Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceeding 10 numbered pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required. Please number pages.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including web sites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

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Advertisements for the *Ontario Mathematics Gazette* should be sent to **Dean A. Murray** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

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▲ Editor's Message

MARILYN HURRELL, EDITOR



I was very pleased to receive a manuscript for this issue entitled "Another Summer's Dalliance: The Musical Group", from an author previously unknown to me, Chantal Buteau. It was inspired by Steve Sparling's "A Summer's Dalliance" in the June 2005 *OMG*. I sent the manuscript to Steve for review and he has written a Letter to the Editor expressing his delight. Who says this is not a rewarding "job"?

This issue also features Duncan and Litwiller's article "The Birthday Problem Extended"; between them they have had over 850 articles published in journals across the US and Canada. Paul Alves has submitted "The Math Apprentice" which many of you may want to try with your classes next year.

"Counting Coots" is an excerpt from Jay Ingram's book *The Velocity of Honey* and describes an interesting experiment on how birds can seem to count. A friend of mine told me about this chapter and I asked her to write it up and submit it for review. I hope you enjoy it.

The columns as always, feature interesting information and teaching tips and just plain mathematical fun. Fred Ferneyhough has now co-authored his last "Technology Corner" column and Mary Bourassa will take it over completely in September. Quoting from Fred: "Please let your readers know how much I have enjoyed my stint with *OMG*". Lynda Ferneyhough keeps the name alive as she has agreed to become an Associate Editor for *OMG*, starting with this issue. Welcome! And Director's Dialogue returns in September.

Enjoy your summer! ▲

Marilyn

▲ President's Message

DAN CHARBONNEAU, PRESIDENT



I am truly honoured and humbled to greet you as President of OAME. In trying to prepare this column I thought I'd look at previous June *Gazette* issues to see what other incoming Presidents had to say. Going through these I was in awe of people who have served as OAME presidents and OAME Board/Executive members, and by those who have contributed articles to the *Gazette*. I began to reflect on how I got involved in OAME and the influence that involvement, and people I have met, have had on my teaching and on my personal and professional growth. I've decided to share that with you as my incoming message.

I graduated from the University of Waterloo in 1986 with a Bachelor of Mathematics/Teaching Option, so before venturing out into teaching I had already been influenced by the likes of Ron Scoins, Ed Anderson and Ron Dunkley. In May 1988, I attended my first OAME conference in Niagara Falls and the one workshop I remember to this day, was presented by Todd Romiens, this year's OAME Lifetime Achievement Award winner, In 1989, I moved back to my hometown of Sudbury. In the early 90's, I was lured by two OAME members, (thanks Jerry Malone and Ron Lewis) to NOMA (Northern Ontario Mathematics Association). I first attended workshops and then later helped others organize local NOMA math workshops and annual conferences.

In the fall of '94, I became a member of the OAME Council as the NOMA Chapter Representative. I have been a member of the Council (now called the Board of Directors) ever since then as either a Chapter Rep, elected member or executive member. Because of this, I have been fortunate to work with and to learn from many influential math visionaries who have served OAME as president. The list, since 1993, includes Ron Lancaster, Todd Romiens, Susan Stuart, Barry Onslow, Judy Crompton, Ralph Connelly, Peter Saarimaki, Mary Howe, Shirley Dalrymple, Marg Warren, Stewart Craven, Gord Cooke and Kathy Kubota-Zarivnij. They have spearheaded change in mathematics, created an OAME vision for learning mathematics and were involved in many OAME publications, which include several OAME *Linking Assessment and Instruction in Mathematics* documents, and OAME projects, the most recent being *Growing Up Mathematically*. I have been mentored by a diverse and talented group of Ontario educators who have served on the OAME Executive with me. I wish to thank them on behalf of ALL the teachers, administrators and students who have benefited from their involvement in mathematics education and who have participated in the workshops they have presented. Their passion for improving mathematics teaching and learning is an inspiration to all of us.

I recently attended the 2006 Leadership Conference on "Learning Systems: The Case of Mathematics-for-Teaching" and I look forward to attending the 33rd OAME conference "Every 1 Counts" in London. It is fitting that these are the themes I take as OAME President. I am a novice at understanding principles of complexity theory and how their intertwining aspects can be applied to student learning and to in-service experiences for teachers. Attending the Leadership Conference gave me better insight about how complexity learning systems enable the emergence of new collective possibilities. I have discovered a great deal about teaching, learning and students, from other educators I have worked with through OAME, NOMA, and my own school. I am a classroom teacher, like many of you, who has to grapple with the new revised curriculum, implement new board policies and initiatives, prepare students for EQAO testing, complete midterm report cards and try to figure out how I'm going to connect with the students in my Grade 10 applied math class. There is much more for me to discover about teaching and about student learning of mathematics. I want to become a better teacher and person so that my colleagues and students know that each and "Every 1 Counts".

Over the past 15 years, I can't believe all the time and effort so many colleagues have contributed to mathematics education in Ontario and to keeping OAME alive and vibrant. I wish to thank them all for their contributions and support. A big thanks to those of you whom I haven't met but who contribute to local OAME chapters by providing teacher math in-service through local and provincial math workshops and conferences. I want to thank you for doing what you do and for being an OAME member.

We have about 1 800 members in OAME. I hope that YOU will become more involved in the association by first becoming an OAME member, if you aren't already, and then by joining your local chapter executive (see the OAME web site), attending local chapter and provincial

OAME conferences, getting your Grade 7 and 8 students involved in the OAME local and provincial Mathematics Olympics, presenting a math workshop for local teachers or by simply dialoguing with fellow teachers about math. I encourage and challenge you to get one other teacher/co-worker/friend to become an OAME member and to bring that person with you to your next chapter meeting or local conference. I was fortunate that someone invited me to get involved.

I am truly honoured to serve you as President of OAME. ▲

▲ Letter to the Editor

Dear Editor:

I am totally excited to see that my article spawned another! One writes an article but often receives little feedback. In addition to the vanity of seeing my name in print there is the added thrill of communicating to others my passion for mathematics.

I had fun writing that article and I am exceedingly pleased to see *Another Summer's Dalliance: the Musical Group*, published in this issue of the *OMG*.

That is the idea of a journal - isn't it?

Steve Sparling

(Author of *A Summer's Dalliance: OMG* June 2005 Vol 43 #4) ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor

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mhurrell@tbaytel.net
for further details.

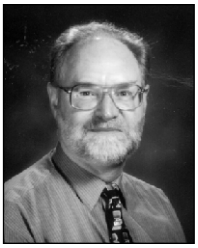
▲ Technology Corner: Create your own Kaleidoscope!

MARY BOURASSA



Mary Bourassa teaches mathematics at Lisgar Collegiate Institute in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She is also an OAME director, president of COMA, gives workshops around the world and is a National Instructor for Texas Instruments.

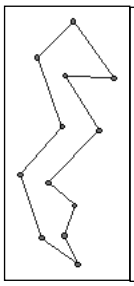
FRED FERNEYHOUGH



Fred Ferneyhough is former head of mathematics at Harold M. Brathwaite SS in Brampton. He retired from there in February 2006. Fred is an avid user of and advocate for the use of technology in the mathematics classroom. He is also an author and tech consultant for several books and a National Instructor for Texas Instruments.

This activity is a lot of fun and contains a lot of mathematics. It is appropriate for students at a wide range of grade levels so the sample questions at the end can be tailored to match your students' level. Whether you are teaching transformations to elementary students or in grade 12 college mathematics, your students will be able to combine art and math using The Geometer's Sketchpad.

1. Using the segment tool create a polygon with at least 12 sides. It should be fairly thin. See Figure 1.
2. Select 3 or 4 adjacent points and under the Construct menu choose Triangle Interior or Quadrilateral Interior (or Ctrl-P). With the interior still selected go to the Display menu and select Color (or right-click and choose Color) then choose one you like.



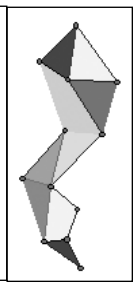
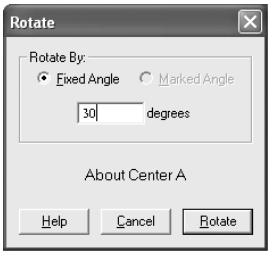


Figure 1. ***Figure 2.***

3. Repeat the process until the entire shape is coloured in. See Figure 2.

4. Select all of the points and hide them by going to the Display menu and selecting Hide.
5. Now we need to recreate the same polygon many times over. We will use the Rotate tool to accomplish this. Select the bottom vertex as the centre of rotation then select the entire polygon by creating a rectangle around it using the selection tool. Go to the Transform menu and select Rotate. A window will pop up allowing you to choose the angle by which to rotate. Use any factor of 360.



6. Repeat the rotation process until you have gone full circle. To accomplish this leave the new polygon selected and go to Transform, Rotate and click on Rotate (or press Enter). This way you can keep the angle fixed. See Figure 3 for the result.

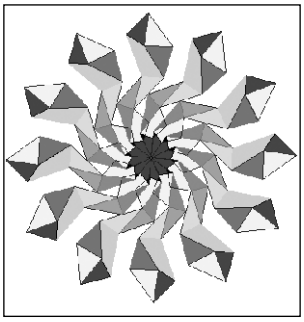


Figure 3.

7. We want our kaleidoscope to move so we need to create circles that will act as paths. Create three circles each with its centre in the middle of the kaleidoscope. One should be a little larger than the kaleidoscope, the second circle should have a radius about two-thirds the radius of the first. The third circle's radius should be about one-third the radius of the first. Be sure that the point defining the length of the circle's radius is a free point, that is, it is not attached to any of the kaleidoscope segments. See Figure 4.

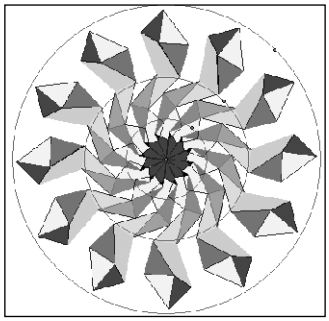


Figure 4.

8. We now need to merge points from the kaleidoscope to the circles. Select the outer vertex of your original polygon and select the largest circle. Go to the Edit menu and select Merge Point to Circle. You should see your kaleidoscope expand to join the circle. If you've lost track of your original polygon go to the Display menu and choose Show All Hidden then choose Hide to get your figure back to where you need it.
9. Repeat to join other points from your original polygon to the other two circles. See Figure 5 for the result.

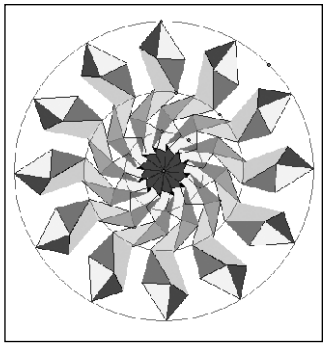
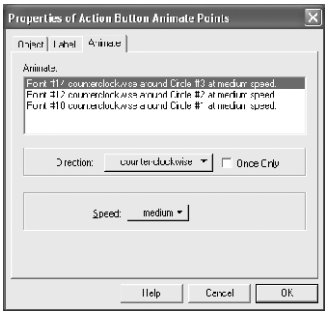


Figure 5.

10. Select the 3 merged points and go to the Edit menu and select Action Button then Animation. A window will pop up. Click OK.
11. Before you click the Animate button, select the 3 circles and any visible points and hide them. See Figure 6.



12. Save your kaleidoscope.
13. Press Animate and enjoy!

Note: when prompted to save when you exit, choose No so that you preserve your original kaleidoscope.

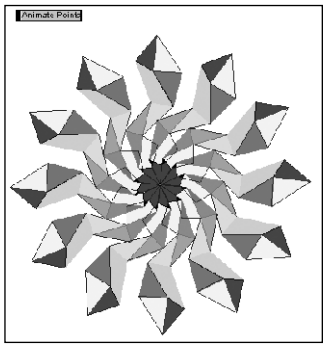


Figure 6.

Possible questions to go with this activity:

a) How many rotations are required for the given angle you chose?

b) Are all parts of the kaleidoscope moving at the same speed?

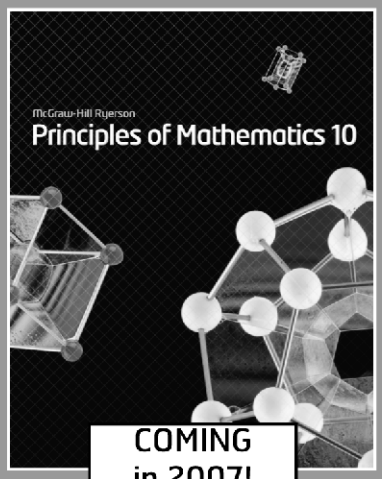
c) Try varying the speed for the Animate button. What changes do you notice?

d) What would be the effect of having fewer circles? More circles? ▲

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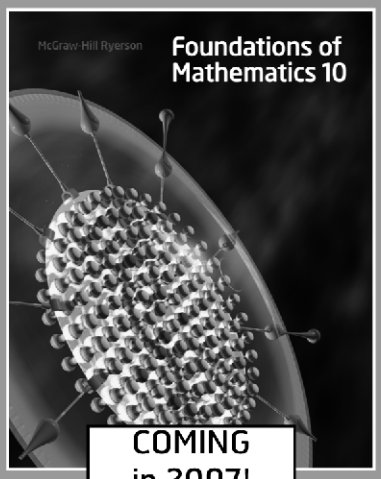
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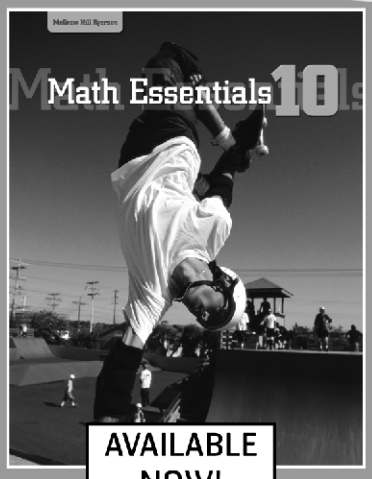
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▲ Hey, It's Elementary! Let the Slow Times Roll!



LYNDA COLGAN

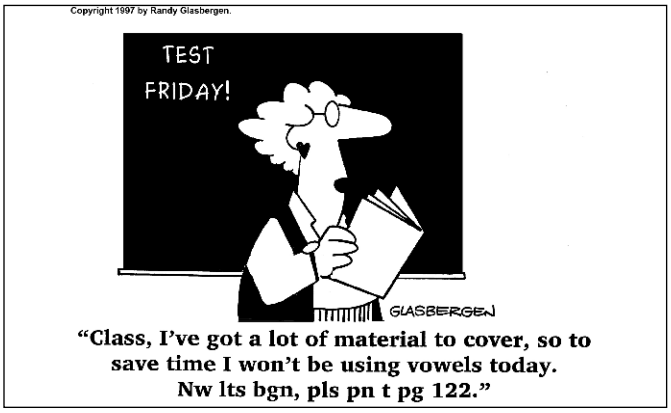
In a series of recent research-based focus groups, for an ice-breaker activity, several groups of teacher candidates were asked to draw themselves in a way that would communicate their new identity as educators. While this task was designed to relax the participants and be a springboard for the questions that followed, it turned out that the answer to this question is probably worthy of a research-based paper of its own.

The teacher candidates drew startlingly similar caricatures in his or her self-portrait. The sketches portrayed harried creatures with bags under their eyes, disheveled clothing, hair askew, glasses cockamamie, mismatched socks around ankles, arms laden with bags (or plastic milk crates) overflowing with books and file folders from which papers hung precariously, running frantically to get through the doors of a classroom just as they were slamming shut.

The message from these early career educators was that to fit in, to be one of the crowd, to fit the implicit membership criteria–you need to be racing around. When you're able to juggle everything fast enough and fill in every available minute in your daytimer–you've made it–you are a successful teacher.

These novice teachers were quick to recognize that their profession (as well as the broader society in which they live) works on this simple equation:

I'm busy = I must be successful

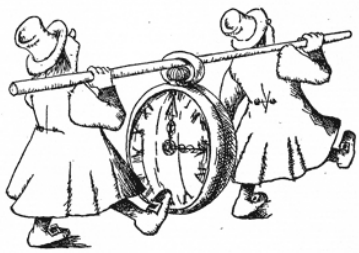


And so, their first priority was to show that they could get up to high speed, "warp speed" or maybe even fast forward mode to prove that they could survive the demands of the profession.



The frenetic images sketched by these young teachers brought to mind Lewis Carroll's *Alice in Wonderland* and the white rabbit who in Chapter One races past Alice and her sister, frantically referring to a pocket watch, saying to itself, "Oh dear! Oh dear! I shall be late!" It seems that the White Rabbit has an appointment with the Queen of Hearts and is in a panic because he is going to be late.

The looming image of the role of time (or the lack thereof) in these beginning teachers' lives also brought to mind Jonathan Swift's 1727 book *Gulliver's Travels*.



You may remember that the Lilliputians thought that the object that Gulliver wore on his hand was his god because he consulted it so often. It was, of course, his watch.

More recently, James Gleick (author of *Chaos and Genius*) produced a scathing study of the cult of speed in his book, *Faster: The Acceleration of Just About Everything*. In *Faster*, Gleick posits that increasingly, time management seems to be the core problem of most people's daily existence. He examines the problem from as many angles as he can find and uses interesting anecdotes to illuminate his arguments. For example, he shows that indices for the preferred length of a soundbite (down from 40 seconds for presidential candidates in 1968 to less than 10 seconds in 1988) have all relentlessly accelerated. "A compression of time characterizes the life of the century now closing," wrote Gleick in 1999. To illustrate, he used the example of overnight mail, itself a fairly recent phenomenon. "In the world before *FedEx*, when 'it' could not absolutely, positively be there overnight, it rarely had to," Gleick explains. "Now that it can, it must." And now, only a few years later, faxes, email, and the Internet have rendered even *FedEx* too slow for many jobs. Gleick asks, "Do

people really want to live at this pace?" Clearly, some do, he concedes, "But what are the costs?"

Around the same time came *Slowness*, a novel about an eclectic group of misfits who travel to a converted 18th-century chateau, where they are to attend a conference of entomologists; a narrator, Milan, and his wife who have gone to the chateau for a weekend getaway; and, in a bit of time travel, a young Chevalier who is seduced there on the same night 200 years ago–by the mysterious Madame de T. Though a relatively short novel, the background notes to the book tell us that it took five years to write (not surprising that the author is Czech and the book is written in French). In his book, author Milan Kundera proposes an equation from existential mathematics: the degree of slowness is directly proportional to the intensity of memory. Kundera wants to make us think about the dangers of the speed that seems to characterize our modern society. Like Gleick (the scientist), Kundera (the sensual romantic) wants us to ask ourselves questions: Where are we headed? And is it worth the price we will have to pay?

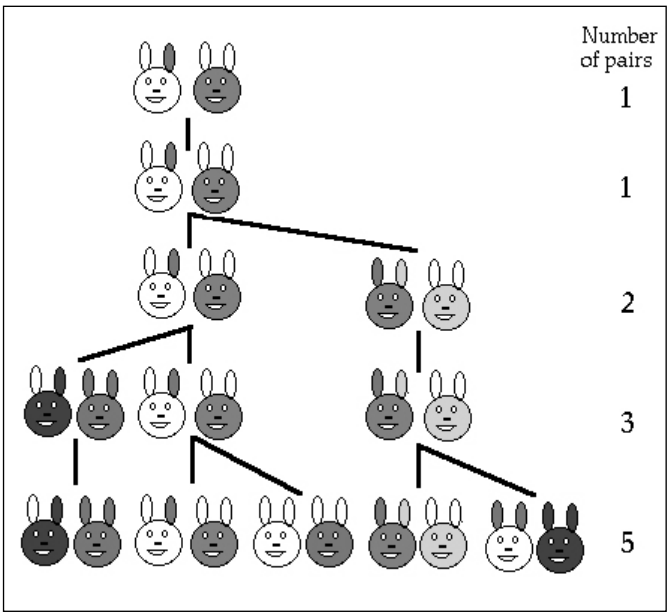
Most recently came, *In Praise of Slow: How a Worldwide Movement is Challenging the Cult of Speed* by Carl Honoré, a highly respected Canadian journalist. By his own admission, the catalyst for this work came when Honoré's life hit the crisis point. The young father decided that he needed to re-examine his priorities during a multitasking frenzy while standing in line waiting to board a plane and saw an advertisement for a "30-second bedtime story". His personal tipping point emerged because the concept actually appealed to him at the time. When Honoré came to his senses, he began to question whether trying to do as many tasks as quickly as possible actually "maximizes the returns". The book is a compendium of various organizations from Italy, Japan, parts of Europe, and North America that advocate a slow philosophy. His investigations reveal the paradox that slower might actually mean faster. A home-cooked meal might take less time to make than ordering pizza; working 30 hours a week instead of 60 might mean an increase in productivity; meditation for an hour could lead to a clearer understanding of a problem than four hours of top-down analysis. His conclusion is that the richer, more meaningful lives we are desperately racing to catch might be right in front of us if we would only pause, take a deep breath, and acknowledge the validity of seemingly time-consuming creative pursuits; things where the process is as important as the finished product.

These ruminations on my students, their disturbing self-portraits and this small sample of time-focused books brought to mind some other rabbits–these ones simultaneously biological and mathematical.

In 1202, Fibonacci investigated the reproduction of rabbits under ideal circumstances. He made some assumptions and restrictions to frame his problem:

- A newly-born pair of rabbits, one male, one female, are put in a field.
- Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits.
- Suppose that our rabbits never die and that the female always produces one new pair (one male, one female) every month from the second month on.

The problem that Fibonacci posed was this: How many pairs will there be in one year?



To answer the problem, Fibonacci explained that:

- At the end of the first month, the original pair of rabbits mate, but there is still one only 1 pair.
- At the end of the second month the original female produces a new pair, so now there are 2 pairs of rabbits in the field.
- At the end of the third month, the original female produces a second pair, making 3 pairs in all in the field.
- At the end of the fourth month, the original female has produced yet another new pair, and the female born two months ago produces her first pair also, making 5 pairs.

The exercise of thinking about the rabbits led me to a consideration of the classic penny problem, e.g., Is it better to be paid $1 000 000 today or to be paid one penny today, two tomorrow, four the third day, and to continue doubling the payment every day for 30 days (one month)?

Of course, both problems are powerful illustrations of two different models of growth. Both problems start with a small quantity but before you know it, there are pennies and rabbits everywhere. There is a powerful message here for mathematics education.

Many school boards, mathematics consultants and administrators have rejected alternative, teacher-led professional development models like Lesson Study and Peer Coaching because they appear to involve a relatively small number of teachers–the educational equivalent of a penny or a pair of rabbits. Sadly, many educational leaders, administrators, and bureaucrats want quick fixes and fail to understand that by being patient enough to invest the time necessary to reap the benefits of individual capacity-building approaches, improved mathematics teaching and learning is achievable.

If we want to achieve fundamental change to the culture of schools, and specifically, the teaching of mathematics, we must adopt long-term strategies for professional development and instructional improvement, and we must acknowledge from the beginning that it is going to be a slow process–but as we saw with a single pair of rabbits or one penny, there exists an amazing growth opportunity because the conditions are right.

Research indicates that changes that are implemented slowly, last; whereas, changes that are suddenly foisted upon schools and teachers tend to be relatively short-lived. In schools that adopted an individual capacity-building approach to professional development, teachers reported that the combination of experiences embodied in these models (e.g., grappling together to understand a concept deeply, developing lessons collaboratively, teaching and reflecting on lessons and debriefing the lesson cooperatively,–all within the teaching/learning environment in which it is taking place) were far better than any series of seminars or workshops that they could possibly attend outside the school.

As educators who may be trying to convince administrators and senior officials about the efficacy of "single penny" approaches, we need to portray a realistic picture of the anticipated outcomes of investment in the process.

One of the misconceptions about Lesson Study for example, is that it is about producing a public library of many different lessons, when in fact it is typical to find that a very small body of lessons is available after one year of considerable human and financial investment. More importantly, true Lesson Study (like Peer Coaching) focuses on how to teach, not what to teach. When teachers meet to coordinate what subject matter will be taught at each grade level, or to correlate provincial expectations to lessons, these may be very important activities–but they are not Lesson Study.

In our desire to *adapt* not *adopt* models from outside our own professional culture by trying to squeeze these square pegs into round holes (possibly a consequence of our eagerness to take the simple parts we like and make them fit our fast-paced, instantly measurable 'results' culture?) we may have inadvertently misrepresented the goals of exemplary, research-based programs such as Lesson Study. The development of lessons is not really the major goal of Lesson Study. The real purpose is for teachers to learn about how children are learning, meaning that any lesson is a byproduct of the larger concern. In fact, the Japanese (who began the Lesson Study tradition) say that the most important outcome of Lesson Study is that it develops eyes through which we can see children.

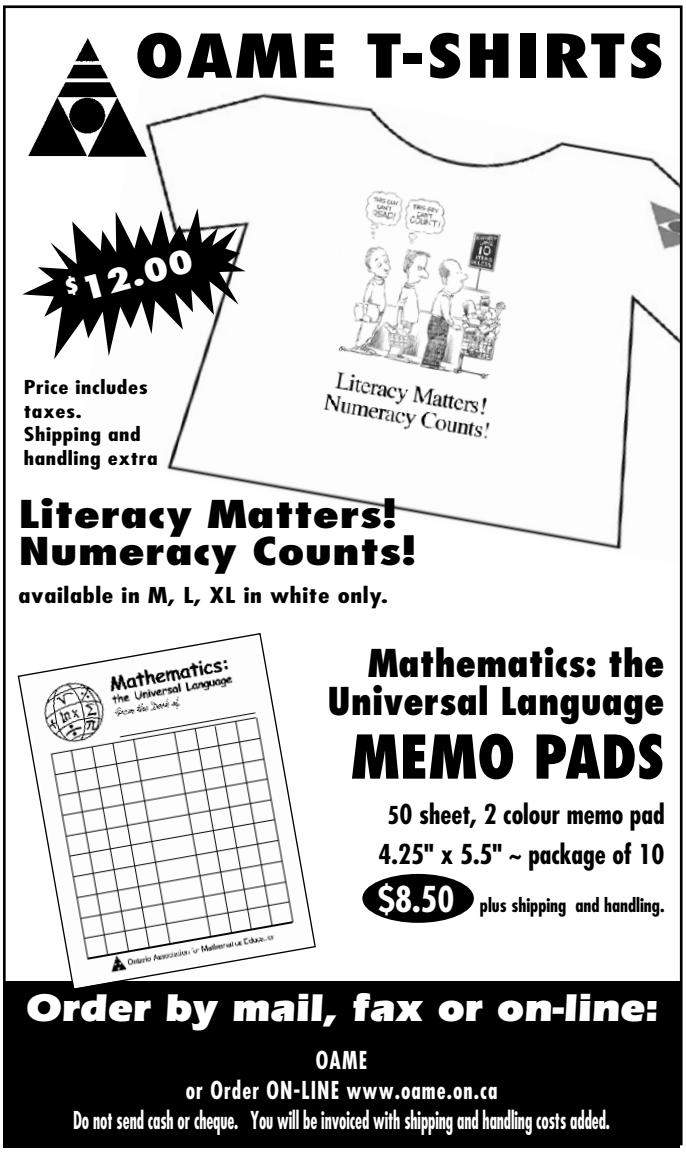
Research suggests that as a consequence of individual teachers learning more about subject matter, students, and teaching and bringing their educational vision to life in the classroom, over time, "the system learns" in much the same way that pennies seem to multiply.

In our hurried world, we may not recognize the benefits of support for teachers to grapple together with the teaching of mathematics so it makes sense to the students. We may not anticipate how long it takes to support teachers to learn how to teach mathematics differently. Progress may appear to occur at a painstakingly slow pace by today's standards.

But perhaps the hourglass by which we measure change needs to be recalibrated so that we lose the "White Rabbit" mentality of running futilely to this one hour session or that. Canadian philosopher Susan Wendell argues that the sheer speed of the contemporary world is disabling. That is, as the pace of life speeds up, more and more otherwise healthy people will be less and less able to cope with and adapt to it.

For many educators, the complexity of teaching and its concomitant, relentless focus on reform will become impossible, even if they are competent and capable in many respects–simply because they will not have the stamina to handle it.

Much of what goes on in the name of professional development does not address the true instructional needs of teachers, but authentic programs like Lesson Study and Peer Coaching are valued and trusted by teachers and are effective change-agents. If we know that the most profound learning experiences take place in slow environments, maybe it's time to start a "Slow PD" movement in Ontario. In an age when we worship the speedy flow of data and the efficient use of time, it is time to remember that some types of knowledge are still best served up slowly, aiding not only digestion, but nourishment. ▲



▲ Friday Special!

JACK WEINER



Fridays at the University of Guelph, five minutes before the end of class, I say, "Pens down, books closed, it's "Friday Special" time!" Then I show on screen an article or cartoon or letter or...something that has to do with math, the "real world" and has absolutely nothing to do with class, assignments, or tests. It is math for fun. Often, it is some flagrant example of how the media abuse math. I hope you use some of these. Make them "Monday Morning" specials or "We need a break!" specials. If one falls flat, blame me. If it works, you take the credit.

Here is a recurring Friday Special theme:

When the media does math, more often than not, the media gets it wrong!

However, you would expect the tech giants of our generation to get it right. Always.

This is in part a story about computer sales mammoth *Dell*. Mostly it's about the power behind the *Dell* machines, *Intel*.

In November, 1994, *Intel* admitted that it had produced a processing chip that **couldn't do the math**. Here is a quote from a Lynchburg College (Lynchburg,Va.) mathematics professor Thomas Nicely that, at the time, brought a problem *Intel* knew about into the open.

"The Pentium floating-point unit is returning erroneous values for certain division operations. For example, 1/824633702441 is calculated incorrectly (all digits beyond the eighth significant digit are in error). This can be verified...by computing (824633702441.0) X (1/824633702441.0), which should equal 1 exactly (within some extremely small rounding error; in general, coprocessor results should contain 19 significant decimal digits). However, the Pentiums tested return 0.999999996274709702 for this calculation."

The magnitude of this error might seem inconsequential but, hey, when you depend on the machine's calculations to send a rocket to Mars, you need accuracy.

Let's not be too harsh. Everyone makes mistakes. I

am especially fallible. Thank goodness I have bright, attentive students ready to correct me at every slip.

However, *Intel* did something unforgivable from a public relations point of view. The company decided it would be too expensive to replace the chips in all the shipped machines with the potentially faulty chip installed. It was up to you, the end of the line customer, to convince *Intel* that your work required arithmetic precision. Only then would your machine be, grudgingly, serviced.

Well, screams of protest prompted *Intel* eventually to do the right thing and every one with a potentially bad chip had the chip replaced on request.

So now please take a look at the accompanying ad, from the late 1990's, for the new generation of *Dell* computers. "Why cruise at 60 (MHz) when you can speed at 75?"



You might think my point has to do with how far we have come in terms of processing speed. We are now at 2GHz, 2.5 GHz, 4GHz!

Not my point at all. At the time, 75 MHz was a substantial improvement over 60. The ad goes on to say (did you read ahead?), "You get 15% more processing power…"

Now, unless there is an incredible coincidence in the curve of processing performance, I think that going from 60 Mhz to 75MHz is a **25%** increase in processing speed.

Let's be fair. The people who write the ads may, by the very nature of the jobs, be people who, in school, shied away from math. Understandable math fallibility. But on the heels of the *Intel* fiasco, you would think the tech folks of these super companies would double check something as important as advertising copy and catch errors like this.

Here is a more scary scenario. Maybe they did. If that's the case, well, oh oh.

Best wishes, Jack

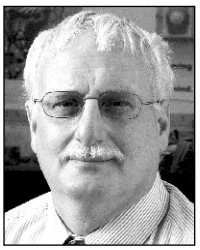
PS For the whole story well told, go to **http://davefaq.com/Opinions/Stupid/Pentium.html**

From that web site, here are two paragraphs that describe in detail *Intel*'s dismissal of the potential problems and in fact their under-estimating that potential. The boldface is mine.

"Intel's initial public response stoked the flames, rather than calmed them: the company set up a fax-back system to brief worried users. The message described the bug as a "subtle flaw" and estimated that the average "spreadsheet user" would encounter the problem only once in every 27 000 years. The idea that Intel wanted to get across was that the rest of the PC was bound to fall apart before your Pentium processor produced an incorrect answer. ***However the users immediately interpreted this as meaning that around 3 spreadsheet users a day worldwide would be getting erroneous results from their spreadsheets, with even more frequent errors for people doing serious scientific work. Most importantly, anyone doing iterative functions, where a variable is repeatedly calculated, could see the inaccuracies snowball through their calculations.***

But above all, the question raised by the newsgroup was, "Why didn't you tell us as soon as you knew that there was a problem, rather than keeping us in the dark?" The second question is invariably "Will you replace my chip?", to which the answer seems to be, "Probably not." ***Unless you can show Intel that you are doing high powered mathematics that needs full double precision figures Intel is unlikely to oblige. To-date we only have two reported examples to draw on: one Pentium user, an undergraduate mathematics student, says that he had his request for a replacement chip turned down, despite the fact that he could be doing these complex calculations on his PC. The other user, using his computer for medical analysis ("If you were going under the knife, would you want to know that the analysis may be wrong?") says that he was put on the list for a replacement after 10 minutes of discussion with an Intel rep."*** ▲

▲ OAME/NCTM JUNE 2006 REPORT



JOHN KERSLEY

Professional development of mathematics teachers is very important to NCTM. Besides the regional conferences and annual meeting, which are listed below, NCTM allows members to bring many workshops right to your computer. Visit the site, **www.nctm.org/eworkshops** for further information and a current listing of the interactive online sessions.

In addition, NCTM offers a multitude of publications for professional development. For further information, contact me regarding these and other NCTM publications or visit **www.nctm.org/catalog**. And, NCTM members have access to NCTM journal articles as well.

In North America there is local, regional, provincial or state, or even international large scale testing of students in mathematics. NCTM has issued a policy statement in regards to this. The question is: what is the role of large scale testing in making significant, high stakes decisions about schools, students and instruction?

NCTM's position: NCTM recognizes the importance of measuring the learning of students and the effectiveness of instruction. Large-scale tests can and should be among several measures that are used to make significant decisions about students and instruction. However, such critical decisions about students and instruction must involve more than the results of any single test. NCTM strongly supports a balance of day-to-day classroom assessments, which help teachers to improve instruction, and external tests that track progress and provide for national and international comparisons.

NCTM has changed a few of its web pages. One very interesting page is entitled "illuminations" at **www.illuminations.nctm.org**. It has improved graphics and new search options making for easy navigation to find resources quickly. Users can search by keyword as well as by band and by topic. This area contains many hundreds of lesson plans and lots of online activities for all mathematics educators from pre-kindergarten to grade 12. Definitely worth a visit!

Web sites of interest to mathematics teachers as suggested by NCTM

Calculus-help.com www.calculus-help.com

- good site for practice or review questions and for test questions,
- links to associated topics and information to make classes interesting

Getsmarter.org www.getsmarter.org

- compare students in math and science worldwide.
- for all grades

CuriousMath.com www.CuriousMath.com

- search the list of topics for fun and trivia

LessonPlanz.com www.lessonplanz.com

- directory of free online lesson plans for all grade levels and subject areas

NCTM Conferences www.nctm.org/meetings

NCTM Regionals

- Chicago "Winds of Change" September 20 -22, 2006
- Phoenix "Show Me the Mathematics—Learning through Representation October 5 – 7, 2006
- Atlantic City "Dive into Mathematics: It's a 'Shore' Thing" October 19 – 21, 2006

NCTM Annual Meeting Next Year in 2007

- Atlanta "Mathematics: Representing the Future" March 21-24, 2007 ▲

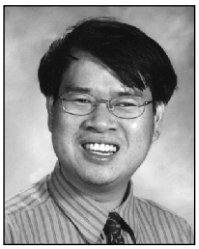
2ND INTERNATIONAL SYMPOSIUM ON MATHEMATICS AND ITS CONNECTION TO THE ARTS AND SCIENCES (MACAS2)

The Organizers of the 2nd International Symposium on Mathematics and its Connection to the Arts and Sciences (MACAS2): Astrid Beckmann, Claus Michelsen & Bharath Sriraman are pleased to announce that the 2nd Symposium is scheduled to be held in Odense, Denmark from May, 29-31, 2007, and will be hosted by Claus Michelsen at The University of Southern Denmark, Odense.

For details, please visit
http://www.macas2.sdu.dk/

▲ Get Interactive!: Web Sites for the Dog Days of Summer

Louis Lim



Louis Lim is a part-time mathematics teacher at Unionville HS and a PhD student at York University. He is currently completing a 3-year term as an editorial panel member of the NCTM's Mathematics Teacher and is a former editor of the OAME Gazette*. In the spring 2005, he was the recipient of OAME's "Exceptional and Creative Teaching in Secondary Mathematics" award.*

With another school year coming to an end (I just finished an amazing decade of teaching!), this edition of *Get Interactive!* contains web sites to browse during the summer. Included are links to an art gallery and to a museum; mathematics postage stamps; and to contemporary mathematics. Whether you teach elementary, secondary, or post secondary school, think about how you can incorporate these treasure troves into your teaching of mathematics. Enjoy!

1. Mathematics Exhibits

The Art of Justin Mullins, **http://www.justinmullins.com/**

I learned about Justin Mullins' "Mathematical Photography" featured in a London, England art gallery from a *Toronto Star* article (March 5, 2006), "It's mathematics, but is it art?" (see Mullins' web site). His art work is described as "made up of simple, black and white digital outputs…his exhibition share qualities of 'elegance and simplicity'." For secondary teachers, I'm confident you'll agree that "Beauty" is appropriately named: $e^{i\pi} + 1 = 0$. For elementary teachers, have a look at "Surrealism". Post "1 + 1 = 3 for large values of 1" in your classroom and you can be sure students will want a mathematical discussion!

The Franklin Museum, **http://fi.edu/math/index.html**

Although intended to supplement a class visit for grades 1-8 to the Franklin Museum in Philadelphia, the web site provides interesting word problems pertaining to sports, trains, science and space. Also included are on-line resources that are independent of visiting the museum. In particular, "Melting Pot Math: Multicultural Problems for the 21st Century" is a wonderful collection of word problems, organized by country, that connects mathematics with social studies. "Neighborhood Math" allows middle school students to solve word problems, such as identifying geometric shapes and figures from photos, applying number sense skills in a pizza restaurant and conducting a survey.

2. Mathematical Postage Stamps

(a) Ten Equations that Changed the World, **http://www.mscs.dal.ca/Faculty/swami1.jpg** and **http://www.mscs.dal.ca/Faculty/swami2.jpg**

Nicaragua created a set of 10 postage stamps featuring mathematical equations. Dr. Srinivasa Swaminathan, professor emeritus at Dalhousie University in Nova Scotia, published *Equations that Changed the World* (1990), which remains significant to me since it renewed my interest as a stamp collector. My favourite stamps contain the equations $1 + 1 = 2$ (Primitive Humans); $a^2 + b^2 = c^2$ (Pythagorean Theorem); $F_1x_1 = F_2x_2$ (Archimedes' Principle); and $e^{\ln N} = N$ (Napier's Equation). The Nicaraguan stamps have tested my patience and perseverance as I only have 20% of the set. Whenever I find an additional stamp for my collection, my excitement is equivalent to adults finding their favourite candy not consumed since childhood.

(b) **http://ublib.buffalo.edu/libraries/asl/exhibits/stamps/** (Select Mathematics and Computation I to IV)

Maiken Naylor of the University of Buffalo maintains a wonderful collection of postage stamps pertinent to mathematics and science. Photos and brief descriptions are included. Stamps include: a proof of the Pythagorean Theorem; Archimedes' contribution that the volume of a sphere is 2/3 the volume of a cylinder; Fermat's Last Theorem, $x^n + y^n = z^n$ has no integer solutions when $n > 2$; Euler's Relation[1], $e^{i\phi} = \cos\phi + i \sin\phi$; Karl Gauss' Complex Number Plane; and the digits of π spiraling on a stamp.

(c) **http://jeff560.tripod.com/**

Florida high school mathematics teacher, Jeff Miller, has compiled an impressive postage stamp database organized by mathematicians and mathematical topics. Included are: 13 stamps to commemorate the World Mathematical Year 2000; Pascal's Triangle, reprinted from *Precious Mirror of Four Elements* (1303); graphs

[1] If we substitute $\phi = \pi$ into the equation $e^{i\phi} = \cos\phi + i \sin\phi$, then we get $e^{i\pi} + 1 = 0$ (Justin Mullins' art work called "Beauty").

of world population growth; fractals; the golden section; the binomial expansion of $(a + b)^2 = a^2 + 2ab + b^2$; arithmetic including $1 + 1 = 2$, $3 – 3 = 0$, $3 – 0 = 3$; and numerous mathematicians including Descartes, Gauss, Euclid, Kovalevskaya[2], and Leibniz.

3. Contemporary Mathematics

(a) Fermat's Last Theorem,
http://www.pbs.org/wgbh/nova/proof/wiles.html

Fermat's Last Theorem, an unsolved mathematical problem for 350 years, was proved in 1995 by Andrew Wiles of Cambridge University. The problem, which states that there are no integer solutions for the equation $x^n + y^n = z^n$, $n > 2$, made headlines in newspapers such as the *New York Times* and the *Globe and Mail*. The web site consists of an interview with Wiles, which conveys that mathematics is still being created today and that it is a humanistic discipline. Intrigued that he could understand the problem as a 10-year old, Wiles made it a life goal to prove it. OAME *Gazette* readers interested in learning more about Fermat's Last Theorem should read Simon Singh's *Fermat Enigma*.

(b) The Four Colour Problem,
http://www.npr.org/templates/story/story.php?storyId=4254287 and **http://www.math.gatech.edu/~thomas/FC/fourcolor.html**

The Four Colour Problem states that any map (even with billions of regions) can be coloured with at most four colours, such that neighbouring regions are coloured differently. First stated in 1852 by Francis Guthrie, a computer was used in 1976 by Appel and Haken to solve the problem. Outputting 1476 graphs, the proof was accepted as correct although unverified by humans. In 1996, Robertson, Sanders, Seymour and Thomas published their proof[3] which reduced the 1476 graphs to 633 graphs. An interview in early 2005 with mathematician Keith Devlin from Stanford University discusses how another computer verified the accuracy of the 633 graphs.

(c) Clay Math Institute Millennium Problems,
http://www.claymath.org/millennium/

The Clay Math Institute identified 7 unsolved problems in the year 2000 that are worth $1 million for each correct solution: Birch and Swinnerton-Dyer Conjecture; Hodge Conjecture; Navier-Stokes Equations; P vs NP; Poincare Conjecture; Riemann Hypothesis; and Yang-Mills Theory. I first heard of the Riemann Hypothesis in undergraduate mathematics and was reintroduced to it in an episode of *Numb3rs* and in the book and motion picture, *A Beautiful Mind*. My understanding is that the solution of the Riemann Hypothesis would determine how the prime numbers are distributed. Such a solution would have consequences for financial institutions which use prime numbers for encryption. Mathematician Louis De Branges de Bourcia of Purdue University in Indiana claims to have proved the Riemann hypothesis **http://en.wikipedia.org/wiki/Louis_De_Branges_de_Bourcia)**.

Reference

Johnson, A. (1994). *Classic math: History topics for the classroom.* Palo Alto, CA: Dale Seymour Publications. ▲

[2] When I was a teacher candidate at Westview Centennial SS in the former North York Board of Education, I stuck my 1st year university calculus notes on a bulletin board in my associate teacher, Doreen Roberts' classroom, hoping students would read it before taking OAC calculus. I got the idea from Sonya Kovalevskaya whose bedroom was wallpapered with her father's calculus notes. She spent many nights reading the notes, which helped when she eventually learned calculus. When Kovalevskaya passed away, her brain was preserved and used to distil the belief of male superiority when her mathematical brain was larger than scientist Hermann von Helmholtz (Johnson, 1994).

[3] Justin Mullins' "Ugliness" is a reproduction of the 633 graphs for the Four Color Problem.

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▲ Photo Math ~ A New Way of Picturing Math

Ron Lancaster
Independent Mathematics Consultant



Ron Lancaster is a Lecturer in Mathematics Education at the Ontario Institute for Studies in Education at the University of Toronto. He is also an independent mathematics consultant and works with teachers, students and educational organizations in North America, Asia and Israel.

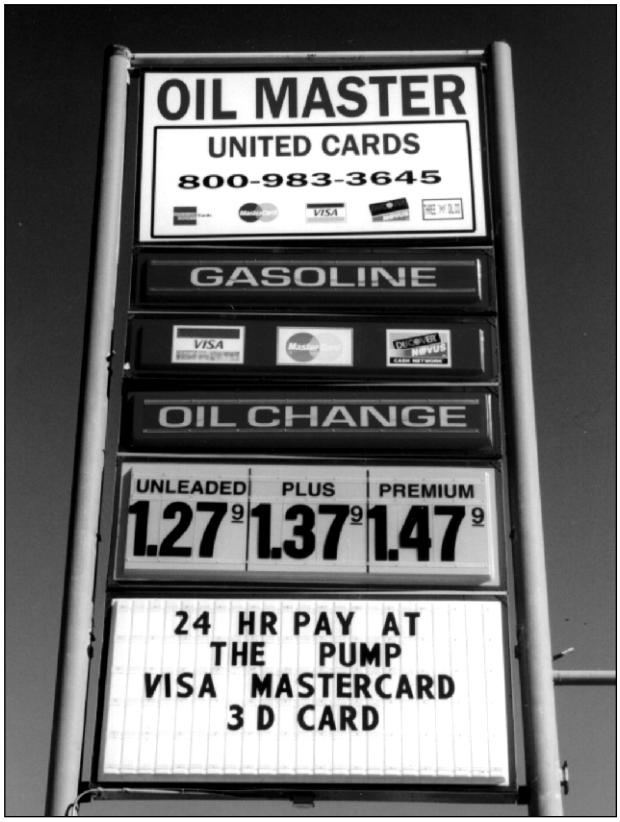
"A picture is worth a thousand words" is an expression that is often used to convey the importance of using images to make a point. The point of this column is to go beyond this notion and promote the idea that in fact "A picture is worth a thousand numbers". Photos with Math is what readers can expect to find in every Photo Math column and the emphasis will be on seeing and doing mathematics everywhere. Teachers may want to assign these questions to their students as part of an overall effort to make connections between the curriculum and the world around us. Establishing these connections is at the heart of the new Ontario curriculum as well as the new NCTM document, *Principles and Standards for School Mathematics*. It is hoped that these questions will inspire students to be more creative and playful and that in the end, students will develop an ability to see the world from a different perspective.

Teachers and students are encouraged to pose mathematical questions related to their own encounters and to submit these problems along with photographs to the author for possible publication in future columns. Photos can be sent via e-mail or by regular mail to the author of this column.

Introductory Comments

These photos were taken during a recent business trip to Kilgore, Texas. Kilgore is located in East Texas in an oil-rich area.

During my trip, I created a Math Trail at St. Luke's Church for teachers attending my workshops. After I prepared the Trail, I spent several hours driving around Kilgore searching for additional mathematical scenes. The following three photographs were taken during that drive.



(a) Someone with a mathematical eye might look at these gasoline prices and view the 9 as an exponent instead of the intended $\frac{9}{10}$ of a cent. Using this exponential perspective, calculate the price of a gallon of gas for each of the three blends.

(b) The expressions given below continue the pattern of prices in the photograph. Find the value of each of these expressions and discuss the manner in which the answers change from one case to the next.

$$1.57^9,\ 1.67^9,\ 1.77^9,\ 1.87^9,\ 1.97^9,$$

$$2.07^9,\ 2.17^9,\ 2.27^9,\ 2.37^9,\ 2.47^9$$

(c) The author of this column once saw a sign that advertised gas for 1.08^9 dollars per gallon. Thinking in terms of exponents, he immediately knew that this was equal to 2 dollars per gallon. How did he know this?

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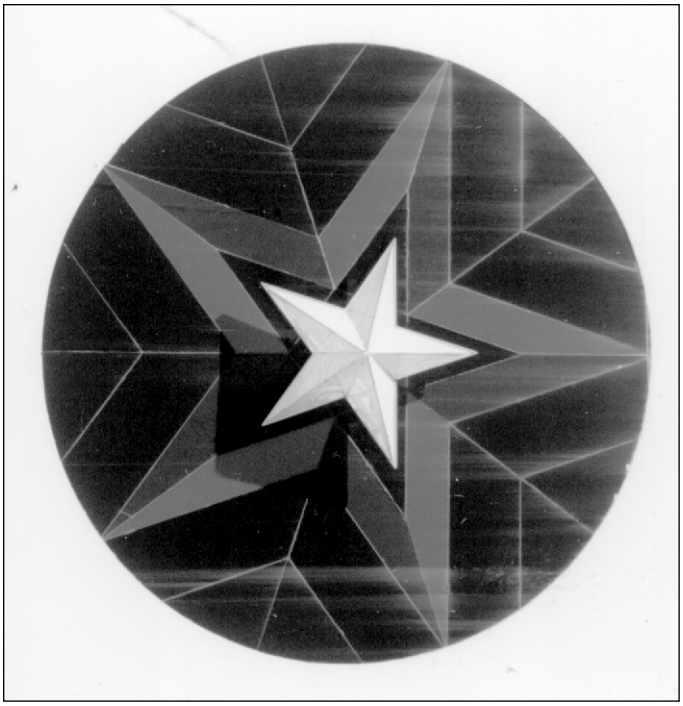


(a) Let L and W represent the length and width of a storage space at A-1 Self Storage. Find L and W for each of the four spaces listed in the photo and record your results in the following chart.

STORAGE SPACE	L	W
1		
2		
3		
4		

(b) Sketch the graph of L versus the storage space type along with the graph of W versus the storage space type. Is it possible to find two simple equations that can be used to produce these graphs?

(c) While it is possible to find equations that can be used to produce the graphs in part (b), it can be said with absolute certainty that these equations are not linear. There is, however, a way of slightly altering the data for the sizes of the spaces in the photograph so that the resulting graphs are perfectly linear. Make this change and then find the equations.



This huge and beautiful geometric design appears on a building in downtown Kilgore. Certain details of the design are sure to be lost during the scanning of the photo and in going from a colour to a black and white image. However, readers should be able to see that the design consists of a large circle, an inscribed pentagram, and a second smaller pentagram similar to the first.

(a) Suppose the radius of the circle is 1 metre. Find the perimeter and the area of the inscribed pentagram.

(b) Suppose the area of the smaller pentagram is one-fifth of the area of the larger pentagram. Is it true that the perimeter of the smaller pentagram is one-fifth of the perimeter of the larger pentagram? Include some calculations to support your answer.

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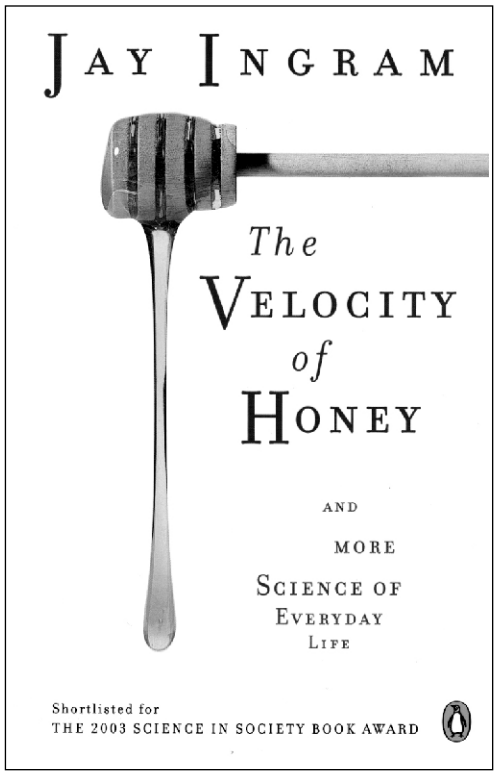
▲ Book Review: *Counting Coots*

Mary (Gartrell) Sakamoto



Mary (Gartrell) Sakamoto has been part of a family involved in education for her entire life. Her parents, Bill and Martha Gartrell both enjoyed careers teaching mathematics and science, mostly in Thunder Bay, ON, with Martha pioneering the teaching of computer science in the late 1960's. Mary taught 3 years in Thunder Bay and then 27 years in the Eastern Townships of Quebec in Lennoxville and in Richmond, retiring as head of department in 2002.

Jay Ingram, who is always so interesting on the Discovery Channel's show *Daily Planet* published *The Velocity of Honey and More Science of Everyday Life* in 2003. Each chapter is written in his fascinating, easy to understand style with an entertaining approach to looking at the world around us. The chapter, *Counting Coots*, caught my attention and gave me pause to think. He first describes the inherent ability of certain birds to distinguish numbers. Coots in particular, need to limit the number of eggs in their clutch to exactly eight, deposit any excess in another nest, and then eliminate ones that have been surreptitiously placed in their own nest. He then goes on to describe controlled experiments to teach numbers to birds and animals. Pigeons can learn that pecking on a certain button forty five times will open a door with food but that a different button requires fifty pecks for the same result. Mr. Ingram then reviews two experiments designed to determine the number sense of very young children, including infants as young as a few days old. It seems we are born knowing the concept of one, two and three. Until the 1980's, the accepted school of thought was that infants are born with no idea about numbers and that the learning process was gradual and time consuming. Both Prentice Starkey, a psychologist at the University of Pennsylvania and Karen Wynn at Yale University, demonstrated that this was not so, and that the very young could not only distinguish between the numbers one, two and three but also count with them. Both used a slide show technique on their young subjects, using concentration time as a measurable indicator. The elements of boredom and surprise were introduced to analyse reactions. Here is a synopsis: Prentice Starkey first studied babies between the ages of four and nine months by sitting them on their mothers' laps facing a screen that displayed a series of dots or other objects. A video camera recorded how long they continued to look directly at the screen. If, for example, they saw slide after slide showing 2 dots, the recorded time steadily declined to an average of 1.9 seconds. As soon as the slide was switched to 3 dots, the gaze was held longer, 2.5 seconds on average. A further experiment linked sound to sight. Babies looked longer at a screen of 2 dots when a soundtrack of 2 drumbeats was being played as opposed to 3 drumbeats. The same result held with 3 dots on the screen and 3 drumbeats being played as opposed to 2. In the 1990's Karen Wynn furthered this study and she concluded amazingly that 4- and 5-month old babies comprehend addition and subtraction within these first 3 numbers. She started from the well known fact that young infants display a strong reaction when something impossible or seemingly magical occurs. Imagine a puppet theatre, some Mickey Mouse puppets, and a screen to hide behind. Mickey Mouse puppets, one by one, can be seen to come on stage, leave the stage, and/or go behind a screen. One Mickey comes on stage, goes behind the screen and the screen is lifted to reveal 1 Mickey. The stare time was measured. The pattern is repeated with 2 puppets, and then with 3. The average stare time was the same in each instance. If however, 2 puppets went behind the screen and only 1 Mickey was there when the screen was lifted, they were startled and looked at the screen a full second longer. The longest stare time occurred when 2 puppets went behind the



screen and 3 were revealed when the screen was lifted. Subtraction also seemed to be understood. Two puppets came on stage. One went behind the screen, but the other left the stage. The stare time when the screen was lifted and 1 Mickey was revealed was measured. This was repeated for 3 coming on stage, 1 leaving and 2 remaining to be revealed behind the screen. Then unexpected results were introduced. The babies showed surprise and a long reaction time to the seemingly impossible 2 – 1 = 2. One Mickey came on stage and went behind the screen. A second came on and then left again. When the screen was lifted two puppets were visible and the babies reacted to the magic. Jay Ingram concludes that he doubts the controversy over infant numeracy is resolved, but he finds the evidence that very young babies have some numerical knowledge is pretty convincing. Since reading the article, I have paid attention to my own speed at determining the number of objects arranged haphazardly. Be it peanuts, pencils, pins, or petunias, up to 3 is instantaneous. More than that and the hesitation and need to double check sets in. Six-tenths of a second is the recorded average time for adults to recognize 1, 2, or 3 dots on a screen. For 4 and 5 dots, the average time increases to eight-tenths and one second respectively and errors creep in as the number of dots increases. Of course the dots on a die or figures on playing cards with their familiar patterns are always recognized instantly. I seem to be average and that is fine by me. The number system that we use is based on groups of 3. We write large numbers with commas or spaces; 82 745 328 619 and read them by billions, then millions, then thousands, and finally, 6 hundred and 19. Even Roman numerals never use a mark more than 3 times in a row. Our natural instincts seem to be used to advantage here and I wonder if this ability with groups of 3 should be utilized more with teaching. We already consider the multiples of 2, 5, and 10 as a comfort zone for students. Treating the 3 times table in the same manner could work to a teacher's advantage. Play up more the divisibility by 3 rule, and keep adding and subtracting 3, 6 and 9. We are always looking for aids to help explain new and challenging concepts. Try making your examples with 3 in mind and it could facilitate your students' ability to understand. Apart from always looking for teaching applications, it is important to remember that continuing education and professional development do not always happen in a university course or in a workshop. Look for this wonderful little book and enjoy all of the chapters. ISBN 0-670-89310-2 ▲

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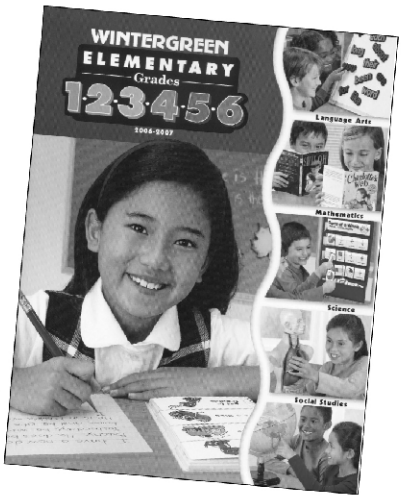


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▲ Union Gas Awards

BY:
DAVE AND SUE HESSEY
EXECUTIVE DIRECTORS
AND JACQUELINE HILL
OAME AWARDS CHAIR

In October 2005, directors of each of the seventy-two school boards in Ontario were invited by the Ontario Association for Mathematics Education (OAME) to nominate up to fifteen employees who have made an outstanding contribution to student learning of mathematics. The three award winners were publicly recognized at the annual OAME conference held at Fanshawe College on May 12 at the Every 1 Celebrates banquet at Western Fair Raceway in London. Together with their parent corporation Duke Energy, Union Gas has a mandate to focus on building leadership capacity in not for profit organizations, encouraging employee volunteerism, and supporting Mathematics and Science Education. Union Gas has sponsored these special awards for the last four years and continues to provide financial support to OAME because of their belief in a strong foundation in Mathematics for Ontario's workforce.

OAME recognizes the critical role that teachers play in creating an atmosphere for learning mathematics. They do this by preparing motivating lessons that often employ manipulatives or technology, by encouraging students to talk with each other to construct their understanding, and by using rich and varied assessments to inform their teaching and improve students' learning. These teachers invariably go the extra "kilometre" to help students individually, to participate in extra-curricular activities, and to communicate with parents. The first two Union Gas awards are specifically for an elementary and for a secondary classroom or special education teacher who demonstrates exemplary mathematics teaching practices and supports inclusive mathematics education. The third Union Gas award recognizes a lead teacher, a department head, a resource teacher, a teacher-consultant, or an administrator who demonstrates a strong commitment to a progressive vision for inclusive mathematics education and leadership that motivates and engages teachers in mathematics professional development.

Special congratulations to the 2006 "Ontario Mathematics Educators of the Year":

1. The Union Gas Award for Teaching Excellence in Mathematics (elementary) awarded to Jessica Reiter, Peel DSB
2. The Union Gas Award for Teaching Excellence in Mathematics (secondary) awarded to Anne Fitton, Ottawa-Carleton DSB
3. The Union Gas Award for Outstanding Leadership in Mathematics Education awarded to Judy Dussiaume, Rainbow DSB

OAME would like to congratulate all the teacher-leaders who have been nominated over the past four years for these special awards, sponsored by Union Gas. They are all deserving of recognition as outstanding mathematics teachers and/or leaders.

Special congratulations to the following Ontario educators who were nominated by their boards in 2006.▲

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▲ Another Summer's Dalliance: The Musical Group



CHANTAL BUTEAU
ASSISTANT PROFESSOR
MATHEMATICS DEPTARTMENT
BROCK UNIVERSITY
ST. CATHERINES ONTARIO

Last June Steve Sparling had a blast with a three-color triangular highlighter. He determined all six possible arrangements by simply interchanging the three caps. That started his rave on groups. After a mathematical discussion, Steve refers the reader to interesting applications of groups such as RSA public key encryption and Crick's discovery of DNA's shape. With the upcoming international symposium in Toronto/Guelph/Waterloo on the composer Iannis Xenakis, Steve's dalliance may continue. Xenakis was indeed a composer who used mathematics; his composition *Nomos Alpha* is a musical realization of ... the group of rotations of the cube onto itself!

Iannis Xenakis: A Composer Who "Mathematized" His Music

Xenakis (1922-2001) is a contemporary composer renowned for using stochastic methods in composition. Raised in Greece, he enrolled in an Engineering program at the Athens Polytechnic and took courses in Mathematics and Music. After World War II, he fled to Paris where he worked with the eminent architect Le Corbusier who later led him to meet with Messiaen, an established composer and music theorist. Both masters had a great influence on Xenakis' musical thought, as later acknowledged by the composer himself.

Xenakis labeled his early music as *Free Stochastic Music*. His approach was, among other things, to attempt reproducing the noise of rioting crowds as he experienced them in his days as an activist during the war. The sound is an aggregate of thousands of isolated sounds following a stochastic law. Some of his later work took a more deterministic tone as illustrated by his Nomos Alpha (1965) for solo cello in which he adopted a different compositional approach based on group theory.

Group of People, Music Group, ... , Well, Let's Just Regroup

Let us revive last summer's dalliance by revisiting the concept of a group. Consider again Steve's equilateral triangular highlighters with the corners labeled Y, R, and G for yellow, red and green respectively. By rotating and reflecting the caps, we obtain six different positions, namely: the identity (no change); a rotation of 60˚; a rotation of 120˚; a reflection keeping the G vertex fixed; and the two other reflections for the R and Y vertices. This actually forms the dihedral group D_3 of order 6.

Another example of a group is the so-called *clock*, with elements the integers from 0 to 11. Adding two of its elements amounts to counting around the clock: e.g., 10 + 2 = 0 and 8 + 5 = 1 (at 8 a.m., meeting you in 5 hours means meeting at 1 p.m.). This is the additive group Z_{12} called *'Z mod 12'*, which turns out to be indeed very musical! If we associate 0 to the note C, 1 to C#, 2 to D, 3 to D#, 4 to E, and so on, up to 11 to B, we can start making music theory with the group Z_{12}. For example, the chord C – E – G is represented by {0,4,7}, and its transposition up to F major (corresponding to "+5" in Z_{12}) is represented by {5,9,0}, which is the same as {0,5,9}. The group Z_{12} is nowadays commonly used in music theory literature.

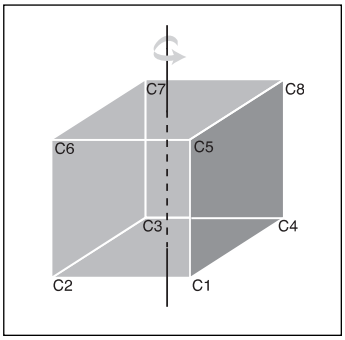
One may ask: what makes Z_{12} and D_3 groups? Each one has an identity element (no cap change in D_3 and 0 in Z_{12}); the combination of any two elements yields an existing (e.g., 7 + 11 = 6 in Z_{12}); each element is invertible (e.g., 5 has inverse 7 in additive Z_{12} since 5 + 7 = 0); and the operation is associative. That is all one needs for a group!

What about the Xenakis' cube in *Nomos Alpha*?

Let's Spin that Cube!

Place a die on a table. There are 6 different possible numbers facing up. For each one of these, there are 4 possible numbers facing you, for a total of corresponding 24 different rotations forming a group.

Xenakis used this group to compose *Nomos Alpha*. To each vertex of a cube, he associated a musical cell, for example a note cloud or a repetition of notes. Since this group is a subgroup of the permutation group, Xenakis represented each rotation as a permutation of the vertices. For example, in the following figure, a 90˚ counterclockwise rotation with respect to the vertical axis maps the cells C1 into C4, C2 into C1, C3 into C2, C4 into C3, etc ..., and this rotation can be represented by



(41238567). For Xenakis, each rotation was the sound of an eight-cell progression element.

Xenakis' Last Twist: A Fibonacci Sequence of Rotations

Having the elements for his composition, Xenakis had to determine the order in which they would appear. He chose to use the Fibonacci sequence to determine that order. Let us recall this celebrated sequence – made even more popular as the encoded last wish of a murdered man scribbled with his own blood on *le Louvre's* waxed floor in Dan Brown's *Da Vinci Code*. Starting with 0 and 1, define the next element of the sequence as the sum 1 + 0 = 1, and continue the process of adding pairs of successive elements: 1 + 1 = 2, 2 + 1 = 3, 3 + 2 = 5, 5 + 3 = 8, 8 + 5 = 13, ... , $a_{n+2} = a_{n+1} + a_n$.

Applying the same process to the group of rotations of the cube, let us first choose two rotations r_0 and r_1. The subsequent rotations of the sequence are defined by composing the two preceding ones: $r_{n+2} = r_{n+1} + r_n$. The group being finite, the sequence will eventually start repeating itself. Take for example the identity rotation and the rotation of 180˚ around the vertical axis as r_0 and r_1. The third rotation of the sequence is again the same 180˚ rotation, the fourth rotation is the identity, and we are back at the start. In this case, we say that the period of the sequence is 3 and the sequence contains 2 different rotations. Depending on the choice of the two initial rotations (the order matters!), the sequence will have different periods and will have a different number of rotations.

Which rotations did Xenakis select for his Nomos Alpha? He chose r_0 = (23146758) and r_1 = (56871243), one of 1128 possibilities (24 first rotation x 24 second rotation x 2 order – 24 for first and second rotations being the same). Interestingly, Moreno Andreatta showed in his Ph.D. thesis (2003) that the Xenakis combination has a maximum period (18) and maximum number of different rotations (13). Can you hear the algebra in *Nomos Alpha*?

A Summer's Dalliance

On June 8-10 of this year, the University of Guelph, University of Toronto and University of Waterloo will host *The Creative and Scientific Legacies of Iannis Xenakis* International Symposium. Experts from all over the world will discuss Xenakis' music. Will they spin and hear the cube sing *Nomos Alpha*?

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▲ One Problem; Many Solutions

Shawn Godin, Consultant: Mathematics Accountability Quality Assurance Division Ottawa-Carleton District School Board



Shawn is currently finishing up a three year term as a consultant with the Ottawa Carleton District School Board. He returns to reality in September as the Head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans.

A good problem is one that engages students with a variety of mathematical backgrounds and can be solved using the tools that they have at their command. This article is about one such problem that I came upon a few years ago.

The first time I encountered this problem was on the *Discovery Channel*. They used to have a segment on their daily science news program in which a mathematician would pose new problems once a week. The problem, in its original form, is as follows:

You have all 13 cards of one suit (from ace to king) arranged in a stack. You perform the following actions:

- *take the top card and put it on the bottom of the deck*
- *take the next card and place it face up on a table*
- *repeat the process until all cards have been placed on the table.*

How should the cards be arranged originally if you want them to come up in the order: Ace, 2, 3, …, Jack, Queen, King?

At the time, I used it with a grade 9 class and a grade 12 class. Recently, I have given it to a group of grade 7 to 12 teachers to solve and then to try with their students.

People attacked the problem with a range of techniques. Here are some possible strategies for solving the problem, and some anecdotes that go with them.

Solution #1: Trial and Error

Take a random arrangement of cards (noting the arrangement) and perform the actions. After all cards have been placed, cards in the correct position on the table must have started in the correct position in the deck. Others were in the incorrect position, so you can try another arrangement taking the new information into account. It sounds messy, but if done correctly you can have a solution with a maximum of 12 tries.

Solution #2: Deduce the Pattern

Since you are flipping up every second card, you can deduce that ace, two, three, four, five and six must go in the second, fourth, sixth, eighth, tenth and twelfth positions. For the remaining cards you have to remember that some cards have been removed, and some spaces remain open.

Anecdote #1

When I gave the problem to the grade 9 class, I allowed them to work in groups and they were to tell me when they had a solution. As a result, there was a competitive air in the classroom. Everyone wanted to be done first. Most of the groups started with solution #2 and called me over before they tried it out, only to show me something that didn't work. They then proceeded with trial and error to get the remaining cards. They messed up with the cards that got passed over the first time. It was amusing to see the looks on their faces as they called me over with great confidence only to have the cards come up Ace, 2, 3, 4, 5, 6, 7, 9, Jack, King, 10, 8, Queen time after time.

Solution #3 : The "Official" Solution

The solution shown on the Discovery Channel was similar in nature to solution #2, except that you keep track of the cards with some sort of system. They used 13 boxes to represent the 13 cards.

Notice that you skip every second card so that you can place ace through 6 as in solution #2.

	A		2		3		4		5		6	

As you reach the end of the deck, the cards that we "skipped" at the start are now at the bottom, so the table can be treated as a closed loop. Note that cards that are now "missing" from the deck (shaded below) won't count in our "skipping". Now we can place 7, 8, 9 and 10.

7	A		2	8	3		4	9	5		6	10

Finally we can repeat the process and complete the solution.

7	A	Q	2	8	3	J	4	9	5	K	6	10

Anecdote #2

When I gave this problem to the group of teachers, it was in a modified form that appeared in *Crux Mathematicorum with Mathematical Mayhem, Volume 28, #5*, (September 2002) as question M51. It was worded:

> *You have a deck with cards numbered 1 through 25. You perform the following operations on the deck:*
>
> - *you place the top card on the bottom of the deck.*
> - *you place the new top card on the bottom of the deck.*
> - *you flip the new top card face up on the table.*
>
> *You continue this process until all cards are face up on the table. Find the order of the cards in the deck if, when the process is performed, the cards get laid out on the table in the order 1, 2, 3, . . . ,25.*

This, in essence, is the same problem. Teachers were supplied with cards numbered 1 through 25. Most of them solved the problem as either solution #3 or as a combination #1 and #2. Afterwards some teachers expressed their concern in not being able to find a more "mathematical" solution. It is interesting that many of the teachers present would have preferred if there were some sort of equation to solve, or it involved some sort of notation. Understanding the problem and finding the answer logically seemed to be "cheating".

Solution #4: Play the Tape Backwards

Start with the cards laid out face up on the table in the order that you want them (Ace, 2, 3, …, King). Then, starting with the King, reverse the process. That is:

- pick up the King and place it face down to start the deck
- take the last card now showing (Queen) and place it face down on the top of the deck
- take the card from the bottom of the deck (King) and put it on the top of the deck
- repeat the process until all cards are back in the deck(so for the next two steps you would place the Jack on top of the deck, then move the Queen from the bottom of the deck to the top).

The cards will now be in the desired order. You have "undone" the process of placing the cards on the table to give the initial order of the deck. This procedure can be tricky, especially when deciding how to start and end.

Anecdote #3

Associated with this solution, I had one of those rare moments in a career when the stars align and something magical happens in a classroom. In the aforementioned grade 9 class, one girl chose to work alone I will call her Wilma). For the first ten minutes or so, Wilma just sat and stared at the cards laid out on her desk. I was starting to wonder if she was thinking, confused or had just gone to a "better place". I kept an eye on Wilma, and noticed that she started picking up the cards as in solution #4, which was thrilling because I hadn't thought of such a succinct solution. When I talked to her, she very quickly convinced me that she knew perfectly well what she was doing. Nobody else in the grade 9 or the grade 12 class came up with that solution. When I gave the problem to the teachers, a couple of them had also used that method.

Solution #5: Use Function Notation

Start with the cards in the deck in the order that you want them to come up, and perform the operation, noting the order that the cards are flipped up. You will find the following:

Deck	Table
A	2
2	4
3	6
4	8
5	10
6	Q
7	A
8	5
9	9
10	K
J	7
Q	3
K	J

We can think of this as an "assigning" function, where if $y = f(x)$, then x is a card in the deck, and y is the card in the same position (numerically) on the table as x was in the deck. Thus the inverse function $x = f^{-1}(y)$ can almost be thought of as card y started out in position x, so if we write the inverse function and then order the table, we get:

Table	Deck		Table	Deck
2	A		A	7
4	2		2	A
6	3		3	Q
8	4		4	2
10	5		5	8
Q	6		6	3
A	7	→	7	J
5	8		8	4
9	9		9	9
K	10		10	5
7	J		J	K
3	Q		Q	6
J	K		K	10

The solution appears in the last column. Basically we have symbolically done solution #4. Another way to think about this process is to start with blank cards, number them 1 to 13 (with a small number in the corner) and go through our process. When you are done "name" the cards (Ace, 2, 3, …, King) as they flip up.

Anecdote #4

When I originally saw the problem, we had been talking about inverse. I doubted that any would think to use function concepts (and they didn't), but I presented it as a solution to them and it became an interesting experience that helped some of them understand the process of taking an inverse.

Solution #6: Use Permutation Notation

Perhaps, if we are to be a bit more mathematically precise, we should talk about permutations rather than functions. So, for our 13 cards, a permutation of the cards can be given by y = (3 7 Q 10 8 2 K 4 A 9 6 J 5).

Then

$$P=\begin{pmatrix} A & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & J & Q & K \\ 3 & 7 & Q & 10 & 8 & 2 & K & 4 & A & 9 & 6 & J & 5 \end{pmatrix}$$

represents the action on any permutation of cards that has the same effect of moving the permutation, in this case (Ace, 2, 3, …, King) to (3, 7, Queen, …, 5). So if we let

$$x=(A \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad J \quad Q \quad K)$$

then we can write this symbolically as y = p(x). In our case the "putting down on the table" process is equivalent to the permutation

$$P=\begin{pmatrix} A & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & J & Q & K \\ 7 & A & Q & 2 & 8 & 3 & J & 4 & 9 & 5 & K & 6 & 10 \end{pmatrix}$$

thus our solution is $P^{-1}(x)$, as $P(P^{-1}(x)) = x$. A couple of intriguing things happen if we look at this way. First, we can see that if the action is performed over and over and over, we will eventually end up back where we started. Thus if we track the Ace, we see

A→7→J→K→10→5→8→4→2→A.

Similarly,

3→Q→6→3

and

9→9.

So if you start with the cards in the order A, 2, …, K and perform the process over and over, you will find:

- the 9 always stays in the 9th position
- after every 3rd time through the process, the 3, 6 and Q are back to their original positions
- after every 9th time through the process the A, 2, 4, 5, 7, 8, 10, J and K are back to their original positions, thus the whole deck is back to its original position.

So, as you can see, one little problem can lead to some nice investigations and insights for students from a wide variety of mathematical backgrounds. You can have as much fun as you please with this problem. Here are some suggestions to get you started:

1. Try using some of the techniques to solve the 1 to 25 problem.
2. Solve methods that involve multiple iterations of the process. For example, in the original problem when the cards are on the table, pick them back up and do the process again and try to get them to come up in order.
3. Invent your own methods, and solve them. ▲

▲ The Birthday Problem Extended

DAVID R. DUNCAN AND BONNIE H. LITWILLER
PROFESSORS OF MATHEMATICS
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Bonnie Litwiller and David Duncan teach mathematics and mathematics education courses at the University of Northern Iowa. They have published over 850 articles in journals for mathematics teachers and have spoken at numerous professional meetings. Each has served as President of the Iowa Council of Teachers of Mathematics and Dr. Litwiller has served on the boards of NCTM, SSMA and AMTE.

A well-known probability experiment concerns the so-called "Birthday Problem": In a randomly chosen group of n persons, what is the probability that at least two persons share the same birthday anniversary?

To solve this problem, the probability that no two people share the same birthday is first calculated. If the n persons were arranged in some order and each of them was to declare his/her birthday, then different sequences of length n are possible (ignore February 29). If all birth dates are considered to be equally likely, each of these sequences is equally likely as well.

To find the number of sequences which contain no matches, again visualize the n people declaring their birth dates in order. The first person may declare any of 365 days. To avoid a match, only 364 dates are available to the second person; 363 are available to the third person; 362 to the fourth person, and so on. The nth person has (365-n+1) or 366-n dates available. The number of ways in which the n people can fail to have a match is (365)(364)(363)(362)…(365 - n + 1). The probability of no matches is thus

$$\frac{(365)(364)(363)(362)...(365-n+1)}{365^n}$$

The probability of **at least** one match is then

$$1-\left[\frac{(365)(364)(363)(362)...(365-n+1)}{365^n}\right]$$

Surprisingly few people are needed in a group in order that the probability of at least one match exceeds $\frac{1}{2}$. For 22 people, the probability of at least one match is

$$1-\left[\frac{(365)(364)(363)(362)...344}{365^n}\right]=1-.5243=.4757;$$

however, the probability of at least one match for 23 people is

$$1-\left[\frac{(365)(364)(363)...(343)}{365^{23}}\right]=1-.4927=.5073$$

Denote 23 as the critical value in the Birthday Problem; that is 23 is the smallest number of persons for which the probability of at least one match exceeds $\frac{1}{2}$.

A similar experiment can be performed with decks of 52 regular playing cards. If each of a group of n persons draws a card from his/her deck, what is the probability that at least two of them will get exactly the same card (same suit, same value)? The same reasoning that was used in the Birthday Problem shows that the probability of at least one match with n people is:

$$1-\left[\frac{(52)(51)(50)...(52-n+1)}{52^n}\right].$$

If n = 8, the probability of at least one match is .4325 while if n = 9, the probability of at least one match is .5193. The critical value for n is therefore 9.

Let us generalize the problem. Suppose that each of n persons has a set of cards numbered consecutively from 1 to r. If each person selects a card at random from his/her deck, what is the probability of at least one match? The same reasoning as that used in the two previous examples leads to the conclusion that the probability of at least one match is:

$$1-\left[\frac{r(r-1)(r-2)...(r-n+1)}{r^n}\right].$$

For a given value of r, what is the critical value for n; that is, what is the smallest value for n so that the probability of at least one match exceeds $\frac{1}{2}$? Table 1 depicts critical values for n:

Table 1

r (number of cards held by each person)	Critical value for n (smallest number for which the probability of at least one match exceeds 1/2)
2	2
3 - 5	3
6 - 9	4 (Note r = 6 is the same situation as if each person rolled one hexahedral die; and r = 8 leads to the situation where each person rolled one octahedral die
10 - 16	5 (r = 12 is the same situation as if each person rolled one dodecahedral die)
17 - 23	6 (r = 20 is the same situation as if each person rolled one icosahedral die)
24 - 32	7
33 - 42	8
43 - 54	9
55 - 68	10
69 - 82	11
83 - 99	12
100 - 116	13
117 - 134	14
135 - 156	15
157 - 178	16
179 - 201	17
202 - 226	18
227 - 252	19
253 - 280	20
281 - 309	21
310 - 340	22
341 - 372	23

As an example of how to read Table 1, suppose that each person in a group draws a card at random from a set of cards numbered 1 through 250. Since for r = 250, the critical value of n is 19, the probability of there being at least one match will exceed $\frac{1}{2}$ (specifically 0.504) if there are 19 or more persons in the group. If there are 18 persons in the group, the probability of at least one match is less than $\frac{1}{2}$ (specifically, 0.466).

Challenges for the Reader:

1. Verify the critical values of n in Table 1 by direct calculation.
2. Extend Table 1 to at least r = 1000.
3. Table 2 reports the number of values of r associated with each critical value of n.

Table 2

Critical Value of n	No. of Values of r
2	1
3	3
4	4
5	7
6	7
7	9
8	10
9	12
10	14
11	14
12	17
13	17
14	18
15	22
16	22
17	23
18	25
19	26
20	28
21	29
22	31
23	32

Conjecture the rate at which column 2 will increase using your results from Challenge 2. ▲

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▲ The Math Apprentice

Paul Alves



Paul Alves teaches math at Fletcher's Meadow Secondary School in Brampton. He will be moving to Stephen Lewis Secondary School in September as head of math and business.

A Little Background

The inspiration for this summative assessment came from the popular reality series *The Apprentice*. For those not familiar with the program, a group of potential candidates vie for a position as an apprentice with Donald Trump. In an episode from a past season the two competing teams were given the job of designing a bottle for a new soft drink. The teams had to come up with a unique design that was at the same time practical. The marketing of the product was also a requirement. Our project was very similar but with better hair and no one was fired.

With this in mind, we set upon our task. Although there are existing tasks that focus on optimizing the design of a package, we wanted to expand on that and incorporate more expectations in our task.

Set-up

Each of the grade nine classes were divided into teams. Groupings of three seemed to be optimal (no pun intended). Each team was then assigned a product. Some of the products were gum, chocolate bar, a new drink, ice cream and so on. The overall goal of the team was to create a package for their assigned product that was aesthetically appealing to consumers but at the same time was economically feasible to mass produce. Each team was then given a project portfolio that contained a handout outlining the design stages involved and target dates (See Figure 1). On the day the project was introduced, teams came up with a team name and brainstormed unique design ideas for their product.

The team would be adding all of their work to the portfolio as well as personal logs that were completed at the end of each period that was devoted to the project.

The same products were given to each class in grade nine. Teams were told that their package design would be competing against a rival design from another grade nine class. We had three academic classes which meant that each product would have at least three competing package designs. A couple of products had four designs, as one class was larger than the others.

Where's the Math?

Each team researched existing products and analyzed their packaging before deciding on an approach for their product. They were asked to come up with at least three possible designs. The volume of the product was decided upon by the team, based on the research they completed. The surface area of each possible design was then determined. A spreadsheet came in handy when determining the surface area, especially when a dimension was changed.

Teams were also given a materials list (See Figure 2) from which they chose the material for their package. The possible choices included aluminum, plastic, cardboard and waxed cardboard. We provided a cost per square centimetre and an initial cost for each type of material. Some materials had two possible suppliers with different costs. Teams had to determine the unit cost per package. Initially, we didn't provide students with a production level for their product. This would be something we would change in the future. For example, when a team calculated the unit cost to manufacture their package, they either calculated something ridiculously large because they were basing production on one package or something very small because they mass produced beyond reason.

The End

At the end of the project, each team was expected to submit a prototype of their package, a poster or presentation piece that marketed the product, and their entire project portfolio. By this time, the portfolio was to include all personal logs, evidence of the various design stages and a detailed formal report summarizing the project from start to end. Teams were also expected to present their package prototype to the class. They were urged to approach this from the point of view of convincing the company (the class) to select their package for the product. They were not to focus on the actual product but the innovative package that they had designed for the product. Each team did their best to trump (I couldn't resist) their peers.

In order to determine which package was the best for each product we decided to let the public decide. A

Product Design Project

Design stages:

The following stages will guide your group from the beginning to the end of the design process. You will be given class time to work through each development stage.

Each group will keep a design portfolio that will track the group's progress. ALL work, ideas, contributions, etc, must be kept in the portfolio. There will be a design log for each group member in the portfolio. At the end of each work session, members will reflect on what was accomplished and how they contributed to the group during that session.

STAGES:

1. Brainstorming
   - ❑ Develop as many design ideas as possible for your product. Your group should try to come up with at least three.
   - ❑ Note the advantages and disadvantages of each design. You don't have to get into the numbers yet. Just note in general the advantages and disadvantages.
   - ❑ Completed by _____.
2. Exploring the Designs
   - ❑ Investigate the details of each design. Decide on dimensions.
   - ❑ Look at different packaging materials and the costs associated with each design.
   - ❑ Completed by _____.
3. Decide on Package Design and Build Prototype
   - ❑ Based on investigation, decide on your final package.
   - ❑ Build model. You may need to bring in materials to help with construction.
   - ❑ Incorporate at least three geometric designs from the geometry unit in the labelling of the package. Don't just draw a rectangle.
   - ❑ Completed by _____.
4. Plan Presentation and Create Report.
   - ❑ Plan out a three-minute presentation including important details regarding cost effectiveness and consumer appeal. Plan who speaks when. Approach it from the standpoint of your group is trying to convince the company (the class) to go with your package design. BE CREATIVE. Add whatever you think that will make your presentation memorable.
   - ❑ Prepare report that summarizes the key points of your design package. Again, approach it from the standpoint of trying to convince the company that this is the package that they pick.
   - ❑ Completed by _____.
5. Organize your portfolio.
   - ❑ Make sure all information is in your portfolio including individual logs, design ideas, calculation rough notes, and final report. Try to organize it so that things are easy to find and identify. **Hand in the portfolio, presentation materials and prototype**.
6. Judging occurs on _____ at the Open House!

Evaluation of the Project:

- The project will be a component of your final exam. (Percentage to be determined).
- As well, you will be asked to complete a *peer evaluation* where you will assess the contributions each member of the team made to the group's effort.

Each person will also be required to complete an *exit interview* INDIVIDUALLY, in which you will discuss the design process and your final conclusions with your Lab Supervisor (teacher). This will take place in June during the Final Assessment period.

Figure 1

Grade 9 Product Design
Cost of Materials List

The following options have been investigated for the cost of different materials. For some materials, a couple of options have been provided.

Package Material	Cost
Aluminum	The Metal Company: \$0.0008/cm^2 plus an initial processing fee of \$1300 Getting Canned: \$0.0007/cm^2 plus an initial processing fee of \$1000
Plastic	The Plastic People: \$0.0005/cm^2 plus an initial processing fee of \$1000 The Green Solution: \$0.0004/cm^2 plus an environmental processing fee of \$1500
Cardboard	Boxes R' Us: \$0.0001/cm^2 plus an initial processing fee of \$700
Waxed Cardboard	Outside the Box: \$0.0002/cm^2 plus an initial processing fee of \$900

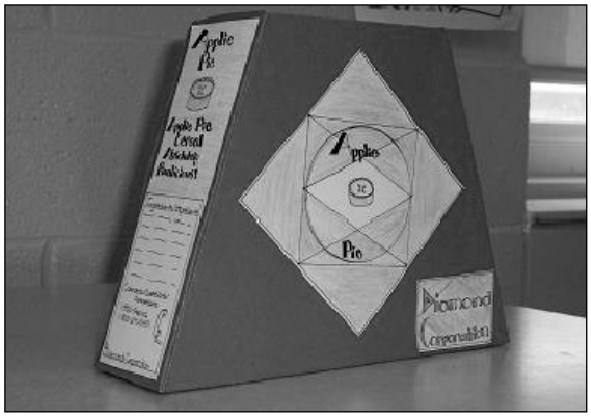
Figure 2

school open house was coming up so we set up displays with the package designs grouped by product. As parents, students, teachers, administrators and anyone else who toured the school made their way through the math area, they voted on the best design.

Evaluation

We wanted the evaluation of the team's work to reflect the overall quality of the project and the contributions that each team member made to the project. We also wanted to evaluate each individual student's understanding of the math behind the project. There were a few components that went into the evaluation: teacher evaluation of the project; peer evaluation; and self-evaluation. All of these played a role in assessing each project and each student's contribution to the project. In order to keep students accountable, interviews were scheduled during the exam cycle. Students were expected to answer specific questions on design decisions and strategies used to determine some of the values in their report.

This project was integrated into the final summative assessements for the course. Its percentage worth was not given on the design stage handout as we were still considering the weight of all the assessment pieces that were to account for the final 30% of each student's mark.





Examples of student work

Possible Changes

As mentioned earlier, one problem that came up was that students were not provided with a level of production for their packages so this led to some interesting answers for the unit costs to produce their packages. This would be something that we would specify in the future.

The voting would not have passed muster with any elections overseer. Since students were roaming the building during the open house, some persuaded friends to vote for their design. In the end, some designs won out based more on who was there rather than the actual design. Democracy can be messy. One alternative we are considering is having the teams present to another class running at the same time preferably not a grade nine class. Hopefully, this would make the judging more objective. We have access to TI Navigator and therefore are considering tabulating the votes electronically rather than a paper ballot.

Another possible modification would be to buddy up with another school in the area and have the students compete against their peers in another school. Perhaps a Math Apprentice Fair (similar to the current data fairs being held around the province) could be held which would then allow teams to see competing designs and to pitch their own design. The judges could be parents, support staff, members of the community and so on. What a great way to reach out to the community and to expose them to the current math curriculum.

In the end the project turned out to be something that engaged the students and the teachers. Students were able to display dramatic, creative and academic talents in a collaborative manner. We would be happy to share resources or get feedback if anyone else wants to run a similar project. School open house was coming up so we set up displays with the package designs grouped by product. As parents, students, teachers, administrators and anyone else who toured the school made their way through the math area, they voted on the best design. ▲

▲ The Criss-Cross Calculation

DUNCAN McDOUGALL



Duncan McDougall has been teaching for 27 years and is a graduate of both Bishop's and McGill Universities. After 13 years in the public school systems of Quebec, Alberta, and British Columbia, he left the classroom for private teaching. Throughout the last 15 years, Duncan has taught all levels of high school mathematics, college and university calculus, and mathematics for elementary school teachers. This rich exposure to diverse programs and curriculae has provided him with some of the ideas, which he is now publishing. Duncan has given several workshops on Rapid Calculation and non-conventional approaches to operations on radicals and fractions. His latest project is developing a special family of polynomials for curve sketching called Diophantine Polynomials. Currently, Duncan owns and operates TutorFind Learning Centre in Victoria, British Columbia.

The Criss-Cross Calculation is a quick and reliable trick to easily determine the number of solutions in a standard system of two linear equations.

The logic behind using this calculation is in understanding what to look for before searching for it. The student should know what type of solution to expect before going through the solution process.

With the criss-cross calculation, students can easily find the number of solutions and not assume all systems have single solutions. Needless to say, one can expect frustration on the part of the student when what they expect to find is not there. I suggest a fast and easy step to be calculated so that the learner is assured that either a single solution exists or it does not.

Consider the system of linear equations $ax + by = c$, $dx + ey = f$ and calculate ae, bd, af, and cd. If $ae \neq bd$, then the system is automatically consistent and so a unique solution exists. If then $ae = bd$, then the system is either inconsistent or dependent. We then examine af and cd. If $ae = bd$ along with $af \neq cd$ the system is inconsistent and if $ae = bd$ along with $af = cd$, then the system is dependent. I refer to the following chart:

Inner Cross	Outer Cross	Type of System	Number of Solutions
$ae \neq bd$		Consistent	One
$ae = bd$	$af \neq cd$ or $bf \neq ce$	Inconsistent	None
$ae = bd$	$af = cd$ or $bf = ce$	Dependent	Infinite

It turns out that it doesn't matter in the inconsistent case whether $af \neq cd$ or $bf \neq ce$, just so long as one of them is not equal. This is also true for $af = cd$ or $bf = ce$ in the dependent case.

Some numerical examples might make this easier to use:

a) $3x - 4y = 12$; $5x + 2y = 10$ — $3 \cdot 2 = 6$; $5 \cdot -4 = -20$; $(6 \neq -20)$ — Conclusion: 1 solution

b) $3x - 4y = 12$; $6x - 8y = 36$ — $3 \cdot -8 = -24$; $6 \cdot -4 = -24$; $(-24 = -24)$; $3 \cdot 36 = 108$; $6 \cdot 12 = 72$; $(72 \neq 108)$ — Conclusion: no solution

c) $3x - 4y = 12$; $6x - 8y = 24$ — $3 \cdot -8 = -24$; $6 \cdot -4 = -24$; $(-24 = -24)$; $3 \cdot 24 = 72$; $6 \cdot 12 = 72$; $(72 = 72)$ — Conclusion: Infinite solutions

In short, the learner can determine both the number and type of roots by performing the criss-cross calculation before searching for a solution. This eliminates frustration, saves time, and teaches the learner that knowing what to look for makes the process of solving systems of linear equations easier.

All of the above discussion, however begs the question as to whether a simple set of calculations can be performed on a standard set of 3 by 3 linear equations Try it! ▲

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