Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceeding 10 numbered pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required. Please number pages.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including web sites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

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The *Ontario Mathematics Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

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Advertisements for the *Ontario Mathematics Gazette* should be sent to **Dean A. Murray** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising Rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARILYN HURRELL, EDITOR



I have a confession to make. Even though I was a partner in it's creation (the Grade 7-8 Shopping Cart problem under the leadership of Chris Suurtamm) and have seen clips and have tried some of the activities at OAME Board Meetings, I had not really paid full attention to the OAME *GUM* project. Shame on me.

However, this term I decided to show it to my classes of preservice math teacher candidates at Lakehead University. Accordingly I previewed it last weekend. I was blown away by the content! There they were, examples of exactly what I was trying to get through to my students! All laid out for them to see and appreciate! The whole continuum in one DVD! Lots of room for discussions! Opportunities for them to see inside real classrooms! Enough said. If you have not viewed this resource yet, do so.

Lynda Colgon's column in this issue is dedicated to John Van De Walle, who passed away suddenly on December 2, 2006. His death has left a hole in the mathematics community and I am grateful to Lynda for honouring him in our journal.

I am pleased that John Grant McLoughlin has returned to the *OMG* with an article in this issue. John used to write a column titled "The Devil's Advocate" for this journal. Two teachers from London ON have submitted an article on a project they gave their classes. I know that there must be many more good ideas out there; why not write up your idea and submit it to me? The *OMG* is always looking for classroom ready material.

The OAME elections have gone electronic this year. Be sure to note the instructions on how to exercise your right to vote for OAME Board members. Deadline is April 13, 2007.

Have a great spring and Happy Pi Day.

Marilyn

▲ President's Message

DAN CHARBONNEAU, PRESIDENT



It is Frank Sinatra time regarding my presidency of OAME (the end is near). Included in this *Gazette* are instructions on how to cast your online votes for elected positions on the Executive and Board of Directors. Make sure to vote between March 16 and April 13. I am very impressed by the slate of candidates and the wealth of experience/background they bring to OAME. I wish to thank all candidates for showing an interest in serving our members. Your new Executive and Board will be announced at the Annual General Meeting being held during the OAME' 007 "Building Bonds" Conference in Barrie in May. This AGM is open to all OAME members, so come join us as we welcome your new president Jacqueline Hill. A special "thank you" and "goodbye" to Executive members Kathy Kubota-Zarivnij, Kevin Maguire, Mary Bourassa, and to Board directors Silvana Simone and Shawn Godin.

Since my last message, several new and/or revised mathematics related resources and links are available online. I'll share a few with you.

Authentic summative assessment tasks, which illustrate how teachers might assess student achievement in key skills and concepts in Grade 7, Grade 8, Grades 9 and 10 Applied, can be downloaded from the Leading Math Success library at **http://www.edu.gov.on.ca/eng/studentsuccess/lms/library.html**. The "My Professional Practise" section of the library offers teachers instructional supports for planning and delivering various components of effective lessons and programs. The newest addition is a video clip on "Simplifying Algebraic Expressions Using Algebra Tiles". Other clips on "Pattern Building", the "Pythagorean Theorem" and "Transformations" can also be found.

As a follow up to the January Regional Meetings held in Kingston, Sudbury, Thunder Bay, London, Halton and Richmond Hill for K-6 Numeracy leaders, the Literacy and Numeracy Secretariat will offer daily web-conferencing in the last week of March. A webcast called "Making Math Accessible to Students" can be viewed at **www.curriculum.org** on March 28 from 4:00 to 5:00 pm. The Ministry of Education and Training will be doing the rollout of the revised Grade 12 Mathematics Curriculum in late March or early April. Regional in-service will also be provided in the spring.

Grade 7 and 8 teachers are reminded that Regional and OAME Chapter playdowns for the Ontario Mathematics Olympics are taking place throughout the province this spring. Teams are comprised of four students: two boys and two girls of whom two are in Grade 7 and two in Grade 8. Check the OAME web site and click on the "Conferences/Events" link for more OMO information and for the OMO contact in your Chapter. Chapter winners will be invited to participate in the provincial OMO being held June 1 and 2 2007 at the University of Ottawa and hosted by COMA.

Many teachers find it difficult to access Board limited funding to attend conferences. This year, the Ministry of Education and Training has forwarded over $20M to OTF and the affiliates for both local and provincial teacher PD. I know that ETFO will have summer institutes for teachers and OECTA is having a "Teachers as Leaders" conference on March 27 and 28. Contact your local and provincial federation for more information on PD opportunities and don't forget to ask for funding in order to attend the OAME conference in Barrie May 3-5. Check the OAME web site to view the program and to register.

I'm finishing up my last president's message to you on a very mild mid-January day in Sudbury. We've had but a few centimetres of snow this winter and our local northern lakes have little or no ice on them. In previous messages, I brought to your attention my concerns about global warming, climate change and the environment in general. Some have commented that this has little to do with math. Having surfed the web for many hours I have found a great deal of available environmental information and data that we can incorporate into our mathematics K-12 programs.

The Patterning, Data Management, Measurement and Number Sense and Numeration strands and many of their expectations, as well as the Process Expectations, can be addressed in each grade using environmental data and issues. Activities exist or can be created using available data whereby students can use their math skills and demonstrate competency in math while at the same time becoming enlightened about the repercussions of climate change and global warming. I have compiled an extensive list of web sites and descriptions of what can be found on these sites for your use in your K-12 math programs. I'd like to thank Patrick Adams and Rob Smith from the Environment Accounts and Statistics Division at Statistics Canada for their contributions and time. I also have

examples of what can be done with some of the data. By the time you receive this *Gazette* message, I will have persuaded Greg Clarke to post the links and descriptions on the OAME web site. I hope you will make use of them and perhaps share with us your lesson or activity so that we can add it to the OAME web site.

It has been an honour to serve as your OAME president.

Warmest regards (not warmest weather),

Dan Charbonneau ▲

▲ Letters to the Editor

To: The Editor of the OAME *Gazette*

My husband, Bob Robinson, and I enjoy reading the *Ontario Mathematics Gazette*. Bob reminisces with each issue and holds the OAME in such high regard.

This has been a difficult year for Bob as he struggles to conquer sinus cancer. He is moving forward well and participated as a speaker at the NorthWest Mathematics conference in Victoria, BC in October, 2006.

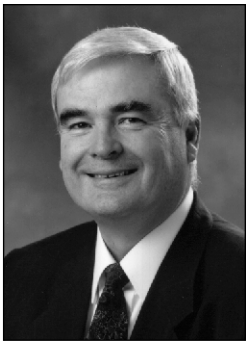
I thought your readers would be interested in knowing that Bob received the *Outstanding Male Mathematics Educator Award for 2006*. I am including below the article that was published in the *Washington Mathematics Journal* which is the official journal for the Washington State Mathematics Council.

Bob fondly recalls all his colleagues in Ontario and the many rewarding professional experiences he had while associated with the OAME of which he is an honorary lifetime member. He also continues to be a mathematics education consultant with Spectrum Educational Supplies.

Thank you, Marilyn, for your excellent editorship of the *Gazette*.

Jo Anne Robinson, President, WSMC No Limit! Consultant

Outstanding Male Mathematics Educator



On August 8th, 2006, Bob Robinson of Everett received the Richard P. Stucky Outstanding Male Mathematics Educator Award for 2006 sponsored by the Washington State Mathematics Council (WSMC). Bob has been an outstanding contributor to mathematics education in Washington and Canada. He has been a classroom teacher and played an active role in the mini-conferences promoting the Essential Academic Learning Requirements established by the Washington State Commission on Student Learning.

Bob was co-chair of the NorthWest Mathematics Conference (2001), and has presented workshops locally, nationally, and internationally. He is a past member of the Board of Directors of the National Council of Teachers of Mathematics (NCTM) as well as an honorary lifetime member of the Ontario Association for Mathematics Education (OAME) in Canada. He is also a lifetime member of the Ontario Mathematics Coordinators' Association (OMCA). Bob also has held leadership roles on a variety of NCTM committees. He has organized affiliate leadership conferences, chaired several regional and national NCTM conferences and was Program Chair for a national NCTM convention in Salt Lake City, UT. Currently he is on the program committee for the 2007 Annual NCSM (National Council of Supervisors of Mathematics) conference in Atlanta, GA to be held in March, 2007.

As the Public Relations Chair for the WSMC he serves on the Board of WSMC. Bob keeps the mathematics community of Washington state well-informed regarding mathematics issues within the state and across the nation. He also promotes WSMC and its publications. Currently, he is the Facilities Chair for the upcoming 2007 NW Math Conference to be held in Bellevue, WA, in October. ▲

Dear Editor:

The *Friday Special* in the December, 2006 *OMG* was provided by guest columnist Suzanne Carere. But instead of her picture at the top of the column, my smiling face was front and centre. Well, here is a wonderful face to put to the name. Introducing George Brown Professor of Nutrition, *Toronto Star* columnist, and TV commentator, Suzanne Carere.



Best wishes, Jack ▲

▲ Technology Corner: Modeling and Geometry with Bridges

MARY BOURASSA



Mary Bourassa teaches mathematics at Lisgar Collegiate Institute in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She is also an OAME vice-president, past-president of COMA, gives workshops around the world and is a National Instructor for Texas Instruments.

Bridges, through their structure and engineering, lend themselves well to the mathematics classroom. They are also accessible to students making the connection to the real-world easy. Arch bridges were first built along the Euphrates River over 5000 years ago. Their remarkably strong yet simple design means that many semi-circular Roman arches built over 2000 years ago are still in use. The same arch shape created the first windows in buildings allowing light in. The geometry of the semi-circular arch is straightforward – they are made of identical isosceles trapezoids cut by expert stone masons and placed together without any mortar. The entire structure did need support until the stone at the very top, the keystone, was in place. The diagram shown in Figure 1 was created using The Geometer's Sketchpad. Some geometry of quadrilaterals is needed to complete this task.

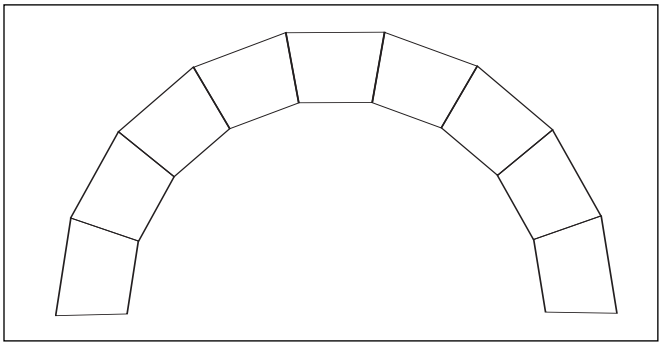


Figure 1

Students must know that the sum of the angles in a quadrilateral is 360°. They can then choose the measure of the two angles at the top and at the bottom based on how many stones they want in their arch.

The arch shown in Figure 1 has 9 stones. If we completed the other (bottom) half of the arch, an 18-sided polygon would be created. Since each interior angle of a regular n-sided polygon measures

$$\frac{(n-2)180^\circ}{n}$$

then each interior angle would measure

$$\frac{(16)180^\circ}{18} = 160^\circ.$$

If we let the measure of each of the angles at the base of the trapezoid be x° (see Figure 2), then we know that

$$\begin{aligned} x + x + 160^\circ &= 360^\circ \\ 2x &= 200^\circ \\ x &= 100^\circ \end{aligned}$$

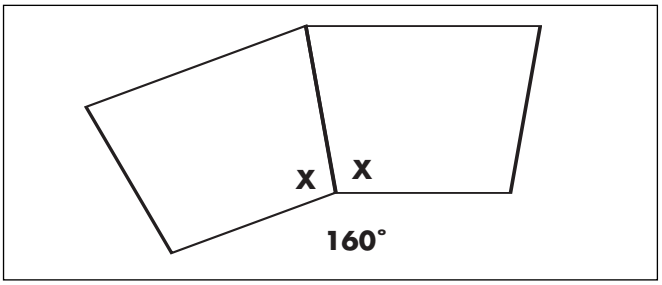


Figure 2

Therefore the angles at the top of the trapezoid will measure $\frac{360^\circ - 2(100^\circ)}{2} = 80^\circ$. Once students have determined the angles required for the number of stones they want, they can construct it using *Sketchpad*.

1. Start by creating a horizontal segment which will be the bottom of the keystone.
2. Mark one end as the center (Transform, Mark Center).
3. Select the segment along with the end points and rotate (Transform, Rotate) by the appropriate angle.
4. Do the same with the other end of the horizontal segment marked as the center but with the negative angle.
5. Connecting the top finishes off the keystone.

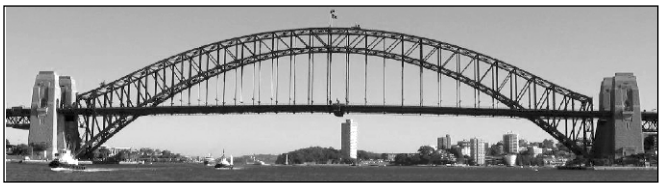
The other stones can all be created by rotating the top and bottom segments – students will have to think about what angle to use to accomplish this. After completing this kind of activity students should be able to answer questions like: If the measure of one of the interior angles of an isosceles trapezoidal stone from a semi-circular arch is 86°, how many stones are in the

arch? (Answer is 45.)

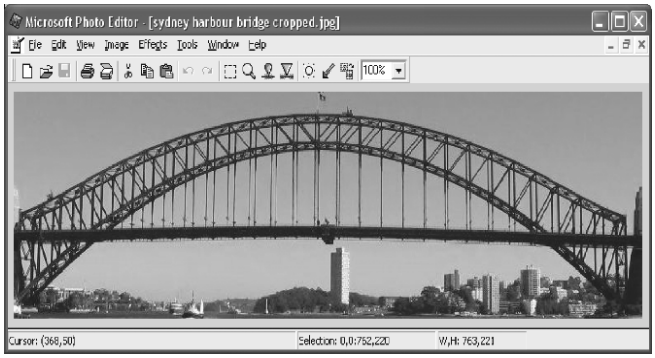
Bridges are also a great source of real-life modeling situations. Many arch bridges are parabolic, others are hyperbolic or semi-circular. Possible questions to start such an activity include:

- How wide/tall could a ship be and still fit under the bridge?
- Is there a relationship between the height and width of this type of bridge?
- What model fits the top arch? Is this true for all arch bridges?
- What other shapes were used in bridge building?

Here is an example: What shape is the lower arch of the Sydney Harbour Bridge?

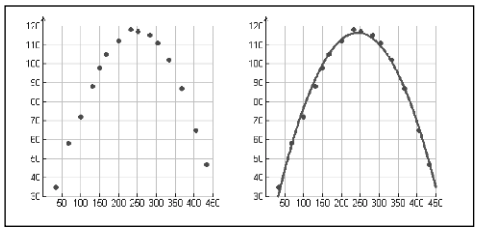


There are many ways to place the picture on a coordinate grid. You can use a graphics editor, *The Geometer's Sketchpad* (paste the picture in and mark points then determine their coordinates) or use *TI-Navigator* to have students mark points, collect the data from them and then send the entire class' data back to them. Using a graphics editor will mean that you have to determine the coordinates of the lower left corner and use those to adjust the y-values of the data you collect.



I entered the x-values in List 1 and the y-values in List 2. As the bottom left corner of my window is at (0,150), I actually need to plot (150 – y) vs x. To accomplish this create a formula for List 3. Make sure your cursor is at the very top of L3 and press ENTER then enter the formula: [L3] = 150 – [L2]. Creating a scatter plot of L1 and L3, we see that the data seems to be quadratic.

x	y
34	115
67	92
99	78
131	62
150	52
166	45
200	38
233	32
251	33
284	35
303	39
334	48
368	63
405	85
433	103



Students should be able to determine a model without using regression. This can be done by adjusting the parameters of $y = a(x - h)^2 + k$ or by using the transformation graphing app can be used on a graphing calculator. They can compare their model (once they convert it to standard form) to the quadratic regression equation yielded by the calculator: $y = 0.001918x^2 + 0.935956x + 2.15505$.

As a follow-up to this activity, students could research the company "BridgeClimb" which takes people up on the top arch of the bridge. Calculating the potential revenue of this company would be very interesting.

Newer cable-stayed bridges are also a great source of questions relating to triangles and trigonometry, especially those where the main support is not vertical (see Figure 3).

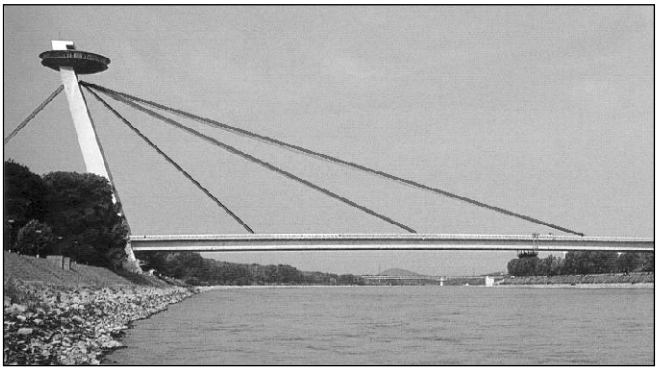


Figure 3

These ideas could be extended into a project that would be a wonderful way of collaborating with physics teachers who can explain all the forces that act on bridges. Students could then build their own bridges and show the mathematics involved in these beautiful designs. ▲

▲ Hey, It's Elementary!
Mathematics Education Loses a Giant



LYNDA COLGAN

John Arthur Van de Walle, 63, of Midlothian, died suddenly at his home on December 2, 2006. John was a professor emeritus at Virginia Commonwealth University, where he worked for 30 years with both pre-service and in-service teachers of all grade levels. John's professional passion was to help teachers teach mathematics through problem-solving that encourages children's conceptual development in mathematics. He was a popular and oft-requested speaker at local, regional and national organizations for teachers, supervisors, and educators. John was greatly admired and respected by all who knew him for his passionate and creative approach to teaching mathematics to children. John graduated cum laude from Bellarmine College in 1965 with a degree in mathematics. He earned his Master's Degree in mathematics at St. Louis University in 1967, and in 1972 he earned his Ph.D. in mathematics education from The Ohio State University. John began his career at VCU in 1973. In May of 2000, he earned VCU's Charles P. Ruch Award for Excellence in Teaching. He served on a variety of committees for the National Council of Teachers of Mathematics and was a member of the NCTM Board of Directors from 1998 to 2001. John was the author of "Elementary and Middle School Mathematics: Teaching Developmentally", currently in its 6th edition. His book is the leading text and resource in the U.S. and Canada for teaching K-8 mathematics. He recently published three grade-band books for classroom teachers. He was co-author on a popular series of textbooks for grades K-6. John's family was the center of his life, and he loved his wife, daughters and granddaughters with unmatched devotion. He approached everything he did in life with zest and enthusiasm. No one enjoyed playing with toys more than John, and his favorite playmates were Maggie, Aidan and Grace. With his wife Sharon, he recently traveled to places they had always wanted to visit - Hawaii, Nova Scotia, Portugal, Alaska, Vancouver and most recently Ireland. John swept all who knew him into his exuberance for any enterprise, and he was well-loved for it by all who knew him. At his church, he volunteered as an altar minister and lector. For many summers, he worked helping to build houses for Habitat for Humanity. Currently he was volunteering by working with teachers and children at his granddaughter's school, Robious Elementary. John will be sorely missed by his family, his friends, and his colleagues. He is survived by his wife of 40 years, Sharon Smith Van de Walle; two daughters, Gretchen Van de Walle and her partner, Jeannine Pinto and their daughter, Aidan Pinto and Bridget Phipps and her husband, Deron and their children, Maggie and Grace Phipps; one brother, Mark J. Van de Walle and his wife, Elizabeth; one sister, Jane Dangler and her husband, Don; and two nephews, Mark and Timothy Van de Walle.



This was the obituary that appeared on the web site of the Bliley Funeral Home, in Richmond, Virginia to mark the sudden death of a giant in the field of mathematics education, John Van de Walle. *The Richmond Times-Dispatch* reported that "Dr. John Arthur Van de Walle, a professor emeritus of education at Virginia Commonwealth University, collapsed while speaking on the phone with his doctor and died in his wife's arms". The article, a tribute to John's life, went on to say that "he was the leading math educator in the country," quoting Dr. Richard Rezba, the director of the VCU Center for Life Sciences Education who often shared teaching techniques with his friend.

As a community, mathematics educators in Ontario owe John Van de Walle an enormous debt of gratitude. More than any other individual, Dr. Van de Walle made the Canada/U.S. border invisible since he was as at-home with and as ready to lend a hand to teachers from Alabama to North Rubber Boot. His influence has been

felt by countless pre-service teacher candidates, in-service course participants and those who are in leadership roles, because of his long reach as textbook author, consultant, workshop presenter, and friend. Most recently, here at home, his work shaped *The Early Math Strategy, The Junior Math Strategy*, as well as the *TIPS* document and *PRISM* initiatives. His writing has made many highly theoretical and obscure research publications accessible and implement-able for front-line teachers, and has formed the foundation for actionable classroom recommendations and instructional look-fors.

John will be remembered for many different things, by many people, from the realistic, research-informed advice in his textbook to his down-to-earth workshop style, hearty laugh and sommelier-savvy; however, the critical mass of educators at all levels will undoubtedly associate the following with Professor Van de Walle: the 3-part lesson structure; authentic problem solving; manipulatives (especially counters for solving the infamous *Condominium Problem*) and his love of and respect for children as mathematics learners.

What better way to honour a mathematics education visionary; advocate for the complexity of elementary mathematics teaching and learning and, most importantly, kids; and, generous and prolific "share-er" of strategies, techniques, and resources than to use his own words. The following excerpts are from a conversation John had with Catherine Kelly, of the Colorado Council of Teachers of Mathematics in November, 2006, very shortly before his death. They are used here with permission.

John Van de Walle on the Three Part Lesson Structure

I've gotten way too much mileage out of that three-part lesson model. I only wanted something that was simple and easy to understand with no jargon. For example, the Connected Mathematics approach of *launch, explore, summarize* is equally good if not essentially the same thing.

What is important in this model is the middle phase (during, *explore*) in which students are working on a problem or task without having been directed by the teacers how to solve the problem. Here is where the teacher must "let go" and really allow the students to make mistakes and develop their own ideas.

In my opinion, good lessons must have this time for students to develop ideas while attempting to solve a problem – the during phase. However, if this is not followed by a rich discussion this approach will almost certainly fail. Too many teachers fail to reserve time for discussion. There is also a skill in developing a classroom atmosphere where students talk to each other, evaluate other responses, and truly discuss ideas. The development of a mathematical community of learners is the key to allowing students to struggle with mathematical ideas.

As Jim Hiebert puts it in the NCTM books *Teaching Mathematics Through Problem Solving*, "problem solving leads to understanding" and understanding should be the ultimate goal every day. Contrast a show-and-tell approach with a problem-based approach. With the former, the teacher is dependent on students paying attention to her wonderful explanation or the one in the book. Even if the student is paying attention, almost certainly the focus is on the directions or rules that the teacher is giving and not on the core mathematical concepts involved. Even the good students know that there is a page of exercises to be done as soon as the explaining is over.

With a problem-based approach the student has nowhere else to turn other than his or her own ideas. As a result, rather than looking for rules, students attempt to make sense of the relevant ideas embedded in the problem or task. Even if the problem is not solved, their own relevant ideas have been engaged. The class discussion that follows will be meaningful and interesting. Ideas are developed and integrated with each learner's existing understanding.

For me it is also extremely important to consider what each approach is saying every day to students. With teacher-directed wonderful explanations, students often view mathematics as a collection of rules that often become confused and have little meaning. Imagine sitting every day in a classroom and trying to acquire a set of rules that really make little sense to you. How boring! In the problem-based classroom students daily experience the most basic fact of mathematics: math makes sense. Further, they come to realize that they are the ones who are capable of making sense of mathematics. Learning mathematics is fun!

John Van de Walle on Mathematical Communication

Language, both written and spoken, is the way that we, as humans, express our own thoughts and experience the thoughts of others. In a problem-based classroom the important language is not that of the textbook but that

VOTE
MARCH 16 - APRIL 13

ALL MEMBERS TAKE NOTE!!
OAME ELECTIONS ARE ONLINE

Annual elections for positions on the Executive and Board of Directors will be online from March 16-April 13. To vote, you must activate your online account on the web site at **www.oame.on.ca** if you have not already done so. Click on "Members Only" and follow the directions to activate your account. You will find your membership number and expiry date on the envelope of your *Gazette*.

From March 16 to April 13 you will find "OAME Elections" in the links on the left side, after you have logged in using your username and password, created when you activated your account. Just follow the instructions. Short biographies for all candidates are included.

If you need assistance, contact the Executive Directors

which reflects students' ideas. By expressing their thoughts students communicate with one another authentically, without being concerned about the right word or the correct way to write the symbolism. This reflective discussion about ideas is where learning occurs.

Do not take this to mean that vocabulary is not important. As with any discipline, mathematics has its own vocabulary and labels for important ideas and conventions. It is important that students acquire this vocabulary so that they can communicate effectively and can read mathematics as they progress through school. But, most importantly, teachers need to allow the ideas to develop first. Only when an idea is clearly developed is it time to apply conventional labels and use mathematical terms and symbols. Students in the fourth grade can slice a 38 by 24 rectangle into two or more parts so as to easily determine how many squares are inside. Only after this approach is well established and makes sense is it reasonable to label the process an application of the distributive property.

I've come to believe that students should write in mathematics classes every day, even in kindergarten and certainly in the upper grades. Writing should be an integral part of solving the problem – words, pictures, and numbers that show how the problem is solved and why the solution makes sense. There are three good reasons for requiring writing as part of solving daily problems. First, writing requires reflective thinking about the ideas involved and thus it enhances learning. Students refine and clarify their ideas and often correct their own errors. Second, writing provides the teacher with a record of student thinking since it is impossible to listen to every student every day. This written record of what is going on in each student's mind is a key to helping students who need assistance and challenging those who are capable. It is the information teachers need to reflect on today's lesson and correctly plan the task for tomorrow. Written work that is occasionally kept can be used to talk with parents and even create grades. Finally, writing is a rehearsal for the discussion phase of the lesson. When all students have been writing about their solutions there is no reason to ask for volunteers to share, a practice that always results in the same three students giving the answers that tend to shut down further discussion. When students have written, any student can be called upon to share what they have been thinking about or how they solved the problem. The written work serves as a script of sorts so that they are not fumbling for words or ideas.

John Van de Walle on Manipulatives

Although the term "manipulatives" is currently the word of choice for tangible materials, I prefer the term "models" to manipulatives. Rather than talk about when they are most valuable, I think the more appropriate issue is to understand what they can and cannot do to help children learn. First we need to understand that kids' learning is a product of reflective thought, not working with their hands. That means that we cannot judge the value of an activity by the presence or absence of a physical model. I think there are lots of excellent activities that are best done without a model. For example, comparing two fractions to decide which is larger: if done with a model, all that is done is to make each fraction and look at them. No thinking occurs about the numbers involved, relationships to 1/2 or 1, the relative size of fractional parts, etc. That is because the model removes the need for such thinking.

Models for various numbers (small whole numbers, multi-digit numbers and decimals, fractions and percents, integers) should be introduced to students early on in the development of these number systems. Models are simply pedagogical conventions that have been invented to represent numbers. They make sense to us because we already understand the ideas that the model represents. For someone beginning to learn about these numbers, their emerging concepts can be tested against these models in the same way a student might ask a teacher if he or she has found the correct answer. If the model fits with the student's ideas, then there is reason to proceed.

For most activities, once models have been introduced and are familiar to students, they should simply be available for student use as the student desires. Of course there are activities where models should be "off limits" as in the fraction comparison example. There also are times when the teacher might require students to use a particular model because it fits best with the activity.

The greatest danger with models, and I believe it is significant, is to directly guide students in the use of a model for getting a computational result. Base-ten blocks for addition and subtraction are one example where we overly direct students so that there is more student attention placed on how to use the blocks than on developing a useful computational procedure. Another example might be showing students how to use two-color counters to model integer computation. In both cases, once it is understood how these models represent the numbers involved, students should work at developing their own procedures so that the thinking is on the procedures involved, not on the models.

John Van de Walle on Calculators

My books have always said, "A calculator should be in or on the desk of every student, every day, from kindergarten through high school." I think of a calculator first as a model for numbers. Imagine skip-counting 100 by steps of 0.1 and then by steps of 0.01. The relative value of these decimal fractions is inescapable.

Of course the calculator can and should be used as a computational tool, primarily to allow for realistic computations that no sane person outside of school would do without a calculator. Students should never be asked to do by hand computations that are more tedious than typical adults would attempt without a calculator.

Of course calculators can be misused. But this is the same issue as manipulatives. The teacher is in control and can decide when students may and may not use them. Why is this so difficult?

It has been said that computational facility should not be an indicator of mathematical thought or sophistication. That means that we should allow all capable children to use calculators to access mathematical ideas without being held back due to weak computational skills.

What I've said mainly applies to four-function and fraction calculators. The value of the graphing calculators is, like computer software, so compelling that it seems to me ethically wrong to not begin using them at least by 8th grade and I believe much earlier than that.

John Van de Walle on Authentic Assessment

In the early 1990s there was much talk about performance assessment. This was good for education and for mathematics education in particular. However, what I find to be amazing is that teachers continue to make some distinction between a performance assessment and an instructional problem or task that requires students to think, struggle, and to develop ideas. Of course there is a place for summative assessment. However, when teachers teach through problem solving there is an almost continuous stream of information coming from students that tell us what it is they understand and how they understand it. I see no distinction between assessment tasks and tasks designed for learning. When students are required to

make sense of a problem, show their thinking, and defend the validity of their results, no test that I know of can give the teacher a better picture of what the student knows.

The 1989 standards document from NCTM said that we should shift away from assessments that can only tell us what students do NOT know and toward assessments that tell us what they DO know. A test that looks only at answers, no matter if in open-response format or multiple choice, can only tell us what students do not know. Problem-based, student-centered teaching is a daily form of performance assessment. Use it!

John Van de Walle on Professional Development

We are still looking for the silver bullet that will solve all of our professional development problems. As a result we search for PD materials, exemplary videos, and expert presenters, and the effects are negligible at best.

As a profession we need to understand that learning about teaching must be ongoing and life long. Second, we must understand that teachers must think deeply and analytically about teaching and reflect on what they are doing in some form of structured mode that is built into their weekly schedules. Leaders and coaches have the task of orchestrating this reflective activity in partnership with other teachers. The common goal of everyone must be student learning.

To this end some form of joint planning involving two or more teachers and a leader or coach if possible should take place regularly. Mutually planned lessons should be taught and then looked at by all involved – via video or live observation or both. Analysis should be on the lesson and not on the teacher.

Ongoing reflection of this sort can take many forms. Lesson study is the complex end of the spectrum with one-to-one coaching at the other. Grade-level meetings can be arranged for teachers to work on a lesson even without a coach. What is most difficult is for a teacher to be self-reflective without anyone to use as a sounding board or a different view of what just went on in the classroom.

Orchestrated reflective activity about planning and teaching is about learning how to learn about teaching. It is not about creating super lessons for the file.

John Van de Walle on Kids

Believe in kids! Give them the opportunity to solve problems and make sense of mathematics. You do not need to tell them everything. Kids are amazing! Given the chance they will develop ideas that you could never have thought of – ideas much more interesting and just as valid as those found in your textbook. Students who are engaged in making sense of mathematics will be less engaged in disrupting your class. In short, let the kids do the mathematics. Let's believe in our kids! Let's give them the opportunity to think!

John was an exemplary educator and mathematics teacher leader whose intellectual energy, creativity, and commitment inspired us all. There is no doubt that we will continue to benefit from his many contributions via print, web and video resources; nonetheless, we will miss him as a colleague. An ancient proverb states that while good men must die, death cannot kill their names. Those words must surely have been written with John Van De Walle in mind.

References

Kelly, C.A. (2006). A Conversation with John Van de Walle. *The Colorado Mathematics Teacher, 39*(2), 7-12. ▲

▲ Upcoming Events

For further information on OAME Chapter Conferences contact the Chapter Representative and for NCTM conferences, contact John Kersely, OAME/NCTM Rep.

See the last page in this *Gazette* for contact information. Also visit **www.oame.on.ca**

OAME Chapter Conferences and Events

March 29 2007	St Basil-the-Great College Toronto 4 - 7:30 pm	TEAMS
April 26 2007	Brock University 4 - 7 pm	GOLDEN
June 1 - 2 2007	OMO 2007 : University of Ottawa	COMA
October 11 2007	University of Waterloo	GVMA
October 19 2007	Annual Conference: St Charles College S.S., Sudbury 4 - 11 pm	NOMA

OAME Annual Conference:

May 3 - 5 2007	Georgian College Barrie, ON "Building Bonds"	MAC^2

NCTM Annual Conference

March 21 - 24 2007	Atlanta, GA Annual Meeting and Exposition	

* Editor's note: Nearly every Chapter will have an OMO playdown in early spring: check the OAME web site for Chapter links and information.

BACKGROUNDER
EWORKSHOP.ON.CA / ATELIER.ON.CA

MEDIA INQUIRIES CONTACT
STEPHANIE WESTLAKE (CONSULTANT)

eworkshop.on.ca / **atelier.on.ca** is a one-of-a-kind, multi-faceted, online, teacher resource available in both English and French versions that address learning challenges specific to students of each language; it combines a series of feature modules addressing numeracy and literacy from kindergarten to grade six with other new initiatives from the Ministry of Education.

This online tool is the best available curriculum-based resource for Ontario teachers; it provides a series of modules related to numeracy, literacy and leadership that use animation, photos and video clips taped in real classrooms to support and demonstrate the latest concepts for teaching. Other key features include printable classroom resource materials and links to additional educational resources. Unlike other stand alone teacher resources, **eworkshop.on.ca** / **atelier.on.ca** also addresses topical concerns including youth oriented health issues such as allergies and daily physical activity, and the site is regularly updated which makes it an essential resource for all elementary teachers who can bookmark the site for frequent visits.

Teachers also have the option to register online to participate in an educational forum and to receive important bulletins with up to the minute news related to education at the elementary level.

The **eworkshop.on.ca** / **atelier.on.ca** resource is the result of a partnership between TVOntario and the Ministry of Education. TVOntario has been a partner in education with the Ontario government since the 1970's, offering programs linked to the Ontario curriculum and developing online expertise through a range of online services that complement its educational television programming.

View the site at **www.eworkshop.on.ca** or **www.atelier.on.ca**. ▲

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▲ Friday Special!

JACK WEINER



Fridays at the University of Guelph, five minutes before the end of class, I say, "Pens down, books closed, it's "Friday Special" time!" Then I show on screen an article or cartoon or letter or...something that has to do with math, the "real world" and has absolutely nothing to do with class, assignments or tests. It is math for fun. Often, it is some flagrant example of how the media abuse math. I hope you use some of these. Make them "Monday Morning" specials or "We need a break!" specials. If one falls flat, blame me. If it works, you take the credit.

It is January 3, 2007 as I write and March something, 2007 as you read. A belated Happy New Year to you all.

In this *Friday Special*, I would like to share an experience I had when, "several years ago" (okay, 1980!), I was teaching a grade 9 class at Parkside High School in Dundas, Ontario.

Joel was working on this two-part question.

1. Solve for x in each of the following:

(a) $3x - 6 = 32$ (b) $3x - 12 = 32$

Here was Joel's COMPLETE solution.

(a) $x = 8$ (b) There is no solution.

Now when I looked over his work, instead of just saying, "WRONG, WRONG, WRONG!", I asked Joel to explain how he arrived at his answers. It turns out that he had a completely logical way of doing math in which his answers were absolutely correct. Of course, he was living in a different mathematical universe than you and I.

I have shared this example at many conferences. I always leave some time for participants to try to find a "consistent" way of solving these problems so that Joel's answers are correct, at least, correct in this alternate universe.

Before reading further, try this yourself. Find a method of solving these problems where the first answer is 8 and the second is "no solution."

**********INSERT THE JEOPARDY THEME******

Okay, are you back? Any success?

Many of you will have discovered the method in Joel's madness. He reasoned in (a) that "thirty SOMETHING minus 6 = 32. Well, 38 – 6 = 32. So x = 8." Oh, in Joel's world, x is a digit.

He continued, "In (b), thirty SOMETHING – 12 must be a number in the 20's or even smaller! It can't be 32! There is no answer!"

WOW! To Joel, x is a digit! He somehow missed the class(es) where we decided that 3x meant 3 times x.

This is not a mathematical mistake. It's a misunderstanding of a notational convention. And we math teachers are at least partly at fault.

Consider this: if r = 3 and $x = \frac{1}{2}$, then $rx = \frac{3}{2}$.

However, if we substitute for r and x directly, we would write $3\frac{1}{2}$. You and I interpret this as $3 + \frac{1}{2}$.

But Joel and all students can ask, "How did times turn to plus?" Darn good question!

By the way, inevitably, some teachers at these conferences find other alternate universes.

(a) $3x - 6 = 32$. Therefore, $3x = 38$. Cancel the 3's and $x = 8$.

The method has to work in (b) as well. So …

(b) $3x - 12 = 32$. Therefore, $3x = 44$. We can't cancel the 3 so there is no solution.

Here is my favourite teacher solution.

(a) $3x - 6 = 32$. Cancel the 3's so that $x - 6 = 2$. Therefore, $x = 8$.

Does this work in (b)?

(b) $3x - 12 = 32$. Cancel the 3's so that $x - 12 = 2$. Now cancel the 2's so that $x - 1 = \ldots$ Wait a minute! There is no longer a right side! The right side has become the **black hole of mathematics**! We can't solve it if there is no right side! No solution!

Math teachers: what a twisted bunch we can be.

Best wishes, Jack ▲

▲ Call for Manuscripts

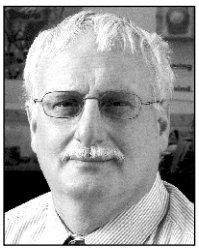
The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor

Marilyn Hurrell
for further details.

▲ OAME/NCTM Report March 2007



JOHN KERSLEY

A few months ago NCTM introduced its next step in the implementation of the NCTM Standards.

It is called, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. In NCTM's view these curriculum focal points are "clusters of related knowledge, skills and concepts" of topics at each grade level that need to be understood as a student develops mathematically. In short they are essentially the students' building blocks in mathematics. Their purpose is to present a way for teachers to focus on the teaching, learning and assessing of mathematics. Together these focal points will result in a connected, coherent body of mathematical knowledge and preparation for future mathematical learning. For further information the web site is **www.nctm.org/focalpoints**. The focal points can be examined on line and, most importantly, can be downloaded in its entirety or by individual sections. NCTM is now considering a task force to continue this for grades 9 through 12.

In 2004, NCTM published a series of books called, *Empowering the Beginning Teacher of Mathematics*. There were three volumes: elementary, middle and high school. As these publications have become quite popular with teachers, NCTM has decide to produce a companion series for teachers' mentors. In the near future expect to see *Empowering Mentors of Teachers of Mathematics: Elementary School*, *Empowering Mentors of Teachers of Mathematics: Middle School*, and *Empowering Mentors of Teachers of Mathematics: High School*.

Current Publications from NCTM

Mathematics Sampler Series There are four books to the series.

- Prekindergarten- Grade 2 provides sample assessments for use prior, during and after mathematics lesson.
- Grades 3 - 5 contains sample assessments sorted by number operations, algebra, geometry, measurement and probability and rubrics
- Grades 6 - 8 contains sample assessments compiled from various sources again sorted by number operations, algebra, geometry, measurement and probability and rubrics
- Grades 9 - 12 provides sample assessments to indicate students' thinking, includes multiple choice, short and extended responses

Mathematics through Paper Folding: an old favourite that is becoming popular again, demonstrating reflections, transformations and symmetry

How to Enrich Geometry Using String Designs: another popular favourite that is also making a comeback. Good for making classrooms attractive, interesting and decorative

Contact me or check the web site, **www.nctm.org** for further details.

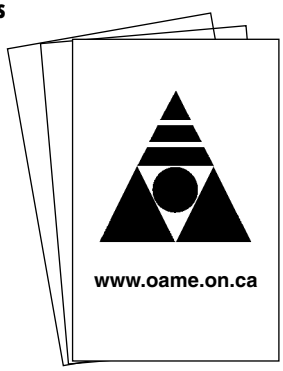
Web Sites of Interest to Mathematics as Suggested by NCTM

A Maths Dictionary for K-6: **www.teachers.ash.org.au/jeather/maths/dictionary.html** Check site for clear, concise definitions and pictorial examples.

Infinite Algebra: **www.kutasoftware.com** Software to create your classroom examples, classroom

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assignments, homework assignments, quizzes, tests, reviews, and exams. Offers free trial.

Math Open Reference: **www.mathopenref.com** Web site on geometry for student learning with interactive tools and animations.

Math Challenge: **www.whitehouse.gov/kids/math** Obvious political connection, but for all ages, lots of problems.

Shelley's Mathematics Articles: **homepage.mac.com/shelleywalsh/MathArt/mathart.htm** Provides much information on all levels of mathematical topics, including history, basic arithmetic algebra, geometry and trigonometry.

NCTM Regional Conferences and Expositions: www.nctm.org/meetings

- Richmond, Virginia, October 11 – 13, 2007
- Kansas City, Missouri, October 25 – 27, 2007
- Houston, Texas, November 29 – December 1, 2007

NCTM Annual Meeting and Exposition

Atlanta "Mathematics: Representing the Future" March 21-24, 2007

There will be over 1000 presentations, seminars or workshops dedicated to offering teachers better ways to meet the needs of students with emphasis placed on the exchange of ideas in teaching and establishing networks of teachers on an international basis. ▲

▲ Call for NCTM Rep

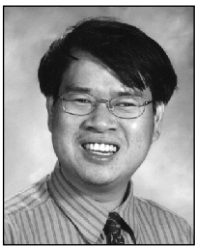
OAME is taking applications for a three-year term as NCTM representative commencing in September 2007.

Job shadowing available now.

For more details, please contact:
Executive Directors
Dave and Sue Hessey
(519) 471-6324
EDs@oame.on.ca

▲ Get Interactive!

Louis Lim



Louis Lim is department head of mathematics at Richmond Hill High School, York Region DSB. Louis' involvement with the OAME include being a former editor of the OAME's Ontario Mathematics Gazette*, presenter at chapter and annual conferences and elected board director. In spring 2005, he was recipient of OAME's "Exceptional and Creative Teaching in Secondary Mathematics" Award.*

Problem Solving Resources

In this edition of *Get Interactive!*, I provide on-line resources for problem solving. Central to learning mathematics, problem solving is one of five process standards of the National Council of Teachers of Mathematics (2000) and one of seven mathematical processes of the revised Ontario mathematics curriculum.

As I reflect on my eleven years as an educator, my professional life has been spent aligning mathematics curriculum, instruction and assessment. While doing so, I have forgotten how much I enjoy pondering about a mathematical exercise or problem. That is one reason why I enrolled in York University's Master of Arts programme for mathematics teachers. One of the courses I am taking is "Problem Solving", taught by Professor Martin Muldoon. I just wish I had more time to solve the bi-weekly problem sets! I think if we expect students to problem solve, then as teachers we should as well. These include engaging in George Polya's Problem Solving process (understand the problem; devise a plan; carry out plan; look back), and formulating our own questions.

Numerous print resources exist for Problem Solving. Professor Muldoon cited Paul Zeitz's *The Art and Craft of Problem Solving*, which contains 660 problems, to "get lost for some of the time…at the conclusion of the trip, the reader will be toughened and happy and ready for more adventures. And he or she will have learned a lot of mathematics" (p. xii). Zeitz's book, requiring at least knowledge of 1st year university calculus, provides many interesting problems to challenge the classroom teacher.

For students, I am thrilled with Shawn Godin's new "What's the Problem?" column in the OAME *Gazette*. Further, each of the NCTM's journals *Teaching Children Mathematics (K-6), Mathematics Teaching in the Middle School (5-10),* and *Mathematics Teacher (8-14)* has a problems department for students.

1.) University of Waterloo Mathematics Contests

The University of Waterloo's Centre for Education in Mathematics and Computing web site, **http://cemc.math.uwaterloo.ca/english/contests/index.shtml**, houses past contests and solutions to grades 7 and 8 Gauss; grade 9 Pascal and Fryer; grade 10 Cayley and Galois; grade 11 Fermat and Hypatia; and grade 12 Euclid and the Canadian Open Mathematics Challenge. The Gauss, Pascal, Cayley, Fermat, and Euclid contests are suitable for all students, while the Fryer, Galois, Hypatia, and the Canadian Open Mathematics Challenge are recommended for our top 25% students. Not only can past contests prepare students for upcoming contests, they can supplement our mathematics programs and Math Clubs.

2.) Word Problems for Kids

Word Problems for Kids, **http://www.stfx.ca/special/mathproblems/**, is a popular site with over 3 000 000 visitors since 1999. The site is maintained by Dr. Ping Wang from the mathematics department at St. Francis Xavier University in Nova Scotia. Each grade, from 5 to 12, contains a collection of 40 to 50 problems, with hints and solutions. I have used some of the problems and am impressed with both the quality and quantity of problems.

3.) Project InterMath

InterMath, **http://intermath.coe.uga.edu/new InterMath /index.htm**, contains an extensive collection of problems for middle school and high school mathematics teachers. Organized into 4 strands, each then subdivides into particular topics: (i) Algebra (patterns, functions, equations, graphing); (ii) Geometry (circles, quadrilaterals, triangles, polygons, 3-D objects); (iii) Number Concept (fractions, decimals, ratio and proportions, percents); and (iv) Data Analysis (statistics, probability). Some of the problems make use of technology (e.g., *Geometer's Sketchpad*, graphing calculators, spreadsheets). Most problems have extensions and external links that provide further information.

4.) Elementary Problem of the Week

The Math Forum has archived *Elementary Problem of the Week* from September 1996 to June 1997, available free at **http://mathforum.org/sum95/ruth/96.97/index.html**. Suitable for grades 3 to 6, each week contains a problem posed, along with student exemplars provided from schools that submitted solutions. Unfortunately, teachers interested in accessing the web site, now called *Math Fundamentals Problem of the Week* must register by paying $25.00.

5.) Ken's Problem of the Week

Ken Duisenberg has been providing problems of the week since 1996, **http://ken.duisenberg.com/potw/**. Problems are referenced to the originators that provide references to additional print resources. Detailed solutions are provided.

6.) Problem of the Week Math Contest

David Rock, dean of the College of Education at Columbus State University, maintains the *Problem of the Week Math Contest* web site, **http://www.colstate.edu/mathcontest/**. Each week, 4 contest problems are posted: Problem of the Week; Elementary Brain Teaser; Middle School Madness; and Algebra in Action. Students are encouraged to submit their work, with acknowledgement on the web site for correct solutions. Pre 2006 problems are available at **http://www.olemiss.edu/mathed/contest/past.htm**.

7.) Math Counts Problem of the Week and Go Figure

This American project provides middle school students with enrichment in mathematics, **http://www.mathcounts.org/**. Archived since 1996, there are many challenging and interesting problems that can promote the problem solving process. The *Go Figure* problems are indexed according to level of difficulty.

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Zeitz, P. (2007). *The art and craft of problem solving (2nd ed.).* Hoboken, NJ: John Wiley & Sons. ▲

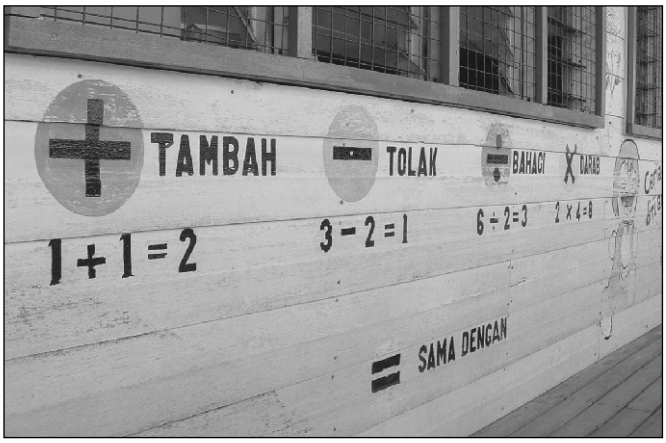
▲ Photo Math - The Fine Art of Viewing the World Through a Mathematical Lens

RON LANCASTER



Ron Lancaster is a Mathematics Lecturer in the Department of Curriculum, Teaching and Learning, the Ontario Institute for Studies in Education (OISE/UT) at the University of Toronto, 252 Bloor St West, Rm 11-162, Toronto Ontario M5S 1V6

Teachers and students are encouraged to put on a pair of mathematical glasses and become a math spotter, someone who sees mathematics everywhere and then possibly poses mathematical questions based on the images taken. Ron Lancaster, the current Editor for Photo Math, welcomes photos with questions for possible publication in future columns. Material can be sent via e-mail or by regular mail to either of the two addresses given above.



Photographer: Roberta Koss, robertamath@earthlink.net
Location: Local school on the Island of Libaran (in the Sulu Sea), Borneo

(a) Are here any errors in the calculations shown on the side of this school?

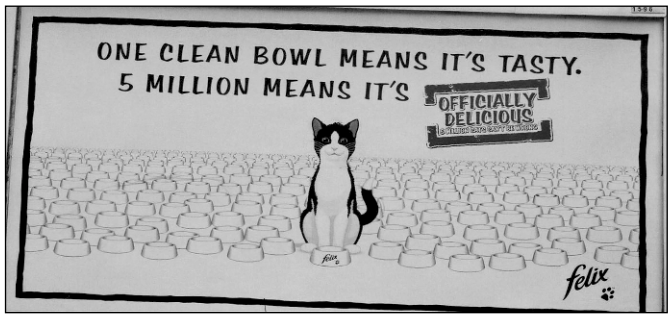
(b) What language is this?



Photographer: Roger Turton, rturton@sbschool.org
Location: Greenfield, Massachusetts

(a) Do all of these rims have rotational symmetry?

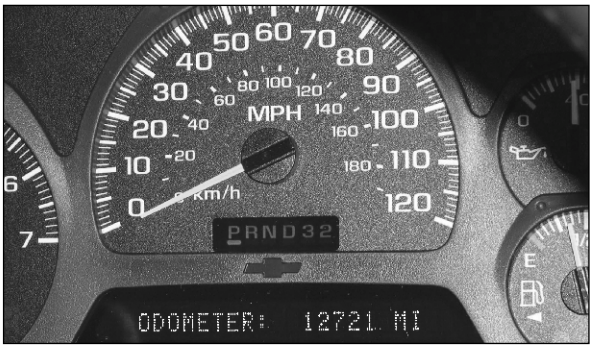
(b) Which rim has the smallest perimeter? Which one has the biggest area?



Photographer: Ron Lancaster, ron2718@nas.net
Location: Bristol, England

(a) Estimate the number of bowls in the billboard.

(b) How much larger would the billboard have to be if the image contained 5 million bowls?



Photographer: Dave Bradley, dave.bradley@granite.k12.ut.us
Location: Salt Lake City, Utah

(a) Aside from being odd, what type of number is displayed on the odometer?

(b) How far will the car have to be driven before this type of number makes another appearance? ▲

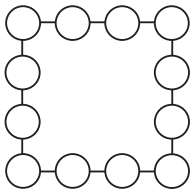
▲ What's the Problem?: Sliding Cards

Shawn Godin



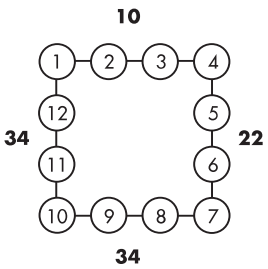
Shawn is the head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans, ON.

Welcome back problem solvers. When last we met I left you with a problem that appeared as M198 from the May 2005 issue of *Crux Mathematicorum with Mathematical Mayhem*. The solution appeared in the May 2006 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem*. web site **http://journals.cms.math.ca/CRUX/**.



Each of the integers from 1 to 12 is to be placed in one of the circles in the figure so that the sum of the integers along each side of the figure is 25. Determine the sum of the four integers placed in the corners.

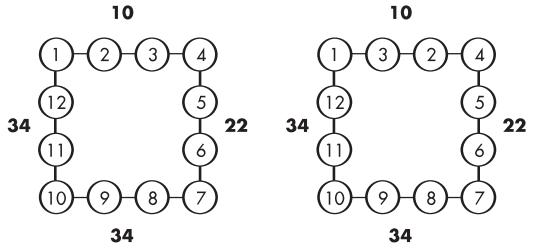
This is one of those problems in which the answer is the least interesting part about it. Most of the students that submitted solutions submitted a configuration that worked and got their sum of the corners from there. When I have given this problem to my students, that is what most of them tend to do. An interesting thing starts to happen when I get them to present these "solutions". It isn't long before they start to see that there are many ways to get a configuration that satisfies the condition. I then ask them how they know that they have the only possible solution. Lets backtrack and look at what is going on.



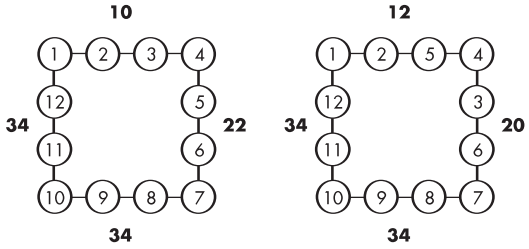
When I give this problem to my students, I like to give them number tiles, or cards numbered 1 through 12 that they can move around and experiment with. Suppose we are using numbered cards and start by arranging them in the obviously wrong pattern (right).

From here we could start randomly moving cards and hope for the best, or we could look for some patterns. Suppose we decide to switch two cards and observe what happens to the sums of the sides. I will refer to cards as being corner cards if they occupy one of the four corners and side cards otherwise. There are only a few different ways that we can switch two cards.

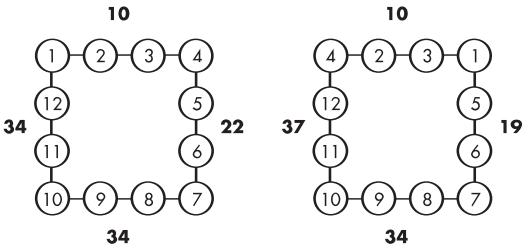
1. Switch two side cards from the same side. This has no effect on any of the sums and isn't of much use (other than to note that if we have a winning combination, we can create a number of other winning combinations by just switching side cards, but this isn't too interesting). So for example if we switched the 2 and 3 the sums would be the same.



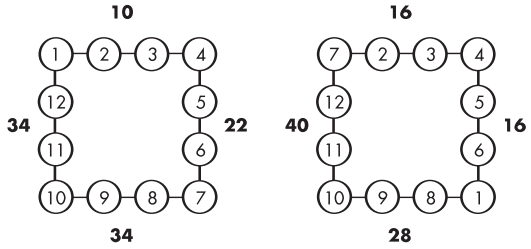
2. Switch two side cards from different sides. In this case the sum on one side goes up and the sum on the other side goes down by the difference between the cards being switched. So if the 3 and 5 were switched, one side would go up by 2 and the other down by 2.



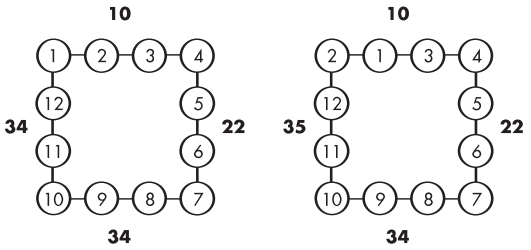
3. Switch two corner cards from the same side. In this case the sum of one side goes up and the sum of the other side goes down by the difference between the two cards being switched. The sum on the side that contains these two corners stays the same. So if the 1 and 4 were switched, one side would go up by 3 and the other down by three.



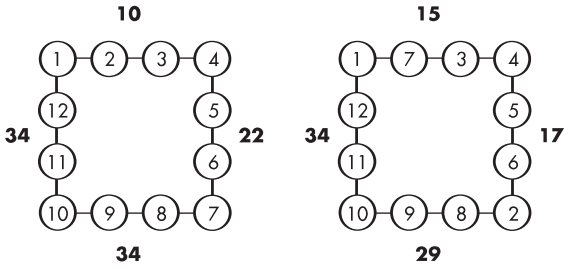
4. Switch two corner cards from different sides (diagonally opposite). In this case the sum on two sides goes up and the sum of the other two sides goes down by the difference between the two cards being switched. If we switched the 1 and the 7, the sum on two sides would go up by 6 and the sum on the other two goes down by 6.



5. Switch a side card with one of one of the corners on the same side. In this case the sum on the side where we made the switch stays the same while one other side goes up or down by the difference between the cards being switched. If we switch the 1 and 2, one side's sum goes up by 1.



6. Switch a side card with one of the corners not on the same side. In this case three sides change, one going one way (up or down) and the two attached to the corner go the other way. For example if we switch the 2 and the 7, one side goes up by 5 and two others go down by 5.



There are a couple of things that we notice at this point. Looking at the first four cases we observe that they make no difference in the sums of the corners and they make no differences to the sum of the sums of the sides. Since the sum of the sums of the sides is 100, it seems that it might be possible to use these four moves to make the sums on the four sides 25, and answer the question. (This will indeed work, I found one with 4 moves. Can you find one with fewer?)

If you look at case 5 and 6 you see that there is an "imbalance" and thus the sum of the sums will change. A little thinking shows that if you have a case where the sides sum to 25, if you want to change the sum of the corners you have to switch a corner with a side, but this changes the overall sum of sums. The only way to bring it back is to do a move that brings the sum of the corners back to what it was originally.

As with many problems, this one has a more insightful solution. If we start with the condition that all the sides have sums of 25, then if we add all 4 sides we get 100. When we do that, each number has been accounted for and the corners have been added twice (once for each side they are on). Since all the numbers from 1 through 12 add up to 78, the four corners must add to 22. If you check out the featured solution from the web site you will note that the solver used this argument, but with a little algebra to illustrate it.

I really like this problem for several reasons:

1. Students at all levels can engage in the problem.
2. There is a nice insightful solution.
3. The problem doesn't have to end with the answer, there are several things that we can explore.
   i) What sums, other than 25, are possible for all 4 sides.
   ii) You can turn this into a game. Starting from a "random" arrangement of the numbers, players see how many moves (of switching 2 cards) it will take to make all 4 sides have the same sum, or some specific sum.

Have fun exploring this problem. For more on this and similar problems you may want to check out the article "Magic Triangles: Beyond the Elementary Idea" by John Grant McLoughlin in the May 2006 issue of *Crux Mathematicorum with Mathematical Mayhem*.

Now for Your Homework

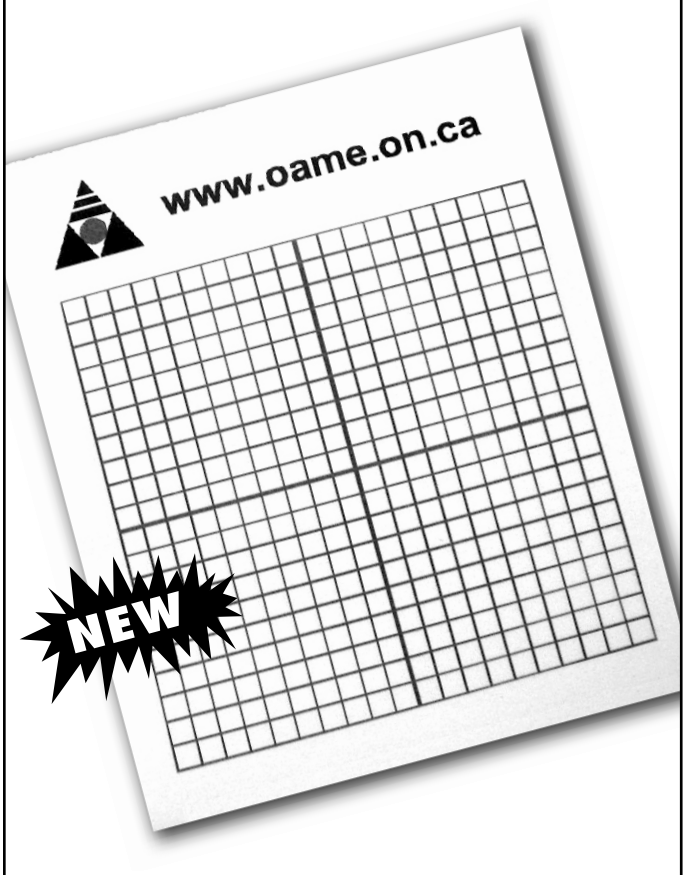
Consider the following procedure for dividing the three-digit number 375 by 8. Write down the number formed by the first two digits, namely, 37. Multiply this by 2 to get 74. Add to this the units digit of 375 (the original

number), obtaining 74 + 5 = 79. Then divide by 8 to get 9 with a remainder of 7. Add this result (9, remainder 7) to the number 37 (the first two digits of 375) to get your answer: 46, remainder 7. Thus, 375 divided by 8 equals 46 with a remainder of 7.

Does this method always work for three digit numbers? Why, or why not?

Until next issue, happy problem solving! ▲

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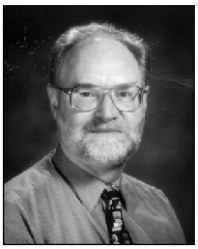
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▲ CAS Corner: White Box vs. Black Box

FRED FERNEYHOUGH



Fred Ferneyhough is former head of mathematics at Harold M. Brathwaite SS in Brampton. He retired from there in February 2006. Fred is an avid user of and advocate for the use of technology in the mathematics classroom. He is also an author and tech consultant for several books and a National Instructor for Texas Instruments.

Through the leadership of Math Coordinator Irene McEvoy, 21 secondary schools in the Peel District School Board have embarked on a journey in the use of CAS (Computer Algebra Systems) with a focus on grades nine and ten applied mathematics. Last year, seven schools were provided with a class set of *TI89* calculators and two days of training. In the current school year, 21 schools were given a class set of *TI89*s and three days of training. By the end of the school year, these *TI89*s will be exchanged for class sets of Texas Instruments' newest device, called *Nspire*.

So, what have they been learning? One of the key concepts that they have been looking at is two methods of using these devices – the white box approach and the black box approach. For this issue, I will look at solving linear equations in grade nine and systems of equations in grade ten as examples of the two approaches.

The black box approach would represent a use where the device is just a tool in the hands of the students. Refer to Figure 1. Students who are learning to solve equations would understand nothing about the method of solving equations if they were taught to use the "Solve" command on the devices. If our objective is to have students understand the algebra in solving equations, this method has failed the students badly.

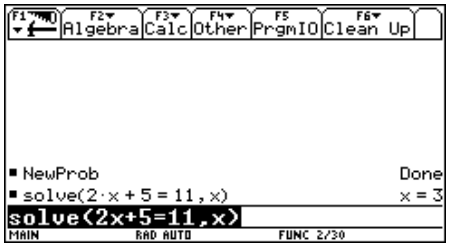


Figure 1

Contrast this with the white box approach. William G. McCallum of the University of Arizona defines a white box as "a transparent device that shows each step of the calculation, although the steps themselves are carried out by the machine". (The reference here is taken from William G. McCallum, University of Arizona, Tuscon, Arizona, in "COMPUTER ALGEBRA SYSTEMS in secondary school mathematics", ©2003, The National Council of Teachers of Mathematics, Reston, Virginia. ISBN 0-87353-531-6).

Using this approach, a student using the device will step through the solution using the same steps that we would expect using pencil-and-paper, with the device doing the mechanical steps. To begin, the student types in the equation and presses [ENTER]. If they understand the ideas behind solve equations, their next step would be to subtract 5 from each side. On the *TI89* or *TI92* devices, pressing [−][5] displays as (2x + 5 = 11) -5 with the result 2x = 6: see Figure 2. Teachers have a lot of problems with the form of the command on the left side of the screen. Students have fewer issues with this. The *TI Nspire* avoids this problem in its output.

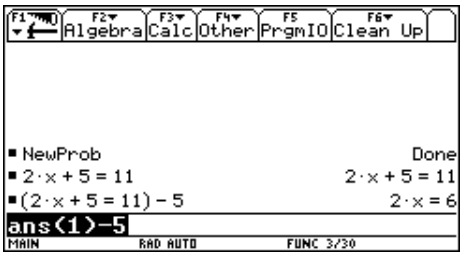


Figure 2

To complete the command, press [÷][2] as shown in Figure 3. Using this approach, the device completes the calculations that the student initiates.

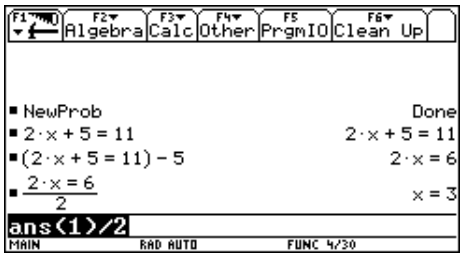


Figure 3

Another advantage of the white box approach is the option of allowing students to make the types of errors commonly done by novices and learn from those errors. For the equation that we have been looking at, many students think that the numeric term "5" can be moved to the other side of the equation by adding 5 to both sides. So, let them try. The resulting equation is obviously incorrect as it is no simpler than the original equation. In fact, it is worse. From here, we can either have the student start over or take the correct step by subtracting 10 from each side in the next line.

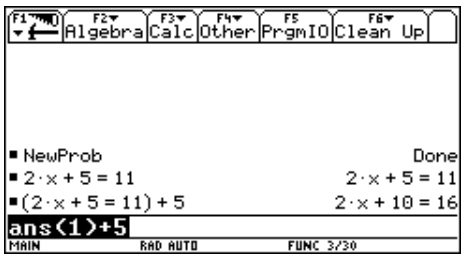


Figure 4

CAS devices can also be used with algebra tiles to give a variety of solution methods. However, it must be noted that the objective remains to develop algebraic skills. I had a great deal of success with this approach and found the students were able to develop the skills they needed with the assistance of the CAS device and tiles.

For systems of equations, the black box approach involves a similar approach as for linear equations. The logical connector "and" is inserted between the two equations. The student presses [ENTER] and the solution is displayed – see Figure 5. However, the student understands absolutely nothing about solving a system of equations other than how to manipulate the device.

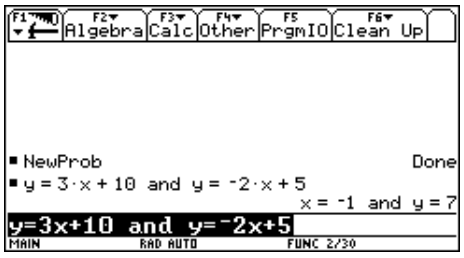


Figure 5

Let's look at a white box approach to this problem. This one is very easily done since the setup lends itself to the method of substitution. In the second equation, substitute 3x+10 for y to get the result "3x + 10 = –2x + 5" – see Figure 6.

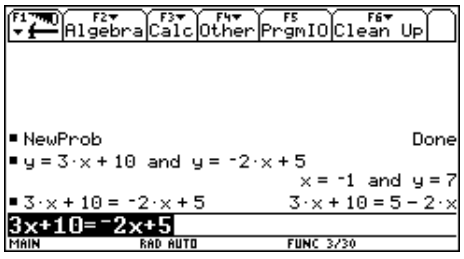


Figure 6

This is simply a linear equation and the student can use the device to solve for x in any way that the teacher prefers. In Figure 7, the white box approach has been used to solve for x. Note the final command where the device has been used to substitute x = –1 into the first equation to get the value of y.

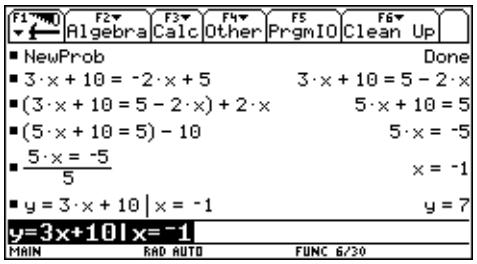


Figure 7

The system presented is rather simplistic, so let's look at something where the student has to isolate a variable as their first step. The system below adds a level of complexity that we need:

3x – 4y – 6 = 0

2x + 3y + 13 = 0

In Figure 8, the white box approach has been used to isolate y. Note that the choice of variable y is completely arbitrary in this example.

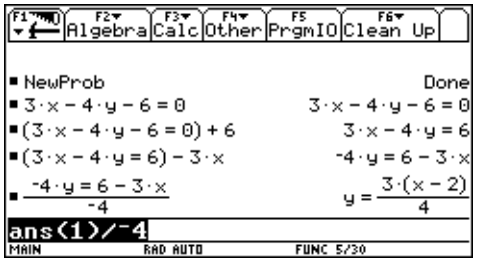


Figure 8

Once y has been expressed in terms of x, the resulting expression is substituted into the second equation in place of y. The result of this work is a linear equation in variable x. The device takes care of the expansion of the factors and simplification of the expression shown in Figure 9.

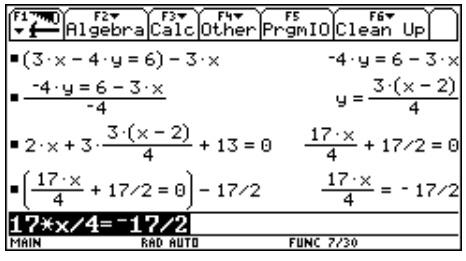


Figure 9

The work to solve this equation is shown in Figure 10. In the final command on this screen, the value of x has been substituted back into the original first equation.

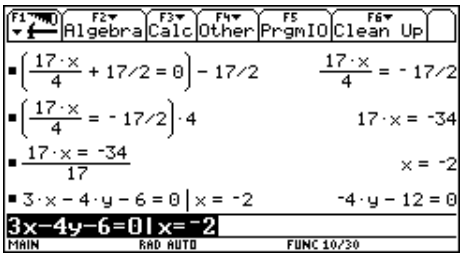


Figure 10

Finally, we solve for variable y as shown in Figure 11. Throughout this example, students will need to follow exactly the same sequence of steps that we would have expected them to do with pencil-and-paper.

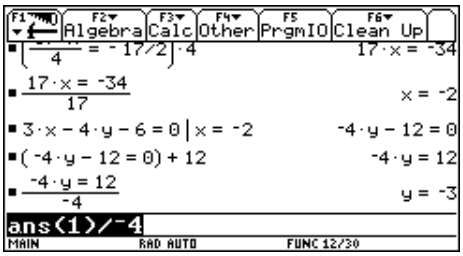


Figure 11

The "Solve" command is a dangerous command to show to students. Once they have seen it, they will not want to go back to the process of stepping through the solution as we would like. If they use the black box approach with the systems of equations shown in Figure 12, the answers will not make any sense to them. (A black box solution just comes up with an answer like magic, no reasoning included.)The white box approach lends itself better to these systems. I never showed the Solve command to my classes until after any assessments pertaining to this topic. However, once they have completed the basic work, I often gave them access to the CAS devices to solve equations using the Solve command.

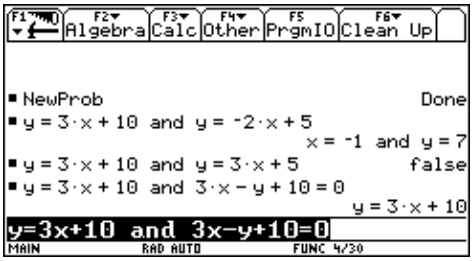
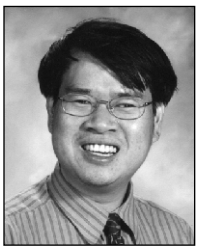


Figure 12

CAS devices are being used in many Ontario schools in several boards. I realize that there are many teachers who strenuously object to the use of these devices for fear that their students will only know how to use the devices. I have found the opposite, that teaching with these devices promotes understanding of algebraic properties. ▲

▲ Audio CD Review: *Math, Music and Mayhem – the Sequel*

Reviewed by: Louis Lim



Louis Lim is department head of mathematics at Richmond Hill High School, York Region DSB. He recently completed a 3-year term as editorial panel member of the NCTM's Mathematics Teacher and is former editor of the OAME's Ontario Mathematics Gazette. In spring 2005, he was recipient of OAME's "Exceptional and Creative Teaching in Secondary Mathematics" Award.

Math, Music and Mayhem – the Sequel is an audio CD consisting of songs, problems, and activities, created by Dave Mitchell. Mitchell is a southwestern Ontario mathematics educator with over 3 decades of teaching experience. This resource is a must for grades 7 to 12 mathematics teachers. Like the original CD (see September 1999 issue of the OAME *Gazette*), the sequel emphasizes the art of teaching. As a mathematics educator for over 10 years, I have searched endlessly for innovative resources. I believe Mitchell's CDs are truly effective since they invite and motivate students to enjoy mathematics.



My favourite part of the sequel CD is the collection of songs, which include *Good Guys/Bad Guys; (a + b)²; and Cosine Law*. Each song can be played several times during class for students to recall the tune, which allows them to remember the mathematical concept, procedure, or formula. Here are the lyrics to the *Cosine Law Song*:

First form of the cosine law has a^2 on the left,
Then after the equal sign, you write down all the rest.
There's b^2 plus c^2 minus 2bc times the cos of angle A.
To find side a or angle A, this formula's OK.

Second form of the cosine law has b^2 on the left,
Then after the equal sign, you write down all the rest.
There's a^2 plus c^2 minus 2ac times the cos of angle B.
To find side b or angle B, on this one we'll agree.

Third form of the cosine law has c^2 on the left,
Then after the equal sign, you write down all the rest.
There's a^2 plus b^2 minus 2ab times the cos of angle C.
To find side c or angle C, this one's for you and me.

Using music addresses a variety of learning styles (e.g., visual and auditory). The 3 forms of the Cosine Law can be written on the chalkboard, and while the song is repeatedly played as closure to the lesson, the teacher can point to the formulas. Hopefully during the next class, students will "complain" that they could not get the song out of their head (be it at dinner or in bed) – then you know they can recite the 3 forms of the Cosine Law! Also, observe your students while they write their unit test – don't be surprised if you see students' lips moving when they write the forms of the Cosine Law!

Mitchell uses his extensive teaching experience of what works to provide analogies and strategies to help students grasp a particular skill or concept. For instance, the *Good Guys/Bad Guys* song describes how to multiply (and divide) integers: If a good guy (+) wins (+), that's good (+); if a good guy (+) loses (–), that's bad (–); if a bad guy (–) wins (+) that's bad (–); if a bad guy (–) loses (–), that's good (+). I must admit I have sung the song on my way to work! Mitchell is not only an innovative mathematician, he is also a talented musician!

Another strength of this resource is the opportunities to problem solve. Although I am familiar with most of the problems provided on the CD (e.g. prime number generator using $n^2 - n + 41$, with n a natural number), I am reminded to use interesting problems to promote thinking and problem solving. One problem is to take any natural number. If it is even, divide by 2. If the number is odd, multiply by 3 then add 1. Repeat the process with each result until you end up with 1. (e.g. $20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$). I like the way that Mitchell presents the problem with an example or two and then challenges students to, in this particular problem, to determine which number from 2 to 100 will

require the most steps. Hence, opportunities not only to think but also to communicate!

I can't wait to talk with my students about Shakuntala Devi, who is able to compute calculations in her head faster than a computer! A *Google* search found 31 500 sites on this "human calculator", who calculated the 23rd root of a 201-digit number in her head in 50 seconds, while a computer took 62 seconds. Truly extraordinary and guaranteed to generate student-student dialogue!

Math, Music and Mayhem – the Sequel is available for $12.00 (shipping included) at **http://www.arithmecode.com**. Buy it – it's level 4+ material! Both on-line and print order formats are available. I hope Mitchell will create *Math, Music and Mayhem – the Sequel*. I do have one complaint for Dave Mitchell: for some reason, I just can't get the Cosine Law out of my head.

Reference

Lim, L. (1999). Announcing the three best kept secrets in math resources: Music, video, and the Equation Editor. *Ontario Mathematics Gazette, 38*(1), 20-21, 23. ▲

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▲ What is Water Pressure? Reflecting on a grade 10 academic performance task

Donna Kotsopoulos

Susan Lavigne



Donna Kotsopoulos is a secondary mathematics teacher in the Thames Valley District School Board. She is also a doctoral candidate at the J. G. Althouse Faculty of Education, University of Western Ontario.



Susan Lavigne is a mathematics teacher with over 30 years experience at both the secondary and post secondary levels. She is currently teaching in the Thames Valley District School Board.

Introduction

One of the challenges we face as mathematics educators is the development of learning opportunities that both address curricular expectations and simultaneously provide a measure of authentic, active, and relevant experience for students. As Jarvis (2006) points out, learning opportunities that engage students in active, reflective, and deep mathematical thinking happen thoughtfully and purposefully. These learning opportunities, as Jarvis suggests, referencing the Expert Panel on student Success in Ontario (2004), "do not just happen magically by rearranging desks and bodies" (p. 35).

In this article, we share a recent quadratic relations performance task, *What is water pressure?* (see Appendix A), developed collaboratively for our grade 10 academic students during the first semester of the 2005 – 2006 school year. We[1] detail the activity and some student responses that highlight some of the challenges that arose for students. We also describe throughout this piece our process of developing this task, demonstrating not only thoughtful and purposeful intent but also the timeliness of a collaborative endeavor. Our intention in this article is to share our process of developing the task, some issues that arose for students and to share the task itself.

[1] *The pronoun 'we' refers to the authors of this paper.*

The Performance Task: *What is Water Pressure?*

In this performance task, students were asked to investigate the water flow from the drinking taps in our school which project water in the form of a parabolic arch. The context for this assignment was set around water pressure and how differing water pressures changes the flow of the water. Therefore, taps with higher pressure have a larger flow of water whereas taps with perhaps lesser pressure have a smaller flow of water from the fountain. We set the 'ideal' tap to have water pressure that generated a water flow that reached no more than 3 cm above the flow guard of the tap (see Appendix A).

Students were expected to determine various forms of the quadratic function that modeled the current flow of the tap they were investigating. They were also asked to model the ideal flow of the tap based on parameters provided within the assignment. The task was open-ended given that each fountain had a different flow-rate. Consequently, we also saw this as a good 'problem' in that there was no apparent or common solution.

The performance task was completed in one 70 minute period, at the end of the quadratics unit, in both of our classes. Students in each class were semi-randomly paired together. Students' names were drawn from a hat. The pairings were semi-random in that, as classroom teachers, we reserved the right to make modifications as required. For example, neither two very strong students nor two very weak students were paired. Only one such modification was necessary in one of the classes.

After reading through the problem as a whole class, the students then moved to the fountain immediately outside of our classroom to brainstorm possible dilemmas that might arise such as the depth of the fountain bowl in relation to where the x-axis might be imagined. Following this discussion, the whole class returned to the classroom for any final questions and to reiterate the assignment expectations. Students then dispersed throughout the building to investigate their assigned fountains. Students had access to their class notes and textbooks. Calculators were permitted. One report was to be submitted per pair on chart-sized graph paper which was provided to the students.

Purposeful, Intentional, and Collaborative

The development of this performance task took approximately 2 months. Our process included the key components identified in the National Council of Teachers of Mathematics (NCTM 1991): we started with an idea, checked it against our course expectations, developed a plan, drafted an assessment approach and completed the task ourselves before implementing the activity. During this time, we also researched the topic of water pressure.

We met numerous times to discuss the task and components of the task. These initial meetings were brief and occurred during the school day during our planning period. We began preliminary drafts of the performance task and the assessment tool to be used. Many versions of the task were exchanged between us via email and in person. When we felt that we were close to a final draft, we met yet again to do a trial-run of the activity to allow us to reflect on any challenges that might arise for students or for ourselves in evaluating open-ended student responses. We also met following the completion of the activity in each of our classes to discuss and compare student responses, particularly those pairs in each class that had been assigned the same water fountain.

Assessment

One of our key considerations was how we would assess students' results from this open ended task. Given that student responses and approaches would vary, we developed a task-specific rubric (see Figure 1) to examine student thinking (Suurtamm, 2005). We felt that this format gave students the most amount of feedback about their work. The task-specific rubric was provided to students and explained at the beginning of the task.

Student Dilemmas

We saw throughout this performance task significant evidence of what the current curriculum guidelines for grades 9 and 10 call "good problem solvers" (Ontario Ministry of Education/OME, 2005, p. 14). Students actively discussed, deliberated, and modified their data as they proceeded to completion. Furthermore, we were pleased to see a number of unanticipated student responses.

After the students were sent off to their own fountains, we were initially surprised at how quickly they returned back to the classroom. We had anticipated that a significant portion of the allotted time would be devoted to time at the fountains because we spent a significant amount of time at the fountain during our trial-run of the task. Once students started to compile their report, however, they reflected on their problem solving

	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
Graphs	☐ draws graph of existing water flow OR ideal water flow ☐ partially labels graph	☐ graph of BOTH existing water flow and ideal water flow ☐ fully labels graphs of both existing water flow and ideal water flow	☐ graph of both existing water flow and ideal water flow in such a way that it is clear that all necessary measurements were accurately taken and incorporated into the graphs ☐ fully labels graphs of both existing water flow and ideal water flow	In addition to Level 3 ☐ draws the graphs with an artistry that clarifies and enhances the understanding of what these graphs represent ☐ explains any adjustments made to measurements
Equations	☐ most equations are incorrect or missing ☐ proper form is infrequently used ☐ little narrative is used	☐ most equations are determined correctly ☐ proper form is usually used ☐ some narrative is used	☐ all equations are determined correctly ☐ proper form is used ☐ appropriate narrative is used	In addition to Level 3 ☐ standard form is used to check the vertex form and factored form equations
Report	☐ a recommendation is made unrelated to the data collected	☐ a recommendation is made based on the data collected	☐ a recommendation is made based on the data collected ☐ reasons are given in support of the recommendation	In addition to Level 3 ☐ the report is written in a style that engages and convinces

Figure 1: What is water pressure rubric.

strategies and recognized that they indeed needed more time at the fountains to gather additional information such as the depth of the bowl, height and width of the flow, and so forth. All of the groups went back to their fountains to gather more data.

One surprising outcome was the way in which students represented their data. We had inaccurately anticipated that students would visualize their graphical representation in the second quadrant of the Cartesian plane - as they saw it! When we discussed this during our trial-run, we agreed that although it would be mathematically more straightforward for students to work in the first quadrant, we had anticipated that the students would not do the transformation independently but rather would work with a graphical model that closely resembled what they saw. Most students surprisingly did the transformation of their water flow to the first quadrant (see Figure 2).

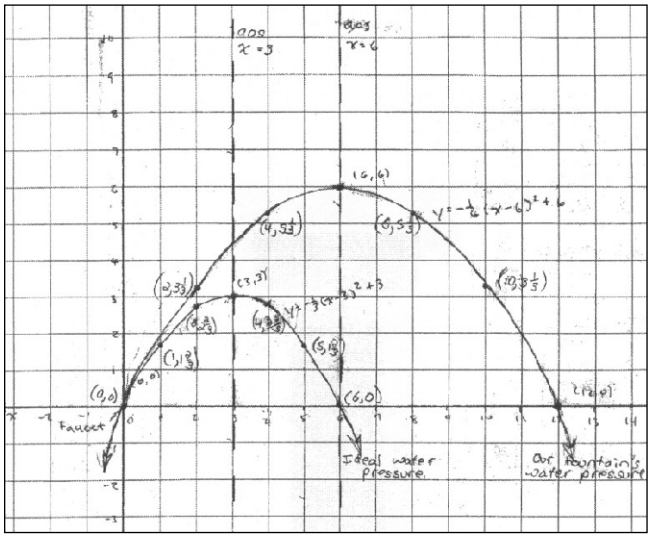


Figure 2: Level 4 example of students' graphical representation of their actual water fountain and their ideal water fountain

In our trial-run, we positioned the x-axis along the bottom of the fountain bowl. Indeed, we suggested this during the whole-group brainstorming. What we found however, is that most groups shifted the x-axis to the height of the fountain guard. Although this made their calculations easier, it also shows significant problem-solving skills at work.

Most students used precise measurements in their calculations rather than rounding to whole numbers, as we had done in our trial-run. The students' approach may be more consistent with the ways in which mathematics is done by "quasi-mathematicians"[2] in real-life context where preciseness is paramount (Shockey, 2006). The students' responses were closer to real-life mathematics than we were.

Concluding Comments

The NCTM suggests that assessments should be valuable tools that enhance student learning while simultaneously informing the pedagogical decisions of teachers (2000, pp. 22-23). Our task was valuable in both of these ways. The task itself provided for a richer and more interesting and active experience for our students. Our collaboration afforded us the opportunity to reflect together on the types of mathematical objects the students constructed and our pedagogical practices and assumptions framing this experience. Our intent in this paper has been to share our resource, but also to share our experiences as educators developing this task to illustrate the timeliness of thoughtful planning and the benefits to the teacher and students that this sort of investment yields.

Appendix A

What is water pressure?

When a tap is opened and water flows from it, it is because the water is "pushed." This "push" is pressure. The speed at which water flows from the opened outlet depends on the amount of "push" or pressure which exists at that time in the system.

What is wrong with high water pressure?

High water pressure has some advantage, such as in firefighting systems. However, in the home plumbing system, it can be damaging because water, with a strong "push" behind it, can erode or wear away many materials and cause leaking water heaters, banging water pipes, dripping faucets, dishwasher and clothes washer noise and breakdown, and leaking water pipes. Probably most important advantage to lower the pressure water pressure is the potential savings to the water bill.

How much can we save with lower water pressure?

Reducing water pressure is directly proportional to the potential savings. For example, a $\frac{1}{3}$ reduction in water will potentially result in a saving of approximately $\frac{1}{3}$ because $\frac{1}{3}$ less water flows at this lower pressure.

[2] By quasi-mathematicians we refer to those individuals who engage in mathematics although they may not themselves perceive themselves to be mathematicians. Schocky (2006), for example, discusses the mathematics of cardiac surgeons.

Your assignment:

Some of the water fountains at our school have lower water pressure than others. Ideally, the water flow should reach a maximum height that is 3 cm above the faucet guard. Attached is a map of our school that identifies the water fountains. You and your partner will be assigned a water fountain to study. Your task is threefold:

1. Determine the equation of the quadratic relation that models the water flow for your fountain. Present this equation in three forms: a) vertex form, b) factored form, and c) standard form.
2. Determine the equation of the quadratic relation that models the ideal water flow for your fountain. Present this equation in vertex form.
3. From this information you must make a recommendation regarding your fountain (e.g., Should the water pressure be increased or decreased?). Present your findings in a single report format. Include a fully labeled graph as part of your report. You have one period to complete this assignment.

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▲ Sizing Up the Metric System

HELENE J. SHERMAN



Helene J. Sherman, E.d. D., is the Associate Dean for Undergraduate Education in the College of Education at the University of Missouri-St. Louis and is a professor of education, specializing in the field of mathematics education. She has taught at the elementary, middle school mathematics, community college and at the university level where she currently teaches undergraduate and graduate courses in mathematics methods for grades K – 8. Dr. Sherman is the co-author of Teaching Children Who Struggle with Mathematics*, published by Pearson/Merrill Prentice Hall in 2005. Dr. Sherman has published in numerous national education journals, among which are* Teaching Children Mathematics, Teaching Mathematics in the Middle Grades, Educational Research Quarterly, Science Scope *and* Science and Children *and the* Ontario Gazette. *She served as the mathematics consultant for Money Math and co-authored three activity books on teaching metric measurement across the curriculum as well as nine mathematics activity guides. Dr. Sherman also directs and co- directs Federal and statewide grant projects in St. Louis area school districts, focusing on ways to implement her research related to K- 8-classroom application of mathematics teaching methods. She regularly presents workshops and talks throughout the country for districts and at academic conferences on methods for teaching mathematics to elementary and middle school children. . Dr. Sherman has won several teaching awards, among which are the 2003 Missouri Governors Award for Excellence in Teaching, 2000 College of Education Distinguished Teaching Award, 1998 Emerson Electric Outstanding Teaching Award and the 1997 Distinguished Alumni Award from the University of Missouri-St. Louis.*

Students can effectively integrate science and mathematics concepts by constructing, experimenting with, and adapting their own measurement tools. As they test and label their products with accurate numerical notation, children learn and practice concepts and skills in both disciplines in an enjoyable and practical manner. Reasoning their way through the following hands-on activities and then using the conclusions in practical situations advances the goals of both the Curriculum and Evaluation Standards for School Mathematics and the National Science Education Standards.

The following ideas focus upon estimation, a powerful tool for learning observation and measurement relationships. Through these activities, students connect science and mathematics not by chance, but in a purposeful manner. They observe an object's physical properties and use whole and decimal numbers to approximate and then record actual lengths, volumes, and masses. When learners move to the application stage by estimating long distances or unfamiliar quantities, they are using scientific thinking by considering variables and hypotheses.

Constructing a Meter Tape

In constructing meter tapes, students visualize units before estimating and measuring sizes of objects. Moreover, the activity allows students to discover the base-10 relationships fundamental to metrics, place value, and numeration.

Using an unmarked strip of adding machine tape that is somewhat longer than 1 m, pairs of students complete the following discovery experience:

Starting at one end of the tape, students place a base-10 rod (a plastic rod measuring 10 cm in length with 1-cm divisions marked) lengthwise on the paper and mark where the rod ends. Then they move the rod so that the end is aligned with the first mark and place another mark where the rod ends. They repeat this process several times until they reach the end of the paper tape.

After marking off the first 10 sections, students orally name each section as "one out of 10" and write 1/10 in each one. Students cut off the rest of the tape.

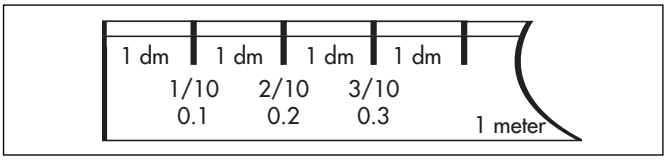


Figure 1. Constructing the Meter Tape

Students number the sections with decimals; this activity lends itself to introducing decimal notation in a meaningful way. They record 1/10, 2/10, … , 10/10. Then they write the appropriate decimal number in each section, beginning with 0.1 and ending with 1.0 (see Figure 1). This activity develops the knowledge that the notations 10/10 and 1.0 represent the same length as the strip of 10 dm. Learners can use this activity to discover that the strip is made up of 10 congruent

sections, a concept fundamental to the metric system and to understanding decimals.

Since students are not able to "discover" names, tell them that the entire length is called a meter, and that the term for each segment that measures one-tenth of the whole is a decimeter. It helps students to relate the prefix deci with the word dime and to connect the quantity of one-tenth for both terms. The symbol "m" and "dm" are written on the paper strips.

One fifth-grade student exclaimed, "Measuring by counting to 10 is much easier than counting to 12 or 36 for inches and yards," while he moved the base-10 rods along his meter tape.

Discovering the Centimeter

To develop an understanding of the length of a centimeter, students continue the activity on the same tape. Still working in pairs, students mark each decimeter off in tenths, using a unit cube or markings on the base-10 rod.

Introduce the word centimeter and its symbol and analyze it for the root word meter and the prefix centi. Students discuss the fact that if the entire tape were marked off, there would be 100 cm in the meter. Compare centimeters to a penny (one-tenth of a dime and one-hundredth of a dollar).

On another piece of paper (because 1 cm is too small for children to write on), students record 1/100 for a centimeter and write the decimal for each one. Thus, students learn to write 0.01, 0.02, 0.03, and 0.04 until they count 10 decimeters on the tape.

Some children may notice that 10 cm fill the same amount of space as 1 dm. Students will develop these relationships over time as they become more adept with the sizes and place values for decimals. Working with these place-value relationships is integral to understanding the future concepts involving whole numbers and algorithms that are throughout the mathematics curriculum.

Table 1. Estimated and Actual Measurements

	ESTIMATE		ACTUAL	
OBJECT	cm	dm	cm	dm
Circumference of wrist				
Length of shoe				
Circumference of head				
Computer width				
Bathtub width				

Have students estimate and then measure their wrist sizes, shoe lengths, and head circumferences, then record the data in Table 1. This table can include objects found at school and at home, such as computers, bathtubs, cars, books, etc. Encourage students to measure different parts of different objects. It is important that students can now estimate by actually visualizing the decimeters and centimeters, developing a sense of what is reasonable from their experiences, not haphazardly guessing numbers. "They can reach intelligent decisions based on both the clear understanding of relationships and the content or application of these relationships" (Greene, 1993).

Students take the strips home to measure actual items in both decimeters and centimeters. Suggest items such as a refrigerator or a television and ask the students to add their own ideas. Students report and compare their measurements at school, so that they share their reasoning with the whole class.

After taking the tapes home to measure with their families, many students reported that their parents really enjoyed measuring with metric units. The parents admitted they didn't know much about metric measuring and thought they wouldn't like it until they found out how quickly and easily they could find the length of something by counting units of tens or tenths. Discuss the variables such as the shape of the measured items and how their shapes were compared with the strip.

Experiments with marble racing can also demonstrate variables and inferences. Students lean a grooved centimeter ruler against a stack of books for elevation and roll a marble from the top of the ruler (see Figure 2).

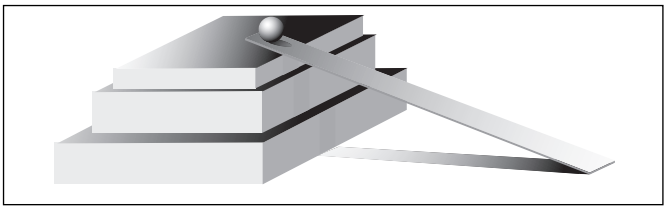


Figure 2. Marble Racing Experiment

They measure and record the distance the marble rolls in centimeters or decimeters. Students then raise or lower the ruler in order to understand the meaning of independent variables. The change in the elevation, the independent variable, can be made several times and measured with a vertical ruler from the surface to the top of the ruler. The distance the marble rolls each time the elevation is changed, the dependent variable, is recorded, and students can determine which elevation causes the marble to roll the farthest.

Discovering Volume

Constructing a cube model enables students to clearly visualize three dimensions and how volume is measured in the metric system. A piece of poster board or tag board works well for this model. Using their own constructed metric tape and colored pens, students outline a 30-cm square on the paper (see Figure 3). The lines are connected to form a large tic-tac-toe shape and the line segments to be cut are marked with a dotted line. Provide a pattern for students to follow as they cut, fold, and tape the sides to form an open box.

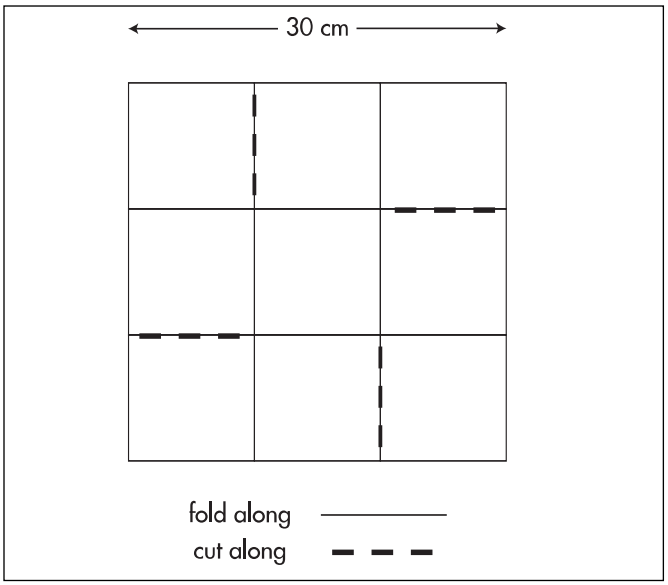


Figure 3. Pattern for 1 000 cm³ box.

Students examine the box to estimate how many cubic centimeter units might fit inside and then decide how to find the actual amount. Most suggest counting, but others object to the time that would take. Students can determine from their construction and by using the decimeter sticks or their meter tape that the area of the bottom of the box is 100 cm^2.

To test this hypothesis, students construct a 100 cm^2 or 1 dm^2 flat and try to fit it into the bottom of the box. Ten centimeter-high flats can be piled inside the box. Students get a feel for the volume concept by estimating and then fitting cubes into the box. Students draw the box, mark its dimensions in centimeters and decimeters, and record that 1 000 cm^3 can fit inside.

After constructing the boxes, a sixth-grade student happily declared, “This box explains what the drawing in my book with all those dotted lines is supposed to mean!”

Mystery Boxes

To practice volume estimation and measurement, play a game in which students examine several small cardboard boxes. Groups of two to three students first estimate each box’s volume by observing and comparing the boxes to the 1,000 cm^3 box. Groups record their volume estimates for the three of the boxes.

After each group estimates the volumes of the three boxes, they make the actual measurement with a metric ruler or by fitting cubic centimeters into the boxes. Groups compare their estimates to the actual measurements and record the differences in Table 2. The group with the smallest total difference wins the game.

Table 2. Mystery Boxes.

BOXES	ESTIMATE	ACTUAL	DIFFERENCE
1	cm	cm	cm
2	cm	cm	cm
3	cm	cm	cm
			Total

Students often depend only on number sizes and not on the processes of problem solving to determine whether they should add or subtract. This activity provides a meaningful way for students to understand what the term difference means and why subtraction is the appropriate operation.

Discovering Liquid Volume

For the study of capacity when measuring liquids, students can use plastic replicas of the 1 000 cm^3 paper boxes, or they can place tin foil inside their paper boxes to prevent water leakage.

As previously done, introduce the term liter for the box during the experience, not prior to it. Point out to students that the liquid capacity of each of these cubic centimeters is (for everyday usage) 1 mL. The reference for the size of a milliliter is made as students handle a container, such as a glass of water, and estimate how many cubic centimeter containers could fill it. “A knowledge of a wide variety of everyday measurement references is the foundation of good measurement sense as well as good number sense.” (Hope, 1989).

Water, Water Everywhere

Estimating liquids requires a different thought process than that for discrete or separate items and so must be carefully developed. In this volume-estimation game, students pour various amounts of water into a 500 mL beaker to provide a visualization from which they can begin estimating and direct measurement.

Give each group of students three cards marked with volume such as 500 mL, 150 mL, and 50 mL. Each group choose one of their cards, and each member takes turns pouring water into an unmarked container until they think they have reached the amount indicated on the card. The members then pour the water into the beaker to obtain the actual reading.

The groups compute the total differences between the estimated amount and the actual reading and enter the data into Table 3. Then they repeat this process for the other two cards. After all three cards are completed, the group adds the differences from each turn, and the group with the smallest total difference wins the game.

Table 3. Water, Water Everywhere.

TARGET	ESTIMATE	ACTUAL	DIFFERENCE
550 mL	mL	mL	mL
150 mL	mL	mL	mL
50 mL	mL	mL	mL
			Total

Students find that they understand that the term difference represents the difference between the estimate and actual measurements. At the end of the activity, one student reflected, "I can now really picture in my head what 500 mL looks like because I had to pour what I thought was correct and discovered it for myself. Just being told to pour a certain amount doesn't make me think as much as deciding by myself".

The Values of Measurement

Constructing and working directly with measurement tools allow students to construct their own concepts of the metric system and its application. Importantly, learners use that knowledge to develop awareness of length, volume, and capacity as well as quantity and place value.

Understanding and enjoying measurement in a stimulating and challenging environment leads to the kind of problem solving, reasoning, communicating, and valuing of science and mathematics we would like to encourage throughout the entire curriculum. Teaching metrics in this manner stimulates students' thought processes and provides interesting and novel approaches that get everyone involved.

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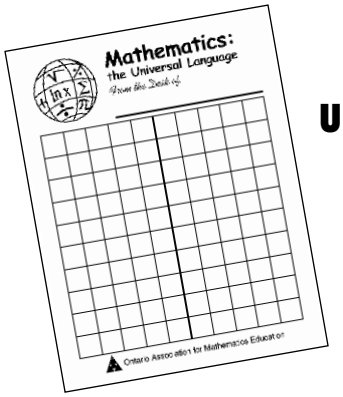


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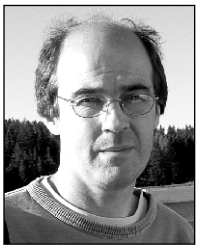
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▲ Playing Games with Mathematics

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John Grant McLoughlin holds a cross-appointment with Mathematics and Education at University of New Brunswick. John's mathematical interests include problem solving, recreational mathematics, and elementary teacher education.

Recreational mathematics offers a wonderful vehicle for developing mathematical thinking. Game playing arouses curiousity about the principles underlying the structures and strategies associated with the games. This is particularly evident when the players are students at a mathematical event such as the University of New Brunswick Math Camp. Four of the games and challenges that were posed at the camp are shared here, followed by an additional problem involving polynomials.

The purpose of the challenges is threefold: to broaden one's knowledge base of recreational mathematical ideas; to engage in playful mathematical activity; and to develop mathematical arguments and proofs in response to the challenges. This latter point is developed in a subsequent piece (Part III) that appears in the October 2006 issue of *Crux with Mayhem* and can be viewed using the link to October 2006 at **http://journals.cms.math.ca/CRUX/v32**. The essence of this article corresponds to Part I that appeared in September 2006. It should be noted that this two-part model was also used at the camp held in May 2006. The challenges were introduced to students in a brief presentation one day. The discussion of the mathematical principles, including insights gained from participating students, took place the next evening.

1. Sim

Six dots are drawn on a piece of paper to form the vertices of a hexagon. Two players are each assigned a colour. The players take turns joining any two of these dots with a line segment, using their assigned colour. The loser is the player who completes a triangle with three of the original six dots as its vertices and with all three edges the same colour.

Challenge: Prove that there must always be a loser (winner).

2. Game of 31

This mental math game involves a running total which starts at zero. Each player has the choice to add 1, 2, 3, 4, 5, or 6 to the total. Players alternate turns. The winner is the player who is able to bring the total to 31.

For example, Player A may choose 6; then Player B chooses 4 to raise the total to 10; Player A adds 2 to get 12; and the game continues, as shown in Figure 1, until Player B wins by adding 5 to 26 to obtain 31.

Player	A	B	A	B	A	B	A	B
Chosen number	6	4	2	4	4	1	5	5
Total	6	10	12	16	20	21	26	31

Figure 1

Challenge: Determine a winning strategy. You may choose to play first or second.

3. Chomp

Counters are placed in a rectangular grid such that one counter appears in each small rectangle. The counter in the bottom left-hand corner is a different colour than the others. Players take turns selecting one counter. If the counter selected occupies the bottom left-hand corner of a rectangle on the grid, all the counters in that rectangle are removed. The object is to force your opponent to select the differently coloured counter (the one in the bottom left-hand corner of the original grid).

Try playing this game with rectangular boards of different sizes.

Challenge: Suppose that you play two games of Chomp in which the boards would be 2 x n or k x k, examples of which are shown. Determine a winning strategy in each case. You may choose to play first or second.

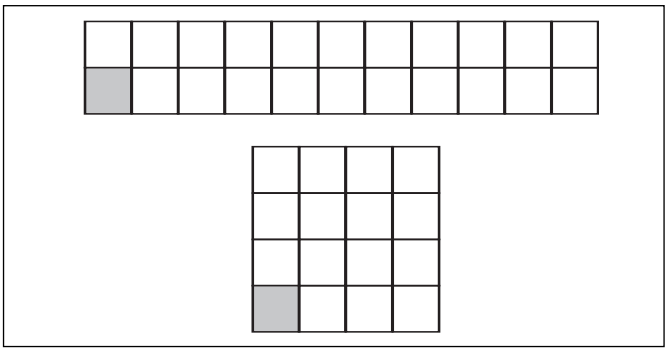


Figure 2

4. Fifteen Finesse

The numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 are available for use in this game. Each number can be used only once. Two players alternate turns selecting one of the available numbers. To win the game, a player must obtain exactly three numbers that sum to 15. (Neither a pair of numbers such as 7 and 8, nor a set of four numbers such as 1, 3, 5, and 6 constitutes a winning combination.) The game ends in a draw if no player is able to acquire three numbers that sum to 15..

Challenge: Explain the underlying structure of the game, and suggest strategies that may help to win a game.

5. A Polynomial in Transition

Consider the polynomial $x^2 + 10x + 20$. Under the following conditions, is it possible to transform this polynomial to become $x^2 + 20x + 10$? Justify your answer.

Conditions:

(i) On each step you may only change the constant term or the coefficient of x (but not both).

(ii) The change must be an increase of 1 or a decrease of 1.

(iii) The change must NOT produce a polynomial that can be factored into the form $(x + m)(x + n)$ where m and n are integers. For example, you could not begin by reducing 10 to 9, since $x^2 + 9x + 20 = (x + 5)(x + 4)$.

The bibliography lists a selection of sources from which the problems and ideas have been developed. As mentioned previously, discussion of the problems is expanded upon in the subsequent article available through the Canadian Mathematical Society (CMS) link. Feedback is welcomed. A pdf copy of Part II can be readily sent to interested readers who contact me via johngm@unb.ca. Enjoy the challenges and continue to play with math!

Acknowledgement: The author is grateful to the Canadian Mathematical Society for permission to reprint (revise) the article, Playing Games with Mathematics (Part I) from Volume 32(5), pages 275 and 276, of Crux Mathematicorum with Mathematical Mayhem.

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▲ Considering Sets of Problem Conditions

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*Bonnie Litwiller and David Duncan teach mathematics and mathematics education courses at the University of Northern Iowa. They have published over 850 articles in journals for mathematics teachers and have spoken at numerous professional meetings. Each has served as President of the Iowa Council of Teachers of Mathematics and Dr. Litwiller has served on the boards of NCTM, SSMA and AMTE.*Puzzles involving numerical relationships have often been used to motivate mathematics students. It is an especially serendipitous occurrence if the puzzle involves a logical "twist". We shall present one such example.

Suppose three people are sharing their small change. The following three conditions hold for numbers of cents possessed by each person.

1. The product of the three amounts is 72.
2. The sum of the three amounts is 35 or less.
3. One of the persons has more change than either of the other two.

Based on this information, can the separate amounts possessed by the three persons be determined? A reasonable first step is to list all possible sets of three positive integers whose products are 72:

1, 1, 72	2, 2, 18
1, 2, 36	2, 3, 12
1, 3, 24	2, 4, 9
1, 4, 18	2, 6, 6
1, 6, 12	3, 3, 8
1, 8, 9	3, 4, 6

Next, let us determine whether the second given condition can be used to eliminate any of those possibilities.

The sums are as follows:

1 + 1 + 72 = 74	2 + 2 + 18 = 22
1 + 2 + 36 = 39	2 + 3 + 12 = 17
1 + 3 + 24 = 28	2 + 4 + 9 = 15
1 + 4 + 18 = 23	2 + 6 + 6 = 14
1 + 6 + 12 = 19	3 + 3 + 8 = 14
1 + 8 + 9 = 18	3 + 4 + 6 = 13

The first two sets of amounts (1, 1, 72 and 1, 2, 36) are impossible since the sums are 74 and 39; this violates Condition 2.

The third condition also eliminates one of the possibilities. Since one of the people has more change than either of the other two, the 2, 6, 6 set must be eliminated. There still remain nine possibilities, with no way to reduce the list further.

Now suppose that a fourth condition is specified: Even if the sum were known, Condition 3 would still be necessary.

Note that this Condition 4 does not itself specify the sum but simply indicates that such knowledge would not be sufficient to identify a unique solution without the aid of Condition 3.

How can this new condition help? First observe that if the sum were known to be any of 13, 15, 17, 18, 19, 22, 23, 28, only one set of amounts would be possible. These are respectively 3, 4, 6; 2, 4, 9; 2, 3, 12; 1, 8, 9; 1, 6, 12; 2, 2, 18; 1, 4, 18; 1, 3, 24. In any of these cases the problem would be uniquely solved without the aid of Condition 3.

The necessity of Condition 3 therefore implies that the sum must be 14. For only that amount could there be two possible solutions, one of them being ruled out by Condition 3. The remaining solution would be the set 3, 3, 8.

These four conditions are not all of the same logical character. The first three directly address the numerical relationship among the amounts, while the fourth specifies a relationship among the previous conditions. The fourth condition is thus a sort of "meta-condition", a notion of great importance in the analysis of any logical structure.

Challenges for the Reader

1. Re-do this problem using a product other than 72 for which all four conditions are involved.
2. Restate this problem in a different context. ▲

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