Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceeding 10 numbered pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required. Please number pages.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including web sites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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▲ About the *Ontario Mathematics Gazette*

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Submission of Advertisements

Advertisements for the *Ontario Mathematics Gazette* should be sent to **Dean A. Murray** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising Rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARILYN HURRELL, EDITOR



This seems to be the issue of Guest writers. This is quite exciting as it means that more people are involved in creating this journal. Fresh points of view and ideas are always welcome.

Why don't you, the readers of the *OMG* get involved also? I am always happy to receive any feed back – good or bad – on any issue of the *OMG*. This helps me to deliver to you what is useful for your particular mathematics needs.

Ron Lancaster's column features photos and questions created by Louis Lim. Fred Fernerhough's *CAS Corner* comes to us this time from Australia and is written by Peter Fox, a TI colleague of Fred's. Chris Suurtamm's *Assessment Corner* uses material supplied to her by students in her classes at University of Ottawa and Jack Weiner teams with Suzanne Carera to investigate *Kelloggs Froot Loops.* I have a Guest Editorial from Walter Whiteley.

I wish you all a very happy Holiday Season.

Marilyn

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

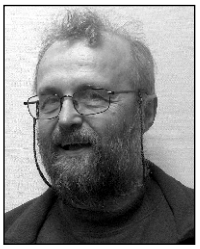
Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor

Marilyn Hurrell for further details.

▲ Guest Editor's Message

Walter Whiteley
Member of the Board of Directors, OAME
Faculty Member, York University



Walter Whiteley teaches in the Mathematics and Statistics Department at York University, and supervises graduate students in Mathematics, Computer Science, and Mathematics Education. He also coordinates a part-time MA program for Teachers of Mathematics at York. His research is in Applied Geometry (working with engineers, computer scientists, biochemists and mathematicians) as well as research in Visual and Spatial Reasoning with mathematics educators. He taught in the Quebec CEGEP system for 20 years (courses between Grade 11 high school and university), before coming to York University. For the last 18 months he has chaired the Council of Ontario Universities Mathematics Review Task Force, coordinating feedback from all the Ontario Universities into the Grade 11-12 mathematic curriculum revisions, and working within the University system to develop revised Admissions Requirements and revised first year courses for September 2008.

Not Another Five Years of Curriculum Review?!#?

As the current cycle of mathematics curriculum review grinds on towards a conclusion, there is a mixture of relief, exhaustion, frustration and concern. After all the intensive efforts, and generous contributions, the process and the results reveal gaps and areas of concern about 'What is missing?', 'What are other jurisdictions doing better?', Can we serve our students better? … .

The Ontario Ministry of Education is set up for a five-year cycle of curriculum revision. What can we do differently over the next four years, so that the next round is qualitatively different than this cycle has been?

For the last 16 months, I have had many conversations with folks involved in the current cycle. Every extended conversation has included some expression of frustration, and agreement that we need to do something different between now and the next revision. The goal is to enable a larger, better-informed, more coherent vision to be embodied in the future curriculum. The report of the Ministry Task Force last spring (the 'Salisbury Report', recommendations 17-20) recognized this need, and recommended the Ministry appoint a 'Mathematics Curriculum Council' to support this. This particular strategy is on hold (the Ministry is uncertain about such Curriculum Councils) and such a Ministry appointed structure might impose significant constraints that would reduce what can be accomplished. (Anyone who has participated in current and previous Ministry efforts, and experienced the unreasonable restrictions on circulating drafts, obtaining further feedback, … will recognize some of these risks.)

Of course, many of us have our own special concerns. I am a geometer, so I ask what has happened to erase geometry from the Grade 11 and 12 curricula? How have the revision process and the experiences in K-10 contributed to this outcome? What are the consequences for a generation of students even more ill equipped to play with the stuff of geometry, including 3-D where many big applications live? Kids enter school living in 3-D and many graduate with weaker skills than those they started with. Can't we do better?

Most of us are tired and ready for a break from 'change'. On the other hand, we are convinced that we need an alternative process to generate some essential improvements for the sake of our students and to support the teachers. After some years of micro-analysis of content for particular courses and destination pathways, under specific, non-mathematical constraints, it is time for a holistic analysis which starts with big ideas, the big picture and major insights drawn from current research and current practices around the world. This possibility is scary – as it could lead to more major changes – though I anticipate it will also reinforce and further evolve the pedagogical and process changes of the last decade.

Others will need to contribute their own insights and concerns. How do we build the varied local observations and concerns into a larger community conversation that will generate a vision and an implementation to bring to the table in the next round? Who can best animate and support the invitations, the gatherings, the exchanges, the research and the writing that could move us forward? How do we develop a large community dedicated to better learning of significant mathematics, over the period K-12 (maybe K-14, and include the universities and colleges)?

Three key organizations that might be able to provide the coordination and support are:

- OAME
- OMCA
- Fields Institute Mathematics Education Forum

Should one of these groups take a leadership role? Should others be invited to join (including for example the Ontario College Mathematics Association – OCMA; some ongoing Ontario Council of Universities Mathematics Review Task Force, some science educators groupings ...)? Should all three groups prepare a joint proposal and approach the ministry?

I sense a consensus on the need and I think there can be a consensus on the larger goals. What we need now are some strategies and tactics to build some structures and coordinate the energy so that we achieve success. The Fields Institute has recently set up an electronic list for discussion of options. Please e-mail me if you want to be involved, now, in the discussions, and we can add you to this list for the short term. Perhaps we can develop proposals for the February meetings of the OAME Directors, the Fields Math Ed Forum, and the OMCA. We seem cursed to live in 'interesting times'. ▲

▲ Call for *Gazette* Editor

The *Ontario Mathematics Gazette* is taking applications for a two-year term as editor commencing with the September 2008 issue.

Job shadowing available now.
For more details, please contact:
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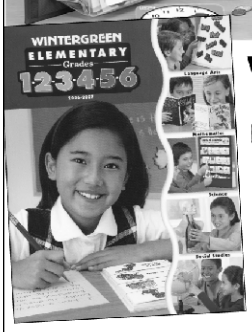


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▲ President's Message

DAN CHARBONNEAU, PRESIDENT



Season's Greetings. I hope that all members will have an opportunity to rest and recharge during the holiday season. It is well deserved as I'm certain you have all been very busy throughout the fall. I am impressed by the commitment shown by so many of our members to the improvement of teaching practices and student learning in this province. Kudos to the classroom teachers and also to those of you who are OAME board members, school board math coordinators or consultants and those who work at the Ministry or the EQAO. If you know of someone who has made a significant contribution to mathematics in your area or provincially, I encourage you to nominate them for one of several OAME awards and/or our Union Gas Math awards. A nomination form for the OAME awards is included in this *Gazette* issue and nomination forms for the Union Gas Awards have been sent to the Directors of Education of every school board; contact them for more information.

As in my prior messages, I encourage each of you to get more involved in your local OAME chapter. This can be accomplished by attending or presenting at your chapter conferences, (many have been held this fall), having your Grade 7 and 8 students participate in the local Ontario Mathematics Olympics play downs or by joining your local chapter executive. In this issue of the *Gazette* you will find a salmon colored insert informing you that this year's OAME elections will be done through on-line voting. Have you ever considered running for an OAME board or executive position? What does this entail? The executive meets four times a year to plan the OAME board meetings, address and respond to math issues that arise both inside and outside the province, set and review the budget, plan the Leadership Conference and oversee any other business/activities related to OAME. The OAME board meets three times a year and is made up of the executive members, fifteen Chapter representatives, and nineteen or so elected directors. As an elected director you would sit on one of several Standing Committees and participate in one of several Working Group Committees. Several hours of committee time at each meeting would allow you to work with committed and enthusiastic fellow educators as a team, developing OAME materials/publications and providing input on various math issues. Our OAME Board meetings would also enable you ample professional development and networking opportunities. Get someone to nominate you before the January deadline.

A wealth of information is provided at our OAME Board meetings. The following updates are samples of the content from our recent October 2006 meeting, some of which may be old news by the time this *Gazette* is published.

The Grade 12 Math curriculum revisions are in their final stages with editing and publishing to follow. You can expect the final document to be out on the last day of winter (March 20, 2007 or so), followed by in-service in the spring. The expected new course, called Calculus and Vectors MCV4U, will have Grade 12 Advanced Functions MHF4U as a prerequisite course that may have to be taken concurrently in some schools, particularly non-semestered schools. The Grade 12U Math of Data Management MDM4U can be taken by students coming from both the Grade 11U Functions MCR3U and the 11M Functions and Applications MCF3M courses.

The Literacy and Numeracy Secretariat has announced that all of its archived web casts are now available to all educators at **http://www.curriculum.org/secretariat/literacy_en.html**. Please check the archives section of this site for links to streaming video and additional resources from past presentations, including the most recent character development web page. Here you will find resources such as keynote presentations, workshop sessions and support material from *Finding Common Ground: Character Development in Ontario Schools, K-12*. Your director of education also has a CD you can copy that contains all past webcasts and related information.

Elementary teachers from K to 6 will have several Ministry math implementation supports and resource documents available to them this winter. A *Guide to Effective Instruction in Math 4-6: Number Sense and Numeracy* will be distributed to boards in December with others in Math 4-6: Geometry, Measurement, Data Management and Patterning and Algebra being distributed as they are ready throughout the winter. Similar *Guides to Effective Instruction in Math K- 3* will also be distributed to boards this winter.

Grade 7 to 10 teachers are encouraged to visit **http://www.edu.gov.on.ca/eng/studentsuccess/lms/library.html** for many math planning guides from Grade

7 to 10 such as the **TIPS4RM** *Targeted Implementation and Planning Support for Revised Mathematics* curriculum that will save teachers hours upon hours of planning. The Ministry is also releasing a Grade 9 and a Grade 11 prototype this fall of a new resource for students and teachers called *Critical Learning and Instructional Path Supports*. Several other "CLIPS" will be released over time.

Teachers of the new Grade 11 **MCF3M** and **MBF3C** math courses should visit the OAME web site at **www.oame.on.ca** to get full units of study and daily lesson plans for these courses, that were developed by a writing team sponsored by OMCA. This visit will save you hundreds of hours of preparation time.

Finally, I wish to invite you to the OAME Leadership Conference *Mathematics: Making Sense of It All* being held in Toronto February 15-17, 2007. A separate registration form was sent to you with the September 2006 *Gazette* or you can visit the OAME web site to download the program and registration form.

In my last message I brought to your attention my concerns about global warming and the environment. I am still researching web sites and resources that I will share with you in my next message in March 2007. I welcome any assistance you can provide on this issue.

Sincerely,

Dan Charbonneau

▲ Upcoming Events

For further information on OAME Chapter Conferences contact the Chapter Representative and for NCTM conferences, contact John Kersely, OAME/NCTM Rep.

See the last page in this *Gazette* for contact information. Also visit the OAME web site.

OAME Chapter Conferences and Events

February 15 - 17 2007	Leadership 2007 "Math: Making Sense of it All"	OAME
February 21 2007	Mini conference Agincourt CI	SAME
April 2007	Spring Conference Brock University	GOLDEN
June 1 - 2 2007	OMO 2007 University of Ottawa	COMA

OAME Annual Conference

May 3 - 5 2007	Georgian College Barrie, ON "Building Bonds"	MAC^2

NCTM Conference

March 21 - 24 2007	Atlanta, GA Annual Meeting and Exposition

▲ Technology Corner: Optimization with *Excel*

MARY BOURASSA



Mary Bourassa teaches mathematics at Lisgar Collegiate Institute in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She is also an OAME vice-president, past-president of COMA, gives workshops around the world and is a National Instructor for Texas Instruments.

Optimization problems can be a student's nemesis or can help them connect mathematics to real life. One way students can discover relationships is through investigations using technology. Here is one such situation:

You have been hired to help improve profits for a food production company. They want to know what dimensions they should use to make a cylindrical can that will require the least materials possible. They have determined that the can could have a volume of 400 ml, 540 ml or 600 ml. Determine the optimum radius and height for each possibility then use this information to make a conjecture about optimum dimensions of a cylinder of fixed volume. Note: 1 ml = 1 cm^3

If the surface area was fixed and we were looking for the cylinder of maximum volume it would make sense for students to build them and fill them to measure the volume. It is however difficult for students to build different sized cylinders with a fixed volume so we will use a spreadsheet to help us. I have found that students have a wide range of abilities with spreadsheets and some time must be spent going over cell references and the creation of formulas using Microsoft *Excel*.

Some basics:

- Each cell in a spreadsheet is referenced by its column letter and row number.
- To enter a formula in a cell, start with an equal sign followed by the formula. For each cell reference in the formula you can click on the cell in question.
- When you copy a formula the cell references are automatically updated.

For example (see Figure 1):

In the Total column, C2 has the formula A2 + A2 * B2. When that cell is copied down to C3, the formula is adjusted to A3 + A3 * B3.

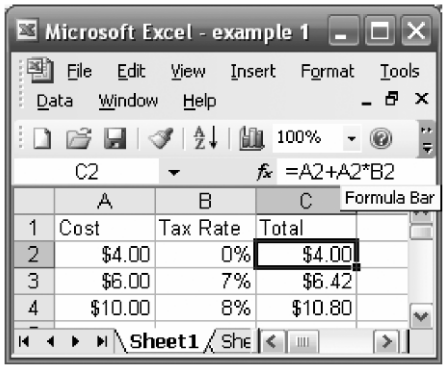


Figure 1

What if you reference an interest rate in a formula? You want that cell to stay fixed in the formula even when you copy the formula to other cells. This fixed reference is called an absolute cell reference and is obtained by placing dollar signs in front of both the letter and number. For example, if cell A1 is fixed, refer to it as A1.

You also need to know:

- The command for taking a square root is SQRT(number). For example, SQRT(4) will return 2.
- To get pi type pi().

The instructions to the students could be "Set up a spreadsheet as shown (Figure 2) then create formulas for the radius and surface area. Copy these down to A12 and C12. Set the number of decimal places for A3 to C12 to 1, by selecting the cells, right-clicking, choosing Format Cells, then under the Number tab, set the number of decimal places" .

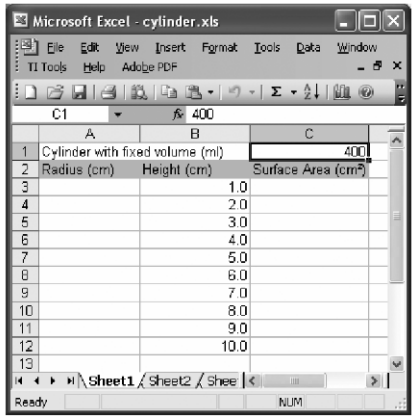


Figure 2

Students may need help rearranging the formula for volume so that the radius is isolated:

$$v = 2\pi r^2 h$$

$$\frac{v}{2\pi h} = r^2.$$

$$\sqrt{\frac{v}{2\pi h}} = r$$

$$\text{or} \quad r = \sqrt{v/(2\pi h)}; \quad r > 0$$

The formula we need in our spreadsheet will therefore be A3 = SQRT(C1/(2*pi()*B3)). Notice that the cell reference for the volume is an absolute reference. Students often forget to put brackets around the denominator.

Students then need to enter the formula to calculate the surface area of the cylinder. Once the formulas have been entered the spreadsheet should look like Figure 3.

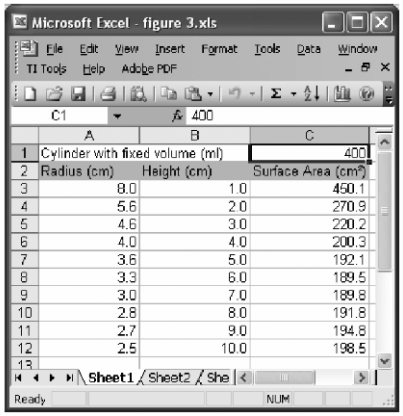


Figure 3

The minimum surface area appears to be where the height is 6 cm but we should narrow in on the optimal dimensions by changing the heights so that we start at 5, end at 7 with increments of 0.1 each time (see Figure 4). One way of doing this is to enter the first three values then select all three and the 18 cells below as Excel will fill those cells by following the established pattern.

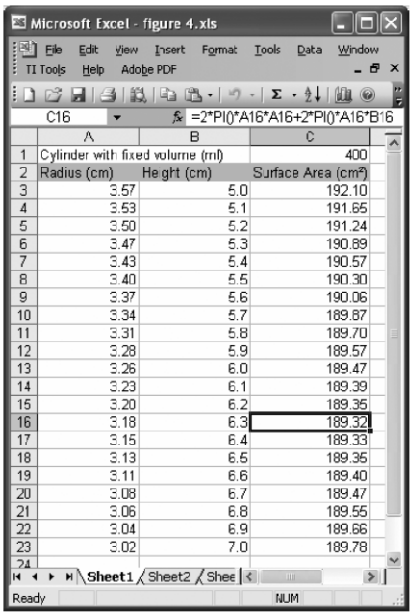


Figure 4

Our new minimum surface area occurs when the height is 6.3 cm and the radius is 3.18 cm.

Students now need to repeat the process for the other two volumes. As the formulas are already set up they only need to change the volume (cell C3) and adjust the heights back to 1.0 – 10.0 to get started. The resulting spreadsheets are shown in figures 5 and 6.

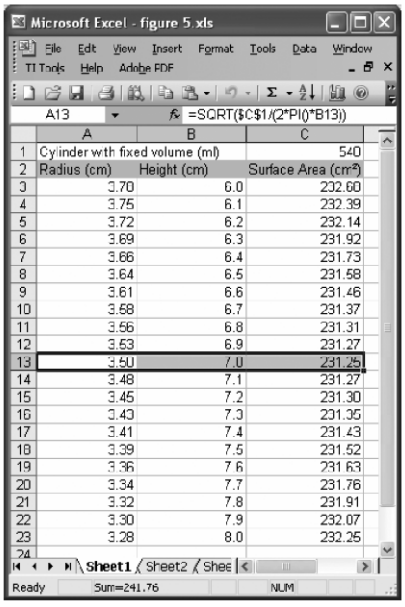


Figure 5

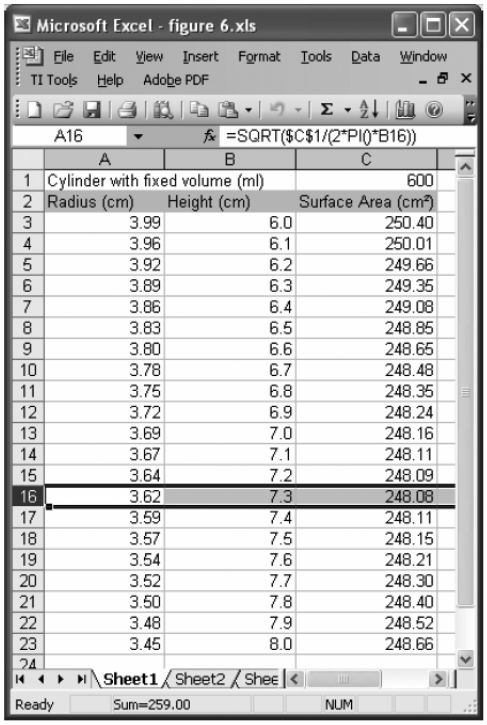


Figure 6

The results can then be summarized in a table and a conclusion drawn:

Volume (ml)	400	540	600
Surface Area (cm?)			
Radius (cm)			
Height (cm)			

Conclusion

For a given volume, the minimum surface area of a cylinder is obtained if _____.

They should find that the minimum surface area is obtained when the height is twice the radius.

If time allows, students can repeat this process for a rectangular prism. As you have three dimensions to work with, set one up as a constant. Start with a fixed volume of 1000 ml and a height of 10 cm.

Using this approach students are able to discover the relationship in question and they will remember it! It can also promote group work if access to a sufficient number of computers is an issue and is a great opportunity for students to communicate their findings through a report or journal entry. ▲

▲ Hey, It's Elementary! The *Pygmalion Effect*



LYNDA COLGAN

"...You see, really and truly, apart from the things anyone can pick up (the dressing and the proper way of speaking, and so on), the difference between a lady and a flower girl is not how she behaves, but how she's treated. I shall always be a flower girl to Professor Higgins, because he always treats me as a flower girl, and always will; but I know I can be a lady to you, because you always treat me as a lady, and always will."

George Bernard Shaw

In this quotation, Eliza Doolittle suggests that a person's place in society is largely a matter of how he or she is treated by others. Those who remember *My Fair Lady*, the Hollywood interpretation of Bernard Shaw's play, *Pygmalion*, will recall how Rex Harrison, the snobbish phonetics professor Henry Higgins agrees to a wager that he can take a flower girl, Eliza Doolittle (Audrey Hepburn), and make her presentable in high society.

This same quotation also appears as the closing statement in Robert Rosenthal and Lenore Jacobson's 1968 publication, *Pygmalion in th Classroom,* to emphasize the fact that students' intellectual development is largely a response to what teachers expect and how those expectations are communicated.

The *Pygmalion* study involved giving teachers false information about the learning potential of certain students in grades one through six in a San Francisco elementary school. Teachers were told that these students had been tested and found to be on the brink of a period of rapid intellectual growth when in reality, the students had been selected at random.

At the end of the experimental period, some of the targeted students – and particularly those in grades one and two – exhibited performance on IQ tests which was superior to the scores of other students of similar ability and superior to what would have been expected of the target students with no intervention.

These results led the researchers to claim that the inflated expectations teachers held for the target students (and, presumably, the teacher behaviors that accompanied those high expectations) actually CAUSED the students to experience accelerated intellectual growth.

Few research studies in the field of education have generated as much attention and controversy among educators, researchers, and the general public as Rosenthal and Jacobson's *Pygmalion* study – most notably in the popular press where many articles began appearing which used the *Pygmalion* findings as a springboard for the claim that perhaps "Johnny can't read" because his teachers don't have faith in his abilities and don't encourage him, particularly if he is poor or a member of a minority group. Only a few articles looked at the positive side, suggesting that teachers and parents might be able to improve children's school performance dramatically by communicating high expectations to them.

In the years since the original *Pygmalion* study was published, a great many additional studies have been undertaken. Several academics examined Rosenthal and Jacobson's study and found enough technical defects serious enough to cast doubt upon the accuracy of its findings. Some replication experiments seemed to confirm the *Pygmalion* findings, and others failed to do so. Other researchers conducted studies which sought to identify the ways that expectations are communicated to students. Meanwhile, the popular press, for the most part, continued to treat the *Pygmalion* findings as gospel and sometimes cast aspersions on America's teachers for the failure of some children to learn, claiming that teachers' low expectations were either creating or sustaining the problem.

Whether one is inclined to accept or doubt the findings of the *Pygmalion* study and other research supporting "self-fulfilling prophecy" effects, it is clear that educators and the general public were and are very interested in the power of teacher expectations on student outcomes.

I was reminded of the original *Pygmalion* study recently when I read the results of an experiment conducted by a Ph.D. student at the University of British Columbia and recently published (October, 2006) in the prestigious journal, *Science*. Ilan Dar-Nimrod, the lead researcher, concluded that women who are told that their gender affects their math skills do poorly on tests, while those told that their math skills depend on how they were raised perform better. "If they think they can't do anything about it – that they belong to a group that is mathematically dumb because of their genes – [women] react by choking, or underperforming."

Like the *Pygmalion* study, Mr. Dar-Nimrod gave 220 female university students tests similar to those used for graduate school entrance exams. Each woman was expected to perform a challenging math section, but not before reading an essay that dealt with gender difference in math. Of the four essays, one argued there was no difference, one argued the difference was genetic and a third argued the difference stemmed from the way girls were taught in elementary school. The fourth essay covered the subject of women in art since it has long been held by researchers that simply reminding a woman of her gender will negatively impact her test performance.

According to Mr. Dar-Nimrod, though the essays were made up, the results were startlingly real, and dramatically demonstrate how malleable people are in their beliefs about and assessments of self. For example, the women who were told that prior experience determined their math ability got twice as many answers right on the exam as those women who were told their genetics were to blame. The women who were simply reminded of their gender also performed worse than those told there was no difference in gender abilities in math.

The study highlights a key concept in psychology known as the "stereotype threat" – a phenomenon in which a person tends to underperform when reminded of a negative label associated with a group to which they belong.

The original *Pygmalion* and the latter-day *Pygmalion*-like study shout one statement: teacher expectations can and do affect students' academic achievement and attitudes. More importantly, research clearly demonstrates that communicating low expectations has more power to limit student achievement than communicating high expectations has to raise student performance.

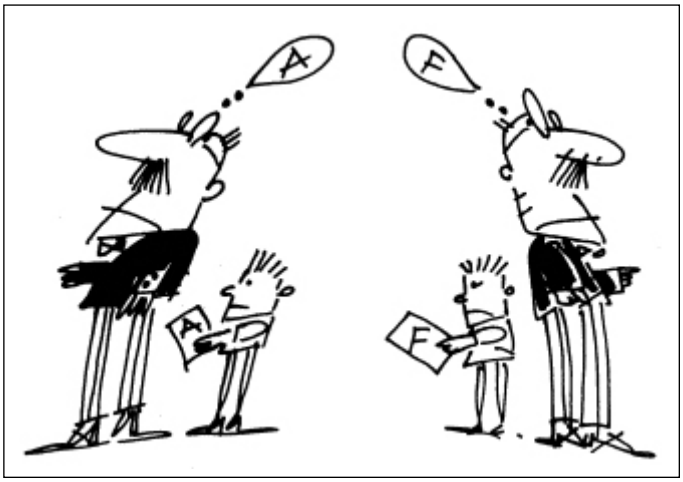
In fact, researchers have identified numerous factors (many unconsciously held) that can lead teachers to hold lower expectations for some students than others. These include:

- GENDER. Lower expectations are often held for older girls–particularly in scientific and technical areas–because of sex role stereotyping.

- SOCIOECONOMIC STATUS. Teachers sometimes hold lower expectations of students from lower SES backgrounds.
- RACE/ETHNICITY. Students from minority races or ethnic groups are sometimes viewed as less capable than Anglo students.
- TYPE OF SCHOOL. Students from either inner city schools or rural schools are sometimes presumed to be less capable than students from suburban schools.
- APPEARANCE. The expense or style of students' clothes and students' grooming habits can influence teachers' expectations.
- ORAL LANGUAGE PATTERNS. The presence of any nonstandard English speaking pattern can sometimes lead teachers to hold lower expectations.
- MESSINESS/DISORGANIZATION. Students whose work areas or assignments are messy are sometimes perceived as having lower ability.
- READINESS. Immaturity or lack of experience may be confused with learning ability, leading to inappropriately low expectations.
- HALO EFFECT. Some teachers generalize from one characteristic a student may have, thereby making unfounded assumptions about the student's overall ability or behavior.
- SEATING POSITION. If students seat themselves at the sides or back of the classroom, some teachers perceive this as a sign of lower learning motivation and/or ability and treat students accordingly.
- NEGATIVE COMMENTS ABOUT STUDENTS. Teachers' expectations are sometimes influenced by the negative comments of other staff members.
- OUTDATED THEORIES. Educational theories which stress the limitations of learners can lead to lowered expectations.
- TRACKING OR LONG-TERM ABILITY GROUPS. Placement in "low" tracks or groups can cause students to be viewed as having less learning potential than they actually have.

Sadly, researchers have found that some teachers interact with students for whom they hold low expectations in such a way as to limit their development and inhibit student growth, including:

- Giving low-expectation students fewer opportunities than high-expectation students to learn new material.
- Waiting less time for low-expectation students to answer than high expectation students.
- Giving low-expectation students answers or calling on someone else rather than trying to improve their responses by giving clues or repeating or rephrasing questions, as they do with high-expectation students.
- Giving low-expectation students inappropriate reinforcement, e.g., giving reinforcement which is not contingent on performance.
- Criticizing low-expectation students for failure more often and more severely than high-expectation students and praising them less frequently for success.
- Failing to give feedback to the public responses of low-expectation students.
- Paying less attention to low-expectation students than high-expectation students, including calling on low-expectation students less often during recitations.
- Seating low-expectation students farther from the teacher than high-expectation students.
- Interacting with low-expectation students more privately than publicly and structuring their activities much more closely.
- Conducting differential administration or grading of tests or assignments, in which high-expectation students–but not low-expectation students–are given the benefit of the doubt in borderline cases.
- Conducting less friendly and responsive interactions with low-expectation students than high-expectation students, including less smiling, positive head nodding, forward leaning, eye contact, etc.
- Giving briefer and less informative feedback to the questions of low-expectation students than those of high-expectation students



- Asking high-expectation students more stimulating, higher cognitive questions than low-expectation students.
- Making less frequent use of effective but time-consuming instructional methods with low-expectation students than with high-expectation students, especially when time is limited.

Given the power of teacher expectations to influence students' learning and their feelings about themselves, it seems important to invest in our educational system by focusing on improving the ways in which teachers form expectations about students and communicate them–especially to students they perceive as having limited potential.

If we believe that the *Pygmalion* effect works for children, then, by extension it should apply to teachers. It may not be as easy as Eliza Doolittle's task of learning to enunciate the rain in Spain falls mainly on the plain, but it may be worth reminding ourselves to:

- Avoid unreliable sources of information about students' learning potential, e.g., social stereotypes, the biases of other teachers, etc.
- Set goals (for individuals, groups, classrooms, and whole schools) in terms of floors (minimally acceptable standards), not ceilings and communicate to students that they have the ability to meet those standards.
- Use heterogeneous grouping and cooperative learning activities whenever possible; these approaches capitalize on students' strengths and take the focus off weaknesses.
- Develop task structures in which students work on different tasks, on tasks that can be pursued in different ways, and on tasks that have no particular right answer. This will minimize harmful comparisons.
- Emphasize that different students are good at different things and let students see that this is true by having them observe one another's products, performances, etc.
- Concentrate on extending warmth, friendliness, and encouragement to all students.
- Monitor student progress closely so as to keep expectations of individuals current.
- Give all students generous amounts of wait-time to formulate their answers during recitations; this will increase participation and improve the quality of responses.
- Stress continuous progress relative to previous levels of mastery, rather than comparisons with statistical norms or other individuals, when giving feedback.
- Focus on giving useful information, not just evaluation of success or failure, when giving feedback.
- Diagnose the learning difficulty and follow through by breaking down the task or re-teach it in a different way, rather than merely repeating the same instruction or giving up, when students do not understand an explanation or demonstration.
- Think in terms of stretching the students' minds by stimulating them and encouraging them to achieve as much as they can, not in terms of "protecting" them from failure or embarrassment.

As Ovid told the story in the tenth book of *Metamorphosis*, the sculptor Pygmalion, a prince of Cyprus, sought to create an ivory statue of the ideal woman. The statue, which he named Galatea, was so beautiful that Pygmalion fell desperately in love with his own creation. He prayed to the goddess Aphrodite to bring Galatea to life. She granted his prayer and the couple lived happily ever after.

The implication of the *Pygmalion* effect for teachers and educational administrators is daunting. It suggests that the performance of your students depends less on them than it does on you – the performance you get from students is no more or less than what you expect, which means you must always expect the best. As Goethe said, "Treat a man as he is and he will remain as he is. Treat a man as he can and should be and he will become as he can and should be".

Selected References

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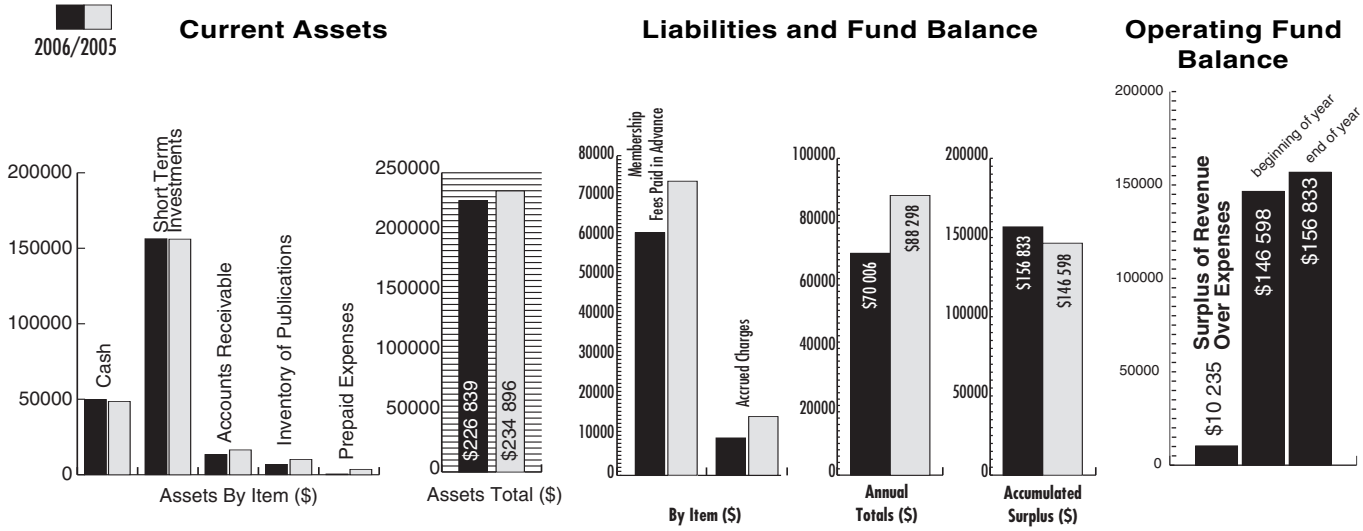
Rosenthal, R., and Jacobson, L. *Pygmalion in the Classroom: Teacher Expectation and Pupils' Intelectual Development*. New York: Holt, Rinehart and Winston, Inc., 1968. ▲

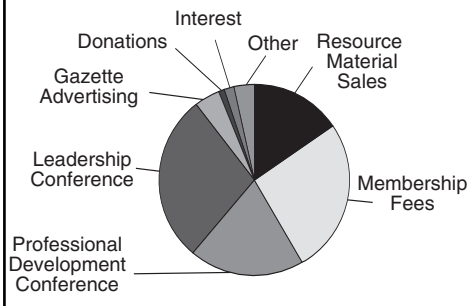
OAME Financial Statement 2005/06

Notice to Readers

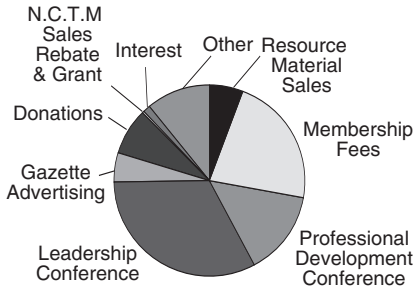
I have compiled the Balance Sheet of the Ontario Association for Mathematics Education (Association Pour L'Enseignement Des Mathematiques) as at August 31, 2006 and the Statement of Operations and Operating Fund Balance for the year then ended, from information provided by management. I have not audited, reviewed or otherwise attempted to verify the accuracy or completeness of such information. Readers are cautioned that these statements may not be appropriate for their purposes.

London, Canada September 15, 2006, PHILIP J. GRIFFIN Chartered Accountant

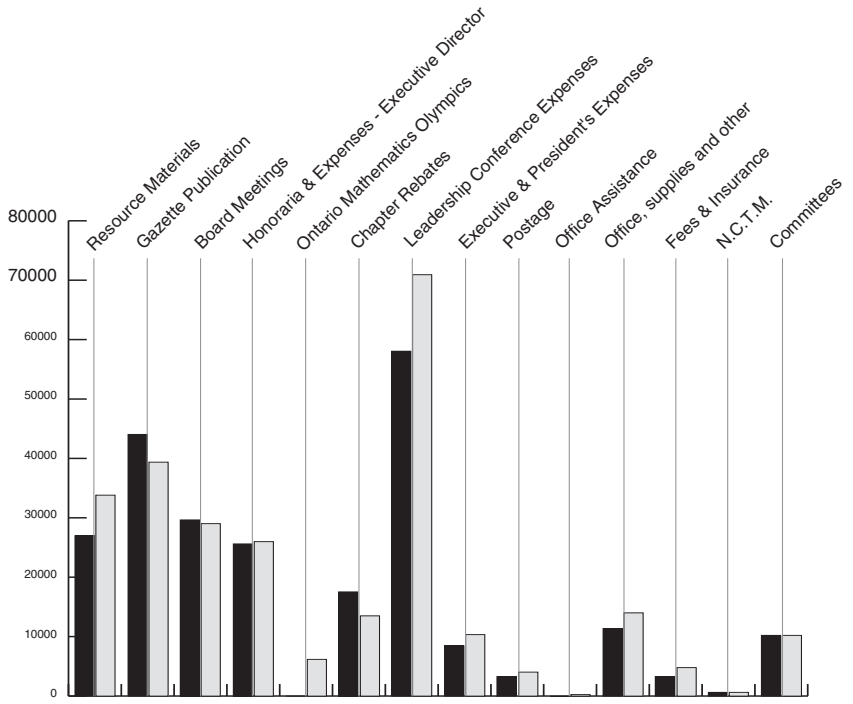




2006 ($249 377)



2005 ($253 080)



▲ Friday Special!

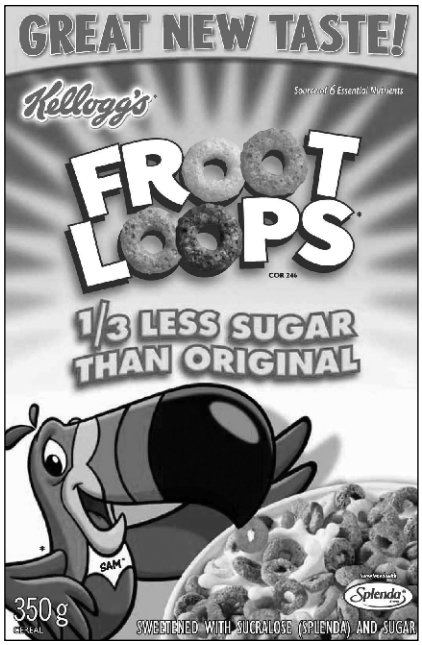
JACK WEINER



Fridays at the University of Guelph, five minutes before the end of class, I say, "Pens down, books closed, it's "Friday Special" time!" Then I show on screen an article or cartoon or letter or...something that has to do with math, the "real world" and has absolutely nothing to do with class, assignments or tests. It is math for fun. Often, it is some flagrant example of how the media abuse math. I hope you use some of these. Make them "Monday Morning" specials or "We need a break!" specials. If one falls flat, blame me. If it works, you take the credit.

Welcome to the December 2006 *Friday Special* and welcome to our guest columnist, Suzanne Carere: **scarere@georgebrown.ca**. Suzanne is a Registered Dietitian and instructor in the Faculty of Hospitality and Tourism Management at George Brown College in Toronto ON. Active in the media, Suzanne spent two years writing for the *Toronto Sun* before starting the column, "*Vice Squad*", last year at the *Toronto Star*. She has also completed two seasons of CBC's *Food Chain* and is called upon frequently as an expert for City TV's *City Pulse News*.

This column first appeared on July 21, 2006 as a *Toronto Star Vice Squad* column and is reprinted with permission of both *The Star* and Suzanne Carere.



Vice Squad: *Froot Loops* 1/3 less sugar: Less sugar, yes, but more calories.

Product: *Froot Loops* 1/3 Less Sugar

Price: $4.49-$5.29 for 350 g box

Total calories: 112 for a 27 g serving

Manufacturer: *Kellogg Canada Inc*.

The position: *Froot Loops* 1/3 Less Sugar gives consumers choice if they are looking to limit their sugar intake.

Top 4 ingredients: Flour mix (corn/white/whole oat flour, sugar, salt), sugar, hydrogenated coconut oil, colour

Nutritional breakdown for a 27g serving: *Froot Loops* 1/3 Less Sugar: 23 g carbohydrate (0.9 g fibre, 8 g sugar, 14g starch), 0.9 g fat (0.3 saturated), 2 g protein, 149 g sodium.

Regular *Froot Loops*: 24 g carbohydrate (0.9 g fibre, 12 g sugar, 11 g starch), 0.5 g fat (0.5 saturated), 1 g protein, 110 g sodium and 110 calories

Analysis: Take a moment to compare the nutrition information of both products and see if anything stands out as being a little ... strange? Now give yourself a point if you found each of the following:

- 1/3 Less Sugar is higher in fat
- 1/3 Less Sugar is higher in sodium
- 1/3 Less Sugar is higher in calories
- 1/3 Less Sugar is only lower in carbohydrates by 1 g

Shocked? I sure was. When scanning the numbers, the first three were probably easy — although no less surprising — to pick out. The last one may have been a little trickier. In the process of reducing the sugar content in this cereal, Kellogg's ended up using an artificial sweetener — sucralose — and more flour. This increased total starch by 3 g and led to a slightly higher sodium, iron, thiamin, niacin and protein content.

Research has shown, however, that some starches (like white bread) actually turn into sugar in the blood faster than table sugar itself. In the end, it's the concentration of fibre that plays a larger role than the number of grams of sugar. To your body, this means there's basically no difference in sugar content between these two cereals.

You don't have to go by my word on this, however. You can ask any dietitian you want — even Johanne Trudeau, director of nutrition for *Kellogg's Canada*. Despite quotes from ***Kelloggs*.ca** stating that they are

dedicated to the health of children and nutrition education, Trudeau was quick to clarify with me that *Kellogg's* 1/3 Less Sugar wasn't formulated with the health of children in mind. Say what? Apparently it was created to meet the consumer demand that moms want their children to consume less sugar.

Stunned, I asked: "Wouldn't that be for health reasons though?" Her answer: "You're going too deep." According to Trudeau, *Kellogg's* didn't take the time to find out why moms wanted their kids to eat less sugar, they simply made a product that would "make moms feel better".

When I pushed her on the fact that both cereals would provide a similar sugar load to the blood, she told me that although she and I might know that, "most kids and moms don't".

Alternative: I know I've been hard in the past on companies that are trying to make healthy products but in the end fall short of their goals. This, however, is an entirely different ball game. If I'm going to treat myself to a high-carbohydrate, low-fibre cereal like this one, I'd rather buy one that doesn't take advantage of consumers' lack of knowledge.

Take it or leave it: I think my work here is done.

Jack back talking to you. I have a suggestion. Bring a few boxes of a healthy cereal – my recommendation is *Post Alphabets*, which probably does not reflect well on my choices! – to class for a snack. Have students carefully measure out the recommended serving size (usually 30 or 35 g). I think most will be VERY surprised to see how small a serving that actually is. You could also compare the nutritional components with the Canada Food Guide recommendations, reinforcing ratios and percents.

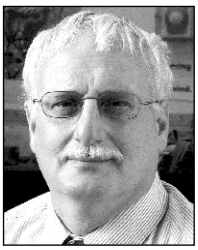
Suzanne adds, "I tell all of my students to look for a cereal that has at least 10% of its carbohydrate content coming from fibre; that is, if there are 25 g of carbohydrate per serving, the amount of fibre must be at least 2.5 g. Fibre slows down digestion and helps to keep you feeling full longer. A cereal like *Special K*, although low in total calories, turns into sugar almost immediately in the body and you're hungry 15 minutes later.

My favorite cereal is *Maple Mini Wheats*, which is about 12% fibre. (By the way, regular *Cheerios* in this game comes out to be higher in fibre than *Multi-Grain Cheerios*!)

Best wishes, happy holidays, and thank you Suzanne!

Jack ▲

▲ OAME/NCTM Report December 2006



JOHN KERSLEY

As we come to the end of the year, 2006, we naturally look toward 2007, a brand new year.

NCTM's Professional Development Focus for 2007 is **Representations**. The theme is, "Show Me the Math: Learning through Representation". The saying is a takeoff on the phrase, "Show me the money", popularized from the movie, *Jerry Maguire*.

The process of representation as stated by Skip Fennell, current NCTM President, includes using models to organize, record and communicate mathematical ideas, and to translate these models to solve problems.

- **Representation and Planning** includes choice of models or materials (representations) to illustrate the mathematical focus of the day's lesson
- **Representation and Teaching** deals with the instructional role of representation,
- **Representation and Assessment** is concerned with student's performance and success of the actual instruction.

As the year continues, look for further information and detail on the NCTM web site, **www.nctm.org**

New from NCTM

A new publication from NCTM is entitled, *Curriculum Focal Points from Prekindergarten through Grade 8 Mathematics :A Quest for Coherence.* Its purpose is to support NCTM's *Principles and Standards.* The authors suggest at each grade level that there be a series of focus points or significant mathematical targets which can be used as a basis for mathematical instruction. It is available via the NCTM catalogue, stock number# 13089, or by downloading free of charge at **www.nctm.org**

At the NCTM Annual Meeting in the spring, NCTM's president, Skip Fennell mentioned that he wants to further develop NCTM's position on research. His aim is to raise awareness of the importance of research to what

is being taught in classrooms and to future educators.

NCTM has published a new series on research entitled *Teachers Engaged in Research: Inquiry into Mathematics Classrooms.* There are 4 volumes to the series.

- *Prekindergarten – Grade 2* contains instructional practices and issues to broaden views of teaching and learning mathematics. Stock number #13042NB2
- *Grades 3 – 5* deals with how students come to know and understand mathematics, and the complexity of teaching the subject. Stock number #13043NB2
- *Grades 6 – 8* contains research activities that had a major impact on conceptions of teaching and direct impact on practice. Stock number #13044NB2
- *Grades 9 – 12* has examples of problems and tasks used by the authors to investigate students' understanding of content, to pedagogical issues, to social issues. Teachers can use these in their classrooms. Stock number #13046NB2.

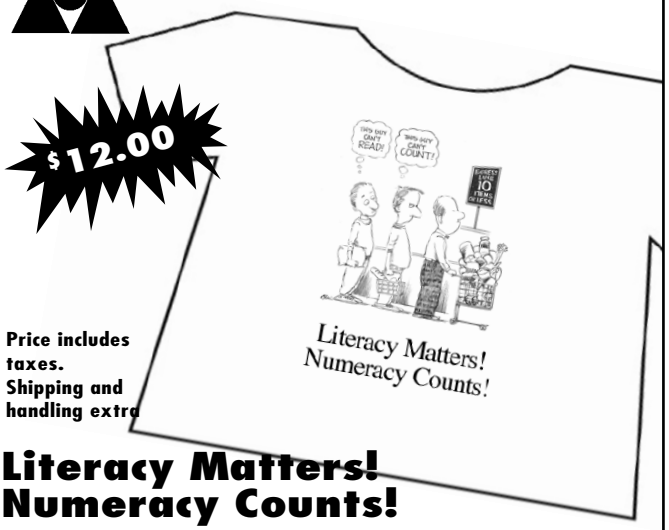
Also from NCTM:

- *Math Here & Now: Math of Pandemic Viruses;* a mathematical activity to examine and predict the spread of viruses and pandemics: **www.nctm.org/mathhereandnow**
- *Hitting the Bull's-Eye: A Dart Game Simulation Using Graphing Calculator;* an article on using darts to teach probability incorporating other mathematical ideas such as geometry, graphing circles, and the Pythagorean theorem: **www.nctm.org/middle**
- *Mathematics Teacher Calendar Problems Online;* a database of calendar problems for Mathematics Teachers subscribers: **www.nctm.org/high**
- *Tips for Teachers:* **www.nctm.org/teachmath/tips.htm**

Web sites of interest to mathematics as suggested by NCTM:

- Education Place's Math Center: links to online resources for K – grade 6. Houghton Mifflin affiliation **www.eduplace.com/math/mhm**
- Need help with algebra? Visit purplemath also includes graphing, matrices, good for review purposes: **www.purplemath.com**
- Smile provides 200 hands-on lesson plans from elementary to secondary: **www.iit.edu/~smile/mathinde.html**
- Math Club offers resources to help middle school and high school teachers establish and run a Math Club also provides a data base for a great variety of other resources: **www.unl.edu/amc/mathclub/index.html**
- NCTM Conferences: **www.nctm.org/meetings**
- NCTM Regional Conference and Exposition
  Richmond, Virginia October 11 – 13, 2007
  Kansas City, Missouri October 25 – 27, 2007
  Houston, Texas November 29 – December 1, 2007
- NCTM Annual Meeting and Exposition
  Atlanta "Mathematics: Representing the Future"
  March 21-24, 2007 ▲

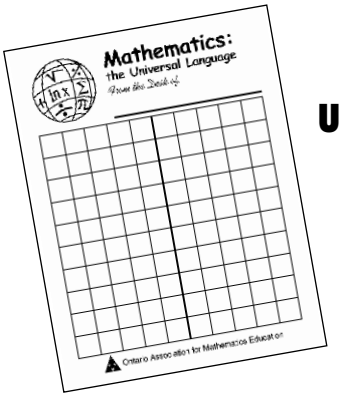
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OAME/AOEM Awards Nomination Process

Deadlines

Nominations for awards to be presented at the Annual Conference must be received by January 31 of the calendar year in which the conference takes place.

Nominations received after January 31 will be considered for awards to be presented during the next three calendar years. A nomination will be allowed to stand for three consecutive years. After that time, the nomination must be renewed. A standing nomination may be amended at any time by the original nominator. The nominator need only inform the chair of the Awards Committee in writing of the proposed changes to the nomination form.

The OAME/AOEM Nomination Form

Write a letter supporting the nomination of your candidate for the indicated award. Provide specific and detailed information about the nominee's activities as they relate to the following areas of consideration:

- demonstrations of excellence and creativity in mathematics education;
- specific contributions to mathematics education in the school, region, province or nation;
- demonstrates leadership in mathematics education in the school, region, province or nation;
- support of and contribution to the activities of OAME/AOEM, and
- contribution to the development of students both within and outside of the classroom.

Please include any additional information that you feel may be relevant to the nomination, but which may not be specifically mentioned above.

The Nomination Cover Page

Complete the Nomination Cover Page and attach it to your letter of recommendation for the nominee.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

OAME/AOEM Life Membership Award

The Life Membership Award is given to recognize a person who has contributed in a significant way to OAME/AOEM; demonstrated outstanding leadership in mathematics education; and has accumulated 10 or more years of membership in OAME/AOEM.

Kenneth D. Fryer Award

In Memory of the late Kenneth D. Fryer, OAME/AOEM, in conjunction with the Grand Valley Mathematics Association, established the Kenneth D. Fryer Award. The award is to be given annually to recognize a mathematics department which fosters collegiality, team work, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Award for Outstanding Contribution to OAME / AOEM and Mathematics in Ontario

This is to recognize an individual who has made significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics

The award is presented annually to recognize an exceptional and creative elementary teacher who demonstrates excellence in mathematics education by contributing to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics

The award is presented annually to recognize an exceptional and creative secondary teacher who demonstrates excellence in mathematics education by contributing to the overall development of students.

AWARD NOMINATION COVER PAGE

INTENDED AWARD *(please check one)*

- ❑ **Life Membership**
- ❑ **Kenneth D. Fryer**
- ❑ **Award for Outstanding Contribution to OAME / AOEM and Mathematics Education in Ontario**
- ❑ **Award for Exceptional and Creative Teaching in Elementary Mathematics**
- ❑ **Award for Exceptional and Creative Teaching in Secondary Mathematics**

Nominee's Information:

_____

Name of Nominee: (Principal and Department Head for K.D. Fryer Award)

_____

_____

School, Organization, or Business Address

Tel: ( ) _____ Fax: ( ) _____

Home Address: _____

_____

Home Phone: ( ) _____

Email: _____

Nominator's Information:

_____

Name of Nominator

_____

_____

School, Organization, or Business Address

Tel: ( ) _____ Fax: ( ) _____

Home Address: _____

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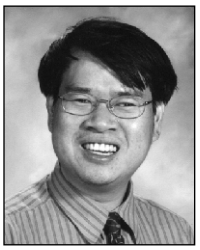
Home Phone: ( ) _____

Email: _____

SEND TO: Jacqueline Hill

▲ Get Interactive!

Louis Lim



Louis Lim is department head of mathematics at Richmond Hill High School, York Region DSB. He recently completed a 3-year term as editorial panel member of the NCTM's Mathematics Teacher and is former editor of the OAME's Ontario Mathematics Gazette. In spring 2005, he was recipient of OAME's "Exceptional and Creative Teaching in Secondary Mathematics" Award.

This edition of "Get Interactive!" focuses on web sites to support teachers with the effective use of technology in elementary and secondary school mathematics. Professional development workshops occurred during the summer on *Tinkerplots, Geometer's Sketchpad 4*, *TI-15* calculator for elementary teachers, and *Computer Algebra Systems* (*TI-89 Titanium* and *Nspire*) for secondary teachers. Further, Texas Instruments hosts sessions on the *TI-84+* graphing calculator, *TI Navigator*, *Fathom!*, *TI-Interactive*, *TI-10* and *TI-15* calculators, along with several sessions on *CAS* and an all-day Ontario Summer Institute. Technology is important in the Ontario mathematics curriculum since specific references are made to graphing technology, dynamic geometry software, and computer algebra systems: **http://education.ti.com/educationportal/activityexchange/activity_list.do?cid=us**

Research

Several teachers in my Additional Qualification Senior Mathematics course this past summer expressed concerns with incorporating technology because of students' weak numeracy skills. One goal was to help them realize effective use of technology can positively influence students' mathematical, procedural and conceptual understanding. I believe we must be able to articulate our practices to students, parents, teaching colleagues and administration. Thus, familiarity with research pertaining to mathematical technology is important.

1. The web site, **http://education.ti.com/educationportal/sites/US/sectionHome/effectivenessoftechnology.html**, maintained by Texas Instruments, is a compilation of research on graphing calculators, the TI-Navigator system, and several action research studies conducted by classroom teachers. Also included are 50 success stories shared by elementary, secondary, and post-secondary educators who have incorporated technology in mathematics.
2. Bernhard Kutzler, from the University of Linz, Austria, shares some of his technology in mathematics education papers at **http://b.kutzler.com/art_eng.htm**. Included is an article on which paper-pencil skills are important to retain with the use of Computer Algebra Systems (CAS). Discussion with colleagues is important as we continue to examine the role of CAS in mathematics. Another article worth highlighting is research on CAS in high school, "The Algebraic Calculator as a Pedagogical Tool for Teaching Mathematics".

Activities

3. The Ontario Ministry of Education funded the Programming Remediation and Intervention for Students in Mathematics (PRISM), which includes Geometer's Sketchpad activities organized for grades 7 to 9 in measurement, geometry, number sense, algebra, and data management. Some of the activities contain QuickTime videos to help with implementation. Included are teacher and parent notes. See **http://www.nearnorthschools.ca/it/PRISMNEO/index.asp**
4. Geometer's Sketchpad in grades 3 to 5? Absolutely! Check out **http://www.dynamicgeometry.com/general_resources/classroom_activities/young_learners/pages/activities_3to5.php**. Premade sketches are included with the activities, along with lesson plans and student worksheets. Activities are correlated with the NCTM's content standards (number sense, algebra, geometry, measurement), and process standards (problem solving, reasoning and proof, and communication).
5. If you're looking for lesson plans and activities using graphing calculators, computer algebra systems, Geometer's Sketchpad, Fathom!, or spreadsheets, be sure to visit **http://mathforum.org/mathtools/** and browse "All Technology Types" (upper right corner of page). Most files contain premade sketches or applets that can save time from having to recreate them on your own. There are also PowerPoint files, with the most impressive (in my opinion) being a

ready-to-use template of the popular *Jeopardy* game on television that teachers simply need to input questions and answers. There are interesting materials for both elementary and secondary mathematics teachers on this web site.

Your Turn!

E-mail me, **louis.lim1@gmail.com**, your favourite web sites. Please also include a brief paragraph on how you have used these web sites in teaching mathematics. I hope to compile them for inclusion in a future edition of Get Interactive! ▲

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▲ Photo Math ~ The Fine Art of Viewing the World Through a Mathematical Lens

RON LANCASTER
INDEPENDENT MATHEMATICS CONSULTANT



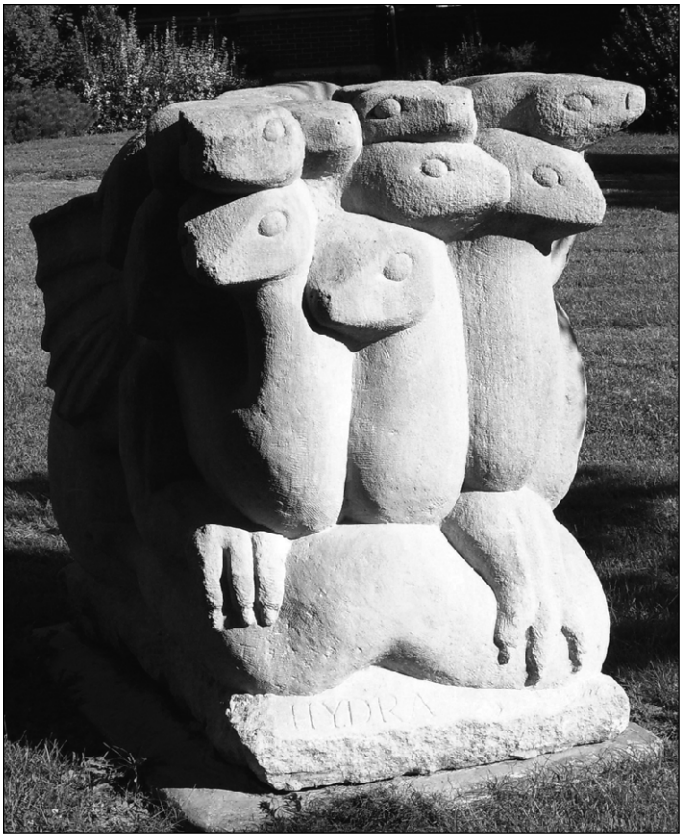
Ron Lancaster is a Lecturer in Mathematics Education at the Ontario Institute for Studies in Education at the University of Toronto. He is also an independent mathematics consultant and works with teachers, students and educational organizations in North America, Asia and Israel.

Teachers and students are encouraged to put on a pair of mathematical glasses and become a math spotter, someone who sees mathematics everywhere and then possibly poses mathematical questions based on the images taken. Ron Lancaster, the current Editor for Photo Math, welcomes photos with questions for possible publication in future columns. Material can be sent via e-mail or by regular mail to Ron at the addresses above.

Introductory Comments

All of the material for this column was created by Louis Lim, Head of Mathematics at Richmond Hill High School. The photographs and the questions are wonderful examples of what can be seen when the world is viewed through a mathematical lens. Many thanks to Louis Lim for submitting this material. Louis can be reached at **louis.lim1@gmail.com**

The following photographs were taken by Louis Lim during a morning walk through the Canadian National Exhibition and the Harbourfront in Toronto. It is amazing what one can mathematically find when least expected!



The Hydra

Just north of the Bandshell in the Canadian National Exhibition are "The Sculptures of the Greek Gods", created in January 1979 by artist E. B. Cox. *The Hydra* is one such sculpture.

(a) The following problem is posed on a plaque next to the statue:

"A monstrous dragon with nine heads originally – however if one head was cut off, two heads grew in its place. This one has 14 heads – so figure how many heads were cut off."

(b) Suppose the dragon has n heads. Determine an expression which would determine how many heads were cut off.



Wind Turbine

There are several Wind Turbines in the Canadian National Exhibition, which capture wind to create electricity. The web site, **http://www.ftexploring.com/energy/wind-enrgy.html**, provides a formula for the wind power:

$P = d \times D^2 \times V^3 \times C$

where d is the air density; D is the diameter of the turbine blade; V is the wind velocity; and C is a constant.

(a) Which variable in the equation of wind power has the greatest impact? Explain.

(b) If the wind velocity triples, by what factor does the wind power increase?

(c) If the diameter of the turbine blade triples, by what factor does the wind power increase?

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Dufferin Gate Arch

The Dufferin Gates is one of the major entrances to the Canadian National Exhibition. The Arch can be modelled by approximating it to a polynomial function.

(a) What type of polynomial function can we use to approximate the model of an equation of the arch?

(b) Determine an equation which approximates the equation of the Dufferin Gates Arch in the photograph.

(c) Explain why we said "approximate" in (a).

(d) A more accurate model makes use of the hyperbolic function, defined by

$\cosh x = \frac{1}{2}e^{x} + \frac{1}{2}e^{-x}$, with $e \doteq 2.718$.

i. Use a graphing calculator to graph

$y_1 = \frac{1}{2}e^{x}$ and $y_2 = \frac{1}{2}e^{-x}$.

ii. Predict what the graph of $y_1 + y_2$ looks like.

iii. Use your graphing calculator to graph $y_1 + y_2$. This combined function is called the hyperbolic function.

iv. Find an equation of the photograph of the Dufferin Gates Arch using the hyperbolic function, expressed in the form $y = \frac{1}{2}e^{ax} + \frac{1}{2}e^{-bx}$, by determining values of a and b.



Pi Harbourfront

The March 2006 edition of "Photo Math" column contained two photographs of π, to celebrate π day on March 14. While walking along the Harbourfront, south of the Rogers Centre, I came across a π statue. Unfortunately, I could not find a plaque which describes this masterpiece – so who knows if π was intended! Here are several applets available on the Internet that may interest your students:

(a) As Ron Lancaster said in his March 2006 column, there is a wealth of resources that pertain to π on the Internet. One is an applet that simulates Buffon's Needle Problem, which involves dropping a needle on a piece of paper and determining the probability it will cross one of the lines on the page. Go to the web site **http://www.mste.uiuc.edu/reese/buffon/bufjava.html** and try the simulation for 1000 drops. What happens to the value of π as you increase the number of drops to 10 000? 50 000?

(b) The web site **http://www.pbs.org/wgbh/nova/archimedes/pi.html** provides a simulation to demonstrate Archimedes method of approximating the value of π: "It finds an approximation of pi by determining the length of the perimeter of a polygon inscribed within a circle (which is less than the circumference of the circle) and the perimeter of a polygon circumscribed outside a circle (which is greater than the circumference). The value of pi lies between those two lengths."

(c) How many digits of π can you memorize? The web site, **http://oldweb.cecm.sfu.ca/pi/yap2.5M.html**, recites over 42 000 digits of π, taking 342 minutes. Questions can be created such as: "Verify that it would take you 133 years (without sleep or food) to recite the world record of 6.4 billion digits of π".

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▲ Competing Assessment Frameworks: Sometimes "Level 1" just doesn't feel right

Chris Suurtamm
Ann Arden*, Jennifer Roy*



Chris Suurtamm is an Assistant Professor of Mathematics Education at the University of Ottawa. She is an experienced mathematics teacher who is committed to helping teachers with the implementation of the new curriculum and assessment practices.

As is often the case in this assessment column, the theme of this article comes from reflections after conversations with teachers. Chris and a colleague were conducting research with beginning teachers, in collaboration with local school boards, to determine how beginning teachers could best be supported. Beginning teachers in our area had completed our questionnaire and we were following this up by meeting with focus groups of about 6 – 8 new teachers after school. It was a great opportunity for beginning teachers to connect with one another and to share some of their challenges and successes. They were very eager. In fact, we found that all we needed to do was ask the question "So, how is it going?" and the conversation began and didn't stop.

When teachers talked about some of the challenges they faced, assessment seemed to always come up. On one particular day, they were talking about grappling with assessment in terms of what they thought was expected of them and what they thought students needed. Specifically, one teacher was talking about a student working on a task similar to one of the exemplars published by the Ontario Ministry of Education. This student regularly struggled, did not often meet with success, and therefore was frequently unmotivated to engage in the work. However, in this case, the student put a great deal of effort into the task and was able to demonstrate some new understanding. Yet, when the teacher compared the student's work to the exemplars, the work was at a Level 1. This new teacher was terrified that giving the student Level 1 would devastate the student and felt that Level 1 did not do justice to the effort that the student had put forth nor did it recognize the progress that the student had made. So, the dilemma for the teacher was whether she should follow the exemplars or follow her heart and find a way to recognize the progress.

This example is a demonstration of the tensions of working within different assessment frameworks and this prompts a brief discussion of three assessment frameworks and how they emerge in daily classroom events (Brookhart, 2001; Natriello, 1987). The current curriculum and assessment model is a model based on a criterion-referenced system. That is, the criteria for obtaining a Level 1, 2, 3, or 4 are clearly delineated and students and teachers know what is expected in student work. Classroom assessment rubrics and other assessment tools all match the criterion laid out in curriculum and assessment standards documents. In Ontario, we refer to Level 3 as the provincial standard and one goal of the system is for all students to reach this provincial standard. A second assessment system is norm-referenced. This may be the one that we experienced in a university course when we were told that our marks were "bell-curved" and our mark was considered with respect to everyone else's. Hence, the system was designed with an understanding that a few will have great success, some will struggle or fail, and most will be in between those two. Basically, this system is designed to describe a student's performance in comparison with the rest of the students. Although our current educational system is criterion-referenced, we see evidence of norm-referencing in such instances as when a student or parent wants to know how the student is doing in comparison with the rest of the class, or when universities request a ranking of students. A third assessment system is self-referenced. This refers to examining student progress, or seeing where a student is now, compared to where he or she was before. As teachers assess students' work they easily recognize when a student has made progress, such as when he or she can apply a skill that they were not able to before, or has increased confidence to draw diagrams or models when solving a problem. These types of assessments are not necessarily measured against a rubric but against the teacher's and student's awareness of previous work.

It is fairly straightforward to see how the self-referencing system and the criterion-referenced system were competing in the anecdote that the beginning teacher related. On the one hand, using self-referencing, the teacher recognized the immense progress that the student had made but using criterion-referencing, she felt she was

** Ann Arden and Jennifer Roy are Master of Arts students at the University of Ottawa and have contributed their ideas to this column.*

unable to acknowledge the full extent of the progress.

Teachers are not the only ones caught up in these assessment quandaries. Students can become confused by the mixed messages of our assessments. One recent graduate of our teacher education program related a story from her practicum. She had been spending many lunch hours working with a struggling student on her understanding of a particular math concept. Previous to this, the student had been meeting with little success and often gave up on assessment activities rather than show what she didn't know. However, on the next test that the class had, the student attempted the problems and many of her solutions showed that she had made remarkable progress. The teacher candidate was elated and saw the student in the hall after she had marked the test and congratulated her on what a great job she had done. The student beamed. However, when the tests were handed back in class, the student received her test, looked at the mark and just put her head down on the desk where it remained for most of the morning. The teacher candidate was perplexed but on later reflection realized that the praise that she gave the student based on her progress may have created a situation where the student assumed a very high mark on the test and was dismayed to find her mark was 70, which though substantially higher than previous marks, was not the A+ that she thought she might have deserved for her time and efforts and the teacher candidate's enthusiasm and praise. The teacher candidate was operating on a self-referencing system of seeing the enormous progress the student had made but using a criterion-referencing system, the student did not see the mark matching the effort. So, what do teachers do?

First, it is important to be aware of these competing assessment perspectives and to be sensitive to the expectations of administrators, teachers, parents, and above all, students. Our current curriculum and assessment model is criterion-referenced, with defined standards of achievement that allow students and parents to see what is expected in student work. However, these standards need to be shared with parents and students. Students, in particular, need opportunities to clearly see what is expected of them and how they are being assessed.

One way to do this is to share exemplar with students or to have students work in groups to discuss (anonymous) students' work. This allows students to see different approaches or to compare the work to an established rubric to help them understand the expectations and think about the quality of their own work. Studies have shown that having students participate in self-assessment can result in students focusing more on feedback from their teacher than their grade and being more honest and reflective about their own work (e.g. Klenowski, 1995).

Teachers also should be aware that self-referencing, or providing formative feedback to students about their progress, is one of the most important aspects of assessment. Although a mark cannot be based solely on effort, it is crucial to recognize a student's hard work and progress with anecdotal feedback and to highlight specific aspects of what the student is doing well. This is especially important if this effort and progress is not yet reflected in a new level of achievement or mark. For instance, in the case above where the student was disappointed with a mark of 70% because the mark did not appear to match the teacher's praise, the teacher might want to refrain from comments like "excellent work" which might set the student up for disappointment if the mark does not appear "excellent" to the student. Instead, the teacher might want to emphasize where the student's efforts met with success such as: "You really explained your thinking well in the last problem" or "All the practice you did on factoring really showed up in question 2". When teachers praise students' successes, rather than merely pointing out errors, students generally want to repeat those actions that lead to success.

When a certain level doesn't "feel" right, an honest, compassionate conference with a student can often be the only way to make sure that his or her fragile self-confidence is not shattered. Students should be reassured that they will have other opportunities to show how much they have improved, as we honour the use of students' "most recent, most consistent" work in determining marks. Helping students become familiar with the criteria and the standards by which work is assessed is expected. Providing students with positive feedback to encourage their progress and being sensitive to their reactions to assessment are crucial to support student learning.

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KEYNOTE SPEAKERS

Avis Glaze
Trevor Brown
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▲ Director's Dialogue

ANN MICHELE STENNING



Ann Michele Stenning is a mathematics teacher at South Secondary School with the Thames Valley District School Board. She is currently completing her Masters of Education in Curriculum Studies. Ann Michele was part of the planning committe for the 2006 OAME conference "Everyone Counts".

This edition of Director's Dialogue highlights the events of the OAME Board of Director's Meeting at the Holiday Inn Select, Toronto Airport on Friday October 20 and Saturday October 21 2006.

Chapter Reps Meeting, Friday, October 20, 2006
2:00 – 6:30 pm

Greg Clarke welcomed new chapter reps from NWOAME, SAME, GVMA and Y^{4}MA. Before beginning a professional development session on Geometer's Sketchpad, chapter reps took turns sharing a new or exciting development within their professional lives. A full report of this Round Table can be found on the OAME web site.

Board Meeting, Friday, October 20, 2006
7:00 – 9:30 pm

Dan Charbonneau, the OAME president, welcomed guest Cathy Lynch, the co-chair of OAME 2006, who presented her report on the event. Cathy presented OAME with their proceeds of the conference. The remaining portion of the board meeting was spent learning about First Nations as presented by Equity Speaker, Dr. Pamela Rose Toulouse: **Honouring and Respecting First Nation Learners: Imbuing Equity in Education.** Pam highlighted the importance of honouring First Nation Learners through an inclusive curricular approach. We examined the unique learning styles of First Nation students that impact and enrich our educational environments.

The remaining part of the evening was spent in curriculum work groups.

JK – 6: Laurie Moher reported that the committee met all day Oct 20 and are developing a math and literature resource.

7 – 9: Connie Quadrini reported that the committee met all day Oct 10 and brainstormed They are developing a curriculum continuum along with lesson plans.

10 – 12: Mary Bourassa reported that members in the grade 10-12 work group discussed a document titled "Math is the Path". The group decided to poll their individual boards to find similar documents that help with course selection. Further discussion followed involving new course changes in Grades 11 and 12.

Board Meeting, Saturday, October 21, 2006
9:00 am – 3:00 pm

Committee meetings were held 9 – 12 noon. Details can be found on the OAME web site.

Ministry of Education/EQAO Reports: Guests Myrna Ingalls, Sandy Dilena and Anthony Azzopardi spoke to Board members.

Highlights:

- Myrna Ingalls suggested teachers visit **http://www.edu.gov.on.ca/eng/studentsuccess/lms/** to view Ministry resources. A new project called Critical Learning Instructional Path Supports (CLIPS) identifies what is required for school success. For example, board members viewed a classroom video designed for CLIPS on sinusoidal waves.
- Anthony Azzopardi presented Senior Mathematics Curriculum Revision details. He announced that implementation of the new grade 11 and 12 courses is due to begin in April 2007. Board members were also shown a draft of Clear Pathways, a chart which shows how each math course relates to one another.
- Sandy Dilena presented observations from the grade 3 and 6 mathematics assessments. In both areas students are struggling to understand measurement and the grade 6 assessment shows a lack of knowledge of order of operations. The school's Item Information Report (IIR), collected from the grade 3 and grade 6 mathematics test can be found on each school's secure web site. Administration dates for the grade 9 mathematics test are posted on the web site. The test will reflect the revised curriculum from 2005. Assessment questions are posted at **www.eqao.com**
- Kathy Kubota-Zarivnij, OAME's past-president, showed board members how her work with the Literacy and Numeracy Secretariat is facilitating professional development. For instance, at **www.curriculum.org/secretariat/calendar.html** there are web casts which can be used at department meetings. There are success stories available at **www.inspirelearning.ca/english/index/htm** as well.

Chapter Reports

CHAMP - Rep: Lynda Ferneyhough:

We are looking forward to this year's round of contests: secondary are being run in Dec and elementary in Feb. CHAMP held a Fall Event on Oct 19 with stimulating sessons and good professional development. We are still looking for a webmaster.

COMA - Rep: Ron Gaudreau:

COMA sponsored a mathematics social for all mathematics teachers in the region on Oct 12. This is a first time initiative by COMA with the purposes of socializing and joining OAME. Our annual Mini-Conference was held on Saturday, November 18, at the University of Ottawa. COMA will be sponsoring the Ontario Math Olympics on June 1-2, 2007 Registration details will be available through Chapter reps in February.

GOLDEN - Rep: Laura Tonin:

We are just starting to plan our spring conference at Brock University for April 2007.

GVMA - Rep: Rob Gleeson:

Congratulations to the math department at Lisgar CI, Ottawa, for winning the Kenneth D. Fryer Award at this year's OAME conference. GVMA plans to continue sponsoring the award along with increasing the award's value. OAME will match GVMA's support. Our fall conference: Mathematics To Inspire You, was on October 19. Teachers are reminded to visit **www.gvma.ca** and to consider ordering some of our publications.

ISOMA - Rep: Diane Lang:

ISOMA held a Fall Conference: Bringing Us Together, on Oct. 25, 2006 at Appleby College.The workshops ranged from use of smartboard technology to hands on projects and classroom ready modules. We are in a re-building mode this year and our main goals are to help people make connections between schools, between elementary and secondary panels, and between the broader mathematics community. ISOMA is also planning to run a regional Junior Math Olympics.

MAC² - Rep: Andrew Morrison:

MAC^2 held a successful Fall Conference that identified and discussed important math concepts that children learn K – 12. Plans are well along for OAME 2007.

NOMA - Rep: Sue Melville:

The NOMA committee organized a fall conference: "Taking the Spook Out of Mathematics" that took place on Friday, October 13th in Sudbury. The conference featured Jack Weiner, Associate Professor at the University of Guelph as keynote speaker and offered workshops for teachers at all levels.

NWOAME - Rep: John Colledanchise:

Maria Casasola and Jack Ludwig created an event called Lighting Mathematical Fires for November 9, 2006, in Thunder Bay. As a NWOAME sponsored event the sessions were designed mostly for primary and junior teachers. The event included guest speaker Emilia Veltri and everyone was be asked to identify what they are looking for from professional organizations like OAME. NWOAME will host a Dinnerlude event for senior elementary and secondary math teachers at a later date.

PRMA - Rep: Sandra Jean Price:

Pine Ridge hosted a successful mini-conference on the GUM project on October 19th.

QSLMA - Rep: Robert Sherk:

QSLMA held their annual fall conference on Nov 9th. There were 6 workshops to cover primary to post secondary interests.

SAME - Rep: Debbie Drape:

SAME is busily planning for their mini-conference on February 21st at Agincourt CI. The Chapter is planning well in advance for their local OMO.

SWOAME - Rep: David Petro:

We have planned a mini conference for the beginning of winter dealing with the Revised 2005 Math document and the revised grade 11 and grade 12 courses. We are also making plans to participate in the OMO with 2 to 3 SWOAME teams.

TEAMS - Rep: Pat Margerm:

TEAMS held a fall mini-conference on October 25 at Eastern Commerce CI. A new format for the evening was tested in which the keynote speaker was replaced with an opportunity to attend a second workshop. The memorial for the late TEAMS president Anne Cirillo will focus on students, in keeping with Anne's work in mathematics education. For more TEAMS details visit **http://oame.on.ca/chapter/TEAMS/**.

WOMA - Rep: Ann Michele Stenning:

WOMA hosted their fall conference on Tuesday, November 28 at Saunders S. S. Participants received a 75 minute session from one of six session choices, access to publishers' displays and dinner. Kathy Kubota-Zarivnij, OAME's past-president and a member of the Literacy and Numeracy Secretariat, spoke during dinner on the topic of Learning Mathematics for Teaching.

Y⁴MA - Rep: Pietrina Orlandi:

Y^{4}MA has been busy at many levels. Preparations have begun for OAME 2008 as committees are beginning to be established. On October 12, 2006, Y^{4}MA hosted their annual Fall Conference at St. Elizabeth CHS entitled "A Really Good Math Conference". ▲

▲ What's the Problem: Endlessly Adding Nines

SHAWN GODIN



Shawn is the head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans.

Welcome back problem solvers. When last we met I left you pondering over the following problem that appeared as problem M13 from the October 2001 issue of *Crux Mathematicorum with Mathematical Mayhem*. The solution appeared in the November 2002 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem*. web site **http://journals.cms.math.ca/CRUX/**

Given $n = 9 + 99 + 999 + \ldots + \underbrace{99\ldots999}_{2001\ 9\text{'s}}$

How many of n's digits are 1's?

Ah, one of the classics. A problem that we could do if we had the time and a big supply of the 3 P's: pencils, paper and patience. Unless you spent the last three months adding, you must be thinking there has to be a better way. So where do we start?

One of the classic problem solving strategies is to solve an easier problem. Sometimes the solution can give us insight into our problem at hand. In this case, instead of adding up to a number made up of 2001 nines, we start with something a bit smaller. Let's see what this gives us.

NUMBER OF TERMS	SUM
1	9
2	9+99=108
3	9+99+999=1107
4	9+99+999+9999=11106
5	9+99+999+9999+99999=111105

I see your hands, I hear you call out, "Oh, oh sir, I've got it!" A pattern is starting to emerge. Looking at the digits a few things seem to be happening, let's summarize.

NUMBER OF TERMS	SUM	NUMBER OF DIGITS IN SUM	NUMBER OF 1'S IN SUM
1	9	1	0
2	108	3	1
3	1107	4	2
4	11106	5	3
5	111105	6	4

So it seems, other than the first case, when we add t terms, the sum has t + 1 digits and there are t – 1 ones in the sum. So it would seem that for our problem, the final answer is 2002 digits long, and contains 2000 ones. Now that wasn't so hard was it?

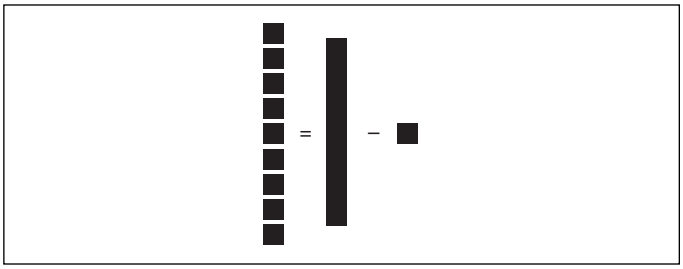
The only problem with our theory is that if we had continued our exploration a little further we would have found a problem.

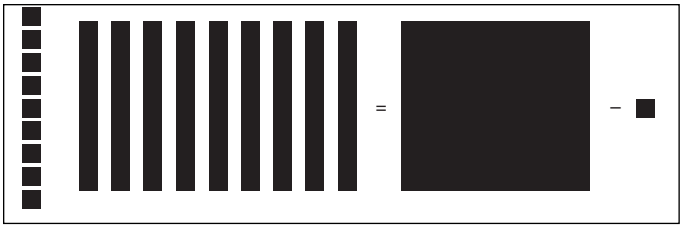
NUMBER OF TERMS	SUM	NUMBER OF DIGITS IN SUM	NUMBER OF 1'S IN SUM
6	1111104	7	5
7	11111103	8	6
8	111111102	9	7
9	1111111101	10	9
10	11111111100	11	9

This seems fine except when we are adding 9 terms. If we reflect on the results, we can see that there is a flaw with our pattern since the leading digits seem to be ones, but the last couple of digits seem to be "counting down". So, at some points those last digits might be ones.

At this point we may want to look at some different strategies. If you have access to technology, you may want to invoke tools such as spreadsheets or Computer Algebra Systems. Using CAS you can completely solve the problem, but this may leave you a little flat since you will gain no understanding as to why.

If we look at manipulatives, base 10 blocks might shed some light. The first two terms seem to imply a pattern. Notice that each term is 1 less than a "new block".





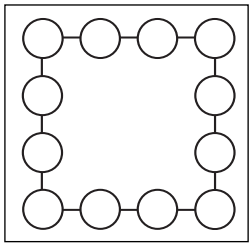
Aha! A bit of insight. Let's rewrite the sum

$$
\begin{aligned}
&9 + 99 + 999 + \ldots + \overbrace{999\ldots99}^{2001\ 9\text{'s}} \\
&= (10-1) + (100-1) + (1000-1) + \ldots + (10^{2001}-1) \\
&= \overbrace{10+100+1000+\ldots+10^{2001}}^{2001\ 1\text{'s}} - (1+1+1+\ldots+1) \\
&= \overbrace{111\ldots10}^{1997\ 1\text{'s}} - 2001 \\
&= \overbrace{111\ldots10}^{1997\ 1\text{'s}}0000 + 11110 - 2001 \\
&= \overbrace{111\ldots10}^{1997\ 1\text{'s}}0000 + 11110 - 2001 \\
&= \overbrace{111\ldots10}^{1997\ 1\text{'s}}0000 + 9109
\end{aligned}
$$

So n has 1998 ones. So we saw how to chase a pattern until it relented, but not be too willing to accept the first pattern we saw. (If you want another example of this, try the prime producing polynomial $f(x)=x^2 - x + 41$ for x = 1, 2, 3…).

Now for Your Homework:

Each of the integers from 1 to 12 is to be placed in one of the circles in the figure so that the sum of the integers along each side of the figure is 25. Determine the sum of the four integers placed in the corners.



Until next issue, happy problem solving.▲

▲ Call for NCTM Rep

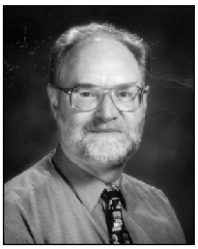
OAME is taking applications for a three-year term as NCTM representative commencing in September 2007.

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▲ CAS Corner: Integrating Technology

FRED FERNEYHOUGH



Fred Ferneyhough is former head of mathematics at Harold M. Brathwaite SS in Brampton. He retired from there in February 2006. Fred is an avid user of and advocate for the use of technology in the mathematics classroom. He is also an author and tech consultant for several books and a National Instructor for Texas Instruments.

GUEST COLUMNIST, PETER FOX
HEAD OF MATHEMATICS
ELISABETH MURDOCH COLLEGE



Peter Fox is a practising classroom teacher (part time) and is also contracted with Texas Instruments to develop the CAS Pilot in New Zealand where he has been working with their teachers, writing curriculum, providing training and advising their Ministry on implementation issues relating to the use of CAS. His involvement in the New Zealand CAS pilot stemmed from his involvement in the CAS pilot in Australia where he worked on examination setting panels, course review, course implementation by teaching a number of CAS classes and a 12 month full time contract working directly with Texas Instruments. More recently Peter has been working on the development of TI-Nspire CAS+.

Recently a good friend of mine purchased one of the original 1980 HDT Brock Commodores. He has spent some time returning it to its original condition. He removed the modern sound system and replaced it with the original AM/FM Cassette deck, not particularly practical, but he is a fanatic. His other car has cruise control, ABS, airbags,



and a myriad of other safety features and creature comforts. This spawns the question, why would you want to turn back the clock and drive such a vehicle?

I asked him what his Commodore was like to drive, the response, "it is an 80's car you know…it has less power, the fuel economy and handling are worse…but it's fun to drive". For some people this nostalgic indulgence delivers an intrinsic pleasure. Of course, for some vehicles in the 1980's had a great deal of technology, so just how far back does one need to go?

Unlike most advances in the motor car industry, some technologies have been met with reserve and wariness. At the introduction of the microwave oven people were concerned about its safe use. We didn't really understand what it was capable of doing, just that cooking might become quicker leaving us with more time to do other things. Now, almost every kitchen has a microwave, but we still use a conventional oven and, of course, the beloved barbeque as applicable.

Unless we wish to isolates ourselves, change and progress is a natural aspect of life, one which requires a degree of adaptation and often involves moving outside our comfort zone. Education is not exempt from this process, in particular, mathematics education. If mathematics is to continue to remain relevant to generation.com it will be dynamic. Our view of what is important in the teaching and learning of mathematics must constantly be reviewed and critiqued, perhaps even more than ever before. This is not to say we should simply accept change for change's sake, we must proceed with care when stakes are high.

Some teachers may recall pre-calculator days when students used trigonometric and logarithmic tables, square root and cube root tables and of course tables for normal (including inverse normal) and other distributions. The majority of teachers currently teaching senior mathematics classes will likely remember pre-graphics calculator days, either as a student or classroom teacher. Life was simple, we knew exactly what students needed to know. Students were required to know how to use the chain rule, product rule and quotient rule. They needed to know how to substitute a number into an expression, sketch graphs, manipulate and solve algebraic expressions, by hand.

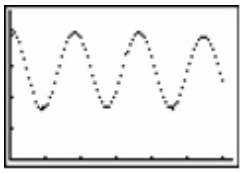
As with the many technologies, the introduction of graphics calculators produced a degree of uncertainty. Press the right combination of buttons and a graph is generated! Do students still need to know how to sketch a graph? This question was coupled with, "now we have to teach them the mathematics and how to use the calculator!"

Examination questions evolved to take into account student access to graphics calculator numerical and graphical functionality; this provided a level of comfort for teachers, students still needed to understand the mathematics. Students' ability or dexterity to sketch/draw a graph without the aid of a calculator can rightly be challenged. Prefacing this challenge must be the question, "what value do we place on this skill, with or without the use of technology in our modern world?" If this and other by hand skills are high on our priority list then assessment needs to reflect this priority, namely school based tasks and examinations that exclude such technology for some components of assessment.

During the early years of graphics calculator use, teachers and students quickly discovered there was a lot more to these devices than at first realised. Initially the discoveries were based around graphing functionality, "you can determine the point of intersection between two functions". New questions were pondered: "Do students need to know how to solve equations simultaneously any more?" Then calculus, "you can draw the derivative of a function" and probability "you can solve problems involving the normal distribution, without tables." Further investigation lead to the development of supplementary programs for graphics calculators that could carry out certain types of algebraic manipulation. Continual discovery, reflection, critique and evaluation are a sign of a healthy curriculum. Assessments continue to adapt and questions/tasks continued to be developed that challenge students understanding regardless of their ability to use a calculator. Uncertainty had given way to discovery.

As greater familiarity and acceptance continued to grow teachers found that graphics calculators could be used as effective learning tools. The calculator can graph the derivative of a function. For example, when teaching students about the derivative of sin(x), students can enter the function in Y_1 and then use Y_2 to sketch the graph of its derivative function. Including a parameter, f(x) = sin(kx), provides the opportunity for students to discover more about the relationship between the function and its derivative. Integrating data-logging technology provides a context for this learning. A simple pendulum combined with a calculator and motion detector provide a rich environment for students to attach meaning to the parameters surrounding the equation: f(x) = asin(n(x – b)) + c, and its derivative.

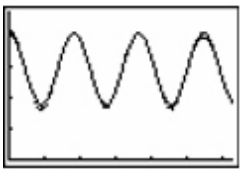
The images shown here include:



- raw data
  Collected using a motion detector connected to a calculator



- equation to model data
  Students relate the parameters they determine to the physical characteristics of the pendulum

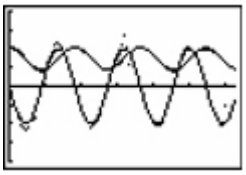


- graph generated by student
  Positive feedback to students about the accuracy of their equation modelling the pendulum's motion

The raw data includes data for the velocity of the pendulum



- derivative of displacement time equation (stored in Y_1)
  Students determine the general shape of the velocity graph, again based on their understanding of the pendulum's motion. Students then use the numerical derivative on the calculator to sketch a graph.

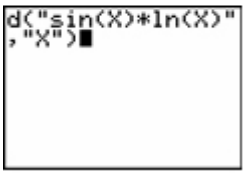


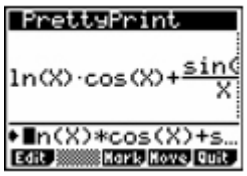
- velocity data and derivative of displacement time, viewed with position time data and graph
  Once again, students receive positive feedback, confirmation of their understanding of the relationship between the pendulum's displacement and velocity.

Most teachers would recognise the richness of potential learning that is inherent in this situation. Discovery has given way to contextual explorations.

Around five years ago, graphics calculators started to incorporate what became popularly known as 'flash' technology. For the Texas Instruments range of calculators this meant the introduction of "Flash Applications" (and their analogues for other makes of graphics calculators) There has been an enormous growth in the development of such flash applications. This has included applications such as Symbolic which enables students to symbolically determine the derivative of a function using a graphics calculator. Quasi-operating systems such as Mirage offer additional calculator functionality such as the determination of 'exact values' for calculations involving surds. The syntax for these expressions is not very elegant and without complementary use of the Pretty Print application, the output requires careful reading. This sort of functionality blurs the boundaries between graphics calculator and a computer algebra system. What about other 'black-box' symbolic manipulation functionality such as factorising, expanding and integrating?

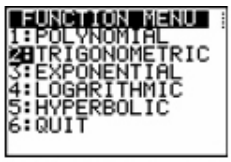




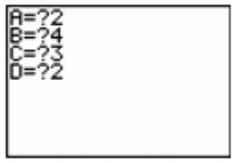


Many students use programs such as Probcheat and Factor 9.These programs lead the user through a series of menus that carry out certain routines of probability and algebra, but what about integral calculus? A simple program (requiring slightly less than 2kB of memory) such as the one described in the following discussion can be used as a simple 'look-up table' for standard anti-derivatives such as those found in the formula sheet for Mathematical Methods examinations.

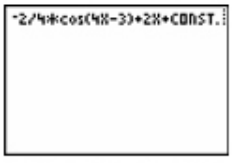
The function menu in this integration program provides students with the basic expressions they would be required to anti-differentiate in the Mathematical Methods course, as well as carry out related definite integrals (real decimal terminals) and draw corresponding enclosed area graphs. Students identify which process and function type their question relates to, for example, in the screen shots shown here, a trigonometric function is selected. The next option is to choose between asin(bx – c) + d and acos(bx – c) + d, enter the parameters then finally choose between an anti-derivative (indefinite integral), definite integral or area.





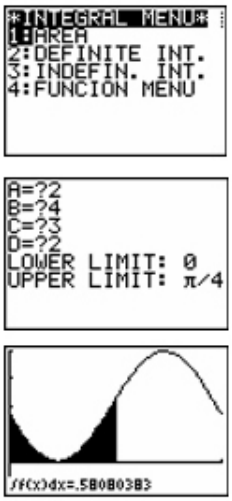






If indefinite integral (anti-derivative) is selected, the result is displayed, with an arbitrary constant The 'area' option determines the (numerical approximate) area bounded

by the function and the x – axis and shades the corresponding region(s).

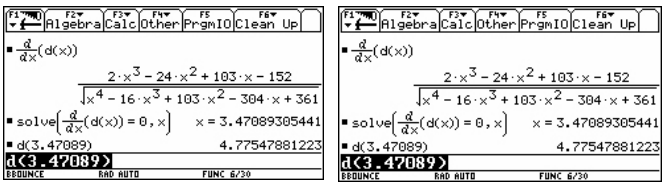


The integration program provides a 'step by step' approach to determining the integral of an expression, but is similar in overall conception to programs such as ProbCheat and Factor 9. These programs do provide students with access to 'black-box' computations which can assist students in solving problems once they have identified the necessary steps. In the interests of student learning, the use of these types of programs calls for a re-evaluation of the use of technology. Given a step backward is not desirable, should a step forward be considered?

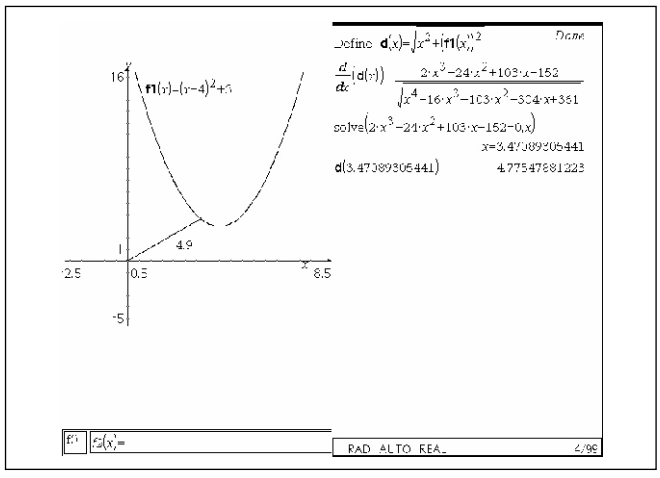
During 2001 - 2005 I was involved in the VCAA Mathematical Methods (CAS) pilot study. Student access to CAS involved us in thinking about some central question with respect to mathematics teaching and learning and challenged us as educators, on our views about what mathematics do our students need to learn?

Much of what I have learned about teaching in a CAS environment can be articulated in a story about Chelsea. A number of students in my first CAS class held strong beliefs about what mathematics should be. These students were very apt and experienced at solving equations requiring basic algebraic manipulation. My typical CAS classes generally began with a challenging question. On this particular occasion students were asked to find the minimum distance between a point on the curve $f(x) = (x - 4)^2 + 3$ and the origin. While most of the students opened their books and switched on their calculators, Chelsea sat looking at the board where the question was written. She did not have her books open, nor did she bother switching on her calculator. At first glance, it appeared she was thinking about the problem and pondering how to solve it. After two or three minutes she had not started and other students were already beginning to determine their answers. I asked Chelsea if she was having any problems, "No" was the response, "I'm done". After some discussion, Chelsea was asked to tell the class how she solved the problem. Chelsea stood at the board, brimming with confidence. "To start with, I would put a point on the curve; call it P (x, y). The distance between this point and the origin can be determined using the distance formula. This would provide a relationship between d, x and y. Using the function, the expression $(x - 4)^2 + 3$ can be substituted for y. This provides an expression for d in terms of x only. This expression can be differentiated and solved equal to zero. This will determine a value for x which should then be substituted into the distance equation…" Chelsea was not interested in the answer, in her eyes she had identified the structure of the solution to the problem and was looking forward to the next challenge.

Not all students are like Chelsea. Nor does Chelsea's story give any insight to what level of CAS should be incorporated into student learning. It does however shed some light on how CAS can shift the focus of mathematics. My experience has been that students in CAS active classrooms are better able to see the 'bigger picture' when attempting to solve problems. This type of learning can be understood by simply looking at the way CAS calculators display 'working'. The following screen shots below show two sequences of computation involved in the question outlined above. The first from a TI Voyage 200 while the 'computer' looking screen image is from a trial version of a forthcoming technology, TI-Nspire. The advantage of this newer platform for the same problem is the ease in which students can generate a dynamic environment that models the question posed to the class. The graphing screen also includes a dynamic geometry environment.



Voyage 200

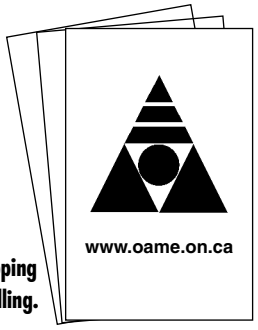


TI-Nspire

Users of current model graphics calculators will notice the syntax and notational improvements in these CAS environments. Given that graphics calculators can do much of these calculations, but in an environment where students need to be able to manipulate applications, programs and obtain accurate syntax for expressions, it is easy to see how these CAS provide a more natural working environment that enables students to readily employ numerical, graphical and analytical approaches to mathematical analysis. The use of technology-free components of assessment play an important role in ensuring that we are able to be confident that important aspects of working mathematically, where facility with mental and/or by-hand approaches is expected, are properly addressed. ▲

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▲ NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2007-2008

Each year, elections are held for a President-Elect, a Vice President (3 year term), and 6 to 9 Directors (3 year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either grade JK–6 or grade 7–12 issues based on their personal experience, expertise and interests. Nominations for these positions are to be submitted by 4:00 p.m., January 19, 2007, to:

Kathy Kubota-Zarivnij

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2007, the Nomination Committee will publish a slate of candidates, including qualifications, for consideration by the membership. Voting will be online in the Member's Only section of the OAME web site www.oame.on.ca, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Barrie, May 2007.

NOMINATION FORM

I nominate:_____

For the position(s) of: ☐ President-Elect ☐ Vice-President ☐ Director

If for Director, area of personal experience/expertise, interest : ☐ grades JK–6 ☐ grades 7–12

Nominee's Address: _____

_____ Email:_____

Telephone: H ( ) _____ W ( )_____

Nominator's Name : _____

Nominator's Address:_____ Email:_____

Telephone: H ( ) _____ W ( )_____

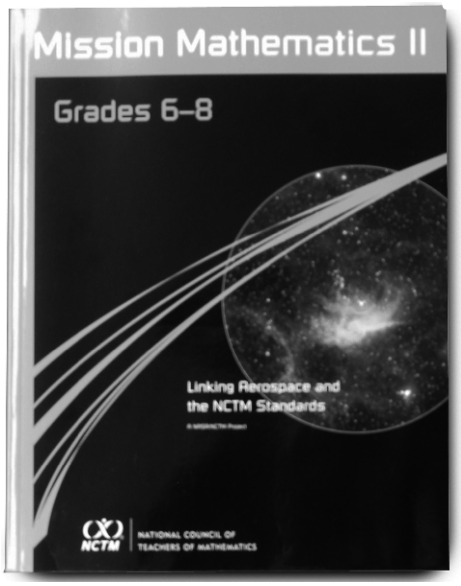
▲ Book Review: *Mission Mathematics II Grades 6-8*

REVIEWED BY JOANNE HARRIS
RETIRED GRADE 8 MATHEMATICS TEACHER



Joanne Harris is a recently retired grade 8 Mathematics teacher. She believes that making connections to the real world, to other subjects and to previous knowledge is one of the keys to helping students learn.

Are you a Middle School teacher looking for a reference book that will offer current and relevant world problems for your students to investigate? Then look no further than the recently revised *Mission Mathematics II* series. There are four books in the series; PreK-Grade 2, Grades 3-5, Grades 6-8 and Grades 9-12. This series of books is intended to supplement, not replace, a standard Mathematics textbook.



Mission Mathematics II Grade 6-8 has five sections and each section has a series of between 2 and 7 activities. Within a section the activities are not meant to stand alone but rather to be used as a series. Topics for the sections include: Our Solar System and Beyond; Space and Speed; Space Rocks; Aeronautics; and finally International Space Station and Other Earth-Orbiting Satellites.

The activities within the sections are designed for students to work in small groups to model the workplace trend towards teamwork. Each of the over 40 activities is presented using the same format. Teacher background comes first and this includes information on important mathematical ideas. Then there is a brief statement about the Mission (the task). The required Materials and Equipment are listed next and for the most part these are materials that are found in any average mathematics classroom. They include such items as grid paper, protractors, calculators or BLM's which are found at the back of the book. Some suggestions for starting or leading into the activity are provided in the section called Launching the Activity. Developing the Activity provides more detail about the task and then Concluding the Activity wraps it up. A limited number of the activities provide suggested extensions.

Throughout the book, in the margins, teachers will find teacher tips on how to set up an activity, or how long an activity might take, or safety issues to be aware of, or web sites to further explore a topic, or assessment tips. Since the book is from NCTM, each activity identifies the NCTM Standards addressed by the Activity. An Ontario teacher can easily use these Standards to identify the appropriate Ontario Curriculum Expectations addressed by the activity.

Within the current Science and Technology curriculum, several sections of the book would be particularly useful for a grade 6 or 7 teacher, especially one teaching in an integrated setting. These would include for the grade 6 teacher, the section on the Solar System and Aerodynamics (Paper Airplanes) and for the grade 7 teacher, the section entitled Space Rocks. If the setting in your school has Mathematics taught independently from Science and Technology be aware that the students will be studying Space, and Characteristics of Flight in grade 6 and the Earth's Crust and types of rocks in grade 7. The Science and Technology teacher may have a supply of rocks you could borrow to conduct the space rocks activities.

Mission Mathematics II Grades 6-8 is an excellent resource for either a classroom library or a school library. Each of the four volumes in the series is designed for a different grade grouping so be certain to look for the one that matches your grade level.

This four part series is available from NCTM. Each volume is priced individually.

Mission Mathematics II Grades 6-8 is $35.95 (US).

▲ Historical Mathematical Stories: Frivolous or Functional?

GLADYS STERENBERG



Gladys Sterenberg is an assistant professor at the University of Lethbridge. Her passion is working alongside teachers to implement stories of mathematics in elementary classrooms.

Introduction

Principles underlying the *Ontario Curriculum grades 1-8: Mathematics, 2005 revised* (Ontario Ministry of Education, 2005) emphasize that students have many learning styles and recognize that a variety of teaching approaches is necessary. Because of limited mathematical backgrounds, many elementary teachers tend to rely on textbook-centred instruction (Ball, 1990). However, since the implementation of the *Curriculum and evaluation standards for school mathematics* (National Council of Teachers of Mathematics, 1989), alternatives to textbook-centred approaches are emerging. One such approach focuses on cross-curricular and integrated learning and involves the use of literature in mathematics classes. Collections of mathematical stories are becoming more available as the current interest in mathematics and literature expands.

Challenged to explore alternative teaching approaches, I decided to investigate historical legends of mathematics with my grade 3/4 students. This seemed to be a good place for integrated learning since we were studying legends as a form of writing in language class. I chose to use the mathematical legend of the Bridges of Königsberg for several reasons. I believed that students would identify geometric properties of various two-dimensional shapes and generate patterns involving addition, subtraction, and multiplication. The mathematical task embedded in the story offered students an opportunity to develop a spatial sense within a problem-solving context.

In addition to addressing the mathematics curriculum strands of geometry and patterning, I wanted to promote positive attitudes towards mathematics. *Ontario curriculum grades 1-8: Mathematics, 2005 revised* states that confidence and a positive disposition towards mathematics can develop when students understand that there are several ways to arrive at the correct answer, when students believe they are capable of finding solutions, and when students are persistent. I chose to offer students a problem that had no solution (something rarely experienced in mathematics classrooms) to encourage them to develop, select, and apply problem-solving strategies that would deepen their mathematical understanding. While this legend is sometimes presented as a problem at the high school level for exploring map theory and topology, I was not interested in having students create formulas for circuits. I wanted my students to grapple with the legend, persist in their investigation of patterns, reason, communicate, and generate alternative solutions.

In this article, I present my attempt to integrate this particular legend into my mathematics class. Building on my experience of using literature and stories in mathematics, I believed this teaching approach would provide students with a context for solving problems. Also, I thought it would help them develop reasoning and communication skills, components common to both curricula. As described below, my students responded in unanticipated ways, experiencing the story as both frivolous and functional.

Telling the Story: The Bridges of Königsberg

All conversation and debate ceased as Herr Gunterling, the Grand Mayor of Königsberg, burst into the classroom. Professor Euler turned towards the noise and waited for an explanation. With a brief nod to the students, the mayor unfurled a large map.

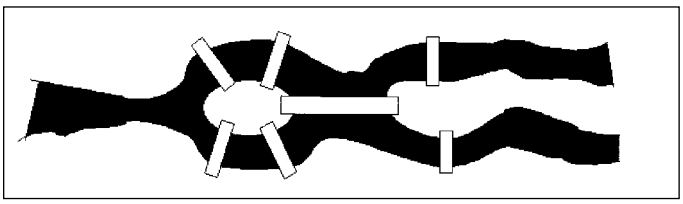
Now the mayor tried to describe the situation for Euler. "Two rivers meet in Königsberg to form the mighty River Pregel. Right where they meet, the River Pregel's channel splits around a large island that is the heart of old Königsberg. Königsberg is a city of seven bridges—two from the south shore of the river to the old city island, two from the north shore to the old city island, and one each from the peninsula between the two rivers to the south shore, the north shore, and the old city island."

Euler held up a hand to interrupt. "I can visualize your city. Peter, please draw it on the board so everyone can see. What is your math problem Herr Mayor?"

"In one month Königsberg will hold its thousand-year

celebration. We'll have a grand parade, which, by legend, must cross every bridge in Königsberg, but may only cross them once. No one can design a parade route that crosses each bridge once and once only. Nor can they tell me if it is even possible." (Haven, 1998, p. 109)

The grade 3/4 students listen intently as I tell the classic story of the seven bridges of Königsberg. Students then set out to solve the 300-year-old problem—to plan a parade route through the city that crossed each bridge once and only once—if it is indeed possible. Their conversations are diverse as they explore the city map.



I interact with the students, enjoying their sense of curiosity. Even when they are unable to find a solution, they sense that this task is important and persist. Comments abound: "It's practically impossible"; "We're almost at the conclusion that you can't do it. We tried it so many ways"; "You either want less or you want more"; "We always have one bridge that we're missing"; and "If we could build a bridge right there, problem solved". One student suggests to his classmates, "A mathematician never gives up".

However, I become nervous when the conversations focus on the role of legends. They begin to believe that the problem has changed over time and perhaps the number of bridges is wrong. One student says, "It's a legend so they did it a thousand years ago but maybe at that time, they didn't have the bridges". I become anxious as their investigations meander into considerations of alternative solutions such as using boats, walking over a frozen river, and catapulting across the water. Is this mathematical activity? Has the story distracted them from the mathematics? Are these experiences of story frivolous or functional?

Making Human Sense

My decision to use a historical story as a teaching tool was based on emerging research. One teaching approach proposed by many educators involves the study of the history of mathematics. Researchers such as Ernest (1991), Fauvel (2000), and Lakatos (1978) believe that this can encourage positive views of mathematics. Teaching through historical mathematical stories offers one way of integrating mathematics and language curricula. Many researchers believe that the use of story can enhance our understanding of mathematics. Borasi and Brown (1985) suggest that a storied approach to learning mathematics promotes natural and spontaneous mathematical learning. Burton (1996) claims:

A narrative approach to mathematics and its pedagogy is consistent with a view of mathematics as being socially derived and with the understanding of mathematics as being socially negotiable…By engaging with the narrative, we place the mathematics in its context and personalize it, making it come alive to the conditions of the time. Context provides meaning. (pp. 32-33)

The integration of literature and mathematics embodies the characters, settings, plots, and problems of life. Stories show our relationship to our world. They are meant to recall our lived lives and to prompt thoughtful consideration. Stories can be used to dramatize mathematics and support rich, emotional, and engaging experiences of mathematics.

Donaldson (1978) suggests that pre-school children can reason in sophisticated ways when tasks are presented in rich contexts that make 'human sense'. She writes:

So long as our thinking is sustained by this kind of human sense, and so long and the conclusion to which the reasoning leads is not in conflict with something which we know or believe or want to believe, we tend to have no difficulty. Thus even pre-school children can frequently reason well about the events in the stories they hear. However, when we move beyond the bounds of human sense there is a dramatic difference. (p. 76)

Mathematics can be described as functional when it contributes to our daily lives and when it is useful. Utilitarian purposes for mathematics are emphasized in the *Ontario curriculum grades 1-8: Mathematics, 2005 revised*. When describing the importance of mathematics, learning is situated within an information- and technology-based society. The function of mathematics education here is to prepare students for their daily lives and future work: "Through mathematical activities that are practical and relevant to their lives, students develop mathematical understanding, problem-solving skills, and related technological skills that they can apply in their daily lives and, eventually, in the workplace" (*Ontario Ministry of Education, 2005*, p. 3). When mathematics is viewed solely in functional terms, it becomes a consumer product. Mathematics may be viewed as a commodity where students are expected to

master mathematical skills and knowledge. This can lead to disconnected thinking as teachers emphasize correct answers through the execution of algorithms and rely on traditional teaching approaches.

While a functional approach can lead to viewing mathematics as a product, the intent of this curriculum seems to focus on contextual thinking. It states, "Mathematics is a powerful learning tool. As students identify relationships between mathematical concepts and everyday situations and make connections between mathematics and other subjects, they develop the ability to use mathematics to extend and apply their knowledge in other curriculum areas, including science, music, and language" (p. 3). Narrative approaches to mathematics are consistent with this intent.

However, my concern that a preoccupation with the story may divert attention away from the mathematics remains. The students in the class seem to be treating the mathematics in a frivolous manner by ignoring it. They are focused on the narrative aspects of the legend and I am unsure of how or when to intervene. How is their engagement with the story connected to their mathematical learning?

Pimm (1992) calls on teachers to "avert the destruction of meaning, of connection and any sense of personal involvement which characterizes more than caricatures the effect of much exposure to school mathematics" (p. 95). He implies that immersion in the relational and contextual aspects of story supports, and perhaps enables, mathematical understanding. When storied aspects of mathematics are returned to the classroom, mathematics becomes interesting and creative. Davis (1996) argues that mathematics becomes an inhumanity when we ignore human experiences: "It is thus that we have created a system that values compliance over creativity, that spawns destructive behavior by destroying our experience, and that conditions learners to reach for the formulaic ahead of the imaginative" (p. 281). However, I continue to wonder how using historical stories as a teaching approach can help students learn mathematics.

Reflecting on the Mathematics

Are my students engaged in mathematical activity? My own concerns lessen as the students continue to work on the problem. Most students begin to consider why the problem is impossible. Emerging patterns are discussed as they grapple with alternative solutions: "If we move the bridges around, we can get it" and "If you close the north west bridge [it works]."

Like Euler they play with the placement of bridges



and began to explore circumstances that create a circuit. They form conjectures involving odd and even numbers: "It is not possible because it is an odd number. It works when it is an even number because they can all pair up but when it is an odd number, they can't." However, counter-examples provide the motivation to search for alternative patterns: "I figured out that it wasn't an odd number because you can do it with an odd number but if you move this bridge up here, it was impossible" and "You can do it with 9, 7, 5, and 11." Students have mathematical insights as they recognize patterns, form generalizations, communicate their ideas, and reason mathematically. The story appeals to the students; creativity and imagination are evident in their actions and comments. Processes of mathematical learning are apparent in this situation. Historical mathematics presented as a story provides these students with opportunities to engage in mathematical problem solving.

Learning through historical stories can be considered both frivolous and functional. Perhaps learning mathematics should be frivolous, not in the sense of being silly and thoughtless, but as it pertains to being playful and lighthearted. Rather than focusing solely on the usefulness of mathematics, maybe our notions of mathematics as functional can be informed by our concept of a mathematical function where a quantity is expressed in relationship to other variables. Mathematical functions show relationships; functional mathematics could be described as being dependent on contextual relationships. Learning through story is functional in the sense of showing relationships to our world. Certainly integrating literature and mathematics supports the idea that learning mathematics is dependent on enjoyment and playfulness in relation to contextual and situational variables. Thus, our experiences of mathematical stories can be both frivolous and functional.

Conclusion

Returning to and expanding the notion that mathematics is socially constructed and negotiated,

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mathematics educators and researchers are promoting new curricula that emphasize the mathematical processes of communication and reasoning. We are beginning to understand that the development of mathematical concepts occurs in a contextual and relational manner and that this context can provide meaning. Using stories in mathematics classrooms enables us to experience the human dimensions of mathematics.

By challenging common misconceptions of mathematics as a disconnected set of rules and procedures to be memorized, and of mathematicians as isolated social loners, stories show mathematics as part of human culture. Perhaps this is the most compelling reason for teaching and learning through historical mathematical stories. Students are interested in stories and using literature provides an alternative approach to teaching mathematics.

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▲ Counting Family Groupings

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Bonnie Litwiller and David Duncan teach mathematics and mathematics education courses at the University of Northern Iowa. They have published over 850 articles in journals for mathematics teachers and have spoken at numerous professional meetings. Each has served as President of the Iowa Council of Teachers of Mathematics and Dr. Litwiller has served on the boards of NCTM, SSMA and AMTE.

Teachers are always looking for settings in which permutations, combinations, and the fundamental principle of counting can be exemplified. We will describe a set of exercises using these notions which stem from a simple social setting. Suppose that three couples are together at a social setting and wish to have pictures taken.

1. If a picture is to be taken of an unordered subset of these six people, in how many ways can this be done?

   To choose such a subset, a separate "in-out" decision must be made concerning each person. The number of ways in which these six decisions can be made is then 2 x 2 x 2 x 2 x 2 x 2 or $2^6 = 64$. But one such subset is the empty set, leaving 63 non-empty subsets (combinations).

2. Re-do (1) with the additional conditions that order matters (permutations). In this case, for each unordered subset the total number of orderings (in a row) must be determined. If a subset has n elements, the number of distinct orderings can be determined as follows, again using the fundamental principle of counting:

   Choose first person: n choices

   Choose second person: (n-1) choices

   .

   .

   .

   Choose nth person: 1 choice.

A subset of n persons can then be ordered in n! ways.

To count the total numbers of permutations, we proceed as follows:

- The set of all six persons can be arranged in 6! ways;
- Each of the six five-person subsets can be arranged in 5! ways:
- Each of the C(6,4) or 15 four-person subsets can be arranged in 4! ways;
- Each of the C(6,3) or 20 three-person subsets can be arranged in 3! ways;
- Each of the C(6,2) or 15 two-person subsets can be arranged in 2! ways;
- Each of the six one-person subsets can be arranged in only one way.

In total, the number of possible ordered subsets that can be arranged for the photograph is:

6! + 6 x 5! + 15 x 4! + 20 x 3! + 15 x 2! + 6 = 1956 – a huge number for only six persons!

3. Suppose that all six are to be photographed in a row, but that couples are to remain together in the ordered arrangement. This problem can be addressed in two ways:

   a. Choose the first person (6 choices):

      Choose the second person (1 choice – must be the first person's spouse);

      Choose the third person (4 choices);

      Choose the fourth person (1 choice);

      Choose the fifth person (2 choices);

      Choose the sixth person (1 choice).

      In total there are 6 x 1 x 4 x 1 x 2 x 1 or 48 ways of arranging these people with couples together.

   b. Arrange the three couples in order (3! or 6 ways) For each couple, order the two people (2 x 2 x 2 or 8 ways).
      As before, there are 6 x 8 or 48 arrangements.
      Note that 6 x 4 x 2 = 2(3) x 2(2) x 2(1) = (3 x 2 x 1) x 23 = 6 x 8.

The reader and the students are encouraged to address similar questions for other groupings of differing numbers of persons. ▲

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▲ A Page of Problems
It's Do or Dice!

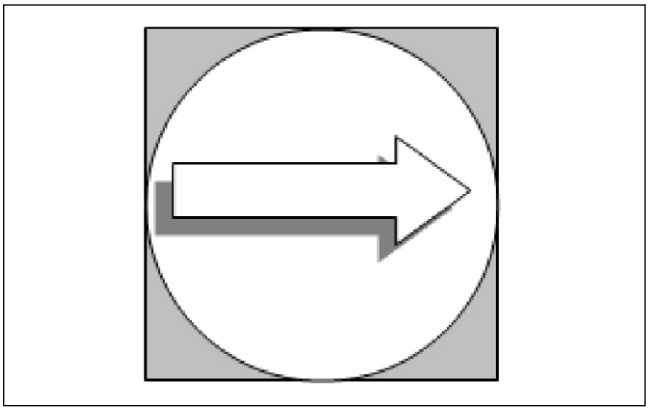
A. Craig Loewen

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Middle School

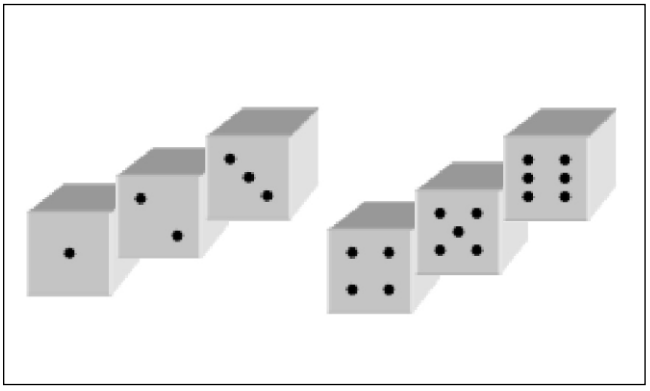
Create a spinner that would give you the same results as

- rolling a six-sided die, and
- rolling two four-sided dice together.



High School

On average, how many times will you have to roll six regular six-sided dice before getting a 1, 2, 3, 4, 5 and 6 all in one roll?



Hint: What is the probability of rolling a 1, 2, 3, 4, 5 and 6 all at once with six dice?

Junior High

Do you have a better chance of rolling an even sum with two regular four-sided dice, or rolling an odd sum with two regular six-sided dice?

Conduct an experiment to test your hypothesis.

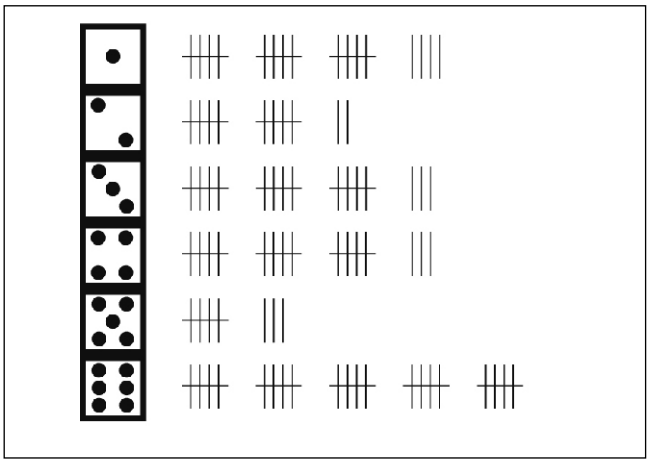


Elementary

Jo rolled her six-sided die 100 times and recorded the results in the tally chart below.

Do you think her die was "fair"?

How do you know?



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