Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceeding 10 numbered pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required. Please number pages.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including websites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

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▲ About the *Ontario Mathematics Gazette*

The *Ontario Mathematics Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

Editor

▲ **Marilyn Hurrell**

Associate Editors

▲ **John Egsgard**

▲ **Todd Romiens**

▲ **Lynda Ferneyhough**

***Abacus* Editor**

▲ **Trevor Brown**

Design, Tech Art, and Production

▲ **Penny Clemens**
Graphic Designer, Cambridge, ON

Printing & Binding

▲ **Pole Printing,** Box 69, 89 King Street East
Forest, ON N0N 1J0 (519) 786-5112

Advertising Manager

▲ **Dean Murray**

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▲ Editor's Message

MARILYN HURRELL, EDITOR



New! New! New!

It is the start of a new school year for most of you and the start of my fourth year as Editor of the OMG. The time has gone by quickly for me; I guess that is what happens when you enjoy what you do.

So much so, that Fred Ferneryhough is back with a new column on Computer Algebra Systems, CAS Corner. Fred is one of our most respected contributors and I look forward, as I am sure you do, to learning about CAS applications for the classroom.

We also have another new column by Shawn Godin, designed to challenge you and your students, titled What's the Problem? I think this fits very well into the OMG and provides some interesting problems for nearly every grade level.

President Dan touches on some social issues which I don't think we can ignore much longer after the hot and humid summer all of us have endured. Lynda Colgon encourages us to be more "user friendly" to children. Mary Bourassa explains some very useful programs to help in creating math documents and papers and Louis Lim provides some great websites to explore. Ann Michele Stenning makes her debut as author of the Directors' Dialogue column. As usual, all the columns, special features and articles are relevant for today's teachers.

At the OAME June 2006 Board meeting, some proposals to "market" OAME to Faculties of Education and other groups, were made. More on this in the December issue. Meanwhile you can contact your Chapter Rep for details.

Marilyn

▲ President's Message

DAN CHARBONNEAU, PRESIDENT



In hopes of making the September 2006 Gazette deadline I find myself trying to put some thoughts together on this fifth consecutive day of 30˚C+ hot and humid July weather. Even Northern Ontario can't escape "Global Warming". I'll have more on this issue later in my message.

As you read this I'm sure you are already overwhelmed with September activities and yet still full of energy and enthusiasm. Although the summer months are a time to recharge I know that many of our OAME members were involved in a variety of professional development activities. Kudos to all lifelong learners who took AQ courses or participated in or presented some other professional development workshops. Many of you were involved in EQAO math assessment or you may have participated in further revisions of the Grade 12 math curriculum. Some of you, along with our friends at OMCA, were finalizing courses of study for the new Grade 11 U/C math course. If you haven't made use of this resource yet, make sure to check out the OAME website and follow the links. You may have been one of those who can't say "no" and volunteered to do curriculum writing/planning or to create assessment tasks for your school or board. Many of you have already spent countless hours preparing for this year's classes. A big thank you to all of you who gave up some of your summer time in one form or another in order to contribute to mathematics education in your school, board or the province. The mathematics community from JK to 12 and in post-secondary institutions is engaged and very much alive in Ontario as we continue to implement our revised math curriculum.

As a follow up to my first President's Message, I encourage and challenge you to get involved in a local chapter of OAME and other math initiatives in the province. In June the Ministry of Education gave significant sums of money (over $20M) to OTF and each affiliate to be used for professional development for teachers. The OAME Executive has sent letters out to the affiliates inviting them to work with OAME in providing PD in the area of mathematics. This could be your chance to get involved. We hope to hear from them shortly. OAME would like to develop some PD that could be used for teacher in-service through local or provincial workshops. The Ministry has also increased the number of PD days by two for the 2006-2007 school year. Although these will focus on Ministry issues and initiatives, I hope these will also include some math in-service.

Many local OAME chapters are planning fall or winter conferences. Why not put your name in as a presenter or volunteer? For a listing of all local and OAME conferences check out the OAME website. The OAME 2007 Planning Committee is putting together a great program. Bring a co-worker with you to the Barrie conference being held in May 2007. If you are looking to take on more of a leadership role in mathematics in your school or board, it is not too early to put in a request to your board to attend the OAME Leadership Conference "Mathematics: Making Sense of It All" being held in Toronto February 15-17, 2007. See this issue of the *Gazette* or the OAME website for more information.

Now that you have been challenged to participate in the mathematics community, I have an even greater challenge for you and fellow educators. After hearing David Suzuki's keynote address at OAME 2005 and, more recently, after seeing Al Gore's "An Inconvenient Truth" at the theatre this summer, I feel compelled to encourage ALL teachers to educate themselves and their students about Global Warming. If we as educators do not bring this issue to the attention of our students, who will? As a teacher of mathematics, I was very impressed with all the data that was presented to support the claims that the Earth is warming. We need to bring these statistics to life in our classes. Did you know that the 10 hottest years ever recorded have all occurred in the last 14 years? I encourage you to first see Al Gore's film, keeping in mind all the data and statistics that are presented, or you may prefer to read one of many David Suzuki books about our planet. Then I challenge you to incorporate some of the statistics on global warming into the data management strand in your math classes. A quick search using "global warming" on the net revealed many great websites. "An Inconvenient Truth" encourages people to visit **www.planetcrisis.net** for video clips and information. The BBC News has put together an on-line 6 part series called "Planet Under Pressure" that addresses issues pertaining to water, energy demand, population, pollution and global warming to name a few. Try **www.news.bbc.co.uk/1/hi/sci/tech/3747724.stm** or do a search using keywords "BBC" and "Planet Under Pressure". There are great

past and present pictures of world sites. The US Environment Protection Agency has a great website for kids at **http://www.epa.gov/globalwarming/kids/**. The World Resource Institute has free, not to be missed, online databases that students from Grade 5 to Grade 12 Data Management can explore at its EarthTrends site **http://earthtrends.wri.org/**. If any of you are involved in writing math textbooks or other math resources, I urge you to include issues around global warming. If any of you wish to share some resources or websites pertaining to global warming, please e-mail me the information and I'll try to get them posted on our OAME website.

We only have ONE planet. I hope I can COUNT on you to help save it.

Best regards,

Dan Charbonneau.

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▲ Letters to the Editor

Dear Editor,

Putting Math to Work

I am looking for Grade 11 or 12 students and teachers in the Greater Toronto Area to participate in a unique mathematics and engineering experiment to update Thomas Young's classic double-slit experiment. Young used light waves 200 years ago to show that waves interfere; this October, we'll re-do his experiment using radio waves. The hands-on project will apply the math that students learn in class linking algebra, geometry and trigonometry to physics. For more information, e-mail Phil at **radio_meteor@yahoo.ca**.

The experiment is supported in part by Sinclair Technologies Inc. of Aurora, ON and Tevelec Ltd. of Mississauga, ON.

Thank you

Phil Eberhardt

[Phil is a recipient of the Science Teachers' Association of Ontario's Irwin Talesnick Excellence in Teaching Award.]

Dear Editor,

I recently joined the OAME, attended my first OAME conference, and received my first issue of the *Gazette*. My first impressions of this organization and the journal are very positive. I especially appreciated the *Abacus* section because it demonstrated how one can bring sophisticated mathematics to even our younger learners.

The article on *One Hundred Ants*, for example, contained an excellent way to help young children develop an understanding of division and divisibility in particular. The set of divisibility rules on the page that followed, however, was somewhat problematic. First, the rule for divisibility by 3 is wrong. According to this rule, the number 127 is divisible by 3!

The correct rule should be:

- A number is divisible by 3 if the sum of the digits is divisible by 3.

Furthermore, the rule for divisibility by 4 is confusing and the rules for divisibility by 2 and 8 are not particularly helpful. On the other hand, there is a divisibility test for powers of 2 which is quite pretty:

- A number is divisible by 2 if the last digit is divisible by 2.

- A number is divisible by 4 if the last 2 digits are divisible by 4
- A number is divisible by 8 if the last 3 digits are divisible by 8

It is a nice pattern which even young children can generalize. Slightly more difficult, but not impossible with somewhat older children, is proving this rule.

Overall, a "thank you" for a most enjoyable summer read!

Eric Golombek,
Principal of Associated Hebrew Schools Toronto

Editor's Note: Others have sent in some comments about this section. Would you please inform Eric [and all readers] that we have noted the comments and we shall be making the necessary adjustments. Trevor Brown, Abacus Editor

▲ Upcoming Events

For further information on OAME Chapter Conferences contact the Chapter Representative and for NCTM conferences, contact John Kersely, OAME/NCTM Rep

See the last page in this Gazette for contact information. Also visit the OAME website.

OAME Chapter Conferences and Events

September 24 2006	Fall mini conference Twin Lakes SS, Orillia ON - 4 - 8:15 pm	MAC^2
October 12 2006	Meet and Greet with Chris Suurtamm St Anthony's Soccer Club, Ottawa ON - 4:15 - 6:30 pm	COMA
October 12 2006	"A Really Good Math Conference" St Elizabeth CHS, Thornhill ON - 4:30 - 7:30 pm	Y^{4}MA
October 13 2006	Annual Conference Lockerby Comp School, Sudbury ON	NOMA
October 18 2006	Fall Mini Conference Mississauga SS, Mississauga, ON	CHAMP
October 19 2006	Fall Conference University of Waterloo	GVMA
October 19 2006	Fall Mini Conference Bowmanville, ON	PRMA
November 9 2006	Fall Mini Conference Loyalist CVI, Kingston ON - 4:30 - 7:30 pm	QSLMA
November 18 2006	Annual Fall Conference SITE Building, U of Ottawa, ON - 8:30 am - 1:30 pm	COMA
February 15 - 17 2007	Leadership 2007 "Math: Making Sense of it All"	OAME
June 1 - 2 2007	OMO 2007 University of Ottawa	COMA

OAME Annual Conference

May 3 - 5 2007	Georgian College, Barrie, ON "Building Bonds"	MAC^2

NCTM Conferences

March 21 - 24 2007	Annual Meeting and Exposition Atlanta, GA	

▲ Technology Corner: Integrating Technology into Documents

MARY BOURASSA



Mary Bourassa teaches mathematics at Lisgar Collegiate Institute in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She is also an OAME vice-president, past-president of COMA, gives workshops around the world and is a National Instructor for Texas Instruments.

The integration of technology in the Ontario mathematics curriculum necessitates that teachers are able to create lessons, worksheets and assessments that help students visualize patterns, charts and graphs as well as calculator and computer screens.

We will look at three ways of making the most of technology in the documents you create. This means that you can ask questions based on graphs when assessing students, can demonstrate mathematically correct form and create worksheets for activities that can easily be followed even when students are working at different paces. The effort you as a teacher put into this will be repaid by being able to ask richer questions and have activities run more smoothly – a big advantage when you have a class of 32 students who would otherwise all be asking you different questions at the same time!

1. *MathType*

MathType (**www.dessci.com**) is software that integrates into *MS Word* that allows you to easily write in mathematically correct form. It also works very well with other word-processing programs, presentation software and web-authoring software. *MS Word* has *Equation Editor* built into it which can do some of what *MathType* does, but *MathType* is far more powerful and a good investment for a school to make.

Here are a few examples of what *MathType* allows you to create:

Write the ratio of stars to planes in simplest form: ☆☆☆☆ ☆☆☆☆ ✈ ✈	Three drawings are shown. What would the next two look like? ☆ ☆☆ ☆☆☆ ☆ ☆☆ ☆
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\log_a x + \log_a y = \log_a (xy)$
$y = \lim_{x \to \infty} \frac{3x^2 - 4x + 5}{1 - 2x^2}$ $= -\frac{3}{2}$	$f(x) = \begin{cases} x^2 - 4 & \text{if } x < 2 \\ -(x-4)^2 & \text{if } x \geq 2 \end{cases}$

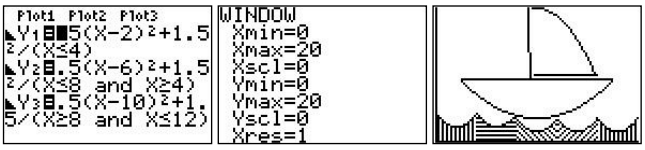
The first two examples use matrices and symbols (Edit menu, Insert Symbol). The last example also uses a matrix to properly align the items. You can choose to align to the equal sign on each line (as in the limit example) by going to the Format menu and choosing the "Align at =" option. Although the menus are easy to use, there are keyboard shortcuts available too. The following table lists a few of the more commonly used ones:

Parentheses (round brackets)	Ctrl + 9
Brackets (square brackets)	Ctrl + [
Braces (curly brackets)	Ctrl + {
Full-sized fraction	Ctrl + F
Square root	Ctrl + R
Superscript (high)	Ctrl + H
Subscript (low)	Ctrl + L
Thin space	Ctrl + Space

You can choose the font and font size along with colour for everything you create in *MathType* and can add your own items to the tabs simply by dragging them up. *MathType* allows you to make mathematical notation look on paper (or on a screen) as it should. Students will therefore see questions written in the form they expect on assessments. I also have all of my lessons word-processed and working with *MathType* makes editing them a breeze.

2. *TI-Connect/TI-SmartView*

TI-Connect software is available free from the TI website **www.education.ti.com**. You need a USB link cable to connect your calculator and computer. Some of this software's features include the ability to get screen shots from your calculator, back up your calculator (do you have the latest operating system?) and upgrade flash components easily.

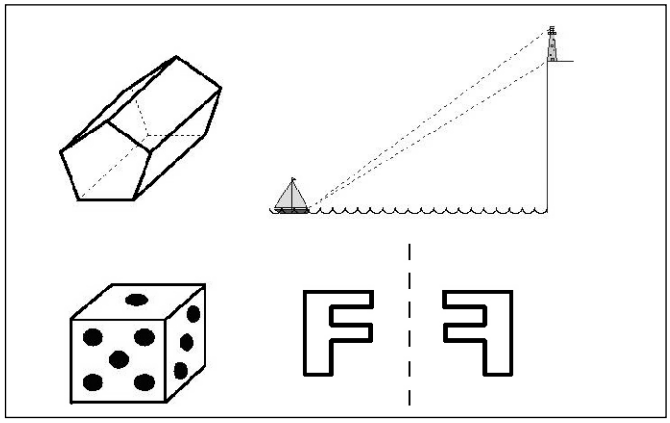


TI-SmartView is the emulator for the *TI-84* and it also allows you to take screen shots which you can then incorporate into your working document. Whether you are using *TI-Connect* or *TI-SmartView*, adding calculator screen shots to activity sheets makes it much easier for students to follow. It takes away some of the button pushing time and allows you to focus more on the mathematics of the activity. You can also include screen shots on assessment pieces. For example, you can take a screen shot in "Match Me" and have students describe how they would have to walk, with respect to the CBR, in order to match the given graph.

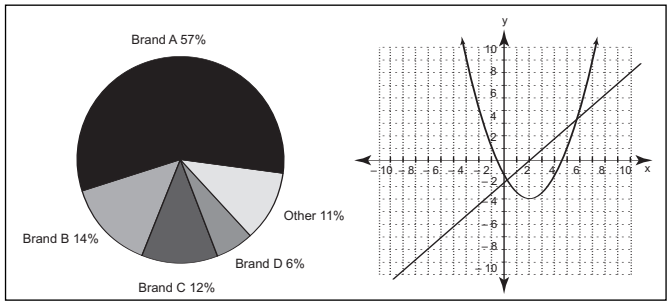
3. *FX Draw*

FX Draw is similar in nature to *MathType* in that it can be incorporated into *MS Word* but its purpose is to produce graphs and diagrams. It hails from Australia and is very inexpensive: **www.efofex.com**.

There is a gallery of ready-to-use diagrams including dice, spinners, circle geometry, isometric drawings, parallel lines, platonic solids, prisms, cylinders, cones, spheres, pyramids and a good selection of trigonometric diagrams. Here are a few examples:



You can also create graphs like:



Other features of *FX Draw* are the ability to easily create Venn diagrams, normal curves, draw scatter plots with lines/curves of best fit (with residuals, if desired) and draw any type of geometrical figure. You can label lines, mark angles and include equations in proper mathematical form. *FX Draw* is simply an amazing tool and I can't imagine working without it!

This software list is quite obviously not comprehensive. Some other notable software packages include *The Geometer's Sketchpad, Fathom!, Tinkerplots* and *Peanut* software (available free from **http://math.exeter.edu/rparris/**). Make the most of all the fabulous resources at hand. ▲

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▲ Hey, It's Elementary! Seeing Ourselves as Children See Us



LYNDA COLGAN

Mathematics is a discipline that enjoys a peculiar property: everybody has some mental image of it." (Furinghetti, 1993)

We live in an age of makeovers. Every evening, television bombards us with programs about re-doing everything from rooms and hairstyles to family dynamics and gardens. The new celebrities on the block come with a tool-kit – from Mike Holmes with his hammer and Debbie Travis with her paint brush to Jo Frost, the supernanny with parenting pointers. In today's world, star power belongs to those revered folks who can transform something, be it ceiling, cowlick or child, from 'bad' to 'good' with efficiency and flair.

In addition to the makeover gurus, there are 'stagers' who prepare homes for sale by focusing on first impressions. These new age experts increase the curb appeal and marketability of a house, transforming it from a 'home' to a product. One such expert, Brenda Rawls, describes it this way: "Staging your house to sell is like entering a beauty contest. And you want to look good and win the contest."

Faux finishes. House fluffing. Each one is about first impressions and making people see what you want them to see, not what may be there right in front of their eyes – the physical and architectural equivalent of spin doctoring.

I got thinking about massive makeovers this summer. I could do without the accoutrements that come with my 53 years, to be sure, but in spite of my silver highlights (no gray hair for me!), wrinkles and bunions, it turns out that it's not me physically that's in the most need of a head-to-toe revamp, it's the profession with which I am most closely affiliated that could benefit from an overhaul.

As mathematics educators, we are frequently confronted with the fallout that is associated with the fact that mathematics and mathematicians have a fairly poor

image among society in general and school children in particular.

It turns out that not only does mathematics have a poor image, but that image has a face and a personality. “Mathematicians have no friends, except mathematicians. They are usually fat, unmarried, aren’t seeing anyone, and have wrinkles in their forehead from thinking so hard.” (See Figure 1.) “[Mathematicians are usually] bald, overweight, unmarried men who wear beards and glasses and lead little or no social life.” (See Figure 2.)

No, I’m not describing myself or anyone I know. The profiles are legit first-hand data from a study conducted several years ago by a group of researchers led by Professor John Berry from the Centre for Teaching Mathematics at Plymouth University. Under Berry’s direction, the team conducted an international comparison of the images of mathematicians held by children in lower secondary schools in Europe; Britain, Finland, Romania and Germany and the USA. Data was collected using a simple questionnaire and from an interview about the picture that the child drew of the way he/she imagined a mathematician to look like and act.

When two groups of children aged 12-13 years in schools in Plymouth (UK) and New York City (USA) were given the task of drawing a mathematician, regardless of the side of the Atlantic Ocean on which the resided, they produced highly generalized images of which Figures 1 and 2 are typical examples.

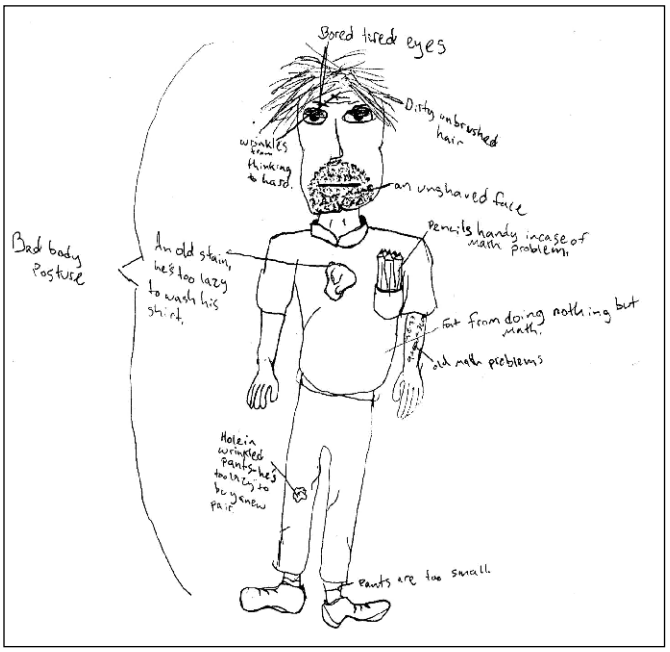


Figure 1.

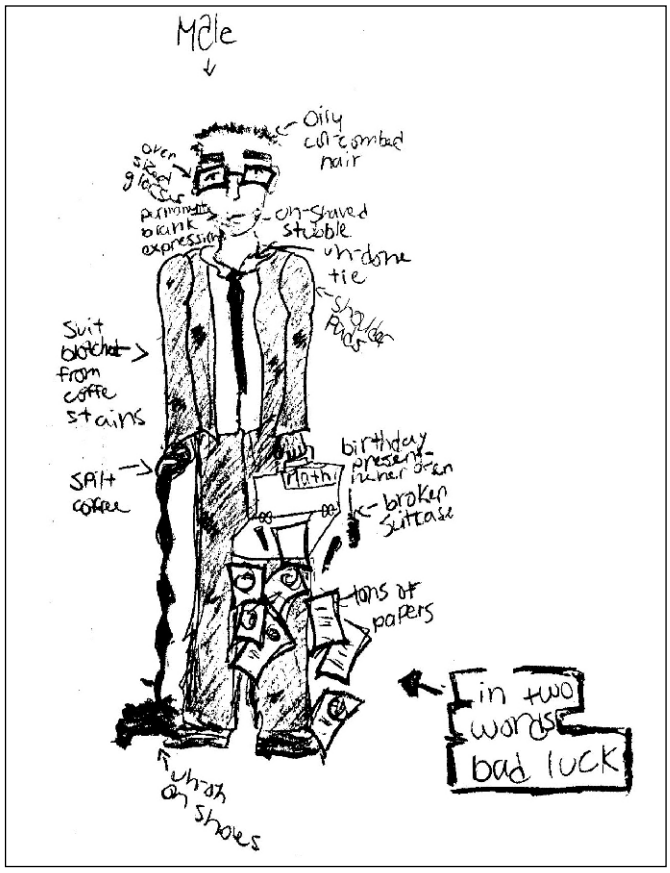


Figure 2.

More disturbing was that according to the report, in Finland several children drew pictures of math teachers holding machine guns. One pupil wrote a caption: “If these sums are wrong, it’s the end of you”.

The researchers suggest that as mathematics educators, we should be very concerned about these images, because the negative mental constructs of mathematicians and mathematics widely held by students tends to have a negative effect on their attitudes to learning mathematics.

More importantly, not only does this ‘image of mathematics’ refer to an internalized picture, it communicates beliefs and attitudes that seem to represent cumulative social experiences lived by the children through school: interactions with parents, peers and the mass media and other influences.

While the pictures are revealing, interviews with the children about their drawings provided deeper conceptions. For example, when pupils were asked why one might need to hire a mathematician, the responses were very shallow. Pupils mentioned teaching and tutoring, doing taxes, working in a bank or shop, or “to solve hard problems” (USA pupils), “to do hard sums”

(UK pupils) [although no specific type of "hard problem" or "hard sum" was ever mentioned]. The study concluded that it does not seem to be at all clear to pupils what it is that mathematicians do and what types of problems they can solve: *"The bottom line for many students is that despite being exposed to mathematics continuously from Kindergarten...the typical [student] cannot connect the value of the study of mathematics with what mathematicians really do. Put differently, students have learned when to "call" or hire a doctor, electrician, geologist, or plumber, but not when to "call" or hire a mathematician".*

Over the past few years, mathematics educators in Ontario have had the opportunity to attend many professional development sessions as extensions of *The Early Math Strategy* and *Junior Math Strategy* as well as the *TIPS* and *PRIME* initiatives. The focus has been on learning about the best ways to teach and assess mathematics. But good curriculum and pedagogy may not be enough.

There is a bigger challenge facing each of us individually and all of us collectively: the total makeover of the public's perceptions about mathematics. When the media (and your neighbours) bemoan the state of math skills among today's schoolchildren, what they are really referring to is basic numeracy, not mathematical thinking. It's up to us to show them the difference, and to prove that mathematics is much more than numbers; is relevant and ubiquitous; and, creative and cool. Perhaps most importantly, it's up to us to show the world that we can laugh at both ourselves and our domain.

The time is ripe. We can be aided and abetted in our global math perception makeover by the hugely successful television drama *NUMB3RS*. We can capitalize on some 80 movies that have a math theme or message, including *Good Will Hunting, Big, The Man Without a Face, Contact* and *A Beautiful Mind*; plays such as *Fermat's Last Tango* (about Andrew Wiles' proof, and featuring songs like *"There's a Big Fat Hole in your Proof"* and *"Math Widow"*), and *Proof*. We can sing the words to songs like Tom Lehrer's *New Math* or *That's Mathematics!* We can write Piaku: Poetry in which the number of syllables in each sentence corresponds to the consecutive digits of Pi.

That's Mathematics

By Tom Lehrer

Counting sheep
When you're trying to sleep,
Being fair
When there's something to share,
Being neat
When you're folding a sheet,
That's mathematics!

When a ball
Bounces off of a wall,
When you cook
From a recipe book,
When you know
How much money you owe,
That's mathematics!

How much gold can you hold in an elephant's ear?
When it's noon on the moon, then what time is it here?
If you could count for a year, would you get to infinity,
Or somewhere in that vicinity?

When you choose
How much postage to use,
When you know
What's the chance it will snow,
When you bet
And you end up in debt,
Oh try as you may,
You just can't get away
From mathematics!

Andrew Wiles gently smiles
Does his thing and voila.
QED we agree and we all shout horrah,
As he confirms what Fermat
Jotted down in that margin
Which could've used some enlargin'.

Tap your feet,
Keepin' time to a beat,
Of a song
While you're singing along,
Harmonize
With the rest of the guys,
Yes, try as you may,
You just can't get away
From mathematics!

The Storm: by Mike Rollins

3	Flash, Rumble
1	Rain
4	Awesome thunder
1	Wind
5	The wind will plunder
9	As did the pirates from on yonder
2	Raid, fall
6	Damn! My ship is sinking
5	The water stinging
3	Quivering
5	The storm moves onward
8	I'm tired, I'm cold, I sink, I die.

Figure 3: Piaku: Poetry

We can reinstitute *Family Math* on a large scale. We can hold *Math Fairs* in our community centres. We can tell (and embellish) math stories around the campfire starting with, "A long time ago, there was a boy named Carl Gauss". We can print glossy fundraising calendars that feature math months: alternate ways of counting and using numbers, focusing on Russian Peasant Multiplication or Mayan Math; geometry in our driveways and carpets; the probability of winning a car during "Roll Up the Rim Month"; topology studies of patterns of closeness and position, or understanding the patterns of motion (especially Santa Claus' path).

We can reach out to the community and host "preview" nights to help viewers of all ages understand the math behind the upcoming episode of *NUMB3RS*. We can hold open forums and help folks to understand how mathematics is behind the weather person's predictions of tomorrow's weather forecast or why probability theory can let us predict the outcomes of elections (usually with remarkable accuracy). We can hold open meetings to explain why stock market analysts use various mathematical theories to try to predict the future behavior of the stock market or how insurance companies use statistics and probability theory to predict the likelihood of an accident during the coming year (and set their premiums accordingly).

One of the ways that I try to bring mathematics into the public eye, creatively, gently and accessibly is through my bi-weekly column in the *Kingston Whig-Standard*. My column, Myths and Milestones, features mathematics prominently. On the "myths" week, I try to dispel a commonly held misconception, such as there is a math gene and during the "milestones" week, I focus on calendar math such as tangrams (for Chinese New Year) or freedom quilts (during Black History Month). As you can see from the proof of the column provided, I try to make the columns hands-on and kitchen-table oriented. It may be a small step, but the column is having a positive impact.


FORUM

Some real da Vinci codes

math magic

earth, air, fire, water

With the number of local newspapers that abound in Ontario and the number of knowledgeable mathematics educators across the province, this type of "public relations" column could appear from Windsor to Cornwall, and Port Stanley to Rainy River. ▲

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▲ Friday Special!

JACK WEINER



Fridays at the University of Guelph, five minutes before the end of class, I say, "Pens down, books closed, it's "Friday Special" time!" Then I show on screen an article or cartoon or letter or…something that has to do with math, the "real world" and has absolutely nothing to do with class, assignments, or tests. It is math for fun. Often, it is some flagrant example of how the media abuse math. I hope you use some of these. Make them "Monday Morning" specials or "We need a break!" specials. If one falls flat, blame me. If it works, you take the credit.

THE DEVIANT "MEDIANT"

PROBLEM: The "mediant" of two fractions, a/b and c/d , where a, b, c, and d are positive real numbers, is defined by the fraction:

$$\text{mediant} = (a + c)/(b + d).$$

Given that $a/b < e/f$ and $c/d < g/h$, prove that

$$(a + c)/(b + d) < (e + g)/(f + h) .$$

SOLUTION: COUNTER-EXAMPLE using the batting averages of Julie and John!!!!

SEASON	JULIE	JOHN
	hits/at bats	hits/at bats
1	40/200 =	100/400 =
2	93/300 =	32/100 =
Cumulative averages	133/500 =	132/500 =

I recently talked to about 200 enthusiastic grade 7 and 8 students at Dundas District, a K through 8 school in Dundas, Ontario (just a little west of McMaster in Hamilton and very close to Parkside High School where I taught for five happy years).

It was May 31. Neither teacher organizer Lesley nor I anticipated when we set the date that the temperature would hover above 30˚C. So we were nervous that everyone would be a little uncomfortable. We were, until we got started. Then… we had fun. If I had to do it over again, I would have taken a break after 30 minutes for a quick walk around the schoolyard. Maybe pull a Ron Lancaster and point out some "math in the world all around them".

We did some math, talked about school and life, and what's ahead in high school. We decided that college, apprenticeships, university were all good options and that career enjoyment was more important in the long run than money.

All in all, a neat 60 minutes.

As part of the "math" portion (we had to do soooooommmmme math), I brought up the "deviant mediant". I prefaced by saying this:

"Okay, this is going to get me in trouble, but, sorry, the evidence is there for all to see. Guys on average are better at sports than girls. They are stronger and better coordinated. Maybe in another universe, males and females would be equally talented or females superior, but here, well …."

"Look, for example, at the statistics from the coed baseball league to which Julie and John belong. Do the math. In season one, Julie has a batting average of 40/200=.200 while John has a batting average of 100/400=.250. John has the better average. Guys are better at sports than girls."

"Season two: Do the arithmetic and you see that Julie's average of 93/300=.310 is no match for John's average of 32/100=.320. Guys are better …."

"So cumulatively over the two seasons, Julie's average of 133/500=.266 is of course lower than John's average of .132/500=.264 again showing that guys are…uh, wait a minute. John's average is supposed to be higher than Julie's."

"Okay, something must be wrong. John has the higher average in both season one and season two. Both John and Julie have the same number of at bats over the two seasons. How can John's cumulative average be lower?"

The Deviant Mediant

The reaction from the students. First and predictably, when at the start I claim guys are better, guys cheer and girls jeer. Then they get caught up in the numbers. By the way, ask them to tell you how batting averages are measured. (Vernon Wells is batting two fifty this year.) Then the genders switch roles for cheering and jeering when we see Julie's winning average over two years.

And we talk about how this could happen.

One point students rarely notice: who gave us permission to add fractions this way: $\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b+d}$, that

is, adding the numerators and adding the denominators? That's precisely what we did to compute the combined season stats.

This incorrect fraction addition contains the seed of the real answer which, usually, at least a few students will discern. Julie's high average of .310 occurs for many more at bats that John's .320. So it carries more "weight" when computing this "deviant mediant". If we took the average of the batting averages, Julie would, over two seasons, have .255 versus John's higher .285.

Then ask the question: "Which is the better measure of average over the two seasons?"

Ask this question too: "Which way would you prefer to add fractions: the right but admittedly a little more difficult way or the deviant mediant?"

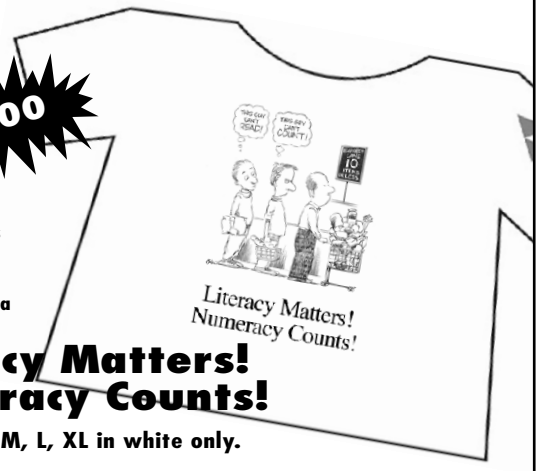
Best wishes, Jack ▲

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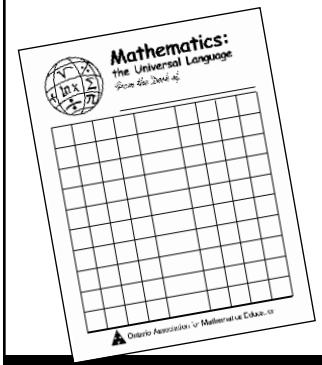
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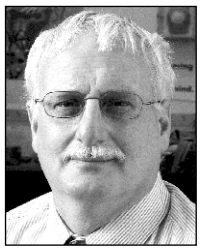
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▲ OAME/NCTM SEPTEMBER 2006 REPORT



JOHN KERSLEY

At the NCTM Annual Meeting in the spring, NCTM welcomed a new president, Francis "Skip" Fennell, a professor of education at McDaniel College in Maryland. In his term as president, he intends to further connections or partnerships with parents and educators of mathematics. He wants greater parental support for mathematical education. In addition, he wants to further develop NCTM's position on research. His aim is to raise awareness of the importance of research to what is being taught in classroom and to future educators.

The 71st NCTM yearbook (2009 edition) is entitled, *Understanding Geometry for a Changing World*. The editors want to explore the essential components of geometrical reasoning and identify effective ways for teachers to help students build significant mathematical understanding. This yearbook is to follow the 70th NCTM yearbook (2008 edition) focusing on algebraic reasoning. If you are interested in submitting an article for this yearbook details can found at **www.nctm.org/publications/yearbook.htm**. The deadline is December 1, 2006.

The following site, **www.ed.gov/teachers/become/about/survivalguide/survguide.pdf** provides useful general and practical tips for new teachers, downloadable in PDF format. It is an US government publication entitled, *Survival Guide for New Teachers.*

NCTM offers a multitude of publications and resources for professional development for all teachers. For example, one resource is called *Reflections*. It features classroom videos that help teachers to analyze and reflect on instructional practices. The site is **www.nctm.org/reflections**. For further information and for other NCTM publications the website is **www.nctm.org/catalog**.

I mentioned in my last column that NCTM has an interesting page entitled "illuminations" at **http://illuminations.nctm.org**. This site offers over 500 lesson plans and over 60 online activities for all

mathematics educators from pre-kindergarten to grade 12. Users can search by keyword as well as by band and by topic. It is worth a visit!

Websites of interest to mathematics as suggested by NCTM

- **Mathematical Applications**
  www.bced.gov.bc/careers/aa/lessons/math.htm
  lessons which help students to connect the skills they are learning in their mathematics classes and specific occupations. Grade 8 to 10 level
- **Cool Math**
  www.coolmath4teachers.com
  provides lesson plans and lots of ideas for teaching math skills for all grades
- **Goodies**
  www.mathgoodies.com/teachers.html
  a free math site offering hundreds of activities K – 12 to teachers, students and parents

NCTM Conferences www.nctm.org/meetings

NCTM Regionals

- Chicago "Winds of Change" September 20 -22, 2006
- Phoenix "Show Me the Mathematics—Learning through Representation" October 5 – 7, 2006
- Atlantic City "Dive into Mathematics: It's a 'Shore' Thing" October 19 – 21, 2006

NCTM Annual Meeting

- Next Year in 2007 Atlanta "Mathematics: Representing the Future" March 21-24, 2007 ▲

▲ Call for NCTM Rep

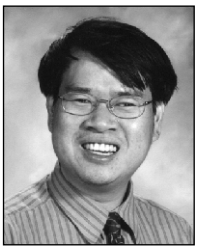
OAME is taking applications for a three-year term as NCTM representative commencing in September 2007.

Job shadowing available now.

For more details, please contact:
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Dave and Sue Hessey
(519) 471-6324
EDs@oame.on.ca

▲ Get Interactive!

Louis Lim



Louis Lim is department head of mathematics at Richmond Hill High School. He recently completed a 3-year term as editorial panel member of the NCTM's Mathematics Teacher and is former editor of the OAME's Ontario Mathematics Gazette. In spring 2005, he was recipient of OAME's "Exceptional and Creative Teaching in Secondary Mathematics" Award.

Another school year begins! This edition of *Get Interactive!* contains two list serves to join, an electronic column to read, a goldmine of activities and lesson plans to access, and an announcement of a mathematician's upcoming 300th birthday to celebrate. Think of how you can incorporate these websites into your teaching of mathematics, regardless of whether you are an elementary, secondary, or post-secondary educator. The information highway has certainly opened new windows and doors to teaching mathematics!

List Serves

1.) Math Forum Internet News
http://mathforum.org/electronic.newsletter/index.html

Math Forum Internet News, a weekly electronic newsletter, is hosted by Drexel University in Philadelphia. To provide a flavour of the 3 or 4 links provided weekly during the past 11 years, I share some websites from volume 11, year 2006: issue 11.3 contains a listing of 269 occupations and the mathematics used; issue 11.2 has "Geometry and Art" (after viewing this link, maybe submit your own found connection to Ron Lancaster's *Photo Math* column); issue 11.15 has "Curious Math", including cool numeracy tricks, and issue 11.22 which links basketball and mathematics. To subscribe, send an e-mail to **majordomo@mathforum.org** and write subscribe newsletter in the body.

2.) The Dollar Stretcher
http://www.stretcher.com/index.cfm

Subscribing to the weekly electronic newsletter *The Dollar Stretcher* provides numerous ways to save and

invest money. Each week, 2 to 4 articles are sent. The good news is that your e-mail is not sold to third parties. Archived articles pertain to automobiles, banking, budgets, consumer protection, credit cards, earning money, insurance, investments, mortgage, taxes, and travel.

As an aside, I recommend David Bach's **http://www.finishrich.com/pages/home.php**. Bach is author of *The Automatic Millionaire* series. He is a highly energetic speaker who shares two golden rules; invest (and diversify) 10% of your gross income using automatic payments, and pay your mortgage bimonthly rather than monthly. Although the books are light reading, you can listen to the free audios located at the bottom of the website. I find that Bach makes so much sense, while highlighting the power (no pun!) of compound interest![1]

Electronic Column

3.) John Allen Paulos' Who Counts http://abcnews.go.com/Technology/WhosCounting/

Be sure to attend OAME 2007 in Barrie since John Allen Paulos will be a keynote speaker! I heard Paulos speak at the NCTM 2000 annual meeting in Chicago and enjoyed his lively presentation and wonderful sense of humour; for instance, what do Winnie The Pooh and Alexander The Great have in common?[2] Paulos is a mathematics professor at Temple University in Philadelphia, who has written best-sellers such as *Innumeracy* and *Once Upon a Number*.

Since the year 2000, Paulos has written a monthly column for **abcnews.com** that is accessible to the general public. Recent columns have linked statistics and probability to the DaVinci Code (June 3, 2006); "Why Medical Studies Are Often Wrong" (August 7, 2005), "Misleading Numbers in the News" (December 5, 2004), and "Why People Vote Like Their Neighbors" (August 1, 2004). I particularly enjoyed "The 9-11 Lottery Coincidence" (October 6, 2002), where Paulos explains the New York State lottery numbers, 9 – 1 – 1, a year after September 11, 2001 should not be viewed as bizarre. Very interesting!

Lesson Plans and Activities

4.) National Council of Teachers of Mathematics' Illuminations http://illuminations.nctm.org/

I believe that the NCTM's *Illuminations* website is one of the most comprehensive collections of lesson plans and activities available on the Internet for preK-12 mathematics teachers. Organized in preK-2; 3-5; 6-8; and 9-12 grade bands, the website is easy to navigate for the busy "teacher-in-the-trench". In March 2006, I took my grade 9 applied mathematics to the computer lab to complete the "Paper Pool" activity. Students explored the applet on the computers after reading *Sticks* by Joan Bauer. I found the lesson plan and activity of top quality (and student interest was high). Have a look – there's bound to be some interesting ready-to-use activities!

Announcing Leonhard Euler's 300 Birthday Celebration!

5.) Euler 2007 http://www.euler-2007.ch/en/index.htm

Be sure to celebrate Leonhard Euler's 300th birthday on April 15, 2007. A website has been devoted to celebrating this extraordinary mathematician[3]. Some of Euler's work now appear in high school, including the Konigsberg Bridges Problem; mathematical notations he standardized, such as f(x), e, $\sum$, π, i; the sum of the vertices and faces subtracted by the edges in any convex polygon is always 2, $v - e + f = 2$, and Euler's formula $e^{ix} = \cos x + i \sin x$, or $e^{i\pi} + 1 = 0$ when $x = \pi$.

I enjoy sharing with students Euler's positive outlook in life. When he became blind in his 40s, he said that he could devote more time to mathematics without outside distractions! ▲

[1] *I agree with Bach that we need to teach students how to save and invest money. I was shocked to recently read about a famous NHL hockey player that my former high school in Belleville graduated who declared bankruptcy. He makes several million dollars a year but his debts far exceed his savings.*

[2] *Winnie The Pooh and Alexander The Great have the same middle name!*

[3] *Please see my June 2006 column for websites of postage stamps that commemorate Euler.*

▲ Photo Math – the fine art of viewing the world through a mathematical lens

RON LANCASTER
INDEPENDENT MATHEMATICS CONSULTANT



Ron Lancaster is a Lecturer in Mathematics Education at the Ontario Institute for Studies in Education at the University of Toronto. He is also an independent mathematics consultant and works with teachers, students and educational organizations in North America, Asia and Israel.

Teachers and students are encouraged to put on a pair of mathematical glasses and become a math spotter, someone who sees mathematics everywhere and then possibly poses mathematical questions based on the images taken. Ron Lancaster, the current Editor for Photo Math, welcomes photos with questions for possible publication in future columns. Material can be sent via e-mail or by regular mail to Ron at the addresses above.

Introductory Comments

One of the major topics of the grade 9 mathematics courses is slope. It is a central idea that plays a major role in these courses and it continues to be important in courses past grade 9 including those in science where rates are dealt with.

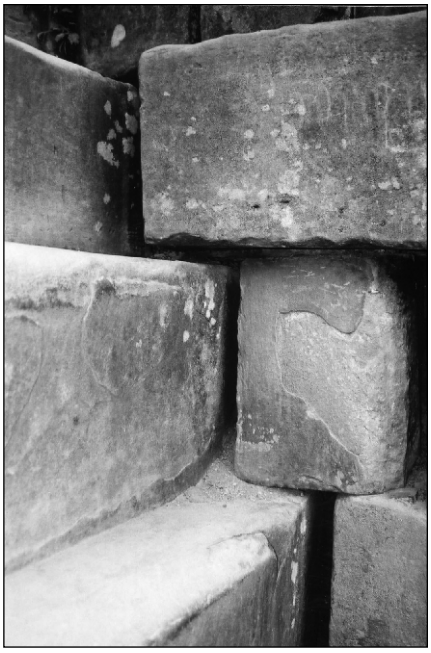
For a number of years teachers have been introducing students to the concept of slope by having them walk in front of a motion detector (a CBR) hooked up to a TI83/84 graphing calculator. Teachers looking for a new angle to supplement this exercise in data collection might want to send their students out into the halls and their homes where they can measure the rise and run for various sets of stairs and then calculate the slopes of these stairs. Students could be asked to compare the slopes within a building (are they the same?) and within a city (is there a range that they lie within?). Students could also be asked to contact municipal authorities to obtain the building code information for the slope of stairs within that community.

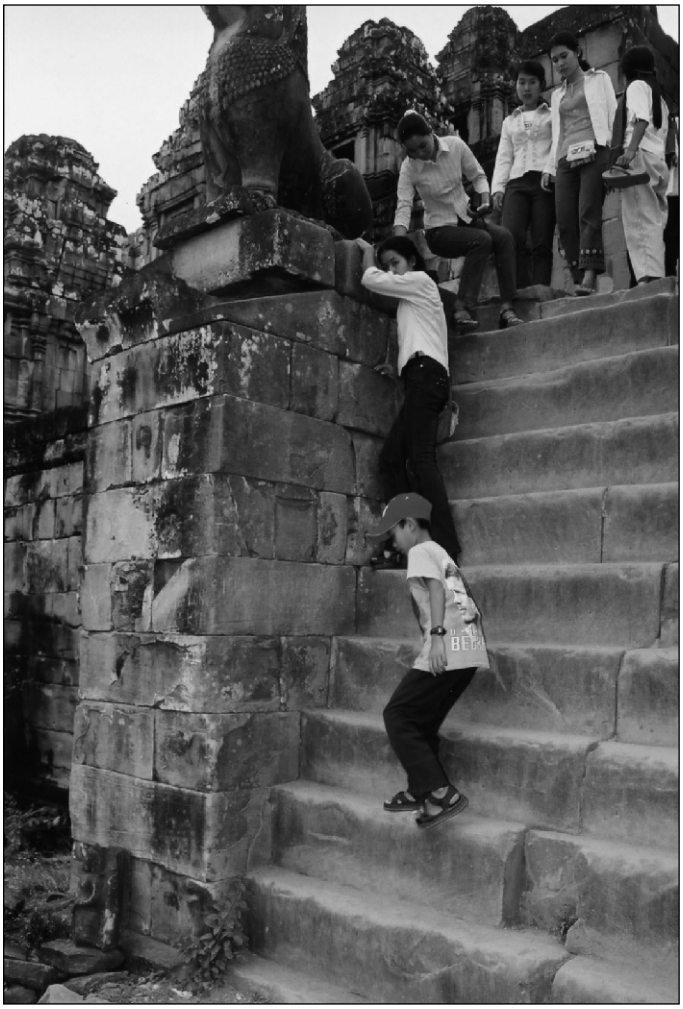
The five photos that appear in this month's column can be used to enhance the activity of measuring the slope of stairs. These photos were taken by Ron Lancaster in Angkar Wat, Cambodia in March 2006. They show a set of steps that is utterly frightening to be on. As you can see everyone is taking their time and no one is making any fast moves up or down. A good question to ask students is why in the world would anyone make the slope of these steps so large. There is a giant Buddha at the top of these stairs and in Cambodia and elsewhere in Asia there is a tradition of never turning your back on the Buddha. By making the steps so steep, people have no choice when going down the steps. They have to go down sideways or else they will fall.

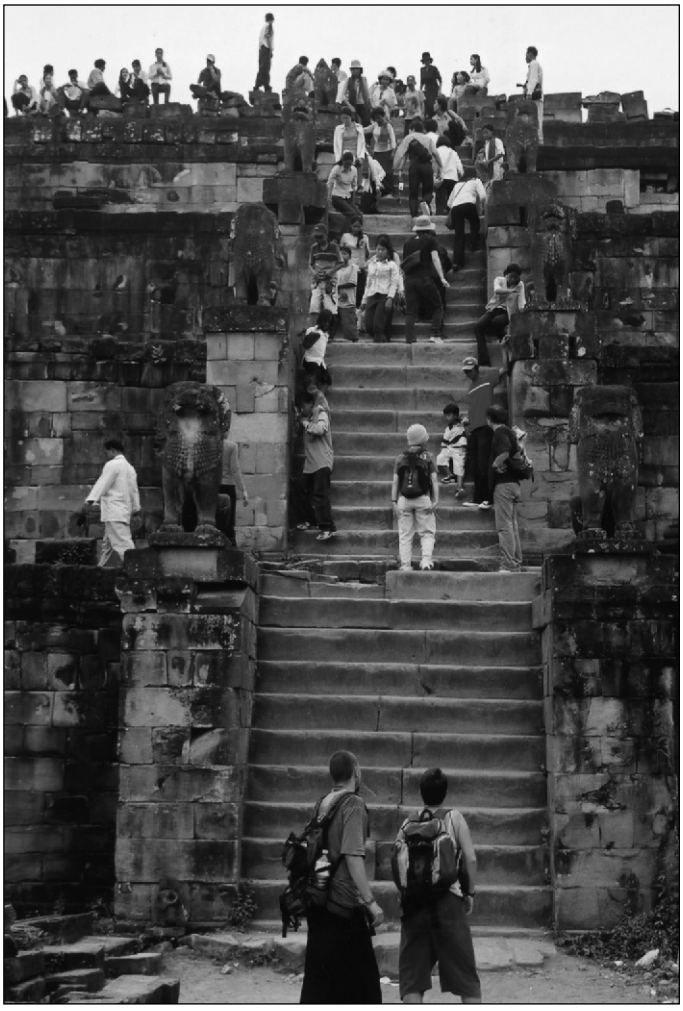
Teachers may want to post these photographs on the walls of their classroom when teaching slope this fall. They will provide students with a connection between slope and culture from the far east.









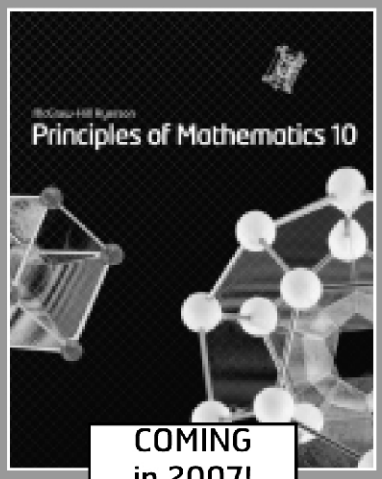


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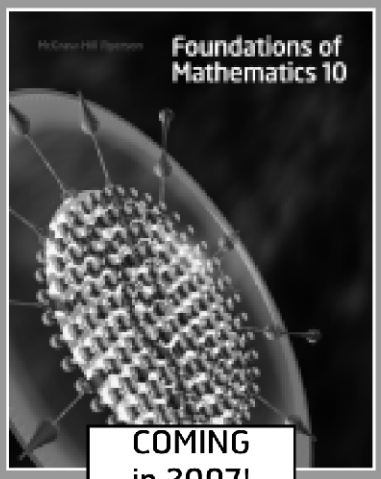
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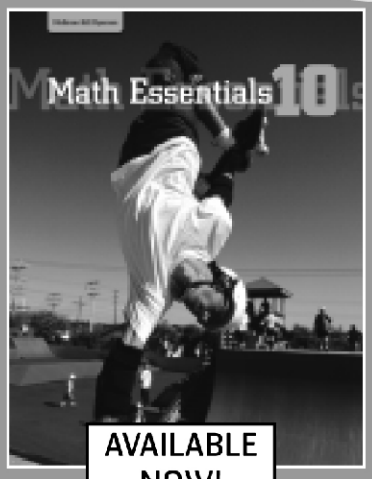
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▲ Director's Dialogue

ANN MICHELE STENNING



Ann Michele Stenning is a mathematics teacher at South Secondary School with the Thames Valley District School Board. She is currently completing her Masters of Education in Curriculum Studies. Ann Michele was part of the planning committe for the 2006 OAME conference "Everyone Counts".

This edition of Director's Dialogue highlights the events of the OAME Board of Director's meeting at the Holiday Inn Select, Toronto Airport on Friday June 9 and Saturday June 10, 2006.

Chapter Reps Meeting: Friday, June 9 2006

Greg Clarke welcomed new chapter reps Diane Lang (ISOMA), Pietrina Orlandi (Y^{4}MA) and returning reps. He thanked Joanne Anas who will be replaced by Ron Gaudreau in October.

The role of the Ontario Math Olympics Chapter Coordinator was discussed. Solutions were offered to prevent last minute registration. Chapter Sharing time followed. One Chapter Rep mentioned the use of etch-a-sketch in classrooms where one knob is the x-axis and one is the y-axis. One other Rep mentioned that their group has a short Professional Development session at each of the chapter executive meetings. Greg Clarke then offered to conduct professional development sessions at future Chapter Rep meetings. Possible topics were discussed including the use of math related segments in movies and TV shows, interactive applets, the use of MCIS for conference registration purposes and the new grade 11 curriculum.

Board of Directors' Meeting: Friday June 9 and Saturday, June 10 2006

Dan Charbonneau welcomed everyone and introduced new board members. He welcomed Jacqueline Hill as President-Elect and Mary Bourassa as VP to the executive. He thanked Kathy Kubota-Zarivnij for her awesome job last year. As the new president of OAME Dan gave a short description of his involvement in the organization. Dan introduced Kevin Maguire who described how OAME would actively search for ways to communicate with the Ministry of Education and Training. Using a new Ad Hoc committee called Guiding Principles for Success or GPS, one rep from each curriculum committee will work together in responding to and informing the Ministry.

Jack Weiner, former chair of Promotions, showed the latest version of the Promotions powerpoint. He demonstrated how to use it with Faculty of Education students in an effort to advertise the advantages of OAME membership.

Highlights: Curriculum Committee Meetings: Fri June 9 2006

JK-6: Priority is to make a teacher package for integration of mathematics and literature. This package would include picture books and the use of traditional storybooks to support mathematical thinking

7-9: It was decided to focus on creating one lesson for grade 7-9 that incorporates the phases of learning as reflected in *Growing Up Mathematically*, (*GUM*); to plan a lesson that reflects GUM, and/or implements the lesson in a classroom, followed by a post lesson debrief by the teachers.

10-12: Update on the most recent changes to the mathematics courses at the Ministry level; plans are to leave the Data Management course relatively unchanged with the introduction of an Advanced Functions course, and the removal of Geometry and Discrete. Calculus will continue to be taught under a new name of Calculus & Vectors.

Jacqueline Hill reported from OMCA on their efforts to build curriculum around the newly announced courses. The group is developing assessment pieces and then will plan the scope and sequences of courses. Some TIPS activities would be integrated into a day-by-day outline along with references to various textbooks currently used. Plans are to release a grade 11 C day-by-day lesson plan by August 15. This would be available on the OMCA website to math coordinators to distribute to teachers.

Chapter Reports

CHAMP - Rep: Lynda Ferneyhough:
We had a very successful contest and Awards Night on June 7th. There was a fun workshop for parents to engage in, refreshments and prizes. Thanks so much to Paul Alves who has run our contests for

many years. We are planning the Fall Event for October 19th.

COMA - Rep: Joanne Anas:

COMA has elected a new executive for 2006-07; Ron Bender (President), Anne Fitton, Joanne Anas (Co-Vice-Presidents), Pat Kehoe, Carrie MacFarland (Co-Secretaries), Cathy Hall (Treasurer), Beata Myhill (Elementary Membership Co-ordinator) ,Melissa Perrotta (Secondary Membership Co-ordinator) and Ron Gaudreau (OAME Chapter Rep). Plans are underway for a Fall Social in October along with the Annual Fall COMA Mini-Conference on Saturday, November 18, 2006 which will be held at the University of Ottawa, S.I.T.E. Building. COMA would like to congratulate Anne Fitton from Sir Wilfrid Laurier SS in Orleans for winning the Union Gas Award for Teaching Excellence in Secondary Mathematics. We are proud to have her in our Chapter.

GOLDEN - Rep: Laura Tonin:

Our annual mini-conference at Brock University in April went very well. The staff at Brock were very accommodating and helpful. Our committee is hoping to attract new members interested in helping organize events.

GVMA - Rep: Barb Vukets:

The GVMA Fall Conference will be held at the University of Waterloo on Oct 19, 2006. Along with a choice of sessions, teachers will have an opportunity to socialize with their colleagues while enjoying a great dinner along with guest speaker Dale Nikkel. GVMA's new website is now available at: **www.gvma.ca**. The site features conference information, links to professional resources, math topics, math resources and general information about the Chapter and it's executive.

ISOMA - Rep: Diane Lang:

The good news is that we are back on track and energized to promote Mathematics Education in 2006-2007. Several schools sent representatives to a meeting in November where the issue of revised Grade 12 curriculum was addressed, and the heads of Mathematics at Toronto area independent schools have met regularly to share ideas. In the fall we will plan a

▲ Nominations for OAME/AOEM Board of Directors 2007-2008

Each year, elections are held for a President-Elect, a Vice President (3 year term), and 6 to 9 Directors (3 year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either grade JK–6 or grade 7–12 issues based on their personal experience, expertise and interests. Nominations for these positions are to be submitted by 4:00 p.m., January 19, 2007, to:

Kathy Kubota-Zarivnij

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2007, the Nomination Committee will publish a slate of candidates, including qualifications, for consideration by the membership. Voting will be online in the Member's Only section of the OAME website www.oame.on.ca, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Barrie, May 2007.

NOMINATION FORM

I nominate:_____

For the position(s) of: ☐ President-Elect ☐ Vice-President ☐ Director

If for Director, area of personal experience/expertise, interest : ☐ grades JK–6 ☐ grades 7–12

Nominee's Address: _____

_____

Telephone: H ( ) _____ W ( )_____

Nominator's Name : _____

Nominator's Address:_____

Telephone: H ( ) _____ W ( )_____

conference and organize to support the Ontario Math Olympics program. Our goal is to get every independent school in Ontario to join OAME. Contact Peter Thompson for membership information.

MAC² - Rep: Andrew Morrison:

We are planning a fall mini-conference that will connect to the theme of OAME 2007, "Building Bonds". The conference website **www.oame2007.ca** is now open and ready to receive Online Session Proposals from potential speakers for the conference.

NOMA - Rep: Sue Melville:

The NOMA committee has already begun planning the annual fall conference to be held on Friday, October 13, 2006 at Lockerby Composite School in Sudbury.

NWOAME - Rep: Maribeth Williams:

After 8 successful OMO playdowns co-chairs Maria Casasola and Margaret Strerz mentored this year's new coordinators. In the Fall our chapter will promote OAME at the Faculty of Education and host an evening mini conference.

PRMA - Rep: Sandra Jean Price:

We are currently planning our annual Fall mini-conference to be held in Bowmanville on October 19th. The conference will focus on the *GUM* project and Mathematics Education K-12. While all teachers may attend, we are primarily focusing on pre-service teachers from Faculties of Education in the area..

QSLMA - Rep: Robert Sherk:

QSLMA has been active in two main areas these past few months. We have promoted the annual provincial conference though our email newsletter. We have also run a very successful local OMO under the guidance of Doug Evans and his curriculum class at Queen's. We will start planning for the fall local mini-conference.

SAME - Rep: Nora Mathe:

SAME hosted the OMO at U of T Scarborough Campus on May 26 and My 27. It was a resounding success. Next years plans will be discussed at our last dinner meeting in June.

SWOAME - Rep: David Petro:

SWOAME has some projects being planned in the future: participation of two SWOAME teams at the OMO 2007; representation of OAME at local boards Summer Institutes at the end of August and preliminary plans are being made for a Fall mini-conference at the University of Windsor.

TEAMS - Rep: Pat Margerm:

We initiated a spring mini-conference this year, and, with 100 participants, it shows a great deal of promise for the future. Keynote speaker was Anthony Meli who presented a mathematical look at the Da Vinci Code. We will be planning a fall and a spring mini-conference during the 2006-07 school year. We will be initiating an award in our late president Anne Cirillo's honour and will make a decision on its format in the fall.

WOMA - Rep: Ann Michele Stenning:

WOMA hosted OAME 2006 : "Every 1 Counts" in May 2006. Many delegates attended the Equity Breakfast to hear Beverly Freedman speak about the hard wiring differences between males and females. Everyone received handouts during an energetic Keynote address by Jack Weiner. The following day, Catherine Fosnot impressed us with her Keynote address which included video of complex discoveries that young students make when using mathematics. We were supported by volunteers and along with great planning by our two co-chairs, Cathy Lynch and Heather Summers, we felt that OAME 2006 was a success.

Y⁴MA - Rep: Pietrina Orlandi:

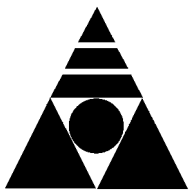
Y⁴MA is currently undergoing a process of renewal. In the spring of 2006, the chapter hosted its A.G.M. and elected a new executive to two year positions. In addition, it put forth a proposal, which was passed by the OAME board of directors, to host the OAME 2008 provincial conference. We are very excited about this opportunity. Currently, the executive is busy organizing it's fall conference, themed, "A Really Good Math Conference!" ▲

▲ Call for Abacus Editor

The Ontario Mathematics *Gazette* is taking applications for a two-year term as *Abacus* editor commencing with the September 2007 issue.

Job shadowing available now.

For more details, please contact:
Executive Directors:
Dave and Sue Hessey



▲ What's the Problem? Introduction and Crossing a River

SHAWN GODIN
HEAD OF MATHEMATICS, BUSINESS AND TECHNOLOGY, CAIRINE WILSON S.S.



Shawn is currently finishing up a three year term as a consultant with the Ottawa Carleton District School Board. He returns to reality in September as the Head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans.

Welcome to a new column! After I submitted my article "One Problem; Many Solutions", which appeared in the June 2006 issue of the *Gazette*, I was talking to the Editor at an OAME board meeting and she tricked me into doing a regular column, so here I am. I have been in love with and involved in mathematical problem solving for many years. For the last five years I have served as the editor of the Mathematical Mayhem section of the Canadian Mathematical Society's international problem solving journal *Crux Mathematicorum with Mathematical Mayhem*. The CMS has just started making public all of the material from Mathematical Mayhem on their website **http://journals.cms.math.ca/CRUX/**. Many of the problems I will discuss in this column appeared in Mathematical Mayhem.

As an introduction, a little history. *Crux Mathematicorum* was created in 1975 by Léo Sauvé and Fred Maskell, professors at Algonquin College in Ottawa, as a way to link the post secondary and secondary communities in Ottawa. It started as *Eureka* primarily with some problems for people to solve as well as the occasional article. In its original form it was sponsored by Algonquin College and COMA. As time went on, the editors started receiving solutions from people outside the Ottawa area and soon from around the world. The Canadian Mathematical Society got involved and the name got changed (there was another established magazine with that title) and the rest is history. *Crux* now receives problems, solutions and articles from around the world and is recognized as a premiere source for problem solving material.

Mathematical Mayhem was started in 1988 by high school students Ravi Vakil and Patrick Surrey as a journal by students for students. As it evolved, the editorial staff consisted of some of the finest mathematics students in Canada. All eight of the founding editors and editors emeriti, represented Canada at some point at the International Mathematical Olympiad. Between them they brought home three gold, seven silver and three bronze medals as well as one honourable mention. They continued to be involved with the Canadian team, acting as deputy leader eight times and leader once. Over the years the level of the mathematics increased as members of the editorial board moved from high school to university, and some on to graduate studies. By the mid 1990's, after the loss of one of their major sponsors, *Mayhem* was close to ending its run. The editor at the time approached the Canadian Mathematical Society, and in 1997 the two journals merged and *Mayhem* began its life within *Crux* as *Crux Mathematicorum with Mathematical Mayhem*.

By this time the content of both journals was very similar. When the last editor's term ended and I started my term we decided to change the format a little. Upon consulting with past *Mayhem* staff members and the editor-in-chief of *Crux*, it was decided that the material in *Mayhem* would be aimed more towards the pre-university crowd. Over the last five years we have worked hard to sculpt the *Mayhem* material to be accessible to a wider audience. The last step in the process was to make the material available over the internet to all, which happened this year.

Let's start by looking at one of the past *Mayhem* problems. This was problem H285 from the April 2001 issue.

> *Four people, A, B, C, D, are on one side of a river. To get across the river they have a rowboat, but it can only fit two people at a time. A, B, C, D, could each row across the river in the boat individually in 1, 2, 5, and 10 minutes respectively. However, when two people are on the boat, the time it takes them to row across the river is the same as the time necessary to row across for the slower of the two people. Assuming that no one can cross without the boat, and everyone is to get across, what is the minimum time for all four people to get across the river?*

You may have seen this problem before, maybe as people crossing a bridge with a single flashlight or some other set-up. I like this problem because you don't need much mathematical sophistication to get into the

problem, but the solution and analysis of the problem can involve some careful thinking.

Take a minute to think about it… Got an answer? Don't peek at the solution! Now hands up, how many people got 19 minutes? Most of you, that is great! No matter what group of people I give this to, either students or teachers, this is the answer that most come up with, and they come up with it fairly quickly. At this point I always answer "that is a great start!". Let's think about this a bit more.

If we look at the people when they travel with someone else we find:

A – always takes the amount of time of the other person

B – only takes 2 minutes if travelling with A, otherwise takes the amount of the other person

C – takes 5 minutes with A or B, but 10 minutes with D

D – takes 10 minutes with everyone

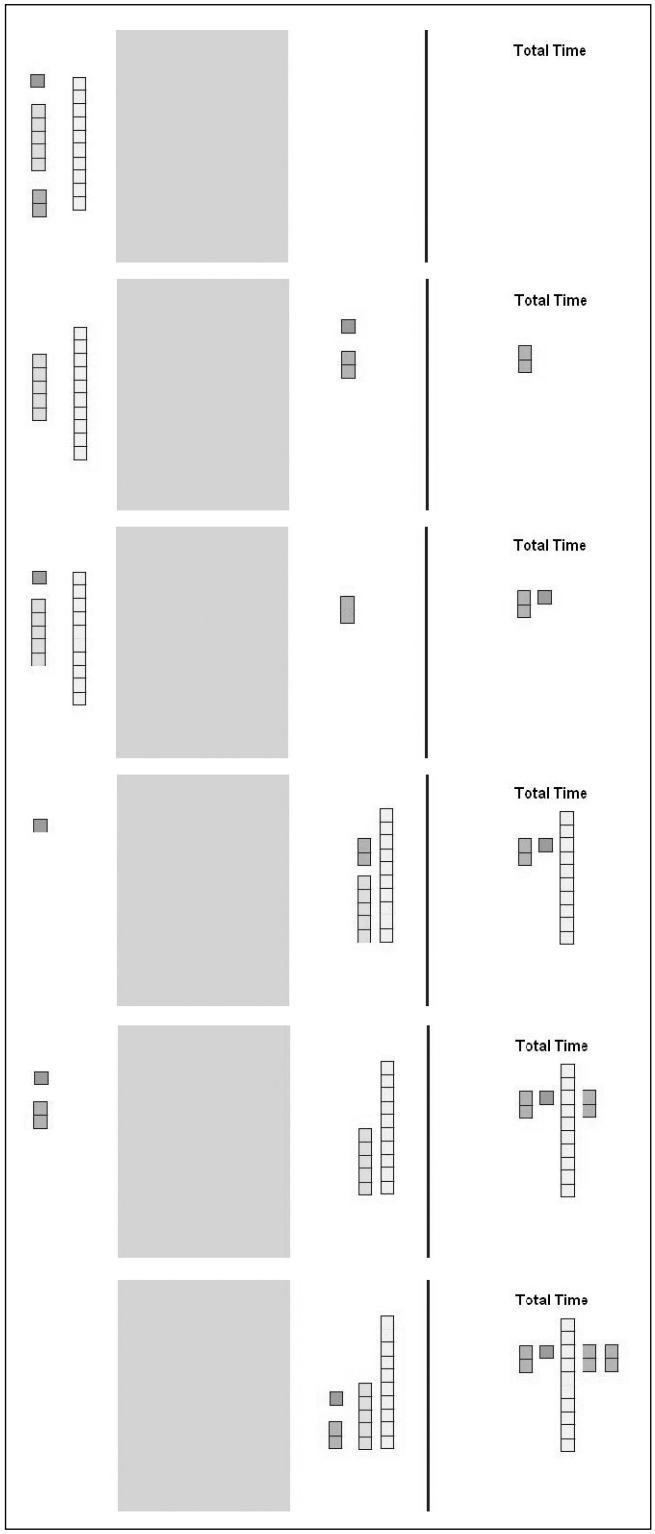
If we look at these times we see that one of the trips must be 10 minutes. There will also be a 5 minute trip unless C and D go together … hmmm…

Most people's logic is something like: A is the fastest, he should make the most possible trips. As a result, most people get something like:

A & B go first (2 minutes), then A comes back (1 minute), followed by A & C (5 minutes), A returns again (1 minute) and finally A & D (10 minutes) for a total of 19 minutes.

If we could somehow get C and D to go together, we would save one of the 5 minute trips. In order to do that A & B go over first, A comes back, C & D go over, B comes back and, finally A & B go over for a total of 17 minutes. It seems a little counterintuitive to send A & B over first with the thought that they both will have to come back.

Students at different levels can tackle the problem in different ways. Manipulatives might help. If linking cubes were used students could uses a blue block for A, a red for B, a green for C and a yellow for D. The cubes could be used to model the physical movement of the people across the river. Also, if you had stacks of one blue, two red, five green and 10 yellow you could use the blocks to keep track of time. So if B & C go together, C has more, so a red "stack" would be added to the total. When you were done, the student would not only have a total for the trip, they would have a visual clue as to "who" was costing them time. A similar model would be slips of paper with "A 1 minute", etc. written on them. The linking cubes solution is modelled in the diagrams below.



For more advanced students, you can start to attack a "proof" that 17 minutes is indeed minimal. Just because we cannot find something faster than 17 minutes, does that mean there isn't anything faster? If

that were the case, many of you would probably have "proved" that 19 minutes was the fastest.

Using some counting techniques, we observe, for the first trip there are $\binom{4}{2} = 6$ ways to pick our first two contestants to cross the river. When they reach the other side there are 2 ways to pick someone to come back. For the next trip there are $\binom{3}{2} = 3$ ways to pick the next two to cross. On the other side we have 3 ways to pick someone to come back. Finally there is only 1 way to bring the final two over. Thus there is a total of 6 x 2 x 3 x 3 x 1 = 108 ways to make the trip. One could do an exhaustive search and show that 17 minutes is indeed the minimum. If certain symmetries were employed, this number could be cut down a bit (for example there is another trip that gives 17 minutes that is "different" from the one described earlier).

Of course, one could use a little logic to cut down our work. If we try to minimize things we could look at the following facts:

1. There are three trips going over, with two people.
2. At least one of these trips must be 10 minutes.
3. The least they could be is 2 minutes.
4. There are two trips coming back, with one person.
5. The least they could be is 1 minute.

Looking at all this information, the absolute minimum trip time (although it might not be possible) is 16 minutes (10 + 2 x 2 + 2 x 1). It doesn't take much to see that this is impossible. Either noting that in this case that B goes "over" twice even though he doesn't come back, or that in order for A to come back twice, he would have to go across on every trip which means that there is never any room in the boat for C. Thus 17 minutes is indeed the minimum time.

The problem leads to many different approaches, as well as, in many cases, an opportunity for the students to take the problem to a different level. It also gives the venue that if students cannot come up with the "proof" themselves, if they have thought about and discussed the problem enough, they will be able to appreciate it. It also leads to the possibility of further research if anyone asks questions such as "What would happen with five people?" or "Is there a different set of times for the four people where the strategy above doesn't give the minimum time?".

I believe that if we want students to become better problem solvers, it is very important for us, their teachers, to also be active problem solvers. As a result of this belief, the structure of the column will help that process. At the end of each column I will present a problem that we will consider next time. I would encourage you to play with the problem, try to solve it and share it with your students. This way you can become involved in the problem solving process and we will, in a sense, become a large professional support group. I would also encourage comments, anecdotes, etc. from the readers so that we can make the column an interactive experience. And now for your homework.

Given

n = 9 + 99 + 999 + ... + 999---9 *(2001 nines)*

How many of n's digits are 1's?

Until next issue, happy problem solving! ▲

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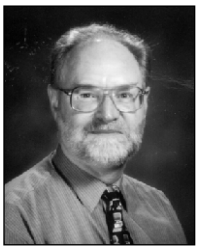
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▲ CAS Corner

FRED FERNEYHOUGH



Fred Ferneyhough is former head of mathematics at Harold M. Brathwaite SS in Brampton. He retired from there in February 2006. Fred is an avid user of and advocate for the use of technology in the mathematics classroom. He is also an author and tech consultant for several books and a National Instructor for Texas Instruments.

Welcome to the *CAS Corner*. After six years of articles pertaining to technology in general, my colleague and friend, Mary Bourassa, has taken over that column.

This is a new column dedicated to the use of CAS (Computer Algebra Systems) in Ontario secondary school mathematics courses. The intent of the column is to demonstrate the appropriate of use of a CAS in the courses we teach.

A CAS is simply a software program that can do the manipulations that we want our students to learn. Using a CAS can be tricky since you need to be careful that it doesn't become either a black box that substitutes for or replaces learning and you may introduce a different problem in learning the syntax of the software.

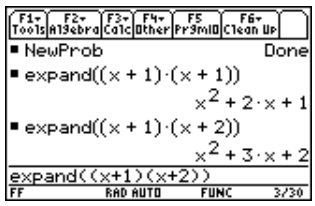
Most of the work that you see in this column will be done on the TI-89 Titanium hand held device from Texas Instruments. I will also be using the new TI Nspire device. However, any of the work that I do here could also be done through other software programs such as Mathmatica®, DERIVE™ or Maple™ or other hand held devices such as the TI-92 Plus or the TI Voyage 200.

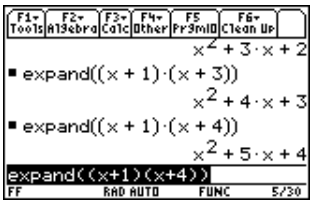
In this column, I hope to introduce CAS as a tool for learning and teaching. I have been fortunate in being in schools where principals supported my requests for these tools and provided the funds to purchase them. Through many attempts and mistakes, I have learned several different ways to use CAS in my classrooms and hope to pass along both. In this issue, I talk about expanding and factoring using the TI-89 Titanium.

Algebraic skills have, for some reason, declined over the past thirty years. I often joke that the students that I taught in grade nine general in 1976 could do more algebra than the students that I might encounter in 2006 in grade ten academic. Regardless of the value that we place on algebraic manipulations, a student lacking in algebraic skills will have limited success in mathematics.

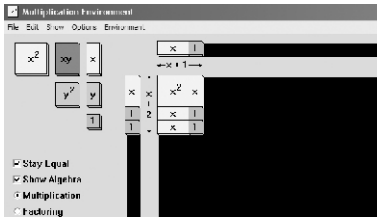
One of the very unusual reactions that I have noticed is that students seem to be very accepting of an answer provided by a technology. In fact, I suspect they will take an answer from a piece of technology ahead of your teaching. If you tell them the exact same thing that the device shows, they may believe you, but only after they see it done on the calculator or computer screen first.

I like to spend a day exploring patterns in expanding as a first step in this topic. Consider the following screens. The expanding operation is accessed by pressing F2 on the device and scrolling down to the Expand feature. Given the results shown on the right, ask your students what would happen if the pattern were extended twice.

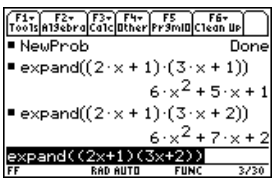




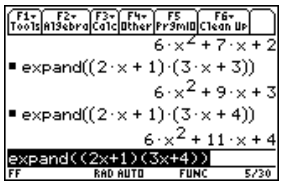
Also, ask if they can explain how the device came up with the answer that it shows. This, in my experience, is one of the best ways of teaching with CAS. Given that they will accept the result on the calculator, I try to get my students to explain whatever results appear. Another tool in your repertoire should be algebra tiles. I think that this is one of the most underutilized manipulatives in mathematics. It takes some work, but I think that it is worth the effort and fits nicely into the statements in the curriculum revisions dealing with a "variety of tools". To do this, you simply place one factor along the top and the second factor along the left side and then attempt to use tiles to create a rectangle representing the answer. The screen shot was done with the Virtual Tiles software program.



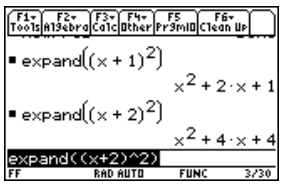
More patterns can be created using the CAS and tiles together. In this pattern, I want my students to detect the changes in the linear term and the constant in each result. Once they have that, I will pose the same two questions: What will the next two problems look like (and what will the result be?). Can you explain what the software did to get this result?

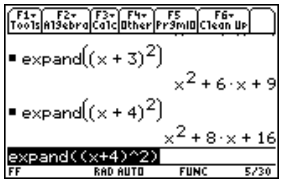


However, now I want to start to have them consider factoring without calling it that. Is there any way that I can look at the result and get back to the factors that created it? One student called this "something like algebraic jeopardy".



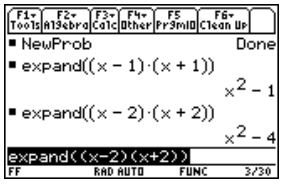
There are standard patterns that we teach in expanding and factoring and these can also be investigated using a CAS and tiles. This screen shows the results from squaring a binomial.



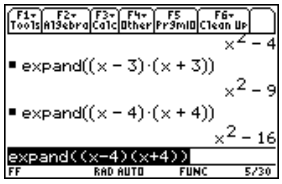


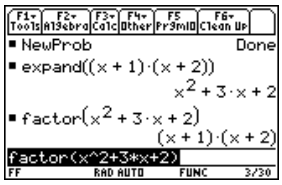
Once again, the same two questions will help consolidate the students' understanding. We can also introduce the vocabulary of algebra and ask why the result can be called a "perfect square trinomial."

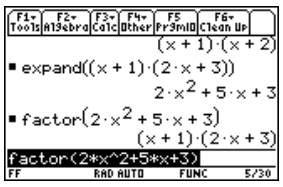
The difference of squares pattern is easily established using the CAS, but is not as easily done using the tiles.

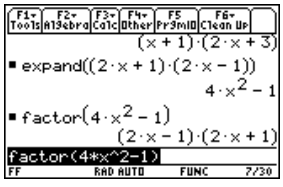


Finally, we can show that the operation called "factoring" simply is the reverse of factoring. In the next three screens, two factors are expanded and then, the result is factored. Even the weakest student will notice that the final result is always identical to the original question. The question about what the device did applies here as well. These screens show the same idea with another example and the difference of squares pattern.









Factoring seems to be one of the most difficult concepts for our students. Using a CAS simply gives them a tool to explore the idea. After such an exploration, the ideas still need to be reviewed and consolidated so that the necessary skills are acquired. ▲

▲ Intermediate Mathematics Olympics May 26-27 2006 Scarborough

Merilyn Fox



Merilyn Fox loves teaching math to grades 7 and 8 at Henry Kelsey Sr. Public School in Scarborough. She was the recipient of the 2003 OAME award for Exceptional and Creative Teaching in Elementary Mathematics. Merilyn is currently on the executive of SAME and a director of OAME.

The Ontario Mathematics Olympics (OMO) for intermediate students is, to all appearances, a simple event; assemble one hundred twenty grade 7 and 8 students together at one location for a day and a half to write a series of exciting, enjoyable, challenging math questions.

In reality, it is a task of monumental proportions. The behind-the-scenes organization begins as much as two years ahead of time, with the application for hosting the event. Jobs such as scouting locations and arranging Olympian tasks take approximately a year. Finding dedicated and responsible volunteers and persuading them they have the time to donate their particular skills, searching for community-minded businesses and individuals who will support the committee with funds or equipment are continuous.

What could persuade a Scarborough group of more or less sane teachers and vice-principals to take on such a daunting task? The fact is, the Math Olympics is an incredibly valuable activity that gives the organizers back more than they put in.

The students who attend the event have been selected by their individual chapters through local competitions. They are a cross section of types of student, but what they have in common is that they are very, very good at mathematics. They are the best in their school. They are the best in their city, and among the best in their province. What they see when they arrive at the competition is a group of over one hundred other students who love math and are excellent at it! They can be themselves with these others, because they share a common passion. This sometimes happens with music students and often with athletes, but seldom do

these talented mathematicians get to shine. The math Olympics gives them that chance. Whether they win or not, the students have an enjoyable time traveling to another part of the province, meeting new friends, and engaging in activities geared to their love of mathematics.

This year, the competition was held at the University of Toronto Scarborough Campus. Contestants, chaperones and volunteers stayed overnight in the residence townhouses, ate and competed in the university facilities. As the teams arrived from around the province, they were greeted with gift bags that included playing cards, sudoku books and t-shirts. After everybody had been shown to their rooms and dropped off their luggage, the students were engaged in a scavenger hunt. Activities such as the hand-cuff problem broke the ice and established the fun atmosphere we were striving for. After dinner, the students had a choice of a dance, deejayed by two Scarborough teachers, or informal games, such as cards and basketball.

The competition began the next morning, immediately after breakfast. This is the point where it became absolutely essential to have an army of volunteers. Three were posted in the halls to direct competitors to their correct location. Six were stationed in the competition rooms, two per room, to handle questions and make quick decisions on anything unforeseen that had to be dealt with. Fifteen were instructed on the intricacies of the marking schemes by our chief markers. One readied a computer for input of data, another readied a different computer for the slide show that would entertain the audience while the last event was being marked and the results tallied.

Students, combined with others on their team or alone, worked on four separate events that took forty-five minutes each. The organizers were very fortunate to have different writers for each section, so there was a good variety of tasks. Some questions were challenging paper and pencil puzzles, others required co-operation and teamwork, plus the use of manipulatives. For a look at these questions, please see the OAME website, where they will be posted.

Our trained markers kept up with the events and after the last question, the team got the results into the computer in short order. It took longer to determine the recipients of the special awards and the three top finishers in every event, since we were trying to recognize as many excellent students as possible.

The day ran smoothly, with just enough glitches to keep us on our toes. There were some problems though. The main one was that our organizing group was too small. We needed two people to be responsible for each job, and sometimes we didn't have enough volunteers to have the luxury of one per job. We are all full-time teachers or vice-principals. Life had an irritating way of throwing us curves such as family needs or job crises. In hindsight, it is apparent that more people were needed, and especially some who have more flexibility in their schedules. Additional resources such as retired teachers, administrators or people seconded to the board would have helped enormously.

The Math Olympics is important to schools which support the event. Parents and the media can see that here are some students who are in the news for all the right reasons. They love being good at school in general and mathematics especially. At last, some good news about our kids, and Scarborough in particular!

At the moment, the Ontario Mathematics Olympics is the final step in the competition. I think that a national final would be an appropriate goal for someone to consider. Other math contests give national results. This would truly be a chance for the province's best students to meet like-minded individuals from across the country, and perhaps travel to a part of the country they haven't seen.

I was co-chair of this year's event. Would I do it again? Absolutely! The rewards are tremendous, not only for the students but also for me. The excitement and the energy of the contestants tell me, more compellingly than any thing else could, that the Intermediate Mathematics Olympics is a worthwhile activity. Now, if I could only find about thirty others of like mind…. ▲

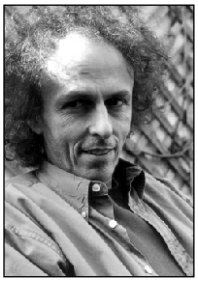


▲ Keynote Speaker at OAME 2007 - Jon Allen Paulos

Lyn Vause



Lyn Vause is a Consultant with the Simcoe Muskoka Catholic Board, presently on secondment to the Ministry of Education. She is also presently working on her doctorate at the Ontario Institute for Studies in Education



Jon Allen Paulos is a professor of mathematics at Temple University in Philadelphia and has written several best-selling books, numerous math related articles for the New York Times and segments for ABC News. He is the Michael Moore of the math world! His writing is thoughtful, provocative and subversive. He uncovers the threat that rampant innumeracy poses to issues of democracy, poverty, war, environmental degradation and economic disparity. He argues that the nervousness that most of the population feels when confronted with large numbers, percents, decimals, fractions and complex bar graphs makes them vulnerable to political, economic and social duplicity and deceit. He is particularly critical of our lack of skepticism when confronted with media reports that can often contradict one another from one day to the other.

One example he gives is,

> *"We are all going to starve," cries one newspaper. "The Earth's population, laid end to end, would stretch to the moon and back eight times. There are simply too many people on the planet for us to survive for long!"*
>
> *"Don't panic," exclaims another. "If every man, woman, and child on Earth were given a cubical room measuring 20 feet per side, all of the apartments would fit inside the Grand Canyon. There's plenty of space for everyone!"*

Which one is correct? Both are. The linear vs. cubical measurement of space is a common misconception in not only young students but in the voting adults who make democratic decisions on population growth based on a hazy understanding of these mathematical concepts.

In a similar vein, he alerts us (or in this case Americans) to the Orwellian reality of illegal, mass wiretappings in the search for 'terrorists'. He questions whether the loss of liberty and privacy is worth the high cost and ineffectiveness of such measures. The volume of the information generated by wiretaps would make them essentially impossible to follow up on because there wouldn't be enough person-power to investigate more than a tiny fraction of the wiretaps. He also states, "Even if an accurate profile of potential terrorists is drawn, the fact that such a vanishingly small percentage of us are terrorists means that the vast majority of the people investigated will be innocent." If the terrorist profile was accurate to 99% (exceedingly high), this would mean it was incorrect 1% of the time. He gives this illustration; "let's further assume that one out of a million American residents has terrorist ties — that's approximately 300 people — and the profile will pick out 99 percent, or 297 of them. Great. But what of the approximately 300 million innocent Americans? The profile will also pick out (only) 1 percent of them. And so these 3 million false positives means that 3 million, innocent people will be caught up in a Kafkaesque dragnet." His point is that it is a huge cost in time, money, resources and privacy to solve a problem that is not yet at the same scale as other problems such as handguns, health care, smoking and the environment which are more significant contributors to premature death in the U.S.

Paulos is both humorous and deadly serious in his quest to rally us out of our shock and awe of all things mathematical so that we can become critical thinkers and less susceptible to the manipulations of political spin-doctoring. He demonstrates that mathematical knowledge is vital to an intelligent understanding of the news that we confront everyday. Our decisions are often emotional responses to first person narratives and media hyperbole rather than a rational response to numerically proven realities. He gives the example of the billions spent on the war on drugs (8 000 American die from cocaine and 6 000 from heroin every year) while 400 000 Americans die every year from smoking and yet the tobacco farmers receive subsidies. This is not a call to eliminate the war on drugs, of course, but just a wake-up call to the misapprehension many people have about the actual problems in our society – some problems are just sexier than others and receive an unfair share of the economic pie.

His "Proposed Math Quiz for Presidential Candidates"

is a potent tool for any politician seeking election, or, really for any voter who wants to ensure they can discriminate between boondoggle and fact. One of his questions, which is shown below, illustrates how easily taxpayers can be misled by a switch from the median to the mean if they don't really understand these terms.

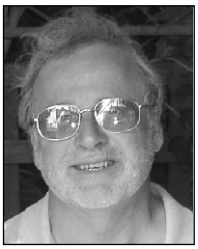
"In Disproportia, a small Midwestern town, the average tax cut per household is $2 200, but the median tax cut is $150. What does this say about the distribution of taxable incomes in the town? If the founder of a high-tech company were to move into the community, which is more likely to rise, the mean or the median tax cut?" (For the answer go to **http://abcnews.go.com/Technology/WhosCounting**.)

Jon Paulos is a unique mathematician. He is a storyteller. He brings mathematics to life while at the same time encouraging our own 'inner mathematician', our rational need for objectivity and critical reasoning in our response to the complexity of the information we now have to deal with on a daily basis. He uses mathematical logic for the cause of social justice and political reform. He reveals how serious a lack of numeracy is in the development of democratic decision-making. His work has important implications for those of us who teach children. As the world becomes more mathematically complex, the necessary level of mathematical acuity will be stretched to the limits. In a time when there is much discussion on critical literacy, Jon Paulos reminds us that such critical literacy is probably not possible without the same level of mathematical literacy.

Jon Paulos will be a keynote speaker at the 2007 OAME Conference to be held in Barrie. Two of Jon's books you may be interested in reading are *A Mathematician Reads the Newspaper* and *Innumeracy: Mathematical Illiteracy and Its Consequences*. You can also read articles of his on the websites of *The New York Times* and *ABC News*. ▲

▲ Algetiles, Anomalies and Complex Numbers

ROLLY MEISEL



Rolly Meisel taught physics and mathematics in the District School Board of Niagara, finishing his career as Head of Mathematics at Ridgeway-Crystal Beach High School. In retirement, he spends some of his time acting as a technical consultant, photographer, and website author for McGraw-Hill Ryerson mathematics texts. He remains a technophile and continues to explore ways to use technology to improve the educational experience for all students.

JAY SPEIJER



Jay Speijer is the Intermediate-Senior Mathematics Consultant for the District School Board of Niagara, where he previously taught mathematics and physics. Prior to teaching he worked in the field of electrical power and is a licensed Professional Engineer of Ontario. He has served on various Ministry of Education initiatives, including Course Profile Project Manager and Lead Writer. Jay is a former President of the Ontario Mathematics Coordinators Association and co-author of a number of McGraw-Hill Ryerson textbooks for Ontario and Atlantic Canada.

Introduction and Intended Audience

Algetiles are useful manipulatives for the illustration of many mathematical processes, including the factoring of trinomials. However, they must be used carefully in order to avoid incorrect results when negative coefficients are present. This article is intended to make teachers aware of the possible incorrect results, introduce a sign test that provides teachers and students with an easy way to check each representation and to suggest an extension that explains the origin of the anomalies, which might be useful as an enrichment or math club activity. Teachers who do not have some experience with algetiles should consider learning what they are and how they are used

by visiting the websites referenced at the end of this article.

Building Rectangular Models to Represent Factoring

When algetiles are used to factor trinomials, the representation builds on the early conceptual development of numeric multiplication that students are exposed to in elementary school: two numeric factors represent the width and length of a rectangle. When the length and width are multiplied together they give the area of the rectangle. For example, 2 and 3 are factors of 6, which can be represented using a rectangle-area model (Figure 1):

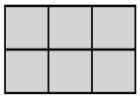


Figure 1

The rectangle-area model can be extended to include expressions that contain variable quantities, such as 2(x + 3) (Figure 2):

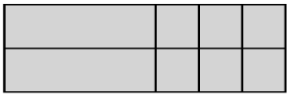


Figure 2

and (x + 1)(x + 2) (Figure 3):

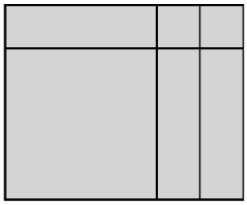


Figure 3

The Need for Addition of Zero Pairs

The model can be extended to algebraic expressions involving negative coefficients. However, one must proceed with caution.

Students who have worked with algetiles to collect like terms will be familiar with the concept of adding zero pairs, as needed, to perform certain operations.

For example, start with 2x, represented by two x-tiles. Ask the student how to subtract 3x from this representation. It is obvious that you do not have enough x-tiles to take away 3 of them. However, you can add a zero pair of x-tiles (Figure 4a), and then, carry out the subtraction (Figure 4b).

Start with 2x, and add a zero pair.

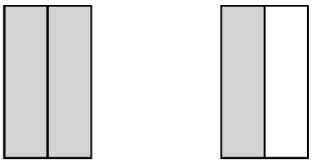


Figure 4a

From the alternate form of 2x, remove 3 positive x-tiles leaving –x as the result:

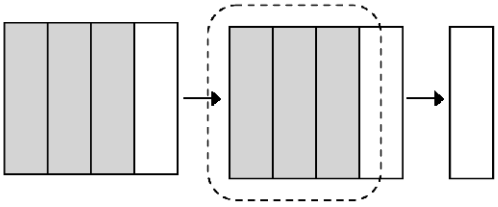


Figure 4b

The zero pair concept is vital for the factoring of trinomials involving negative coefficients. However, if not handled carefully, some interesting and seemingly incorrect results can occur. To illustrate, two examples that work properly will be shown, followed by an example that produces incorrect results.

Example 1: Factoring Trinomials with Negative Coefficients

Consider the trinomial $x^2 – 5x + 6$. Call this Example 1 for reference. This trinomial can be factored: $x^2 – 5x + 6 = (x – 3)(x – 2)$

One x^2 tile, five negative x tiles, and six positive unit tiles are used to create a representation using algetiles like this (Figure 5):

Example 1

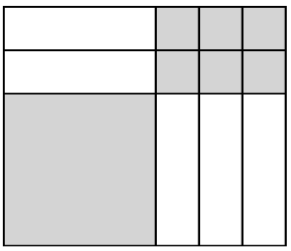


Figure 5

Example 2: Factoring a Trinomial When a Zero Pair is Required

Sometimes a zero pair must be added to make the representation work. Consider the trinomial $x^2 – x – 2$ as Example 2. In this case, we have one x^2 tile, one negative x tile, and two negative unit tiles to work with (Figure 6):

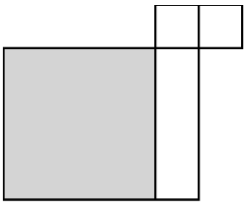


Figure 6

In order to create the proper representation, add a zero pair, e.g., one positive x tile and one negative x tile (Figure 7):

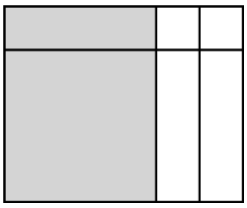


Figure 7

The algetiles show that the correct factored form is: $x^2 - x - 2 = (x - 2)(x + 1)$

Example 3: Adding a Zero Pair That Produces an Incorrect Result

However, sometimes adding a zero pair produces an incorrect result. Consider the trinomial $x^2 - 2x + 3$ as Example 3. Simple inspection shows that this trinomial is not factorable. Using algetiles, we have one x^2 tile, two negative x tiles and three positive unit tiles to work with (Figure 8):

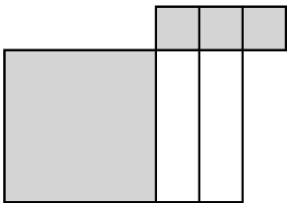


Figure 8

Add a zero pair to complete the rectangle (Figure 9):

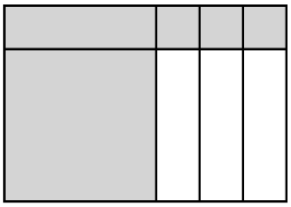


Figure 9

The representation implies that the factors are $(x - 3)$ and $(x + 1)$, which is clearly incorrect.

Using a Sign Test to Identify Incorrect Representations

It turns out that there is a straightforward method to determine when the representation does not reflect a real trinomial. The factors suggested by the representation imply that the constant term in the trinomial should be $(-3)(1) = -3$. However, the three unit tiles are positive. Comparing the predicted constant term from the factors to the constant term in the representation constitutes a "sign test" for the representation. In this case, the sign test shows that the representation is inconsistent, and hence that the factors are not correct.

If we inspect Example 1, we notice that it passes the sign test. The factors imply a constant term of $(-2)(-3) = 6$. This matches the 6 positive unit tiles in the representation (Figure 5, repeated here).

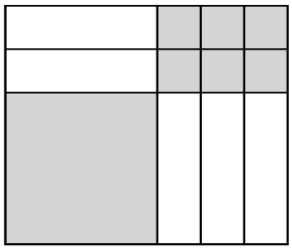


Figure 5

Oddly enough, adding a zero pair to Example 1, and rearranging the tiles can produce an incorrect result (Figure 10).

Example 1B

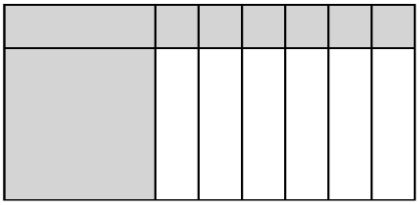


Figure 10

This representation implies that the factors are $x^2 - 5x + 6 = (x - 6)(x + 1)$, which is clearly incorrect. Note, however, that this time the representation fails the sign test. We will call this Example 1B for later reference.

For completeness, note that Example 2 passes the sign test, indicating that the representation is correct for the given trinomial.

The foregoing argument provides a way for students to check whether adding a zero pair has produced a correct or incorrect representation using a simple sign test.

Extension: Algetiles as Complex Numbers

As an interesting extension, consider introducing an alternative interpretation of the algetiles using complex numbers. This extension might be useful as an enrichment or math club activity, but is not intended for general classroom use.

Let the former positive x tile represent ix, and the former negative x tile represent –ix. Unit tiles will still represent real numbers. The dimensions of the former x^2 become ix by ix. The area will represent

$$\begin{aligned} |(ix)|^2 &= |(ix)|\,|(ix)| \\ &= |x|\,|x| \\ &= x^2 \end{aligned}$$

as before.

Let us return to Example 3 (Figure 9, repeated here).

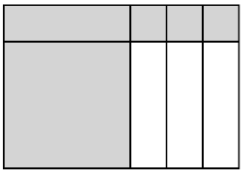


Figure 9

This example failed the sign test. Apply the complex interpretation:

$$\begin{aligned} x^2 - 2ix + 3 &= (x + i)(x - 3i) \\ &= x^2 + ix - 3ix - 3i^2 \\ &= x^2 - 2ix + 3 \end{aligned}$$

which is correct.

In a similar manner Example 1B becomes: $x^2 - 5ix + 6 = (x - 6i)(x + i)$ which is also correct.

Example 4: Using the Complex Interpretation to Introduce Complex Roots

Although this extension is not generally useful at the intermediate level, one simple example could be of interest. Consider $x^2 + 1$ as Example 4. This is often used as an example of a non-factorable binomial, as opposed to $x^2 - 1$, or alternatively as part of a quadratic equation with complex roots. Using algetiles, we have one x^2 tile and one unit tile to work with (Figure 11):

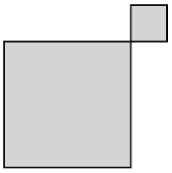


Figure 11

Add a zero pair to complete the rectangle (Figure 12):

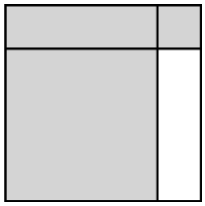


Figure 12

This implies that the factors are $x^2 + 1 = (x + i)(x - i)$, which is correct. In addition, if this is introduced as the quadratic equation $x^2 + 1 = 0$, these factors produce the correct complex roots for the equation.

Summary

Algetiles must be used carefully in order to avoid incorrect results when negative coefficients are present. The sign test, as described in Example 3, provides teachers and students with an easy way to check each representation of a trinomial using algetiles. Having said this, one could argue that using algetiles to introduce the factoring of trinomials in early high school is most beneficial when working with positive coefficients only, since this builds on concepts introduced in elementary school. Once a basic understanding of factoring has been achieved, the model is less pedagogically helpful when negative coefficients are introduced. One problem to overcome is the counter-intuitivity of the physical (e.g. length) representation of expressions like x – 3. With the added complications of zero pairs and the aforementioned sign test, the instructional effectiveness of the model can become hindered to the point of distraction. On the other hand, using algetiles to examine the factors of trinomial expressions involving negative coefficients may serve as a rich exercise in senior mathematics when complex numbers are involved.

References:

http://www.phschool.com/professional_development/teaching_tools/pdf/using_algebra_tiles.pdf for the algetile novice, using algetiles effectively.

http://www.learner.org/channel/workshops/algebra/workshop5/lessonplan1b.html for an introduction to representing trinomials with algetiles.

http://argyll.epsb.ca/jreed/math9/strand2/2210.htm for factoring polynomials with tiles. ▲

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KEYNOTE SPEAKERS

Avis Glaze
Trevor Brown
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Jon Paulos
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▲ OAME 2006 Award Recipients



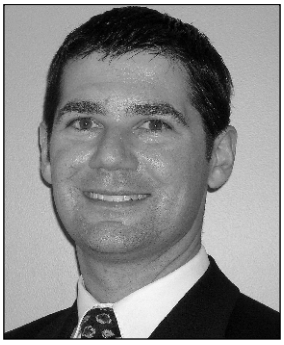
Life Member Award:

How many people can say they have dedicated 42 years of their lives to the teaching profession? Our OAME Life Member award, Todd Romiens has accomplished this amazing feat. In his 42 years in education he has been a math teacher, department head, consultant, textbook writer and editor. He has been involved in curriculum writing, and is presently a professor at the Faculty of Education at the University of Windsor. Todd has served as councellor, president, past president of OAME and as editor of the Gazette. He organized and ran the 1993 OAME annual conference. He continues to spread a wealth of experience as a speaker across North America. Rumour has it that he was the longest standing president of SWOAME (a chapter of OAME) in their history! In closing I would like to read a quote from one of the letters of support for his nomination "Todd truly believes (and I daresay) as Plato did) that no culture is complete without an appreciation for the beauty of math". Congratulations Todd.



The OAME Award for Exceptional Creative Teaching in Secondary Mathematics:

Kelly Blair has been a program leader since the year 2000 in the subject areas of Mathematics/Business and Law. Kelly works at the school, board and provincial levels to enhance the lives of the students in the classroom. She uses technology and manipulatives to make lessons real and meaningful for her students. She continually asks her students to reflect on their learning. Kelly has been involved at the Ministry level in the development of the Targeted Implementation and Planning Supports for students in grades 7, 8 and 9 applied. She continues to be a great speaker here at our OAME annual conferences. In the words of one of her nominators "She is a tremendous role model and inspiration for all teachers in our school" It is with great pleasure that OAME gives this award to Kelly.



Award for Outstanding Contribution to OAME:

Through his involvement with the Ministry of education, this individual has played an important role in the promotion of mathematics in the province of Ontario. From a Lead role in the Early Math Strategies to his position in the policy and programs branch of the Ministry, Peter Sovran has been instrumental in the implementation, and development of the Early Math Strategy. He was tireless in his conviction that good mathematics change would occur when teacher training was significant, on-going, and accessible and practical for teachers. And he made it happen!

To close with a quote from one of his letters of support "Peter has done more to promote mathematics in the elementary system than any other person in recent memory… no one could possibly be as qualified as he is to receive this award". Those are really strong words, that really ring true for Peter. Well done Peter!



The Union Gas Award for Teaching Excellence in Elementary Mathematics:1

OAME and Union Gas have teamed up to present some more awards. The Union Gas Award for Teaching Excellence in Elementary Mathematics is given to an elementary classroom or special education teacher who demonstrates exemplary mathematics teaching practices and supports inclusive mathematics education. This year's winner comes to us from the Peel District School Board. Please help me recognize Jessica Reiter for all that she does for mathematics education. Jessica has taught JK – right through grade 5 in both England and Ontario. She is currently teaching in a grade 3 classroom. She has been called a transformational leader in her school. The students in Jessica's class love mathematics. It has been said that they complain if math is not happening that day! Jessica keeps on top of the cutting edge research in mathematics. She shares her growing knowledge and

enthusiasm with the rest of her staff by modeling lessons in a meaningful and engaging way. I would like to close with a quote from one of Jessica's grade three students that came in as part of the nomination package "Mrs Reiter makes math enjoyable. She lets us use markers and big paper. She gave us questions that we had to think about so we could get better. If we don't think we can do math at the beginning of the year, she makes us be able to".

Along with her students let's celebrate Jessica's achievements!



The Union Gas Award for Teaching Excellence in Secondary Mathematics:

Anne Fitton comes to us from the Ottawa-Carleton District School Board. In the words of one of her nominators Anne is an enthusiastic, dedicated teacher, who has been actively involved in her school, and at the board level. She has developed many computer based technology activities, sometimes for courses that she isn't even teaching. At the board level Anne has served on the Executive of the Math Subject Council for 8 years. Her greatest strength is in the classroom. She constantly strives to improve her teaching. Anne continually develops and seeks varied resources so that the lessons that she presents are rich and meaningful. She is respected and admired by other educators. Well done Anne!



The Union Gas Award for Teaching Excellence in Leadership:

This year's winner has contributed to the changing face of math education in the province of Ontario. If you were to think of one person who truly knows the "ins and outs" of Geometer's Sketchpad, this individual would come to mind. As the leader of the Lighthouse Project in her Rainbow District School Board, she has demonstrated a vision, and, the dedication and determination to make the vision a reality. Judy Dussiume has countless energy in all that she does. She is a true leader. The supporting letters for her nomination came from far and wide throughout the province. I would like to read to you some of the quotes that came from the letters of support to the nomination.

"As a coordinator she demonstrates a passion for both her subject and for teaching."

"Nowhere in the province could there be a more deserving candidate."

"The students and teachers of Rainbow are a mirror of Judy's excellence."

"Her creative uses of technology, especially Geometer's Sketchpad are admired throughout the province and beyond."

"She casts a vision, builds teams, and leads change."

It is a great honour for us to celebrate all that Judy brings to mathematics and the province of Ontario!



Kenneth D. Fryer Award:

This award is given to a mathematics department that fosters collegiality, team work, and excellent classroom teaching. They must contribute to the overall development of students and demonstrate leadership in the mathematics education community. This year's recipients come to us from 29 Lisgar Street in Ottawa. They are the teachers of the Lisgar Collegiate Institute. Under the direction of their department head Tania Asselstine, the department has flourished. A quote from a grade 11 student who attends Lisgar Collegiate: "The Lisgar Math Department was phenomenal before the SMART board, but the new technology brought the department to the next level. Imagine giving teachers who can already mesmerize a class with quadratic equations and mathematical applications for an entire period with only a piece of stone and some chalk, a large powerful technological wonder." Congratulations Team! ▲

OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration:

- demonstrations of excellence and creativity in mathematics education;
- specific contributions to mathematics education in the school, region, province or nation;
- demonstrates leadership in mathematics education in the school, region, province or nation;
- support of and contribution to the activities of OAME/AOEM, and
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

OAME/AOEM Life Membership Award

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and has accumulated 10 or more years of membership in OAME/AOEM.

Kenneth D. Fryer Award

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Award for Outstanding Contribution to OAME / AOEM and Mathematics in Ontario

This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics

The award is presented annually to recognize an exceptional and creative elementary teacher who demonstrates excellence in mathematics education by contributing to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics

The award is presented annually to recognize an exceptional and creative secondary teacher who demonstrates excellence in mathematics education by contributing to the overall development of students.

AWARD NOMINATION COVER PAGE

INTENDED AWARD *(please check one)*

- ❑ **Life Membership**
- ❑ **Kenneth D. Fryer**
- ❑ **Award for Outstanding Contribution to OAME / AOEM and Mathematics Education in Ontario**
- ❑ **Award for Exceptional and Creative Teaching in Elementary Mathematics**
- ❑ **Award for Exceptional and Creative Teaching in Secondary Mathematics**

Nominee's Information:

_____

Name of Nominee (Principal and Department Head for K.D. Fryer Award)

_____

_____

School, Organization, or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

Nominator's Information:

_____

Name of Nominator

_____

_____

School, Organization, or Business Address

Telephone Number ( ) _____

Fax Number ( ) _____

Home Address _____

_____

Home Phone ( ) _____

SEND TO: Jacqueline Hill

▲ Archimedes' "Pie"; an Old Idea, a New Tool†

KAMEAL MINA
AND JAMIE PYPER



Kameal Mina is a recent graduate of the Althouse Faculty of Education at the University of Western Ontario. He is the recipient of the 2006 Laura and Edward McLeish award for outstanding academic achievement in mathematics as a teaching subject at the Intermediate/Senior level.



Jamie Pyper is a former department head of mathematics, currently seconded from the Thames Valley District School Board to the Esso Centre for Mathematics Education (www.edu.uwo.ca/essocentre) and the Faculty of Education, UWO. Jamie is a J/I and I/S mathematics education instructor, and the coordinator and practicum advisor for the concurrent mathematics and education program at UWO.

Introduction and Historical Perspective

Archimedes' "Pie"
Around 240 B.C., Archimedes asked his wife to prepare a pie for him. She did her best to make it fully rounded and she succeeded. When Archimedes was about to eat, he started looking at the pie and asked himself: "if this pie were a square, I would know its perimeter quickly, but as it is a circle, now what would its circumference be?" Archimedes stopped eating and dedicated himself to solving the problem of the circle. The pie is preserved now in the Louvre in Paris, France

This story, while fictitious, is a great hook for introducing the "Archimedes' Pie" manipulative. This manipulative is used in the process of finding the value of π‡, which is $\frac{C}{d}$ or $\frac{4A}{d^2}$ ±, where C is the circle circumference, A is the circle area and d is the circle diameter. It is thought that the symbol π is used to identify this ratio, as it is the first letter of the Greek word "περιφερεια", which means circumference.

Squaring the circle is to find the side length of a square that has the same area as a circle with diameter, d. This is a problem that is old in history (Berggren et al, 2000). Around 1650 B.C., an ancient Egyptian called Ahmes, wrote a papyrus that shows the instructions for squaring the circle. Schepler (1950) believes that the idea itself dates back to about 3400 B.C. The value given in the Ahmes papyrus is $\pi = \frac{256}{81} \div 3.16049387$, making a relative error of about 0.6% compared with the reference value.

Around 950 B.C., the Hebrews (Zebrowski, 1999) used the value $\pi = 3$ (relative error of –4.5%), as mentioned in I Kings vii 23 and II Chron. iv 2, in the Bible. In fact this ratio of 3 would be accurate if the shape were a hexagon, not a circle.

The Babylonians, Indians, Chinese, Greek, and the Romans contributed to the estimation of π, with different accuracies. Different values for π have been registered throughout history. Around 240 B.C., the Greek mathematician Archimedes (287-212 BC) followed a different approach in determining π. Archimedes inscribed and circumscribed a 96-sided polygon to a circle with unit diameter. He explained that the circle circumference ranges between the perimeters of the two polygons. He found π to be in the range: $\frac{223}{71} < \pi < \frac{22}{7}$

Taking the upper limit $\pi = \frac{22}{7}$, the relative error is about 0.04%. This value is considered to be the most accurate estimation determined from historical investigations. At that time, numbers were only represented as fractions. With the increased use of decimals, and calculators today, in decimal notation, Archimedes' range of values would be: $3.14084507 < \pi < 3.142857143$.

Now, π has been calculated with more than 400 billion digits after the decimal point, using high-speed computers (Schepler, 1950).

Idea and Structure of the Activity

Manipulatives can be defined as educational tools used to help students visualize abstract concepts in mathematics and science. Regarding the circle, students

† *Keywords: multiculturalism, constructivism, self-esteem, individual variations, history, math.*

‡ *The value of π stored in our calculators ($\pi = 3.141592653...$) will be considered as a reference for any discussion in the present article. The relative error is defined as* $E = \frac{\pi_{cal} - \pi}{\pi}\%$, *where π_{cal} is the estimated value of π by any researcher.*

± *Reminder: the circle area is* $A = \pi\left(\frac{d}{2}\right)^2$.

use 3.14 as a value for π, and rarely, it seems, do they appreciate that π is an irrational number (Steuding, 2005), nor do they appreciate the rich history of π.

The motivation for the activity capitalizes on the constructivism theory of learning, inherent human curiosity, and desire for ownership in learning. Students come into our classrooms with such different, unique, and complex mathematical memories. So, with a cool story, lots of past experiences, and a new tool to play with (and hence investigate with), we explore circles for a value of π.

The activity is a traditionally straightforward measurement of various items. Students will take their own measurements of the circle circumference and diameter, list them in a table, and determine an average value for their own findings. Students' past experiences will bring a rich texture to the class discussion of accuracy and precision, and measurements errors and approximations. Lots of inconsistencies and differences in the numbers and calculations will be seen.

The Tool

The tool is composed of a transparent strip of plastic (transparency) that can be rolled as a circle. The flexibility of the plastic strip allows it to curl easily. The strip is graduated in two directions. One direction measures the diameter of the circle and the other measures its circumference, as shown in Photo 1. Although Archimedes followed a theoretical analysis based on the knowledge of the dimensions of a certain type of triangle (e.g. equilateral triangle), the present activity requires students to take measurements by themselves, and avoid the theoretical analysis.

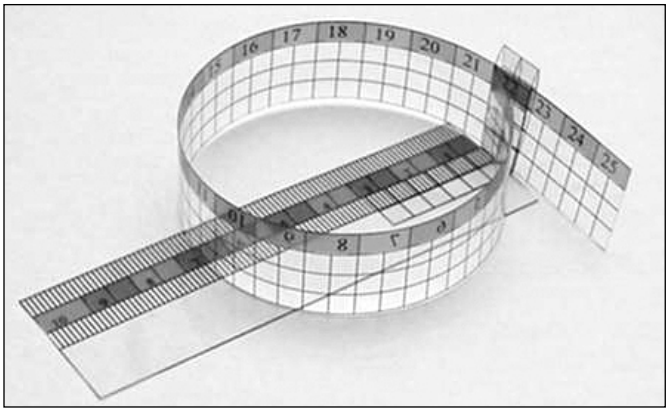


Photo 1 Archimedes' Pie

What Should the Student Do?

With the "Archimedes' Pie", students will take measurements of the circle's diameter and circumference and create a numerical model, as shown in Table 1. For measurement purposes, the circumference piece of Archimedes' Pie is graduated in 5mm units, while the diameter piece is graduated in 1mm units.

Circle Circumference, [cm]	Circle diameter, [cm]	$\pi = \frac{C}{d}$	Error (%)
15	4.75	3.1579	0.5
17	5.3	3.2075	2.1
19	5.9	3.2203	2.5
21	6.6	3.1818	1.3
23	7.05	3.2624	3.8
Average Value of π =		3.206	2.1

Table 1 Circle circumference, diameter, and the corresponding ratio.

The values filled in Table 1 are sample measurements by the author. For example, when the circumference was 210 mm, as shown in Photo 2, the corresponding diameter was found to be about 66 mm, as shown in Photo 3. Dividing 210 by 66, we get $\pi = \frac{210}{66} \cong 3.18$. It is clear from Table 1 that the maximum error is about 3.8%.

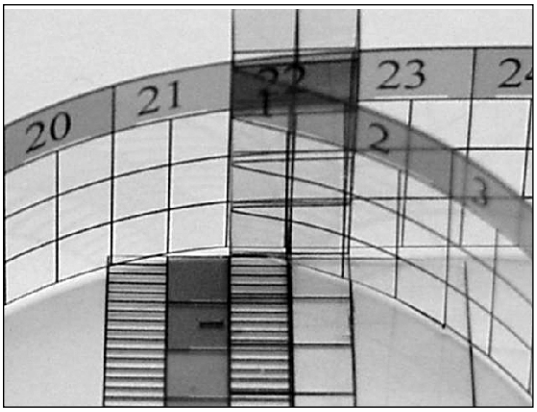


Photo 2 Circumference was adjusted to 210 mm.

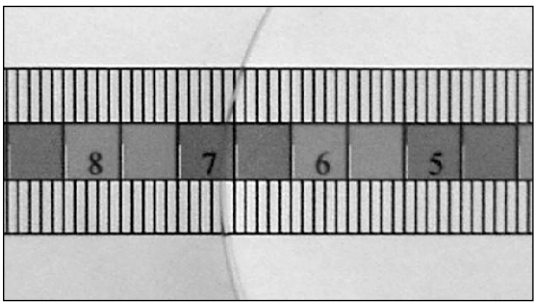


Photo 3 Corresponding diameter was found to be 66 mm.

A rich extension to the "Archimedes' Pie" is to have students discover the measurement error issue, perform the calculations, attempt to find measurement methods to reduce this error, and participate in a large or small group discussion (providing opportunities for assessing verbal and written communication if desired).

Classroom/field Measurements

In order to validate the use of the Archimedes' Pie, classroom and field measurements were taken. First, this tool was tested with preservice Intermediate/Senior math teachers in a faculty of education. The preservice teachers were assigned five small learning groups and they were asked to take their own measurements and estimate an average value for π. Table 2 shows the values obtained from the preservice teachers.

Group number	Estimated π	Error (%)
1	3.39	7.9
2	3.24	3.1
3	3.32	5.7
4	3.4	8.2
5	3.18	1.2
Average value of	= 3.306	5.2

Table 2 Estimation of π by preservice teachers.

It is clear that the maximum error in Table 2 is about 8.2%, which is relatively high. The fifth group, however, gave the most accurate measurement with a relative error of about 1.2%. The error of the average value is 5.2%. This led to slight modification in the design, which was tested later with high school students. Also, preservice teachers suggested the use of solid cylinders and simply measure its circumference and diameter rather than circles drawn on papers.

The manipulative was tested later with grade nine academic students (it can be used with applied classes as well) in two classes (one class of twenty four students and the other of twenty five students) in a high school. The students in each class were divided into five small learning groups. Each group included four to six students. The students were hooked with the fictitious story about Archimedes and were given the tables to fill them out. They were asked to have each member in the group work individually on the manipulative. Students were instructed in the use of the tool, as it is tender and can be torn apart easily. Students were also taught to take the most accurate measurements possible. It took about 30 minutes to work with the manipulative for each class.

Individual variations regarding the student's ability were observed in both measurements and calculations. For example, some students had accurate measurements (they cared about reading the correct diameter value) and correct division of the circumference and diameter. Others had accurate measurements, but incorrect calculations. One class of students gave an overall average value of about 3.39 (relative error of about 7.8%), while the other class of students determined an overall average value of 3.18 (relative error of about 1.2%).

It was noticed that two factors affect the accuracy of the results. The first, which will be discussed below the accuracy of the tool itself. The second is how careful students are in taking measurements and calculating their value of π. The high school students were engaged and focused during the investigation. Archimedes' Pie can be used as an introduction to the circle to grades 8, and 9. Also it can be used as a review for grades 10, 11, and 12.

Benefits and Cautions of the Activity

Successful opportunities for students to develop understanding may also encourage students to take ownership of mathematical concepts as they use their experiences in different situations. The material used to manufacture this activity is soft plastic, which is safe for all students. The plastic strip is graduated in order to allow multiple measurements; otherwise, one student might think that π has only one valid value for a certain diameter and circumference. The manipulative is coloured in blue for one unit and red for the next, to help differentiate measurements easily. Alternatively, black and white copies would also be acceptable, thereby making it an inexpensive tool to produce on the photocopier.

The maximum error in any of the measurements listed in Table 1 reached 3.8%, while the error based on the average value was found to be 2.1%. This activity was hand-made and hence does not allow great accuracy in the measurements. Other designs and materials may be used for greater accuracy, with the provision of specialized equipment.

Conclusion:

The Archimedes' Pie tool provides an opportunity to appreciate the historical development of the mathematical evaluation of π. This helps the student to live for an instant in the past, where current technologies were not available. The student may imagine how

difficult it was to construct the basic rules in geometry, such as finding the area and the circumference of the circle, that they use and take for granted in their math classes today. Students can also appreciate the use of the decimal system and calculators that contain a relatively accurate value for π, avoiding the exclusive use of fractions in calculations.

As it is intended to rely on the concept of constructivism, students are given the chance to build their own understanding of the idea that the circle circumference is proportional to its diameter through the constant value π. Students are also taught to be careful in taking their measurements and should understand that it is natural to have errors in measurements.

Manipulatives may also help us teach concepts without the feeling of guilt of indoctrination. In this manipulative, allowing students to take measurements and conducting simple division calculations may eliminate the idea of imposing the value of value of π we know. Another benefit concerns the respect and attitude towards other cultures regarding the development of mathematical concepts. For our culturally diverse classroom communities, this historical perspective may strengthen interpersonal connections and individual or collective self-esteem. Students also can imagine how much different life was for those who lived many centuries ago, without enjoying the technologies and advances in science we own today.

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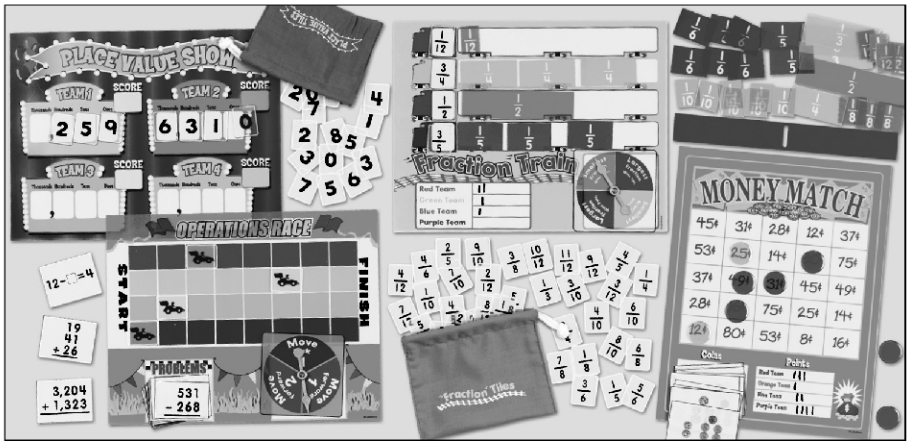
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