Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceeding 10 numbered pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including websites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

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Advertisements for the *Ontario Mathematics Gazette* should be sent to **Dean A. Murray** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

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▲ Editor's Message

MARILYN HURRELL, EDITOR



This issue of the *OMG* brings about two columnist changes. Fred Ferneyhough has turned his column, *Technology Corner*, over to Mary Bourassa. Mary is very involved in teaching with technology and is on the Board of OAME. We welcome her and wish Fred a happy and healthy retirement. *Get Interactive!* which was created by and written by Kevin Maguire for two years, is now in the very capable hands of Louis Lim, my predecessor as Editor of the *OMG*. Welcome back Louis, and all the best to Kevin.

When I received Lynda Colgon's column for this issue, I was already in the midst of obtaining permission from the Virginia Math Teachers' Association to reprint their cover photo and solution to a Sudoku puzzle. (Permission granted). I had first heard of this craze last year when the Vancouver Sun published a puzzle on the front page and promised one every issue thereafter. I told Lynda we were on the same page and that I would publish both hers and my takes on this puzzle. (Lynda goes on to explore other interesting puzzles in her column). A friend who had just returned from Britain brought me a book and my 9 and 11 year old granddaughters and I struggled to solve one of the puzzles. Then their mother found them a simpler version for children, with more missing squares filled in, so we are now 15 - 0 for puzzles solved. I also have seen a CD of puzzles in a local book store. So, jump on board and try out the puzzle on the cover. Unlike Lynda who has left you to suffer, I have included the solution on page 34.

Have a great holiday season.

Marilyn

▲ President's Message
Knowing Mathematics for Teaching Mathematics

KATHY KUBOTA-ZARIVNIJ



Kathy Kubota-Zarivnij is currently program coordinator of mathematics for TCDSB, senior elementary mathematics textbook author, mathematics AQ course instructor, and doctoral candidate. She has been an OAME director for 5 years and served on the OAME Executive for 3 years. She has a variety of mathematics education experiences, such as: elementary school administrator; in-service and pre-service mathematics teacher, and mathematics curriculum and program consultant, writer, and/or workshop presenter for different organizations, such as CCC, EQAO, MoE, NCTM, OAME, OCC, and TLP.

How's your school year going so far? What mathematics have you been doing lately? Are you thinking that you were doing mathematics when you were teaching mathematics lessons? When you read mathematics teacher's guides and student textbook pages, were you engaging in some mathematical thinking? When you were making sense of student problem solutions, how were you using mathematics to understand student learning? Or, when was the last time you solved mathematics problems yourself, for your teaching, for your learning or for the simple intrigue and joy of it?

In this, my third president's message, I'd like to spend some time with you, doing and thinking about mathematics within the context of the ideas highlighted in my previous president's messages. In my first president's message, I introduced the five key conditions of a complex learning system (e.g., internal diversity, redundancy, decentralized control, organized randomness, neighbour interactions), in terms of the structure and activities of the OAME community. In my second president's message, I introduced and outlined the purpose, structure, and content of OAME's latest professional development resource, *Growing Up Mathematically*.

In this *President's Message*, I am inviting you to engage fully in mathematical thinking and doing, as a means of furthering your experiences and insight into the range of K to 12 mathematics. John Mason (2005) explains full mathematical engagement as "actively doing such things as jotting own ideas, doing tasks and constructing your own examples, trying to make connections, getting involved in detail, standing back to get the big picture, explaining what you are doing or trying to do to someone else; being prepared to struggle; acknowledging feelings" (p. x). Also, by doing mathematics yourself, rather than reading and hearing about mathematics, you will have the immediate experience upon which to consider your own mathematics content and pedagogical knowledge needed for teaching mathematics to students. So, get some colleagues together from your division or department and let's do math!

Let's Do Math! Developing a K to 12 Mathematics Sense To Understand Student Learning

Here is a problem to solve. It has been popularized throughout our mathematics community, during professional learning sessions about Japanese lesson design and lesson study led by Akihiko Takahashi and Makoto Yoshida. Consider the questions and comments below as my voice in our mathematical conversation. Here's the problem:

- 36 children are in the class. There are 8 more boys than girls.
- How many boys? How many girls? Show several different solutions.

Understand the Problem

What does the problem ask you to do? What are the mathematical ideas and details in the problem? What relationships between the ideas and details do you need to pay attention to as you plan to solve the problem?

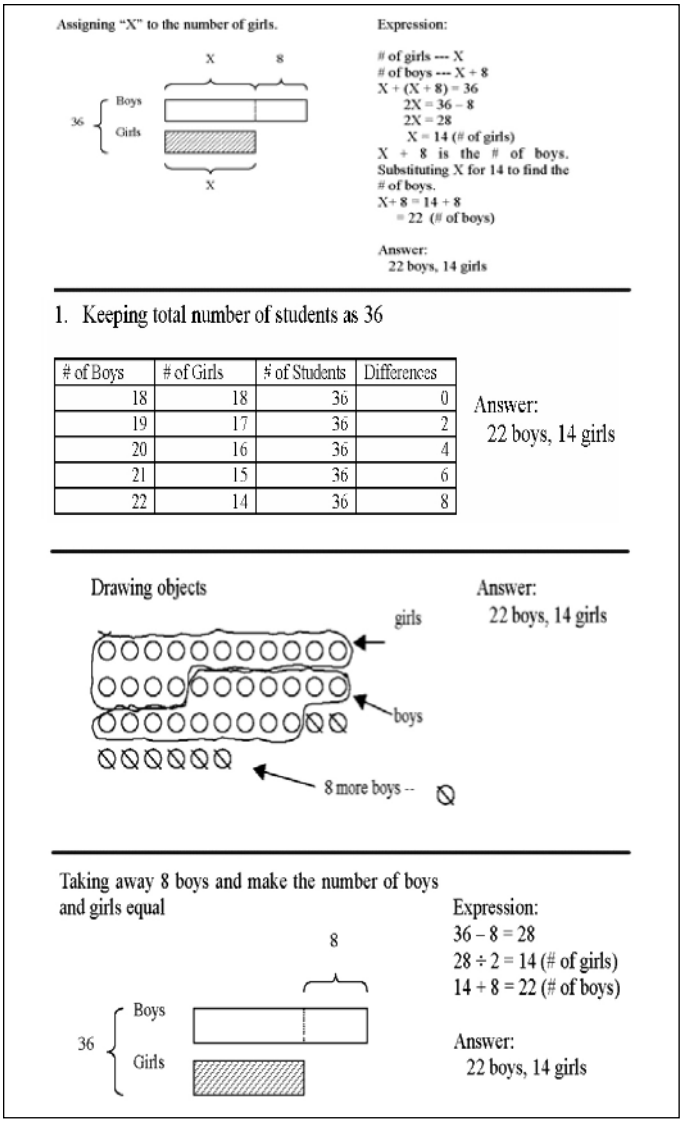
Predicting the Range of Possible Solutions

What solutions are you imagining to be possible as you make sense of the problem? It turns out that solutions to this problem include a range of mathematics from grade 2 (solve problems involving the addition and subtraction of two-digit numbers with and without regrouping; collect and organize primary data in charts) to grade 10 (solve systems of two linear equations involving two variables, using the algebraic method of substitution or elimination).

So, now that you know the range of possibility, what are some of the different solutions to this problem?

Developing Multiple Solutions

What did you do? Did you provide a numerical answer, one solution, or different solutions within the



predicted range of possibilities? What different ways did you represent your thinking? Though there is one numerical answer, there are at least nine mathematically different solutions, that I have seen. Keep talking, sharing mathematical thinking, and building on each other's ideas to develop several different solutions.

Assessment for Learning

If you have your students (elementary or secondary) or additional colleagues at your school solve this problem, notice the mathematics in their solutions. Think about:

- How is the mathematics similar and different in the solutions developed?
- How were mathematical representations (e.g., concrete models, labeled diagrams, mathematical symbols and terms) used in developing clear, precise, and persuasive solutions?
- Which mathematical ideas and strategies were used more frequently and less frequently?

Organizing Solutions for Learning

I have read and heard a shift towards highlighting the sharing and discussion of solutions as being the most significant aspect for mathematics learning through problem solving. As a result of your observations of the mathematics used in the solutions, it is important that the teacher strategically organizes the sharing of solutions for building collective and individual understanding of mathematics concepts, procedures, and/or strategies. Think about:

- How will you decide how many and which solutions will be shared?
- How will you visually organize the solutions so that they can be viewed
- What mathematics will be the focus for discussion in each solution?

How would you organize the sharing of these 4 solutions (Takahashi, 2004) for grade 6 to 9 students? What mathematics learning would be explicit through your strategic sharing?

Using Complexity Theory to Improve the Conditions for Student Learning

This 36 children problem seems uninteresting if we were to only focus on calculating the numerical answer. However, it becomes intriguing when the problem is focused on developing multiple solutions, thus opening it up enough to allow for diverse possibilities. Yet, the problem is sufficiently constrained with problem details (e.g., 8 more boys than girls, 36 children in the class) to ensure that the mathematical ideas are too diverse and can be interrelated. When the teacher knows the mathematical possibilities in the problems posed to students, he/she can determine whether or not significant mathematics learning can emerge.

Davis & Simmt (2003, 2005) suggest that these key conditions have a particular relevance to the mathematical work of a teacher. So, think about these five conditions as they relate to mathematics teaching and learning within a classroom and within teacher professional learning models.

Internal Diversity refers to the pool of possibility that your students and colleagues have when they are developing and comparing solutions to a problem. Such diversity is the basis of their collective capacity for new learning through problem solving. If they are expected to produce solutions

using the same method at the same time, the outcome of the problem solving would not be a productive one.

Internal Redundancy refers to the importance of having enough common mathematics knowledge among the students and teacher to develop mathematical understandings together. In such a robust learning system, if a few students or the teacher do not contribute to the class' mathematical understanding, other students could contribute productively. Some redundancies among participants in a collective include learning actions that are automatized. For example, in a classroom that focuses on reasoning and mathematical communication, it is routine that students and the teacher pose questions like, "How do you know? Why is that? How is your solution similar and different to the other solution?" In fact, the class' diversity can be their source of intelligence only if some of the students' and teacher's ideas and actions are sufficiently alike.

Neighbor Interactions doesn't refer to group work seating. It refers to ensuring that there is sufficient density of mathematical ideas shared among the learners for the possibility of new ideas. This sharing includes oral, visual, modeled, and dramatized modes of communication. Collective and individual mathematics knowledge emerges as new ideas are blended with old ones and opens up spaces for more effective learning.

Organized Randomness is another condition. In a complex learning system, there's always a bit of randomness. Some of that randomness is ignored by the system—which is to say, it doesn't really affect what the system does. Other bits of randomness come to be really important, such as unexpected observation, the sudden insight. Intelligent learning systems, it seems, take advantage of more of these random events, and they're able to do so because they have strategies to organize such events. This idea applies to the differentiated student responses to the 36 children problem. The noticing and coordination of sharing solutions for mathematics learning is productive when it is strategically organized.

Decentralized Control refers to the idea of abandoning the top-down model of centralized management in favor of a shared and distributed sort of control. Intelligent collective action can't be directed or instructed into existence at either the individual or the collective level. Both in the classroom and in professional learning settings, the students and teachers need to negotiate the parameters and possibilities needed for their learning, such as time lines, the focus of discussions, and including links to personal experiences, dilemmas and curiosities, rather than solely focusing on the content in power point slides, textbooks, and worksheets.

Therefore, if mathematical ideas and questions are not plentiful; if the diversity of student and teacher thinking is not shared explicitly, interactively and continuously; if the learning goals and student achievement is perceived to be in the control of the teacher, then it is highly probable that the learning of the collective and the individuals in the collective will be stifled and underdeveloped.

So, think about how such principles of complex learning systems can be used to structure mathematics learning experiences and increase the capacity for developing deeper conceptual understanding of mathematics. through problem solving.

Learning and Knowing Mathematics for Teaching Mathematics

You think about and do mathematics in different ways: when you were teaching mathematics lessons, when you read mathematics teacher's guides, student textbook pages and/or student responses to math questions, when you were trying to make sense of student responses to problems. In all these ways, you were using and learning mathematics to understand student learning for teaching.

Ball (2003, 2005) explained that it takes considerable mathematical knowledge and reasoning to understand the ways that particular mathematical ideas relate to learning and learners. Teaching is a form of mathematical work, as it involves a steady stream of mathematical problems that teachers must solve (e.g., choosing mathematical representations that are developmentally appropriate for student learning, understanding and relating different student responses to subsequent teaching strategies, choosing mathematics tasks that are challenging and accessible). So, there is a professional knowledge of mathematics for teaching that is tailored to the work teachers do with curriculum materials, instruction, and students.

So, think about how our mathematics work together through my president's message contributes to your learning of mathematics for teaching.

Continuing Our Collective Work to Improve Student Learning and Achievement

OAME's collective knowledge includes our knowing of

mathematics pedagogical content, K to 12. OAME's collective practices includes our advocacy for effective mathematics pedagogical strategies. We have come together in unities (e.g., OAME board, OAME chapters, curriculum writers and reviewers, learning teams in schools and school boards) that have potentialities that are not represented by the individual learners themselves. It is through the individual work of the OAME members and the collective practices of the OAME community that our goal to improve student achievement through improved instruction is being realized.

Come join our mathematics learning community at our upcoming 2006 Leadership Conference. This conference focuses on making sense of the ideas of complex learning systems and applying these ideas to practical and strategic actions for improving mathematics teaching and learning. I hope to meet you there!+



KKZ

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SUDOKU

Play on this grid

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

Instructions

■ There is really only one rule:
Fill in the grid so that every row, every column and every 3x3 box contains digits 1 through 9.

■ This is a row:

	6		1		4		5	

Each row is 9 cells wide. A filled-in row must have one of each digit. Each digit appears only once per row. There are 9 rows per grid.

■ This is a column:

2
8
7
5

Each column is 9 cells tall. A filled-in column must have one of each digit. Each digit appears only once per column. There are 9 columns per grid.

■ This is a box:

	6	
		8
2		

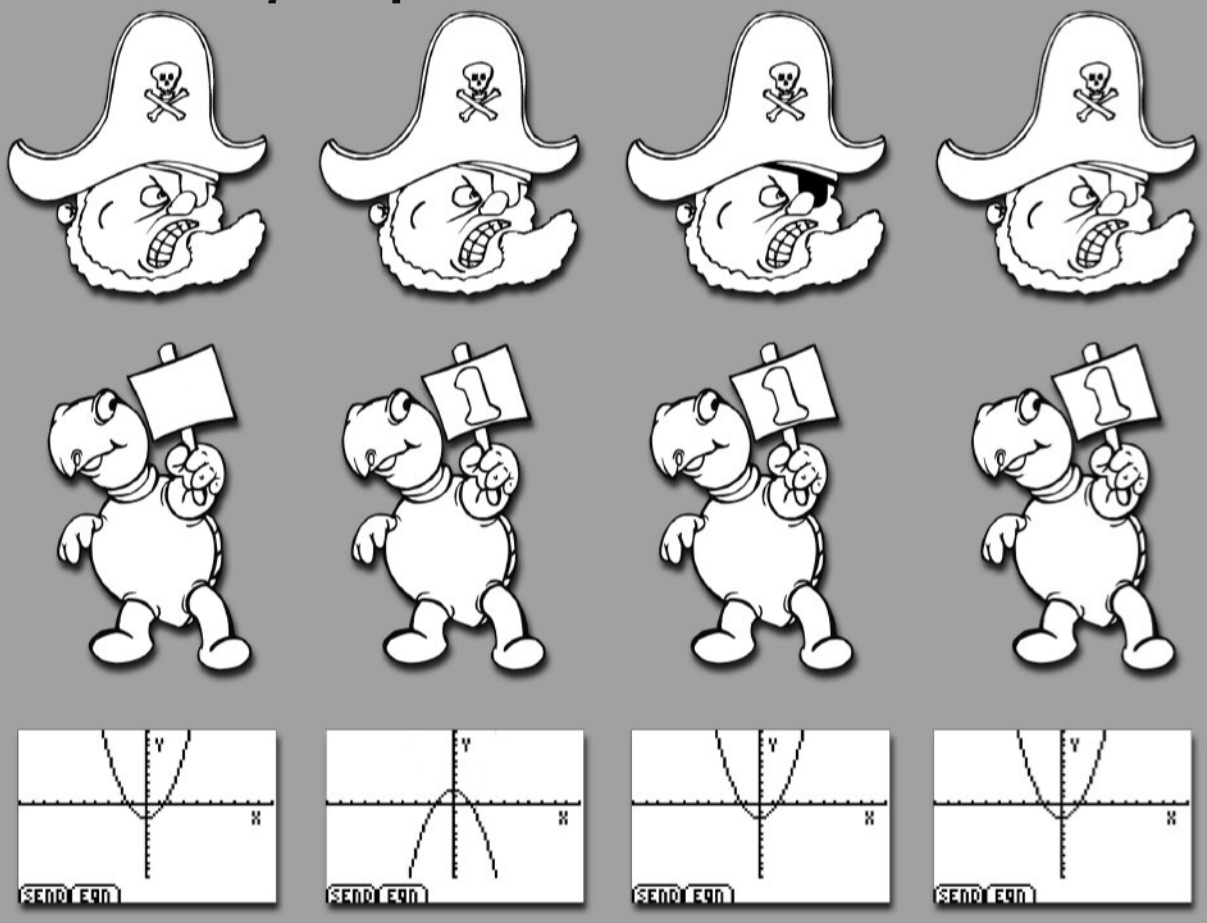
Each box contains 9 cells in a 3x3 format. A filled-in box must have one of each digit. Each digit appears only once per box. There are 9 boxes per grid.

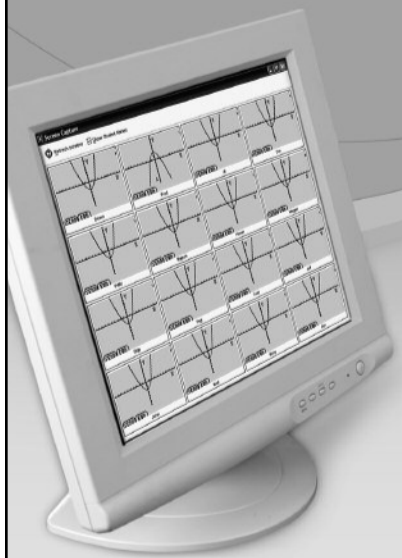
■ You can't change the digits already provided in the grid.

■ Every puzzle has just one correct solution.

Solution on page 34

Quick, can you spot the difference?





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▲ Guest Editorial ~ Whose Mathematics Is It?

Mary Lou Kestell
The Literacy and Numeracy Secretariat
Ontario Ministry of Education



In this *Guest Editorial*, I want to examine some beliefs that I have encountered in the past ten months while working in the Literacy and Numeracy Secretariat and while interacting with leaders in education — specifically those in the Ontario mathematics community. The Secretariat is a strategic arm of the provincial Liberal government that said to taxpayers, "We will have 75% of 12 year olds achieving at standard in reading, writing and mathematics by 2008. Our goal is to raise the bar and close the gap in public education". The Ontario mathematics community to which I refer is composed of colleagues attending OAME annual and chapter mini-conferences, Canadian Mathematics Society meetings, Summer Institutes, Frontier College gatherings, Ministry of Education Regional Office Meetings with directors, superintendents, principals and curriculum leaders and OAME Board meetings.

On October 19th results from the 2004-2005 province-wide assessments in reading, writing and math were released. When I came home from work the next day my husband said, "Help me with this math. The newspaper says there are about 180 000 students at each grade level in Ontario schools. You and your colleagues are Student Achievement Officers. There are about 45 of you. How can 45 people improve the achievement of 180 000 x 12 people?" I laughed. I called my friend and colleague, Kathy Kubota-Zarivnij and asked my husband to ask her the same question. She laughed too.

Kathy and I talked and agreed that, while it has not been articulated clearly or loudly, there is only one way to come even close to achieving our goal: we must turn the heads of all Ontarians to the task of helping all of our children learn to learn. We must all help all of them learn to read, to write, to do math and to think critically. We must do it as directors, superintendents, principals, teachers, students, parents, caregivers, community members, friends and neighbours. We must turn our eyes to the students and ask what each and every one of us can do to make our students intelligent and successful speakers, writers, learners and problem solvers — thinkers in the 21st century. We, especially those of us in education, must act as caring and responsible citizens to bring knowledge, articulation, democracy and civility to our communities.

To quote the Secretariat's Chief Student Achievement Officer, Dr. Avis Glaze, "Public education is a public good. We must deliver on our promise to educate all learners successfully. We are engaged in the most important of enterprises — that of public and universal education. As each generation matures, intensifies, let us not be guilty of romancing mediocrity. One of the resources children have is time — and that time in elementary and secondary schools is finite. Neither they nor we can afford to waste that time. Every moment counts when we are engaged in the process of educating young people".

So as I pick up the challenge of the work at the Secretariat, I, of course, focus on what we can do to engage the education community in working to help children be successful in math.

The NCTM Curriculum and Evaluation Standards for School Mathematics advocates "mathematics for all" as a central idea in education reform, but this slogan can be used to champion two very different views of mathematics and how it should be taught. As Jay Hill observes:

1) mathematics as a culture in and of itself into which the student is to be "enculturated" (as mentioned by Robert Davis in "The Culture of Mathematics and the Culture of Schools") or
2) mathematics as an invention of the student who is inextricably linked to his or her own culture.

What is the better alternative in teaching mathematics – "ideology or reality?". The first has certain aesthetic appeal, especially for those of us who found early success in mathematics and studied it at university. However, the latter view is a more effective vision for raising the level of mathematics education in this province. It is the second view of mathematics that is represented in the Ontario curriculum documents and the Expert Panel reports on numeracy and mathematics. It is the second view that we must engage our students, teachers, principals and superintendents in to be successful at bringing our discipline to the masses. We have generations of people who have been exposed to the theoretical world of math in our schools and openly

show disdain and/or anxiety when the "Math" word is mentioned in polite company. Very few people in Ontario's general public are proud and eager to share stories of their experiences with mathematics learning. What good is mathematics for all if only an elite few actually have use for it in the future? By focusing upon the theoretical and ideological aspects of the mathematics culture, we separate the subject from the very reality it is used to explain. Whose math is it anyway?

Students bring intuitive knowledge of mathematics through their everyday interaction with their local community and culture. They use their prior knowledge and through interactions with the teacher, other students and mathematics build further knowledge. For this learning, students do not depend on explicit teacher instruction; students (individually and collectively) have the capacity and propensity to model, reason and make connections to mathematical thinking. In fact, students use physical, pictorial, numerical, or symbolic representations to show their mathematical ideas, strategies and solutions to problems. Yet, students' socially constructed mathematical knowledge is focused and facilitated by the teacher.

In a problem-centred classroom, the teacher facilitates the interaction between the students and the mathematics rather than acting as a disseminator of knowledge. This requires that teachers know mathematics and can hear the different thinking that students apply. Then they can coach students to communicate and make mathematical structures and processes explicit. This honours individual thought and engages every student in learning. Students encounter problems and identify the mathematical thinking that models the situation. They move back and forth between the mathematical concepts and their interaction with their perceptions and interpretations of the environment.

The learning continues from connecting to and building conceptual understanding to the analysis of algorithms and viewing more efficient ways to engage in doing the math. An algorithm is a structured series of procedures that can be used across problems regardless of the numbers. Algorithms promote accuracy and are seen to be efficient. There is a common misconception that the algorithms one learned as a child are the only acceptable ones to be passed on to the next generation of learners, or that an algorithm presented in a single math textbook — one that functions simply to represent some thinking — is the only one accepted by the teacher.

Consider the two algorithms below. Analyze each. Then try to justify why one is valued more than the other.

Traditional algorithm

```
  1  1
  1 358
+ 908
  2 266
```

Partial sums addition algorithm

```
  1 358
+ 908
  1 000
  1 200
     50
+    16
  2 266
```

What matters most is that the child can understand and owns the knowledge when he or she walks away from a lesson. I have said, after working seven years at EQAO and seeing thousands of papers from grade 3

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and grade 6 students, "The subtraction algorithm is harmful to a child's mathematical health. And the long division algorithm is so dangerous that when it is forced upon the students, it becomes the time many of them decide they will never be a part of the 'math crew' and they are out of there forever". On the provincial tests, we see work from students who have been taught algorithms but do not have power over them. They carry "1s" (ones) all over the place and in ways that reveal deep misconceptions about what operations processes the algorithms are meant to represent.

In my presentation at the 2005 OAME conference, I had a gentleman in the group who was doing PhD research comparing Canadian math, science and engineering education to that in Germany. He told us that he had thought that science and engineering would be completely different in the two countries but that he had expected math to be the same. Then he found out that we and the Germans teach completely different algorithms. After we huddled and figured out why his algorithms worked, we talked about why one was better than the other. We concluded that an algorithm is only as good as the meaning and understanding that it brings to its user. If you don't follow the logic, you are likely to forget how to use it. We also learned that with children from many different cultures in our midst, we ought to honour all algorithms that make sense to the user and encourage students to share all kinds of algorithms from their own families.

In Lev Vygotsky's work on how children learn, we can find statements such as, "Thought is not merely expressed in words; it comes into existence through them. The speech structures mastered by the child become the basic structures of his thinking. The structure of the language one habitually uses influences the way she perceives her environment". This says very loudly to me that we need to hear students talking about their thinking in math class, sharing their strategies, learning to re-present their thinking and learning all along the way. This is mathematics that matters to their culture, allows them to connect to it, value it and want to learn more. It becomes a tool of their success.

By focussing upon individual cultures' interpretations of mathematical concepts, the curriculum can build upon the pre-existing foundation of knowledge owned by students. Students can then develop their understanding of mathematics within their culture (the culture that will mold the rest of their lives) and make the field of mathematics truly for all.

This is a rallying cry. Let us set the lens on student success. Let it be our goal to have students say, "I love math. Give me more". Let us be so good at our jobs that students demand richer math courses — opting for more math rather than dropping out. It is up to leaders in the mathematics community to bring together the teachers to learn mathematics for teaching mathematics. This whole initiative is about empowering all educators to help children learn. Let us focus on research and student achievement data to see what to do next. Let us turn that data into information: using processes and tools of mathematics to gather, organize, sort, synthesize and analyze what students need from us in order to succeed. Let us adopt Premier McGuinty's goal of having 75% of them there by 2008.

Ron Edmonds, founder of the Effective Schools Movement said many years ago, "We can, whenever and wherever we choose, successfully teach all children… . We already know more than we need to in order to do that. Whether or not we do it must finally depend upon how we feel about the fact that we haven't so far".

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▲ Letters to the Editor

Dear Editor,

I was thrilled to read the articles by Jack Weiner and Lyn Vause in your September 2005 issue of the *Gazette*. Lyn reviewed *Young Mathematicians At Work: Constructing Multiplication and Division* by Catherine Twomey Fosnot and Maatenn Dolk. Both Jack Weiner and Catherine Twomey Fosnot will be keynote speakers at **OAME 2006: Every 1 Counts**.

Thanks goes out to *OMG* for highlighting these two individuals. The articles help introduce these amazing instructors to OAME members.

We look forward to seeing all of you in London for OAME 2006. Watch for registration to open in early February.

Sincerely, Ann Michele Stenning
OAME 2006 Social Convenor ▲

Dear [Editor]

I finally got round to reading the September 2005 issue of the *Gazette* last week. (Being retired now I have trouble finding time to do everything I would like to do!) and I just wanted you to know that I look forward to reading [Lynda Colgon's column] in each issue. I particularly enjoyed this last one and I guess that's what prompted me to write. For many years I took WOMA teams to the Math Olympics and so remember watching [Lynda] in action in Scarborough in 1996 and again in Kingston in 2000. Although retired from teaching Grade 4, I still continue teaching mathematics on a volunteer basis and try to stay current with mathematics teaching ideas. I hope [Lynda] finds the time and the enthusiasm to continue submitting articles to the *Gazette*.

Sincerely, Marg Kemp ▲

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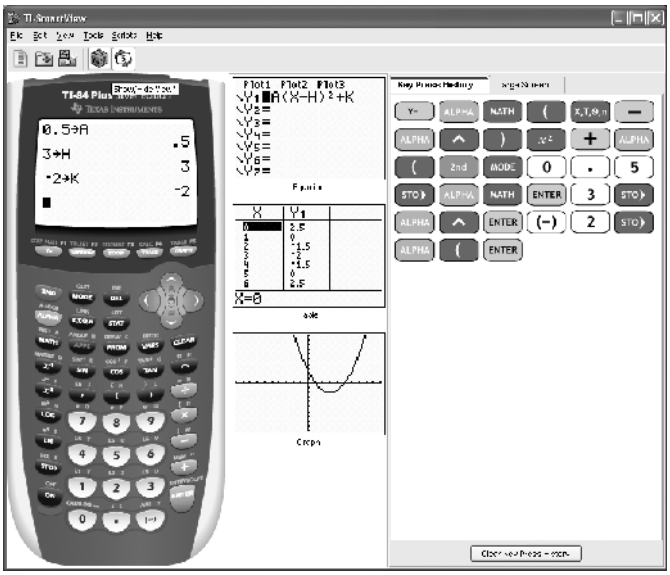
▲ Technology Corner: TI-SmartView

MARY BOURASSA



Mary Bourassa teaches mathematics at Lisgar Collegiate Institute in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She is also an OAME director, president of COMA, gives workshops around the world and is a National Instructor for Texas Instruments.

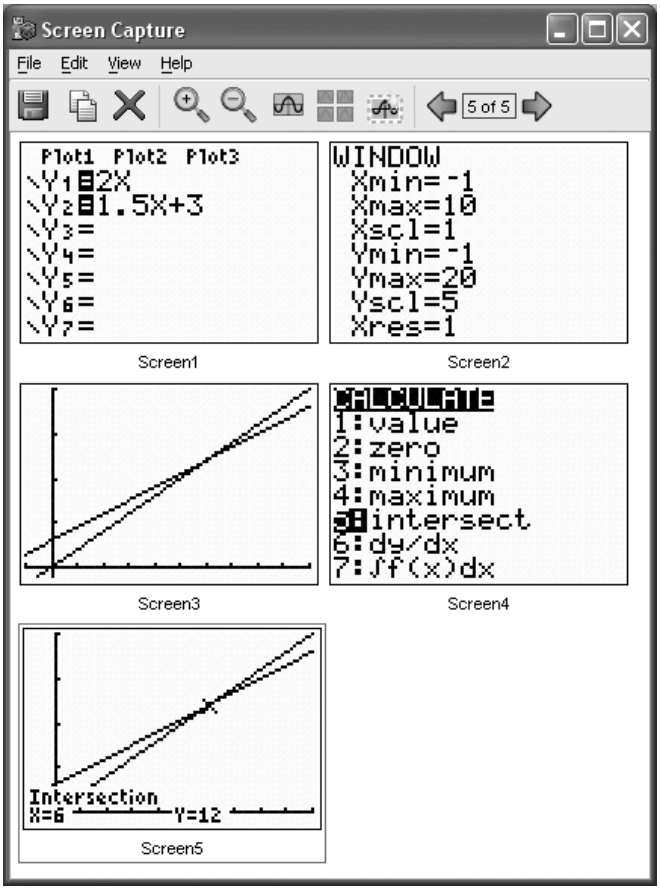
Texas Instruments recently released emulator software for the TI-84 family of graphing calculators. This software program, named TI-SmartView, has a number of excellent features for the classroom teacher with access to a computer and projector. Instead of having to connect an overhead graphing calculator to the viewscreen palette on an overhead projector and only have the screen of the teacher calculator visible, you can now press the keys directly on the calculator emulator. This means that you no longer have to dance back and forth between the calculator and the poster to demonstrate which keys are being pressed. The following screenshot shows many of the features of TI-SmartView.



You can choose to see in View^3 mode which shows the equation editor, table(s) of values and graph all at once! This can also be hidden as there will be times when you won't want your students to see all of that

information right away. You can also show a large version of the calculator's screen so that even the students at the back of the room can see what is being done.

The Key Press History shows all of the keys that have been pressed – an excellent way of helping those students who have fallen a step or two behind. They even match the colours of the keys on the calculator. This history can also be copied and pasted into other documents to help you provide better instructions on worksheets.



TI-SmartView also gives you the ability to easily take screen shots without having to connect a calculator to your computer. No more wires to worry about! The screen shots can then be dragged into another program, such as Word.

If you don't have time to finish the activity you are doing one day, you can save the state of the emulator so that you can start where you left off the next period. You can save as many states as you like so that you are always ready for the next class.

Scripts allow you to save a series of commands which will be executed in that order when run, rather like a video of the keys being pressed. A script can be run at regular speed, in slow motion or step-by-step. This allows you to set up a complicated process ahead of time and frees you up so that you can walk around the room as your students are stepping through the process. There are a number of scripts that come with the software.

If you happen to be fortunate enough to teach with a SMART board or other interactive white board, TI-SmartView is that much cooler! You can now actually push the button on your calculator as the students do the same.

This emulator is very powerful and changes the dynamics of teaching with a graphing calculator. And best of all, you can change the colour of the faceplate just like you can on a real TI-84! ▲

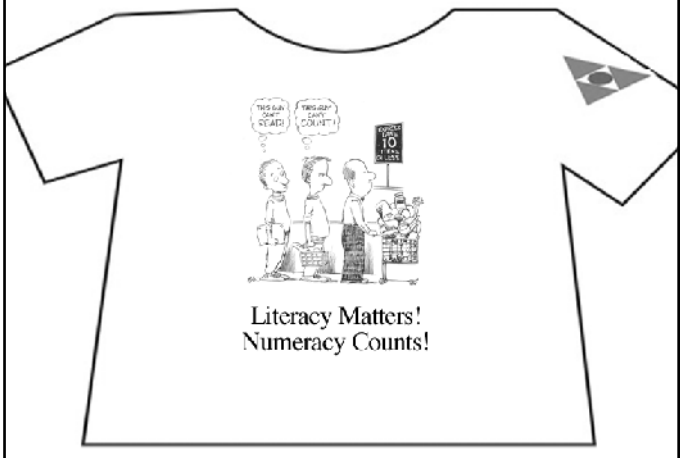
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OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration:

- demonstrations of excellence and creativity in mathematics education;
- specific contributions to mathematics education in the school, region, province or nation;
- demonstrates leadership in mathematics education in the school, region, province or nation;
- support of and contribution to the activities of OAME/AOEM, and
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

OAME/AOEM Life Membership Award

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and has accumulated 10 or more years of membership in OAME/AOEM.

Kenneth D. Fryer Award

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Award for Outstanding Contribution to OAME / AOEM and Mathematics in Ontario

This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics

The award is presented annually to recognize an exceptional and creative elementary teacher who demonstrates excellence in mathematics education by contributing to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics

The award is presented annually to recognize an exceptional and creative secondary teacher who demonstrates excellence in mathematics education by contributing to the overall development of students.

AWARD NOMINATION COVER PAGE

INTENDED AWARD *(please check one)*

- ❑ **Life Membership**
- ❑ **Kenneth D. Fryer**
- ❑ **Award for Outstanding Contribution to OAME / AOEM and Mathematics Education in Ontario**
- ❑ **Award for Exceptional and Creative Teaching in Elementary Mathematics**
- ❑ **Award for Exceptional and Creative Teaching in Secondary Mathematics**

Nominee's Information:

_____

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

_____

_____

School, Organization, or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

Nominator's Information:

_____

Name of Nominator

_____

_____

School, Organization, or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

SEND TO: Jacqueline Hill

▲ Hey It's Elementary: Mathemagical Funderstanding Capitalizing On The Mathematics Puzzle Craze In Our Classrooms



LYNDA COLGAN

I saw it being done openly by a clerk in a book store in town just the other evening. Not too long before that, I saw it being done surreptitiously at a corner table in *Tim Horton's.* I myself have done it in full view at the beach. And according to many reports, a lot of people find that they are so addicted to it, that a day just isn't complete without doing it at least once. The question is: are you too ready to sudoku?

If you are still scratching your head wondering what all the fuss is about, I'll state it simply. People everywhere of all ages, shapes and sizes are doing … of all things … math (well, logic and reasoning) in public places. And they're doing it for fun.

Sudoku is based on a game called Latin Squares invented by 18th-century mathematician Leonhard Euler. This type of puzzle has been around in its present form since the late 1970s, when it was introduced in the magazine "Math Puzzles and Logic Problems". It first appeared as a sudoku (sue-DOE-koo) puzzle in April 1984 in Japan with a title, which can be translated as "the numbers must be single" or "the numbers must occur only once".

The beauty of sudoku is that people of all ages can do it because the rules are simple. All you need is a few minutes (about 20 should do it), a pencil (and eraser) and some concentration. The classic grid layout of the sudoku is reminiscent of other newspaper puzzles like crosswords and chess problems. A typical puzzle consists of a nine-by-nine grid. Some of the spaces contain numbers; most of the spaces are blank. The player's goal is to fill in the blanks with digits from 1 to 9 so that each row, each column, and each of the nine three-by-three blocks making up the grid contains just one of each of the nine digits. The grid has numbers, but nothing has to add up to anything else. Depending on the number of clues and the size of the grid, sudoku puzzles can be fairly easy or extremely difficult to solve.

Here are three puzzles for you to try. Remember the object of sudoku is to fill in the grid so that every row, every column, and every 3 x 3 box contains the digits 1 through 9. You can't change the digits already provided in the grid. Every puzzle has just one correct solution. There's no right or wrong place to start a sudoku puzzle. The best way to learn is simply to jump in and play. Use your mental reasoning, process of elimination, logic, lots of patience, and trial and error to work out which numbers go where.

			1			7	4	
	5			9			3	2
		6	7			9		
4			8					
	2						1	
					9			5
		4			7	3		
7	3			2			6	
	6	5			4			

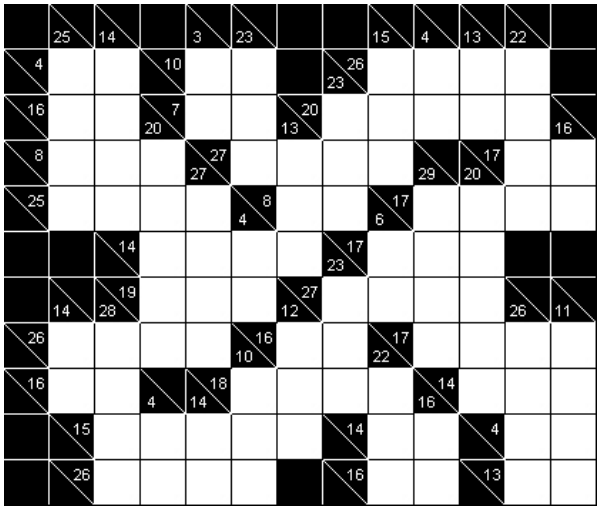
	8				7			
5	2	4					6	3
						5		8
	5	1						4
			5	8	6			
9						6	3	
8		9						
3	6					4	8	9
			6				5	

				2	3	4		
	6	3		9	8			
4			5					
	2	5		8			7	3
	1						5	
6	4			5		1	9	
					5			8
			9	7		3	6	
		6	8	3				

The good news is that teachers across the continent are cutting sudoku puzzles out of the newspapers and surfing the web daily to download problems of different difficulty levels to use in the classroom. The great news is that the sudoku phenomenon is being embraced by large numbers of elementary school children. They may never know that they are learning perseverance and applying high-level problem-solving skills. All they want and need to know is that they are having fun!

With all the focus on curriculum expectations, levels of achievement, province-wide testing and the rigorous curriculum, math fun has not been a priority for many educators. But, it seems that the outside world may be giving us a push to return to including some mathemagical funderstanding in our programs.

In addition to Sudoku, most national, regional and local newspapers have expanded their standard puzzle page to include more than standard crosswords. You don't have to look far to find *Math Squares, Magic Squares,* or *Cross Sum Puzzles* to keep you challenged during your morning coffee.



Cross Sums™ is a puzzle that is solved much like a crossword puzzle, using digits instead of letters. Where an entry in a regular crossword may form a word, the digits in a Cross Sum entry add up to a sum. The sums or clues, are embedded in the dark squared of the puzzle immediately to the left of an across number or immediately above a down number. Important! Only digits 1 through 9 are used and no digit may appear more than once in any one number.

23	6	19	2	15
10		1		
	5			9
4	12		8	
11		7		3

Can you finish off the 5 by 5 magic square above? Remember that each of the numbers 1 to 25 is used once and only once.

To capitalize on the puzzle craze, and get children of all ages to do some mathematics when they least expect it, we can also turn to the history books. Here are a few funderful and funtastic examples that I hope you will try with your students. Physicians remind us that laughter is often the best medicine. Maybe an injection of fun and laughter into the math class is exactly what the doctor ordered!

Algorithms for Multiplication During Ancient and Medieval Times

Most of us probably learned to multiply large numbers this way:

$$\begin{array}{r} 86 \\ \times\ 57 \\ \hline 602 \\ +\ 4300 \\ \hline 4902 \end{array}$$

Once we learned our basic multiplication facts, we were taught the school way for multiplying big numbers. For many of us, this algorithm is a quick and relatively simple series of steps that we follow automatically.

What most of us never learned was that there are many other ways to multiply big numbers. One of these methods is often called the Russian peasant algorithm or sometimes Medieval Multiplying. You don't need the complete times table to use Russian peasant multiplication because the whole process is made up of doubling numbers, cutting them in half, and adding them up. The rules are simple:

1. Write each number at the head of a column.
2. Double the number in the first column, and halve the number in the second column.
3. If the number in the second column is odd, divide it by two and drop the remainder.
4. If the number in the second column is even, cross out that entire row.
5. Keep doubling, halving, and crossing out until the number in the second column is 1.
6. Add up the remaining numbers in the first column. The total is the product of your original numbers.

Let's multiply 57 by 86 as an example:

~~57~~	~~86~~
114	43
228	21
~~456~~	~~10~~
912	5
~~1824~~	~~2~~
+ 3648	1
4902	

Historical records tell us that real Russian peasants may have kept track of their doublings with bowls of pebbles, instead of columns of numbers.

Russian peasants weren't the only ones to use this method of multiplication. The ancient Egyptians invented a similar process thousands of years earlier, and computers are still using related methods today.

Egyptian Multiplication

The Rhind Papyrus is a part of the largest surviving mathematical scroll and dates to 1832 B.C. The papyrus shows how Egyptian engineers calculated the proportions of pyramids. It also shows how the Egyptians did addition, subtraction, some division and multiplication. The surprising thing about their system was that they only multiplied and divided by two. Here is how to multiply 41 by 59 the Egyptian way:

41	**59**
1	59
2	118
4	236
8	472
16	944
32	1888

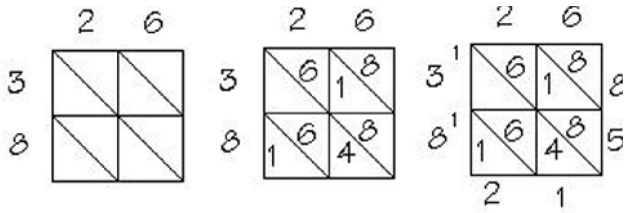
Double the numbers in each column. Stop at 32 since 64 is bigger than 41.

1	59	✓
2	118	
4	236	
8	472	✓
16	944	
32	1888	✓
	2419	

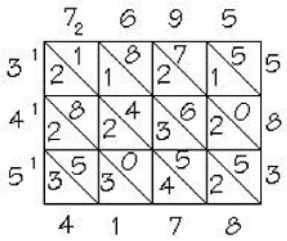
Since 41 = 32 + 8 + 1, we sum the numbers that are the appropriate rows to get the final product.

Gelosia Multiplication, Lattice Multiplication And Napier's Bones

With the introduction of Arabic numerals, Europeans began to compute more and more complex calculations. One example is long multiplication, for which the jalousia (or gelosia) method was used. The method is so called after the iron grill, or jalousia, Italian men would place over their windows to keep strangers from staring at their wives. Here is how you would multiply 26 times 83 using the gelosia method.



Step 1. Write the numbers on the edges starting at the bottom left hand corner and continuing in a clockwise way. Step 2. Multiply all possible combinations and put the answers into the square like in a times table. Step 3. Now add along the diagonals. Then you can read the answer from the bottom around counter clock wise as 2158. The little ones are the carried numbers for the adding.



Here's how to multiply 543 x 7695 using gelosia.

Lattice Multiplication

The lattice method of multiplication appears in the first printed arithmetic book, printed in Treviso (Italy) in 1478. It shows this exact method and a variation as well as some variations of the long multiplication algorithm commonly taught today.

Lattice multiplication and variations of today's standard long multiplication were introduced into Europe by Fibonacci (whose correct name is Leonardo of Pisa). He was an Italian who learned about Arabic numerals from a Moorish teacher in North Africa. As we saw with Medieval Multiplying and the Egyptian method, before the Hindu-Arabic system was used in Europe, multiplication was often done with counters because Roman numerals were ill-suited to calculation and very few people knew how to multiply.

In lattice multiplication, the partial products are laid out in a lattice and adding along the diagonals gives the answer to the multiplication.

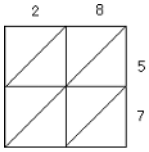
The lattice method of multiplication is illustrated by

the following examples.

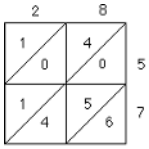
Example 1

28 x 57

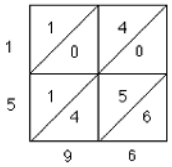
As 28 and 57 have two digits each a lattice is set out with two columns and two rows. The diagonals are drawn in each cell as shown below. 28 is written above the lattice with 2 above the first column and 8 above the second. 57 is written to the right of the lattice with 5 along the first row and 7 along the second.



The partial products of these digits are set out in the corresponding cells with the tens above the diagonal and ones below. eg. the partial products in this case are 5 x 8 (= 40), 5 x 2 (= 10), 7 x 8 (= 56) and 7 x 2 (= 14).These are set out as shown below.



The sum along each diagonal is then recorded as shown below and these digits 1, 5, 9 and 6 form the answer to the multiplication.

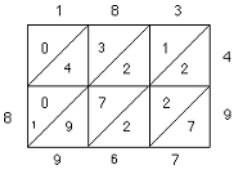


Thus 28 x 57 = 1596

Example 2

183 x 49

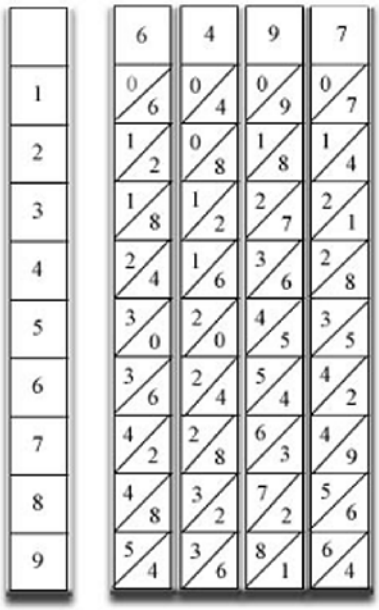
The lattice set out for this multiplication will have 3 columns and two rows as 183 has 3 digits. As before the numbers are set out as shown below and the partial products are written down in their respective positions. The numbers along the diagonals are added to give the answer.



Note that in this example adding along the third diagonal gives 19 which needs 1 to be carried to the diagonal above it. Therefore the addition should begin with the lowest diagonal (the product of the ones from the two numbers). The same principle lies behind Napier's rods.

Napier's Rods

John Napier's rods (1617), also known as Napier's bones, are one of his most important contributions to the world of mathematics and alongside William Oughtred's invention of the Slide Rule (1615), represents one of the most revolutionary developments in calculation devices since the abacus. The rods were basically multiplication tables inscribed on sticks of wood or bone. In addition to multiplication, the rods were also used in taking square roots and cube roots.



9	X	6(1000)	=	54000
9	X	4(100)	=	+3600
9	X	9(10)	=	+810
9	X	7(1)	=	+64
9	**X**	**6497**	=	**58474**

By looking at the representation of Napier's rods on the left you'll notice each rod is inscribed with a multiplication table of a number from 0 to 9. The rod to the extreme left, with numbers from 0 to 9, is used as an index for quick reference. By placing any of the rods side by side, one is able to multiply larger numbers with equal ease. In this case, add the thousands, hundreds, tens and single units to the numbers appearing in row 9, as per the example.

There are lots of matemagical ways to multiply. the first rule for all of them is: have fun. ▲

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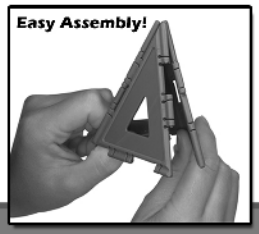
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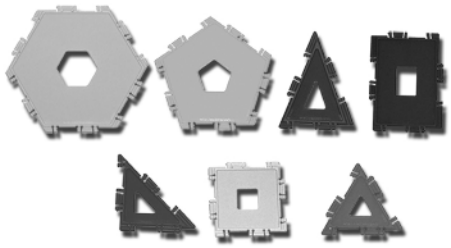


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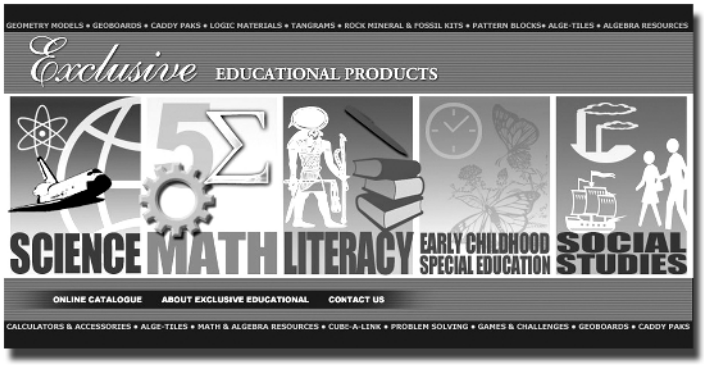
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▲ Friday Special!

JACK WEINER



Fridays at the University of Guelph, five minutes before the end of class, I say, "Pens down, books closed, it's "Friday Special" time!" Then I show on screen an article or cartoon or letter or…something that has to do with math (well, usually), the "real world"(well, most often), and has absolutely nothing to do with class, assignments, or tests (always!) It is math for fun. Often, it is some flagrant example of how the media abuse math. I hope you use some of these. Make them "Monday Morning" specials or "We need a break!" specials. If one falls flat, blame me. If it works, you take the credit.

There was an interesting and rather sad story about six months ago that involved a minor consumer revolt. An elderly gentleman (that's middle-aged hubris talking) was incensed that the price of gas at the pumps had shot up to the stratospheric heights of 91 cents per litre. He filled his tank and, with excellent numeracy skills, determined the cost of his purchase at 70 cents per litre. He paid this amount and, to the amazement and chagrin of the employee at the kiosk, drove off.

Yes, the employee wrote down and reported the license number to the police. The authorities in turn had no choice but to charge the gentleman with theft.

Thankfully, sane heads prevailed. The fella's son, upon hearing of the details, came forward and paid the difference. All was forgiven. Well, 91 cents per litre now sounds like a bargain. It didn't and doesn't take us a long to become inured to a new reality. What I don't understand is that, as I write, the price per barrel of oil is now below pre-Katrina prices and yet the price per litre of gas is definitely way up from pre-Katerina prices. We went from 70 cents to $1.40 and are relieved to be paying a just under a buck a litre.

With this background, I thought it would be timely to share this photograph, reprinted from the *Toronto Star*.



A gas station at Jarvis and Isabella Sts. sells gas for 14.4 cents a litre in April 1975. As Toronto prices hit an average yesterday of 91.5 cents, one small business owner who uses her car daily said, "I'm outraged. Don't get me going."

Show this to your classes and ask your students for the evidence that the picture is ancient!

Reason 1) Gas per litre at 14.4 cents!

Reason 2) The 1970's vintage cars!

Reason 3) The sign hailing "Canada's First Metric Gas Station"!

Of course, none of these reasons can be considered conclusive. Again, ask why.

Reason 2) It could just be a coincidence that the 2005 photograph caught only 1970's cars in its composition!

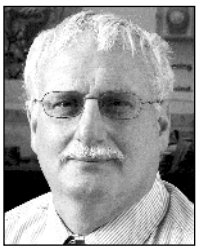
Reason 3) The station might have continued to display a commemorative sign of that significant distinction! And last…

Reason 1) Many gas stations have signs that are not able to post prices over $1 per litre. That station could be selling at $1.144 per litre!

Did I miss anything?

Best wishes for the holiday season, Jack ▲

▲ OAME/NCTM DECEMBER 2005 REPORT



JOHN KERSLEY

NCTM's Theme for 2005-2006 is Entitled "Assessing to Learn and Learning to Assess"

In brief, the focus is assessment. See **www.nctm.org/focus**. In her NCTM President's Address, Cathy Seeley said, "When we assess to learn, we seek information that allows teachers to find out what students know in order to improve their mathematics learning. This means learning to assess all the kinds of mathematical knowledge we identify as important. By assessing to learn and learning to assess, we can maximize the positive impact of assessment on students' learning without unnecessary negative consequences." and further adds, "Teachers and students can benefit from assessment results that tell what a student knows and that identify a student's potential misunderstandings. The most useful assessment results for directly influencing students' learning are those that are immediate and specific." For her full address on this issue, again go to **www.nctm.org/focus** and scroll down.

NCTM resources for assessment include:

- *Mathematics Assessment Sampler Grades 3-5* Stock #12937C10
- *Assessment Standards for School Mathematics* Stock #593C10
- *Mathematics Assessments: Myths, Models, Good Questions and Practical Suggestions* Stock #505C10

I strongly suggest that one carefully read Chris Suurtamm's columns on assessment ideas, theories, and practices which have been published since 2002 in issues of this publication, the *Ontario Mathematics Gazette*. Her column is entitled "Assessment Corner". She has lots to offer, to think about and to do.

New NCTM publications: (www.nctm.org/catalog)

- *Mission Mathematics Series* Prekindergarten-grade 2, grades 3-5, grades 5-8, grades 9-12
- *Navigating through Measurement Series* grades 3-5, grades 6-8 grades 9-12
- *Navigating through Problem Solving and Reasoning in Grade 4* Statistical Questions from the Classroom.

In order to encourage new members, it is very common for companies to offer incentives to get people to join. As mentioned in my last report in the September issue, NCTM is offering rebates to affiliates for members who join NCTM through the affiliates. In addition, NCTM is offering a $10 gift certificate for every five new members recruited and a chance in a draw for further prizes. One can join on line at **www.nctm.org/membership/application** and include the sponsoring member's name and member ID.

Become an Advocate for Mathematics Education

The Advocacy page at **www.nctm.org** (look for "to be an advocate" on the NCTM's website) allows one the chance to actively support mathematics education, to speak out on mathematical issues and to take part in policymaking process. I might also add that OAME is always looking for people to share their views on

▲ UPCOMING EVENTS

For further information on OAME Chapter Conferences contact the Chapter Representative and for NCTM conferences, contact John Kersely, OAME/NCTM Rep. See the last page in this *Gazette* for contact information.

Chapters now have websites that are linked to the OAME website: see these for full information on upcoming events.

OAME Chapter Conferences and Events

Dec 8 2005	Math Marketplace Philip Peacock CHS	CHAMP
Early February 2006	mini conference	SAME
March 2006	mini conference	TEAMS
May 26/27 2006	OMO 2006 U of T: Scarborough Campus	SAME

OAME Annual Conference:

May 11 - 13 2006	Fanshawe College "Everyone Counts"	WOMA

NCTM Annual Conference

April 26 - 29 2006	St Louis MO	

mathematics education, especially relating to mathematics in Ontario. Any one wishing to do so, contact Marilyn Hurrell, editor of this publication, the *Ontario Mathematics Gazette* (see page 1)

NCTM is introducing a new Outreach Program

TI in partnership with NCTM and "Numb3RS", the Paramount Network Television Series are beginning a new venture designed to support teachers in providing interesting classroom activities developed by NCTM and Texas Instruments. For more information the website is **www.nctm.org/news/2005_09numb3rs.htm**

NCTM supports the Professional Development of mathematics teachers in three areas:

1. EVENTS: NCTM regional conferences, the NCTM annual conference and a listing of affiliates conferences **www.nctm.org/affiliates/confernces.htm**
2. RESOURCES: Both print and electronic resources are available. The important NCTM resource is the *Principles and Standards for School Mathematics*. Simply go to **www.nctm.org** and explore. For print material, try **www.nctm.org/catalog**
3. SERVICES: NCTM offers professional development services for teachers and teacher education programs.For more information, the website is **www.nctm.org/profdev**

Websites of Interest to Mathematics

- A+ Compass **www.apluscompass.com** A precision math compass for drawing geometric constructions on whiteboards, overheads or desktop.
- Math Matters **www.mathmattersonline.com** math challenges for grades 2 to 5
- Middle Web **www.middleweb.com/mw/aaAboutMW.html**

Resources for schools, educators and parents working to improve achievement for students.

- One mathematical Cat **www.onemathematicalcat.org** Teachers have access to over a hundred web arithmetic, algebra and calculus exercises with printable worksheets and solutions.

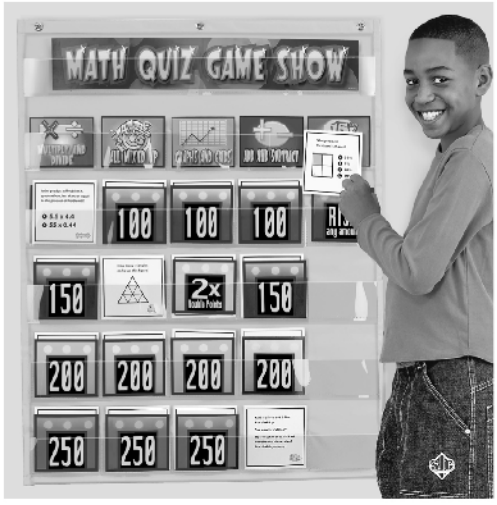
NCTM Annual Meeting

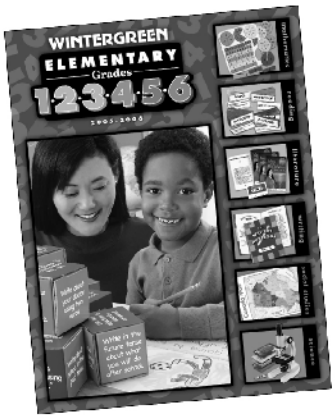
ST. Louis **Asking Questions–Generating Solutions** April 26-29, 2006 ▲

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▲ Get Interactive!

Louis Lim



Louis Lim is a part-time mathematics teacher at Unionville HS and a PhD student at York University. He is currently an editorial panel member of the NCTM's Mathematics Teacher *and is a former editor of the OAME* Gazette*. In the spring 2005, he was the recipient of OAME's "Exceptional and Creative Teaching in Secondary Mathematics".*

I am excited to take over the *Get Interactive!* column. Below are websites that focus on using the media (movies, television, newspapers) to make mathematical connections. While viewing the websites, think about its potential to the teaching and learning of mathematics.

Numb3rs, currently in its 2nd season on CBS television (Fridays, 10:00 pm), features FBI agent Don Eppes working in collaboration with his mathematician brother, Charlie, to solve crimes (based on actual FBI cases). The purpose of *Numb3rs*, endorsed by the NCTM and Texas Instruments, is to portray mathematics as useful and important to the general public. Lesson plans, based on the mathematics in the television program, are available on the website for grades 7-12 teachers during the week of the actual airing of each episode. Teachers can sign up on-line for a free teacher kit, including a poster of "We All Use Math Everyday" as well as receiving up to 75 stickers (bookmark shaped) to distribute to students. Monday math classes can begin with a class discussion on the latest episode of *Numb3rs*! **http://www.cbs.com/primetime/numb3rs/ti/**

The **Math in the Movies** website contains an extensive collection of motion pictures, shown at movie theatres (now available on DVD), that contain scenes pertaining to mathematics. At the time of writing this column, *Proof* is in movie theatres. Particular movie snippets that can be shown to students include: *I.Q.* (Zeno's paradox), *Man Without a Face* (finding the centre of a circle; *Pythagorean Theorem*; volume), and *Sneakers* (cryptography). **http://world.std.com/~reinhold/mathmovies.html**

If you have read the *USA Today* newspaper, you are probably familiar with the "Infographs" and "Snapshots" that appear in the bottom corner of the first page to each section of the newspaper. This website (see below) contains 20 lesson plans (grades 5-12) that include teacher notes, student handouts, transparency versions of the "Infographs" and "Snapshots", correlations to the NCTM's standards, instructional strategies, solutions, and suggestions for incorporating technology and assessment. The focus is on interpreting and analyzing charts, graphs, and figures. **http://www.usatoday.com/educate/mathtoday/**

Lesson plans have been created from newspaper articles published in the *New York Times*. A collection of 50 mathematics lesson plans, suitable for grades 6-12, are provided. Students read the actual newspaper article and then answer a series of questions – a great way to promote literacy in mathematics! The lesson plans contain opportunities for group work, are interdisciplinary in nature, and contain extension activities. Hopefully, students will seek out mathematical connections when they read newspapers on their own. With the holidays quickly approaching, why not incorporate "Getting Green for Christmas" (December 18, 2003)? **http://www.nytimes.com/learning/teachers/lessons/mathematics.html** ▲

Ontario Association for Mathematics Education
PUBLICATION

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▲ Photo Math - A New Way of Picturing Math

RON LANCASTER
INDEPENDENT MATHEMATICS CONSULTANT



Ron Lancaster is a Lecturer in Mathematics Education at the Ontario Institute for Studies in Education at the University of Toronto. He is also an independent mathematics consultant and works with teachers, students and educational organizations in North America, Asia and Israel.

"A picture is worth a thousand words" is an expression that is often used to convey the importance of using images to make a point. The point of this column is to go beyond this notion and promote the idea that in fact "A picture is worth a thousand numbers". Photos with Math is what readers can expect to find in every Photo Math column and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario curriculum as well as the new NCTM document, *Principles and Standards for School Mathematics*. It is hoped that these questions will inspire students to be more creative and playful and that in the end, students will develop an ability to see the world from a different perspective.

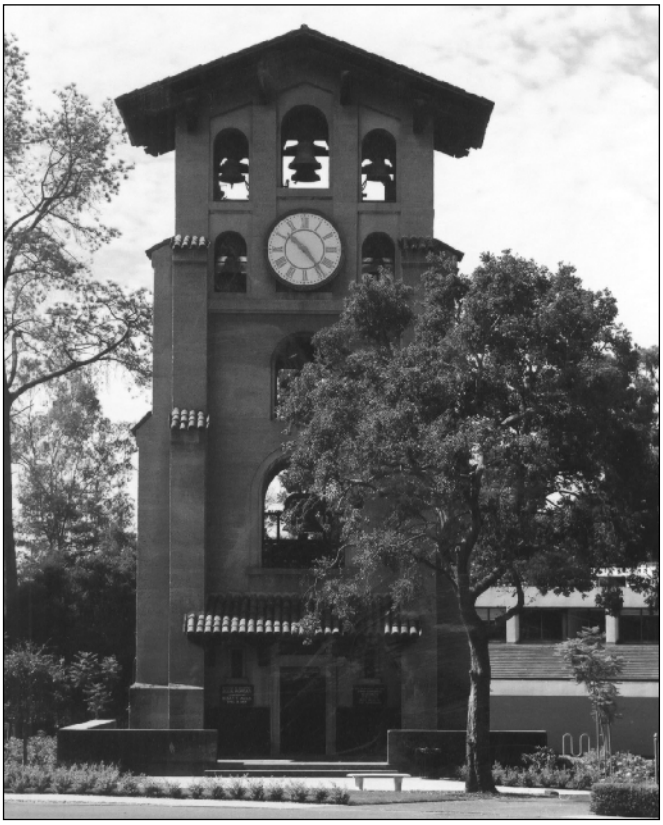
Teachers and students are encouraged to pose mathematical questions related to their own encounters and to submit these problems along with photographs to the author for possible publication in future columns. Photos can be sent via e-mail or by regular mail to the author of this column.

Introductory Comments

Two summers ago I was a participant in the Master Class on Sketchpad, a week long conference organized by Key Curriculum Press and held on the campus of Mills College in Oakland, California. The course was taught by Nick Jackiw and Nathalie Sinclair. It was an outstanding professional experience and I learned so many new things about using Sketchpad.

During my free time I created a Math Trail on the campus of Mills for the other participants. The photos and questions that appear in this column are from the Mills College Math Trail. The entire Math Trail can be downloaded by visiting the following website: **http://www.keypress.com/pdc/institutes/highlights/MathTrail.pdf**

Question 1



Photograph of El Campanil Clock Tower on the campus of Mills College, Oakland, California.

Walk back across the bridge, cross the oval and walk to the El Campanil Clock Tower. This stunning structure was designed by Julia Morgan, one of America's most important architects. She designed hundreds of churches, office buildings, homes, stores, hospitals, and educational buildings in San Francisco following the earthquake in 1906. Morgan designed a number of buildings for William Randolph Hearst, including the Hearst Castle in San Simeon, California.

Stand back from the tower so that you can see the clock on the tower.

(a) Relative to the current time, when was the last time that the minute and hour hands were on top of each

other? When will the hands coincide next?

(b) Relative to the current time, when was the last time that the angle between the minute and hour hands was ? When is the next time that the angle will be ?

(c) Using Roman Numerals the number 4 is written as IV. Why do most clocks, including this one, use IIII to write the number 4?

(d) As noted on the plaque, El Campanil was dedicated by Susan T. Mills in the year MDCCCCIV. How many years ago was this?

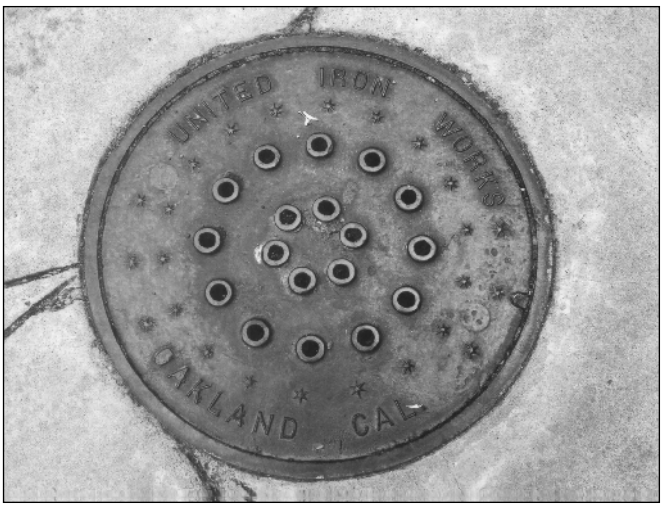
Question 2



Photograph of plaque for El Campanil Clock Tower on the campus of Mills College, Oakland, California.

On what day of the week did the dedication of the building take place?

Question 3



Photograph of a manhole cover on the campus of Mills College, Oakland, California.

Return to El Campanil and walk to the right side of the front of Mills Hall. On the sidewalk locate a manhole cover that has two rings of small holes on it along with a ring of small stars.

(a) Count the number of small holes in the innermost ring and the number of holes in the second ring. Record your numbers in Table 1.

Table 1

ring	number of holes
1	
2	

(b) Establish several patterns in the number of holes per ring and then decide how many holes one might use if a ring (larger than the other two) were to be added to the existing ones.

(c) Suppose that the manufacturer of this manhole used the pattern of simply doubling the number of holes to determine how many holes should be used per ring. Using this pattern would it ever be possible to make a manhole that has a total of

(i) 42 holes

(ii) 99 holes

(iii) 762 holes

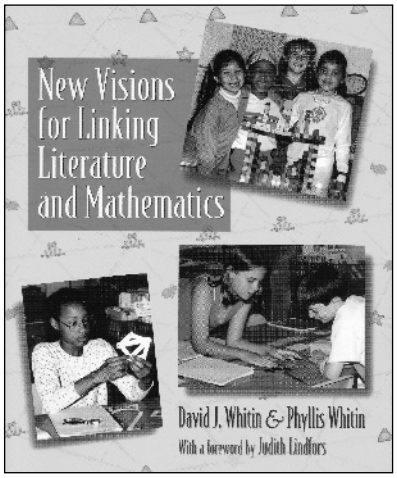
For information about a wonderful mathematical problem involving manholes ("Why are manholes round?"), go to the following website.

The Manhole Puzzle **http://orion.math.iastate.edu/burkardt/puzzles/manhole_puzzle.html** ▲

▲ BOOK REVIEW: *NEW VISIONS FOR LINKING LITERATURE AND MATHEMATICS*

REVIEWED BY: PAT MARGERM

So much to teach! So little time! Teachers often look for ideas for providing students with experiences for learning curriculum expectations within the reality of the time limits of the class day. Integration is often seen as a way of meeting this goal. In addition, many elementary teachers respond to the idea of applying their enjoyment of children's literature to teaching mathematics. Recommendations for the use of children's literature in teaching mathematics appear in both *Teaching and Learning Mathematics – The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario*, 2004 (page 27) and *Early Math Strategy – The Report of the Expert Panel on Early Math in Ontario*, 2003 (pages 24-25). OAME's project, *Growing Up Mathematically*, also includes examples of using children's literature as a strategy for setting the context in mathematics lessons. These are some of the reasons that make a resource that links literature and mathematics very attractive.



David Whitlin is an experienced author. This is the second book that David and Phyllis Whitin have written that has been jointly published by both the National Council of Teachers of English (NCTE) and the National Council of Teachers of Mathematics (NCTM). The first book was *Math is Language Too*. David has had several articles published in NCTM's journal, *Teaching Children Mathematics*, and is the author of an impressive list of books including;

- A Mathematical Passage: Strategies for Promoting Inquiry in Grades 4-6 (Paperback)
- Inquiry at the Window: Pursuing the Wonders of Learners
- Mathematics in the Making: Authoring Ideas in Primary Classrooms
- It's the Story that Counts: More Children's Books for Mathematical Learning, K-6
- Read Any Good Math Lately?: Children's Books for Mathematical Learning, K-6
- Living and Learning Mathematics: Stories and Strategies for Supporting Mathematical Learning

So what does this book have that might be useful to teachers?

Once upon a time picture books with links to mathematics were less common. Now there are many websites and resources that list recommendations of picture books that can be linked to mathematics. The problem of linking literature and mathematics has shifted from a lack of suitable books to the selection of suitable titles. What criteria can be used in choosing picture books? These resources offer a solution to the second problem by suggesting the following criteria:

1. Mathematical integrity e.g., accurate mathematics that is accessible to the reader and is presented in a believable context
2. Potential for varied response
3. An aesthetic dimension
4. Racial, cultural, and gender inclusiveness

The choice of these criteria is justified through the identification of parallels in NCTE and NCTM standards. The authors develop confidence in understanding and applying the criteria in the first chapter "Criteria for Selecting Math-Related Books". The chapter begins with a thorough analysis of one book, *If You Hopped Like a Frog*, which meets all of the criteria. Then understanding of each criteria is further developed through comparisons of books which do and do not meet each criteria. Finally the chapter ends with a summary and a list of books which do and do not meet the criteria.

The rest of the book is organized with the following chapter titles:

1. Books for a Wide Range of Ages
2. Problem Posing with Children's Literature
3. Using the Strategy of Book Pairs
4. Best Books for Exploring

Four of the five chapters contain vignettes of lessons based on selected titles, samples of student work and lists of book title suggestions. The chapters are followed by five appendices which provide additional suggestions for mathematical investigations related to picture books.

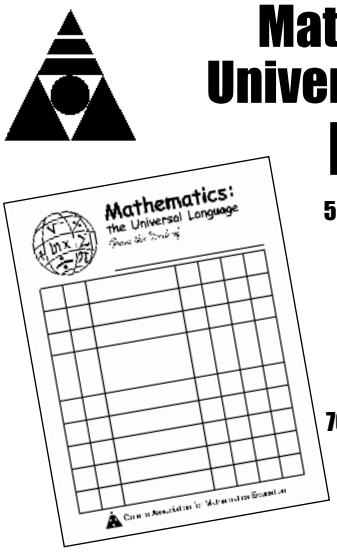
While it appears that the mathematics presented in

the book would be familiar to teachers in Ontario (e.g., proportional reasoning which is now explicitly identified in the revised curriculum), it is unfortunate that most of the book seems to focus on mathematics at the primary/junior level. More teachers are reporting success with the use of picture books in math classes at the intermediate and secondary levels.

Since the authors analyze both "old favourites" such as *The Doorbell Rang* (1986) and *Ten Black Dots* (1986), as well as more recent titles such as *First Day in Grapes* (2002) and *One is a Snail, Ten is a Crab* (2003), readers may be inspired to revisit their familiar books from a new perspective as well as be encouraged to seek out new titles.

Readers may wish to have an opportunity to explore the ideas and/or resources presented in the book before making a decision to purchase it. Here are three suggestions for sampling the ideas in the book.

1. Four lessons that reference David Whitin's ideas for connecting mathematics and literature can be accessed through the following website: **http://www.readwritethink.org/lessons/author_detail.asp?authorid=151&lessonid=817**
2. A list of recommended book pairs for linking math and literacy can be viewed at the following website: **http://www.journal.naeyc.org/btj/200503/06whitin.asp**
3. A sample chapter from the book can be read on the following website: **http://www.ncte.org/library/files/Store/Books/Sample/33487Intro_Chap01.pdf** ▲



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▲ Reflections of OMO 2005



HEATHER BOYCHUK
CO-CHAIR OF OMO 2005
SECONDARY SCHOOL TEACHER

The Northern Ontario Mathematics Association (NOMA) was pleased to host the 2005 Ontario Mathematics Olympics (OMO) in Sudbury. Almost 30 teams participated in the events that took place June 3 and 4 at Laurentian University and Science North.

On Friday June 3, 110 mathletes and 60 coaches, parents, and volunteers began to arrive at Laurentian University in Sudbury. After a BBQ dinner, the mathletes and coaches went by bus to *Science North* where they saw an *IMAX* film and experienced a virtual voyage. The mathletes also took measurements in preparation for the Math Trail Event for the next day. Students returned to Laurentian residences for a snack and for the technology preparation session. Teams practiced using the TI-83+ and CBR's. Students were so excited it was hard to convince them to turn in for the night!

On Saturday, the day began with a breakfast buffet, then the mathletes went by bus to *Science North* for their morning events. Parents and coaches were sent off to Dynamic Earth, where they had the opportunity visit an underground tunnel and explore the history of mining. They put themselves in the places of miners from 1900's. The experience included a simulated dynamite blast, various demonstrations and an exploration of today's telemining technologies

Meanwhile, the mathletes took part in two events at *Science North*. One event was the Math Trail which took teams throughout the Science Centre. Students measured, graphed, estimated, performed experiments and hunted for clues to complete the math trail. While some teams were completing the math trail, others were doing the individual paper and pencil event which included a manipulative component. The mathletes also had the opportunity to see "Wings Over the North" and 4D adventure, one of *Science North*'s amazing attractions.

For the afternoon, students returned to Laurentian University and completed the final two events. One event involved a relay and logic puzzle (see page 27), the other event was the CBR challenge. Mathletes had to walk in front of a motion sensor to create or match a variety of graphs. After the events were over and we were anxiously waiting for the final results, the mathletes occupied themselves with extra large OAME cards. In fact all weekend, Poker seemed to be a popular activity! Several magic tricks were demonstrated by students. There was never a dull moment!

The last event of the day was a tie-breaker challenge in which teams had to complete an equation solving puzzle in the fasted time! 3...2...1...GO! It was silly and exciting! Try to decode the message (see page 27) to find out where OMO 2006 will be held.

The awards ceremony and video concluded the 2005 Ontario Mathematics Olympics. The video which highlights all the activities of the two day can be seen on the OAME website. The bronze medals were given to Emma Barrett, Nissi Wei, John Liu and Kefan Xie of SAME (Scarborough Association for Mathematics Education). In second place, earning the silver medals from ISOMA (Independent Schools of Ontario Mathematics Association) were: Ruby Lung, Caroline Chin, Jonathan Tam and Jeremy Leung. The trophy for first place, along with the gold medals were awarded to mathletes from PRMA (Pine Ridge Mathematics Assocation). This team consisted of Brie Crerar, Lindsay Dimppoulos, Casey MacLean and Danian Misiweicz.

We would sincerely like to thank our many sponsors: Union Gas, OTIP, INCO, Texas Instruments, Laking Toyota, Staples, Gougeon Insurance, Scholar's Choice, Copy Copy, Spectrum, Nelson, CCS, McGraw-Hill and Falconbridge for making OMO 2005 such a great success!

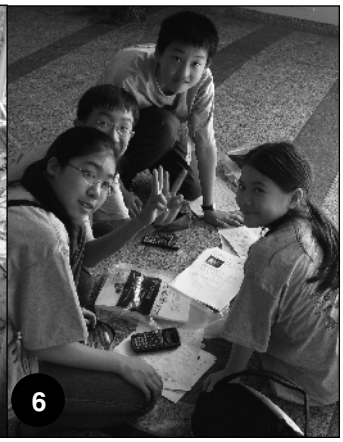












1. Mathletes taking measurements in preparation for the Math Trail.

2. Bed of Nails, one stop along the Math Trail.

3. Look at the hexagons, another station along along the Math Trail.

4. Teams march through the "oldest rock in the world" at Science North.

5. Parents, Coaches, Volunteers & Mathletes experience Science North.

6. Teams work diligently to complete the Math Trail Task.

OMO-LOGIC PUZZLE

You have been invited to the Math Olympics Banquet! The banquet is in a private room with a secret entrance. To enter the room you must know the secret code to get in. The key pad is shown below. In order to unlock the door you must press the buttons in alphabetical order to get to the secret room! There are hidden letters behind each button from A to P. You know where the vowels are but the rest of the letters are hidden. Use the clues to find the button letters.

Place the letters B, C, D, F, G, H, J , K, L, M, N, P in the empty squares of the grid according to the following clues.

	O		
A			
			I
		E	

CLUE 1: H and B are in the _____ row.

CLUE 2: F is in the _____column.

CLUE 3: _____ are in corners.

CLUE 4: P is in the _____ column but not in the third row.

CLUE 5: L is in the same row as _____.

CLUE 6: C is in the same column as _____.

CLUE 7: _____ contains two consecutive letters of the alphabet.

CLUE 8: K is in the _____ row and third column.

Decode the Message to Find Out Where OMO 2006 Will be Held...

The letters shown in the table each represent a different number.

Letter	A	B	C	D	E
Number					
Letter	F	G	H	I	J
Number					
Letter	K	L	M	N	O
Number					
Letter	P	Q	R	S	T
Number					
Letter	U	V	W	X	Y
Number					
Letter	Z				
Number					

Solve the following equations to find the value of each letter.

$A + 3 = 7$	$B - 5 = 11$	$4C = 20$	$\frac{D}{3} = 9$
$E - 8 = 16$	$10F = 120$	$G - 17 = 8$	$H + 12 = 19$
$\frac{I}{9} = 5$	$13J = 39$	$10K + 2 = 22$	$18 + L = 24$
$5M = 130$	$N - 19 = 1$	$\frac{O}{6} = 14$	$2P + 5 = 23$
$2Q - 8 = 18$	$14R = 112$	$-18 + S = 3$	$T + 22 = 23$
$26U = 286$	$3V - 4 = 50$	$\frac{W}{5} = 8$	$X + 25 = 39$
$7Y = 105$	$5Z + 3 = 88$		

Decode the message:

21 4 26 24 (21 5 4 8 16 84 8 84 11 25 7)

7 84 21 1 21 84 26 84 2006! ▲

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▲ Gone to Ghana for the Summer

TARA TASKER



*Tara Tasker is the creator of the non-profit website **www.bonfieldpublicschool.com** which provides free resources for the Ontario Curriculum. She is currently teaching a grade 5/6 class at H.A. Halbert Junior Public School in TDSB.*

Introduction

Project Overseas is a joint professional development initiative between the Canadian Teachers' Federation and the Canadian International Development Agency. It started in 1962 with two Canadian teachers travelling to Nigeria. This year, forty-eight teachers from across Canada travelled to twelve developing countries in Africa, Asia and the Carribean. I had the good fortune to be chosen as part of a team of six to travel to Ghana in West Africa.

Ghana

Portuguese traders arrived in Ghana in 1471 to mine gold, and eventually established a headquarters for the slave trade. The area became known as the Gold Coast and various European powers established forts along the coast of Ghana. The British took control of the Gold Coast in 1874 and established Accra as the capital in 1876. Under the leadership of Kwame Nkrumah, Ghana was the first black African colony to gain independence from Britain and became a republic on July 1, 1960. The name Ghana comes from one of the great inland trading empires that flourished in West Africa from the fourth to the eleventh century.

Ghana is half the size of the Yukon, spanning 238 540 square kilometers and three different climate zones. The southern forest belt is warm and humid, the coastal belt is warm and dry, and the northern grassland is hot and dry. More than twenty million people live in the ten regions of Ghana. English is the official language of Ghana; however, more than fourteen indigenous African languages are also spoken.

Mathematics in Ghana

In Ghana, each Canadian worked with a local Ghanian co-tutor to plan and offer instruction to local teachers in two different locations; Takoradi, the capital of the Western Region and Cape Coast, capital of the Central Region. The teachers ranged from twenty three to fifty five years of age and had two to thirty five years of experience. Most teachers had high school followed by teacher training college. About 25 percent of the teachers in our workshops had a university degree, some had only basic school education and a handful had graduate degrees. In both locations only 25 percent of the participants were women. The Ghanian teachers reported class sizes ranging from sixteen to ninety six students. A starting salary would be about $150 Canadian. (70 percent of teachers around the world live below the poverty line: Canadian Teachers Federation.)

The workshop consisted of five days (Monday to Friday) from 8:30 a.m. until 5:30 p.m.. There were two breaks and a two-hour lunch. I had the pleasure of teaching the JSS (equivalent of grades seven to nine in Canada) Math and Science for Special Women. The Ghana National Association of Teachers (GNAT) felt that creating a workshop for women who taught math and science to encourage their participation in math and science, was important. In Ghana, mathematics and science are still viewed as "male" subjects and there are very few female teachers in these subject areas.

The Republic of Ghana JSS Mathematics syllabus was published in September 2001. It specifies that ten thirty minute periods should be allocated for math per week. It also recommends that these periods be organized into five double periods. The syllabus indicates that assessment should be weighted as 30 percent knowledge and understanding and 70 percent application of knowledge. The procedure for grading is as follows:

A	B	C	D	E	F
100-80	79-70	69-60	59-45	44-35	34 and below
excellent	very good	good	pass (credit)	weak	very weak

Report cards often communicate a student's ranking within the class. For example, in mathematics a JSS student might find 44/156 on their report card, which means they scored 44th out of 156 students. Report cards have limited space for comments; however, the syllabus recommends the following comments to inform students about their learning:

- keep it up
- has improved
- could do better

• hard working

• not serious in class

The syllabus recommends continuous assessment which is defined as projects, class tests, homework and terminal tests (a summative test) to determine a student's mark. The objectives outlined in the syllabus are considered criteria that are to be achieved by the pupil and therefore exams are also criterion-referenced.

The body of the syllabus, where the "content" for each is specified, is organized into a table with the following headings:

Unit	Specific Objectives	Content	Teaching and Learning Activities	Evaluation

The units are numerically sequenced. For example, 1.2 would be JSS 1 (grade 7) unit 2, whereas 2.4 would be JSS 2 (grade 8) unit 4. It would then list the specific objectives concerning the content. Teaching and learning activity suggestions are made along with evaluation ideas. It is very well organized and easy to interpret and follow.

Many of the workshops focussed on content rather than pedagogy. GNAT understands that there are many teachers who are unqualified or have difficulty with the content themselves. However, my co-tutors and I tried to provide examples of good pedagogy as well. This theory was well received. One participant wrote that she will "change methods of teaching for better understanding". However, this change to constructivist learning will be difficult. There are two factors that are preventing its adoption. The first is the lack of manipulatives. While in Ghana, we had the opportunity to visit a number of rural schools north of Accra (the capital). Figure 1 is a picture of a typical rural school.

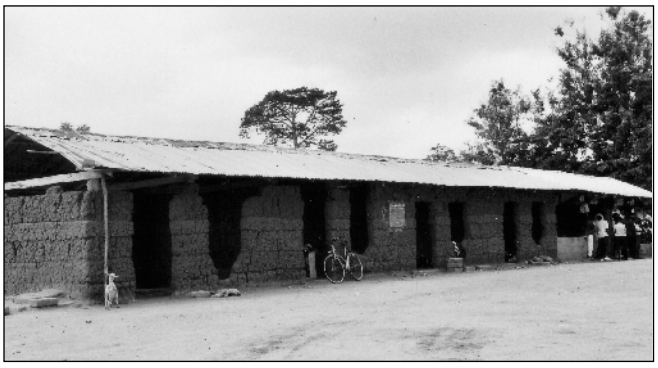


Figure 1: A rural school in Ghana. Please note the shadows of doors and windows and the patchwork roof made of plastic and metal.

The 4m x 4m classrooms will be filled with forty to fifty students. There are giant potholes in the floors and the blackboards are rubbed down to the plywood. This school has no electricity, no running water and no bathrooms. Seeing goats in the school yard grazing on what little grass there might be is common. Facilities, not manipulatives, are high on the wish lists for rural communities. This community, with help from international aid assistance, GNAT, and fundraising, is fortunate to have raised enough money to build a new school.

The second factor making the adoption of the constructivist approach difficult is how teacher performance is evaluated. Every month, an inspector visits a school to evaluate the teachers. Teacher performance is based on how many exercises (questions) have been assigned, completed and marked. The more exercises, the better the teacher. As a result, many teachers in my workshop asked me how they could do math experiments and still meet the requirement of assigning so many exercises.

Back in Canada

I returned to Canada August 9 after 30 hours of transit and 40 hours without sleep. When I returned to my classroom, I thought about how lucky I was to live and teach in Canada. Children in Ghana are entitled to a free education under the constitution, however access to that education might be challenging depending on geography, qualified teachers and educational resources. Unfortunately, 60 percent of Ghanian students do not enter JSS (grade 7). Despite the poverty, poor facilities and lack of resources, I was amazed at the dedication and ingenuity of the Ghanian teachers. They sacrificed a week of their holidays away from their families and were absent from their second job to attend a 5-day workshop to learn. It truly made me proud to be a mathematics teacher. ▲

▲ Building the Base of Communication: Using Word Walls to Build Mathematical Language

SANDRA HIMANN



Sandra Himann is a grade 5/6 teacher with the Waterloo Region District School Board. Sandra is passionate about teaching mathematics, and learning with her students.

Ontario Curriculum Connection

In an investigative mathematics program, as outlined in *The Ontario Curriculum, Grades 1-8* (Ontario Ministry of Education, 2005), students are active inquirers of mathematics. Throughout the five strands of curriculum expectations are woven "…a set of seven expectations that describe the mathematical processes students need to learn and apply as they work to achieve the expectations…" (Ontario Ministry of Education, 2005, p.11). One of the seven processes that support learning in mathematics is communication. Recognizing that all of the processes are important and interconnected, problem solving and communication are identified as having "…strong links to all of the other processes…" (Ontario Ministry of Education, 2005, p.11). Effective communication requires students to share their mathematical ideas visually, orally and in writing. In addition, effective communication enhances student understanding of relationships, further develops mathematical arguments and helps students to reflect upon and clarify their understanding of mathematical ideas. Using a word wall featuring mathematical words, prominently displayed in the classroom, is one way to support effective communication.

As students begin to investigate the world of mathematics, they develop new skills and practice existing ones. Assessment and evaluation of student achievement is completed for the purpose of supporting student learning. According to the achievement chart outlined in *The Ontario Curriculum, Grades 1-8* (Ministry of Education, 2005, p. 22) evaluation of student learning includes four categories of knowledge and skills. These categories are interconnected and include knowledge and understanding, thinking, communication, and application. In the area of communication, students are evaluated on their ability to express and organize their ideas and mathematical thinking, their ability to communicate for various audiences, and their use of conventions, vocabulary and related terminology. Recognizing that communication of mathematical ideas can be shared in a variety of ways, effective written communication requires precision of language. The creative use of a word wall featuring mathematical words is one way to increase the knowledge, understanding and use of mathematical language.

NCTM Connection

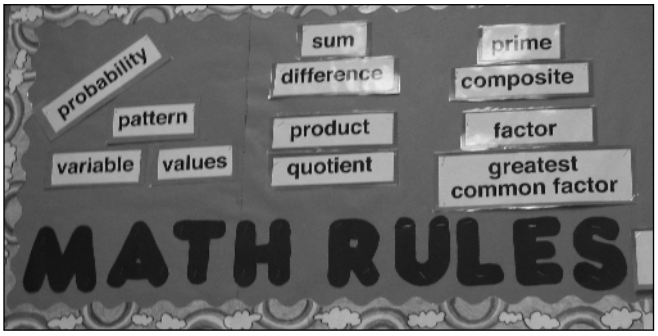
The National Council of Teachers of Mathematics (NCTM) developed the *Principles and Standards for School Mathematics* (2000). One of the standards included at all levels of mathematics education, from pre-kindergarten to grade twelve, is the ability to communicate ideas. Through the use of mathematical symbols, pictures and language students share ideas and clarify their understanding of mathematical concepts. Knowledge of mathematical words, their meanings and usage is one component of effective communication. A word wall featuring mathematical words is a way to enhance students' knowledge of mathematical language, and help students develop a high standard of mathematical communication.

Beginning in the primary years, students use pictures, words and numbers to convey their mathematical ideas. In the late primary and early junior years students continue to use pictures, words and numbers to communicate their thinking, incorporating more complex mathematical terminology. From grade 6 through to the end of high school, the concepts and ideas students are taught become more complex, and require more advanced use of conventional mathematics terminology for thorough communication. The ultimate goal is to develop all students' communication skills so that they use symbols, numbers and words in well-constructed mathematical arguments.

Classroom Connection

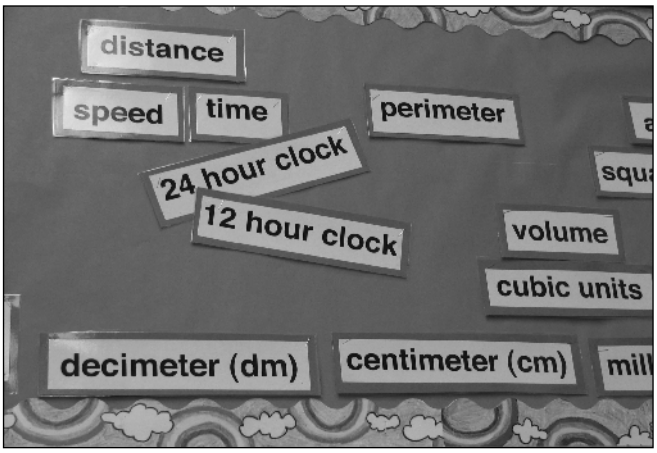
It was early in the school year, a few years back, when I realized that the students in my grade 6 class were struggling with mathematical communication. When completing basic tasks and practicing skills, they could cope. But, when asked to share their conceptual understanding through oral and written communication the results were less than satisfactory. Although encouraged to communicate their ideas using oral, visual

and written communication, their ideas displayed simple illustrations and very little use of mathematical vocabulary. After further investigation it became clear that mathematical terminology was not part of their everyday language. In an effort to increase their knowledge, understanding and use of mathematical words, we created a word wall featuring mathematical words.



We began by posting and reviewing words from the concepts we had already studied. Then, every few days we returned to the wall to inspect the words posted, discuss their meanings, give examples where possible, and contemplate their relationship to other words on the wall. As the year continued and we progressed through each unit of study, more words were added to the wall and reviewed every few days. By mid May, EQAO assessment time, most students had obtained a general knowledge of mathematical language, and had increased its use in their oral and written communication. The word wall, featuring mathematical words and the weekly review of mathematics vocabulary had made an impact. More confident in their knowledge and understanding of mathematical vocabulary, the students spoke using proper terminology and communicated their written understanding of math ideas, using math vocabulary, with more frequency.

As the next school year rolled around, a few changes were made to the word wall. To enhance understanding of the words and their interdependent relationships, each word was colour-coded according to its mathematical strand, and arranged in a small relational group. For example, product and quotient were blue for the Number Sense and Numeration strand, and were placed side by side as these answers had a relationship with each other. Distance was green for the Measurement strand, and was placed above time and speed in a triangular format to help students remember the relationship among these three words.



Sometimes strand colours were intermingled when words from one strand applied in another as well. For example, 'equal' can be used to compare two sides of a numerical equation, and used in reference to objects on a balance scale. As the look of the word wall changed, so did the lessons associated with the wall. The word wall, featuring mathematical words, had become a 'work in progress'.

Initially, reviewing the words was teacher directed; I would stand near the wall, select five words each time and ask the students for definitions, examples or relationships. The students who knew the vocabulary were keen participants, reinforcing their knowledge with each answer. The students who were struggling were not as eager to partake and would, at times, disengage from the activity. As a result, more adjustments were made to ensure everyone was actively involved in the learning process. At times, the words on the wall became challenges for individuals, pairs and full group activities. More students liked this idea and became involved in this fun environment.

From there I realized standard old games that all of us have grown up with would also work to reinforce learning. Games like Bingo and Twenty Questions, creatively adapted to reflect content would be a lot of fun and educational. So, in September, I plan to continue to model mathematical language through problem solving and math congresses, highlight and review word wall words using some of the challenges from previous years, and include adapted games to reinforce knowledge, understanding and the use of mathematical words. It seems the word wall in my classroom has become part of the learning environment. Like water, its use changes with each new wave of students, each new grade and each new idea. For example, the more students I work with who are learning English as a second language, the

more I realize their need to understand the vocabulary before they can use it to communicate their mathematical thinking. My ultimate goal is to actively engage all my students, build a base of mathematics vocabulary for use in communicating ideas, and have fun at the same time.

Word Wall Words

The following are some of the words displayed on the word wall in my grade six classroom.

Number Sense and Numeration - whole number, decimal, fraction, percent, ratio, sum, difference, product, quotient, expanded form, standard form, word form, prime number, composite number, factor, multiple, least common multiple, greatest common factor, numerator, denominator, proper fraction, improper fraction, mixed number, equal

Measurement - distance, perimeter, square units, volume, cubic units, mass, time, 12-hour clock, 24-hour clock, speed, money, (and a large number of measurement units), equal

Geometry - 2-dimensional, net, polygon, congruent, similar, obtuse, right-angled, acute, scalene, isosceles, equilateral, symmetry, order of rotational symmetry, 3-dimensional, polyhedron, prism, pyramid,

Data Management and Probability - data, tally chart, table, intervals, X-axis, Y-axis, line graph, bar graph, line plot, pie graph, stem and leaf plot, mean, mode, median, range, chance, probability, tree diagram,

Patterning and Algebra - pattern, variables, growth patterns, T-table

Student Activities

The following games can be played as a whole class, in small groups or in pairs. The games are presented as samples, and can be altered according to classroom needs.

Bingo – Students use a standard Bingo card, and write one word from the word wall in each Bingo space. Definitions or examples are given for a specific word and students mark the appropriate vocabulary word on their Bingo card. A free space is given in the middle. A winner can be declared for a line, four corners, an 'X', a 'T' or the full card.

Twenty Questions – The game leader (student or teacher) selects a mathematical term from the word wall, unknown to the rest of the class. Students ask insightful questions in an attempt to discover the word (Teachers may need to spend some time discussing good questioning techniques). Only the answer 'yes' or 'no' can be given by the game leader. Points can be awarded based on the number of questions required to discover the word or the activity can be used cooperatively, praising the class as a whole.

Mystery Word – In preparation for the game, students (partners) select a word from the word wall, and develop three to five clues for the word. The first clue is vague and the last clue is very specific (Teacher modeling may be needed). The number of clues needed to solve the mystery word determines points. If the word is solved after the first clue, the player might get 10 points. But, if the word is solved after the fifth clue, the word may only be worth 2 points. Once all students have finished writing their cards, the game can begin.

Teacher Note - Many existing games can be used to increase knowledge, understanding and use of mathematical words. Depending on your class, students may enjoy the challenge of creating their own word wall games, then try them out on the rest of the class. A little creativity, a few ideas and a little game set up is all that is required for a highly successful, and useful word wall in any classroom.

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Ontario Ministry of Education. (2005). *The Ontario curriculum grades 1-8: Mathematics*. Retrieved August 27, 2005, from http://www.edu.gov.on.ca.

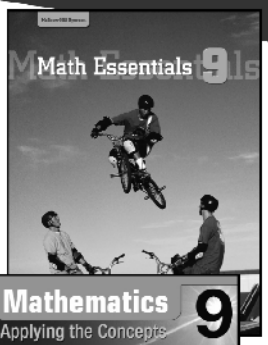
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics, Inc. ▲

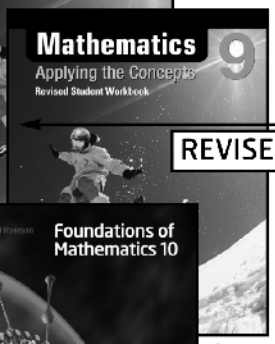
9	6	3	1	7	4	2	5	8
1	7	8	3	2	5	6	4	9
2	5	4	6	8	9	7	3	1
8	2	1	4	3	7	5	9	6
4	9	6	8	5	2	3	1	7
7	3	5	9	6	1	8	2	4
5	8	9	7	1	3	4	6	2
3	1	7	2	4	6	9	8	5
6	4	2	5	9	8	1	7	3

PUZZLE SOLUTION

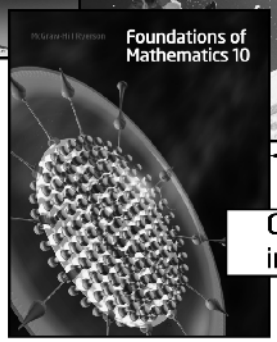
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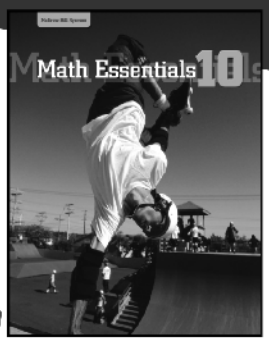
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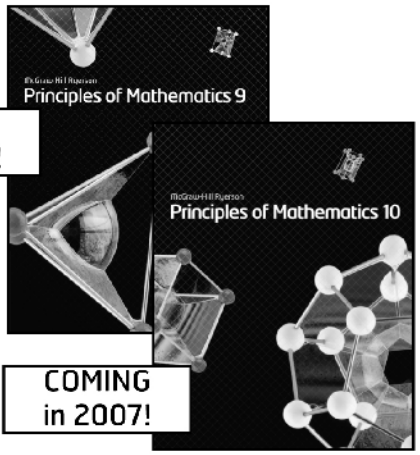
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▲ Nutty Bolt Math

TIM SIBBALD



Tim Sibbald currently balances his time between challenging his students and challenging himself in OISE's doctoral program.

Often when math teachers mention hardware they mean graphing calculators, protractors or rulers. However, a golden opportunity awaits for an inexpensive trip to the local hardware store. The simple nut and bolt provide an opportunity to explore proportion, linear versus nonlinear graphs and linear relationships in meaningful ways (MPM1D, MFM2P). They also provide a gateway to metric/imperial conversion (MFM2P, MAP4C).

A nut and bolt are seemingly simple objects that are available from any hardware store. However, they may not be quite as simple as passive thinking tends to suggest. Firstly, there are both metric and imperial nuts and bolts with different fundamental measurements. Secondly, for imperial nuts and bolts, there are two varieties of threads called Unified National Coarse (UNC) and Unified National Fine (UNF). There are other varieties (such as British standards) but they are not typically stocked in Ontario stores. The most commonly available versions are detailed in Table 1.

Table 1 Commonly Available Bolts

Diameter (Inches)	Threads Per Inch	
	UNC	UNF
3/16	24	32
1/4	20	28
5/16	18	24
3/8	16	24
7/16	14	20
1/2	13	20

Table 1.

If you have a variety of hardware, simply developing this table is interesting. Students will assume that there should only be one thread count for a given diameter and it is inevitable that discussion will lead to someone checking their results. Depending on the accuracy of your rulers (1/16 in. being key), you may want to mark the bolts according to their official diameter and just have students determine the number of threads per inch.

Suppose students make a graph, using Table 1, to show the number of threads per inch versus the bolt diameter (shown in Figure 1). Are the graphs linear? The UNF graph is not linear but there may be some debate about the UNC graph. It might be argued that the UNF is not linear because of the steps that arise from keeping the number of threads per inch the same for 5/16 and 3/8. In the case of UNC a convincing way to communicate the nonlinearity is to calculate the first difference. This directly ties into the grade nine academic and grade ten applied curricula.

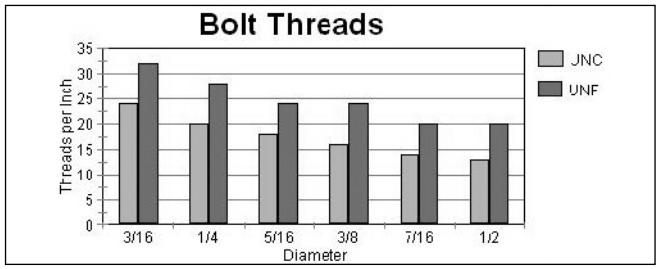


Figure 1.

After discussing the issue of whether to join the dots with a smooth curve, students can answer a variety of graph reading questions including interpolation and extrapolation. How many threads per inch would they predict for a bolt that had a diameter of 3/4 of an inch? If you are teaching interpolation, and want a minor fraction connection; What would be the number of UNC threads for a bolt that is 13/64 in diameter? You can also reverse the prediction and ask what the diameter would be if a UNC (or UNF) bolt had 28 threads to the inch.

An important aspect of the information is to ask how far a nut actually moves when it is turned one revolution. The answer is the reciprocal of the number of threads per inch and this answer is in units of inches e.g., 1/(threads per inch) = inches per thread). However, student responses are usually uncertain providing the teacher an additional opportunity! Two ways to explore the question are proportion and graphing. Proportion emerges because one can measure how far a nut moves after many turns and then calculate the result of one turn, or any other number of turns. For example, a nut moves 0.75 inches in 10 turns, how far does it move in 6 turns? How many turns will it take to move the nut 1.2 inches?

Graphing “nut position” versus “number of turns” generates a line where intercept and slope concepts can be explored. It is helpful to specify the way the measurements are to be made. The simplest approach is to measure from

the thread side of the bolthead to the nut. This reflects the thickness of the material being held. However, with experience, this choice can become a point of discussion within the class. It does not indicate how far the bolt head will stick out above the material. How could one measure so that the graph would reflect this? One could also measure from the bottom of the bolt to reflect how far out of the material the end of the bolt sticks.

If a bolt is threaded all the way to the bolthead then the y-intercept will be zero. This means that the nut is flush against the bolthead to start with and the material thickness is zero. Most longer bolts have a section with no threads between the bolthead and nut. The length of the gap is then the y-intercept and indicates the minimum thickness of material required.

The slope of a graph of "nut position" versus "number of turns" shows how far the nut moves along the bolt with a single turn. It may, in fact, be necessary to use five turns at a time (a run of 5) for measuring accuracy, but dividing the measurement (the rise) by five (rise/run) can convert this to the distance for a single turn (the slope!). Challenge your students to relate this to the "threads per inch" concept! Surely, the number of threads and the movement of the nut have to be related. The answer they will be seeking is that the nut moves according to the reciprocal of the threads per inch. If there are 20 threads per inch then the slope will indicate that the nut moves 1/20 inches per turn.

The concepts of domain and range can be introduced naturally using the length of the bolt. Different lengths are available and this can be used to explore the link between the domain and the range for the graph of the line. Specifically, domain indicates the number of threads available and the range is the minimum and maximum thicknesses of the material. Keep in mind, that for practical reasons, the nut has to be on the bolt! The effect of washers on the domain and range could be developed for enrichment.

There are a variety of ways to link nuts and bolts to measurement conversions. There are straight forward conversions, such as asking how long a 2 inch bolt is in cm, or a 3.5 cm bolt is in inches. A more challenging ("tips") question might be: If bolts are only available in 0.5 cm increments and you need to bolt two pieces of 3/4 inch thick wood, how long a metric bolt do you need? This question may result in a discussion of the width of the nut since it needs to be on the bolt. The answer seems to be (3/4 + 3/4)x2.54 or 3.81 cm, but, if you specify nuts are 0.4 cm then a 4.2 cm bolt is needed which gets rounded up to 4.5 cm.

Revisit Table 1 for a moment. Imagine that the same type of table exists for metric hardware. What would the numbers be? This requires conversion to metric and then rounding to the nearest millimetre. This is reminiscent of many textbook questions that require calculating paint or carpeting costs that must be rounded to suit the size of a can or the width of a carpet roll.

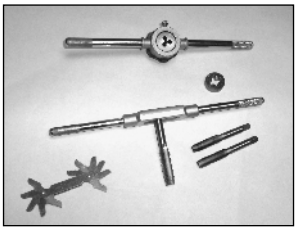
Within the classroom, the use of hardware has worked well. It provides a realistic context for lessons while facilitating mathematical concepts of proportion and graphing. There are numerous ways to develop questions around the hardware that provide further applications of the mathematical concepts that are learned. Having used the ideas in this article several times I come back to it wondering if I have mastered the "nuts and bolts" of it or if I should "spring" something new on my students (Hooke's law being linear....).

Sidebar - What do the markings mean?

A communication task I have used is to describe the markings on boltheads precisely! Many hexagonal bolts, available for household use, have three line segments separated by 120˚ that touch the outer vertices of the bolt head. This is the most common marking to find in hardware stores. It means that the bolt is made of medium carbon steel, has been quenched (submerged in oil or water while hot) and tempered. Bolts with this marking that are 2 to 3 inches long can withstand 60 000 psi (pounds per square inch). Interesting to tie this concept to the area of the bolt shaft! An engineering application for the area of a circle!

Sidebar - What are taps and dies?

When machinists want to produce a thread they will typically use a tap and die, which thread a nut and bolt respectively. These are fluted tools with relatively sharp teeth. The flutes allow a place for excess material to go as the threads are carved.



In the figure, the die is the circular piece shown at the top in a work tool and just below before being put in the tool. At the bottom is a tap in a work tool and two other taps that are loose. The item at the bottom left is a thread gauge used to measure thread sizes in a convenient way. Each piece has teeth spaced at a specific number per inch and these mesh with the threads on a bolt that are spaced the same. You will likely find that your shop teachers will loan you a sample of these tools if you explain what you are doing. ▲

then no, it won't. However, if the g.c.f. is not 1, then we should proceed.

Applying Euclid's Algorithm again, we have

$$\begin{array}{r} 1 \\ x^2+x-12\overline{)\,x^2-x-6} \\ \underline{x^2+x-12} \\ -2x+6 \end{array} \qquad \begin{array}{r} -\frac{1}{2}x-2 \\ -2x+6\overline{)\,x^2+x-12} \\ \underline{x^2-3x} \\ 4x-12 \\ \underline{4x-12} \\ 0 \end{array}$$

Here $-2x + 6 \neq 1$ necessarily and so (x-3) has to be common to both $x^2 - x - 6$ and $x^2 + x - 12$. If students are having difficulty with factoring, half the work of reducing is now already done. That is, we know the expression can be reduced and we know the term needed to start the process.

The above is another way of approaching a common algebra problem, but it is not for every student. Some may have forgotten the division algorithm altogether while others would rather try their luck at factoring. On the other hand, some students will have added to their understanding of how things work and become more independent learners. They may even realize the power of division and Euclid's Algorithm. A high-energy honors class may appreciate the power of reducing fractions without a calculator and the natural lead into calculating lowest common multiples using

$$\text{l.c.m.}\,(a,b) = \frac{axb}{\text{g.c.f.}(a,b)}$$

This is the method used for calculating l.c.m. in Asia. However, a teacher who is going back to the basics may appreciate another use of the division algorithm.

In short, the calculation of the greatest common factor is simple and direct and knowing that g.c.f. of any two numbers is 1 explains many steps. This partner need not be silent anymore. ▲



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▲ GCF: The Silent Partner

DUNCAN MCDOUGALL



Duncan McDougall has been teaching for 27 years and is a graduate of both Bishop's and McGill Universities. After 13 years in the public school systems of Quebec, Alberta, and British Columbia, he left the classroom for private teaching. Throughout the last 15 years, Duncan has taught all levels of high school mathematics, college and university calculus, and mathematics for elementary school teachers. This rich exposure to diverse programs and curriculae has provided him with some of the ideas, which he is now publishing. Duncan has given several workshops on Rapid Calculation and non-conventional approaches to operations on radicals and fractions. His latest project is developing a special family of polynomials for curve sketching called Diophantine Polynomials. Currently, Duncan owns and operates TutorFind Learning Centre in Victoria, British Columbia.

Why can't we reduce $\frac{3}{7}$? Why do we multiply 2 x 3 for the lowest common denominator of $\frac{1}{2}$ *and* $\frac{1}{3}$, but but not multiply 2 x 6 for the l.c.d. of $\frac{1}{2}$ *and* $\frac{1}{6}$? Why can't we combine $\sqrt{5}+\sqrt{7}$? Why do we rationalize $\frac{1}{\sqrt{6}}$ immediately, but not $\frac{2}{\sqrt{6}}$? Why is the lowest common multiple of sin x and cos x their product? Why can we "cross-multiply" to get equivalent terms for $\frac{1}{x+h}-\frac{1}{x}$?

The answer of course is a greatest common factor of 1. As in the true nature of a silent partner, the greatest common factor is always there when we need it, but never in the forefront. A g.c.f. of 1 is almost always taken for granted although without it, we couldn't justify our steps.

A g.c.f. of 1 is the reason why we can or cannot proceed with some of math's most fundamental steps. It is our back-up and solid answer to all of the above questions: Quite simply we cannot reduce $\frac{3}{7}$ because g.c.f. (3,7) = 1. We multiply 2 x 3 for a common denominator because g.c.f. (2,3) = 1, but g.c.f. (2,6) = 2. We cannot combine $\sqrt{3}+\sqrt{7}$ because g.c.f. (3,7) = 1 and we rationalize $\frac{1}{\sqrt{6}}$ right away but not $\frac{2}{\sqrt{6}}$ because g.c.f. (1,6) = 1 while g.c.f. (2,6)≠1. The same holds true for common denominators in higher stages of mathematics like trigonometry and calculus.

For example, g.c.f. (sin x, cos x) = 1 and g.c.f. (x + h,x) = 1 as well.

My argument is that the nature of and calculation of the greatest common factor should be much more dominant in the curriculum and hence in the classroom because it is so fundamental. It's time to reinforce basic skills so students can understand why they do things the way they do and then match this understanding with basic skills such as the division algorithm.

Now let's pursue our discussion of g.c.f. by examining its role when reducing fractions and rational expressions later in the curriculum. For example, reduce $\frac{57}{95}$ to lowest terms without a calculator. Many students would first wonder whether it was possible, and even if it were, how then to proceed. The quickest method is to calculate g.c.f. (57,95) using Euclid's Algorithm.

Step 1) Divide the smaller number into the larger keeping track of the remainder.

$$\begin{array}{r} 1 \\ 57\overline{)95} \\ \underline{57} \\ 38 \end{array}$$

Step 2) Divide the remainder (38) into the previous divisor (57) again keeping track of the remainder.

$$\begin{array}{r} 1 \\ 38\overline{)57} \\ \underline{38} \\ 19 \end{array}$$

Step 3) Repeat step 2 until the remainder is zero.

$$\begin{array}{r} 2 \\ 19\overline{)38} \\ \underline{38} \\ 0 \end{array}$$

Step 4) The divisor which yields a remainder of zero is our g.c.f.

Here g.c.f. (57,95) = 19 and so to reduce $\frac{57}{95}$, we simply divide top and bottom by 19. This gives $\frac{57}{95}\div\frac{19}{19}=\frac{3}{5}$.

Some may think this skill is no longer necessary until we run into rational expressions in Grades 10 and 11. For example, reduce $\frac{x^2-x-6}{x^2+x-12}$ to lowest terms. Here, we are instructed to factor both top and bottom and then reduce. Logically it would be better to know whether factoring is worthwhile, as the expression may not reduce after all.

Finding the g.c.f. for polynomials reinforces the division algorithm and answers a key question. Will the expression reduce at all? If g.c.f. $(x^2-x-6, x^2+x-12)=1$

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