teachers are trying to make sense of assessment but are sometimes running into difficulty. This article highlights some of the changes that have occurred in mathematics teaching, learning and assessment, and then specifically discusses some of the assessment issues that teachers are grappling with.

Background

Changes in mathematics teaching and learning over the past few decades have grown out of a concern over learners' limited understandings of mathematics as taught from the perspective of mathematics as a set of procedures, rules and algorithms (NCTM, 1989). New perspectives on mathematics education and current research prompts a shift on the focus of mathematics teaching to help students develop a conceptual understanding of mathematics to accompany the development of procedures (Hiebert, & Lefevre, 1986). Teachers are encouraged to build classroom learning environments that support the development of mathematical reasoning through collaborative problem solving. Thus, a teacher's practices would foster active engagement of students in defining and solving complex problems, and discussing, defending, and debating their various solutions and mathematical ideas. Students are expected to not only solve problems but also, to understand what they are doing, to explain their methods, to adapt their thinking, and to follow the explanation of others. The intent is to support the development of students who view mathematics as a sensible endeavour, who have a range of strategies for mathematical work and who can employ these in a flexible and eventually efficient manner (Carpenter, Franke, Jacobs, Fennema & Empson, 1997).

As classroom instruction and activity change in mathematics, so too must assessment. Traditional methods of assessment such as paper-and-pencil tests may be well-suited for assessing the application of procedures, rules and algorithms from mathematics in the textbook but other forms of assessment provide opportunities for students to explore problem-solving tasks and to show their full range of mathematical capabilities. As well, assessment should be embedded in instruction and teachers assess students over time as they are engaged in mathematical activity. Therefore, assessing students through portfolios, investigations, performance tasks, presentations and projects values the learning process and allows students to show what they can do.

However, these assessment methods and the use of this type of assessment information may be unfamiliar to some teachers. As such, teachers are often expected to use these assessment methods without truly having an understanding of the reasons for using them, ways to use them and how to use the information that these forms of assessment provide. Consequently teachers often lean on prescribed assessment procedures and rules to guide their assessment decisions. In some sense, this leaning on assessment rules parallels the way students, who have not had the time or opportunities to develop a conceptual understanding of mathematics, rely on rules and procedures to perform on a test. It also prompts the development of misunderstandings about assessment that further complicates matters. The following addresses several issues that appear to cause confusion as teachers use new assessment methods.

Using Rubrics

Rubrics are an assessment tool with a specific purpose of describing stages of what good work looks like and are not designed to be used with every assessment strategy. When a student is given a test or quiz of knowledge of procedures then the test or quiz is probably best marked with a marking scheme rather than a rubric. However, when students are given a complex problem-solving task that provides the opportunity for a variety of methods of solutions that may be qualitatively different, a marking scheme will not suffice. In this case, a rubric is a good way to show the assessment criteria and to display different qualities that might be inherent in student solutions. The rubric should be made specific to the task at hand so that the rubric has meaning for both the teacher and student.

Achievement Levels and Numeric Marks

The requirement to use a variety of assessment techniques implies that teachers will collect different types of assessment data. When observing, a teacher may be making written anecdotal comments. When a student writes a test, a teacher may use a marking scheme to score it which may result in a numeric mark. When assessing a performance task the teacher may use a rubric to assess written work as well as their observations of the student at work. Hence, assessment data may include numeric marks, levels from a rubric, anecdotal comments, and information from a checklist. A teacher needs to use all of this information as well as his or her own professional judgment to combine this data to indicate the student's overall achievement of the curriculum expectations.

Formative Assessment

Formative assessment refers to the gathering and use of information about students' ongoing learning by both teachers and students to modify teaching and learning activities and to diagnose the concepts and misconceptions and decide what comes next. An effective formative assessment program requires more than just the addition of a few observations and an extra quiz in the middle of a unit. Formative assessment may include observations, conferencing, questioning, quizzes, progress reports on projects, journal entries, or the submission of written work. Formative assessment recognizes the goals of the unit and provides appropriate check-points where learning can be assessed using appropriate assessment strategies and tools. It prompts teachers and students to examine student work with a view to next steps. Although student work may be marked and recorded, the marks are merely indications of student progress at that point in time. The feedback in the form of comments and next steps and the opportunities for improvement will make the difference in student learning. Formative assessment can make significant change in student achievement by providing students with meaningful feedback throughout the learning process.

Valuing the Student

If assessment is to be used to improve learning then student work should be viewed in terms of what this work suggests that the student does well rather than merely what the student is struggling with. Quite often, student work is viewed in terms of what the student doesn't understand rather than focusing on what the student does understand. It is important to recognize the effort that students have put into their work and the progress that they have made. All too often, in this era of accountability, we use a deficit model to justify why the student received a level 2 rather than a level 3 rather than highlighting what the student did understand.

Organizing Assessment Information

Mathematics is a complex activity that includes both content and mathematical processes. Assessing a students' understanding of mathematics should use multiple forms of assessment strategies. Thus a balanced assessment program would address many content strands, incorporate the range of mathematical processes, and use a variety of assessment methods. However, when organizing and recording assessment data, a teacher will need to choose one of the three possibilities: content, process, or assessment type as an organizing structure. Hence, a teacher may choose to organize by content strand, mathematical process (or in Ontario, Achievement Chart category), or by type of assessment. In any case, once a teacher chooses the organizing structure, attention still needs to be paid to be sure that the other two components, content, process, or type of assessment, also remain balanced. Organizing, recording, and assimilating assessment data is a challenging task. The task is even more complex because content strands such as number sense, measurement, geometry, algebra, or data management, are not necessarily distinct. Process categories such as problem solving and communication are also not distinct and separating assessment data into such categories is difficult.

Conclusions

Quality assessment is a multifaceted process. It is understandable that teachers grapple with new ways to assess. For some, changes in assessment have appeared quickly and there may not have been time to develop a sound understanding of quality assessment. In many cases, busy teachers found quick fixes to make sense of rubrics, performance tasks, new report cards, and achievement levels and categories. Some may feel that they are making these changes because they are mandated rather than changing their assessment practices because they make sense for students. However, focusing on providing students with meaningful mathematical activity and quality feedback may prompt a move away from a reliance on prescribed assessment procedures towards sound assessment that assists students and teachers to improve learning.

References

Carpenter, T., Franke, M. L., Jacobs, V., Fennema, E. & Empson, S. (1997). A longitudinal study of intervention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education, 29*, 3-20.

Hiebert, J., & Lefevre, P. (1986). *Conceptual and procedural knowledge in mathematics: An introductory analysis.* In J. Hiebert (Ed.), "Conceptual and procedural knowledge: The case of mathematics" (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum.

National Council of Teachers of Mathematics [NCTM]. (1989). *Curriculum and evaluation standards for school mathematics.* Virginia: NCTM. ▲

Ontario Association for Mathematics Education
PUBLICATION

Linking Assessment & Instruction in Mathematics: PRIMARY YEARS

- ▲ Eleven field-tested Assessment Activities with Blackline Masters
- ▲ Sample student solutions illustrating Levels of Performance
- ▲ Notes to Teachers
- ▲ All investigations now keyed to Specific Expectations of The Ontario Curriculum

This 168-page document is available at the following rates: $16.00 per copy; Bulk price: $336.00/box (28 copies $12 each). (OAME members $12.00 for first copy, state membership number.) Shipping and handling costs extra.

Order by mail, fax or on-line:

OAME
or Order ON-LINE www.oame.on.ca
Do not send cash or cheque. You will be invoiced with shipping and handling costs added.

▲ Photo Math - A New Way of Picturing Math

RON LANCASTER
INDEPENDENT MATHEMATICS CONSULTANT



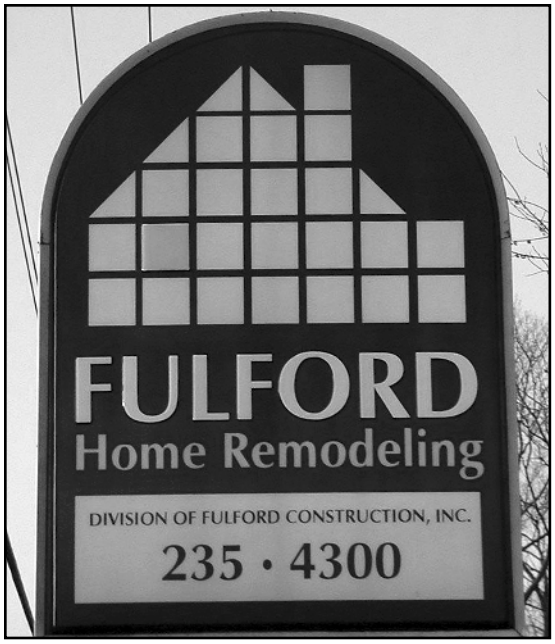
Ron Lancaster is a Lecturer in Mathematics Education at the Ontario Institute for Studies in Education at the University of Toronto. He is also an independent mathematics consultant and works with teachers, students and educational organizations in North America, Asia and Israel.

"A picture is worth a thousand words" is an expression that is often used to convey the importance of using images to make a point. The point of this column is to go beyond this notion and promote the idea that in fact "A picture is worth a thousand numbers". Photos with Math is what readers can expect to find in every Photo Math column and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario curriculum as well as the NCTM document, *Principles and Standards for School Mathematics*. It is hoped that these questions will inspire students to be more creative and playful and that in the end, students will develop an ability to see the world from a different perspective.

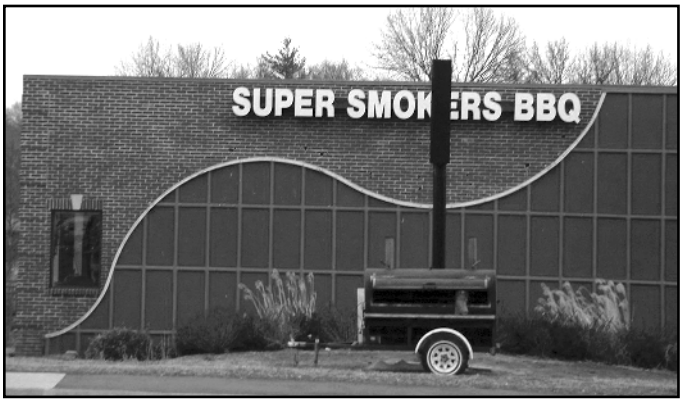
Teachers and students are encouraged to pose mathematical questions related to their own encounters and to submit these problems along with photographs to the author for possible publication in future columns. Photos can be sent via e-mail or by regular mail to the author of this column.

Introductory Comments

The beautiful photographs and the questions for this issue of Photo Math were submitted by **Mike Rust** and **Curtis James**. Both Mike and Curtis teach mathematics at Clayton High School in Clayton, Missouri. I was on the campus of this public school recently to speak at a conference and I was impressed with the academic tone and the wonderful collection of public art installed around the campus.



Photograph 1: Photograph of sign in front of Fulford Home Remodeling in Swansea, Illinois.



Photograph 2: Photograph of Super Smoker BBQ in St. Louis, Missouri.



Photograph 3: Photograph of a Taco Bell restaurant in St. Louis, Missouri.

©2005, Ron Lancaster. Any commercial use or any use outside of a teacher's classroom requires permission from the author.

Photograph 1

(a) If the sides of the squares are 15 cm long calculate the area of the design.

(b) What is the ratio of squares to triangles?

(c) Create an original design using the same shapes.

(d) What is the ratio of squares to triangles in your design?

(e) Calculate the area of your design.

Photograph 2

The following questions involve the idea of drawing a set of axes directly on Photograph 2 and then thinking of the curve that appears on the side of the building as being the graph of a function.

(a) Is it possible to position the x-axis so that the graph has one real root? Is there more than one way of doing this? Is it possible to position the x-axis so that the graph has two real roots or three or four?

(b) Assuming that the height of every one of the rectangular panels is 1 metre, estimate the combined area of the full and partial panels that lie under the curve from the start of the curve to the end.

(c) Position the x-axis so that it is at ground level. The origin can be located anywhere. Select points on the curve, determine their coordinates by making measurements directly on the photograph and then find a possible equation of the curve (you might want to try using the cubic and quadratic regression commands on the TI83/84).

(d) This question will require the use of a topic from Integral Calculus. While not part of the Ontario curriculum, this question may be of interest to students with an intrinsic interest in mathematics.

Find the area under the curve between its endpoints and compare your answer with the estimate from part (b). Should these answers be the same?

Photograph 3

What is the common name for the curve formed by the roof? How is this name related to the name of the restaurant? Can this be considered to be a mathematical play on words or a mathematical pun? ▲

NOTICE: *Photo Math Contest Deadline Extended* to November 30, 2005. See photographs and submission details as previously published in the *Ontario Mathematics Gazette* Vol 43#4 (pages 14, 15) and on the OAME website www.oame.on.ca.

▲ Up, Down, Sideways: Difficulties of a Daughter and Dad in Learning Mathematics

DANIEL H. JARVIS, PHD (ABD)
PHD CANDIDATE/GRADUATE TEACHING ASSISTANT
J. G. ALTHOUSE FACULTY OF EDUCATION
THE UNIVERSITY OF WESTERN ONTARIO



Dan Jarvis is a mathematics and visual arts educator who has taught in both Thailand and north-eastern Ontario. As a PhD candidate at the University of Western Ontario, Dan is presently focusing his doctoral research on curriculum negotiation and professional development models for mathematics educators.

Introduction

One of the complex routines that comprise the teaching profession is the communication between the school and the home. This article, drawing on contemporary research and personal anecdote, will provide mathematics educators with further insight into some common perceptions held by parents[1], and strategies for more effectively communicating and working with these important stakeholders to increase the overall quality of students' mathematics experience.

As a mathematics educator who was very interested in the Esso Family Math Program[2], I had asked the organizers about the possibility of participating in one of the local groups with my 4-year-old daughter. We were welcomed and encouraged to attend, and I was also asked if I would consider keeping a brief reflective journal throughout the six sessions, as a form of ongoing research. Having been regularly involved in educational research, I had no qualms about this request and looked forward to recording impressions of the program as Clara and I experienced it together. What I did not expect was my own emotional response, as a parent (and teacher), to the vignette which I will describe shortly. My feelings that night, and throughout the program as I had more time to think about this and other incidents, have helped me to reconsider the nature and importance of risk-taking and mistake-making, and how these two activities relate to the teaching and learning of mathematics.

The Teacher-Parent Connection

Research conducted throughout the past few decades has shown that parental involvement in children's schooling is one of the most important factors regarding academic success (*Expert Panel on Early Math in Ontario, 2003*, pages 17-18). While many of these studies have focused primarily on the mother's influence, a recent study by Flouri and Buchanan (2004, page 150) found that the father's, involvement also has a significant long-term effect in a child's school achievement. Encouraging both mothers and fathers to take active roles in their children's learning is therefore an important goal for all teachers. However, certain formidable obstacles exist in this undertaking.

In many parents' minds, mathematics is still predominantly regarded as the memorization of number facts and related computational skills (Onslow, Edmunds, Adams, Waters, & Chapple, 2002, page 7). Moreover, many parents are unfamiliar, and therefore rather uncomfortable with the content of today's standards-based, K-8 mathematics curriculum (e.g., new topics such as probability and statistics). Even when parents earnestly want to assist their children with homework assignments, many are unsure how to do so (Whiteford, 1998, pages 65-66); and they may be rather uneasy with the multiple teaching and learning strategies that are now used in many mathematics classrooms (Bay-Williams & Meyer, 2003, pages 54-55; *Expert Panel on Student Success in Ontario, 2004*, page 80). For example, they may not easily relate to elements of cooperative group work, increased communication and problem solving, the use of manipulatives and technology, and varied assessment, since their own mathematics education likely did not include any or many of these components. This is not to say that these strategies and resources are always used properly, or that they guarantee successful learning (see for example, "Concrete Materials", *Consortium of Ontario School Boards, 2003*, Section 1,

[1] While home support may include parents, guardians, and/or other caregivers, the author has used the term "parent" throughout for clarity.

[2] The Esso Family Math Project is a "community-based program for families who would like their children to experience success in math, yet who may find it difficult to obtain help in facilitating their children's mathematical development." The author would like to thank Dr. Barry Onslow, the FM Project Director, for his valued input on this paper, and the many members of the FM team for their encouragement regarding our FM experience. For more information and excellent resources, visit their website at: http://www.edu.uwo.ca/essofamilymath/ .

page 29). The quality of instruction will largely depend, as always, on the quality of the instructor.

Clearly, facilitating a positive home-school connection between teachers and parents represents a meaningful yet difficult task. What, then, can teachers do to bridge this gap in such a way as to encourage and educate parents, thereby supporting student learning? Fortunately, a growing body of qualitative research is emerging in this area. Drawing from a series of case studies, Anderson (1998, page 336) suggests that teachers should: acknowledge and validate parental contributions and reduce their claim to authority; ask parents to think about what sorts of mathematical activities they already take part in at home with their children; ensure, as much as possible, that materials flow from home to school and vice versa, and should extract important general principles and specific mathematical strategies from the literature and from conversations with parents about student learning and progress.

In an attempt to assist parents, teachers have successfully implemented various ideas and programs such as colourful information booklets regarding mathematics; regular math problems featured in school newsletters; web-based calendars and resources; take-home math practice bags; math music CDs and videos; games and puzzles; math walks or field trips; children's books that have mathematical connections; inviting parents to attend class as observers or assistants, and parent/family math nights and programs (Colgan, 2002; Kloosterman, 1998; Onslow, Edmunds, Chapple et al., 2002; Ontario Ministry of Education, 2003; Peressini, 1998). These initiatives and innovative resources have been shown to assist parents in becoming more involved in their children's learning. But perhaps even more crucial for this home-school connection to be effective is that teachers communicate to parents how their children must be allowed to *regularly struggle with math problems* in order to develop as learners.

Risk-Taking and Mistake-Making

The National Council of Teachers of Mathematics clearly emphasized in their *Principles and Standards* document that "effective mathematics teaching requires understanding what students know and need to learn and then *challenging* and supporting them to learn it well" (2000, page 16, emphasis added). Students of mathematics must be adequately challenged in day-to-day learning activities. It is only in this way that children will be able to grapple with, what Piaget referred to as, "disequilibrium" or "cognitive dissonance", and in so doing, reinforce existing neural connections and also develop new connections and deeper mathematical understanding. Martinez and Martinez (2003, pages 29-30) likewise discuss the recognition, and powerful potential, of "eustress" (good stress) - as opposed to "distress" - in helping children learn mathematics. Chatterley and Peck (1995) describe how even the most well-intentioned teacher can rob students of mathematical learning: "If we understand the process necessary to provide the referents within the minds of our students, we will cease to mentally cripple them by being overly kind and sympathetic and by helping too much and often far too soon" (page 436).

Notwithstanding the need for adequate challenge, teachers need to explain to parents that they not overly swamp their children with learning tasks that are too far above, or ahead of them (Turner & Meyer, 2004, page 312). Tomlinson (2001) notes that "when a student continues to work on understandings and skills already mastered, little if any new learning takes place. On the other hand, if tasks are far ahead of a student's current point of mastery, frustration results and learning does not" (page 8). Both extremes are obviously to be avoided - the secret, therefore, lies in finding the right combination of, what I like to refer to as, *applied mystery and mastery*.

The Russian psychologist, Lev Vygotsky (1930/1978), developed an insightful educational theory which he referred to as the Zone of Proximal Development (ZPD) - the difference between the learner's capacity to solve problems on his/her own, and his/her capacity to solve them with external support. According to this theory, the teacher's or parent's role is to provide appropriate instructional *scaffolding* and relational support in order to maximize student achievement within his/her ZPD. Adequate and appropriate scaffolding allows teachers and parents to extend and maximize the degree of challenge for children, while still encouraging them to take risks and to learn new concepts through their initial and important failures.

The following real-life vignette provides insight into one parent's perspective, and might assist teachers in their attempts to communicate these realities and related strategies to parents.

Vignette: Up, Down, Sideways

As Clara and I walked into the public library, there was a sense of excitement and anticipation as we looked around the room of new faces. We found two empty chairs at one of the side tables and introduced ourselves to the other children and adults already seated next to

us. We both noticed the many interesting books, posters, and manipulatives to be found around the space, and therefore became curious to know just what kind of experiences were in store for us. After about ten minutes, the leader invited us to come up and take some pizza, drinks, and dessert. Following dinner, everyone was formally welcomed and then the leader began the program with various planned activities. At one point during the evening, the children were all asked to come forward and sit on the carpet in front of the leader, so as to share their understandings of patterning.

> ***Leader:*** *Alright now, boys and girls, let's have a look at these bears and see if we can't find a pattern. Notice that the first bear is standing straight with his head up and feet down. But look, the second bear is upside down with his head at the bottom and feet at the top. And who can describe the third bear?*
>
> ***Kulchurika:*** *It's laying down, sort of - sideways, like.*
>
> ***Leader:*** *Okay, right. So, let's say it together while I point to each of the eight bears - Up, down, sideways, up, down, sideways, up, down - who thinks they know what bear would come next if this pattern continues? Yes, Clara?*
>
> ***Clara:*** (hand held high, with much enthusiasm) *Up?!*
>
> ***Leader:*** *Well, that's a good suggestion, Clara. Let's look at the pattern again to check your answer. Say it with me - up, down, sideways, up, down, sideways, up, down, up - does that seem right, do you think?* (Brief pause - Clara shakes her head, knowing there's a problem. She does not offer a correction, but looks back at me with confused, anxious, and saddened eyes.) *Does anyone else have another idea?*
>
> ***Aize:*** *I think it's supposed to be sideways.* (They recite it together, as a group, trying this new idea.)
>
> ***Leader:*** *Well, what does everyone think?* (general affirmation) *Yes, I think we've discovered the pattern!*

Discussion: Conceptual and Perceptual Difficulties

Clara was not unlike other students of the same age in terms of conceptual difficulties experienced throughout the Family Math program. Referring back to my journal notes, I see that she not only struggled with the bear orientations (patterning), but also with the conservation of length (the misinterpretation of "spread-out" counters being of *greater number* than "closely-aligned" ones) and capacity (the possibility of shorter, but wider, containers holding more water than taller ones): and a general difficulty in estimating objects exceeding the number 10. All of these quite normal conceptual challenges were eventually understood as her mathematical thinking developed through time and with practice, and I should note that Clara has enjoyed and excelled in mathematics in formalized schooling. What's important here, though, is not her eventual understanding of concepts, but the actual fact that she was allowed, and encouraged, to *struggle* with challenging mathematics in a caring environment, such as that provided by the Family Math leader that evening.

Let's return again to the bear patterning incident described in the vignette. Given that it is a relatively rare occurrence for an adult to be observing his/her own child volunteer an answer during a public forum, perhaps I should not be all that concerned about my reaction. However, I wonder if it is not somehow connected to a significant commentary on parental involvement in general. At that particular moment - when Clara offered an incorrect response and then looked back at me - although I felt very confident with the curriculum and reform-oriented strategies, and supported by leaders and volunteers that night, it was my reaction as a parent to her first public, academic "failure", which really quite surprised me. I realized that even with all of my mathematical training and background, along with my years of teaching and learning experiences, my initial and automatic response was still to feel somewhat embarrassed - to look on the error with disappointment and perhaps judgment, rather than view it as an *opportunity for growth and new learning*. As a mathematics instructor, I had not responded in this way with hundreds of students over the years, when they had volunteered incorrect answers in my classrooms. And yet, with my own child, my *affective*, or emotional, response seemed to have overcome my well-reasoned and academically-seasoned *reflective* one. While I did not mention any of this initial reaction to Clara, and even though we worked on patterning together (at her request) throughout the coming week until that particular aspect was well understood, this personal revelation both bothered and intrigued me. I had been confronted with a form of parental pride which, while often being a strong and healthy emotion, in this instance was found to conflict with clear thinking about thinking - a strange case of *virtuous vice*.

Most importantly, perhaps, I must now ask myself the following question: Would I have allowed Clara to struggle with the mathematics and to wrestle with her own *cognitive dissonance* if the two of us had been

working on her mathematics homework together (e.g., not corrected her immediately, but rather listened and asked good questions to help her clarify her own thinking)? Quite often, it is this sort of cognitive tension, so very necessary for learning, which can be, and often is, avoided by parents, and even by some teachers, who do not understand the educational benefits of students taking risks and making mistakes.

Conclusion: Understanding Patterns in Math and Mind

In retrospect, Clara most likely wanted to please her father, gain affirmation from the Family Math leader, and be accepted by the other children that evening - one could argue that these are quite natural feelings. But more than this, I wonder if she just really wanted to understand the mathematics - to solve the puzzle before her. Perhaps it was this curiosity that originally drove her hand up, or later, drove her to practice until she more fully understood this new piece of math curriculum.

What if the leader had disregarded her answer completely or worse yet, commented on its lack of correctness in jest? What if the parent she glanced back at had not forced a smile and been quite so successful in masking his inner disappointment? What if the very first time she dared to take a public risk in a mathematics learning environment, she met with blatant disapproval or even ridicule? Would she ever have tried again? On the other hand, if Clara is daily encouraged to take risks in solving problems, by both her teacher at school and by her caregivers at home, she is far more likely to take further risks, to communicate her ideas to others, to develop deeper understandings of key concepts, and to simply enjoy the wonders of mathematics.

Dad has since learned to welcome risk-taking during shared mathematics times and to anticipate Clara's, and his own, initial errors with more of an attitude of enthusiasm. He's tried his best to convey this to his daughter by listening more carefully, asking good questions, and appreciating all of Clara's answers, since her incorrect answers often convey more information about her actual mathematical understanding than do her correct ones. More than anything, perhaps, he's learned that one can always be proud of a brave attempt and a willingness to continually learn from one's mistakes.

As teachers, we can share these types of stories with parents, making them aware that it is natural for them to want to shield their children from frustration and errors. By doing so, we help parents to recognize the tangible benefits that arise from children taking risks and persevering with a difficult concept or problem. Teachers and parents have the exciting opportunity and responsibility to translate these simple aphorisms into successful teaching and learning experiences for all children.

References

Anderson, A. G. (1998). Parents as partners: Supporting children's mathematics learning prior to school. *Teaching Children Mathematics, 4*(6), 331-337.

Bay-Williams, J. M., & Meyer, M. R. (2003). Help! Parents and standards-based mathematics. *Principal Leadership, 3*(7), 54-60.

Chatterley, L., & Peck, D. (1995). We're crippling our kids with kindness!! *Journal of Mathematical Behavior, 14*(4), 429-436.

Colgan, L. (2002). Mathematics at home. *Ontario Mathematics Gazette, 41*(1), 12-15.

Consortium of Ontario School Boards. (2003). *Targeted implementation and planning supports: Grades 7, 8, and 9 Applied mathematics.* Toronto, ON: Queen's Printer for Ontario. Retrieved July 15, 2005, from http://www.curriculum.org/occ/tips/index.shtml

Expert Panel on Early Math in Ontario. (2003). Early math strategy: The report of the expert panel on early math in Ontario. Toronto, ON: Ontario Ministry of Education. Retrieved July 15, 2005, from http://www.edu.gov.on. ca/eng/document/reports/math/index.html

Expert Panel on Student Success in Ontario. (2004). *Leading math success: Mathematical literacy, Grades 7-12: The report of the expert panel on student success in Ontario.* Toronto, ON: Ontario Ministry of Education. Retrieved July 15, 2005, from http://www.edu.gov.on.ca/eng/document/reports/numeracy/index.html

Flouri, E., & Buchanan, A. (2004). Early father's and mother's involvement and child's later educational outcomes. *British Journal of Educational Psychology, 74,* 141-153.

Kloosterman, P. (1998). Parent involvement in elementary problem solving. *School Science and Mathematics, 98*(4), 205-210.

Martinez, J. G. R., & Martinez, N. C. (2003). Raising middle school math standards without raising anxiety. *Middle School Journal, 34*(4), 27-35.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA.

Onslow, B., Edmunds, G., Adams, L., Waters, J., & Chapple, N. (2002). Children and their parents: Learning math together and having fun. *Child & Family, 6*(1), 6-14.

Onslow, B., Edmunds, G., Chapple, N., Forsythe, J., Burakari-Adera, R., & Waters, J. (2002). Parents: The logical link for supporting children's mathematical understanding (Part 1). *Ontario Mathematics Gazette, 40*(4), 27-31.

Ontario Ministry of Education. (2003). *Helping your child learn math: A parent's guide - The Ontario early math strategy.* Toronto, ON: Author. Retrieved July 15, 2005, from http://www.edu.gov.on.ca /eng/document/brochure/earlymath/index.html

Peressini, D. D. (1998). The portrayal of parents in the school mathematics reform literature: Locating the context for parental involvement. *Journal for Research in Mathematics Education, 29*(5), 555-582.

Tomlinson, C. A. (2001). *How to differentiate instruction in mixed-ability classrooms (2nd ed.).* Alexandria, VA: Association for Supervision and Curriculum Development.

Turner, J. C., & Meyer, D. K. (2004). A classroom perspective on the principle of moderate challenge in mathematics. *The Journal of Educational Research, 97*(6), 311-318.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press. (Original work published 1930.)

Whiteford, T. (1998). Math for moms and dads. *Educational Leadership, 55*(8), 64-66. ▲

▲ Supports for Effective Implementation of the Revised Grade 9 / 10 Curriculum: The Role of Manipulatives in Making Algebra Meaningful

ANN KAJANDER



Ann Kajander is an Assistant Professor of Mathematics Education at Lakehead University. She is an experienced mathematics teacher who is committed to the development of deep mathematical understanding in students as well as teachers.

The recent revisions of the Ontario Mathematics Curriculum (Ministry of Education, 2005) have taken further steps towards encouraging conceptual understanding by more explicitly embedding the *Process Standards* (NCTM, 2000) in the expectations. For many of us, trained in a methodology and value system which included a vision of mathematics as an efficient set of methods for generating answers, this evolution continues to be a challenge.

There are many concerns for teachers at the classroom level when moving to a conceptual learning basis. The feeling of some is that such learning "takes too long", is messy, noisy and disorganized, and at worst, results in manipulatives being thrown around the classroom!

While deep learning takes longer initially, it saves time in the long run because students are able to retain more of what they really understand. Observations about lack of longer term retention are *exactly* why we need to move towards more student centered, pedagogically suitable, and conceptually based models of learning, a.k.a. the "messy" ones. Our curriculum now explicitly states that aids such as algebra tiles are to be used (MOE, 2005). Put bluntly, learning based purely on procedural methods has little meaning or long term retention for most students, and does not facilitate the depth of understanding needed to use math for any worthwhile endeavor outside (or inside) of school. Even more bluntly, it's boring, demeaning, and encourages the all-too-prevalent perception that mathematics is difficult, useless and unintelligible.

Instead the current vision is that mathematical ideas make sense, can be agreed upon generally based on concrete investigation with understanding, and that mathematics is a useful tool to support higher level learning in many domains.

To follow are some tried and true suggestions for introducing algebra with understanding in grades 9 and 10, which have been successful in "real" classrooms even with students who have not learned with manipulatives before. The use of algebra tiles to investigate algebraic rules like "FOIL" or factoring techniques are outlined based on my own classroom experiences doing so. It's all about starting at the beginning…

Getting Started

Algebra tiles are a direct generalization of base 10 blocks and are relatively natural for children to use based on previous experiences developing mathematical operations concretely. The converse is also true; students who have never seen them before need to gain initial understanding of modeling basic operations and number properties using hands on investigations. And yes, they may need to spend a few minutes building a tower or robot before getting started.

Students who have not learned multiplication using an array or measurement model may need to review this understanding. For example, we can understand 2 x 3 as two groups of three, or three groups of two, or as an *array that is two by three*.

Using the latter model, we can relate it to measurement, area, and factors and products. Two and three are *factors* of six, and six is the *product* of two and three.

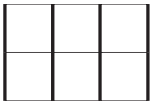


Figure 1. Two times three with an area model.

Moving on to calculations such as 23 x 14, we can also think of the product as an area. Using base 10 blocks to model this product generates many computational methods as well as the traditional one. We see that the product is really (20 + 3)(10 + 4) and that the products can be grouped into areas, or calculated separately.

For example,

23	23
x 14	x 14
12	200
80	30
30	80
200	12
322	322

are two perfectly correct ways to multiply these numbers. Each individual product corresponds to an area in the diagram in Figure 2.

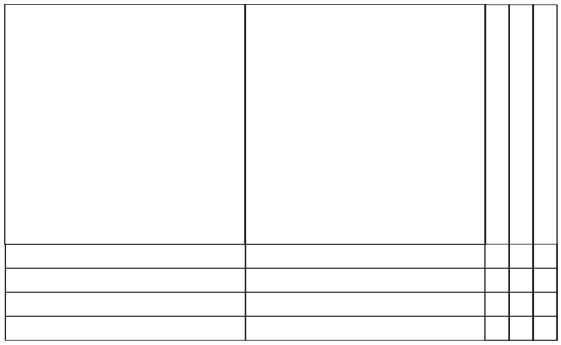


Figure 2. A model for 23 x 14 using base 10 blocks. The large squares are 10 x 10 or 100, the long rectangles are 1 x 10 or 10, and the small squares are 1x 1 or 1. All of the numbers in the calculations – 200, 80, 30 and 12 – can be found in regions of the rectangle, and are generated by the individual products. For example, 20 x 10 is 200.

Reviewing such previous work may be necessary for students who have not learned this way before. Students can enhance both their understanding of operations as well as numbers by comparing notes with each other on such methods. They are also learning (or reviewing) skills for exploring, thinking, and communicating mathematically. Such introductory activity is engaging, active, and not difficult, so it may set a positive tone in your classroom.

Deep understanding of number and operation directly supports subsequent algebraic understanding. We are now ready to introduce a new tile, which we imagine to be a "stretchable" one. Several models can be used to support the introduction of the tile of unknown length or area.

Choose algebra tiles that are different in color from the base 10 tiles used so far. I like to suggest that these new tiles are a bit magical in that we need to understand that their size can change. We can turn the lights down and imagine them growing very, very big, or getting very, very tiny. The "long" x tiles change simultaneously as do the square x^2 tiles. See Figure 3.

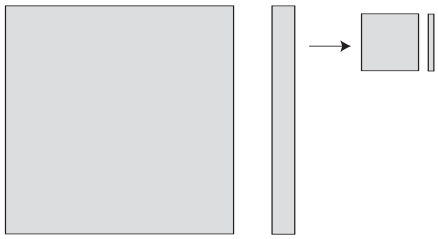


Figure 3. Visualizations of x^2 and x: these 'magic' tiles can be any size, but all dimensions change together.

Another way to see this concretely is to tape pieces of paper to the (vertical) overhead screen to represent the constant base 10 numbers. Then the algebra tiles can be placed on the overhead projector glass, and the overhead projector moved in and out. As a result of the motion of the projector, we see the size of the *number* tiles staying *constant* on the screen, and the *variable* tiles *varying* as the projector moves farther and closer. These powerful visual images help sustain lasting images of the roles of the tiles and the concept of a variable.

Working with a Variable

The understanding of the notion of a variable is one of the most fundamental developmental concepts at the intermediate level. I have found that misunderstanding the concept of a variable is also responsible for many student errors in procedural application. For example, a "typical" grade 9 error is "simplifying" 3(x + 2) as 3x + 2, as many of us have seen repeatedly. Working with algebra tiles allows students to determine their own procedure or rule; we see clearly in Figure 4 that multiplying x + 2 by 3 gives 3x + 6.

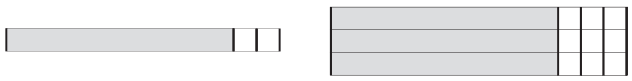


Figure 4. Investigations of x + 2 and 3(x + 2).

Students who are previously unfamiliar with algebra tiles might do well to spend a class investigating problems similar to the one illustrated in Figure 4, perhaps handing in a report of their findings and generalizations at the end. Such investigation takes no longer than a traditional lesson on expanding brackets, but makes more sense. The introduction of negative numbers can then be done with negative algebra tiles, or even just be built procedurally on this conceptual introduction with the positive tiles.

Moving to Binomials

My students laugh when I tell them I really don't remember what "FOIL" stands for, and that they don't need it anyway. Rather, we will develop the methods in a

way that is based on sense making, not acronyms.

Referring back to examples such as 23 x 14 (review by rebuilding, as necessary), and reviewing the idea that x times x is x^2 (show the algebra tile, and review with a 10 by 10 tile as necessary), I usually spend one or more class periods at the grade 9 level with kids working in small groups on carefully sequenced problems like the ones to follow. Students are asked to model the operation with algebra tiles, and then use the model to write the expanded version of the expression. Again, they can be encouraged to generalize their findings.

For example, ask students to model:

x + 3

2(x + 3)

2x

3(x + 4)

x(x + 4)

Stop here and compare notes (see Figure 5).

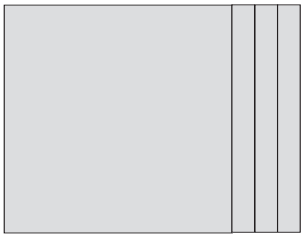


Figure 5. "Seeing" $x(x+ 3)$ as $x^2 + 3x$.

Students continue to work:

(x + 2) x

(x + 2) (x + 4)

Share again (See Figure 6).

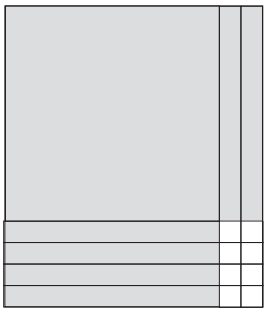


Figure 6. "Seeing" (x + 2) (x + 4) as $x^2 + 6x + 8$. Note that the 6x term is really the sum of the 4x and the 2x, an important point for learning factoring! (For ease of black and white printing, the variable tiles here are shown shaded, and the constant tiles are shown while. In reality, they would be coloured a different colour. Positive tiles are used here, as in all diagrams in this paper.)

Students can then continue with more examples similar to the one in Figure 6, until generalization is clear. When we "see" the model in our mind, it becomes obvious why the 2x and 4x pieces in the example in Figure 6 are added to give the 6x term. But what is even more wonderful is how transparent the move to learning factoring subsequently becomes.

Developing Concepts of Factoring in Grades 9 and 10

I have experienced great frustration over the last few years "reviewing" factoring in grade 11, because it feels brand new. Retention from grade 10 seems so minimal. This situation is an example of where the "extra" time used to learn something conceptually is really a time saver in the long run. I have found that introducing the factoring in grade 10 as exactly the same concept (just with the reverse information) as the binomial expansion just described makes it highly accessible to students. In either case we have a rectangle – it's just a question of whether we know the side lengths or the area.

One way to handle the introduction of simple factoring is to give students "mystery" bags of tile. Different groups receive different bags, and they switch. The challenge is to see which bags have pieces that can be made into rectangles, *and* to find the terms representing the sides. For example, a bag with an x squared piece, five x tiles, and 6 units easily yields a rectangle whose dimensions are x+3 and x+2. Students are asked to draw as many solutions as possible in the time period, and to generalize as much as possible. Personally I like to throw in a few bags that do *not* make rectangles as well. The next class can begin with sharing insights. How do we *know* which ones will work to create rectangles (that is, rectangles whose second side is not simply one)? Can we predict without drawing what the side length (factors) will be? A little encouragement will lead students quite quickly to the central idea that the coefficient of the "x" term must be split in two groups, and the product of the number of x tiles in each group yields the constant term (see Figure 7).

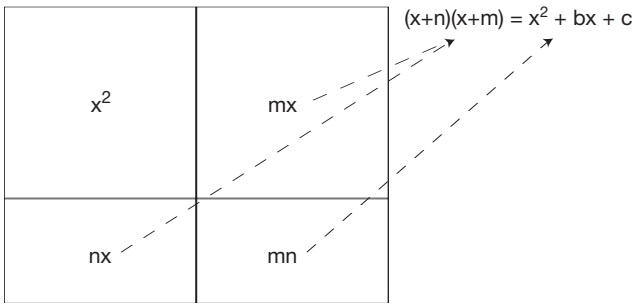


Figure 7. The x terms must be split into two groups, of size n and m, where n and m add to give b, and multiply to give c. This is hard to understand in a rule, but easier to visualize and construct with the tiles.

Summary

While working with concrete materials may require a little extra time initially, the long term benefits are more that worth it in my experience. Students learning in this manner are learning more than how to expand or factor; they are learning to *do* mathematics and to *think mathematically*. Such skills do not in fact get absorbed by osmosis by watching a teacher. They are active processes which must be developed in that messy, seemingly disorganized manner that characterizes true learning.

The Next Challenge

Are you ready to bring some algebra tiles along to your next department meeting to investigate factoring by decomposition?! It's a little "messier", but it can be done! Stay tuned for future articles to get some tips, or share your own solution by sending it in to the *Gazette*! Of course, you could always pose the problem to your students! Here's one to begin – try factoring $2x^2 + 11x + 15$. Now, *can you generalize*?!

References

Ministry of Education (2005). *The Ontario Curriculum, Grades 9 and 10.* Toronto: Queen's Printer for Ontario.

National Council of Teachers of Mathematics (2000). *Principles and Standards of School Mathematics*. Reston, VA: NCTM. ▲

OAME T-SHIRTS

$12.00

"Literacy Matters! Numeracy Counts!"

available in M, L, XL in white only.

Price includes taxes. Shipping and handling extra



ORDER FROM: OAME

Or order ON-LINE www.oame.on.ca

NEW! Visa or MasterCard now accepted.
Please provide: Cardholder Name, Card No. and Expiry Date

▲ An Equation Game

MICHELLE TRIGGS, TEMECULA VALLEY USD

"An Equation Game" *by Michelle Triggs, reprinted with permission from the* ComMuniCator, *journal of the California Mathematics Council (Volume 29, Number 3) 2005.*

CONCEPTS: Algebra, Mathematical Reasoning

SKILLS: Finding equations, given the slopes and y-intercepts

GRADES: 9–12

MATERIALS: Scratch paper, two different sets of 24 game markers, two paper clips, Student Activity Sheet (page 30)

BACKGROUND

Students work in pairs and play a game involving linear equations. Students locate an equation with the slope and y-intercept they select on the game board provided. The object of the game is to cover five squares in a row.

DIRECTIONS

Distribute the Student Activity Sheet containing the game board, two different sets of 24 game markers, and two paper clips. Explain the rules of the game.

- The first player chooses one of the slopes and one of the y-intercepts located at the bottom of the game board and marks them with the paper clips.
- The player finds the correct equation on the game board and places a marker on the square.
- The next player then moves only one of the paper clips, finds the correct equation, and places a marker on the square.
- Students take turns and if a player makes a mistake, that player loses a turn.
- The game ends when a player has five markers in a row in any direction.

VARIATIONS

- Generate random values for the slopes and y-intercepts by creating spinners for each.
- Create other game boards with different linear equations.

An Equation Game

by Michelle Triggs

Each player takes a turn and uses the paper clips to select a slope and y-intercept. He/she then finds the correct equation on the gameboard and places a marker on the square. The next player then moves one of the paper clips and repeats the process. A player who makes a mistake loses a turn. The first player with five markers in a row, in any direction, wins.

$4x + 3y = -6$	$y = -2$	$-2x + 2y = -1$	$4x + 6y = -3$	$-x + 2y = 4$	$8x + 6y = 3$
$x + y = -2$	$-x + 2y = -2$	$-3x + 4y = -2$	$2x + 2y = -1$	$-2x + y = -1$	$2x + 3y = -3$
$2x + 3y = 6$	$-x + y = 2$	$4x + 3y = 6$	$y = \frac{1}{2}$	$-3x + 4y = 8$	$y = 1$
$-x + 2y = -4$	$-3x + 4y = -4$	$x + y = 2$	$-x + 2y = 2$	$8x + 6y = -3$	$-x + y = -1$
$-x + y = 1$	$4x + 3y = 3$	$y = -\frac{1}{2}$	$-2x + y = -2$	$2x + 3y = -6$	$-2x + y = 2$
$-3x + 4y = -8$	$x + y = 1$	$-x + 2y = 1$	$-3x + 4y = 4$	$-x + y = -2$	$-4x + 2y = 1$
$4x + 3y = -3$	$y = -1$	$-2x + 2y = 1$	$4x + 6y = 3$	$-2x + y = 1$	$2x + 3y = 3$
$x + y = -1$	$-x + 2y = -1$	$-3x + 4y = 2$	$2x + 2y = 1$	$-4x + 2y = -1$	$y = 2$

Slope							
$\frac{-4}{3}$	-1	$\frac{-2}{3}$	0	$\frac{1}{2}$	$\frac{3}{4}$	1	2

y-intercept					
-2	-1	$\frac{-1}{2}$	$\frac{1}{2}$	1	2

▲ Learning Algebra with Computer Algebra Systems in Our Revised Ontario Math Curriculum

TOM STEINKE, KEVIN SPRY, FRED FERNEYHOUGH

__Kevin Spry__ is on leave from the Upper Grand District School Board and currently serving as a consultant with Texas Instruments. Kevin was an author and tech consultant for Pearson. __Tom Steinke__ is Coordinator of Mathematics for the Ottawa Carlton Catholic District School Board and an author for Nelson. __Fred Ferneyhough__ is former mathematics head at Harold M. Brathwaite SS in Peel and an author and tech consultant for several books.

Curriculum Story

Ontario has been, and continues to be, a leader in Canada and North America in implementing learning technologies in secondary mathematics. Our 1999 and 2000 grades 9 – 12 mathematics curricula explicitly mandated the use of graphing calculators (e.g., TI-83/84), dynamic geometry software (*GSP*), and dynamic statistics software (*Fathom*). As you read this article, the revised grades 9 and 10 mathematics curriculum will be coming to life in our Ontario classrooms. This revised grades 9 and 10 curriculum references Computer Algebra Systems (CAS). This means that Ontario is the first large jurisdiction in North America to include the use of CAS. At the recent USACAS conference in Atlanta, Georgia, over 100 educators from around the world heard about Ontario's curriculum through presentations by Carolyn Kieran of the Université du Québec à Montréal. The Ontario CAS story had many of the participants green with envy!

Below are some grades 9 and 10 expectations from the revised curriculum that references CAS:

MFM1P (page 40)

- add and subtract polynomials involving the same variable up to degree three [e.g., $(2x + 1) + (x^2 – 3x + 4)$], using a variety of tools (e.g., algebra tiles, **computer algebra systems**, paper and pencil)
- multiply a polynomial by a monomial involving the same variable to give results up to degree three [e.g., $(2x)(3x)$, $2x(x + 3)$], using a variety of tools (e.g., algebra tiles, drawings, **computer algebra systems**, paper and pencil)
- solve first-degree equations with non-fractional coefficients, using a variety of tools (e.g., **computer algebra systems**, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies) (Sample problem: Solve $2x + 7 = 6x – 1$ using the balance analogy.)

MFM2P (page 58)

- expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials [e.g., $(2x + 3)(x + 4)$] or the square of a binomial [e.g., $(x + 3)^2$], using a variety of tools (e.g., algebra tiles, diagrams, **computer algebra systems**, paper and pencil) and strategies (e.g. patterning)
- factor binomials (e.g., $4x^2 + 8x$) and trinomials (e.g., $3x^2 + 9x – 15$) involving one variable up to degree two, by determining a common factor using a variety of tools (e.g., algebra tiles, **computer algebra systems**, paper and pencil) and strategies (e.g., patterning)
- factor simple trinomials of the form $x^2 + bx + c$ (e.g., $x^2 + 7x + 10$, $x^2 + 2x – 8$), using a variety of tools (e.g., algebra tiles, **computer algebra systems**, paper and pencil) and strategies (e.g., patterning)
- Similar expectations exist for MPM1D and MPM2D

Summer Workshop Story

During the past summer, a series of 2-day Teachers Teaching with Technology Summer CAS Workshops took place across Ontario – Ottawa, Sudbury and Brampton. At these workshops, participants experienced the power and potential of the TI-89 and TI-Voyage 200 (handheld CAS) as thinking tools to support improved student learning. Expanding the repertoire of teachers to support learning in the problematic area of algebra was a large focus of the summer workshops. In the words of the new curriculum, these teachers experienced first-hand what learning algebra "using a variety of tools" including CAS and algebra tiles looked and felt like. The curriculum pushes us to move beyond merely using paper and pencil, to include CAS and algebra tiles. This has huge potential to help more students make sense of algebra.

The participants also came to realize that the TI-89 and TI-Voyage 200 did everything their familiar TI-83/84 graphing calculators could do.

There is a growing body of research around the impact of CAS on struggling learners in mathematics

(Cedillo & Kieran, 2005; Drijvers, 2005; Heid & Edwards, 2001; Heid, Blume, Hollebrands, & Piez, 2002; Kieran & Saldanha, 2005; Lagrange, 2005; Zbiek, 2005) that supports the inclusion of CAS in our curriculum. At the summer workshops, participants who were already using CAS (see Mother Teresa High School Story below), validated the research claims around the potential of CAS based on their experiences with CAS in their classrooms.

Mother Teresa High School Story

One school community that recently embraced CAS is Mother Teresa High School in Ottawa. The math department at Mother Teresa attended the Ottawa CAS Summer Workshop en mass and shared some of their experiences with the group. In the spring of 2005, Alex Belloni, the Math Department Head at Mother Teresa, got funding for a class set of 30 TI-89's from her administration. She also got release time support from Pat Kehoe, an OCCSB Supporting Student Success Itinerant Teacher, for her teachers to learn about the TI-89. During May and June of 2005, Alex and her fellow math department teachers used their TI-89's with their grades 9, 10 and 11 math classes. Alex and Pat also attended the USACAS 2005 Conference in Atlanta and learned how educators around the world are using CAS.

Natalie Robinson, a teacher at Mother Teresa H.S. writes:

> *"When I introduced the concepts of expanding and factoring to my grade 10 applied students we used the TI-89s to create a pattern chart. Every single student in the class was actually doing their work! Many felt confident enough to brag about how they could predict the last two or three in each series. It was very rewarding to have these students so engaged in June!!! The students had very few complaints or worries about using the new TI-89s when they saw how similar all the commands are to a regular computer. They jumped right in, and many were trouble-shooting their neighbours."*

Zana Omazic, another teacher at Mother Teresa H.S. writes:

> *"I teach grade nine and I know they struggle with operations with polynomials. So, CAS will be a great tool in presenting that topic in a bit more fun and interactive way. Now they can "ask" the calculator*



why x+x is 2x and not x^2 (instead of me)".

Alex Belloni, Math Department Head at Mother Teresa H.S. writes:

"Even though we just began exploring this technology at Mother Teresa H.S., I see a tremendous potential in the use of CAS with our high school students. From a teacher's point of view, CAS has been an extension of my mind and arms. I find it is an excellent didactic tool to help students discover algebra patterns and algebraic properties as an alternative to the board and chalk. CAS allows students to be as relaxed and very engaged in learning algebra as I have rarely seen before. They are eager to learn and don't seem afraid to make mistakes. Students have a very natural disposition to technology and have learned to trust it. In my opinion, CAS has opened new doors in students understanding of mathematics."

Harold M. Brathwaite Secondary School Story

Another school that has initiated the use of CAS is Harold M. Brathwaite SS in Brampton. Some grade nine applied classes used TI-92 Plus devices while learning basic algebra and equations. Former department head Fred Ferneyhough used the CAS devices along with algebra tiles to teach the concepts and found that students not only better engaged in the classes but did much better on tests than in the past. As with the story above, he found that he was able to teach the concepts better and was able to show students why some of their misconceptions were incorrect. Later in the semester, when these same students needed to use equations in a variety of activities, they were able to recall their skills and apply them in the new context.

How to Get Started

Tom Steinke, the grades 7 – 12 math consultant for the OCCSB, secured funding from his Student Success Leader to purchase two TI-89 Titanium ViewScreen Kits for each of his 14 high schools as well as 3 student TI-89 Titanium calculators for each of the schools. Plans are underway to further fund the acquisition of more TI-89's as well as providing teacher release time for teachers to collaboratively learn how to effectively use CAS in their grades 9 and 10 classrooms. The ultimate goal is to have at least one class set of 30 TI-89's or TI-Voyage 200's in every OCCSB secondary school.

Your starting point needs to be your school district's Student Success Leader. They have the financial means and the mandate to support getting teachers and students the tools they need to ensure student success in mathematics. Investing in CAS and teacher professional learning around the effective use of CAS should be one critical part of every board's efforts to improve student success in mathematics.

Collectively, we, as an Ontario mathematics education community, should also advocate for the Ministry of Education to centrally and explicitly fund the purchasing of at least one class set of CAS handheld devices (TI-89 or TI-Voyage 200) per high school school, to ensure that all our students across the province have access to the learning potential of these powerful tools.

If your school or district has sufficient access to computers in the classrooms, you should also investigate the potential of CAS software such as Derive, Maple, MathCad, Mathmatica, and many more. One caution in using software that is too sophisticated is that the tool used for CAS should be developmentally appropriate for young learners of mathematics. Many of these tools are designed for expert users.

What's Coming Up

Recognizing that, like in 1999, when "new" technologies were included in the curriculum, there is now a need for collaborative professional learning opportunities around the effective use of CAS through OAME Chapter Mini-Conferences, the upcoming OAME Conferences, and school and school districts. Some OAME Chapters have already included special CAS presentations in their local conferences and more plans are underway for the upcoming year. Let us know if there is interest in your area!

Last November, a CAS Summit was held in Ottawa and drew together educators from across Ontario and also the United States. Preparations are now underway to hold a Follow-Up CAS Summit this fall in Toronto. Stay tuned for details!

Exciting Times!

In Ontario we have a strong tradition of teachers being leaders in implementing new and innovative technologies and strategies in their classrooms. Our revised curriculum gives us a wonderful opportunity to continue that tradition, by embracing the learning potential of CAS for the benefit of all our students!

"My dream as a department head is to create a mathematical culture in our school where cutting edge technology is part of the process of

discovering and understanding the subject. I believe it is our duty and the students' right to nothing less. I wait with much anticipation for the new experiences, challenges and ideas that the use of CAS will bring in the new school year. I have no doubt how exciting and rewarding they will be for students and teachers alike."

Alex Belloni, Math Department Head at Mother Teresa H.S.

CAS Resources:

NCTM Mathematics Teacher, November 2002 **http://my.nctm.org/store/ECat/product.asp?id=12442**

Computer Algebra Systems in Secondary School Mathematics Education **http://my.nctm.org/store/ECat/product.asp?id=12339**

CAS Research References

Cedillo, C. & Kieran, C. (2005). Initiating Students into Algebra with Symbol-Manipulating Calculators. In J. Frey, A. Cuoco, C. Kieran, L. McMullin, & R. Zbiek (Eds.), *Computer Algebra Systems in Secondary School Mathematics Education* (pp. 195-196). Reston, Virginia: National Council of Teachers of Mathematics.

Drijvers, P. (2005). Algebra on Screen, on Paper, and in the Mind. In J. Frey, A. Cuoco, C. Kieran, L. McMullin, & R. Zbiek (Eds.), *Computer Algebra Systems in Secondary School Mathematics Education* (pp. 241-267). Reston, Virginia: National Council of Teachers of Mathematics.

Heid, M. K., & Edwards, M. T. (2001). Computer Algebra Systems: Revolution or Retrofit for Today's Mathematics Classrooms? Theory into Practice, 40(2), 128-136.

Heid, M. K., Blume, G. W., Hollebrands, K., & Piez, P. (2002). Computer Algebra Systems in Mathematics Instruction: Implications form Research. *Mathematics Teacher, 95*(8), 586-591.

Kieran, C., & Saldanha, L. (2005). Designing tasks for the co-development of conceptual and technical knowledge in CAS activity: An example from factoring. In K. Heid & G. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Syntheses, cases, and perspectives.* Greenwich, CT: Information Age Publishing.

Lagrange, J. (2005). Learning Techniques and Concepts Using CAS: A Practical and Theoretical Reflection. In J. Frey, A. Cuoco, C. Kieran, L. McMullin, & R. Zbiek (Eds.), *Computer Algebra Systems in Secondary School Mathematics Education* (pp. 269-184). Reston, Virginia: National Council of Teachers of Mathematics.

Zbiek, R. M. (2005). Using Research to Influence Teaching and Learning with Computer Algebra Systems. In J. Frey, A. Cuoco, C. Kieran, L. McMullin, & R. Zbiek (Eds.), *Computer Algebra Systems in Secondary School Mathematics Education* (pp. 197-216). Reston, Virginia: National Council of Teachers of Mathematics.

Web CAS Resources

- CAS Articles from Berhard Kutzler **http://b.kutzler.com/art_eng.htm**
- Research with CAS – Texas Instruments **http://education.ti.com/us/resources/research/partc.html**
- CAS-Cat Project - University of Melbourne **http://extranet.edfac.unimelb.edu.au/DSME/CAS-CAT/**
- Curriculum and Assessment in an Age of Computer Algebra Systems **http://www.ericdigests.org/2003-1/age.htm**
- CAME – Computer Algebra in Mathematics Education **http://www.lonklab.ac.uk/came/** ▲



The Grand Valley Mathematics Association

Since 1972

Thirty-four years of Service to Mathematics Education

http://home.golden.net/~gvma E-mail: gvmath@golden.net

GVMA PUBLICATIONS & PI PINS

EVALUATION INSTRUMENTS FOR MBF3C MATH OF PERSONAL FINANCE	$10
ACTIVITIES FOR INTERMEDIATE MATHEMATICS	$10
GREATEST HITS	$ 8
RECREATIONAL MATH PROBLEMS	$ 5
A HISTORY OF THE ASSOCIATION	$ 5
SPECIAL *One copy of* **EVERY** *publication for only*	$25
PI Pins – Actual size 2" by 2". View a sample pin on the GVMA website	$1.25

FALL CONFERENCE AT THE UNIVERSITY OF WATERLOO
THURSDAY, OCTOBER 20, 16:00 – 21:00

▲ OAME Awards, 2005

JUDY DUSSIAUME

Judy Dussiaume is a Co-ordinator with the Rainbow DSB, a former VP of OAME, and past chair of the Awards Committee.

UNION GAS AWARDS

Union Gas Award for Teaching Excellence in Elementary Mathematics



Felicity Laudisa of the Toronto DSB works at Davisville Junior Public School. She has been a teacher for seventeen years providing an environment for learning that encourages students to be problem-solvers, decision-makers, and life-long learners. One of her strengths is ensuring regular communication with families. Congratulations to Felicity for her contributions to mathematics education and for winning the Union Gas Award for Teaching Excellence in Elementary Mathematics.

Union Gas Award for Teaching Excellence in Secondary Mathematics



Ron Gaudreau is currently a math teacher at Sir Wilfred Laurier S.S. in the Ottawa-Carleton DSB. Ron has also taught a number of subjects including science and computer science. He has been a department head of math, computer science, technology and music. This well-rounded individual also spent time in Germany as a vice-principal and math teacher. Ron is known to many of us for his work as a consultant and an active member of OAME, both regionally through COMA and provincially. Ron's principal says that "Ron is the type of teacher an administrator loves to have on staff. He is wonderful with students in and out of his classroom, works well and supports his fellow teachers, and is dedicated to the profession". Congratulations Ron – we are very pleased to present to you the Union Gas Award for Teaching Excellence in Secondary Mathematics.

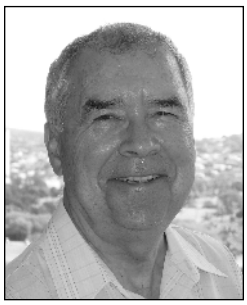
The Union Gas Award for Outstanding Leadership in Mathematics Education



Anna Jupp is a Family of Schools Instructional Leader for Elementary Mathematics. Anna's nominator stated that "Watching Anna work with educators and students is to see a gifted, compassionate knowledgeable teacher and mentor. She is able to relate to the everyday experiences of the classroom teacher who is struggling to synthesize knowledge, skill, applications and pedagogy." Anna has worked on many provincial projects including the Early Math Strategy Expert Panel Report and Technical Guide. Anna is also currently a member of the OAME executive. Congratulations Anna!

OAME AWARDS

OAME Life Membership Award



Eric Muller was a Professor of Mathematics at Brock University and the director of the Concurrent B.Sc./B.Ed. Program. He retired from these roles in June of 2004 but we think he will never retire as a mathematics educator and thus it is very appropriate to award Dr. Muller with a Life Membership for OAME. Some of you may have experienced "Math

▲ Upcoming Events

For further information on OAME Chapter Conferences contact the Chapter Representative and for NCTM conferences, contact John Kersely, OAME/NCTM Rep. See the last page in this *Gazette* for contact information.

Chapters now have websites that are linked to the OAME website: see these for full information on upcoming events.

OAME Chapter Conferences and Events

October 13 2005	Philip Pocock CHS Mississauga	CHAMP
October 20 2005	University of Waterloo	GVMA
October 27 2005	St Augustine CHS	Y4MA
November 17,18 2005	Travelodge Airlane Hotel MathFest Thunder Bay	NWOAME

OAME Annual Conference:

May 11 - 13 2006	Fanshawe College "Everyone Counts"	WOMA

NCTM Annual Conference

April 26 - 29 2006	St Louis MO

Trails" for Niagara Falls or Welland Canal or the Canadian Virtual Math Trail. Or perhaps you have read one of Eric's contributions to the OAME Mathematics Gazette. Or you might have recently used one of his Learning Objects. For over 15 years, Eric organized two mini-conferences every year for the Golden Section of OAME. His contributions to mathematics education have been tremendous. Eric is truly deserving of this award – he has given so much of his life to mathematics education.

Kenneth D. Fryer Award

The **mathematics department of Central Peel S.S.** is the winner of the Kenneth D. Fryer Award which



recognizes a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community. Fred Ferneyhough as past department head, worked with a group of people that he saw continually discussing teaching methods, sharing ideas and growing together. New technologies were embraced and promoted through sharing within the board, at OAME conferences and T3 conferences. As a secondary department they made connections to feeder schools in a variety of ways including Math Olympiads. Members of this department coached many school teams and above all students found these teachers to be fun, yet demanding so that they enjoyed mathematics while they learned about mathematics.

Outstanding Contribution



Gord Banks is the recipient of the OAME Award for Outstanding Contribution. Gord is a retired math educator but continues to volunteer his time and efforts. Gord and his wife Lorraine served as co-chairs of registration for the OAME 2003 Annual Conference. Gord demonstrates a life long love for mathematics and unselfishly contributes his time and expertise. A former student says that "Mr. Banks was exceptional at fostering mathematics education at O'Neill CVI, and in Ontario and Canada as well; he meets every possible qualification one would want for an outstanding teacher." Another former student says that "Mr. Banks always puts others before himself in order to guide someone in the right direction." And yet another remarks " … despite being retired for nearly a decade, Gord Banks', mark is still unmistakable." Even in retirement, Gord continues to present workshops, to help organize Durham DSB regional Math Awards, and to contribute to other events. Congratulations for your exceptional contributions and for winning this award.

Exceptional and Creative Teaching Secondary



Louis Lim, as the former editor of the OAME *Gazette,* contributed many articles to this journal and continues to do so. He obviously enjoys mathematics and loves to share his enthusiasm through his writing, his workshops and his work with students and staff at Unionville High School. His principal states that Louis "demonstrates a passion for student learning, … organizes assemblies for students to hear speakers on the subject of mathematics … and conducts action research in the applied program". A parent writes that her son "… enjoys Mr. Lim's teaching style, his sense of humour and the fact that he shows a real interest in each and every student … Mr. Lim taught math in such a way that it resonated with my son, and applied teaching strategies that made what can often be a difficult subject into one that was interesting as well as challenging. My son virtually 'blossomed' under Mr. Lim's guidance". Louis' exceptional and creative teaching is supported with his energy evidenced by still another student's comments: "It didn't matter how much energy I saved for the last period of the day, I could never match the amount Mr. Lim put in!" Congratulations to Louis Lim on winning the Award for Exceptional and Creative Teaching for Secondary Mathematics. ▲

▲ *Growing Up Mathematically:* A New OAME Resource



The *Growing Up Mathematically* (also know as GUM) resource package is an interactive professional development program, designed for Ontario elementary and secondary schools. *Growing Up Mathematically* is based upon the simple idea of one student growing up through the years and what his / her experience in mathematics should be over those years. *Growing Up Mathematically* is a set of resources including a vision statement, a DVD video, classroom experiences spanning the grades K – 12 on a CD, and supporting professional development materials. These resources can be used as stand-alone pieces or more powerfully as a set of connected components that depend and build upon each other. The vision statement, video, and professional development materials might be used by principals, consultants, coordinators, and other mathematics leaders to promote conversations between and amongst groups of teachers and/or parents about teaching and learning mathematics. The classroom experiences and sample student work are intended to be used by teachers and their students as living examples of what mathematics looks like, sounds like, and feels like according to the vision articulated by OAME.

Why did OAME create *Growing Up Mathematically*?

Teachers have a good understanding of what stage children are at in their mathematical development in the grade(s) they teach. Teachers also have a solid knowledge of what the curriculum requires and what types of classroom experiences are appropriate for students at that grade. OAME sees learning mathematics as a continuum that occurs across all grades levels. This continuum includes the development of skill, knowledge, and process. Students must experience learning as connected and teachers can only make this happen if they have an awareness of what happens before and after their grade level. Elementary teachers often make assumptions about what they should be "preparing" students for in high school and secondary teachers just as often make assumptions about what kinds of experiences students have had in elementary schools.

Growing Up Mathematically brings to life the continuum of learning mathematical processes through glimpses into real classrooms in the video. Teachers can identify and converse about how the use of technology, manipulatives, communication, reasoning and problem solving becomes more sophisticated as students grow up through the years. The classroom activities incorporate the development of mathematical skills, knowledge, and processes.

The Components of *Growing Up Mathematically*

A Vision for Learning Mathematics

OAME's Vision for Learning Mathematics is based on principles and beliefs that are consistent with those expressed in current Ministry of Education curriculum documents.

Professional Development Guide

The vision statement, video, classroom learning experiences, and assessment ideas have all been designed to promote rich conversation among teachers. The professional development component of this resource has been developed to inspire and focus such conversation and is a valuable tool for principals, coordinators, consultants, department heads, and teacher groups interested in professional growth. The professional development component of *Growing Up Mathematically* examines the following topics:

- Engaging Students in Learning Mathematics
- Developing Mathematical Understanding
- Developing Mathematical Communication
- Consolidating and Sharing
- Conversations about Student Work

For convenience, a powerpoint is provided on the CD, to assist mathematics leaders in facilitating discussion around the above topics.

Grade-Specific Learning Tasks and Student Work (CD)

This resource contains classroom-ready learning tasks for Kindergarten, Grade 1, Grade 5, Grades 7/8, Grade 9, Grade 11 College, and Grade 11 University/College in both WORD and PDF format. OAME believes that the primary purpose of assessment is to gather information about where students are in their learning and development. This information should be used to give feedback to individual students about their strengths and weaknesses and to identify what the next steps should be in the learning process. For this reason, the examples of student work included in this resource have not been assigned levels. Instead, assessment ideas and comments have been provided, and where appropriate, next steps have been suggested.

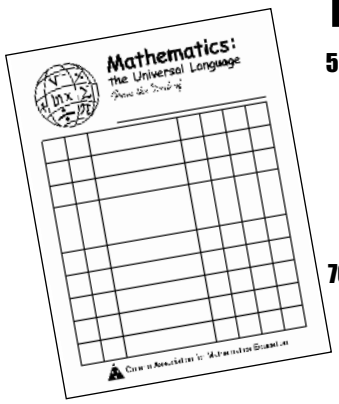
The Video (DVD)

The *Growing Up Mathematically* video focuses on the process components of mathematics and demonstrates the growing sophistication with which learners use materials, technology, vocabulary, and problem-solving skills. It also provides examples of exemplary classroom teaching skills, including context setting, questioning, grouping, and assessing. The video was filmed in five Ontario classrooms, ranging from Kindergarten to Grade 11.

Order on-line at **www.oame.on.ca** for $25.00 each plus shipping and handling. For orders over 50 copies at $22.00 each plus shipping and handling, contact **EDs@oame.on.ca** or 519-471-6324. ▲

Mathematics: the Universal Language

MEMO PADS



50 sheet, 2 colour memo pad

4.25" x 5.5" ~ package of 10

$8.50

plus shipping and handling.

Order through OAME

70 Chestnut Crt, London N6K 4J5

phone / fax (519) 471-6324

or Order ON-LINE

www.oame.on.ca

▲ Director's Dialogue

JACQUELINE HILL



Jacqueline Hill is a consultant with the Durham DSB. Her involvement in the mathematics education community includes Course Profile writing, Workshop Leader, Pine Ridge Math Association President, and writer for Nelson Thomson Learning.

Welcome to the Winner's Corner!

All you have to do is read the *Director's Dialogue* and answer a question about the material and you could be a winner! Winners who correctly answered the question in the June 2005 *OMG* include: Joanne Harris from Southhampton, Antionette Payne and Paul Alves from Durham, and Kathy Pilon from Perth.

This edition of Director's Dialogue highlights events at the Friday and Saturday, June 10 and 11 2005 OAME Board of Directors' meeting at the Holiday Inn Select, Toronto Airport.

Chapter Reps Meeting Friday June 10 2005 6:30 - 7:45 pm.

a) Greg Clarke welcomed new chapter reps: from GOLDEN, Laura Tonin and from WOMA, Anne Michele Stenning.
b) Promotion cases will not be sent out in the future as these are very costly, but membership brochures will be available.
c) Sue Hessey pointed out to chapter reps that all executive members of each chapter must be OAME members.
d) Success Stories: each chapter rep mentioned one good thing their chapter had done this year or is planning to do next year. Top of the list were miniconferences, which give the math community a chance to learn and share ideas. Most chapters also hosted OMO events.
e) Website: Greg took the chapter reps through some updates(such as on-line shopping) and helped individuals get on-line. He mentioned the 'members only' section, and that the *Gazette* is there in PDF format.

Board of Directors' Meeting: Fri June 10: 8 - 9:30 pm and Sat June 11 9am - 4 pm

a) President Kathy Kubota-Jarivnij welcomed all members. Introductions were made. Kathy led the board though an introductory talk on Complexity. She helped us understand that the next board meetings will revolve around Complexity Theory. She challenged us to look up Complexity on the internet and gave the website **www.complexityandeducationualberta.ca** and to look for these titles *The Hidden Connections* and *Emergence*.

b) Kathy introduced the completed *Growing Up Mathematically* or *"GUM"* project. Each board member was given a complete package and Kathy took us through the CD. The board members were then given 2 tasks to experience the activities in the Project.

Other Highlights Include:

Kathy presented a Complexity activity involving the question "There are 36 students in the class. There are 8 more boys than girls." Groups worked on all the different solutions for the number of boys and girls that could be in that class. Then as a large group they were sorted by grade and solutions were classified from simple to complex.

Committee Reports

1) **Constitution:** Bill Otto reported:
   a. They are working on many motions.
   b. They are moving towards on-line voting.
   c. They will create a template for chapter constitutions
2) **Equity:** Jacqueline Hill reported:
   a. The drafts for the equity poster are ready.
   b. Joyce Tonner has made a power point presentation on equity in Math.
   c. Jacqueline offered to produce four *Gazette* articles based on this presentation.
   d. OAME's vision of math equity:
      i) A learning environment where all students do, see, hear and touch mathematics in a profound and meaningful way.
      ii) The learning of mathematics is supported through the active and ongoing involvement of students, parents, educators and the community at large.
3) **Futures:** Dan Charbonneau reported:
   a. They are creating a letter/pamphlet to try to get long term sponsorship for events such as OMO and the Annual Conference.
   b. They are working on an OMO Guideline Book and are updating the conference binder.
4) **Promotions:** Jack Weiner reported:
   a. They are looking at corporate sponsor ideas.
   b. They want to develop more OAME items such as dice , cards, shirts.
   c. They plan for promotion packages that board members will be sharing around the province.
   d. They are working on an outline for the *GUM* project and how to share it. e.g., at miniconferences.
5) **Publications:** Joyce Tonner reported:
   a. Sue Hessey and Marilyn Hurrell will investigate the cost of a colour cover for the *Abacus*.
   b. They will promote more articles in the *Gazette* on how to use the new curriculm. e.g., a Call for Submissions.
   c. They have made an equity liaison.

OMO 2005: NOMA hosted a very successful Olympics with 24 teams. Heather Boychuk and Dan Charbonneau showed the OMO video which will be posted on the website.

OAME 2005: Kevin Maguire thanked his committee for all of their hard work. Sue Hessey and Greg Clarke were thanked for their countless hours of help with the Annual Conference.

Leadership 2006: K-12 math. Research-based Math. The theme will be Complexity Theory and Math Education. An open invitation will be extended to Dalton McGinty, Gerrard Kennedy and Avis Glaze to drop in any time to the conference. 180 delegates will be the maximum.

Sharing Ideas:

1) Greg Clarke had a website update.
2) Jacqueline Hill updated the group on the Think Literacy Phase 2 project.
3) Dave Petro showed us some of the materials from his Ministry writing of the E-LEARNING Geometry course. As well he shared websites on math on the radio. Try: **http://www.bbc.co.uk/radio4/science/5numbers.shtml**.

Chapter Reports: Please refer to the OAME website: **www.oame.on.ca** for full chapter reports and contacts.

Closing Remarks: E-mail your response to this skill-testing question to **jacqueline_hill@durham.edu.on.ca** "What is all the rage at Leadership 2006?"

Yours in Mathematical fun,

Jacqueline

2005-2006 OAME BOARD

Direct General Inquiries to:
EXECUTIVE DIRECTORS
Dave & Sue Hessey

PRESIDENT
Kathy Kubota-Zarivnij

PAST-PRESIDENT
Gordon Cooke

PRESIDENT-ELECT
Dan Charbonneau

VICE-PRESIDENTS
Jacqueline Hill

Anna Jupp

Kevin Maguire

CHAPTERS & REPRESENTATIVES
CHAMP - **Lynda Ferneyhough**

COMA - **Joanne Anas**

GOLDEN - **Laura Tonin**

GVMA - **Barb Vukets**

ISOMA - **Chris Kirkpatrick**

MAC^2 - **Andrew Morrison**

NOMA - **Sue Melville**

NWOAME - **Maribeth Williams**

PINE RIDGE - **Sandra Price**

QSLMA - **Robert Sherk**

SAME - **Nora Mathe**

SWOAME - **David Petro**

TEAMS - **Pat Margerm**

WOMA - **Ann Michele Stenning**

Y^{4}MA - **Rosey Mastrofrancesco**

DIRECTORS - TERM EXPIRING 2006
Kirsten Boucher

Laurie Moher

Dean Murray

Joyce Tonner

Lyn Vause

Jack Weiner

DIRECTORS - TERM EXPIRING 2007
Beverly Farahani

Fred Ferneyhough

Shawn Godin

Connie Quadrini

Silvana Simone

Ann Yeager (On leave from OAME till June 30/05)

DIRECTORS - TERM EXPIRING 2008
Mary Bourassa

Heather Boychuk

Charlotte Cutajar

Merilyn Fox

William Otto

Tara Tasker

Blanche Thomas

NCTM REPRESENTATIVE
John Kersley

WEBSITE COORDINATOR
Greg Clarke

GAZETTE EDITOR
Marilyn Hurrell

ABACUS EDITOR
Trevor Brown

OAME WEBSITE
www.oame.on.ca