Submission of Articles

The *Ontario Mathematics Gazette (OMG)* is looking for articles, news and "good ideas" useful to mathematics teachers or mathematics teacher education. We are seeking submissions from mathematics teachers K-12 and tertiary, that describe teaching mathematics content in an innovative way.

The criteria for manuscript submission to the *OMG* includes the following:

- The ideas/activities are of interest to the readership.
- The article/ideas are not widely known or practised and are innovative.
- The mathematics content is appropriate for the readership.
- The mathematics content is accurate.
- The article is well written and grammatically correct.
- The article/ideas are free of sexual, ethnic, racial and other bias.
- The article has not been published previously, nor is out for review by other publications.

Articles must be original. Your article must be word processed, preferably using *MS WORD*, double spaced on one side with wide margins, not exceeding 10 numbered pages of text and prepared according to the *Publication Manual of the American Psychological Association Fifth Edition*. Diagrams and figures should be drawn by computer if possible or neatly drawn in black ink in camera-ready form. Embedded images should also be submitted separately in jpeg or tif format. Proof of the photographers' permission is required. For photos of students, parent or guardian permission is required. Please number pages.

You must submit **4 hard copies** of your article to the Editor **Marilyn Hurrell** whose address appears on page 2. Your name may not appear anywhere in your article, including web sites. Your name and full mailing address and e-mail address must be included on a separate sheet. Your article will be sent out for blind review. If it is accepted as is, or with major/minor revisions or rejected, you will be notified. Once an article is accepted for publication, computer files of text, original graphics and photos will be required. E-mail submissions are welcome but must be followed by hard copies.

The Editor reserves the right to edit manuscripts before they are published. Once an article is published it becomes the property of OAME.

PERMISSION TO REPRINT: Permission to reprint any part of this publication for instructional use or for inclusion in an affiliate publication must be obtained through the Editor Marilyn Hurrell. Full credit must be given to the author and to the *Ontario Mathematics Gazette.*

▲ Table of Contents

Up Front

▲ Editor's Message / Marilyn Hurrell 2
▲ President's Message / Anna Jupp 3
▲ From the Editor's Desk / Marilyn Hurrell 4
▲ Letters to the Editor 5

Regular Features

▲ The *Abacus* . . . . . .8 page insert
▲ Technology Corner: Gapminder / Mary Bourassa 6
▲ Hey, It's Elementary! RSVP... You're Invited to Learn About Family Math and to Come to Queens / Lynda Colgan 8
▲ OAME/NCTM Report / John Kersley 11
▲ Get Interactive! Sharing Internet Videos / Louis Lim 13
▲ What's the Problem? Alphametrics / Shawn Godin 14
▲ CAS Corner: User Defined Functions on a CAS Device / Fred Ferneyhough 15
▲ Photo Math – A New Way of Picturing Math / Ron Lancaster 18
▲ Formative Assessment: Questioning, Listening and Responding / Chris Suurtamm, Adrian Jones 20
▲ Director's Dialogue / Ann Michele Stenning 38

Special Features

▲ Book Review: *Big Ideas for Growing Mathematicians* by Ann Kajander / Reviewed By Laurie Moher 17
▲ OAME Leadership Conference 2008 / Sue Hessey, Executive Director 22
▲ College Mathematics Project 2007 / Graham Orpwood, Margaret Sinclair, Laurel Schollen . . .

Articles

▲ Establishing Families of Planar Figures Through the Principle of Cavalieri / Jerome Proulx 25
▲ Using E-STAT to Prepare for the Teachable Moment: Teacher Candidates Demonstrate How Easy It Is. / Doug Franks, Mike McCabe, Barbara Olmsted 30
▲ Mathematical Mini Me's: What Would Dr. Evil Think? / Maria Casasola 35

Notices

▲ OAME Memo Pads 5
▲ OAME T-shirts 6
▲ OAME OMCA Free Resources 8
▲ OAME Playing Cards 12
▲ OAME Elementary Sticky Chart Pads 17
▲ OAME Growing Up Mathematically (GUM) 24
▲ OAME Sticky Graph Pads 30
▲ 2007-2008 OAME Board . . . . . .inside back cover

Advertisements

▲ Pearson Education . . . . . .Inside Front Cover
▲ Wintergreen 12
▲ Thomson Nelson Learning . . . . . Back Cover

The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

▲ About the *Ontario Mathematics Gazette*

The *Ontario Mathematics Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

Editor

▲ **Marilyn Hurrell**

Associate Editors

▲ **John Egsgard**

▲ **Lynda Ferneyhough**

***Abacus* Co-editors**

▲ **Mary Lou Kestell**

▲ **Kath Kubota-Zarivnij**

Design and Production

▲ **Penny Clemens**
Graphic Designer

Printing & Binding

▲ **Pole Printing,** Box 69, 89 King Street East
Forest, ON N0N 1J0 (519) 786-5112

Advertising Manager

▲ **Robert Sherk**

Submission of Advertisements

Advertisements for the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising Rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARILYN HURRELL, EDITOR



This issue of the *OMG* marks the end of my Editorship of the *OMG*. Five years ago, I took over from Louis Lim and followed the successful formula established by him and Jack Weiner and Todd Romiens and John Egsgard and..... many Editors before them. That was to gather articles, columns and other special features which would be of interest and useful to the teachers of Ontario, and to present them in a quality journal. Twenty issues later, I believe I have achieved that goal.

Many others joined me on that journey and I thank them all, from the production team to the reviewers and especially to the authors.

This will also be the final issue for Fred Ferneyhough, who said he was leaving two years ago, but then came back to write the "CAS Corner". Fred will now pursue other retirement projects. Also, this is Louis Lim's last column for "Get Interactive", as he assumes a new editorial role with NCTM. John Kersley will write one more NCTM column in September, before turning this task over to our new NCTM representative Laurie Moher.

It is my pleasure to introduce the new Editor of the *OMG*, Stewart Craven. He will take over for the September 2008 issue. Stewart began teaching History and Mathematics in Toronto in 1986. He rose through the ranks to become a Consultant in Mathematics, and then Program Coordinator, Mathematics and Numeracy for the TDSB from 1999 to 2007. Presently he is associated with the Science Centre in Toronto.

He has been involved in several organizations including the Fields Institute, the Ontario Mathematics Coordinator Association (OMCA), both the CMS and the CMESG, and of course OAME. Stewart was president of OAME (2003-4) when I began my editorship and I enjoyed many meetings with him at the helm. He helped design and coordinate the production of the GUM (Growing Up Mathematically) project and resulting CD and DVD. He has been a speaker and Keynote at many conferences across Canada and the USA. He spent time at TVO and was a writer and host for *The Virtual Classroom* 1998 - 1999.

I am sure that you, as well as I, will look forward to the September 2008 *OMG*, and the new look of it under Stewart and his editorial team.

All the best for a safe, warm and relaxing summer break. It has been a pleasure to serve you.

Marilyn

▲ President's Message

ANNA JUPP, PRESIDENT



Anna has been working as an Instructional Leader of Mathematics with the Toronto District School Board for the past 10 years. She supports K-8 teachers though the development and facilitation of professional learning opportunities, resource development, and consultation. Anna has contributed to various Ministry initiatives including the Expert Panel Reports for the Early Math Strategy and Education for All. Anna is also a mathematics AQ course instructor with OISE and has been part of an author team for primary math programs . She has served as a Board Director since 2001 and 3 years as Vice President before becoming President. Anna was the recipient of the 2005 Union Gas Award for Outstanding Leadership in Mathematics.

Connecting the dots. Who would have thought that it was so important?

How far we've come. A look at the list

The last several years have been very eventful for mathematics education in Ontario. As mathematicians, educators, parents, life-long learners, we have experienced significant changes in what and how mathematics is taught and learned. Ministry-led reviews have brought about a more robust program and higher expectations for Kindergarten students. A revised Grade 1- 8 curriculum has put an emphasis on the processes through which students acquire and apply the big ideas of mathematics. OAME played an integral role in the discussions and debates that led to revised curricula, new courses, and adjusted pathways in the secondary panel. Faculties of education have adapted programs so that pre-service teachers are better equipped with both the content and pedagogy necessary to meet the needs of our students. Through the efforts of the Ministry of Education, the Literacy/Numeracy Secretariat, OAME, OMCA, teacher federations, and school boards across the province, Ontario teachers have come to experience exceptional professional learning opportunities led by exemplary colleagues as well as some of the most noteworthy researchers and educators from around the world. More change has come with a surge in the use of technology both as a vehicle for teaching mathematics and as a learning tool. In recent years, mathematics as a disciple has captured the attention of decision makers and has begun to receive the financial support required to impose and sustain change and growth in our schools.

What's the connection to connecting the dots?

We are at a point on this journey of mathematics education reform where analysis and reflection are essential for further progress. So where lies the connection between continued success in mathematics education in Ontario and the process of relationship building and collaboration?

An analysis of what appears to be a simple activity of joining a series of dots reveals something more complex. From the onset, the connecting task requires the skills of prediction and visualization. Knowing how to proceed to link the dots demands the identification of patterns. Other higher order thinking skills such as application, analysis, synthesis, and evaluation are all called upon at different points of the process. The connection of all the dots reveals the final image.

Replace these dots now with the conditions, participants, stakeholders, patterns, cause and effect relationships, and decisions that are all the artifacts of mathematics education in Ontario to date and the relationship is clear. The connection between each of these variables is vital in order to see the big picture. The study of the successes and errors of the past reveal for me the importance of making connections and all the skills required in the process. The maintenance of current relationships and the establishment of new connections will support the sustenance of future growth and higher achievement for our students in mathematics.

What are these connections about and how do they slowly reveal the big picture?

Educator-to-Educator Connections

Never has there been a better time for educators who work with students at different stages of the continuum of learning, to come together. Discussions about what important mathematics students should be learning at different stages and grades, as well as student needs, have an enormous, positive impact on programs and resources. Teachers benefit from sharing proven learning and instructional strategies used in different panels and divisions that improve student understanding and achievement. OAME through the collective work of its members provides an ideal forum for such work.

Subject-to-Subject Connections

Furthering cross-curricular connections will not only allow teachers to take the principles and strategies that work in other subject areas and apply them to the teaching of mathematics, but will also help students make their own connections. The OAME vision statement affirms that "Learners are better able to transfer and to use knowledge meaningfully when new ideas are presented within rich and meaningful learning contexts, and when learners make connections both within and outside of school".

Stakeholder-to-Stakeholder connections

The OAME vision statement also identifies the importance of building relationships amongst different groups. "The learning of mathematics is supported through the active and ongoing involvement of students, parents, educators, and the community at large. Learning is enhanced when all participants in the process are active partners within a community of learners (e.g., parents, educators, students, and the community)".

We need to continue to work alongside school and board administrators, parents, taxpayers, the Ministry of Education, and teacher federations to establish common beliefs, attitudes and goals for mathematics education in Ontario.

Over the next year, I plan to engage in the process of connecting the dots so that I may identify the success and challenges of our efforts to improve mathematics education in Ontario. In my future President's messages I will explore and share some strategies for furthering, sustaining, and building new and old connections. As president of OAME, I am honoured to have the opportunity to serve and support you and other mathematics educators across the province and look forward to connecting with you throughout the course of my term.

Anna ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

▲ From the Editor's Desk

During my five years as editor of the *OMG*, I have been fortunate to develop working relationships with editors of many other mathematics journals. One of these is Gene Potter, the Editor of *Confluence*, the journal of the Mathematics Educators of Greater St Louis (Missouri), or MEGSL.

MEGSL promotes Pi Day as one of their themes. I was invited to their NCTM Regional Conference in 2006, called Pi-Paloosa. I mentioned in the March 2008 issue of the *OMG* their web site, devoted to Pi-Day activities. I was delighted when Gene sent the following email. to describe what occurred in St Louis last March 13. May this be an inspiration to all of you. Thanks Gene.

"Hi Marilyn:

I spent Pi Day at the St. Louis Science Center, from 9:30A to 4:30P, assisting little guys with their Pi Day headbands - circumference v. hat size (diameter), adding links to the Pi Chain (the digits of pi are endless), helping dads helping their middle school student and/or high school student solving the best-buy pizza problem, or the "How Many Cheese Balls are in Here" problem, or "How Many Cans of Soda are in Here (large cyclinder)" problem. I should have documented the different approaches. Engineers had the most difficulties, I think. A very, very young guy (5 years old maybe) looked at the pizza problem and said "just count the pepperoni pieces" which works just fine because they are about evenly spaced. With the soda problem almost no one considers scaling as a possible strategy.

What is pi? Or, you might need to explain "the number that represents the ratio of circumference to diameter" by crawling on the floor mapping out the diameter and marking it off on the circle. Then, there was the Tower of Hanoi puzzle which is one of the permanent displays. And, at 1:59 pm on Pi Day, the Pi Moment, alarms go off, we have the Moon Pie drop from 17 meters up and parachute Baby Einstein down to the crowd (it's Albert's B-day also). Then there was the Pi Ball Drop emphasizing surface area and volume of a sphere using pi. The theme was "Pi is Here."

Pat Kennedy, Pi Day Chair for MEGSL, organized and coordinated efforts between MEGSL and the SLSC, and enlisted people to donate Pi Words and charted the letter-length. Pat also managed the Pi Chain and helped the very young color large pi symbosl and post them around the halls. Several kids participated in the Pi

Treasure Hunt which Pat also encouraged. Pat helped with almost everything. Carol and Barb dedicated their skilled efforts helping kids make Pi Day bracelets. Everyone got a Pi Day sticker with the symbol pi and circled with many of the digits of pi.

Cathy, we ran off your Pi Day Sudoku sheets and many took them home to work.

We took a break to have lunch with the SLSC staff - "all things circular or cylindrical". Wonderful and yummy. SLSC Gift shop was selling Pi Day t-shirts and caps. After a wonderful day of meeting people from several countries, Carol, Barb and I needed dinner. Where to go? Let's go to Pietro's on the Hill to have a little picatta with a glass or two of Pinot Gregio.

During the day, a gentleman from Russia said that they celebrate Archimedes Day on the last Sunday in April and Pi Approximation Day on July 22. Another gentleman shared the idea that in China 355 divided by 113 was used for pi and Chuang-Chih was the famous mathematician that first computed it around 200 AD. A young lady from Lafayette HS said she could recite 150 digits of pi. She recited about sixty and then was interrupted by her friends. On rare moments, I would comment on the magical force of pi in that after 359 digits of pi, one finds 360, the number of degrees in a circle. I also heard several stories about people holding Pi Day celebrations at Penn State, Washington, California, Maine, Kentucky, etc.

All in all, I think that MEGSL/SLSC helped to create a positive attitude towards mathematics. It all starts with a positive attitude.

Best regards, Gene Potter"
pottergene@msn.com ▲



▲ Letters to the Editor

Dear Editor,

I received my copy of the March 2008 *Gazette* last week and I wanted to thank you for dealing with the errors that crept into my December 2007 article titled "Financial Literacy and the Rule of 72: (Some Intriguing Applications Grades 10 - 12)".

I appreciated your recognition of the problem in your March editorial as well as the complete reprinting of my submission, as it was meant to appear.

Thanks again for your thoroughness,

John Kitney ▲

Dear Editor,

In the March 2008 Issue of the *Gazette* are two articles which I would like to comment on.

The first is the one on Financial Literacy by John Kitney.

I agree with him that our more academic students are badly served by the curriculum, in that they receive little, if any education on the concepts needed to invest knowledgeably. I have always taken the time to teach the ideas associated with compound interest, and especially the advantages of investing in a tax sheltered environment such as RRSP's from an early age. I still take a lesson a year in the Math Challenge @ Western program to explain this topic. I think that the absence of a unit on this topic in the Mathematics curriculum for the university-bound students is reprehensible, and should be corrected.

The other article which caught my attention was that concerning the introduction of Trigonometric functions. For a long time many of us have been using the definition of trigonometric functions as a point on the unit circle. The point P is $(\cos\theta, \sin\theta)$ on the circle $x^2 + y^2 = 1$, with θ as the angle between the x-axis and the line OP. This immediately gives the information stated in the article and the identity $\cos^2\theta + \sin^2\theta = 1$. It also leads to all the trigonometric functions, and as a special case by considering the first quadrant only, the study and solution of triangles. Trigonometry is no longer used primarily for the solution of triangles, navigation and surveying as in the 19th century. Our students need a more modern definition of the trigonometric functions today, and I find it amazing that the Ontario grade 10 mathematics curriculum specifies such an outdated introduction to the

subject. It took me many years to understand that there are not two trigonometries, one for solving triangles and one for functions, and we are still perpetuating this anachronism in our curriculum.

I highly recommend the book *Trigonometric Delights* by Maor Eli. Princeton: ISBN 0-691-05754-0.

Yours Sincerely,
Tom Griffiths
tgriffit@uwo.ca ▲

OAME T-SHIRTS

I Love Math

Available in M, L, XL 4-colour design in white only.





NEW

Math will rock your world!

Available in M, L, XL red & white design in black only.

Literacy Matters! Numeracy Counts!

Available in M, L, XL in white only.



$15.00

$15 each. Price includes taxes. Shipping and handling extra.

Order by mail, fax or on-line:

OAME
or Order ON-LINE www.oame.on.ca
Do not send cash or cheque. You will be invoiced with shipping and handling costs added.

▲ Technology Corner: Gapminder

MARY BOURASSA



Mary Bourassa teaches mathematics at Lisgar Collegiate Institute in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She was also past-president of COMA, gives workshops around the world and is a National Instructor for Texas Instruments.

A friend of mine told me about a web site (**http://www.gapminder.org/** - see Figure 1) that has lots of data presented in a very cool way. I was fascinated by what I saw and upon sharing it with other math teachers locally had such a good response that I thought it was worthy of a wider audience.



Figure 1

Quoting from the web site: "Gapminder is a non-profit venture promoting sustainable global development and achievement of the United Nations Millennium Development Goals by increased use and understanding of statistics and other information about social, economic and environmental development at local, national and global levels." Along with having excellent Gapcasts and Video Lectures on topics that are pertinent to our everyday life, there is Gapminder 2006 which is powered by Trendalyzer . These graphs are what made me go "Wow!". Let's take a look.

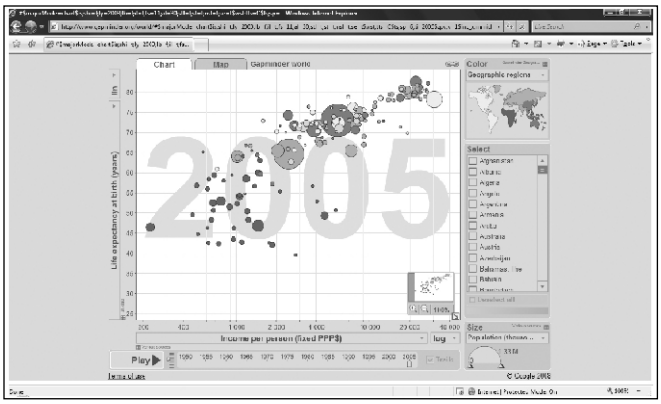


Figure 2

The graph shown (Figure 2) has colours representing different geographic regions: America (yellow), Europe & Central Asia (orange), Sub-Saharan Africa (blue), Middle East & North Africa (green) and East Asia & Pacific (red). The size of each circle is relative to that country's population so China has the largest circle. As you scroll your mouse over each data point, the country's name appears, the data values are shown on the axes and its population is displayed in the lower right of the window. This graph shows Life expectancy at birth against Income per person. The fairly large dot representing the United States (relatively large population) is to the far right showing a high income per person and it is near the top because there is a high life expectancy. Most data is available for 1950 – 2005. Observe the difference between the graph from 2005 and the one from 1950 (Figure 3).

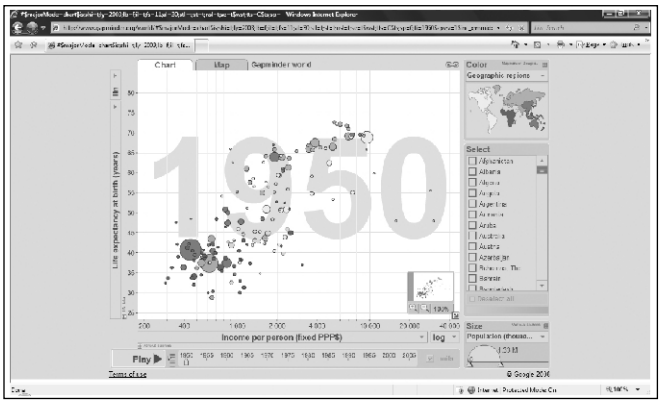


Figure 3

You might notice a "Play" button below the graph. This allows you to animate across the time axis from the year you have currently selected to the year 2005 (or the year with the latest data). Seeing the data move is quite powerful. You can also select one or more countries from the list on the right so that those points are labelled and the rest of the data becomes shaded (Figure 4). If you animate across time with countries selected you can also show the data trail.

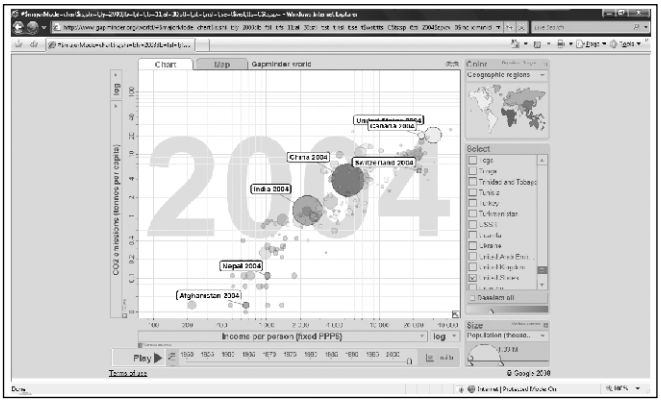


Figure 4

The options for each axis are: Income per person, Infant mortality rate, Children per woman, Life expectancy at birth, CO_2 emissions, and several subcategories in each of Population, Health, Economy, Environment, Technology and Education. The horizontal axis also has Time as an option. The variable controlling the size of the dot can be changed so different ways of looking at data truly abound.

This is also a great application of linear and logarithmic scales. The following graph (Figure 5) shows Mobile phone users against Income per person. The number of mobile phone users per 100 people can easily be shown using a linear scale. Luxembourg topped out at 106 mobile phone for every 100 people (in 2002 – imagine what it is today!) with Hong Kong close behind at 94 almost confirming my observation that everyone in Hong Kong has a cell phone!

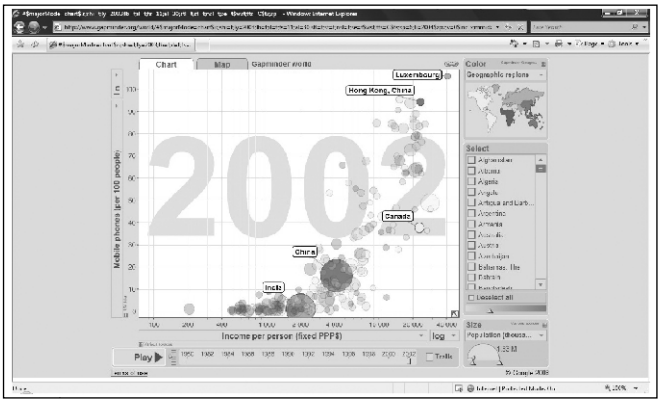


Figure 5

However, if we graph population with a linear scale (Figure 6), we do not get much from this graph other than the fact that China and India have much more people than all other countries. Canada is just one of a jumble of dots near the bottom of the graph. In fact,

Canada is located at roughly the same height as Monaco even though Canada's population is more than 1000 times that of Monaco!

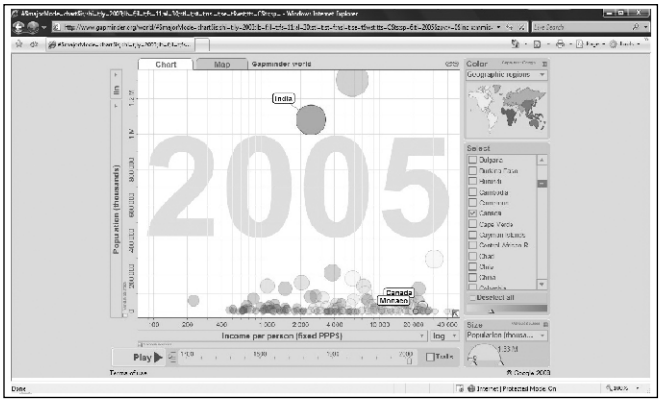


Figure 6

If we change the scale to logarithmic (Figure 7), we get a much better sense of the relative populations while still being able to see India and China.

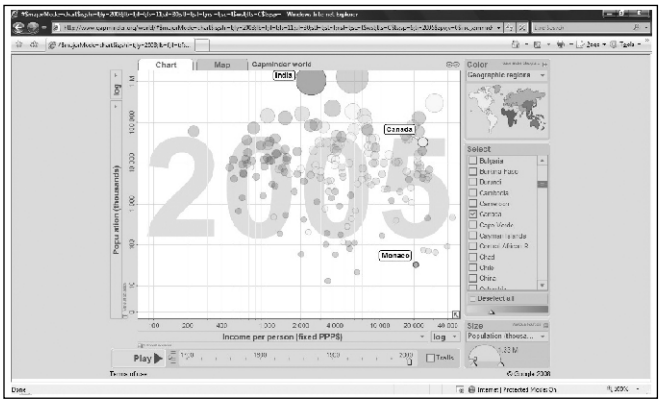


Figure 7

Every time I "play" with the data provided in these graphs, I learn something new. The applications are not just for a Data Management class but run the gamut of grades and subjects. Enjoy exploring with this incredible tool! ▲

NEW FREE RESOURCES AVAILABLE

For all grade 12 Mathematics courses

Visit www.oame.on.ca for resources developed by OAME and OMCA

funded by the Ministry of Education.
More than 100 Ontario Educators participated in this project.
More lessons and assessments have been added for Semester 2. Check for updates regularly.

▲ Hey, It's Elementary! RSVP... You're Invited to Learn About Family Math and to Come to Queens



LYNDA COLGAN

There is a beautiful and gentle folk song by James Taylor that begins with the line, "whenever I see your smiling face, I have to smile myself". Taylor's tune played on a continuous loop in my mind and I found myself smiling giddily last Thursday evening and well into Friday as a direct result of seeing all the smiling faces at Park Dale Public School in Belleville taking part in the school's first annual FAMILY MATH night on March 27th.



Jackie (left) and Janaki (right) with 2 students from Park Dale

Two elementary teacher candidates from the Queen's Faculty of Education, Jackie McLaren and Janaki Nair invited me to this wonderful community event because they had played a major role in planning the many mathematics activities and games that parents and children were playing together in the hallways, library and gymnasium of the school.

On arrival, I was greeted by two Grade 6 students who invited me to guess the number of jelly beans that filled a jar. Then they gave me a mathematics scavenger hunt form and directed me to the snack table where I could choose among 1/2 bagel bites, 1/4 banana slices, 1/8 orange segments or other fractional delights to provide a healthy energy boost before hitting the mathematics tasks.

Everywhere I looked, families were engaged in meaningful mathematics and they were having fun using simple, (mostly) everyday materials to solve problems and tangle with thinking challenges. Toothpicks, dominos, dice, playing cards, measuring cups, and colourful geometric puzzle pieces were central to the activities.

As I walked past one mat in the gym, I heard a young voice call out excitedly, "Look, I made a polyhedron." She

then engaged me in conversation for several minutes about the number of faces, edges, and vertices in this particular 3-dimensional shape.



Top left: Michelle and Noah. Top right: Paul and Kathleen. Bottom Left: Mrs. Lee (Park Dale teacher) Brooke and Kelly (mother). Bottom right: Alex and Julie.



Chelsea

Her enthusiasm, pride and knowledge was obvious. As I walked away, eager to see all of the stations in action, I said, "Maybe one day you could be an engineer." She scrutinized her polyhedron and all of the building blocks around her and confidently replied, "Yes. I could."

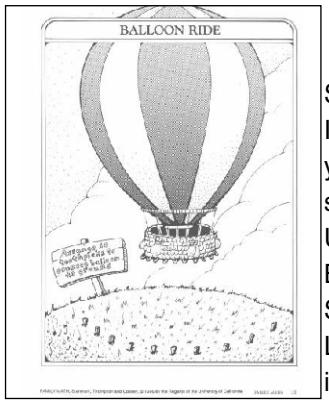
That response brought another smile and a flood of warm memories. I first learned about FAMILY MATH when I was a young teacher with the Scarborough Board of Education. One of the board mathematics consultants, Shirley Fairfield, had attended a week-long FAMILY MATH "train the trainer" program at the Lawrence Hall of Science in Berkeley, California and came back to Scarborough determined to spread the word about this wonderful initiative. I was lucky enough to take Shirley's course to learn the skills and acquire the resources to lead my own FAMILY MATH workshops. "Balloon Ride" (shown below) continues to be one of my favourite strategy games. Though Shirley herself is long retired, her legacy lives on in the many Scarborough schools and community organizations who continue to offer imaginative FAMILY MATH programs in a variety of formats to bring mathematics into family recreation and learning time.


Balloon Ride

Why

How

More Ideas





From LHS web site.

Following in Shirley's footsteps, I have over the years attended several courses at the University of California at Berkeley's Lawrence Hall of Science (LHS). On one hand, LHS is an amazing and invitational education resource centre for preschool through high school science and mathematics teachers. On the other hand, LHS is a public science center that offers exciting hands-on experiences for learners of all ages. This museum and teaching centre was established in 1968 to honour Ernest O. Lawrence, the university's first Nobel laureate. Since then, the Lawrence Hall of Science has become famous because of its exemplary and far-reaching outreach programs, including FAMILY MATH.

The mission statement for the Lawrence Hall of Science is simple: to inspire and foster learning of science and mathematics for all. To achieve its mission, faculty from the mathematics and science departments in collaboration with mathematics and science educators work to advance interest, skills, and knowledge by engaging students and their families in exciting science and mathematics at LHS as well as in their classrooms, schools, and communities; designing and disseminating effective instructional materials and exhibits; and, creating innovative learning and leadership opportunities for educators.

It was to achieve this mission that the FAMILY MATH program was born at LHS. FAMILY MATH is built on the

assumption that everyone can learn and enjoy mathematics. FAMILY MATH honours and celebrates the fact that parents and other family members are their children's first and most influential teachers and strives to generate programs and resources to help parents feel as comfortable with their 1,2,3's as they are with their A, B, C's so that they are positive role models. Thus, FAMILY MATH nights are intended to be challenging and engaging learning experiences for adults and children alike and participants can expect to encounter problems that are loosely connected to the school curriculum, including algebra, probability, statistics, estimation, logic, geometry, and measurement.

In many cities around the world, FAMILY MATH programs are proudly passing along the LHS torch, organizing programs for families (usually an hour-and-a-half to two hours per session, once a week for four to six weeks) in order to reduce mathematics anxiety and increase mathematical literacy across generations.

One of the most successful FAMILY MATH initiatives in Ontario is the partnership between the Faculty of Education at the University of Western Ontario and Literacy Link South Central. In libraries and community centres from St. Thomas and Woodstock to Tillsonburg and Ingersoll, FAMILY MATH programs are thriving.




From University of Western Ontario Family Math site

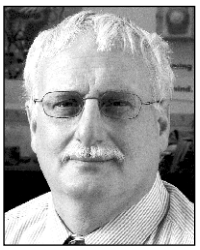
Supported by funding from the Imperial Oil Charitable Foundation, CitiFinancial, Texas Instruments and Investing in Children, two coordinators help committed volunteers (including teachers, parents, university students, retired persons, and other community members) from local towns to implement FAMILY MATH programs in keeping with the spirit of the original vision of the LHS.

What does a typical FAMILY MATH program look like? The same group of families meet together for a six week course. "Welcome" activities are posted around the room so that families can engage with math as soon as they arrive: graphing height vs. arm span, reading children's literature books that have a math message, or playing collaborative games with blocks. When every family has arrived, the evening officially begins with a simple community meal so that all instructors, volunteers and families can meet, mingle and talk. After dinner, the families participate in collaborative problem-solving activities, rotating through a number of "stations" each of which has its own focus: a math problem, book, song, or picture chosen to meet the grade and comfort level of children and parents. At the end of the evening, families are given ideas and suggestions for supporting and extending the math concepts at home using everyday objects such as newspapers, buttons, tape measures, calculators, bottle caps or coins. On the last evening, there is a celebration. Families participate in more fun activities that culminate in a graduation ceremony, complete with certificates of achievement. The best part is that the program is free (or very low cost) for families so that no one is excluded because they cannot afford the course.

The impact of well-organized FAMILY MATH programs is powerful, as this parent attests: "I have struggled with math my entire life. I fell behind as a young child and never did catch up. Now my children are suffering the same fate. I was convinced that I could never learn math, and it took all I had to attend this class. The most important thing I learned [during the FAMILY MATH program] is that I'm not stupid or math disabled in some way. I just needed to learn a different way than other people. And my kids are the same way. Now I can work with them and help spare them the trauma I had to face as someone who was perceived as dumb at math. I want the best for my kids, and I know that mastering math is a big part of school and job success. This class really changed how we look at math in our family".

I'd like James Taylor's "smiling faces" to play forever in my mind in response to happy and poignant reactions to mathematical epiphanies. Margaret Mead once said, "Never doubt that a small group of thoughtful, committed citizens can change the world. Indeed, it is the only thing that ever has". If you are a teacher, parent council member, mathematics-passionate university student or community member, or church outreach or literacy volunteer who is interested in learning more about FAMILY MATH so that you can offer a program in your neighbourhood, I am willing to run a short, six-session course over the summer: just in time for a fall implementation. There will be no charge for the course other than the cost of materials (approximately $50.00) and times and dates will be negotiated based on response. If you are interested, please contact me at **Lynda.Colgan@queensu.ca**. Mathematician Siméon Poisson once said, "Life is good for only two things, discovering mathematics and teaching mathematics". Come to the course and find out for yourself why I think he is right. ▲

▲ OAME/NCTM Report JUNE 2008



JOHN KERSLEY

For the year 2007-2008 NCTM's Professional Development focus is data analysis and probability. NCTM's title is "Becoming Certain about Uncertainty: Data Analysis and Probability". The rational behind this chose of a focus is that since our world has access to increasing amounts of information, there is an important need of knowing how to deal with the information. Thus, data analysis and probability have become more important in our mathematics curriculum. The intent of this focus is to help mathematical educators understand what data analysis and probability is all about, including collecting, organizing, displaying data, using appropriate statistical analytic tools and developing/evaluating inferences and predictions and the understanding and using basic probability concepts. In all grade levels students' learning must include hands on experience with data ranging from concrete to abstract forms. In addition, data analysis and probability stretches across and connects to all parts of the mathematical curriculum: number, measurement, algebra and geometry.

In its support of this focus of the year NCTM offers a variety of resources. The following is a short list of them. A complete list can be found at **www.nctm.org** and click on elementary, middle or high school resources at the bottom right of the page under "Focus on Data/ Probability".

For Elementary Grades

- ***Look at Me Lessons*** engage students in the exploration of logical and numerical relationships while collecting and displaying data.
- ***Can It Be? Lesson*** focuses on connections between mathematics and children's literature.
- ***Developing Data-Graph Comprehension in Grades K-8, Second Edition***
  This publication includes background information leading students to make connections between data and graphs.

For Middle Grades

- ***Sticks and Stones Lesson*** allows students to investigate probability via collection of data, examination of likelihood of various moves, and use of basic ideas of expected value to determine the average number of moves needed to win a game.
- ***Harry Potter and the Magic of Mathematics*** The article illustrates how teachers can use the Harry Potter book series to teach linear modeling and probability to their students.
- ***Navigating through Data Analysis in Grades 6-8***
  This publication illustrates the general notion of statistics as a process, while prompting discussions of increasingly complex mathematical issues. It provides ways to extend and deepen students' knowledge of data analysis, including comparison of data sets with equal and unequal numbers of elements and analysis of data involving two variables.

For High School

- ***Real-Life Traffic Data: A Vehicle for Statistical Exploration*** This article examines the mathematics that can result from having students gather and examine real-time data. It also presents ideas for research projects that use real-time data.
- ***Explorations with Chance Lesson*** promotes the analysis of fairness of certain games by examining the probability of outcomes. Students are given opportunities to predict results, play games, and calculate probabilities.
- ***Navigating through Probability in Grades 9-12***
  This publication provides strategies for introducing students to two different approaches to probability, within real-world contexts. Interpretation of probability as relative frequency over the long run and analyzing outcomes abstractly allows students to arrive at empirical and theoretical probabilities.

Web Sites of Interest to Mathematics as Suggested by NCTM

- **www.exemplars.com/materails/samples**
  Free K-12 mathematics tasks. Each task comes with a rubric and samples of students work
- **www.onlinemathcourses.org** online mathematics education courses at the graduate level designed for mathematics teachers
- **www.igetitmath.com** Math coach chronicle is a free bimonthly electronic newspaper linking mathematics teachers from K – 8. Issues deal with

specific math topics and includes activities, games and practical classroom strategies

- **www.spacetime.us/arcade** free mathematics games and puzzles. Users can play games or download. Geared to elementary students.
- **serc.carleton.edu/sp/library/datasim/index.html** teaching with data simulations. Site helps students visualize statistical concepts and see processes behind the gathering, analysis and interpretation of statistics.

NCTM Professional Development Opportunities

Events are available at **www.nctm.org/profdev** This site also includes resources, (books, online and articles) for the focus of the year 2007 – 2008: 'Becoming Certain about Uncertainty: Data Analysis and Probability'.

A complete list of workshops on line can be found at **www.nctm.org/workshops.aspx**

NCTM Conferences www.nctm.org/meetings

- NCTM Annual Meeting and Exposition
  Washington, D.C. April 22 – 25, 2009
- NCTM Regional Conferences and Exposition for 2008
  Oklahoma City, Oklahoma October 2, 3, 2008
  Cleveland, Ohio October 16, 17, 2008
  Reno, Nevada November 6,7 2008

Affiliate Conferences

A list of affiliate conferences can be found at **www.nctm.org/affiliates/conferences.htm** ▲

CUSTOM DESIGNED PLAYING CARDS

- large 7" x 4.5" deck of 52 cards
- plus 4 extra cards with card tricks and math games on them for K-12 use in classrooms.
- 58 cards will be enclosed in a "tuck" box.



$12 per deck plus shipping and handling.

Order by mail, fax or on-line:

OAME
or Order ON-LINE www.oame.on.ca
Do not send cash or cheque. You will be invoiced with shipping and handling costs added.

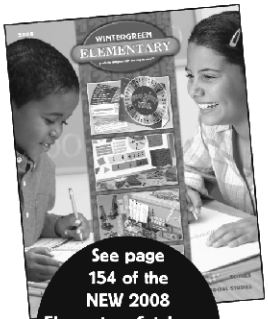
WINTERGREEN helps students visualize math concepts!

WINTERGREEN ELEMENTARY
products designed with learning in mind™

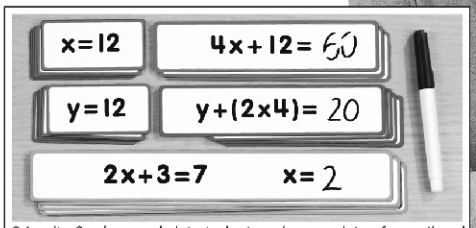
Grade Level 4 5 6

EQUATION SOLVER ACTIVITY CENTRE

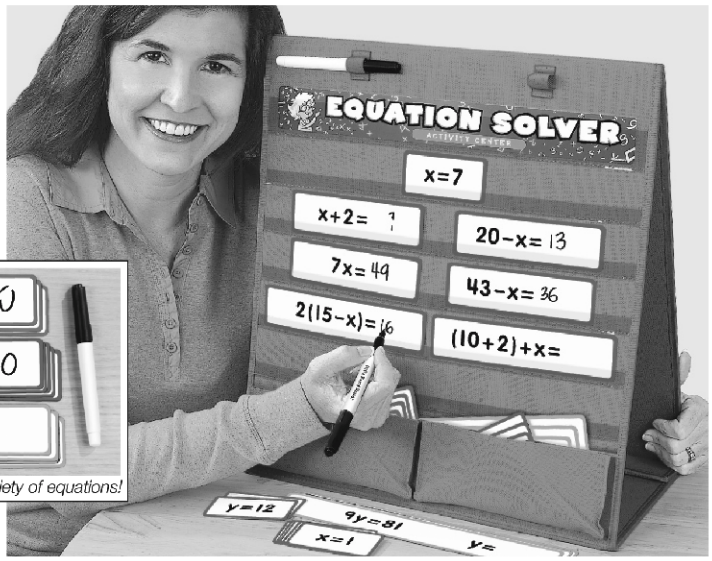
Introduce algebraic thinking—with our ready-to-use centre that makes challenging concepts a breeze to understand! Simply slide a variable card into the top of our tent-based easel and add write & wipe equation cards…then students solve each problem by plugging in the variable! The colour-coded cards let you create a new set of problems every time, and the answers are right on back—so the centre is perfect for independent practice *or* teacher-guided lessons. With 84 cards, 2 markers and a freestanding, 16½" x 18" easel.



See page 154 of the NEW 2008 Elementary Catalogue for details. Call for your FREE catalogue today!



84 write & wipe cards let students solve a variety of equations!



Targets standards in these areas:
- Using algebraic thinking to solve problems
- Solving for variables
- Solving equations

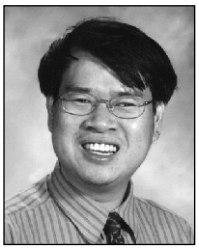
Telephone FREE: 1-800-268-1268

Online: www.WINTERGREEN.ca

Fax FREE: 1-800-567-8054

▲ Get Interactive! Sharing Internet Videos

Louis Lim



Louis Lim is department head of mathematics at Richmond Hill High School, York Region DSB, and Additional Qualifications instructor with Queen's University and York University. Louis' involvement with the OAME includes former editor of the Ontario Mathematics Gazette, presenter at chapter and annual conferences; and elected board of director. In spring 2005, he was recipient of the OAME's "Exceptional and Creative Teaching in Secondary Mathematics" Award.

June 2008 marks my final issue as columnist of "Get Interactive!", having written the column for the past 2 years. I am stepping down as I have been appointed editor of the "Media Clips" department of the National Council of Teachers of Mathematics' *Mathematics Teacher* journal. Coincidently, I resigned as OAME *Gazette* editor back in 2003 to represent Canadian mathematics educators on the *Mathematics Teacher's* editorial panel, a 3-year term which I immensely enjoyed. I have tough shoes to fill as "Media Clips", like "Mathematical Lens" are both very popular departments of the NCTM journal and are the brainchild of my good friend, colleague, and mentor, Ron Lancaster.

1.) Mathemagician Art Benjamin



One of the most dynamic, mind boggling, and humorous presentations I have attended in my 12 years of teaching, was at the Texas Instruments International T^3 conference during last year's March Break in Chicago. Dr. Art Benjamin, mathematics professor at Harvey Mudd College in Claremont, California, combines his love of mathematics and magic. He performs all the computations in his head, faster than pushing the buttons on a calculator! Unlike a magician, Benjamin explains the calculations in his head (e.g., how to square a 2 digit number; calculate the day of the week you were born). I retrieved a similar presentation from the Internet, **http://www.ted.com/talks/view/id/199**. Everything is explained in his book, *Secrets of Mental Math* (2006; ISBN 0-307-33840-1). He says that with enough practice, we can impress our own students, which hopefully can improve their liking of mathematics. Math is really about patterns!

2.) "Ma and Pa Kettle Math"

This classic, from the 1951 *Ma and Pa Kettle Back on the Farm* is a 2 minute 14 second video segment that really gets us to think about incorrect mathematical reasoning to arrive at a correct final answer. Do 2 wrongs really make a right? So, is the right answer of how to divide 25% evenly among 5 people 5% or 14%? There is potential for some engaging class discussion! **http://video.google.com/videoplay?docid=7106559846794044495**

3.) Fermat's Last Theorem and Andrew Wiles

I remember the excitement of reading the front page headline of the Globe and Mail back in 1993 announcing that Fermat's Last Theorem has been proved. The entire documentary is available on the YouTube in 5 parts. I highlight the first and last part, **http://www.youtube.com/watch?v=qiGOxGEbaik** and **http://www.youtube.com/watch?v=9WTcZr-_PWM&feature=related**. An emotional Andrew Wiles, professor of mathematics at Princeton conveys the rich mathematical history and the challenges he faced to solve a famous problem in our lifetime.

Final Remarks

Please sign up for the free *Math Forum Electronic Newsletter*. It is sent weekly to your e-mail and highlights about 3 web sites that you may find interesting in teaching and learning mathematics. The link, **http://mathforum.org/electronic.newsletter/**, also contains an archive of newsletters dating back to 1996.

On another note, I came across a compilation of Math Blogs, **http://www.blogged.com/directory/education/math**. It was here that I located the "Ma and Pa Kettle Math". I created a blog as an on-line journal of my training as a runner and walker several months ago[1]. Take it from me, it's easy to set-up and maintain. You choose from a template and then add features, requiring no computer programming skills!

It's been a pleasure to serve the *Gazette* readership by writing this column. I leave you with "You Might Be a Mathematician If…" **http://swiss.csail.mit.edu/~jaffer/YMBAMI** Bye!

Louis Lim

[1] My Blog is located at http://lim314159.blogspot.com/ if you're interested.

▲ What's the Problem?: Alphametrics

Shawn Godin



Shawn is the head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans, ON.

Welcome back problem solvers. Last issue I left you with the following problem.

The letters A, B, C, D, E, F, G and H represent distinct digits. Determine their values given the two products below are true. (Note the first digit of a number must be non-zero)

$$\begin{array}{r} ABCD \\ \times\ E \\ \hline DCBA \end{array} \qquad \begin{array}{r} BFDG \\ \times\ G \\ \hline GDFB \end{array}$$

This problem appeared as M273 from the December 2006 issue of *Crux Mathematicorum with Mathematical Mayhem* and solution appeared in the December 2007 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem* web site **http://journals.cms.math.ca/CRUX/**

This problem lends itself nicely to some experimentation. Especially the second product since the four digit number BFDG is being multiplied by one of its own digits G. Students could experiment with some random four digit numbers, such as:

2374 x 4 = 9496
3912 x 2 = 7824
5106 x 6 = 30636

It should soon become evident that the choices for B and G are limited. Since the product must be four digits as well B x G < 10. When we look at the product, we see that B x G plus any carry from the previous product must be G. This leads to, since neither of B or G can be zero, B = 1.

Thus we have

$$\begin{array}{r} 1\ FDG \\ \times\ G \\ \hline GDF\ 1 \end{array}$$

So G x G must end with 1. The only possibilities are 1 and 9, but since B = 1, we must have G = 9. So the second product must be

$$\begin{array}{r} 1\ FD\ 9 \\ \times\ 9 \\ \hline 9\ DF\ 1 \end{array}$$

Now looking at the thousands digits, we see that there cannot be a carry from the product 9 x F. The only time this happens is for 0 and 1, so F = 0 and the second product is now

$$\begin{array}{r} 1\ 0D\ 9 \\ \times\ 9 \\ \hline 9\ D0\ 1 \end{array}$$

Finally, focusing on the last unknown digit, we see that 9 x 9 = 81, so there is a carryover of 8 being added to the product of 9 x D. The result of that has a unit digit of, zero, so we know that the units digit of the product 9 x D must end in 2, thus D = 8 and the second product is

$$\begin{array}{r} 1\ 08\ 9 \\ \times\ 9 \\ \hline 9\ 80\ 1 \end{array}$$

Turning to the first product we have

$$\begin{array}{r} A1C\ 8 \\ \times\ E \\ \hline 8\ C1A \end{array}$$

Looking at the thousands digit again we conclude that E x A ≤ 8. From the remaining digits we see that the only possibilities are A = 2, E = 3 or A = 3, E = 2 or A = 2, E = 4 or A = 4, E = 2. From the units digit of the product, we see (by trying the different possibilities) that E = 4 and A = 2. Thus the product is then

$$\begin{array}{r} 2\ 1C\ 8 \\ \times\ 4 \\ \hline 8\ C1\ 2 \end{array}$$

Concentrating on the tens digit we see that 4 x C + 3 (the carry from 8 x 4) must be 11, 21, or 31. The only possibility is C = 7, which gives

$$\begin{array}{r} 2\ 17\ 8 \\ \times\ 4 \\ \hline 8\ 71\ 2 \end{array}$$

If this type of problem appeals to you, you might want to check out the *Journal of Recreational Mathematics*. Each issue has articles on recreational mathematics, as well as some problems posed by the readers and a section called alphametics which is made up of problems like the one above. The section has also led to 3 books of these problems. You can check out information about the journal and the books at **Baywood.com**.

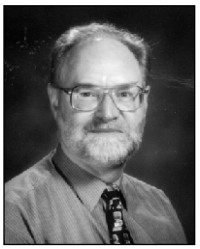
Now for your homework.

The surface areas of the six faces of a rectangular prism are 1254 cm^2, 1254 cm^2, 770 cm^2, 770 cm^2, 1995 cm^2, 1995 cm^2. Determine the volume of the prism.

Until next time, have a great summer and happy problem solving! ▲

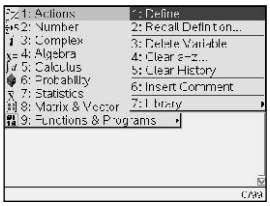
▲ CAS Corner: User Defined Functions on a CAS Device

FRED FERNEYHOUGH

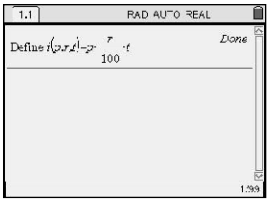


Fred Ferneyhough is former head of mathematics at Harold M. Brathwaite SS in Brampton. He retired from there in February 2006. Fred is an avid user of and advocate for the use of technology in the mathematics classroom. He is also an author and tech consultant for several books and a National Instructor for Texas Instruments.

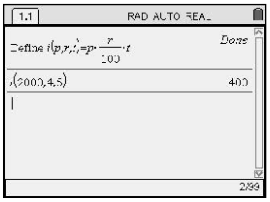
One feature of CAS devices such as the TI Nspire™ CAS or the TI-89 is the ability for the user to define a function that they can use for their own purposes. To access this feature, press (home) followed by (1): Actions and (1) :: Define. Normally, we use this for just regular functions.



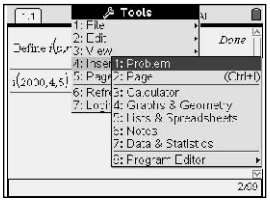
In this example, the simple interest formula is being defined. The function name is "i" for interest and the function has three parameters for principal, rate and time.



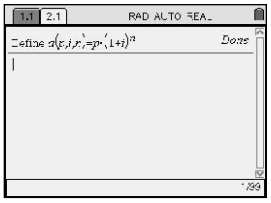
Once the function is defined, the user only needs to substitute values for p, r and t into the brackets following the function name, and the software evaluates the expression. So, in this screen the interest on an investment of $2000 at 4% simple interest for 5 years is $400.



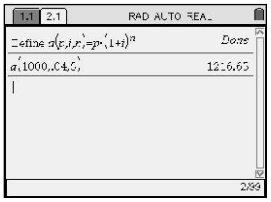
Let's start a new problem. Press (ctrl) followed by (home) to open up the Tools menu. Choose (4): for Insert and choose (1) : Problem. This inserts a new problem into the document. All variables and functions from problem 1 will not be recognized in this problem.



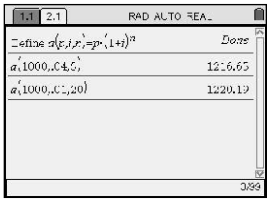
Add a new Calculator application page. In this problem, we will define another function involving interest, only this one is compound interest. The amount is defined in terms of the principal, the interest rate and the number of interest periods.



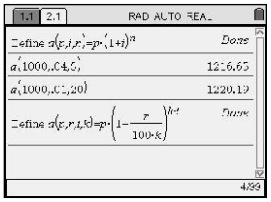
So, if you have $1 000.00 to invest at 4% per annum compounded annually for five years, we can use this function to find the amount. Note that the user has to interpret the numbers as they insert them, in exactly the same way that students have to change things as they use the formula. The interest rate is 4% but the student must enter 0.04.



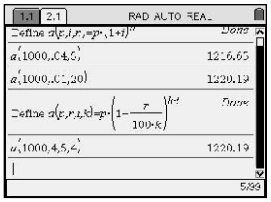
What if we leave the principal and time the same but change the interest rate to 4% per annum compounded quarterly? The user has to interpret the values to input for interest rate and the number of interest periods as shown.



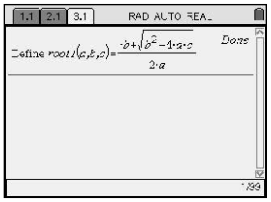
So, let's re-define the function. Add another parameter, k, to represent the number of compounding periods per year. We'll use k in the formula to change the interest rate and also multiply the time by the number of periods per year. Notice also that the rate has been changed to a decimal.



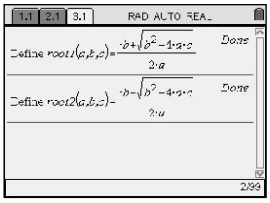
Now, the user does not have to worry about the changing any numbers in the formula. Simply enter the given information and press (ctrl) followed by (enter). The idea here would be have the students come up with these formulae on their own. Providing them with the formulae does not teach them anything.



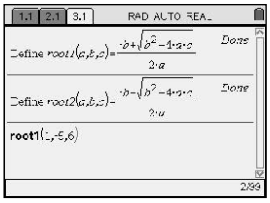
Start another new problem and add a Calculator application page. The quadratic formula is another good use for a user defined function. In this screen, the first root is defined in terms of a, b and c, the coefficients of a quadratic in standard form.



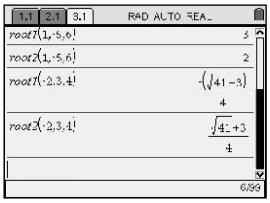
The second root is very similar. In fact, you can move up to the previous expression, press (enter), and simply edit the expression.



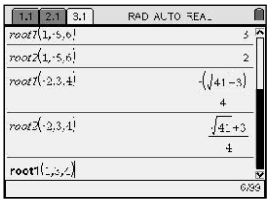
For the quadratic equation $x^2 - 5x + 6 = 0$, the user inserts the constants 1, -5 and 6 into the function and presses (enter).



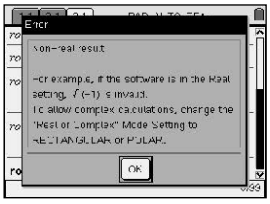
Both of the solutions to the first equation are shown at the top of this screen. The two roots for a second equation are shown solved in exact form.



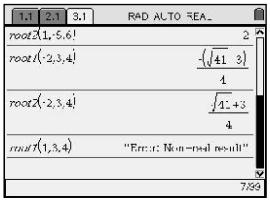
An interesting result occurs when you attempt to use the two formulae for a quadratic equation where there are no real roots. Such a set is shown in the substitution in the first root.



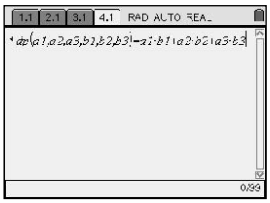
When (enter) is pressed, the software immediately displays an error message.



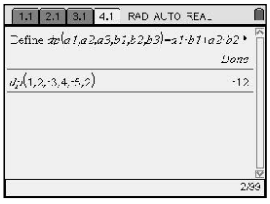
When the error message is cleared, a similar message appears as the result for the operation.



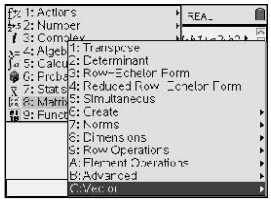
In the new Calculus and Vectors course, students have to calculate the dot product and cross product of algebraic vectors in three space. This screen shows a user defined function called “dp” which can be used to find the dot product. The right side is the sum of the products of corresponding components.



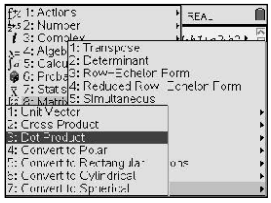
For the two vectors $\bar{a} = (1, 2, -3)$ and $\bar{b} = (4, -5, 2)$, the six values are typed into the function. This really does not save students a lot of time, but it does set up the cross product in a few steps.



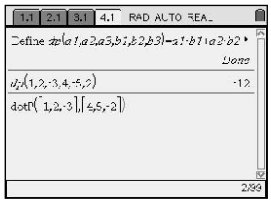
The device does have a dot product features. Press (menu) and choose (8) Matrix and Vectors. Choose sub-menu (c) Vector.



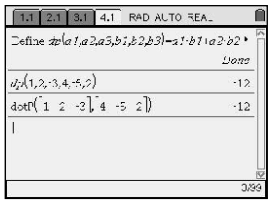
This opens up a menu where you can find both the Dot Product and Cross Product features.



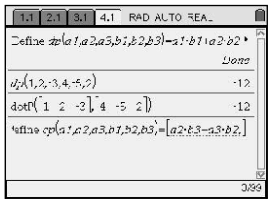
To use the Dot Product, you must press (ctrl) followed by (!) to get square brackets. The components for each vector are entered separated by commas. You also need a comma between the two vectors.



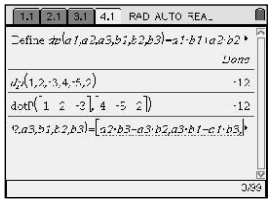
Press (enter) to get the result. Notice that the phrase “dot product” has been reduced to “dotP”. Typing in this phrase lets us avoid the menus.



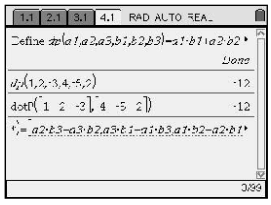
Now, let’s move on to the cross product. This is a formula that can be confusing, so a user defined function may be of value. The first component of the function is shown in this screen.



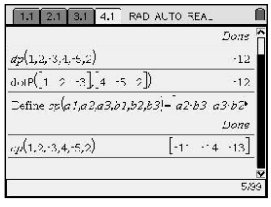
The second component is separated from the first with a comma.



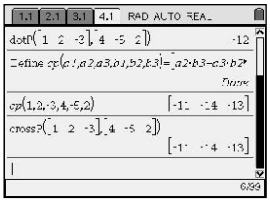
The final component is shown and the square brackets are completed.



Using the new function with the two vectors above gives us the components of a vector in three space that is perpendicular to each of the given vectors.



Using the menus, we can find the same result.

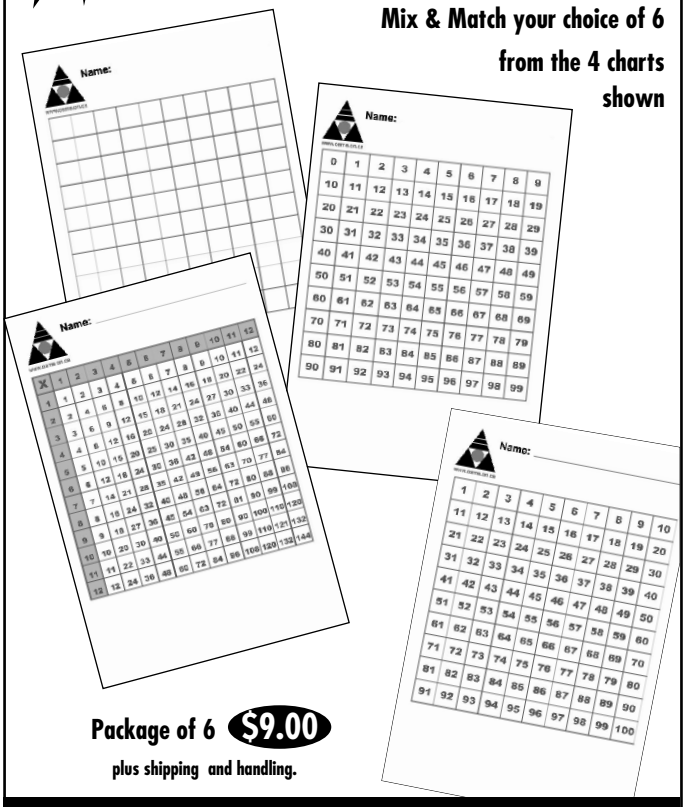


The advantage to using these functions is to reduce the tedium in mathematics. A formula such as the one for the cross product can reduce the number of numerical errors that are so easily made by students (and teachers) and allow us to interpret the results geometrically. ▲

OAME Elementary Sticky Chart Pads

NEW

Great for Elementary or Secondary Student Use
4 colours • 4" x 6" • 25 sheets per pad
Mix & Match your choice of 6
from the 4 charts shown



Package of 6 $9.00
plus shipping and handling.

Order by mail, fax or on-line:

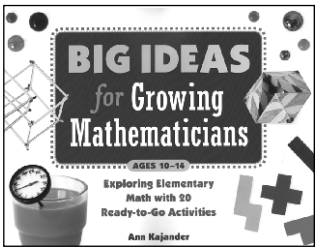
OAME
or Order ON-LINE www.oame.on.ca
Do not send cash or cheque. You will be invoiced with shipping and handling costs added.

▲ Book Review: *Big Ideas for Growing Mathematicians* by Ann Kajander

Reviewed by Laurie Moher

Laurie is currently a Literacy Coach with the Kawartha Pine Ridge District School Board. She is currently serving her fourth term on the board of directors for OAME.

This book follows Ann Kajander's first book, *Big Ideas for Small Mathematicians* which is a collection of twenty-two activities for students aged 6-11 (reviewed in the December 2003 issue of the *Gazette*). *Big Ideas for Growing Mathematicians* offers a collection of twenty classroom tested mathematics activities geared to students aged 10-14. These activities cover a range of math topics and some have been designed to help students investigate and understand the concepts behind procedures they encounter in mathematics class.



The book features a visually appealing, consistent layout that makes it easy to follow and use. The activities use readily available materials or in some cases provide the materials within the book itself. This book utilizes the same structure as the first book and includes the following sections:

- **The Big Idea:** In this section the author gives a brief statement summarizing the concept being investigated during the activity or a question to guide the investigation. For example, in the activity 'Toothpick Rectangles' the big idea is summarized as, "some rectangles are bigger than others, even if the perimeter is the same". In this same section, the author gives a visual summary of the content areas and the process skills used in the activity as well as any prerequisite knowledge and skills students need for the activity. In this book the content areas would be the strands addressed (e.g., patterning) and the process skills would include those from the Ontario Curriculum (e.g., reasoning) in addition to other skills such as visualizing, creativity and collaboration.
- **The Mathematical Idea:** As the heading suggests, this section gives mathematical background

information pertaining to the upcoming task. It also includes possible solutions to the task where appropriate. The author has given a good deal of information in the 'Mathematical Idea' sections and given it in a way that is clear and supported with helpful examples.

- **Making it Work:** This section includes the objectives for the activity, the materials and preparation needed and the procedure to follow in order to do the activity. In the toothpick activity previously mentioned, the objectives are as follows: "Children examine the areas and perimeters of rectangles constructed from toothpicks. They also investigate the relationship between area and perimeter". At the end of this section, there is a boxed feature entitled, 'Helpful Terms'. This is the mathematical vocabulary that students need to use and learn during the activity. It should also be noted that a glossary of terms from all of the activities can be found at the back of the book.
- **Suggestions:** Here the author often gives suggestions for helping students who are struggling to get started with the mathematics of the activity. At times she gives tips for success with the materials or presentation of the activity.
- **Assessment:** This section usually offers general suggestions for encouraging students to communicate their thinking/reasoning strategies. In some cases it is suggested that no formal assessment is needed.
- **Extension Activities:** For every activity there is at least one suggestion for where the teacher may want to go next with the students if they wish to do a similar or new investigation building on what they have learned in the current one.

Each activity includes its own instruction sheet that gives a step-by-step explanation of what the students will do. Any masters or templates that need to be copied for the students are also included. This will save teachers who wish to use this book a great deal of preparation time.

In the classroom, these investigations could be used individually when they fit with expectations being addressed during a unit of study. They could also be used as a way to involve parents in some engaging math homework. They are a ready-made collection of activities for a Math Fair or Family Math Night or for a Math Club. This book would be a nice addition to a school's resource library or a parent resource library.
ISBN-13: 978-1-56976-212-7; ISBN-10: 1-56976-212-0 ▲

▲ Photo Math - The Fine Art of Viewing the World Through a Mathematical Lens

RON LANCASTER



Ron Lancaster is a Mathematics Lecturer in the Department of Curriculum, Teaching and Learning, the Ontario Institute for Studies in Education (OISE/UT) at the University of Toronto, 252 Bloor St West, Rm 11-162, Toronto Ontario M5S 1V6

Teachers and students are encouraged to put on a pair of mathematical glasses and become a math spotter, someone who sees mathematics everywhere and then possibly poses mathematical questions based on the images taken. Ron Lancaster, the current Editor for Photo Math, welcomes photos with questions for possible publication in future columns. Material can be sent via e-mail or by regular mail to either of the two addresses given above.

Question 1

In Israel licence plates for cars consist of seven numbers (Photograph 1) and phone numbers, not including the area code, (Photograph 2) consist of seven numbers too.

(a) Assuming all combinations are permitted, determine how many licence plates and phone numbers are available for use.

(b) What is the probability that a person who owns a car and a phone, has the same seven-digit number for both? Assume that the numbers for licence plates and phone numbers are assigned at random.

(c) How many examples are there for which the sum of the two seven digits numbers is 9,999,999?

©2008 Ron Lancaster. Any commercial use or any use outside of a teacher's classroom requires permission from the author.



Photograph 1



Photograph 2



Photograph 3

©2008 Ron Lancaster. Any commercial use or any use outside of a teacher's classroom requires permission from the author.



Photograph 4

Question 2

(a) Given the distance to the three cities in Photograph 3, do you have enough information to identify the precise location where this photograph was taken?

(b) Note that the three numbers shown on the sign are all prime. If you were to drive in the opposite direction, will there be other times when the three numbers will again all be prime?

(c) The three numbers in the photograph are terms in an arithmetic sequence and as noted earlier, they are all prime. Find four terms of an arithmetic sequence that are all prime.

Question 3

Photograph 4 was taken on Highway 6, a new toll road in Israel that is based on the technology of Highway 407 in Ontario.

Visit the web sites given below and obtain data to answer the following questions. If these two sites do not provide you with enough data, use a search engine to find other sites.

(a) Compare the speed limits for both highways. Compare the time it would take to drive from one end to the other at the speed limit for both highways.

(b) Which toll road is more expensive to use?

- Highway 6, Yitzhak Rabin Cross-Israel Highway, Israel http://www.roadtraffic-technology.com/projects/highway_6/
- Welcome to the 407 ETR: Keeping you ahead of the curve http://www.407etr.com/ ▲

▲ Formative assessment: Questioning, Listening and Responding

CHRIS SUURTAMM

ADRIAN JONES
GRADUATE STUDENT IN MATHEMATICS EDUCATION AT THE FACULTY OF EDUCATION, UNIVERSITY OF OTTAWA



Dr. Christine Suurtamm is an Assistant Professor at the University of Ottawa.

This article positions classroom questioning, listening and responding as being central to developing students' mathematical thinking. Further, the interactive feedback that occurs through reflective classroom dialogue resulting from thoughtful questioning and responding is a large component of formative assessment as it informs both students and teachers about next steps (Black & Wiliam, 2006). The information that teachers can gather about what students know and think, as well as how they learn, can transform pedagogy to create opportunities for student success. The following discussion argues for the importance of thoughtful teacher questioning, listening, and responding and draws on our work with a teacher, Terry, in one of the case studies of promising practices in the Curriculum Implementation in Intermediate Mathematics (CIIM) research project (Suurtamm & Graves, 2007) to provide examples.

Questioning

Most teachers pose a variety of questions to students in a number of different ways. These questions have a range of purposes, many of which help to engage students in the flow of the lesson. However, not all questions are equal. While some questions merely evoke the responses that teachers have in mind as the next step to the teacher's solution, other questions can be designed to probe student thinking and check for understanding. These questions often require pre-planning and reflection and teachers need to spend time on framing questions that solicit demonstrations of student thinking and understanding.

Some of these types of questions can grow out of an initial question. For instance, Terry, a teacher we observed in a classroom showed the class a model of a relation using a series of linking cube towers. The first tower had 4 cubes, the second tower had 7 cubes, the third had 10 cubes, and the fourth had 13 cubes. She asked the students "What kind of relation is this?" After the students had offered responses, Terry then encouraged the students to offer more information about the relationship and their understanding of it by using questions such as "Why is it linear?" and "How do you know?". As the students offered more ideas as to what the relationship meant she asked the students to summarize their understanding in simple terms: "If you were in grade 3 or grade 4, and I said 'why is this a linear relationship' and you didn't know anything about slope or first differences or anything, how would you explain it, do you think?"

The aim of such questions is for students to think, respond and then think again about the responses. This creates a dialogue in the class that generates new ideas, perspectives, and multiple ways of thinking about a particular concept. As Terry went on to have students build models of linear and then quadratic relationships she encouraged her students to consider not only what they were doing and how they were doing it, but to question why by thinking about what makes a relation linear or quadratic.

Another instance of probing student thinking was in a discussion about the equivalence of the forms (standard and factored) of a quadratic equation. Terry asked the question of the class "When I say that this is equal to that… What am I saying? What does that mean that those two things are equal?" She paraphrased a student response and then asked: "… how could I verify, how could I prove that they were equal?"

Within Terry's class, we saw that students freely offered ideas and suggestions to the 'why' prompts and openly conjectured patterns and connections that they were seeing. They responded to Terry's expectation to "be prepared to explain how you figured it out".

Listening

Along with using probing questions to elicit student responses, teachers also need to be attentive to the responses. How teachers listen and the responses that they give tell students whether the teacher understands their thinking.

Davis (1996) discusses three modes of listening: evaluative, interpretive, and hermeneutic listening. Simply put, evaluative listening is when the teacher hears the student response and judges the response as correct or incorrect and then continues to move forward with his or her intended lesson plan. Most classrooms have many instances of evaluative listening. It goes something like this: "What is the square root of 9?" and the student either responds with the correct answer and is met with a positive response from the teacher or the student responds with an incorrect answer and is met with a negative response.

Interpretive listening, on the other hand, attempts to get at understanding what the students are thinking when they respond to a question. As such, the teacher presents opportunities for students to communicate their thinking, and teacher and students alike listen to the responses to understand the meaning and to connect the ideas to other responses and ways of thinking. Hermeneutic listening goes beyond this to engage listener and speaker, student and teacher, as equal participants in negotiating collective meaning. Incorporating more instances of interpretive (and hermeneutic) listening in a class encourages student thinking and provides opportunities for meaningful on-going assessment.

Terry followed her questions with instances of interpretive listening as she would often paraphrase a student response or ask another student to paraphrase a student response. She would constantly ask her students to explain or think of additional information to add. Examples of questions that reflect her interpretive listening include "Why are we doing this?", "What do you think?" and referring to different models presented with "Is there anything else that you notice?" Terry often used these questions to prompt students to listen to other student responses and to expand their understanding of particular concepts.

Responding

As seen in the above examples it is hard to separate questioning, listening, and responding as they tend to have a natural flow when teachers are posing meaningful questions and listening and building on student responses. It is also evident that this process requires great attention on the teacher's part. Teachers have to listen carefully to formulate meaningful responses that would show an understanding and may also challenge students to extend their thinking. As well, increased participation by students requires that teachers take all answers seriously.

Terry encouraged all of her students to respond to questions, and would work with their responses, whether their answers were correct or incorrect. As an example, Terry presented a model of a relationship and asked the class what type of relationship the model represented. Although it was a quadratic model, one student offered a response that it might be exponential. Terry paraphrased the response:

"Good thinking! Sam is thinking about the exponential relationships where we had x in the exponent. Those were non-linear relationships. It's not that this time, but good thinking."

She then went on to probe for other responses and then together the class looked at reasons why they would know the model was quadratic. Terry also used responses from students as an opportunity to get them to think about what went wrong and why. As an example of this, Terry spoke to a group of students who had made a mistake with their calculations of a warm-up problem. She asked them: "… what would have been a clue when you did QuadReg that something was off [with their data points]?" Later on she asked "So how would you have known that there was a problem maybe before you actually did it?". In these cases, Terry insisted that the students themselves find the cause of the problem and identify ways to check for similar errors in the future.

What Teachers Learn From This

Teachers learn a great deal about their students' prior knowledge, misconceptions, and current understanding of mathematical ideas through creating a dialogue with thought-provoking questions, attentive listening, and responses that develop the conversation. During a debriefing session after one of Terry's lessons, she discussed the question raised by one of her students about whether the same value of x should be used in the x^2 and the x terms of a quadratic, pointing to her students' developing notion of variables.

> *"But it just shows they haven't fully formed the concept of that variable… that they haven't quite really understood that the mathematical expression is really a relationship between those two variables."*

Teachers also use the responses of students to adjust their pedagogical moves. For instance, Terry became aware of her students' confusion at the end of one class when some of the students said that they had difficulty with the process of using colours to represent

continued on page 40

▲ OAME Leadership Conference 2008

BY SUE HESSEY, EXECUTIVE DIRECTOR
PICTURES BY DAN CHARBONNEAU, PAST PRESIDENT

The OAME Leadership Conference 2008, "Investigating Pathways to Understanding Mathematics" was held Feb 21-23 at the Holiday Inn Select, Toronto. On Thursday night, 170 delegates and workshop presenters from across Ontario met to listen to and engage in the opening keynote activity. Dr Akihiko Takahashi from the University of Chicago explained the Japanese approach to understanding mathematics, "Teaching Through Problem Solving". Most Japanese texts begin with a problem-solving activity so that students are challenged to use their previous learning to attack unknown problems to develop understandings of mathematics. Delegates experienced some of these problem- solving activities based on the ideas of an open-ended approach. On Friday night, following a buffet banquet, George Gadanidis and troupe entertained with songs and poetry. This keynote looked at mathematics through the lens of performance. Check out **mathfest.ca** for details on the upcoming Math Performance Festival and Contest for grade 4 - 6 students. Receptions followed both keynotes so that delegates could network informally with other Ontario mathematics leaders. Friday noon, delegates had a chance to talk to exhibitors from 16 companies during lunch and see/purchase their latest resources. The many door prizes and contributions made to the delegates' bags by the exhibitors were much appreciated.

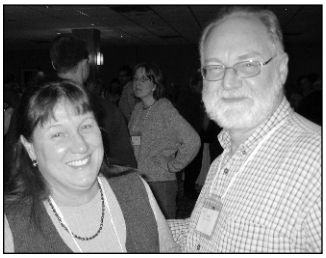
On Friday and Saturday, delegates participated for seven hours in one of two specific workshops, looking in depth at pathways in their chosen division, to help them better understand mathematics. The Elementary Workshop looked further at investigating Japanese approaches for understanding mathematics, lead by Dr Takahashi and facilitated by Pat Margerm and Kathy Kubota-Zarivnij. They explored some of the big ideas by using English translations of the most popular Japanese textbook series. Specifically they explored addition and subtraction, multiplying, dividing and quantitative relationships using the number line. Lastly they represented problems visually, in order to connect to algebra. Participants used manipulatives to try problems and were each given one of the Japanese texts to take home.

The Secondary Workgroup looked at many new emerging technologies, led by Fred and Lynda Ferneyhough. The use of handheld devices and software has changed the way students learn math and how teachers present content and activities to them. These tools allow multiple representations of a problem and solutions can reflect several approaches. Participants had the opportunity to try out TInspire, TI Navigator and Smartboards. These technologies appeal to students raised in a technology rich environment and can often spark interest in the process of understanding mathematics.

When asked what aspect of the conference they found most valuable, delegates responded with comments like:

- Talking & thinking about math
- WOW! – problem solving talk was outstanding
- Appreciated seeing connections between grade 1 and high school
- Many new approaches to developing a deeper understanding of math concepts
- Informal professional dialogue, sharing with colleagues, networking
- Leaders prepared, thorough, welcoming, supportive
- Extended time to delve into a topic, continuity of same presenters for 2 days
- Superb technology information, chance to try before purchasing

Don't miss out next year on this exceptional professional development opportunity. Plan now to attend Leadership 2009 at the Holiday Inn Select, Feb 19-21, 2009. Look for the flyer with your September *Gazette*. ▲






▲ College Mathematics Project 2007

GRAHAM ORPWOOD

MARGARET SINCLAIR

LAUREL SCHOLLEN



Graham Orpwood is Professor Emeritus at York University Faculty of Education and the Project Manager for the College Mathematics Project. He is a former science teacher and was the founding Director of the York/Seneca Institute for Mathematics, Science and Technology Education (YSIMSTE). His research interests include curriculum and assessment and the uses of research in policy deliberation.



Margaret Sinclair is an Associate Professor of Mathematics Education in the Faculty of Education at York University. Her research focuses on the use of technology in elementary and secondary school mathematics and on the design and use of mathematics learning objects



Laurel Schollen is the Dean of Applied Science and Engineering Technology at Seneca College and Director of the College Mathematics Project. She is also co-Director of the York/Seneca Institute for Mathematics, Science and Technology Education (YSIMSTE) since its founding in 1999.

More than one-third of all first semester Ontario college students who take a mathematics course either fail or barely scrape through with a D, thus placing themselves "at risk" of not completing their chosen program. That is the troubling conclusion of the College Mathematics Project (CMP) annual report, recently released by Seneca College. And if we look separately at those who have graduated from Ontario secondary schools within the past few years and those who are either older or have come from other jurisdictions, the news does not get any better: 36.5% of the recent Ontario graduates are at risk, compared with 24% of the second group.

The report is not all bad news however: it also recounts the major efforts being made by all colleges and school boards to increase student success and the focus of the project's recommendations is entirely on how we can work together to this end. This update is designed to highlight the major CMP findings and suggestions for action – readers who would like more information can download the report free at **www.ysimste.ca**.

CMP conducted a pilot study in 2005-2006 which was reported a year ago in *Ontario Mathematics Gazette* (vol. 45, #4, pp. 33-34). In 2007, the study was repeated with 6 GTA colleges (Centennial, George Brown, Georgian, Humber, Seneca and Sheridan) in collaboration with the corresponding Distrct School Boards. CMP was undertaken by researchers from the York/Seneca Institute for Mathematics, Science and Technology Education (YSIMSTE) based at Seneca College and supported by the School-College-Work-Initiative (SCWI) and its GTA Regional Planning Team.

Here, then, in a nutshell, is the story of the College Mathematics Project 2007.

What CMP Did

In 2007, CMP reviewed the files of over 20,000 students entering the 6 GTA colleges in the Fall of 2006. Of these, 10,743 (53%) took a first semester mathematics course and, of this group, 7,538 (70%) were recent Ontario graduates (ROGs). Four of the six colleges tested students after admission and placed them either in a regular college-level mathematics course or, where their backgrounds required it, into a preparatory or remedial mathematics course. Achievement in these courses was examined separately.

We also linked the students' college files with their secondary school transcripts (from their college applications) so that we could relate college achievement both with the "mathematics pathway" they had taken through secondary school and with the identity of the boards and schools from which the students came. As far as we know, CMP is the first study ever to give systematic feedback to boards and secondary schools about the postsecondary success of their graduates. (Readers in a GTA Board can find out about the college mathematics achievement of their school's

graduates by consulting with their board's mathematics coordinator who has access to the CMP database.)

What CMP Found

As noted at the beginning of this article, CMP found that 66% of the overall cohort achieved good grades (A, B & C) in first-semester college mathematics, while 34% achieved D, F or withdrew (resulting in CMP labeling them "at risk"). These results were similar for students in both Business and Technology programs in college. Other highlights of the research are as follows:

- Over 40% of students taking MAP4C and who go on to take college mathematics were found to be at risk; this dropped to 30% if the students took MCT4C.
- Grade 11 courses selection was also found to be important, with nearly 50% of those who took MBF3C becoming at risk in college, compared with 20-30% of those taking MCF3M or MCR3U.
- Over 50% of those who had taken Grades 9 and 10 Applied mathematics were at risk in college, compared with fewer than 30% of those who had taken Academic Mathematics.

How CMP Used the Results

This research raised as many questions as it did conclusions. CMP does not seek to point fingers or lay blame but to help colleges and schools find solutions and improve student success. We convened a Deliberative Forum in October 2007 and invited mathematics specialists and student success leaders from colleges and school boards, along with the Ministry and other interested provincial organizations. The CMP team presented the research data and analyses and both college and school board representatives described initiatives already under way aimed at increasing student success. The Forum then broke into four discussion groups focused on mathematics, guidance, student success, and provincial/board/college policy to respond to questions raised by the research and to suggest useful ways forward. These deliberations were very productive and the final CMP recommendations are based on what we learned at this Forum.

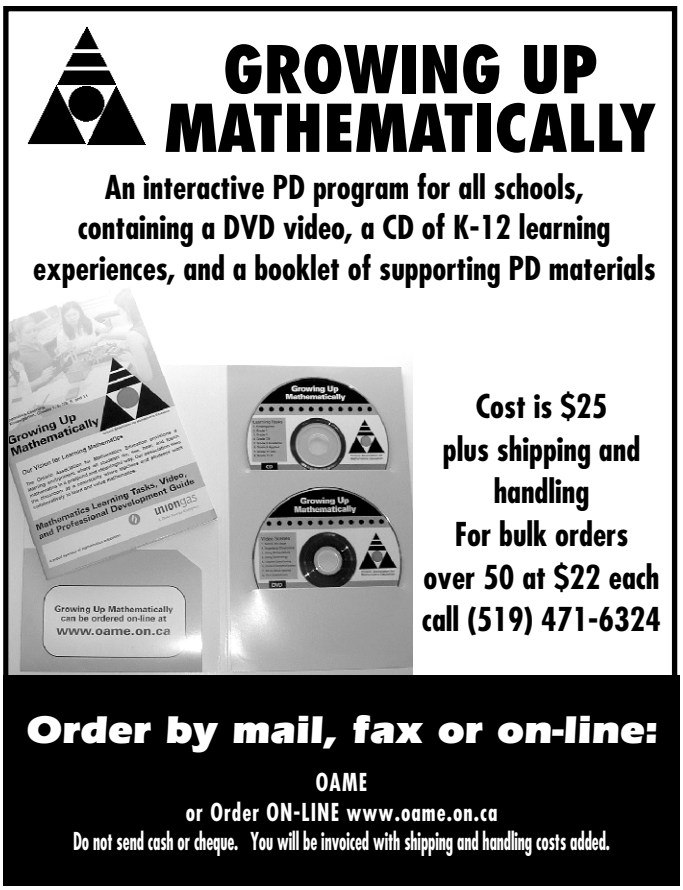
What CMP Recommended

There are two general conclusions of CMP and eight concrete recommendations, two each for students and parents, colleges and college faculty, schools and teachers, and the Government of Ontario. The general conclusions of CMP are that student achievement in first-semester mathematics in Ontario colleges needs to be significantly improved and that this requires action by all stakeholders.

The recommendations call for: students to select courses most appropriate for their postsecondary goals and to inform themselves early in their secondary school careers about colleges and college programs, colleges to strengthen their commitment to student success and to inform themselves better about effective strategies, schools to place equal value on college as university destinations for students and to show this by ensuring that all college-preparation courses are available to students, and the Government to adopt a K-16 vision of student success (i.e. to see students' postsecondary achievement as part of the overall goal of the school system).

Where CMP is Going Now

CMP has recently received funding from the Ministry of Education to continue through 2008. It has expanded to include 11 colleges corresponding to over 50% of the overall provincial college enrolment and will expand the range of research questions to be studied. Ultimately, we would like to be able to include all colleges and school boards and be able to provide feedback to every secondary school in the Province. We also look forward to reporting at the OAME conference and in the *Gazette* on a regular basis. ▲



▲ Establishing Families of Planar Figures Through the Principle of Cavalieri[1]

Jérôme Proulx



Jérôme Proulx is Assistant Professor of Mathematics Education at the University of Ottawa. He is interested in exploring school mathematics concepts for teacher education practices and his principal research interests are geared toward the development of innovative and alternative approaches for the preparation and the professional development of mathematics teachers.

Aspects of the history of mathematics have a capacity to trigger motivation and interest for learning (and even teaching!) mathematics. It can provide insights, amazements, curiosity, fun and also a humanness to mathematics, a discipline often seen by many as dry, formal and ... not very human! In addition, for a mathematics educator, the history of mathematics can also provide and trigger other aspects of rich interest: it can inspire new ways of making sense, perceiving and conceptualizing a specific topic and its concepts. Being attuned to and informed by the history of mathematics can inform and provide insightful resources to enrich our approaches to and understandings of a topic, as it can keep alive earlier rich intuitions and ways of working on school mathematical concepts. Obviously, the goal would never be to reproduce history in our teaching, but rather to see how some aspects of the history of mathematics can provide a rich ground of (mathematical) inspiration concerning the concepts themselves.

In this article, I offer an illustration of such an inspiration through the presentation and discussion of a historical mathematical principle, the principle of Cavalieri, and how it offers insights for approaching and making sense of the area of planar figures. This historical-mathematical principle facilitates the establishment of strong connections between planar figures, not through relating them by algebraic transformations of their formulas, but through geometrical transformations, comparisons and reasoning.

[1] Many of the ideas present in this paper were developed through discussions with a colleague of mine, David Pimm from the University of Alberta, to whom I am thankful.

Cavalieri's Principle

Bonaventura Cavalieri (1598–1647) was an Italian mathematician whose work is often mentioned in accounts of the emergence in Europe of infinitesimal calculus and infinitesimal calculations in the early part of the seventeenth century. Cavalieri is particularly known through one of his "theorems," the Cavalieri's principle, that reads:

> *If between the same parallels any two plane figures are constructed, and if in them, any straight lines being drawn equidistant from the parallels, the included portions of any one of these lines are equal, the plane figures are also equal to one another; and if between the same parallel planes any solid figures are constructed, and if in them, any planes being drawn equidistant from the parallel planes, the included plane figures out of any one of the planes so drawn are equal, the solid figures are likewise equal to one another. […] The figures so compared let us call analogues, the solid as well as the plane, [...]*

Gray (1987, p. 13) expresses a two-dimensional version of the Cavalieri's principle as follows:

> *The principle asserts that two plane figures have the same area if they are between the same parallels, and any line drawn parallel to the two given lines cuts off equal chords in each figure.*

Figure 1 offers an illustration of the Cavalieri's principle with two plane figures:

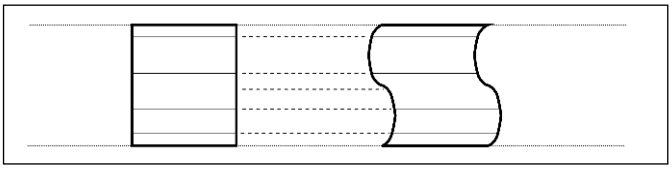


Figure 1. An illustration of Cavalieri's principle in two dimensions

[2] Those interested in some historical accounts will be interested in knowing that "when Cavalieri found his thoery in 1635, Chinese mathematicians had used the theory for more than one millennium" (He, 2004, p. 9 – see article for more details). As well, as one of the anonymous reviewers mentioned, Archimedes knew the Hat-Box theorem for comparing cones, cylinders and spheres using a 3D version of the principle. In fact, it is not clear if Cavalieri's principle was principally developed in a 2D or a 3D context, since Cavalieri's statement is in both dimensions. In this article, I focus only on issues of 2D and planar figures, but I refer the reader to Claude Janvier's work (e.g., 1992a, 1992b, 1994, 1997) who has done similar work concerning volumes of solids and 3D – work that has had a profound influence on these present inquiries about area.

Informally, what the principle asserts is that if you cut each polygon horizontally at the same height and that each chord obtained are of equal length, then the two polygons are of the same area. In order to make the comparison, both polygons need to have the same height; if not, the comparison appears not possible since one of the polygons would be cut at a place where there would not be any of the other polygon left to cut.[2]

Plane Geometry and Cavalieri

The Cavalieri principle, when linked to the study of area, can allow for qualitative and comparative assertions of figures, and in that sense appears helpful to establish geometric links and relate planar figures together. For example, using the particular case of a parallelogram and its corresponding rectangle (with the same base) can lead to an insightful application of the principle, where both figures lie between the same parallels (see Figure 2).

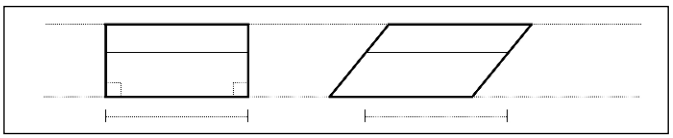


Figure 2. Cavalieri's principle at work with a rectangle and a parallelogram

For every cross-section, the lines have equal length. Therefore, by the Cavalieri principle, both figures have the same area. Indeed, any parallel line drawn would cut chords of equal length in each figure.

This suggests an interesting association between parallelograms and rectangles, where any parallelogram has a single associated rectangle (seen as a rectangular parallelogram) that has the same base, same height and, as a result, same area. This leads to the understanding that any parallelogram is a member of a family of parallelograms that are more or less "oblique" or skew with the same height and equal cross-sections all the way up. The degree of obliqueness of a chosen parallelogram within the family makes no difference in terms of area. Figure 3 shows the creation of members of one family of equivalent parallelograms[3]. Working geometrically with Cavalieri allows seeing that all figures within the same family have the same area. In other words, all parallelograms of the same base and same height have the same area.

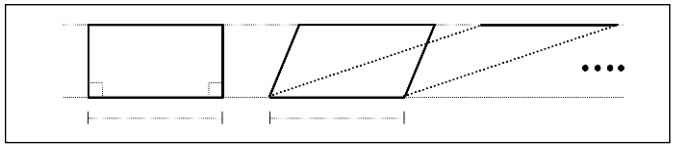


Figure 3. Creating a family of parallelograms

It should also be noted that there is a unique rectangle in the above family, where the rectangle can be taken as a possible reference point to which all parallelograms are associated. Therefore, if needed, all calculations of the area of a parallelogram can be reduced or transformed to the calculation of the area of the associated rectangle in its family (that is, with the same base and same height).

Rhombuses and Squares

Similar considerations can be drawn for rhombuses and squares. In a family of parallelograms (for example, the one in Figure 3), there is the presence of two equivalent rhombuses, one left-facing and one right-facing. Consequently, any rhombus belongs to a unique family of parallelograms and so can be associated with a single rectangle of the same area, that is, a rectangle that has the same base and the same height as the rhombus. However, not all families of parallelograms contain a rhombus. For example, if the shorter side of a parallelogram is selected as the base of the parallelogram and the height is larger than this length, then no rhombus can be found in this family. Thus, all rhombuses are associated with a family of parallelograms, but not all families of parallelograms contain a rhombus. (If one wants to obtain or know the rhombus of equivalent area to a parallelogram, one uniquely needs to play with the orientation of having the longer bases between the parallel chords to establish the family.) And, when the "rectangle" of a family is a square, there are no two equivalent rhombuses in the family since it is the square itself.

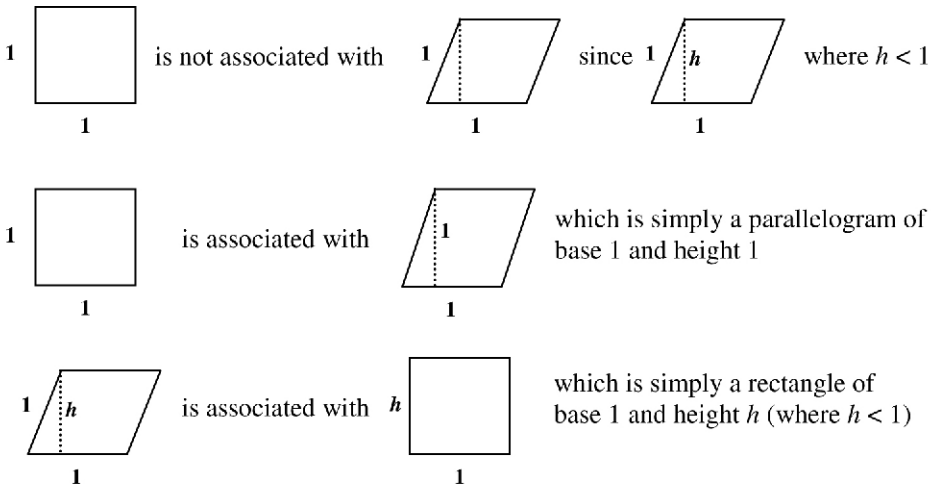
One insight Cavalieri offers here, is a sensitivity to related figures drawn between the same pair of parallels (that is, of same height), which draws attention to the notion of "family" of figures resembling one another in terms of some attributes (i.e., same base and same height, and same chord length all around). These figures become associated. This association leads also to a possible autonomy in the selection of the "simplest" member of that family, in order, for example, to calculate its area – often this simplest form can be seen as rectangular in character.

[3] These sorts of illustrations/demonstrations of equivalences within families could easily and insightfully be made using a Dynamic Geometry Software (e.g., Geometer's SketchPad) in order to keep constant the area while moving the bases along parallel lines.

Thinking box – mathematical reflection #1:

Thinking of squares and rhombuses in terms of the Cavalieri's principle appears interesting, but also puzzling since these are normally associated with the usual phrasing of "a square is a rhombus, with angles at 90°". This is still true here, but it becomes a little bit trickier in regard to families of parallelograms, since rhombuses and squares are not necessarily directly linked in these, which can lead to false assumptions concerning their respective areas. In a word, a square is not the associated "rectangle" of the rhombus (except if the rhombus is square).

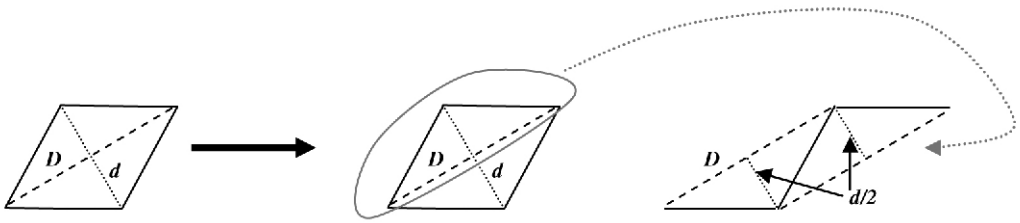
For example, in terms of families of parallelogram, the square with a side of length 1 cannot be compared to a rhombus with a side of length one, since the height of the rhombus would be smaller than 1 and would therefore not be associated to the square as it would be of a smaller height and so of a smaller area. The rhombus of side 1 would be associated with a rectangle of base length of 1 but of height of same length as the rhombus, which would not be the square.



Thinking box – mathematical reflection #2:

Understood in that way, a rhombus becomes defined as a parallelogram that has its base and side of same length (but not of same height). Hence, seeing rhombuses in terms of parallelograms raises concerns in terms of the "data" that is usually given to calculate its area. Whereas the diagonals of the rhombus are usually given (or have to be found to apply adequately the formula), seeing the rhombus as a parallelogram leads one to expect that its "base" and "height" be provided – a practice that appears quite unusual for rhombuses.

This said, even if only diagonals continue to being given, Cavalieri's "thinking" in terms of families can still lead to interesting results without resorting to the rhombus formula with its diagonals. Here is an illustration:



What one ends up with is a parallelogram with a base of length D and of height d/2, which has the same area and can be transformed in its associated rectangle of dimensions D and d/2.

Cavalieri's Principle and Trapezoids

Another quadrilateral studied is the trapezoid. Cavalieri provides a means to identify families of trapezoids, which always contains a "rectangular" trapezoid (that is, a trapezoid where one side is perpendicular to the base), that could be seen as the simplest form (see Figure 4). There are again two such trapezoids, one left-facing and one right-facing.

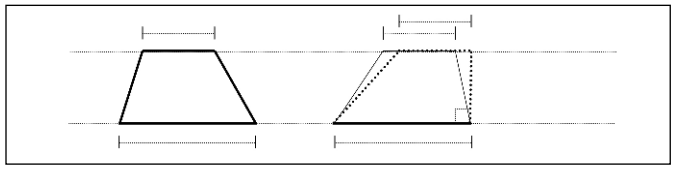


Figure 4. A family of trapezoids with same height and same base

This perspective of family affords a way of interconnecting all the different sorts of trapezoids (scalene, rectangular, isosceles) together. As a family of trapezoids gets established, one can notice that a trapezoid is indeed defined as "a quadrilateral with two sides parallel" (Wolfram MathWorld web site) which takes into account not only standard trapezoids that are typically offered (see Figure 5), but also any quadrilateral that has a pair of opposite sides that are parallel, which are indeed trapezoids, without necessarily making it a parallelogram (see Figure 6)[4]. In that sense, the family of trapezoids is composed of any quadrilateral that has the same height and a pair of opposite and parallel sides, making the family of trapezoids look like the following (Figure 7).

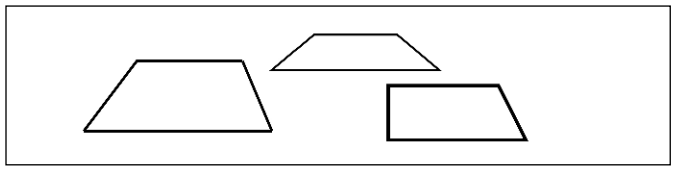


Figure 5. Some examples of standard trapezoids



Figure 6. Another type of trapezoid

*[4] I used the expressions "standard" and "typically offered" because from the glance that I gave to textbooks and web sites, I only found one web site that offered a picture different from the ones offered in Figure 5 and along the type presented in Figure 6 (**http://id.mind.net/~zona/mmts/geometrySection/commonShapes/trapezoid/trapezoid.html**). It seems indeed that this type of trapezoid is not a form often studied.*

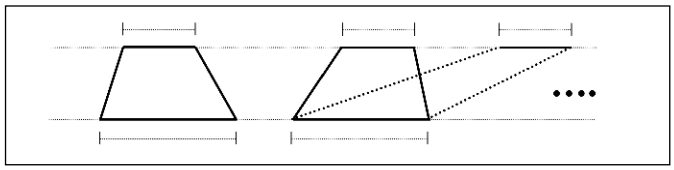


Figure 7. A family of trapezoids with non-standard ones

No family of equivalent trapezoids contains a rectangle. However, there is a fixed relationship between a trapezoid and its associated rectangle. By rotating the rectangular trapezoid about the midpoint of the remaining slant side, a rectangle that is the double of the trapezoid is produced (see Figure 8), establishing an interesting and significant relationship between trapezoids and rectangles. Trapezoids become perceived as the half of a related rectangle that has the same height and a base equal to the sum of both the trapezoid's bases[5].

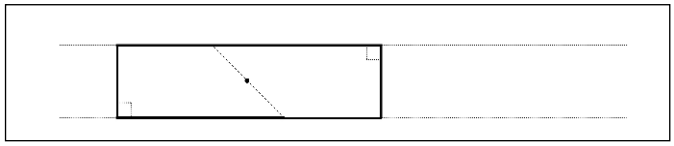


Figure 8. Two copies of a right trapezoid creating a rectangle

Cavalieri's Principle and Triangles

Cavalieri's principle can also be applied to triangles and enables connection of any given triangle with a family of area-equivalent triangles. As Figure 9 shows, all the triangles in this family are equivalent to a right-angled triangle (again, there are two of those).

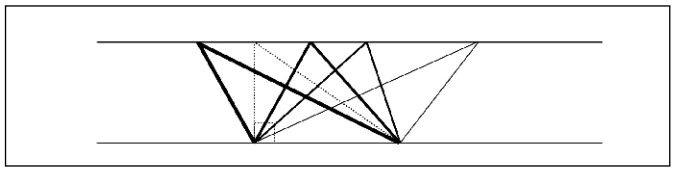


Figure 9. An infinite family of Cavalieri-equivalent triangles

As with trapezoids, there are no rectangles in this family either. But, again, it is possible to establish a relationship between the right-angled triangle and its associated rectangle, which is once more the double of it

[5] As well, as one reviewer noted, there is always an isosceles trapezoid ( ) in every family and this standard form could be cut in two identical pieces along its middle/mirror line ( ) in order to subsequently drag one of these pieces to create a rectangle with bases being the average of both small and long bases (→). However, here, the idea of the 1:2 ratio is to some extent lost. This said, both transformations could lead to an interesting exploration in regard to how both ways of transforming the trapezoid end up being equivalent.

(obtained by rotation about the mid-point of the hypotenuse – see Figure 10). As some people do (e.g., Jamski, 1978), this rotation or double of the triangle can lead triangles to be called "half rectangles," a connotation that emphasizes the relationship existing between a triangle and its associated rectangle[6].

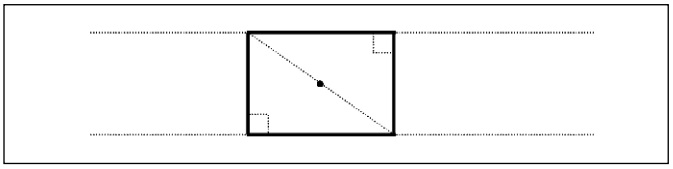


Figure 10. Two copies of a right-angled triangle forming a rectangle

Thinking box – mathematical reflection #3:

This conceptualization of triangles within a family that ties together all area-equivalent triangles (that is, with same base and same height) appears mathematically powerful, especially concerning non-standard forms (ones that are to be decomposed into other simpler forms to evaluate their area). As one knows, any polygon is always decomposable in triangles, whatever its form. Therefore, because a triangle can be related through its family to a simpler form, it can, for example, permit an easier calculation of all the different elements of decomposition which leads to a more accessible assessment of the area of a given polygon. In a word, any polygon is decomposable in triangles and these triangles can be transformed in simpler forms, rendering more accessible issues of decomposable or unusual polygons.

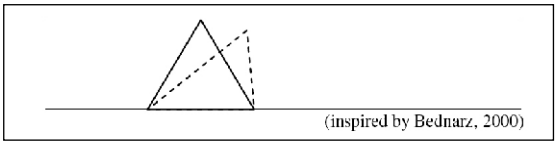
Using Cavalieri to Establish Relationships and Ratios

The last two examples of trapezoid and triangle have indicated how Cavalieri can be used with repetition of a particular figure, in order to focus on its fixed relationship with a more familiar figure – in this case, a rectangle. Cavalieri's principle therefore highlights families and relationships between planar figures. Earlier on, when discussing rectangles, parallelograms, squares and rhombuses, the term "associated" meant that two figures belonged to a Cavalieri-area-equivalent family, and hence the area relationship was 1:1. However, in the latter two examples with trapezoids and triangles, the notion of "associated figure" was extended to one which in each case the areas were in the ratio of 1:2. Hence, Cavalieri enables the establishment of 1:1 relations among rectangles, parallelograms and rhombi, and 2:1 relations between rectangles and trapezoids/triangles. These ratios and families relate and connect different figures together through geometric links, not necessarily through formulas or calculations (since nothing was calculated in these examples), and appear as fascinating geometric links between planar figures.

Thinking box – mathematical reflection #4:

In addition to offer means of interrelating figures and establishing the equivalence of their area, Cavalieri's "thinking" also offers the possibility of making qualitative comparisons that end up in "greater" or "lesser" area of a figure in contrast to another.

For example, if one is faced with the following two triangles with the same base, one need not resort



to calculations, as the family association can directly be called upon which automatically leads to the comparison that the second (dotted) triangle has a smaller area than the other. Similar comparisons could be made with triangles of same height but of different bases.

Conclusion

The historical principle of Cavalieri, approached in the way offered in this paper, provides appealing geometrical insights to make sense of the area of planar figures and to establish geometrical relationships and comparisons between the different figures. These insights appear mathematically powerful as they offer tools for reasoning about planar figures and their area in terms of relationships and connections concerning their attributes; it tightens the links existing between planar figures with respect to their area.

In addition to offer these insights, the principle of Cavalieri was brought in as an illustration of the range of possibilities and insights that can be drawn from specific developments that have occurred in the history of mathematics. Through its use, the history of

[6] As was said for the trapezoids, there is the presence in this triangle family of an isosceles triangle that could be cut along its median/"mirror" to create a rectangle through dragging one of its pieces.

mathematics can surely motivate and raise students' curiosity toward specific concepts and topics. Furthermore, it can also stimulate profound mathematical thoughts in us, mathematics educators; thoughts that, in addition to fostering motivation and curiosity, have the potential to foster mathematical reasoning and understanding in our students.

References

Bednarz, N. (2000). *MAT 3224, Didactique II: Document de référence, didactique de la mesure et questionnement.* Montreal, QC: Université du Québec à Montréal, Département de Mathématiques.

Cavalieri, B. (1635). *Geometria indivisibilibus continuorum nova quadam ratione promota.* Bologna: Clemente Ferroni. (A method for the determination of a new geometry of continuous indivisibles.)

Gray, J. (1987). *Unit 9: The route to the calculus* (MA 290: Topics in the history of mathematics). Milton Keynes, Bucks: The Open University.

He, J.-H. (2004). Zu-Geng's axiom vs Cavalieri's theory. *Applied Mathematics and Computation, 152*, 9-15.

Jamski, W. D. (1978). So your students know about area? *The Arithmetic Teacher, 26*(4), 37.

Janvier, C. (1992a). Le volume comme instrument de conceptualisation de l'espace. *Topologie Structurale, 18*, 63-75.

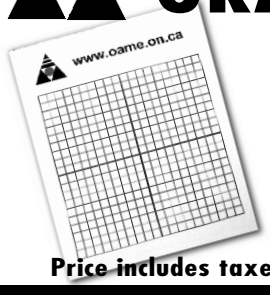
Janvier, C. (1992b). *Le volume, mais où sont les formules? Un vidéo sur l'enseignement des mathématiques au secondaire* [VHS/color/33mins.]. Mont-Royal, QC: Éditions Modulo.

Janvier, C. (1994). *Le volume, mais où sont les formules – document d'accompagnement.* Mont-Royal, QC: Éditions Modulo.

Janvier, C. (1997). Grandeur et mesure: la place des formules à partir de l'exemple du volume. *Bulletin de l'Association Mathématique du Québec, 37*(3), 28-41

Wolfram Research. Mathworld: The web's most extensive mathematics resource. **http://mathworld.wolfram.com** ▲

OAME STICKY GRAPH PADS



GREAT FOR STUDENT USE!

3 colours, 2.5" x 3"

50 sheets per pad

$9.00 for 10 pads

Price includes taxes. Shipping and handling extra.

Order by mail, fax or on-line:

OAME

or Order ON-LINE www.oame.on.ca

Do not send cash or cheque. You will be invoiced with shipping and handling costs added.

▲ Using E-STAT to Prepare for the Teachable Moment: Teacher Candidates Demonstrate How Easy It Is.

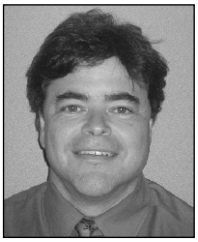
DOUG FRANKS

MIKE McCABE

BARBARA OLMSTED



Doug Franks is an associate professor in the Faculty of Education at Nipissing University, teaching junior/intermediate mathematics. He is also chair of Nipissing's Master of Education program. His research interests include applications of technology in mathematics education, and mathematics professional learning communities.



Mike currently teaches mathematics and health and physical education in the concurrent teacher education programme at Nipissing's University's Brantford campus. His research focuses on parents assisting their children with mathematics homework and the development of critical reflective practices skills in teachers.



Barbara Olmsted is an Assistant Professor in the Faculty of Education, teaching Junior/Intermediate Health and Physical Education to pre-service teachers. Dr. Olmsted also teaches Leadership in the new Bachelor of Physical and Health Education degree program. Her research interests include the engagement of pre-service teachers in appropriate health education practices and the development of inclusive strategies to enhance physical education participation in elementary school children.

Introduction

Imagine if you will, the typical Monday morning as you prepare for the upcoming day and week with your students in grades 7 and 8. You are busy at your desk

as the students start to trickle in. They are excited and full of tales about their past weekend. Through the dull murmur of the increasing number of voices, you hear two of your students discussing a party they had attended on the Saturday night. They start to rehash the events and this ultimately leads to stories about certain students and their behaviours while "partying". Although you are not sure of the exact details, you manage to understand that there was alcohol and marijuana at the party. Also, there was talk of sexual activity.

What now? Should you intervene? How? What will you say?

Every teacher talks of the 'teachable moment' and wishes they were more prepared to deal with it when the opportunity presents itself. It is important to you to be more than an adult voice warning the students of the pitfalls of such behaviours. You feel as though you should be more armed with the facts before you intervene. Instead, you tell the party-going students that their discussion is not appropriate and to "quiet down!" They respond by saying, "It's no big deal, Miss. Everybody is doing it!" You look at them sternly and think to yourself, "Can this be true? Are all my young, apparently not-so-naive students engaging in these risk behaviours?" How can you find out how many of your students are engaging in such behaviour?

The Health Behaviour of School-Aged Children (HBSC) has data housed on Statistics Canada's web site **http://estat.statcan.ca/content/english/hbsc/hbsc_start.shtml** which contains within it an electronic time series database (E-STAT) from over 250 different surveys updated annually from 35 countries all over the world. This web site includes health and lifestyle data gathered from children aged 11, 13 and 15, and is organized according to topics such as personal injuries, food choices, personal relationships and risk behaviours including tobacco, alcohol and drug use and sexual experience. The core of the information demands an understanding of various strands of the mathematics curriculum (with a strong emphasis on data management) and the use of technology. There are also direct links to the health curriculum. The recently redesigned web site can now prepare you for this and future 'teachable moments'. It can also help our students understand the magnitude of their involvement in health-related issues and provide the vehicle for their participation in character education, all the while increasing their understanding of mathematics.

This paper is the result of current research conducted within a teacher education programme. Faculty of Education students were introduced to the data on the Statistics Canada web site and were asked to create Microsoft PowerPoint® presentations linking the data to the Ontario Mathematics **http://www.edu.gov.on.ca/eng/curriculum/elementary/math.html** and Health curriculum **http://www.edu.gov.on.ca/eng/curriculum/elementary/health.html**.

Literature Review

Technology in Education

The Canadian Teachers' Federation (2003) identified that teachers felt ill-prepared to implement computer technology in their classrooms. As with many initiatives in education, there is the sense that to evoke significant change, implementation should take place at the teacher candidate level. In a survey given to teacher candidates, Waller (2006) found that nearly 94% were eager to integrate technology into their practice teaching lessons. As stated by Bahr, Shaha, Farnsworth, Lewis, and Benson (2004), the opportunity to use technology in practice teaching enhanced teacher candidates' attitudes towards further use in a teaching setting.

Interdisciplinary Teaching

With an increased demand on teachers to meet curricular expectations comes an increased importance on the development of interdisciplinary planning and implementation. This may be particularly relevant for teachers who are responsible for all subject areas within their classrooms. As demonstrated by Bullock, Park, Snow, and Rodriguez (1999) there is some resistance from teacher candidates regarding the concept of interdisciplinary planning. The authors propose that this is due to the experiences these teacher candidates had as students, namely being taught in a discipline specific manner. A number of studies (Spalding (2002); Akerson & Flanigan (2000); Bullock et al. (1999)) dealt with Faculties' efforts to model effective interdisciplinary instruction. Spalding found the three most frequent responses given for the utility of interdisciplinary learning were as follows: 1) important for student learning, 2) important for understanding the subject area, and 3) relevance to "real life".

Statistics Canada Web Site

Lundy (1998) and Lundy and Graham (2002) have used E-STAT with their teacher candidates to investigate the effectiveness of an inquiry-based model as it related to the social context of the catchment areas of their teaching placements. The authors noted that E-STAT

provided an excellent resource of reliable data that is applicable to use in a variety of subject areas and immediately applicable to classroom use. It also informed students of necessary social issues related to teaching in today's classrooms.

Methodology

Two hundred ninety-six teacher candidates, working in pairs, were provided with a health topic related to the HSBC data. These topics were comprehensive and directly linked to the four strands of the Ontario health curriculum (substance use and abuse, personal safety and injury prevention, human growth and development, and healthy eating). Each PowerPoint® presentation included the following:

1. An Introduction to the topic.
2. A list of the specific expectations from the Ontario mathematics curriculum and Ontario health curriculum.
3. A visual display of at least two different graphic representations of their data using either the graphing capabilities within E-STAT or some other graphing software.
4. A list of discussion-based questions they could be incorporated into the particular grade of their choice (between grades 4-10). These questions were meant to challenge the students' understanding of the mathematics as well as to stimulate discussion around critical issues and character education.

Through surveys and focus group interviews, the teacher candidates provided feedback as to the effectiveness of the resource. Their feedback resulted in two significant occurrences: a revamping of the current HBSC web site to become more teacher-friendly, and the development of a new web site **www.nipissingu/education/iteachhealth** to assist teachers in integrating mathematics with seemingly marginal subjects such as Health.

The Questionnaire

A total of 240 out a potential 296 (81.1%) teacher candidates responded to questionnaire. The questionnaire consisted of demographics, assignment-based, and practicum-based sections. The assignment-based section was a 50-item survey, using a 5-point Likert scale, of the students' views of the assignment—its meaningfulness, and the effectiveness of implementation of the HBSC data. The practical-based section of the questionnaire dealt with the teacher candidates use (if any) of the database during their practice teaching placements. They were asked to identify the nature of their use of the HBSC database (subject area, setting and context) and whether they experienced barriers to its use (computer availability, internet access).

Focus Group Interview

Seventeen students took part in three focus group interviews. The protocol for these interviews was based on the SWOT model—Strengths, Weaknesses, Opportunities, and Threats of the use of the HBSC data in creating their presentations. The results of both the questionnaires and the interviews were then merged where appropriate to tell a meaningful story about the various elements of the project. These acted as the impetus for the creation of a new web site (as presented in the following sections) and the revamping of the HBSC data on E-STAT

The 'Outcomes and Discussion' section below will focus on the products of the students' efforts, namely their PowerPoint® presentations and the resultant changes to the E-STAT resource based upon the feedback they provided.

Outcomes and Discussion

With regard to the original question pondered by the teacher in the opening scenario, ("Are all middle-school-aged students engaging in such risk behaviours?"), a quick perusal of the HSBC data can help teachers prepare for much needed lessons for these vulnerable students. Once these data are accessed, organized, displayed and understood, students and teachers can lead discussion or debate regarding the social implications of their findings. In addition to the basic background knowledge provided by the data, there is great potential for the middle school students to complete their own investigations of critical issues that are perhaps of special interest to them: (substance use, nutrition, personal feelings, bullying, leisure time, friends…). This allows teachers the opportunity to take learning far beyond the scope of the traditional mathematics classroom, incorporating character education issues while having the students display their understanding of data management.

This meets the specific expectation from the Ontario Mathematics Curriculum data management and probability strand,

> *Grade 6: collect and organize discrete or continuous primary data and secondary data (e.g., electronic data from web sites such as E-STAT or Census At Schools) and display the data in charts, tables, and graphs (including continuous line graphs) that have appropriate*

titles, labels (e.g., appropriate units marked on the axes), and scales (e.g., with appropriate increments) that suit the range and distribution of the data, using a variety of tools (e.g., graph paper, spreadsheets, dynamic statistical software) (Ontario Mathematics Curriculum, 2005).

and from the Ontario Health Curriculum substance use and abuse sub-strand expectations.

Grade 6: determine influences (e.g., interpersonal, personal, legal, economic) on the use and abuse of tobacco and other drugs (e.g., alcohol, cannabis, LSD) and consider them as part of a decision-making process to make healthy choices (Ontario Health and Physical Education Curriculum, 1997).

Access to the data opens the possibility of creating the "teachable moment" through discussions related to the social and personal impact of risk behaviours as well as potential strategies to combat the pressures that adolescents face in today's world. The topics of discussion provide a natural lead into the development of resistance techniques and assertiveness skills as required in the health curriculum (e.g., Grade 7: Demonstrate resistance techniques and assertiveness skills to deal with peer pressure) (Ontario Health and Physical Education Curriculum, 1997).

The figures and table below are taken from the teacher candidates' presentations. As previously mentioned, the students had the option of using the E-STAT graphing capabilities or exporting the data to other software.

Each display provides teachers the opportunity to ask mathematics-relevant questions to ensure students have the necessary prerequisite knowledge to participate in the discussions and subsequent lessons mentioned above. Also, students are required to recognize and display all essential elements of graphing (e.g., titles, axis, labels, legends…).

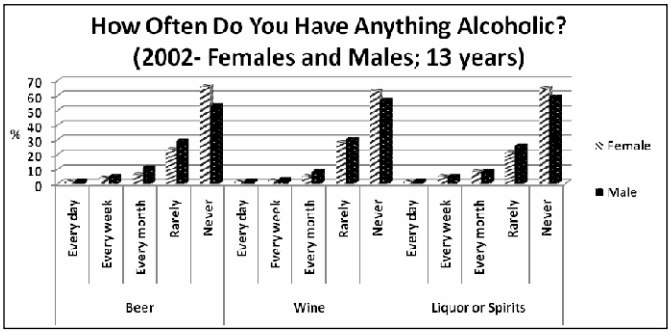


Figure 1. How often do you drink anything alcoholic? (Canadian males and females- 13 years.)

In addressing the issue of alcohol consumption in 13 year olds (Figure 1), this student teacher has displayed data by gender, exporting the information from the E-STAT web site to Excel®. This can lead to discussion comparing the frequency of Canadian male and female consumption and possible reasons for differences. Similar data are also available for 11 year olds and 15 year olds, providing the opportunity for longitudinal analysis and potential reasons for changes in frequencies. Linking such data to recent literature, (e.g., newspaper articles or social movements such as Mothers Against Drunk Driving (MADD)) can bring to bear the implications of alcohol consumption. The presentation on the **www.nipissingu/iteachhealth.ca** web site provides additional discussion questions for teachers use.

In addition to Canadian students, the HSBC data provides information for 35 countries worldwide. This, again, provides potential for discussion, this time incorporating social studies issues. Figure 2. demonstrates the ability to export the data and display it using Excel® graphing capabilities.

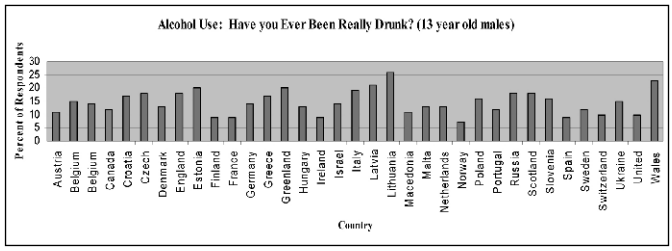


Figure 2. Alcohol Use: Have you ever been really drunk (13 year old males- all countries)

Alternative strategies to displaying data include tables identifying categories of interest and the potential for limiting the time period for information (past 12 months as indicated in Table 1 below). This allows students to make sense of the data with or without a graphic representation. The tables are provided within the HBSC database and can easily be exported.

Table 1 Have You Used or Taken Cannibis in the Past 12 Months?

QUESTION: HAVE YOU USED OR TAKEN CANNABIS IN THE PAST 12 MONTHS?

Geography = Canada | Sex = Females
Age group = 15 years | Time period = In the last 12 months

Student response	Never	1 or 2 times	3 to 5 times	6 to 9 times	10 to 19 times	20 to 39 times	40 times or more
2002	62%	12%	7%	5%	5%	4%	4%

Of course, the issue of peer influence plays a significant role in adolescents' decision making. Some of the questions within the HBSC data direct the focus on peer behaviour. This allows teachers to help students

make connections between peer group behaviour and personal decision making. Understanding the data in Figure 3, displayed in a pie graph provided within the database, leads potentially to students comparing their peer group to those of the rest of Canadian girls their age. Teachers could use the issue of tobacco use to introduce the concept of converting whole numbers to fractions and percent, having the students link their peer group numbers to the graphs, (e.g., "Of your closest 20 friends, determine which category you would best fit into on the pie graph"). This targets expectations from the operation sense sub-strand of the curriculum, (e.g., grade 8: use estimation when solving problems involving operations with whole numbers, decimals, percents, integers, and fractions to help judge a reasonable solution). (Ontario Mathematics Curriculum, 2005).

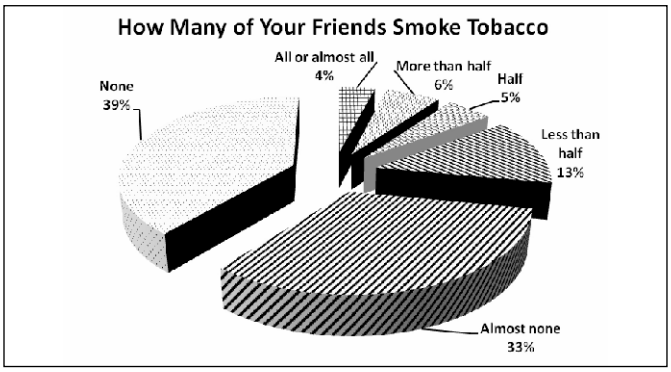


Figure 3. How Many of Your Friends Smoke Tobacco? (Canadian Females-13 yrs, 2002)

Conclusion

Most teachers identify time as the limiting resource when preparing for and implementing lessons. When health-related topics such as substance use and abuse and human growth and development suddenly present themselves as "teachable moments" it is our role as educators to be prepared to address the subject with as much reliable information as possible. Our profession's increased reliance upon technology makes the Internet an invaluable resource and provides potential to, at the very least, find the available data and formulate it into meaningful lessons. If we can integrate a number of subjects and build a social conscience regarding issues such as the ones presented here, even better. The HBSC data on the E-STAT web site provides an ideal tool to use an understanding of mathematics as the basis for the development of these lessons.

The participants in this research told us, through their presentations, that the Statistics Canada data are extremely enticing for use in their classroom. They also told us how we needed to transform the sites into useful, readily available lessons to assist in their efforts to integrate subjects such as mathematics, health and social studies. To this end, efforts have been made to heed the teachers' message. The continued partnership between Statistics Canada and our Faculty of Education's program has now resulted in the development of related parallel teaching resources. On the E-STAT web site, **http://estat.statcan.ca/content/english/hbsc/hbsc_start.shtml**, the health data have been collated to 'best fit' health curriculum according to a number of broad categories such as risk behaviour, friends and relationships, and bullying and fighting, to name a few. On our university's web site, technology driven Microsoft PowerPoint® presentations directly related to these topics have been created and make the data even more readily available to teachers.

References

Ackerson, V.L., & Flanigan, J. (2000). Preparing teacher candidates to use an interdisciplinary approach to science and language arts instruction. *Journal of Science Teacher Education, 11*, 345-362.

Bahr, D.L., Shaha, S.H., Farnsworth, B.J., Lewis, V.K., & Benson, L.F. (2004). Preparing tomorrow's teachers to use technology: Attitudinal impacts of technology- supported field experience on pre-service teacher candidates. *Journal of Instructional Psychology, 31*, 88-97.

Bullock, P., Park, V., & Snow, J. (2002). Redefining interdisciplinary curriculum: A journey of collaboration and change in secondary teacher education. *Interchange, 33*, 159-182.

Canadian Teacher's Federation. (2003). *An investigation of the integration of information and communication technology into Canadian school and classrooms.* Canadian Teacher's Federation. Retrieved August 11, 2006, from **http://www.ctf-fce.ca/bilingual/pubs/ctfreport/ictenglish.pdf**

Lundy, J. (1998). Pre-service teachers explore their practicum communities: Mapping & graphing the social context of schooling using E-STAT. *Canadian Social Studies, 32*, 130-134.

Lundy, J., & Graham, R. (2002). Using Statistics Canada data and lesson plans in senior elementary grades. *School Libraries in Canada, 22*, 15-17.

Ministry of Education and Training. (1998) *The Ontario Curriculum, Grades 1-8: Health and Physical Education.* Toronto: Author.

Spalding, E. (2002). Of organelles and octagons: What do preservice secondary teachers learn from interdisciplinary teaching? *Teacher and Teacher Education, 18*, 699-714.

Waller, K. (2006). *The ITeach laptop learning program computer integration study: A descriptive overview of pre-service teacher use of computer technology during the practice teaching experience.* Unpublished manuscript. ▲

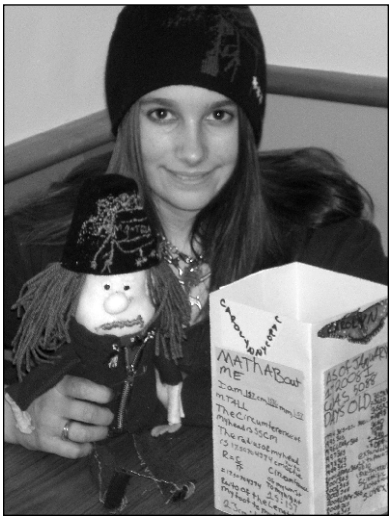
▲ Mathematical Mini Me's: What Would Dr. Evil Think?

MARIA CASASOLA

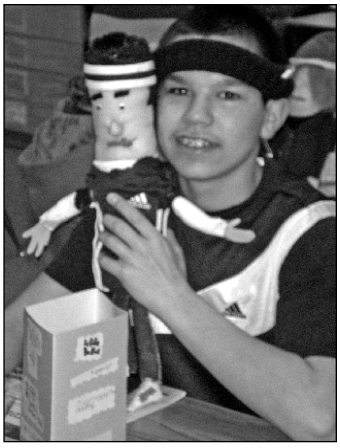
Maria Casasola is a grade eight math and art teacher at Claude E. Garton School in Thunder Bay.

This is the third time I have been asked to have my grade eight students create mathematical centerpieces for the NWOAME conference. I have had over two years to think about this and try to come up with an idea that would engage students and hopefully impress the math teachers in Northwestern Ontario who would be our audience. I generated, dismissed and quite simply forgot a plethora of ideas. As much as I liked how we had merged art and math in the past, I wanted to do something different. The deadline was getting closer and still there was no muse sitting on my shoulder whispering in my ear with the perfect solution.

I didn't realize that I was surrounded by muses every day in my classroom. Granted they are far too large and noisy to sit on my shoulders. All I had to do was let the students be my inspiration. We were in the middle of their Fathers of Confederation puppet plays when an idea started to formulate. I had already taught nearly 60 students how to sew the noses, ears and lips – which is quite a feat since I am largely self-taught. They already know how to safely use the low-melt hot glue guns to put clothing together. I knew what we would do. They would use the art techniques they learned making their puppets for history and apply them to make a self-portrait puppet or action figure.



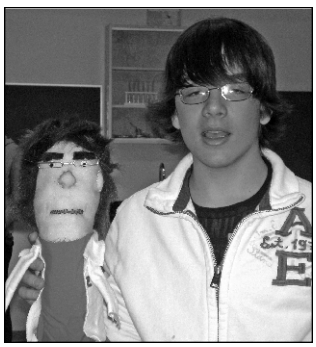
Since the idea was to be a self-portrait, they also needed to represent themselves mathematically. The next problem was how to display the math about them in such a way that the centerpieces could stand up and not fall into somebody's lunch. Did Michelangelo have to think about these things? I adapted an idea in a book to make a paper cube. Well, it really wasn't a cube and my grade eights would immediately question it being called a cube. I of course am pleased when they question. I want them to be skeptics who need proof before they accept any statement as truth. We decided to name it an open rectangular prism. While we are skeptical, we are working towards efficiency. So in the end we conveniently called it an O.R.P.



It was beginning to look like we might actually pull this off. The remaining issue was what should I have them do for their mathematical self-portrait? I wrestled with this idea for a long time and I'm not sure that I got it entirely correct. I determined that, for the panels that they needed to create, half would be directed and tied directly to the grade eight curriculum that we already covered. In contrast, the remaining half would be open ended. I didn't know what would happen with those portions, but I was willing to see where the math took us.

Our mathematical journey was fairly predictable in many cases and depended upon students' abilities and interests. The highlight for me was when one boy in my class decided to figure out what percentage of the Canadian population his family of five was. In a game of mathematical one-upmanship his seating partner went on to determine what percentage of the world population his family of four was. It became a learning opportunity for the use of technology to show really large and small numbers. It created a real need for use of scientific notation. Some of the best trips are the ones you never plan.

I share with you the planning sheets, rubric and photographs not as a model of perfection but rather as record of what we did. I am always thrilled with the successes of my students. They worked really hard and approached this task with seriousness. I think that they did a fabulous job. Perhaps they will act as your mathematical muse and inspire you to take some chances.



Mini-Me: A Math Self-Portrait

Create a self-portrait puppet/action figure. Say hello to your own little Mathematical Mini Me. Accompany it with a write up explaining your "personal math".

Pay attention to the details. It needs to be like you, as well as delight your viewers and intrigue their mathematical minds.

Math

Level 1	Level 2	Level 3	Level 4
• gives inappropriate but partially complete explanations • rarely uses appropriate mathematical terminology	• gives appropriate but incomplete explanations • sometimes uses appropriate mathematical terminology and symbols	• gives appropriate and complete explanations • usually uses appropriate mathematical terminology and symbols	• gives appropriate and complete explanations showing many connections • always uses appropriate mathematical terminology and symbols

Art

• applies few of the skills, concepts and techniques taught (sewing, nylon stuffing, facial features, hair construction, clothing design, hands, feet, glue application) • communicates math ideas in limited and incomplete ways • centerpiece is only visually appealing from a single view through the limited use of design principles and details • uses critical and creative thinking processes with limited effectiveness	• applies some of the skills, concepts and techniques taught (sewing, nylon stuffing, facial features, hair construction, clothing design, hands, feet, glue application) • communicates math ideas in complete but simple ways • centerpiece is visually appealing from some views through a partial use of design principles and details • uses critical and creative thinking processes with some effectiveness	• applies most of the skills, concepts and techniques taught (sewing, nylon stuffing, facial features, hair construction, clothing design, hands, feet, glue application) • communicates math ideas in complete ways with increasing sophistication • centerpiece is visually appealing up close and from a distance through the use of design principles and details • uses critical and creative thinking processes with considerable effectiveness	• applies all (or almost all) of the skills, concepts and techniques taught (sewing, nylon stuffing, facial features, hair construction, clothing design, hands, feet, glue application) • communicates math ideas in complex and well-developed ways • centerpiece is visually appealing up close and from a distance through the complex use of design principles and details • uses critical and creative thinking processes with a high degree of effectiveness

Get Stuffed – Be Your Own Math Action Figure







Mini-Me: A Math Self-Portrait Template

Use pictures, numbers and words to show:

- How many days have you been alive?
- Use January 1, 2008 as the cut-off date

Now that you have the answer, express that number using :

- Scientific Notation
- Expanded Form using Powers of 10

Use pictures, numbers and words to show the physical you:

- How tall are you in mm, cm, and m?
- What is the circumference of your head?
- Use the circumference to determine the radius of your head.
- What is the ratio of your wrist circumference to your height?
- What is the ratio of the length of your foot to your height?

Half of the template is directed and reviewed some concepts. The remaining panels are more open ended allowing students more choice

---

Use pictures, numbers and words to show the math about the people who are important to you:

- Math about your family:
- Math about your friends.

Use pictures, numbers and words to show your favourites:

- What is your favourite number?
- Express it at least three different ways.
- Explain why it is your favourite number:

The mathematical ideas can be easily changed depending upon your focus.

Mini-Me: A Math Self-Portrait Art Supplies

- Knee-highs
- Stuffing
- Embroidery thread (red, skin tone)
- Google eyes
- Yarn
- Faux fur
- Old clothes
- Low-melt glue guns
- Low-melt glue sticks
- Clear nail polish to stop runs
- Card stock to make open rectangular prism (O.R.P.)
- Interesting paper
- Unlimited imagination and perseverance!!!

Mini-Me: A Math Self-Portrait Open Rectangular Prism (O.R.P.) Instructions

1. Use two sheets of the same size paper. We used card stock. As long as you don't have to support anything, then even regular paper will do. Fold one side 3 cm shorter than the other side. This will make a tab that extends out 3 cm on one side.
2. Fold the long side over the short side of both sheets of paper, making tabs.
3. On one of the folded papers, place a small amount of glue along the small folded tab, next to the crease but not in it.
4. Place the non-folded edge of the second sheet of paper square into the crease and fold the glue-covered tab over this sheet of paper. Press flat until the glue holds. Repeat with the other side.
5. Before continuing allow the glue to dry completely. Once the glue has dried, the O.R.P. can be collapsed flat for easier storage.



▲ Director's Dialogue

ANN MICHELE STENNING



Ann Michele Stenning is a mathematics teacher at South Secondary School with the Thames Valley District School Board. Having finished her Masters of Education degree in Curriculum Studies, she is currently assisting other parent volunteers develop weekly literacy activities which she and other moms provide to "at risk" students.

Board of Director's Meeting, Feb. 8-9, 2008, Holiday Inn Select, Toronto Airport

Friday, Feb. 8, 2008

OAME president, Jacqueline Hill, welcomed the OAME board members and thanked the committee chairs for their effort during the past year. John Kersley, the NCTM representative, shared some resources, namely a book of math cartoons. Tom Boland, a member of the promotions committee, showed the board members where to find a powerpoint presentation on the benefits of OAME membership. He also noted other features of OAME's web site, namely, the OAME road show and the GUM project. Bill Otto, the chair of the constitution committee, began a discussion on OAME's future direction. Of specific interest was identifying OAME's strengths, obstacles, concerns and recommendations for OAME. Suggestions for the future included the use of technology to support districts that want or need technology like Adobe Connect and Skype. Other board members suggested that OAME use Podcasts and wikipedia as well as the member's only section of the OAME web site to provide professional development to interested teachers.

Saturday, Feb. 9, 2008 Board Meeting

Professional Development:

Kathy Kubota and Mary-Lou presented mathematical ideas which demonstrated complexity. Kathy and Mary-Lou had board members take a number, double it, add 6, double that, subtract 4, divide 4, then subtract 2. The result would equal the original number. In the classroom, Kathy noted that the number students select indicate a student's comfort level with numbers. To further develop number sense, Kathy urged classroom teachers have students try this exercise using integers and fractions. Lastly, this question could also be used to introduce algebra by writing the solution to this problem using a variable. The result would look like {2(2x+6)-4}divided by 4 then -2.

Another problem posed to board members was the Legs Problem. There are 44 legs in the room. People are sitting on stools around one table with no empty stools. Each stool has 3 legs. One table has 4 legs. How many people are around the table? Show 2 different solutions. Through the use of the varied approaches to the solution, Kathy identified solutions that included repeated addition, repeated subtraction, table of values and algebra. Mary Lou stressed the importance of naming these strategies for students. Kathy and Mary Lou were thanked by Anna Jupp for making complexity less complex. For pictures of these multiple solutions, visit the OAME web site and look for the link titled Ontario Bansho: Interpreting Complexity Theory.

Once committee meetings finished, board members were introduced to Carol Matsumoto, the Canadian affiliated services committee (ASC) representative with NCTM. Board members learned of a Canadian resources page, found under the link titled lessons and resources on the NCTM web site. Carol discussed the possibility of planning a Canadian leaders conference through NCTM. NCTM's web site provides complete details on the benefits and restrictions of NCTM membership.

Curriculum Committees:

Committee chairs shared the following highlights:

JK-6: They discussed activities with Rekenrek and ideas to insert into the bundled Abacus.

7-9: The committee shared lessons, reviewed, and edited around Math in Flight. They will pilot with 2-3 teachers/levels. Student samples will be reviwed in June. They may include video. They lessons are in the MATCH template and focus on processes and DI. They will polish the project in June.

10-12: They revised some tasks in GUM 2. Some tasks received a second edit, others a first review.

CHAPTER REPORTS

CHAMP - Rep: Lynda Ferneyhough:

CHAMP had a very successful miniconference in October. Plans are already underway for next year's fall conference. We also ran a challenging contest. We had more schools participating this year. Thanks to Sonia Dwarka for her incredible dedication in organizing and marking the contests. We continue to hold our meetings at the Dufferin-

Peel Catholic School Board offices on the first Thurs. of the month. We would welcome anyone interested in becoming a part of the executive. Contact Lynda at FMSS 905-495-2675 for more information.

COMA - Rep: Pat Kehoe:

Our annual Mini-Conference was held on Saturday, November 17, at the University of Ottawa. The conference was re-named this year as the Ronald C. Bender Memorial Conference in honor of our former COMA president who died this past year. Plans are under way at the University of Ottawa to sponsor a scholarship in Ron's memory. The conference was very successful with over 100 teachers attending. Sixteen workshops were offered from Primary to Secondary divisions. Our OAME 2009 chairpersons, Paul Gautreau and Cathy Hall, continue to work tirelessly on plans for next year's provincial conference. Any COMA members who would like to join the organizing committee are encouraged to contact either of the Chairpersons.

GOLDEN - Rep:Laura Tonin:

We are starting to organize the annual conference for 2010. Our planning committee is selected and we are starting to discuss presenters and hospitality ideas.

GVMA - Rep: Rob Gleeson:

No report.

ISOMA - Rep: Diane Lang:

Our chapter hosted its Fall Conference, “Celebrating Sharing” on November 22. Over seventy teachers participated. We had wonderful sessions ranging from Math and Music to Gambling to Sharing Ideas about the New Grade 12 Courses. Conference participants provided some excellent suggestions for future activities and directions.

MAC^2 - Rep: Danielle Blair:

MAC^2 is currently establishing a Financial Mathematics Award for qualifying graduating students from the Simcoe County, Simcoe Muskoka Catholic, and Trillium Lakeland Boards. The award, which will look beyond achievement to include attitude, excitement for subject and leadership, is planned to be available starting next year and will be a yearly award offered on behalf of MAC^2 to support deserving students. Finally, MAC^2 is working on plans to host a “Roadshow” parent presentation that will be made available to parent groups/parent councils of the three boards covered by our chapter. A subcommittee is currently working on clarifying the MAC^2 mission statement in order to infuse our vision into the presentations.

NOMA - Rep: Sue Melville:

No report.

NWOAME - Rep: Tom Boland:

Our Math Olympics team is also busy planning our regional event and plan to once again bring our winners to the Provincial OMO. We are pleased to have some new members coming out to our regular meetings and anticipate new and renewed interest in the organization as a result of the upcoming conference.

PRMA - Rep: Sandra Jean Price:

The Pine Ridge Math Association is very proud to be hosting OMO 2008 on May 30th and 31st. Facilities at Trent University in Peterborough have been booked, and the committee chairs have been established. Jacqueline Hill, Fred Pulfer, Ruth Cornale, Kevina Morrison, Manuel Salvati, Charles Wyszkowski, Bernie Beales and many others are busy planning the events for students and coaches. We are looking forward to this exciting weekend.

QSLMA - Rep: Robert Sherk:

This past term, the QSLMA group has hosted a fall mini-conference and continued to promote OAME activities through our email list serve. Our conference was a success with approximately 70 people attending. On Saturday morning, Nov. 10th, at Queen's University, we had a program with 10 workshops, keynote speakers Rod and Anne Yeager and an luncheon as well. Thanks to the Queen's MSTE group for co-sponsoring the event.

SAME - Rep: Debbie Drape:

Two goals that we had for this year were to increase membership and to try to get more members on the executive. We have a number of very enthusiastic math-oriented educators who have joined our executive team and their assistance and input during the preparation of our mini-conference was greatly appreciated. As for general membership, we have encouraged participants at our conference to become members and have provided the membership registration form with our conference information. We will

continue to promote membership at all our events.

SWOAME - Rep: Joe Goulet:

No report.

TEAMS - Rep: Laurissa Werhun:

Discussions for the location of OMO 2011 have already begun. That's planning ahead! The constitution is close to being finalized. The TEAMS executive continues to provide a web newsletter that is sent to all current TEAMS members with information about upcoming events as well as some recommended resources such as web links.

WOMA - Rep: Ann Michele Stenning:

WOMA's fall mini-conference entitled "Smarten Up" was attended by over a hundred educators. As in the past, publishers were in attendance to display their products while participants enjoyed complimentary snacks and drinks. Conference attendees learned of ways to implement Smart Boards into their classrooms. A $10 fee was charged and supper was included.

Y^{4}MA - Rep: Ada Cartaginese:

Y^{4}MA has been busy planning the annual OAME conference, "The Path is Made by Walking". ▲

continued from page 21

first and second differences in the making of manipulative models for various relationships. Terry responded to these comments by recognizing that the colour scheme for the models was getting in the way of understanding for some students and began the next class with acknowledging that parts of the activities during the previous class on quadratics may have been confusing for some and she provided a detailed sheet to help clarify ideas and sort out confusion caused at the end of the previous class. She also reflected that when she does this activity with another class she will concentrate on building the models first and worry about the colours later.

Summary Comments

Questioning, listening, and responding to student thinking form a major component of formative assessment as this process provides valuable information about student thinking and understanding as well as reflection on pedagogical moves. However, the task of incorporating these processes in a classroom is not an easy one. We saw that Terry had created a classroom environment that encouraged discourse, conjecture, questioning, and collaboration to advance and enhance student thinking through a multiplicity of rich experiences.

Rich experiences require planning. Teachers need to carefully think about what questions to pose, to listen attentively, and respond in ways that value student solutions and continue to probe student thinking. Such activities place the emphasis on making student thinking public and provides space for students to be active in their learning, to investigate and explore mathematical ideas and to communicate and extend those ideas.

References

Black, P., & Wiliam, D. (2006). Developing a theory of formative assessment. In J. Gardner (Ed.), *Assessment and learning* (pp. 81-100). London: Sage.

Davis, B. (1996). *Teaching mathematics: Towards a sound alternative.* New York: Garland Publishing.

Suurtamm, C., & Graves, B. (2007), *Curriculum Implementation in Intermediate Mathematics* (CIIM): Research Report. Toronto, ON : Queen's Printer.▲

2008-2009 OAME Board

Direct General Inquiries to:
EXECUTIVE DIRECTORS
Dave & Sue Hessey

PRESIDENT
Anna Jupp

PAST-PRESIDENT
Jacqueline Hill

PRESIDENT-ELECT
Beverly Farahani

VICE-PRESIDENTS
Gordon Cooke

Lynda Ferneyhough

Anne Yeager

CHAPTERS & REPRESENTATIVES
CHAMP: **Shaun Knowles**

COMA - **Pat Kehoe**

GOLDEN - **Laura Tonin**

GVMA - **Robert Gleeson**

ISOMA - **Diane Lang**

MAC^2 - **Michele Walma**

NOMA - **Carol Lemmon**

NWOAME - **Tom Boland**

PINE RIDGE - **Sandra Jean Price**

QSLMA - **Robert Sherk**

SAME - **Debbie Drape**

SWOAME - **Nicholas Chevalier**

TEAMS - **Laurissa Werhun**

WOMA - **Ann Michele Stenning**

Y^{4}MA - **Ada Cartaginese**

DIRECTORS -TERM EXPIRING 2009
Anna D'Armento

Anna Dutfield

Paul Gautreau

Duncan Leblanc

Laurie Moher

Walter Whiteley

DIRECTORS -TERM EXPIRING 2010
Dennis Caron

Glynnis Fleming

Cathy Hall

Connie Quadrini

Silvana Simone

Trish Steele

DIRECTORS -TERM EXPIRING 2011
Paul Alves

Jan Crofoot

Merilyn Fox

Sue Melville

Bill Otto

Dwight Stead

Lynn Strangway

NCTM REP UNTIL AUG 31/08
John Kersley

NCTM REP AFTER SEPT 1/08
Laurie Moher

WEB SITE CO-ORDINATOR
Gregory Clarke

GAZETTE EDITOR
Marilyn Hurrell (Until June 30/08)

Stewart Craven (After July 1, 08)

ABACUS CO-EDS
Mary Lou Kestell

Kathy Kubota-zarivnij

OAME web site www.oame.on.ca