Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Fifth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Stewart Craven, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Stewart Craven at **numeratecitizen@mac.com**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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Editor

▲ **Stewart Craven**

Associate Editors

▲ **Anna Dutfield**

▲ **Shirley Dalrymple**

▲ **Tom Steinke**

***Abacus* Co-Editors**

▲ **Mary Lou Kestell**

▲ **Kathy Kubota-Zarivnij**

Design and Production

▲ **Penny Clemens,** Graphic Designer

Printing & Binding

▲ **Pole Printing,** Box 69, 89 King Street East
Forest, ON N0N 1J0 (519) 786-5112

Advertising Manager

▲ **Robert Sherk**

Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising Rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

STEWART CRAVEN, EDITOR



My students and I are now at the beginning of the winter/spring semester at the Ontario Science Centre Science School. As expected, this new crop of young people is alive with curiosity, enthusiasm, and creativity. My colleagues and I are fortunate to be able to work with such talented students, all of whom share the desire to know as much about science and mathematics as they possibly can.

This term, I decided to take a different approach to introducing my Vectors and Calculus course to the class. Because this group of students had the opportunity to get to know each other over the course of a four-day orientation, I was confident that I could immerse them in an activity that required a great deal of trust and co-operation. Late Friday afternoon, I projected the following statement onto an overhead screen:

> *Be it resolved that the teaching and learning of mathematics beyond a superficial understanding of arithmetic should be confined to the realm of a small elite, and hence, other citizens should not be burdened with mathematics education beyond grade school (i.e., Grade 8).*

This statement served as an introduction to a debate in which two opposing groups of ten were to prepare opening and closing remarks over the weekend. It was my assumption that both groups would have preferred to argue *against* the proposition, but I couldn't have been more wrong! Given that both teams wanted to argue *in favour* of the proposition, a coin toss was employed to break the deadlock.

The debate was magical. Both sides put forward well thought-out arguments that highlighted a plethora of key issues with respect to mathematics education. For example, the "pro" side argued that teaching students who had little or no interest in mathematics was an immense waste of resources. They also presented the idea that many adults claim that mathematics beyond arithmetic plays no role in their day-to-day lives. To establish this point, one student surveyed 20 adults (she met these people at a ski resort) and discovered that 90 percent of them could not remember a single mathematical concept from high school! In other words, why should society waste its time teaching ideas that have no relevance to the lives of most people? The "pro" side also argued that even if people were to study mathematics up to the end of high school, unlike "riding a bicycle" —a skill that you retain for life—most people would

flounder if they had to take a university mathematics course even two or three years later.

Students on the "con" side of the debate argued that people need mathematics to be employable in many jobs. (Jobs they mentioned included engineer, biologist, physician, physicist, mathematician, and actuary). They also took issue with the idea that mathematics is the only subject deemed irrelevant to our daily lives because most people have relegated much of what they learned in history, geography, and science class to the dust heap as well! They further argued that people, perhaps subconsciously, draw on their knowledge of a broad range of ideas, including mathematics, to make informed decisions and to better understand and appreciate the world around them.

We did not declare a winner of the debate, but we did continue our discussion of the place of mathematics in our lives with respect to decision making; political analysis; scientific, financial, and historical understanding; job performance; aesthetics; and recreation.

I would like to provide one more example of a student response to mathematics. Scott was a student in my fall semester class. From the moment I met him, I knew that he was special. I have never witnessed a student observe a mathematics demonstration with such rapt attention. (It was a demonstration of Cabri 3D). My final examination for Scott's class included the following question:

> *Suppose you are having a conversation with a student in Grade 10. Provide a convincing argument for this student that the Grade 12 Functions course is a "must." Include in your discussion, three applications outside the classroom for the functions that we have studied.*

Scott had already "aced" the exam by the time he reached this question, so he decided to take a philosophical approach. His eloquent response was three pages long and included this thought:

> *I do not believe education [has] its meaning in its content, or even necessarily [in] the skills one must develop. Our educational system is designed to "bring kids up to speed" on* **humanity**.

I couldn't agree more, except that Scott is perhaps more optimistic than I in his use of the verb "is." If our educational system isn't already designed "to 'bring kids up to speed' on humanity," it certainly should be.

It's clear that our students benefit from a multi-faceted approach to mathematics that includes an emotional side. Scott's last sentence in his essay comes from his heart:

> *The meaning of life* is proportional *to Mathematical Understanding* [and] **inversely** proportional *to Ignorance.* ▲

▲ Letters to the Editor

I just finished reading the OMG while travelling from London, England to Toronto. ... I must say that the issue is impressive. I like the theme of equity and differentiated instruction, the great article on combining calculus and vectors especially. I really liked your editorial piece, [but] noticed an error in the statement of Euler's Equation. Probably you've already got feedback on that. The "–" should be a "+".

...The articles on differentiated instruction, StatsCan math resources, and Calculus-Vectors integration so impressed me that I ... sent them on to the teachers at my son's school. I think the recommended book in the DI article should serve as a PD resource for the staff.

Congratulations on a really good publication.

Gord Doctorow

Editor's Note: This was one of a couple of e-mails I received regarding my egregious error. (The "–" should have been a "+" in Euler's equation.) I'd like to be able to say that I was just checking to see if anyone reads my editorials, but... ▲

▲ President's Message

ANNA JUPP, PRESIDENT



Anna has been working as an Instructional Leader of Mathematics with the Toronto District School Board for the past 10 years. She supports K–8 teachers though the development and facilitation of professional learning opportunities, resource development, and consultation. Anna has contributed to various Ministry initiatives, including the Expert Panel Reports for the Early Math Strategy and Education for All. Anna is also a mathematics AQ course instructor with OISE and has been part of an author team for primary math programs. She has served as a Board Director since 2001 and 3 years as Vice-President before becoming President. Anna was the recipient of the 2005 Union Gas Award for Outstanding Leadership in Mathematics.

Embracing Change in Mathematics

Over the past several months, we have experienced much change locally, nationally, and internationally. The election and inauguration of Barack Obama as President of the United States has changed what we thought was possible. With the shifting economy throughout the world, priorities and spending habits have adjusted to address financial uncertainty. Here, at the OAME Board of Directors, spring brings with it a change in presidency and makeup of the Board. In this, my last message as president, I invite you to look ahead and consider the benefits of embracing change in the teaching and learning of mathematics.

Change and the Unknown

In planning our math courses, programs, and lessons, we must go beyond the knowledge and skills outlined in the Ontario curriculum and consider the demands of the changing world in order to ensure that we are meeting the needs of the twenty-first-century learner. It is the current reality that we, as classroom teachers and math educators, are preparing students for jobs that currently don't exist. In front of us, we have individuals who will be expected to use technology that has yet to be invented.

What the future will bring is unknown, but what we do know is that critical thinkers, effective communicators, and technologically literate citizens will be better equipped to meet the challenges of the unknown. We must ask ourselves: *What are the math skills that students will need in the future? What important mathematics will they need to know and be able to do in order to be successful? Which classroom experiences will be most beneficial for the learning of mathematics?* I direct you to the OAME Vision and Guiding Principles as a guide for making content and instructional decisions.

A Vision for Learning Mathematics

The learning of mathematics should be based upon the following beliefs and principles:

a) Learning is structured around the key concepts of mathematics.

b) Learners build on prior knowledge and understanding and work at a developmentally appropriate level to construct new personal meaning.

c) Learners are better able to transfer and use knowledge meaningfully when new ideas are presented within rich and meaningful learning contexts, and when learners make connections both within and outside of school.

d) The learning process is enhanced when new ideas are presented in a variety of ways and when all learners have access to "hands-on" and "minds-on" activities and opportunities. This includes the use of manipulatives, computers, calculators, and hand-held data-collection devices.

e) Learning is enhanced when students experience a variety of instructional approaches, ranging from direct instruction to inquiry-based learning.

f) Learners require opportunities to interact with each other during the learning process, to teach others, and to explain and justify the processes of mathematics. Learning is enhanced when the learners are encouraged to communicate their understanding.

g) Effective mathematics education requires strategies that engage and motivate learners, that help students develop positive attitudes about themselves and about mathematics, and that encourage learners to make positive contributions to the learning environment.

h) Learning is enhanced when students and teachers understand learning expectations and assessment standards and participate actively in assessing and evaluating their personal growth.

i) Learning is enhanced when teaching, coaching, assisting, supporting, and feedback is planned and designed to improve specific learning and mental processing skills and to accommodate different learning styles and needs.

j) Learning is enhanced when teachers plan and reflect together.

k) Learning is enhanced when all participants in the process are active partners within a community of learners (e.g., parents/guardians/caregivers, educators, students, and the community).

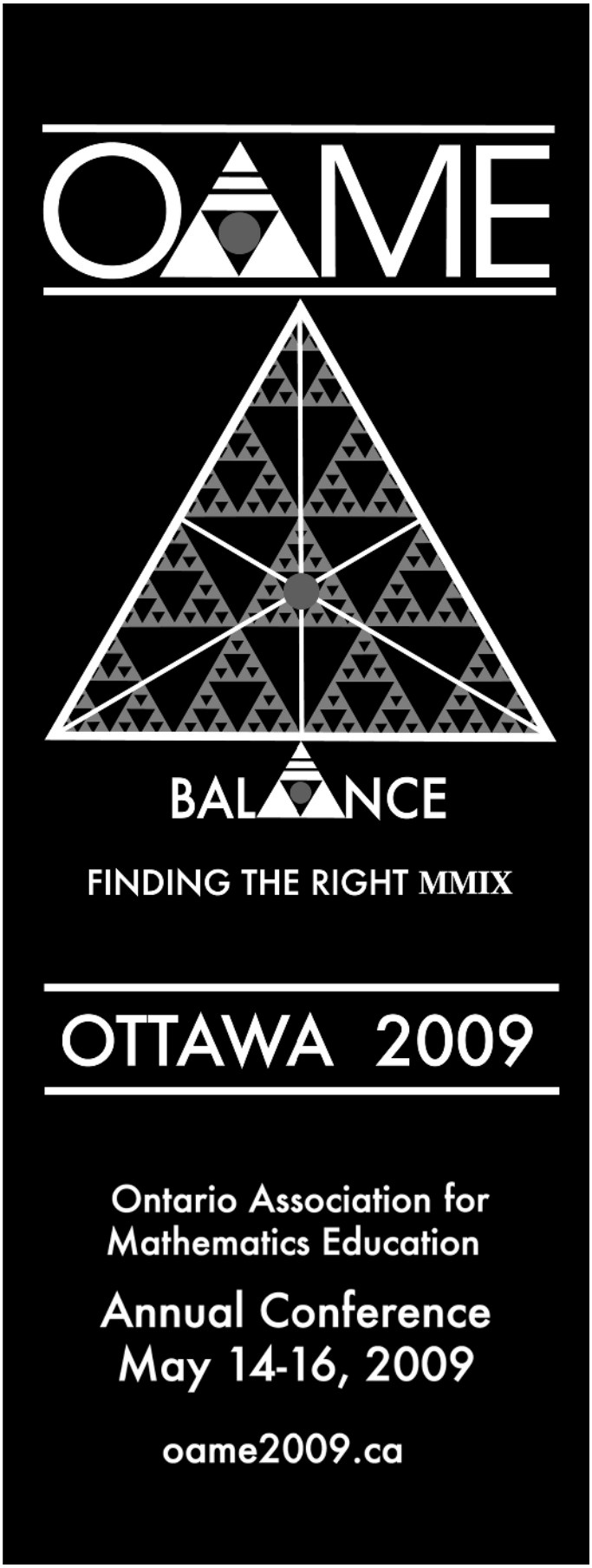
Trying on Change

I call on you to try to implement instructional strategies that may be new or unfamiliar, but that will potentially meet the diverse needs of your students. Consider Bansho as a way of consolidating learning and building communication skills. Practise teaching through problem solving, so that students think critically about the concepts and skills they are learning. Investigate new technologies and how they might be used to enhance the teaching and learning of mathematics in your grade or course, thus providing students with skills that can be taken beyond the classroom and into the future.

Change through Connections

Change is exciting, but not simple. When considering change, students' needs must be kept at the forefront. Collaboration with colleagues, participation in professional development opportunities, and risk taking will bring excitement, new approaches, and the formation of new partnerships and better mathematics learning environments.

It has been my pleasure to serve you as OAME president. For me, it has been a year of change, together with ongoing opportunities for learning, collaboration, and personal challenge. I thank the members of the Board and Executive, especially the executive directors, for their support and encouragement. I also thank all of you in the math community who continue to share my excitement and passion for mathematics. ▲



▲ Technology Corner – Connecting Manipulatives and Technology

MARY BOURASSA



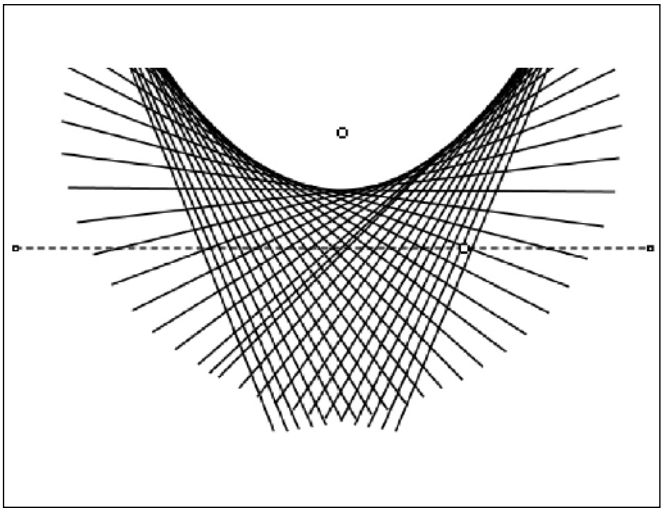
Mary Bourassa teaches mathematics at Lisgar Collegiate Institute in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. A past-president of COMA, Mary gives workshops around the world and is a National Instructor for Texas Instruments.

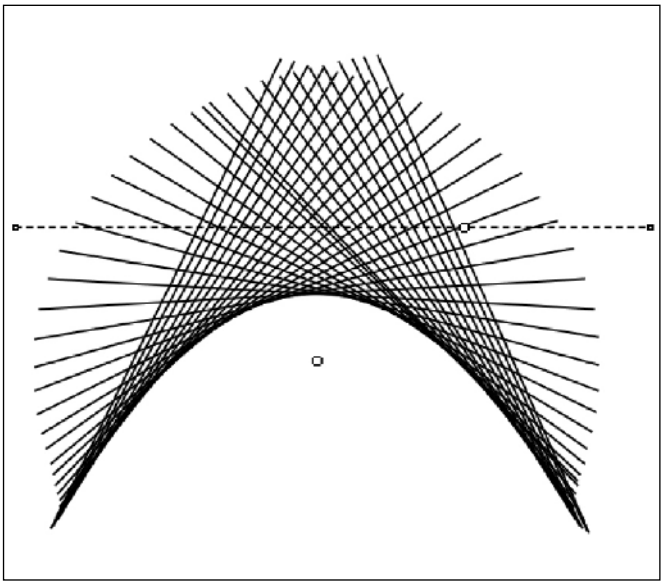
Although I strongly believe in the power of technology to help students discover mathematics and further their understanding of a wide variety of topics, manipulatives also continue to hold an important place in our curricula. Many students need these tools to help them grasp concepts or transition from concrete information to more abstract generalizations.

During first semester, I introduced the geometric definition of the parabola to my Grade 10 gifted class. As in previous years, each student received a piece of waxed paper and repeatedly folded one edge to a fixed point. Although it is not part of the Grade 10 academic curriculum, this activity was a great success. It was amazing to hear my students' exclamations when they figured out what they were creating. I followed this up with an activity using technology, in which the students constructed the locus of a fixed point and a line.

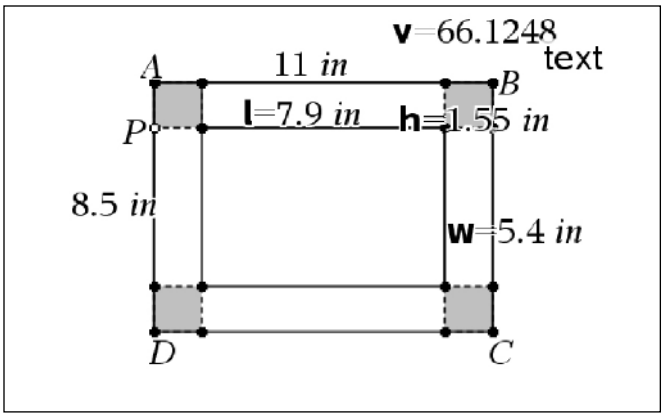
Because a locus is not something that the students had previously heard of, we demonstrated several examples before starting the TI-Nspire™ portion of the activity. I chose one student, Ivy, and asked another eight or nine students to stand equidistant from Ivy. The locus of the points (students) created a circle. Another student, John, then stood up, and I told the others to now stand equidistant to Ivy and John. The locus now created a perpendicular bisector. The students also created a parabola, with one student as the focus and the wall as the directrix.

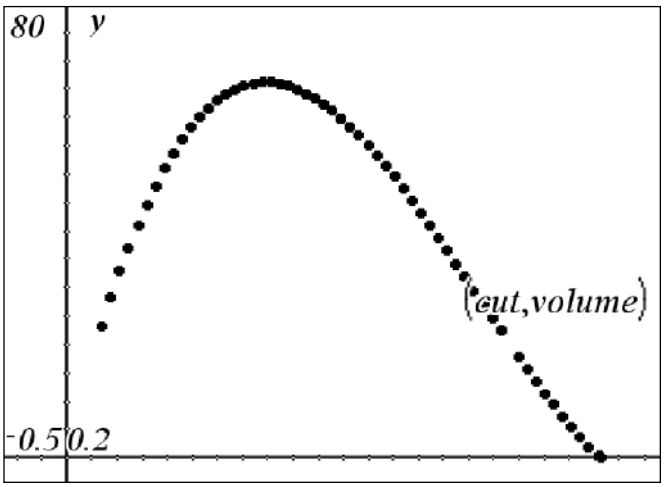
When students subsequently construct the locus using TI-Nspire or The Geometer's Sketchpad, they completely understand what is happening and what the lines represent. The advantage of technology is that they can move the focus and see the effect instantaneously.

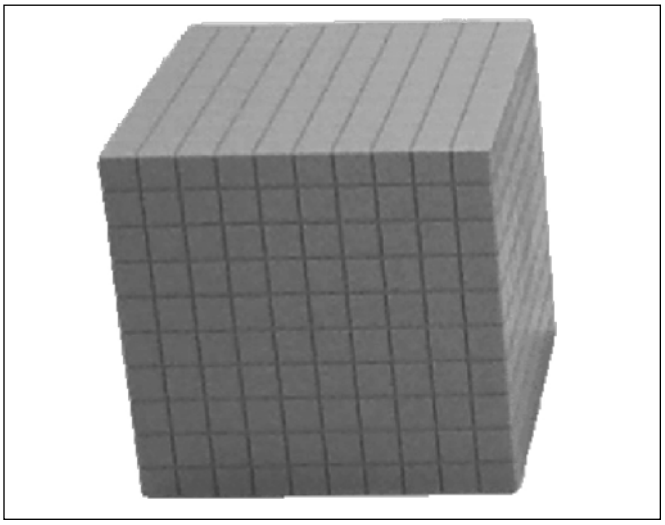




Whether students are in Grade 9 or in Grade 12, the "exploring the box" problem is significantly easier if students create their own boxes. Given a piece of paper of fixed size, students must cut out congruent squares from the corners, then fold up the sides to create an open-topped box. Fred Ferneyhough created a wonderful version of this activity for TI-Nspire, but the physical creation of boxes makes the entire problem more meaningful. Students also like to compete to see whose box actually holds the largest volume. Following this up with the activity using technology allows students to rapidly collect lots of data for the various possible sizes of the square. Depending on the grade level, students can narrow in on a numerical solution, model the data and find the maximum, or use calculus to determine the optimum value.







The Painted Cube activity is another example where manipulatives greatly enhance the activity. Given a cube of side length 10 that is submerged in a can of paint, students must determine the number of individual cubes with paint on 0, 1, 2, and 3 sides. To accomplish this, they look at patterns starting with a cube of side length 3. Having cube-a-links available for groups to construct the cubes is invaluable, as many students have great difficulty visualizing a three-dimensional object. Once students have collected data for several cubes, they can enter the data on a graphing calculator or on a spreadsheet. Looking at the resulting scatter plots for side length versus the number of cubes with paint on *x* sides, students can determine appropriate models, which can be used to extrapolate to a cube of side length 10.

Combining the power of technology with the hands-on value of manipulatives creates a rich learning environment for our students and helps them better understand and enjoy mathematics. ▲

CARTOON CONTEST

Create a cartoon based on a mathematical object or concept (1, 2, 3, or 4 frames)

Winning entries will be published in the Gazette

All artwork and captions must be original.

OPEN TO: Group #1 - any elementary school student
Group #2 - any secondary school student

GROUPS JUDGED SEPARATELY: Winning entries will be published in the *Ontario Mathematics Gazette* starting with the June 2009 issue (and in the subsequent 4 issues).

DEADLINE FOR SUBMISSIONS: March 31, 2009

SUBMIT ENTRIES as jpegs to: Stewart Craven

ENTRIES MUST INCLUDE: Student Name, Grade, Present Teacher, Student Contact Information, School and School Address/Phone Number

▲ OAME/NCTM Report March 2009

LAURIE MOHER

Laurie is an Instructional Coach Facilitator for Literacy and Numeracy (JK–8) with the Kawartha Pine Ridge DSB. She has served on the Board of Directors of OAME for 12 years.

In his January/February President's Message, Henry (Hank) Kepner talked about the importance of an ongoing commitment to professional development and urged mathematics teachers to be intentional about their professional development in the coming year. He further encouraged teachers to create a clear, well-defined professional development plan, and suggested that teachers consider:

- improving their knowledge of mathematical content
- expanding their knowledge of instructional strategies that will promote student learning and engagement
- exploring the appropriate use of various classroom assessments

Once teachers have a professional development plan, NCTM offers several resources to assist them in meeting their professional development goals.

Enhanced Journal Articles

Many schools in Ontario are using Professional Learning Communities (PLCs) as a way of coming together as a staff (or as a hub of teachers from different schools) for the purpose of learning together. NCTM's Enhanced Journal Articles can be a valuable resource for such groups. Each Enhanced Article provides a link to the journal article (with options to "sign in," "join NCTM," or "purchase"), followed by a rationale, suggestions for use, procedures, discussion questions, and extensions. There is a wide selection of enhanced journal articles available for all of NCTM's journal publications, including ON-Math. These enhanced articles serve as models for using journal articles as starting points for professional development.

NCTM Position Statements

NCTM has produced position statements on a variety of topics. Of particular interest to OAME members who attended the 2009 Leadership Conference is the September 2008 position statement entitled *Algebra: What, When, and for Whom.* The position statement entitled *Teaching Mathematics to English Language Learners* relates to NCTM's PD Focus of the Year, Equity: All Means ALL. Other position statement topics include Closing the Achievement Gap; Assessment; Early Childhood Mathematics; Equity; and Technology. These can be found by following the link in the President's Corner entitled "President's Messages," followed by the sidebar link "Position Statements." NCTM position statements can serve as a stimulus for discussion among teachers, or as a resource for groups of teachers who wish to develop their own definitions or position statements on a particular topic.

NCTM E-Workshops

NCTM is offering a number of 90-minute interactive E-Workshops in an effort to encourage groups of teachers from far-reaching locations to come together to learn about a variety of topics. These workshops provide an opportunity for teachers to learn, share, discuss, and ask questions in a collaborative setting. Teachers go away from each workshop with at least one task to try with their students. They are then offered an opportunity to come together for a second 90-minute follow-up session where they discuss the impact of the activities on student learning and plan further activities. Some sessions are offered as one-time three-hour workshops.

The workshops are organized around specific topics and often target particular grade levels. (NCTM has recently added sessions for K–5 teachers.) Some of the topics for March include:

- Implementing the Algebra Standards in Grades 9–12
- Meeting the Needs of All Students
- Math Games for the Classroom in Grades 6–8
- Teaching with Curriculum Focal Points in Grades 3–5

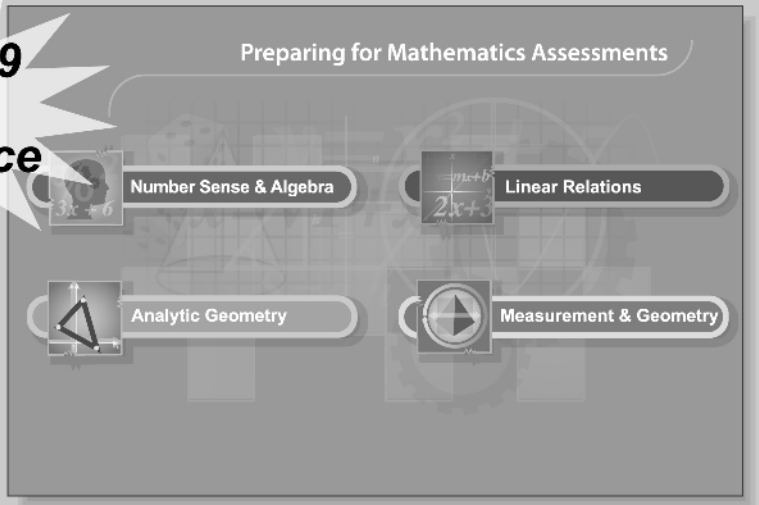
NCTM E-Workshops cost $149 and are delivered via a combination of Internet and phone. With a projection screen and speakerphone, a group of teachers can participate in a session with the payment of a single fee. ▲

Preparing for EQAO Math Assessments

This software is designed to prepare students for the grade 9 academic and applied EQAO math assessments. Randomized sample questions with detailed interactive and model solutions are included.

Grade 9 EQAO Resource

The program contains 74 activities, organized into four strands:



Free Trial Available At: www.nectar.ca/mathAssessment_demo/



Helps students improve *mathematical reasoning*, *analytical*, *computational*, and *problem solving* skills.

Teachers can use the program for:

- *Individualized learning by assigning specific questions to students with skill deficits;*
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- *Assessment purposes;*
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▲ What's the Problem? It's Hip to Be Square!

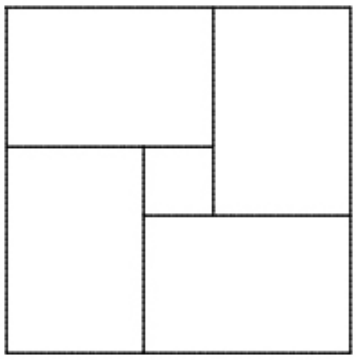
SHAWN GODIN



Shawn is the head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans, ON.

Welcome back problem solvers! Last issue, I left you with the following problem:

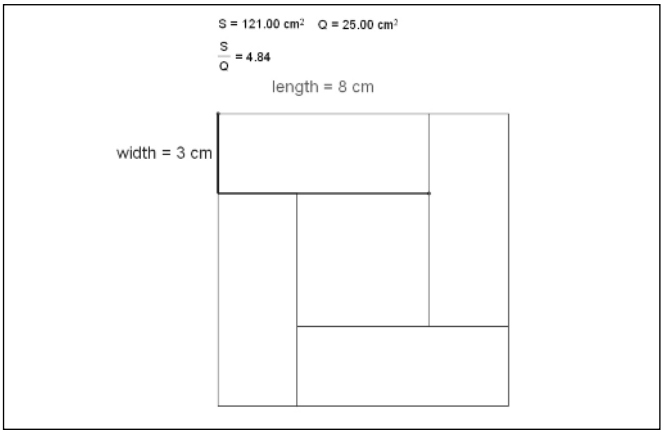
Four congruent rectangles are arranged in a square pattern so that they enclose a smaller square. Let S be the area of the outer square and let Q be the area of the inner square. If the area of the outer square is 9 times the area of the inner square, determine the ratio of the sides of the rectangles.



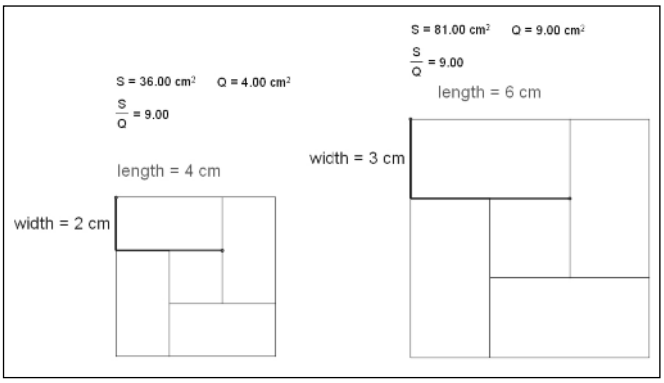
This problem appeared as M310 from the October 2007 issue of *Crux Mathematicorum with Mathematical Mayhem*, and its solution appeared in the September 2008 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem* website at **http://journals.cms.math.ca/CRUX/**.

Like many mathematical problems, this one cries out for experimentation. This might involve drawing some examples with pencil and paper or constructing some examples with geoboards or linking cubes. I tend to be drawn to technological solutions, so I constructed a sketch with The Geometer's Sketchpad. You can access this sketch by e-mailing me at **shawn.godin@ocdsb.ca**.

My sketch reproduces the diagram from the problem. The length and width of the top rectangle are changeable, but have integral length. The areas of the outer and inner squares are measured and the ratio is calculated.



It doesn't take much experimenting before you come across a solution (or possibly many). As you can see below, rectangles with dimensions 2 cm by 4 cm and 3 cm by 6 cm both give the desired ratio of areas. Thus, the ratio of the sides of the rectangles is 2:1.



Are we done? We could be, but there are a couple of important questions we should ask:

1. Is this the only solution?
2. What is the relationship between the ratio of the sides of the rectangles and the ratio of the areas of the squares?

To answer these questions, we need to go back to our sketches or models. It shouldn't be hard to see that the side of the large square is the sum of the length and the width of the rectangles, while the side of the small square is their difference.

Thus, we have $S = (l + w)^2$ and $Q = (l - w)^2$. Hence, $\dfrac{(l + w)^2}{(l - w)^2} = 9$,

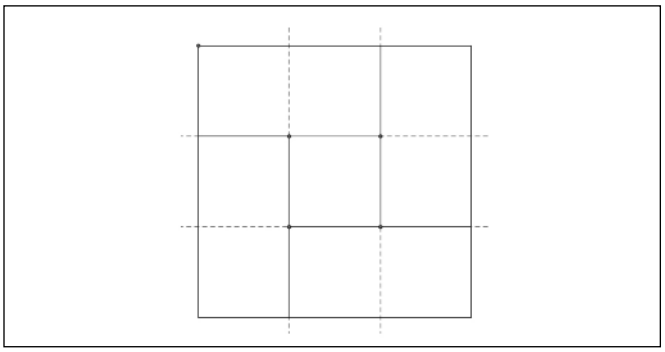
but since the length and width are both positive, and because we can assume, without loss of generality, that the length is bigger than the width, we have, after taking square roots,

$$\frac{l+w}{l-w} = 3$$
$$l + w = 3l - 3w$$
$$4w = 2l$$
$$l = 2w$$

Thus, we have shown that the only way that we can get the ratio of the areas of the squares to be 9 is if the ratio of the sides of the rectangles is 2:1. In the process, we have also seen how the areas of the squares are related to the sides of the rectangles.

Let's look at one more solution before we get to the homework, shall we? Above, I have presented a very standard and "satisfying" algebraic solution/proof of the problem. But consider the following solution:

By the symmetry of the diagram, the smaller square is centred in the larger square. Since the area of the larger square is 9 times the area of the smaller square, the sides of the smaller square are $\frac{1}{3}$ the length of the larger square. As a result, if the sides of the smaller square are extended, they trisect the sides of the larger square, and show that the ratio of the sides of the rectangles must be 2:1. Consequently, the diagram below could serve as a "proof without words (or algebra)."



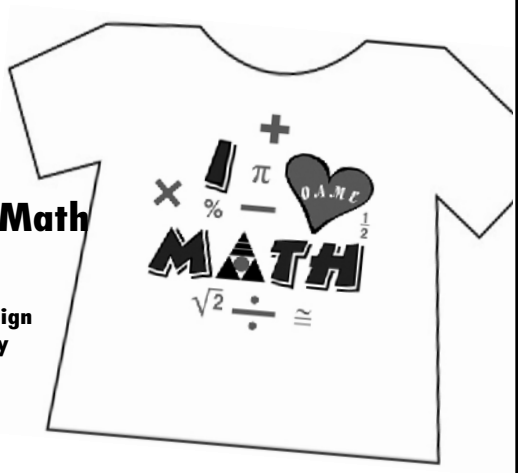
Which solution do you prefer?

Now, for your homework!

Let a, b, and c be non-zero digits. A student takes the fraction $\frac{ab}{ca}$, where ab and ca represent the two-digit integers $10a + b$ and $10c + a$, and applies a (false) cancellation law, cancelling the a from the numerator with the a from the denominator. For example, if $a = 6$, $b = 5$ and $c = 2$, the student would obtain $\frac{65}{26} = \frac{5}{2}$ (by cancelling the 6's). Determine all triples (a,b,c) for which this student actually obtains the correct answer.

Until next time, happy problem solving! ▲

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▲ Hey, It's Elementary! Using Bloom's Taxonomy to Prepare an Assessment Blueprint Using the Ontario Mathematics Grade 3 Curriculum: A Look at the "Thinking" Distribution of the Expectations

LYNDA COLGAN

AND EMILY STEPHEN



Lynda is celebrating her 32nd year as an educator. She has taught at all levels—elementary, secondary, and post-secondary—and has mentored fellow teachers and more recently, teacher candidates and graduate students. Her first teaching award, The Marshall McLuhan Distinguished Teacher Award, was given in recognition of her "Learning Partner Initiative," in which elementary children attended professional development sessions with their classroom teacher to learn about innovative uses of technology in mathematics.



Emily, an M.Ed. candidate in Curriculum and Instruction at Queen's University, is interested in studying how the verbs used in curriculum documents influence instructional and assessment decision making. Emily is certified to teach at the Primary and Junior levels in Ontario and Florida. Currently, when she is not immersed in her graduate work, Emily tutors children in mathematics and literacy.

Introduction

A ***blueprint*** is a type of paper-based reproduction (usually of a technical drawing), documenting an architectural or an engineering design. Blueprints are much more detailed drawings than simple floor plans: they are exact, detailed, scaled drawings of plans and establish quality through the precise articulation of exacting specifications for a home, building, or structure. In addition, blueprints are comprehensive and convey the "big picture" by including plans for the foundation and information on footings and framing; front, side, and rear elevations; roof plan; electrical layout and kitchen cabinet layout; and construction details.

Although the traditional use of the word *blueprint* has a long history and a specific definition, more recently, the term *blueprint* has come to be used more generally to refer to ***any*** detailed plan; thus, the phrase ***assessment blueprint*** is used to describe a document that details the specifications for developing a comprehensive plan to guide the collection of an appropriate and rich set of student data.

Like their architectural counterparts, *assessment blueprints* are constructed according to exacting "industry" standards and specifications and communicate precise measurements and dimensions. Unlike their architectural counterparts (which, although rich in measurements and dimensions, lack the step-by-step directions to build the house), *assessment blueprints* are instructional as well as illuminative tools that serve to align planning, instruction, and student assessment to educational curriculum and process standards in order to continually advance students' learning.

Background

As an assignment for a graduate course called *The Assessment of Student Growth and Achievement*, which I teach at the Faculty of Education at Queen's University, I asked my M.Ed. candidates to select a taxonomy of learning from a menu that included the original Bloom's Taxonomy (Bloom et al.), the Revised Bloom's Taxonomy (Bloom and Anderson), SOLO (Biggs and Collis), and ICE (Robert Wilson and Susan Fostaty-Young), and then prepare an assessment blueprint for a teaching unit from an Ontario curriculum document of their choice.

I include classes and assignments about assessment blueprints because research has suggested that the use of an assessment blueprint is enormously helpful in planning and implementing purposeful and appropriate classroom assessments. Moreover, it has been suggested that the use of assessment blueprints can help teachers meet the professional expectations implicit in curriculum policy documents.

Typically, an assessment blueprint lists overall and specific expectations in the left-hand columns and the taxonomy thinking levels that are being used across the top row. In the example below, overall expectation one and its two specific expectations represent "knowledge" behaviours. The task, then, is to identify assessment tasks that help us measure those expectations. I have

Overall Expectations	Specific Expectations	Items Selected to Assess Knowledge Objectives	Items Selected to Assess Comprehension Objectives	Items Selected to Assess Application Objectives	Items Selected to Assess Analysis Objectives	Items Selected to Assess Synthesis Objectives	Items Selected to Assess Evaluation Objectives	Weighting
Expectation 1: The students will know...	Expectation 1-1: Name...	supplied response						1
	Expectation 1-2: List...	matching, true/false						1
Expectation 2: The students will use...	Expectation 2-1: Translate...			supplied response				1
	Expectation 2-2: Practice...			multiple choice				2
Expectation 3: The students will create...	Expectation 3-1: Produce...					performance assessment		3
Expectation 4: The students will rate...	Expectation 4-1: Assess...						open-ended task	3
	Expectation 4-2: Choose...						open-ended task	2

Figure 1. Example of an Assessment Blueprint.

suggested a supplied response, matching, and true/false questions, since each of these item types is useful for measuring lower-order student "knowledge." Since overall expectation one seeks lower-order outcomes, I would weight these questions lower than expectations three through four, which represent higher-order outcomes. Continuing with the example, expectation four represents an "evaluation" behaviour. For this example, I have selected an open-ended exercise as an appropriate assessment type to measure these higher-order student abilities.

Most often, the emphasis given to each row and column is determined by the percentage of instructional time spent on the corresponding topic/thinking level. However, this is not always the case. Sometimes, a foundation topic requires only a small percentage of the instructional time, but is crucial to later learning. Hence, greater emphasis will be given than what would have been the case had percentage time alone been used to determine the number of items to place in each row, column, and cell.

One of my supervisees, Emily Stephen—who completed her teacher education at St. Petersburg College in Tarpon Springs, Florida, in an "Especially for Canadians" program in which there was a heavy emphasis on Bloom in all curriculum and assessment courses—elected to pursue this topic for her major assignment for the course. With her permission, I share the results of her thorough and thoughtful work with *Gazette* readers because I think it may be of great interest to elementary mathematics teachers and teacher leaders across the province. Why? Her expectation analysis, based on a comparison of the verbs used in our curriculum document with those in Bloom's original taxonomy, illustrates the distribution of "thinking" across all strands for Grade 3 Mathematics. Her table and pie graph illustrate that almost 75% of the Grade 3 expectations fall under the umbrella of the first three "thinking" categories (Knowledge, Comprehension, and Application), with approximately 25% of the expectations spread out over the three higher-order thinking categories (Analysis, Synthesis, and Evaluation). If this summary of curriculum expectations (as viewed from the perspective of their "thinking" demands) were implemented literally in the classroom, nearly three-quarters of instructional time and opportunities to learn and 75% of all assessment activities would focus on knowledge, comprehension, and application, while just 25% of all instructional time and assessment would be geared toward analysis, synthesis, and evaluation exercises.

To understand how Emily came to this conclusion, I present a brief background of Bloom's Taxonomy, as well as the Revised Taxonomy, a discussion of the value of assessment blueprints, and a summary of Emily's findings for your consideration. I believe that Emily's Expectation Summary provides one interesting perspective on our curriculum and could help to explain why so many teachers, parents, and students continue to struggle with the typology of the elusive "Level 4" question.

Bloom et al - Original Taxonomy of Educational Objectives in the Cognitive Domain

Cognitive Dimension

Cognitive Process					
1.0 Knowledge	2.0 Comprehension	3.0 Application	4.0 Analysis	5.0 Synthesis	6.0 Evaluation
					Judge
					Criticize
				Compose	Prove
				Propose	Decide
			Classify	Produce	Assess
			Diagram	Invent	Revise
		Exhibit	Outline	Imagine	Appraise
		Apply	Compare	Formulate	Estimate
	Express	Dramatize	Group	Create	Rate
	Retell	Solve	Arrange	Design	Evaluate
Tell	Describe	Employ	Contrast	Predict	Determine
Recount	Identify	Practice	Examine	Construct	Conclude
Relate	Translate	Compute	Inventory	Improve	
Specify	Estimate		Subdivide	Develop	
Identify	Restate		Interpret	Rearrange	
Know	Recognize		Categorize		
Collect	Locate		Dissect		
Cite	Summarize		Analyze		
Repeat	Explain				
Recall	Report				
Define	Convert				
Enumerate	Discuss				
List					
Name					
Label					

(Heward, 2006, p. 535)

Figure 2. Original Taxonomy of Educational Objectives in the Cognitive Domain from 1956 (Bloom et al.).

A Review of Benjamin Bloom and His Colleagues' Original Taxonomy of Educational Objectives in the Cognitive Domain (1956)

Serving as the Associate Director of the Board of Examinations at the University of Chicago, in 1948—although some argue 1949—Benjamin Bloom and his colleagues (comprising diverse educators and measurement specialists, including Max D. Englehart, E. Furst, W.H. Hill, and D.R. Krathwohl), devised a taxonomy for the classification of educational objectives in three domains: 1) cognitive, 2) affective, and 3) psychomotor. While all three taxonomies have been widely published, adapted, and applied, Ontario educators are likely to be most familiar with the taxonomy that was developed for the cognitive domain. (As an aside, although the taxonomy is commonly referred to as Bloom's Taxonomy, the creation of the taxonomy was a collaborative effort and should more accurately be called Bloom et al.'s Taxonomy).

Although the cognitive taxonomy has been published in more than 20 languages and has been applied to countless innovative curriculum and measurement projects, it is interesting to note that Bloom and his colleagues saw their framework as a work in progress because from their perspective, it was never entirely finished. The framework was initially created so that educators from various universities could share information and generate a comprehensive test bank comprising different multiple-choice test items designed to measure a range of objectives indicated by standard vocabulary. The original goal was to reduce the laborious task of developing a multitude of new and

comprehensive test items on an annual basis for standardized examinations on common topics.

To this end, Bloom's Taxonomy can be described succinctly as a "six-level classification system that uses observed student behaviour to infer the level of cognitive achievement" from the general and concrete to the complex and abstract. The intention of using precise vocabulary to specify student behaviour from general to abstract was so that teachers could guide instruction and confirm learning. (Please note that the original term "behaviour" as used by Bloom can be replaced with the term "cognitive process." Many people have confused the word "behaviour" as used in the context of learning with the theory of behaviourism.)

The six cognitive domains that appear in Bloom's original cognitive taxonomy are: Knowledge, Comprehension, Application, Analysis, Synthesis, and Evaluation. (Please see Figure 2.) At the Knowledge and Comprehension levels, students can identify, summarize, define, and explain a concept or fact, but are not expected to relate the information to any other concept or material. At the Application level, students independently identify and use a general principle or abstract concept in a new concrete situation. At the Analysis level, students can break down communication or a concept into its elements or parts and clearly identify relationships between or among these elements or parts. For instance, students can recognize unstated inferences, check the reliability of a hypothesis, and differentiate the use of propaganda techniques to understand the fundamental structure, effect, or theoretical basis of the content. At the Synthesis level, students put parts together to form a unique communicative whole to produce a plan or set of operations. Thus, students will show an understanding of abstract relationships, classification schemes, hypotheses, or the preliminary discovery of principles or generalizations. At the Evaluation level, students are able to make judgments about the value of something for a determined purpose, based on internal and external evidence.

It is important to note that Bloom's Taxonomy is assumed to be a "cumulative hierarchy" in that mastery of the lower levels (Knowledge and Comprehension) is believed to be necessary in order to achieve mastery at the higher levels (Application, Analysis, Synthesis, and Evaluation).

The original authors believed that the taxonomy could be applied in four distinct ways; i.e., to:

1) create a common language about learning goals to communicate across people, subject matter, and grade level
2) determine the specific meaning of broad education goals for a particular course or curriculum, based on national, state, provincial, and local standards
3) align educational objectives with instruction, activities, and assessments in a unit, course, or curriculum
4) provide a range of educational possibilities (instruction, activities, and assessment) for a narrow course or curriculum

To this end, Bloom's cognitive taxonomy has been one of the most universally applied frameworks in education.

A Review of Lorin Anderson and Her Colleagues' Revision (2001) of Benjamin Bloom and His Colleagues' Taxonomy of Educational Objectives in the Cognitive Domain

The revision of Benjamin Bloom's original taxonomy of educational objectives in the cognitive domain was initiated by Lorin Anderson, who, in 1994, edited a book called *Bloom's Taxonomy: A Forty-Year Retrospective.* Between 1995 and 2000, Anderson and a group of colleagues comprising cognitive psychologists; curriculum experts; and testing, measurement and assessment experts met twice each year to complete their revision of Bloom's original taxonomy. This revision is entitled *A Taxonomy for Learning, Teaching, and Assessing.*

The Anderson team's revisions reflect advances in cognitive psychology over the last half-century, the language of 19 other research-based taxonomies of learning, and the terminology being used in contemporary curricula.

Unlike Bloom, whose cognitive taxonomy was one-dimensional, Anderson devised a two-dimensional taxonomy by devising a Taxonomy Table organized around two domains: Cognition and Knowledge. The Knowledge domain, depicted vertically, operates horizontally and consists of four dimensions identified from lower to higher levels of thinking: Factual, Conceptual, Procedural, and Metacognitive. Similar to Bloom's taxonomy, the cognitive domain is depicted horizontally, but operates vertically and consists of six cognitive levels: Remember, Understand, Apply, Analyze, Evaluate, and Create. (Please see Figure 3.)

Anderson et al - Revised Taxonomy of Educational Objectives in the Cognitive Domain

Cognitive Dimension

KNOWLEDGE PROCESS	Cognitive Process					
	1.0 Remember	2.0 Understand	3.0 Apply	4.0 Analyze	5.0 Evaluate	6.0 Create
	1.1Recalling a.Identifying	**2.0Interpreting** a.Clarifying b.Paraphrasing c.Representing d.Translating	**3.1Executing** a.Carrying out	**4.1Differentiating** a.Discriminating b.Selecting c.Distinguishing d.Focusing	**5.1Checking** a.Coordinating b.Detecting c.Monitoring d.Testing	**6.1Generating** a.Hypothesizing
	1.2Recognizing a.Retrieving	**2.1Exemplifying** a.Illustrating b.Instantiating	**3.2Implementing** a.Using	**4.2Organizing** a.Finding b.Coherence c.Integrating d.Outlining e.Parsing f.Structuring	**5.2Critiquing** a.Judging	**6.2Planning** a.Designing
		2.2Classifying a.Categorizing b.Subsuming		**4.3Attributing** a.Deconstructing		**6.3Producing** a.Constructing
		2.3Summarizing a.Abstracting b.Generalizing				
		2.4Inferring a.Concluding b.Extrapolating c.Interpolating d.Predicting				
		2.5Comparing a.Contrasting b.Mapping c.Matching				
		2.6Explaining a.Constructing				

(Mayer, 2002, p. 228-232)

Figure 3. Revised Taxonomy of Educational Objectives in the Cognitive Domain (Anderson et al.).

Factual knowledge refers to the basic elements students must know to be acquainted with a discipline or solve a problem in the discipline. Conceptual knowledge refers to the understanding of interrelationships among the basic elements within a larger structure that allow them to work together. Procedural knowledge refers to how to do something, methods of inquiry, and decisive factors for using skills, algorithms, techniques, and methods. Finally, metacognitive knowledge can be understood as knowledge of cognition, as well as the awareness and knowledge of one's own learning.

In contrast to Bloom's original taxonomy, the cognitive dimension in Anderson's taxonomy does not function hierarchically because her team posits that higher-order thinking can take place within each of the six cognitive levels of thinking. At Remembering, students obtain relevant knowledge from long-term memory. At Understanding, students make meaning from instructional messages, including oral, written, and graphic communication. At Applying, students use a procedure in a new or given situation. At Recognizing, students break material into parts and determine how these parts relate to one another in structure or purpose. At Evaluating, students make judgments based on criteria and standards. Finally, at Creating, students put elements or parts together to make a logical or functional whole by reorganizing elements or parts into a new structure or pattern.

Although both taxonomies are similar in the number of cognitive domains, four of the levels of cognitive thinking have been changed. Bloom's Knowledge level was changed to "Remember" in Anderson's taxonomy; Bloom's Comprehension level was changed to "Understand" in Anderson's taxonomy; Bloom's sixth level, Evaluation, now identified as "Evaluate," was demoted from the sixth to the fifth level of thinking in Anderson's taxonomy; and Bloom's highest level was changed to "Create" in Anderson's taxonomy.

Not only have the levels of thinking in the cognitive domain been altered in Anderson's revision, but the nouns in Bloom's original taxonomy have also been replaced by verbs (to represent thinking as an active process).

According to the literature, Anderson intended for three Taxonomy Tables to be created: one for educational objectives identified in a curriculum document, one for instruction, and one for assessment. Objectives are taken from a curriculum, teacher instruction, and planned student assessment, and placed in the corresponding Taxonomy Table cell by looking at the cognitive and knowledge dimension. Anderson intended that the three Taxonomy Tables be fully aligned, or at the very least, partially aligned in a single row (Knowledge domain) or in a single column (Cognitive domain). If complete or partial alignment does not exist, this informs educators that adjustments must be made to their instruction or assessment, based on where the objective is placed in the Taxonomy Table.

Implications of the Taxonomies for the Classroom

According to Earl (2003) and the literature on these taxonomies, educational curriculum objectives should be aligned with instruction and assessment to maximize student growth and achievement. Alignment refers to the degree of correspondence among objectives, planning, instruction, and assessment.

When Bloom and Anderson constructed their taxonomies, their intention was for students to take their acquired knowledge (retention or rote learning) and use it in a variety of new situations (transfer or meaningful learning). They also intended that their taxonomies be used as planning tools for modifying curriculum standards and differentiating curriculum. In a study reported by Noble (2004), both taxonomies were used by sixteen Kindergarten to Grade 6 teachers as a tool for planning curriculum units for differentiated learning centres over eighteen months in two elementary schools. It was found that Bloom's Revised Taxonomy helped teachers better meet student learning needs and address their different intellectual strengths, and helped students reflect on their own learning (including how they believe they learn best) and on the importance of respecting other classmates' learning strengths. Anderson's taxonomy helped teachers challenge students at the highest cognitive level, meet different levels of student ability, and facilitate student success. In all, both taxonomies helped the teachers in this study create a differentiated curriculum that supported more meaningful student learning for all students.

Curriculum Standards

The Ontario Mathematics curriculum standards, also known as curriculum expectations and processes, are intended—and expected—to be used by teachers to plan and deliver instruction and to assess student learning. Explicit curriculum standards help educators focus their attention and efforts on what they deem worthwhile for students to learn by clearly articulating what should be taught and assessed.

Curriculum expectations, like all educational objectives, have a similar grammatical format: subject (student); verb (content expected to be taught by the teacher and learned by the student); and object (what the student is expected to do with or to the content). An expectation is normally phrased in the following way: "The student will be able to (or learn to) divide whole objects and sets of objects into equal parts, and identify the parts, using fractional names, without using numbers in standard fractional notation." In broad brushstrokes, expectations or objectives state the intended learning outcome for the student (following appropriate instruction), to be verified through assessment.

A Description of Assessment Blueprints and Their Uses

An assessment cycle consists of five parts: 1) knowing the purpose of, and need for, a clear assessment strategy; 2) deciding on what type of information is needed (qualitative or quantitative); 3) selecting a consistent and reliable assessment instrument to measure the information; 4) evaluating the data and assessment process; and 5) reporting the results of the data effectively.

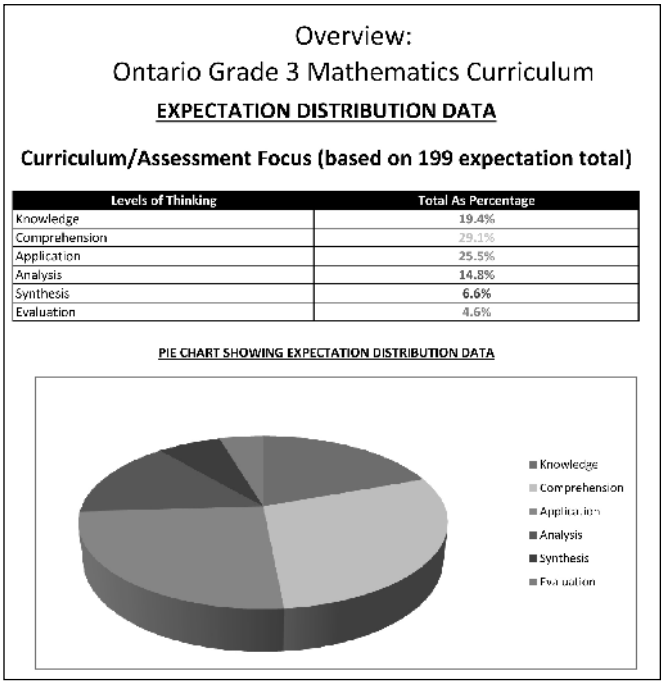
For the purposes of this exercise, the data to be collected about a student requires an objective organizer, which, in this case, is the classification of objectives under the six levels of cognition of either

Assessment Blueprint: Ontario Grade 3 Mathematics Curriculum

		Levels of Thinking					
Strand / Substrand	Expectations	Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation
Mathematical Process Expectations							
Problem Solving	3m1			X		X	
Reasoning and Proving	3m2			X		X	
Reflecting	3m3		X	X	X		
Selecting Tools &Computational Strategies	3m4	X		X	X		
Connecting	3m5			X	X		
Representing	3m6		X	X	X	X	
Communicating	3m7	X	X				
Number Sense and Numeration							
Overall Expectations	3m8	X	X	X	X		
	3m9		X	X			
	3m10		X	X			
Quantity Relationships	3m11		X		X		
	3m12	X		X			
	3m13	X	X				
	3m14					X	
	3m15		X	X			
	3m16		X				
	3m17	X			X		
	3m18		X				
	3m19	X	X	X			
	3m20			X			
Counting	3m21	X	X				
	3m22	X	X				
Operational Sense	3m23			X			X
	3m24	X	X			X	
	3m25		X	X			X
	3m26	X	X			X	
	3m27	X	X	X			
	3m28	X		X			X
Measurement							
Overall Expectations	3m29	X	X	X			
	3m30		X	X	X		
Attributes, Units, and Measurement Sense	3m31	X	X	X			
	3m32					X	
	3m33	X	X	X			
	3m34	X	X		X		
	3m35	X			X		
	3m36	X	X	X			
	3m37	X	X	X			
	3m38				X		
	3m39	X	X	X			
	3m40	X	X	X	X		
Measurement Relationships	3m41				X		X
	3m42			X	X		
	3m43		X		X		
	3m44			X	X		
	3m45		X	X			
	3m46			X	X		
Geometry and Spatial Sense							
Overall Expectations	3m47				X		
	3m48		X		X		
	3m49	X	X	X			
Geometric Properties	3m50	X		X			
	3m51	X			X		
	3m52		X	X	X		
	3m53			X	X		
	3m54		X	X		X	
Geometric Relationships							
Geometric Relationships	3m55			X	X		
	3m56		X		X		
	3m57	X	X			X	
	3m58	X	X				
	3m59	X	X		X		
Location and Movement	3m60		X				
	3m61	X		X			
	3m62		X			X	
Patterning and Algebra							
Overall Expectations	3m63		X			X	
	3m64		X	X	X		
Patterns and Relationships	3m65	X		X		X	
	3m66	X	X	X			
	3m67						
	3m68					X	
	3m69		X	X			
	3m70		X	X			
Expressions and Equality	3m71			X			X
	3m72		X	X			X
	3m73	X		X			
	3m74	X		X			
Data Management and Probability							
Overall Expectations	3m75	X	X	X	X		
	3m76	X	X		X		
	3m77		X	X			
Collection and Organization of Data	3m78		X	X			
	3m79	X		X			X
	3m80	X	X		X		
Data Relationships	3m81	X	X				
	3m82		X				X
	3m83	X	X	X			
Probability	3m84		X	X	X		X
	3m85		X	X	X		
Total		38	57	50	29	13	9
Percentage		19.4%	29.1%	25.5%	14.8%	6.6%	4.6%

Figure 4. Assessment Blueprint: Ontario Grade 3 Curriculum.

Bloom's original taxonomy, or under the six domains of cognition and the four levels of knowledge of Anderson's revised taxonomy. Bloom's original taxonomy, with Anderson's revised taxonomy in mind, was used to create a partial assessment blueprint for the Ontario Grade 3 Mathematics curriculum document. In a complete assessment blueprint, the row and column totals should reflect the emphasis given to each expectation (row totals) and each thinking level (column totals). In this table, Bloom's (1956) six hierarchical domains of cognitive development have been presented horizontally (column totals), and the Ontario curriculum's mandated expectations, under each strand and sub-strand, are presented vertically. There are no weightings assigned; all curriculum expectations are considered to be equal and are placed into categories solely on the basis of the verb that communicates the expectation. The chart matches each of the 196 expectations in the Ontario Grade 3 Mathematics curriculum, with a level of thinking expected of students as delineated in the taxonomy, and a percent is computed for each of Bloom's categories of thinking.



Overview:
Ontario Grade 3 Mathematics Curriculum

EXPECTATION DISTRIBUTION DATA

Curriculum/Assessment Focus (based on 199 expectation total)

Levels of Thinking	Total As Percentage
Knowledge	19.4%
Comprehension	29.1%
Application	25.5%
Analysis	14.8%
Synthesis	6.6%
Evaluation	4.6%

PIE CHART SHOWING EXPECTATION DISTRIBUTION DATA

Figure 5. Ontario Grade 3 Mathematics curriculum expectation distribution data.

As we see by looking at the chart and graph that summarize Emily's analysis of the language of the expectations in our Grade 3 Mathematics curriculum, there appears to be a heavy emphasis (73.3%) on knowledge-, comprehension-, and application-type goals.

In Conclusion

A quotation by Arthur Costa (*OMNI Magazine, 1992*) comes to mind:

> *We have not succeeded in answering all of your problems. The answers we have found only serve to raise a whole new set of questions. In some ways, we feel we are as confused as ever, but we believe we are confused on a higher level and about more important things.*

The "thinking" distribution summarized in the pie chart raises many questions, one of which is about the extent to which the expectations may allow for a full range of qualitatively distinct demonstrations of understanding that are assessable using the Achievement Chart, given that our learning goals are heavily clustered toward one-half of the taxonomy.

Many interpretations could be based on this data; however, if it is integral that planning, instruction, and assessment be aligned with curriculum standards as Earl (2003), Bloom (1956), and Anderson (2001) suggest, then it would appear from Emily's course-based, individual analysis, that the Ontario Grade 3 Mathematics document does not incorporate enough expectations (which, by extension, translates into instructional time, as well as opportunities to learn and assessment experiences) aimed at critical, higher-order thinking.

It can be argued that the focus between Kindergarten and Grade 3 is the acquisition of foundational skills and processes—one possible explanation for the preponderance of information-based expectations. Nonetheless, an informed examination of the Grades 6 through 8 curriculum documents would be both worthwhile and interesting as an important first step toward future professional development and curriculum development endeavours. If continuous improvement in all we do for mathematics education is the "big idea," then we will never run out of things to do and ways to do them.

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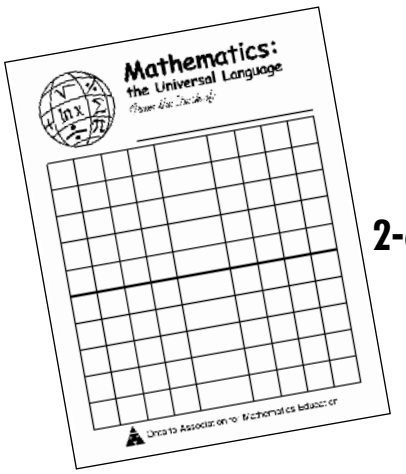
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P.S. It's great to be back writing this regular column for the Gazette. Thanks to many of you for your get-well wishes during the difficult health challenges that I faced between October 2007 and June 2008: the reason for my long hiatus. Your support and encouragement were enormously appreciated and contributed to my recovery. I hope to be a vital part of the Ontario mathematics community for many years to come, and I am aware of the enormous privilege that I have in being a regular columnist for this highly respected, long-running publication. ▲

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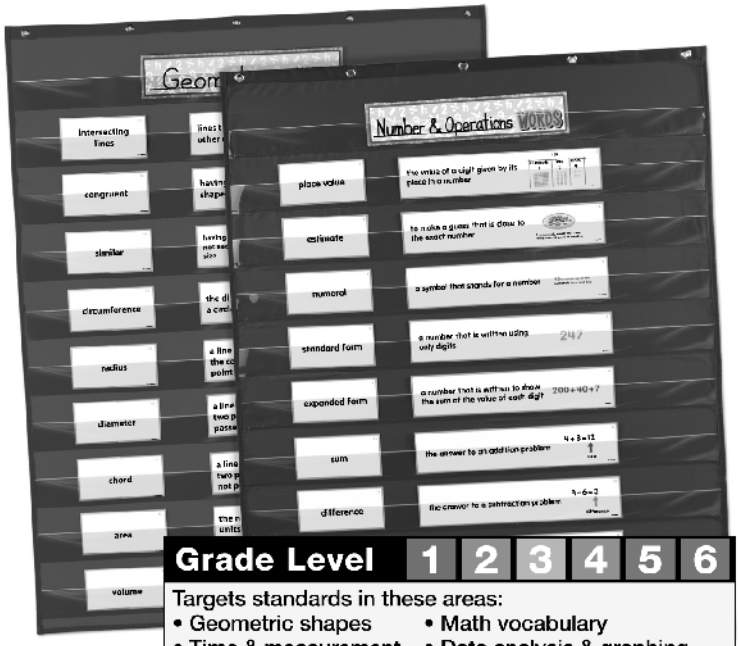
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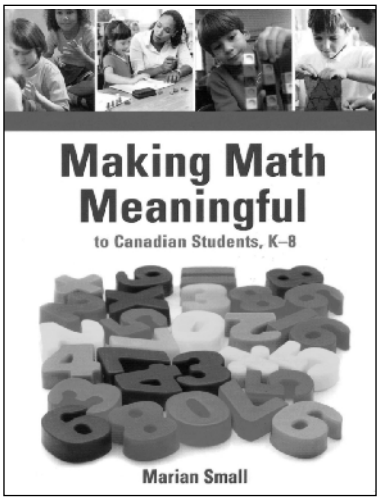
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▲ Book Review: *Making Math Meaningful to Canadian Students, K–8*

Reviewed by
Kathy Kubota-Zarivnij,
OAME Abacus Co-Editor

What does a teacher need to know and be able to do, specifically, to teach mathematics effectively? It's not just about knowing how to do the mathematics. It's knowing math in ways that are usable in the practice of teaching. Knowing mathematics for teaching emphasizes the interplay of mathematics and pedagogy. Any teacher of mathematics would agree that it takes considerable mathematical knowledge and reasoning to understand the ways in which particular mathematical ideas relate to learning and learners.



Hence, mathematics teaching is a form of complex mathematical work, as it involves a steady stream of mathematical problems that teachers must solve, such as choosing mathematical representations that are developmentally appropriate for student learning; understanding students' mathematical thinking, especially their misconceptions; and choosing mathematics tasks that are challenging and accessible. How can a teacher learn to teach mathematics effectively? Is there any professional resource that can help mathematics educators develop the depth and breadth of their mathematics knowledge for teaching? I believe that there is a Canadian solution to these problems—*Making Math Meaningful to Canadian Students, K–8*.

Making Math Meaningful to Canadian Students, K–8 is a comprehensive, readable, and practical mathematics education resource. It was written by Dr. Marian Small, a Canadian mathematics education professor who has spent over 30 years working in—and thinking about—mathematics education, subsequently translating her research, knowledge, and insights into a text resource for those who engage in mathematics teaching and learning. Unlike other popular mathematics text resources, this resource has a Canadian focus. A considerable amount of content in this text is from Canadian research, Canadian resources, and Canadian curriculum from various regions of the country. Findings from international mathematics education research are also included throughout the text.

This resource takes several positions with respect to mathematics teaching and learning that are consistent with the OAME vision of mathematics education. Here are just a few of those positions:

- "Mathematics is learned best when students are actively engaged in constructing their own understandings. This is only likely to happen in classrooms that emphasize rich problem solving and the exchange of many approaches to mathematical situations, and that give attention to and value students' mathematical reasoning" (Small, 2008, p. 1).
- Instruction should be focused on key mathematical ideas and key mathematical processes. Small explains that "the thinking behind this approach is that, if we can connect a new idea being taught to related ideas that have been previously learned, it is more likely that the new knowledge will be assimilated… Despite increasing abstraction as students go up the grades, and shifts in focus from grade to grade, key ideas remain fundamentally simple" (p. 18).
- Teaching through problem solving has many benefits for student learning (p. 38):
  - "...students have the chance to build essential connections between what math is, why it is needed, and how it is applied."
  - "Problem solving builds perseverance."
  - "Problem solving builds confidence, maximizes the potential for understanding, and allows for differences in style and approach."
  - "Problems can provide practice, both with concepts and with skills."
  - "A problem-solving approach provides students with better insight into what mathematics is all about."

Making Math Meaningful to Canadian Students, K–8 is organized in several ways that readers from different perspectives and educational contexts will appreciate. The beginning chapters include "How Students Learn Mathematics and What Mathematics We Want Them to Learn," "Focusing Instruction on Key Ideas and Key

Processes," "Focus on Problem Solving," and "Focus on Communication," while the ending chapters focus on "Assessment and Evaluation," "Planning Instruction," and "Developing a Teaching Style." The middle chapters are devoted to particular strands or aspects of a strand:

- Number Sense and Numeration: Mathematics for the Younger Learner; Developing Early Operation Concepts; Representing Multi-Digit Whole Numbers; Computation with Whole Numbers; Fractions; Decimals; Ratio, Rate, and Percent; Integers
- Geometry and Spatial Sense: 3–D and 2–D Shapes; Location and Movement
- Measurement: Length and Area; Measuring Capacity, Volume, Mass, Time, and Angle
- Data Management and Probability: Data Display and Analysis; Collecting and Describing Data; Probability
- Patterning and Algebra: Patterning and Algebraic Thinking

It may appear that these chapter topics are similar to other popular mathematics text resources, but this resource is uniquely arranged, detailed, and pragmatic. Each mathematics content (strand) chapter has the following features:

- a problem to engage the reader mathematically at the start of each chapter
- an outline of the main ideas, "In a Nutshell"
- highlighted mathematical principles (e.g., numeration principles, fraction principles)
- mathematical problems and activities that can be used directly in the classroom (LOTS!!!)
- sample student responses to problems and tasks
- suggestions for manipulatives, technology, and children's literature
- descriptions of common errors and misconceptions, accompanied by suggested teaching strategies
- assessment considerations
- a reference list that includes significant mathematics education research

As a non-fiction reader who has certain feng shui requirements for engaging with text, I was compelled to do a final flip through the 696 pages of this all-inclusive professional learning resource. I was struck by the obvious features that make it readable and engaging. Each chapter is colour marked for accessibility. Each page is crisply divided into two-column text, with colour-coded levels of headings to facilitate scanning. Diagrams are large enough, coloured, and detailed with description. Student responses are in "real" student writing, indicating authenticity, and tables are used liberally to help the reader notice key ideas and see relationships among ideas. All of my feng shui non-fiction reading requirements were met and even surpassed!

So, HURRAY for Dr. Marian Small! Canadian mathematics educators now have a thorough mathematics education resource that can be used for a variety of purposes (e.g., as a personal reference text, as a pre-service or in-service teacher education course text, or as a professional learning resource). *Making Math Meaningful to Canadian Students, K–8* shouldn't be read from cover to cover; it should be used for teaching and learning from cover to cover! ▲

ABACUS: A COMPILATION OF VOLUME 46

This resource for teachers includes the concepts of addition, subtraction, multiplication, and division of whole numbers and decimals, alternative algorithms, mental math strategies, manipulatives, and links to literature. It will be useful for exploring these aspects of teaching and learning personally or with a team of teachers. Includes one two-coloured bead string.

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▲ Data Rocks – Modelling Data from Statistics Canada and Extrapolation in Excel: The Case of Life Expectancy (or Will Men Catch Up to Women?)

YVES SAINT-PIERRE, STATISTICS CANADA



Yves Saint-Pierre has worked at Statistics Canada in many capacities since 1990: questionnaire design, data editing and imputation, analysis, and publication. He has been sharing his knowledge with schools since 2002.

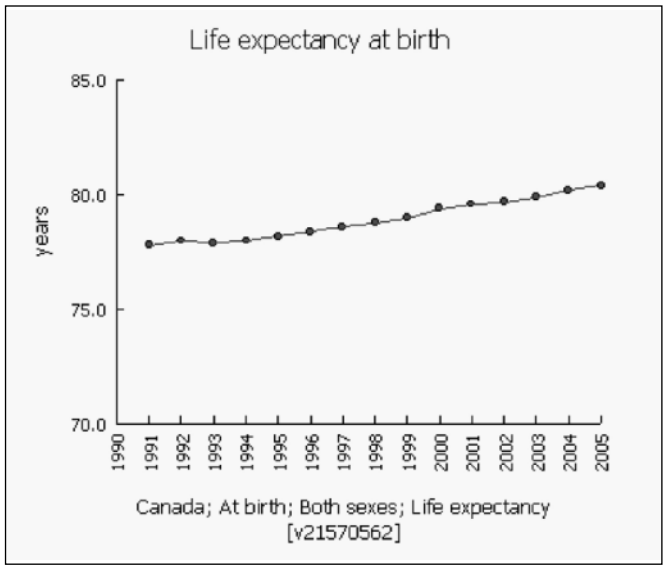
In this article, I will focus on one of the function modelling lessons available from Statistics Canada's Learning Resources **www.statcan.gc.ca/edu/index-eng.htm** and show how you can work through it, using Excel. This lesson models life expectancy as a linear function: first, by looking at life expectancy for the whole population, and then by examining the different trends in the life expectancies of men and women.

The lesson asks students to retrieve data from E-STAT[1] and then transfer the E-STAT data onto a spreadsheet and make a few transformations. All of the instructions necessary to get the data from E-STAT are provided.

It turns out that the data on life expectancy at birth follows a pattern that behaves like a linear function from 1979 to 1999 (and indeed in the years after that, from 1990 to 2005, as evidenced in a different dataset). So we could ask ourselves: If present trends continue, in what year will the average Canadian (or Ontarian or Newfoundlander or...) live to be 100 years old?

Well, let's see the data as it's displayed in E-STAT!

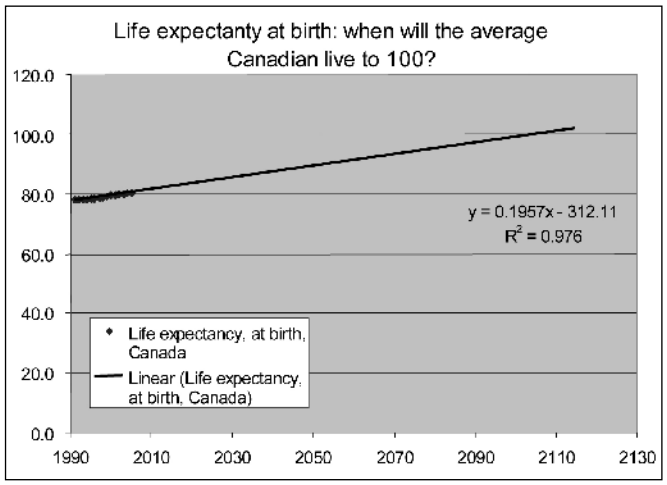
[1] *E-STAT is an enormous data warehouse giving free access to data to the education sector (students and teachers). In brief, if you'd like to search for time series on this or any other topic from the Learning Resources page at* ***www.statcan.gc.ca/edu/index-eng.htm****, click on E-STAT, read the licence, and click on Accept and Enter > Search CANSIM in E-STAT > type in the word « expectancy » or the table number 102-0511 for data from 1990 to 2005, or 102-0025 for data from 1979 to 1999 (the data used in the lesson plan).*



Source: Statistics Canada, CANSIM in E-STAT, table 102-0511.

Now, using Excel to display the same points on an **XY (scatter)** plot, you can easily **add a trend line**, choosing a linear model. Then specify two things in the options for that line:

1) Show the equation of the line and the R-squared.
2) Forecast forward some years. (I'm betting we'll need at least 100 more years, and I'm adding a few more.)

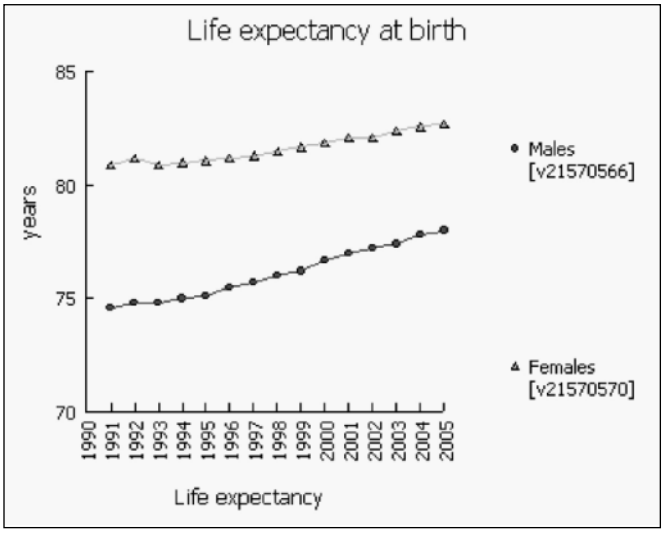


Source: Statistics Canada, CANSIM in E-STAT, table 102-0511.

By golly, yes! Looks like 2106!

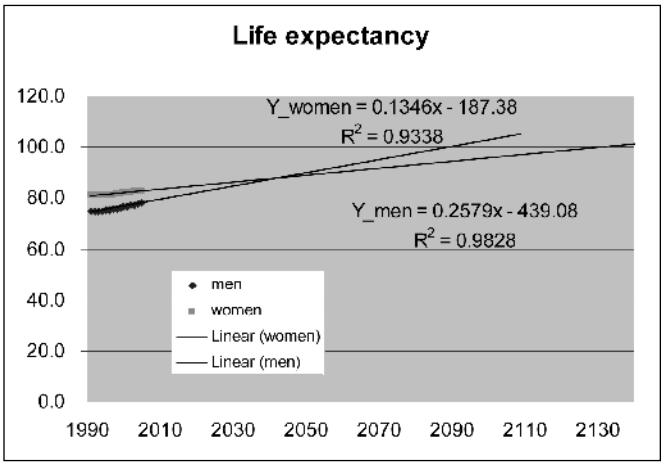
In the graph above, *y* is the life expectancy and *x* is the year. Would we have predicted the same had we used the older data series of 1979 to 1999? Using that data (see footnote 1) and the same method as in the graph above, I get y = 0.1932x - 307.2. You do the math! (Hint: The answer is quite close. It seems that life expectancy is not one of those things that "moves in mysterious ways!")

And by the way, aren't you curious about how men and women compare? See the trend lines below for their respective life expectancies from 1991 to 2005.



Source: Statistics Canada, CANSIM in E-STAT, table 102-0511.

When will their respective curves cross? What group will be the first to reach the average age of 100?



Source: Statistics Canada, CANSIM in E-STAT, table 102-0511.

Well, ***if*** *present trends continue*, the life expectancy of men will catch up to that of women in 2041, and the average Joe will live to be 100 in 2091, 45 years **before** the average Jane. Hmmm. These are interesting applications of linear modelling. I wonder if they are realistic?

Think this through with your students! What factors might affect whether or not present trends will continue? What is the trend of life expectancy over a much longer term (say, 400 years)? This is an opportunity to discuss lifestyle with your students. Could the lifestyles of today's children and youth (e.g., eating habits, exercise or lack thereof) mean that they could be the first generation to have a **shorter** life span than their parents? Why not take this opportunity to reflect on the health habits that contribute to longevity?!

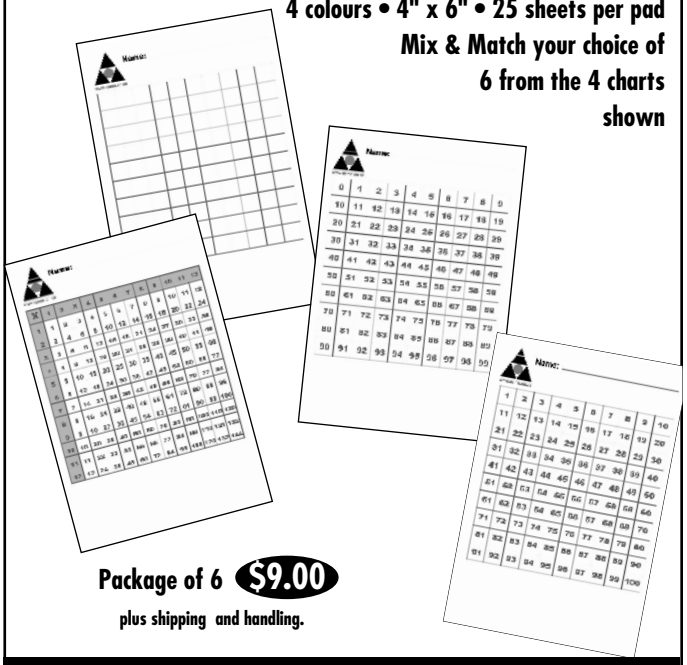
More math ideas: You could ask teams of students to use the same data series in E-STAT to model life expectancy for different provinces and then compare the results.

This linear function lesson is part of our Function Modelling Resource, where you can find examples and lessons for many different types of functions.[2]

Remember that we are available to answer questions about this or anything else related to Statistics Canada. To contact us, just e-mail: **mdm4u@statcan.gc.ca**. ▲

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[2] To find this page on our website, please start with the Learning Resources website at ***www.statcan.gc.ca/edu/index-eng.htm*** *and then click on Teachers > Resources by school subject > Mathematics > Grades 9 to 12 > Key Resources (default tab) > Function modelling using secondary data from E-STAT.*

▲ Software Review – Autograph 3: A New Approach to Dynamic Software... from Great Britain

Douglas Butler
Betty Temmer



Douglas Butler has taught mathematics at the secondary level in the UK for most of his career. In addition to working with the Autograph development team, he now concentrates on helping teachers make the most of technology in their teaching,



Betty Temmer has been a secondary math teacher with the Grand Erie District School Board throughout most her career. Retired from the classroom (but not from mathematics), she is presently involved in training teachers in a variety of new technology initiatives.

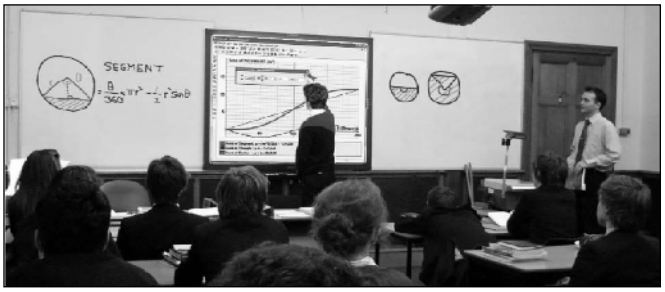


Figure 1. Autograph in use in Oundle School in the UK, where it was born.

A dynamic math software program from across the pond is finding its way into secondary schools and colleges in Canada, as well as many other countries around the world. Autograph uses selectable, dependent objects to study coordinate geometry in both 2-D and 3-D, dynamic Euclidean geometry, vectors, probability and statistics, and much more. Its intuitive interface and compatibility with interactive whiteboards and tablets make it worth investigating. The software is available as a free 30-day download at **www.autograph-math.com**.

To help you get started, here are some key tips worth checking out.

TIP 1: LEVEL OF OPERATION

Once Autograph is loaded, you are asked to choose either *Standard* level or *Advanced* level. *Standard* level has a simplified interface and is more suitable for intermediate grades. The *Advanced* level allows for more sophisticated topics (e.g., radian measure, derivatives) and is geared toward senior students.

TIP 2: PAGE TYPE

Autograph opens to a 2-D graphing page, but there are three page types available. Examples of each follow.

- Statistics and Probability
- 2-D Graphing page
- 3-D Graphing page

TIP 3: WHITEBOARD MODE

Click on this icon in the top toolbar if you are using Autograph on the big screen, as in Figure 1.

In this mode, all lines are thickened, and the typeface for the axes and equation lists are enlarged for ease of viewing from anywhere in the classroom. Two other useful options in this mode are “no shift for multiple select” and the on-screen mathematical keyboard (Figure 2).

You can set this up in *View => Preferences => Whiteboard*.



Figure 2. The on-screen keyboard allows for input without the regular keyboard.

TIP 4: STATISTICS PAGE – ANALYZING RAW DATA

Click on this icon to open up a new Stats page. At all times, the right-click button gives options appropriate to the current selection.

Select “Raw Data” to open up the raw data dialogue box (Figure 3). Here you can paste data as a single column from a spreadsheet or create a data set by sampling from one of Autograph’s built-in distributions.

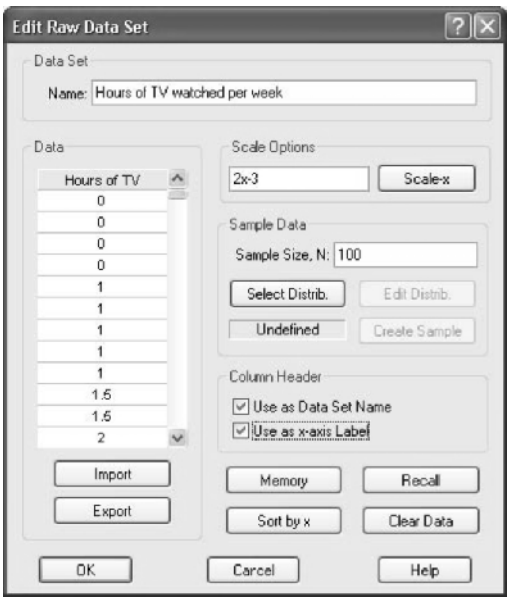


Figure 3. The raw data edit box.

Once a data set is entered, press OK, and the data set is listed at the bottom left of the Autograph page (Figure 5). From raw data, you can create a dot plot or box plot, but to create additional diagrams, the data needs to be grouped. Right-click and choose "Group Data Set" to bring up the dialogue box shown in Figure 4.

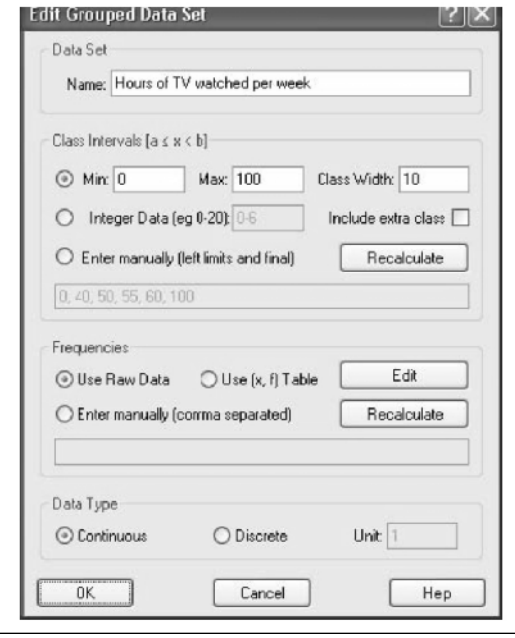


Figure 4. The grouped data edit box.

Once the data is grouped, a histogram, a cumulative frequency diagram, and a table of values can all be created (Figure 5).

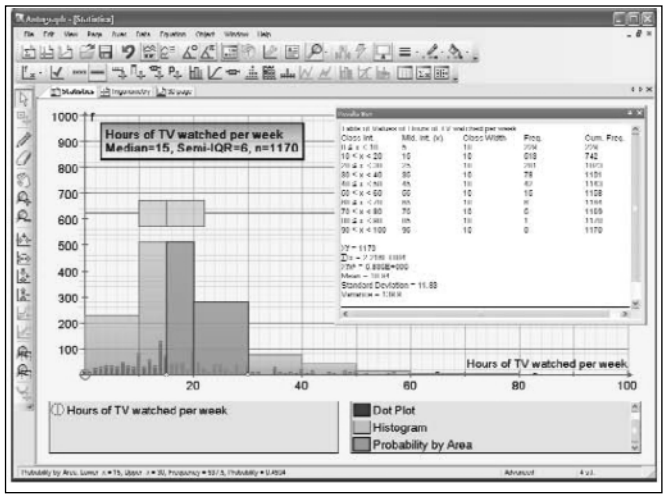


Figure 5. Exploring raw data in Autograph.

TIP 5: STATISTICS PAGE – EXPLORING PROBABILITY

The Advanced Level right-click menu also offers a set of built-in probability distributions (Figure 6).

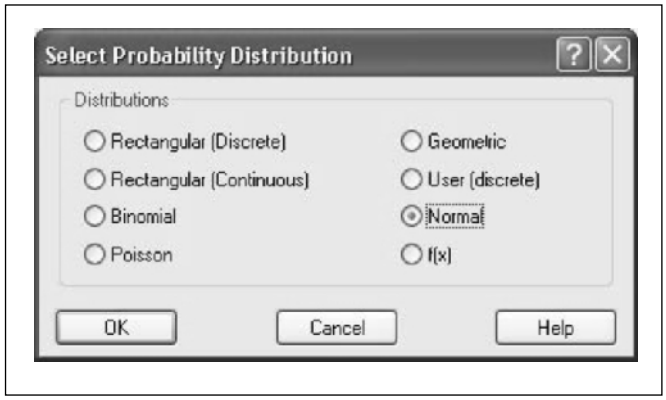


Figure 6. Probability distributions.

Once the distribution type is chosen, appropriate parameters are entered (e.g., *n* and *p* for the Binomial). These can also be set to fit an underlying raw data set.

The animation controller can be used to animate any of these parameters, giving a dynamic view of how the distributions behave.

Figure 7 shows two normal distributions and a visualization of type-1 and type-2 errors in hypothesis testing.

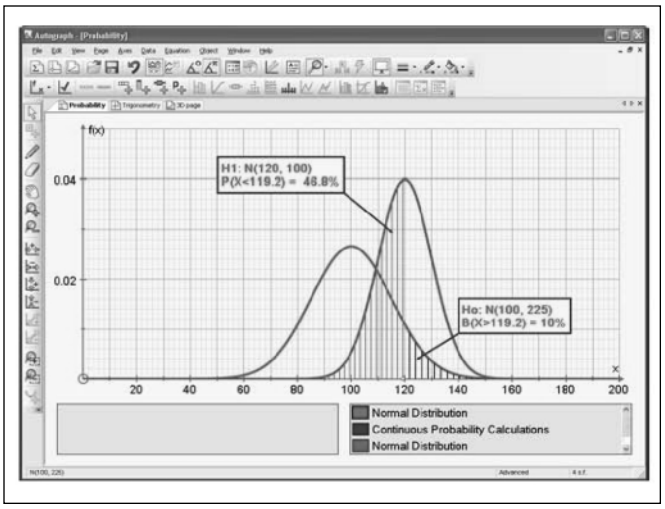


Figure 7. Hypothesis testing—the visual approach.

TIP 6: 2-D GRAPHING PAGE – ENTERING EQUATIONS

On a 2-D page, press or "Enter" to open the equation entry dialogue box (Figure 8) below.

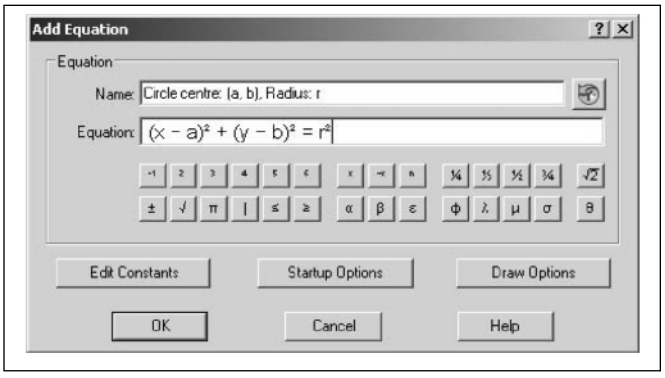


Figure 8. Equation entry dialogue box.

Autograph uses a unique, single-line text entry, which can be typed in or copied and pasted from elsewhere in a wide variety of forms:

$y = ax + b$

$2x - 3y \leq 6$

$x^2 - 3y^2 + xy + x + y = 0$

$x = asinbt, y = bcosmt$

$r = asinbt$

$y' + 2y = sinx$

$y'' + 2\lambda y' + n^2x = 0$

Up to 2 functions can be defined using y=f(x) notation by using f(g(the command from the toolbar. This is great for operations with functions or for transformation of functions.

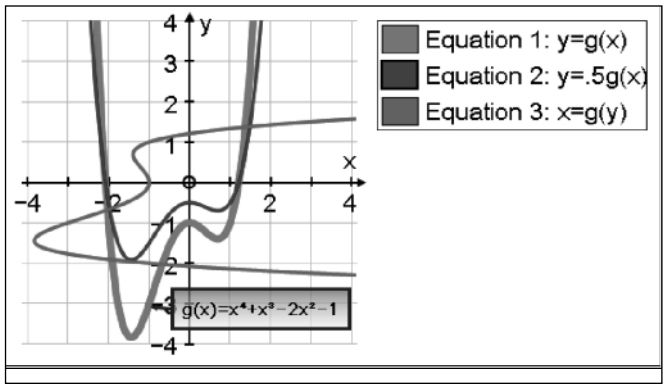


*Figure 9. T*ransformations of y=g(x).

TIP 7: THINGS TO DO WITH ENTERED EQUATIONS

Press the "slow plot" button to make plotting proceed slowly—perfect for teaching. Press Spacebar any time to pause and continue plotting.

Use the scribble tool to anticipate or otherwise draw on the page. (You'll find many uses for this tool!)

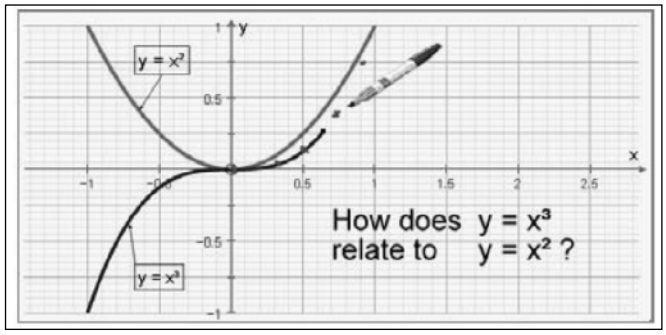


Figure 10. Creative use of the scribble tool and the slow plot.

Use this button (Advanced Level only) to create the first derivative of the equation that is selected in the list. This is particularly effective when "slow plot" is on.

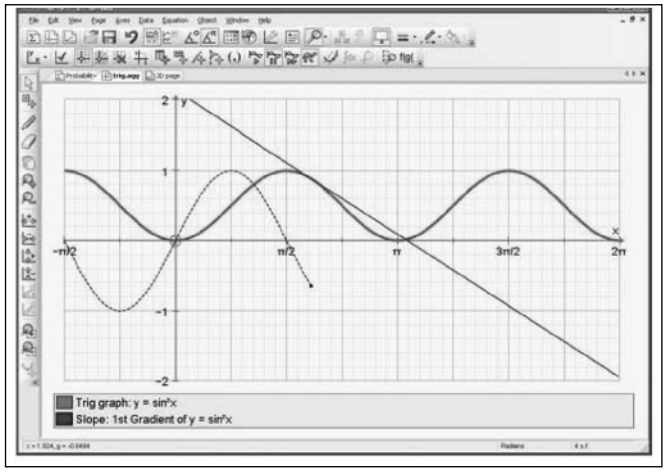


Figure 11. Seeing the link between y= sin²x and its derivative.

TIP 8: USING THE CONSTANT CONTROLLER

If any coordinates (or a point or a vector) or

equation on a page includes constants, these constants can be varied, using the controller.

In Figure 12, an equation has been manipulated to fit over the imported image of a bouncing ball.

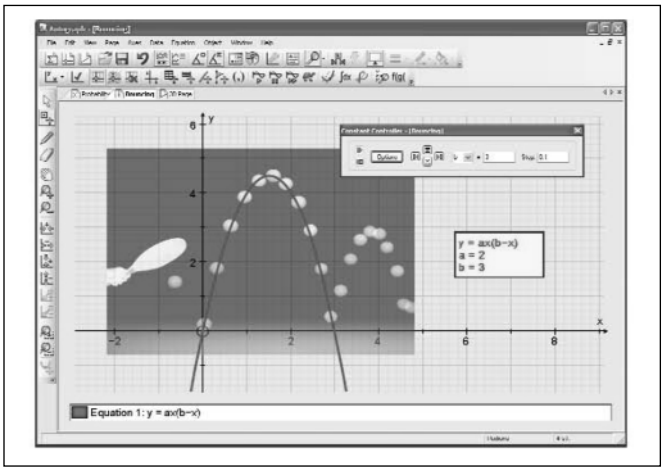
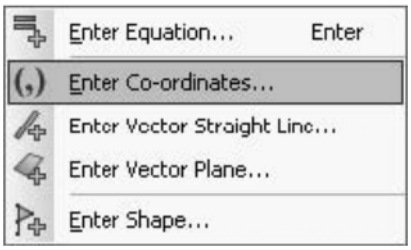


Figure 12. Using the constant controller to fit a parabola.

Also shown in Figure 12 is a dynamic text box indicating the current equation and current values of the constants.

TIP 9: GETTING STARTED IN 3-D

On opening a 3-D page, the Advanced Level right-click menu is:



A bounding cube is required; otherwise, a plane would fill the entire screen. You can use the select tool outside of the box to drag the scene around. Hold "Ctrl" while you drag to move the "camera" in and out!

The background colour is black by default (to display 3-D surfaces best in fully blended colour), but this can be altered using the buttons.

Points can be added, either by entering their coordinates, or by clicking within the 3-D cube. They are placed halfway through the cube, and can then be dragged around afterward to reposition.

TIP 10: USING POINTS IN 3-D

With two points selected, a line or vector can be drawn. Vector operations including addition, subtraction, cross-product, etc., can be effectively illustrated.

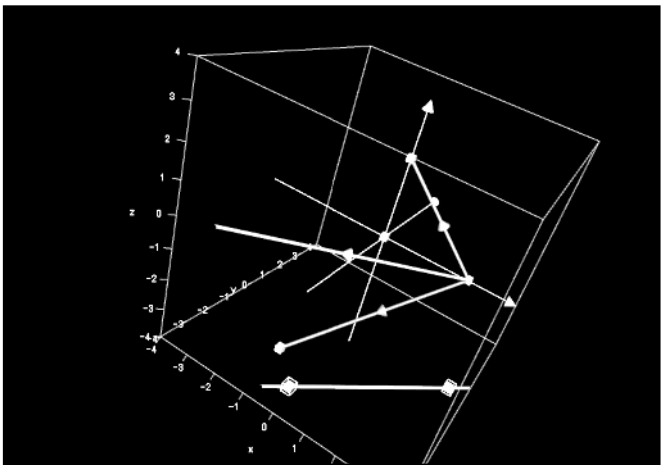


Figure 13. Vector addition in 3-D.

With 3 points selected, a plane can be drawn.

Equations of selected objects are given.

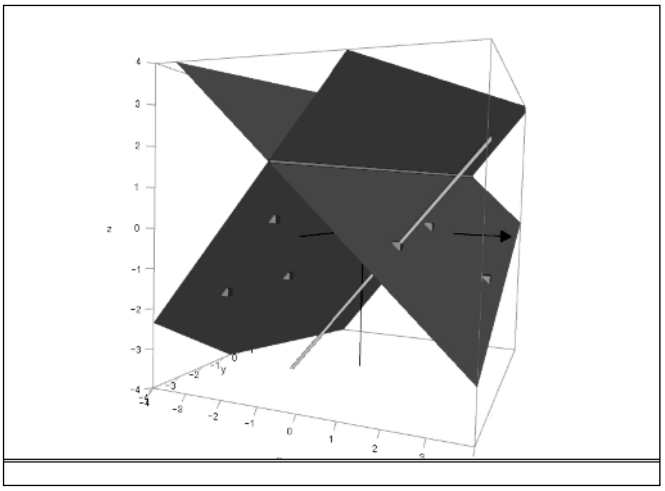


Figure 14. Intersecting Planes.

Plane: –1.715x+0.669y+1.353z=1
Line of Intersection:
(x, y, z) = (0.7564)+ λ(–0.3714, –0.9284, –0.01154)
Angle between planes: 1.423 *radians*

Calculations available in 3-D include points of intersection, closest distances, angles between lines and/or planes, midpoint, etc.

TIP 11: EQUATIONS IN 3-D

As in 2-D, equations can be entered in a variety of forms, including implicit, explicit, polar (cylindrical or spherical), and parametric.

In Figure 15, y = sinx was entered, using the "plot as 2-D" option.

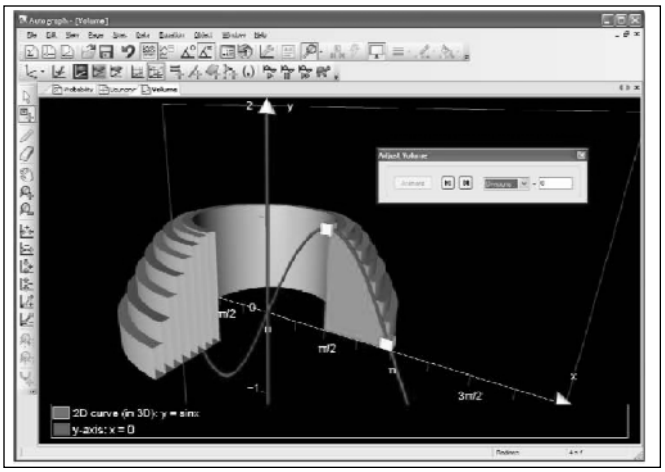


Figure 15. Volumes of revolution about any axis.

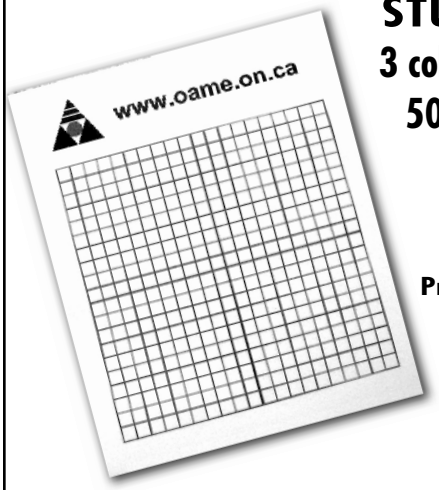
TIP 12: AUTOGRAPH RESOURCES

If you're interested in learning more:

- Web-based tutorials (these are awesome!), lesson plans, student worksheets, and a User Forum can be found on the Autograph website at **www.autograph-math.com**.
- Attend an Autograph session at the annual OAME Conference in Ottawa this May! ▲

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▲ Engaging Students with Problems and Investigations

ED BARBEAU



Ed Barbeau is professor emeritus at the University of Toronto and served for many years on the OAME Board of Directors. Currently, he operates a problems correspondence program, Olymon, for secondary students and gives talks to school students. He also has lots of material on his website: ***www.math.utoronto.ca/barbeau****.*

You may have noticed the following phenomenon. Having taught a unit of the syllabus and given your students a number of exercises to practise on, a number of students fail to retain the material or simply do not "get it." Sometimes, however, with a little intervention, it seems as though the light dawns, and the material becomes secure. What is it that suddenly makes clear to a student that which was previously not understood? What can be done to bring the student to this point of revelation?

I believe that a significant determinant of success or failure in mathematics is the way a student looks at a situation and is able make it his or her own. Even when the student is diligent and goes through the work, he or she may lack fluency, flexibility, or insight. The difficulty seems to be that many textbook exercises are dry and predictable, so that even though students can work through such exercises at a superficial level, there is nothing about these exercises that draws attention to underlying structure and excites higher cognitive levels of strategy and perspective.

I am asking the reader to help me with a little investigation. I have amassed a number of problems that I have tried with students of various ages and that I hope you might take on board and try with your students. Several of these problems can be found on my website: **www.math.utoronto.ca/barbeau**. Many of them are suitable for elementary students, but they can be used with students at different levels, often inviting different sorts of responses. After you have tried one or more of these problems with your students, I am asking that you share with me (and the readers of the *Gazette*) how they worked for you.

Specifically,

1) What was the age and grade level of the students to whom you gave the problem? How was the problem presented?
2) Was it assigned to students in a group or individually? Did the students work on it in class or at home?
3) How did the students react to the problem? Did they seem to enjoy working on it? Did they experience success with it? Did it encourage discussion?
4) Was the problem useful in developing any particular skills?
5) Which curricular goals did the problem support?

Here is a four-part example. Please consider this example on your own or with colleagues before looking at my comments.

1) You and your students might have noted the singular fact that 2 x 2 = 2 + 2. Is it possible to find other pairs of numbers for which the sum is equal to the product? (By "numbers," we can mean positive integers, integers, fractions, or real numbers.)
2) Continuing with this theme, we ask for two pairs of positive integers for which the sum of the numbers in each pair is equal to the product of the numbers in the other pair. For more senior students, we can broaden the question to integers that could be non-positive.
3) We can generalize the question in 1) by giving a geometric setting. We might observe that the area and the perimeter of a 4 x 4 square are equal. Are there other rectangles with integer side lengths that have the same area and perimeter? Another form of this question is to ask for pairs of integers whose product is equal to twice the sum.
4) Analogous to 2), we can ask for two pairs of positive integers such that the product of the numbers of each pair is equal to twice the sum of the numbers of the other pair. Again, we could extend the question to other types of numbers.

Comments

These comments are not intended to be a lesson guide, but rather, are meant to indicate some of the issues that might arise and some points that might be brought into the discussion, where appropriate. My experience is that it is difficult to predict what will happen with a specific group, and often, one is pleasantly surprised.

1) Students might be asked whether there are other pairs of integers that have the same sum and product. A useful insight is that the sum of two positive integers does not exceed twice the larger. To lead up to this, you might ask whether there are examples other than (2, 2) where both positive integers are the same. This insight leads to the conclusion that neither of the integers can exceed 2. (Do you see why?) On the other hand, what is the relation between sum and product when one of the integers is 1?

Students working through this may realize that the product of two positive integers does not always exceed the sum. At the same time, they may better understand why, when the product is larger, it can be considerably larger than the sum.

What happens if we allow fractions? To oil the wheels, one might ask for examples, in particular, if one of the numbers is 3. Since adding any positive number to 3 will give a result exceeding 3, the second number should be greater than 1. Have the students try some possibilities, and try to narrow in on when the sum of 3 and another number is less than the product, and when it is greater than the product. Is the second number likely to be less than or greater than 2?

Students at the high school level may be inclined to try algebra, although this is not really necessary. However, there is something to be learned. In solving the equation $xy = x + y$ for integers, we can "complete the rectangle," thus:

$$0 = xy - x - y = (x-1)(y-1) - 1$$

and solve $(x - 1)(y - 1) = 1$. It would be interesting to know how many students spontaneously hit on this technique.

The process could evolve in different ways. Students would likely solve for one variable in terms of the other, solving for y in terms of x:

$$y = \frac{x}{x-1}$$

This reinforces that 1 cannot be a member of the pair, but also makes it clear that every other number x can be paired with a corresponding y to make the product and sum equal. (Students might check this directly.) Writing the equation in the form

$$y = 1 + \frac{1}{x-1}$$

allows the graph to be readily sketched. What are the asymptotes? What does the graph tell us about integer pairs (x,y) that work? As x increases, the corresponding value of y decreases. Is this to be expected?

However, the form $(x-1)(y-1)=1$ engenders another perspective. It tells us that if we subtract 1 from each member of the pair, we get reciprocals, so that $x-1=t$ and $y-1=\frac{1}{t}$ for some number t. Thus, any pair with an equal sum and product has the form

$$(x,y)=\left(1+t,1+\frac{1}{t}\right)$$

for non-zero real t. Conversely, it is a simple exercise to check that every pair of this form yields the same sum and product. So we now have an exhaustive parametric answer to our questions.

Moreover, this form of the equation tells us at once that the graph of acceptable (x,y) is a rectangular hyperbola, namely the standard hyperbola $xy=1$ with the centre translated from (0,0) to (1, 1).

2) In looking for two pairs of positive integers for which the sum of each pair is equal to the product of the other, the key observation is that when the sum and product of each pair are unequal, for one of the pairs, the product is less than the sum. When can the sum of two positive integers exceed the product? Once this question has been answered, then some trial and error yields all the possibilities.

More senior students may again try to "algebratize" the problem, and set up the system

$$x+y=uv;\ u+v=yx.$$

A little manipulation yields that

$$(x-1)(y-1)+(u-1)(v-1)=2$$

to be solved for positive integers.

Again, we can extend the range of the problem and ask for solutions in integers that are not necessarily positive or in fractions. What could a student do? A good strategy is to get some examples. For example, setting $x=0$ forces $v=-u$ and $y=-u^2$, so that $(x,y;u,v)=(0,-u^2;u,-u)$ is a solution for any number u. Similarly, $x=-1$ leads to $-1=uv+u+v$, which simplifies to $1=(u+1)(v+1)$. If, say, $u=-1$, then $y+v=1$ and we are led to $(x,y;u,v)=(-1,y;-1,1-y)$.

More generally, eliminating y from the pair of equations, $x+y=uv$ and $u+v=xy$, leads to $x^2=xuv-u-v$. How might this be analyzed? You may have taught students to "complete the square"; one can also "complete the rectangle." To set this up, multiply the equation by x to get

$$x^3=x^2uv-xu-xv=(xu-1)(xv-1)-1$$

or

$$x^3+1=(xu-1)(xv-1)$$

Now one can try specific values of x and factorizations of x^3+1 to produce some examples of pairs.

3) The rectangle problem is surprisingly rich, and there are a number of approaches that students might use that are more or less geometrical, numerical, algebraic, and combinatoric. Quite a bit of insight can be given by constructing a table of differences between area and perimeter for various pairs (a,b) of positive integers.

Students may enjoy solving the same problem for triangles with integer side lengths. They could be asked to look at right triangles first, where two examples can be easily found. There is, of course, the decision regarding the best formula for the area.

Since perimeter varies linearly as the dimensions of similar figures and area as the square of the dimensions, the relationship between perimeter and area is not invariant with respect to similarity. Another way of formulating the problem is to ask for shapes of rectangles or triangles for which a system of units can be defined to make their side lengths integers and the area equal to the perimeter. For example, a square has the requisite property; just make the unit of measurement such that each side is 4 units long.

If we allow real numbers for the measurements, students might be asked to consider this question:

> *Suppose that you are given a geometric figure; is it always possible to define a system of units to make the numerical values of the perimeter and area equal?*

Discussion

I hope that this has given you some notion of what might or could transpire in considering these problems. While the details will vary from group to group, as will the depth to which it is practical to go, one might hope that the investigation will lead to a structural understanding of the situation.

In particular, where algebra is involved, the goal is to help students understand that an algebraic equation is a bearer of information, but that the form of the equation governs which information is readily accessible. It is not just mastery of technique that is important to a student proceeding to a higher level, but the strategic uses to which the student is capable of putting his or her skills. Accordingly, it is important to challenge students to go beyond just the basics.

Other Questions

The following are further examples that involve basic material and that encourage students to look at the material's substance:

1. Magic Squares

a) Form the nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9 into a 3 x 3 square array, such that the sums of each row, of each column, and of each diagonal are all the same.

(Look for different approaches that students might have. A productive observation is that there are exactly eight ways to have three distinct digits sum to 15; in how many of these does a given digit occur? What number should go in the middle? Why?)

b) Form the product of the numbers in each row. Add the products for the three rows. Do the same for the columns. What do you notice?

c) Consider the digits in each row as making up a three-digit number. Add the squares of the three-digit numbers. Now suppose that the digits in each row are taken in reverse order to make up three-digit numbers and that these numbers are squared and added. What do you notice? Do the same for the columns. Do the same for the diagonals (two diagonals broken) in both directions.

2 a) Use the digits from 1 to 8 inclusive, each exactly once, to form two numbers. Take their difference. What is the smallest possible value of this difference?

b) Answer the same question, except with the eight digits 1, 2, 3, 4, 6, 7, 8, 9.

3. The digits from 1 to 9 inclusive are each used exactly once to make up three numbers in the ratio 1:3:5. What are the three numbers?

4. The ten digits from 0 to 9 inclusive are each used exactly once to make up three numbers A, B, and C for which $A + B = C$ and $A < B$. What are the possibilities?

(A useful way to begin is to consider the number of digits that each of the three numbers could possibly have.)

5. A number and its square together involve each of the nine digits from 1 to 9 inclusive exactly once. What are the possibilities?

6. Find sets of three positive integers, such that when you add 1 to the product of any two of them, you get a perfect square. An example is the set (1, 8, 15), since $1 \times 8 + 1 = 9 = 3^2$, $1 \times 15 + 1 = 16 = 4^2$, and $8 \times 15 + 1 = 121 = 11^2$. How many sets can you find that, like the example, have the additional property that the difference between the two smallest numbers is equal to the difference between the two largest numbers?

(This problem has the advantage of showing that there are several different forms a family of solutions might take. It would be interesting to see how complicated students might make their initial examples. Will some realize that solutions are easy to find if we make one of the three numbers equal to 0? If this happens, you would then exclude this possibility. If individual solutions are found, then get the students to look for patterns that might allow for the determination of infinite families. For middle or early secondary students with some algebra, they should formulate an algebraic expression for the family and verify that it works.)

7. Two perfect squares have four digits when written in base 10. Each digit of the larger square is equal to the corresponding digit of the smaller square increased by a fixed amount. What are the possibilities?

(This problem can take advantage of a useful way of calculating the difference between two squares wrapped up in the formula $m^2 - n^2 = (m + n)(m - n)$. In words, the rule is "the difference between two squares is equal to the sum of the roots times the difference of the roots of the squares." We can justify this geometrically. In this case, the difference of the squares is a multiple of 1111, which, in turn, is the product of the primes 11 and 101.)

Geometry

Although elementary students are not learning formal geometry, it is important to get them thinking geometrically and to give them experience with geometric constructs. Here are three problems that you might try. Let me remind you that a *quadrilateral* is a closed plane geometric figure bounded by four line segments. It is said to be *convex* if any two points in it are joined by a line segment that lies entirely within it. Examples of quadrilaterals are squares, rectangles, and trapezoids, but there are other quadrilaterals with no two sides parallel. A *parallelogram* is a quadrilateral for which both pairs of opposite sides are parallel; squares and rectangles are particular examples of parallelograms.

1. Let *ABCD* be a quadrilateral and let *K, L, M, N* be the respective midpoints of the sides *AB, BC, CD, DA*. Join *KL, LM, MN, NK*. It turns out that *KLMN* is a

parallelogram whose area is equal to half that of the quadrilateral. This can be seen by cutting off the "ears," triangles *AKN, BLK, CML,* and *DNM*, and covering the parallelogram *KLMN*. Students might try to do this with a quadrilateral cut out of Bristol board.

2. Let *ABCD* be a quadrilateral and *K, L, M, N* be the midpoints defined in the previous problem. Join *KM* and *LN* and let them intersect at the point *O*. Then the quadrilateral is partitioned into four smaller quadrilaterals *AKON, BLOK, CMOL, DNOM*. The sum of the areas of two of these smaller quadrilaterals that do not have a common side is equal to the sum of the areas of the other two smaller quadrilaterals. In other words, the sum of the areas of the quadrilaterals *AKON* and *CMOL* is equal to the sum of the areas of the quadrilaterals *BLOK* and *DNOM*.

   (All the students need to know is that two triangles on equal bases and between the same heights have the same area. Encourage imagination and discourage computation and the use of formulae. One might hope that someone will think to join the vertices of the quadrilateral to *O*. Did this happen with your group?)

3. Let *A, B, C, D* be four distinct points on a page. There are six pairs of these points: *(A, B), (A, C), (A, D), (B, C), (B, D), (C, D)*, and we can measure the distances between each of these pairs. Normally, we will get six different numbers, but there are some configurations where there will be fewer different numbers. Find all the configurations in which you get only two different numbers.

   (This should be a question that is easy to get into. An example that most classes hit on right away is the set of four vertices of a square; this helps define the problem for those unclear about what is being asked. In all, there are six possibilities that are more or less difficult to find. It is a remarkable group that finds all six, but most will find two or three more.)

I look forward to hearing from you about these investigations and about others that you have done with your students. My e-mail address is **barbeau@math.utoronto.ca**. ▲

▲ Go Fly a Kite: Using Diagonals to Classify Quadrilaterals

CORNELIS (KEES) DE GROOT



Cornelis (Kees) de Groot, Ph.D., is an associate professor of secondary mathematics education at the University of Rhode Island, Kingston, RI. His interests are related to learning transitions in mathematics, performance assessment, and teacher education. Dr. de Groot is editor of Secondary Math Exemplars.

Classifying quadrilaterals and other polygons is an important task in the learning of geometry. This kind of activity requires that students not only know the parts and properties of quadrilateral figures, but that they also know how these parts and properties are interrelated within and among the figures. Pierre van Hiele (1986) identified the level at which this activity takes place as informal deduction (see Figure 1). At this level, students do not produce formal proofs and often are convinced by measuring and verifying. Activities at the level of informal deduction are necessary to assist students in the transition to the level of deduction, which is the level at which much high school geometry instruction takes place. According to van Hiele, the attainment of levels is not age-related, and thus, we need to do these sorts of activities with anyone who is at Level 2. This, then, includes our high school students. I have done the activity I present in this article with students from Grades 5–12 and with teachers.

Level 0 (Basic Level): Visualization; Students recognize figures as total entities (triangles, squares), but do not recognize properties of these figures (right angles in a square).

Level 1 (Analysis): Students analyze component parts of the figures (opposite angles of parallelograms are congruent), but interrelationships between figures and properties cannot be explained.

Level 2 (Informal Deduction): Students can establish interrelationships of properties within figures (in a quadrilateral, opposite sides being parallel necessitates opposite angles being congruent) and among figures (a square is a rectangle because it has all the properties of a rectangle). Informal proofs can be followed, but students do not see how the logical order could be altered, nor do they see how to construct a proof starting from different or unfamiliar premises.

Level 3 (Deduction): At this level, the significance of deduction as a way of establishing geometric theory within an axiom system is understood. The interrelationship and role of undefined terms, axioms, definitions, theorems, and formal proof is seen. The possibility of developing a proof in more than one way is seen.

Level 4 (Rigor): Students at this level can compare different axiom systems (non-Euclidean geometry can be studied). Geometry is seen in the abstract with a high degree of rigor, even without concrete examples.

Figure 1. Van Hiele: Levels of Geometric Thought. From **euler.slu.edu/teachmaterial/Van_Hiele_Model_of_Geometr.html**

I was inspired to create this activity by my experiences as a child, when I would construct a kite from two sticks, twine, and paper. To construct a kite, I first made sure that I had a longer stick and a shorter stick to form the spars. I used a piece of twine to determine the middle of the shorter stick by matching the length of the stick with twine, folding it in half, and using that piece to mark the middle. Then, using twine, I would tie the shorter stick onto the longer stick so that the middle of the shorter stick was above the middle of the longer stick and the two sticks were perpendicular to each other. Next, I would take twine and connect it to each end of each stick so that a quadrilateral figure (kite) was made. I would then cover this area with paper, and glue flaps over the twine perimeter.

Asking the Inverse Question

In many cases, especially at lower van Hiele levels, we have students work from the figures to the parts and properties. To assist students in the development of their mathematical reasoning, it is important that they be asked to invert this approach (Freudenthal, 1978). In this article, I propose an activity that addresses the classification of quadrilaterals by reasoning from the properties of the diagonals of quadrilaterals to the shape that belongs to them. So, instead of working toward the idea that a square is a figure in which the diagonals are equal and are perpendicular bisectors, we invert the task as follows:

> *When you think of four-sided figures (quadrilaterals), you might think of a kite. A kite is actually a special kind of quadrilateral. As you know, the skeleton of a traditional-looking kite is made with two intersecting sticks (spars) and some twine going from end to end. The two sticks (spars) are called the* diagonals of the quadrilateral.
>
> ***Find a systematic way of determining the maximum number of different quadrilateral shapes you can design this way.***
>
> *Remember, all you have is "sticks and twine." Decide on what sorts of things you can change so that the design is different from any one you have designed up to this point. Report on your method of investigation, and evaluate how sure you are that you have found every possible different quadrilateral. If there is something special about a given quadrilateral design, then give it a name. Justify your choice of name, or demonstrate that the name that you have chosen is appropriate.*

Materials

- Plastic straws (flat coffee stirrers are preferred) or flat dry pasta, such as linguini (if allowed)
- Scissors
- Rulers
- Protractors
- Geometric software, such as The Geometer's Sketchpad

Discussion

The key to this task is the emphasis on a systematic approach to finding all possibilities. How will a student know that all possible scenarios have been determined? The following is a way to scaffold this for students who may need assistance in exploring the possibilities systematically. (I have found that many are not inclined to do this without some prompting.)

Even though students may think that a certain quadrilateral has been created, they must verify or prove their conjecture. Allow students to do this at van Hiele Level 2, but promote Level 3 by posing probing questions. Using concrete materials or computer software, such as The Geometer's Sketchpad, allows almost all students to enter this problem in a way that encourages exploration. Dry spaghetti (if allowed) or coffee stir straws are very useful, since these can be easily broken or cut into different sizes. These materials can represent the spars (diagonals), as well as the twine

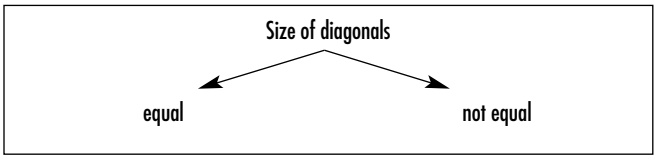
(sides). Equality of measures can be done by comparison or by using a ruler. The use of software such as The Geometer's Sketchpad may help students (at van Hiele Level 2) move from concrete reality to a medium in which they can design, measure, label, and verify. At a more advanced van Hiele level (3), students should prove their findings. I have provided one example proof below.

Scaffolding a Systematic Approach to Quadrilateral Classification

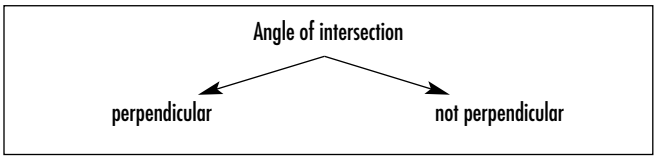
There are fundamentally three attributes that can be varied for diagonals:

1. their relative size
2. the angle of intersection
3. how they intersect relative to each other

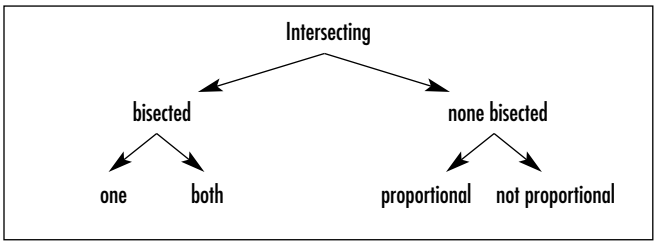
For relative size, we can say that the two diagonals are either equal or they are not:



For the angle of intersection, it may be helpful to notice that it must be between 0 and 180 degrees and that a clever division may be to say that it is either 90 degrees or it is not:



How two diagonals can intersect proves to be the more complicated issue. This is represented as follows:



This schema also provides a very nice hierarchical classification system in which all three elements come together as a conceptual tree diagram (See Figure 2).

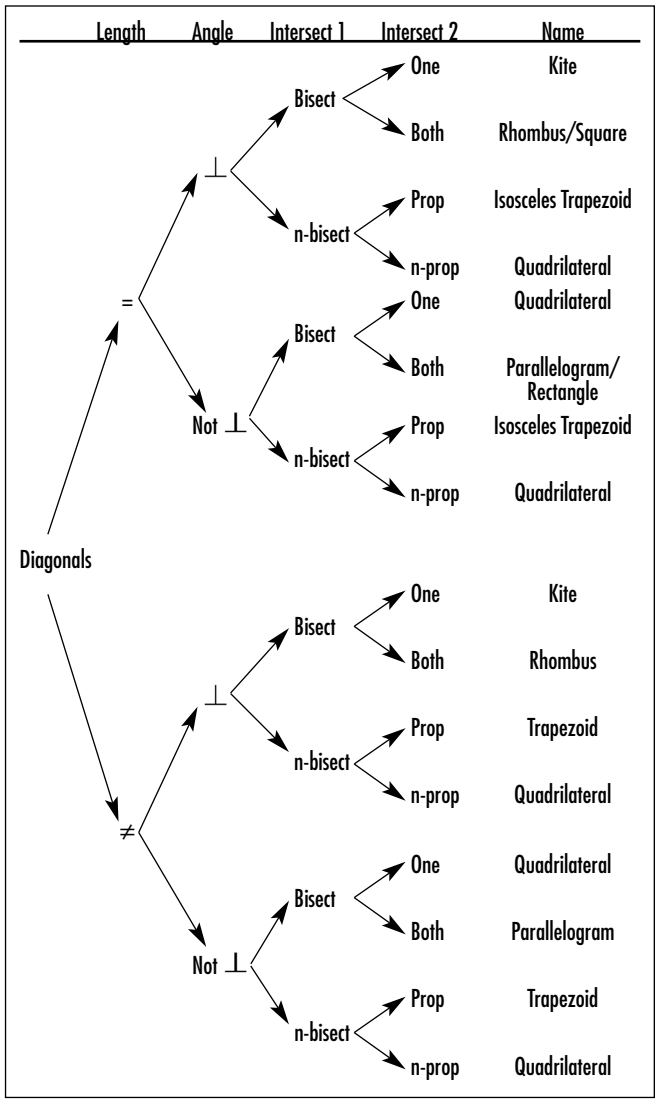


Figure 2. A systematic approach to quadrilateral classification.

The system in Figure 2 is represented pictorially in Figures 3, 4, 5, and 6.

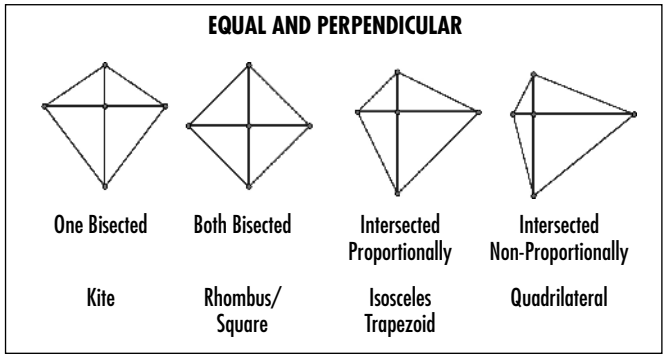


Figure 3. Equal and perpendicular diagonals.

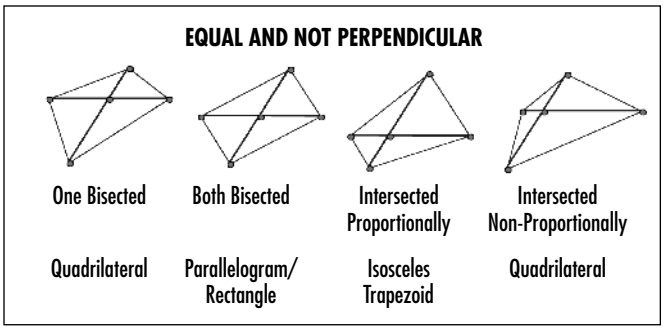


Figure 4. Equal and not perpendicular diagonals.

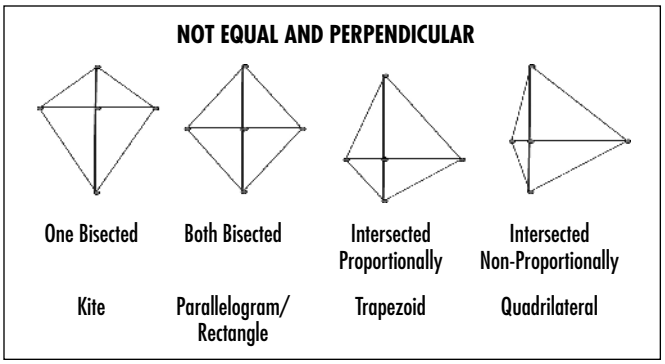


Figure 5. Not equal and perpendicular diagonals.

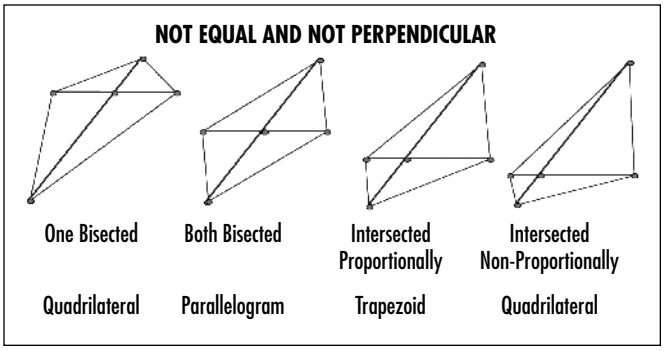


Figure 6. Not equal and not perpendicular diagonals.

The Case of Two Diagonals Intersecting Proportionally

In my experience, the case of two diagonals intersecting proportionally is rarely obvious to students. During the exploration, some prompting from the teacher is required. Even in doing this activity with teachers, I find that this is not an immediate thought. It is the trapezoid that makes this activity all the more interesting and rich. For example, after this activity, one can think of a rectangle as a trapezoid. To show that a quadrilateral is a trapezoid, we need to establish that one pair of opposite sides is parallel. Let's explore this further.

In order to demonstrate that two opposite sides are parallel, we consider one of the two diagonals to be a transversal of the pair of opposite sides we wish to show parallel. In the given situation, we cannot directly deduce similarity from the proportionality of the corresponding diagonal segments (see Figure 7). We will see that two similar triangles are formed, with these corresponding parallel sides as bases. In other cases, where the proportionality factor equals one, two congruent triangles are formed with parallel pairs of opposite sides, and we get results such as rectangles, squares, and parallelograms (see Figures 3, 4, 5, and 6). When neither diagonal bisects and is not segmented in proportion, we do not obtain a special quadrilateral, just a generic one.

We will now consider a deductive proof that the quadrilateral will have to be a trapezoid when the two diagonals intersect proportionally at van Hiele Level 3, using the law of cosines and sines.

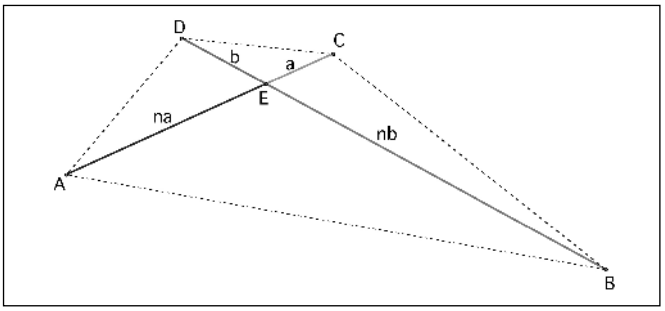


Figure 7. Diagonals intersecting proportionally.

In Figure 7, the diagonals $\overline{AC}$ and $\overline{BL}$ intersect proportionally in *E*. To show that the figure is a trapezoid, we need to show that $\overline{DC}$ and $\overline{AB}$ are parallel. This, in turn, can be shown by proving that either $\angle EAB \cong \angle ECD$ or by showing $\angle EBA \cong \angle EDC$. This then will show that $\overleftrightarrow{AC}$ or $\overleftrightarrow{DB}$ is a transversal of $\overleftrightarrow{DC}$ *and* $\overleftrightarrow{AB}$ with congruent alternate interior angles, thus showing that $\overleftrightarrow{DC}$ *and* $\overleftrightarrow{AB}$ are parallel.

Step 1

Use the law of cosines to show that $\overleftrightarrow{DC}$ *and* $\overleftrightarrow{AB}$ are proportional by the same factor *n*.

$$DC^2 = a^2 + b^2 - 2ab\cos(\angle CED)$$
$$AB^2 = (na)^2 + (nb)^2 - 2(na)(nb)\cos(\angle BEA)$$
$$= n^2\left[a^2 + b^2 - 2ab\cos(\angle BEA)\right]$$

Because $\angle CED$ *and* $\angle BEA$ are vertical angles, they are congruent. From this follows that: $AB^2 = n^2DC^2$ and thus that $AB = nDC$.

Step 2

Use the law of sines in both triangles to show that $\angle EAB \cong \angle ECD$.

(1) $\frac{\sin(\angle CED)}{DC} = \frac{\sin(\angle ECD)}{b}$

$$\Rightarrow \sin(\angle CED) = \frac{DC}{b}\sin(\angle ECD)$$

(2) $\frac{\sin(\angle BEA)}{nDC} = \frac{\sin(\angle EAB)}{nb}$

$$\Rightarrow \sin(\angle BEA) = \frac{nDC}{nb}\sin(\angle EAB)$$

$$= \frac{DC}{b}\sin(\angle EAB)$$

From (1) and (2) follows that $\angle ECD = \angle EAB$.

QED

Alternatively, we can reason as follows in Step 2: Because all three pairs of corresponding sides of the two triangles are in the same proportion, the two triangles must be similar. Similar triangles have congruent corresponding angles. In essence, the law of sines represents this relationship.

I hope that you and your students will gain much pleasure from this activity. In this case, posing the inverse question creates a more challenging environment. Students at van Hiele Level 2 are sometimes easily discouraged when the tasks given to them are at Level 3. According to van Hiele and others, no matter what the age or grade level of the students, it is necessary to deliberately design activities that will help them transition through these levels. This does not appear to happen "naturally."

I have found that this activity assists with the Level 2 to Level 3 transition.

References

Freudenthal, H. (1978). *Weeding and sowing: Preface to a science of mathematical education.* Dordrecht, Netherlands: D. Reidel Publishing Company.

Van Hiele, P.M. (1986). *Structure and Insight.* Orlando, FL: Academic Press. ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

▲ Directors' Dialogue

ANN MICHELE STENNING



Ann Michele Stenning is a mathematics teacher at South Secondary School with the Thames Valley District School Board. Having finished her Masters of Education degree in Curriculum Studies, she is currently assisting other parent volunteers with the development of weekly literacy activities, which she and other moms provide to at-risk students.

OAME Board of Directors Meeting February 6–7, 2009

Friday, February 6, 2009

The Board of Directors meeting began with remarks by Anna Jupp, OAME's president. She reflected on change and how it influences our practice as teachers. To illustrate this fact, she showed the video "Shift Happens," available at **www.youtube.com**. Afterward, a member of the Constitution Committee directed members to consider changes to OAME's Board. Members discussed the appropriate size of the Executive and Board of Directors, whether members should be locally or provincially elected, and whether there should be a balance of elementary and secondary educators on the Board.

Saturday, February 7, 2009

The Equity Committee met over breakfast. In planning for the OAME annual conference, Sue Melville reported that Kelly-Lee Assinewe of the N'Swakamok Friendship Centre Alternative School in Sudbury will speak about alternative approaches to educating First Nations students during the Equity breakfast. She also announced the members that would be featured during the panel discussion: Shaun Knowles, Barb Guglielmi, and Kelly-Lee Assinewe. The Equity Committee devised a list of questions for the three guests. Future plans for the committee include a study of Marion Small's new book on questioning and differentiated instruction. Other ideas include the development of differentiated assessments and a sample grade book.

OAME executive director Dave Hessey began a presentation of OAME's finances. Dave told members that OAME expenses are currently below budget; Connie Quadrini, one of the annual conference co-chairs from 2008, shared that last year's OAME conference yielded a profit. Executive director Sue Hessey announced that some issues of the *Abacus*, the elementary math

magazine of OAME, would be shrink-wrapped and sold for $10. These lessons would focus on adding, subtracting, multiplying, and dividing numbers. In addition to the lessons, the purchaser would receive a strand of beads in red and white OAME colours, to be used as a manipulative. Sue also noted that purchasers would be allowed to make multiple copies of these lessons to facilitate teaching and learning. Greg Clarke, OAME's webmaster, stated that the website's appearance will change and that the site will become easier to use. Jacqueline Hill, OAME's past-president, announced that all of OAME's positions had been acclaimed and that elections were not necessary this year. The positions are as follows:

President-Elect: David Zimmer

Vice-President: Bill Otto

Directors (K–6): Urvi Shah, Marilyn Hurrell, Sonia Ellison, and Lyla Kadlec

Directors (Grades 7–12): Sandra Jean Price, Rob McLeish, and Pat Kehoe

Their biographies will be posted on the Online Election section of the website from March 15 to April 1. Anna Jupp thanked the following people for their contribution, commitment, and time: Jacqueline Hill, Gord Cooke, Anna D'Armento, Anna Dutfield, Paul Gautreau, Duncan LeBlanc, Walter Whitely, and Diane Lang.

Members were reminded that OMO 2009 will be held May 29–30 and that OAME 2010 will be held May 13–15. Voting confirmed that SWOAME would host OAME 2011.

Robert Sherk, Publications Committee chair, reported that the deadline for the submission of articles for the June edition of the *Gazette* is April 10, 2009. Articles should be submitted in both MS Word and PDF formats. Stewart Craven, editor of the *Gazette*, announced that he would not be renewing his term as editor when it expires in June 2010. An ad for the editor's position will be placed in the June 2009 issue of the *Gazette*. Plans are underway for a feature issue on games, tentatively scheduled for December 2009.

In anticipation of the 40th anniversary of OAME in 2012, OAME's past-presidents Shirley Dalrymple, Peter Saaramaki, and Don Attridge, joined the committee meeting to share their ideas for the 40th Anniversary Edition of the *Gazette*. One idea is that this issue may extend past print articles and photos to include video clips on the website that tell the story of the first 40 years of OAME. Each chapter will be invited to submit a brief written history of the chapter for inclusion in this issue. The target publication date would be mid-May 2012. Anyone who would like to contribute in any way to this special issue should contact Peter Saarimaki

CHAPTER REPORTS

CHAMP – Rep: Shaun Knowles:

Thanks to all who came out to make our FALL CHAMP EVENT a success. Participants enjoyed the breakout sessions, food, prizes, and guest speaker. Once again, we appreciate your support and efforts. Looking ahead, CHAMP is busy planning secondary contests, OMO trials, Grades 7 and 8 Contests, and an Award Ceremony. Visit our website to find the details for these events. We continue to hold our meetings at the Dufferin-Peel Catholic District School Board offices on the first Thursday of the month. You are welcome to join our exciting and enthusiastic executive team.

COMA – Rep: Pat Kehoe:

Our annual Ronald C. Bender Memorial Mini-Conference was held on Saturday, November 22 at the University of Ottawa. The conference was very successful, with over 100 teachers attending. Sixteen workshops were offered, covering topics from JK to Grade 12 Mathematics. Many thanks to our sponsor, the University of Ottawa. Upcoming events include our local Math Olympics to be held April 25. COMA is also proud to be hosting this year's provincial OAME Conference (May 14–16) in our nation's capital, during the annual Tulip Festival. The theme of the conference is Balance: Finding the Right MMIX, and our chairpersons, Paul Gautreau and Cathy Hall, continue to work tirelessly with their subcommittees to make this conference an exciting one for all participants! Check the OAME website for more details.

GOLDEN – Rep: Laura Tonin:

Our committee is hard at work planning the 2010 annual conference. It will take place May 13–15, 2010, at Brock University. In addition, we are hosting our annual mini-conference on April 23 at Brock University. There will be two workshops, followed by dinner.

GVMA – Rep: Rob Gleeson:

No report.

ISOMA – Rep: Diane Lang:

ISOMA thanks Branksome Hall School and Edith Louie for hosting our Fall Conference, LEAP, in November. The conference was well attended by math teachers in elementary and secondary classrooms and by teacher candidates from OISE. Participants had their choice of attending two workshops from 17 different selections, and everyone was treated to an entertaining talk on Chance by Professor Jeffrey Rosenthal following dinner. Bishop Strachan School is hosting ISOMA's Middle School Math Olympics on Saturday, April 18. If your school is interested in attending, contact Carly Ziniuk at Bishop Strachan to register or to find out how you can help with this event. Finally, we are delighted to welcome Urvi Shah from Trinity College School, who will take over as ISOMA Chapter Representative to OAME this Spring. We are also happy to welcome Robert Tong to the ISOMA

Executive; we look forward to the new ideas that Robert and Urvi will bring to our activities. If you are interested in joining the Executive, please contact us. Finally, I wish to express my gratitude to Sue and Dave, the dedicated and hard-working OAME Board members, and in particular, the members of the ISOMA Executive, for making my term as ISOMA Chapter Rep such an enjoyable and worthwhile experience.

MAC² – Rep: Michele Walma:

MAC² had a successful fall mini-conference! Thank you to Kathy Kubota-Zarivnij for her keynote address on the use of Bansho as an instructional strategy for the teaching and learning of mathematics. Following her keynote address, breakout sessions took place. These sessions modelled what Bansho could look like in each of the divisions. Seventy people attended the mini-conference. Due to the success of the conference, our chapter is looking at offering the same mini-conference in the Spring!

NOMA – Rep: Carol Lemmon:

We are working hard at setting up a P.A. schedule for teachers to attend in mini-conference format. Our focus will be on topics of interest to new teachers or teachers new to math. After-school sessions and after-dinner sessions will be available. Plans for regional playdowns for the Math Olympics are starting, with hopes of running the playdowns at Lasalle Secondary School at the beginning of May.

NWOAME – Rep: Tom Boland:

NWOAME has had a busy and exciting first half of the year. In the Fall, we hosted a Meet-and-Greet Dinner Social, where over 70 members and guests came to find out more about our organization. We used the evening to kick off a poster and logo contest, to provide information about NWOAME and OAME, and to preview upcoming events. We also listened to a keynote address from guest speaker Walter Whitely (York University) on "How to Think Like a Mathematician." We have planned a "Pi Week" in mid-March, during which we will be hosting a variety of events focusing on different aspects of Pi. We are looking forward to a high level of involvement from students, teachers, and parents. NWOAME membership has grown considerably, and we continue to look forward to busy, exciting times as we do our part to get teachers talking about math!

PRMA – Rep: Sandra Jean Price:

The Pine Ridge Math Association is working diligently on preparations for hosting OMO 2009 on May 29 and 30 at Trent University in Peterborough. We are hoping that it will be as successful as last year's event, also held at Trent. The theme, once again, is "Eat, Sleep, Math: Life's Highway Essentials." We are currently looking for sponsors to support this year's event. We are also preparing for our local playdowns. We have tentatively scheduled two days in early May in Durham and one day in Peterborough for playdowns .

QSLMA – Rep: Robert Sherk:

The main activity this term is to canvass local math heads' councils and garner support for planning future local mini-conferences, a local OMO, and possibly, the annual conference in 2012.

SAME – Rep: Debbie Drape:

SAME is pleased to announce that we are well on our way to meeting some of the goals that we have been working on over the past two years. We have increased the number on our Executive with some dynamic, wonderful mathematics educators. One of our new members will be working on getting our dormant website up and running again. Several other new Executive members have stepped right in to help with the workload and have brought with them fresh eyes and new ideas. This year, SAME has been able to go back to hosting two mini-conferences, which was the norm for many years in the past.

SWOAME – Rep: Nicholas Chevalier:

No report.

TEAMS – Rep: Laurissa Werhun:

The TEAMS Executive was delighted to find out that TEAMS is officially hosting OMO 2011. If any TEAMS members are interested in helping with the organization of this event, please contact the co-chairs. Do you know a Grade 7 or Grade 8 student who would enjoy problem solving in a group setting? We are planning our TEAMS Math Olympics, to be held on March 28, at the Catholic Education Centre, 80 Sheppard Ave. E. Winning students go on to the Ontario Math Olympics in Peterborough. If you are interested in fielding a single student or a team of four students (one male and one female in Grade 7, and one male and one female in Grade 8), please contact Michele Goveia or Anthony Meli. TEAMS held a very successful mini-conference on November 6, 2008 at Dante Alighieri Academy (Beatrice Campus) and Bathurst Heights Learning Centre, on Linking Assessment and Instruction. Over 200 members attended. Door prizes included the sponsorship of one member to attend the OAME conference in May and another to attend the OAME Leadership Conference in February. The TEAMS Executive is looking forward to encouraging elementary teachers to become more involved. Please come out to our meetings. We meet monthly at the Catholic Education Centre, and we look forward to seeing you there.

WOMA – Rep: Ann Michele Stenning:

WOMA hosted its annual fall mini-conference in November, with higher than normal attendance. Our dinner speaker was Sherry Mantyka from St. John's, Newfoundland, an educator and author of *The Math Plague*.

Y⁴MA – Rep: Ada Cartaginese:

No report. ▲

▲ 2008-2009 OAME BOARD

OAME website www.oame.on.ca

Direct General Inquiries to:
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NWOAME - **Tom Boland**

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