Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership.
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceeding 10 numbered pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required. Please number pages.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including web sites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

PERMISSION TO REPRINT: Permission to reprint any part of this publication for instructional use or for inclusion in an affiliate publication must be obtained through the Editor Marilyn Hurrell. Full credit must be given to the author and to the *Ontario Mathematics Gazette.*

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

▲ About the *Ontario Mathematics Gazette*

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Submission of Advertisements

Advertisements for the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising Rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARILYN HURRELL, EDITOR



In the sprit of the holiday season, I want to thank the people who help to make the *OMG* happen.

When I receive a manuscript for possible publication in the *OMG*, I select 3 reviewers who have stated interest and expertise in the topic and grade level of the manuscript. I send these reviewers a sheet that asks them various criteria and they respond both on this sheet and on the manuscript itself.

The reviewing process is very important and the suggestions of the reviewers serve to make a good manuscript better. Some manuscripts need revision, others are accepted right away. Most take about 6 weeks to "polish" but I have had a few that have taken up to a year for approval.

I would like to send thanks all the reviewers of the *OMG*. They are:

Kirsten Boucher, Alan Chan, Dr. Ed Barbeau, Dr. Lynda Colgon, Barbara Canton, Prof. Dave Cosh, Stewart Craven, Dr. Ralph Connelly, Ruth Dawson, Chris Dearling, John Egsgard, Pat Grew, Tom Griffiths, Dave Handley, Elaine Harvey, Joanne Harris, Jeff Irvine, Anna Jupp, Dr. Ann Kajander, John Kerseley, Peter Komlos, Murray Major, Pat Margerm, Jamie Pyper, Louis Lim, Todd Romiens, Dr. Geoff Roulet, Walker Schofield, Jay Speijer, Roly Meisel, Dr. Christine Suurtamm, David Shepherd, Shirley Scott, Dr. Margaret Sinclair, Robert Sherk, Mary and Lawrie Sakamoto, Prof. Jack Weiner, Prof Kathy Kubota-Zarivnij, Mary Bourassa, Fred and Lynda Ferneyhough, Laurie Moher, Andrew Morrison, Merilyn Fox and Duncan LeBlanc,

Thank you also to the columnists AnnMichele, Fred, Mary, Lynda, Shawn, Chris, Ron, John and Louis, to Associate Editors John and Lynda, and to ED's Sue and Dave Hessey, to authors of articles, Robert, Advertising Manager, and finally to Penny Clemens who puts it all together.

See you in the new year!

Don't forget Pi day March 14

Marilyn

▲ President's Message

JACQUELINE HILL, PRESIDENT
E-MAIL: hill_jacqueline@durham.edu.on.ca



Tis the Season One and All!

As teachers our year is in full swing. As OAME Board members our committee work is in full swing. I hope you get a chance this holiday season to enjoy the things you love.

I would like to think you had a chance to attend your local chapter's mini-conference or fall offerings this year. Coming up we have the OAME Leadership Conference that will be held on February 21 and 22, 2008. There are two themes this year: first a K-8 focus on Japanese Lesson Study, second a grade 9-12 focus on Emerging Technologies. Our Annual Conference is being hosted by Y^{4}MA in Markham this year. The dates are May 8-10, 2008. Math is the Path is the theme for this conference and the organizing committee has gone to great lengths to ensure that teachers and coordinators alike will have many workshops to attend that meet their individual needs. The last initiative for this school year is the Ontario Mathematics Olympics that is being hosted by the PRMA chapter. The event takes place at Trent University on May 30th and 31st of 2008. This is a great challenge and inspiring time for our young grade 7 and 8 mathematicians.

Differentiated Instruction

A few phrases to keep in mind when you think about Differentiated Instruction are;

1) There are no two students who learn at the same pace or solve problems in exactly the same way or have the same interests or have learned the same skills
2) If students aren't learning the way we teach…then we need to teach them the way they learn. (Dunn and Dunn)
3) Give the students lots and lots of choice, in the process, product, and content
4) "…A philosophy that enables teachers to plan strategically in order to reach the needs of all the diverse learners…to achieve target standards… [A philosophy]…to reach the unique needs of all learners." (Gregory & Chapman)
5) "Curriculum tells teachers what to teach, while differentiated instruction tells teachers how to teach it to a range of learners by employing a variety of teaching approaches." (Education for All, 2005, p. 14)

I would like to leave you with what I feel is a very nice math problem. As written it does not have any "what if questions attached" but it is a great problem if you are looking to see the thought process of your students.

Question: Using the last four digits of a phone number, how many times will the first three digits add up to the final digit? (e.g., 2316 is 2 + 3 + 1 = 6, that is, the first three digits add up to the last digit,6. Explain your answer using two different methods.

Until next issue,
Yours in Mathematical fun,
Jacqueline Hill ▲

NEW
FREE RESOURCES AVAILABLE

For all grade 12 Mathematics courses
Visit www.oame.on.ca for resources developed by OAME and OMCA
funded by the Ministry of Education.
More than 100 Ontario Educators participated in this project.
More lessons will be added for Semester 2.
Check for updates regularly.

▲ Upcoming Events

For further information on OAME Chapter Conferences contact the Chapter Representative and for NCTM conferences, contact John Kersely, OAME/NCTM Rep.

See the last page in this *Gazette* for contact information. Also visit **www.oame.on.ca**

OAME Chapter Conferences and Events

Feb 21 - 23 2008	Leadership Holiday Inn Toronto Airport	OAME
Feb 28, 29 & March 1, 2008	Regional Conference "Focus on Math: Success in Our Classrooms" Travelodge Airlane Hotel, Thunder Bay ON	NWOAME
Early May 2008	Spring mini conference Brock University	GOLDEN
May 30, 31 2008	OMO: Trent University	PRMA

OAME Annual Conference

May 8 - 10 2008	Sheraton Parkway Toronto N Richmond Hill	Y^{4}MA

NCTM Regional Conferences

April 9 - 12 2008	Annual Meeting and Exposition "Becoming Certain About Uncertainty" Salt Lake City UT	

RECOGNIZE AN EXCEPTIONAL LEADER IN MATHEMATICS EDUCATION



The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate enhances the learning environment, providing opportunity for students to do, see, hear and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him.

OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page on the next page or contact the Executive Directors for an electronic version (oame.hessey@sympatico.ca).
2. Provide up to 3 letters of support, of no more than 2 pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate *primarily* to the specific award. You may also include the following areas for consideration:
   - demonstration of excellence and creativity in mathematics education
   - leadership in mathematics education in the school, region and province
   - support of and involvement in OAME/AOEM
   - contribution to the development of students both within and outside of the classroom
   - additional relevant information not specifically mentioned
3. Include the nominee's Curriculum Vitae of up to 5 pages.
4. Send the completed cover page and supporting documents to: Dave and Sue Hessey, Executive Directors OAME

Deadlines: Nominations are due by January 31, 2008. The awards will be presented at the Annual Conference in May, 2008.

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated 10 or more years of membership in OAME/AOEM.

Award for Outstanding contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancing mathematics education in Ontario.

Kenneth D. Fryer Award* recognizes a mathematics department which fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

*This award was established in memory of the late Kenneth D. Fryer and is co-sponsored by the Grand Valley Mathematics Association. Curriculum Vitae not required.

AWARD NOMINATION COVER PAGE

INTENDED AWARD *(please check one)*

- ❑ **Life Membership**
- ❑ **Outstanding Contribution to OAME / AOEM**
- ❑ **Exceptional and Creative Teaching in Elementary Mathematics**
- ❑ **Exceptional and Creative Teaching in Secondary Mathematics**
- ❑ **Leadership in Mathematics Education**
- ❑ **Kenneth D. Fryer**

Nominee's Information:

Name of Nominee: (Principal and Department Head for K.D. Fryer Award)

School, Organization, or Business Address

Tel: ( ) _____ Fax: ( ) _____

Home Address: _____

Home Phone: ( ) _____

Email: _____

Nominator's Information:

Name of Nominator

School, Organization, or Business Address

Tel: ( ) _____ Fax: ( ) _____

Home Address: _____

Home Phone: ( ) _____

Email: _____

SEND with supporting documents by Jan 31 to: Dave and Sue Hessey, Executive Directors.
Contact the Executive Directors for an electronic copy of this form.

▲ Letters to the Editor

Dear Editor,

I am grateful to [the *OMG* [Vol 46#1 Sept 2007] for taking an interest in my work. I am always thrilled when someone deems my articles worthy of publication. As my objective is to help as many students as possible, I am achieving this goal by reaching out to teachers. You make this happen by reviewing and editing articles and by understanding the objectives of the author. So, thank you and your reviewers very much for publishing my latest article on polynomials and for all the work that you put into it.

Duncan McDougall
Victoria BC ▲

Editor's Note: The following letter was sent to me by Suzanne Alejandre, in regards to Louis Lim's column Get Interactive which originally appeared in the OMG March 2007 [Vol 45#3] and was reprinted with permission in the Sept 2007 issue of the California ComMuniCator. This is followed by Louis' response.

Dear Louis Lim,

Please let me introduce myself. I am Suzanne Alejandre, on staff at the Math Forum @ Drexel and formally a middle school mathematics and computer teacher for many years in Rialto, California. I have been a CMC member for many years and also a huge fan of the *ComMuniCator [March 2007]*. In fact, one of the reasons I've maintained my CMC membership is so that I could continue to receive that excellent publication even though I now live in Pennsylvania.

Yesterday I received the most recent issue of the *ComMuniCator* and as I was reading through it in the evening I noticed your column on Page 44, Problem Solving Resources. When I got to this section I was saddened by your words:

> *4) Elementary Problem of the Week The Math Forum has archived Elementary Problem of the Week, from September 1996 to June 1997, available free at mathforum.org/sum95/ruth/96.97/index.html. Suitable for grades 3 to 6, each week contains a problem posed, along with student exemplars provided from schools that submitted solutions. Unfortunately, teachers interested in accessing the web site, now called Math Fundamentals Problem of the Week must register by paying $25.00.*

The words that saddened me were “unfortunately” and “must”. I actually was a fellow participant of Ruth Carver’s in that Summer Institute of 1995 when she started the Elementary Problem of the Week as her project. That summer my project was to start the Tessellation Tutorials which was the start of many pages that I wrote that became known as Suzanne's Mathematics Lessons. They continue to be freely available on the Math Forum's site: **http://mathforum.org/alejandre/**

In 1995, the Math Forum was still an NSF-funded project (National Science Foundation) and it was during those NSF-funded years that many of the services that remain freely available were developed. We have continued to develop those original services and added more so that now these are among the free services we offer:

- Ask Dr. Math
- Teacher2Teacher
- Teacher Exchange
- Math Forum Internet Library
- Math Forum Internet Newsletter
- Math Tools
- Discussions
- NSDL Online Workshops (a new resource)
- Tools for Building Math Concepts Workshops (a new resource)
- and ... this area of the Problems of the Week Teacher Documents: **http://mathforum.org/pow/teacher/**

The one service, Problems of the Week, is the area of the Math Forum that now requires membership.

Is it “unfortunate” to charge a teacher $25.00 to be able to receive 80 new problems each year (and access to 20 “Library” problems during the summer)? If you were to purchase a book of resources, how much would that cost? Many teachers are happy to pay that amount (and more for access to our full Library of problems) because

(1) they are good problems

(2) they know that they are supporting the Math Forum!

Again, I'm sorry that you view the Math Forum’s attempt to sustain itself as unfortunate.

Sincerely, Suzanne Alejandre
Educational Resource and Service Developer
The Math Forum @ Drexel
http://mathforum.org/ ▲

Dear Editor,

I was thrilled to receive a copy of the September 2007 *ComMuniCator*, a journal from California, which reprinted my *OMG* Vol 45#3 March 2007 “Get Interactive” column. Recall that this edition of my column featured web sites that teachers could use to promote problem solving with their students.

I appreciate the opportunity to respond to Ms. Alejandre’s letter. As you know, my column features web sites I, as the columnist, believe that K-12 teachers may find useful in their classrooms. As I continue to spend hours “surfing the Net”, I must be selective in which sites to feature. My hope is each column is written in a positive and enthusiastic tone. Thus, I am disappointed to have read Ms. Alejandre’s letter as it simply focuses on two words “unfortunately” and “must”, rather than realize that readers of the OAME *Gazette* may go to the Math Forum as a result of my column.

As an educator who has hosted 15 teacher candidates, I continue to hear first hand that some newly minted teachers struggle to make ends meet. In fact, some have debts in the tens of thousands of dollars from their post-secondary education. When “surfing the net”, one expects the information highway to be free and accessible – not to see something you want and then have to pay a registration fee (regardless of the amount). Hence, I stand by my words of “unfortunately” and “must”. It is indeed unfortunate that teachers need to pay money to access the word problems when lots are available for free on the Internet. If this is optional, then I do apologize for saying “must” and would retract that with “may”.

I do hope Ms. Alejandre can appreciate my perspective, while always remembering that I do think highly of the Math Forum (hence, my rationale to feature it in my column). If the Math Forum is a non-profit organization as I am led to believe, then it is inappropriate to talk about how much I would pay for resource books since these are published for profit. Perhaps it is time to recruit sponsors (I noticed Texas Instruments is one) in order to once again offer the problem solving feature for free. Although “many teachers are happy to pay that amount”, some are unable to pay that amount and that is not equitable for them.

Sincerely, Louis Lim
OMG “Get Interactive” Columnist ▲

▲ Technology Corner: TI-Nspire

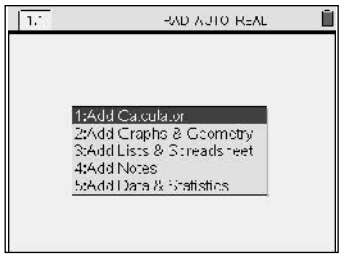
MARY BOURASSA

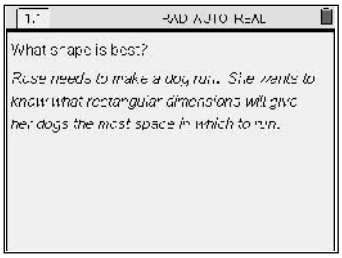


Mary Bourassa teaches mathematics at Lisgar Collegiate Institute in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She was also past-president of COMA, gives workshops around the world and is a National Instructor for Texas Instruments.

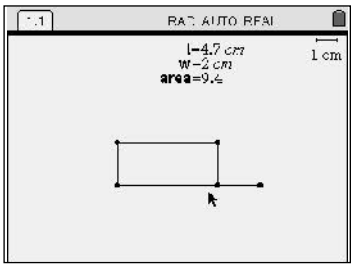
TI-Nspire is the latest learning technology from Texas Instruments. There is a handheld as well as software in both the CAS and non-CAS version. Fred Ferneyhough has been showing you some of the ways to use the CAS version. I will provide an overview of the different applications included with TI-Nspire and a step-by-step of one of its great features.

There are five types of pages that you can create with TI-Nspire: Calculator, Graphs & Geometry, Lists & Spreadsheet, Notes and Data & Statistics. Briefly, the Calculator application allows you to define and evaluate mathematical expressions. The Graphs & Geometry (G&G) application allows you to create graphs and geometric constructions on the same page. The Lists & Spreadsheet application works like a spreadsheet but has the advantage of being able to connect to values on a G&G page or a Data& Satistics page. The Notes application is a text editing tool that allows you to set up problems or activities which you can then share with students (an example of a Notes page is shown). The Data & Statistics application, the latest addition to TI-Nspire, is a powerful statistical tool. If you do not have the Data & Statistics tool, download the latest TI-Nspire operating system.



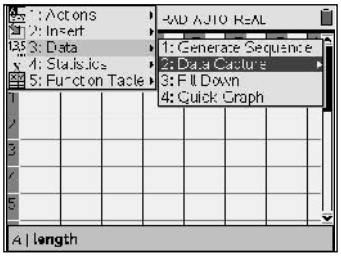


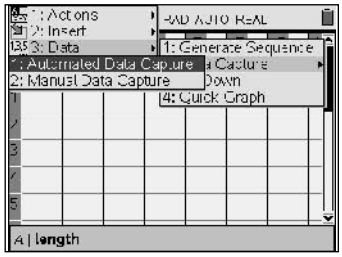
One of the very powerful features of TI-Nspire is the ability to capture data. We will see how this is accomplished using the example of determining the dimensions that produce the rectangle of maximum area for a fixed perimeter.



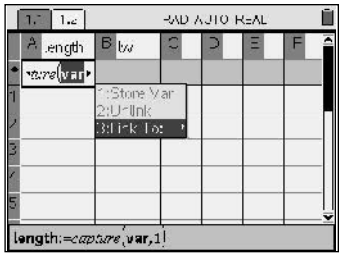
I have completed the construction so that I can drag the point on the segment along it to create rectangles of different areas while the perimeter remains constant. It is important to note that the measurements for length and width that you see on the screen are attached to the variables 'l' and 'w' and the area is stored in a variable of the same name.

The next step is to collect the data. Start by inserting a new Lists & Spreadsheet page by pressing ctrl I. We will name the first two columns 'length' and 'lw', respectively. In the formula cell for column A press **Menu > Data > Data Capture > Automated Data Capture**.

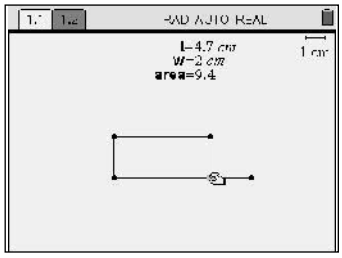




You now have to specify the variable to capture. Press **var > Link To** and select the variable 'l' from the list as that is the variable used in the G&G page to store the length measurement. Press **enter** to execute the command.



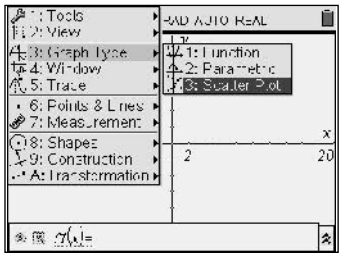
Repeat the process for the next column, linking it to variable 'area'. When you have completed the entries in the formula cells you will notice that the current values for length and area appear. We want to capture many more data values so we will return to the G&G page by pressing **Ctrl ◄**.



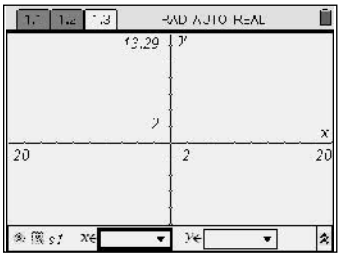
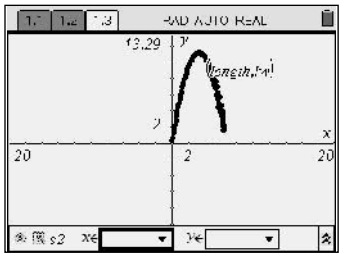
Move the pointer so that the point on the segment is selected; the pointer will change to an open hand. Grab the point by holding down the click key; the hand will close. Now, as you drag that point to change the size of the

rectangle, the values of the length and area will be stored in the spreadsheet. Collecting data is as simple as that! Press **esc** when you have finished.

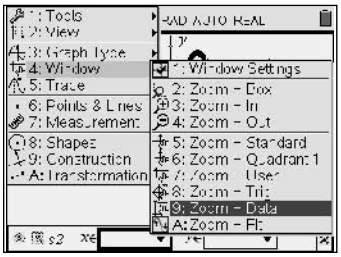
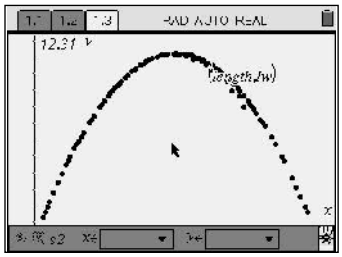
Looking at page 1.2 we now see lots of values for length and area! To plot these values insert another G&G page (Ctrl I). **Press menu > Graph Type > Scatter Plot**.

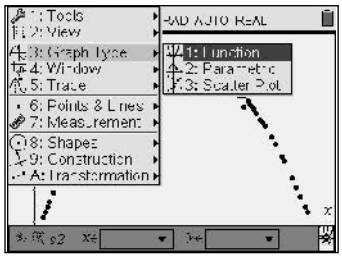
You now have to select the independent and dependent variables from the drop down menus at the bottom of the screen. Press **tab** to move from x to y. As soon as you have selected your variables, the scatter plot appears.

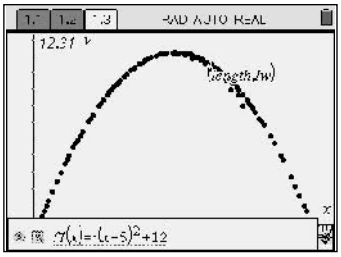
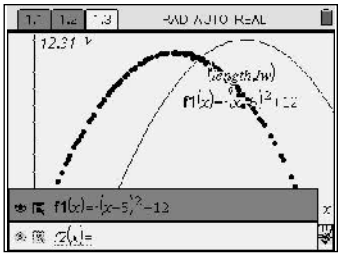
To adjust your window to better fit the data, press **Menu > Window > Zoom – Data**.

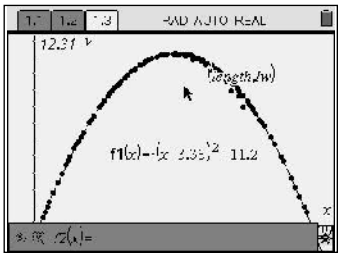
The data appears quadratic. Let's model it. Press **Menu > Graph Type > Function**. The function input line will appear at the bottom of the screen. I have typed in a function for f1(x).

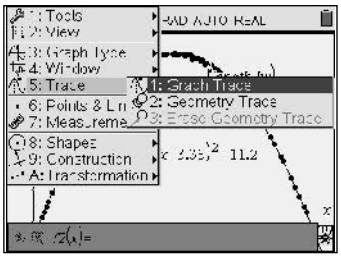
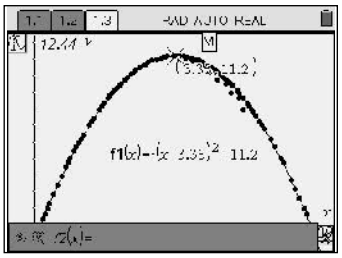


To adjust it simply grab the parabola near the vertex and move it on top of the scatter plot. If you grab one of the arms of the parabola, you can adjust the vertical stretch as well.

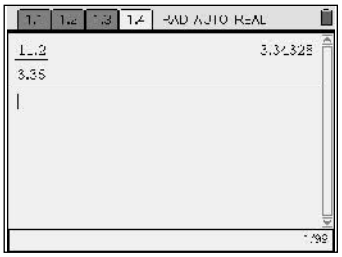



I am going to hide the scatter plot now so that the curve is easier to see. If you trace (**Menu > Trace > Graph Trace**) along the parabola there are a couple of very cool features to notice. Firstly, the window adjusts so that you can see the top of the curve which had been hidden. Secondly, when you get to the maximum point, an uppercase M appears!



So, without using regression we have found that the maximum area occurs when the rectangle has a length of 3.35. Calculating the corresponding width reveals a value that is practically identical. Sharing results within a class should lead to the conclusion that a square produces the maximum area for a given perimeter.



I hope you can see some of the power of TI-Nspire and can now start to think about how to apply it in your classroom. ▲

▲ Call for NCTM Rep

OAME is taking applications for a three-year term as NCTM representative commencing in September 2008.

Job shadowing available now.
For more details, please contact:
Executive Directors Dave & Sue Hessey

▲ Hey, It's Elementary! Using Digit-al Computers: Employing the Sensory Mathematics Sense-Making Tools at Our Fingertips



LYNDA COLGAN

On Saturday September 15th, I hosted 166 future elementary school teachers at a one-day conference called "Are Your Math Smarts Better than a 5th Grader's?" There were speakers from across the province who conducted workshops on many topics related to teaching and learning mathematics in contemporary classrooms. By the end of the day, the participants had solved problems using geoboards and learning carpets; learned how to incorporate Statistics Canada data into measurement and graphing activities; found intriguing patterns in a Chinese Checker board and a magic square; and, in addition to learning a few math raps and songs, singing some arias from the math operetta Sohcahtoh performed by a troupe of amazing puppets, also mastered some tactile techniques to take advantage of those sophisticated digit-al computers you and I commonly call fingers.



Many of us were taught to associate counting on our fingers with either cheating or failing to know our number facts. I vividly remember Miss Patterson, my grade 3 teacher at Bruce Public School asking all members of the class to lay their fingers flat on the top of the desk while she asked us one by one, row by row to answer number fact questions or compute answers to mental math exercises. Sadly, my own lasting image of her is one of a hawkish-like creature peering over the top of cat eye-framed glasses, scanning the room for the slightest finger movements, poised and primed to swoop down on her prey at the minutest movement of muscle.

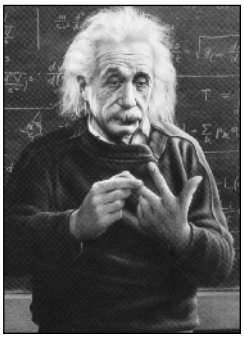
In yet another example of "everything old is new again" or things coming full circle, it turns out that here and now in 2007, using one's fingers for math is not only O.K., but actively encouraged by educators. When a child uses her fingers to do arithmetic, we should not assume that her understanding is immature–she is, in fact, activating the math part of the brain.

Brian Butterworth, a Professor at University College, London, England is currently working with colleagues on the neuropsychology and the genetics of mathematical abilities. His unique research offers a completely original picture of how our brains do math.

As Butterworth explains, one of the reasons the brain was a mystery for so long is that it is protectively encased by the skull. Under ordinary circumstances one does not have easy access to see the brain. With contributions from science, engineering and mathematics, came the advent of imaging tools for seeing inside the skull–CAT scans and MRIs, meaning that it is now possible to generate three-dimensional images of the brain that not only allow static views but also movies that show the dynamic behavior of the brain.

By producing movies of brain activity from subjects "doing math", Butterworth has generated data that he believes suggests that the left parietal lobe of the brain is where math happens. We all know that some of us are good at math and some of us are not; however, Professor Butterworth contends that the reason a person falters at math is usually not because of the wrong gene or "engine part" in the left parietal lobe, but because he or she has not fully developed the mathematics sense that we were born with.

Butterworth uses an example to illustrate his point. Imaging tells us that the left parietal lobe is also where fingers are registered in our brain–a fact that Butterworth believes not only can explain the reason we naturally count on our fingers, but cries out for why classroom instruction should use our built-in digital computers to enhance computational fluency. Butterworth points out that just as Einstein's brain has been found to have unusually densely packed cells in the left parietal lobe, so musicians have been shown to have a bigger and more elaborately connected motor cortex–the region of the brain that controls hand and finger movements–than non-musicians. This is not a sign that these people (e.g., Einstein or musicians) are predisposed towards math or music, but, argues Butterworth, simply illustrates the fact that that

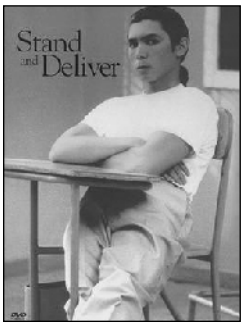


finger practice–with numbers or notes–makes perfect.

One of the strategies that Butterworth suggests to keep the left parietal lobe dense is to try to find new ways to do old things.

Do you remember learning the multiplication tables? For most people, these are learned, not as abstract patterns of number but through language, almost like a poem, and it is not unusual for people to recite the table in order to find the answer to a mathematical problem.

There are however, more brain-based ways to learn how to multiply. I was reminded of this the other day when one of our Keynote Speakers, Dave Mitchell, showed a clip from the movie, Stand and Deliver. In this 1988 classic, a true story based on the life of dedicated mathematics teacher, Jaime Escalante, a group of dropout-prone students are inspired by Mr. Escalante in a Pygmalion-like turnaround. Except this unique teacher doesn't teach grammar, posture and manners à la Henry Higgins in My Fair Lady, but rather challenges his students to learn calculus to build up their self-esteem. This cadre of tough guys and street kids does so well that, sadly, they are accused of cheating on a nation-wide Advanced Calculus Placement examination.





In an early scene from the film, mathematics teacher Jaime Escalante confronts Chuco, a defiant gang member during his class at Garfield High School in East Los Angeles. The encounter goes like this…

Escalante: *Ohh. You know the times tables?*

Chuco: *I know the ones. ..twos. ..three.*

[On "three" Chuco makes a very impolite finger gesture to Escalante]

Escalante: *Finger Man. I heard about you. Are you The Finger Man?*

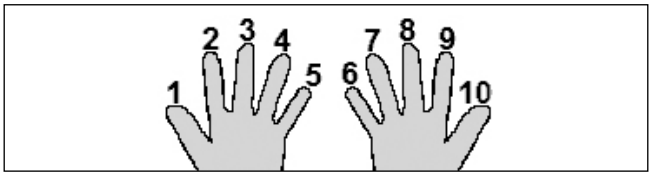
Well, just so you know, I'm the Finger Man, too. Do you know what I can do? [Escalante is holding his hands up to the class, moving his fingers in time with each number fact.] I know how to multiply by nine! Nine times three. What you got? Twenty-seven. Six times nine.

One, two, three, four, five, six. What you got? Fifty-four.

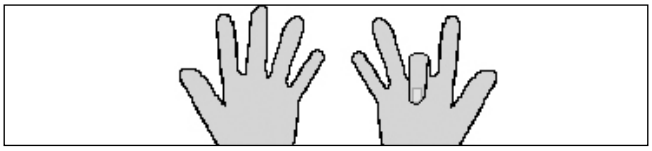
You wanna hard one? How about eight times nine? One, two, three, four, five, six, seven, eight. What do you got? Seventy-two.

To capture Chuco's attention, Escalante was using a well-known finger-multiplication strategy: multiplying by nines by using his fingers as a visual display.

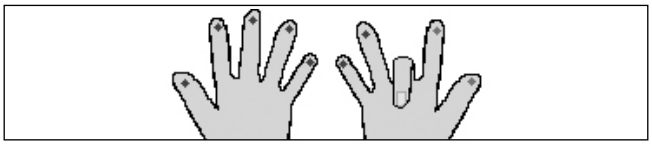
Hold both your hands up with palms facing you. Number the fingers from left to right as one to ten.



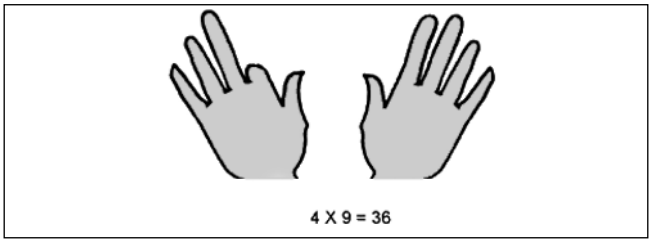
Now hold down the finger of the number you want to multiply by nine. In this example, we are going to multiply by eight, so the eighth finger is held down.



The fingers to the left as the tens and the fingers to the right are units. In this example, there are seven fingers to the left and two to the right which makes 72.

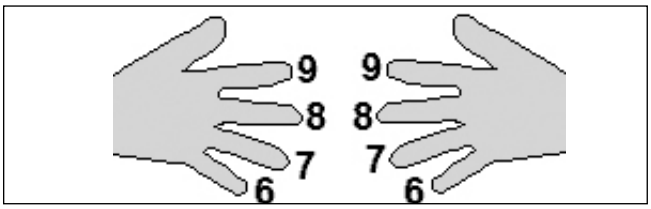


Here is how you would multiply 4X9 using your fingers.

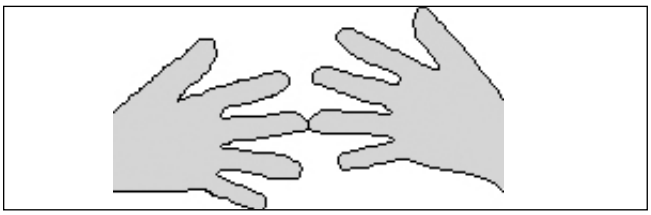


You can also use finger math to multiply by six, seven, eight or nine. This particular method does not apply to multiplying by one to five and is limited to computing products from 6 x 6 to 9 x 9. It is said that this method was widely used in medieval Europe.

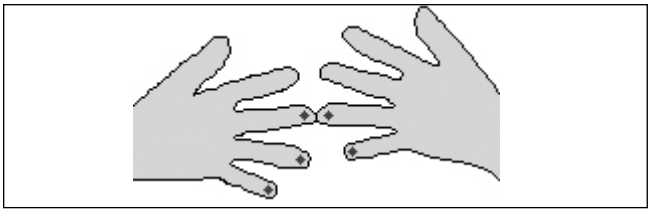
Hold your hands with palms facing you and fingers towards each other. Number the fingers on each hand from bottom to top as six to nine. (The thumb could be ten, but it's easy to multiply by ten, so don't bother!)



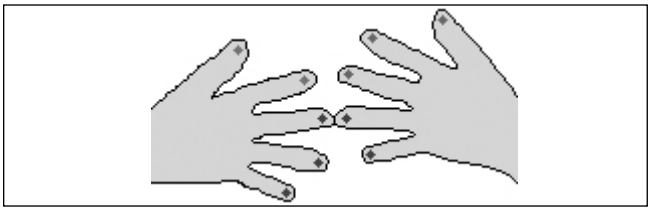
Now touch the two fingers of the numbers that you want to multiply. You may need to twist your hand slightly to do this. This example is multiplying eight by seven, so the finger number 8 touches the finger number 7 on the other hand.



Now for the calculation. First you add the fingers below and including the touching ones. These are the tens. Here there are five of them, three on one hand and two on the other, so that makes 5 x 10 = 50.



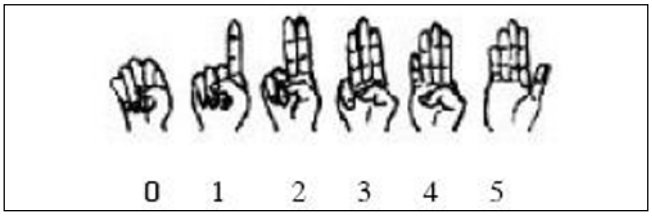
Then you multiply the fingers on the left hand above the touching ones with the fingers on the right hand above the touching ones. These are the units. Here there are two fingers on the left, and three fingers on the right, so that makes 2 x 3 = 6. Add this to the 50 you've already got, and that makes 56.



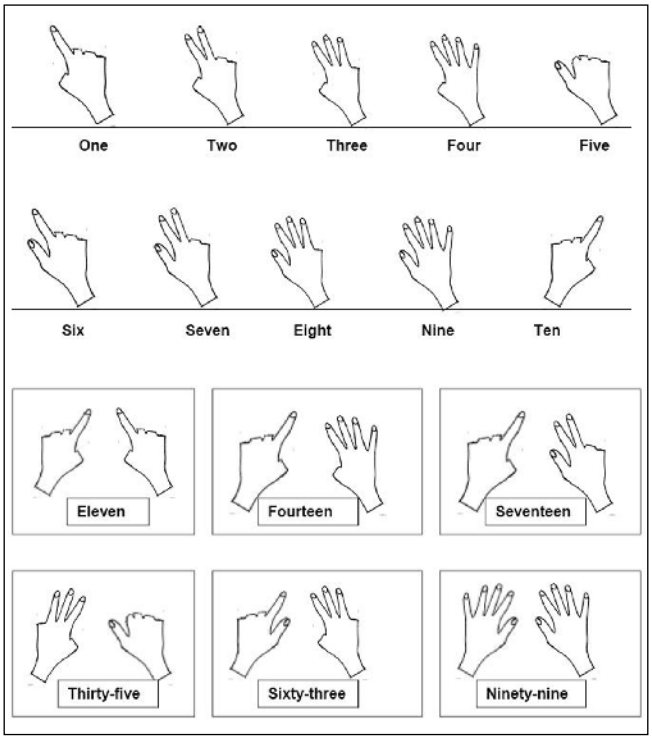
In the example above, we separated the tens and units. What happens if you try to multiply 6 x 6? You should find that the tens figure is 2 x 10 = 20, and the units figure is 6 + 6=12. Added together, this makes 36, which is correct.

At the conference, we also learned that although children around the world count on their fingers from a very early age, there are cultural differences in the ways in which the fingers are used.

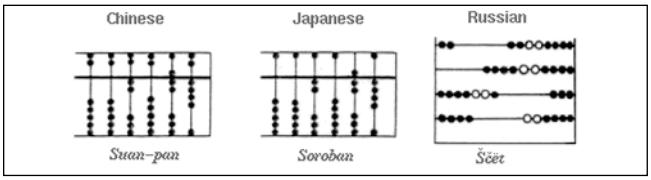
In North America, children typically show numbers by raising in succession, fingers from the index finger to the little finger and then raising the thumb. Two hands are required to get to ten, but the finger pattern does not provide a visual clue about the number ten being special in terms of place value.



Many Asian children are taught to use their fingers differently to count–in fact, they are taught to use their hands and fingers to simulate an abacus (or more accurately, a soroban).

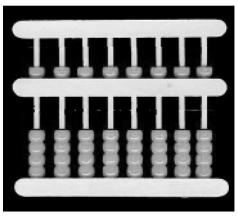


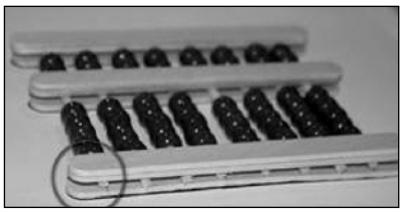
Historians believe that the Sumerians were the first to develop the abacus in around 2700 B.C. By the 5th century B.C., the Zapotecs of Central America, the Greeks and the Romans had all developed some form of abacus, and it is presumed, because of their similarities that the Roman abacus was introduced to China at some point.



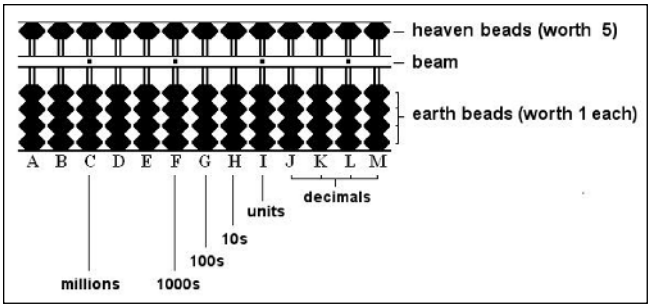
The Japanese changed the ancient Chinese abacus to better reflect the base 10 number system (and at the same time simulate what children were doing on their fingers).

The only way to learn to use a Japanese abacus (known as a soroban) is to practice moving the beads. You can make a simple soroban with 4 popsicle sticks, some bamboo skewers, a few pony beads and some glue.

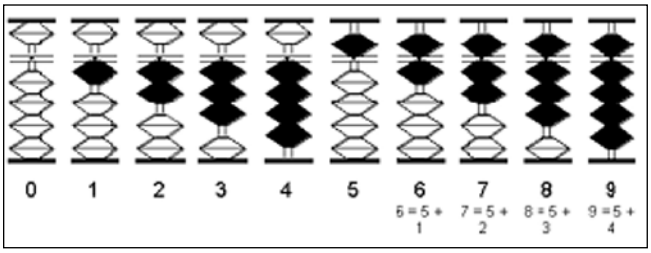




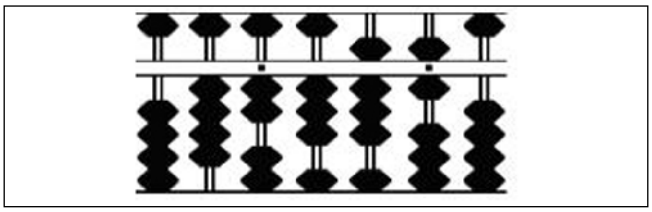
To use your soroban for computations, you will need to know how to represent numbers on it.



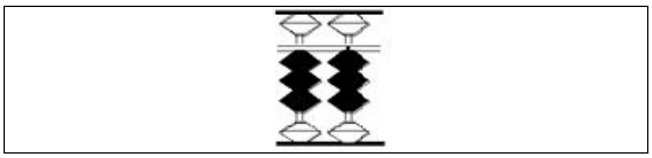
The numbers from 0 through 9 are shown on the soroban below (note that only the units or ones rods is shown in this illustration).



What number is this? If you said 42, 386, you are correct.

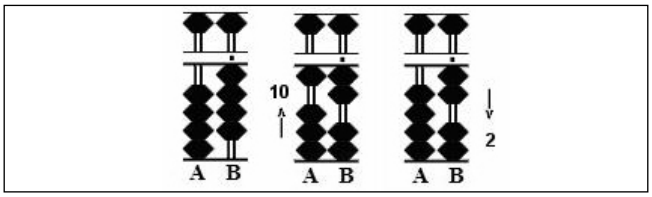


You can also use the soroban for addition. Our first task is to add 33 + 65. Begin by showing the number 33 on your soroban.

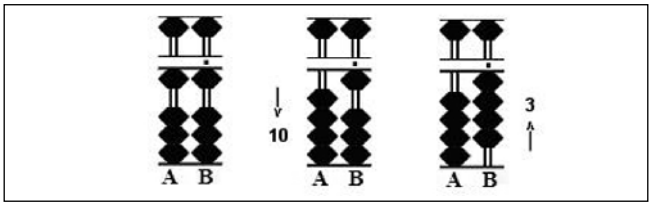


Move the beads to add on 65. First move 50 down and 10 up, pinching the beads together, and then 5 down. The total is 98.

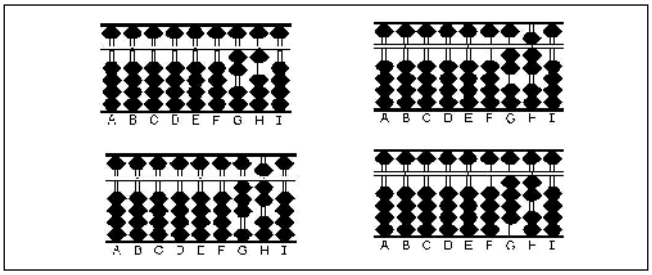
To find 4 + 8 on the soroban, you need to think 4 plus 10 subtract two, as shown below. Why?

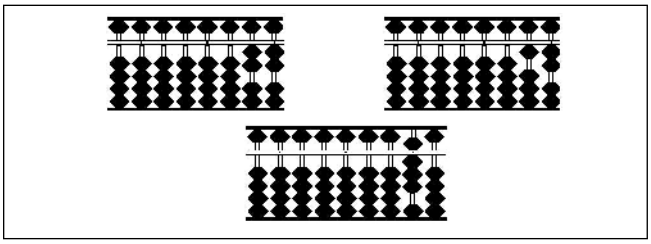


Similarly, to subtract 7 from 11 on the soroban, you need to think 11 minus 10 plus three. Why?

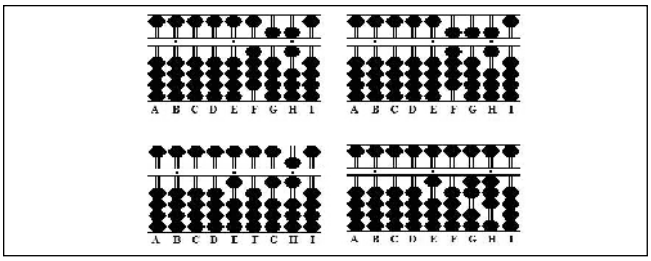


The same principle applies to all computations. In the examples that follow, the bead movements take you through the steps to compute 21 + 6 + 15 and 22-14, respectively.

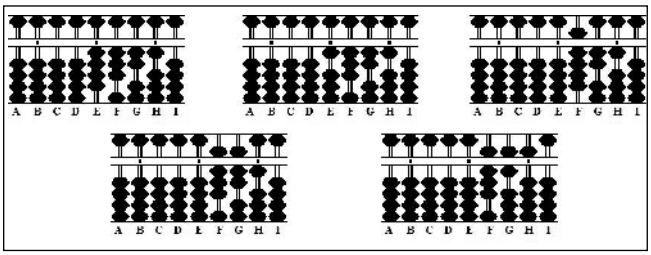




Position the beads to show 456 on your soroban. Follow the illustrations to see how to add 567 to 456.



As a final example, use the beads on the soroban to find the difference when you subtract 3456 from 4321. Use the illustration to confirm your solution.



Many skilled soroban experts are able to add 15 three digit numbers in under 10 seconds with the soroban–less than the time it would take to say them all! The 2002 champion of soroban mental calculation added 30 three digit numbers in 6 seconds.

So, the next time you're given an arithmetic problem, get out those fingers (or the next best thing to them, a soroban), and do the mathemagic. ▲

▲ Call for Manuscripts

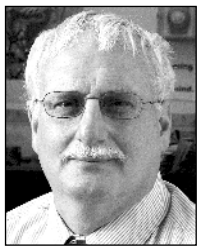
The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor

Marilyn Hurrell for further details.

▲ OAME/NCTM Report December 2007



JOHN KERSLEY

In my last column (September 2007 issue of the *Gazette*) I mentioned that NCTM has updated its home page web site, **www.nctm.org**. Here important events, position statements, focus of the year information, the latest resources and the president's message are listed. NCTM is, of course, well known for its journals and its multitude of publications, (see **www.nctm.org/catalog**) but it offers a wealth of other resources. My most useful NCTM site is **www.nctm.org/resources**. The following is a brief description of the material at this site:

- **Resources for Canadian Mathematics Teachers** A listing of all Canadian math associations and a link to their home page, including ours, OAME
- **Lessons and Resources** for elementary, middle and high school
- **Family Corner** Resources for parents and families to help students learn math.
- **Tips for Teachers** homework, testing, grading communication with parents, tools and technology etc
- **Reflections** A video based web site for professional development of teachers, focusing on algebra from grades 3 - 8
- **Illuminations** A rich source of lessons and activities covering a whole range of topics and grades.
- **Figure This** A host of mathematical challenges and information for parents, students and teachers
- **Student Math Notes** Great resources for students and teachers grades 5-10.

It is worth the time to visit and explore. You will definitely find lots of material for your classroom.

A final note: NCTM has another site, **www.nctm.org/eresources**, but this I leave for you to explore and for me to comment on in a future column.

Web sites of interest to mathematics as suggested by NCTM

- **PBS Teachers www.pbs.org/teachers** Click on math, select grade and topic, then explore Lots of material for educators including effective ways of using media and technology in the classroom, thousands of lesson plans, videos, blogs, etc and opportunities for professional development
- **Intermath www.intermath.coe.uga.edu** The University of Georgia provides workshops and technological support for grades 4-8 teachers
- **The Fun Works www.thefunworks.org** This is a really different site. It is a collection of career explorations for students (ages 11 – 15) using their interests such as music, sports, arts and crafts, medicine, law, technology, engineering, science and mathematics. It also has links to thousands of sites.
- **National Library of Virtual Manipulatives www.nlvm.usu.edu/en/nav/vlibrary.html** This is a free collection of digital activities including games, tutorials, puzzles and digital depictions for all grades. Lots to explore and something for everyone.
- **Thinking Blocks www.thinkingblocks.com** This is an interactive math program to help students learn to model and solve multi-step math problems by guided practice, videos and assessment

NCTM Conferences www.nctm.org/meetings

- NCTM Annual Meeting and Exposition
- Salt Lake City, Utah "Becoming Certain About Uncertainty" April 9 – 12, 2008

NCTM Regional Conferences and Exposition for 2008

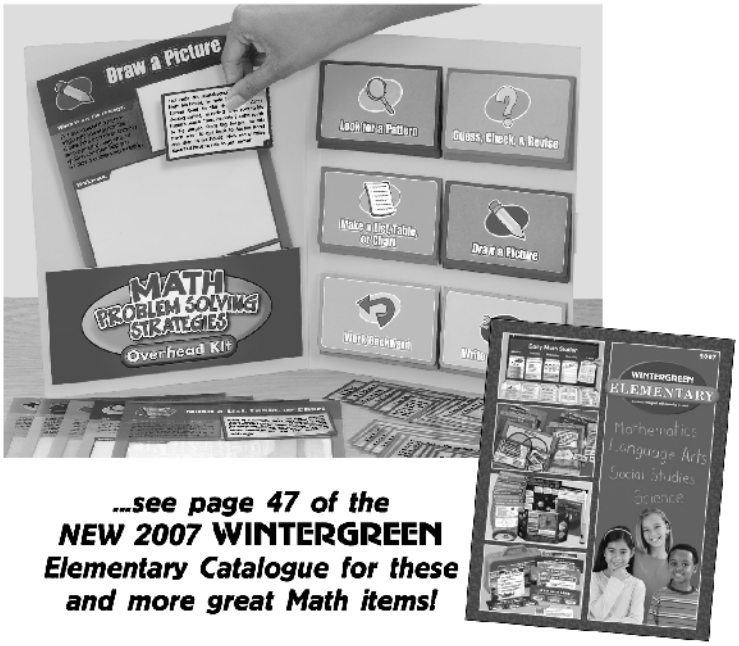
- Oklahoma City, Oklahoma October 2, 3, 2008
- Cleveland, Ohio October 16, 17, 2008
- Reno, Nevada November 6,7 2008

Affiliate Conferences

A list of affiliate conferences can be found at **www.nctm.org/affiliates/conferences.htm** ▲

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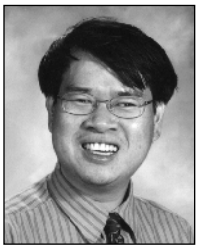
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▲ Get Interactive! Making Connections

Louis Lim, with Son Bui, Jeff Ford, Christine O'Hara, Dan MacKinnon, Jen Rowden, and Whitney Woloshyn



Louis Lim is department head of mathematics at Richmond Hill High School, York Region DSB, and Additional Qualifications instructor with Queen's University and York University. Louis' involvement with the OAME includes former editor of the Ontario Mathematics Gazette, presenter at chapter and annual conferences; and elected board of director. In spring 2005, he was recipient of the OAME's "Exceptional and Creative Teaching in Secondary Mathematics" Award.

In the next two issues of "Get Interactive", I feature contributions from my Honour Specialist Mathematics class, taught on-line in the summer 2007 for Queen's University. My class consisted of 12 future leaders in mathematics education. One of their assignments was to identify and describe three web sites that fit the connections theme of the September 2007 issue of this column. I have selected one submission from each teacher to share with the broader mathematics education community (half are featured in this issue, the remaining in March 2008). I thank my Honour Specialist students for their excellent submissions.

1.) Earth Math

Contributed by Son Bui, Hamilton Wentworth District School Board

The Earth Math web site, **http://earthmath.kennesaw.edu/main_site/algebra_with_equations.htm** allows students to solve problems and analyze phenomena that exist in our day-to-day lives. Most of the applications are focused on environmental issues and concerns we confront daily—in our neighbourhoods, cities and towns, and in the countryside and wilderness areas. Issues addressed include population growth, oil and coal consumption, air pollution, water availability, food supply, and urban and rural development. The activities address topics such as equations, functions, and calculus. Included are explanations for perquisite skills needed for the activities.

2.) Mathematics and Geography

Contributed by Jeff Ford, Upper Canada District School Board

How many people are in the world? The answer changes every millisecond as shown at **http://math.berkeley.edu/~galen/popclk.html**. Students will be fascinated as they watch world population figures change right before their eyes. There is a freeze/unfreeze button that allows for pausing the population at any time. **http://www.ibiblio.org/ lunarbin/worldpop** compares the current world population with archived world population figures and allows you to choose the date. **http://www.census. gov/ipc/www/popclockworld.html** shows a stationary world population figure that can be updated by simply refreshing the browser. This last site includes monthly world population predictions shown below the current population figure. There are numerous curricular connections, which include graphing, linear and non-linear relations, scatter plots and line of best fit, and applications of statistics.

3.) Math and the Real World

Contributed by Christine O'Hara, Kawartha Pines District School Board

http://oncampus.richmond.edu/academics/education/projects/webquests/mathworld/index.html is a web quest in which students navigate through various links to see how mathematics is used in the real-world. Students will have created a children's book about how math is used in everyday life after they have completed their web quest. During the first part, students look at various sites to review how to brainstorm and concept map. In the second stage they learn how mathematics is used in such tasks as making chocolate fudge, measuring blood pressure, sports statistics, taking the shortest route, building roller coasters, and how to get the most for your money. This web quest, suitable for grades 5-7 students, makes curriculum connections to statistics and probability, number sense and algebra, and measurement.

4.) Origami and Mathematics

Contributed by Dan MacKinnon, Ottawa Carleton DSB

Origami is a wonderful way to get students interested in a variety of problems that include geometry, measurement, and combinatorics. Through origami, mathematics is linked to cultural practices, artistic expression, and even engineering applications. There are many online resources that explore mathematics and

origami, many which have an educational focus. Some starting points are Eric Anderson's page on math and paper folding, http://www.paperfolding.com/math/; Tomas Hull's math and origami page, **http://www.merrimack.edu/~thull/origamimath.html**; and Robert Lang's site **http://www.langorigami.com/**. There are some film clips of origami at the Green Fuse Film web site **http://www.greenfusefilms.com/**.

5.) Math and Art

Contributed by Jen Rowden, Durham District School Board

Many people cannot see the relationship that exists between mathematics and visual art. The web site, **http://faculty.gvsu.edu/goldenj/mathinart.html**, helps teachers to make several connections between these two subjects, including kaleidoscopes and the golden rectangle. It provides lesson ideas, mostly focusing on grades 1-6 but can be adapted to work for even the senior grades. One connection is a tutorial focusing on tessellations, which is organized by history, lesson, and theory. Fractals are another important math/art connection that is covered well by the web site **http://www.district87.org/staff/hallk/firstwebpage/fractals.htm#fractalsmathorart**. The site can be used as a self-directed lesson for students to follow, providing them with a complete guide including introduction, resources, and evaluation.

6.) Math, Product Identification and Security

Contributed by Whitney Woloshyn, Albert College, Belleville, ON

How does the computer know you entered your SIN incorrectly? Why can't people input random credit card numbers? Why has the ISBN recently changed from 10 to 13 digits and how does it work? The University of Cambridge site **http://nrich.maths.org/public/viewer.php?obj_id=2036** introduces instances when we see numbers every day and connects to math topics such as number theory, proof by induction, divisibility, and modulus arithmetic. This site has many opportunities for problem solving. A high school lesson on exploring credit cards, UPC and ISBN bar codes is found at **http://illuminations.nctm.org/LessonDetail.aspx?id=L693**. A 2-minute video introduction to UPC bar codes is located at **http://www.howstuffworks.com/upc.htm**, while a 2-page summary of a variety of coded numbers can be found at **http://www.pims.math.ca/pi/issue3/page05-06.pdf**. ▲

▲ What's the Problem: Fivefold Quadratics

SHAWN GODIN



Shawn is the head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans, ON.

Welcome back problem solvers. Last issue I left you with the following problem.

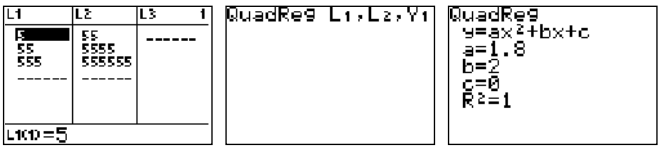
Find a quadratic polynomial f(x) such that, if n is a positive integer consisting of the digit 5 repeated k times, then f(n) consists of the digit 5 repeated 2k times. (For example, f(555) = 555555.)

This problem appeared as M256 from the September 2006 issue of *Crux Mathematicorum with Mathematical Mayhem* and solution appeared in the September 2007 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem* website **http://journals.cms.math.ca/CRUX/**

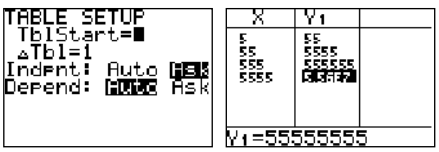
The problem statement allows us to create a table of values.

x	y
5	55
55	5 555
555	555 555

Since we know we are after a quadratic, we can enter these values into a graphing calculator, use the quadratic regression to get the quadratic of best fit.



We can see that the function $f(x) = 1.8x^2 + 2x$ matches our three points perfectly. We could use the table "ask" feature introduced in the September 2007 *OMG* in Mary Bourassa's column to check to see if it works for more of our values.



It becomes apparent that technology has some limitations in this case. If we continue, our answers will be written in scientific notation and we will not be able to tell if the pattern does indeed continue. Even if you had access to CAS you could verify many cases, but does that show that it always works?

One of the fundamental concepts in mathematics is the idea of proof. Unlike the court of law with its "beyond a reasonable doubt", mathematical proofs are absolute. Many mathematical proofs have their own beauty for their simplicity and elegance. Hungarian mathematician Paul Erdos use to joke that God had a book with all the most beautiful mathematical proofs in it that he kept hidden from us. When Erdos would see a nice proof he would comment that it was almost good enough for The Book.

One of the best known elegant proofs is Euclid's proof that there are infinitely many prime numbers. The proof assumes that there are finitely many primes and then shows that this is impossible. I strongly encourage you to look this proof up. Another nice proof you would enjoy is the proof that $\sqrt{2}$ is irrational. Email me if you are interested in either of these proofs.

For our case we would like to prove, absolutely, that the function $f(x) = 1.8x^2 + 2x$ has the property that

$$f\left(\overbrace{5555...55}^{\text{k 5's}}\right) = \overbrace{5555...55}^{\text{2k 5's}}.$$

Proofs take a bit of "playing about". In this case two, seemingly irrelevant, facts give us the proof:

1. We can write 1.8 in fractional form as $\frac{9}{5}$.
2. There is a common factor of x in our function.

Using these facts we can rewrite our function as $f(x) = x\left(\frac{9}{5}x + 2\right)$. Thus if we use $x = \overbrace{5555...55}^{\text{k 5's}}$ we get

$$\begin{aligned} f\left(\overbrace{5555...55}^{\text{k 5's}}\right) &= \overbrace{5555...55}^{\text{k 5's}}\left(\frac{9}{5} \times \overbrace{5555...55}^{\text{k 5's}} + 2\right) \\ &= \overbrace{5555...55}^{\text{k 5's}}\left(\overbrace{9999...99}^{\text{k 9's}} + 2\right) \\ &= \overbrace{5555...55}^{\text{k 5's}}\left(\overbrace{1000...001}^{\text{k–1 5's}}\right) \\ &= \overbrace{5555...55}^{\text{k 5's}}\left(10^n + 1\right) \\ &= \overbrace{5555...55}^{\text{2k 5's}} \end{aligned}$$

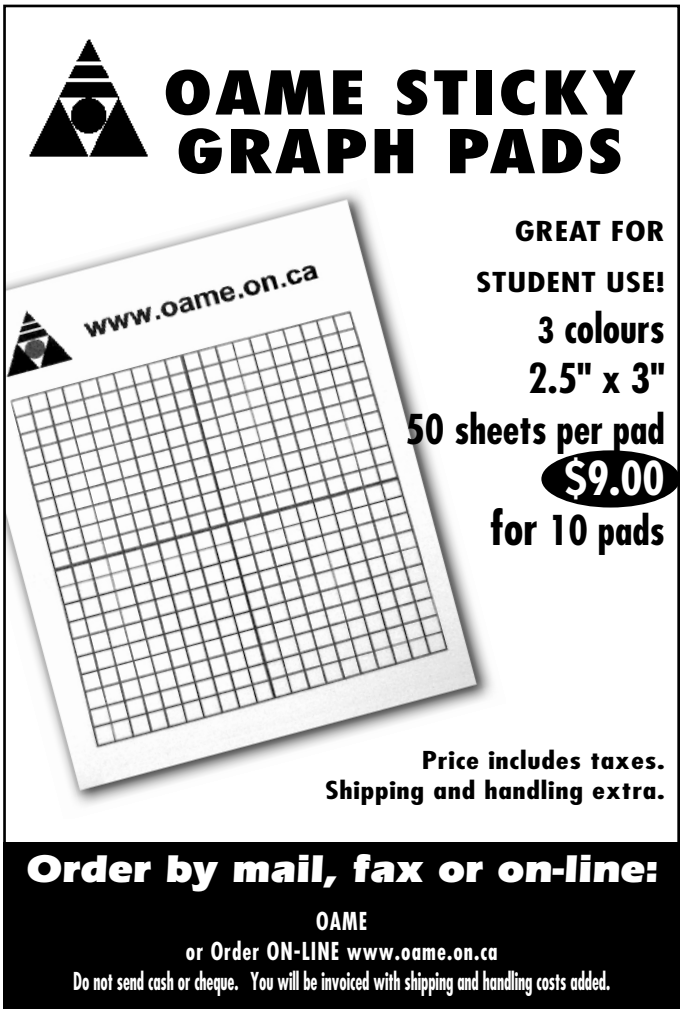
The evaluation above shows that no matter what positive integer made just of 5's we use, f(x) will be made just of 5's with twice as many digits. Sometimes a specific instance of a general statement makes the general statement easier to understand. Let's restate our proof above using the specific case of x = 55555.

$$\begin{aligned} f(55555) &= 55555\left(\frac{9}{5} \times 55555 + 2\right) \\ &= 55555(99999 + 2) \\ &= 55555(1000001) \\ &= 55555\left(10^5 + 1\right) \\ &= 5555500000 + 55555 \\ &= 5555555555 \end{aligned}$$

If you enjoyed this problem you will enjoy M267 from the November 2006 issue. Now for your homework.

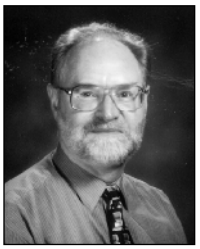
A mouse eats his way through a 3 x 3 x 3 cube of cheese by tunneling through all of the 27 1 x 1 x 1 sub-cubes. If he starts at one of the corner sub-cubes and always moves onto an uneaten adjacent sub-cube, can he finish at the center of the cube? (Assume that he can tunnel through walls but not edges or corners.)

Until next time, happy problem solving! ▲



▲ CAS Corner: Expanding Patterns on CAS

FRED FERNEYHOUGH



Fred Ferneyhough is former head of mathematics at Harold M. Brathwaite SS in Brampton. He retired from there in February 2006. Fred is an avid user of and advocate for the use of technology in the mathematics classroom. He is also an author and tech consultant for several books and a National Instructor for Texas Instruments.

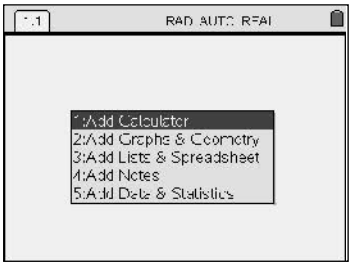
In the first article in this series, expanding and factoring work on a CAS were covered together. Since that time, I have had the opportunity to add more detail to these topics. (You could say that I expanded on the work.) In this issue, we'll cover expanding polynomials and look at factoring patterns on CAS in March 2008.

A CAS tool can be used to help students explore patterns in expanding. In fact, this is the one area of mathematics that many teachers cite as the topics that they are most hesitant to use a CAS for. Hopefully, this will help them see more than just the "black box" and see the idea of patterning to aid understanding.

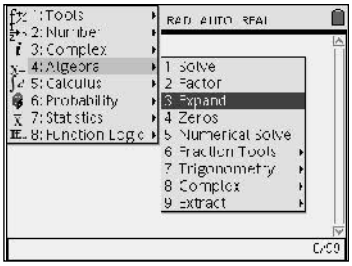
Let's start with some simple expanding patterns and then move on to the difference of squares, the perfect square and the products of binomials. The order is not important, but I would encourage that you use a CAS with algebra tiles wherever possible.

Note: The screens that you see here are from the new TI-nspire CAS, but could easily be done on the home application on the TI89 Titanium, the TI Voyage 200 or the TI 92 Plus.

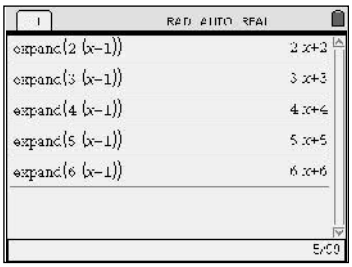
To begin, start a new document press (home) and choose (6) for New Document. You might get a prompt asking if you wish to save the previous document. Once you have answered that prompt, press (1) to open a new Calculator page.



First, we'll need to find the expand command in the menu. Press (menu) for menu followed by (4) for Algebra and (3) for Expand. The command "expand()" will appear on the screen.



Complete the command to read "expand(2(x+1)) and press • . The device adds the multiplication symbol between the factors. The results are shown on the right of the screen. One of the best features of using a CAS is that your students can generate a large number of examples very quickly.



Given the five examples in the screen above, I would ask my class to answer two questions.

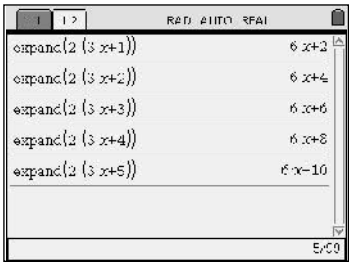
1) Can you explain the pattern in these examples? Explain patterns in both the questions and answers.
2) Can you predict what the results will be for the next two examples?

These two questions will be repeated throughout the development of skills.

The use of algebra tiles could be introduced at this juncture. I have found that the work with the two tools helps consolidate the students' skills.

By the way, the TI-nspire CAS device can do copy and paste in the same way that you would with Windows©. Move up to an expression that you want to copy. When it is highlighted, press (ctrl) followed by (c). This will copy the expression. Move to the next line and press (ctrl) followed by (v) to paste the expression into the line. Then, cursor back to any values that you wish to change, press (clear) to delete the characters and type in the new values. If you need to select a particular bit of text, move to the front of the expression, press and hold the (shift) key while pressing ▸ on the Nav Pad (the donut). The device also allows others Windows© shortcuts that you may be familiar with.

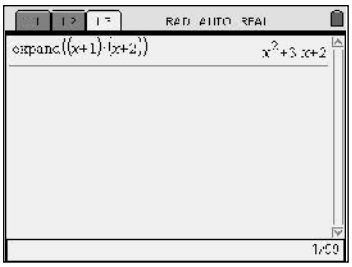
Next, we'll need to increase the difficulty level for the next set of problems. Press (home) followed by (1) for



Calculator to start a new page.

This set begins with the same first problem as the previous set but changes the coefficient of the first term in each factor. Again, I'd want my class to answer the same two questions as before.

If I'm confident that my class has picked up the pattern, I could then assign a few examples for them to work on either independently or in pairs, without the CAS device.

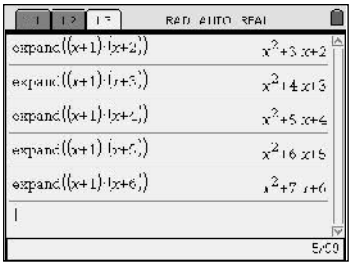


Let's move on to simple trinomial expansions. Press ⌂ followed by 1 to open a new page. I'm sure that all teachers have tried the following set of questions:

1 How is the coefficient of 3 in the result related to the numbers in the factors?

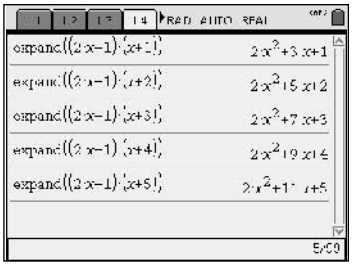
2 How is the constant of 2 in the result related to the numbers in the factors?

3 Do you think that this pattern will work in another example?



There's only one way to find out – generate more examples. In this set, I have changed only one factor by increasing the value of the constant each time. I'm hoping that students will verify the pattern that they conjectured in the previous step, as well as answer the same two questions posed earlier. This set of examples provides a very good point to begin the use of algebra tiles if you haven't done so already.

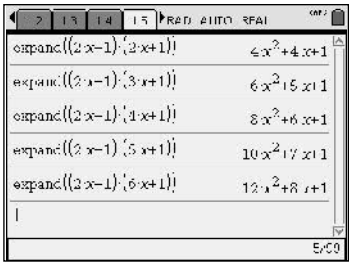
At this point, it would be wise to consolidate the pattern that students will have expressed – that the sum of the two constants in the factors produces the coefficient of the middle term and that the product of these values produces the constant in the result. Students will likely need some printed examples to attempt with the CAS device to anchor their knowledge. While algebra tiles can be used to support the development of these skills, I do not advise using algebra tiles with negative values.



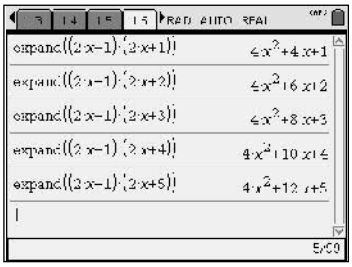
If we increase the difficulty level one notch at a time, students have a much better chance of success in noticing patterns. In this screen, I have changed the coefficient of x to two in each of the first factors and allowed the constant in the second factor to increase by one in each problem.

This is definitely one type of problem where the use of algebra tiles supports the development of concepts and skills.

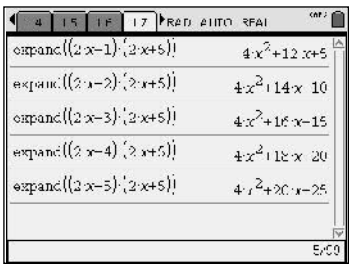
It would be easy for students to notice the patterns in the constants and the patterns in the linear terms and predict the results in the next two problems. There is always value in students observing such patterns, and this is not to be discouraged. However, what is needed is for them to expand the pattern observed above regarding the products and sums, incorporating the new factor of two.



If we increase the coefficient of the variable in the second factor, we will have another pattern for students to detect, but more importantly, the results, along with a demonstration using algebra tiles, can consolidate the observations above.

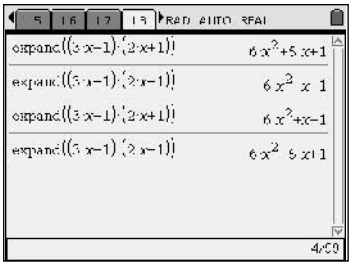


What remains is to generalize this result before moving on to special patterns. In this screen, the constant in the second factor increases by one with each new problem. If students have picked up the pattern above, they should be able to determine if these examples support and verify that pattern.

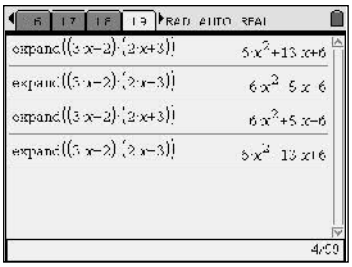


Finally, we vary the final occurrence of the unit value by increasing the constant in the first factor. While there are still linear patterns in the results, we need students to use these results to verify the product and sum observations made above.

We could also mix up positive and negative values to ensure that the pattern can be generalized even further. In this case, the constants have remained the same but the signs have been permuted to cover all possibilities.

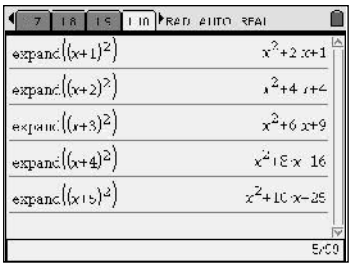


Repeating this pattern for another set of similar examples is again, quick and easy with a CAS device. There is the added bonus of some new patterns for students to notice in these examples.



As before, a worksheet to provide practice with more generic problems would help students follow up on the development of skills and concepts.

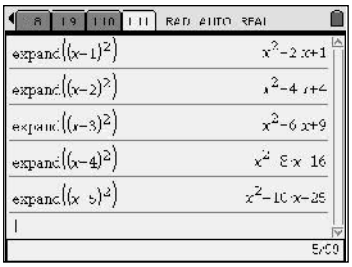
Let's move on to the perfect square expanding pattern. Press (home) followed 1 to open another new Calculator page.



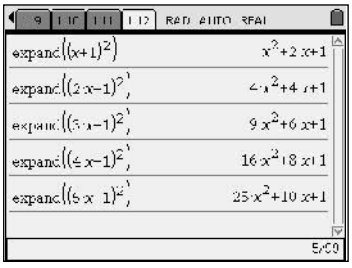
We are hoping that students will see something similar in this pattern. This time, we also want them to pick up on the pattern in the linear term.

As before, this set of examples provides an ideal point to use algebra tiles.

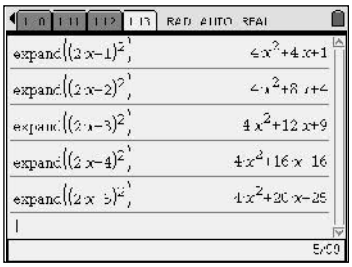
The same two questions should be answered, plus we want them to explore the same examples but replace the addition of terms with subtraction. The results are shown here, but the students should generate these on their own.



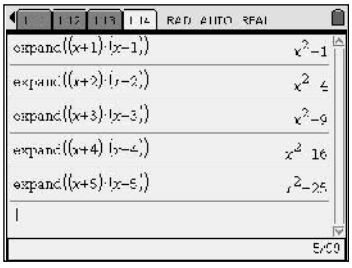
To increase the level of difficulty, we should introduce a coefficient with the variable. Press (home) followed by 1 to open a new calculator page. The next set will resemble the previous examples, but this time, the coefficients are being changed on each line. Again, being able to generate examples quickly is a very convenient feature of a CAS. Students should pick up patterns and be able to answer the same two questions.



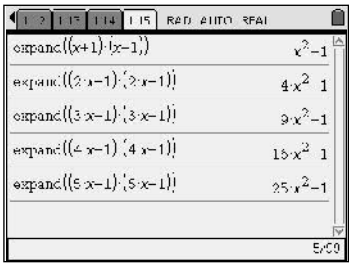
Before moving on to another pattern, extend this pattern one more time. Keeping the coefficient of the variable term constant and increasing the constant terms provides examples that extend the pattern.



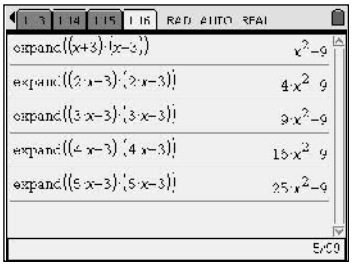
The final "special" pattern is the Difference of Squares. At this point, it is hoped that students' experience with expanding the products of binomials would have provided sufficient experience to explain why each of these products is "missing" a middle or linear term.



To increase the level of difficulty, leave the constant term as 1 and vary the coefficient of the first term. It is hoped that students will comment upon the fact that these examples are very similar to those in the previous set.



To wrap this section up, we should attempt to show students more generic problems. Let's fix the constant at 3 and vary the coefficient. In this way, they will have seen a variety of combinations.



As before, practice sheets to be done without the CAS will assist students with the consolidation of skills. ▲

▲ Assessment Corner: Using a Variety of Assessment Strategies: What the Data Tell Us

ANN ARDEN & DR. CHRIS SUURTAMM



Dr. Christine Suurtamm is an Assistant Professor at the University of Ottawa.

One of the dominant messages in the research literature and in the NCTM assessment standards (1995) with regard to effective mathematics assessment is the need for teachers to use a variety of assessment methods. This message is also a central element in the Ontario curriculum documents. This article uses the data collected in the Curriculum Implementation in Intermediate Math (CIIM) research project to frame a discussion about the use of a variety of assessment strategies. The CIIM research project examines the implementation of the Ontario curriculum in Grades 7 – 10. The project is designed to provide information about how the curriculum is understood and taught. It describes mathematics classroom practices, helps to suggest areas of implementation that might need further support, and highlights the types of supports that are reported to be most helpful to teachers. The project is in its final phase of case studies but already the results of the first year have provided much information (Suurtamm & Graves, 2007)[1]. The first phase included analysis of data collected through focus group interviews with leaders in mathematics education, a web-based questionnaire for intermediate teachers across the province, and focus group and individual interviews with teachers.

Assessment Practices

Teachers across the province were invited to respond to the questionnaire and our questionnaire data represents the responses of 1096 teachers of Grades 7 – 10 from 42 Ontario school boards. The following is a discussion of the results of items that are related to assessment practices. In the questionnaire, teachers were asked about the assessment methods they use to get a sense of their students' understanding of mathematics and the assessment methods they use to determine students' marks in mathematics. In both cases, teachers reported that tests and quizzes were the most widely used forms of assessment. Other assessment methods, such as performance tasks, homework, observations, and student responses in class were also reported to be used but there were smaller percentages of teachers reporting their use. This provides evidence that some teachers are using a variety of assessment methods but that many teachers rely heavily on tests and quizzes. Table 1 reports the percentage of teachers in each course who reported that they use tests a lot to determine a student's mark.

	A lot
Grade 10 Academic	82%
Grade 9 Academic	70%
Grade 10 Applied	53%
Grade 9 Applied	53%
Grade 8	43%
Grade 7	41%
Grade 9 Essential/Locally Developed	40%
Grade 10 Essential/Locally Developed	35%

Table 1: The extent to which teachers use the tests to determine a student's mark

These data suggest that teachers of Grade 9 and 10 Locally Developed courses and teachers of Grades 7 and 8 do not rely as heavily on tests as those teachers of the other Grade 9 and 10 courses, particularly at the Academic level. This might suggest that for some courses, a wider variety of assessment strategies are used, perhaps to address the needs of the students who are in the classroom by providing varied opportunities to show what they know and can do.

Using a variety of assessment strategies is important for a number of reasons. One is that it addresses the needs of a variety of learners, and another is that this provides multiple opportunities for students to show what they know and can do. For instance, students may not perform well in a test-taking situation and using a variety of assessment tasks provides other opportunities. However, another important reason to use a variety of assessment forms is that a paper-and-pencil test or quiz is not a suitable assessment form for assessing many of the mathematics curriculum expectations.

Examining the Curriculum

All Ontario mathematics curriculum documents include the process expectations of problem solving,

[1] The report of the first year results is available on-line through the web site: http://www.edu.gov.on.ca/eng/studentsuccess/lms/

reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, communicating. Many of these process expectations cannot be thoroughly assessed using only paper-and-pencil tasks but also need to include observation of problem solving and investigations, listening and responding to students' mathematical ideas through conferencing or observing student interactions, or require tasks or projects that occur over a longer time period than can be afforded by a traditional test.

However, it is not just the process curriculum expectations that may prompt a variety of instructional and assessment strategies. An examination of the verbs in the overall and specific curriculum expectations suggest that students will do such things as pose problems, describe, explain, determine through investigation, or use a variety of tools and strategies. For instance, in Grade 7 measurement, a specific expectation is that

Students will:

determine, through investigation using a variety of tools and strategies (e.g., decomposing right prisms; stacking congruent layers of concrete materials to form a right prism), the relationship between the height, the area of the base, and the volume of right prisms with simple polygonal bases (e.g. parallelograms, trapezoids), and generalize to develop the formula (i.e., Volume - area of base x height) (Ontario Ministry of Education, 2005a, p. 101).

What do instructional activities look like that address this expectation? And, how would a teacher assess whether a student is able to determine, through investigation, the relationship between the height, area of the base and the volume of right prisms that have different polygonal bases? It would seem as though the teacher would have to provide an assessment opportunity that would allow students to demonstrate that they could determine such relationships through investigation. This would mean that the teacher might be observing such investigations, or having students work in groups and record the details of their investigation rather than merely focusing on whether the student has been able to remember the formula for the volume of a right prism.

Expectations that prompt investigation are found throughout all of the intermediate mathematics curriculum documents. In the Grade 9 Academic course, two of the three overall expectations in the Measurement and Geometry strand require students to demonstrate what they can do through investigation.

Students will:

determine, through investigation, the optimal values of various measurements;

solve problems involving the measurements of two-dimensional shapes and the surface areas and volumes of three-dimensional figures;

verify, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems. (OME, 2005b, p. 36)

If teachers are to assess how students are able to determine optimal values or determine properties through investigation, they need assessment strategies that allow them to observe or examine how students investigate mathematical ideas. Other examples can be found in the Grade 10 Academic course, where students are asked to investigate mathematical ideas using a variety of tools such as dynamic geometry software in the trigonometry strand.

Therefore, using a variety of assessment strategies is not just connected to the notions that students learn in different ways and that they need to be given a variety of opportunities to show what they know and can do. The mathematical activity that is embedded in the curriculum expectations calls for a variety of forms of assessment.

Challenges

However, teachers may not be as comfortable with alternate forms of assessment as they are with the use of tests. Table 2 shows the questionnaire responses of teachers when they were asked how comfortable they were with using a variety of assessment strategies.

	Very comfortable
Grade 9 Essential/Locally Developed	48%
Grade 8	48%
Grade 10 Essential/Locally Developed	44%
Grade 9 Academic	42%
Grade 10 Academic	42%
Grade 7	39%
Grade 9 Applied	33%
Grade 10 Applied	32%

Table 2: Teachers' comfort with using a variety of assessment strategies

The above data suggest that teachers may need further support in the development and use of different

assessment strategies. As well, there is further data provided in the responses to the final question on the questionnaire that asked teachers to provide additional comments that could be related to any aspect of teaching intermediate math. Some of these responses provided insight into teachers' use of a variety of assessment forms. Some participants suggest that finding time to develop other forms of assessment is challenging. As one respondent explained,

> *For some of the questions concerning the time I have to complete certain tasks... I use my own time a great deal. I complete all research at home in the evenings and on weekends. I see students daily throughout my lunch hour for Math resource. I would love to be able to use a variety of assessments, but seem to have no time to prepare other assessments besides tests. (Participant 06127)*

Other issues include the class time that alternative assessments such as observation require such as this teacher's comment.

> *Discussion can be useful, but realistically I don't usually have time to get around to interview and observe every student and make comments for each task so a worksheet keeps them on task and fills in the gaps I miss. (Participant 06430)*

Others suggest that they need to have more professional development opportunities to learn to develop and use other forms of assessment as these two respondents report.

> *My board has done so much in the way of PD for manipulatives. The PD I am looking for is more student based. I'd like to learn more about how to use group work and rich assessment tasks in my classroom. (Participant 22013)*

> *I think it would be helpful to have good examples of performance assessment tasks for the course, a better idea of how many to give and a timeline for them. (Participant 06952)*

Concluding comments

Using a variety of assessment methods helps to ensure that all students have an opportunity to demonstrate their learning and avoids a reliance on only paper-and-pencil tasks such as tests and quizzes which tend to emphasize merely knowledge recall and procedural learning (Shepard, 2001). The use of a variety of forms of assessment that both elicit and contribute to developing mathematical understanding and the full range of mathematical processes is critical in mathematics classrooms.

Observations from the CIIM research project suggest that while quizzes and tests are still the most frequently used form of assessment both in getting a sense of students' understanding and in determining students' marks, some teachers are also using other forms of assessment such as performance tasks, homework, observations, and student responses in class. Comments made in the math leader focus groups and in response to the open-ended items on the questionnaire also confirm this as they suggest that while teachers are moving toward using more assessment forms, there is still a strong reliance on tests and quizzes. Some reasons for this are suggested including a lack of time available to develop new forms of assessment and a lack of familiarity and comfort with new forms of assessment. However, teachers also express a need and desire for more professional development with regard to newer forms of assessment and our data suggest that such professional development and dialogue with colleagues is an important component of developing new ideas about assessment.

References

National Council of Teachers of Mathematics (NCTM). (1995). *Assessment standards for school mathematics.* Reston, VA: NCTM.

Ontario Ministry of Education. (2005a). *The Ontario curriculum Grades 1-8 mathematics revised.* Toronto: Queen's Printer for Ontario.

Ontario Ministry of Education. (2005b). *The Ontario curriculum Grades 9 and 10 mathematics revised*. Toronto: Queen's Printer for Ontario.

Shepard, L. A. (2001). The role of classroom assessment in teaching and learning. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 1066-1101). Washington, DC: American Educational Research Association.

Suurtamm, C., & Graves, B. (2007), *Curriculum implementation in intermediate mathematics (CIIM) : Research report.* ▲

▲ VOTE ONLINE

To elect Executive and Directors for the OAME Board of Directors, please vote online at **www.oame.on.ca** from **Mar 1- Apr 10, 2008**.
Click on member's only to activate your account if you have not already done so. If you require a written ballot, please contact in writing by Feb 1, 2008 Dave and Sue Hessey, Exec. Directors

▲ Photo Math - The Fine Art of Viewing the World Through a Mathematical Lens

Ron Lancaster
35 Oak Knoll Drive, Hamilton, ON L8S 4C2



Ron Lancaster is a Mathematics Lecturer in the Department of Curriculum, Teaching and Learning, the Ontario Institute for Studies in Education (OISE/UT) at the University of Toronto

Teachers and students are encouraged to put on a pair of mathematical glasses and become a math spotter, someone who sees mathematics everywhere and then possibly poses mathematical questions based on the images taken. Ron Lancaster, the current Editor for Photo Math, welcomes photos with questions for possible publication in future columns. Material can be sent via e-mail or by regular mail to either of the two addresses given above.

These photographs were taken on the same day in June 2007 and within a few minutes of each other.

(a) How much more expensive is it to buy Senza Piombo (unleaded) gas or Diesel fuel at the station in Photograph 1 than at the station in Photograph 2? Express these answers in euros, as a percentage and as a ratio. Are the ratios the same? Should they be the same?

(b) Use the information on the pump in Photograph 3 to calculate the cost of the purchase? Did you get the same price that is shown on the pump? Should you get the same answer?



Photograph 1



Photograph 2



Photograph 3

©2007, Ron Lancaster. Any commercial use or any use outside of a teacher's classroom requires permission from the author.

OAME Financial Statement 2006/07

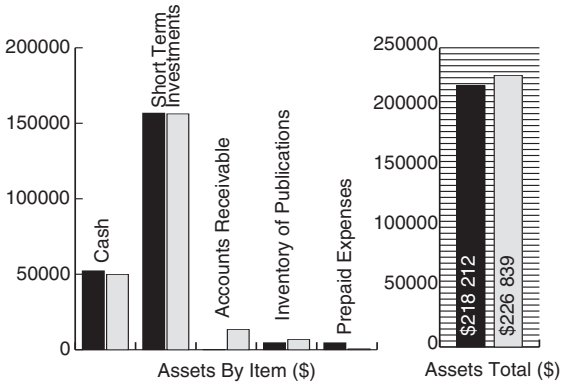
Notice to Readers

I have compiled the Balance Sheet of the Ontario Association for Mathematics Education (Association Pour L'Enseignement Des Mathematiques) as at August 31, 2007 and the Statement of Operations and Operating Fund Balance for the year then ended, from information provided by management. I have not audited, reviewed or otherwise attempted to verify the accuracy or completeness of such information. Readers are cautioned that these statements may not be appropriate for their purposes.

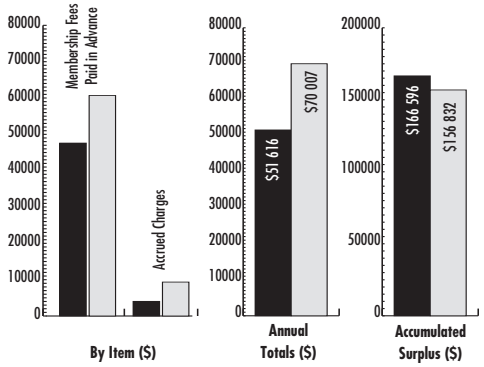
London, Canada September 29, 2007, PHILIP J. GRIFFIN Chartered Accountant

2007/2006

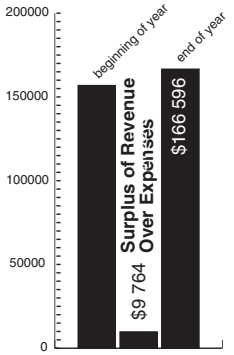
Current Assets



Liabilities and Fund Balance



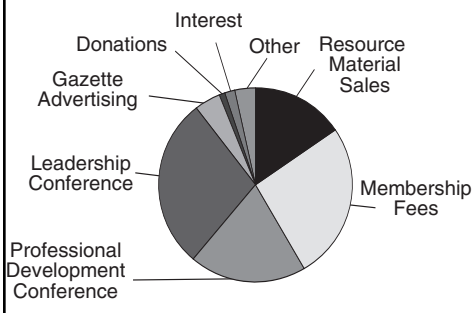
Operating Fund Balance



Revenue

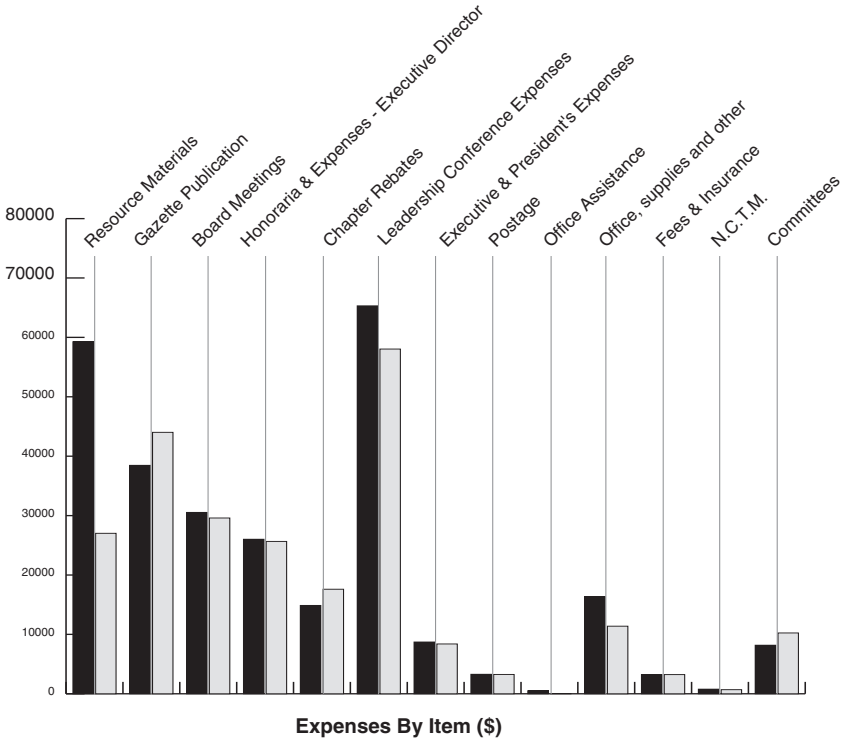


2007 ($231 944)



2006 ($249 377)

Expenditures



▲ Technology in University Mathematics Instruction? At Brock, Yes!



CHANTAL BUTEAU
DEPARTMENT OF MATHEMATICS
BROCK UNIVERSITY

Excerpts reprinted with permission of the Canadian Mathematical Society: H. Ben-El-Mechaiekh, C. Buteau, W. Ralph: MICA: A Novel Direction in Undergraduate Mathematics Teaching, *in Canadian Mathematical Society Notes, V.39 (6), pp. 9-11.*

Why do mathematics students at Brock University choose to spend hours of extra time on mathematics projects? The reason is the new MICA program. MICA stands for Mathematics Integrated with Computers and Applications: a "hands on" approach to teaching mathematics, making extensive use of technology and remarkably increasing the level of students' engagement (and enrolment: it tripled since MICA's start in 2001). The MICA reform, grounded in years of pioneering the integration of technology into the curriculum spearheaded by Eric Muller, consisted in introducing three core MICA courses (a total of 2.5 credits) and revising "traditional" courses in light of the MICA principles (Brock Teaching, 2001) — for example, technologies such as Maple, Journey Through Calculus (Ralph, 1999), Geometer's SketchPad, and SAS were fully imbedded into Calculus, Linear Algebra, ODE's, Optimization, Cryptography, Statistics, etc.... In the core MICA courses, students explore, model and program (in Visual Basic.NET, Maple, or C++) fundamental mathematical concepts.

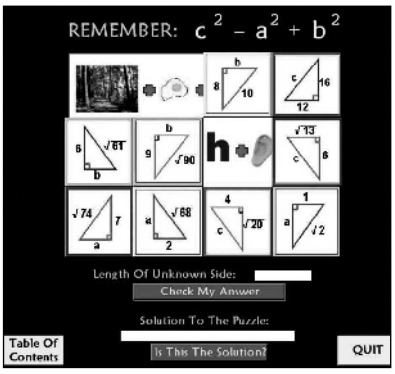
Why Use Computers in University Mathematics Teaching?

Exploration in mathematics is severely limited by the use of purely analytical tools. The tight choreography of traditional programs around problems solvable "by hand" inevitably imposes a "canned" and regimented approach. Technology offsets the rigidity of a traditional program by providing an endless supply of problems having both analytic and computational aspects. Many of the final projects by first year MICA students are original investigations of problems of their own invention. For example, two first year students developed an unusual novel algorithm for generating Pythagorean triples. They wrote a computer program to test their conjecture and succeeded in verifying the first few cases algebraically. It was particularly gratifying to see their motivation and dedication driven more by the thrill of the discovery rather than the project's grade.

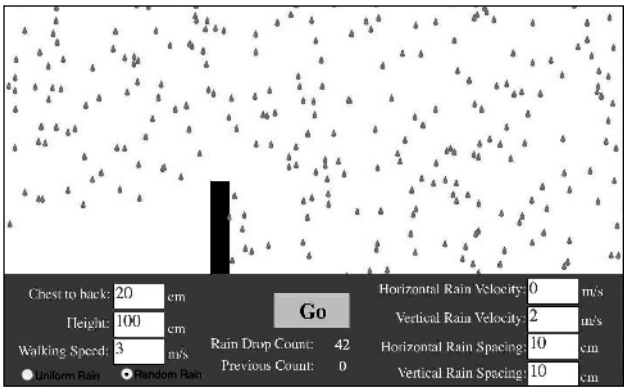
As noted, technology can enable students' engagement into a new level of creative discovery; it also plays a role in preparing students for new concepts. In a recent second year MICA lab, students were writing programs aimed at preparing them for the statement of the Central Limit Theorem. When performing sums of a strongly skewed random variable, many asked with some amazement why the curve always drifted into a symmetric shape. These same students would shortly be exposed to the full treatment of the theorem, including proof, in the statistics class. But they are already interested in the result and have experienced this wonderful phenomenon using computer programs they entirely wrote. In short, they have experienced a situation where they naturally raised the question before being shown the result. This principle is fundamental to the MICA philosophy.

On-Line MICA Student Projects

Go see original on-line first year and second year student projects (students individually choose their own topic) at www.brocku.ca/mathematics/studentprojects. Have a look and play (yes, for free!) with Lindsay's Pythagorean Concentration Game (in Exploring the Pythagorean Theorem teaching project) or empirically answer Matthew and Kylie's question "Is it better to walk or run in the rain?" (in the Running in the Rain exploratory project). Know that MICA student projects have been shown at mathematics education conferences, including the recent International Commission on Mathematical Instruction (ICMI) Study 17 Conference: Digital technologies in mathematics education—Rethinking the terrain (Buteau & Muller, 2006).



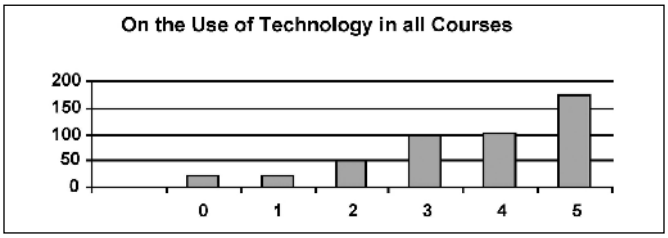
Pythagorean Concentration Game in Exploring the Pythagorean Theorem MICA teaching project created in 2004 by first year concurrent (MICA/Education) student Lindsay Claes.

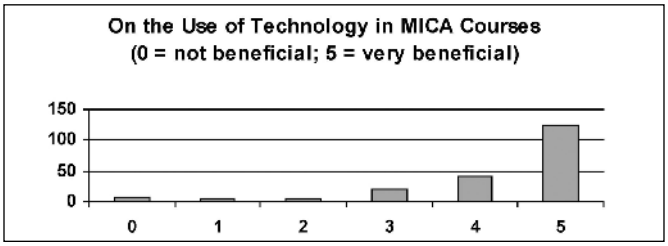


Running in the Rain MICA exploratory project created in 2005 by second-year MICA student Kylie Maheu and third year concurrent (MICA/Education) student Matthew Lillie.

But Do Students Appreciate the Use of Technology in Their Learning of Mathematics?

In a recent survey[1], students remarkably rated as positively beneficial (77.74% of responses) the use of technology in their mathematics courses (79.36% when restricted to mathematics majors), and they overwhelmingly rated the use of technology in core project-based MICA courses as beneficial (91.13% of responses).





More About MICA

MICA offers four streams (Pure Mathematics, Applied and Computational Mathematics, Mathematics Education, Statistics) providing ample opportunity to prepare for pure/applied graduate studies with a choice of more theoretical/applied courses or a teaching career[2], yet also recognizing the reality that for many, a B.Sc. in Mathematics is a terminal degree. Our graduates have been given the additional computational/modeling/simulation perspective; a clear advantage corroborated by the very positive feedback we regularly receive from supervisors and employers of our former students. Every year, a number of our students are accepted in outstanding graduate programs with funding from NSERC or OGS.

If you wish to know more about MICA program, view our recent Canadian Mathematical Society Notes article (October 2007) at **http://journals.cms.math.ca/Notes/**, or see (Ben-El-Mechaiekh et al, 2007). You may also directly contact the author.

[1] The survey is part of an internal assessment of the MICA program coinciding with the graduation of the first cohort. A total of 347 students were polled.

[2] The formation of future teachers is a cornerstone of the Department of Mathematics' mission at Brock. This is reflected by the development of numerous programs or courses for all levels of school teaching. Appropriate technologies such as Geometer's SketchPad are used in appropriate courses.

References

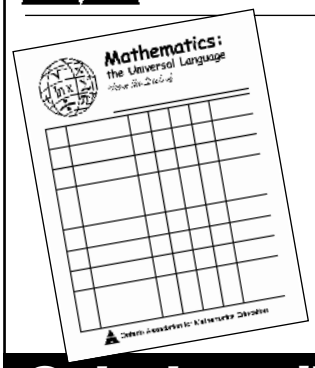
Ben-El-Mechaiekh, H., Buteau, C., Ralph, W. (2007). MICA: A Novel Direction in Undergraduate Mathematics Teaching, in *Canadian Mathematical Society Notes*, V.29 (no.6).

Brock Teaching (2001): URL : **http://www.brocku.ca /ctl/pdf/Brock_Teaching_1.pdf**

Buteau, C. & Muller, E. (2006). *Examples of undergraduate student achievement in a technological learning environment.* Project presentation at the International Commission on Mathematical Instruction (ICMI) Study 17 Conference: Digital technologies in mathematics education—Rethinking the terrain, Hanoi University of Technology, Hanoi, Vietnam.

Ralph, B, (1999): *Journey Through Calculus*, Brookes/Cole ▲

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▲ Carleton-Ottawa Mathematics Association Ontario Mathematics Olympics June 1 & 2, 2007

CO-CHAIRS OF OMO 2007
JUDY LOSCHMANN
CATHY HALL



Judy Loschmann (left) is an Intermediate teacher with the Carleton-Ottawa District School Board. She is OMO Coordinator for COMA and has chaired COMA's Regional Math Olympics for the past 6 years. This year, she served as Co-Chair of OMO 2007... and had a blast doing it!

Cathy Hall (right) is a retired teacher (Ottawa-Carleton District School Board) who is currently involved in the development and in-service of new math resources. She serves as Treasurer for COMA and is an OAME Director. According to her, Co-Chairing OMO 2007 was an incredible experience!

It was a hot night in June. The city was Ottawa. People had travelled from all over Ontario to battle for the much-coveted trophy. When we looked up into the stands, all we could see was a sea of red shirts! Was it hockey play-off time? Yes! Had the Ottawa Senators made it the playoffs? You bet! But that wasn't what was happening here. This was OMO 2007 – the Ontario Mathematics Olympics.

Thirty-one teams of "mathletes" representing all but one OAME chapter participated in the events that took place June 1st and 2nd at the University of Ottawa. Who were these mathletes? They were the top 124 Grade 7 & 8 math students from all across the province. They were selected based on their ability to think divergently, to work well in teams, and to operate under tight time constraints when problem solving. These young mathletes were models of commitment, discipline, and enthusiasm in the world of math.

The COMA chapter hosted OMO 2007. This weekend event of social interaction and math competition was organized by an enthusiastic and energetic committee of thirteen members, together with volunteers from COMA Council and St. Matthew High School students. It was a wonderful opportunity for us to work with a stellar group of individuals, all in one place for the love of MATH!

The earliest teams registered before noon on Friday, but most teams arrived between 3:00 and 5:00 p.m. Right away, team members and their supervisors picked up their well-stocked math loot bags. Items such as calculators, tangrams, caribiners, playing cards, puzzles, and Pi pins were packed in colourful pencil cases for the mathletes and plastic portfolios for the coaches and chaperones. Everyone also received a bright red OMO 2007 t-shirt and an OMO water bottle before visiting the Photo Zone to have a team picture taken by our talented photographers. After picking up their room keys, they went to check out their accommodations. Everyone stayed in the university residence, with most in 2-bedroom air-conditioned suites.

After settling into their rooms, many teams took advantage of the Meet'n'Greet and Math Trail activities. The official welcome took place at 5:30, with greetings delivered by COMA, OAME (Jacqueline Hill), and University of Ottawa (Dr. Victor LeBlanc). Following this, the participants enjoyed a tasty BBQ at the University Centre. This provided everyone with the energy needed for the Historical Ottawa Walking Tour which was very well received by teams, coaches, and chaperones. The hour and a half walk, on this hot, steamy night, gave the teams a flavour of our beautiful city. Not only did the participants have a chance to sightsee, but they learned some intriguing historical facts about Canada's capital. The "haunted story" segments of the guided walk proved to be especially popular!

When everyone returned from the walk around 9:30, they picked up a snack bag and headed up to the residence lounges for a variety of activities manned by committee members (learning magic tricks, playing cards and board games, solving "wordies", and listening to more story-telling). After all this activity, everyone was more than ready for lights-out at 11 p.m.!

The Olympic competition began on Saturday morning after breakfast. All contest events took place on the University of Ottawa campus. The morning included three different 4-person Team Events (40 minutes each). These events were run concurrently, with each team working through a 3-room rotation over the course of the morning. Each contest room was overseen by OMO

committee members (proctors, markers), as well as two student volunteers (guides, runners, and troubleshooters).

As well as the official event rooms, there was also a room designated as "Headquarters". Here, one committee member input computer scoring data, while another created an OMO 2007 video to entertain the audience while the last event was being marked and the overall results tallied. As well, everyone knew they could head to this room at any time throughout the day with any questions or concerns. The last, but not least important room, was a comfortable lounge where coaches and chaperones had the opportunity to network or just simply relax!

Following lunch, the contestants rotated through two different Pairs Events of 30 minutes each (SOMA^3, Let's Network). The mathletes found these hands-on and puzzle events to be a welcome break, as many of them told us "their brains were full" after a very challenging morning of Pythagorean Patterns, Math Rally, and The Amazing Race. Throughout the day's events, the mathletes exhibited exemplary team work and co-operation!

While the final scores were being tallied, the teams participated in a probability game ("Skunk") with over 200 people playing at one time! Following this, there was time for our OMO 2007 video and lots of great door prizes for mathletes, coaches, and chaperones. Once the event marking and tallying was done, it was time for the awards ceremony.

To recognize each and every one of the mathletes as winners, they were given a commemorative OMO 2007 dog tag. Medallions were awarded to the top team in each of the 5 Olympic activities. OMO 2007 "Olympic" medals, along with a trophy stand and a gift bag, were awarded to each mathlete in the top three teams overall:

1st Place – ISOMA (Team 8)

2nd Place – PRMA (Team 17)

3rd Place – ISOMA (Team 9)

In our opinion, OMO 2007 was a great success! It meant a lot to us, our first time organizing this competition, to receive words of thanks and appreciation – both during the event and afterwards, in the form of emails from coaches. It was especially rewarding to see the positive attitude and measure of enthusiasm exhibited by the OMO mathletes as they rose to the challenges of the competition. These students are truly our future math leaders!

Reflecting on it now, as we sit collaborating on this article, we can still feel the energy and remember THE FUN! Thanks, everyone, for all your encouragement and support!

So, if you haven't already gotten your teams up and running for your chapter playdowns, it's not too late. Check out the OMO link on the OAME website for a look at the highlights and activities from this year's competition. Then start preparing your teams for what promises to be a jam-packed weekend at OMO 2008 -- being hosted by PRMA at Trent University in Peterborough. Good luck, everyone! ▲



Problem solvers at work... four brains are better than one when trying to unravel Pythagorean Patterns!



Setting off on a Historical/Haunted Walk of Ottawa



Math is COOL!!!

▲ Vancouver 2010 Educational Web Site

The following news special came to me in September 2007 and I thought it worth sharing with you. Log onto the web site, choose the Culture and Education tab. It seems the web site is under construction and there is not much there...yet, Just keep this in mind for the coming few years. - Marilyn

Vancouver 2010 to connect with students and teachers across Canada through an online education portal: vancouver2010.com/edu

VICTORIA, BC, Sept. 25 /CNW/ - Students and teachers in classrooms across Canada now have a direct link to Vancouver 2010, the positive influence of the Olympic and Paralympic Movements and the three pillars of sport, culture and sustainability through an online, bilingual e-magazine ('e-zine') and portal located at vancouver2010.com/edu. This new portal on the Internet, known as /EDU, serves as the foundation for what is expected to become a lively online interactive community - one that will connect teachers, students (from kindergarten through grade 12) and schools, while celebrating the spirit of the 2010 Winter Games.

In its initial stages, the /EDU portal at **vancouver2010.com/edu** will target Canadian schools, teachers and students looking to build content and maximize opportunities to showcase local and national education resources. In the coming weeks, a teachers' forum moderated by the University of British Columbia will come online at /EDU. Also among its many offerings, /EDU provides links to programs affiliated with non-profit organizations including the Canadian Olympic Committee (COC) and the Canadian Paralympic Committee (CPC), as well as Canadian museums, environmental groups and arts and culture organizations.

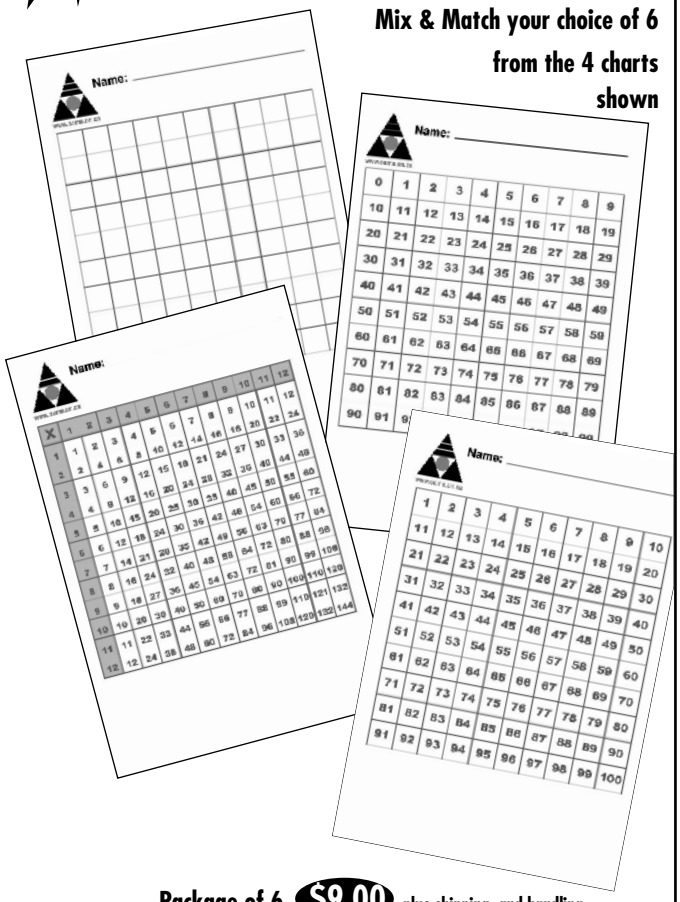
Education programs are integral to the mandate of the Vancouver Organizing Committee for the 2010 Olympic and Paralympic Winter Games (VANOC). With the support of the BC Ministry of Education, VANOC is working with representatives from each of the 13 ministries responsible for education in Canada. Each ministry has agreed to work with VANOC to inform and involve teachers in the program; identify the best education resources in sport, culture and sustainability; and collaborate on the development of new educational resources as opportunities arise.

"Vancouver 2010 creates a climate of possibility that helps us discover the greatness inside ourselves," said John Furlong, VANOC Chief Executive Officer. "Through the Vancouver 2010 website, /EDU will connect students to students, teachers to teachers and schools to schools from across Canada. It will provide them with Olympic and Paralympic resources to help them set their own podiums and identify their own dreams." ▲

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▲ A Shape Quest Through Story

CHANTEL MULDER AND GLADYS STERENBERG



Chantel Mulder is an education student at the University of Lethbridge. She has a passion for high school mathematics and will pursue a lifetime of learning through teaching mathematics.



Gladys Sterenberg is an assistant professor in the Faculty of Education at the University of Lethbridge and the Editor of delta-k. She is interested in narrative ways of knowing mathematics and is currently researching Aboriginal stories of mathematics.

This article first appeared in delta-k, the Journal of the Mathematics Council of the Alberta Teacher's Association, in the June 2007 (Vol 44#2) issue and is reprinted with permission.

Once Upon a Time

The popularity of teaching with picture books in elementary mathematics classrooms is increasing as teachers make connections between literature and mathematics. Since picture books can be read in one sitting, they can offer a springboard into mathematical inquiry. Textbook series and teaching resources provide many pedagogical ideas for integrating picture books into mathematics classrooms and some teachers are introducing picture books into their classroom in an attempt to garner interest, increase motivation and improve mathematical sense-making. The visual nature of these books has aesthetic appeal for many students.

This article explores how students in a Grade 1 classroom interacted with the picture book *The Greedy Triangle*, written by Marilyn Burns (1994). As teachers, we were interested in how the story could be used to investigate different polygons, how the students would engage in the story and the colourful illustrations, and how related tasks could enhance mathematical understanding for students. Chantel developed a lesson plan and Gladys expanded on this work when she taught the lesson to a class of Grade 1 students as a visiting professor. What follows is a summary of the book, a description of the tasks offered to students and a reflection on the mathematical learning of the students as they engaged in the tasks.

The Storyline

The Greedy Triangle, written by Marilyn Burns and illustrated by Gordon Silveria, is about a triangle that is unsatisfied with its shape and tired of having only three sides. Initially, the triangle goes to a shapeshifter to be changed into a quadrilateral. Then the triangle asks the local shapeshifter to add more sides and angles until it doesn't know which side is up. After many trips to the shapeshifter, the triangle learns that being a triangle is the best after all.

Storytelling

We started by considering the general and specific outcomes listed in the Alberta Program of Studies. The ones we chose to work on focused on exploring and classifying circles, triangles, and rectangles according to their properties. Since we wanted to emphasize process skills of communication and reasoning, we decided to design student tasks in relation to the book *The Greedy Triangle*.

Gladys began the lesson began by playing a game. Students were asked to guess an answer to the statement, "I spy with my little eye something that is a triangle." This sparked a rich discussion on the attributes of classroom objects. The first suggestion was made by a student who tried to form a triangle with his fingers. I asked him to explain why he thought this shape was a triangle. Another pointed to designs on the carpet. Yet another identified the wings of a butterfly in a poster. At each suggestion, Gladys asked students to explain why they thought the shapes were triangles. When one student mentioned that the points on the maple leaf on the Canadian flag could make a triangle, other students were quick to dispute this conjecture. This led to much discussion about the characteristics of triangles. This initial task seemed to provide students with an opportunity to identify and describe triangles in their classroom environment.

During this game, Gladys was sitting with her hand on her hip, thus forming a triangle with her arm. None of the students guessed her answer and when she offered it to them, they recognized the shape, and many tried to form a triangle using their arms, legs, and fingers. This activity helped prepared them for the description in the book of the triangle's favourite thing to do. As a triangle, it could "slip into place when people put their hands on their hips."

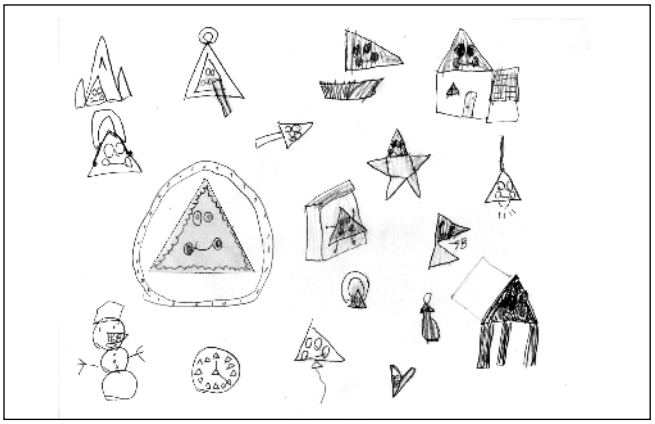
After this game, Gladys read the story with the students. The illustrations were instrumental in fostering conversations about the shapes and their attributes. Students were asked to predict what shape the triangle might change into. Much discussion occurred when new terms were introduced. The students especially enjoyed the section on the quadrilateral's experiences and were quick to point out many examples of quadrilaterals around the classroom. Students remained motivated to interact with the story throughout the entire reading of it.

Following the story, students were given a sheet of paper with a triangle glued to it. These triangles were arranged in a variety of orientations. Chantel had noticed that many students thought about shapes in a particular way. For example, when asked to draw a triangle, students often crafted an equilateral triangle with a horizontal base. A misconception can occur when students see shapes "on their side" or "upside down" and believe that these shapes are somehow "wrong." Chantel wanted the students to draw on their experiences of reading the book and realize that if they saw a triangle "upside down," it was not incorrect and, indeed, it was the same object. Students were instructed to draw a picture incorporating the shape and use as many triangles, quadrilaterals, and other shapes as possible in their drawings.

So the Story Goes

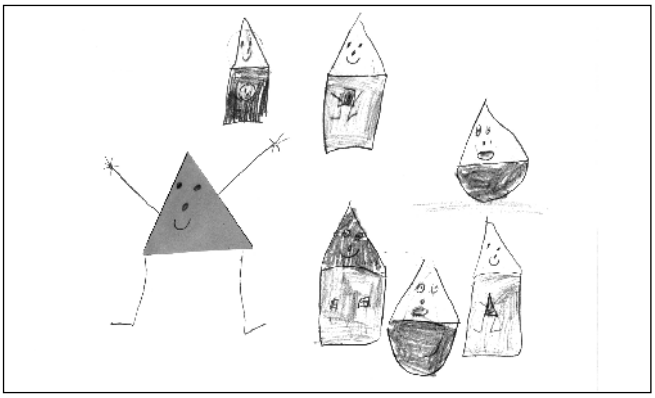
Students were quite engaged in this task and created interesting drawings. However, the most important part of this task was how students talked about their shapes. Gladys was able to ask students to classify their shapes and explain their thinking. Again, this fostered rich conversations and allowed Gladys to assess student understanding of the attributes of the shapes.

Jared was very articulate about his drawing:



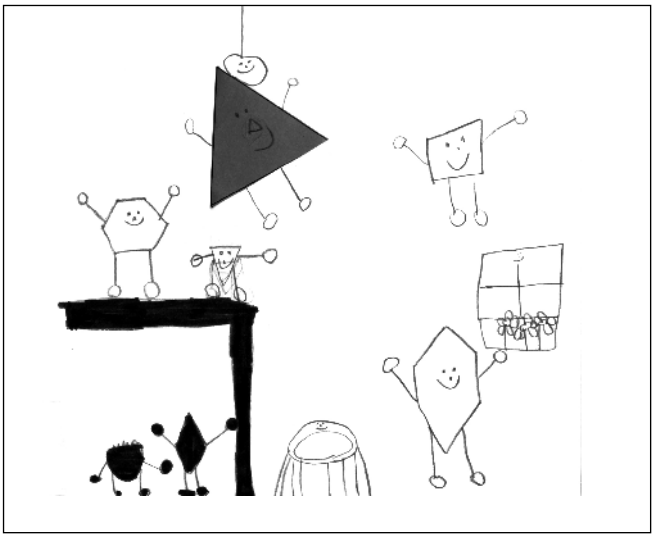
He talked about where he saw triangles in places outside of the classroom and was particularly proud of showing Gladys that shapes can exist inside other shapes. He was able to tell her why the triangles that looked different were still called triangles. While Gladys missed it at the time, she could have asked him about the 3D box in the centre of the picture. This might have extended his understanding about the difference between 2D shapes and 3D objects.

Shondra emphasized that she was drawing friends for her triangle. She wanted these friends to be different so she made them with triangular faces but added quadrilaterals and "partial" circles to her shapes.



Gladys talked with her about semi-circles and squares and rectangles, and Shondra was beginning to make mathematical distinctions between these shapes.

Andrew wanted to include as many shapes as he could in his picture. He especially liked hexagons and enjoyed talking about how he was making the sides join together so that they were "even." He described the L shape as a half a square, and we talked a bit about what he was seeing. His drawing showed his experimentation with drawing the shapes, but this was a mere artifact of a much richer conversation about shapes.

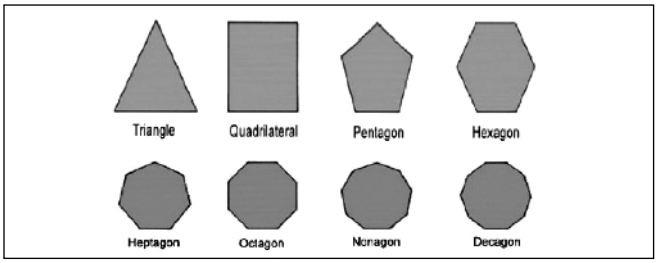


As students completed their drawings, they were offered an opportunity to choose another sheet with a shape and draw a picture using only this particular shape. Chantel wanted students to be able to identify hexagons, pentagons, and rectangles in their environment and distinguish these from triangles. This was quite effective.

This lesson was extremely successful in motivating students to talk and think like mathematicians. During a 45-minute period, these students remained captivated by the tasks and the story. Their excitement was contagious.

As we reflect on this lesson, we believe that follow-up lessons might be fruitful. These lessons could include

- using geoboards to experiment with different shapes,
- examining the different polygons presented in the book in three dimensions,
- making a collage of shapes found in magazines,
- identifying shapes at home by completing an activity with a parent (eg, have a worksheet with the following shapes on it and ask parents to help their child find the shapes in their home and draw them) or



- integrating the story with other content areas such as health (eg, extending this assignment to teach our students about self-acceptance and how important it is for students to accept themselves and others for their uniqueness).

For us, the most exciting parts of this lesson were the conversations that the students engaged in. The Greedy Triangle provided a context for students to see geometric shapes all around the world and to investigate the concept of angles and sides. The story was especially helpful in connecting geometric shapes to shapes found in the students' environment. The integration of literature encouraged students to share their understandings of mathematical concepts. Student engagement and learning were enhanced.

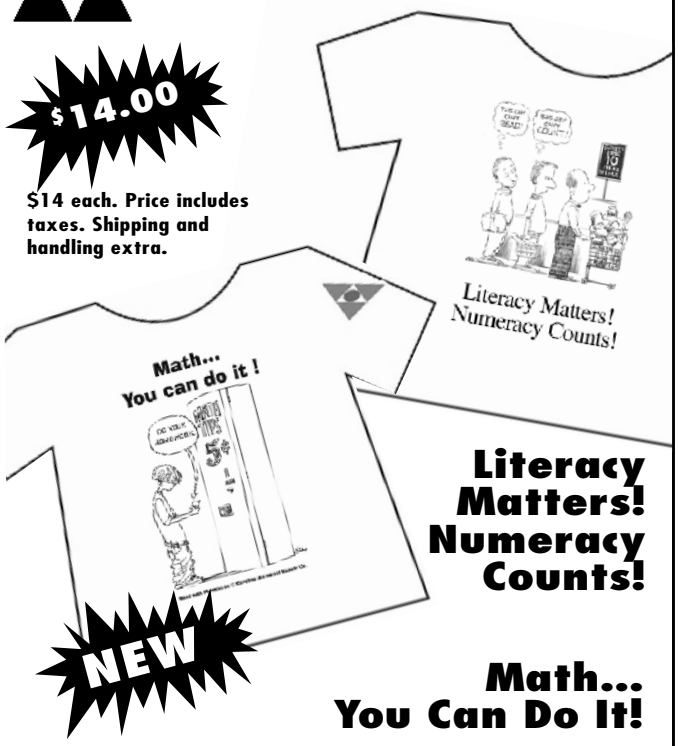
References

Burns, M. 1994. *The Greedy Triangle*. New York: Scholastic. ▲

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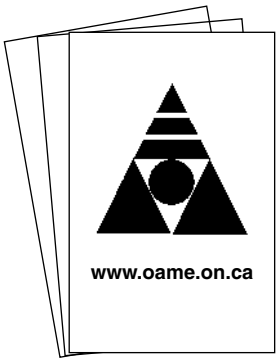
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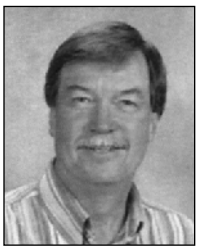
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▲ Financial Literacy and the Rule of 72 (Some Intriguing Applications, Grades 10-12)

JOHN KITNEY



John Kitney recently retired from Bayridge Secondary School in Kingston Ontario and was Head of Mathematics. John is committed to working with teachers to integrate financial literacy in the classroom.

The integration of financial literacy with the elementary and secondary curricula is a matter that deserves greater attention. The lives of our students will be increasingly affected by the monetary framework on which our society is built. Their successes, which include financial independence, will be determined to a considerable extent by their degree of fiscal understanding. My personal interest in this subject was piqued 6 or 7 years ago with the opportunity to participate in a summer program involving business as well as fellow mathematics teachers. The focus of the in-service was a resource called "Taking Stock In Your Future". At that time the sponsoring parties included the Investor Education Fund (Ontario Securities Commission), OISE/UT and the Toronto Stock Exchange. In this article I propose to share some applications that relate to this free resource which provides meaningful starting points for an understanding of compound interest and building personal wealth.

The most instructive part of my experience with the "Taking Stock …" program has been the opportunity to work with similarly motivated business teachers and share our two perspectives on topics in financial literacy. I found myself eager to understand the various ratios and indicators used to assess investment types and relative performance, along with their curious names. I also found that, under the financial literacy umbrella, there was a great deal of mathematics that was easily accessible to students as early as the junior and intermediate levels. To this end the Investor Education Fund continues to support the development of resources for use in the classroom as well as background and reference material for teachers. The website: **www.investorED.ca** is an excellent place to start. "Teacher's Corner" is a link to updates on the Fund's latest classroom initiatives and a range of supplementary activities. Workshops are available, at no cost, to introduce teachers to the Investor Education mandate with any of the three main resources:

1) Taking Stock In Your Future: Intermediate
2) Taking Stock In Your Future: Senior
3) Fair Play

One of many ideas that connects the business and mathematics curricula has the intriguing title of "Rule of 72". As with many other concepts that we present to our students from different perspectives, here is one of those ideas that I had not encountered as a math teacher, in such simplicity. At the same time, I found that many business teachers had either forgotten or rarely taken the chance to fully investigate the mathematical implications or limitations of this little gem. The accessibility of the graphing calculator (TI-83 series or equivalent) in our high schools and the emphasis on a functions approach to mathematics in the Ontario secondary curriculum provide an ideal framework for a discussion of this relationship.

The Rule of 72

A single payment compound interest investment will double its initial value if the product of the interest rate and the number of conversion periods is approximately 72 (using the interest rate as the percentage for each conversion period). That is to say, $1000 invested at 8% compounded annually will become about $2000 in 9 years. If this interest rate is changed to 12% then the "doubling time" is about 6 years. This immediately begs the question of whether or not we can determine the exact time required to precisely double the principal of a compound interest investment. In the familiar formula, $A = P(1 + i)^n$, this means we want to calculate the value of the exponent, n, when $(1 + i)^n$ is equal to 2. Even without an understanding of logarithms our grade 10 students can solve for the desired exponent by using the intersection of two functions. The doubling time at 12% p.a. compounded annually is the value of x that satisfies both $y = 1.12^x$ and $y = 2$ at the same time. A graphing calculator allows an easy determination of the point of intersection by entering the functions under [Y =] then selecting "5:intersect" in the menu from [2nd] CALC (found on the [TRACE] key). Repeating this process for a range of interest rates we can closely examine the Rule of 72. The chart below shows six such "intersections". It was set up using domain [–1, 20] and range [0, 5] and

was inspired by Exploratory Activity B (Activity 1.6) in the Taking Stock In Your Future: Senior[1]. The first calculator display (Fig. 1) shows six exponential functions along with the line y = 2. The second display shows the table of values for $y = 1.04^x$ (Fig 2).

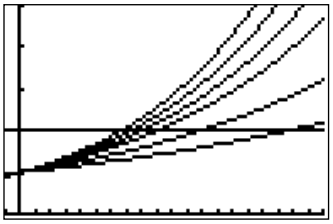


Fig 1

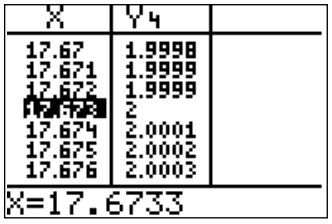


Fig 2

i %	x	Doubling time (n)	i x n
4	17.6730	17.67	70.7
5	14.2067	14.01	70.1
6	11.8956	11.90	71.4
7	10.2448	10.24	71.7
8	9.0065	9.01	72.1
9	8.0432	8.04	72.4
10	7.2725	7.27	72.7
11	6.6419	6.64	73.0
12	6.1163	6.12	73.2

Chart 1 Is i x n always equal to 72?

We can conclude, from the chart, that the Rule of 72 works "best" when the rate of interest is close to 8%. Of course, there is a certain amount of variation in the product i x n (using interest rates from 4% to 12%) but none-the-less it is a "good" rule.

At this point the grade 11 student might be interested in examining the amount of variation in the final amount (A) of an investment if the Rule of 72 was strictly applied. The following chart uses the compound interest formula, $A = P(1 + i)^n$, \$1000 for the principal, P, various rates of interest, i, and values of n such that i x n = 72.

Chart 2 Examples of "doubling" when i x n = 72

i %	n	(1+ i)^n *	\$ A
4	18	$(1.04)^{18}$	2 025.82
5	14.4	$(1.08)^{14.4}$	2 018.95
6	12	$(1.06)^{12}$	2 012.20
7	10.29	$(1.07)^{10.29}$	2 006.13
8	9	$(1.08)^{9}$	1 999.00
9	8	$(1.09)^{8}$	1 992.56
10	7.2	$(1.10)^{7.2}$	1 986.22
11	6.55	$(1.11)^{6.55}$	1 980.91
12	6	$(1.12)^{6}$	1 973.82

[1] Kelly et al, Taking Stock In Your Future: Senior, Investor Education Fund, 2002, (page 38).

*Students who are interested in the programming option can create a short algorithm on the TI-83 calculator that will input the interest rate, i , and provide the value A(\$).

Once again the Rule of 72 has optimum values between 7 and 8% and there is also some variation over the range of percentages that we have chosen to examine (a variation of 1.3% at our extremes of 4 and 12 percent).

A more sophisticated treatment of this topic will fit very nicely with the recently revised curriculum for grade 12 Advanced Functions (MHF 4U) as it provides an excellent application of logarithms. This involves a comparison of two functions, each one related to a chart of data that we have investigated above. The functions define doubling time (n) in each of the two ways that we have discussed. The first can be seen to be in the family of rectangular hyperbolas where n is a function of i and n x i = 72. In other words, . The second function will produce the exact values of doubling time, n, from a ratio of logarithms, also dependent on i. We are interested in a comparison of the shape of these functions, where exactly they might intersect, and how quickly they deviate from each other over the domain of input, i.

Students in the new MHF course will learn the following manipulations in their unit on logarithms.

If $a = b^y$ then $\log y = \log_b a$, but we can also observe that $a = \log b^y$, $a = y \log b$, so $y = \frac{\log a}{\log b}$ and this is considerably more "calculator ready" than using a = by.

We can now develop the log function that relates to the doubling time of our investment at various interest rates. We want to examine when $2 = (1 + i)^n$ or $n = \log 2 \div \log (1 + i)$. Recall that i is the percentage rate as a decimal. With the syntax of the graphing calculator we will use the [Y =] feature to input the following:

$$Y1 = \frac{\log(2)}{\log\left(1+\frac{x}{100}\right)}$$

as well as the function that we are comparing it to,

$$Y2 = \frac{72}{x}$$ (see footnote[2])

Using a domain ("window") of x values [0, 20] and a range [0, 40] the graphs are only barely distinguishable (Fig. 3 & 4). On the other hand, we know that they are not identical because of our previous investigations (chart 1). Using the [ZOOM] menu (2: Zoom In) two or three times with the cursor (+) located near the value

[2] x represents the interest rate as the percentage value, for both functions: 4 for 4%, 5 for 5% etc.

x = 8 (for 8%) on the curve(s), we can quickly begin to see the variation between the functions.

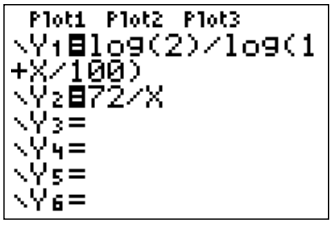


Figure 3

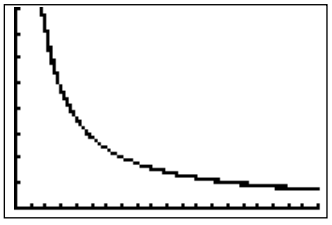


Figure 4

Another idea would be to graph the "difference function" which is very conveniently generated using Y3 = Y1 – Y2. This requires the use of [VARS] followed by the [Y-VARS] key, 1:Function, etc. Using domain [0, 20] and range [-1, 1] to get started, we can see where the difference between the Y1 and Y2 functions is minimal (values for Y3 near zero). The x-value of the point of intersection of the two original functions is the "zero" of the difference function, Y3 (Fig. 5 & 6). This value, x = 7.8468715, can be determined with the [2nd] CALC feature (using the [TRACE] key) and selecting "2: zero" from the subsequent menu. The point of interest on the parent functions (Y1 and Y2) is approximately (7.85, 9.12).

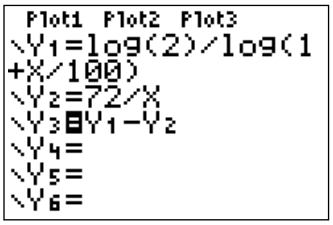


Figure 5

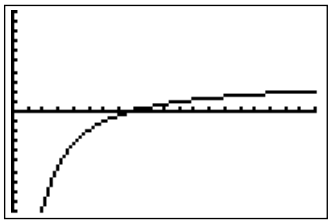


Figure 6

So we have now determined that the Rule of 72 works beautifully at 7.85% and surprising well in the neighbourhood, we can see yet again that it is a pretty good approximation over a reasonable domain of commonly used rates.

If we return to our grade 11 investigation (chart 2) where the investment was $1000, we can check the rate of 7.8469% p.a. with conversion periods, n = 72 ÷ 7.8469 = 9.1756. This represents an annual compounding of interest for just for just over 9 years.

$$A = 1000(1.078469)^{9.1756}$$

$$= 2.000.0001$$

while, of course, 7.8469 x 9.1756 = 72.0000.

The mathematics of investment affords many opportunities for applying high school math skills of patterning, formula manipulation and graphical analysis. In this discussion we have examined one instance where a simple memory aid from the business sphere (the Rule of 72) provides an opportunity to apply the skills of mathematical modelling. The use of graphing technology provides easy access to the visual displays of the respective functions and makes some theoretical concepts very real.

The Taking Stock In Your Future guide is a very helpful starting point for investigations of this type. It provides a range of activities that allow discussion and analysis at various entry levels. In "Part 1: Building Personal Wealth as a Component of the Curriculum" teachers can find a careful review of concepts as well as an introduction to the language of investment for those who are less familiar.

These resources (along with workshops to interested groups) are available free of charge from the Investor Education Fund **www.investorED.ca**. ▲



▲ An Averaging Anomaly: Increasing Means Without Increasing Data Scores

BONNIE H. LITWILLER AND DAVID R. DUNCAN
PROFESSORS OF MATHEMATICS
UNIVERSITY OF NORTHERN IOWA



*Bonnie Litwiller and David Duncan teach mathematics and mathematics education courses at the University of Northern Iowa. They have published over 850 articles in journals for mathematics teachers and have spoken at numerous professional meetings. Each has served as President of the Iowa Council of Teachers of Mathematics and Dr. Litwiller has served on the boards of NCTM, SSMA and AMTE.*Puzzles involving numerical relationships have often been used to motivate mathematics students. It is an especially serendipitous occurrence if the puzzle involves a logical "twist". We shall present one such example.Mathematics teachers are always looking for settings in which basic statistical measures can be computed and analyzed. We will present an anomalous situation involving arithmetic means.

Suppose that there are two apartment complexes, A and B. At a given time, the mean age of the Complex A residents is greater than the mean age of the Complex B residents. Then a single residency change occurred; a single member of A moves into B. When mean ages are re-computed for this new grouping, it is found that both means (for A and for B) have increased from their earlier values, even though no individual ages had changed during this short time span.

How could this anomalous result have occurred? Suppose that initially Complex A has 20 residents with a mean age of 41.0; Complex B has 12 residents with a mean age of 32.0. Suppose further that the resident to be transplanted is 35 years old.

Recall that Mean = Sum of Data / Size of Set, and equivalently, Sum = (Mean)(Size). Thus, before the single resident's move,

Sum (Complex A) = (41)(20) = 820 and

Sum (Complex B) = (32)(12) = 384

After the move:

$$\text{Mean (Complex A)} = \frac{820-35}{20-1} = \frac{785}{19} = 41.3$$

$$\text{Mean (Complex B)} = \frac{384+35}{12+1} = \frac{419}{13} = 32.2$$

Although the mean increases are small, both did in fact occur. Note that if the 35-year-old had instead moved from B to A, both means would have decreased.

The readers and their students are encouraged to find other settings in which a switch of a single element from one set to another cause the means of both sets to increase or to decrease. ▲

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▲ Directors' Dialogue

ANN MICHELE STENNING



Ann Michele Stenning is a mathematics teacher at South Secondary School with the Thames Valley District School Board. Having finished her Masters of Education degree in Curriculum Studies, she is currently assisting other parent volunteers develop weekly literacy activities which she and other moms provide to "at risk" students.

This edition of Director's Dialogue highlights the events of the OAME Board of Director's meeting at the Holiday Inn Select, Toronto Airport on Friday, October 12 and Saturday, October 13, 2007.

Chapter Rep Meeting, October 12, 2007

- Greg Clarke demonstrated Smart Board technology and how it can be used with a laptop.
- Reps accessed virtual manipulatives via the National Library of Virtual Manipulatives
- Fred Ferneyhough introduced Mimio as an alternative to Smart Board.
- Lynda and Fed Ferneyhough used TI-Nspire for Grade 9 Applied to Grade 12 Calculus and in courses from Grade 9 Applied to Grade 12 Calculus & Vectors.

(Minutes of the full meeting are available to members on the OAME web site).

Chapter Reports

CHAMP - Rep: Lynda Ferneyhough:

We are a hosting Calculus: The Musical! at Fletcher's Meadow S.S. on Dec. 11th at 4:30. For those who missed it at the OAME banquet last year, this is a golden opportunity to enjoy math set to music, and math history presented in comedy sketches.

COMA - Rep: Pat Kehoe:

The COMA community suffered a huge loss this spring with the passing of our president Ron Bender. Ron was a committed educator and advocate for mathematics teachers. He was the first recipient of the Prime Minister's Award for Teaching Excellence and was actively involved throughout his career in our association

GOLDEN - Rep: Laura Tonin:

Our committee has been keeping busy by starting to plan our spring mini-conference. Our theme this year is Math: Just for the Fun of it. It will either be at the end of April or beginning of May at Brock University. We are also in the beginning stages of planning for OAME 2010.

GVMA - Rep: Rob Gleeson:

We have a new T-Shirt out entitled 'The Math Reference Shirt'. It comes in many colours and sizes. Check out HYPERLINK "http://www.gvma.ca" www.gvma.ca for details.

ISOMA - Rep: Diane Lang:

We wish to thank the OMO organizers for a really fun time in Ottawa. Word is getting out - schools in our chapter are already asking how they can participate in the Math Olympics event for 2008. Charlotte Danard from Branksome Hall School was awarded one of three 2007 René Descartes Medals from the University of Waterloo's René Descartes Foundation for the Advancement of Mathematics.

MAC² - Rep: Danielle Blair:

Our fall conference titled "Differentiated Instruction in Mathematics: How it Can Really Happen", featured Dr. Marian Small. Teachers and a small handful of parents were provided with practical examples from K - 10. MAC² is beginning work on a one hour "roadshow" parent presentation that will be offered to interested parent councils/groups through local school boards. Further plans also include a spring mini-conference.

NOMA - Rep: Sue Melville:

Future efforts will be devoted to attracting new NOMA members as we have lost three very experienced NOMA executive committee members in one year due to retirement, a change in job and family responsibilities.

NWOAME - Rep: Tom Boland:

After a bit of a summer break, NWOAME has resumed regular meetings and has planned a mini-conference in Thunder Bay for Feb 28 through March 1, 2008. We are in the process of inviting speakers and presenters and are very pleased to have Jacqueline Hill as our keynote speaker. Other business matters on the go include Math Olympic preparations and membership recruitment.

PRMA - Rep: Sandra Jean Price:

On October 2nd, PRMA hosted a Fall Social appropriately named "Laughs and Suds"

featuring Don Fraser as our speaker. Teachers were given a chance to eat, mingle, shop at several publisher displays, and solve a variety of problems. Our chapter is looking forward to hosting OMO 2008 at Trent University on May 30th and 31st.

QSLMA - Rep: Robert Sherk:

The QSLMA executive held our fall mini-conference on November 10th at Queen's Faculty of Education in Kingston. We changed to a Saturday morning event with two workshops, lunch and keynote. As well, our membership secretary, Amy Baker, represented and promoted OAME at a local conference for elementary teacher candidates.

SAME - Rep: Debbie Drape:

Our main goal for the 2007-2008 school year is to increase membership. Our mini-conference is being planned for February 15th, a PA day for area schools Hopefully this will attract more people and increase membership. Discussions about our local math Olympics have already begun. Teams of writers for the questions have been established and various sites for play-downs are being considered.

SWOAME - Rep: Joe Goulet:

We are planning a mini SWOAME conference which will include a joint Mathematics and Guidance Presentation/Working session. The goal is to increase awareness of the various pathways available to students.

TEAMS - Rep: Laurissa Werhun:

TEAMS has decided to sponsor one member to attend the OAME Leadership Conference in February and one member to attend the OAME Conference in May. As well, TEAMS has already started planning for the TEAMS Math Olympiad to be held Saturday April 12, 2008 at the Toronto Catholic Education Centre and the executive has reviewed organizing OMO 2011. Other efforts are focused on communicating more information to our members and a web newsletter has been established.

WOMA - Rep: Ann Michele Stenning:

A mini conference will occur at Sherwood Fox elementary school in London in late November or early December. .

Y^{4}MA - Rep: Ada Cartaginese:

Y^{4}MA is very busy planning OAME 2008: The Path is Made by Walking. It will be held on May 8-10, 2008 at the Sheraton Parkway Toronto North in Richmond Hill. To find more information about the conference visit **http://oame.on.ca/OAME2008**. ▲

▲ Nominations for OAME/AOEM Board of Directors 2008-2009

Each year, elections are held for a President-Elect, a Vice President (3 year term), and 6 to 9 Directors (3 year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either grade JK–6 or grade 7–12 issues based on their personal experience, expertise and interests. Nominations for these positions are to be submitted by 4:00 p.m., January 18, 2008, to:

Dan Charbonneau, Past President

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2008, the Nomination Committee will publish a slate of candidates, including qualifications, for consideration by the membership. Voting will be by mail, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Toronto, May 2008.

NOMINATION FORM

I nominate:_____

For the position(s) of: ☐ President-Elect ☐ Vice-President ☐ Director

If for Director, area of personal experience/expertise, interest : ☐ grades JK–6 ☐ grades 7–12

Nominee's Address:_____

_____

Telephone: H ( ) _____ W ( ) _____

Nominator's Name :_____

Nominator's Address: _____

Telephone: H ( ) _____ W ( ) _____

2007-2008 OAME Board

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QSLMA - **Robert Sherk**

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WOMA - **Ann Michele Stenning**

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