Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership.
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceeding 10 numbered pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required. Please number pages.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including web sites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

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Submission of Advertisements

Advertisements for the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising Rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARILYN HURRELL, EDITOR



This is the fifth time I have welcomed you back to another teaching year and another Volume of the *Gazette*, and my wishes are always the same: enjoy this new school year and all the opportunities it brings.

This issue of the *Gazette* brings some changes in contributors. The *Abacus* section has new Co Editors in Kathy Kubota-Zarivnij amd Mary Lou Kestell, both past presidents of OAME and both of whom work for the Numeracy and Literacy Secretariat. You will notice the new look and format of the *Abacus* as they begin with a focus issue on Addition.

Jack Weiner has completed his tenure as columnist. See *From the Editor's Desk* for Jack's departing remarks.

OAME awards were given out at the Annual Conference last May in Barrie ON. Congratulations to all the winners. You will find their bios on pages 21-24 Look these awards over and perhaps think of whom you could nominate for an award next year; I know there are many talented and hard working teachers out there who deserve some recognition. You will find nomination forms in the December 2007 issue of the *Gazette* or online at **www.oame.ca**

Words of Wisdom:

"The dream begins with a teacher who believes in you, who tugs and pushes and leads you to the next plateau, sometimes poking you with a sharp stick called 'truth". - Dan Rather

Marilyn

▲ President's Message

JACQUELINE HILL, PRESIDENT



Happy Start to a New School Year One and All.

I hope you found yourself this summer relaxing and enjoying the great outdoors.

With a new school year comes a renewed sense of responsibility to the students we teach. My president's message is truly "From the Classroom and Beyond" and I hope that I can help the grassroots teacher with some of the initiatives that float around the education circles.

On our OAME front, local chapters are offering a variety of mini-conferences this fall. Just visit the OAME web site to check out the conferences nearest you. The OAME Leadership Conference will be held on February 21 and 22, 2008. The themes are: Emerging Technologies in grades 9–12, and in grades 1-8 Japanese Bansho (Japanese Lesson Study). The Annual Conference is being hosted by the Y^{4}MA chapter and will be held in Markham this year May 1, 2, 2008 . The Ontario Math Olympics will be held at Trent University, hosted by the PRMA chapter, for the next two years. The 2008 dates are May 8, 9 and 10. The 2009 dates are May 29 and 30.

At our June Board meeting we started to explore Differentiated Instruction in more detail. A related powerpoint can be found on the OAME web site if you are interested. To visualize what Differentiated Instruction is, picture teaching in the one room school house 150 years ago. The teacher needed to consider what they would do before, during and to assess the lessons. Here is a quick list of before, during and assessing.

Before (strategies to form groups)

- students show five, three, or zero fingers to indicate their prior knowledge of the topic
- administer a mini pre-test
- four corners activity to indicate what students know
- observe students complete a related task and take anecdotal notes
- allow a variety of options and approaches for students to demonstrate what they know (e.g., manipulatives, drawing)

During (How the different groups work diligently)

- decide whether to group students based on ability (homogeneous vs mixed groupings)
- consider size of groupings" pairs, triads,etc.
- consider personality traits, gender, attendance, etc. to maximize productivity
- allow flexibility in group members if needed
- group according to learning style
- allow choice in approach to practice activities: kinesthetic, technology, etc.
- regroup students and/or scaffolding and/or individual attention according to formative assessment (timely and specific feedback)
- involve students in self-reflection to mark own progress (metacognition)

Assessing (Now what to do with the products)

- clearly identify expected outcomes or "look-fors"
- allow for opportunity for students to demonstrate depth and breadth of mastery by providing range of problems/activities/samples
- allow students to design final product as long as it meets expected outcomes and level of work quality identified by teacher
- with the choice of product comes the choice of potential opportunity for marks
- self-assessment of personal progress and subject mastery
- peer assessment of teamwork (not of each other's aptitude!)
- portfolios

Until next issue,

Yours in Mathematical fun,

Jacqueline Hill

▲ From the Editor's Desk

I first met Jack Weiner at a *Focus Four* Conference in Toronto in the early 1980's. I attended his workshop and was captivated by his enthusiastic delivery of how to plan a math trail for Grade 12 General Level students. I joined GVMA at his suggestion and that led to my involvement with OAME as a whole.

At an OAME Board meeting in 2003, Jack gave me a draft copy of his book, *The Mathematics Survival Kit - How to Get an A in Math* and not only did I read it over, I still have it and sneak a peek at topics in it when I need them for tutoring. This book was reviewed by Louis Lim in the June 2004 *OMG*.

Last fall. I had the opportunity to log onto ®*Maplesoft*, and have a preview the e-version of *MSK*, facilitated by Tina George and Jack. This resource is aimed at high school students and particularly those who have gone on to university and have forgotten some of the mathematics they need for first year calculus, science and business courses. This is an interesting way for students to review using the technology that is such a big part of their learning.

Well, it seems Jack is gaining more success in this field as he pursues this phase of his teaching career. This results in a loss to the *OMG* of a fine columnist. I'll wish Jack all the best, and let him explain.

Marilyn

Hello everyone:

I am afraid I must take a leave of absence, for now anyway, from writing the *Friday Special* column. However, abandoning the *FS* will in no way increase my leisure time.

("Leisure: a seven letter word meaning ????")

In partnership with *Maplesoft* and (if funding permits) the University of Ontario Institute of Technology, I am chairing an exciting research in teaching project at the University of Guelph. The task: integrate the best of technology with the best of traditional teaching in math courses and later in physics, chemistry, computing, economics and more.

Throughout, data will be collected and analyses and correlations will be carried out. I am confident the project will produce, and the data will confirm, better, deeper, more sustained learning in our students. (We ran a beta version of this approach in the winter semester, with some astounding results.)

Watch for reports of our progress.

If you want to contact me, I am always available by email, snail mail, and phone.

Good luck. I wish the very best to all of you and all of OUR students. It takes a village to raise a child. It takes a profession to nurture a student.

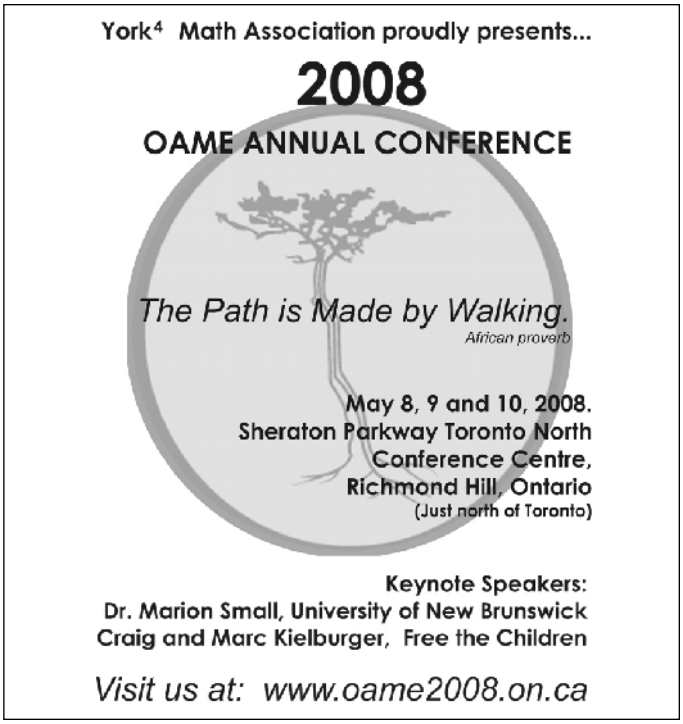
Jack Weiner, Associate Professor
Department of Mathematics and Statistics
University of Guelph

▲ Letter to the Editor

Dear Marilyn

Thank you for the March and June 2007 issues of the *Ontario Mathematics Gazette*. I was very impressed with the general layout of your journal. In particular, I enjoyed the integration of both elementary and secondary materials. Your mathematical/pedagogical balance is excellent, providing innovative content for a wide range of teaching levels. Finally, from an editor's point of view, I enjoyed the well-written and grammatically sound writing.

Yours truly
David Tambellini, Editor *Vector*
BC Association of Mathematics Teachers ▲



▲ Technology Corner: Features of the TI-84

MARY BOURASSA

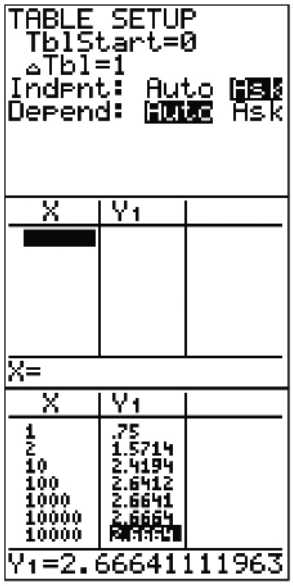


Mary Bourassa teaches mathematics at Lisgar Collegiate Institute in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She was also past-president of COMA, gives workshops around the world and is a National Instructor for Texas Instruments.

As a new school year gets going I thought it would be a good idea to take another look at the most commonly used technology at the high school level: the TI-84 and TI-83 families of graphing calculators. There are several features of the calculators that are not widely known. Read along and see how many of them you use!

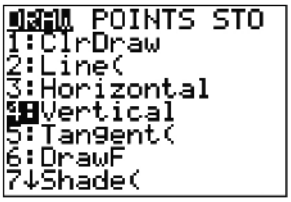
Auto vs. Ask

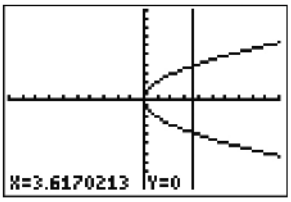
When looking at a table of values you have the option of starting at a particular value with a fixed increment or you can enter values of your choosing for the independent variable. This can be controlled through the TBLSET command found by pressing [2nd][WINDOW] . In this window you can set the starting value for the table as well as the increment. However, if you wish to enter your own values for the independent variable, change the next line to Ask instead of Auto. Now when you go to your table of values ([2nd][GRAPH]) you will see an empty table of values with a prompt at the bottom of the screen ready for you to enter an x-value. When you type in a value, the corresponding y-value will be calculated and displayed. This is a great tool when looking at limits. If you set the independent variable to Auto and the dependent variable to Ask, it will generate the x-values based on the table setup. Each corresponding y-value will appear when you press [ENTER] with the cursor on that dependent value's cell. Students can use this feature to check their work as they go when calculating tables of values.



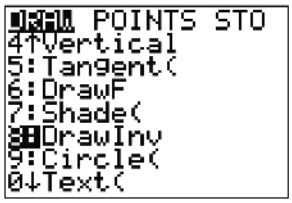
Drawing

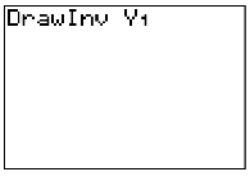
There are a couple of interesting commands available when introducing functions and their inverses. The calculator can draw a vertical line which you can then sweep across the screen. From the graph screen press [2nd][PRGM] then choose 4:Vertical. A vertical line will appear and if you use the right and left arrow keys, you can move it thus allowing students to see the vertical line test in action.

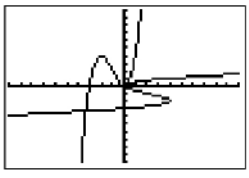


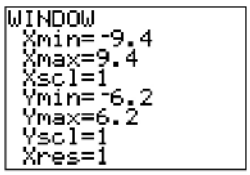


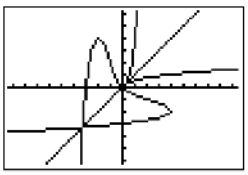
When exploring the relationship between a function and its inverse it would be nice to be able to graph the inverse even if it is not a function. To do this press [2nd][PRGM] then choose 8:DrawInv. The DrawInv command will be placed on the home screen. You need to specify which function's inverse to draw. Press [VARS] , arrow right to [Y-VARS], choose 1:Function then select the equation in which you have stored your function. When you press [ENTER] the inverse will be drawn. By adding the function y = x, the symmetry of the graphs should become clear. You may want to square up your window to make the relationship more evident. You can use the fact that there are 94 pixels across and 62 pixels down to get a good window.





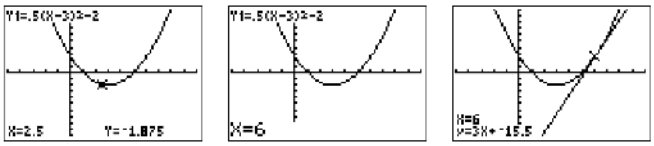






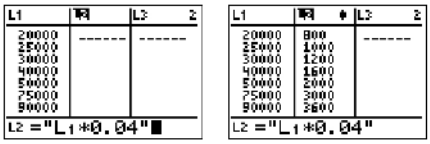
In the same DRAW menu, option 5 allows you to draw a tangent to a curve and displays the equation of the tangent. I show this to my students when I have them create their own question (and solution) involving

finding the equation of two or more tangents that intersect at a point that is not on the curve. When you choose the tangent command the graph is displayed so that you can choose the coordinates of the point of tangency. You can do this using the arrow keys or you can type in a value for the x-coordinate. The tangent is then drawn and its equation is displayed. It is important to note that this tangent was drawn so its equation is not stored in the equation editor. To get rid of anything drawn using a command in the DRAW menu, press 2nd PRGM and choose option 1:ClrDraw.



Keeping Formulas

Lists are a great way of working with data and the ability to use formulas in lists allows students to explore the data. Having the ability to keep a formula in a list is a great feature of the TI-84. To accomplish this you need to place quotes around your formula. For example, if you want to calculate 4% of List 1 and store the result in List 2, the formula for List 2 will be "L1 * 0.04". If you do not include the quotes the values for List 2 will be calculated but you will not be able to edit the formula. By adding the quotes you can go back to the formula and change it to 5% if you want to. The formula I'm using is very simple but it becomes even more useful when the formula is longer. This strategy is also useful if you need to keep track of what formula is in each list.

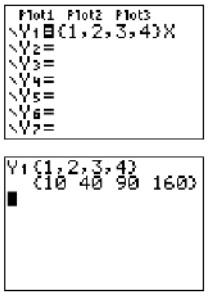


Lists in Equations

It is possible to define multiple equations in one using a list. One way of doing this is to include the list in your equation. For example, if you are looking at slope you could enter the values 1, 2, 3, 4 in List 1 then define your equation as Y1 = L1 * x. The calculator will graph y = x, y = 2x, y = 3x and y = 4x in order.



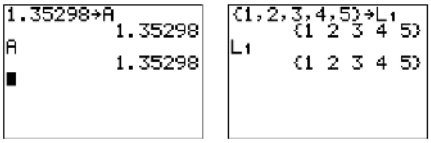
Another way of using a list in an equation is to define the list as part of the equation. To produce the same graphs as in the previous example you would define the equation as Y1 = {1,2,3,4}x. Entering a group of numbers in brace brackets creates a list and each element will be used in the equation in the order it appears.



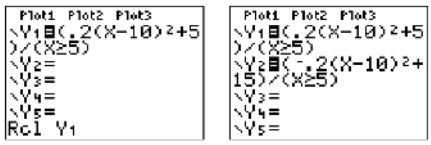
You can also use the same strategy to calculate multiple values of an equation. If you pull up the equation you are interested in and follow it with a list, the respective y-values will be produced.

Store and Recall

The STO▶ key allows you to save values to the calculator's memory. For example, if the result of a calculation was 1.35298 you can press STO▶ followed by a letter, using the ALPHA key. You can use that value by substituting the chosen letter in any calculation. In the same way you can store data into a list.



The recall key (2nd STO▶) can be particularly useful in the equation editor. Using it allows you to paste one equation into a new one. Start by entering an equation in Y1. Move the cursor to the right of the equal sign for Y2 and press STO▶ then VARS , arrow right to [Y-VARS], choose 1:Function then 1:Y1 and press ENTER . The command will be displayed at the bottom of the screen, as shown, and once you press ENTER , Y1 and Y2 will display the same equation. Y2 can then be edited as needed.

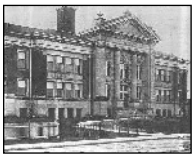


These are just a few ways of getting the most out of your graphing calculators. By knowing how to use these tools to their full extent, we can more easily bring the mathematics that we want our students to learn to the forefront. ▲

▲ Hey, It's Elementary! Miss Thompson was right...you can't do the math if all you see are squiggles



LYNDA COLGAN



Unfortunately, I missed my high school reunion in May–although given the major renovations to the stately old building I remember, there would have been few opportunities to reminisce at 'my' locker or in 'my' seat in 'my' classroom. I am an alumnus of Riverdale C.I. in Toronto–a venerable school that is celebrating its 100th anniversary this year.

Many people and events conspired to make my years at Riverdale rich and memorable, and I will be forever grateful to the many wonderful teachers who helped me to grow and learn, shaping not only who I am today, but who I aspire to become. The list is long and appreciative memories of these amazing individuals often come to mind, but ever present in the continuous loop are Pierre Saarimaki, Jane Robinson, Emily Morino, Jim Rowney, John Ford, Grace Garbutt and Kay Hoey.

When I was a student at RCI, one had very few options when it came to course selection. One could choose between instrumental music, vocal music or art and Latin and/or Greek. In order to take ancient Greek (with the legendary Miss Thompson), one had to attend early morning classes (7:45 a.m.) because there was no room in the regular timetable for her course offerings–an obstacle to which few students objected.

One very notable day, I recall that Miss Thompson arrived at our Grade 12 mathematics class. There were muted giggles as some students suggested that her age (imagine Professor Minerva McGonagall) had made Miss Thompson forget where to go (although in retrospect, she was probably the age I am today). Mistaken classroom identity was decidedly not the case and this legendary teacher proceeded to deliver a lesson I have never forgotten.

"Many of you have not studied Greek, and your mathematics teacher and I agree that this could be an impediment to your mathematics career," she said, peering over the top of her round gold-rimmed spectacles. "I am here today to teach you the names of the Greek letters–not just the obvious ones like alpha, beta and pi, but the less common psi, xi, tau and omega," she continued, "because it is important that you learn not just to recognize these letters, but how to pronounce them." As we stared at her in the way that only dumbfounded 16 year-olds can, she explained sensibly, "You'll be needing to read mathematics out loud and to yourself. It's impossible to think about formulas and what operations and processes are required if ø, ϕ, Ω, ψ, and Σ are all just variations of a squiggle to you." And, so the lesson began.

It turns out that my years of Latin classes with Miss Hoey and that one invaluable lesson with Miss Thompson had enormous and long-lasting impact in my love of and success in language and mathematics.

THE GREEK ALPHABET

Greek letter that look like Roman letters are not used as mathematical symbols for example. A is capital a, not capital α. The letter χ is pronounced "kye", to rhyme with "sky"; φ, ψ and ξ may ryhme with either "sky" or "tea".

A	alpha	N	nu
B	beta	Ξ	xi
Γ	gamma	O	omnicron
Δ	delta	Π	pi
E	epsilon	P	rho
Z	zeta	Σ	sigma
H	eta	T	tau
Θ	theta	ϒ	upsilon
I	iota	Φ	phi
K	kappa	X	chi
Λ	lambda	Ψ	psi
M	mu	Ω	omega

One of the principle reasons that this particular lesson came to mind so vividly is that I was discussing the issue of reading mathematics recently with one of my graduate students, Svitlana Voytsekhovska, who is interested in the challenges that students face when they are confronted with mathematics text in its many genres.

Like so many teachers, Svitlana had heard many of her colleagues say things like, "My students can't do math problems because they can't read math problems," or "if they'd just test math instead of language, my students would do great on standardized tests." These experiences led her to come to Queen's from the Ukraine to study what makes reading mathematics so difficult and to describe some research-based strategies that teachers can use in their classrooms to ameliorate the situation.

In her research, Svitlana had come across an interesting approach to creating a math word wall as well

as a number of other strategies. This column is based on some of what I have learned from her so far, and I hope to share much more, with her permission, after she finishes her research.

PREFIXES & MATHEMATICS		
Uni/mono	one	unit, unicycle, monopoly, inify, unique
bi/di/du	two	binomial, binary, bisect, dioxide, dual
tri	three	tricycle, trianlge, trillion, triple, triathlete, triplet
quad/tetra	four	quadratic, quadrant, quarter, quadruple, tetrahedron
pent/quint	five	pentagon, pentathlon, quintet, quintuplet
hex	six	hexagon, hexameter
hept/sept	seven	heptagon, heptameter, September, septuagenarian
oct	eight	octet, octopus, October, octane, octagon, octave
non/nove	nine	nonagenarian, November, nonagon
dec/deci	ten	December, decimal, decade, decobel, decathlon, decimeter
cent/hec	100	century, cent, centimeter, centennial, hectare
milli/kilo	1000	million, millimeter, kilometer, kilogram, kilowatt
myriad	10,000	myriad

SIZE & AMOUNT PREFIXES		
micro	small	microscopic, microfilm, microphone, microscopic, microcosm
macro	large	macroscope, macrobiotic, macocosm
mega	huge	megalith, magnificent, magnum, magnitude
equi	equal	equal, equator, equation, equidistant, equity
hyper	excessive	hyperbole, hyperactive, hyperextension, hypercritical
omni	all	omnivorous, omnipotent, monipresences, omnibus
pene	almost	peninsula, penultimate, penumbra
poly	many	polygon, polyester, polyglot, polysyllabic
super	greater	superior, Superman, supernova, superpower, supernatural
ultra	beyond	ultramodern, ultrasonic, ultraviolet

TEACH CONTENT VOCABULARY

Students learn vocabulary through repeated exposure to words. Instruction need to be a process of letting students figure out meaning within the context of your subject. Always start with asking students to explain what is going on in the text as they are uncovering meaning of the word. Have students represent the words in a variety of ways: visually, antonyms/synonyms, identify where they have heard it before or parts of the word before. Use strategies like word walls, vocabulary trees, vocabulary bookmarks, word maps, etc. Beck, McKeown& Kucan (2002)

LATIN ROOTS		
annu-enni	year	annual, biennial, centennial
cede/ceed	go	recede, antecede, succeed
flex/flect	bend	flexible, reflex
form	shape	form, formation, transform
multi	many	multiply, multitude, multiple
var	different	vary, variable, variety

GREEK ROOTS		
gon	angle	pentagon, hexagon, polygon, octagon,
gram	written	diagram, grammar, epigram, telegram
graph	write	graphic, pictograph, photograph, telegraph
meter	measuring device	thermometer, barometer, odometer

Among others, Mary Lee Barton and Robert Marzano, experts in reading instruction and assessment, respectively, have developed a number of resources to help all teachers to integrate reading to learn strategies into their particular discipline, regardless of grade.

While these educational leaders agree that almost all elementary teachers have received instruction in reading to learn strategies as part of their language curriculum pre-service and in-service teacher education, they point to research that suggests that the strategies (though not constrained by age or subject content) are used predominantly within the context of language activities in the classroom.

They cite other research that states that on average, students have 6 to 8 years of reading instruction and at least 12 years of writing instruction as part of their language program throughout elementary/secondary schooling; therefore, reading and writing in and of themselves do not seem to be the problem.

The real problem is that students need to learn how to read mathematics in the same way they learn how to read a novel or a poem, listen to music, or view a painting; therefore, these experts are urging teachers of all subjects, but especially teachers of mathematics to learn about and implement reading to learn strategies in the context of the regular mathematics classroom curriculum. In order to comprehend the texts of certain disciplines, readers must often develop reading skills peculiar to those disciplines. In particular, students must learn to read mathematically.

Barton and Marzano are quick to emphasize that although mathematics teachers don't need to become reading specialists in order to help students read mathematics text in its many incarnations, they do need to recognize that students need teachers' help to read in the broad spectrum of mathematical contexts.

As a first step, they contend that teachers must help students use explicit, taught strategies to acquire vocabulary and read mathematics for meaning.

A brief summary of some of their research-based resources does little justice to the breadth and depth of their work, but perhaps these 'teasers' will be just the inspiration for more of us to turn to their books and try their ideas. Here goes…

Remind students to read mathematics, whether in their textbook or on activity sheet, SLOWLY, and emphasize that, unlike other genres, like fiction, in mathematics, usually every word counts–in other words,

mathematics tends to be an adjective- and adverb-free zone.

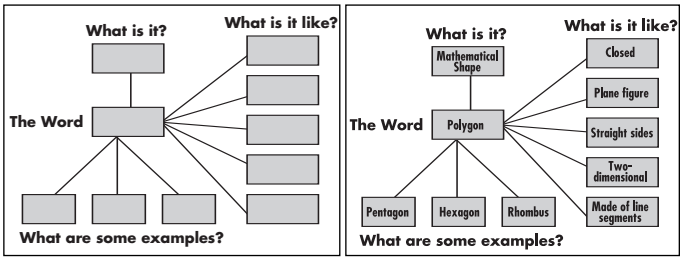
CATEGORIES OF RESEARCH BASED STRATEGIES FOR READING TO LEARN	EXAMPLES
ANTICIPATORY ACTIVITIES Practices that elicit curiosity, provoke questions, access recall of newly learned information, activating students' prior/background knowledge about a subject, topic, or concept.	• KWL • Anticipation Guides • I Used to Think, But Now I Know • Reciprocal teaching
THINK ALOUD & SHARED READING: Think aloud: A deliberate selection of text in which the teacher reads publicly to the class or more small group (usually to emphasize a concept or as a modle for learning). Text may or may not be visible to the students. Shared reading- Like the read aloud, a deliberate selection of text in which the teacher reads publicly to the class or small group but negotiates meaning, vocabulary, skill to be learned in an interactive process with the students. In shared reading, text should be visible to the students.	• Modeling writer's craft with literature • Teaching rules of notice of a textbook: headings, captions etc. • Teaching voice or tone through read aloud • Teaching visual text such as: graphs, tables, illustrations, maps • Using context to decode vocabulary • Getting unstuck strategies • Subject area organization / reprensentaion focus lessons
QUESTIONING & RECIPROCAL TEACHING Effective questioning involves bridging levels of critical thinking for teaches to monitor and guide students understanding in reading, writing, listening, and reflecting, and for students to self monitor their understanding of text and language which is unfamiliar.	• Blooms Taxonomy for generating effective questions • Reciprocal teaching • QAR • QtA • Stump the Teacher
NOTE TAKING & NOTEMAKING Note taking: Taking and using notes effectively from lecture or class discussion. Note making: Recording notes from text sources to make sence, clarify, and better understand what is read.	• Cornell Notes • Double entry journal • Q/A journals
GRAPHICS ORGANIZERS Visual display tools for promoting, extending, prompting students for understanding concepts, relationships, rules in and across subject areas.	• Concept Maps • Thinking maps • Tree diagrams and flow charts
VOCABULARY INSTRUCTION Strategies that focus on the study of words through in-depth decoding and application and repeated exposure. Not rote memorization.	• Word maps • Linear arrays • Vocabulary cartoons
WRITING TO LEARN Writing can assist students in processing, reflecting and uses three different kinds of knowledge: declarlative, procedural & conditional (Fisher & Frey, 2004)	• Learning logs • Quick writes • Response Journals

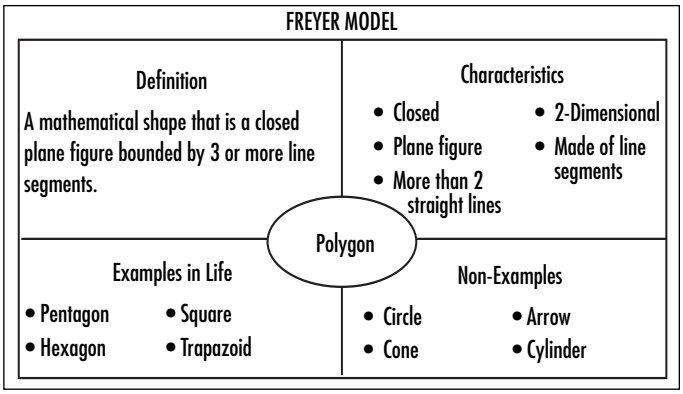
Remind students that diagrams, symbols, tables, graphs and illustrations are not just to brighten up the page–they are an alternate form of text, called visual text that requires special skill sets to be decoded in their own way. More importantly, visual text, in general, contributes positively to one's understanding and solution of the problem.

Remind students not to be passive readers. They should have a tool kit ready to help them to read–paper, pencil, calculator and simple manipulatives. They need to make the mathematics, whatever the genre, their own, using as many techniques as possible to mark up and personalize the text in order to extract meaning from it.

Research suggests that repeated exposure to vocabulary is the best way to learn words; however, embarking on vocabulary as formal word study may also enhance vocabulary attainment. Experts suggest that vocabulary instruction be adapted to include visual representations, synonym and antonym work, word hunts, word walls, and word sorts.

Here are two examples of research-based vocabulary strategies that work: Concept Mapping and the Freyer Model.





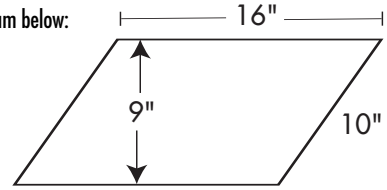
The development of a student's power to use mathematics involves learning the signs, symbols and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking (NCTM Standards, 1989 p. 6).

While experts agree that students should not be taught recipe approaches to employ as a first start in problem solving (e.g., total means you probably have to

add something), graphic organizers, especially KNWS, have been shown to be effective in helping students to learn how to extract information from mathematics text in order to problem solve. Here is just such as example.

K-N-W-S			
K	N	W	S
What facts do I KNOW from the information in the problem?	Whick information do I NOT need?	What does the problem ask me to find?	What STRATEG/ operations / tools will I use to solve the problem?

Look at the parrallelogram below:



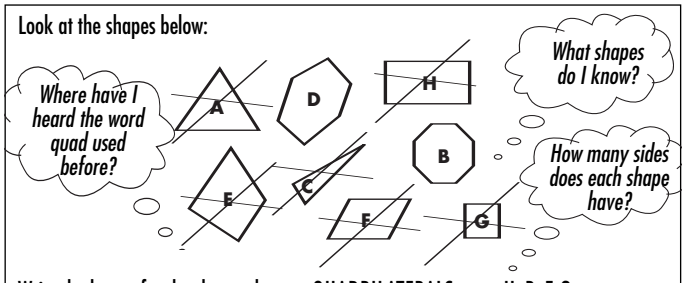
What is the area, in square inches, of the parallelogram?

Area of parallelogram = base x height. Show all work.

K-N-W-S			
K	N	W	S
What facts do I KNOW from the information in the problem?	Whick information do I NOT need?	What does the problem ask me to find?	What STRATEG/ operations / tools will I use to solve the problem?
It is a parallelogram. The area of a parallelogram is Base x Height. The base is can also be the top. Height is up and down. The base = 16 in. The height = 9 in.	10 in is neither the base or the height, it is the length of a side.	Find the area of the parallelogram.	16 x 9 = 134 inches.

Thinking 'aloud' (whether to the class or to oneself) and marking text (not just underlining words) have also been shown to be effective decoding strategies and helpful first steps in problem solving.

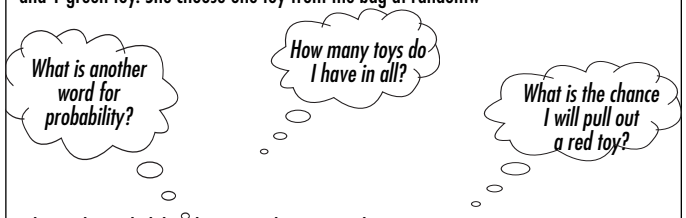
Look at the shapes below:



Write the letters for the shapes that are QUADRILATERALS. H, B, F, G

Write the letter for the shapes that is a HEXAGON. _____

Janie bought a bag of toys for her kitten. the bag contains 3 red toys, 2 yellow toys, and 1 green toy. She choose one toy from the bag at randomw



What is the probability that Janie chooses a red toy? _____

What is the probability that Janie chooses a green toy? _____

With the advent of the original NCTM Standards and the various incarnations of the Ontario Mathematics Curriculum, communication has been touted as an important process strand–one that has the potential to give teachers' enormous insight into student understanding. However much of the focus has been on writing mathematically (e.g., journals and explanations) and using 'math-talk' (e.g., evaluating alternate solutions) in the classroom. Our next challenge, as committed mathematics educators is to learn how we can improve mathematical reading comprehension by looking to our colleagues who are experts in reading-to-learn strategies. A reader with higher level reading skills will most likely be able to comprehend mathematical text better, and a student who is able to read mathematical text easier and with greater comprehension will likely have fewer mathematical anxieties–and that doesn't sound 'all Greek' to me.

References

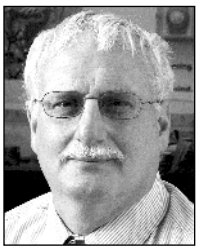
Barton, Mary Lee & Heidema, Clare. 2002. *Teaching Reading in Mathematics.* Colorado: MCREL.

Billmeyer, Rachel & Barton, Mary Lee. 2002. *Teaching Reading in the Content Areas: If Not Me, Then Who?* Alexandria: ASCD.

Harvey, Stephanie & Goudvis, Anne. 2000. *Strategies That Work.* Ontario: Stenhouse Publishers.

Tovani, Cris. 2000. *I Read It, But I Don't Get It.* Maine: Stenhouse Publishers. ▲

▲ OAME/NCTM Report September 2007



JOHN KERSLEY

NCTM, like any other company, updates its web site from time to time. If you haven't visited NCTM's home page at **www.nctm.org**, it is worth a visit. NCTM redesigned its web site to access resources for all grade levels more quickly. In addition, there are listed important NCTM events, new publications, the current focus of the year information, position statements, research clips, etc. The result is a colourful home page that has lots of information and links to NCTM resources without being too crowded and difficult to use.

For me the site, **www.nctm.org/resources** is very useful. From here I can easily access all sections of NCTM including lessons and resources for elementary, middle and high schools, tips for teachers, teaching opportunities, illuminations, students math notes and resources for Canadian mathematics teachers. Take the time to visit the web sites of the other provincial mathematics organizations.

Current Publications from NCTM

- *Mathematics Assessment Literacy: Concepts and Terms in Large Scale Assessment*
  This booklet is divided into 2 parts: Part 1 terminology used in talking about assessment; Part 2 applications of these terms leading to a discussion of standardized tests.

New publications of the very popular Navigation Series

- *Navigating through Mathematical Connections in Grades 9 – 12*
  The book shows ways of connecting mathematical topics including mathematical modeling and different representations in solving problems.
- *Navigating through Number and Operations in Grades 6 – 8*
  The book deals with ways to consolidate students understanding of integers, rational numbers, decimals, per cents and proportionality.
- *Navigating through Number and Operations in Grades 9 – 12*
  The book's activities deal with rational and irrational numbers, investigate properties of integers and complex numbers and practical applications in real life.

New NCTM Series

- *Teachers Engaged in Research*
  With examples, these books demonstrate how teachers understand students' reasoning processes and thus, learn how to ask appropriate questions, make useful comments and show support.

  There are 4 in the series

  Teachers Engaged in Research PreK – 2

  Teachers Engaged in Research 3 – 5

  Teachers Engaged in Research 6 – 8 and

  Teachers Engaged in Research 9 – 12

Consulate **www.nctm.org/catalog** or me for further details.

Web sites of interest to mathematics as suggested by NCTM

- MATH LEAGUE Resources for helping students in grades 4 to 8. Topics includes whole numbers, decimals, data statistics, fractions, geometry and ratio proportion. **www.mathleague.com/help/help.htm**
- MATH PLAYGROUND Math Playground is a fun, action-packed site for elementary and middle school students. Practice your math skills, play a logic game, and have some fun! **www.mathplayground.com**
- KUTA SOFTWARE Customized algebra worksheets, unlimited questions, multiple choice and free response questions also links to other sites. **www.kutasoftware.com/nctm-ia1.html**
- MATH GOODIES A place for math help featuring interactive lessons, worksheets, and homework help. Over 500 pages of activities for students, educators and parents. **www.mathgoodies.com**
- GEOGEBRA Free, dynamic mathematics software integrating geometry, algebra and calculus. Create geometric constructions with points, vectors, segments, lines and functions working with variables and derivatives. **www.geogebra.org**

NCTM Conferences www.nctm.org/meetings

NCTM Regional Conference and Exposition

- Richmond, Virginia "Exploring Mathematical Horizons" October 11 – 12, 2007
- Kansas City, Missouri "Jazzed about Math" October 25 – 26, 2007
- Houston, Texas "Energizing Mathematics" November 29 – 30, 2007

NCTM Annual Meeting and Exposition

- Salt Lake City, Utah "Becoming Certain About Uncertainty" April 9 – 12, 2008

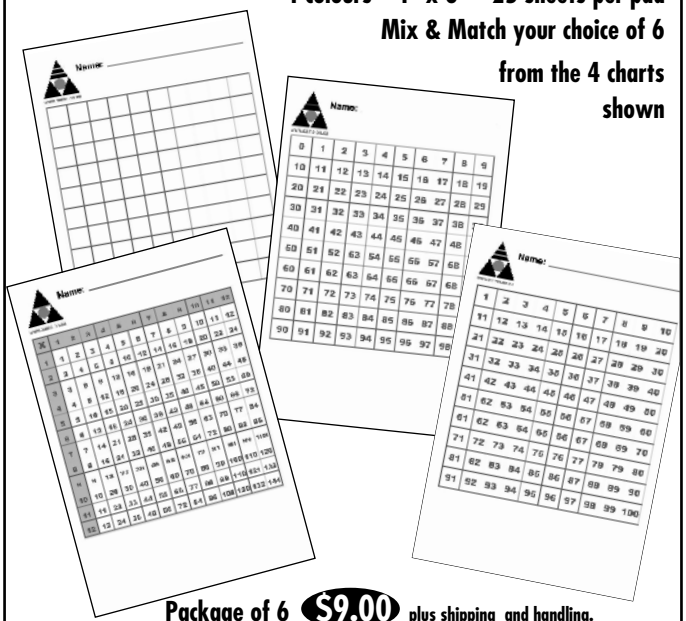
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A list of affiliate conferences can be found at **www.nctm.org/affiliates/conferences.htm** ▲

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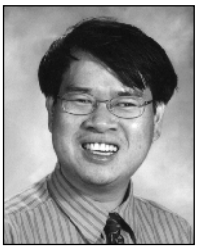
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▲ Get Interactive! Making Connections

Louis Lim



Louis Lim is department head of mathematics at Richmond Hill High School, York Region DSB. Louis' involvement with the OAME includes being a former editor of the Ontario Mathematics Gazette*, presenter at chapter and annual conferences and member of OAME board of directors. In spring 2005, he was recipient of OAME's "Exceptional and Creative Teaching in Secondary Mathematics" Award.*

Making connections is such an important theme in mathematics that it has been identified as one of the five process standards of the National Council of Teachers of Mathematics (NCTM) as well as one of seven mathematical processes of the Ontario mathematics curriculum. As a classroom teacher, I know first hand how challenging it is to answer students' "Why do we have to learn this?" and "When are we ever going to use this?", questions. It is for that reason that both Judith Covington and I, co-editors of the November 2006 issue of the NCTM's *Mathematics Teacher* journal, sought to answer these questions with our focus issue, "Making Mathematics Meaningful". I encourage you to read our focus issue as there are interesting articles such as "How Far Up Am I? The Mathematics of Stadium Seating" and "Building a Career Mathematics File: Challenging Students to Find the Importance of Mathematics in a Variety of Occupations".

1.) Mathematics and Nature

The web sites, **http://www.mcs.surrey.ac.uk /Personal/R.Knott/Fibonacci/** and **http://www.world-mysteries.com/sci_17.htm**, provide a wonderful link between mathematics and nature. Students can be given the first few terms of the Fibonacci number sequence (which is "nature's numbering system") and asked to determine the next few terms. Then, connections can be made to nature, (e.g., number of petals in flowers; number of spirals in pinecones and pineapples), and lots of curricular connections, regardless of grade level, including recursion, Pascal's Triangle, and ratios and proportions.

2.) Mathematics and Art

How do artists make use of geometry? *What's the Problem? Geometry & Art Through the Ages*, **http://www.ima-art.org/education/IMA_education/home.asp?levelid=1**, explores how artwork from the Indianapolis Museum of Art uses lines, polygons, and volume: "Geometry is a way of seeing the world". Mathematical concepts include perspective drawings, parallel and perpendicular line, symmetry, and tessellations.

My friend, Mario Soster, is a fan of Escher Art. *The Mathematical Art of M. C. Escher*, located at **http://www.mathacademy.com/pr/minitext/escher/**, connects Escher's work with mathematics. Curricular connections include Euclidean and non-Euclidean geometries, tessellations, polyhedra, space and paradoxes.

3. Mathematics and Cryptography

Cryptography, the science of codes, has applications to banking (see **http://www.ams.org/ams/mathnews/banking.html**), transactions over the Internet, and in wars. An introduction to the subject, including its rich history, can be found at **http://www.antilles.k12.vi.us/math/cryptotut/intro.htm**.The site, **http://www.math.washington.edu/~koblitz/crlogia.html**, provides some suggestions on how to incorporate cryptography in the teaching of mathematics. Curricular connections include prime numbers, factoring, matrices (code and decode messages), and frequency analysis of letters in the alphabet.

4.) Mathematics and the Environment

The OAME executive provided me with a handout of web sites pertaining to the environment and asked me to include a link in each issue of the OAME *Gazette* for Volume 46. As I write my column on July 3, I flipped through the *Toronto Star* and noticed these articles: "Ponds issue global warning" (page A3); "Fallout from a dirty bomb" (page A12); "Watchdog warns of chemicals in water" (page A14); and "Lush Canadian fields but many fewer farmers" (page AA8). Before submitting my column to Editor Hurrell on 07-07-07, there is currently a 24-hour *Live Earth Concert*, organized by Al Gore. With 150 music acts in 8 major cities in the world, the intent is to promote the environment to an audience of 2 billion people. Thus, the environment is a very timely topic. Here is the web site I want to highlight, which makes mathematical connections:

The web site, **http://www.zerofootprintoffsets.com/calculator.aspx**, allows students to determine how many tonnes of carbon dioxide, CO_2, they emit into the atmosphere per year through their lifestyle. Before the activity, students should ask their parents/guardians their average monthly electricity, natural gas, heating oil, propane costs, as well as what percentage of electricity comes from renewable resources. There is much opportunity for students to analyze and interpret their data (either individually or as a class), taking into account the grade level.

I surfed the Internet and located another web site to calculate one's ecological footprint. The *Ecological Footprint Quiz*, **http://www.ecofoot.org/**, asks for 14 questions and compares the number of global hectares you need for your lifestyle (food, mobility, shelter, goods/services) and compares you to the average person in your country and in the world[1].

Connections make mathematics teaching and learning interesting. More to come…

Reference

Covington, J., & Lim, L. (Eds.). (2006, November). Making mathematics meaningful [Focus issue]. *Mathematics Teacher, 100*(4).

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[1] Even though I am below the average Canadian (5.1 global hectares compared to 8.8 global hectares), I was alarmed to learn if everyone in the world lived like myself, we would require 2.8 planets. Ouch! Now to make some changes…



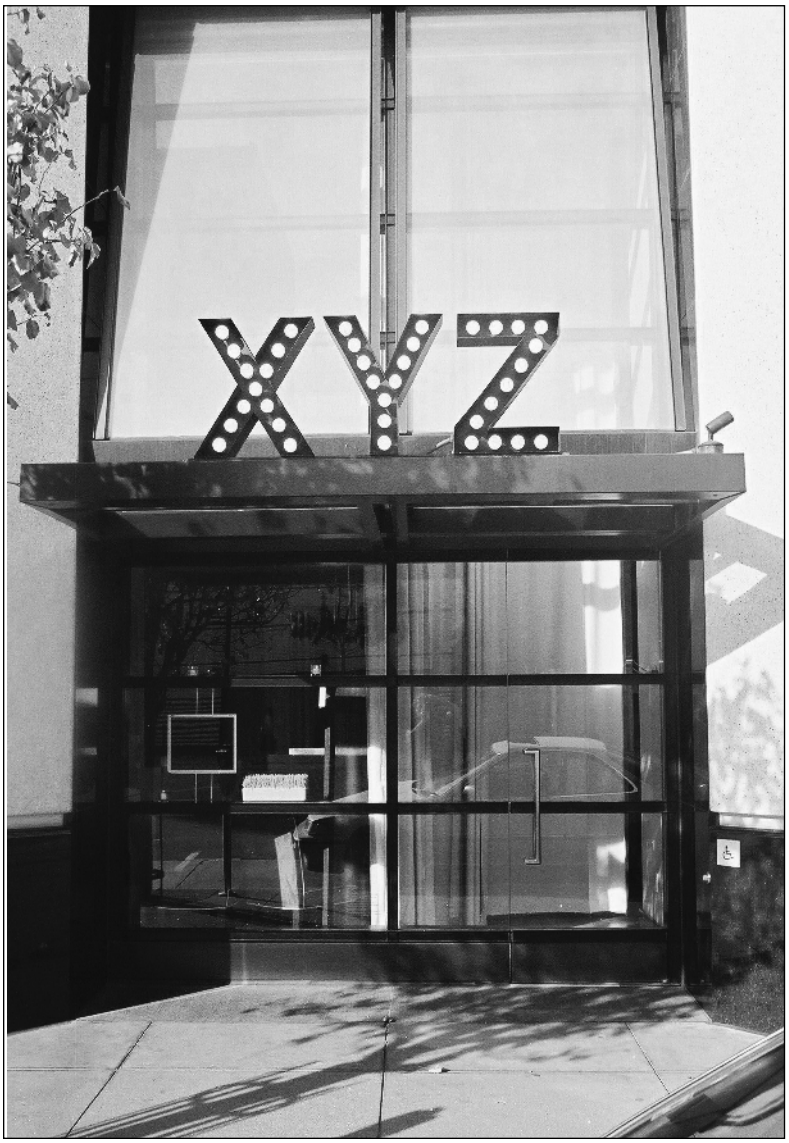
▲ Photo Math - The Fine Art of Viewing the World Through a Mathematical Lens

RON LANCASTER



Ron Lancaster is a Mathematics Lecturer in the Department of Curriculum, Teaching and Learning, the Ontario Institute for Studies in Education (OISE/UT) at the University of Toronto, 252 Bloor St West, Rm 11-162, Toronto Ontario M5S 1V6

Teachers and students are encouraged to put on a pair of mathematical glasses and become a math spotter, someone who sees mathematics everywhere and then possibly poses mathematical questions based on the images taken. Ron Lancaster, the current Editor for Photo Math, welcomes photos with questions for possible publication in future columns. Material can be sent via e-mail or by regular mail to either of the two addresses given above.



Photograph 1. Photographer: Ron Lancaster. Location: San Francisco, California.

(a) This business is known locally as The XYZ in the W. What type of business is this?

(b) How many lights have been used for each of the three letters? Suppose that the company that designed these letters has produced letters (exact same height, approximately the same width) for every letter of the alphabet. How many lights would be used for each letter? What is the average number of lights per letter?

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Photograph 2. Photographer: Ron Lancaster. Location: San Francisco, California.

(a) As noted in the lower left corner of the billboard, CBS has a permit to install a sign on this wall that is 30 feet by 50 feet. Measure the length and width of this billboard either directly on the photo using a ruler or by pasting the photograph into a dynamic geometry software such as Sketchpad or Cabri. Is the ratio of the width to length equal to 30:50? If not, do you have enough information to know if the sign could have been longer or higher or both?

(b) Assume that the size of the billboard is 30 feet by 60 feet. Outline what steps you could follow in order to estimate the height of the building and the height of the truck parked on the street.

(c) If you knew the height of the truck, how you could you use this information to decide if the billboard is 30 feet by 60 feet?

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▲ What's the Problem?: Shooting Hoops

SHAWN GODIN



Shawn is the head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans, ON.

Welcome back to a new school year problem solvers. Last issue I left you with the following problem.

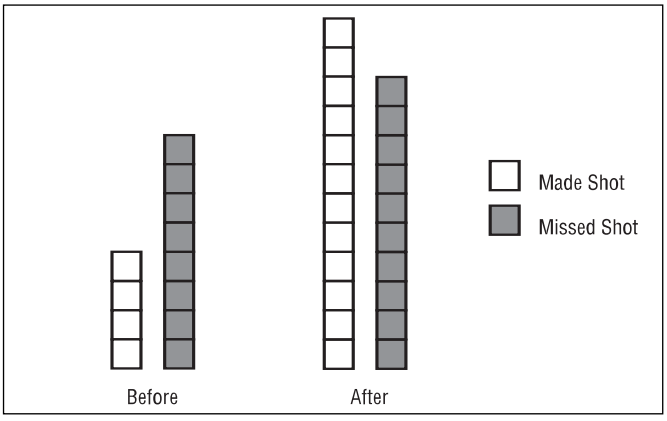
By the end of the season, Samuel had made more than 50% of his foul shots, even though at the start of the season his average was below 50%. Show that there was a time during the season when his average was exactly 50%.

This problem appeared as M202a from the September 2005 issue of *Crux Mathematicorum with Mathematical Mayhem* and solution appeared in the September 2006 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem* web site **http://journals.cms.math.ca/CRUX/**.

This is another problem where a little experimentation might give us some insight. Suppose we flip a coin 20 times and keep track of the number of heads and tails. We can use these numbers to simulate our problem with either heads or tails as the number of baskets. I flipped a coin 20 times and got the table below.

TRIAL	OUTCOME	HEADS		TAILS	
		TOTAL	%	TOTAL	%
1	H	1	100	0	0
2	T	1	50	1	50
3	T	1	33	2	67
4	H	2	50	2	50
5	T	2	40	3	60
6	T	2	33	4	67
7	H	3	43	4	57
8	T	3	38	5	63
9	T	3	33	6	67
10	H	4	40	6	60
11	H	5	45	6	55
12	H	6	50	6	50
13	T	6	46	7	54
14	H	7	50	7	50
15	H	8	53	7	47
16	T	8	50	8	50
17	H	9	53	8	47
18	H	10	56	8	44
19	H	11	58	8	42
20	T	11	55	9	45

We notice that for both the heads and tails we never skip over 50% going either way. We can model this situation using manipulatives. Using linking cubes, one colour for a made shot and a different colour for a missed shot we can see that, if Samuel starts with less than 50% of his foul shots made, it means that he made fewer shots than he missed. Similarly, by the end of the season he made more shots than he missed. The only way the "made" pile can pass this "missed" pile is for the piles to have the same number at some point, which means that Samuel would have made 50% of his shots at that point. Have your students try this out to convince themselves.



For a more formal proof, let's assume that at some point we jump over the 50% point. So after n shots we have made m baskets with

$$\frac{m}{n} < \frac{1}{2}.$$

After one more shot, and made basket, we pass the 50% mark, so

$$\frac{m+1}{n+1} > \frac{1}{2}.$$

Since the denominators in both sets of inequalities are positive, we can multiply the first inequality by 2n and the second by 2(n+1) to get

$$2m < n$$

$$2m + 2 > n + 1$$

Subtracting 1 from each side of the second inequality allows us to put both inequalities together as

$$2m < n < 2m + 1.$$

Recall that m is the number of made shots and n is the total number of shots, just before we were above 50%. Both of these values are integers, but 2m and 2m+1 are consecutive integers. Thus n, an integer, must be strictly in between two consecutive integers which is impossible. So our assumption, that we skipped over 50%, is impossible. A nice proof by contradiction that, even if they cannot reproduce it, students possibly down to grade 9 or younger may be able to follow if they have had the chance to play with the manipulatives first.

You may want to try the following extension, which originally appeared as part b) of our featured problem.

For what other percentages p can you be certain that the average was exactly p at some time when you know only that the average was below p and the above p at a later time?

This problem was a modification of a problem that appeared on a Putnam contest. The Putnam contest is the premiere math contest written by university students in North America. Ian VanderBurgh of The Centre for Education in Mathematics and Computing at the University of Waterloo wrote a wonderful article about the original Putnam problem in his column The Problem of the Month in the April 2006 issue of Crux Mathematicorum with Mathematical Mayhem. The column is available directly from the link

http://journals.cms.math.ca/cgi-bin/vault/public/view/CRUXv32n3/body/PDF/page147-148.pdf?file=page147-148 or by going to the journal page and searching out the issue. Now for your homework:

Find a quadratic polynomial f(x) such that, if n is a positive integer consisting of the digit 5 repeated k times, then f(n) consists of the digit 5 repeated 2k times. (For example, f(555) = 555555.)

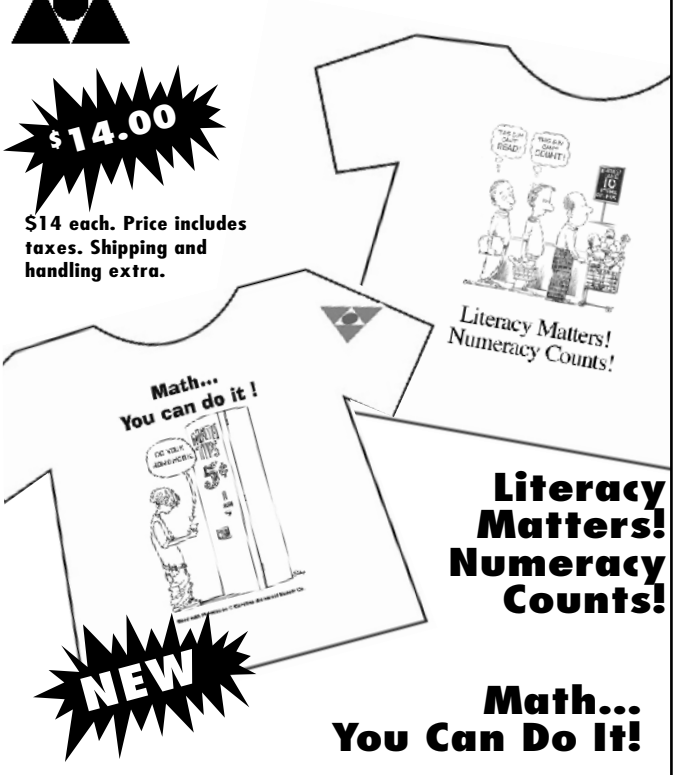
Until next time, happy problem solving!▲

▲ Call for NCTM Rep

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Dave and Sue Hessey

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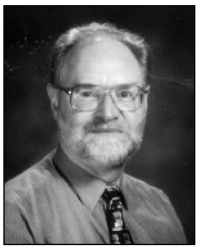
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▲ CAS Corner: Basic Algebra with a CAS

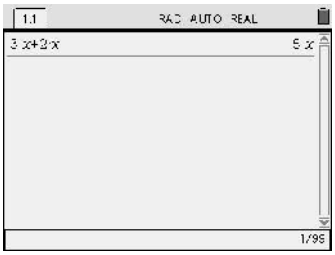
FRED FERNEYHOUGH



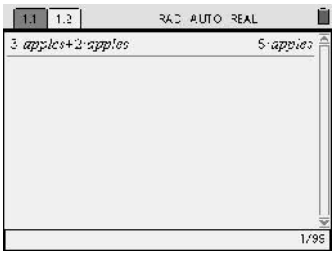
Fred Ferneyhough is former head of mathematics at Harold M. Brathwaite SS in Brampton. He retired from there in February 2006. Fred is an avid user of and advocate for the use of technology in the mathematics classroom. He is also an author and tech consultant for several books and a National Instructor for Texas Instruments.

During the last few years of my teaching career, I had the opportunity to use CAS devices at my last three schools. Over time, I developed a pattern of teaching that benefited students learning basic algebra. Although I cannot adequately represent it here, all of the work that is shown was supplemented with algebra tiles, and I worked with tiles wherever possible.

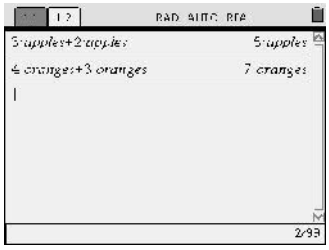
In this article, let's look at the very basics of algebra, adding like terms. An activity such as the one shown to the right is good for several reasons. First, kids will accept the answer since a technological device produced the result. Second if you can get kids to accept the answer, this exercise can be turned into a communication problem – "Can you explain how the calculator produced this answer?"



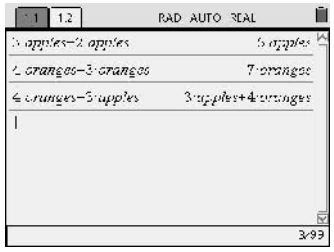
However, we can do even better. Let's try starting with the screen to the right. In this example, the variable is "apples" rather than a single letter. The statement makes sense to students seeing this for the first time since it is in a context that they understand. The only drawback is that the device adds in the dot representing multiplication and we may need to take a moment to explain this.



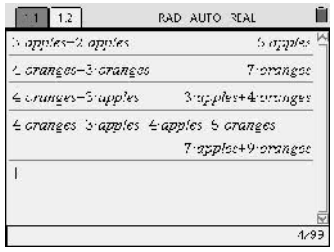
We can also try another "variable". In this case, "oranges" has been used for this purpose.



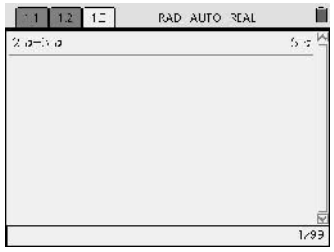
Mixing variables does not make a lot of sense to kids, just as it shouldn't when looking at an algebraic expression. It's amusing when the students almost become indignant that we'd ask such a question. At the same time, they have recognized the idea of like terms.



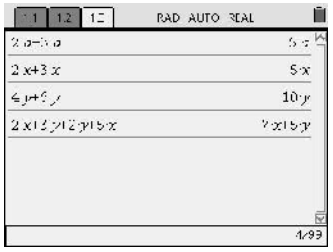
This type of example also makes some sense to them and reinforces the idea of like terms.



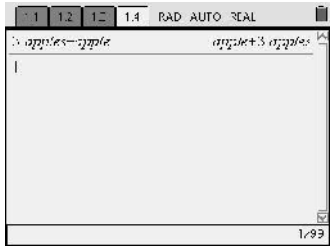
By this point in time, I'm hoping that at least one student is getting tired of typing in the words "apples" and "oranges". If this happens, I can offer to be a nice guy and ask if they'd like to allow one letter to represent a variable, say "a" for "apples". Now, we have come full circle. We're back to the type of expression that we started with in the first screen, but now, the expression has a context that the students understand – and they think that they've gotten away with something by asking for the shortcut.



I can continue my lesson as I might have in the beginning using whatever variables I wish. However, now there is a basic understanding that each letter stands for some quantity that can be defined by the student.



Since someone is going to ask, the device cannot distinguish between the plural and singular of a "variable". The answer is "no, it cannot". Let's not expect too much! ▲



▲ '007 is the Year for Building Math Bonds

CHERYL McGINNIS



Cheryl is a grade six teacher just finishing her sixth year of teaching at Huronia Centennial Public School (Simcoe County DSB) in Elmvale. In her first year of teaching, Cheryl was a volunteer at the first provincial OAME conference hosted in Barrie (2002 - Year of the Palindrome), and this year was the co-chair of the Hospitality Committee for OAME '007 - Building Bonds

The year 2007 marked a very important year for the Math Association for Cottage Country. For this small, but mighty group, that meant hosting the Provincial OAME Annual Conference on May 3rd, 4th, and 5th. It was unanimously decided that 2007 was the year for James Bond, for very obvious reasons! What grew from this theme was very symbolic in nature – the creation and nurturing of bonds within the realm of teaching mathematics – thus the title of our conference – **Building Bonds '007**.

Planning for this conference started well over two years ago, with many seasoned volunteers returning to share their expertise from hosting the provincial conference in 2002: The Year of the Palindrome. One thing was certain: the organizing committee wanted to raise the bar and present a well-run conference, while catering to the needs of everyone concerned – publishers, volunteers, presenters, delegates, and a new group in the circuit – the parent community. **Building Bonds '007** Provincial Conference was to provide a variety of rich learning experiences and opportunities for enthusiastic people working together within a nurturing math community.

With one year to go, committee members attended the 2006 conference in London, Ontario. We met with the London group and received very valuable feedback. Our promotions group did an excellent job designing logos and preparing pamphlets, signage, power-point presentations, and magnets to entice delegate hopefuls for the following year. Presenters and publishers were invited to attend and sign up on-line.

For the next few months, committee members met regularly as a whole team and in smaller groups, organizing a variety of important details. Facilities were booked at Georgian College in Barrie to house the conference. Money was budgeted and allocated for each event. A program was designed, both electronically and in hard copy with all the necessary information. Promotions continued, with members going to the community for support, both financially and to gather gifts for delegates, presenters and parents. Registration gathered information for the people who were to present workshops and those attending them. Exhibits communicated with publishers and organized tables and spaces for displays. Hospitality started meeting with groups at the college to organize availability of food and snacks on site for approximately 1200 attendees. They ordered speakers' gifts and planned events for interested participants. Volunteers were recruited from York's Teacher Candidates from the Barrie and Richmond Hill sites. Finally, preparations began for the introduction of a Parents' Conference, to take place on the Saturday morning.

As the year passed (and so quickly) the conference began to take form. Under the very capable leadership of Lyn Vause and Joyce Smith, little wrinkles were ironed out one by one.

On the morning of Wednesday, May 2nd, the twenty core committee members and ninety-five volunteers congregated to set up for the conference for 979 attendees and 208 presenters. The registration centre was opened, signage and balloons were posted, publishers set up, and finally, delegates arrived. A "Mix and Mingle" at the Last Class restaurant started off the social events with snacks, drinks, games, and music.

Over the following two and a half days, participants had the opportunity to see a variety of prolific key note speakers. Inspiring messages came from Dr. Avis Glaze, Chief Student Achievement Officer of Ontario and CEO of the Secretariat. The next keynote speaker was Jon Paulos, a professor of mathematics at Temple University in Philadelphia. On Friday, well known friend and presenter Trevor Brown, Director of Mathematics Education at OISE/ University of Toronto delighted the Intermediate and Senior delegates, and Dr. Sue Willis, the Dean of the Faculty of Education, Monash University, in Australia, shared insights with a primary and junior focus. Nazneen Dindar, Vice-Principal at Roland Michener P.S. in Ajax was the keynote speaker at the Equity Lunch.

Many expert presenters delivered workshops on a

variety of math topics relating to assessment, integration, technology, literacy, and core topics. Included were a Texas Instruments Mini-Conference Technology Playroom, and a Photo Math Gallery Exposition. Workshops were very well received by the delegates and enthusiasm for teaching math was renewed as math talk amongst colleagues reverberated through the hallways.

On Thursday evening, McGraw-Hill sponsored a lovely Wine and Cheese at Last Class with special music provided by Evening Blues (Karen and Greg Parnell). Some delegates headed to Casino Rama for an evening of intrigue and daring! Another first also occurred this evening in the form of a Murder Mystery called Licence to Kill. Greg Clarke, our dauntless chapter president, developed a plot with many twists and turns when a young education student from Georgian College meets his end on the fender of a car. Participants were sent to look for clues around campus: the scene of the crime in the parking lot, the evidence room and video taped interrogation in *Summer Suites*, the library computers, Tim Hortons, Last Class, to name a few. Teams had to piece together the events leading up to that fateful night and find the murderer(s), so-called friends of the deceased! Everyone involved had a fabulous time and many thanks go out to Greg and his organizing committee, York volunteers who played the roles of the characters, and the mysterious professor in the trench coat! (NOTE: No actual teacher candidates were harmed in the making of this mystery.)

Friday evening was a star studded event from right out of the movies. The title of the OAME banquet was "The World is Not Enough". Approximately one hundred twenty guests were treated to a delicious meal at the Barrie Country Club. People thoroughly enjoyed visiting with James Bond and his leading lady and had the opportunity to sip a martini as well! Following dinner, several worthy men and women received distinguished awards.

Our committee was particularly thrilled and proud to share the evening with Bill Morrison as he received his Lifetime Achievement Award. Bill is a most deserving recipient who has been an innovative leader in mathematics education and in OAME, both past and present. Congratulations Bill! The evening concluded with *Calculus The Musical*, an appropriate way to end a lovely evening, enjoyed by all in attendance.

As part of the committee's commitment to building math bonds, Andrew Morrison enthusiastically organized the first Parent Conference, in conjunction with the provincial conference. Two hundred fifteen parents registered and the event was opened with an address by Kathleen Wynne, Minister of Education. Keynote speakers, Jack Weiner and Don Fraser entertained, yet evoked thought as to how best support our children's mathematics education in the home. Workshops for parents of students from kindergarten to grade twelve were ably presented by Linda Adams, Anna-Marie Boulding, Susan Stuart, Marie Heffernan, Louise Robitaille, Jennifer Brown, Karen Pond, Jacqueline Hill, Connie Quadrini, Ron Lancaster, and Jack Weiner. Thanks to the Simcoe Muskoka Catholic District Board and the Simcoe County District School Board for sponsorship of this event. Last Class, Georgian College, provided a delicious lunch and snacks for everyone. The Parent Conference provided a very positive experience in linking education and home by building a strong bond between the two.

Thus, **Building Bonds '007** came to a close. Exhausted, yet satisfied, organizers cleaned up, packed their bags and headed for home and a well deserved rest.

The organizing committee is particularly grateful to Georgian College and Patty Bradley for being so accommodating in the planning and implementing of the conference. It took much organization and expertise to produce a conference of this size. Sincere thanks and gratitude is extended to Mick Kingston and Katie Saito at Last Class – they made our worries about food simply evaporate as everyone was treated to an abundance of tasty treats. Thank you to Christine McSorley from the Georgian Dining Room and Mike King from Chartwells for lunches and meals provided to the delegates. Finally, sincere thanks are extended to all Georgian College staff that assisted in any way to make the conference run smoothly. It was much appreciated.

We had many volunteers who donned their red t-shirts to help delegates with directions, publishers to carry boxes, committee members with a variety of tasks (from blowing up balloons to decorating for events). These people (the majority of them were student teachers from York University) went above and beyond the call of duty and were truly appreciated by all. Thanks to Betty Nelsons for providing a variety of excellent meals for this hardworking group.

The success of an event such as this was reflected in the teamwork that occurred during every step of the planning phase, and continued well after the conference ended. The Math Association for Cottage Country is a

very special group of people, who are dedicated to math education and who also enjoy each other's company very much. A phenomenal amount of work was done by everyone, in the midst of regular jobs, family commitments and well-deserved retirement. The backbone of our committee is a select group of experienced, enthusiastic, and well-loved retired teachers! As the conference loomed closer, the closeness of the committee became more and more evident. Everyone worked as a cohesive group, supporting and encouraging each other through challenging times. Food and drink was always in abundance at meetings, and a great comfort in times of stress! Laughter often rang out across the table. This was a unique group of professionals from different boards, stages in careers and lives. It was an honour to have been able to work on this committee with this unique, awesome, and hardworking group. Congratulations on a job well done – Joyce Smith, Lyn Vause, Jack LeSage, Bill Morrison, Brad Hilliard, Jeff Irwin, Dave Davidson, Tracey Nastasiuk, Trish Steele, Diane Vetter, Sheila Garriock, Andrew Morrison, Greg Clarke, John Egsgard, Danielle Blair, Cheryl McGinnis, Judy Blaney, Jennifer Brown, Betty Nelsons, Ursula Irwin and all committee members who worked together to make the conference so successful.

It is the committee's desire to see our work carry on. Monies raised from this conference will be used to strengthen the bonds created in those first few days of May, 2007, by providing learning opportunities for all integral parts of the mathematics community – teachers, parents, and students alike. ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor

Marilyn Hurrell for further details.

Letters to the Editor always welcome!

▲ 2007 OAME Award Recipients

JACQUELINE HILL



The back bar is left-to-right: Edmond Aube, Debbie Allen, Moira Courtenay, Rosie Tut, Naheed Habibzadah. The left bar is front-to-back: Lynda Ferneyhough, Lynda Sanderson, Sabrina Noguchi, Rachel Ng. The right bar is front-to-back: Andrea May, Lorna Cameron, Mukesh Goel. Inset: Hilary Boychuck.

Kenneth D. Fryer Award:

In memory of the late Kenneth D. Fryer, OAME in conjunction with the Grand Valley Mathematics Association, established this award to be given to a Mathematics Department that fosters collegiality, team work, excellent classroom teaching; contributes to the overall development of students and demonstrates leadership in the mathematics education community. This is a huge bill to fill. This year's winning department has only been in operation for 3 years. In the words of one of the nominators "this department has shown exemplary leadership, creativity, flexibility and professionalism. And under the leadership of Lynda Ferneyhough this team of awesome educators has proved to be at the leading edge when it comes to innovation". In this department, informal mentoring and professional development goes on every day. This math department works hand in hand with other departments in the school to staff the "Counting on You" program and works to ensure that all students are taught at their level and achieve success.

It is with great excitement that the teachers of the math department from **Fletcher's Meadow Secondary School** in Brampton under the leadership of their department head Lynda Ferneyhough have won the Kenneth D. Fryer Award.

Life Member Award:

This award is given to the individual who has contributed in a significant way to OAME. This individual must have demonstrated outstanding leadership in mathematics education and have accumulated 10 or more years of membership in OAME.



Let's start with Leadership. Our OAME Life member award winner, Bill Morrison, has a leadership style that legends are made of. He became an OAME member in 1973 and was a founding member of the MAC^2 chapter in 1995. He has served as chapter president not once, but twice. He has co-chaired the provincial Math Olympics, and has always had a hand in leading his chapter teams in their OMO preparations. He has presented at various OAME annual conferences as well as at MAC^2 mini-conferences. He was the Mathematics department head at Barrie North Collegiate Institute in 1998 when the department won the very prestigious Kenneth D. Fryer Award for an outstanding mathematics department.

Bill had numerous letters of support to accompany his nomination and I would just like to close with a few quotes from these letters.

Jack LeSage writes "Bill is our Rock of Gibraltar at MAC^2".

Dave Davidson a Life Member and former president writes "Here's to Bill. Let me know where I sign".

John Egsgard, a Life Member, former OAME president and former NCTM president writes "Bill has helped keep MAC^2 going. He took on the president's role (the second time) even though he was already retired from teaching. And he was a strong contributor to the success of the OAME Annual Conference in 2002".

Well Done Bill!

The OAME Award for Exceptional Creative Teaching in Secondary Mathematics:

Rob McLeish is the Mathematics and Computer Science department head at Sir Robert Borden High School in Ottawa. He works tirelessly with teachers, students and parents to ensure the success of students. Eight years ago Robert Started a Mastery program for the students in his school. This program has helped with student mastery, the understanding and retention of materials. Rob is devoted to staying on top of new teaching methods and tools, from algebra tiles to graphing calculators.



Several of his current and former students had these words to say about Mr McLeish: "I was fortunate enough to enter into high school, and walk immediately in to Mr McLeish's classroom for my homeroom for the year. I hadn't had the greatest instruction in my previous math classes, and the idea of high school math was completely daunting to me. However Mr McLeish demonstrated immediately that he would stop at nothing to be the greatest teacher I ever had."

Another student writes "And my favourite thing about Mr McLeish isn't even the fact that his teaching gives complete and thorough understanding or that he is always there for his students and colleagues. It is his quality of assurance. Never once have I seen him use notes or open a textbook to teach a lesson, but I can trust that he will most definitely teach the right material and in the best possible way".

It is with great pleasure that OAME gives this award to Rob McLeish.

The OAME Award for Exceptional Creative Teaching in Elementary Mathematics:



In the words of one of her nominators "Shelagh McCarthy is an outstanding and creative teacher in all areas of the curriculum, and is particularly passionate in her goal to have all students believe that they can be successful in mathematics. Shelagh differentiates her math program by utilizing the students' multiple intelligences and learning styles to help her students develop conceptual mathematical understanding of the math processes".

Shelagh is a leader in mathematics in both her school and board. She currently teaches at Morison Public

School in Deep River Ontario. Well done Shelagh.

Award for Outstanding Contribution to OAME:



Dr Lynda Colgan is an associate Professor at the Faculty of Education at Queen's University in Kingston. Lynda's passion for mathematics is felt province wide. Lynda has contributed to OAME locally by being a long time member of the SAME chapter. She was chapter president and also helped in the organizing committee for the OAME Annual conference in Toronto.

Her contributions to OAME provincially include being a coordinator of the provincial math Olympics not once but twice, a regular columnist in the OAME *Gazette*, as well as a contributor of many articles to the *Gazette*. She is a long time speaker at annual and regional OAME conferences.

Letters of support for her nomination came from faculty members, the Dean of Queens University, a newspaper editor, a loyal reader, former faculty of education teacher candidates, a publisher, and several math collegues.

But I must end with a quote from her nominator who is a past OAME president, and an OAME Life Member. He states "I have known Lynda since she was an extremely active and energetic student in my grade 12 mathematics class in the late 1970's. In the intervening years I have had opportunities to see Lynda mature and grow mathematically, to see her at work, and to actually work with her".

Those are really strong words, and with this, OAME is delighted that Lynda has won this award.

OAME and Union Gas have teamed up to present three more awards.

The Union Gas Award for Teaching Excellence in Elementary Mathematics:

This award is given to an elementary classroom or special education teacher who demonstrates exemplary mathematics teaching practices and supports inclusive mathematics education. This year's winner comes to us from the Waterloo Region District School Board. We are happy to recognize Christine MacTavish.

In the words of one of her nominator, "Christine is an outstanding teacher who is dedicated to ensuring success for all students by creating an environment where they are engaged and excited about, learning mathematics". Christine shares her expertise in teaching at board sponsored events and OAME conferences. Christine is a grade 3 teacher and because of her passion for mathematics and the students she teaches, she was selected as a demonstration classroom teacher. In this capacity Christine shares her best practices with other teachers in her board.



It is with great pleasure that OAME and Union Gas gives this award to Christine MacTavish.

The Union Gas Award for Teaching Excellence in Secondary Mathematics:



Paul Alves comes to us from the Peel District School Board. He is the head of mathematics at his school, but firstly he is a teacher of mathematics. He has been described as a magician who allows the magic to spill over into the classroom. It has also been said that he puts an incredible amount of thought into his lesson preparation and is constantly trying to improve the classroom learning environment for his students. He is an advocate of technology use in the classroom and is well versed in manipulatives, graphing calculators and computer algebra systems. He works tirelessly at the school, board and provincial level, and at the national level. Congratualtions Paul!

The Union Gas Award for Outstanding Leadership in Mathematics Education:

Bonnie Jaakkimainen is a principal at Brookmede Public School in the Peel District School Board. Bonnie has been described as a principal with passion. She has passion for excellence in mathematics education. She believes in inclusive mathematics education and leadership. The letters of support that were sent in to accompany Bonnie's nomination show that she is well respected by her staff, students and parents.

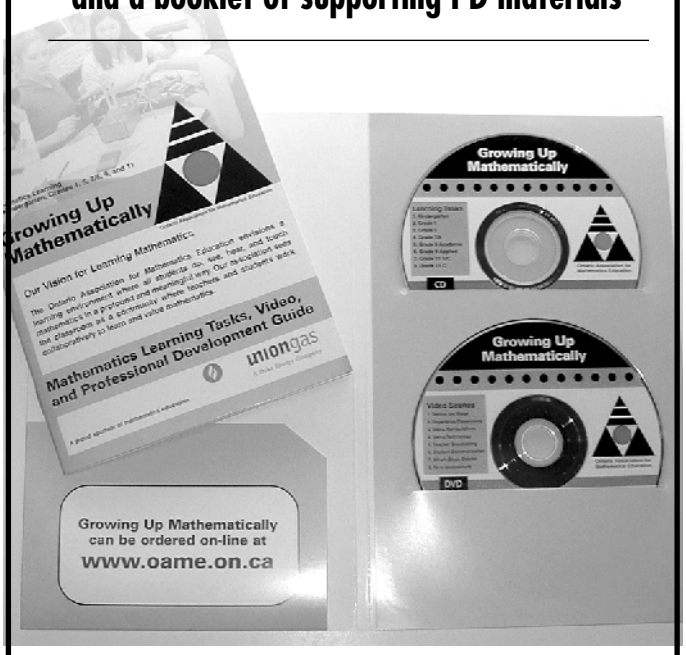


She encourages and supports her staff in many math initiatives. She does not just sit by idly and send them out for professional development. As one person wrote "she walks the talk…as a life long learner". A second quote I would like to share with you is "Bonnie is an exceptional leader of mathematics. She has a clear vision of how children learn and she shares this with others".

It is with great pleasure that OAME and Union Gas gives this award to Bonnie. ▲

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▲ Book Review: *The Man Who Counted*

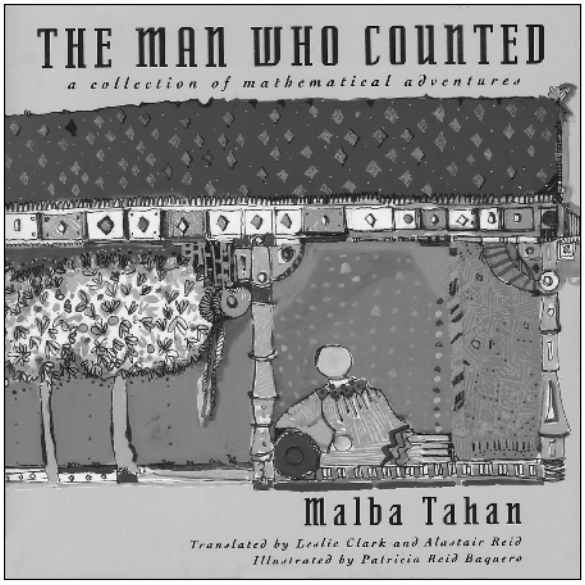
REVIEWED BY MERILYN FOX



Merilyn Fox is a long – time math and English teacher at the Intermediate level. She is currently an Instructional Leader in mathematics and literacy with the TDSB. Her hobbies are reading, music and playing with her grandchildren.

The Man Who Counted, a collection of mathematical adventures by Malba Tahan, translated by Leslie Clark and Alistair Reid, illustrated by Patricia Reid Baquero. Published by W.W. Norton & Company, translation © 1993. 244 pages

Malba Tahan, full name Ali Iezid Izz-Edim ibn-Salim Hanak Malba Tahan, was a fictitious Persian scholar, devout Muslim and observer of human kind. According to the internet, he was the creation and frequent pen name of Brazilian author Júlio César de Mello e Souza. The book purports to have been written "in the nineteenth



day of the moon of Ramadan, 1321", but was actually originally written in Portuguese in Brazil in 1949. There have been many editions and it has been, and still is, being translated into many languages. It is of general interest to anyone who enjoys reading math fiction in the form of puzzles and patterns. It is also of particular interest to teachers in the intermediate classroom, who

could use the book as a stimulating part of teaching problem solving.

The Man Who Counted is a collection of tales following the adventures of the main character and calculator, Beremiz Samir, as told by his friend and narrator, Malba Tahan. Most of the tales involve mathematical stories, puzzles, and interesting facts loosely linked by the adventures of the main character.

The opening of the book tells us how Malba Tahan met Beremiz Samir while he was traveling from Samarra. Samir caught his attention by calling out large numbers from his seat on a rock by the side of the road. Curious, Tahan asked for an explanation. It turns out that Samir was counting – swarms of bees, leaves on trees, sheep in a flock. Tahan was so impressed with his prodigious ability, that he invited Samir to accompany him to Baghdad, certain he would find employment with a vizier or caliph.

Each of the following chapters tells how Samir used his mathematical abilities to solve disputes, enlighten people and find acceptable solutions to seemingly insurmountable problems. The short chapters pose the challenge and give us Samir's solution, along with his reasoning.

Many of the problems will be familiar to you, but they are presented in a fresh new setting as part of a larger story. For example, the first incident is a variation of the inheritance dispute. In this version, a father has died and left his three sons 35 camels, with instructions that half should go to the oldest son, a third to the second son, and a ninth to the youngest. Beremiz adds Tahan's camel to the herd, making all the fractions possible. When the thrilled brothers realize they are all better off than they would have been, they accept Samir's solution, giving the oldest brother 18 camels, the second 12 and the third 4. This adds up to 34 camels, leaving one (his own) to Tahan and one to Samir as reward for his solution.

My favourite story, new to me, was a neat little discussion of ratios. The two travellers encountered a third man who had barely escaped with his life when bandits attacked and looted his caravan. The man, Salem Nasair, asked for food, and promised to pay for it upon their arrival in Baghdad. Tahan put up his three loaves of bread, and Samir offered his five loaves. Nasair promised them eight gold pieces. When they arrived, Nasair gave Tahan three gold pieces and Samir five gold pieces. Samir objected! He believed he should get seven gold pieces compared to Tahan's one. He reasoned that they had divided all eight loaves into three pieces each, or twenty-four pieces. Samir's five loaves were divided into fifteen pieces, of which he ate eight; therefore he gave seven pieces to Nasair. Tahan's three loaves were divided into nine pieces, of which he ate eight, contributing one piece to Nasair. Of course, after demonstrating the irrefutable logic of his answer, he split the eight gold pieces evenly with his friend, four each.

As the two friends journey, they encounter many more chances to demonstrate Samir's mathematical genius. Among the puzzles you may recognize are the grains of rice on the chessboard, told in an unusual way. Another is a version of the problem in which some people always lie and some always tell the truth. The task is to determine what question to ask to determine who is who. These were made more enjoyable by being in the context of a larger adventure. Of course there are envious courtiers who try to trip up Samir by setting the most difficult math questions for him. He always succeeds in solving them, with modesty and good humour which attract many admirers. Among his admirers is his pupil, the demure and modest Telassim, whom he marries, and they are very happy.

When Samir is asked to give assistance in several of the problems, the people he is helping have already tried to find solutions and the dispute is about which of their solutions is correct. One of the great things about this book is that the author dissects the faulty reasoning, and then explains the line of thinking that will lead to agreement. This is an engaging way of demonstrating some problem – solving strategies.

Samir also muses about patterns and relationships. He even states, that "using four fours, we can get any number whatsoever" (p.39), for example, $44 - 44 = 0$, $44 \div 44 = 1$, and so on. I can see myself challenging my intermediate math students with that one!

The book ends with the sacking of Baghdad by a grandson of Genghis Khan. Our hero, however, has moved to Constantinople with his love and his friend, and all is well.

The format of the book lends itself to being read aloud at the beginning or end of a math class. The chapters are short, averaging about seven pages each, including drawings. You could read up to the posing of the problem, then have a discussion or work on a solution together. The solution is always given in the story. You can simply decide when you want to reveal it. Or not! Sometimes, the struggle is an important part of learning. Students need to learn how to get unstuck, and this book provides them with an incentive to try. I recommend it. ▲

▲ Financial Concepts and Applications (Taking Stock in Your Future)

CHRISTINE ALUM

JOHN KITNEY

John Kitney recently retired from Bayridge Secondary School and was Head of Mathematics. John is committed to working with teachers to integrate financial literacy in the classroom.

Chris Allum is Investor Education Specialist at the Investor Education Fund and has been involved with the Taking Stock program since its inception.

Financial literacy and numeracy are embedded in our Mathematics curriculum. The importance of teaching financial literacy is clear in an age where people are required to take more responsibility for their own financial future thanks to declining pensionable jobs and a weaker social safety net. Despite this, there is no compulsory course on the subject in the curriculum.

In mathematics curricula, students learn financial concepts and develop the skills they need to manage their finances. The reality is that many students do not make the connection between the concepts taught and their application to real-life. One way to help students make this connection is through the "Taking Stock in Your Future" program – a free program to teach money management. This program provides teacher resources that are Curriculum Services Canada approved and classroom-ready. The resources explore financial literacy as the overarching theme within a mathematical context.

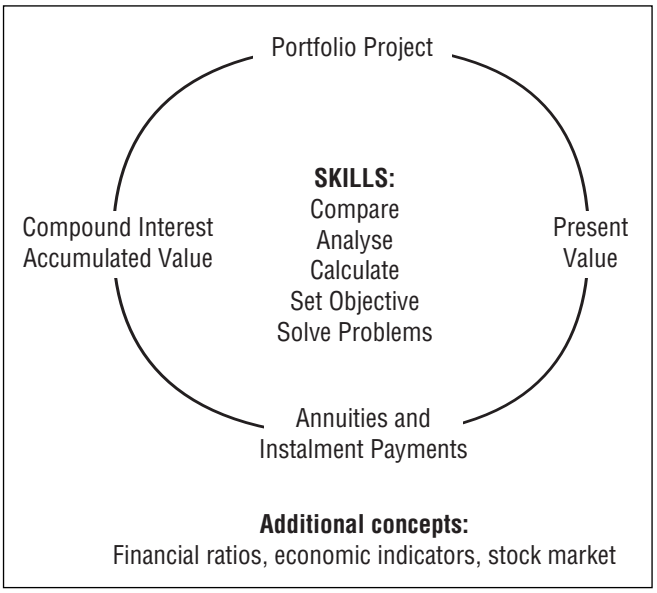
In the *Taking Stock In Your Future Senior Guide*, the student activities revolve around three 'big ideas' in mathematics finance: (i) compound interest and accumulated value, (ii) present value, and (iii) annuities and instalment payments. These math concepts lay the foundation for the Investment Portfolio Project, an exercise where students can apply their skills to create the portfolio.

In the portfolio project, students create an investor profile, calculate monthly savings available to help achieve the investor's goals, develop an asset allocation model, make investment decisions and recommendations and monitor the investments. Students use the math concepts learned to:

- solve problems involving compound interest and accumulated value
- analyse financial data
- review trends in economic indicators
- compare and evaluate financial products
- set objectives to meet goals
- devise a financial strategy to meet goals

In addition, students use technology to:

- manage data
- perform calculations
- research information



The skills students develop are transferable to other aspects of their lives. By working through the concepts and applying them, students can see how useful the information really is. Students who have worked on this project often say it's the best project they've ever worked on in high school.

Contact the Investor Education Fund for these FREE resources and to learn how to implement the *Taking Stock in Your Future* program in your school. Email fundmail@investorED.ca or visit our web site, **www.investorED.ca** for more information. ▲

▲ Solving Polynomial Equations – Reducing the Number of Factors

DUNCAN E. McDOUGALL

RYAN WILLOUGHBY



Duncan McDougall has been teaching for 27 years and is a graduate of both Bishop's and McGill Universities. After 13 years in the public school systems of Quebec, Alberta, and British Columbia, he left the classroom for private teaching. Throughout the last 15 years, Duncan has taught all levels of high school mathematics, college and university calculus, and mathematics for elementary school teachers. This rich exposure to diverse programs and curriculae has provided him with some of the ideas, which he is now publishing. Duncan has given several workshops on Rapid Calculation and non-conventional approaches to operations on radicals and fractions. His latest project is developing a special family of polynomials for curve sketching called Diophantine Polynomials. Currently, Duncan owns and operates TutorFind Learning Centre in Victoria, British Columbia.

This article first appeared in delta-k*, Vol. 44#2 June 2007 Journal of the Mathematics Council of the Alberta Teachers' Association and is reprinted with permission of* delta-k.

The following article describes a sequence of steps designed to reduce to a minimum the number of eligible factors when solving analytically polynomial functions with integral roots and coefficients. Our objective is to list all possible factors and then select only those factors that satisfy certain criteria. This is done by incorporating Descartes' Rule of Signs [Editor's Note: see **www.purplemath.com/modules/drofsign.html**] and the factors of the sum of the numerical coefficients of the given polynomial. The algorithm consists of the following steps:

1. Apply Descartes' Rule of Signs to determine possible numbers of positive or negative real numbers and/or complex roots.
2. Find the sum of the numerical coefficients of the polynomial.
3. List all the factors of this sum then add 1 to each factor. This becomes set B.
4. List all the factors of the constant of the polynomial (Integral Factor Theorem). This becomes set A.
5. Find the intersection of sets A and B, listing only the common elements.
6. Use step number 1 to focus on the number of positive or negative real roots.

Using a variety of examples, let us examine the usefulness of this algorithm.

CUBICS:

Example 1: Solve $x^3 – 5x^2 – 8x + 12 = 0$

Step 1: There are two variations in sign for p(x); Case 1) two positive real roots and one negative real root; or, Case 2) no positive real roots, one negative real root, and two complex roots.

Step 2: The sum of the numerical coefficients, 1 – 5 – 8 + 12, is zero. Since the sum is zero, the number 1 is a root (Remainder Theorem) and is a factor of p(x) (Integral Factor Theorem). We go immediately to either long or synthetic division to find the other two roots. This gives a quotient of $x^2 – 4x$ 12 or $(x – 6)(x + 2)$. We go no further because we have all three factors; $(x – 1)$, $(x – 6)$, and $(x + 2)$ and thus the roots 1, 6, and –2.

Example 2: Solve $x^3 – 4x^2 + x + 6 = 0$

Step 1: There are two variations in sign for p(x); Case 1) there are two positive real roots and one negative real root; or, Case 2) no positive real roots, one negative real root, and two complex roots.

Step 2: a) $1 – 4 + 1 + 6 = + 4 \neq 0$, so 1 is not a root

b) 1 + 1= 2 and -4 + 6 = 2. Since these sums match, (Remainder Theorem), -1 is a root and x + 1 is a factor.

We go immediately to long or synthetic division in order to obtain the other two factors $(x – 2)$ and $(x – 3)$. We go no further because we have all three factors; $(x + 1)$, $(x – 2)$ and $(x – 3)$.

The above two examples serve to show the importance of immediately looking for 1 or –1 as a factor. However, not all polynomials contain these factors, and so we continue with the next example.

Example 3: Solve $x^3 – 4x^2 + 11x + 30 = 0$

Step 1: There are two variations on sign for p(x); Case 1) there are two positive real roots and one negative real root; or, Case 2) no positive real roots, one negative real root, and two complex roots.

Step 2: $1 - 4 - 11 + 30 = 16 \neq 0 \therefore$ 1 is not a root and $1 - 11 \neq -4 + 30 \therefore$ -1 is not a root

Step 3: From step 2, 16 is the sum and its factors are -1, -2,-4, -8, -16 and 1, 2, 4, 8, 16. Adding 1 to each factor 0, -1, -3, -7, -15 and 2, 3, 5, 9, 17.

Set B = {3, -1, -3, -7, -15, 2, 3, 5, 9, 17}.

Step 4: From the Integral Factor Theorem, the factors of 30 are placed in Set A;

Set A = {-1, -2, -3, -5, -6, -10, -15, -30}
{1, 2, 3, 5, 6, 10, 15, 30}

Step 5: The intersection of the two sets gives {-1, -3, -15, 2, 3, 5}. Since –1 has already been eliminated, (Step 2), the list of possible factors is really {-3, -15, 2, 3, 5}.

Step 6: From Step 1, only one of the elements –3 and –15 can be a factor, while two of the three elements 2, 3, and 5 are factors.

If we compare the original number of factors of 30, (16) to the elements in A∩B, we have narrowed it down to 5. Incidentally, the probability of selecting the correct negative root is 1/2, while the probability of selecting the correct positive root is 2/3. Trying the positive roots in ascending order where the probability is higher, we have p(2) = 20. The other roots become 5 and –3, (both in the final list).

QUARTICS:

When we get into the realm of even-numbered polynomials, the rule for 1 works but the rule for –1 does not. However, everything else holds. We now examine two more examples, one of which emphasizes the strength of Descartes' Rule of Signs:

Example 4: $x^4 + 5x^2 + 1 = 0$

Step 1: There are no variations in p(x), so there are no positive real roots. The same is true in p(-x), so there are no negative real roots.

We conclude that there are four complex numbers, then move on!

Example 5: $x^4 + 6x^3 + x^2 - 24x - 20 = 0$

Step 1: There is one variation in sign for p(x), so there we have the following possibilities; Case 1) one positive real root and three negative real roots; or, Case 2) one positive real root, one negative real root, and two complex roots.

Now $p(-x) = x^4 - 6x^3 + x^2 + 24x - 20$.

Since there are three variations in sign, we will look for three negative real roots.

Step 2: The sum of the numerical coefficients is $-36 \neq 0 \therefore$ 1 is not a root.

Step 3: The factors of the sum 36 are -1, -2, -3, -4, -6, -9, -12, -36 and 1, 2, 3, 4, 6, 9, 12, 18, 36.

Adding 1 to each factor gives Set B: {0, -1, -2, -3, -5, -8, -11, -17, -35} {2, 3, 4, 5, 7, 10, 13, 19, 37}

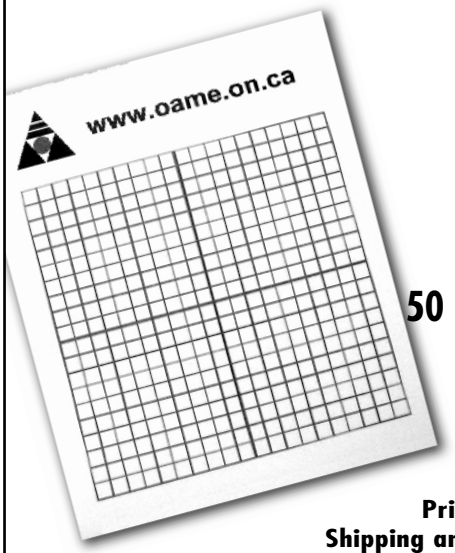
Step 4: The factors of the constant 20 gives Set A: {-1, -2, -4, -5, -10, -20} {1, 2, 4, 5, 10, 20}

Step 5: A∩B = {-1, -2, -5, 2, 4, 5, 10}

Step 6: From Step 1, there are three negative real roots, so -1, -2, and -5 qualify, and since p(2) = 0, we have all four roots.

Naturally, five examples do not make the case for all polynomial functions, as we have not explored rational and irrational roots. However, for the monic polynomial with integral roots, we have a method that cuts down the guessing of factors in order to solve a polynomial function. The combination of Descartes' Rule of Signs along with the intersection of two sets reduces remarkably the number of possible factors to be considered. This in turn reduces time spent on any one question and reduces the frustration from guessing, when solving analytically. ▲

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▲ Street Math

MIKE JACOBS



Mike Jacobs is a Math consultant for the Durham Catholic District School Board. Previously he was Curriculum Chair of Math at Father Leo J. Austin C.S.S. in Whitby, Ontario. He also taught 'maths' for ten years in Yorkshire, England.

Have you ever stopped to think about how closely the math you learned in school (especially elementary) correlates to the math that you use in real life? Try these two questions:

1) You have two cheques to deposit, one for $298 and one for $309. How much do you deposit in total?

2) What is the cost of 5 CDs at $8.99 each?

Now ask yourself, how did you get the answers? Even better, give these questions to your students and ask them how they get their answers. For the first question, did you align the numbers in columns and add the digits starting with the units? Or did you mentally add the hundreds, then the tens and then the units? Perhaps you used $2 from the $309 and in effect did $300 + $307. I am convinced that there are more ways of doing these problems than I have actually taught. I also know that the way I do these does not involve the traditional algorithms that I was taught when I was at elementary school in England. This strikes me as curious.

I am not alone in abandoning my learned algorithms when doing mental math. Similar problems were presented to elementary teachers at our Board (Durham Catholic DSB) and the vast majority used a non-standard approach. Keith Devlin, in his excellent book *The Math Instinct*, refers to the work of Nũnes, Schliemann and Carraher in highlighting the math used by young Brazilian street vendors and how this differed markedly from the math that they where taught in school. This street math reminded me of a former student of mine who refused to do the simplest of money addition problems in class. However I saw the same student out of school working in the local street markets and being able to add five or more prices mentally and work out the change after the items had been paid for.

Why is this? Why would a student, whom I have seen do multi-step problems in his head, believe that he cannot do easier problems in class? Perhaps it lies in the notion that our approach at teaching number sense has been algorithm-driven at the expense of getting our students to develop a good repertoire of mental math strategies. This certainly was my experience as a student when I learned algorithms long before I learned about mental math strategies or even before I learned about how to make a good estimate. To be honest, this is probably how I first taught as well. Now I realize that when it comes to teaching number sense, maybe algorithms are the last thing that should be taught. It is interesting to note that most adults do not use algorithms in everyday life.

Now don't get me wrong. I am not a mathematical anarchist who is against rules and algorithms of any type. What I do believe, however, is that for these algorithms to be understood and used effectively with meaning, there are many other skills that first need to be in place. Students who are being asked to do addition and subtraction must fully understand place value and how to partition numbers. Students who are being asked to do multiplication and division must fully understand addition and subtraction. Research by Kamii and Dominick provides strong evidence that algorithms actually work against the development of place value and number sense in students. In fact, students who had not been taught any algorithms (but instead had been encouraged to invent and refine their own strategies) were significantly more successful than students who had been taught an algorithm. As Pascal said, "We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others."

The *First Steps in Mathematics* framework that we have trained all teachers in at eight of the elementary schools in our Board is an extremely useful tool for placing students on a mathematical developmental continuum. It allows teachers to see, irrespective of the curriculum demands, if students are ready to understand certain concepts and algorithms. In the example right a student has answered the question simply by counting '2+3' and then '3+4' on his fingers. The student has no sense of the quantity '75' nor whether or not his answer is reasonable. The question has to be asked, 'Should any student need to use the standard algorithm for adding two 2-digit numbers?' I would suggest not, and that introducing a standard algorithm too early and for 'easy' numbers is counter-productive. As Fosnot and Dolk have noted, "Using algorithms, the same series of steps with all

$$\begin{array}{r} 32 \\ +43 \\ \hline 75 \end{array}$$

problems, is antithetical to calculating with good number sense."

Algorithms become problematic for students because of the myriad of often subtle rules that must be followed for success. These rules only make sense when the student has developed a thorough working knowledge of such big ideas as place value and operations. Think how confusing the 'rules' for a division algorithm can be if a student has no understanding of place value.

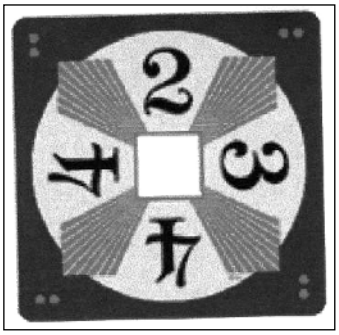
$$8\overline{)2456}\quad 307$$

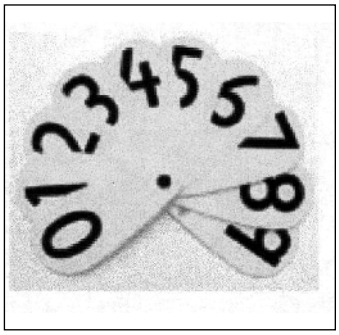
OK, so 8s into 2 doesn't go which is kinda weird 'cos my teacher said that 2 means 2 thousands and I would've thought that there must be at least a few 8s in 2000. Anyway, I know I don't put the '0' down for the 'no 8s' as that's what you do if you get a 0 to start with and now you say 8s into 24. That's 3 which for some reason we have to put above the 4. Now we say 8s into 5. This is 0 but we do put the 0 down this time as it's in the middle of the number and carry the remainder 5 and now say 8s into 56. This is something that finally makes sense to me as I know that that is 7… or is it 8? Hmmm…

Other problems will arise if students are only exposed to algorithms and not mental math strategies. The standard algorithm for finding the cost of the CDs in the initial example is inefficient compared to simply thinking '5 items at $9 gives $45 now subtract 5¢ to get $44.95'. John Van De Walle[7] noted that invented strategies are more concerned with quantities rather than most algorithms which are digit-orientated. He also noted that invented strategies are flexible and change with the numbers involved; algorithms are rigid, however. By encouraging, modelling and practising mental math and self-invented strategies, students will be able develop a repertoire of skills that will allow them the flexibility to deal with real life situations.

How can this be done? I would suggest that we start believing that math is a social activity and a fantastic opportunity for students to feed off each others' ideas. Give your students a verbal question and ask them to display their answers using 'number fans'. At a glance you will see who has the correct answer and can then ask these students to explain what they did. By listening to how others approach a problem, students will realize that there is more than one way to crack an egg. Given sufficient opportunity to practice (the National Numeracy Strategy in the U.K. recommends 10 minutes daily) students will begin to develop the flexible repertoire of mental math skills that they so desperately need.

The *24 Game* is another fantastic resource for improving mental math skills that your students will enjoy using. Students have to use all four of the numbers on a given card exactly once, but with any operations, to get the answer 24. If the numbers are 2, 3, 4 and 4 they could say (4 – 2) x (3 x 4). (Is there another way?) A more difficult example has the numbers 1, 6, 4 and 3. I'll leave this one for you to answer!





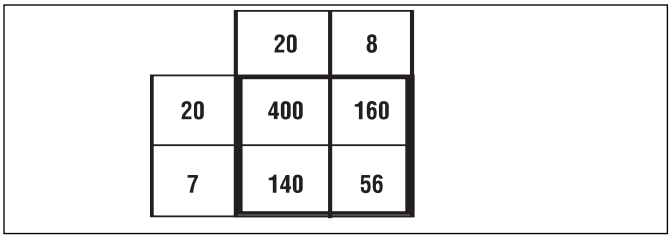
As students become proficient in their mental math skills, we can increase the complexity of the questions so that they might need to use informal pencil-and-paper methods to jot down their workings. This is useful because sometimes there are too many numbers to hold in our head. For example, to do 45 x 14 it might be easier to think of 90 x 7 and to jot these numbers down on paper before figuring out that 630 is the answer. I have often seen carpenters and joiners use informal jottings on the wood they are using to figure out a problem. Eventually the questions we give will be too complex and perhaps it is only at this point that we should introduce formal written algorithm.

What makes a good algorithm though? Is there any one algorithm that is the Holy Grail of long multiplication so that it will be the most efficient, and yet understood by all? I doubt it, but would love to know otherwise. To me, a good algorithm has to:

1) be efficient and simple to use
2) be fully understood by the user
3) connect with the user's previous and future knowledge
4) provide the right answer!

This does mean that one algorithm might be good for one student yet bad for another. We should bear in mind, also, that algorithms can be culturally biased. If we just consider multiplication, compare our 'traditional' algorithm with these others below.

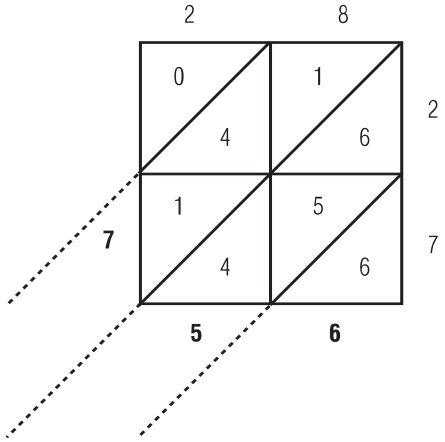
a) Multiplication by use of the Distributive Property: (see how this connects to the Array method)



$= 20 (20 + 8) + 7 (20 + 8)$
$= 20 \times 20 + 20 \times 8 + 7 \times 20 + 7 \times 8$
Hence $28 \times 27 = 400 + 160 + 140 + 56$
$= 756$

$$\begin{array}{r} 28 \\ \times 27 \\ \hline 56 \\ 140 \\ 160 \\ 400 \\ \hline 756 \end{array}$$

b) Elizabethan, Lattice or Gelosia Multiplication (Napier's Bones):



c) Egyptian Multiplication:

$1 \times 7 = 27$
$2 \times 27 = 54$
$4 \times 27 = 108$
$8 \times 27 = 216$
$16 \times 27 = 432$

Hence $28 \times 27 = (4 \times 27) + (8 \times 27) + (16 \times 27)$
$= 108 + 216 + 432$
$= 756$

d) Russian Peasant Multiplication:

~~28~~	~~27~~
~~14~~	~~54~~
7	108
3	216
1	432

Hence $28 \times 27 = 108 + 216 + 432$
$= 756$

Can you see why this works? Is it a good algorithm?

e) Informal Multiplication by Non-Standard Partitioning:

$$\begin{aligned} 28 \times 27 &= (30 \times 27) - (2 \times 27) \\ &= (3 \times 270) - (54) \\ &= 810 - 54 \\ &= 756 \end{aligned}$$

My father, an E.A. in a school in the south of England, was taught the Russian peasant method by a student with special needs. If your students find some of these methods confusing please remember that this does not mean that they are bad algorithms; they are just not 'right' for those students. In my role as Math Coach for the Board I have shown these different methods to students and teachers; not only do they find the different algorithms fascinating but in most cases, they often prefer one of the 'new' methods. Indeed, the reason why these are often preferred is because they make sense to students. I would perhaps go so far as to suggest that the array method should be the first multiplication algorithm that students learn: it is efficient (certainly for up to three-digit by two-digit multiplication), is understood by the students (providing that they truly understand the operation of multiplication), connects with students' previous knowledge (as it is a direct generalization of multiplication that has been modelled with base ten blocks) and can connect with their future learning of multiplication of decimals, fractions and binomials. I am certain that once students fully understand the array method they will be ready to understand the other algorithms.

Having developed, learned and mastered a variety of algorithms (up to four-digit by two-digit as required in the Ontario Curriculum), it is important that we help our students realize when and where they should use them. In years gone by, algorithms were extremely important as they were the best way of quickly finding the answer to lengthy calculations. Nowadays, we would mainly use technology for such questions and we would check the validity of our result mentally by using rounding techniques. Most situations in the real world can be

solved if students have a good repertoire of mental math skills and informal pencil-and-paper techniques. Therefore perhaps we should be honing these 'street math' skills in our students.

References

Devlin, Keith (2005). *The Math Instinct.* New York: Thunder's Mouth Press.

Nũnes, T., Schliemann, A. & Carraher, D. W., (1993). *Street mathematics and school mathematics.* New York: Cambridge University Press.

Northcote, Maria & McIntosh, Alistair (1998). What Mathematics do adults really do in everyday life? *Australian Primary Mathematics Teacher Vol. 4 No. 1*, pp 19-21.

Kamii, C. & Dominick, A. (1998). The harmful effects of algorithms in grades 1-4. In L.J. Morrow (Ed.), *The teaching and learning of algorithms in school mathematics.* (pp130-140). Reston, VA: National Council of Teachers of Mathematics.

Willis, Sue et al. Western Australian Department of Education and Training. *First Steps in Mathematics: Number Sense.* Canada: Pearson Professional Learning

Fosnot, Catherine Twomey & Dolk, Maarten (2001). *Young mathematicians at work: constructing multiplication and division.* Portsmouth, NH: Heinemann

Van de Walle, John (2001). *Elementary and middle school mathematics: teaching developmentally*. New York: Pearson Education. ▲

▲ Upcoming Events

For further information on OAME Chapter Conferences contact the Chapter Representative and for NCTM conferences, contact John Kersely, OAME/NCTM Rep.

See the last page in this *Gazette* for contact information. Also visit **www.oame.on.ca**

OAME Chapter Conferences and Events

October 2 2007	Fall Social TBA	PRMA
October 11 2007	University of Waterloo	GVMA
October 17 2007	Fall Mini Conference TBA	ISOMA
October 18 2007	Conference TBA	CHAMP
October 19 2007	Annual Conference: St Charles College S.S. Sudbury 4 - 11 pm	NOMA
October 2007	Fall Social TBA	COMA
November 2007	Fall Conference TBA	WOMA
February 21 - 23, 08	Leadership	OAME
Feb 28, 29 March 1 2008	Regional Conference Travelodge Airlane Hotel Thunder Bay ON	NWOAME
May 31 and June 1, 08	OMO: Trent University	PRMA

OAME Annual Conference

May 8 - 10 2008	Sheraton Parkway Toronto N Richmond Hill	Y4MA

NCTM Regional Conferences

October 11 - 13 2007	Richmond VA	"Exploring Mathematical Horizons"
October 25 - 27 2007	Kansas City MO	"Jazzed About Math"
November 29 - 30 2007	Houston TX	"Energizing Mathematica"
April 9 - 12 2008	Annual Meeting and Exposition	"Becoming Certain About

▲ The Peel CAS Journey

PAUL ALVES

IRENE McEVOY



Paul Alves is the head of Mathematics and Business at Stephen Lewis Secondary School in Brampton.



Irene is currently an Education Officer at the Ministry of Education; Former Coordinator of Mathematical Literacy, department head and teacher in the Peel DSB; OAME director; Involved in development of 7-10 TIPS4RM, 7-9 Math TIPS and a member of the Expert Panel for Leading Math Success report, Grades 7-12 and presenter at many workshops and conferences including OAME annual conferences.

Grade 10 Students responding to questions regarding using CAS – November 2006

"You have more chances of being perfect and at the same time you're using your brain too." [1]

"You have to use your own problem solving skills." [1]

The Journey Begins

In the revised mathematics curriculum there is an emphasis on using a variety of strategies and tools to develop understanding. In particular, the expectations that address algebra make specific reference to using Computer Algebra Systems (CAS). CAS is not a new tool but its inclusion in specific expectations has posed a challenge for teachers. How do we as educators appropriately use this as a thinking tool rather than a "black box"[2]?

It was this challenge that initiated an action research project involving eight schools in the Peel District School Board in April 2006. The schools were each given a set of TI-89 calculators and two after school training sessions. The participants were expected to conduct action research into the effective use of CAS, especially for students who traditionally would struggle with algebra in Grade 9 and 10 Applied courses. Teachers posed

research questions such as: "Does the use of Computer Algebra Systems improve student learning of algebra, particularly the struggling learners?" The initial pilot encouraged us to further study the effective use of CAS and in particular the need to explore, in more depth, our instructional strategies with calculator-based learning. This prompted an invitation to involve Tom Steinke from the Ottawa Carleton Catholic District School Board in order to learn from his experiences using this technology with teachers in his district. At his suggestion, we decided to videotape a Peel teacher in a Grade 10 Applied math classroom using CAS with students. This would allow an opportunity to open up a teacher's classroom, not only for those involved in this study but for future examination as well.

A highlight of the project was working with Dr. Carolyn Kieran, professor at the University of Quebec at Montreal, a leading international researcher in the field of CAS technology. Her full-day inservice focused on appropriate tasks and effective teaching methods with technology. Teacher questioning and student dialogue played an important focus in the activities and discussions.

With the groundwork laid, the next stage of the project began. This included the participation of 63 teachers from 21 schools in the board. In November 2006 those schools were invited to a session after school where they received a class set of 25 TI-89 calculators and instruction on their use. The parameters of the action research project were shared. This would include a full day of professional learning in December 2006 with Dr. Kieran; viewing and reflecting upon the videotaped lesson; and making connections with the research and practice. In preparation for this session, teachers were provided with three research articles for background knowledge and to generate ideas for discussion as we venture in this new platform for learning and doing mathematics.

Crafting the Lessons – Paul, Irene, Tom and Carolyn

One of the initial challenges was deciding upon the focus of learning to create the lesson for the video. Research identifies that using CAS as a learning tool has the potential to improve algebraic reasoning for students, a topic they often find problematic particularly in applied courses. Use of this technology has also been demonstrated to increase student confidence – a critical factor in achievement. Using CAS to expand and factor expressions is an area that has been explored and many lessons exist incorporating CAS. One of the lessons would address this content from the Grade 10 Applied program. However, there was a strong desire to explore other areas of the curriculum where student learning could be enhanced.

Knowing that one of the more challenging topics in the Grade 10 Applied program is solving systems of linear equations by using the method of substitution, we set out to craft a lesson using the TIPS MATCH template that would incorporate the use of this tool and also focus on the mathematical processes, in particular Reflecting, Reasoning and Proving.

After the creation of the initial lesson, feedback was provided by everyone involved. The lesson changed dramatically from beginning to end as we considered obstacles that would arise in student learning. A focus on the importance of teacher questioning influenced the overall design of the lesson.

An impromptu lesson study evolved as we struggled in crafting an activity that would harness the power of the calculator to assist in student understanding. It was exciting to share strategies and ideas as we anticipated student responses to the original lesson.

The substitution method requires students to select the most appropriate variable to isolate and then substitute into the other equation. One of our first challenges was to provide students with an initial activity where this selection was made more obvious (see Revised Lesson Warmup Problem in Figure 1) thus preparing them to later deal with more complex systems.

Original Lesson Warmup Problem	Revised Lesson Warmup Problem
Warmup: Find the point of intersection (solve the system) graphically. $y = 1/2\,x - 1$ $y = -3x + 4$	Warm-up: Solve the systems: a) $y + 2 = 10$ and $x + y = 12$ b) $3x + 2y = 33$ and $2x = x + 7$

Figure 1

CAS assisted students in the sense making process. For example, once students became comfortable in selecting the appropriate variable to isolate, and using CAS to substitute, they had to rationalize the result provided (see Figure 2).

Students were then expected to show all their steps in solving the systems to demonstrate their algebraic reasoning whether they completed the task using CAS or paper and pencil (see Figure 3).

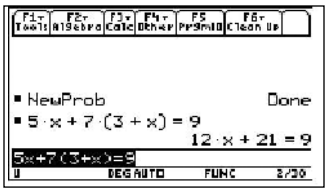


Figure 2

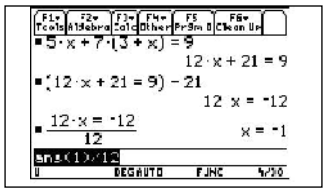


Figure 3

Once students were comfortable with the algorithm for solving a system by substitution, students were presented with "An Interesting Problem". Students were divided into groups and asked to solve the following systems (see Figure 4).

System A	System B	System C
x + 2y = 3 ; 4x + 5y = 6	x + 3y = 5 ; 7x + 9y = 11	-2x + y = 4 ; 7x + 10y = 13

Figure 4

Students were surprised to discover that the solution sets to all three systems were identical. They were challenged to determine what pattern was inherent in the given systems and to complete the system given the equation 3x + 4y = 5 that would yield the same solution set.

The second lesson on expanding binomials didn't require as much revision but still provided us with glimpses into unintended student difficulties arising simply from the manner in which a question is framed. The intention of the lesson was for students to determine a pattern by using the Expand command with a series of binomials (see Figure 5).

MULTIPLYING TWO BINOMIALS

Expand using your calculator:

a) (x + 1)(x + 1) = _____ b) (x + 2)(x + 1) = _____

c) (x + 1)(x + 2) = _____ d) (x + 2)(x + 2) = _____

e) (x + 1)(x + 3) = _____ f) (x + 2)(x + 3) = _____

g) (x + 1)(x + 4) = _____ h) (x + 2)(x + 4) = _____

Look back on the questions you've completed. Look for a pattern. Write down the pattern you've observed.

Use your pattern to complete the next two examples. Check with the calculator.

i) (x + 1)(x + 9) = _____ j) (x + 2)(x + 8) = _____

Figure 5

Leon, one of Paul's students, focussed on the sequence of values in the expanded form of the products in the left hand column (2x, 3x, 4x, 5x and 1, 2, 3, 4) and hypothesized that the next answer would follow the sequence yielding a response of $x^2 + 6x + 5$. Similarly, when he examined the results of the right hand column (3x, 4x, 5x, 6x and 2, 4, 6, 8) he once again hypothesized that the next answer would follow the sequence and be $x^2 + 7x + 10$ (see Figure 6). We recognized that the selection of questions had misled this student to examine the pattern between questions rather than within the given factors.

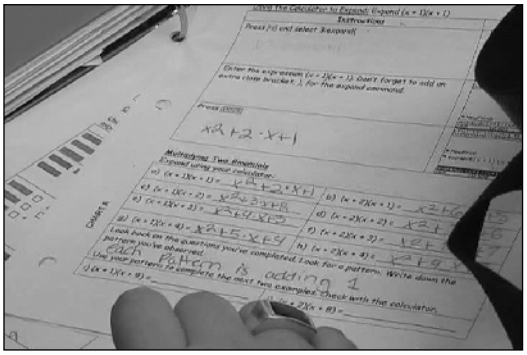


Figure 6

One drawback of the 'Expand' command is the fact that the output is the simplified result and you don't see the fully expanded form of the product. When students were given the question (2x + 3)(3x + 4), many had difficulty trying to resolve the result with their hypothesis on expanding binomials. Many just used the sum and product pattern ignoring the first term in each binomial. In an attempt to address this issue, the Symbolic Math Guide was used to assist students in seeing the expanded binomials (see Figure 7). The Symbolic Math Guide application allowed the students to expand and see the result of that operation and then to simplify.

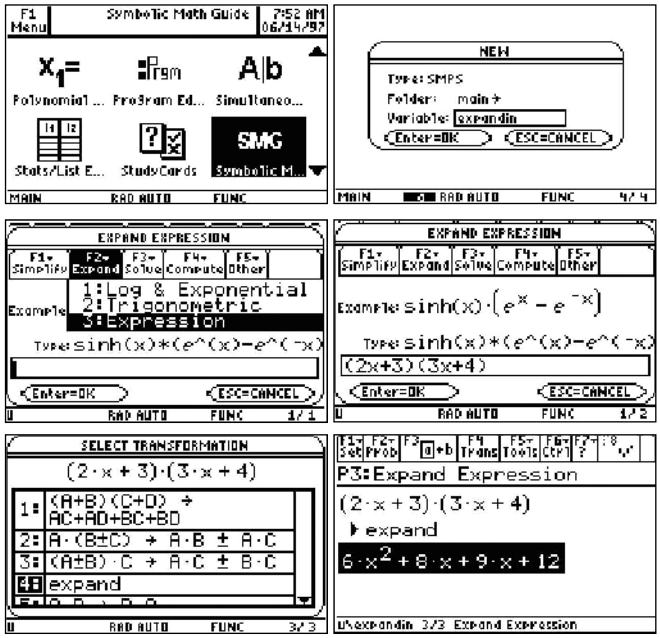


Figure 7

The Journey Continues...

Schools participating in the pilot project met in June 2007 to share their findings and activities they created. The pilot project was expanded to include all schools in the board starting in the fall of 2007. The sharing focussed on the challenges that teaching with any technology presents to the classroom teacher but more so on the benefits to student engagement, confidence and learning. The group agreed that CAS is a tool not to be used in isolation but was to be supported with the use of manipulatives and paper and pencil techniques. The technology allowed students to explore patterns and see connections more quickly. It was intriguing to observe, that once students sensed a pattern, they preferred paper and pencil techniques as more efficient than the calculator. The calculator then became a tool to verify their solutions and allowed students with weak numeric skills to engage in the learning of a concept appropriate to that grade level. Some teachers even observed students enjoying algebra and saying it was fun.

Like any innovation, the use of CAS has had and will continue to have an impact on how students learn and how teachers teach. The overall goal of the project has been to discover how to make that impact as positive as possible on the symbiotic relationship of learning and teaching.

As with all technology, we recognize that the journey is not over. We continue to explore the effective use of CAS in our classrooms to ensure that it is a "white box" [2] rather a "black box" [2]. We want it to be a vehicle for student understanding and an investigative tool rather than simply a means to an end.

Implications for Our Work as Educators

CAS does not relieve students of their mathematical learning obligations.

"The power of the calculator lies in its ability to allow students to hypothesize and then confirm or question that hypothesis much quicker than before. They can then investigate further the underlying concepts that the calculator provided." [1]

There is a parallel in this journey as with the graphing calculator in re-examining curriculum expectations, the goal of mathematics learning and teacher behaviour. CAS provides opportunities to change approaches in algebraic instruction – away from the technical symbolic manipulation, towards pattern recognition and conceptual understanding of calculator generated output.

The opportunity to engage student interest and affect their attitude in learning is a key factor that needs careful examination.

References:

Ed. Fey, Cuoco, et al. (2003) *Computer Algebra Systems in Secondary School Mathematics Education,* Reston, VA: NCTM.

Martinez-Cruz, Armando M. and Contreras, Jose N. (2002) Changing the Goal: An Adventure in Problem Solving, Problem Posing, and Symbolic Meaning with a TI-92. *Mathematics Teacher, 95*(8), 593-597.

Howell, Mark. (2000) High School and Beyond – Rising to the CAS Challenge. *Mathematics Education Dialogues.* April, 10.

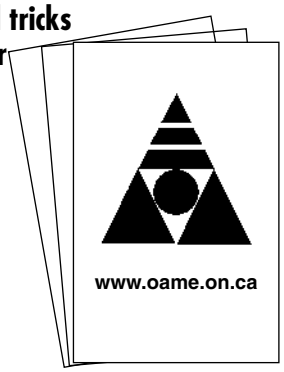
Pierce, Robyn U. and Stacey, Kaye C. (2002) Algebraic Insight: The Algebra Needed to Use Computer Algebra Systems. *Mathematics Teacher.*

Jakucyn, Natalie and Kerr, Kenneth E. (2002) Getting Started with a CAS: Our Story. *Mathematics Teacher, 95(*8), 628-632.

Heid, M. Kathleen and Edwards, Michael Todd. (2001) Computer Algebra Systems: Revolution or Retrofit for Today's Mathematics Classrooms? *Theory into Practice, 40*(2), 128-136. ▲

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[1] *Quotes from students and teacher from video: Peel DSB CAS Project, 2006*

[2] *McCallum, W.G. (2003). Thinking out of the Box. Computer Algebra Systems in Secondary School Mathematics Education, NCTM, 73-86.*

▲ Directors' Dialogue

ANN MICHELE STENNING



Ann Michele Stenning is a mathematics teacher at South Secondary School with the Thames Valley District School Board. She is currently completing her Masters of Education in Curriculum Studies. Ann Michele was part of the planning committe for the 2006 OAME conference "Everyone Counts".

This edition of Director's Dialogue highlights the events of the OAME Board of Director's meeting at the Holiday Inn Select, Toronto Airport on Friday, June 8 and Saturday, June 9, 2007.

Chapter Rep Meeting, June 8, 2007

Greg Clarke began the meeting by welcoming members and introducing everyone to the features of the OAME web site. Greg highlighted the ability of OAME members to access the electronic version of the Gazette and chapter reps were shown how to immediately send membership renewal reminders. Chapters can also use MCIS to facilitate registration for mini-conferences.

Board Member Meeting, June 8, 2007

The new OAME president, Jacqueline Hill, introduced the new board members and then presented an OAME promotions power point highlighting the advantages of being an OAME member. This power point presentation is intended to be shown to non-OAME members in Colleges of Education, Board offices and schools.

John Kersley, the NCTM rep, shared various resources that are available for purchase through NCTM. These resources include problem solving activities, historical case studies and books on how to mentor new educators.

Jacqueline began her presentation on differentiated instruction by showing how terms like SRA, constructivism and multiple intelligences are all terms that can be used to describe differentiated instruction. Differentiated instruction focuses on using a variety of approaches to achieve curriculum outcomes. She presented a framework: curriculum factors versus student factors. Curriculum factors include content (input), process and product while student factors include readiness, interest, and learning profile. Board members were told of various resources available on differentiated instruction: Reach Every Student through Differentiated Instruction from the Ministry of Education and noted authors, namely Gayle Gregory and Carol Ann Tomlinson, were mentioned

Equity Committee Breakfast, June 8, 2007

This year's OAME 2008 conference keynote speakers will be Craig and Mark Keilberger, speaking on Social Justice.

The Equity Committee will spend this year exploring how differentiated instruction addresses equity issues in the classroom. This exploration will be done using two book studies. Titles will be confirmed later.

Board Meeting, June 9 2007

Greg Clarke, OAME's webmaster, presented the features of the OAME web site to the board members and then work within Individual Standing Committees including Publications, Futures, Promotions and Constitution, began.

Publications Standing Committee: Lynda Ferneyhough reminded members that there is a set protocol for answering requests for publication in the *Gazette*. This is found on the first page of every issue. *The Abacus*, new editors Kathy Kubota-Zarivnij and Mary Lou Kestell will have a focus topic for each issue this year: addition & representing numbers, subtraction, multiplication and division. There are plans to include research summaries, references, math problems, common errors made by students, quotes, book reviews, math and literature links and manipulatives. Educators are invited to send in their submissions to the *Abacus* and *Gazette*.

Futures Standing Committee: OMO for 2008 until 2011 will be hosted by PRMA, PRMA, MAC and TEAMS respectively. The OAME annual conference for 2008 and 2009 will be hosted by Y^4MA and COMA respectively, with an intent to host 2010 and 2011 by GOLDEN and SWOAME respectively.

Promotions Standing Committee: Charlotte Cutajar reported that descriptions of GUM Workshops will be revised. The promotion of OAME in the community at large was discussed which lead to suggestions of offering parent workshops and web site links for parents and/or students. Brainstorming occurred on the topic of how to increase membership.

Continuation of Board Meeting, Saturday, June 9, 2007

Thanks were given to Dean Murray and Trevor Brown for their contributions to the mathematical community. Dean Murray was recognized for his role as Gazette advertising manager for the past 12 years. He has been a GVMA member since 1975 and he currently sits on their executive. He also won the Prime Minister's Award for teaching excellence. Now retired, he is working at the University of Waterloo with the Canadian Mathematics Competition. Trevor Brown was recognized as the editor of the Abacus for the last three years and as a former co-editor of the Abacus in the 1980s with Lorna Morrow. He is a well known author, speaker, instructor and developer and reviewer.

Both Dean and Trevor have stepped down from their involvement with the OMG.

JK-6 Curriculum Committee: Anna Dutfield reported that after reviewing what the committee had accomplished over the last year, it was decided to continue working on the "5 Books, 25 Lessons" resource using the same 5 books we had decided upon: 100 *Hungry Ants, 10 Black Dots, The Grouchy Ladybug, Three Pigs, One Wolf and Seven Magic Shapes,* and *TheDoorbell Rang*. New writing partnerships were assigned and a deadline for the first lesson for each book was set for the October meeting

Grade 7-9 Curriculum Committee: Charlotte Cutajar reported that each team shared their work so far with math and flight. The grade 7 team has a summative task and will create a TIPS lesson for each part of their lesson. The grade 8 team shared 5 TIPS lessons including a contest context using material on histograms and scatterplots. The grade 9 team has developed lessons on data management.

Grade 10-12 Curriculum Committee: Dan Charbonneau reported on the idea of producing a one page flyer that would be suitable for new teachers. This idea may be discussed at a later date. Meanwhile, the current goal of the group is to develop workshop material that can be presented at OAME annual conferences. At the next meeting, curriculum group members will list aspects of a good workshop, list five workshop titles that you would attend, and upload any lesson suitable for presentation purposes. Jacqueline Hill, OAME's president, announced that Ministry officials are having OMCA and OAME oversee summer writing for the six courses in grade 12. Writers have been asked to

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provide material for 30% of each course. Early writing for MCV, MDM and MHF will occur at Math Camp and continue throughout the summer. MEL, MCT and MAP will be written in July. 30% of the course work needs to be written by September The approach used in TIPS will guide the creation of new material with the course material posted on the OAME web site by August 20th.

GUM revisions are planned for the summer. One team will write two tasks for grade 12 in the Kingston area and a second team will write two grade 10 tasks in the Sudbury, Manitoulin area

Chapter Reports

CHAMP - Rep: Lynda Ferneyhough

We finished the CHAMP contests and were delighted to have 22 schools in Peel participate, 21 in Dufferin Peel and 7 in Halton Catholic. The top contest winners will be honoured at an Awards Night. Thanks to all who contributed to a most successful contest. CHAMP sent two teams to OMO. We have started planning our fall event for October 18th.

COMA - Rep: Pat Kehoe:

On May 14th COMA held their annual Spring Forums. The secondary forum provided teachers with an opportunity to see, and hear presentations about, new curriculum resources. The elementary forum was an activity-based workshop on the new curriculum materials that have been released by the Ministry for junior division teachers. After coordinating a very successful local Math Olympics in April with 60 teams, Judy Loschmann and Cathy Hill and their committee were very excited to be hosting the Ontario Math Olympics in Ottawa on June 2nd. Everyone at COMA is also very excited about hosting the OAME annual conference in 2009! The planning committee is up and running, Committee Chairs are in place and facilities at Carleton University have been booked. COMA held its AGM in May and elected a new executive for 2007-8 COMA would like to congratulate Rob McLeish from Sir Robert Borden High School in Ottawa for winning the Union Gas Award for Teaching Excellence in Secondary Mathematics. We are proud to have him in our Chapter.

GOLDEN - Rep: Laura Tonin:

Our annual mini-conference at Brock University went well. Our theme this year was Visual Mathematics and participants enjoyed attending two workshops on various topics. We also had a delicious dinner with an amazing dessert table (the chocolate cheesecake was top notch). Our committee is now starting to plan for OAME 2010. We've chosen our chairs and the conference will be held at Brock University. The committee would like to wish our president Ed Haines a happy retirement! Ed will continue to be involved with OAME as he is one of the chairs for the 2010 conference.

GVMA - Rep: Rob Gleeson:

We have set up the program of the October 2007 Mini Conference. We will be looking at Memories of K.D. Fryer, the 50th Anniversary of U. of Waterloo and the 40th Anniversary of the Faculty of Mathematics at U of Waterloo. We are looking at creating exams for the new 11 and 12 curriculums. We are also looking at creating a T-Shirt for GVMA distribution.

ISOMA - Rep: Diane Lang:

ISOMA has two events to report. First, a very successful and enjoyable Math Olympics was held at Crescent School. Thanks to Mark Suckling and his team of volunteers for their excellent organization. This was truly a cooperative venture as teachers from several other schools - St. Andrew's College, Appleby College, Havergal College, Branksome Hall - submitted activities to help Mark create the program for the day. Over 100 students participated, and three teams were selected to compete at OMO 2007. Our second event was a conference call meeting of the Executive to begin the planning of our Fall 2007 Mini Conference, tentatively set for Wed., Oct. 17, 2007.

MAC² - Rep: Danielle Blair:

The 2006/07 school year has been a very busy year for MAC²! In the fall, September 28, 2006, MAC2 hosted a mini-conference at Georgian College. A great dinner was followed by a set of workshops by Jennifer Brown, Connie Quadrini, Gina MicoMonaco and Fred Ferneyhough in which a mathematical concept was explored from Grades 1 to 12. Then in the spring of 2007, MAC² hosted the Annual OAME conference, "Building Bonds", in Barrie May 3rd to the 5th. The conference was a huge success thanks to the dedication and hard work of a small but committed committee.[Editor's note: see writeup on page 19.] Currently, MAC² is wrapping up details from the Annual Conference and discussing goals for next year. For 2007/08 MAC² is planning a fall-mini conference and looking at opportunities to further our commitment of developing a collaborative mathematics community with parents.

NOMA - Rep: Sue Melville:

Representatives from NOMA who attended

the OAME conference in Barrie came back raving about the high quality of the sessions they attended. We have already invited some of those speakers to present at our NOMA fall conference, scheduled for Friday,October 19, 2007 in Sudbury. Interested speakers should contact Sue Melville or Heather Boychuk The other highlight of our spring activities was the NOMA Math Olympics, held at Lasalle Secondary School on Saturday, May 12th. Twenty teams representing 8 local schools vied for medals. We are delighted to send two teams on to the provincial matholympics.. Future plans for NOMA include inviting teachers from northern communities to participate on our committee.

NWOAME - Rep: Tom Boland:

We were pleased to have over 100 students competing in the NWOAME Regional OMO. Four students went on to represent the region at the Provincial OMO in Ottawa. NWOAME has several new voices as a result of attracting new members to the organizing committee. We are currently brainstorming about ways to increase awareness and general membership. NWOAME will hold a conference Feb 28, 29 and March 1 2008 at the Travelodge Airlane Hotel in Thunder Bay.

PRMA - Rep: Sandra Jean Price:

PRMA held its local OMO playdowns at Durham College on May 15th. The chapter will be sponsoring two teams to go to the provincial OMO this year. We are looking forward to hosting the OMO provincial event at Trent University in 2008 and 2009. Our executive is looking ahead to the fall and planning a social with Don Fraser as a keynote. The tentative date for this event is October 2nd, 2007. We hope that the event will be attended by both teachers and pre-service teachers from Faculties of Education in our area. The chapter is also looking into ways we can recognize contributions to the mathematics community within our region.

QSLMA - Rep: Robert Sherk:

This term QSLMA has promoted attendance at the leadership and provincial annual conferences through our web site and email newsletter. There was a very good representation of our group at both events with more than 10 people attending each from the Limestone Board. We also held a local OMO event here in April and sent a team to the provincial event. Thanks to Geoff Roulet and Linda Colgan and their team of Queen's Faculty of Ed students for organizing this event. We are now beginning to plan our Fall mini conference for November 2007 .

SAME - Rep: Debbie Drape:

Last time I gave a report SAME was busily planning for our mini-conference held on February 21st. This year t-shirts were purchased for the executive to wear plus for the student volunteers. It was a great idea and we had an opportunity to wear the shirts again during our local playdowns. The teams that were sent to the provincial OMO did not place but Ottawa had a well organized event and fun was had by all. Two goals that we had for this year were to increase membership and to try to get more members on the executive. . We will continue to promote membership at our mini-conference which we are working on now, to be held sometime in the fall (date TBD).

SWOAME - Rep: Joe Goulet:

SWOAME hosted its annual math Olympics, which saw over 70 participants compete. The three winning teams move onto the provincial competition. We are working on initiatives to more smoothly implement the new grade 12 curriculum.

TEAMS - Rep: Pat Margerm:

Teams for the provincial OMO were selected through the local TEAMS OMO competition on April 14. The first annual awards, in memory of the late president, Anne Cirillo, were presented at this competition by members of her family. Discussions for the location of OMO 2011 have already begun. That's planning ahead! The tentative date for the fall mini-conference is October 25, 2007. Laurissa Werhun continues to provide a web newsletter that is sent to all current TEAMS members with information about upcoming events as well as some recommended resources such as weblinks.

WOMA - Rep: Ann Michele Stenning:

This spring WOMA hosted a mini-conference with a focus on Smart Boards. Publishers and local educational supply stores were in attendance to display their products. Conference participants attended two sessions of their choice and then listened to Kathy Kabota-Zarivnij over supper. A fall conference will be planned for November.

Y^4MA - Rep: TBA Y^4MA is busy planning OAME 2008 to be held May 8-10 at the Sheraton Parkway Toronto North in Richmond Hill. Co-chairs are Connie Quadrini and Gina Micomonaco. ▲

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