Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Stewart Craven, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Stewart Craven at **numeratecitizen@mac.com**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera-ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising Rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

STEWART CRAVEN, EDITOR



A teacher's career is far from uneventful. There are many happenings that will be etched in your memory forever. Recently, I was once again privileged to bear witness to the emergence of a group problem-solving episode that was truly magical. It started quite innocently with a group of 26 Ontario Science Centre Science School students who were attending our communications workshop at Geneva Park near Orillia. The teachers try to find clever ways to group the students (e.g., asking the students to arrange themselves in order of birthdate, without speaking). We decided on this occasion to challenge the students to find a partner whose badge number, when summed with theirs, would be the same as that of every other pair of students. (The badge numbers ran from 1 to 28, inclusive.) This problem would have been simple, except that students 7 and 18 were not in attendance!

All 26 students huddled together and enthusiastically exchanged ideas. They soon realized that they had to use a "next best" solution, whereby one pair of students would have a different total from the rest. Since we actually needed 5 groups of 4 students and 2 groups of 3 students, we then challenged each group to get the same total (no mention of badge numbers in this case), regardless of size. What ensued was simply amazing. For 15 minutes, the students crowded around each other. Ideas for strategies were repeatedly offered, tested, and rejected until the students concluded that there was nothing in the rules that said they had to use their actual badge numbers to number off in a way that would lead to success. In essence, they worked backwards! They chose the total 100 and then made sure that everyone had a unique number.

It is not often that one has the opportunity to observe a large group in which everyone participates, in which leadership is distributed across the group, and in which success is achieved and the solution is innovative. It is not an understatement for me to say that this spontaneous session will certainly become one of my fondest memories as a math teacher. ▲

▲ President's Message

BEVERLY FARAHANI, PRESIDENT



Beverly Farahani is Head of Mathematics at Kingston Collegiate and Vocational Institute. She is also a T³ Instructor and was on the Minister's Task Force.

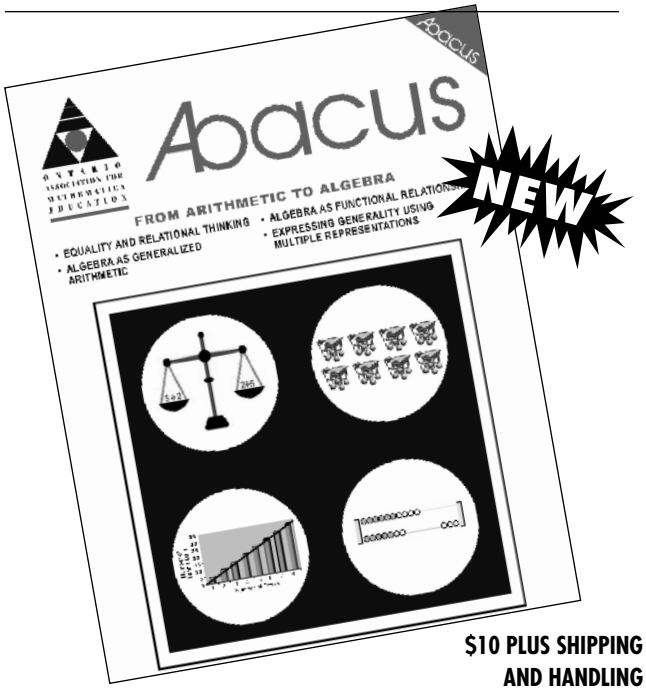
Time flies when you are having fun! It's hard to believe that my journey as OAME president is almost over and that this will be my final president's message. The year has gone by quickly, and I would like to thank everyone for their enduring support. Your impact can never be fully articulated!

We all make a big difference in the lives of our students. We are the ones who are there with our students on their mathematical journeys; some may stay closer to home, while others will travel to mathematical places we have never been. Wherever their journeys take them, we hope that our students are well on their way to a lifetime of learning.

We can be a role model for this. By participating in professional development, attending a conference, or even taking a course, students (and parents) can see that we, too, have a love of learning. Don't be surprised when a parent approaches you to congratulate you on the completion of a professional development course. (Yes, we are a topic of conversation at home!) Parents know that their children will benefit from our new-found knowledge, and our students can look forward to working on a new idea that we've had a chance to play with. Students also appreciate knowing that we, too, are out of our comfort zone when trying something new.

As you proceed through your mathematical path in life, remember: "Life is a journey, not a guided tour." ▲

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▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

In Memoriam
JOAN ROSAMOND ROUTLEDGE (Née Mix)

September 18, 1920–November 25, 2009

Backwoods school teacher, mother, textbook author, Descartes medalist, and OAME President 1975–76



Joan Mix began life in the flamboyant 1920s. Her father's thriving woolen import business funded a custom-built home in Toronto's fashionable Rosedale district. There were private schools for the children, summers at Lake Simcoe, trips to New York, and short excursions to visit Great Aunt Maggie in Pottageville.

At an early age, Joan became a voracious reader, frequently visiting the Toronto Public Library. Several librarians became very fond of their young visitor and began to surreptitiously set aside books that they thought Joan would enjoy. Later, she was to thrive under the strong academic focus at St. Clement's School.

The family's fortunes suffered in the depression and subsequent war. Demand for woolen imports collapsed, the business faltered, the house was sold, and Joan's father died when she was only 15. Joan's long-standing dream of a university education was no longer affordable.

Nonetheless, through working two summers for Eaton's catalogue, Joan was able to afford Teacher's College. In 1938, she arrived in the Muskoka hamlet of Brandy Lake to teach in a one-room school. Joan loved the work, the Muskoka woods in autumn, skating on the frozen lakes in winter, and the spring breakup.

But Joan was needed at home and so returned to teach in Toronto. Her first class included a very precocious young Peter Gzowski. Joan soon met her future husband, Herb, and within two months, they were engaged. But if they were to marry right away, Joan would have to quit her teaching job, and Herb's depression-era income would barely be adequate to support them. The wedding was therefore put on hold while they built a house on Herb's parents' farm property in King Township. They dug the basement using only hand tools, cut pine trees for structural timbers, and did almost all of the construction themselves.

In the meantime, Herb joined the army. Although he survived, Joan's brother Donovan disappeared somewhere over the North Sea while returning from a mission over Europe.

With the end of the war came children. It was a challenging time. Winter storms often isolated the family in King Township, and during the spring thaw, roads within 3 km of the house were impassable.

While the children were young, Joan indulged her love of teaching by supervising the elementary Sunday school at the Aurora United Church. She also chaired the School Board for the one-room school where her children began their education. Mary Scott, the first teacher that the board hired during Joan's term of office, fondly remembers Joan's solid support.

By the time Elvis Presley was gyrating his hips on the Ed Sullivan Show, work restrictions for married women had been lifted and the children were becoming less dependent, so Joan resumed her career. After a year of substitute teaching, she took a full-time position teaching Grade 1 in Richmond Hill. With the extra income, the couple was able to set aside $2500 to purchase an island on Georgian Bay. Joan spent a part of every summer there for the rest of her life.

Joan's next priority was to further her education. She enrolled in the Queen's University extension program, specifically designed to enable mature teachers to complete an undergraduate degree. She began with correspondence courses, sometimes supplemented with evening or weekend lectures in nearby Oshawa. By the mid-1960s, Joan's children were old enough for her to attend summer school at the university's home campus in Kingston. John Coleman, Head of the Mathematics Department, had hired a young Irish immigrant, Jim Whitley, to lead the department's activities in teacher training and high school liaison. Joan enrolled in one of Jim's first summer school courses. A superb teacher and motivator, Jim was instrumental in helping Joan broaden her mathematical perspective.

In the summer of 1968, Joan finally obtained her cherished graduation diploma. Jim Whitley had made a point of being in the platform party, and gave her a rousing cheer as the Chancellor clapped her on the shoulder and instructed her to "Rise, Bachelor of Arts."

By that time, Joan's career had begun to flourish. She soon co-authored a series of elementary mathematics texts for the Gage Publishing Company and continued to develop related teaching aids for many years thereafter.

With the amalgamation of smaller school boards, new opportunities developed. Joan spent the latter part of her career as a Master Teacher and then worked as a Consultant for York Region.

In 1974, Joan was awarded a Descartes Medal by the University of Waterloo for outstanding contributions to mathematics education. She cherished that award. With it came a standing invitation to attend the annual award dinner and presentation ceremony. She was a faithful attendee. When she became too old make the drive herself, her children drove her when they could. In later years, Steve Brown, the host and master of ceremonies, was particularly thoughtful in accommodating her special needs.

Joan also played a leading role in the Ontario Association for Mathematics Education. She was elected President of OAME in 1975, was awarded a life membership in OEA, and then became a Life Member of OAME in 1986, the year after she retired. She continued to be professionally active well into her retirement.

Joan was among the first generation of women to have both a family and a successful professional career. She is survived by three children, four grandchildren, and five great-grandchildren. ▲

▲ What's the Problem? Looking Out for Number One

SHAWN GODIN



Shawn is the head of Mathematics, Business and Technology at Cairine Wilson S.S. in Orleans, ON. The web page for this column is ***whats-the-problem.wikispaces.com****.*

Welcome back, problem solvers! Last issue, I left you with the following problem:

> Clovis likes to take an outing in the natural numbers. Each day, he starts with a natural number of his choice, the biggest possible. Then, during the day, he passes from number to number, using the following rules. Suppose the sequence of numbers is currently at n.
>
> 1) If n is divisible by 3 without remainder, then the next number is $n/3$.
> 2) If the remainder after dividing n by 3 is 1, then the next number is $2n + 1$.
> 3) If the remainder after dividing n by 3 is 2, then the next number is $2n – 1$.
> 4) If $n = 1$, then the sequence stops.
>
> Over the years that he has played this game, Clovis noticed that, whatever the starting number, the sequence always ended at 1. Will this always happen?

This problem appeared as Problem 2 from the 2006 Concours de l'Association Mathématique du Québec. The contest appeared in the Skoliad Corner of the March 2008 issue of *Crux Mathematicorum with Mathematical Mayhem*, and the solution appeared in the February 2009 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem* website at the following address: **journals.cms.math.ca/CRUX/**. Contests from l'Association Mathématique du Québec can be found at: **newton.mat.ulaval.ca/amq/Concours.html**.

This problem begs to be played with. If you haven't had a chance to play with this, I encourage you to do so now. By experimenting, we may (by chance) hit on a counter- example, or more likely, we will get a feel for the rules and start seeing some patterns.

If we start with 44 (remainder of 2), we get: 44 → 87 → 29 → 57 → 19 → 39 → 13 → 27 → 9 → 3 → 1. If we start with 100, we get: 100 → 201 → 67 → 135 → 45 → 15 → 5 → 9 → 3 → 1. In both cases, we see that the pattern does arrive at one, but in a seemingly random fashion. A closer look will reveal some patterns that can be explored further.

First, let's look at the rules. Rule 1 causes the next number to be smaller, but Rules 2 and 3 will make the next number larger. Even though we are dividing by 3 in Rule 1 and only multiplying by 2 in Rules 2 and 3, it would appear that we would be doing more multiplying than dividing, so it seems reasonable that the sequence could get to a point where it increases, rather than decreases.

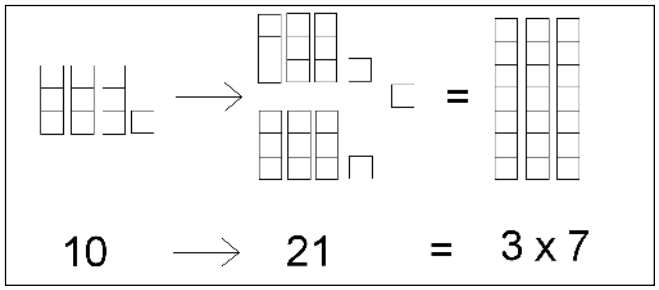
Looking at the sequences generated above and the rules that were used, we see that any time Rule 2 or 3 was used, it was immediately followed by Rule 1:

44 (3) 87 (1) 29 (3) 57 (1) 19 (2) 39 (1) 13 (2) 27 (1) 9 (1) 3 (1) 1

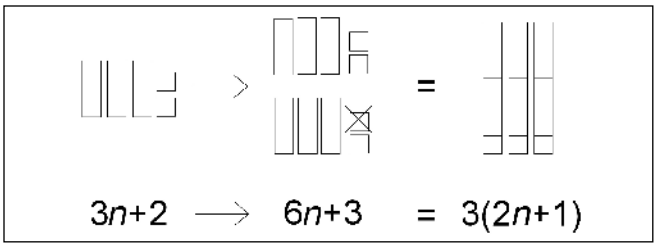
100 (2) 201 (1) 67 (2) 135 (1) 45 (1) 15 (1) 5 (3) 9 (1) 3 (1) 1.

If this pattern holds up, the overall nature of the sequence will be decreasing. This can be seen if we look at every second number in each sequence and notice that these subsequences are decreasing. For example, our first sequence **44** → 8*7* → **29** → *57* → **19** → *39* → **13** → *27* → **9** → *3* → **1** breaks into two subsequences 44, 29, 19, 13, 9, 1 and 87, 57, 39, 27, 3, both of which are decreasing.

We can see what happens with Rules 2 and 3 by using some manipulatives to help illustrate it. If we start with a number that leaves a remainder of 1 when divided by 3, such as 10, when we apply Rule 2, we see that the new number will always be a multiple of 3, so that Rule 1 will be applied next and the resulting number, in this case, 7, will be less than the number we started with.



Similarly, we can show that when Rule 3 is applied, the resulting number will be a multiple of 3. This can be done algebraically, or by using algebra tiles, as shown below.



Thus, whenever Rule 2 or 3 is applied, it will immediately be followed by Rule 1, and this new number will be less than the number we started with. So, since every second term is smaller and the rules will not allow us to have a negative or fractional result, we must continue decreasing until we hit 1 and the sequence stops.

I suspect that this problem was inspired by an unsolved problem in mathematics called the $3x + 1$ problem (also known by many other names). The problem deals with a sequence defined very much like the one in the problem we are investigating.

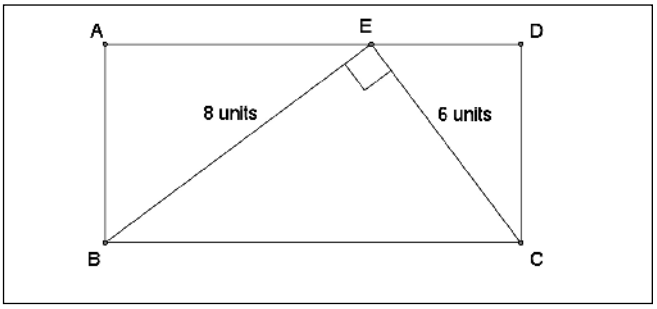
Suppose the sequence of numbers is currently at n.

1) If n is even, then the next number is $n/2$.
2) If n is odd, then the next number is $3n + 1$.

Notice that this is similar to our problem, except that we are multiplying by 3 and dividing by 2. It would seem that this would grow forever, but for every value that has been tried, the sequence ends up at 1, then cycles forever $1 \to 4 \to 2 \to 1 \to 4 \to \ldots$. Mathematicians have not yet been able to prove that this happens for every possible starting number. Maybe one of your students will be the first to prove or disprove the conjecture that it always ends at 1! Good luck!

Now, for your homework!

Determine the area of the rectangle ABCD.



Until next time, happy problem solving! ▲

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In Memoriam
MONA McGREGOR

Former Co-Secretary/Treasurer of OATM and OAME

By Don & Carol Attridge
(with contributions from Dave Davidson, John Egsgard, and Jack LeSage)

It is with heartfelt sadness that we mourn the passing (this past September) of our friend and mentor, Mona McGregor.



Mona was "Mom," who, along with her loving partner, Morley, birthed and raised the clan of OAME members through transitions from OATM (Ontario Association for Teachers of Mathematics), its amalgamation with OMC (Ontario Mathematics Commission), and finally, OAME (Ontario Association for Mathematics Education).

Away from home, at council meetings and conferences, Mona always kept her eye and intuition focused on the members. If anyone needed an aspirin, a bandage, a bowl of soup, or a sympathetic shoulder, Mona was there. She was very inclusive of new members.

As co-secretary/treasurer, she was a nurturing influence, complementing Morley's perceptive political antennae. Together, they promoted the force of mathematics education in Ontario and the coordination of its educators. As a former elementary school secretary (and we all know who runs the schools!), Mona was a politician/educator in her own right. Before the advent of computers, Mona took minutes of meetings by hand, and once home again in Sarnia, transcribed them on her own typewriter. Membership lists were also done by hand and copied with the use of carbon paper. The job was immense, and it is due to Mona and Morley's diligence that we now have archives and a history of OAME, which will be used for the 40th anniversary edition of the *Gazette*.

Mona and Morley were excellent hosts at their home on Bright Street in Sarnia, and they warmly welcomed all who dropped by for a visit. When they celebrated their 50th wedding anniversary, they hosted the party at the Ramada Inn (the location of our OATM/OAME Council meetings in Toronto). The majority of invitees had some connection to mathematics.

OAME recognized and honoured both Mona and Morley with the creation of two annual awards—The Morley McGregor Award for Excellence in the Teaching of Mathematics in Elementary Schools and The Mona McGregor Award for Contribution to the Teaching of Mathematics by a Person Outside the World of Education.

After retirement and right up until her death, Mona often attended conferences, maintaining her personal interest in the people and the organization. We are greatly indebted to Mona for her hard work, her record keeping, and her nurturing of our organization and its members. ▲

▲ Data Rocks – *Census at School* and Piecewise Modelling

Yves Saint-Pierre
Statistics Canada



Yves Saint-Pierre has worked at Statistics Canada in many capacities since 1990: questionnaire design, data editing and imputation, analysis, and publication. He has been sharing his knowledge with schools since 2002.

Census at School remains a great tool for teaching all of your data management curriculum from Grades 4 to 12. New this year: We have made video tutorials that explain various aspects of this project. There are 16 videos in all, covering topics from registering as a teacher and creating a class to preparing your class, filling in the survey, and finally, analyzing your class data in Excel, Fathom, or Tinker Plots. Another tutorial demonstrates how to import a random sample from the International Random Sampler. You can see these videos on YouTube, TeacherTube, or on the "Teachers" page of the *Census at School* website **www.censusatschool.ca**.

Census at School for all ages! (Well, ...for most grades.) While a large portion of the participants in *Census at School* comprises students aged 12 and 13 (50.2% in the year 2008–2009), the project can be used to teach higher-order statistics, such as correlation and the effects of sample size. For example, using our International Random Sampler, your students could get samples of data of Canadian students from previous years and calculate means and percentage distributions from those samples. Each student can get his or her own random samples of between 10 and 200 students. The theory says that the sample means should converge to the population mean as sample size increases. The *Census at School* website has everything needed to test this theory. Besides the random sampler, the Data and Results page also includes summary tables for each question of the *Census at School* for every year since we started this project in Canada. This concept is spelled out in a lesson plan entitled "The Effect of Sample Size on Data Analysis." Look for it under Learning Activities > By math concept > ... under the bold title, **Sampling**.

A new lesson applying **piecewise modelling using multiple functions** was recently released on our website. Look for it on our Function Modelling page at **www.statcan.gc.ca/learningresources** > Teachers > Mathematics > Grades 9 to 12 > Function modelling using secondary data from E-STAT under **Combined functions**). One of our readers suggested that this could be used as a summative assignment after students have learned linear, quadratic, and exponential functions. Another reader questioned the point of fitting different functions to various parts of a time series. Let me expand on that:

Overall, the Federal Debt models with an exponential function. The general logic of growth at a fairly regular pace for most economic indicators signals that it should be an exponential function of time over a long period. But this series is different because we have data for a long period (1867–2008) and because the federal debt has medium-term trends that are influenced by specific events (e.g., great wars, great depressions, the baby boom), as well as by shifts in political climate. Those can be easily mapped along this series, and your students will be applying models to a real-life topic of great interest in these changing economic times.

Economists model economic indicators as functions of other economic indicators, but these models are not studied until the Masters' level. Here is a chance for your Grade 11 or Grade 12 students to take a peek at some of the other factors influencing the debt, using time as a proxy for these other events. We have broken the long series down into ten periods that your students may want to study beyond the mathematical model to find out why the debt behaves in this way during a particular period.

Finally, as always, please do not hesitate to write to me with comments or questions. I welcome your input on StatCan projects and resources for education. In particular, I would like to hear your ideas for creating more online material. On which additional topics would you like to see video tutorials? Are there other data sets that you have used to teach modelling? Do you have exemplary student projects that I would enjoy seeing? If so, please don't hesitate to contact me! ▲

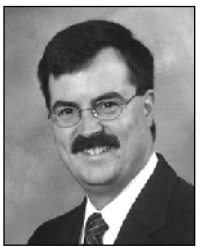
▲ Web 2.0 – Working in the Clouds... TAG, You're It!

GREG CLARKE

ROSS ISENEGGER



Greg Clarke is the Secondary Programme Coordinator for the Simcoe Muskoka CDSB as well as the OAME webmaster. Currently, he is on assignment with the Ministry of Education (CLIPS and GAINS projects). Check out CLIPS at ***oame.on.ca/CLIPS****.*



Ross Isenegger is the Numeracy Coordinator with the Near North DSB. For the past few years, he has been working on the Ministry of Education's CLIPS project. His blog can be found at ***mathfest.blogspot.com****.*

Increasingly, computer users are no longer thinking of their work as being related to a particular device; they now expect to be able to do their work from any Internet-capable device. A good example of this is e-mail. People now expect to be able to check their e-mail from any Internet-connected computer or from their SmartPhones. Increasingly, even word-processing documents are being treated this way. Whether it's through Microsoft's Live Office service, Google Docs, Zoho.com, or the ability to access server space over the Internet, documents can now be accessible from anywhere. Not only is this convenient, but it is also efficient, particularly for collaborative work. For example, it allows the authors of this column to work on drafts without worrying about the version control issues caused by e-mailing documents back and forth.

In the last year, a number of features have been added to **Google Docs**, and our next column will explore some of those features in more detail. However, the core function of Google Docs is to provide an online creation and storage place for the three main types of documents most commonly used today—text documents, spreadsheets, and presentations. Like other Google applications, Google Docs runs completely within a web browser and requires no special software to be installed. It provides a fully functional office suite that can import and export files in Open Office, as well as in MS Word, RTF, HTML or PDF formats. The documents themselves are stored on the Web and are accessible from any computer that can connect to the Internet. There are many options in terms of sharing and privacy controls with Google Docs. For example, you can set the rights so that no one else can read or find a particular document (the default setting), you can set it to be shared with a few colleagues, or you can make it widely accessible to anyone who has the URL. You can also set it up to allow others to collaborate with you or to have editing privileges, or you can set it for view-only.

Here are some easy steps to follow to get started with **Google Docs**:

1) Go to **docs.google.com** and log in with your Google ID and password.
2) Click the "Create new" button, and choose from Document, Spreadsheet, or Presentation.
3) Add information to your new file. You'll notice that the menu system and the formatting controls are pretty standard fare and that most regular things you can do in any standard office suite, you can do with this. You can also rename the file and save it from time to time, although it does save automatically.
4) To share the document, spreadsheet, or presentation with others, click on the "Share" button at the top right, and invite people to join you, or use one of the other options to publish or publicize your work and invite others to view it.

The practice of employing third-party services to store data and documents online is an example of a phenomenon known as *Cloud Computing*. The advantages are many: You are no longer tied down to a specific machine; you have access to files anywhere, provided that a connection is available; and sharing those files with others and collaborating with others can be done much more simply than in the past. Also, since most cloud computing occurs in a browser, it is platform independent, meaning that files can be accessed equally well through Windows, Mac, or a Linux-based machine. The obvious disadvantage is that to access your documents, you need a connection, and in areas with slow or unreliable Internet, this is a factor to consider. However, Google Docs and other online office suites now have features that allow you to download your online documents to your computer for backup, as well as methods to work on them offline, then sync them again once you are reconnected to the Internet.

People are not just storing documents in "the Cloud." Other popular services include calendaring (e.g., Google Calendar, Microsoft Exchange), photo sharing (e.g., Flickr), video sharing (e.g., YouTube, TeacherTube), messaging (e.g., Twitter, Skype), and music promotion (e.g., mySpace). The benefits of having your calendar and contacts available to you on a variety of platforms, not only for viewing, but also for editing, is powerful indeed. If you are interested in sharing photos, music, and videos, there is no comparison between what a professional service can offer and what you might be able to cobble together on your own website. In our last column, we wrote about wikis—web pages that are easy to modify and update and that can be made accessible to anyone on any computer connected to the Internet. Sounds a lot like cloud computing!

Another example of cloud computing—and something that has huge benefits when stored in the cloud— is bookmarking! If you use different computers (e.g., one at work and one at home, or if the one at your school is locked down so you can't save information on the hard drive), then storing your bookmarks online is a huge advantage. You will have access to your bookmarks from any machine you use, whenever you are connected.

In typical Web 2.0 fashion, something very powerful happens when a community of users starts collecting bookmarks online and sharing them. All of a sudden, you have access to an accumulation of experience about valuable sites. In 2003, del.icio.us pioneered the tagging of bookmarks and coined the term *social bookmarking* (the term *bookmarking* because you use it to keep track of websites that you want to be able to come back to, and *social* because your bookmarks can be shared). This site has largely retreated from its smart-alecky URL to the more easily spelled **delicious.com**. Other popular social bookmarking services include Furl, Connotea, Stumbleupon, and Diigo.

A "tag" is a keyword or search word, and you attach tags to every website that you bookmark. Often, you will want to attach multiple tags to a website so that you have a better idea of the types of information and resources that are available. For example, the following are the tags that one might use when bookmarking the OAME website: OAME, mathematics, education, Ontario, teachers, *Gazette*, *Abacus*, chapters, OMO, CLIPS. When you want to find a bookmark that you've saved, you just search for a keyword that you have attached to it.

Tagging is a way of personally organizing the information you collect in a way that makes sense to you. There is no right way or wrong way to catalogue the pages you wish to save; just use a method that makes sense to you, and that you'll understand when you come back to use the information. Internet users have become used to tagging all sorts of resources—photographs on Flickr, videos on YouTube, or blog posts. Notice that this process is de-professionalized. You do not hire a librarian to catalogue your resources; it is what is referred to as a *folksonomy*. In general, it works quite well, especially if there is a critical mass of users adding information to the system.

To sign up for a free **delicious.com** account, go to the website and click "Join Now." To bookmark a site that you stumble upon, copy its address at the top of your browser, open a new tab to **delicious.com**, and click "Save a new bookmark." Paste the address into the URL field and click "Next." You can fill out or change the web page's title, add optional notes, and enter tags separated by spaces. You can even send the bookmark to one of the people in your network. An even easier way to tag pages is to use the add-on for Firefox or Internet Explorer. Click "Help" for more details. Once installed, you will have a tag icon on your menu bars that you simply click on in order to add a web page to your bookmarks.

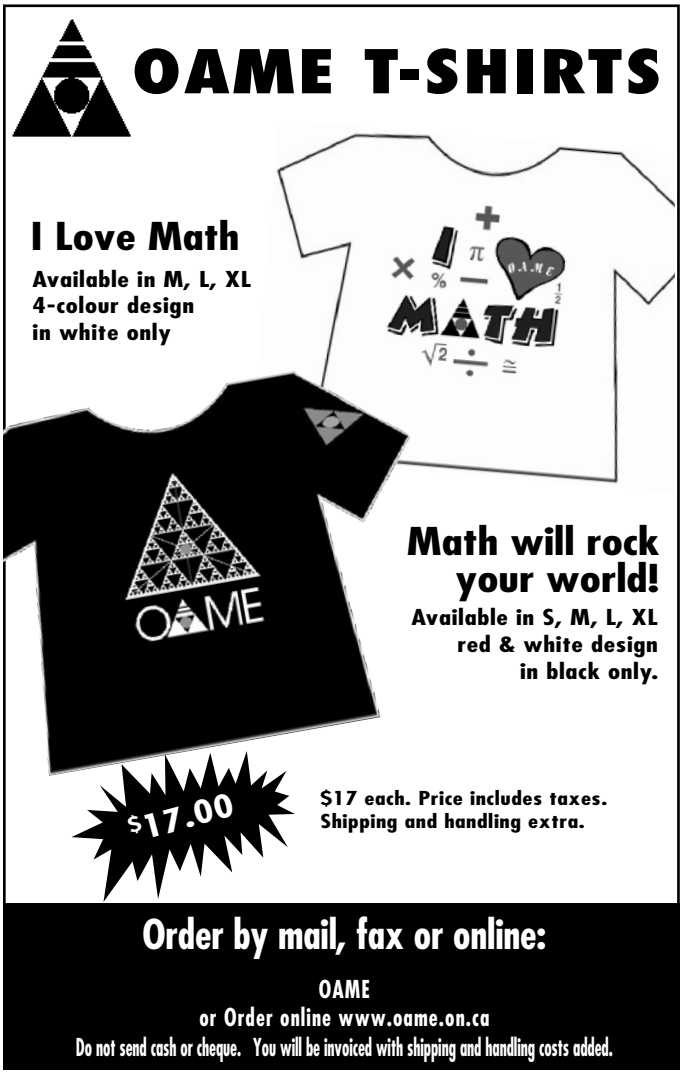
You don't have to create an account to take advantage of other people's research. You can do a search of public bookmarks. For example, try searching "web2.0 math." Currently, the first result is the wiki page that accompanies this series at **mathfest.wikispaces.com/Web2.0+For+Math+Educators**. You can see that seven people have tagged it. Clicking on the number 7, you can then see who those people are. For example, gblibrary has bookmarked the page and noted that it includes videos and definitions, as well as apps. Clicking on gblibrary's link, you can see that there are other interesting Web 2.0 links that you might like to explore. You may even want to add this user to your network. If you want to try to follow just one user, why not try **delicious.com/dougpete** (Doug Peterson of the Greater Essex County District School Board). Doug keeps an eclectic collection of links related to K–12 education and technology.

In every article in this series, we have talked about adding feeds to our Google Reader account. Luckily, you can add feeds to your reader to alert you to new bookmarks added by a user, new bookmarks tagged with

a specific word, or new bookmarks tagged with a specific word by a specific user. At the bottom of any display that corresponds to an RSS feed, you will see the telltale orange RSS icon. Hovering over that link should display its URL in the status bar (e.g., **feeds.delicious.com/v2/rss/dougpete/math?count=15**), which should confirm that you are subscribing to the correct feed, in this case, to Doug Peterson's math feed. If you already subscribe to Doug's blog, then this is a redundant step, since Doug automatically publishes a daily blog post of his additions to **delicious.com**.

Teachers can use the power of **delicious.com** by having students tag relevant sites in a common way, perhaps by choosing an obscure tag like "GBSS–MDM4U." Depending on the topic, some use **delicious**, and not Google, for their first topic search.

Whether you are using documents, bookmarks, photographs, or other resources, we hope that we have inspired you to try "getting cloudy." ▲



▲ Technology Corner — Math Nspired

Mary Bourassa

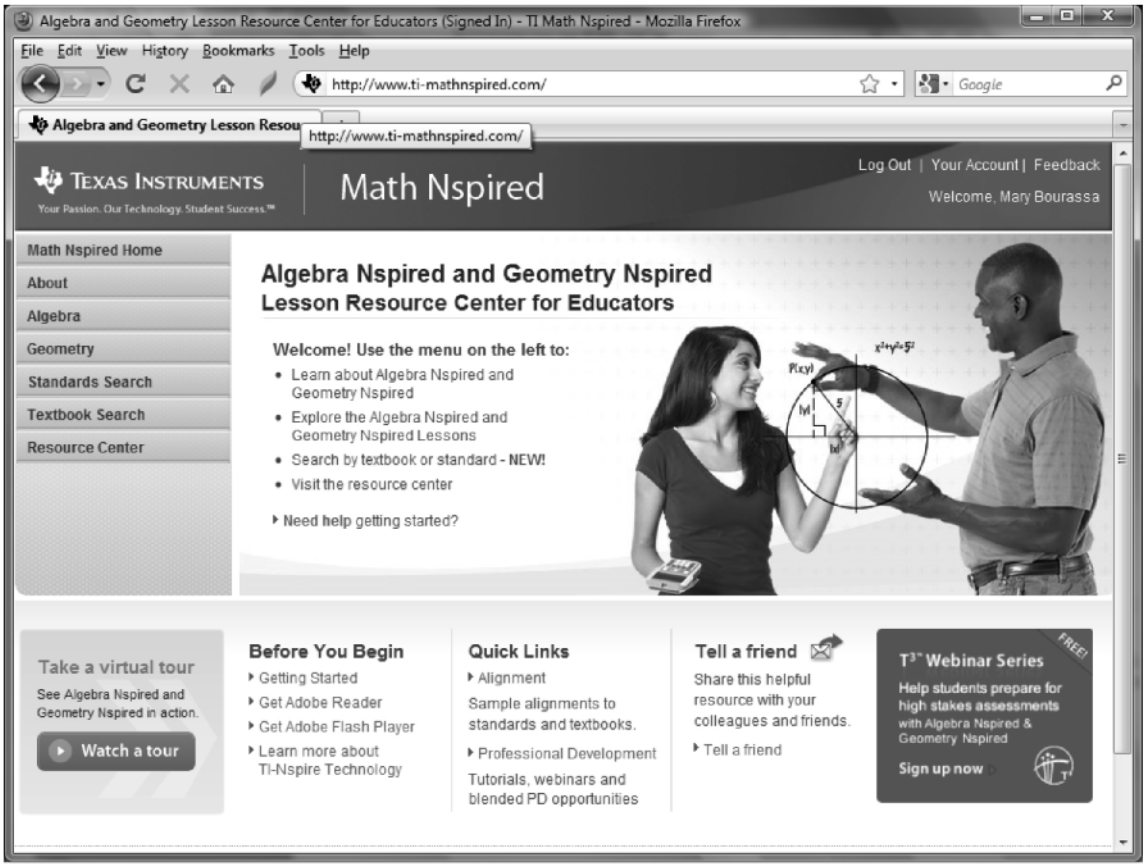


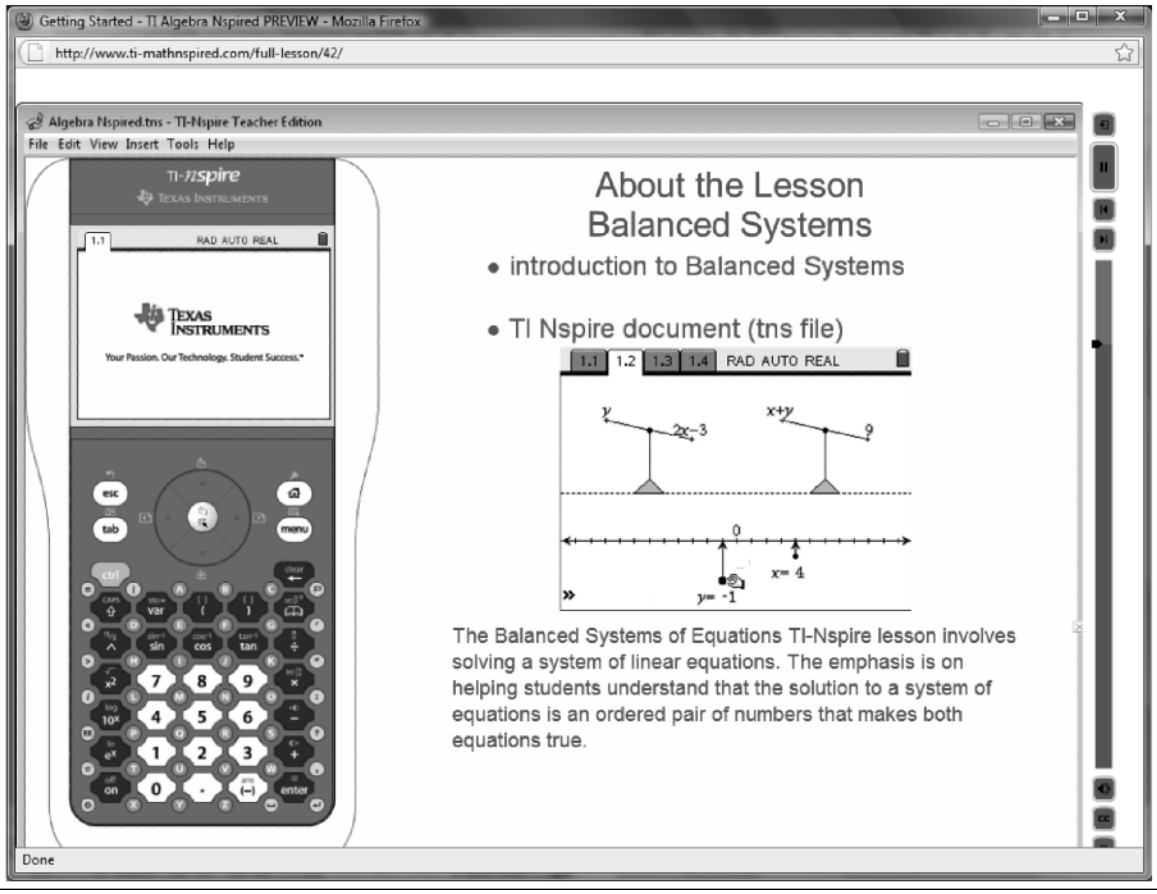
Currently on maternity leave #4, Mary has been teaching mathematics and computer science for the past 15 years. She is a strong advocate for the appropriate use of technology in the classroom. Mary has presented workshops to teachers around the world, has been involved in authoring high school mathematics textbooks, and is a past Vice-President of OAME and Past-President of COMA. Winner of NCTM's Future Leader Award, member of a Kenneth D. Fryer Award-winning mathematics department, and 2009 recipient of the Award for Exceptional and Creative Teaching in Secondary Mathematics, Mary continually strives to discover new and better ways of helping students learn and love mathematics.

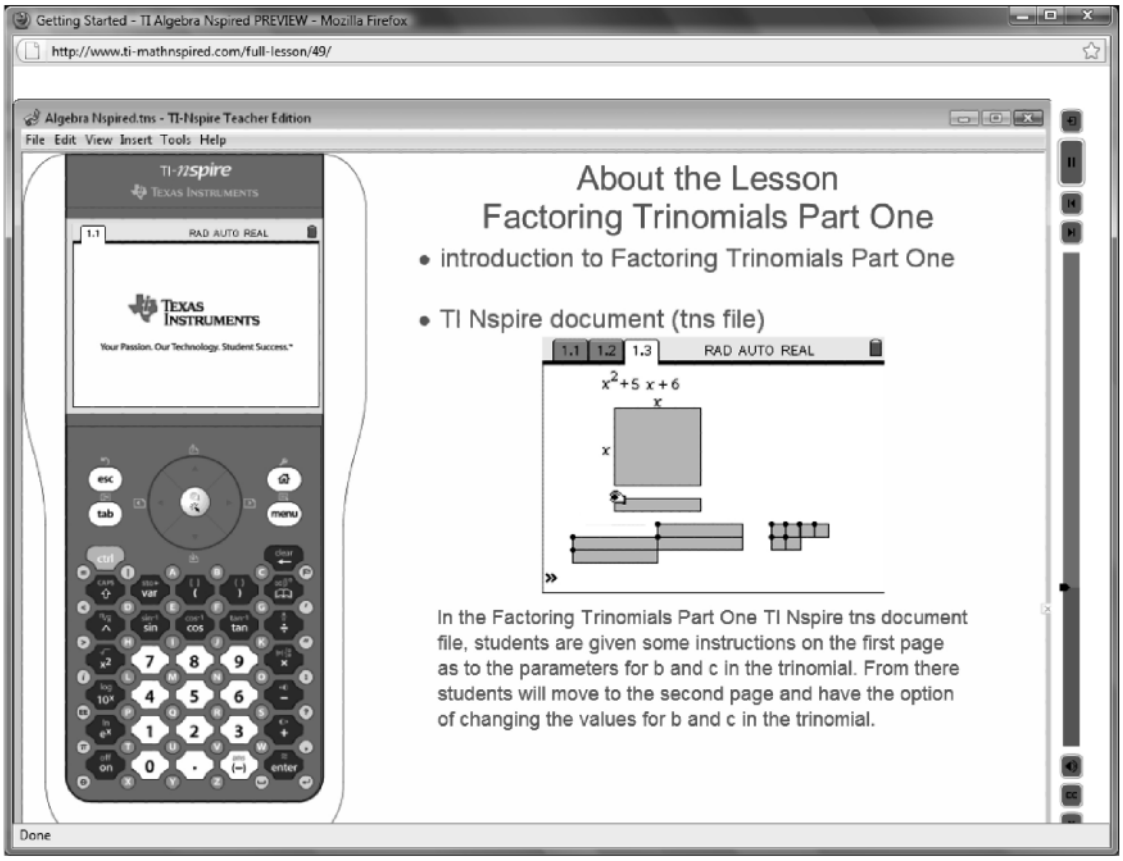
Texas Instruments has always provided resources to help educators use TI technology to its full potential. The latest developments from TI include "Algebra Nspired" and "Geometry Nspired" lessons, which have been created for use with TI-Nspire™ and TI-Nspire CAS™ technology. The goal is to develop conceptual understanding of a mathematical topic through a series of lessons. Although there are a variety of lesson types, the most notable are action/consequence lessons. These lessons allow students to manipulate an object, observe the consequence(s), and reflect upon the mathematical implications, thus promoting classroom discussion and a deeper understanding of the mathematics involved. Each classroom-ready lesson comes with teacher notes (which include teacher tips and tech tips), a TI-Nspire document, a student activity document that asks students to answer meaningful questions, and built-in tutorials for just-in-time learning of basic TI-Nspire skills.

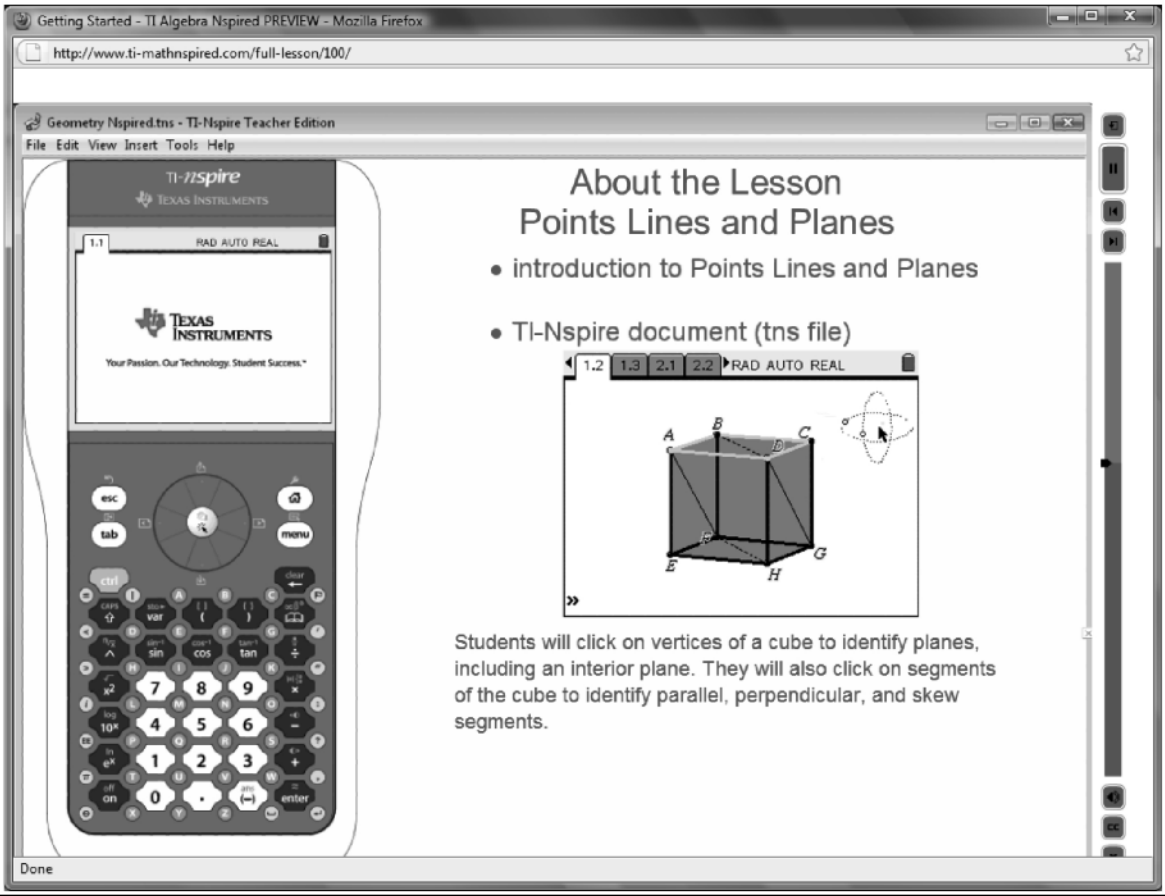
You do need to log in to the Math Nspired website at **www.ti-mathnspired.com**, which is intended for teacher use.

Lessons are separated into Algebra topics and Geometry topics, but you can also search by standard. Ontario standards are included, so you can easily search a particular grade to see whether there is a lesson that


Algebra and Geometry Lesson Resource Center for Educators (Signed In) - TI Math Nspired - Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://www.ti-mathnspired.com/
Algebra and Geometry Lesson Resou
http://www.ti-mathnspired.com/
Texas Instruments
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Welcome, Mary Bourassa
Math Nspired Home
About
Algebra
Geometry
Standards Search
Textbook Search
Resource Center
Algebra Nspired and Geometry Nspired
Lesson Resource Center for Educators
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Share this helpful resource with your colleagues and friends.
Tell a friend
T³ Webinar Series
FREE!
Help students prepare for high stakes assessments with Algebra Nspired & Geometry Nspired
Sign up now


Getting Started - TI Algebra Nspired PREVIEW - Mozilla Firefox
http://www.ti-mathnspired.com/full-lesson/42/
Algebra Nspired.tns - TI-Nspire Teacher Edition
File Edit View Insert Tools Help
TI-nspire
Texas Instruments
1.1 RAD AUTO REAL
Texas Instruments
Your Passion. Our Technology. Student Success.™
About the Lesson
Balanced Systems
introduction to Balanced Systems
TI Nspire document (tns file)
1.1 1.2 1.3 1.4 RAD AUTO REAL
y
2x−3
x+y
9
0
x= 4
y= -1
The Balanced Systems of Equations TI-Nspire lesson involves solving a system of linear equations. The emphasis is on helping students understand that the solution to a system of equations is an ordered pair of numbers that makes both equations true.
Done


Getting Started - TI Algebra Nspired PREVIEW - Mozilla Firefox
http://www.ti-mathnspired.com/full-lesson/49/
Algebra Nspired.tns - TI-Nspire Teacher Edition
File Edit View Insert Tools Help
TI-nspire
Texas Instruments
RAD AUTO REAL
Texas Instruments
Your Passion. Our Technology. Student Success.™
About the Lesson
Factoring Trinomials Part One
• introduction to Factoring Trinomials Part One
• TI Nspire document (tns file)
1.1 1.2 1.3 RAD AUTO REAL
x²+5 x+6
In the Factoring Trinomials Part One TI Nspire tns document file, students are given some instructions on the first page as to the parameters for b and c in the trinomial. From there students will move to the second page and have the option of changing the values for b and c in the trinomial.
Done


Getting Started - TI Algebra Nspired PREVIEW - Mozilla Firefox
http://www.ti-mathnspired.com/full-lesson/100/
Geometry Nspired.tns - TI-Nspire Teacher Edition
File Edit View Insert Tools Help
TI-nspire
Texas Instruments
RAD AUTO REAL
Texas Instruments
Your Passion. Our Technology. Student Success.™
About the Lesson
Points Lines and Planes
• introduction to Points Lines and Planes
• TI-Nspire document (tns file)
1.2 1.3 2.1 2.2 RAD AUTO REAL
A B C D E F G H
Students will click on vertices of a cube to identify planes, including an interior plane. They will also click on segments of the cube to identify parallel, perpendicular, and skew segments.
Done

matches a specific curriculum expectation.

Each lesson features an online video preview that explains the concept being addressed and takes you through the lesson. Note that this is self-contained; you do not need to take out your TI-Nspire to view the lesson. This allows teachers to quickly see what the lesson involves. Here are a few examples:

Math Nspired uses TI technology as technology should be used: It allows students to "discover" mathematical concepts and provides them with a strong understanding of why the mathematics they are learning works the way it does. Students learn by doing and are required to think about what is happening and why. These lessons help to create an engaged classroom where students can conjecture, discuss, and fully understand mathematics. I encourage anyone using TI-Nspire technology to take a look at **Math Nspired**. ▲

ABACUS: A COMPILATION OF VOLUME 46



$10 PLUS SHIPPING AND HANDLING

This resource for teachers includes the concepts of addition, subtraction, multiplication, and division of whole numbers and decimals, alternative algorithms, mental math strategies, manipulatives, and links to literature. It will be useful for exploring these aspects of teaching and learning personally or with a team of teachers. Includes one two-coloured bead string.

Order by mail, fax or online:

OAME

or Order online www.oame.on.ca

Do not send cash or cheque. You will be invoiced with shipping and handling costs added.

▲ OAME/NCTM Report March 2010

LAURIE MOHER



Laurie is an Instructional Coach Facilitator for Literacy and Numeracy (JK–8) with the Kawartha Pine Ridge DSB. She has served on the Board of Directors of OAME for 12 years.

What's New at NCTM?

Whether you are looking for professional development opportunities, resources, or information on hot topics in mathematics, the way to get the most current updates is to visit the "New and Noteworthy" section of the NCTM home page. This page features links to many of the items mentioned above, including the twice-monthly newsletter *Summing Up*.

While exploring the links in the "New and Noteworthy" section of the website, I found two resource areas that may be of interest to you. If you click on the heading "Lessons and Resources" in the navigation bar at the top of the home page, you will have access to a variety of useful tools and strategies. These lessons and resources are presented in three broad groupings along the left side of the page: Elementary School, Middle School, and High School. Within these sections, there are a number of other general categories. I chose to examine two.

The first area I explored was the "Resource Collections" area. Here, I found many resources such as NCTM journal articles, family resources, and student activities organized around important topics in mathematics, including Differentiated Instruction, Assessment, Reflection, Interaction, Equity, and Intervention.

I then looked at "Interaction Resources" to get an idea of what was offered in this area. This is a collection of resources designed to assist teachers in interacting with other teachers and with parents and families. In the section entitled "Interact with Other Teachers," you can join NCTM's Facebook page. You can also read articles written by other teachers and review a variety of teachers' viewpoints on topics related to mathematics. In the "Interact with Parents and Families" section, you will

find resources such as a free *Family Guide*, tips for sending home *Families Ask* flyers, and more. Further down this page, you will again find separate headings and resources relating to Elementary School, Middle School, and High School. I discovered some great ideas and resources in this section, many of which are available to non-members as well as members; however, resources like the journal articles must be purchased if you are a non-member.

Returning to the "Lessons and Resources" page, I looked at "Teaching Tips" and found topics such as Keep up on Math Skills over Winter Break, Differentiated Learning, Preparing for Tests, Alternate Forms of Assessment, Reflection Improves Instruction, and INTERACT for IMPACT. Within this section, I took a closer look at "Tips for Keepin' up on Math Skills over Winter Break." The target audience for this section is parents and families. It includes activity suggestions, as well as some links to interactive tools and/or graphic organizers. Many of these ideas would make a great addition to a school or classroom newsletter.

Both "Resource Collections'" and "Teaching Tips" contain a wealth of resources and are worth taking a look at.

Seeking Some Mathematical Fun in the Sun?

Registration is now open for NCTM's 2010 Annual Meeting in San Diego, California, which runs from April 21–24. You can check out the online meeting preview at **www.nctm.org**. The theme for the meeting this year is "Connections: Concepts and Context." Register by March 12 and save up to 25%!

Are You Due to Renew?

Did you know that every time a new member joins NCTM online or renews his or her NCTM membership online and selects OAME as the Affiliate, OAME receives a rebate? When an individual joins or renews his or her membership online, the member is given the opportunity to select an Affiliate to receive the rebate. Each time that OAME is selected, it receives $3 for a renewing (full) membership and $5 for a new (full) membership. (This does not apply to K–8 school memberships and works only with online transactions.) Encourage your colleagues to join or to renew their NCTM memberships online, selecting OAME as the Affiliate! ▲

▲ Hey, It's Elementary – Count Yourself In!!!

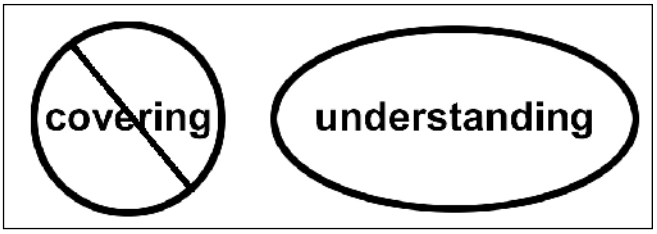
LYNDA COLGAN



Lynda is celebrating her 32nd year as an educator. She has taught at all levels—elementary, secondary, and post-secondary—and has mentored fellow teachers and more recently, teacher candidates and graduate students. Her first teaching award, The Marshall McLuhan Distinguished Teacher Award, was given in recognition of her "Learning Partner Initiative," in which elementary children attended professional development sessions with their classroom teacher to learn about innovative uses of technology in mathematics.

On December 1, 2009, then Education Minister Kathleen Wynne made a major announcement. She had released a discussion paper (*Supporting Learning and Teaching in Ontario's Elementary Schools*, available for download at **www.edu.gov.on.ca/eng/document/discussi/SLTpaper.pdf**) that proposed major, sweeping changes to Ontario's elementary curriculum. Wynne said that a top priority was to make the curriculum engaging and relevant and focused on "big ideas." She addressed the issue of overcrowding in today's curriculum and suggested that the revised curriculum would mean going deeper on fewer topics, not trying to cover too much material, and providing more time for practising math fundamentals.

In short, the discussion paper argues that by increasing depth through concentrated study of integral concepts and by focusing attention on learners' understanding, the development of process skills, and proficiency, there will be a contemporaneous mastery of concepts and skills.



The Ontario Curriculum Council (a group of parents, educators, and professors, whose membership changes

annually and which is currently chaired by Ruth Bauman) will report back this Spring with advice on how to revise the elementary curriculum to address these concerns. The Council's response, in part, will be informed by the voices of teachers, students, parents, and administrators, all of whom have been invited to complete an online survey.

For example, teachers are invited to respond to the following questions about the "overcrowding" of the curriculum:

1. Based on your experience as an educator:
   a) Do you think the elementary curriculum is "overcrowded"? *Yes / No*
   b) Please explain what "overcrowded" means to you.
2. Learning priorities listed (in alphabetical order) by research participants:
   - *Character education/citizenship education/social skills*
   - *Creative thinking skills*
   - *Critical literacy*
   - *Environmental education*
   - *Equity/diversity/inclusiveness*
   - *Global perspective*
   - *Healthy choices and active living*
   - *Learning skills and work habits*
   - *Literacy*
   - *Numeracy*
   - *Problem-solving skills*
   - *Technologies/Information technology.*

   a) Referring to the themes listed [above], please identify, in order of priority, the three themes that you think need to be focused on to a greater degree in the elementary curriculum.
   b) Why are these your top choices?
   c) What would you add to the themes listed above? Why?
   d) Of the themes listed, which one(s) call for the most support for implementation?
3. In your opinion, how could the curriculum be made more engaging for all students (e.g., relevance of material, balance of topics, meeting students' needs)?
4. How might more flexibility be built into the curriculum so that teachers can tailor delivery of the curriculum to the needs of their students and thereby help to improve student learning?
5. In working with the curriculum:
   a) What strategies have you used to create an integrated program of study?
   b) What advice would you provide to the working group on effective integration of subjects?
   c) How can the curriculum be most effectively aligned with new policies and initiatives?
6. As noted [in the document], the current curriculum review process involves extensive consultation and writing by educators from across the province to develop current, relevant, and age-appropriate subject-based curriculum for Ontario students. What advice would you provide to inform future revisions of the curriculum that will meet the needs of students in the year 2020?
7. What is needed to support successful implementation of the curriculum, now and in the future?
8. What additional comments or suggestions would you like to share about the elementary curriculum?

This opportunity for input into the decision-making process is enormously important, and one would hope that school councils, individual school teaching staffs, administration teams, student leaders, and instructors at Ontario's Faculties of Education are making concrete plans to hold local focus groups and discussion meetings to plan their responses to the Elementary Working Group of the Ontario Curriculum Council, so that as many voices as possible can be heard and considered in these extraordinarily important deliberations.

The issues identified by the Elementary Working Group (i.e., overcrowding and lack of relevance in current curriculum policy documents) were arrived at through a public survey, focus groups with teachers, and a substantive literature review of recent trends in curriculum and practice in jurisdictions around the world.

It should come as no surprise that many provinces, states, and countries have come to similar conclusions after reflection about and research into implementation of the sweeping changes introduced in response to the NCTM Curriculum *Standards*. In published studies, common themes about the reform mathematics curriculum have included the issue of too many topics (the curriculum is "a mile wide") and criticisms of "not enough depth" and "a hodgepodge of topics with little coherence." Concomitant with these criticisms has been the call to "vertically align topics." Another criticism has been that the statement of curriculum goals is vague and

"not sufficiently detailed to guide classroom curriculum." The complementary constructive suggestion is that "standards should be expressed succinctly, coherently, and with optimum brevity."

What is missing in Ontario's discussion paper that has been a feature of many other extensive curriculum reviews is the notion of rigour. Many individual states, as well as the National Mathematics Advisory Panel (chaired by Deborah Lowenberg Ball), have called for enhanced rigour in the mathematics curriculum, beginning in Grade 5.

To address the issues of specificity and rigour, several states, including Florida, California, Indiana, and Massachusetts, called upon leading mathematics educators Dr. Barbara Reyes, Dr. Jane Schielack (author of the NCTM *Focal Points*), Dr. Alan Ginsburg, and Dr. James Milgram to lead the revision process of their state-wide curricula. Many concrete and dramatic changes have resulted. The following table shows the stunning decrease in the number of grade-level expectations (GLEs) for mathematics when comparing the "old" document (introduced after the 1998 NCTM *Standards*) with the "new" document (currently in final draft form).

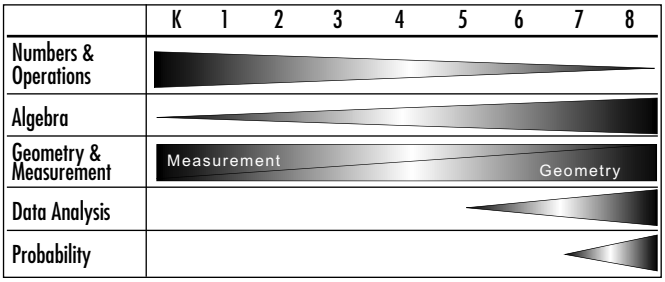
Grade Level	Number of Old GLEs	Number of New Benchmarks
K	67	11
1st	78	14
2nd	84	21
3rd	88	17
4th	89	21
5th	77	23
6th	78	19
7th	89	22
8th	93	19

An important feature of the revised curriculum is that there is significantly less repetition from year to year. In fact, using Florida as an example, there are three "Big Ideas" for each grade. The "Big Ideas" do not address the same topics for each grade, recognizing that at each level there are certain skills that must be honed to prepare students for more rigorous instruction as they move to the next grade. For example, in Kindergarten, these are the three "Big Ideas": *Represent, compare, and order whole numbers and joint and separate sets; Describe shapes and space;* and *Order objects by measurable attributes*. For Grade 4, the "Big Ideas" are: *Develop quick recall of multiplication facts and related division facts and fluency with whole number multiplication; Develop an understanding of decimals, including the connection between fractions and decimals;* and *Develop an understanding of area and determine the area of two-dimensional shapes.* For Grade 8, the "Big Ideas" are: *Analyze and represent linear functions, and solve linear equations and systems of linear equations; Analyze two- and three-dimensional figures by using distance and angle;* and *Analyze and summarize data sets.*

One of the ways in which the number of expectations per grade has been achieved in many states is that more abstract concepts have been moved to higher grades. This helps to reduce the number of benchmarks required for each grade level, thereby facilitating in-depth learning.

As we see in the diagram below, in Florida's revised curriculum, there is a heavy focus on Number and Operations in the early grades as students learn to work with numbers, numerals, and basic operations. In contrast, there is a minor focus on algebra in the early grades—mainly addressed in relation to "patterns." Going up through the grade levels, this trend reverses as students begin to explore more complex algebra concepts.

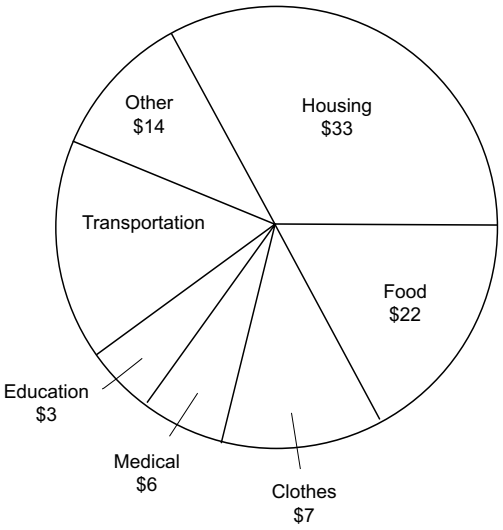
Notice that requirements for more abstract topics, such as probability, have been moved to higher grades, where they can be learned more efficiently and with deeper conceptual understanding.



Not only are there drastic changes to the numbers of grade-level expectations, but there are significant changes to wording and illustrations of meaning for each grade-level expectation. The following table features changes to wording of the curriculum expectations in Florida's mathematics curriculum.

ORIGINAL CURRICULUM POLICY STATEMENT	REVISED CURRICULUM POLICY STATEMENT
Measurement	
Knows about measurement of time, including A.M. and P.M., clocks and calendars.	Tells time to the nearest minute and determines amount of time elapsed.
Data Management	
Analyzes real-world data and makes predictions of larger populations by applying formulas to calculate measures of central tendency and dispersion, using the sample population data, and using appropriate technology, including calculators and computers.	Calculates and interprets measures of variance and standard deviation. Uses these measures to make comparisons among sets of data.

Another important component of the suggested revisions to curriculum documents is the inclusion of specific examples that illustrate concrete ways in which the teacher can actively de-emphasize mechanistic mathematics and use scaffolding to "get at" conceptual understanding. For instance, the first example demonstrates that low-level interpretation questions are being asked of Grade 6 students in their analysis of a circle graph:



(a) What is the cost of transportation?

(b) For each $100 a parent spends raising a child to age 18, how much more is spent on housing and clothes than on education?

(c) TEST PREP. For each $300 spent, estimate how much is spent for food and clothes?
1) $329 2) $90 3) $29 4) $130

(d) Which costs are about twice as much as the cost of education? Five times as much? Eleven times as much?

In contrast, the curriculum document shows how the same type of circle graph can be used to elicit higher-order responses about more mathematically significant concepts:

The pie chart represents the amount of money collected by various stalls at a fun fair.



(a) What fraction of the total amount of money was collected by the games stalls?

(b) What was the total amount of money collected by the various stalls?

(c) How much money was collected by the music stalls?

(d) What was the ratio of the money collected by the food stalls to the money collected by the handicraft stalls?

Perhaps one of the most radical, yet practical, facets to the revisions of many state mathematics curricula, like Florida's, is the explicit attention given to differentiated instruction and the student with special needs. For each "Big Idea" and its supporting ideas, "access points" are provided. "Access points" are written specifically for students with significant learning disabilities, accessing the general education curriculum and reflecting the core intent of the standards with reduced levels of complexity. The three levels of complexity include participatory (Pa), supported (Su), and independent (In), with the participatory level being the least complex. While the student's Individual Education Plan (IEP) team is responsible for making the determination as to which students will be working at which access points, the curriculum document provides practical support to the generalist classroom teacher who is working diligently to support the needs of all students. For example, the Grade 5 Mathematics Benchmark and Access Points (Algebra: Big Idea 2) are specified in this way:

Benchmark:

The student will represent addition and subtraction of decimals and fractions with like and unlike denominators, using models, place value, or properties.

Access Points:

In. The student will express, represent, and use fractions, including halves, fourths, and thirds, as parts of a whole and as parts of a set, using number names.

Su. The student will identify the relationship between half and whole.

Pa. The student will respond to a prompt to identify a specified part of a whole.

While I have focused on Florida's revised mathematics curriculum (largely in part because one of my graduate students, Emily Stephen, received her B.Ed. training in Florida, and I have been impressed by the depth of her pedagogical content knowledge), it would have been possible to feature many other recent examples of revisions across Canada and the United States. I intend to complete the online survey, as requested by the Minister of Education, in response to my reading of *Supporting Learning and Teaching in Ontario's Elementary Schools*, and I hope to organize a group response from the elementary curriculum instructors at the Faculty of Education at Queen's University. I hope that you will do the same.

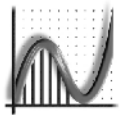
In closing, I intend to be self-indulgent. I would like to share my "wish list" for a revised mathematics curriculum for Ontario, based on my experiences as an educator for pre-service teachers. Based on the assumption that my teacher candidates are representative, I can say with confidence that our current curriculum document is not being used as a guide for quality instruction, but rather, as a reference to mention in lesson plans, to be then put back on the shelf. The new document needs to be the "go to" resource that informs and guides daily practice and that provides the practical "look-fors" that teachers need to help in the selection of instructional and assessment tasks. I hope that the new document will include not only content standards, but also illustrative tasks, examples of student work, and a commentary for teachers. Together, these components will be the teacher's guide concerning what to teach, how thoroughly to treat a topic, and what some instances of student work that meet the content standard might look like. I hope to see a large number of problems of low, moderate, and high levels of complexity to illustrate the curriculum expectations. The inclusion of exemplars, as well as samples of student work and scoring rubrics, would help to illustrate the range of student performance levels at each grade level. Commentary accompanying each problem should discuss the extent to which the problem requires the student to show procedural and/or conceptual understanding, apply knowledge to familiar and unfamiliar situations, reason, generalize, communicate in mathematical terms, and make connections. Imagine how such features could not only inform and guide instruction, but how they could also ensure the careful alignment of instruction and assessment!

I also hope that the new document will be designed to reflect an obvious balance of concepts, skills, and problem solving. I hope, too, that the curriculum will stress rigorous concept development, that it will present realistic and relevant tasks, and that it will keep a strong emphasis on computational skills.

Research tells us that extensive re-teaching and review were common in the previous curriculum and although well intentioned, resulted in minimal learning for too many students; hence, I hope that the revised mathematics curriculum eliminates extensive repetition.

Finally, I hope that those who are charged with reforming Ontario's mathematics curriculum keep their eyes firmly on the vista ahead, and that they find the key to unlocking the chains that tie them to the past:

Do not confine your children to your own learning, for they were born in another time. – Hebrew Proverb



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▲ The Beauty of Catalan Numbers

JEN WOODCOCK



Jennifer is a math teacher at Westside S.S. in Orangeville, ON. She recently completed the M.A. in Mathematics for Teachers program at York University.

> *"The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful."* – Henri Poincaré (as cited in Huntley, 1970, p. 1)

Many of today's high school students would have difficulty relating to this quotation. We work hard to help students see the value of (applied) mathematics as a tool for understanding the complex problems facing our modern world in areas such as modelling the spread of disease pandemics or analyzing the interplay of factors affecting global warming. We strive to convince them that mathematics forms the fundamental basis of the technology they enjoy, yet take for granted: their cellphones, iPods, the Internet. While most will begrudgingly acknowledge the usefulness of math, few—and I mean very few—would say they perceive mathematics as intrinsically interesting or beautiful.

Historically, math educators have been split on the practicality, or even the possibility, of incorporating the aesthetic quality of mathematics into classroom lessons (Sinclair, 2009). Many have argued that it is frivolous or even inaccessible to students who perhaps lack a deep knowledge of the fundamental skills. In fact, as Sinclair (2009) remarks, even Poincaré subscribed to the view that the beauty of mathematics is reserved for only an elite few. But recently, this view has started to shift (Higginson, 2006).

> *I don't want to convince you that mathematics is useful. It is, but utility is not the only criterion for value to humanity. Above all, I want to convince you that mathematics is beautiful, surprising, enjoyable and interesting. In fact, mathematics is the closest that we humans get to true magic. How else to describe the patterns in our heads that—by some mysterious agency—capture patterns of the universe around us? – Ian Stewart (pp. 1–2)*

As teachers, we have an opportunity, even an obligation, to expose our students to beautiful mathematical patterns (notice the emphasis placed on understanding patterns in the Algebra Standard of the *NCTM Principles and Standards for School Mathematics*, 2000). Through these lessons, students may not only develop a deeper understanding of the discipline, but may also come to understand what motivates mathematicians. It's sad that many secondary school students are unaware of what mathematicians actually do. [As an aside, I suppose this view isn't surprising, since only a short time ago, I sat in a Master's level Education course, where a group of grad students didn't fare much better.] Math is often perceived to be a "dead" subject, rarely associated with creativity, excitement, and growth.

It was this reflection that provided the impetus for the development of an activity designed to allow my students to experience the creativity, excitement, and beauty that I find in doing mathematics. The big idea was to find a way to get students thinking and feeling like mathematicians. I needed a topic that was readily accessible and easily understood without a lot of teacher direction so that the students would be freed to think creatively and work collaboratively. I wanted an activity that provided opportunities for scaffolding and the use of manipulatives as needed in order to differentiate instruction and accommodate various learning styles. Finally, I sought out a problem that was tied to an active area of mathematical research, that would expose students to "what mathematicians do," and that would open the door for further exploration.

> *A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.* – G.H. Hardy (p. 84)

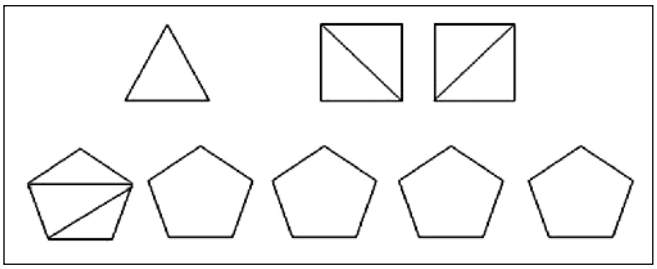
If a mathematician is a "maker of patterns," then through this exercise, the students become the mathematicians Hardy describes. Consider the following investigations. Working in groups, the tasks in each case are to predict what will come next in the sequence, to check these conjectures by extending the pattern, and then to devise a suitable way of organizing and presenting these findings to the class. These fit well with the mathematical process expectations of problem solving, reasoning and proving, and communicating, as

outlined in both the elementary and the secondary Ontario mathematics curriculum documents.

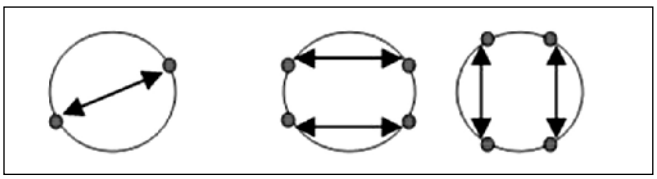
Problem #1: Two students, Saajida and Thomas, are sharing some candies. To be fair, their teacher says that each student should end up with the same number of candies. Saajida, who is placing the pieces one by one into two piles, decides to impose her own rule as well: At no time will she allow Thomas' pile to contain more candies than her pile does. In how many ways can Saajida follow both rules while distributing the candy?

Total number of candies	Number for each student	Distribution Pattern (S=Saajida, T=Thomas)	Number of Patterns
2	1	ST	1
4	2	SSTT, STST	2
6			

Problem #2: In how many ways can a regular polygon be split into triangles using non-crossing diagonal lines?



Problem #3: An even number of people are sitting around a circular table. In how many ways can these people shake hands if: a) all handshakes must occur simultaneously, and b) handshakes are not allowed to cross over one another?



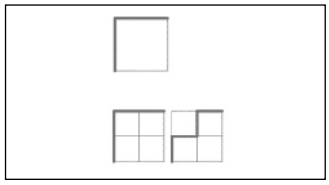
Each of these problems would be accessible to even very young math students. The teacher can vary the amount of prompting students receive; for example, the diagrams for Problem #2 strongly hint at the number of solutions for the pentagon case. On the other hand, omitting those diagrams as shown in Problem #3 makes the task more challenging. These questions lend themselves to a variety of problem-solving approaches. For example, manipulatives might be helpful for solving Problem #1, SMART Board™ technology might assist students in playing with options for Problem #2, and acting it out could help kinaesthetic learners arrive at a solution to Problem #3. Interestingly, the final pattern obtained is the same for all three of these problems. Yet even more fascinating are the mathematical ideas that underlie this pattern. We can explore these through a more detailed look at how a fourth problem might be presented, this time in an MDM4U class. As before, the investigation's starting point is deceptively simple:

Problem #4: Using an n x n grid, explore the pattern(s) created when paths are drawn according to the following guidelines:

1) Paths begin at (0, 0) and end at (n, n).
2) They consist of steps travelling only in the north or east direction.
3) Paths may never pass below the line $y = x$, the main diagonal of the grid.

(Note: In the literature, these paths are known as Dyck paths, where the word "Dyck" is the last name of the German mathematician Walther von Dyck. Dyck path is pronounced so that it rhymes with "bike path.")

To check understanding of the task, we run through the 1 x 1 and 2 x 2 cases as a class.



n	Number of paths
1	1
2	2
3	
4	

n	Number of paths
1	1
2	2
3	5
4	14

After the students have been divided into groups and have been provided with large sheets of grid paper and markers, they set to work extending the pattern. Before they attempt each stage, I ask them to predict how many paths they will find. Is there a strategy they can employ to ensure that they don't miss any paths? In my experience, most groups don't seem to have difficulty coming up with the five paths for $n = 3$, but the task becomes more difficult at each stage thereafter. In particular, I find it useful to have the students stop once they are confident in their answer for $n = 4$. Groups are asked to share their findings, and in particular, to explain their reasoning behind the conclusion that they have found all the required paths. Invariably, groups' answers will differ, both in their path totals and in their explanations. Through the ensuing discussion and particularly in the face of the compelling visual evidence, consensus is reached, at least with respect to the total

number of paths. At this stage, the summary chart on the board shows the following pattern:

- *How might we predict the number of paths for* n *= 5?*
- *Can we use the data we've found to conjecture an algebraic formula describing the pattern we see?*
- *How will we know if our predictions are correct?*
- *How can we sort, classify, and/or organize these paths?*

Thinking takes time. So does playing with mathematical objects. Rather than rushing to give out answers, students are left to ponder these questions for homework that night. They should come prepared to discuss their results with their classmates at the start of the next day's class.

Students are often surprised to discover that there are 42 paths for the 5 x 5 grid. So, we have the sequence 1, 2, 5, 14, 42,…. Whether students are able to conjecture an accurate formula or not, there is value in their struggle and discussion. As with many mathematical problems, there is more than one valid solution. But what happens if the students are stuck at this stage? Turns out, mathematicians get stuck all the time! After discussions with colleagues, what might a mathematician do next? It would be important to find out if others had looked at this problem before. If we consider mathematics as a tree, each part grows from, and hence extends, previous knowledge (Kennedy, 1995). Searching math journals to find this previous knowledge can be a daunting prospect, but fortunately, we have at our disposal a fabulous resource, a piece of technology that gives us an immediate answer. Using the computer, we turn to the Online Encyclopedia of Integer Sequences (OEIS) at **www.research.att.com/~njas/sequences/**, a database containing approximately 160 000 sequences collected by Neil Sloane over the past 40+ years. [**Side note:** This site includes many interesting features, including a puzzles page, where visitors may try to figure out the rule behind a given sequence (perhaps a fun class warm-up activity). Or perhaps, students would enjoy discovering sequence properties, using the feature that allows them to listen to a sequence played on a piano.] We type in our sequence, and lo and behold, we find that this sequence has been studied before and that these numbers are called *Catalan numbers*. Although not as well-known as the Fibonacci sequence, the Catalan numbers are arguably more interesting.

Catalan Objects

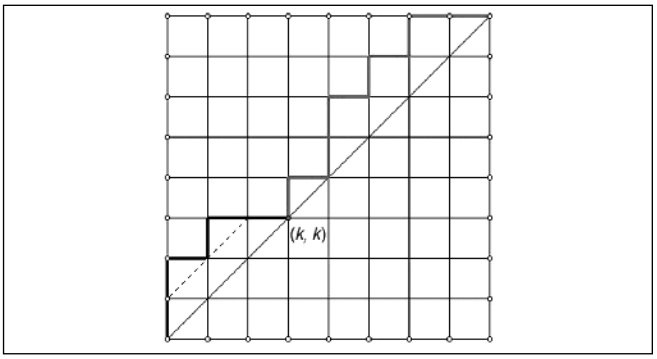
As indicated on the OEIS, there are numerous combinatorial objects that are enumerated by the Catalan numbers; in fact, this sequence is the most frequently searched of all on the OEIS website! Stanley (2010) continues to compile a list that now includes over 170 Catalan objects, including those that we encountered in our four investigations. As his website indicates, continuing to explore the various Catalan objects is an area of ongoing mathematical research; there is certainly still much to discover! Now, back to our paths….

Recurrence Relation

n	Number of paths (C_n)	
0	1	$C_0 = 1$ (by definition)
1	1	$(C_0)(C_0) = 1$
2	2	$(C_0)(C_1) + (C_1)(C_0) = 1 + 1 = 2$
3	5	$(C_0)(C_2) + (C_1)(C_1) + (C_2)(C_0) = 2 + 1 + 2 = 5$
4	14	$(C_0)(C_3) + (C_1)(C_2) + (C_2)(C_1) + (C_3)(C_0) = 5 + 2 + 2 + 5 = 14$
5	42	$(C_0)(C_4) + (C_1)(C_3) + (C_2)(C_2) + (C_3)(C_1) + (C_4)(C_0) = 14 + 5 + 4 + 5 + 14 = 42$

If we begin in the same manner as the OEIS entry and consider $C_0 = 1$, corresponding to the existence of one empty path for $n = 0$, then we can see a pattern that forms a recurrence relation for the sequence. One of my students observed that this process of systematically multiplying the combinations and then adding the products together reminded him of polynomial multiplication. We can introduce summation notation to describe the pattern more concisely. If $C_0 = C_1 = 1$, then for $n \geq 2$, $c_n = \sum_{k=1}^{n} c_{k-1} c_{n-k}$.

Why does this recurrence relation make sense in terms of our paths? Here is a combinatorial argument that may be especially relevant for students in MDM4U who have studied counting problems using the addition and multiplication principles:



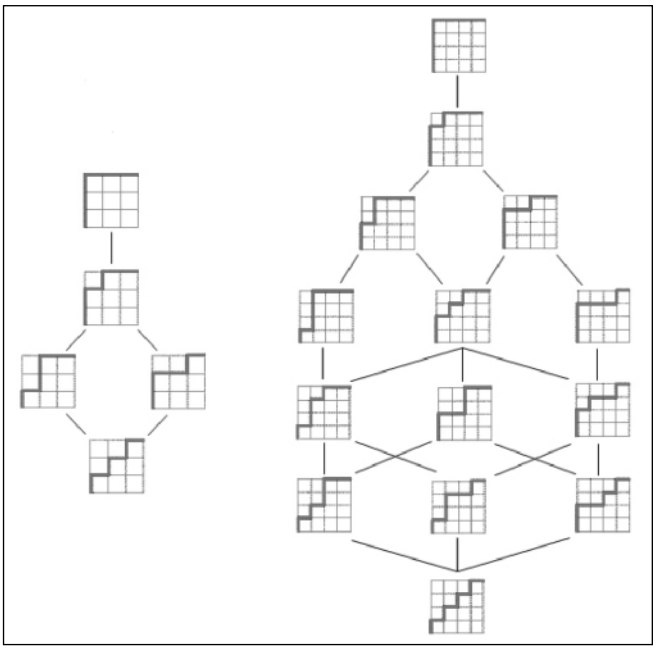
Consider an n x n grid path of the type in Problem 4, and say that the point (k, k) represents the place where the path first touches the diagonal. We know that the first step on the path must be in a north direction and that the segment just before the path touches the point (k, k) must be an east segment. In between those two segments is a sub-path consisting of $k - 1$ north steps and $k - 1$ east steps which does not pass below the line segment connecting (0, 1) to (k-1, k). Since this sub-path remains above the segment indicated by the dashed line in the figure, we can conclude that the number of ways of connecting (0, 1) to (k-1, k) is C_{k-1}. This is also the number of ways of connecting (0, 0) to (k, k) because the first and last steps in this path are fixed, as indicated above. Now beyond point (k, k), we have a path to (n, n), which does not pass below the diagonal. Since there are n-k north steps and n-k east steps in this portion, there are C_{n-k} paths from (k, k) to (n, n) which do not pass below the diagonal. The multiplication principle says that since there are C_{k-1} choices for the first part of the path and C_{n-k} choices for the second part of the path, then there will be $C_{k-1}C_{n-k}$ choices for the paths that first touch the diagonal at the point (k, k). However, the path can first reach the diagonal either when $k = 1$ or when $k = 2$, or…, or when $k = n$, since the kth east step can occur anywhere from the first to the nth east step. Since these cases are disjoint, we apply the addition principle to combine them. Therefore, the total number of paths is given by: (set of paths for $k = 1$) + (set of paths for $k = 2$) + ... + (set of paths for k = n) = $\sum_{k=1}^{n} c_{k-1}c_{n-k}$.

Explicit Formula

The notion that there is more than one way to solve a mathematical problem is another big idea illustrated by this investigation. In addition to the recurrence relation, students might appreciate knowing that mathematicians have also worked out an explicit formula for the Catalan numbers, $c_n = \frac{1}{n+1}\binom{2n}{n}$. We refer the interested reader to the addendum of this paper for a justification of this result.

Classifying and Organizing Paths

Recall that students were asked to determine ways to classify and/or organize the paths. These processes give us a starting point for discussing current research and for further appreciating the aesthetic quality inherent in this area of combinatorics. It is interesting to discover that students "see" these paths in different ways. While some focus on the north and east steps of the path itself, others may focus on the partition above the path or even the number of squares between the path and the diagonal. One organizational strategy commonly used by mathematicians is to partially order the set of paths for each n by inclusion. In this method, Path A lies below or is "less than" Path B if they are of the same length, and Path A does not cross over Path B if the graphs are superimposed. Note that it is possible for the paths to coincide at some points. These pairs of paths are called comparable. Conversely, if one path does cross over the other when superimposed, these pairs are incomparable. Stunning patterns are created when the paths, ordered in this way, are combined into a diagram, known as a Hasse diagram. The results for $n = 3$ and $n = 4$ are illustrated in the figure shown. Lines are drawn to indicate comparable paths.



Students immediately notice the obvious symmetry and self-similarity that is present. In addition, it's interesting for students to know that mathematicians are still busy studying the sub-patterns within these partially ordered sets and are still trying to understand the underlying structures. What seems like such a simple object at first leads to some fascinating and exciting, not to mention beautiful, mathematics. There are many possibilities for students to explore some of the sub-patterns on their own as an extension to the activity.

Sinclair (2006) outlines three roles that the aesthetic can play in a math classroom. First, it can serve as a motivator. Mathematics is intrinsically interesting and

beautiful. In fact, Sinclair (2006) claims that mathematicians are most often attracted to solving problems that have aesthetic qualities such as "connectedness, fruitfulness, apparent simplicity, visual appeal and surprise" (p. 71). In this activity, while students do not get to select the original question, they do have considerable freedom in choosing in which direction(s) to take the problem and in choosing how tenaciously they will pursue answers. Certainly, these problems have visual appeal and allow for the discovery of interesting connections. And it is most intriguing that the enumeration of these seemingly different objects leads to the same integer sequence.

Aesthetic considerations can also serve a generative function. Mathematics is about patterns. Noticing order and relationships can lead students to take solutions in one direction over another, following up on those notions that they perceive to be most pleasing. Many times, this occurs on a subconscious level; one solution just "feels" better than another. Of course, students are most likely to persist with a problem if they believe it will lead to a satisfying result. From this standpoint, the inherent symmetry and patterns in the lattice paths make them ideal mathematical objects to explore.

Finally, Sinclair (2006) describes the evaluative role of the aesthetic. Mathematicians constantly judge solutions, assessing them using criteria such as elegance and clarity. Students, too, need to be provided with opportunities for justifying the value of their mathematical preferences. In discussions during this activity, students are asked to reflect on why they might prefer one solution over another (e.g., whether to describe Catalan numbers using the recurrence relation or by the explicit formula). Through presenting their findings to the class and listening to the presentations of others, students have the opportunity to think about and evaluate problem-solving strategies and methods of organizing and displaying data. It seems that if we design mathematics investigations with an aesthetic viewpoint in mind, the key mathematical processes are quite naturally embedded. While some may argue that beauty is in the eye of the beholder, we owe it to our students to give them opportunities to see mathematics the same way a mathematician does—as a beautiful, dynamic, exciting enterprise. These activities have provided my students with a starting point for that journey.

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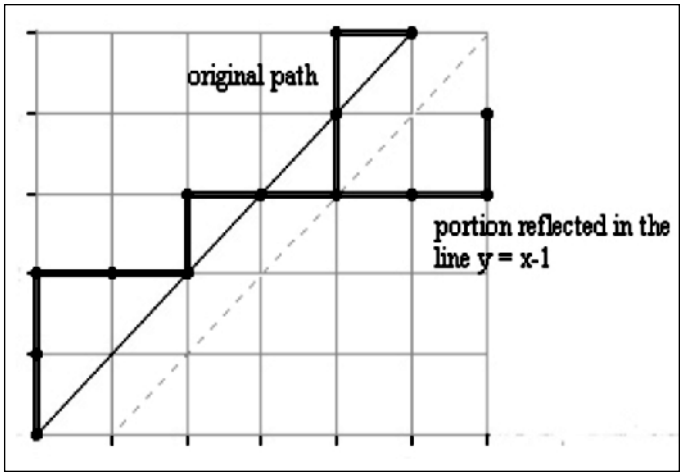
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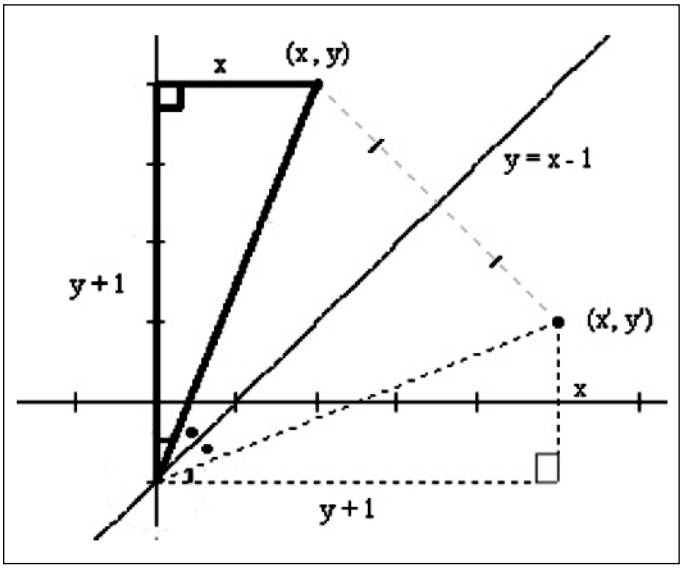
Addendum

We can justify this formula by using a well-known combinatorial argument (Loehr, 2004) and some elementary geometry. There are a total of $\binom{2n}{n}$ paths from (0, 0) to (*n, n*) with north and east steps in an *n* x *n* grid, if these are the only directions we are permitted to travel, since we choose *n* steps in the north direction and then the remainder must travel east. For the moment, suppose that we consider paths that cross *below* the line *y = x*. There must be a first place where the path crosses below the main diagonal. Take the portion of the path



after the first "bad" step and interchange the north and east steps; this is equivalent to reflecting this portion of the path in the line $y = x - 1$, as shown in the corresponding figure. Now we have a situation where, for the $2n$ steps, $n + 1$ are in one direction and $n - 1$ are in the other direction. In fact, this reflection maps the point (x, y) to the point $(x', y') = (y + 1, x - 1)$, as shown geometrically below.

The overall effect, then, is to take $(n, n) \rightarrow (n + 1, n - 1)$ or to create an $(n + 1)$ x $(n - 1)$ grid. On such a grid, there are $\binom{2n}{n-1}$ ways of choosing a path. Since each path that crosses the diagonal can be uniquely transformed like this, there is a 1:1 correspondence.



Therefore, the number of "bad" paths is given by $\binom{2n}{n-1}$. Consequently, the number of northeastern paths on an n x n grid that do **not** pass below the main diagonal can be determined by subtracting:

$$\binom{2n}{n} - \binom{2n}{n-1} = \frac{(2n)!}{n!n!} - \frac{(2n)!}{(n-1)!(n+1)!}$$

$$= \frac{(2n)!}{n!(n-1)!}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= \frac{(2n)!}{n!(n-1)!}\left(\frac{(n+1)-n}{n(n+1)}\right)$$

$$= \frac{1}{n+1}\left(\frac{(2n)!}{n!n!}\right)$$

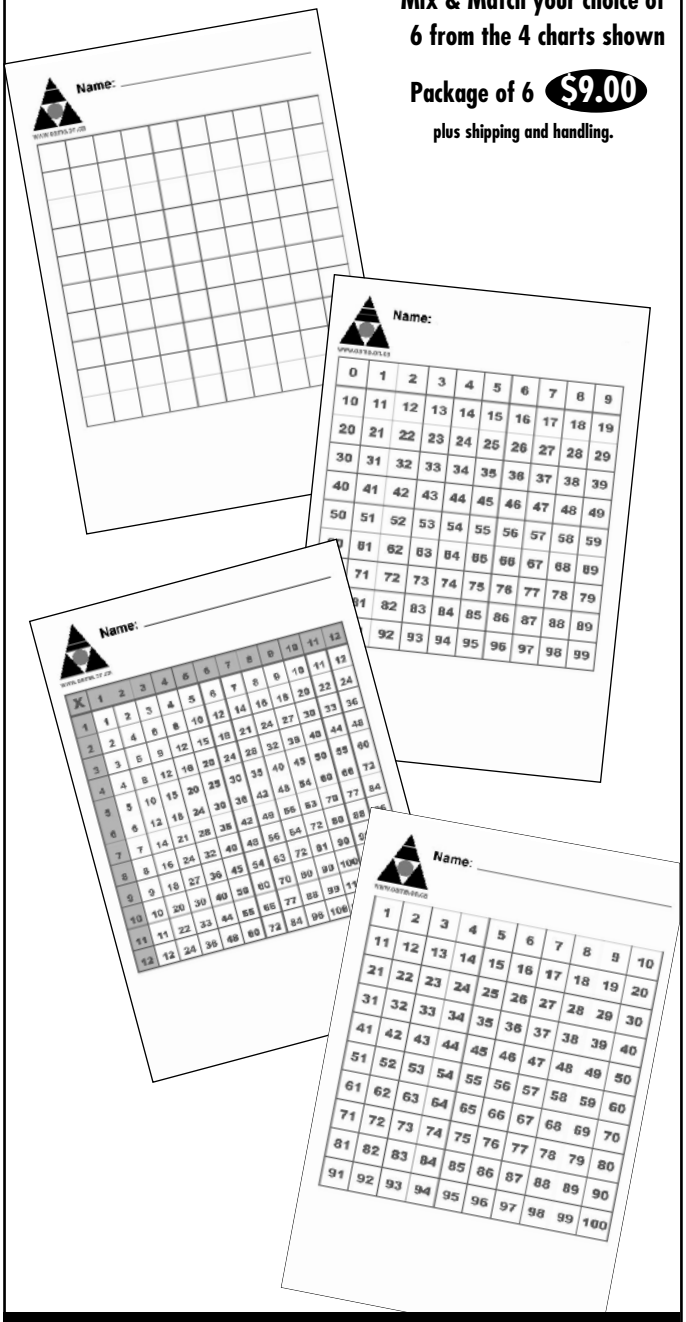
$$= \frac{1}{n+1}\binom{2n}{n}$$

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▲ The Golden Ratio and the Spira Mirabilis

SANDRA PULVER

Sandra Pulver is a professor of Mathematics at Pace University, New York, where she has taught for 44 years. Active in professional organizations at the city, state, and national levels, she has been a coordinator and host of the annual Greater Metropolitan New York Math Fair, a forum for outstanding high school students to display original work in mathematics, and for 18 years, Sandra has served as a grand judge in the mathematics category of the International Science and Engineering Fair, which showcases the creative and innovative research projects of the world's brightest young scholars from the United States and more than 25 other countries.

Having taught mathematics to students of university level for more than 44 years, I find myself still facing the difficulty that confronted me when I first started to teach: How to instill in college minds, the best—perhaps the only—enduring motive for learning mathematics, which, in my view, is the aesthetic motive.

The appreciation of beauty in any form can be developed. This involves presenting the learner with objects of beauty for his or her appraisal, in this instance, with a collection of mathematical formulas, properties, and diagrams of aesthetic appeal.

A favourite object of beauty for mathematicians, scientists, and architects alike is the *golden ratio*, also called the *golden mean* or the *divine proportion*. It is perhaps the most inspiring concept in the realm of mathematics. Not only is this remarkable ratio prevalent throughout nature, but many works of art, like Leonardo da Vinci's "Mona Lisa," also make use of the golden ratio. The ancient Chinese made most of their doors and windows in golden rectangles and triangles, and it is known that the ancient Greeks were aware of the golden ratio's existence, as Euclid's book II of "Elements" makes reference to it.

The numerous connections that this ratio has with so many other mathematical concepts, as well as its prevalence in nature, are enough to fascinate not only scientists and mathematicians, but ordinary laymen as well.

Given the line segment below,



and partitioning it such that: $\frac{u}{v} = \frac{u+v}{u}$, where u or $v \neq 0$, let $\phi = \frac{u}{v}$. Observe that $1 + \frac{1}{\phi} = 1 + \frac{v}{u} = \frac{u}{v} = \phi$.

From this, we can formulate the equation: $1 + \frac{1}{\phi} = \phi$.

Multiplying both sides by f yields the quadratic equation: $\phi^2 - \phi - 1 = 0$.

Solving this equation, we get:

$$\phi = \frac{1+\sqrt{5}}{2} = 1.6180339887... \quad \text{and}$$

$$\phi = \frac{1-\sqrt{5}}{2} = -.6180339887...$$

The first value, 1.6180339887…, is the golden ratio. Note that the decimal portion of each solution of the quadratic is identical and that the sum of the two numbers is 1.

The golden ratio is directly related to the Fibonacci sequence. The ratio of each pair of numbers in this sequence is extremely close to the value of the golden ratio, ϕ. As the numbers in the series get larger, the ratio becomes closer and closer to the number 1.6180339887..., which equals ϕ.

$$\frac{1}{1} = 1$$

$$\frac{2}{1} = 2$$

$$\frac{3}{2} = 1.5$$

$$\frac{5}{3} = 1.6666...$$

$$\frac{8}{5} = 1.6$$

$$\frac{13}{8} = 1.625$$

.....

$$\frac{34}{21} = 1.61904$$

.....

$$\frac{144}{89} = 1.61798$$

After the 40th number in the sequence, the ratio is accurate to 15 decimal places. As the number in the Fibonacci sequence approaches infinity, the limit of successive Fibonacci ratios is the golden ratio, or ϕ .

$$\lim_{n\to\infty}\frac{f_{n+1}}{f_n}=\phi$$

Phi and the Fibonacci sequence are also related in another way. One must first observe that:

$$\phi = 1+\frac{1}{\phi}$$

This is because:

$$1.618033988 = 1+\frac{1}{1.618033988}$$
$$1.618033988 = 1+0.618033988$$
$$1.618033988 = 1.618033988$$

Since this expression is true, it is also equal to:

$$\phi = 1+\frac{1}{\phi} = 1+\frac{1}{1+\frac{1}{\phi}} = 1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\phi}}} = 1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+...}}}}$$

By looking at each part of the infinite continued fractions, the relationship between the Fibonacci sequence and ϕ is evident.

$$1+\frac{1}{1}\to 2$$

$$1+\frac{1}{1+\frac{1}{1}}\to\frac{3}{2}$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}\to\frac{5}{3}$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}}\to\frac{8}{5}$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}}}\to\frac{13}{8}$$

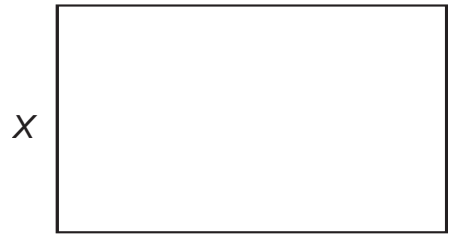
$$\vdots$$

In addition, if

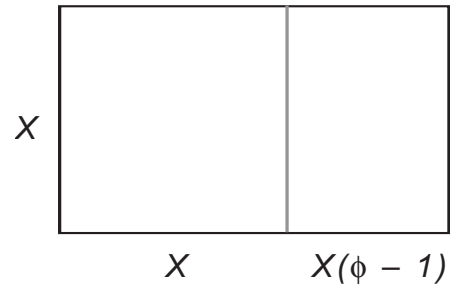
$$x=\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+.....}}}}},$$

then, $x=\sqrt{1+x}$. Therefore, $x^2 = 1 + x$ and $x^2 - x - 1 = 0$ and therefore, $x = \phi$.

One of the most prominent geometric manifestations of this golden mean is the golden rectangle, the length and width of which are in the golden ratio, as shown below:



The rectangle can be partitioned as follows:



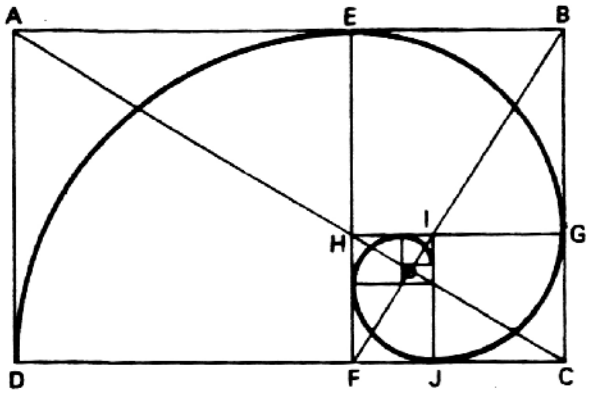
By dividing the golden rectangle to form a square with the side equal to its width (above), we not only get a square, but we get another golden rectangle. This process can be repeated, making this an infinitely self-generating figure. The infinite generation of golden rectangles also generates another figure of aesthetic beauty, the *logarithmic spiral*.

The *logarithmic spiral*, one of the most beautiful mathematical curves, is also called the *equiangular spiral* or the *spira mirabilis*. In studying this *spira mirabilis*, we find that the golden section and the Fibonacci sequence of Leonardo are also associated with it. The appeal of the curve's form is also perhaps related to the fact that it is a familiar sight in the world of nature.

There is no doubt that Greek architects and sculptors incorporated this ratio into their artifacts. The dimensions of the Parthenon in Athens, built in the fifth century BC, can be fitted almost exactly into a golden rectangle. (Since Phidias, a famous Greek sculptor, also made much use of the golden ratio, it was decided that the

Greek letter ϕ, the initial letter of Phidias's name, should be adopted to designate the golden ratio.) A rectangle, the sides of which are in the golden ratio, $\phi : 1$, is called a golden rectangle. Its shape appears to have aesthetic attractions above all other rectangles.

To construct an equiangular spiral, begin with a rectangle ABCD in which AB:BC = ϕ:1. Locate E, the golden cut of AB, and draw EF perpendicular to AB, cutting off from the rectangle the square AEFD. The remaining rectangle, EBCF, is also a golden rectangle. If from this, the square EBGH is lopped off, the remaining figure, HGCF, is also a golden rectangle. This can be repeated indefinitely until the limiting rectangle O, indistinguishable from a point, is reached.



The limiting point, O, is called the *pole* of the equiangular spiral. The sides of the rectangle are tangent to the curve. Alternate golden cuts on the rectangular spiral ABCFHI (e.g., the lines GH and IJ) lie on the diagonals AC and BF. The diagonals AC and BF are mutually perpendicular. $\frac{AO}{OB} = \frac{OB}{OC} = \frac{OC}{OF}$. There are an infinite number of similar triangles (triangles ABC, BCF,…), each being one-half of a golden rectangle.

Another interesting property of the logarithmic spiral is that however different two segments of the curve may be in size, they are not different in shape. The spiral is without a terminal point: It may grow outward (or inward) indefinitely, but its shape remains unchanged. This is one of the points of contact between the Fibonacci sequence and the world of nature. For example, the successive chambers of the nautilus seashell are built on a framework of a logarithmic spiral, and this fundamental mathematical property of the logarithmic spiral corresponds precisely to the biological principle that governs the growth of the mollusk's shell. This principle is the simplest possible: the size increases, but the shape is unaltered. The mollusk's shell grows longer and wider to accommodate the growing animal, but the shell grows by the accretion of material, thus it accumulates rather than grows. (It grows at one end only, each increment of length being balanced by a proportional increase in radius, so that its form is unchanged.) The only mathematical curve to follow this pattern of growth is the logarithmic spiral, which Bernoulli described as the *spira mirabilis*. Photographs of the internal structure of minute fossils of nautilus-like mollusks, plentiful 400 million years ago, reveal that this identical spiral structure has persisted for millions of years, with small variations that make dating possible. The mathematical beauty of the *logarithmic spiral* is uncontested, and the immediate sensory pleasure evoked by its beauty is a common human experience.

The *logarithmic spiral* is a spiral whose polar equation is given by

$$r = \alpha e^{b\theta} \text{ or } \theta = \frac{1}{b} ln\left(\frac{r}{\alpha}\right),$$

hence, the name "*logarithmic spiral*," where r is the distance from the origin to a point on the spiral and θ is the angle from the x-axis. It can also be expressed parametrically as:

$$x = r\cos\theta = \alpha e^{b\theta}\cos\theta$$
$$y = r\sin\theta = \alpha e^{b\theta}\sin\theta,$$

where α is a positive real constant that determines the size of the spiral, while b controls how tightly and in which direction it is wrapped. For $b > 0$, the spiral expands with increasing θ, and for $b < 0$, it contracts; the only difference between positive and negative b is whether the spiral opens to the left or to the right.

The spiral has been known by a variety of names that correspond to one or other of its features. Descartes, who studied it in 1638, called it the *equiangular spiral*. It has also been called the *geometrical spiral* because of the angle at which a radius increases in geometrical progression as its polar angle increases in arithmetical progression. Halley, noting that the lengths of the segments cut off from a fixed radius by successive turns of the curve were continued proportions, named it the *proportional spiral*. Jacob Bernoulli (1654–1705) was so fascinated by the mathematical beauty of the curve that he asked that it be engraved on his tombstone. His tombstone, in the Munster church in Basel, Switzerland, was decorated with a logarithmic spiral that contains the following Latin text: *eadem mutata resurgo*, meaning "although changed, still remaining the same." Hence, the curve is also called the *Bernoulli spiral*. For organisms in which growth is proportional to size, the curve bears the

name *growth spiral*, while Bernoulli, himself, called it the *spira mirabilis*.

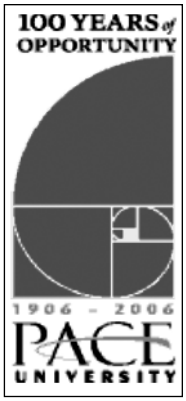
The logarithmic spiral has a much longer history than the science of mathematics. This beautiful curve has not only been studied by mathematicians for hundreds of years, but is also represented in nature in flora and fauna and in the genealogy of the drone bee, conifers, and the shells of mollusks. The spiral can also be found in the spider web, in the pattern of florets of the common daisy, and in the arrangement of seeds in the sunflower. In most sunflowers, you will see two sets of logarithmic spirals intertwined, one being right-handed and the other left-handed, with each floret belonging to both spirals. Comparable arrangements of opposing spirals are found in the pinecone and in the pineapple. The numbers of spirals in each direction in the flower heads are almost always neighbouring Fibonacci numbers, causing logarithmic spirals to be formed. When the seeds are formed, each one moves out directly from the central stem in a straight line. The only active area is the growing tip, and the seeds get bigger only once they have appeared. Each seed appears closest to the central dot, a 0.618 (phi) of a turn from the angle at which the last seed appeared.

Plants also reflect the golden ratio or the logarithmic spiral in the number of leaves, the arrangement of the leaves around the stems, the locations of the leaves, and the number of petals. Leaf edges in some plants (e.g., begonias) roughly follow logarithmic spirals.

Hawks approach their prey in a logarithmic spiral: Their sharpest view is at an angle to their direction of flight; this angle is the same as the spiral's angle. Insects, like the night moth, approach a light source in a logarithmic spiral because they are used to having the light source at a constant angle to their flight path. Usually, the sun is the only light source and flying that way will result in a practically straight line. This is known as the *equiangular spiral*.

The angle any tangent to the curve makes with a tangent to a circle at the same radius, known as the pitch angle, is constant and results in a logarithmic spiral being self-similar. In other words, any part of it looks like any other part (though possibly rotated). The arms of spiral galaxies are roughly logarithmic spirals, and our own galaxy, the Milky Way, is believed to have four major arms, each of which is a logarithmic spiral with pitch of about 12°. The arms of tropical cyclones, such as hurricanes, also form logarithmic spirals.

Many artists have consciously and unconsciously incorporated the logarithmic spiral into their work. My university recently reached its centennial and changed its logo for the year to the logarithmic spiral, since this curve is so appealing to the human eye.



No single property of either the golden ratio or the logarithmic spiral is responsible for its unusual beauty. The vast array of aesthetic properties we have observed, whether in symmetry, convergence, or self-generation, are quite likely to be the reasons for the use of the golden ratio and the logarithmic spiral by mathematicians, artists, musicians, and others over the centuries. The various aspects are complexly and systematically linked and intermingled and serve to make the aesthetic appeal of ϕ unparalleled and unmatched.

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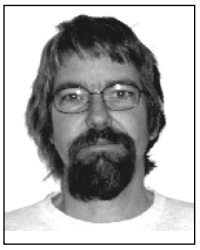
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▲ Impossible Problem?

Tim Sibbald

Amy Pritchard



Tim Sibbald is a secondary math teacher with the Thames Valley District School Board. He recently completed his doctorate on research pertaining to self-efficacy during reflective practices.



Amy Pritchard is currently in her third year of the Concurrent Mathematics and Education Program at the University of Western Ontario.

The following problem is posed at **www.unusualmuseums.org/trig** under the title of "Impossible?? Trig Problem."

Two ladders of lengths 20 ft. and 25 ft. are leaning against each wall of a narrow alley (see Figure 1). Each ladder meets an opposite corner of the alley. The ladders meet at a point 10 ft. above the ground. What is the width of the alley?

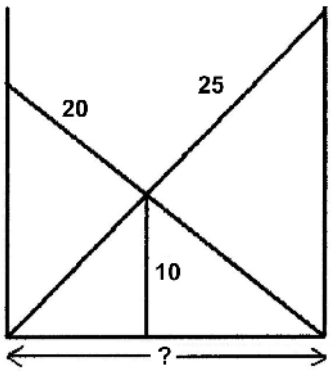


Figure 1: Ladders in an alley

There is no doubt that this is a tricky problem, but is it impossible? Students can be led to address this issue using the *Geometer's Sketchpad* (see Figure 2). Construct a line AC and two perpendicular lines AD and CB. Construct a circle about A, construct its point of intersection with line CB, and display the length of AB. Do the same to display the length of CD. Construct the point of intersection between AB and CD and then join the point of intersection with a perpendicular line segment to AC. In this construction, the lengths of AB, CD, EF, and AC need to be displayed so that lines can be dragged until the required measurements are achieved.

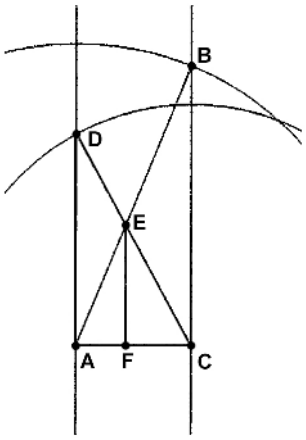


Figure 2: Geometer's Sketchpad construction

If CB is moved, then ladder AB remains the correct length, but ladder CD gets lengthened, which obliges point D to be moved along line AD in order to restore the correct length of the ladder. In other words, finding the solution requires a series of two steps to adjust one variable and then correct another. However, the process does work, and a width of 9.65 cm can be obtained.

This problem can be tackled algebraically and provides an opportunity to contrast algebraic and numerical solutions. In the triangle, we can set up similar triangle ratios in two pairs of similar triangles in order to develop an algebraic expression to evaluate the unknown length. (Note that letters are taken from Figure 2, but numerical values from Figure 1.) First, triangle AFE and ACB are similar, so that

$$\frac{AF}{AC} = \frac{FE}{CB} \text{ leading to } AF = \frac{10x}{\sqrt{25^2 - x^2}}.$$

The second pair of similar triangles is CFE and CAD, which means

$$\frac{CF}{CA} = \frac{FE}{AD} \text{ but } \frac{CF}{CA} = \frac{CA - AF}{CA} = 1 - \frac{AF}{x}$$

$$\text{leading to } x - AF = \frac{10x}{\sqrt{20^2 - x^2}}.$$

Inserting the first equation for AF into the last equation involving AF, and removing a common factor of x, yields:

$$1-\frac{10}{\sqrt{25^2-x^2}}=\frac{10x}{\sqrt{20^2-x^2}}.$$

The solution to this equation can be found by graphing the left and right sides of the equation to find the point of intersection. This results in a value of 9.475.

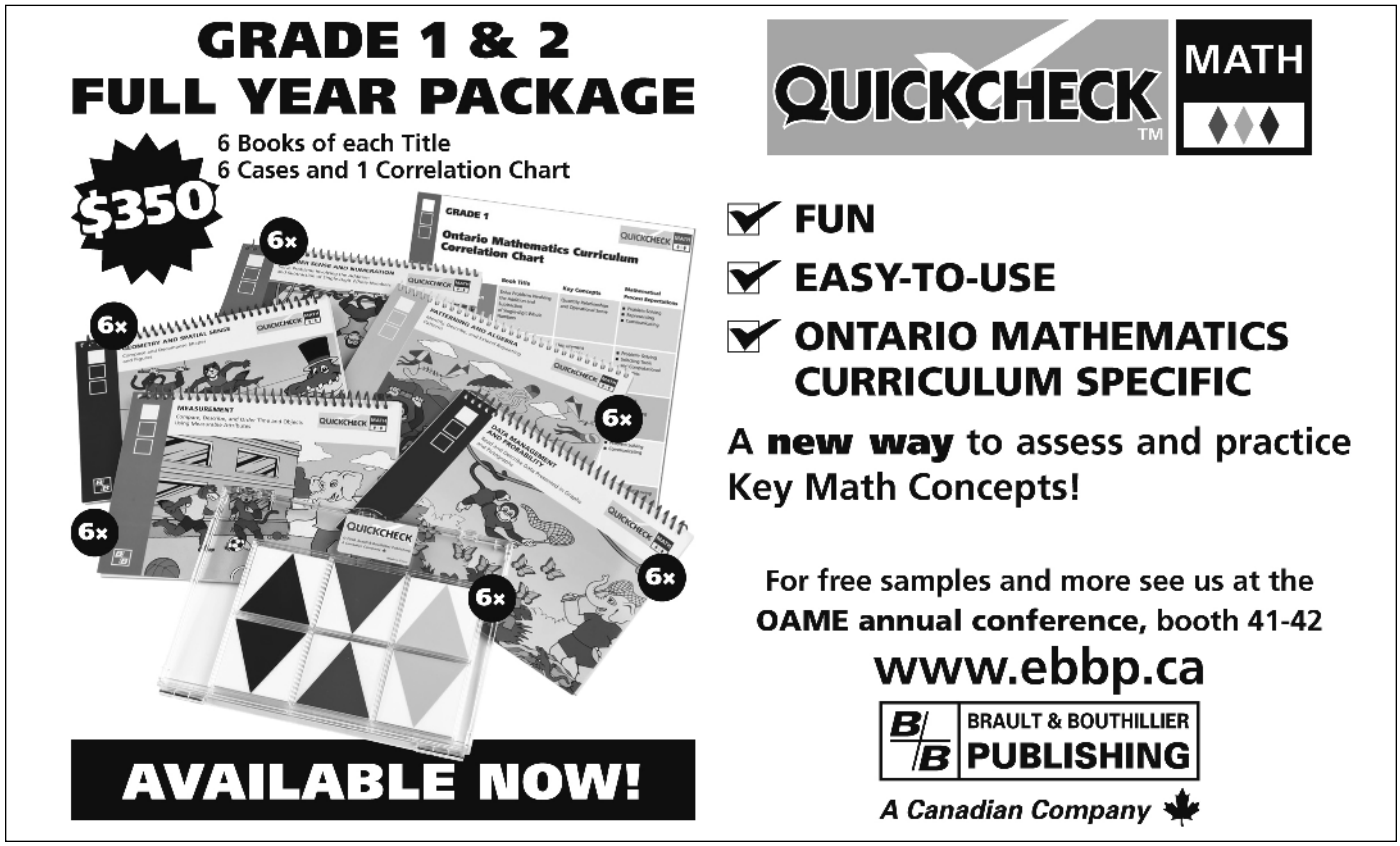
This result can be resolved as a polynomial. Specifically, the radicals can be brought to one side, the expression can be squared, and after isolating the radicals and squaring again, a polynomial can be obtained. The principal is demonstrated below:

$$\frac{1}{10}=\frac{1}{\sqrt{u}}+\frac{1}{\sqrt{v}} \qquad \frac{1}{100}=\frac{1}{u}+\frac{2}{\sqrt{uv}}+\frac{1}{v}$$

$$\frac{1}{100}-\frac{1}{u}-\frac{1}{v}=\frac{2}{\sqrt{uv}} \qquad \left(\frac{1}{100}-\frac{1}{u}-\frac{1}{v}\right)^2=\frac{4}{uv}.$$

Some "play" with this last expression makes it clear that the common denominator can be resolved, but requires multiplying through the expression by $(uv)^2$, which—because u and v are both quadratic in x—is an eighth-degree polynomial. Moreover, anyone who attempts this approach soon realizes that the coefficients are very large because the values 10, 20, and 25, when raised to exponents of four or more, are large. (It is evident that the 1/100, when squared, will give 1/10000.) Ultimately, it becomes clear that a numerical solution may be as useful as an exact value and that this particular problem's "teachable moment" occurs when a numerical solution is seen to be preferable to what would otherwise be an "impossible" solution. ▲





▲ Directors' Dialogue

ANN MICHELE STENNING



Ann Michele Stenning is a mathematics teacher at South Secondary School with the Thames Valley District School Board. Having finished her Masters of Education degree in Curriculum Studies, she is currently assisting other parent volunteers with the development of weekly literacy activities, which she and other moms provide to at-risk students.

OAME Board of Directors Meeting February 5–6, 2010

Friday, February 5, 2010

- Beverly Farahani welcomed everyone and thanked Futures Committee chairs Connie Quadrini and Sandra Jean Price for completing the OAME Annual Conference Planner and the Ontario Math Olympics Planner.
- Beverly Farahani, Glynnis Fleming, and Jacqueline Hill introduced the Curriculum Review Project to Board members. The evening was spent discussing the gaps in understanding at the end of Grades 3, 6, and 10.
- At the end of the evening, Anna Jupp encouraged all Board members to vote in OAME's electronic election. Voting will remain open to all OAME members until April 15; the list of candidates and their biographies can be viewed online. The President-Elect is Connie Quadrini (acclaimed), with two Vice-Presidents to be elected. Glynnis Fleming, Cathy Hall, Ken Maheu, and Susan Pitre are the candidates running for the three positions of K–6 Director. Mary Bourassa, Barry Ferguson, Robert Sherk, Silvana Simone, Tim Sibbald, and Len Rak are running for the three directorship positions covering Grades 7–12.
- Board members had a glimpse of *Abacus 46*, which is available on the OAME website and at upcoming conferences for $10.

Minutes from Standing Committees Saturday, February 6, 2010

Equity Committee

The Equity Committee breakfast was lead by Sonia Ellison, who reported that the Equity Breakfast, usually offered at the annual conference, is to be replaced by a session at the conference. The committee is interested in developing lessons on equity and finding teachers who are already addressing issues involving equity in their classrooms.

Publications Committee

- Robert Sherk reported that access to the Members Only portion of OAME's website will be available to some pre-service teachers on a trial basis. This may lead to increased membership.
- Paul Alves is developing a user satisfaction survey that asks Faculty of Education students to assess OAME's website from a pre-service candidate's perspective.
- Work continues on plans to celebrate OAME's 40th Anniversary. In doing so, the 40th Anniversary Committee is looking to detail the evolution of mathematics at the elementary and high school levels from 1965 to 2010.
- The first two packages of four *Abacus* articles ("Math Operations" and "From Arithmetic to Algebra") are selling at $10 per package. The focus of the third package will be fractions.
- The *Ontario Mathematics Gazette* will soon have a new Editor. Marion Small will take over as Editor, beginning with the September 2010 issue.
- MCIS registration is now open for OAME 2010, including links to book accommodation at Brock University residence or at a choice of local hotels. Please note that the special rate for OAME accommodation ends April 12.
- OMO 2010 coordinators will be able to register their teams electronically this year through the OAME website. Chapters must still pay by cheque, however.
- The Publications Committee was encouraged to continue working with the Canadian Mathematics Society (CMS) and to work toward developing an association with other provinces.

Futures Committee

- Connie Quadrini reported that members discussed the possibility of allowing all students access to the student conference rate. However, it was decided that

only students with OAME membership be allowed to pay the student conference rate.

- Student safety at future Ontario Mathematic Olympics (OMO) events was discussed and will continue to be an issue of great importance at all OMO events.
- The OAME Annual Conference Planner was introduced to members of the Futures Committee for review. Greg Clarke will be posting both the OAME and OMO planners on the Members Only section of the OAME website.
- Yearly meetings for the OMO and OAME past, present, and future co-chairs will continue as well.
- TEAMS will host OMO 2011 and QSLMA will host OAME 2012.

Promotions Committee

- Merilyn Fox reported that new promotional items are available. Sue Hessey, OAME's Executive Director, has legal-sized notepads with a cardboard backing. In each package, there are ten sheets, which include the header: "Math, the Universal Language," along with OAME's website address. 2010 Leadership delegates will receive a half-memo pad, and either an OAME pencil or a decal, plus a membership flyer. OAME 2010 delegates will receive the legal-sized notepad and an OAME pen. Both new items were developed by the Promotions Committee. There are also 60 T-shirts with the new Sierpinski design available for sale. Another design, using the fractal/Sierpinski logo from the Ottawa conference, has been created by Dave Hessey, OAME's Executive Director; this newest design is not yet available, however.
- The Promotions Committee intends to promote OAME to math education professors at the various Faculties of Education in the early Fall. This package will contain an updated PowerPoint (the road show), new membership brochures, pencils, some half-memo pads, secondary graph pads, some *Gazettes*, a deck of cards containing OAME's logo, and a T-shirt for the professor to use as he or she wishes.

Constitution Committee

- Bill Otto reported that committee members are looking at revising OAME's current committee structure and aligning OAME's Board year with the school year.

Awards Committee

- Anne Yeager reported that congratulation letters for award recipients, nominators, and other nominees had been discussed and edited. The criteria for the two Staff Awards were discussed, and a checklist was developed to assist with the determination of these awards.

Curriculum Committees

JK–8

- Laurie Moher reported that members considered offers to partner with a Math for Kids website and Caribou Online Mathematics Contests. Curriculum committee members found that the online contest supported OAME's vision of mathematics. Therefore, OAME's website will provide a link to this contest. The only link found not working was the link to the Kangaroo Problem Database.
- The Curriculum Committee also viewed the Esso Family Math Videos and found them to be useful to parents and teachers. Members suggested that a viewing guide be developed. The guide would, for example, encourage parents to read the book several times before exploring the math with their children in order to increase enjoyment of the story and aid in story comprehension. Parents would also be cautioned not to do all of the math in one reading, but instead, to revisit the book several times, with a different focus each time. For teachers, the viewing guide would include tips on how to use the video as a resource during a Family Math Night.
- Jan Crofoot reported on the gaps that Curriculum Committee members have noticed in student understanding of mathematics. These comments, which address "Questions for Educators and Stakeholder Groups" regarding the elementary curriculum, will be reported to the Ministry of Education.

9–12

- Dwight Stead reported that the Committee identified problems with student understanding in college technology programs after taking MAP4C. Parents and students may not know the difference between the courses; perhaps only one course is needed. There is concern from MDM4U teachers that students coming from MCF3M are not performing as well as those students who enrolled before the last revision. The source of this problem may have more to do with skills deficits than with depth of mathematics knowledge.

- Our secondary curriculum promotes the idea that every student should take Calculus. Perhaps the emphasis should be shifted to Statistics. For further discussion on this topic, members can view the TED Talk Video at **www.ted.com/talks/lang/eng/arthur_benjamin_s_formula_for_changing_math_education.html**. ▲

▲ Vote Mar 1– Apr 15

ALL MEMBERS TAKE NOTE!! OAME ELECTIONS ARE ONLINE

Annual elections for positions on the Executive and Board of Directors will be online from March 1- April 15. To vote, you must activate your online account on the website at **www.oame.on.ca** if you have not already done so. Click on "Members Only" and follow the directions to activate your account. You will find your membership number and expiry date on the envelope of your *Gazette*.

From March 1 to April 15, you will find "OAME Elections" in the links on the left side, after you have logged in using your username and password, which you created when you activated your account. Just follow the instructions. Short biographies for all candidates are included. If you need assistance, contact the Executive Directors by phone, at 519 471 6324, or by e-mail: **EDs@oame.on.ca**

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▲ Functions Graffiti

STEWART CRAVEN, EDITOR



I thought it would be nice to highlight an example of student work at the end of this issue of the *Gazette*. The assignment that I provided to my students is as follows:

Use Geometers Sketchpad™ (or a comparable program) to recreate an image of graffiti. You can choose any one of the five photographs provided. The goal is to create a sketch that is superimposed on the photograph. The closer the two images align, the better your problem-solving skills will be demonstrated.

These are the minimum requirements:

1. *At least **five** different "base" functions must be used.*
2. *At least **one** function from each of these categories must be used— polynomial/rational, logarithmic/ exponential, or trigonometric.*
3. *Transformations must be employed—horizontal and vertical translations, stretches, and/or compressions.*
4. *A list of **all functions** used must be submitted.*
5. *Functions in x and y are permitted.*

The submission shown here was one of the best, but having said that, the quality of all of the submissions was excellent. Obviously, one cannot reproduce every curve in this complex photograph, but this student chose the most prominent curves. She used in excess of 50 different functions and certainly covered the function categories as required. The following are five of the functions that she used. Notice the functions in both *x* and *y* and the very clever method she used for restricting domains! This piece of work was not only comprehensive, but was also very carefully done to ensure that the curves appear continuous and smooth. As a teacher, it is good to know that your students really do understand functions before they move on to Calculus and Vectors!

$$U(x) = \left(-3.25 \bullet (x-2.8)^2 - 3.5\right) + \left(\frac{1}{10^{10}}\right) \bullet ln(x-2.5) \bullet ln(3.197-x)$$

$$F(x) = 9.8^{-x-.04} + 5.5 + \left(\frac{1}{10^{10}}\right) \bullet ln(x+0.97) \bullet ln(0.4-x)$$

$$P(y) = \sqrt{4 - \frac{4.8 \bullet (y-6.4)^2}{8}} - 2.6$$

$$U_4(y) = \left(2 \bullet cos\left(0.63 \bullet (y-1.71)\right) - 3.8\right) + \left(\frac{1}{10^{10}}\right) \bullet ln(y+2.4) \bullet ln(2.7-y)$$

$$S(x) = \left(0.5 \bullet (0.9 \bullet x - 0.3)^4 - (0.9 \bullet x - 0.4)^2 - 5\right) + \left(\frac{1}{10^{10}}\right) \bullet ln(x+1.1) \bullet ln(1.08-x)$$

