Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Fifth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Stewart Craven, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Stewart Craven at **numeratecitizen@mac.com**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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▲ Editor's Message

STEWART CRAVEN, EDITOR



A little over a week ago, I returned home from the fourth Canadian Mathematics Education Forum (CMEF) held in Vancouver. Working groups representing every corner of Canada congregated to continue their work regarding important issues in mathematics education. The areas of focus were wide and varied. They included:

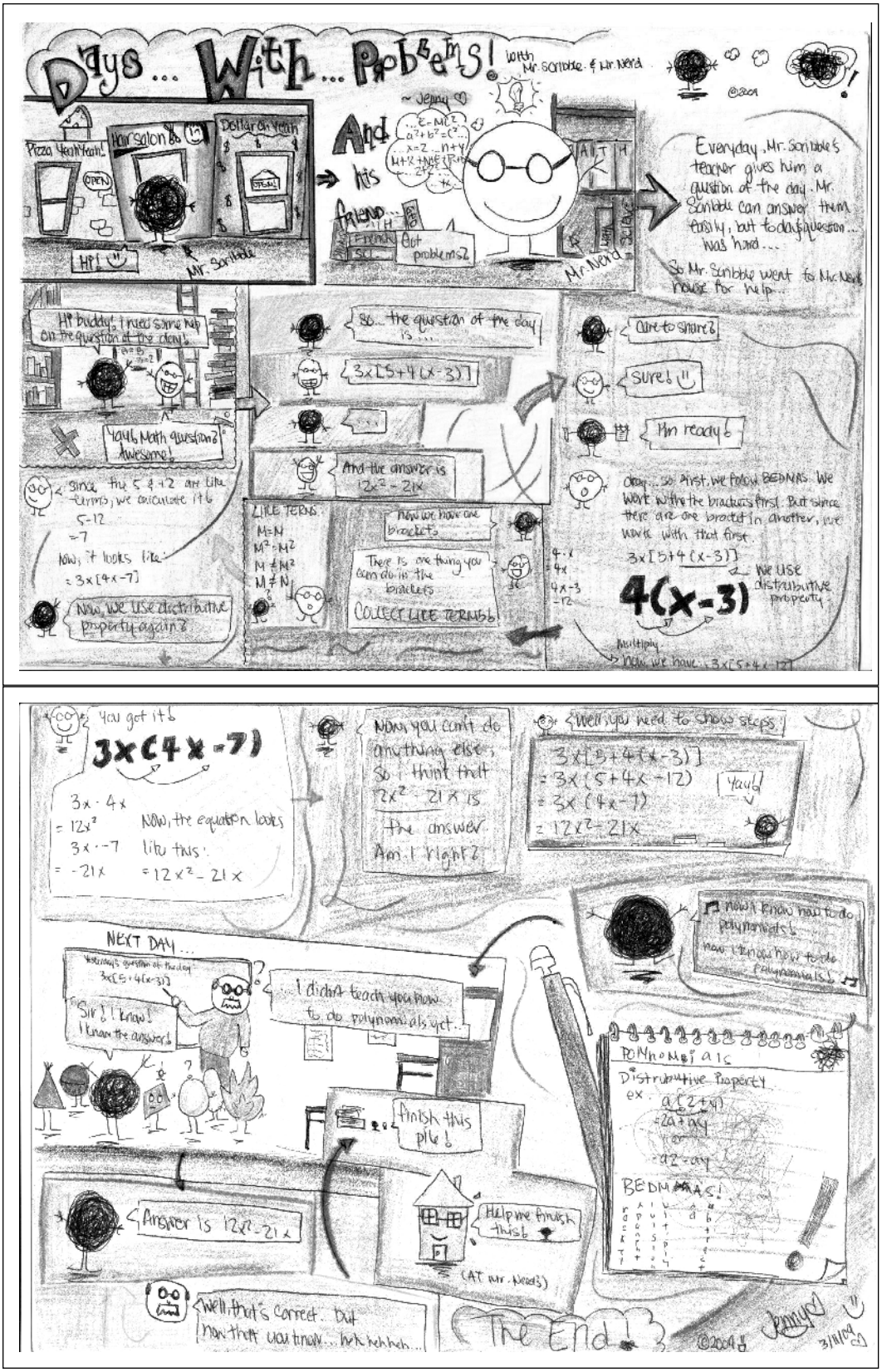
- Assessing for Problem Solving Development
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- Significant Statistics
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- Textbook Design

The working group that I was associated with—Psychology and Philosophy of Mathematics—explored what led each of us to enthusiastically embrace the study of mathematics. We discussed issues such as the importance of mathematics for all citizens and debated the value of introducing mathematical symbols and talismans to very young children in an effort to establish positive associations toward learning, doing, and appreciating mathematics. This conversation evoked radically diverse opinions and feelings.

This is only the beginning of the work that a small cadre of us is committed to carrying forward. It is important that meetings such as CMEF continue to take place so that people from all parts of the country can dialogue and explore important issues in mathematics education. I commend the CMS and the other sponsoring partners for facilitating our discussions.

On a different note, I would like to thank and

congratulate all of the students who submitted cartoons for the cartoon contest. The first one to be published is a cartoon by Jenny Lee from Pierre Elliott Trudeau High School in Markham. Congratulations Jenny! ▲



▲ President's Message

BEVERLY FARAHANI, PRESIDENT



Beverly Farahani is Head of Mathematics at Kingston Collegiate and Vocational Institute. She is also a T3 Instructor and was on the Minister's Task Force.

Writing my first President's Message has made me think about many things. Should I look at big ideas, pedagogy, or some other research topic? In the end, looking at the classroom seemed most appropriate.

By the time you read this message, the school year will be over. I hope that you will take some time to reflect on how your year went. A teacher can make a big difference in the life of a student!

As a classroom teacher, my main focus is my students and how they are progressing mathematically. Many things—from pedagogy to teacher knowledge, teacher beliefs, and student life—influence what happens in a math classroom.

Think back to the first time you taught a particular topic or class. You planned the lesson, hoping that everything would go well. Sometimes the lesson was great, while at other times, it was less than stellar! Each time you teach a topic, you become more adept at anticipating what questions students might ask or what misconceptions they will need to overcome to have a better understanding of the topic. Often, talking to other teachers or attending professional development sessions can generate new ideas or alternative methods, and while being outside your comfort zone is sometimes scary, it is very important for a teacher! It can remind you of how students might feel in your classroom. I am constantly reminded that "Life is a journey, not a guided tour." Eventually, students need to feel that they can travel through mathematics without having you hold their hands as tightly or as often as you did in the beginning.

In a supportive, motivational learning environment, students tend to ask more questions. In fact, they may end up delving deeper into a concept than you first intended! A colleague once shared the following story: A Grade 12 student asked, "Can we work backwards?" The teacher, knowing full well what the student was talking about, but unsure if the other students understood, asked, "What do you mean by that?" As the student clarified her question, many other students started to nod in agreement—they, too, wanted to know! The teacher gave an example of what the students considered to be a final answer, and then asked, "Where did it come from?" Students called out an acceptable response. Before the teacher could comment, a voice from the back of the room altered the answer slightly, and the class agreed that the small change improved the class response. Consequently, the students felt that they had discovered something special because they now had ownership of the mathematical concept.

I think often of the following comment made by a student:

> *A math class has to be a place where it is encouraged that you learn more than you know; a place where doing whatever it takes to advance learning (asking questions, seeking help, etc.) is given priority. No question is ignored simply because it disrupts the flow of the class. The teacher does what is in the students' best interest, not their own.*

I hope that your classroom is a place where mathematical thought and processes flourish, as such an environment enhances student understanding, thus helping students meet the demands of the curriculum. ▲

▲ Call for *Gazette* Editor

The *Ontario Mathematics Gazette* is taking applications for a two-year term as editor commencing with the September 2010 issue.

Job shadowing available now.

For more details, please contact:
Executive Directors:
Dave and Sue Hessey



▲ What's the Problem? Funky Fractions

Shawn Godin



Shawn is the head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans, ON. The web page for this column is ***whats-the-problem.wikispaces.com****.*

Welcome back, problem solvers! I have just set up a WikiSpaces web page for this column. You can comment on the problems and solutions, send in your own solutions, or download any files that I mention in the column (e.g., this column's spreadsheet). You can check out the wiki at **whats-the-problem.wikispaces.com**.

Last issue, I left you with the following problem:

Let *a*, *b*, and *c* be non-zero digits. A student takes the fraction $\frac{ab}{ca}$, where *ab* and *ca* represent the two-digit integers $10a + b$ and $10c + a$, and applies a (false) cancellation law, cancelling the *a* from the numerator with the *a* from the denominator. For example, if $a = 6$, $b = 5$ and $c = 2$, the student would obtain $\frac{65}{26} = \frac{5}{2}$ (by cancelling the 6's). Determine all triples (*a,b,c*) for which this student actually obtains the correct answer.

This problem appeared as M325 from the December 2007 issue of *Crux Mathematicorum with Mathematical Mayhem*, and its solution appeared in the November 2008 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem* website at **journals.cms.math.ca/CRUX/**.

This problem lends itself nicely to a computer exploration. A computer programmer could probably write a dozen lines of code that would yield a solution. I'll start out with the spreadsheet from the Open Office Suite, which is available free at **www.openoffice.org**.

Start by setting up columns for *a, b, c*, $\frac{ab}{ca}$ and $\frac{b}{c}$. The first few entries are show below.

	A	B	C	D	E
1	a	b	c	ab/ca	b/c
2	1	1	1	=(10*A2+B2)/(10*C2+A2)	= B2/C2
3	=A2	=B2	=C2+1		

Notice that I have used formulas in columns A and B. This way, if I change the entry in A2 or B2, the whole column will change. Filling down, we get the following:

	A	B	C	D	E
1	a	b	c	ab/ca	b/c
2	1	1	1	1	1
3	1	1	2	0.52	0.5
4	1	1	3	0.35	0.33
5	1	1	4	0.27	0.25
6	1	1	5	0.22	0.2
7	1	1	6	0.18	0.17
8	1	1	7	0.15	0.14
9	1	1	8	0.14	0.13
10	1	1	9	0.12	0.11

From our first step, we can see one solution $\frac{11}{11} = \frac{1}{1}$. Changing the values in A2 and B2, we can go through all the possibilities. For example, if we make $a = 6$ and $b = 5$, we find the solution that was presented in the problem.

	A	B	C	D	E
1	a	b	c	ab/ca	b/c
2	6	5	1	4.06	5
3	6	5	2	2.5	2.5
4	6	5	3	1.81	1.67
5	6	5	4	1.41	1.25
6	6	5	5	1.16	1
7	6	5	6	0.98	0.83
8	6	5	7	0.86	0.71
9	6	5	8	0.76	0.63
10	6	5	9	0.68	0.56

If we were to go through all of the possibilities for a and b, we would find all of the solutions.

We can approach the problem another way by using algebra. We can represent the problem algebraically as $\frac{10a+b}{10c+a} = \frac{b}{c}$. Manipulating this equation, we can put it into a more useful form:

$$c(10a+b) = b(10c+a)$$
$$10ac + bc = 10bc + ab$$
$$10ac - 10bc = ab - bc$$
$$10c(a-b) = b(a-c)$$
$$2 \bullet 5c(a-b) = b(a-c)$$

In this form, we can easily see that if $a = b = c$, the condition is satisfied, and we get nine (trivial) solutions. Note that as 5 is prime and $1 \le a,b,c \le 9$, then either $b = 5$ or $a - c = 5$.

If $b = 5$, we get

$$\begin{aligned} 2 \bullet 5c(a-b) &= b(a-c) \\ 2 \bullet 5c(a-5) &= 5(a-c) \\ 2c(a-5) &= a-c \\ 2ac-10c+c &= a \\ c(2a-9) &= a \\ c &= \frac{a}{2a-9} \end{aligned}$$

Since c must be positive, and we have already taken care of the case of $a = b = c = 5$, we must have $a > 5$. Trying the remaining values, we find two solutions, the original $\frac{65}{26} = \frac{5}{2}$ and a new solution $\frac{95}{19} = \frac{5}{1}$. Using a similar argument with $a - c = 5$, we get the last two solutions, $\frac{64}{16} = \frac{4}{1}$ and $\frac{98}{49} = \frac{8}{4}$.

But here's an interesting challenge: Can you, at this point, extend this to fractions that have numbers with more digits? (That should keep you busy over the summer!)

On top of that, I am still going to give you some **homework**!

The standings in the first round of the FIFA World Cup, Group B, are shown in the table below. Knowing that each team played each of the others exactly once, use the given information to determine the score of each match.

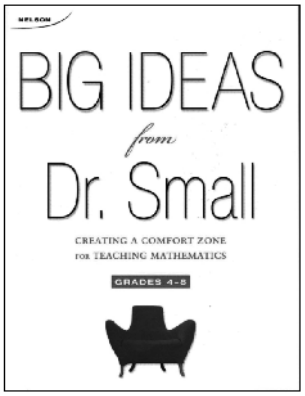
	Wins	Losses	Ties	Goals For	Goals Against
England	2	0	1	5	2
Sweden	1	0	2	3	2
Paraguay	1	2	0	2	2
Trinidad and Tobago	0	2	1	0	4

Until next time, happy problem solving! ▲

▲ Book Review: *Big Ideas from Dr. Small*

REVIEWED BY
KATHY KUBOTA-ZARIVNIJ
OAME ABACUS CO-EDITOR

How did you learn mathematics when you were in elementary school? Many of us learned mathematics as a set of disconnected concepts, skills, and facts, yet we need to understand mathematics differently if we are to effectively teach it through problem solving within a framework that is coherent and connected. "A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the 'big ideas' of mathematics—that is, the interrelated concepts that form a framework for learning mathematics in a coherent way" (Ontario Ministry of Education, 2005, p. 4).



What does it mean to teach to the big ideas? If you ask any group of educators for examples of big ideas, you'll get a variety of answers. Some will suggest a topic like quantity relationships or probability. Some will say that big ideas are important curriculum expectations, while others will suggest overall curriculum expectations like "construct three-dimensional figures, using two-dimensional shapes." Although all of these are important, Randall (2005) would say that none of these ideas are sufficiently robust to qualify as a "big idea" in mathematics.

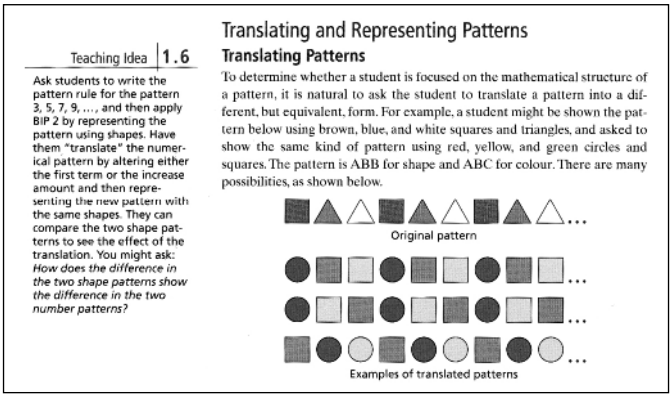
Randall (2005) explains that "a *Big Idea* is a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p. 10). He says that when we truly understand an aspect of mathematics, we see how it is connected or related to other aspects of mathematics that we know, and thus, we develop a deeper understanding of mathematics. Randall identifies 21 Big Ideas that encompass all five strands of mathematics. For example, Big Idea #9 is

"Patterns: Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways" (p. 17). Following a big idea, Randall provides specific details using "examples of mathematical understandings" from relevant strands. For example, Big Idea #9 is exemplified from number and geometry strands as follows: "...known elements in a pattern can be used to predict other elements… some sequences of geometric objects change in predictable ways" (p. 17).

Dr. Marian Small takes a similar, yet somewhat different approach in her latest book, *Big Ideas from Dr. Small, Grades 4 to 8*. Small (2009) explains that "despite increasing abstraction as students go up the grades, and shifts in focus from grade to grade, big ideas remain fundamentally simple and are the same from grade to grade." Organized by the five strands of mathematics, Small (2009) identifies about 4 to 6 big ideas for 18 important topics: Patterns; Algebra; Numbers Beyond 1000; Number Theory; Whole-Number Operations; Fractions; Decimals; Proportional Thinking (Rate, Ratio, and Percent); Integers; Shapes and Their Properties; Location and Movement; Measurement Fundamentals; Length and Area; Capacity, Volume, and Mass; Measuring Angles; Collecting and Describing Data; Displaying and Analyzing Data; and Probability. This resource is focused on mathematics typically taught in Grades 4 through 8. An example, Big Ideas for Patterns (p. 1), follows:

1. Patterns represent identified regularities. There is always an element of repetition, whether the same items repeat or whether a "transformation," for example, adding 1, repeats.
2. The mathematical structure of a pattern can be represented in a variety of ways.
3. Some ways of displaying data highlight patterns.
4. Many geometric attributes, measurements, and calculations involving numbers are simplified by using patterns.

Small (2009) then unpacks the details of these big ideas, using vivid pictorial illustrations and specific mathematical examples. For "Big Ideas for Patterns" (p. 4), additional details include types of patterns (complex repeating patterns, recursive patterns); extending and describing patterns; translating and representing patterns; creating patterns; and using patterns to develop mathematical concepts. As well, grade-specific teaching ideas related to those details are provided. These mathematical details are very helpful to both novice and experienced mathematics educators (e.g., classroom teachers, special education teachers, consultants, coaches) with respect to preparing lessons or reviewing the mathematics of an upcoming unit of study.



Translating and Representing Patterns

Translating Patterns

To determine whether a student is focused on the mathematical structure of a pattern, it is natural to ask the student to translate a pattern into a different, but equivalent, form. For example, a student might be shown the pattern below using brown, blue, and white squares and triangles, and asked to show the same kind of pattern using red, yellow, and green circles and squares. The pattern is ABB for shape and ABC for colour. There are many possibilities, as shown below.

Original pattern

Examples of translated patterns

Teaching Idea 1.6

Ask students to write the pattern rule for the pattern 3, 5, 7, 9, …, and then apply BIP 2 by representing the pattern using shapes. Have them "translate" the numerical pattern by altering either the first term or the increase amount and then representing the new pattern with the same shapes. They can compare the two shape patterns to see the effect of the translation. You might ask: *How does the difference in the two shape patterns show the difference in the two number patterns?*

Some chapters include lists of principles, or important points, that will support mathematics educators in understanding specific aspects of the topic. Such specifics are important for analyzing and/or designing individual lessons or a sequence of lessons and can help answer questions such as: *What is the real mathematical focus of this lesson? What should the next lesson entail? Is this lesson problem really relevant to the learning goal of the lesson?* As well, a glossary comprising mathematical terms related to the big ideas is included at the back of the resource.

Overall, *Big Ideas from Dr. Small, Grades 4 to 8* is a mathematics resource that will support any educator in improving his or her knowledge of mathematics for teaching through the lens of mathematical big ideas and principles (Ball, 2000). This resource reflects several important instructional considerations, including teaching and learning mathematics conceptually; the importance of multiple representations of mathematical concepts using manipulatives and diagrams; and the significance of a range of multiple solutions, strategies, and invented algorithms. Use of this resource can help teachers of mathematics find mathematical details quickly, study a mathematics topic carefully, or make better decisions about which mathematics task or problem to use based on its relevance to the big ideas.

Mathematics teachers have been working on the premise that we should plan both instruction and assessment by considering the big ideas. Now we have a resource that will effectively support our work in this way.

References:

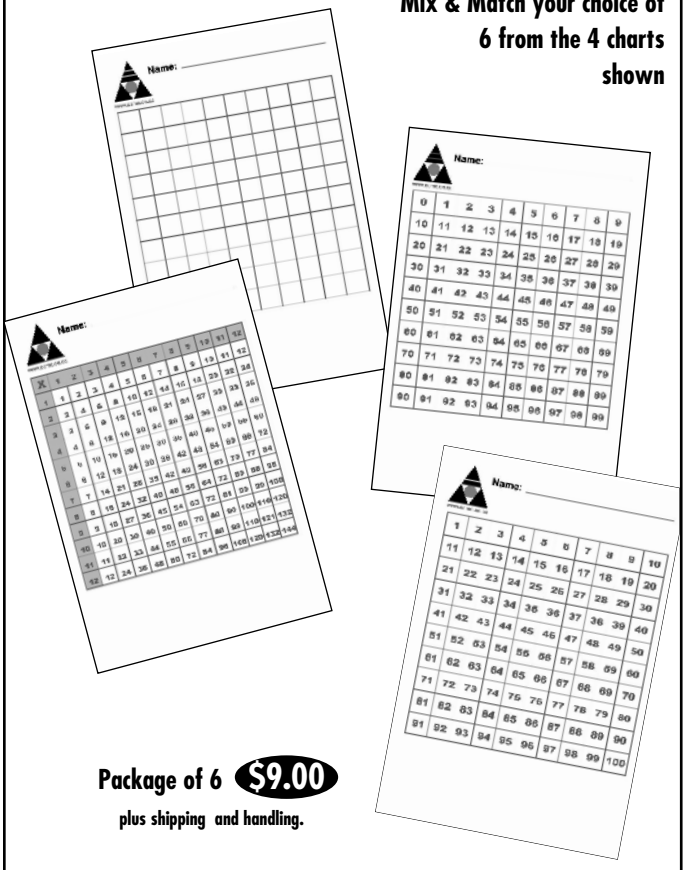
Ball, D., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83–104). Westport, CT: Ablex.

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Randall, C. (2005). Big ideas and understandings as foundations for elementary and middle school mathematics. J*ournal of Mathematics Education Leadership,* (7)3, pp. 9–24. ▲

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▲ Data Rocks – Finding Data on Social Justice at StatCan

YVES SAINT-PIERRE, STATISTICS CANADA



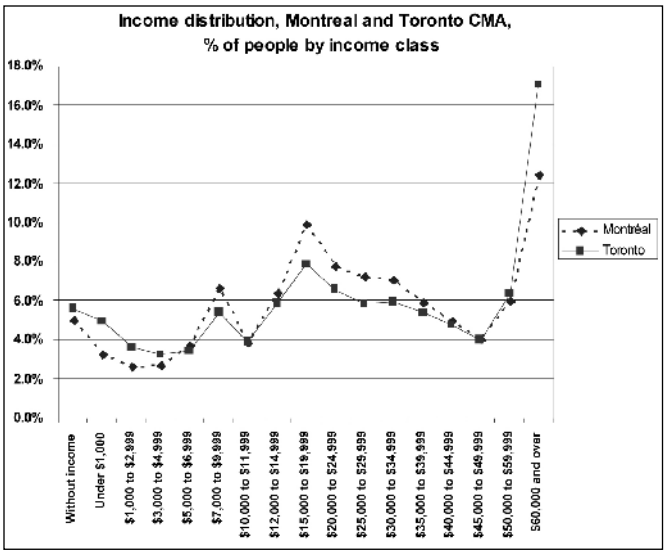
Yves Saint-Pierre has worked at Statistics Canada in many capacities since 1990: questionnaire design, data editing and imputation, analysis, and publication. He has been sharing his knowledge with schools since 2002.

I've been working with schools in the province of Quebec for the last seven years, and for some of that time, with your Quebec colleagues who offer the course on quantitative methods in social sciences in colleges. As far as I can tell, this course is quite similar to the MDM4U course. Since January, I have been looking at themes of social justice in mathematics in preparation for a working group that I will co-lead at the CMESG (Canadian Mathematics Education Study Group) conference in June.

Most people are familiar with Statistics Canada because of the Census, but our numbers cover a lot of ground. Statistics Canada runs 350 other surveys that contribute data to CANSIM, our main data warehouse, along with the Census in E-STAT, which is now available to every teacher and student with just one click.

Among the topics relating to social justice is inequality, which is sometimes related to ethnic origin or gender. Inequality can manifest itself in differences in income, health, or education outcomes. There are many ways to think about inequality, and this article is not intended to cover all of them, but rather, to provide one example of an analysis that was done with data from Statistics Canada and to launch an invitation.

In this article, I will go through the exercise of starting an analysis of data on one theme in social justice (income inequality), and I'll try to show you my thought process, complete with questions, hesitations, and "aha"s. It is possible to find data on individual income for various levels of geography: provinces; counties; municipalities; large urban centres, called Census Metropolitan Areas (CMAs); and their neighbourhoods, called Census Tracts. Let's look at income distribution for Canada's two largest CMAs—Toronto and Montreal. Both are large urban areas composed of a number of municipalities.



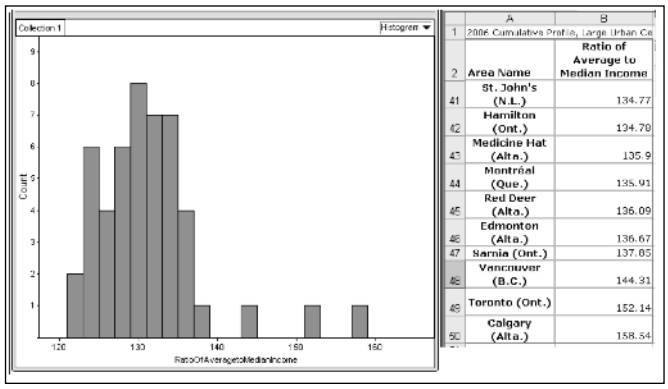
Source: Statistics Canada, Income in 2005 of individuals 15 years and over, (table) Census 2006 (48 Census Metropolitan Areas and Census Tracts) (database), using E-STAT. Accessed March 10, 2009.

Figure 1. Income distribution, Montreal and Toronto CMA.

This graph tells us that Toronto has a greater percentage of people with incomes either below $5000 or over $60 000. By contrast, Montreal has greater percentages of people in the middle ranges of income.

A crude way to look at income inequality inside a given geographic area is to look at the ratio between the mean income and the median income. This is a measure that is easily accessible to your students. In a normal distribution, the mean and median would be equal. They would also be equal in a unimodal distribution (in a theoretical world where everyone had the same income) and in a distribution in which there were exactly the same number of people in each income category, which would be a good theoretical justification for not using this crude measure. But since, in practice, income distributions are skewed, the ratio of the average to the median is a quick way of looking at that skewness, and hence, inequality. In a skewed distribution, the mean will be very different from the median. So a higher ratio of average income to median income in an area will indicate greater relative income inequality in that area.

Figure 2 is a representation of the ratio of average income to median income for all of the CMAs in Canada. The manipulation of dividing the average by the median is easily done in E-STAT. Figure 2 is a composite picture, with the histogram in Fathom and part of the data table in Excel pasted beside it.



Source: Statistics Canada, average income and median income in 2005 of individuals 15 years and over, (table) Census 2006 (48 Census Metropolitan Areas and Census Tracts) (database), using E-STAT. Accessed March 10, 2009.

Figure 2. Ratio of average income to median income for each of 48 large urban areas (CMAs). Histogram showing the number of large urban areas by classes of this ratio.

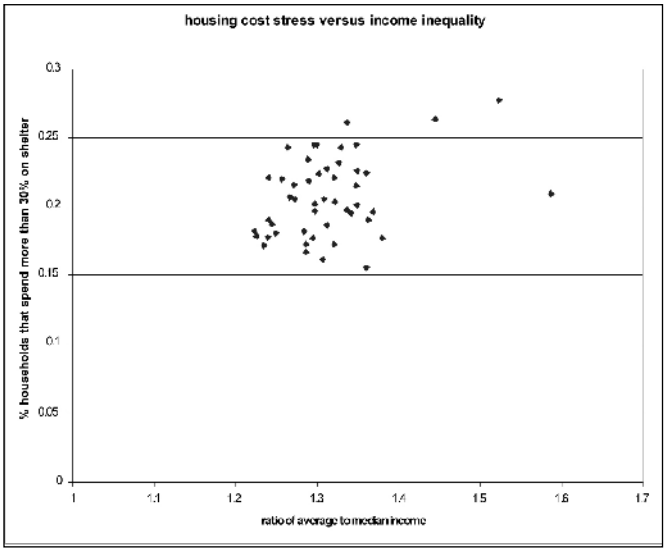
This graph shows us that in most of the 48 CMAs in Canada, the ratio of the average to the median income falls between 120% and 140%. The three outliers are Vancouver, Toronto, and Calgary.

From this analysis, it would make sense to recreate the graph of income distribution, as in Figure 1, to compare the above three urban areas. Note that the above analysis used income before tax to measure inequality. It would make sense to redo the whole analysis with after-tax income. But I'll leave that up to you! (Don't hesitate to call or write for instructions.)

Now, why is this a social justice issue? One could ask whether income inequality can give rise to crime (with some people envying the riches of the more well-off), can get people into dire financial straits (because of higher property values), or whether income inequality might have other consequences. Income inequality can also drive policies from the various levels of government that are designed to offset the effects of inequality. One may find evidence of such policies and effects.

One way of measuring financial difficulty is to calculate the percentage of people who spend more than 30% of their income on housing. This variable is available in the Census and is obtained by adding the number of owner households who spend between 30% and 99% of their income on shelter to the number of renter households who spend between 30% and 99% of their income on shelter and dividing that by the total number of dwellings.

Is there a relationship between our crude measure of income inequality and the percentage of households experiencing financial difficulty?

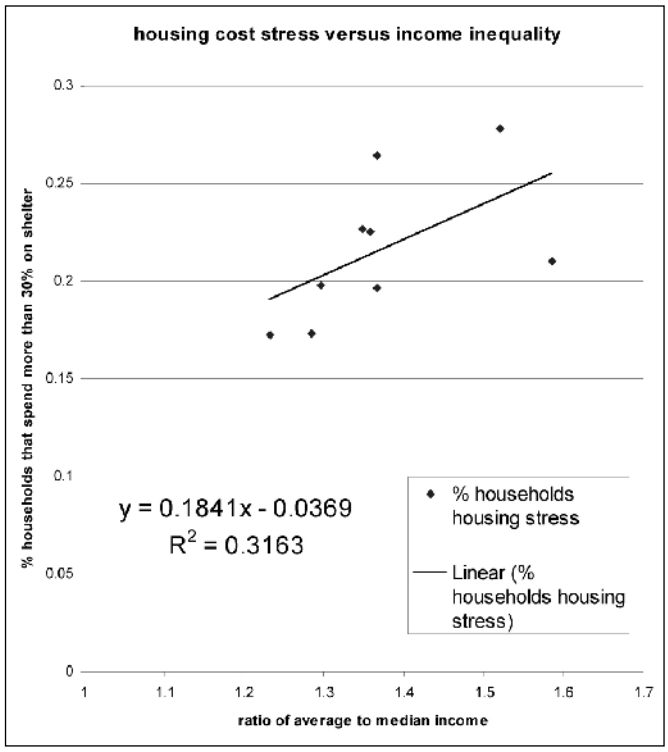


Source: Statistics Canada, average and median income and % of households spending more than 30% on shelter in 2005 of individuals 15 years and over, (table) Census 2006 (48 Census Metropolitan Areas and Census Tracts) (database), using E-STAT. Accessed March 10, 2009.

Figure 3. Income stress versus income inequality.

Hmmm . . . There seems to be some relationship between the two variables, although perhaps not a very strong one, and it seems to pertain only to what appear to be outliers. On the other hand, if the outliers are called Toronto, Vancouver, and Calgary, as we saw earlier, well then, maybe they are not outliers! In any case, to put a regression line through those points would require that we consider each point as equal, yet some urban areas here have populations of 50 000, while others are home to several million. So, in order for the regression to be valid, one should give each city the weight of its population.

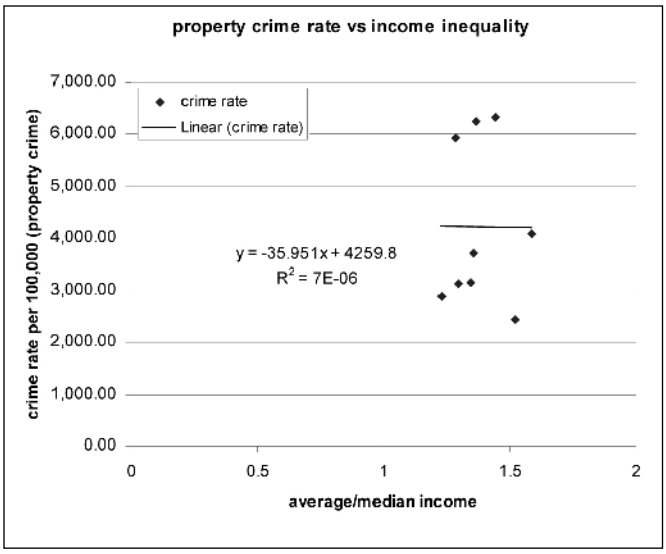
Alternatively, we could look at just a subset of the CMAs, taking only those with a population of more than 500 000 people. In that case, the relative population weights of the urban areas are more equal, and we could put a regression line through the points.



Source: Statistics Canada, average and median income and % of households spending more than 30% on shelter in 2005 of individuals 15 years and over, (table) Census 2006 (48 Census Metropolitan Areas and Census Tracts) (database), using E-STAT. Accessed March 10, 2009.

Figure 4. Income stress versus income inequality, large urban areas.

Now, looking at crime rates for CMAs in 2005 (since that is the year for which income is measured in the 2006 Census), we could also try to see if there is a relationship between those two phenomena. Let's look again, but this time, only at the urban areas with a population of 500 000 or more.



Source: Statistics Canada, average and median income and % of households spending more than 30% on shelter in 2005 of individuals 15 years and over, (table) Census 2006 (48 Census Metropolitan Areas and Census Tracts) (database), using E-STAT. Also, CANSIM table 252-0013, accessed in E-STAT March 17, 2009.

Figure 5. Rate of property crime versus income inequality.

No correlation at all between inequality in income before tax and the rate of property crime! What does it mean? Well, there goes one hypothesis out the window!

What about other crimes (homicide, for example)? Looking again at the larger urban areas...

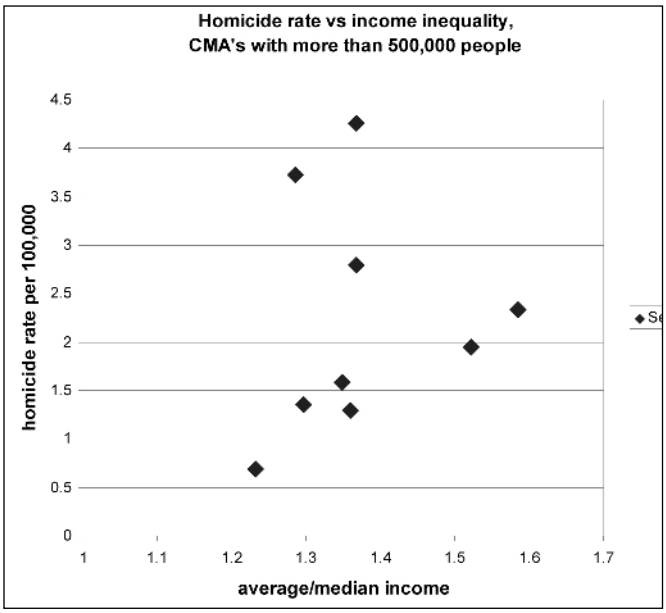


Figure 6. Homicide rate versus income inequality, large urban areas.

We can see a weak relationship between homicide and income inequality. If there were reasons to exclude Winnipeg and Edmonton (the two urban areas with the highest homicide rates in 2005), the relationship would be much stronger. Such a reason could be some external factor that affects the homicide rate but that is not linked to income. Newspaper articles (2005) found on the Web mention gang-related homicide for both urban areas, and so excluding those urban areas may be reasonable if one wishes to look at the relationship between inequality and crime, as the presence of gangs may not be so greatly influenced by income disparity. *Should* homicide rates be related to income disparity? Obviously, those points would need to be discussed at greater length, and this is beyond the scope of this article.

The above data show how quickly one can, given good tools, find and manipulate data for investigating topics related to social justice. And if getting and using the data is easy, your students can focus on the important parts: the math, asking the right questions, and taking the time to discuss the issues.

Our tools today: Statistics Canada's E-STAT, Microsoft Excel, and Fathom.

Your contribution as teachers: Encourage your students, and teach them ways of critical thinking that will help them discern between plausible and spurious relationships between variables.

Some of you may be interested in integrating situations of social justice in your math teaching, and Statistics Canada can be of help. I am preparing a resource—a list of data sources that would allow your students to investigate various social justice issues. If you have an opinion on what you would like from us, send it to me at even if you are not teaching the MDM4U class. The resource could be similar to the one we have on function modelling, or it may take a different shape. Any suggestions?

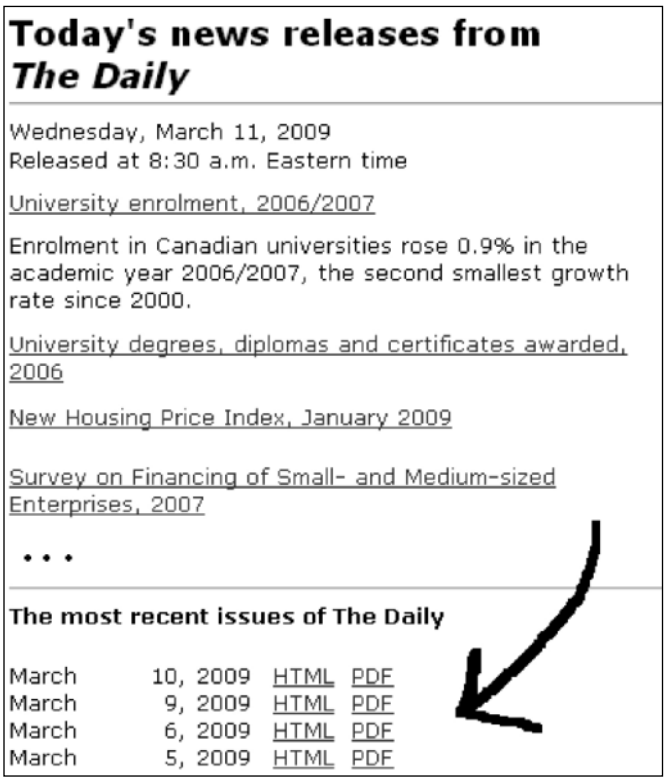
On a different note, students are frequently unaware of the breadth of topics available when they are trying to choose data for a research project. There are literally thousands of hypotheses one could make, of stories one could tell. Students may not know that there is data on a topic that interests them. To that end, please see the following post in this column: Searching for a Topic for the MDM4U Culminating Project: A Helpful Routine.

Searching for a Topic for the MDM4U Culminating Project: A Helpful Routine

Students can—and of course, should—select a topic of their own choosing for their culminating project. But where to start? One easy routine can give students a

sense of the breadth of topics from which they could choose. I recommend to teachers of MDM4U that they implement the following practice: Once a week, have students check the Statistics Canada *Daily* (**www.statcan.gc.ca/dai-quo/index-eng.htm**). The titles of the day are displayed, and the links at the bottom of the page provide access to the titles of the four previous days (see Figure 1).

Most weeks, this will take only a minute or two, although sometimes, a title will resonate with a student who may want to find out more by reading the related article, which might take all of five minutes! The cumulative effect of this weekly habit, if started in September, will be a smorgasbord of possibilities when it's time for students to select their topics!



Today's news releases from *The Daily*

Wednesday, March 11, 2009
Released at 8:30 a.m. Eastern time

University enrolment, 2006/2007

Enrolment in Canadian universities rose 0.9% in the academic year 2006/2007, the second smallest growth rate since 2000.

University degrees, diplomas and certificates awarded, 2006

New Housing Price Index, January 2009

Survey on Financing of Small- and Medium-sized Enterprises, 2007

• • •

The most recent issues of The Daily

March	10, 2009	HTML	PDF
March	9, 2009	HTML	PDF
March	6, 2009	HTML	PDF
March	5, 2009	HTML	PDF

Figure 1. Sample post from *The Daily*.

This routine could also become a whole-class activity. The teacher could use a projector to lead the exercise, opening the links to topics that students find interesting or investigating those links where it's not clear from the title what topic is being covered.

Teachers: Try this routine for yourselves until the end of this term. This will give you the chance to test first-hand how useful it is before September, at which time you could propose it to your students. ▲

▲ *You Want What? When?* A Case for Better Homework

Iain Brodie



Iain Brodie has taught every grade from 1 to 6 in his 19 years with the Toronto District School Board. He is a strong advocate for cross-curricular integration and likes to create meaningful unions of math, poetry, art, science, social studies, and media.

"Oh, no. I have homework!"

Whether you are a parent or a teacher, you have probably heard these words before, maybe even slipping from between your own tightly clenched teeth. I do not know of any other subject in education that has caused so much passionate debate. *Homework is good. Homework is bad. Homework is necessary for success. Homework is taking over our lives!* Believe it or not, this debate has been waxing and waning for a long time. In the United States and Canada, we have seen a rise and fall in the official amount of homework deemed appropriate for over a century.

Many have rung the alarm bells of impending systemic failure, including the authors of *A Nation at Risk*, an American report published by the National Commission on Excellence in Education in 1983. In Ontario, it's difficult to forget the concerted efforts of Education Minister John Snobelen and his successors, who, along with Premier Mike Harris, tried to convince the electorate that the Ontario education system was underachieving. Historically, it is pressures such as these that contribute to the feeling that we need to do more, that students need to work harder, and that we must pile on the homework. This is inevitably followed by a backlash against the quantity (and sometimes the quality) of the work being sent home, and the amount of homework is once again curtailed.

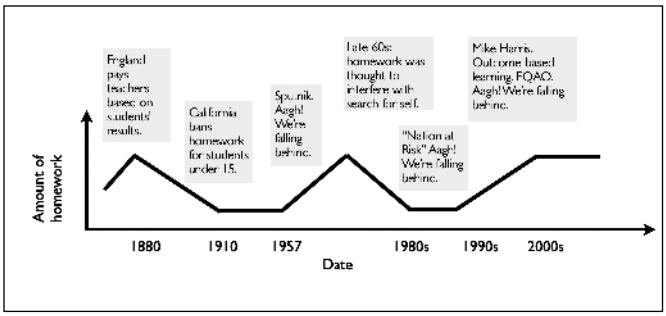


Figure 1: A barely accurate graphic of the ebb and flow of homework.

So what are we to do? Are we to ride out the homework waves like educational surfers waiting for the next "big one"? Or do we learn from current research about what is effective for students and good for families? I vote firmly for the latter, and I think that mathematics educators have a role in educating parents and teachers about a new era of homework, one in which there is no ebb and flow, no excesses, no bans.

What the Research Does and Doesn't Say

Fortunately, there is now a lot of research that can guide us in our efforts to avoid the homework waves.

Homework doesn't really affect achievement at the elementary level (K–6).

I get more flak when I talk about this than about any other homework topic. There have been many studies, some as far back as the 1980s (Ziegler, 1986; Cooper, 1989 & 2006) that have corroborated this, but mention this in a staff room or to some parents, and you will get an earful!

Consider the following: Teacher A regularly sends home math facts for students to practise, while teacher B practises the same math facts with students at school, with the same regularity. Which set of students will have the advantage in mathematical problem solving? Neither side has the advantage. The only real difference is where and when the students are doing the work. Hence, homework at the elementary level appears to be more about resource allocation than about raising achievement. (*Author ducks and takes cover.*)

Homework can improve achievement at the intermediate and secondary levels, but more than certain amounts are counter-productive.

This is straight from the pro-homework side of the homework debate: More than about an hour at the intermediate level to more than two hours at the Grade 12 level actually *reduces* students' achievement levels (Cooper, 1989). However, students with multiple teachers often end up with homework amounts above these levels. In many cases, such overloads could have been resolved through better communication and planning among teachers.

It should go without saying that a large amount of homework for primary—and most junior—students is not a good idea. (*Author peeks out, looks around.*)

At present, there is not enough research to support the notion that doing homework for its own sake produces a work ethic.

While there is some evidence that homework in the early grades helps children get accustomed to homework, at present, there is not enough evidence to show that daily homework helps students develop self-discipline or the habit of getting things done carefully and on time. When a study is finally done on this, I think that it will find that the child's personality and the extent of parental involvement will have more to do with establishing a work ethic than will the actual assignments. (*Author looks to see whether there are any nodding heads yet.*)

Homework has more effect on achievement if it is marked and commented on.

No arguments here, right? This is the simple feedback loop of learning. (*Is it safe to come out now?*)

Children achieve better when their parents are involved in their schooling.

Those who support homework could point to homework as the easiest way to involve parents in their children's schooling, while those who oppose homework might point to rising homework levels and the number of families who lack sufficient time to become involved. Of course, there are also more moderate forces. One research synthesis study (Patall, et al., 2008) found that parental involvement in homework—as opposed to parental involvement with the school and schooling—led to some interesting correlations. Parents, it seems, help their children quite well with reading and homework skills, but actually have a negative effect or no effect on mathematics learning. I would venture to speculate that we might have to educate parents, too, if we are going to involve them in this manner. Not a bad idea, perhaps.

While the pro-homework side can use these findings to argue for continued, regular homework, the anti-homework side can use these same findings to argue for reduced and/or family-involved homework. Personally, I would like to put forth an argument for good, *high-quality*

homework that balances the needs of both the student and the family.

Toward Better Homework

If we are to formulate a new homework policy that fits the needs of this generation and those to come, we must take the following factors and challenges into account:

1. No matter which side of the homework fence you are on, students need to practise what they've learned; they need to prepare/study for upcoming events; they need to complete unfinished classroom work; and they need the chance to explore, innovate, and create.
2. Parents need to be kept in the educational loop.
3. Time is precious, and there are many demands on students and their families today that did not exist previously.

In *The Case For and Against Homework* (Marzano & Pickering, 2007), the authors make a well-reasoned case for not abandoning homework entirely. Essentially, they advocate for improving homework quality by assigning *purposeful* homework. Assignments need to be designed so that most students will finish them and so that all levels of learners will find them sufficiently challenging to be interesting. Parents need to be involved in such a way that they do not have to act as teachers or police officers. Finally, the amount of homework needs to be monitored, taking into account the students' ages and the demands of other teachers that may be involved.

Most of these challenges can be addressed by focusing on three main areas:

1. Communication
2. Asking the question: *When do I **really** need it back?*
3. Thinking beyond the textbook or worksheet

Communication

Generally, any communication between parent and teacher is good. Regular communication is better. Focused communication is better still. I admit that I give my own children's school newsletter only a cursory glance, but the monthly newsletters from their teachers are gold. We get to know what is going on, what is coming up in the way of tests and projects, and we get a brief summary of the units they are covering in class. While this type of communication requires quite a bit of organization, it is easy to do and is a real benefit to busy parents who need to plan ahead for study time, music lessons, sporting events, down time, and so on. Parents who know what their children are doing and why they are doing it are more likely to help than hinder your efforts.

Most of our math resource documents at the elementary level contain "letters to parents" or some such reproducible item. Just sending one of these home along with the regular math homework will greatly assist parents and will start you on your way to becoming a great communicator. This is a good first step for any of us who want to reach out to parents, and most of the work is already done for us.

I go a little farther with my own class, sending out a weekly newsletter with the week's homework, an update on what we are doing, and/or a tutorial on how to work with the students at home. For example, when I send home geometry homework, I include diagrams, vocabulary, and explanations in the newsletter. If there is something procedural involved, I explain the procedure. As a result, I have fewer playground conferences with parents about "how we do math today." You can check out some of my newsletters at **www.schools.tdsb.on.ca/standrews**. From there, choose the following path from the menu on the left: Our Classrooms → Kindergarten–Grade 3 → Mr. Brodie.

If you really want an amazing example of full-out communication, though, talk to a high school teacher (or an elementary teacher) who is using a *moodle*. A moodle is an awesome communication tool for keeping everyone in contact on a daily basis or better. Students are in contact with teachers who are in contact with each other. Teachers are able to coordinate their assignments so that they don't step on each other's homework toes, and online homework help is there for the taking so that students and parents can go to the moodle as a first option. Very, very cool, and evidently, as easy to set up as an e-mail account!

When Do I *Really* Need It Back?

According to the current TDSB homework policy, students must be given blocks of time in which to finish their homework. At first blush, this would seem to kill off the completion and "drill and practice" homework, but really, it just changes it a little. The first thing you need to do is relax. Your students got some of it done in class today. They won't all have time tonight because one student is at hockey, another is at gymnastics, and a third has piano or Scouts. You could even let them catch up in class tomorrow or check up on their work later in the week. As long as it gets seen and commented on, it's okay. Really! And here's an idea from Alfie Kohn (1999): There is no proof that the best practice of commenting on homework has to come from the teacher. Try a

sharing circle or a pair/share. Students tend to be more exacting judges of something like a math journal than are teachers, who generally give the benefit of the doubt. Try it! You might be surprised.

Thinking Beyond the Textbook or Worksheet

There are four types of homework identified in most studies and in the TDSB Homework Policy: completion, preparation, practice and application, and extension/creative. Of the four types, completion is least favoured by students, while the least objectionable was drill and practice (Cameron & Bartel, 2008, p. 4). So much for opening up their minds! Attitudes toward projects and preparing for tests fared only slightly better than completion work. We need to do better for our students so they can do better.

It doesn't take much to incorporate Marzano's advice (2006/07) and think beyond the textbook or worksheet:

Completion and Practice Work

When do you really need to have your students complete all those exercises? How many exercises, and by when? Would it really hurt if they were completed over a few days instead of overnight? How about an installment plan, or an amount based on need? Research done in the 1980s showed that a little extra work for underachieving high school students helps them achieve as much as average students who do no extra work at home (Ziegler, 1986).

When sending home practice work, be sure to let the parents know the reason for the practice. If you are asking students to drill tedious math facts, give them a clear goal to reach for.

Application and Preparation Work

When you give notice of an upcoming test, give plenty of warning and hand out study sheets. This is something I learned from my own children's Grade 3 teacher. It's so much easier to help your child when you know what it is that they have to study. In math, we're lucky that the types of things we test are procedural, conceptual, or application based. Students rarely benefit from tests that are based solely or largely on content.

The following ideas demonstrate how tapping into other curriculum areas can produce homework assignments that are both interesting and motivating. None of these require parents to be teachers; they simply invite them to be partners:

- Have students survey a local building and then sketch it for geometry. Have students build a model of the sketch, using Polydron, cardboard, or paper.
- For social studies, have students mine data relating to families, and use that data in data management lessons.
- Ask students to make a grocery list of the food they would like to eat over the next week. Have them bring the list to class and calculate the cost of the food. Bring in flyers so that students can look for bargains. (Ask how they know something is a bargain.) Have students compare the items on their lists with the recommendations in Canada's Food Guide. Ask them to graphically represent the relative wisdom of their choices.
- Have each student measure the dimensions of the walls of his or her room and bring in the measurements to calculate how much paint would be needed to cover them.
- Have older students figure out how to make their homes "wheelchair accessible." Ask: *What length would the ramps need to be to prevent the slopes from being too steep? Could the same be done at school or has it already been done? How?* As a bonus, discuss equity—in math class. Cool!

Homework time together can be fun and math-filled. Homework assignments like these can be used to practise a skill or concept that has just been learned or to review one before an assessment. While they do require a bit of thinking and planning to create, one such assignment per week for primary or junior students would be more than sufficient. Intermediate and senior students will need something on the meatier side, so you might try looking at a resource like *Math That Matters* by David Stocker, available from the Canadian Centre for Policy Alternatives. It's full of math ideas from a social justice perspective.

One way to give families greater control is to come up with a "Monthly Mathematics Menu" of activities with a little of each type of homework from which to choose. Let each family choose, with certain minimums required. This would provide a solution for those parents who are always asking for extra work for their children, while controlling the load for other students.

Projects

Recent research has shown that parents dislike projects less than other types of homework (Cameron & Bartel, 2008, p. 4). However, it's important to ensure that the project is well defined and won't take over the family's life. Geometry is a great subject for a creative/extension project, especially when combined

with art. Or how about adding some measurement to some kitchen chemistry for a combined science and math project? Just beware of the type of project that requires a lot of time or that might require a lot of parental help or direction. Remember that unless the students are given the opportunity to present and explain their findings in their own words, you can't really tell who has done the work for a project that has been done outside of school. Consider the following vignette:

Parent A: (*carrying a project out from the school*) Yes!

Parent B: What was your mark?

Parent A: Got another A!

Parent B: (*Confused*) *Another* A?

Parent A: Yup. This is the third time I've handed in this same project. Now that my youngest has used it, I can finally recycle it!

A Question of Time or a Question of Quality?

Almost all of the studies done so far correlate the amount of *time* spent on homework against results. Nowhere, it seems, have researchers considered the *quality* of the homework assigned. I'm not sure that they could. But consider recent TIMSS (Trends in International Math and Science Study) and PISA (Programme for International Student Assessment) results: On the 2003 TIMSS, as a general trend, countries that did better assigned less homework, and among the best-scoring countries on the 2006 PISA were those that reported the least amount of homework being assigned. In contrast, those countries that assigned excessive homework had, by and large, poorer results. However, the question neither asked nor answered is this: *What **kind** of homework are students from the more successful countries doing?* I suggest the very real possibility that it is of much higher quality and that it is more focused. Are there any researchers out there willing to find out for us?

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▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

▲ Hey, It's Elementary! Large-Scale Assessment Hits Faculties of Education: The TEDS-M Study

LYNDA COLGAN



Lynda is celebrating her 32nd year as an educator. She has taught at all levels—elementary, secondary, and post-secondary—and has mentored fellow teachers and more recently, teacher candidates and graduate students. Her first teaching award, The Marshall McLuhan Distinguished Teacher Award, was given in recognition of her "Learning Partner Initiative," in which elementary children attended professional development sessions with their classroom teacher to learn about innovative uses of technology in mathematics.

In September 1992, when I was first hired as Mathematics Coordinator for the Board of Education for the City of Scarborough, I arrived to find consultants Shirley Fairfield and Sue Gordon well into the initial planning stages for a city-wide review of Grade 3 and Grade 6 Mathematics. The review, an assessment of mathematics to be conducted in all Grade 3 and Grade 6 classrooms in all of the board's 167 schools, was a huge undertaking requiring an enormous investment of money and personnel. The review was planned, developed, implemented, and analyzed in collaboration with the Scarborough Board's Research Department, which, at the time, was under the leadership of Dr. Lorna Earl, an internationally respected leader in assessment and evaluation. There were several research associates assigned to this large-scale project, but Clara Ho was the "face" of the study on a day-to-day basis as the instruments were generated, field-tested, marked, and revised.

As I reflect back on that experience, I now understand why *The Scarborough Review of Grades 3 & 6 Mathematics* was ahead of its time: It was authentic. To keep the quantitative researchers happy and satisfy requirements about the reliability and validity of the instruments, the review included a small number of multiple-choice questions. But the main body of the review instrument consisted of a good balance of short-answer and open-ended questions, and most importantly, an observation component. Teachers administered the observation component in a one-on-one setting, using a standardized script and manipulatives (e.g., pattern blocks, colour tiles, string, calculators), while recording the students' responses and solutions. There were also teacher, student, administrator, and parent questionnaires. Teams of elementary teachers were involved in the scoring over the summer, and the interpretation and dissemination of the results was the work of Dr. Earl and her team in collaboration with the mathematics team. This endeavour generated highly detailed school-specific and board-wide reports, as well as a set of actionable recommendations.

As a system-wide coordinator, I found the data to be invaluable. It enabled me to design appropriate professional development on the basis of need, and it underscored the power of local research that is well-designed, thoughtfully administered, and appropriately reported.

The mathematics review for Grades 3 and 6 became the catalyst for other system-wide assessments in Scarborough, including one for Grade 12 mathematics. Once more, under Lorna Earl's leadership, the secondary team—consisting of Peter Harrison, David Johnston, Vince Lo, Peter Wei, and me—developed a comprehensive set of instruments that included paper-and-pencil items that spanned the continuum from multiple-choice to open-ended. Particularly innovative was the component in which pairs of Grade 12 students worked on a discovery investigation that required the use of the computer (specifically, an Excel program) to analyze and interpret a set of real-world data (about fish populations, tree growth, or the spread of AIDS) that generated logistic functions.

Once again, the results were highly illuminating and informative and guided the development of a number of board-wide and school-based professional development foci, as well as resource acquisition and development.

I also remember that it was through these two processes that I learned the meaning of a phrase that a colleague from the former North York Board, Peter Skillen, had frequently used: *It behooves a leader not to get too far in front of the troops, lest s/he be considered the enemy.*

The time—1992—was early in the history of large-scale assessment in Ontario. When the mathematics reviews were announced, several teachers and administrators expressed suspicion and concern. *Why was the review **really** being done? Who would see the*

results? Who would be responsible if the results fell short? Some fretted about the impact on low-achieving students in their neighbourhoods: namely, immigrant children, English Language Learners, and children disadvantaged by context or circumstance. *Would these children be compared to children less burdened by cognitive or cultural challenges?* But, for me, the most difficult pill to swallow was the question about whose side my team and I were really on. Were we actually complicit partners in an attempt to undermine the professionalism of teachers and administrators?

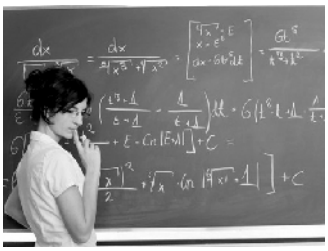
Today, some 17 years later, we are closer to having a culture of assessment in Ontario. The EQAO large-scale assessments happen every year, and although there are minor bumps in the road every now and then (e.g., complaints from individuals who object, based on some philosophical belief or stance), the tests are more routine, and there are few who wouldn't argue that the benefits far outweigh the detriments. Huge, targeted, Ministry of Education initiatives (like *TIPS* and the *Early Math Strategy*); the creation of the Literacy and Numeracy Secretariat; and creative and important spin-offs from individuals (e.g., Wendy Hill's *Learning Carpet*) and publishers (e.g., Nelson's *Prime*) have had many positive effects in classrooms across Ontario.

I am hopeful that the results of a large international study conducted about one year ago will be every bit as constructive and informative for faculties of education as EQAO and the Scarborough Reviews have been for mathematics education leaders, administrators, classroom teachers, and most importantly, students. In April 2008, our faculty at Queen's University participated in what I believe to be an important study that should be of interest to mathematics educators at all levels—the TEDS-M.

TEDS-M

What is the TEDS-M? TEDS-M is an acronym for *Teacher Education and Development Study in Mathematics*. This research study, led by principal researcher Dr. Maria Teresa Tatto from Michigan State University, was developed, piloted, administered, scored, and analyzed by teams from the National Science Foundation (NSF), the International Association for the Evaluation of Educational Achievement (IEA), Michigan State University, and the Australian Council for Educational Research (ACER). In Canada, the study was supported financially by, and conducted under the auspices of, the Council of Ministers of Education, Canada (CMEC) and supported by Statistics Canada.

The TEDS-M was conducted at faculties of education in 17 countries: Botswana, Canada, Chile, Georgia, Germany, Malaysia, Norway, Oman, Philippines, Poland, Russia, Singapore, Spain, Switzerland, Chinese Taipei, Thailand, and the United States. The overall goal of the study was to find better ways to help teachers learn what they need to know in order to teach mathematics well at the elementary and secondary school levels. In pursuing this goal, the study endeavoured to speak to diverse audiences.



The TEDS-M study sought to answer the following research questions:

- *What is the impact of mathematics-related policies for quality assurance and the accreditation of teacher education programs on teacher education institutions, programs, and outcomes?*
- *How do national or program policies influence the recruitment, preparation, graduation, and retention of teachers of mathematics?*
- *What are the characteristics of teacher education policies, institutions, and programs that lead to high levels of mathematics knowledge and knowledge of mathematics pedagogy in future teachers?*
- *What relationship is there between the beliefs about mathematics of teacher educators and those of future teachers?*
- *What kinds of practicum arrangements and school experiences are most effective in preparing future mathematics teachers?*
- *What are the costs of programs in different settings?*

Figure 1, below, illustrates the conceptual model for the TEDS-M.

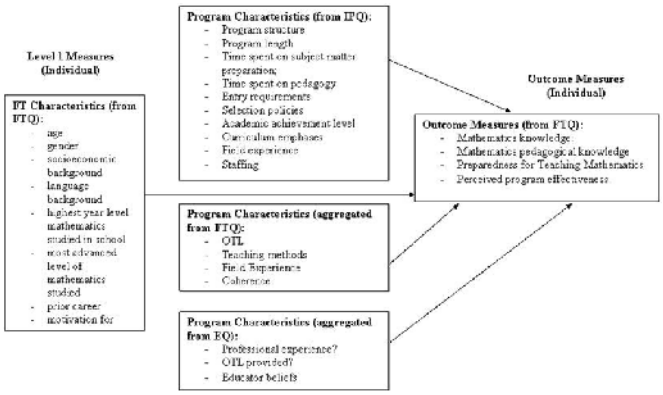


Figure 1. Interrelationships among variables examined in TEDS-M (from the TEDS-M Conceptual Framework).

The rationale for the study came from an extensive review of the literature and rested on the following broad assumptions:

1. *Teacher education is understood and implemented differently across national settings and even between institutions within the same country.*
2. *Teacher education and teacher learning are complex, contested processes.*
3. *Knowledge of the content to be taught is a crucial factor influencing the quality of teaching.*
4. *Teacher education requires understanding of and addressing how teachers should think about mathematics, teaching, and learning.*
5. *Knowledge for teaching involves consideration of the situational contexts [in which] teachers will teach.*
6. *Teacher education embodies a developmental logic of how teachers acquire professional knowledge for the teaching of mathematics and other subjects.*
7. *Teacher education is assumed to be linked to student achievement, but this relationship is poorly understood.*

The study consisted of questionnaires and interviews with policy makers (depending on the country, national leaders, or, in Canada's case, provincial leaders), as well as a K–12 mathematics curriculum analysis; questionnaires for faculty of education administrators, including the collection of documents (courses of study for mathematics education courses and ancillary materials such as assignments and required textbooks); questionnaires for mathematics education instructors (both tenured and adjunct); and questionnaires for teacher candidates (separated into concurrent or consecutive and primary/junior or secondary cohorts).



The teacher candidate questionnaire required two hours to complete and included items that measured background, opportunity-to-learn (OTL), and beliefs. For elementary teacher candidates, the instruments included a total of 70 items, and for secondary teacher candidates, 49 items. The instruments were designed so that on each, approximately two-thirds of the items measured mathematics knowledge (algebra, geometry, number, and data), and approximately one-third of the items measured mathematics pedagogy knowledge (curriculum, planning, and enacting).

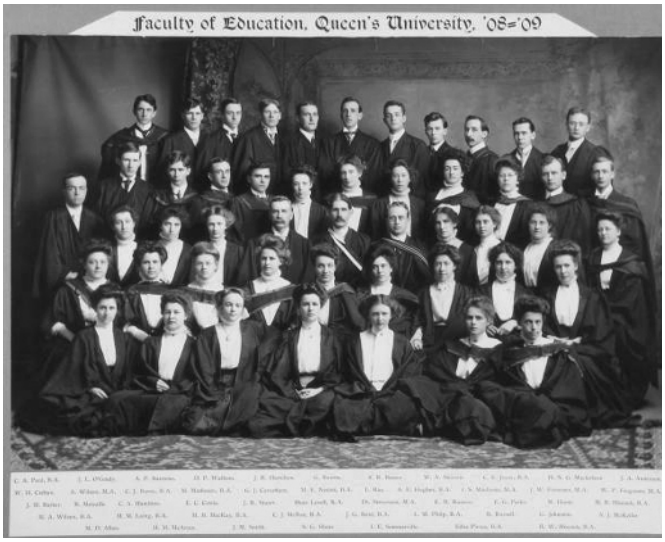
At the national level (or, in Canada's case, the provincial level), researchers gathered data about:

1. *the policies that regulate and influence the design and delivery of mathematics teacher education for elementary and lower secondary teachers*
2. *the ways in which the countries' distinctive political, historical, and cultural contexts shape mathematics teaching and learning and how these influence policy and practice in mathematics teacher education*
3. *the consequences of policy for the development of standards for degrees, coverage of topics, and certification practices/the consequences of policies for the recruitment, selection, and preparation of future mathematics teachers*
4. *what it would take (in terms of resources, capacity, and costs) to improve the preparation of average mathematics teachers*



Through the institutional questionnaire, researchers gathered data that would provide information about:

1. *[the] kinds of institutional and field-based opportunities [that] are provided for future elementary and secondary mathematics teachers*
2. *the ways in which the expectations, curriculum, and standards are enacted*
3. *the qualifications and prior experiences of the university mathematics lecturers/professors/teachers and teacher educators responsible for the implementation of these programs*
4. *factors that might explain the impact of particular routes, programs, and practices on the mathematics knowledge of future teachers of mathematics.*



Portrait of the Queen's Faculty of Education 1908–1909. Used with permission from the University of British Columbia Archives.

Through the teacher educator questionnaires, data was collected from mathematics instructors and teacher educators about their background, their mathematics teaching knowledge, the materials they used in instruction, their espoused theories for teacher change, and their expectations for their teacher candidates. Parts of the Teacher Educator Questionnaire were similar to the Future Teacher Questionnaire, since the correspondence and differences between faculty and future teachers (on such things as knowledge, pedagogy, and beliefs) as an indicator of coherence and its possible effects on teacher education outcomes were examined.

1. *What is the content knowledge of mathematics that future teachers are expected to acquire across the participating countries?*
2. *What is the depth of understanding that they are intended to achieve?*
3. *a) What beliefs about teaching and learning mathematics are promoted by teacher education programs?*

   b) What beliefs do [future teachers] hold about teaching and learning mathematics at the end of their preparation?
4. *What is the knowledge for mathematics teaching (e.g., of the content, pedagogy, curriculum, and attitudes) [that] future elementary and lower secondary mathematics teachers actually have at the end of their teacher preparation and once they are considered "ready to teach"?*
5. *What other characteristics help explain their ability to master this knowledge?*



The teacher candidate questionnaire asked prospective teachers about their:

- academic background
- mathematics content knowledge
- knowledge of mathematics-specific pedagogy and general pedagogy
- beliefs concerning students' dispositions and abilities when learning mathematics
- beliefs about purposes for learning mathematics
- perception of their own level of preparedness to teach mathematics effectively

Preliminary numbers indicate that 15 163 elementary teacher candidates and 9389 secondary teacher candidates participated from some 500 institutions. Some 4837 teacher educators (elementary, intermediate, and secondary mathematics educators) participated. In Canada, 35 faculties of education and some 1290 elementary and 1250 secondary teacher candidates completed the questionnaires.

Like the early Scarborough mathematics reviews, the TEDS-M study is a pioneering study. To date, no empirical cross-national study based on probability samples has analyzed how education systems prepare teachers of mathematics (or any other subject, for that matter), nor has any study identified the explicit and implicit expectations regarding what teachers of mathematics should know and be able to do as a result of this preparation. The goal of the TEDS-M study is to enable countries to compare themselves to other countries in terms of what they expect of their future teachers of mathematics and their ability to help students learn the subject matter. Countries can also explore the opportunities that other nations give their future teachers in order to meet these expectations. These comparisons should provide valuable insight to policymakers and teacher educators regarding diverse teacher education strategies in specific settings.

As with those early Scarborough reviews and the first EQAO implementations, there were challenges in conducting the study. At our faculty, in spite of the fact that CMEC had been involved in the development of the study for some five years prior to its implementation and the research had been cleared by Queen's General Ethics Review Board, a few people (not mathematics educators) publicly questioned the credentials of the IEA and the ethics of conducting such a study—an unanticipated contradiction at an institution committed to research!

In informal conversations with the TEDS-M coordinators at other faculties, I learned that some individuals worried about what would become of their courses of study. *Could an individual be identified if excerpts from the syllabus were published? Could the syllabus that they had carefully crafted over the years become public domain? Could they be forced to follow a standard syllabus if the study found that theirs somehow fell short?*

Some teacher candidates worried that somehow their "achievement" score could come back to haunt them during the job search—in spite of the fact that all students were identified by number, only one person (not a faculty member) had the "key" to match the names to the numbers, and there were no plans to have individual results reported.

As the TEDS-M coordinator for Queen's, I was required to follow very stringent scripts and procedures. Using a stratified random sample, some 200 of our teacher candidates were selected by Statistics Canada to participate; however, there was no way to "require" any individual to complete the instrument during the formal two-hour test administration session. As a result, only 22 teacher candidates showed up—about average for Ontario, according to the inspector from the IEA, who came to report on the fidelity to the administration process. Could the time of year—April—have been to blame? No course credit was given for participation. Did this devalue the study in the view of the teacher candidates? What if a pizza lunch had been included? Would more teacher candidates have participated? No doubt, the sample size will limit some analyses and will be a point of contention when the results are published.

Teaching pre-service teacher candidates is the most difficult job I have had in my 33 years as an educator. I strive to improve my courses every year, based on student feedback, reflection, and professional reading, but I am hungry for more information and creative ideas. What do my colleagues at other faculties of education do? What do their courses of study and classes look like? What could I adapt for use at Queen's? I, for one, am looking forward to the TEDS-M reports (due in January 2010) and what I hope will be a new opportunity for purpose-driven collaboration among mathematics educators at faculties of education across Canada as we interpret, understand, and act upon the results.

References

Tatto, M.T., Schwille, J., Senk, S., Ingvarson, L., Peck, R., & Rowley, G. (2008). *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics. Conceptual framework*. East Lansing, MI: Teacher Education and Development International Study Center, College of Education, Michigan State University.

All images, except the Queen's Faculty of Education portrait, are from Google Images. The Queen's portrait is from the University of British Columbia archives. ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor
for further details.

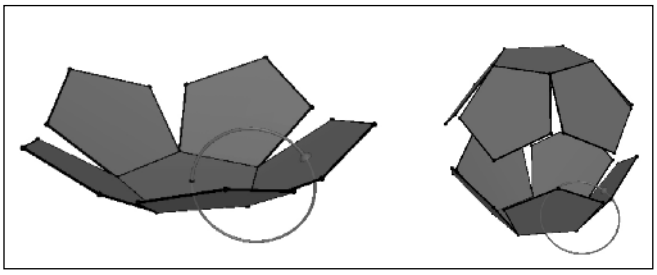
▲ Software Review – Cabri 3D: A New Dimension in Interactive Geometry Software

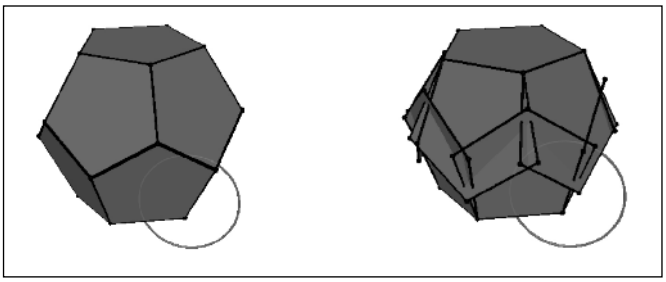
KATE MACKRELL

Cabri 3D is an interactive geometry software (like Geometer's Sketchpad) in which a 3D environment containing objects such as points, lines, planes, and polyhedra is represented on a 2D screen. It's a bit like a 3D version of Cabri Junior on the TI calculators, but with the advantage of the power of the computer. Cabri 3D also won a major award for educational software in the United Kingdom (BETT, 2007).

When I first saw Cabri 3D after its release in September 2004, I thought "Mmm... don't think I can do much with that." At that point, I was perfectly at home moving in a 3D world and thinking in a rectilinear 3D world, but was uneasy in any 3D space not nicely defined by horizontals, verticals, and rectangular prisms. Problems in 3D geometry were just too abstract. Four years on, I'm deeply immersed in Cabri 3D. I feel a bit like I've found the starship Enterprise and am encountering "space—the final frontier." And much to my surprise, ordinary everyday space has turned out to be just as interesting and exotic as space at the far-flung reaches of the galaxy, not to mention somewhat more accessible! The vague blur that was 3D geometry is being replaced by a web of increasingly rich connections and insights.

In describing what Cabri 3D has to offer to Ontario teachers, I'm going to start by referring to Stewart's editorial in the last issue of the *Gazette*. Do you remember Scott, who was spellbound by Cabri 3D? This happened while I was demonstrating the construction of a dodecahedron by a method that involved rotating regular pentagons and then reflecting them through a point. Here are a few screenshots of the dodecahedron folding up:



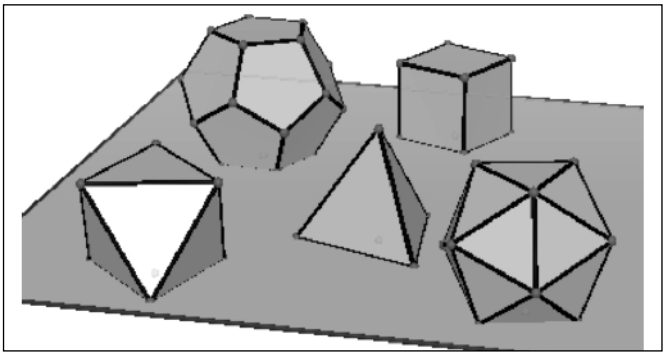


I refuse to give away the final image—it's just too wonderful to briefly glance at in this format. Go to my website (**idisk.mac.com/katemackrell-Public/Cabri3D/index.htm**), make sure you download the free plug-in that enables you to view and manipulate Cabri 3D files that have been embedded in web pages, and then follow the links to Polyhedra. The screenshots above have been taken from the second file, on "foldup polyhedra." Manipulate this figure, fold it up even further, and enjoy the result!

This, to me, is much of what Cabri 3D is about: using curriculum mathematics (polygons, perpendiculars, polyhedra, transformations, etc.) in novel ways to come up with beautiful and surprising results.

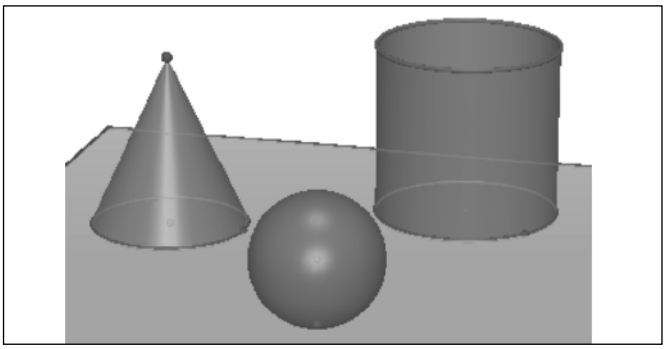
The following provides an idea of some of the curriculum mathematics that Cabri 3D directly addresses.

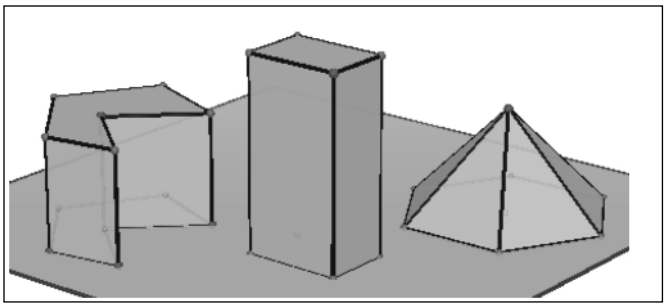
3D Shapes



All sorts of shapes are available.

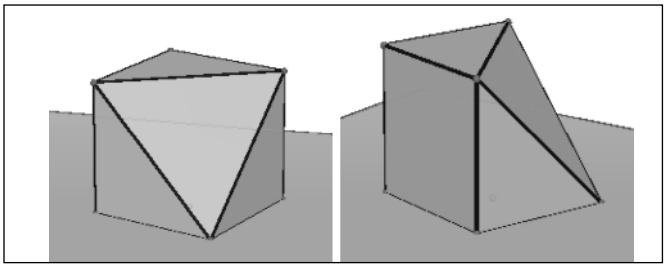
These shapes took less than a minute to create.

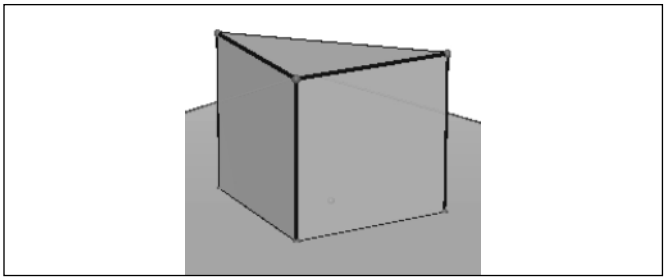


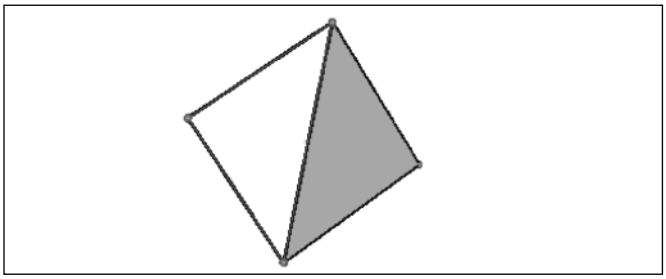


Shapes can be viewed from different angles.

What do you think this shape represents?

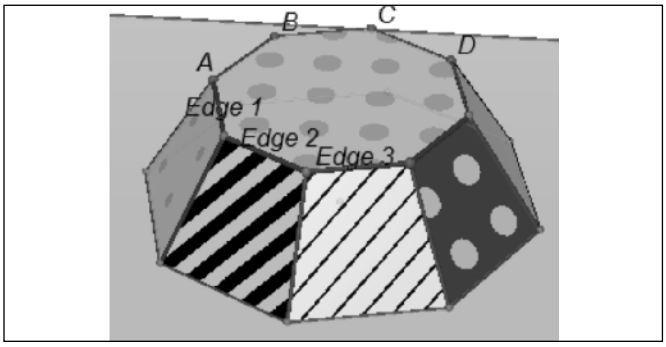




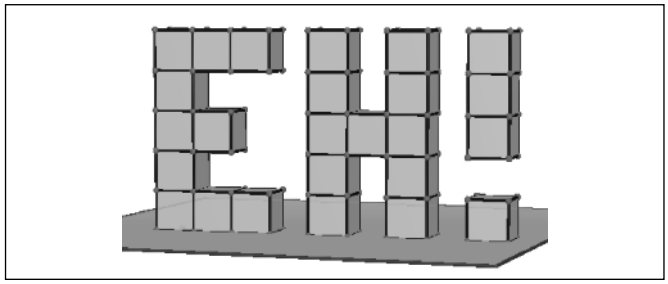


View from above

Faces, edges, and vertices can be coloured or labelled in different ways to facilitate counting.

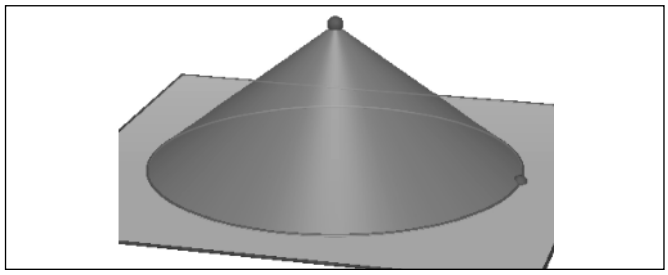


Shapes are easy to create (and move), come in unlimited numbers, and are not constrained by gravity.

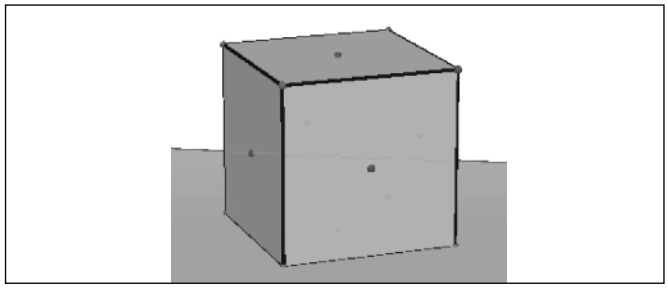


The dimensions of shapes can be varied, and connections can be made between and among different shapes.

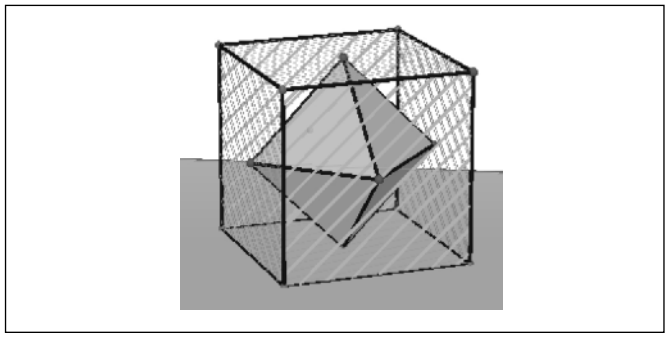
Drag the two red points to change the shape of this cone. When does the cone become a point, a segment (almost), a circle?



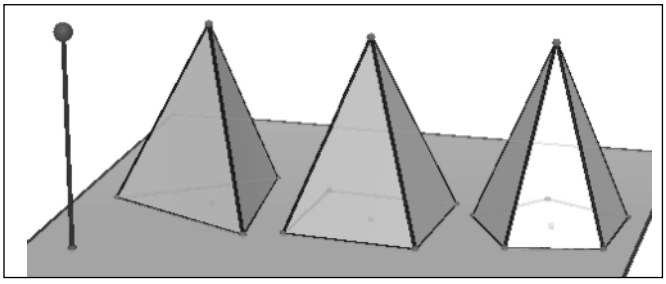
Shapes are not limited by opacity: a polyhedron has been constructed with the middle of each face of the cube as its vertices. An exercise in visualization is to describe this polyhedron.

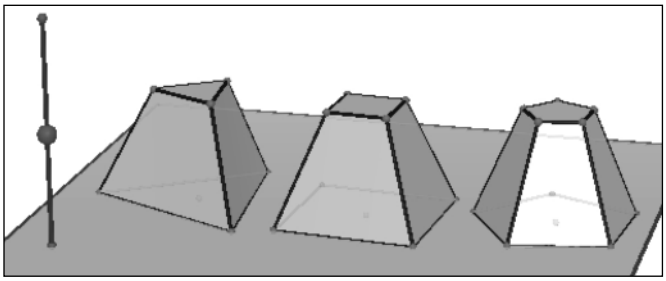


You can now see through the cube to see the octahedron inside it.

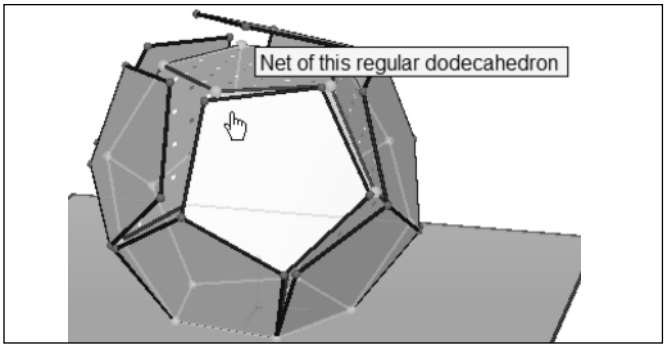


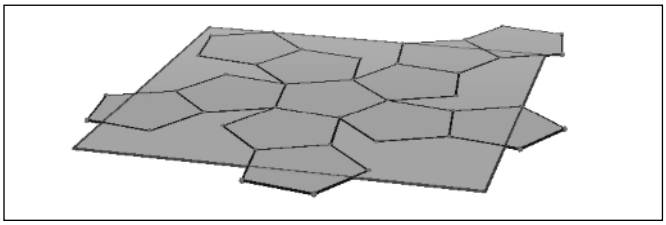
Drag the large red point downward to truncate the pyramids shown. How does this change the number of faces, edges, and vertices that each shape has? Why?

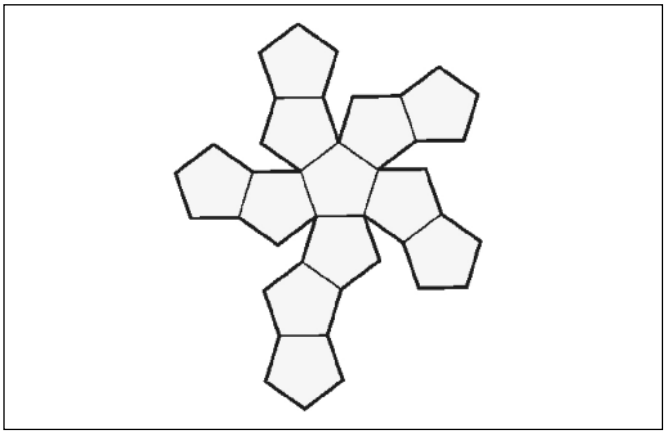




Nets of shapes can be quickly obtained.

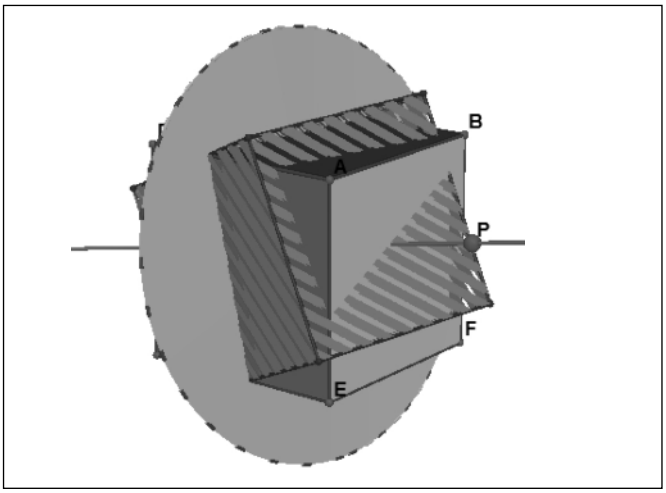






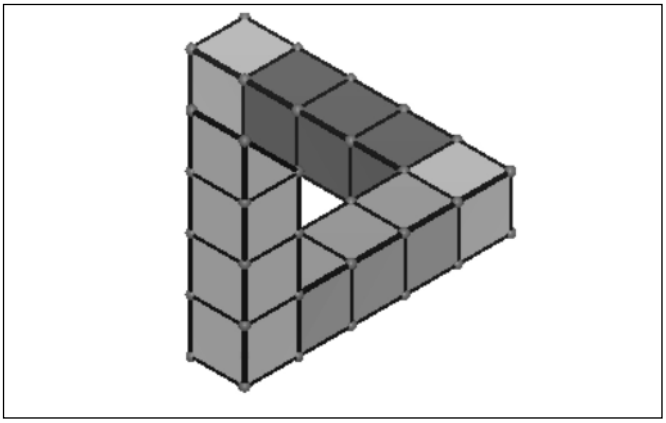
Symmetries can be explored.

The cube is reflected in the blue disk. Move the disk so that the cube and its image coincide. In how many different positions is this possible? What does this say about the symmetry of the cube?



Different representations of shapes can be explored.

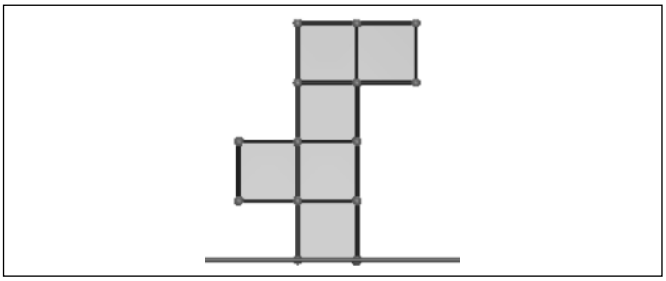
Try constructing this shape in an isometric view.



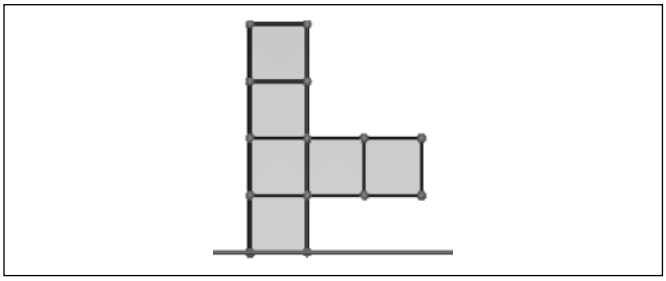
Here are front, side, and top views of a collection of cubes.

Create the original structure.

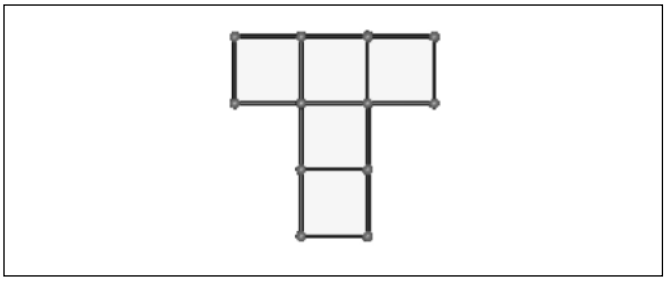
Front



Side



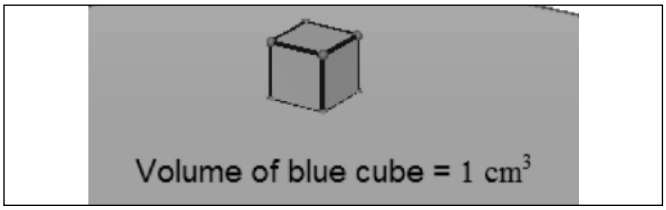
Top



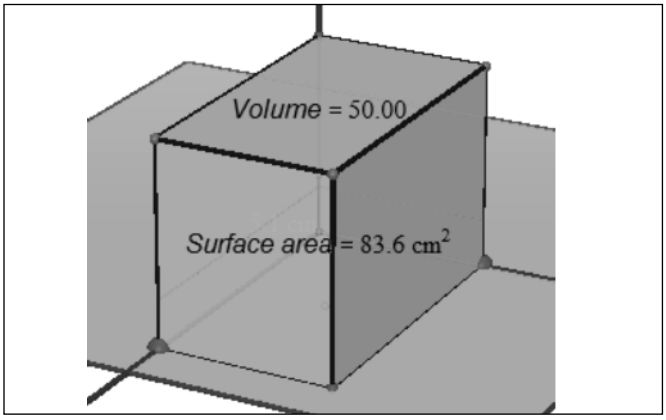
Volume and Surface Area

Estimate how many cubes you would need to build a rectangular prism 3 cm high, 4 cm wide, and 2 cm long. Now add cubes to build this prism. How accurate was your estimate?

How many different ways could you build a rectangular prism made from 24 cubes?

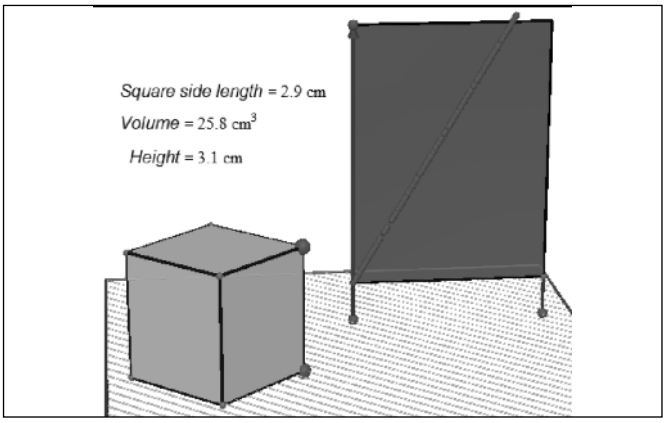


The volume of this rectangular prism is fixed. Drag the red points to find the minimum surface area. Now measure the dimensions of the prism. What do you notice?



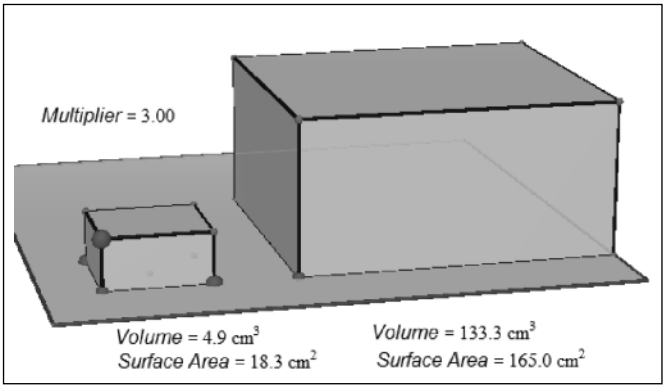
Objects can be directly linked to graphs.

The graph shows the volume of a square-based pyramid against height. How would the graph change if it represented volume against the length of the base?

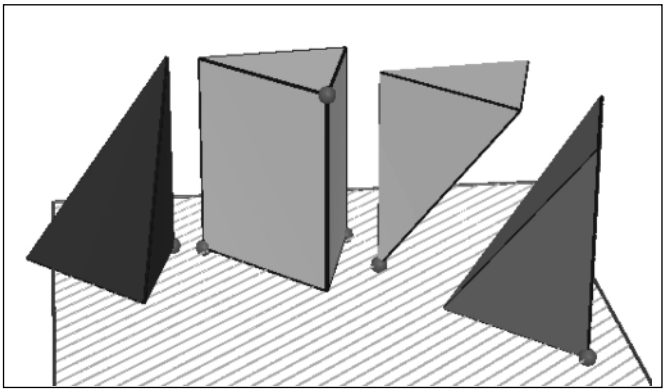


What is the relationship between the multiplier and the surface areas of these two rectangular prisms? What about the volume?

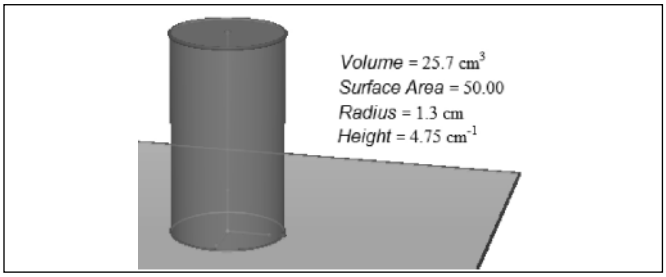
Does this depend on the dimensions of the prism?



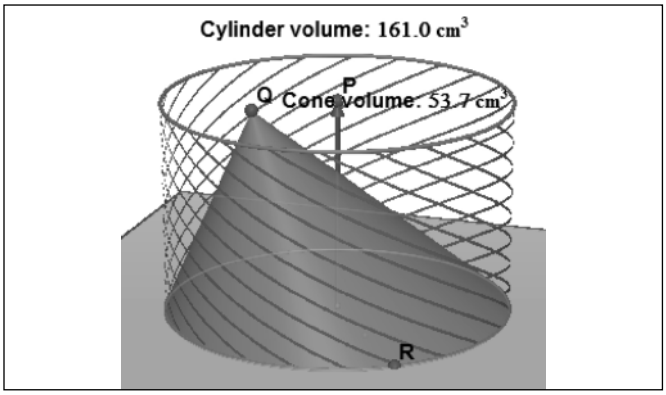
How are the following three pyramids and the prism related?



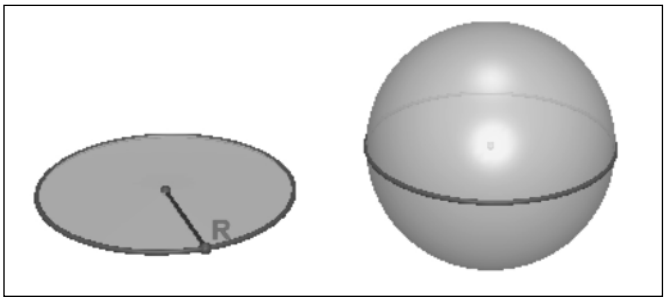
This cylinder has a fixed surface area. What is the largest volume that you can get? How are height and radius related when the volume is largest?



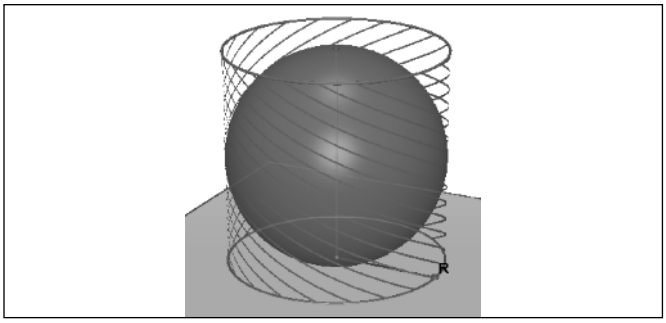
How are the volumes of the cylinder and the cone related?



The circle and the sphere have the same radius. Estimate a relationship between their areas and then use measurement to find a precise relationship.



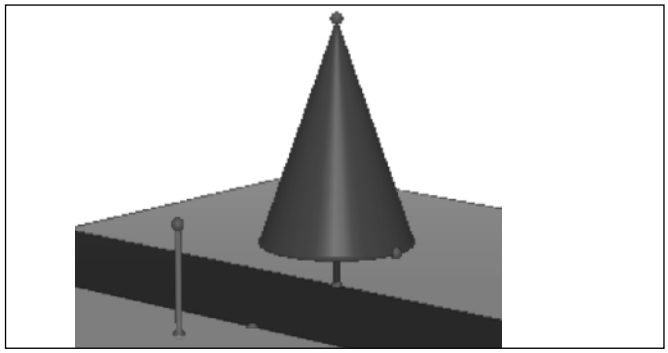
The sphere just fits inside the cylinder. Estimate, and then measure, to find a relationship between their surface areas and volumes.



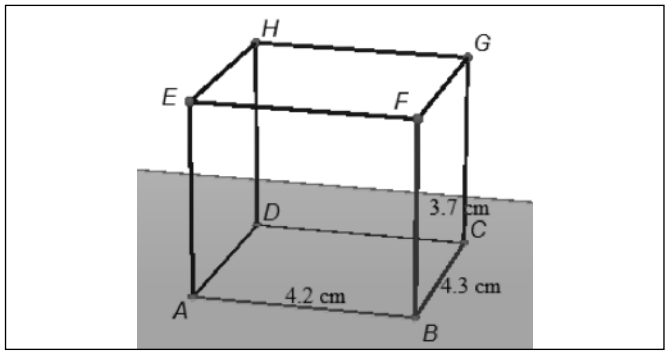
Applications of the Pythagorean Theorem and Trigonometry in 3D

The vertical segment represents an observer on one side of a deep river, who wants to measure the width of the river and the height of the tree opposite.

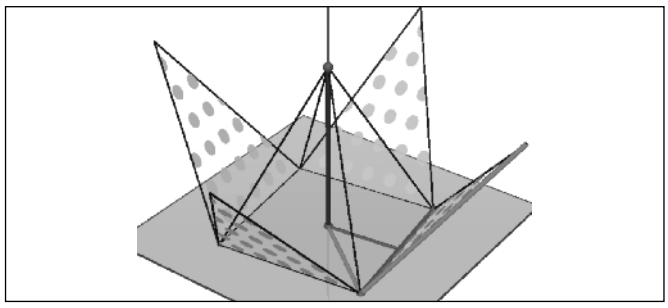
Make measurements corresponding to those that the observer could make without crossing the river, and work out the width and height. Check your answers by measuring.



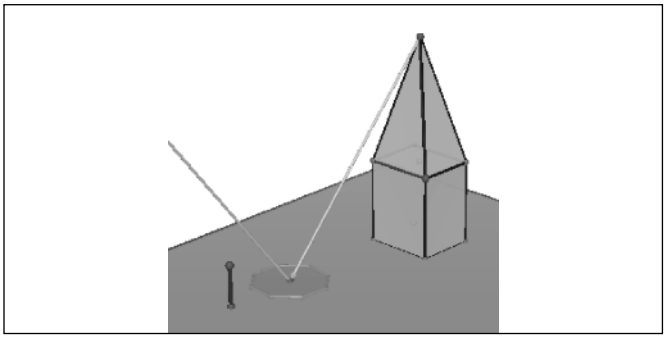
Create segment DB and use the Calculator to find its length. Now check by measuring. Now construct segment DF (one of the diagonals of the cube) and calculate its length. Can you find a calculation that uses only the side lengths of the cube? Change the dimensions of the cube. Does your calculation still work? What can you conclude?



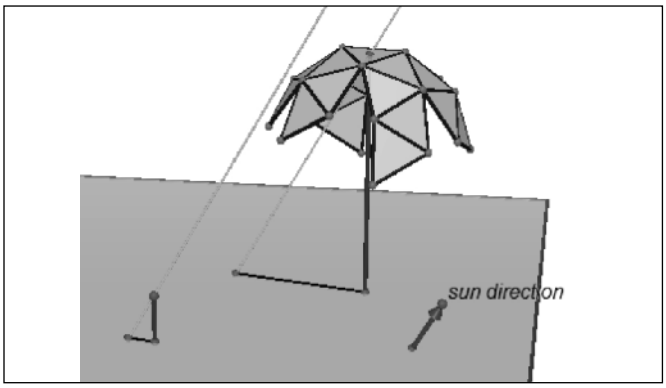
Given the height and base length of this square-based pyramid, how much tape will you need to tape together the edges of the net?



An observer is using a mirror to help measure the height of the building shown. Move the mirror so that the observer sees the reflection of the top of the building. How could the observer now make further measurements and hence work out the height of the building? Check by measuring the height of the building.



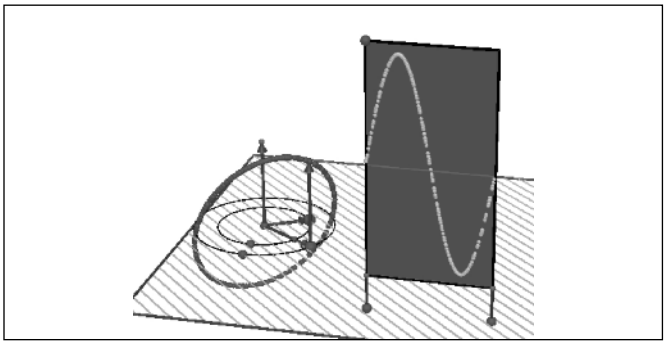
The black segments represent the shadows of the trunk of a tree and of a 1-m high stick. How could you work out the height of the tree without measuring it directly?



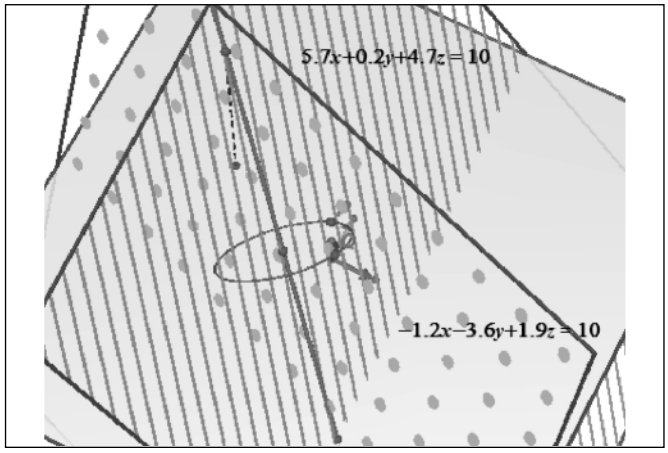
Grade 12 Geometry and Algebra of Vectors

The two (identical) vertical vectors are the cross-product of the horizontal vectors. The tip of one vector has been traced, and the vertical displacement has been graphed.

What does this tell you about the cross-product?



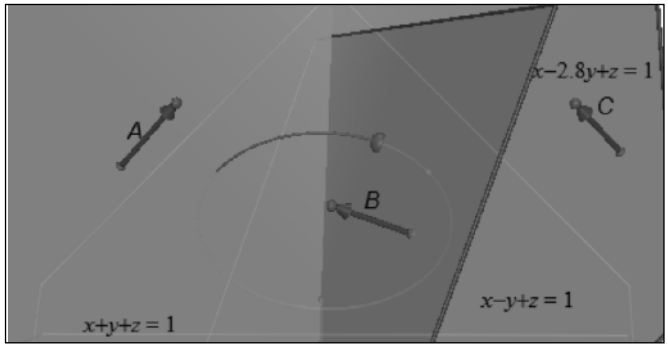
This diagram shows two of the possible planes that intersect in the blue line. How are their equations related to the equations of the line?



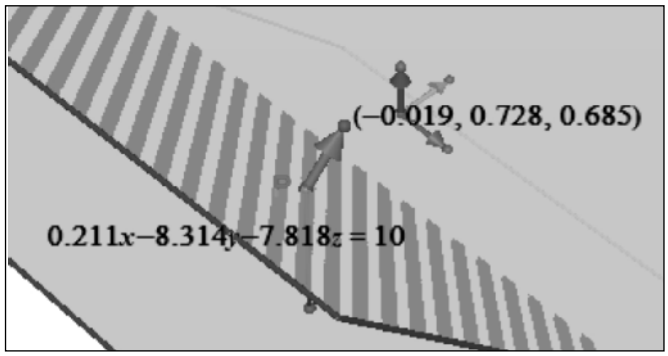
These three planes have been constructed so that they meet in a line. **A**, **B**, and **C** are the normal vectors to the planes.

Find **A** • (**B** x **C**). What does this imply about the normal vectors to the planes?

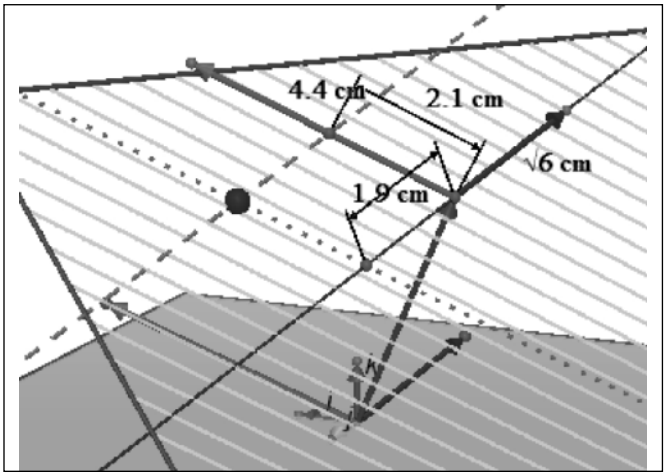
What happens when three lines meet in a point?



Drag the red points to change the orientation and position of the plane. Can you find a relationship among the equation of the plane, the components of the normal vector, and the coordinates of point P?

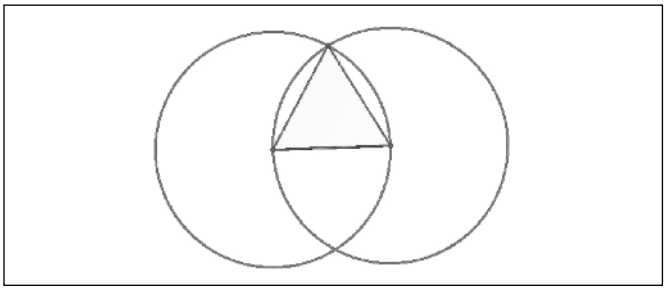


A plane has been defined containing the pink and blue vectors and passing through the tip of the green vector. Verify by construction that the blue point can be found by adding the green vector and a linear combination of the pink and blue vectors.



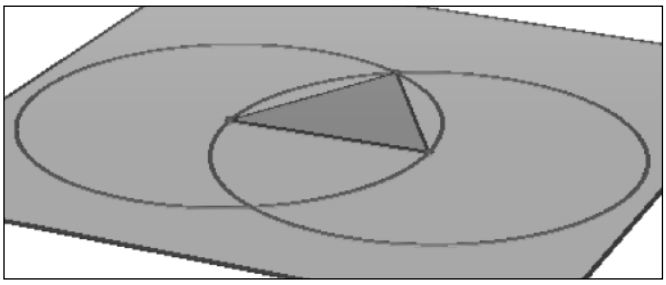
Cabri 3D can also be used for plane geometry—with a new perspective!

An equilateral triangle has been constructed on the base plane, which is seen from above. As with any interactive geometry software, the points defining it may be dragged, but the triangle will remain equilateral.

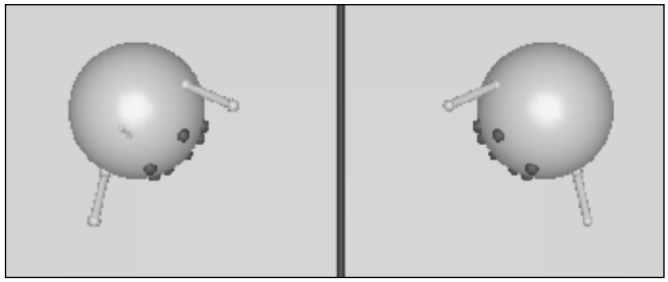


The view angle has changed: Is the triangle still equilateral?

What does this say about the way our brain processes information from a piece of paper?

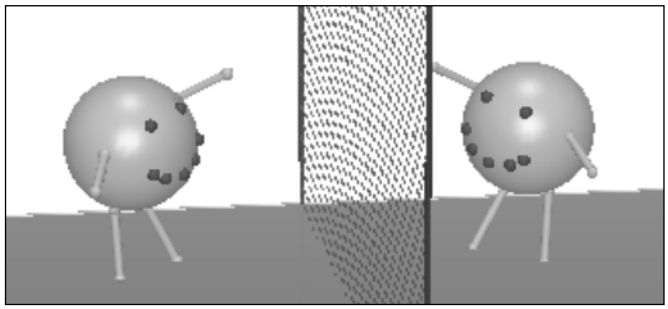


This shows the reflection of various objects in a line.



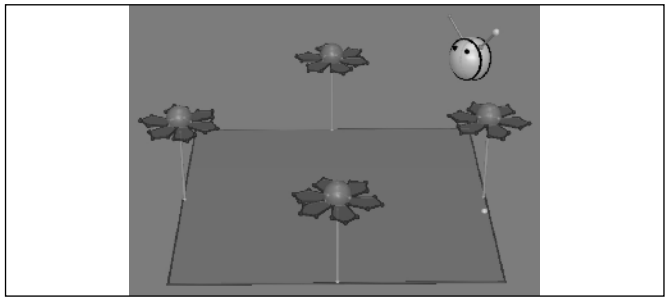
Well, actually, in a plane!

This can mediate between the everyday understanding of reflection and the 2D mathematical definition of reflection.

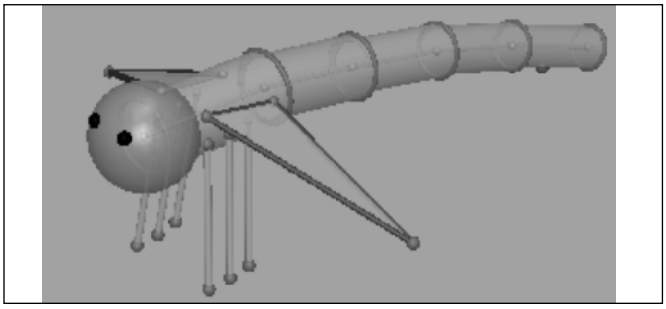


Then there are the areas of the curriculum where Cabri 3D gives an opportunity to either apply curriculum mathematics or to extend it. My research focuses on using Cabri 3D as an environment in which students can choose their own mathematical goals. In a recent project with two Grade 8 classes, students were asked to create a moving insect. In doing so, they needed to extend their understanding of basic geometric concepts into 3D. For example, a circle in 3D cannot be defined by just two points, and a perpendicular is sometimes a plane rather than a line. Students also needed to use transformations—reflections (in a point, line, and plane), rotations, and translations—in order to create motion. Students found the task challenging, but most students were deeply engaged and very pleased with what they succeeded in creating. Here are screenshots from some of their work:

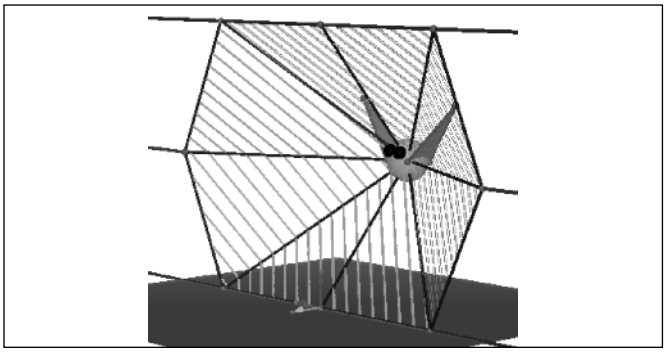
The bee flies in a series of arcs between the flowers, flapping its wings as it flies.



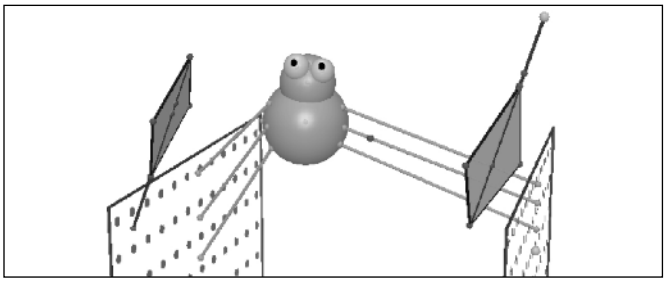
The dragonfly moves up and down, flapping its wings and moving its feet as it flies.



The bug flaps its wings and moves to and fro as it struggles to get free of the web.

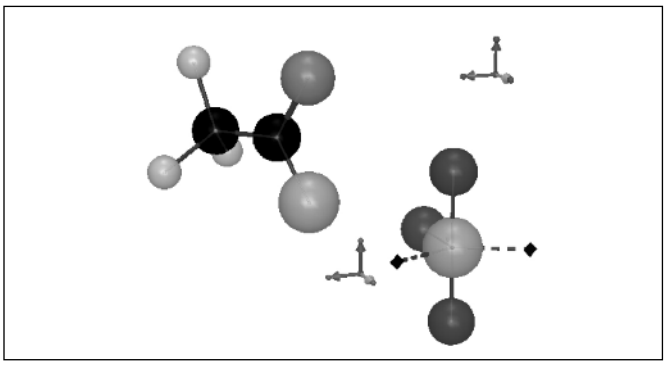


The bats rotate up and down, pushing the insect to and fro.



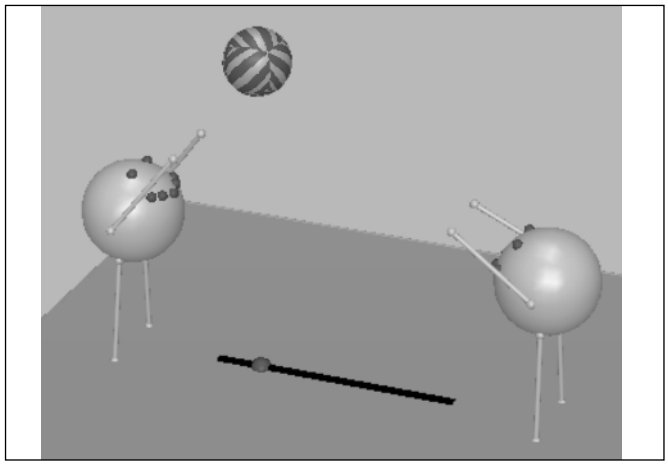
Cabri 3D is a powerful tool in any aspect of the school curriculum in which 3D modelling is useful. The screenshots below show some applications in science.

Two chemistry molecules modelled by a student at the Ontario Science Centre school

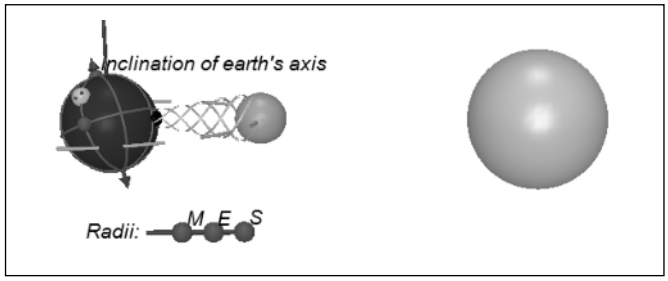


Motion under gravity

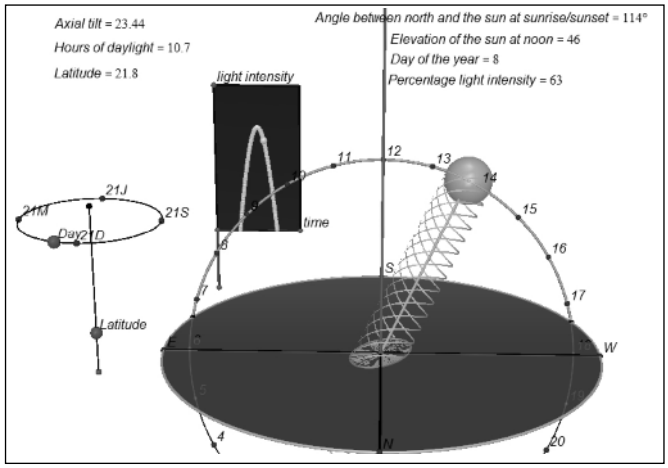
(The speed as well as the path of the ball models that of a real ball.)



Solar eclipse (below)



Path of the sun in the sky and relative light intensity for different days of the year and latitude



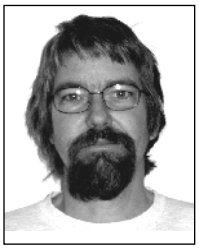
Please see my website **idisk.mac.com/katemackrell-Public/Cabri3D/index.htm** for more possibilities, including resources to get started with Cabri 3D, to create objects in motion, polyhedra, astronomical models, etc., and to access the Cabri 3D files from most of the above screenshots, which can be directly manipulated on the web page.

Have fun! ▲

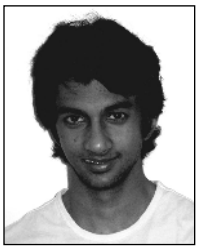
▲ Quadratic Rational Functions

TIM SIBBALD

AND OMAR GILANI



Tim Sibbald is a secondary math teacher with the Thames Valley District School Board. He recently completed his doctorate on research pertaining to self-efficacy during reflective practices.



Omar Gilani is a Grade 12 student in the Thames Valley District School Board. He plans to pursue post-secondary studies in science.

Rational function expectations in the advanced functions course (MHF4U) use one linear function divided by another (a "linear rational"). However, more complex rational functions present a broad range of teaching opportunities that allow teachers to extend the curriculum. This article considers an exploratory approach to extend the expectations regarding "linear rational functions" to quadratic rational functions. The approach was developed and tested in the classroom and found to be an excellent exploratory approach to the topic. To this end, the article includes both a student's perspective and the theory that will provide a resource to teachers.

The exploratory approach considers a quadratic function divided by another quadratic. The vertical asymptotes are maintained in a set position while different groups of students are given the task of investigating specific cases identified according to the position of the zeros of the function. The function, with vertical asymptotes fixed at $x = \pm 5$, is of the form:

$$f(x) = \frac{(x-a)(x-b)}{(x-5)(x+5)}$$

In this expression, the values of a and b are real values. They are assigned to groups of students in a manner that ensures that different characteristic behaviours will be observed. However, to provide an overall picture and resource for teachers, the article leaves these two values as variables. The choice of the vertical asymptotes is for convenience, as is having the horizontal asymptote at $y = 1$. The analysis that is used here can be generalized for other positions of the vertical and horizontal asymptote.

The approach used in this paper to examine the relationship does not require calculus. It uses the quadratic formula and discriminant to explore the number of values of x that generate a particular value of y. Inspection suggests that this may be 0, 1, or 2 and that, for particular values of a and b, the relationship passes through the horizontal asymptote before approaching it. Before delving into the theory, however, let's consider the view of a student who was actively involved in the exploration.

Student Perspective

As a student going into the assignment, I was expecting a fairly routine, run-of-the-mill exercise where the class would gain hands-on experience with the graphs of quadratic rational functions. The quadratic rational functions seemed quite complicated as we were introduced to them, but were found to follow a consistent pattern of laws and definitions; much like other mathematics. The class was given a task to determine the behaviours of quadratic function where the values of "a" and "b" were slightly varied. Definitions that we focused on were the horizontal asymptote, vertical asymptotes, and the x-intercepts. Knowing that graphs do not cross vertical asymptotes, the majority of the class assumed that the definition would also hold true for the horizontal asymptote. As the class began to graph [the] provided functions, slowly some questions arose about possible errors in the graphing calculators or error[s] on the parts [sic] of the students because the graph seemed to cross the horizontal asymptote at a certain point, a phenomenon which, at the time, was thought to be impossible. As we were given more time to explore these "errors," the class began to open up to the idea of having discovered something that went against what they thought. Personally, it was this realization that truly motivated me to explore more behaviours of my function and, later, become involved in this article.

Through this investigation, the class was able to grasp and fully understand the meaning of a horizontal asymptote, as "x" gets big (positive or negative), the graph approaches a fixed value. The class realized that when "x" was not big, the asymptote could be crossed. It was noticeable that this idea did not catch on instantly, as this was an extremely distant possibility in the psyche of the average student in the class. Once this finding reached a consensus around the classroom,

the investigation instantly gained legitimacy, and many students became very interested in the different phenomena of these functions.

Horizontal Asymptote

The horizontal asymptote is $y = 1$, and most students assume that the function cannot cross the asymptote. Subsequently, they find that it can cross in some instances, and discussion about the meaning of approaching $\pm\infty$ ensues. The function crosses the asymptote if $y = 1$, in which case, solving for x gives

$$x = \frac{25 + ab}{a + b}$$

which indicates that the function crosses the horizontal asymptote, except when $a = -b$. This corresponds to $f(x)$ being even and symmetric (see Figure 1). Checking that the value of x inserted in $f(x)$ actually gives a value of one is a worthwhile algebra exercise. It can also be cast as $(x - 5)(x + 5) = (x - a)(x - b)$, and substitution using the above formula resembles an algebraic equation of the type solved when teaching the rational functions unit.

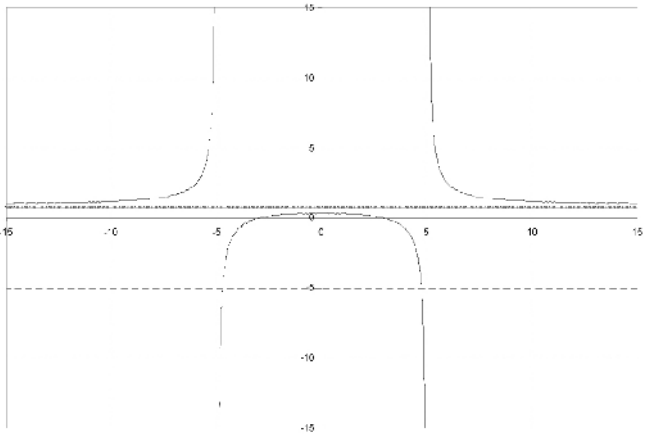


Figure 1: Symmetric case.

Students can solve the location where the function crosses the horizontal asymptote for specific examples. This sets up the opportunity to raise the question of how it can be that the function crosses the asymptote only once. This is an important detail that students need to carefully consider. If the function crosses the horizontal asymptote in between the two vertical asymptotes (see Figure 2), then the central approaches to the vertical asymptotes have opposite signs; in addition, the two external approaches to the horizontal asymptote are opposites (one from below and one from above).

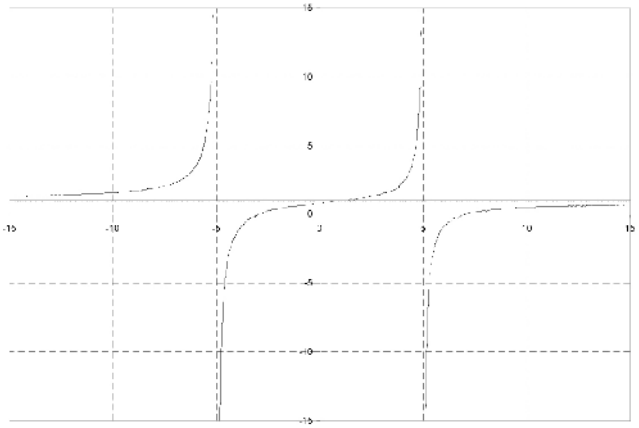


Figure 2: One zero between the asymptotes.

When the point where the function crosses the horizontal asymptote is in the portion of the domain with $x > 5$ or $x < 5$ (see Figure 3), then the values of a and b are not between the values of the asymptotes. In these cases, the function values, as one moves from the asymptotes with $x < 5$ and $x > 5$, either both decrease from $+\infty$ or both increase from $-\infty$. However, one side simply converges (strictly increasing or decreasing) to the horizontal asymptote, while the other crosses the asymptote, reaches a minimum or maximum value, and then converges to the horizontal asymptote from the opposite direction. (Note the change of scale on the x-axis in Figure 3 to highlight this.) The existence of the minimum/maximum outside the vertical asymptotes was quite surprising to students, and considerable effort was spent establishing that this is not an oddity!

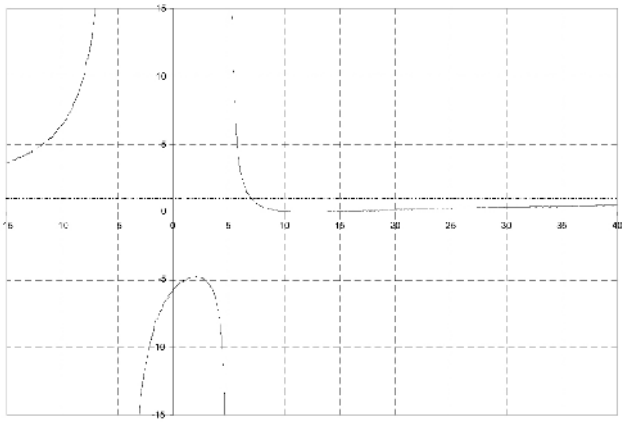


Figure 3: Zeros outside the asymptotes.

The Maximum and Minimum

The maximum and minimum can be found, using a quadratic analysis. The key is to imagine having a specific value of y in mind and solving for x. Since

$$y(x^2-25)=(x-a)(x-b)$$

(here the value of y is considered to be fixed), this means that

$$(y-1)x^2+(a+b)x-ab-25y=0$$

This can be solved using the quadratic formula for x (although students would need to be examining equations with specific values of a and b):

$$x=\frac{-(a+b)\pm\sqrt{(a+b)^2+4(ab+25y)(y-1)}}{2(y-1)}$$

The y values for the maximum and minimum can be found because they correspond to the case where the solution gives only one answer. In other words, the discriminant has to be exactly zero. Notice, however, that the discriminant is a quadratic equation in y, which can be rearranged to:

$$100y^2+(4ab-100)y+(a-b)^2=0$$

Hence, the y values for the maximum and minimum values are:

$$y=\frac{25-ab\pm\sqrt{(ab)^2-25(a^2+b^2)+25^2}}{50}$$

An important investigative aspect arose when the authors tried to evaluate this numerically, using a spreadsheet. The value of the discriminant can be negative due to round-off errors; a value of approximately $-1\text{x}10^{-14}$ was observed. This can be easily fixed by adding a small positive number or by using an absolute value. However, if a teacher chooses to use spreadsheets, he or she needs to be aware of this interesting practical problem and be prepared to help students.

Experiments

Student exploration of quadratic rational functions can be extended through the use of explorations that look at dynamic properties as a and/or b is changed. In our class, student explorations were used as the basis for brief presentations that emphasized the academic sharing of findings. Students were encouraged to tell the class what they tried and whether or not it worked, in order to encourage further thinking about the ideas. What follows is an explanation of three ideas that students can explore. There are, no doubt, other ideas, but these suffice to demonstrate the opportunity that the topic affords teachers.

Is asymmetry related to zero positions?

Some explorations with graphing calculators highlighted that the way in which functions approach the horizontal asymptote may depend on the values of a and b. To explore this, students were encouraged to choose a fixed value of b and then measure asymmetry while varying the value a. The notion of measuring asymmetry was new to the students, so a simple measure $g(a) = f(10) - f(-10)$ was suggested. Students understood that this function would have to be zero if there was symmetry, and that bigger differences would suggest more asymmetry.

The theory behind this idea is that for a fixed value of b,

$$g(a)=\frac{(10-a)(10-b)}{(10-5)(10+5)}-\frac{(-10-a)(-10-b)}{(-10-5)(-10+5)}=\frac{-20(a+b)}{75}$$

The asymmetry function turns out to be linear. Students can appreciate that this is indicative of the zero shifting its influence from one side of the function to the other. However, it is a challenge for them to appreciate that it is not an exact measure and simply indicative of what is occurring.

Does the local minimum relate to the zero positions?

The y-value for the local minimum depends in a complicated way upon the location of the zeros a and b. The formula was provided earlier, and there is no clear way to simplify this. However, students considered the symmetric case where $a = b$ and examined how $f(0)$ varies as a is changed. In effect, they generated the function

$$g(a)=\frac{a^2}{25}\quad a\neq 5,-5$$

This meant that the exploration required that they discover that the function they were investigating had a domain restriction. Several students expressed surprise that the function was not linear, while others found it intriguing that a clearly recognizable shape emerged.

Is the height of the local minimum dependent on its position?

This question is intended to be examined numerically, using a graphing calculator. Creating an organized method for moving the local maximum and examining its height is the real issue. One approach is to consider the case where $a = b$, which has a theoretical solution where the minimum y value as it depends on a is

$$Min\ y(a)=1-\frac{a^2}{25}$$

Students can generate a graph and may guess the shape. If they are efficient, they might be able to do a quadratic regression on a calculator and find the solution.

Another approach is to set $b = 0$ and then vary the value of a. Due to symmetry, it is necessary to consider only positive values of a, and it must be less than 5 in order to have a local minimum. This produces a graph with a domain restriction, $x > 5$, and a horizontal asymptote, $y = 1$. When a is small, the x-coordinate is large and the y-coordinate is close to one. On the other hand, when a is close to 5, the x-coordinate approaches 5 and the y-coordinate approaches .5, with the actual coordinate (5, .5) not being feasible because $x = 5$ is a domain restriction. The positions of the local minimum, generated using $0 < a < 5$ (see Figure 4), show how the height of the minimum varies with the x-coordinate.

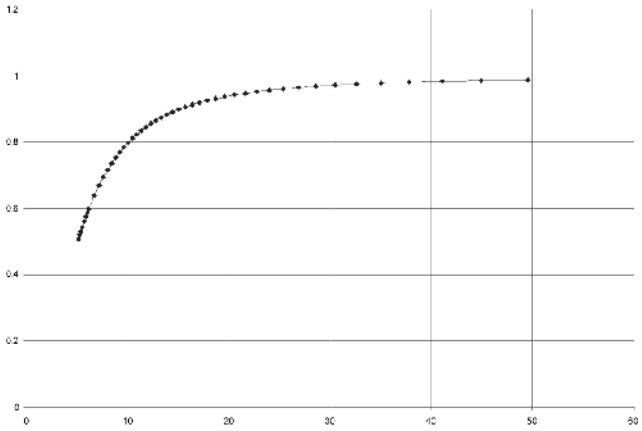


Figure 4: Position of minimum.

Calculus

The utility of calculus is downplayed in this article because the focus is on pre-calculus methods. However, it is interesting to note that the problem is arguably more difficult if one uses calculus. Applying the quotient rule provides a cubic expression divided by a quartic, and while it can be simplified, the process is tedious. An alternative method is to use logarithms to produce a sum and then take the derivative to get:

$$\frac{f'(x)}{f(x)} = \frac{1}{x-a} + \frac{1}{x-b} - \frac{1}{x-5} - \frac{1}{x+5}$$

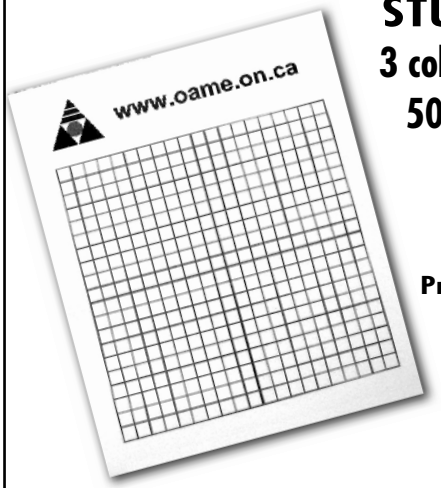
To find the maximums and minimums, one can set the derivative to zero and solve the right-hand side; however, this provides a cubic equation. An important tie-in to the rational functions theory is that multiplying through the right-hand side by $f(x)$ demonstrates that the denominators on the right represent removable singularities (also called removable discontinuities or holes).

It is a matter of opinion whether calculus is more or less efficient than the quadratic pre-calculus method presented in this paper. Perhaps what is more insightful is to combine the ideas to create a conceptual bridge between the two courses.

Conclusion

The use of a directed investigative approach to quadratic rational functions provides a beautiful extension to the curriculum. It is a unique opportunity to create a classroom environment in which students are intrigued by the variety of mathematical behaviours they see emerging. In addition, many experiments reinforce curriculum ideas of domain restrictions, discontinuities, and algebra. The ideas presented in this paper highlight student enthusiasm, the theoretical features teachers need to help them develop good pedagogical methods, and experimental approaches that can be used to develop student presentations. ▲

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▲ OAME Leadership Conference 2009: Developing Perspectives on Learning and Teaching Mathematics

The Conference at a Glance from a Delegate and President's Point of View

Opening Keynote Address: Dr. John Mason
"Using Our Natural Powers to Make Mathematical Sense and Sense of Mathematics"

- John Mason opened the conference by setting the stage for problem solving, reflection, and questioning. In his very personable way, John shared with us his expertise and vast experience in the teaching and learning of mathematics.
- Words and phrases that still resonate with me include:
  outer task, inner task, meta task, conjecturing, convincing, imagining, permissible change, freedom and constraint, specializing, the "problematicity" of tasks, mathematics as a social domain
- To see a video of this keynote address, go to the "Members Only" section under "Files and Links" on the OAME website.

Reception and Social

- This provided a great opportunity to chat with colleagues from across the province.
- I had a chance to talk with workshop leaders and hear more about John Mason's experiences in England.
- There was great food that hit the spot!
- This was a wonderful start to the conference!

Friday Workgroups:
"Developing Algebraic Thinking, K–8"

- This session was led by Dr. John Mason, with our very own Kathy Kabota-Zarivnji and Pat Margerm acting as workshop facilitators.
- Delegates were solving problems, thinking algebraically, and making, modifying, testing, and parking conjectures to learn about students' development of algebraic thinking.
- Throughout the day, we were challenged by problems (often not wanting to let them go until they were solved and analyzed) and inspired by the knowledge of John Mason.

Connecting Practice, Research, and People in Grades 7–12 Mathematics

- Myrna Ingalls and Irene McEvoy from the Ministry of Education led this session, connecting delegates with the latest theoretical frameworks that underpin current Ministry initiatives.
- Purposeful practice was the focus; the stages, tools, and resources that support student learning were also examined.
- We connected through the use of interactive whiteboards and other technologies to experience the latest tools and resources.

Publishers/Suppliers Luncheon

- I shared a meal and experiences with colleagues from different parts of Ontario.
- There were plenty of draw prizes from publishers and suppliers.
- There was lots of time to shop around and ask questions at the display tables.

Friday Banquet and Keynote Address

- The Holiday Inn provided yet another great meal.
- Paul Rosen, a motivational speaker, was truly inspiring—before and during his presentation.
- Through laughter and tears, we learned the power of determination, hope, hard work, and a positive attitude.
- Paul talked about hockey. I was thinking teaching.
- I had a chance to wear an Olympic gold medal!





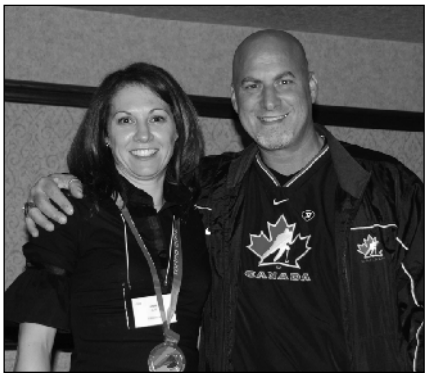

Saturday Workgroups

- I returned on Saturday morning feeling very comfortable. It was another chance to study, investigate, question, and try.
- I felt connected to the workshop leaders and co-delegates after spending so much time with them.
- I ate more food before heading home.

This was yet another fantastic OAME Leadership conference! Thanks to Sue Hessey for taking care of everything from the program and registrations to hotel details and conference bags, and special thanks to Dave Hessey for looking after all the other details (big and small) that needed to be done to make the conference a success.

Looking forward to Leadership 2010,
Anna Jupp

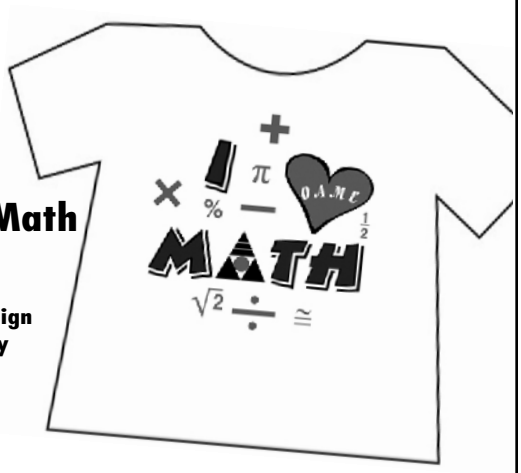
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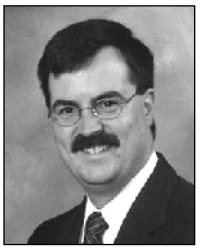
▲ Web 2.0 – An Introduction

Greg Clarke

Ross Isenegger



Greg Clarke is the Secondary Programme Coordinator for the Simcoe Muskoka CDSB as well as the OAME webmaster. Currently, he is on assignment with the Ministry of Education (CLIPS and GAINS projects). Check out CLIPS at oame.on.ca/CLIPS.



Ross Isenegger is the Numeracy Coordinator with the Near North DSB. For the past few years, he has been working on the Ministry of Education's CLIPS project. His blog can be found at ***mathfest.blogspot.com****.*

Have you heard of Web 2.0?

"What is this Web 2.0 you speak of?"

"I don't even know what Web 1.0 is supposed to be!"

"Is this related to a new and improved Spiderman?"

These are just some of the responses you might hear if you were to ask about Web 2.0.

The purpose of this series of articles, which will be published in the next few issues of the *Gazette*, is to introduce the reader to Web 2.0. In so doing, we will describe some innovative tools available on the Internet, explain how to use these tools for professional growth, and suggest how some of them could fit into your classroom. A companion website to these articles is hosted on the mathfest wiki at **mathfest.wikispaces.com/Web2.0+For+Math +Educators**.

This article will start off with a simple definition of Web 2.0 and an introduction to the first and most powerful tool, the *reader*, or *RSS aggregator*.

What is Web 2.0? A simple definition of Web 2.0 is "the Read-Write Web," as noted by Will Richardson. You can link to Will Richardson's blog at **www.weblogged.com**. To see an interview with Will Richardson, refer to the list of Links and References at the end of this article.

When the World Wide Web (now referred to as Web 1.0) first arrived on the Internet, most of the web pages were static pages. Each page was a separate document that linked to another, and then another, and each had to be created and maintained on a web server and uploaded and re-uploaded after changes were made by webmasters or people with technical know-how and access to the right logins and passwords.

Today, much of the content on the web is dynamic. Changes can be made easily, and not just by those with specific technical training. Much of the content, knowledge, and thought recorded on the Internet today is user generated (i.e., created by Internet users from around the world who have creations to share and ideas to communicate). No training required. This world—where anyone can upload pictures, make comments, post new ideas and thoughts on the latest issues, and take part in a larger learning community—is the basis for what is known as Web 2.0. The Web 2.0 for Math Educators wiki contains more information about the philosophical underpinnings of Web 2.0 and includes a link to a marvelous video by Michael Wesch called "The Machine is Us/ing us."

Let's Learn to Read in the World of Web 2.0.

With so much new information being posted each day, how do you know where to look or what websites to start out with? How do you know which sites are reliable or how to find newly created sites? How will you know when to go back to your favourite websites to check for something new?

These are all questions that plague the first-time adventurer in the land of Web 2.0, but luckily, there is a handy Internet application that will become the mainstay of our Web 2.0 journey. That application is the *RSS reader*, sometimes known as an *RSS aggregator*. This handy little application allows you to subscribe to content you want to read and keeps track of what you have already read and what you have yet to read. The content that can be subscribed to includes blog posts, wiki updates, podcasts, photo sharing and video streams, news services, classified ads, and Google blog search results. New content gets delivered directly to your RSS reader, making it your one-stop shopping location for news and information.

Time to log in. Enough of the theory; let's get to work and set things up! Here are the steps we are going to take:

1. Set up a Google account.
2. Log in to the Google Reader website.
3. Load some default feeds into the Reader.

4. Customize the feeds so that you get the type of information you're most interested in when you go to Google Reader each day.

For this series of articles, we are going to be using Google Reader as our RSS reader. (Other readers include PageFlakes and NetVibes, as well as those built into browsers like Internet Explorer and Flock.) In order to use Google Reader, you will need a Google account, which provides access to the whole suite of Google applications like Gmail, Google Docs, and MyMaps.

Step 1: Set up a Google account.

If you already have a Gmail account, then you have a Google account from which you can access the Google Reader program, so you can jump directly to Step 2. If you do not have a Gmail account, we suggest that you create a Google account for yourself now. You might consider adopting a persona and name (e.g., GregTheTeacher or MrEye) that you can use for all your education-related Web 2.0 activity.

Go to **www.google.ca**, click on "Gmail" (at the top left), and then on "Sign up for Gmail." Once your account is created, go to **reader.google.com**, click "Create an account," and use your new Gmail address to set up a Google account. Once all of your accounts are set up and authorized, you are ready to begin.

*One of the great things about most Web 2.0 applications is that they work completely within your favourite browser and require no installations. Hence, they will work equally well at home or at school on the computer that Computer Services has so thoughtfully locked down so your students cannot "mess it up." That being said, there are some that are more user friendly than others, and, perhaps, not surprisingly, Microsoft's Internet Explorer (IE) has its own built-in RSS reader that it wants you to use. Google Reader can still be used with IE, but adding new feeds to it sometimes takes one extra step. Don't worry—we'll make that clear. The browser of choice for the writers of this series is FireFox (**www.firefox.com**), which is freeware that can be installed completely without charge (on machines that are not locked down). FireFox has the added bonus of having some free add-ons that make using these Web 2.0 tools even easier.*

Step 2: Log in to the Google Reader website.

If you have not already done so, log in to your Google account. Once you are logged in, you can navigate from one Google application to another without having to log in again. You can jump from the Google search page to your Gmail to the Google Reader application. Depending on what screen you are on, the link to Google Reader will either be directly at the top of the page (click **Reader**) or under a category called **More**. You can access Google Reader directly at **www.google.com/reader**. However, your reader will be an empty shell without subscriptions; luckily, we have some suggestions. (See the next step.)

Step 3: Load some default feeds into the Reader.

Go to **mathfest.wikispaces.com/Web+2.0+RSS** and scroll past the videos introducing Google Reader to the bottom section entitled "Good Starting Point," where you will find instructions like these:

1. Click on the Starter.xml file and save it to your computer. (Save it, don't open it—unless you are a computer geek who likes to decipher Extensible Markup Language!)
2. Log in to your Reader account at **www.google.com/reader**.
3. Click "Settings" (at the top right).
4. Click "Import/Export."
5. Click "Choose File," then navigate to where Starter.xml is saved and double-click on it.
6. Click "Upload."
7. Once the subscriptions are imported, click "Back to Google Reader."
8. You should see that the subscriptions show up on the left, organized into folders called blogs, del.icio.us, Education, etc.

Step 4: Customize the feeds so that you get the type of information you're most interested in when you go to Google Reader each day.

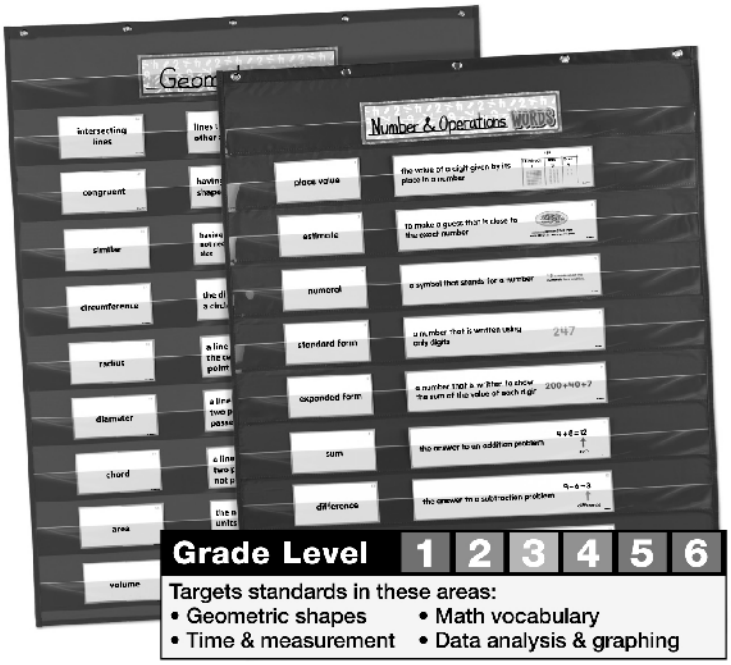
1. Next to the Subscriptions heading on the left, there is a drop-down menu. For now, select "Show All."
2. Expand a folder and click on one of your new subscriptions. The posts will appear in the main reader pane. Click "Show: all items" at the top left of the reader pane, if no items are displayed.
3. Show options are found at the top right of the main reader pane. Posts can either be displayed as a **List** or they can be **Expanded** to show some or all of their content.
4. Click on the title to expand an item, if you are in List View. Click on the title again to visit the page where the item is located.
5. Click "Mark All As Read" to unflag posts in the current view (for a subscription, for a folder, or for all items).

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6. Click "Feed Settings" to unsubscribe from a feed, or to sort or classify the feed differently.
7. One way to add feeds is to click "Add a Subscription" and type in a search term (e.g., "Ontario math education"). There are other ways to add feeds, but we will discuss these in the next article. If you can't wait, have a look at the videos at **mathfest.wikispaces.com/Web+2.0+RSS**.

Next Steps

Now that you've got Google Reader set up with the types of information that you're interested in reading about, bookmark the website and come back to it on a regular basis, perhaps once a day or a few times each week. Browse through the new content that you see each time you visit, and check out the links for those items that spark your interest.

For more details and links to videos and much more (including a sneak peek at tools and topics that will be covered in upcoming columns), check out the companion wiki hosted at **mathfest.wikispaces.com/Web2.0 +For+Math +Educators**. Feel free to use the Discussion link at the wiki to contact the authors of this article if you have any questions, comments, or suggestions.

Links and References

CommonCraft RSS in Simple Language embedded at **mathfest.wikispaces.com/Web+2.0+RSS**

Google Reader videos embedded at **mathfest.wikispaces.com/Web+2.0+RSS**

Interview with Will Richardson at EDUGAINS Conference (17 minutes) **media.curriculum.org/curriculum/GAINS/WRichardsonFeb1308.wmv** or on Youtube

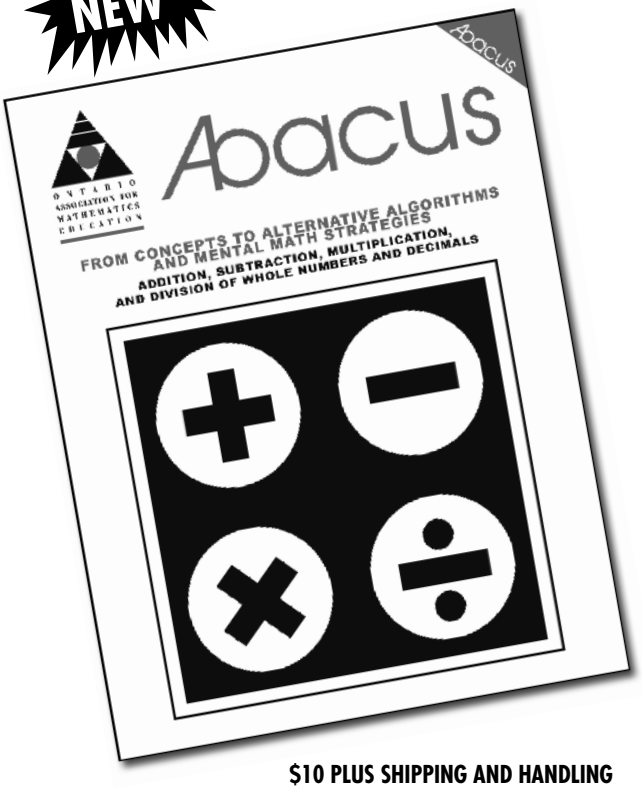
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Will Richardson's Blog **www.weblogg-ed.com** ▲

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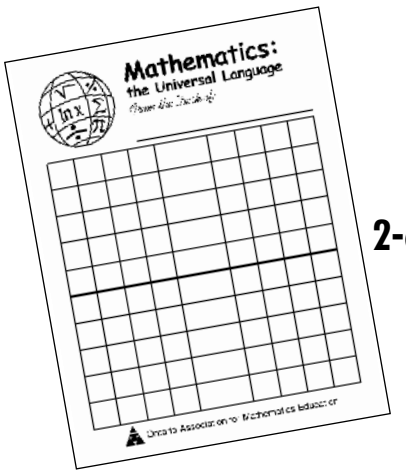
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